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On when to base Event Trees and Fault Trees on Probability Models and Frequentist Probabilities in Quantitative Risk Assessments

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Abstract: This paper discusses the analysis approach when using event trees and fault trees in a quantitative risk assessment context. The basic question raised is when to introduce probability models and frequentist probabilities (chances) instead of using direct probability assignments for the events of the trees. We argue that such models should only be used if the key quantities of interest of the risk assessment are frequentist probabilities and when systematic information updating is important for meeting the aim of the analysis. An example of an event tree related to the analysis of an LNG (Liquefied Natural Gas) plant illustrates the analysis and discussion.

Keywords: *Risk, uncertainties, probability models, chances, event tree*

1. Introduction

A common perspective on risk is the so-called triplet definition based on Kaplan and Garrick [5]:

> Risk is equal to the triplet (s_i, p_i, c_i), where s_i is the *i*th scenario, p_i is the probability of that scenario, and c_i is the consequence of the *i*th scenario, i = 1,2, …N.

For unique situations, the probabilities are interpreted as subjective probabilities (also referred to as knowledge-based or judgmental probabilities), whereas, if repeated similar situations can be generated, the probabilities p_i have to be understood as frequentist probabilities (also referred to as chances). In the following we will use the subscript f to indicate when the frequentist interpretation is adopted. These probabilities are unknown, and subjective probabilities are used to express the (epistemic) uncertainties about the true value of the frequentist probabilities. The framework then established is referred to as the probability of frequency approach to risk assessment.

In practice, however, it is not obvious which of these two approaches should be adopted. When is the situation unique? Consider the event tree in Figure 1 representing release scenarios for an LNG (Liquefied Natural Gas) plant (Vinnem [8]). Are these events unique or can we justify the construction of chance models based on these events reflecting variation in the phenomena studied? What approach should we take? This is the issue of the present paper. We seek to establish some guidelines for when these two approaches should be used. An example case based on the event tree of Figure 1 will be used to illustrate these two approaches and highlight the differences. It is straightforward to adjust the analysis to fault trees.

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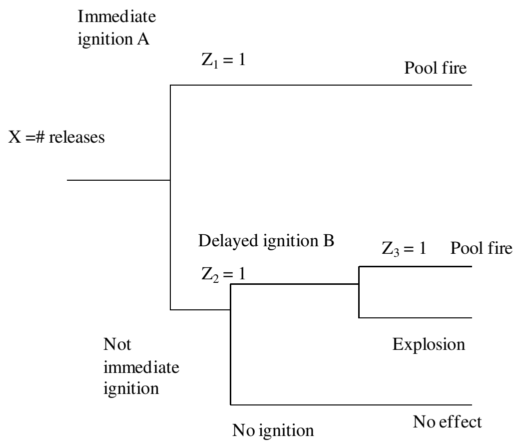

Figure 1: Example of event tree for an LNG case

The issue discussed is related to the distinction between aleatory and epistemic uncertainties in risk assessment (Helton [3], Winkler [9]). The probability/chance models reflect the aleatory uncertainties, whereas the subjective probabilities express the epistemic uncertainties.

The paper is organized as follows. Firstly, in Section 2, we study the event tree in Figure 1 assuming the situation is unique. We summarize the features of this case and discuss its pros and cons. Then in Section 3 we consider analogously the case with probability models introduced. The final Section 4 provides some conclusions. The paper is partly based on Aven [2].

2. The Unique Event Case

The aim here is to implement the Kaplan and Garrick [5] scheme for the examples shown in Figure 1 for the unique event case. Let

X = number of releases (which is approximately equal to 1 if a release occurs and 0 otherwise, as we ignore the probability of two releases in the period studied)
$Z_1 = I(A)$ (I is the indicator function which is equal to 1 if the argument is true and 0 otherwise), A: Immediate ignition
$Z_2 = I(B)$, B: Delayed ignition
Z_3 = I(pool fire)
$Z = (Z_1, Z_2, Z_3)$.

We see that if a release occurs, it can either result in a pool fire, an explosion or no effect, depending on the results of the branching events, immediate ignition, and delayed ignition. The model provides four scenarios:

s_1: release - A - pool fire
s_2: release - not A – B - pool fire (flash fire)
s_3: release - not A – B - explosion
s_4: release - not A - not B - no effect.

The quantities X and Z are unknown, and knowledge-based (subjective) probabilities are used to express the uncertainties (degree of belief). Suppose the following assignments have been made given the background knowledge K of the analysts:

$P(X= 1) = EX = 0.005$

$P(Z_1 = 1) = P(A) = 0.3$
$P(Z_2 = 1 | Z_1 = 0) = P(B | \text{not } A) = 0.2$
$P(Z_3 = 1 | Z_1 = 0, Z_2 = 1) = P(\text{pool fire} | \text{not } A, B) = 0.4$.

To interpret these numbers, consider for example, $P(Z_1 = 1)$. We have $P(Z_1 = 1) = P(A|K) = 0.3$, which means that the analysts consider the uncertainty of immediate ignition occurring (given a release) to be the same as drawing a red ball out of an urn which comprises ten balls of which three are red (Lindley [6]).

From this input we can use simple probability calculus to compute knowledge-based probabilities of the various releases; for example, we find

$P(s_3) = 0.005 \cdot 0.7 \cdot 0.2 \cdot 0.6 = 4.2 \cdot 10^{-4}$.

This illustrates the analysis; now let us reflect on the suitability of this approach in the risk analysis context.

Discussion

This way of conducting the analysis is simple, and all quantities introduced are well-defined and understandable. The probabilities produced are expressing the analysts' (experts') uncertainty (degree of belief) conditional on the background knowledge K of the analysis. The assigned probabilities, for example $P(A) = 0.3$, could be based on data; for example, we may have a situation where we have 3 "successes" out of 10 observations. Hence we derive at $P(A) = 0.3$, where A is the "success" event. This is our (*i.e.*, the analyst's) assessment of uncertainty about A. This probability is not an estimate of an underlying true probability $p=P_f(A)$ as in the frequentist setting, but an assessment of uncertainty related to the occurrence of A.

The probability P(A) could alternatively have been established on the basis of *analyst judgements using all sources of information*. This is a method commonly adopted when data are absent or are only partially relevant to the assessment endpoint. A number of uncertain exposure and risk assessment situations are in this category. The responsibility for summarising the state of knowledge, producing the written rationale, and specifying the probability distribution rests with the analyst (Kaplan [4], Aven [1]). In some cases formal expert elicitation can also be adopted to assign the probabilities. Formal expert elicitation may be undertaken when little relevant data can be made available and when it is likely that the judgement of the analyst will be subject to scrutiny, resulting for example in costly project delays. Formal expert elicitation could be very expensive, so a justification for when to adopt such a procedure is required (Kaplan [4], Aven [1]).

The main problem with this approach is that there exists no formal procedure for systematically incorporating information and distinguishing between variation and lack of knowledge (epistemic uncertainties). Say that we, at a specific point in time, have assigned $P(A) = 0.3$ by a direct argument, and then need to adjust this number as a result of getting some new data relevant for this event. How should we then update our probability? The answer using the direct assignment approach is simply to perform a new direct assignment. Obviously such a method could lead to inconsistencies and the production of some rather arbitrary numbers in some cases. The Bayesian machinery, as will be considered in the next section, is much better in this respect as it provides a well-established and justified approach for how to carry out such updates.

The use of modelling could of course improve the assignment process with respect to consistency. In the next section we cover the case when probability models can be justified. But first let us summarise the method studied in the present section, its features and its pros and cons (Tables 1-2).

Table 1: Summary of Features of the Unique Event Case

Feature	Explanation
Quantities of interest	Occurrence of scenarios s_i
Uncertainty description for these	$P(s_i\|K)$
Model	Event tree, for example $s_3 = g(X,Z_1,Z_2,Z_3) = X(1- Z_1) Z_2 (1-Z_3)$
Unknown model parameters/quantities	X, Z_1,Z_2,Z_3
Uncertainty description for these	$P(X=1\|K)$, $P(Z_i= 1\|K)$, i=1,2,3
Method for these assignments	Data, analyst judgement using all sources of information, formal expert elicitation, modelling

Table 2: Summary of *Pros* and *Cons* of Method for Unique Event Case

Pros	***Cons***
Simple to conduct	Difficult to ensure consistency
All quantities well-defined and can be given easily understandable interpretations	Lack a procedure for taking into account new information
	The numbers assigned may be difficult to assign (numbers may be seen as somewhat arbitrary)

3. The Case with Probability Models introduced (Probability of Frequency Approach)

Now let us assume that the analysts can justify the introduction of a probability model with parameters as follows (see Figure 2):

$$q_0 = E_f[X]$$
$$q_1 = P_f(A)$$
$$q_2 = P_f(B|\text{ not } A)$$
$$q_3 = P_f(\text{pool fire}|\text{ not } A,B).$$

For q_1, q_2 and q_3 it is tacitly assumed that the frequentist probabilities/chances are conditional on the occurrence of a release. To interpret the parameters we need to construct infinite populations of similar situations to the one studied. For example, q_1 represents the fraction of times immediate ignition occurs in the case of a release if the situation is repeated over and over again.

If we know all parameter values we can calculate the probabilities of the various scenarios using standard probability calculus. However, all parameters are unknown and we use knowledge-based (subjective) probabilities to express the analysts' uncertainties about the true value of these parameters.

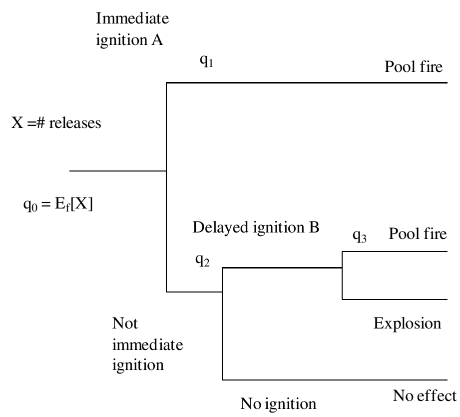

Figure 2: Event Tree for the LNG case, based on Probability Models

Let us concentrate our focus on the relative frequency probability of the scenario s_2 or s_3 occurring; let us call this frequentist probability r. From the above analysis, we have established a relationship (model) between this quantity and the underlying model parameters: q_0, q_1, q_2 and q_3:

$$r = P_f(s_2) + P_f(s_3) = q_0 [(1- q_1) q_2 q_3 + (1- q_1) q_2 (1-q_3)] = q_0 (1- q_1) q_2.$$

We next establish uncertainty distributions on the q_i parameters and use the event tree model to propagate these uncertainties to an uncertainty distribution for r. A numerical example will explain the ideas.

Let us first consider q_0, the expected number of releases. As an estimate of q_0, we used 0.005. To reflect uncertainties we use a subjective probability distribution. This distribution may, for example, be a beta-distribution, a triangular distribution or a uniform distribution. For this case we will simply assume that the analyst specifies a uniform distribution on the interval [0.003, 0.007], which means that the analyst is confident that the true q_0 lies in this interval, and that his/her degree of belief that q_0 lies in the interval [0.003, 0.005] is the same as [0.005, 0.007] (50%). We make similar assumptions for the other parameters. See overview in Table 3.

Table 3: Knowledge-based probabilities for the parameters q_0, q_1, q_2 and q_3

Parameter	**Distribution type**	**Interval**
q_0	Uniform	[0.003,0.007]
q_1	Uniform	[0.2,0.4]
q_2	Uniform	[0.1,0.3]
q_3	Uniform	[0.1,0.7]

Using these distributions and assuming "independent" distributions for the q_i parameters, we can calculate the knowledge-based distributions for r. Independence here means that if, for example, we know that q_2 is equal to 0.12 (say), this would not affect our uncertainty assessment of q_3 (say).

To establish the output distributions using analytical formulae is difficult. It is easier to use Monte Carlo simulation, and this is the common approach for performing this type of uncertainty assessment. Random numbers for each parameter are drawn, and, using the formula $r = q_0 (1- q_1) q_2$, we obtain the associated uncertainty distribution of r, shown in Table 4 and Figure 3. Note that these values are estimates of the probabilities given by the input of the Monte Carlo simulations: the uniform distributions and the formula $r = q_0$

$(1- q_1)\, q_2$. The estimation error is small as the number of replications is large. Hence, there is a knowledge-based probability of 44% that the chance of at least one fatality is in the interval (0.04%, 0.07%].

Table 4: Knowledge-based Probabilities P, for r

Interval for r	**Interval for r. Reformulated intervals (%) (x 10^{-2})**	**Simulated probability**
≤0.0002	≤ 0.02	0.00
(0.0002, 0.0004]	(0.02, 0.04]	0.12
(0.0004, 0.0007]	(0.04, 0.07]	0.44
(0.0007, 0.0010]	(0.07, 0.10]	0.28
(0.0010, 0.0013]	(0.10, 0.13]	0.13
(0.0013, 0.0016]	(0.13, 0.16]	0.03
> 0.0016	> 0.16	0.00

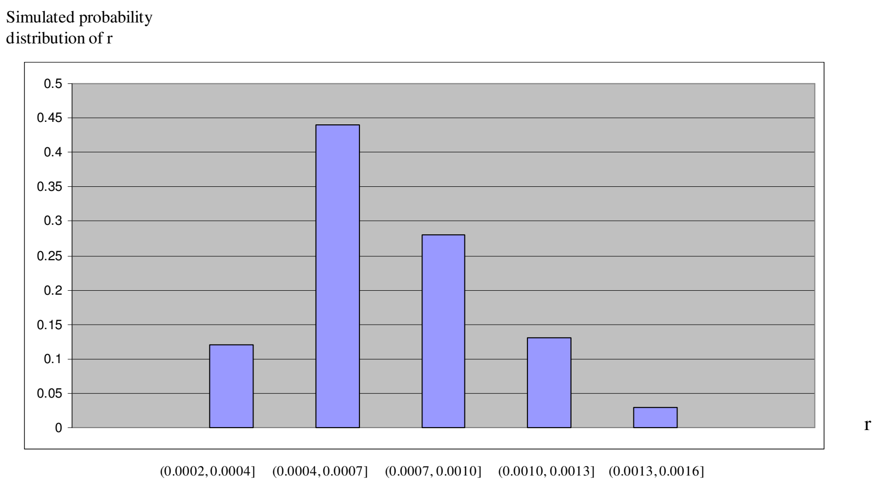

Figure 3: Knowledge-based Probabilities P for r based on Table 3

Discussion

The probability of frequency approach is theoretically appealing. It is in line with Bayesian theory. The idea is to first establish probability models that adequately represent the aleatory uncertainties, *i.e.,* the inherent variability of the phenomena studied. The epistemic uncertainties, reflecting incomplete knowledge or lack of knowledge about the values of the parameters of the models, are then represented by prior subjective probability distributions. When new data on the phenomena studied become available, Bayes' formula is used to update the representation of the epistemic uncertainties in terms of the posterior distributions. Finally, the predictive distributions of the quantities of interest, the observables, are derived by applying the law of total probability. The predictive distributions are epistemic, but they also reflect the inherent variability of the phenomena being studied, *i.e.,* the aleatory uncertainties.

However, in practice the method is not so easy to implement. Following this approach, the analysts are to express the epistemic uncertainties about the parameters of the probability models using subjective probabilities. In practice, it could be difficult to perform a complete uncertainty analysis within this setting. In theory, an uncertainty distribution on the total model and parameter space should be established, which is hard to do for complex cases with hundreds of parameters. If the uncertainty analyses do not cover all parameters, it is difficult to interpret the produced uncertainties.

It is obviously a challenge in practice to establish the epistemic distributions, as indicated above. However, more important is the conceptual issues. Introducing the chances means two levels of uncertainty, and one may question what is gained by this second level. The standard answer would be that we need to establish the probability models with the associated parameters to be able to apply the Bayesian machinery for ensuring consistency in the probability assignments and in the updating of probabilities in the case that new information becomes available. For many types of applications, such updating is important, in particular for risk assessments in an operational phase. However, for many cases (like the LNG case), such an updating is not considered essential, as the assessments are carried out at particular points in time to support specific decisions at these points. The assessment process is not of the form typically implemented when using Bayes' formula.

This approach presumes that a probability model can be justified. The key point is that we can generate an infinite, *i.e.*, in practice a large, number of similar situations to the one studied. For the LNG case, we need to think about similar years, or weeks or plants, but we quickly see that defining such a large population could be difficult – what should be fixed and what should be allowed to vary to generate the aleatory uncertainties? Defining the population that generates the aleatory uncertainties is critical for the proper understanding of what the parameters express. It is obvious that if we cannot provide meaningful interpretations of the parameters, the uncertainty analysis of the parameters will lose its importance as the numbers generated lack basis.

As noted by Singpurwalla [7], p. 17, the concept of frequentist probabilities "is applicable to only those situations for which we can conceive of a repeatable experiment". This excludes many situations and events. Think of the rise of the sea level over the next 20 years, the guilt or innocence of an accused individual, or the occurrence or not of a disease in a specific person with a specific history. Probability models cannot easily be defined. In our LNG case, it is easier to think about repeatability, but the definition of the large population is not obvious. The analysts need to make it clear what this repeatability means. In practice, it is seldom seen that such clarifications are made.

Tables 5 and 6 summarise the main features and the pros and cons of the probability of frequency approach.

Table 5: Summary of features of the Probability of Frequency Approach

Feature	**Explanation**
Quantities of interest	$P_f(s_i)$, for example $r = P_f(s_2) + P_f(s_3)$
Uncertainty description for these	Knowledge-based distribution of these frequentist probabilities (chances)
Model	For example $r = q_0 (1- q_1) q_2$
Unknown model parameters/quantities	q_i, i = 0,1,2,3
Uncertainty description for these	$P(q_i \leq q\| K)$, i = 0,1,2,3
Method for these assignments	Bayesian analysis

Table 6: Summary of *Pros and Cons* of the Probability of Frequency Approach

Pros	***Cons***
The method has a strong theoretical foundation	Quite complex to conduct
It provides a basis for ensuring consistency	Could be difficult to interpret the parameters of the probability models
It makes it possible to systematically take into account new information	Difficult to carry out complete uncertainty analyses

4. Conclusions

The issue is then, in a real case, which approach should be adopted? To answer this question, one has to clarify what are really the key quantities of interest. If it is clear that these quantities are frequentist probabilities, the probability model approach – *i.e.,* the probability of frequency approach − should be adopted. If it is not clear what the key quantities of interest are, the following question needs to be asked: is it important to have at hand a framework where new information can be systematically incorporated? If the answer is yes, the probability of frequency should be adopted, provided that frequentist probabilities can be justified. In all other cases, the unique event case should be adopted. Figure 4 shows the different alternatives.

Following these guidelines, the probability of frequency approach must be justified. The analysts need to make some reflections – are these criteria met? − before proceeding to the next stage of the analysis process – introducing the probability model with parameters. Such reflections are not often seen in practice.

The author of the present paper has good experience with using the unique event approach in many cases, for example for situations similar to the LNG case studied in this paper. The probability assignment processes are then rather simple, and the results achieved have been considered informative for supporting the decision making. Of course, due attention also has to be paid to the background knowledge (assumptions) that the assessment is based on. The results must always be seen in light of the background knowledge. The need for reflecting on the background knowledge also applies of course to the probability of frequency approach.

When applying the unique event approach, it is possible to use Bayesian analysis for a specific event. Say that a frequentist probability p is introduced for the event A, meaning that p is the fraction of times A occurs if we consider an infinite (large) number of similar situations to the one studied. Then the predictive probability to be used in the overall analysis is

$$P(A) = \int P(A|p)\, dH(p) = \int p\, dH(p) = E[p]$$

Where, H is the subjective probability distribution over p.

Having said this, it is clear that there exist situations (as shown by Figure 4) that benefit from introducing the more comprehensive set-up of the full probability of frequency approach. This approach has its attractive features, as discussed in Section 3, the main one being that the Bayesian machinery can be applied. Especially for application in the operational phase, this machinery is very useful as it allows for a systematic incorporation of new information.

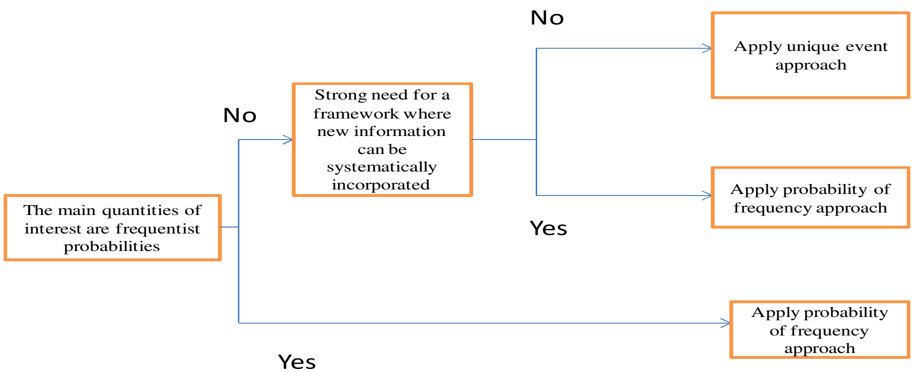

Figure 4: Schematic Procedure for selecting Analysis Approach

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