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# Oil price models and their impact on project economics

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Master thesis

By

Lorentz Aspen

Industrial Economics



University of Stavanger

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Lorentz Aspen

Stavanger 15.06.2011

## Abstract

When assessing values of petroleum projects, a key parameter is the oil price at which the extracted petroleum can be sold at. Many companies use a corporate planning price in their calculations. This price is more often estimated by a fixed price model than a more realistic model. History has shown that the oil price for the past forty years has been anything but stable and is constantly reacting to many different factors, such as war, politic upheavals, speculation and also to industry occurrences such as refinery constraints, oil spills, and discoveries. Consequently all of these events influence the balance in supply and demand, where the imbalance and future outlook is reflected in the price of oil. The choice of using a fixed price level to value projects, fails to embrace the volatility and uncertainty in the oil price and will subsequent lead too poor project evaluation as these features are not reflected in a projects value.

The work in this thesis has been to investigate and compare the behaviour and the uncertainty of four price models which offer different levels of detail and complexity; Fixed Price, Geometric Brownian Motion, Mean Reversion and a System Thinking approach. Using system thinking has not yet been popularized in price modelling. Much effort has therefore gone into establish and refine this model as the level of complexity and detail in this approach requires a reasonable amount of data and understanding.

To compare and evaluate how the models impact project economics, three realistic petroleum projects with different attributes were selected to perform analysis on; Knarr (Norway), Tawke (Kurdistan); Tiber (USA).

The selected price models, all but one, show to contribute to over 50% of the total uncertainty in a projects value. As a consequence, a price model used in project economics should therefore have uncertainty associated to it in order to reflect the possible values different price scenarios could impose in a project.

Two of the models are chosen as recommended models from this thesis work; The Mean Reversion (MR) model and the System Thinking (ST) approach. The Mean Reversion model used here offers a larger uncertainty range, but fails to embrace an increasing trend in the price. The System Thinking Model shows an increasing trend and has a reasonable uncertainty range; however it fails to embrace lower price levels.

The uncertainty in price models is highly influential to the valuation of projects. It is therefore vital to implement a realistic price model with uncertainty when assessing projects. The choice of model should be approached with respect to historic data and attuned for present and future outlook.

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## 1 Introduction

*“We make 30 years prognosis of the oil price,  
not knowing that we cannot even predict the price next summer”  
Nassim N. Taleb*

A key element in estimating net present values and the expected lifetime of petroleum projects is the price one can obtain for the recoverable petroleum product when selling it to the market. Previous studies affirm that the price obtained for the petroleum product affect the financial result of a project more than any other input parameter<sup>1</sup>.

Petroleum is sold in markets through spot prices and future contracts, where suppliers usually hedge themselves towards large price fluctuations by trade-off between contracted price and spot prices. By looking at the nominal price for oil the last 40 years, there is a growing trend for the price of oil, herein major fluctuations, peaks and dips. These are often traced back to events such as war, politic upheavals, speculation and also to industry occurrences such as refinery constraints, oil spills, and discoveries. Consequently all of these events influence the balance in supply and demand, where the imbalance and future outlook is reflected in the price of oil. These events are difficult to model, but since their impact is grave, a price model should embrace this volatility in a price projection. However this may seldom be the case in Exploration & Production companies in the petroleum industry. Price models are used in estimating value and lifetime of projects. Usually a fixed and quite conservative estimate of the price is chosen than more realistic models. Implications of this choice could be over- and underestimating project-values, leading to an incorrect portfolio ranking and may also lead to premature abandonment of mature projects or making non-optimized development decisions. An Industry performance study by Merrow<sup>2</sup> on over 1000 E&P projects, revealed that many projects failed to deliver the performance they promised.

Begg & Smit (2007) showed by using sensitivity analysis on price models that a Net Present Value-range of 3bn to 5bn dollars could be experienced in petroleum projects. This is partially explained by the uncertainty of subsurface quantities of projects, but mostly reflected in the large span of possible petroleum prices. They suggest a holistic approach is needed in assessing projects because of the large uncertainties experienced.

In this thesis a comparison and refinement of selected price models is undertaken. The price models chosen have different characteristics and presents different levels of detail and complexity. They are categorized in three levels, where level 1 is a simple model and level 2 and 3 offers more complexity and variability in increasing order of complexity:

- Level 1: Single price models.  
This level of modelling is limited to choosing a single price for all years. It can also be used as a variation such as having a transition period form current spot price to a long term “planning” price. This could also include a high and low price for stress testing purposes. A model in this level is the fixed price model (FP).
- Level 2: Stochastic price models.

This type of modelling shows the possibility to capture the volatility in the future price of petroleum by using historic data. Two models in this level are the Mean Reversion (MR) model and Geometric Brownian Motion (GBM) model.

- Level 3: Stochastic- and system thinking approach price-models.

For this level a system thinking model is used. System thinking requires a more holistic type of modelling. This means to model how a system influences a price more than looking solely on the price. By establishing a graphic model together with stochastic input data, a simulation of price movements over a given time is obtained. This approach allows the user to adjust all the parameters and the possibility to adjust the level of detail. A good model should embrace all characteristics found in the oil price. Because of the high uncertainty found in the price, any attempt to predict the exact price movement over time would be futile.

Introducing system thinking as a way to model the oil price has not been popularized yet. This approach offers a new way of modelling compared to the more established models. The system thinking approach gives the choice of constructing a model at different level of detail and complexity according to the desire of the user. However, a greater complexity in the model requires a deeper understanding of the petroleum market and industry. In addition, the amount of data needed is extensive. A drawback of the data requirement is the possible lack of data or quality to it. To create a good model by using system thinking approach certain conditions must be covered. The model must replicate how the petroleum market and industry works and embrace the fundamental influence patterns and understand how these ultimately affect the oil price.

However, the superior goal of this thesis is not to predict the “correct” oil price. It is to understand the uncertainty that originates from an oil price model and how the price models and their associated uncertainty affect project economics in the petroleum industry.

## 2 Crude oil; prices, history, trade and impacts

*“The Present is the living sum-total of the Past”*

*Thomas Carlyle*

Crude oil has emerged as one of the biggest commodity markets in the world<sup>3</sup> and has been traded since 1861. Up until 1970, petroleum was traded like any other commodity; buying and selling petroleum was in pure physical terms at spot prices. Today, crude oil is sold through a variety of contract arrangements such as futures, options, forwards and in spot transactions. This chapter will focus on how markets operate, the many ways petroleum is traded, historic events and price development and the general impact petroleum has globally. First, it is important to understand the historic movement of the price and the characteristics of it. There are also some historical events which will be reviewed. Through history, oil has increased its value as a resource, both for countries and companies. It has thus become at centre of attention for conflicts and control. The historic events addressed have still today a large influence on oil price. Then petroleum markets will be introduced, showing the market mechanisms and the ways they are employed. In an ever developing and unpredictable world it can also be important to point out the major impact which petroleum prices have on the industry and if not the global development and its economy.

### 2.1 Characteristics of the oil price

The oil price can be described by four main characteristics<sup>26</sup>:

- High volatility (fluctuations)
- Price jumps larger than what can be considered as “typical” fluctuations
- Almost normal distributions of % annual changes
- Tendency to revert to a long term mean

The graph in Figure 1 shows an increasing trend in the price for almost every year the last decade, except for the jump experienced in 2008.

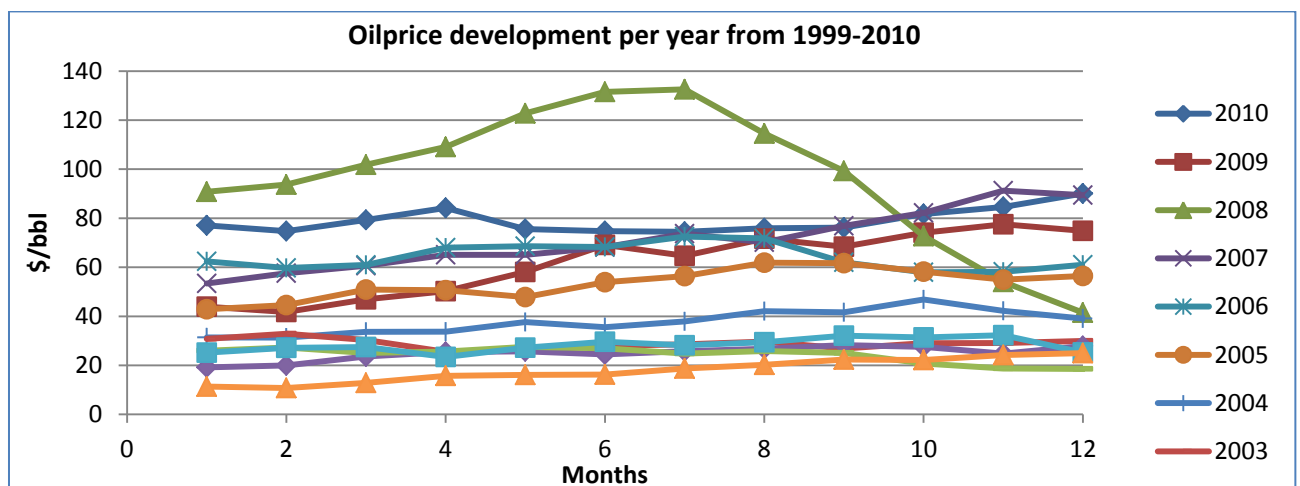


Figure 1 Oil price movements per year for the last decade, monthly average price

Looking at longer timespans, the oil price also shows a quite remarkable tendency to fluctuate around a mean trend. In Appendix A-D, a simple graphic analysis for four different time periods

show that the oil price has a clear tendency to fluctuate around a mean, slightly increasing trend, except for the period of the last 5 years, where the peak of 2008 disrupted the trend.

## 2.2 Important historic events

Historic events have proven important to our present petroleum trade. The reviewed events are believed to be important as they through history show great impact on the industry and the price of petroleum; and still have today. All four events have been deliberately targeted to create an impact or to be an instrument to influence the price. Either to create stability, transparency, control or free market mechanisms.

### 2.2.1 The division of Standard Oil

Standard Oil was the largest company in the world until 1911. In 1904 it controlled 91% of the production and 85% of the down-stream industry in the US<sup>4</sup>. Most of the end-product produced in these early years of petroleum was kerosene, where approximately 55% were exported. Controlling that much of the total market gave Standard Oil close to monopolistic control and competitors were timely forced out of business or acquired. In 1909 the US Department of Justice sued Standard Oil under federal anti-trust law for sustaining a monopoly and restraining interstate commerce. The result was the division of Standard Oil into 34 standalone companies in the up-, mid- and downstream industry<sup>4</sup>. The total size of Standard Oil was enormous. Some of the companies that emerged from the division are today some of the biggest companies in the world. I.e. ExxonMobil (both Exxon and Mobil where previous Standard Oil companies.), Chevron, Conoco (now ConocoPhillips), Amoco, (now merged into BP) and Marathon. The sheer size of these companies combined today would almost be unfathomable. The break-up of Standard Oil must be seen as a step to free the market mechanisms for oil production, refining and trading. If Standard Oil would be left to continue as it were it could create a monopolistic market in the world's largest producer and consumer country at the time.

### 2.2.2 The founding of OPEC

A very important historic event which still influences the world today was the founding of OPEC, a cartel made by sovereign petroleum exporting states. Talks between the oil producing countries Venezuela and Iran were commenced as early as 1949, but it was not until a political spark from the US to discriminate overseas oil supply in favour of Mexico and Canada that brought together Iran, Iraq, Kuwait, Saudi-Arabia and Venezuela in Baghdad in 1960 to form OPEC<sup>4</sup>. Today OPEC is a major player in the petroleum market and is considered to be a cartel working for their member's interest. According to their own statutes, their mission is as follows:

*"The mission of the Organization of the Petroleum Exporting Countries (OPEC) is to coordinate and unify the petroleum policies of its Member Countries and ensure the stabilization of oil markets in order to secure an efficient, economic and regular supply of petroleum to consumers, a steady income to producers and a fair return on capital for those investing in the petroleum industry"<sup>5</sup>.*

OPEC is believed to control 77,2% of the world's total proved reserves<sup>6</sup>, these numbers are based on data provided by OPEC countries themselves and since a majority of the operating companies are governmentally owned, the data can be considered somewhat biased. However OPEC share of the world's proven reserve should still considered being large. Today OPEC consists of 12 member countries and they are shown with their geographical locations and their joining year in Figure 2.

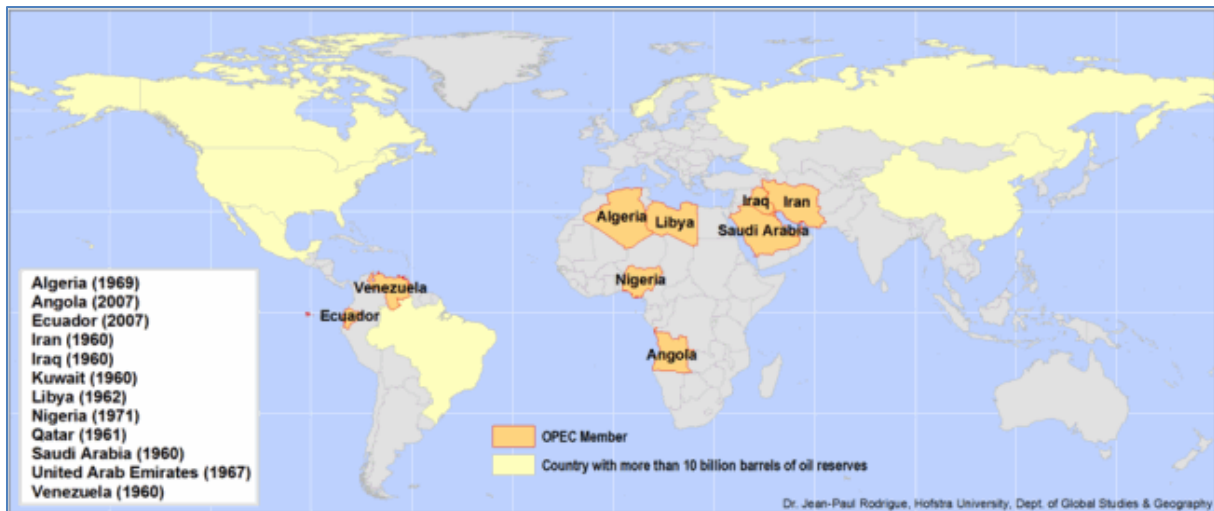


Figure 2 Map of OPEC members. © Jean-Paul Rodrigue, Hofstra University

### 2.2.3 The International Petroleum Act and the establishing of IEA

A vital event for OECD countries was the International Petroleum Act and the establishing of the International Energy Agency (IEA). The IEA was formed in the framework of Organization for Economic Co-operation and Development (OECD) as a reaction to the oil embargo launched by OPEC in connection with the Yom Kippur war which consequently led to the oil crisis of 1973. The International Petroleum Act requires IEA member countries to maintain total oil stock levels equivalent to minimum 90 days of the previous year's net imports. The initial role of IEA was to help members to coordinate a collective response to major disruptions in oil supply by releasing emergency oil stocks to the markets<sup>7</sup>. During its history, the IEA have intervened two times by releasing oil into the markets; 1991 during the Gulf War and in 2005 after hurricane Katrina affected US production, by releasing 2 million barrels per day for a month<sup>8</sup>. Research shows that reported OECD inventory levels, and more so, the US inventory level of petroleum products together with the Strategic Petroleum Reserve (SPR) in the US, strongly affect the fluctuations of the price of oil<sup>9</sup>. The inventories in US & OECD countries are reported weekly and these play a role as price markers in the market. For the WTI price the Strategic Petroleum Reserve (SPR) has been thought to have a considerable effect, but the price has shown to be more affected by the total stock of crude and petroleum products in the short run and the SPR for the long run<sup>17</sup>.

### 2.2.4 The Introduction of derivatives and futures market

In the 1970's, deregulation saw a dramatically increase in the degree of price uncertainty in energy markets, prompting the development of the first exchange-traded energy derivative securities<sup>10</sup>. This emerged as an instrument for industry players to manage and diversify price risk and to help raise capital. The markets were fashioned after similar commodity markets and helped promote market transparency and greater liquidity in trading<sup>11</sup>. The key attribute of derivatives is their leverage. They provide an efficient means of offsetting potential loss among hedgers and transferring risk from hedgers to speculators. The leverage and low trading costs in these markets attract speculators, and as their presence increases, so does the amount of information impounded into the market price<sup>10</sup>. These effects ultimately influence the underlying commodity price through arbitrage activity, leading to a more broadly based market in which the current spot price corresponds more closely to its true value. Because this price



influences production, storage, and consumption decisions, derivatives markets contribute to the efficient allocation of resources in the economy<sup>10</sup>. The WTI price was introduced in futures trading in 1983 at the New York Mercantile Exchange (NYMEX) and by 1990 there were 10 active oil futures contracts trading worldwide, with a combined daily volume 1.3 times more than the total oil demand<sup>11</sup>. Recent years, trade has been around 7 times larger than the total oil demand<sup>11</sup>. Both hedgers (commercial traders) and speculators (non-commercial traders) need to be present for a smooth operation of this market. But recently, the increasing presence of speculators, as seen in Figure 3, has been a subject of concern which could impose regulatory actions by governments, as the Dodd-Frank Act in the US<sup>12</sup>.

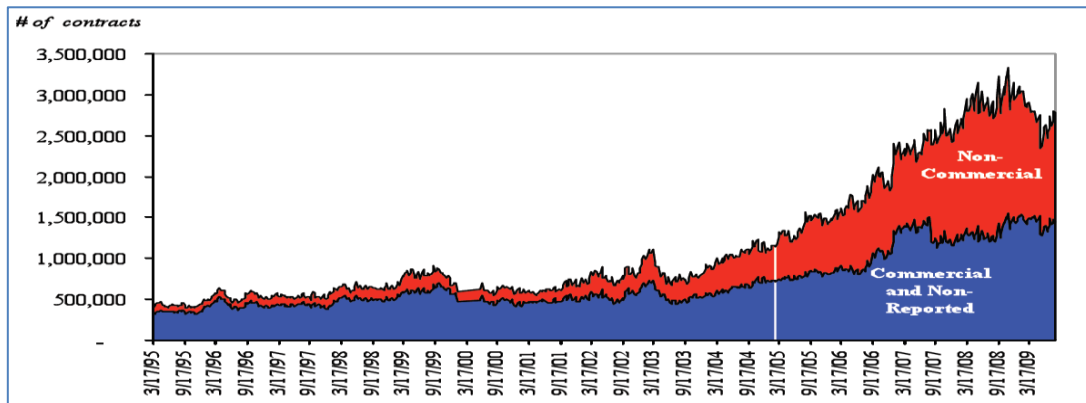


Figure 3 Number of future contracts traded at NYMEX futures. ©CFTC Commitment of Traders Reports. Medlock & Jaffe 2009.

While the division of Standard Oil was aimed to free the apparent control of the price, the three later events were set out to establish a form of price control or risk lowering by directly interfering with the supply and demand mechanisms in the markets. Inventory levels and OPEC's production and spare capacity are highly influential for the direction of the price. The future and spot market react almost instantaneously when EIA releases its weekly report. Studies show that inventory levels correlates to almost 92% of the variation in the WTI price<sup>17</sup>. IEA recently urged its members to increase the production and stated that they would use every tool available to influence the price path of oil to a more sustainable level for maintaining a steady economic growth<sup>13</sup>. Then again OPEC, depending on their member's national budgets and the oil price needed to fulfil them, can be of either great help or of great adversity. OPEC's production rate can be seen as the prime instrument for OPEC's short term price stimulus. The introduction of derivatives market has been studied a great deal in recent years. Especially after the remarkable price variations experienced in 2008 and the increased number of non-commercial players in the market. Although blame has been placed at some<sup>14</sup>, there is no consensus that these markets *alone* drove the price in this period.



## 2.3 Oil-markets and trading

*“We simply attempt to be fearful when others are greedy  
and to be greedy only when others are fearful”*  
Warren Buffett

### 2.3.1 Market mechanisms

The futures market is not generally used to supply physical volumes of oil, but more as a mechanism of risk distribution. These mechanisms play an important role in providing pricing information and trends to markets. The general price movements or trends in futures prices are compared to the expected future spot prices. A futures contract is a contract between two parties which promises to deliver a certain volume, to a certain price, at a certain time in the future. The seller of the contract will make a profit if the price decreases, while the buyer will make a profit if there is an increase in price. The time of the contract is called a maturity time this is usually 1, 2, 6 or 12 months. There are two types of market- labels as to how futures prices are related to the expected futures spot price; normal backwardation and contango<sup>3</sup>. Normal backwardation refers to the situation when the futures price is lower than the spot price. Contango refers to when the futures price is higher than the spot price. An example can be seen in Figure 4, where a 12 month contract is displayed both in a normal backwardation market and in a contango market as it approaches maturity.

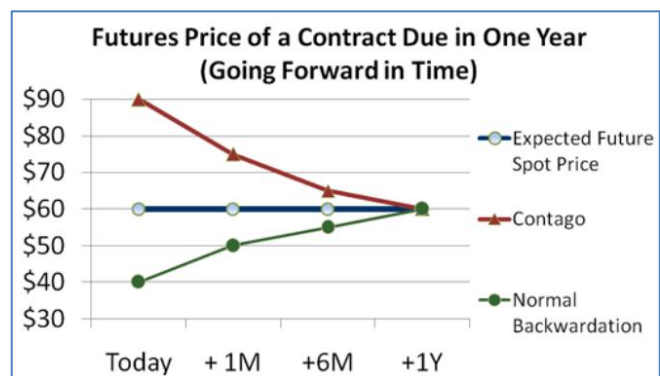


Figure 4 Futures Price of a contract due in one year. ©Investopedia 2007

A closely related type of contract is a forward contract. Forwards contracts are much like a futures contract, but forward contracts are not traded on the exchange, nor are they standardized. An option gives the possibility to trade in price differences, without exercising the right to buy the underlying stock or contract.

### 2.3.2 Crude markers and trade

The pricing of crude oils has become increasingly transparent through the use of marker crudes or crude assays the main criteria for marker crude or assay is for it to be sold in sufficient volumes to provide liquidity in the physical market as well as having similar physical qualities of alternative crudes. All in all, there are over 150 available crudes being traded and the price are adjusted generally by a formula approach where a marker crude is used as the base and then a quality differential (premium/discount) as well as a demand/supply (premium/discount) is added depending on the crude being traded. The most widely used marker crudes are:

- West Texas Intermediate (WTI – USA)
- Brent (Europe and Africa)
- OPEC basket (OPEC countries)

- Tapis and Dubai (in Asia)

The marker crude provides pricing information. WTI for example, does this through its use on the New York Metals Exchange (NYMEX) as the basis of a futures contract. The volumes of futures trading may be equivalent to many hundreds of millions of barrels per day, much more than the daily physical WTI productions and consumption<sup>15</sup>. A futures contract for crude oil is a promise to deliver a given quantity of crude oil but this rarely occurs. Participants are more interested in taking a position on the price of the crude oil. The position long will be when there is an expected growth (contango) and the position short where there is an expected fall in price (normal backwardation). Futures markets are a financial instrument to distribute risk among participants with the side effect of providing transparency on the pricing of crude oil. The Brent marker however, offers pricing information based more on the physical trading of oil through spot and forward trading. It also offers futures trading, but not to the same extent as WTI. Thus, in times of tight supply, this premium will rise and gradually drag up the marker crude price, whilst in times of surplus supply, a reduced premium or even a discount will drag down the marker crude price. Marker crudes can be considered as indicators of what is happening in regional markets. Of course big changes, announcement or events that can significantly influence crude supply levels will sometimes result a large step change in the prices of crude markers. It is this very complexity in markets which makes it very difficult to determine a theoretical price as part of regulation in markets because there may be a perception that because the theoretical price is different from the market price that the market price is for some reason unfair, showing that oil prices may simply not reflect the underlying fundamentals of supply and demand<sup>16</sup>.

## 2.4 Oil price movements

*“We learn geology the morning after the earthquake”*

*Ralph W. Emerson*

The lows and highs of oil price fluctuations can often be traced back to many factors such as political upheavals, wars, excess supply compared to demand, extreme climate conditions, stocks and hedge-funds, refinery capacities, transport availability, competition from other energy sources, emission and environmental concerns<sup>17</sup>. They all have a role in determining the final price charged to consumers and the role that each of these factors play can change over time. As seen in Figure 5, the oil price displays a volatile movement in the early years of trading, when the industrial use of refined products from petroleum started. During the first half and into the second half of 20<sup>th</sup> century the price can be seen as fairly stable. Even two world wars did not cause much a noticeable effect to the price movement.

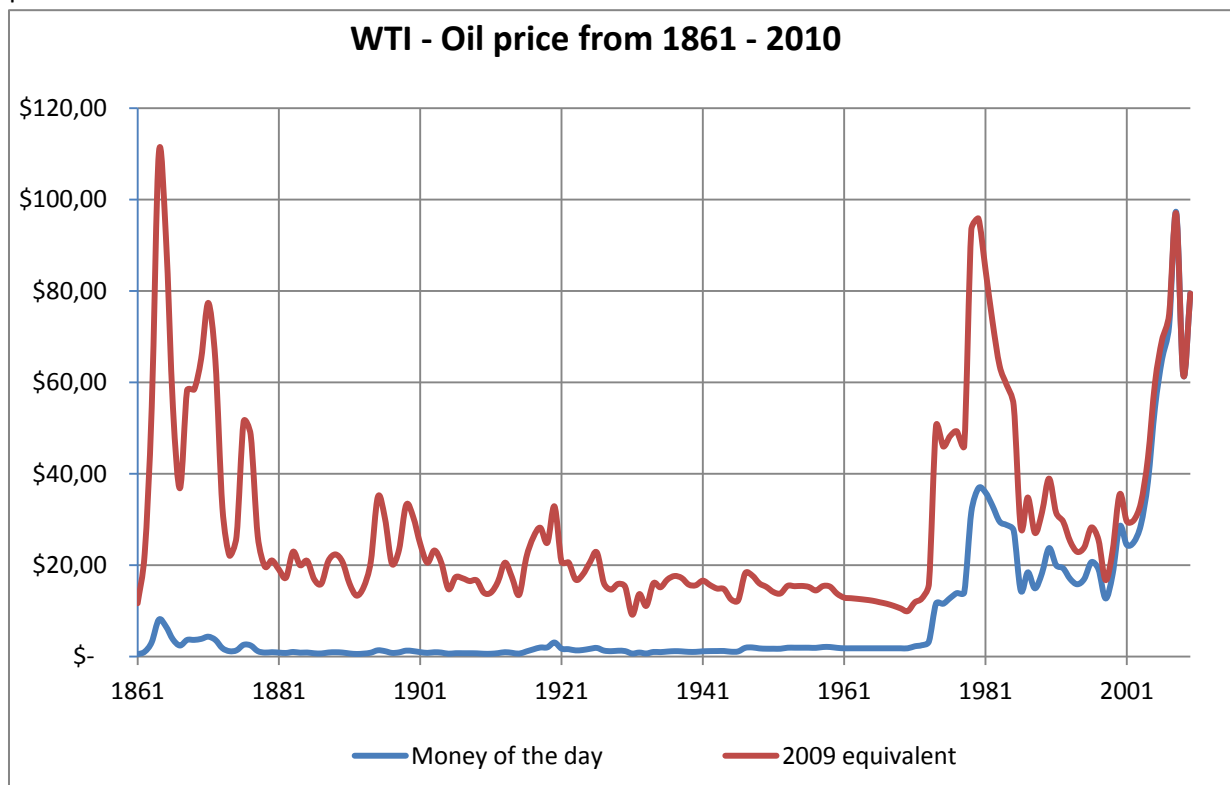


Figure 5 WTI Oil price from 1861-2010, average yearly price.

For the last 40 years however, the price has been anything but stable. Figure 6 below shows that the variations in price for the last 40 years are significant and for a commodity as vital as oil, the effect on society and industry is huge.

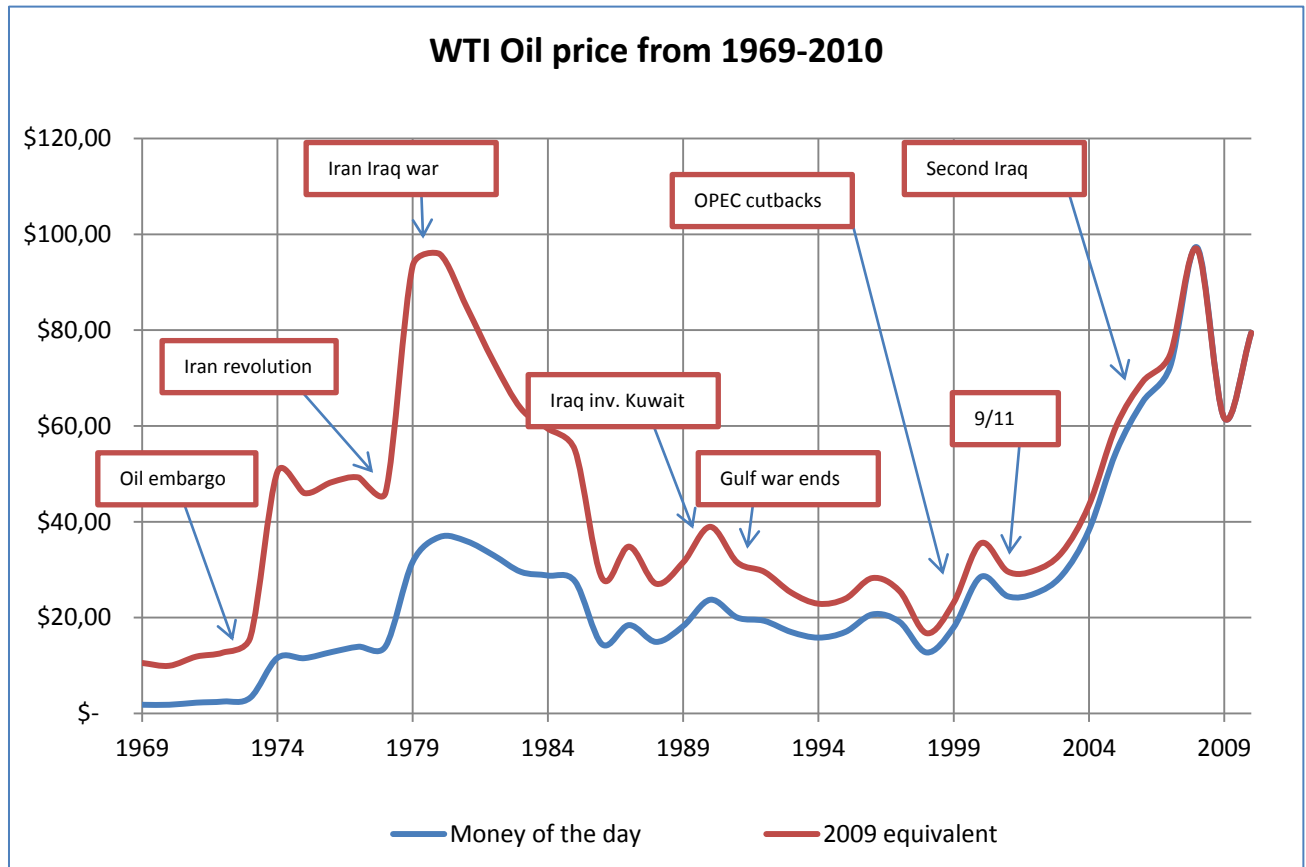


Figure 6 WTI Oil Price from 1969-2010, average yearly price

All these events have a different level of predictability and level of impact. An event such as 9/11 cannot be considered as predictable; however the retaliation from the US would not come as a shock to the world. Black Swans is a term set out in the book "The Black Swan" by the author Nassim Nicholas Taleb which is defined as events that are unpredictable with large consequences and can often only be explained "post-mortem"<sup>18</sup>. The oil price has, as shown in Figure 6, reacted to predictable and unpredictable events during history with great consequences, where predictable events with could affect the petroleum industry in some way usually incur a premium for this. The recent Libyan revolt can be considered a black swan. However, though the levels of crude oil stock in the US was increasing, and the Middle East crisis is currently not affecting production in any major oil producing country apart from Libya, the oil prices are still high on what is believed to be a fear premium. It is the anticipation of what the markets believe will occur in the Middle East that seems to cause the market prices to be high<sup>19</sup>.

## 2.5 Present reserves and future exploration for conventional oil

The amount of reserves not yet discovered is a number which receives much interest. Many academics and E&P companies state that all of the easy oil is already discovered and that the number of undiscovered basins/plays left in the world with recoverable petroleum reserves is miniscule and might be too costly to produce. Hubbert proposed in 1956 the peak oil concept when he made a forecast of ultimate recovery of crude oil for the US and the world. The original curve made by Hubbert is shown in Figure 7. After Hubbert's presentation, there have been countless debates over the timing of peak world conventional oil production rate and ultimate recovery<sup>20</sup>. The amount of undiscovered resources is highly argued and the estimates produced have high uncertainty. In Figure 8 the graph shows the many predictions of peak oil. A recent study combining the use of a mathematical modelling technique based on regression from historical production data using Hubbert's logistic model and a normal distribution model together with multiple-experts analysis have concluded that there are large uncertainties tied to the ultimately recoverable resources in the world.

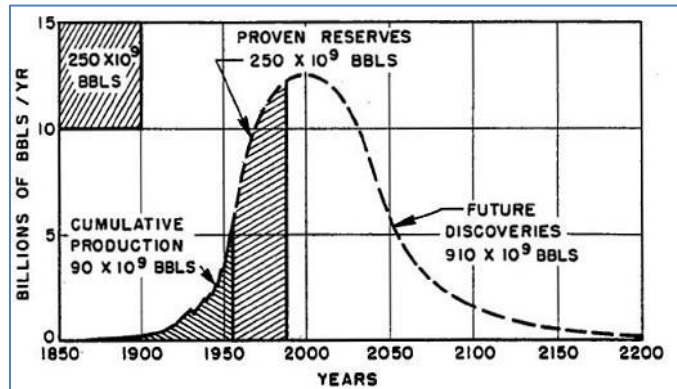


Figure 7 Hubbert's original curve predicting peak oil.  
©HubbertPeak.com

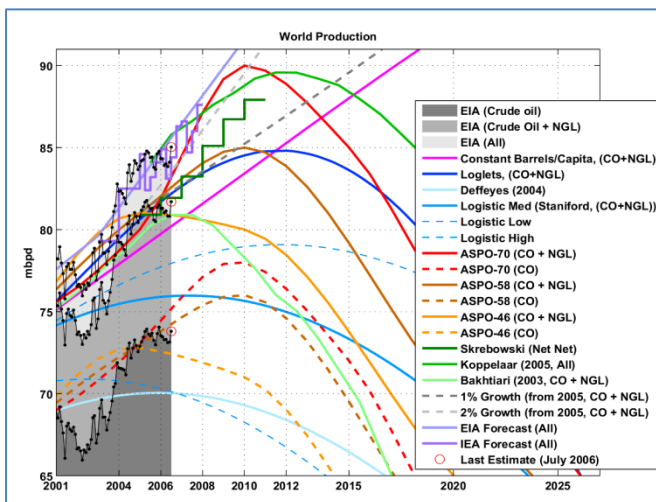


Figure 8 Display of several peak oil predictions. ©The OilDrum

$0.7 \cdot 10^{12}$  barrels and  $P5 = 0.4 \cdot 10^{12}$  barrels and  $P95 = 1.2 \cdot 10^{12}$  barrels<sup>21</sup> (These numbers are of undiscovered resources).

However, higher prices will encourage more exploration, increasing the amount of operating rigs and consequently the chance for finding new reserves will increase. High prices will also make smaller or *standalone* discoveries feasible thus adding them to the total number of proven reserves to be extracted. A projection of future discoveries were done by Association for the Study of Peak Oil (ASPO), their findings represented in Figure 9.

The report's best estimate is  $2.9 \cdot 10^{12}$  barrels with  $P10 = 1.8 \cdot 10^{12}$  barrels and  $P90 = 4.4 \cdot 10^{12}$  barrels. Because of some conservative assumptions done in the modelling, the uncertainty is considered larger than stated, and that it would also be in the upside of the  $P90$ <sup>20</sup> (These numbers are ultimately recoverable resources, total of proven and undiscovered). However US Geological Survey estimates are far narrower and lower, they have their best estimate at

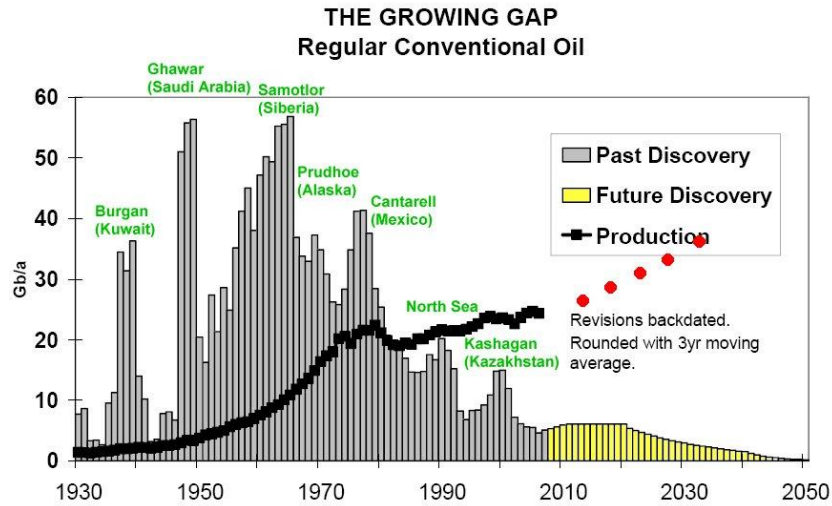


Figure 9 Discoveries of petroleum, past and future, for conventional oil. ©ASPO

ASPO’s projection for future discoveries is, according to them, optimistic and reaches a plateau around 7.000 MM bbl per year before declining after 2021. This is the equivalent of finding 14 giant oil fields<sup>1</sup> per year for 10 years. These numbers are also based for conventional oil<sup>2</sup>. As there is a general consensus in that most of the easy oil is already found, new areas and different types of plays are investigated for extractable reserves.

There is also an indication of increased rig activity related to higher petroleum prices. A graph displaying a reactive movement of rig activity versus the WTI oil price is shown in Figure 10.

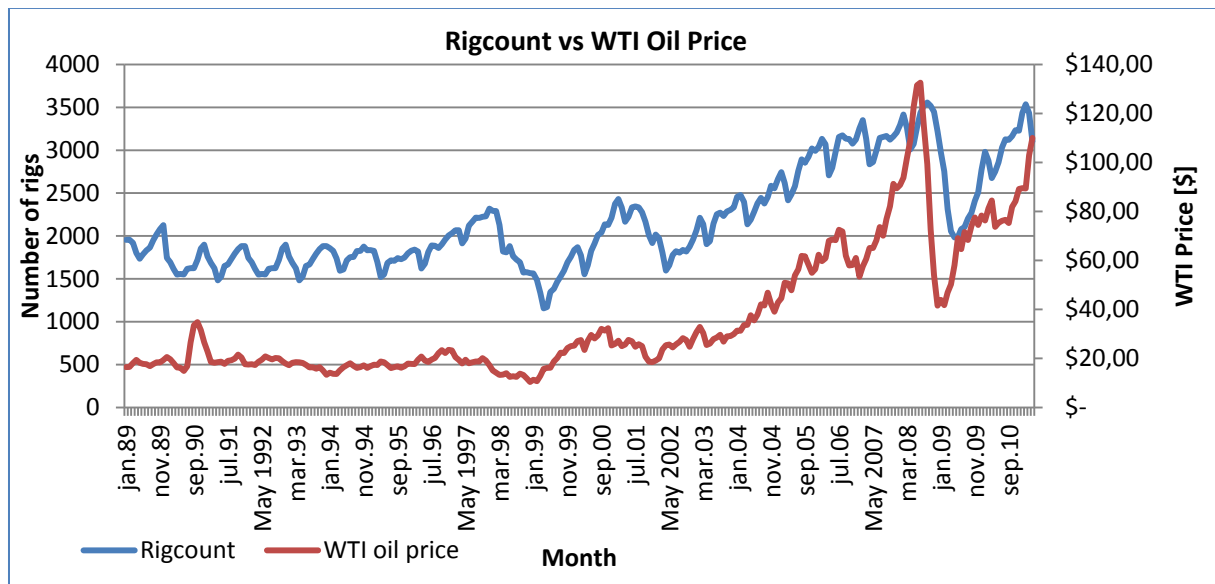


Figure 10 Rigcount vs WTI Oil Price. Rigcount data ©BakerHughes.

The increased number of rigs can mean either increasing exploration activity, initiating work-over on wells in existing fields or further development of existing fields. And an outcome of this is may be the addition of new reserves or increased production or recovery from existing fields.

<sup>1</sup> Giant oil field is defined as an oil field with more than 500 MM bbl of extractable reserves

<sup>2</sup> Conventional oil is liquid oil produced either through reservoir drive or by pumping

## 2.6 Impact of petroleum

Petroleum has grown into a key element for the industrialization and development of countries. It provides fuel for machinery within core industries, fuel for air-, land- and sea-transport and energy generation in the form of heat and light. Today there still is no real substitute which covers all of the attributes of petroleum at a reasonable price and efficiency level. It has thus become an extremely valuable resource which the control of and access to, is highly sought after. There are many predictions on how the demand for oil will change. The only consensus found is that it will increase, but with what factor is still not agreed upon. Demand for oil is primarily driven by growth and by reactions to the oil price. Oil is still considered as a prime fuel for economic growth and the IEA considers the demand for oil will increase to about 99 MM bbl per day in 2035<sup>22</sup>. In the same publication the IEA predicts all growth will be from non-OECD countries, primarily from China and India in this period of time. A prediction of future price of oil, states that the global economy is now experiencing the “China-India bump”, before an anticipated “Africa-bump” will succeed it sometime after<sup>23</sup>.

There is however a balance which needs to be maintained between economic growth and the price of oil<sup>23</sup>. A graphic display of this balance is shown in Figure 11. The platform represents the total amount of players in the petroleum industry; it acts as a scale, trying to balance out World GDP versus the Oil Price. A high price of oil will limit the global growth, and a low price of oil will induce a too sharp rise in the global growth. Both scenarios may create severe problems for the industry. The petroleum industry needs a price level at which it can affordably extract, produce and sell petroleum. But it also needs a price level which supports an adequate demand. This demand is highly influenced by the economic growth in the world.

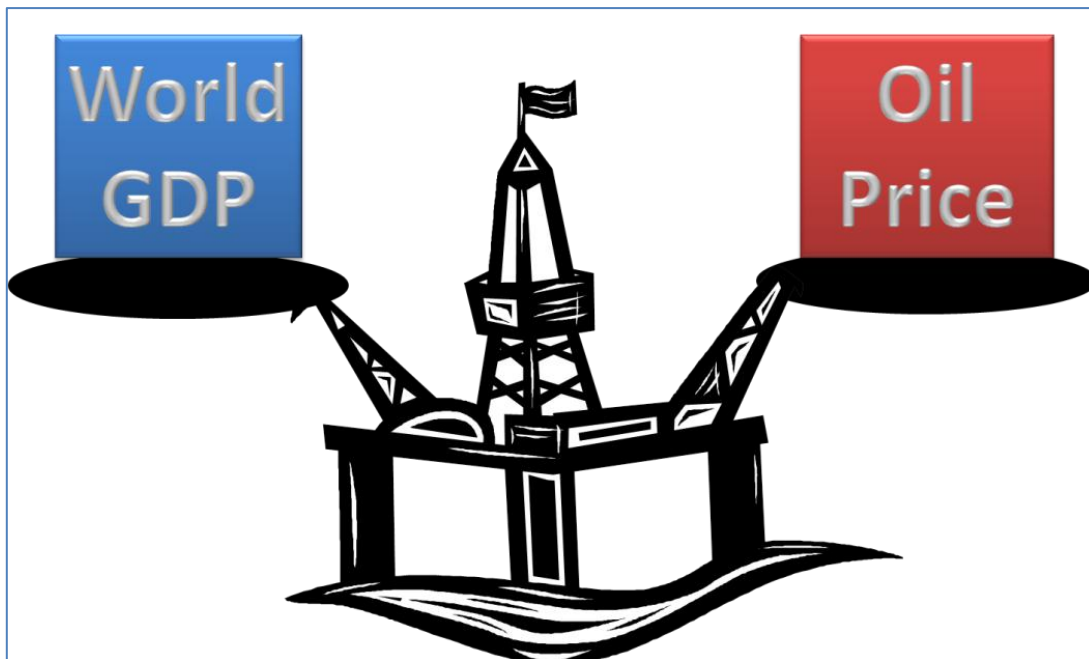


Figure 11 Graphic display of the petroleum industry balancing World GDP versus Oil Price

## 3 Price modelling

*“All models are wrong, some models are useful”*

*George E.P. Box*

### 3.1 Modelling purpose

The purpose of a price model is to investigate how a price behaves and understand the uncertainty which arise from its behaviour. Models of petroleum prices can be developed by using only market data as basis, or include factors beyond the markets. By assuming that all factors influencing the price is reflected in both spot and future prices, one can choose mathematical models which, by making estimates of historical data try to predict future price movements. For stock price returns, a Geometric Brownian Motion (GBM) model is often used, and for commodities a Mean Reversion (MR) model has been found to be preferable, due to a tendency of prices to revert to a long-term mean. A more holistic approach to modelling would be to implement a system thinking approach. System thinking will, instead of looking directly to the historic movement of the price, try to establish the interdependences which affect the movement of the price. System thinking therefore leads to a model which tries to replicate the industry trade movements and what dependencies these are subjected to and how. This type of modelling would require more data depending on how extensive the model is built. A drawback of this requirement is the lack of quality or certainty about some of the data. These uncertainties could then propagate in the simulations and consequently create large fluctuations. Thus there is a great value in the ability to identify and extract relevant input information.

It is also necessary to look at other factors in addition to price investigations. There have been significant changes in areas such as market mechanisms, trade patterns and supply and demand, especially in the past 40 years. These may have had a great impact on the price. It will therefore seem natural to focus more on the latter years when applying models, as data from these years would carry a better resemblance to the present market.



## 3.2 Single price models

### 3.2.1 Fixed Price model

A fixed price model is a model which utilizes a single estimated price. Its use in project economics is regarded as extensive, mostly as a long-term planning price. The choice of price is decided internally in the company and usually kept confidential. What mechanisms are used to set the price may just be as secret as the price set by the companies themselves, but it is reasonable to expect that historical data, futures markets, general market assumptions provided by leading energy organizations (IEA, EIA, OPEC and such) plays a vital role in setting the price. A survey performed by Pareto<sup>3</sup> Research among 22 oil companies concluded that the average planning price for 2010 was \$70 per bbl and the hurdle rate for new projects was on average \$55 per bbl<sup>24</sup>. The survey also provided the low, high and average planning price used in both major and independent E&P companies. An interesting point from the survey is that independent companies usually plan with higher oil prices than major companies. The diagram in Figure 12 represents the results from this survey. The planning price is what they expect the obtainable price for petroleum will be for the given year and further, it will however update itself from year to year. This is also shown in Figure 12, as the average planning price increased from 2009 to 2010.

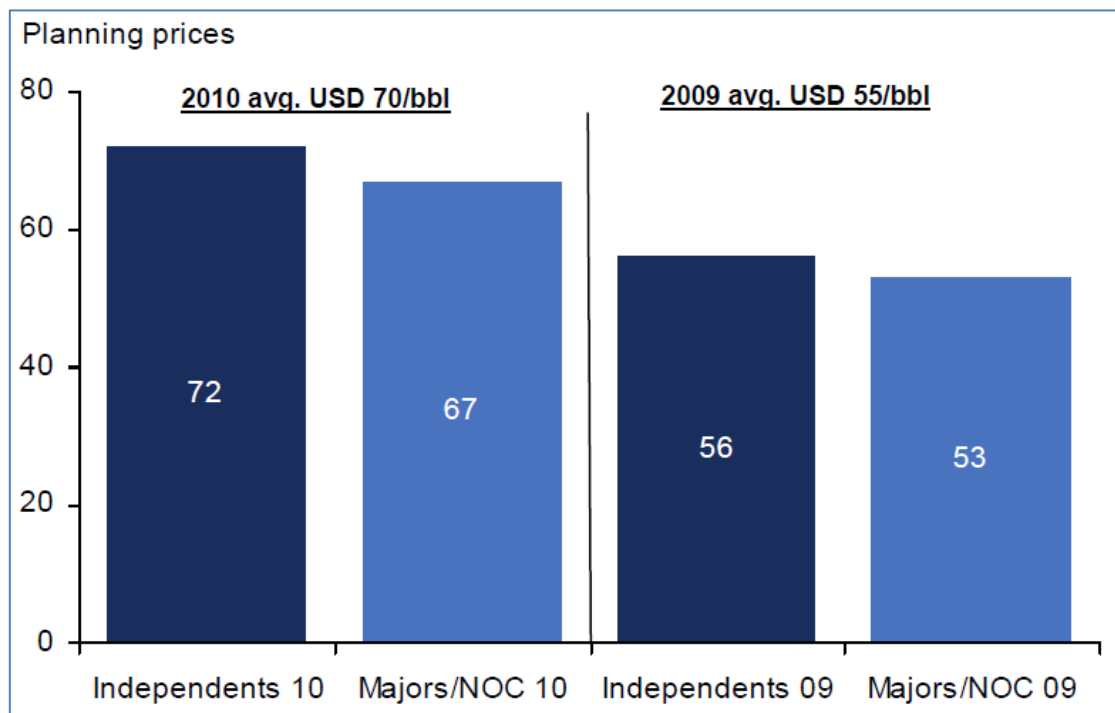


Figure 12: Planning prices for major and independent oil companies. ©Pareto Research

Characteristic of a FP model:

- No inclusion of uncertainty
- Variations in the price can be
  - Transition from current spot price to fixed price

<sup>3</sup> Pareto is a leading and independent Norwegian financial broker house

- Choice of setting high and low prices, or optimistic and pessimistic level (this is however not uncertainty modelling, but stress testing.)

### 3.2.2 Fixed Price equation

$$P_t = P \quad \text{3.2-1}$$

Where the price,  $P$ , is the chosen price level for a given year,  $t$ .

### 3.2.3 Fixed Price estimation

There is no consensus or established technique on how an estimation of fixed price is performed. However, guiding statements from major industry players, analysts and agencies such as OPEC, EIA, and IEA may have a large impact on the estimate.

### 3.3 Stochastic price models

#### 3.3.1 Geometric Brownian Motion

The Geometric Brownian Motion (GBM) is a central model in finance, and it is widely used in modelling stock returns. The equation was introduced in 1965 by Paul Samuelson as a revised version of the Arithmetic Brownian Motion. The Arithmetic Brownian Motion has the ability to produce negative values which would be invalid in many applications, especially in price modelling. The GBM does not have this characteristic because of its lognormal features. The GBM has been used as a fundamental assumption in the famous Capital Asset Pricing Model (CAPM)<sup>3</sup>. The CAPM is used to determine a theoretically appropriate rate of return of an asset<sup>25</sup>. GBM is also used as factor in the Black- Scholes- model to model stock prices and is the most used model for stock price behaviour.

The following characteristics are identified for the Geometric Brownian Motion:

- The expected change is independent of the value of the process
- The GBM follows a Markov-process, where the future movement is only dependent on the last value.
- GBM does only produce positive values
- Volatility and drift are assumed constant

#### 3.3.2 Geometric Brownian Motion equation

$$\frac{dP}{P} = \alpha dt + \sigma \varepsilon \sqrt{dt} \quad 3.3-1$$

Equation 3.3-1 can be interpreted as follows:

Return = Drift Effect + Volatility Effect

or

Relative Price Change = Expected Trend + Uncertainty Component

**P** is the price and **dP** is the differential price changes and  $\alpha$  is the drift coefficient. A positive  $\alpha$  results in an increasing trend and a negative  $\alpha$  in a decreasing trend. The  $\sigma$  is the standard deviation,  $\varepsilon$  is the standard normal distribution and **dt** is the differential time change.

Solving (3.3-1) analytically by Ito's lemma from stochastic calculus<sup>26</sup> yields:

$$P_{t+1} = P_t * e^{\left\{ \left( \alpha - \frac{1}{2}\sigma^2 \right) \Delta t + \sigma \varepsilon \sqrt{\Delta t} \right\}} \quad 3.3-2$$

#### 3.3.3 Geometric Brownian Motion parameter estimation

The estimation of the parameters is done by the following equations<sup>26</sup>:

$$(\alpha - 0,5\sigma^2) = \frac{\sum_{t=2}^n (\ln(P_t) - \ln(P_{t-1}))}{n-1} \quad 3.3-3$$

Or

$$\alpha = \frac{\sum_{t=2}^n (\ln(P_t) - \ln(P_{t-1}))}{n-1} + 0,5\sigma^2 \quad 3.3-4$$

$$\sigma = \sqrt{\frac{\sum_{t=2}^n ((\ln(P_t) - \ln(P_{t-1})) - \frac{\sum_{t=2}^n (\ln(P_t) - \ln(P_{t-1}))}{n-1})^2}{n-1}} \quad 3.3-5$$

Where,  $n$ , refers to sample size and,  $P$ , to historical prices,  $t=2$  refers to the first year with possibility to extract difference between two years of data.

### 3.3.4 Mean Reversion

The basic form of the Mean Reversion Process is also known as the Ornstein-Uhlenbeck Process<sup>27</sup>. This process attempts to address one of the limitations for commodity price modelling in the Geometric Brownian Motion. GBM does not recognize a dependency in the price fluctuations over time, leaving every movement in price totally dependent of the previous step<sup>28</sup>. The mean reverting process is one of the main properties that have been systematically incorporated in the recent literature on commodity pricing modelling, because of its ability to include the key characteristics of commodity price behaviour<sup>3</sup>.

The following characteristics are identified for a mean reversion process<sup>26</sup>:

- Produces only positive values
- Simple, can be solved analytically. Easy to estimate parameters from historical data.
- Independent of the units of price
- Good representation of the behaviour of commodity markets.
- Revert around a trend. The change from time step to time step is dependent of each other, which identifies it as a Markov process.
- The confidence bounds converges as the variance converges to  $\sigma^2/2\eta$  as time increases

### 3.3.5 Mean Reversion equation

By assuming that the logarithm of the oil price follows an Arithmetic Ornstein-Uhlenbeck process, Schwartz proposed a model that has become known as the “Schwartz Model 1”. The geometric Ornstein-Uhlenbeck process is given by the equation:

$$\frac{dP}{P} = \eta(\ln P^* - \ln P) * dt + \sigma \epsilon \sqrt{dt} \quad 3.3-6$$

Equation 3.3-6 can be interpreted as follows:

Return = Mean Reversion Effect + Volatility Effect

or

Relative Price Change = Expected Trend + Uncertainty Component

$dP$  is the differential price change and  $P$  is the price at some instant.  $\eta$  is the mean reversion rate or the speed at which the price tends to revert back to the mean.  $\sigma$  is the standard deviation of the

assumed normal distribution of the volatility term, and  $\epsilon$  is the standard normal distribution. The parameter  $dt$  is the differential time change.  $P^*$  is defined as the long term equilibrium price.

### 3.3.6 Mean reversion parameter estimation

The parameters of the mean reversion process are derived from a linear regression of the following data<sup>26</sup>:

$$\ln(P_t) - \ln(P_{t-1}) \quad 3.3-7$$

versus

$$\ln(P_{t-1}) \quad 3.3-8$$

which results in:

$$\ln P_{t+1} - \ln P_t = a + b * \ln P_t + \epsilon_{Regression} \quad 3.3-9$$

From this regression the estimation of parameters can be performed by the following equations:

$$\eta = -\ln(1 + b) \quad 3.3-10$$

$$\sigma = \epsilon_{Regression} \sqrt{\frac{2 * \ln(1+b)}{(1+b)^2 - 1}} \quad 3.3-11$$

$$P^* = e^{\left(-\frac{a}{b} + \frac{\sigma^2}{2\eta}\right)} \quad 3.3-12$$

### 3.4 Stochastic- and system thinking approach price-models

#### 3.4.1 System Thinking model - Background

System thinking is a computer-aided approach to policy analysis and design. Its applications would be to investigate dynamic problems in complex social, managerial, economic and or ecological systems, literally any dynamic system which can be characterized by interdependence, interaction, feedback and circular causality<sup>29</sup>. System thinking includes several tenets<sup>30</sup> for what a model should embrace:

- Interdependence of objects and their attributes - independent elements can never constitute a system
- Holism - emergent properties not possible to detect by analysis should be possible to define by a holistic approach
- Goal seeking - system interaction must result in some goal or final state
- Inputs and Outputs - in a closed system inputs are determined once and constant; in an open system additional inputs are admitted from the environment
- Transformation of inputs into outputs - this is the process by which the goals are obtained
- Entropy - the amount of disorder or randomness present in any system
- Regulation - a method of feedback is necessary for the system to operate predictably
- Hierarchy - complex wholes are made up of smaller subsystems
- Differentiation - specialized units perform specialized functions
- Equifinality - alternative ways of attaining the same objectives (convergence)
- Multifinality - attaining alternative objectives from the same inputs (divergence)

The field developed initially from the work of Jay W. Forrester. His seminal book *Industrial Dynamics* from 1961 is still a significant statement of philosophy and methodology in the field<sup>31</sup>. It has now grown from considering corporate and industrial problems to include Research & Development management, urban studies, commodity cycles and growth dynamics. It is now applied in economics, public policy, environmental studies, defence and theory building in social sciences.

The system thinking approach to model building requires the following<sup>32</sup>:

- Defining problems dynamically, in terms of graphs over time.
- Thinking of all concepts in the real system as continuous quantities interconnected in loops of information feedback and circular causality.
- Identifying independent stocks or accumulations (levels) in the system and their inflows and outflows (rates).
- Formulating a behavioural model capable of reproducing, by itself, the dynamic problem of concern. The model would usually be a computer simulation model expressed in nonlinear equations, but is occasionally left un-quantified as a diagram capturing the stock-and-flow/causal feedback structure of the system.

Forrester's original work stressed a continuous approach, but increasingly modern applications of system dynamics contain a mix of discrete differential equations and continuous differential or integral equations. Some practitioners associated with the field of system dynamics work on the mathematics of such structures, including the theory and mechanics of computer simulation, analysis and simplification of dynamic systems, policy optimization, dynamical systems theory, and complex nonlinear dynamics and deterministic chaos.

### 3.4.2 System Thinking – Basic model

Conceptually, the feedback concept is at the heart of the system thinking approach. Diagrams of loops of information feedback and circular causality are tools for conceptualizing the structure of a complex system and for communicating model-based insights. Intuitively, a feedback loop exists when information resulting from some action travels through a system and eventually returns in some form to its point of origin, potentially influencing future action. The loops can generate both negative and positive feedback back to their origin thus generating all manner of dynamic patterns.

The loop concept underlying feedback and circular causality by itself is not enough, however. Complex systems change over time. A crucial requirement for a powerful view of a dynamic system is the ability of a formal model to change the strengths of influences as conditions change.

In a system of equations, this ability to shift loop dominance comes about endogenously from nonlinearities in the system<sup>33</sup>.

For example, the S-shaped dynamic behaviour of the classic logistic growth model:  $\frac{dP}{dt} = \alpha P - \beta P^2$  can be seen as the consequence of a shift in loop dominance from a positive, self-reinforcing feedback loop ( $\alpha P$ ) producing exponential growth to a negative balancing feedback loop ( $-\beta P^2$ ) that brings the system to its eventual goal<sup>33</sup>. Only nonlinear models can endogenously alter their active or dominant structure and shift loop dominance. From a feedback perspective, the ability of nonlinearities to generate shifts in loop dominance and capture the shifting nature of reality is the fundamental reason for advocating nonlinear models of social system behaviour.<sup>33</sup>

The concept of endogenous change is fundamental to the system thinking approach. Corrective responses are also not modelled as functions of time, but are dependent on conditions within the system. Time by itself is not seen as a cause. Theory building and policy analysis are significantly affected by this endogenous perspective. The effort is to uncover the sources of system behaviour that exist within the structure of the system itself.

These ideas are captured in Forrester's (1969) organizing framework for system structure:

- Closed boundary
  - Feedback loops
    - Levels
    - Rates
      - Goal
      - Observed condition
      - Discrepancy
      - Desired action

The closed boundary signals the endogenous point of view. The word closed here does not refer to open and closed systems in the general system sense, but rather refers to the effort to view a system as causally closed. The goal is to assemble a formal structure that can by itself reproduce the essential characteristics of a dynamic problem.

Feedback thinking can be interpreted as a consequence of the effort to capture dynamics within a closed boundary. Without causal loops, all variables must trace the sources of their variation ultimately outside a system. Assuming instead that the causes of all significant behaviour in the system are contained within some closed causal boundary forces causal influences to feed back upon

themselves, forming causal loops. Feedback loops enable the endogenous point of view and give it structure<sup>34</sup>.

Stocks (inventory levels) and the flows that affect them are essential components of the system structure. A map of causal influences and feedback loops is not enough to determine the dynamic behaviour of a system. A constant inflow yields a linearly rising stock; a linearly rising inflow yields a stock rising along a parabolic path etc. Stocks are the memory of a dynamic system and are the sources of its disequilibrium and dynamic behaviour<sup>34</sup>.

### 3.4.3 System thinking approach to price modelling

For system thinking, a more holistic approach to modelling is required. This means to investigate all factors able to influence the price path, not only the price path itself, and model the interdependencies that exist between the factors. The possibility of using nonlinear feedback loops in systems thinking together with stochastic variables generates a dynamic model which can replicate numerous real world situations. System thinking has not been widely adopted for price modelling. Sterman (2000) presented a framework for commodities modelling in his book *Business Dynamics: System Thinking and Modelling for a complex world*<sup>35</sup>. The Sterman-model consists of four superior components interlinked; Production, Capacity, Demand and Price. These are shown in relation to one another in Figure 13 and are explained below.

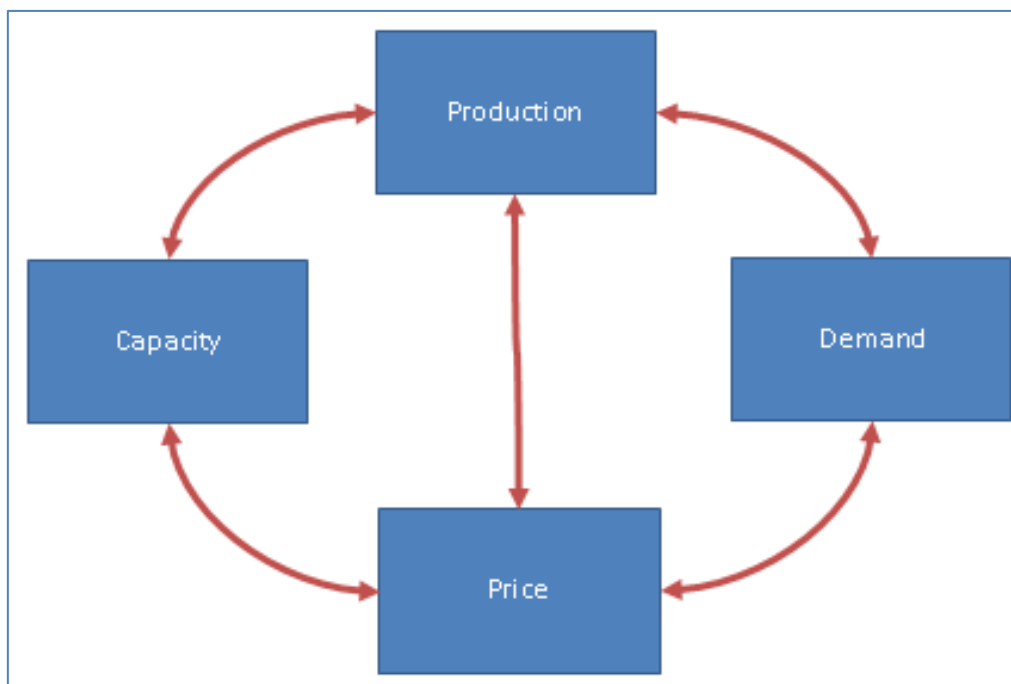


Figure 13 Simplified diagram of the Sterman-model for commodities

Capacity allows for production, while shrinking inventory might require the building of new capacity. Similarly, prices for a commodity might indicate that new capacity will be profitable. Capacity is needed for production, while the production rate will influence the decision of whether to replace capacity. The relationships between production, demand and price come from the fundamental law of supply and demand<sup>35</sup>. However the simplified box diagram in Figure 8 obscures the finer details present in the Sterman model. It is essential to look at each part in detail to get a true picture of commodity markets.



### 3.4.4 System Thinking equation

Mathematically, the basic structure of a formal system thinking computer simulation model is a system of coupled, nonlinear, first-order differential equations<sup>33</sup>,

$$\frac{d}{dt} \mathbf{x}(t) = \mathbf{f}(\mathbf{x}, \mathbf{p}) \quad \mathbf{3.4-1}$$

In equation 3.5-1,  $\mathbf{x}$  is a vector of levels (which is either on the form as stocks or variables from node inputs),  $\mathbf{p}$  is a set of parameters, and  $\mathbf{f}$  is a nonlinear function.

Simulation of such systems is easily accomplished by partitioning simulated time into discrete intervals of length  $dt$  and stepping the system through time one  $dt$  at a time. Each state variable is computed from its previous value and its net rate of change  $\mathbf{x}'(t)$ <sup>33</sup>:

$$\mathbf{x}(t) = \mathbf{x}(t - dt) + dt * \mathbf{x}'(t - dt) \quad \mathbf{3.4-2}$$

The computation interval  $dt$  is selected small enough to have no noticeable effect on the patterns of dynamic behaviour exhibited by the node, although a too small  $dt$  will minimize the effect on extremities in the model.

## 4 Project Economics

*“There are so many men who can figure costs, and so few who can measure values”*

*Anonymous*

In this chapter the technical and economic data which is used in project economics are addressed. When a project is assessed for performance, different types of metrics are used. These metrics are presented in the end of the chapter.

Project economics for oilfields is in very general terms based on finding the best estimate of the Original Oil In Place (OOIP), assessing the recovery factor and then running an expected production profile. This will yield estimates for the yearly production of a field. Calculations are then made to derive the gross revenue, expected capital expenditures (CAPEX), expected operating expenditures (OPEX), and adjusted for tax and depreciation. This will give sufficient data to employ the metrics of choice and perform analysis of the field and its economic potential.

A point in project economics which is vital to address, is the failure of projects to return on the predicted technical and economic metrics that formed the basis of the investment decision<sup>36</sup>. Merrow<sup>2</sup> performed a study over 1000 E&P projects, with CAPEX ranging from \$1 million - \$3 billion. By defining 3 criteria, where failing 2 resulted in “disaster”, he showed that 13% of projects were disaster and for projects with CAPEX larger than \$1 billion over 50% were “disasters”. The criteria were as follows:

- >40% cost growth
- >40% time slippage
- 1<sup>st</sup> year operability < 50% of plan

Failure to meet the investment criteria and achieve the performance level set in the beginning of a project can be related to several issues. Begg & Bratvold (2004) argues that the root cause of the failure of many projects to achieve their optimal performance is uncertainty, in its broadest sense, which leads to over-estimating returns or under-estimating the risks of loss<sup>36</sup>.

When assessing a project, it will therefore seem vital to include the uncertainty associated with the factors which are being used. Only then can a project’s up- and downside be properly assessed and accounted for in future decisions.

## 4.1 Technical data

The size and quality of a field is assessed by its physical quantities. This assessment is usually conducted by interpreting seismic data, well logs, formation tests, flow studies, reservoir conditions etc. A model of the fields' reservoir can then be created by appropriate software. This provides a better reservoir characterization and quantity assessment. An example model in 3-D is shown in Figure 14.

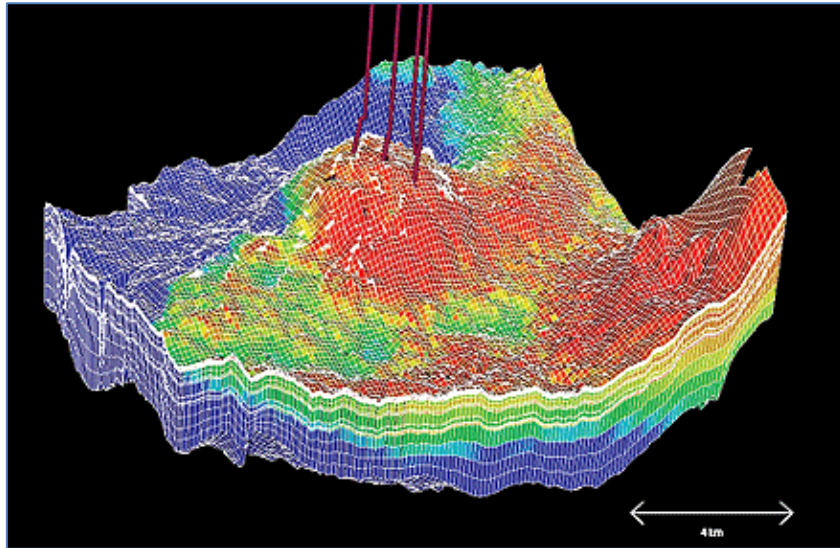


Figure 14 Reservoir model made by computer modelling. ©BG group

### 4.1.1 Original Oil in Place

The original oil in place (OOIP) is defined as the volume oil initially present in a reservoir before extraction. The OOIP is calculated by the following equation and factors:

$$OOIP = \frac{\text{Gross Rock Volume} * \text{Porosity} * (1 - \text{Water saturation})}{\text{Volume Factor}} \quad 4.1-1$$

Where,

Gross Rock Volume: Total volume of the reservoir

Porosity: Fluid filled porosity of the rock

Water Saturation: Water filled part of the porosity (residual water)

Volume Factor: Difference between reservoir and standard conditions

The OOIP gives the total amount of petroleum present in the reservoir.

### 4.1.2 Recoverable reserves

Not all of the total reserves found in the reservoir can be extracted, thus the recoverable reserves is adjusted by a field specific recovery factor:

$$\text{Recoverable Reserves} = \text{STOOIP} * \text{Recovery factor} \quad 4.1-2$$

The recovery factor is dynamic during a project's lifetime dependent on the following:

- Changes in reservoir characteristics, such as subsidence in the reservoir, loss of natural drive

- Employed IOR/EOR methods. Increased Oil Recovery (IOR) and Enhanced Oil Recovery (EOR) are various techniques to extract more oil from a reservoir.
- Recovery incentives provided by the resource owner. Governments may give incentives over tax schemes or demand strategies for extraction which may result in a change of recovery factor.

For example the Ekofisk field on the Norwegian Continental Shelf had an expected recovery factor at 17% at the start of production. Today it is estimated that a recovery factor of 50% is achieved<sup>37</sup>.

### 4.1.3 Production profile

The production profile is a graphic display of the expected production one will obtain from a field. The profile will be dynamic during the lifetime of a project and adjusted accordingly to incentives available to the producer. For example higher prices would be an incentive to prolong the life of a project, sustaining the tail life of a project for a longer period of time. Regulations, EOR/IOR improvements and implementations, field characteristics, prices and petroleum markets could be other reasons for a more dynamic profile.

The general production profile seen in Figure 15, constitutes of 3 parts.

- Build up
- Plateau
- Decline

**Build up:** In this period the field is starting production and the production will increase as more and more of the producing wells come online. Depending on the field and the desired number of production wells needed, the time ranges typically from 1-5 years.

**Plateau:** When all producing wells are online, plateau production is reached and is contained as long as there is energy in the wells to uphold the production capacity.

**Decline:** The natural energy in the reservoir is dissipating and production declines. This section can be prolonged depending on the available IOR/EOR methods for the specific field and is also very dependent on the initial size of the field. A large field will be able to produce commercially further into the decline, because of volumes still being of sufficient amount to be produced economically.

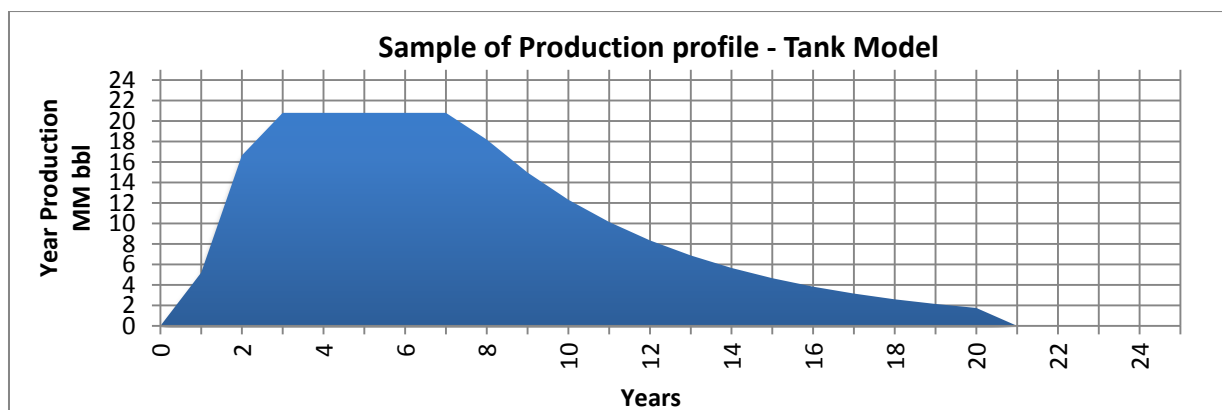


Figure 15 Sample of Field Production profile, Tank model

## 4.2 Economic data

### 4.2.1 Gross revenue

Gross revenue represents the total monetary amount received for selling petroleum at the wellhead at the specific field. Gross revenue calculations are done as follows:

$$GR = P_t * Q_t \quad 4.2-1$$

Where,

**GR** = Gross Revenue

**P<sub>t</sub>** = Average price in year, t, from oil price model [\\$]

**Q<sub>t</sub>** = Production in year, t, from production profile [bbl]

### 4.2.2 Capital expenditure

Capital expenditures represent the total expenditure needed to reach the desired exploitation and operation of a specific field. It will include investments such as production facilities, template instalment, and operation facilities. Capex is accounted for once at 1<sup>st</sup> of January in the year production starts.

$$Capex(t) = IF(t = t_{start}; Capex; 0) \quad 4.2-2$$

Where,

**Capex** = Capex in year t<sub>start</sub> [\\$]

### 4.2.3 Operation expenditures

Operation expenditures represents the total expenditure needed to uphold the desired exploitation and operation of a specific field. This would be costs such as rig-rates, wages and operation costs. Opex is divided into fixed and variable operational cost and will be set as proportional to the actual production each year. Opex will be accounted at the end of the year.

$$Opex(t) = FC + VC * Q_t \quad 4.2-3$$

Where,

**FC** = Fixed cost, constant per year [\\$]

**VC** = Cost per barrel produced [\$/bbl]

**Q<sub>t</sub>** = Production in year, t, from production profile [bbl]

#### 4.2.4 Depreciation

Depreciation is used to calculate the decline of value of assets. Straight line depreciation

$$D_t = \frac{Capex}{n} \quad 4.2-4$$

Where,

$D_t$  = Depreciation in year, t.

$n$  = Number of years of depreciation.

#### 4.2.5 Tax

The petroleum sector is among the most heavily taxed and the impact of taxation on contractual relationships, asset selection, behavioural incentives, the dynamics of demand and supply and financial position of the various parties involved. In upstream oil and gas, total government take, which is the government share in economic profits, globally varies from about 40% to well over 90%<sup>38</sup>.

##### 4.2.5.1 Taxable Income

Taxable income is the part of a capital cash flow for which there will be imposed a tax. Approved deductions and depreciations are usually incurred before a capital cash flow can be taxed. Taxable income can be calculated as follows:

$$TI_t = IF (CCF < 0; GRR_t - CAPEX - OPEX_t - D_f + CCF_{t-1}; GR_t - CAPEX - OPEX_t - D_t) \quad 4.2-5$$

Where,

$TI$  = Taxable income

$CCF$  = Capital Cash Flow

$GRR$  = Gross Revenue Return

$D_f$  = Depreciation

$Capex$  = Capital expenditures

$Opex$  = Operating expenditure

$t$  = year

##### 4.2.5.2 Tax paid

Tax paid is the amount paid of the taxable income at the given tax rate. Tax paid in year, t, can be calculated as follows:

$$T_t = IF (TI_t < 0; 0; TI_t * TR) \quad 4.2-6$$

Where,

$T_t$  = Tax paid in year, t.

$TR$  = Tax rate per year

#### 4.2.6 Net cash-flow

The net cash flow, is the undiscounted capital which remains after all income and expenses are accounted for. The undiscounted cash-flow in year, t, is calculated as follows:

$$NCF_t = GR_t - CAPEX - OPEX_t - T_t \quad 4.2-7$$

Where,

$NCF_t$  = Net cash-flow in year, t.

#### 4.2.7 Discounted cash-flow

Discounted cash-flow is calculated by discounting the undiscounted cash flow in time by using a discount rate. Many factors influence the choice of discount-rate used by companies, but it is influenced by the cost of capital and fiscal regimes. The rate may be adjusted upwards for risk-measures. The discount factors usually ranges from 0-30 % and can be calculated by:

$$DCF_t = \frac{NCF_t}{(1+DR)^t} \quad 4.2-8$$

Where,

$NCF_t$  = Net cash-flow in year, t.

$DCF_t$  = Discounted cash-flow in year, t.

$DR$  = Discount rate

## 4.3 Metrics of economic performance

### 4.3.1 Net Present Value - NPV

The net present value is widely used and is a well-known metric. It calculates the present value of all future cash-flows and includes time value of money and can therefore be seen as a project's total value at present terms. The NPV is calculated by summing the discounted cash flow for the lifetime of a field. A positive NPV will be a signal to invest in the project. NPV has the following characteristics<sup>39</sup>:

- Consistent metric over projects with different characteristics by using time value of cash flows.
- Gives greatest weights to early cash flows. Higher discount rates give higher weight to the earlier cash flow.
- Continues to charge interest after investment capital is recovered.
- Biased in favour of larger projects.

$$NPV = \sum_{t=1}^n DCF_t \quad 4.3-1$$

### 4.3.2 Investment Efficiency - IE

A simple metric found by dividing the NVP on CAPEX and can be seen as an efficiency ratio. A higher index will be a signal of higher profitability. Characteristics to this metric are as follows<sup>39</sup>:

- Adjusts the NPV for size of investment
- Favours investments with low initial capital outlay and large NPV's
- Measurement of the efficiency of capital spent.

$$IE = \frac{NPV}{CAPEX} \quad 4.3-2$$

### 4.3.3 Internal Rate of Return - IRR

The internal rate of return is also a widely used metric and is found by calculating the discount rate when the NPV reaches zero. This can give multiple solutions, thus IRR is an unstable metric. The IRR is also specific for each project it is calculated to. If the IRR surpass the cost of capital the project is seen as profitable and the higher the gap the more robust a project will be. Characteristics of the IRR are as follows<sup>39</sup>:

- Profit indicator which is independent of the size of the investment
- The estimated return is sensitive to errors in estimating requirements and net cash flows in the early years of the project.
- IRR cannot be computed if the cash flows contain all positive or all negative values, or if there is no pay out.



- It is biased in favour of projects with low initial investment and early cash returns because revenue early in the project life influences it the most.
- No direct measure is supplied regarding the absolute size of the profit generated, particularly in long-life projects.
- The project is charged a cost at the IRR value, not the average discount rate.
- Multiple rates of return are possible. A solution may exist for every sign change.
- It will give meaningless results in many acceleration projects. Acceleration projects are defined as projects which enlarges the earliest cash flows to satisfy urgent need for cash.

$$\sum_{t=1}^n \frac{NCF_t}{(1+DR)^t} - NCF_0 = 0 \xrightarrow{\text{yields}} IRR \quad 4.3-3$$

#### 4.3.4 Hurdle rate

A hurdle rate can be interpreted as the price per bbl needed to commence a project. This rate is used more as a metric in order to review the long term trend on new investments. The hurdle rate for a project is simpler to calculate and predict than the oil price in short term because it is based on cost levels with lower uncertainties. Using it as a metric towards price models, it can be a good indicator for initiating new projects in new areas or to implement new technology on existing fields. The hurdle rate can be regarded as a rule of thumb for entering projects or initiating new developments. An example can be seen in Figure 16. Based on data from Wood-MacKenzie, Reuters and IEA, the hurdle rate range for different projects and countries are displayed here. This show for example, by entering the Norwegian Continental Shelf (NCS) one can expect a cost level around \$25-\$65 per bbl. If the selected price model does not support a higher price level than this range, it would not be advisable to invest in projects in this region.

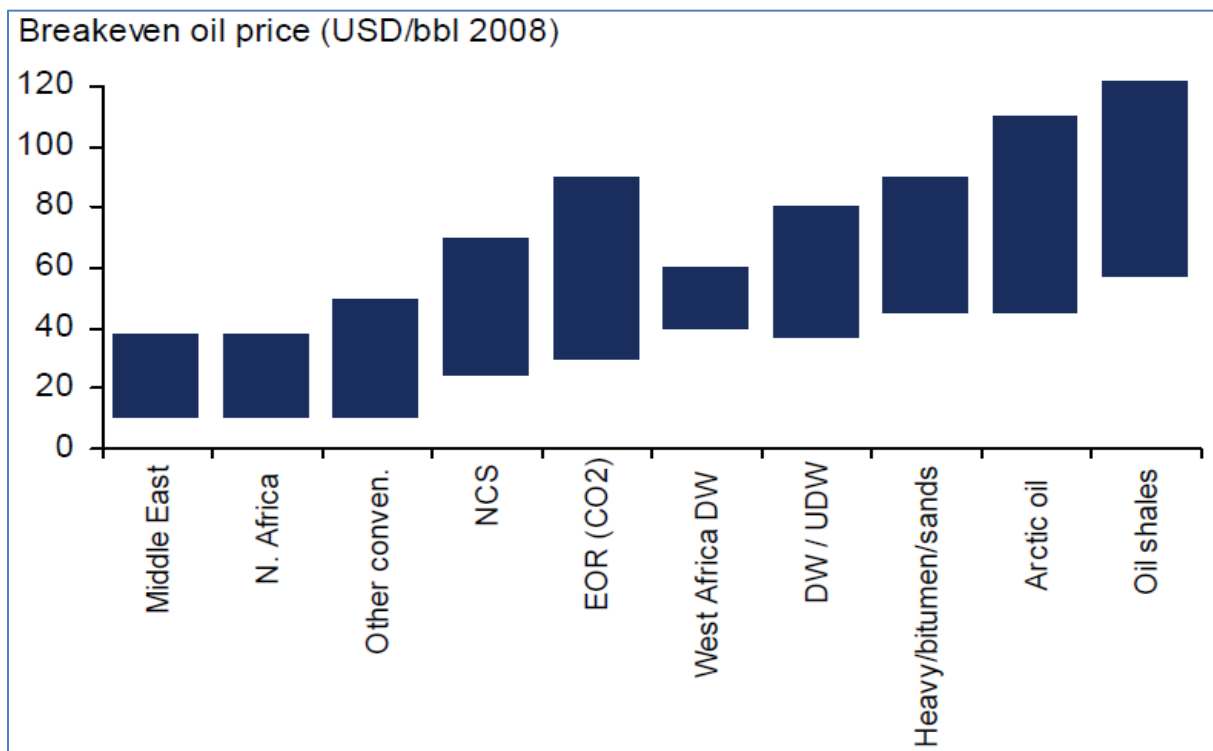


Figure 16: Hurdle rate ranges for different projects and areas. ©Pareto Research

## 5 Selection of fields

Three fields have been selected to investigate the impact of the different price models. The fields have different characteristics and are situated in different geographic locations in the world. As these fields are only used as example for studying the effect of the price models, the field data used are retrieved from the operating companies and governmental agencies where available. In case of missing or lack of data, available data from similar projects have been used. For simplicity in the analyses, the following assumptions are made:

- Only considers income from crude production. Gas and NGL are discarded.
- All crude are sold to the simulated WTI price.
- All fields are analysed in view of the main stakeholder and operator of each field.
- Tax regime is set to a simple percentage of profit system.
- All fields produce all of the calculated recoverable resources in accordance to the production profile created by the input variables.
- Abandonment of the field is set at the consecutive year of the year where production reaches zero.
- Abandonment costs are discarded.

A project will have uncertainty in both technical and economic data. To investigate the impact of different price models and the effect of the uncertainty from the models, two simulation scenarios have been chosen:

- **1 - Open Parameter:** All input data; technical, economic and the oil price, are variables. This scenario reflects total project uncertainty
- **2 - Fixed Parameter:** No input is variable, except for oil price, which consequently reflects the uncertainty created by the different price models

The following fields were chosen and are presented in the following sections:

- **6.1 Knarr, Norway. The field is analysed as a holding of the operator BG Group (45%). Offshore field, most likely 69 MM bbl recoverable reserves**
- **6.2 Tawke, Northern Iraq. The field is analysed as a holding of the operator DNO (55%). Onshore field, most likely 230 MM bbl recoverable reserves**
- **6.3 Tiber, USA (GoM). The field is analysed as a holding of the operator BP (66%). Offshore field, most likely 750 MM bbl recoverable reserves**

## 5.1 Knarr

Knarr is an offshore oilfield with some gas situated at the North-Tampen area, 120 km west of the Norwegian coast. It was approved by the Norwegian Government in 2011 for development. The reservoir is at approximately 4000 meters vertical depth and at a water depth of 400 meters. It is planned to produce the field with a FPSO from well templates, further development can be done into a smaller field, Knarr west, at a later stage. The field will be operated by BG Group who owns 45% of the field and the total operation time is estimated to be from 6 to 20 years. According to the Plan for Development and Operation for Knarr which was presented to the Norwegian government, the owners of the field estimates a NPV of \$ 1.35 Billion dollar (2010) and a hurdle rate at 47 USD per bbl<sup>40</sup>.

Risk factors identified:

Type Classification \	Low	Medium	High
Technology			
Economical			
Political			
Operational			

Table 1 Risk table for Knarr

All in all, Knarr is a low risk field, however large uncertainties regarding lifetime of the field may pose economical concern and the hard weather in the area may cause difficulties for the FPSO to operate as planned. Politically, Norway is very stable and transparent. Development of the field should not pose any major challenges technology-wise.

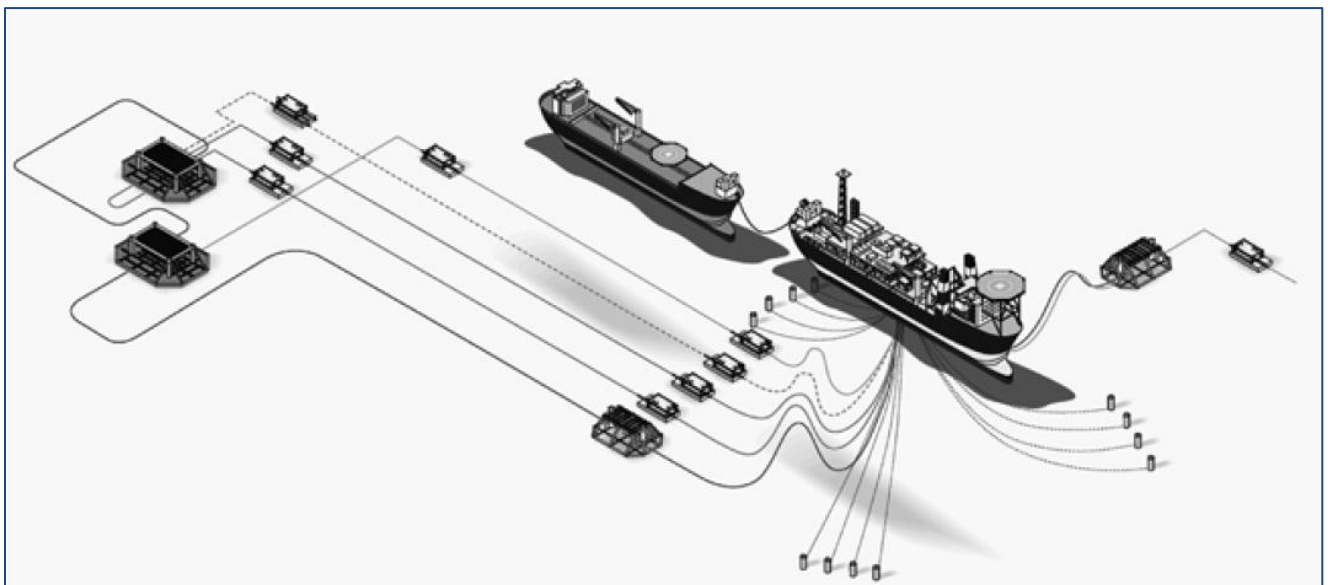


Figure 17 Schematic of the development of Knarr. Source: Plan for Development and Operation for Knarr

The following inputs for production variables are used:

		Input PDFs			
		Real.	Min	ML	Max
$N_p$	Reserves, MMbbl	69,0000	52,836	69	100
$y_R$	Length of Ramp Up (to plateau), yrs	3,0000	2	3	4
$q_P$	Yearly Plateau Rate, MMbbl/yr	11,0000	7,00	11,00	22,00
$P$	Fraction reserves produced at end plateau	0,6000	0,40	0,60	0,70
$q_L$	Field Economic Rate Limit, MMbbls/yr	0,6900	0,48	0,69	0,90

Table 2 Knarr production variables

An expected production profile of the field based on most likely estimates:

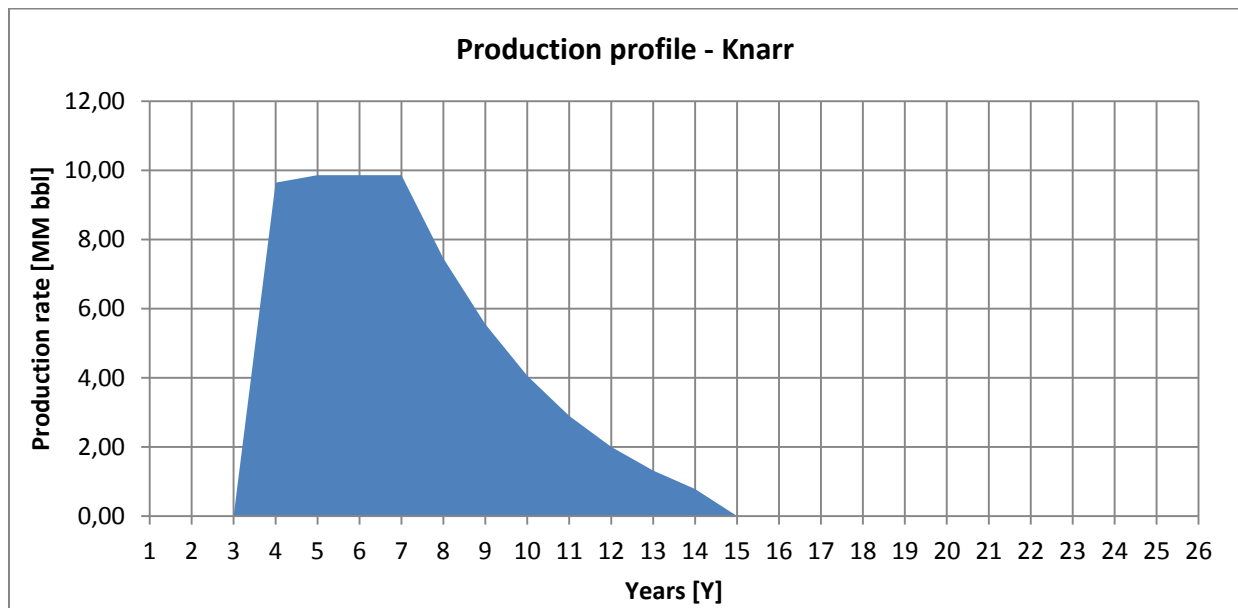


Figure 18 Production profile for Knarr.

The following inputs for economic variable are used:

		Input PDFs		
		P5	P50	P90
Discount Rate	7,0 %			
Oil Reserves	69,00 MM bbl			
Variable Opex	32,00 per bbl	25	32	40
Fixed Opex	116,00 (\$MM)/year	80	116	140
Capex (Development Cost)	1050,00 (\$MM)capital	850	1050	1250
Start-up Year	2,5			
Years after initial investments		2	2,5	3
Probability		0,3	0,6	0,1
PSC Share	0,45 share			
Depreciation Years (SL, n years)	6			
Income Tax	70 %			

Table 3 Knarr economic variables

## 5.2 Tawke

The Tawke field was discovered in 2006 and is an oilfield situated in the north of Iraq which is controlled by Kurdish government. The field produces from two reservoirs which comprise of fractured carbonates systems. The reservoirs are at approximately depths of 2000 to 3000 meters. The field is operated by DNO International who owns a 55% share of the field. The oil is sold by trucks and will also be sold through a northern pipeline exporting oil to Turkey.

Risk factors identified:

Type Classification \	Low	Medium	High
Technology			
Economical			
Political			
Operational			

Table 4 Risk table for Tawke

The field does not impose large investments and since it is an onshore development, the development costs are low. Technology-wise carbonate systems are complex and dynamic and may therefore pose some challenges during production, but not in a major scale. The political environment in the region is strenuous and agreements could change abruptly as seen fit by the ruling government. Operational issues would be failure to export oil and conflicts in the area which can halt the production or export.



Figure 19 Wells at Tawke field ©DNO

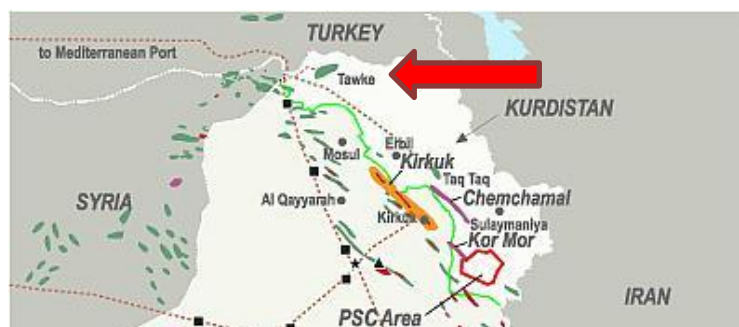


Figure 20 Map of Tawke field, up north in Iraq. ©energy365dino.co.uk

The following inputs for production are used:

		Input PDFs			
		Real.	Min	ML	Max
$N_p$	Reserves, MMbbl	181,1007	150	230	370
$y_R$	Length of Ramp Up (to plateau), yrs	3,000	2	3	4
$q_P$	Yearly Plateau Rate, MMbbl/yr	18,5000	12,00	18,50	33,00
$P$	Fraction reserves produced at end plateau	0,60	0,40	0,60	0,70
$q_L$	Field Economic Rate Limit, MMbbls/yr	0,27	0,18	0,27	0,36

Table 5 Tawke production variables

An expected production profile of the field based on most likely estimates:

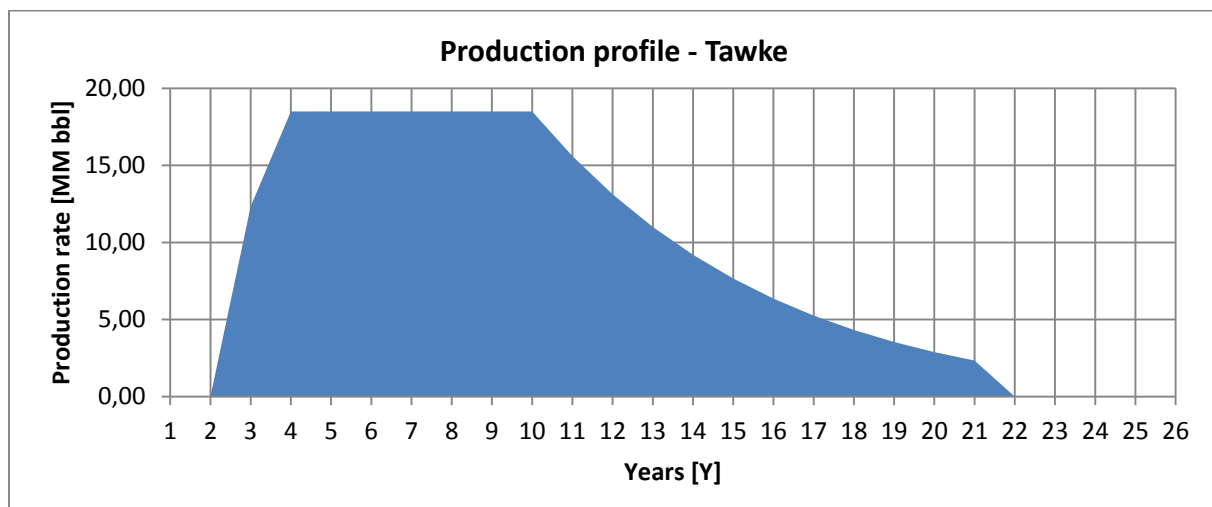


Figure 21 Production profile for Tawke

The following inputs for economic variables are used:

			Input PDFs		
			P5	P50	P90
			0,05	0,5	0,95
Discount Rate	10,0 %				
Oil Reserves	230,00	MM bbl			
Variable Opex	20,00	per bbl	10	20	30
Fixed Opex	116,00	(\$MM)/year	90	116	130
Capex (Development Cost)	410,00	(\$MM)capital	250	335	410
Start-up Year	2				
Years after initial investments			1	2	3
Probability			0,3	0,6	0,1
PSC Share	0,55	share			
Depreciation Years (SL, n years)	6				
Income Tax	75 %				

Figure 22 Tawke economic variables

### 5.3 Tiber

The Tiber field was discovered in September of 2009 and is a deep-water offshore oilfield. It is located 480 km south west of New Orleans in the Gulf of Mexico and is considered to be a giant field<sup>4</sup>. The OOIP is considered to be around 4-6 billion barrels of oil. The major owner of the field is BP who possesses 66%. The field is technically challenging as it is at water depths around 1260 meters and the reservoir are underneath salt accumulations at around 10000 meters total vertical depths. As of date there is no plan for development as this field was where the Macondo well was drilled and a serious blowout caused the explosion and sinking of a semi-submersible platform killing 11 people and caused a major oil spill. For simulation purposes the development options and costs are set as identical to the similar Thunder Horse field in Gulf of Mexico

Risk factors identified:

Type \ Classification	Low	Medium	High
Technology			
Economical			
Political			
Operational			

Table 6 Risk table for Tiber

The field contains giant reserves, but it is challenging to extract. State of the art technology and skill would be required in order to produce a field at these depths. Because of the high investments needed to set up a production facility at the field, the risk factor is set to medium. Politically there is no large risk as the US government is considered stable and transparent. For operations there is a risk regarding the weather systems present in the Gulf of Mexico such as hurricanes.



Figure 23 Picture of Thunder Horse production and drilling platform in the Gulf of Mexico. ©GVA

<sup>4</sup> Giant oil field are fields with more than 500 MM bbl of recoverable resources



The following inputs for production are used:

		Input PDFs			
		Real.	Min	ML	Max
$N_p$	Reserves, MMbbl	750,0000	600	750	900
$y_R$	Length of Ramp Up (to plateau), yrs	5,0000	4	5	7
$q_P$	Yearly Plateau Rate, MMbbl/yr	50,0000	20,00	50,00	100,00
$P$	Fraction reserves produced at end plateau	0,6000	0,40	0,60	0,70
$q_L$	Field Economic Rate Limit, MMbbls/yr	0,6000	0,38	0,60	0,75

Table 7 Tiber production variables

An expected production profile of the field based on most likely estimates:

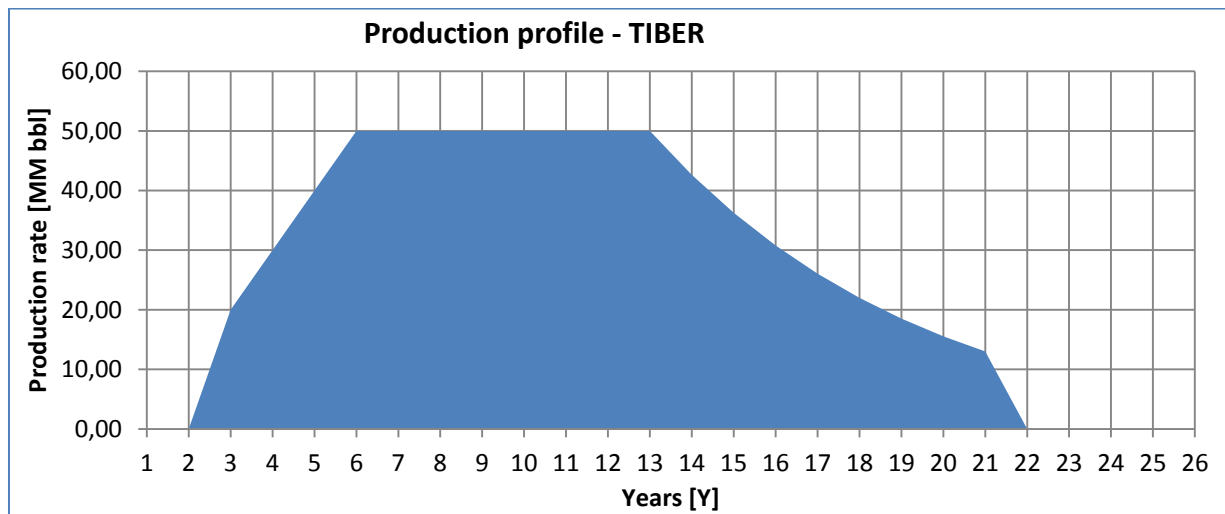


Figure 24 Production profile for Tiber

The following inputs for economic variables are used:

		Input PDFs		
		P5	P50	P90
		0,05	0,5	0,95
Discount Rate	10,0 %			
Oil Reserves	750,00 MM bbl			
Variable Opex	45,00 per bbl	30	45	60
Fixed Opex	120,00 (\$MM)/year	100	120	140
Capex (Development Cost)	4125,00 (\$MM)capital	3000	4125	5500
Start-up Year	2			
Years after initial investments		1	1,5	2
Probability		0,3	0,6	0,1
PSC Share	0,66 share			
Depreciation Years (SL, n years)	20			
Income Tax	40 %			

Table 8 Tiber Economic variables



## 6 Estimation & refinement of models

*“The only man I know who behaves sensibly is my tailor; he takes my measurements anew each time he sees me. The rest go on with their old measurements and expect me to fit them”*

*George B. Shaw*

A total of four models are investigated and they have been separated into three different levels. They are presented in levels of detail and complexity, where level 1 is a simple model and level 2 and 3 offers more complexity and variability:

- Level 1: Single price models.  
Selected model: Fixed Price (FP)
- Level 2: Stochastic price models.  
Selected models: Geometric Brownian Motion (GBM)  
Mean Reversion (MR)
- Level 3: Stochastic- and system thinking approach price-models.  
Selected model: System Thinking (ST)

The time period chosen for collecting data for the Geometric Brownian Motion and Mean Reversion is set to 40 years back in time (1969). This time period will cover important events such as the early years of OPEC and it will also cover the introduction of the IEA Petroleum Act and the introduction of derivatives and futures trading for energy. Also by inspecting the graph in Figure 25, Begg and Smit (2007) showed that there is a tendency of increased variability in the price from 1935 and onwards, however the large variations starts around 1970 and have continued since.

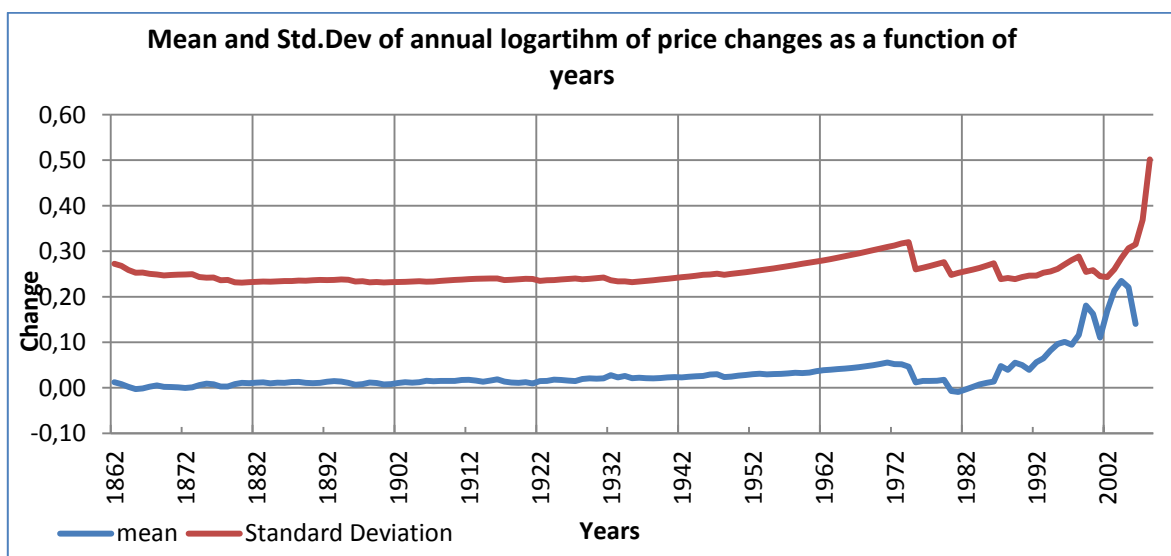


Figure 25 Change in mean and standard deviation of annual logarithm of price changes as a function of number or years data used. Begg and Smit (2007)

## 6.1 Single Price models

### 6.1.1 Fixed Price

For this thesis the estimation of Fixed Price is done with support in the research performed by Pareto Research<sup>24</sup>. This second hand (stock analysts) knowledge may prove to be the best available as first hand (companies) knowledge is not openly available.

The following parameters in Table 9 were used in the Fixed Price model:

Model	Mean	Low	Average	High
Fixed Price	67,50	55,00	70,00	80,00

Table 9 Estimation parameters Fixed Price model. [\$/bbl]

For the simulations the average value of 70 \$/bbl is used. Total projection/simulation time was set to 20 years.

## 6.2 Stochastic price models

### 6.2.1 Geometric Brownian Motion

The parameters are estimated from historic WTI oil price data (Appendix E). WTI is the chosen marker and start value in year 0 is set at December of 2009 at \$ 74,48 per bbl. Total simulation time is set to be 20 years. The  $dt$  is set at 1/1 creating a yearly price in the simulations. By performing parameter estimation on the WTI oil prices from 1969 until 2009, parameters were estimated in a section-weighted method. The estimation of parameters can also be performed by other methods (Begg & Smit ,2007). This method was chosen to give a larger weight to the recent sections of price data.

By dividing time periods by smaller and smaller sections by 5 years (i.e. last 40 years, last 35 years, last 25 years etc.), 8 different sections were created and different weights were distributed to each section. The weighted averages were calculated and standard deviations of the total selection were chosen as input parameters. The  $\alpha-0,5\sigma^2$  was found by using Equation 3.3-3 and  $\sigma$  was found by using Equation 3.3-5. The data is displayed in Table 10. The distribution of  $\alpha$  is shown in Figure 26 Distribution of GBM drift factor,  $\alpha$ , and the distribution of  $\sigma$  is shown in Figure 27 Distribution of GBM Volatility factor,  $\sigma$ .

Section	$\alpha-0,5\sigma^2$	$\sigma$	Weight
40	0,044	0,306	0,05
35	0,058	0,260	0,10
30	-0,014	0,248	0,10
25	0,015	0,268	0,10
20	0,034	0,243	0,10
15	0,066	0,261	0,15
10	0,098	0,258	0,15
5	0,070	0,307	0,25
<b>Weighted Mean</b>	<b>0,054</b>	<b>0,272</b>	
<b>St.dev of samples</b>	<b>0,035</b>	<b>0,024</b>	

Table 10 Parameter estimation, GBM

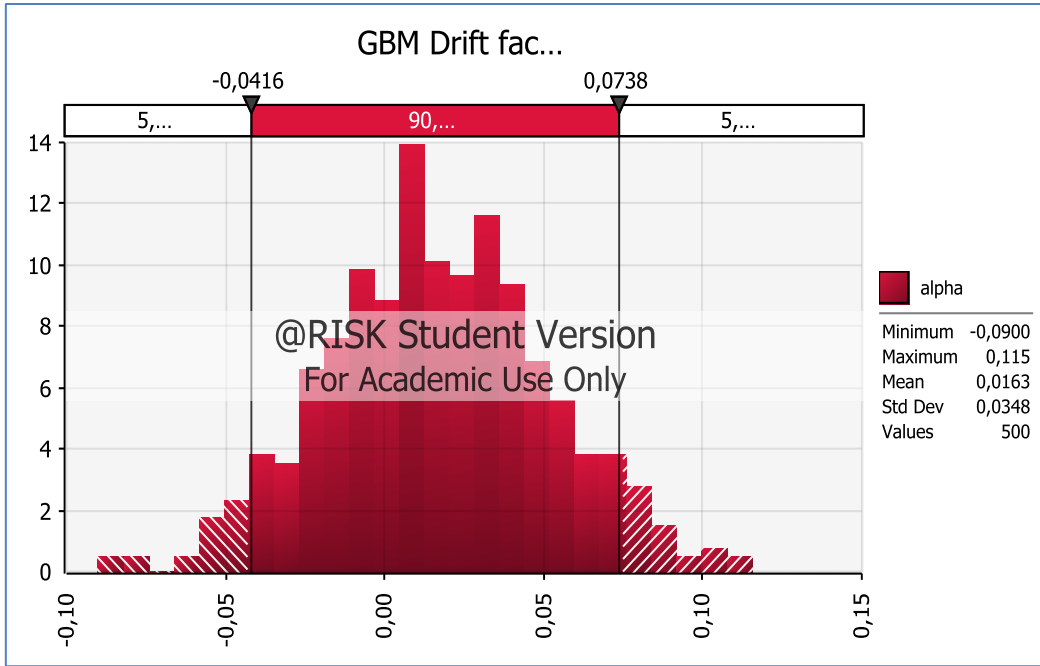


Figure 26 Distribution of GBM drift factor,  $\alpha$

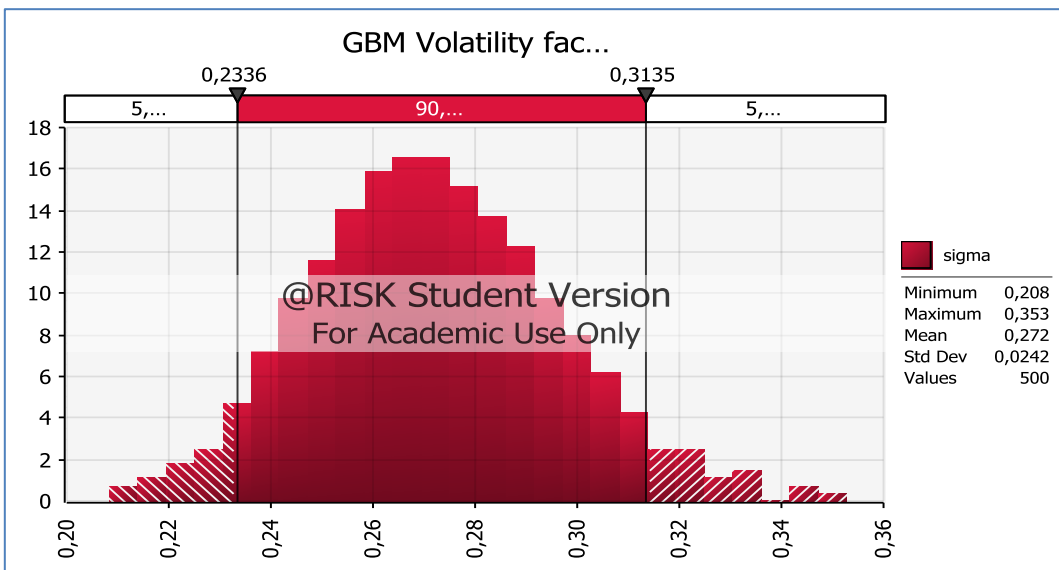


Figure 27 Distribution of GBM Volatility factor,  $\sigma$

### 6.2.2 Mean Reversion

The parameters are estimated from historic WTI oil price data (Appendix E). WTI is the chosen marker and start value in year 0 is set at December of 2009 at \$ 74,48 per bbl. Total simulation time is set to be 20 years. The dt is set at 1/1 creating a yearly price in the simulations. By performing regression on the WTI oil prices from 1969 until 2009 the regression factors were estimated in a section-weighted method. This method was chosen to give a larger weight to the recent sections of price data. By dividing time periods by smaller and smaller sections by 5 years (i.e. last 40 years, last 35 years, last 25 years etc), 8 different sections were created and different weights were distributed to each section. The weighted averages were calculated and the standard deviations of the total selection. The regression was performed in Excel with the use of Equations 3.3-7, 3.3-8 and 3.3-9. The regression parameters are displayed in Table 11.

Section	A	B	Std.dev	Weight
40	-0,277	0,089	0,301	0,05
35	-0,492	0,144	0,313	0,10
30	-0,585	0,159	0,260	0,10
25	-0,463	0,124	0,255	0,10
20	-0,564	0,169	0,232	0,10
15	-0,467	0,145	0,253	0,15
10	-0,242	-0,032	0,268	0,15
5	-0,022	0,030	0,318	0,25
<b>Weighted mean</b>	<b>-0,336</b>	<b>0,088</b>	<b>0,279</b>	
<b>Stdev of samples</b>	<b>0,193</b>	<b>0,071</b>	<b>0,032</b>	

Table 11 Parameter estimation, MR

The parameters were then calculated by the use of Equations: 3.3-10, 3.3-11 and 3.3-12. The parameters are presented in Table 12. F was set to 75 as the long term mean price with a standard deviation of 5. Meaning that for a single run the price process would revert to a single price from the distribution in Figure 30 below.

Input Params		
$P_0$	74,480	\$
$\eta$	0,063	per year
H	7,198	
$\sigma$	0,262	per year
$F=P^*$	75,00	
$P'$	75,12	
dt	1	year
window	20	years

Table 12 Input parameters for Mean reversion

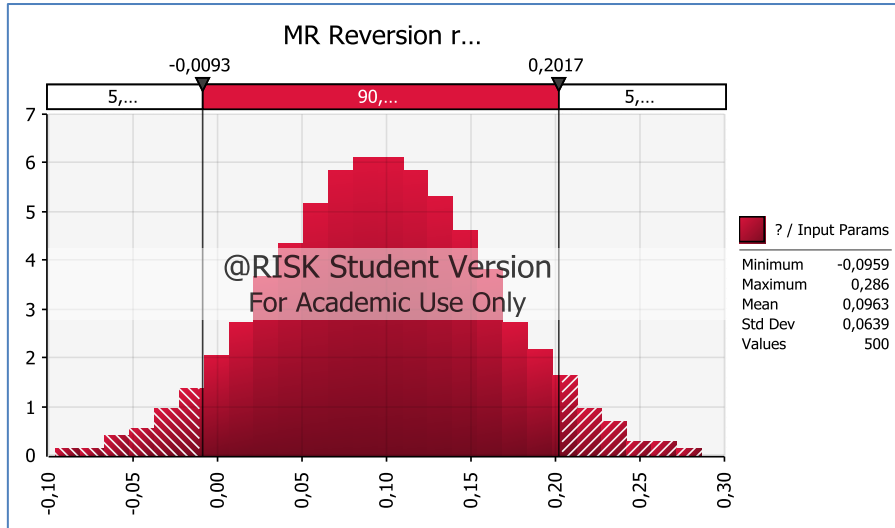


Figure 28 Distribution of MR Reversion rate

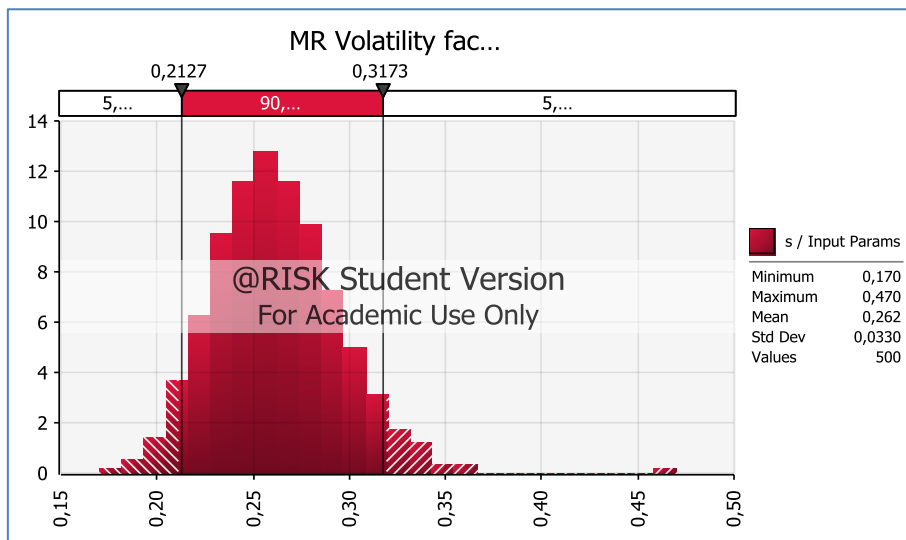


Figure 29 Distribution of MR Volatility factor

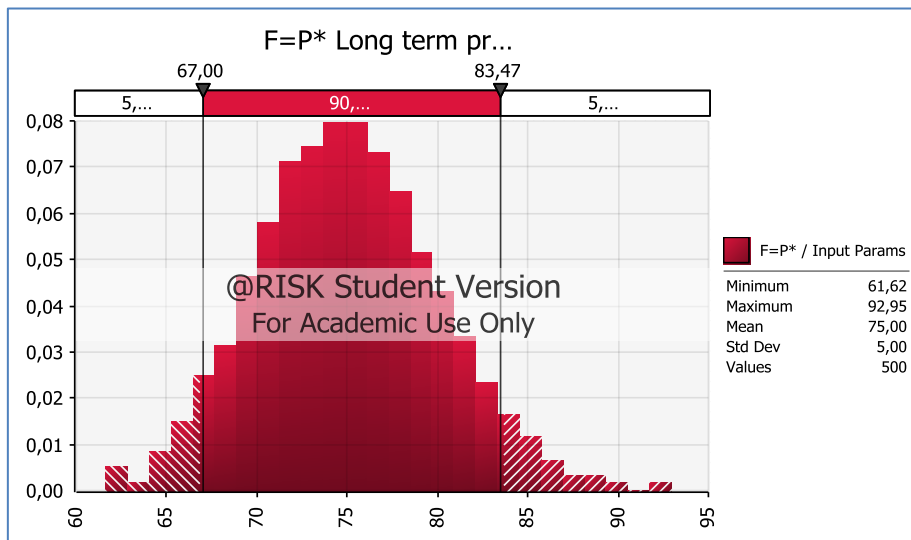


Figure 30 Distribution of F, long term Price

## 6.3 Stochastic- and system thinking approach price-models

### 6.3.1 System Thinking

The System Thinking model, outlay and data, employed in this thesis is developed by Dr. William Strauss of FutureMetrics<sup>41</sup>. During this thesis-work, alterations have been made to the model as seen fit for purpose. Alterations include updating of datasets, creating sub-model to replicate black swan events, adjusting for refinery capacity, adjusting influence patterns of the oil price, total reserves and exploration adjustment and adjusting the input variables. The model software used is Stella<sup>®</sup> provided by Isee Systems and it is comprehensive for this type of modelling. The dynamic linking with spread sheets in MS Excel enables more options for variables and interaction with continuously new data. For generating data for the simulations, Palisade @Risk has been used in MS Excel. For each simulation in Stella, new data from the variables were generated by @Risk and imported into Stella. After generating 500 simulations, the results were imported into MS Excel and @Risk was used to perform statistical analysis on the data.

The model constructed in Stella consists of 8 subparts reflecting the major parts of the total oil flow system and its interdependencies. The main part of the model (Blue colour) is where the main boundaries of the model are set, from reservoir to consumption. Key assumptions to the model:

- Time-unit is set to month, and start of simulation is set to December 2009
- Price unit is \$ per bbl and the chosen marker is WTI in 2009 US dollars.
- Volumetric-unit is set to  $10^6$  bbl, equal to MM bbl, equal to million barrels.
- All data used in the model is primarily based on total resources available in the world and on market data provided by
  - BP Statistical review
  - EIA
  - IEA
  - OPEC
  - USGS
  - Various papers where indicated
- Total simulation/projection time is set to be 240 months, 20 years.
- The dt in the model is set to  $\frac{1}{4}$ , which represents a calculation point 4 times a month, meaning every week. This was chosen because reporting of key input variables are done at this rate. The elasticities are also adjusted for this rate. A smaller dt (per day,  $\frac{1}{30}$ ) would create too many calculation points such as extremities would be absorbed in the model. For a larger dt, (each month,  $\frac{1}{1}$ ) the extremities in the model creates an improbable behaviour of the price paths.
- All elasticities used in input models are based historic data provided by Simmons & Company. Sample outputs and inputs of the elasticities used, are shown in Appendix F - K.

Sub-models presented in their respective colours from the model in Figure 31 and Figure 32:

- **Undiscovered oil**
- **Base oil flow**
- **Demand**
- **Inventory**
- **Efficiency**
- **Price**
- **Development and extraction**
- **Black swans**

### 6.3.1.1 Explanation of units in Stella

In the model constructed in Stella, different types of modules have been used. Typically it is nodes with different characteristics and operations which interact during the simulation.

Stock



This node has a reservoir/conveyor function. The reservoir type has an initial value and increases or decreases during simulations depending on the in- and outflow it is connected with. The conveyor type is also a reservoir, but with a transit time for when the units are ready for the output connection.

Flow



The flow nodes regulate in- and outflow from stocks and conveyors. It can either be uni- or bi-flow depending on its function in the model. Typically price regulation requires a bi-flow model and producing oil from a reservoir requires a uni-flow model.

Converter



This node can be used as a factor of input or a graphic tool for use in other nodes providing a comprehensive interaction and influence system between inputs.

Action Connector



These are arches connecting the desired nodes with Converters, Stocks and Flows showing the impact and relation between them.

A simple box diagram of the complete model is shown in Figure 31. The arrows show how the sub-models are linked to each other.

In Figure 32, the complete model as constructed in Stella is shown. Displayed here are all the different stocks, nodes and flows, and also how they are interlinked with the action connectors.



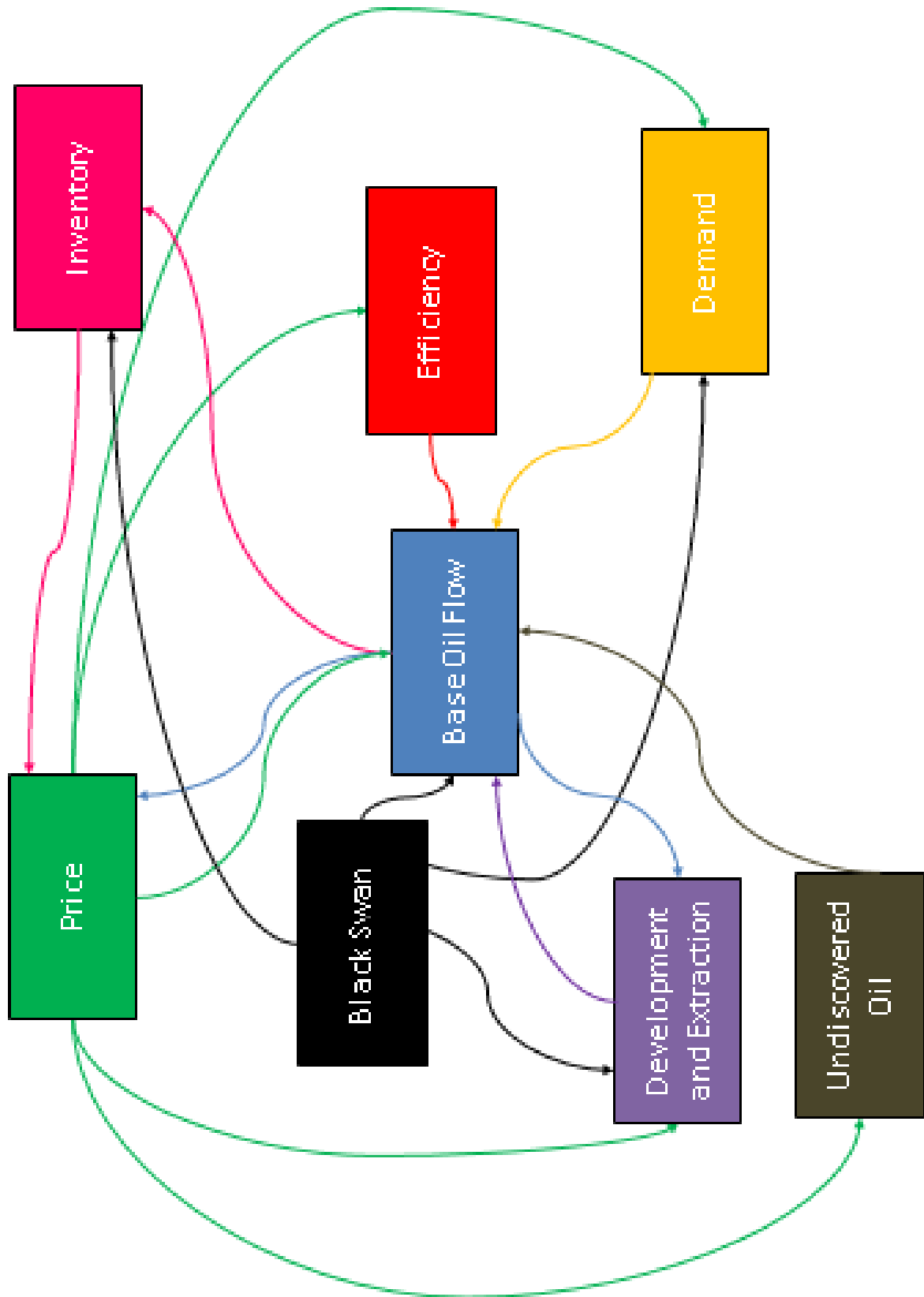


Figure 31 Simple box diagram of model created in Stella

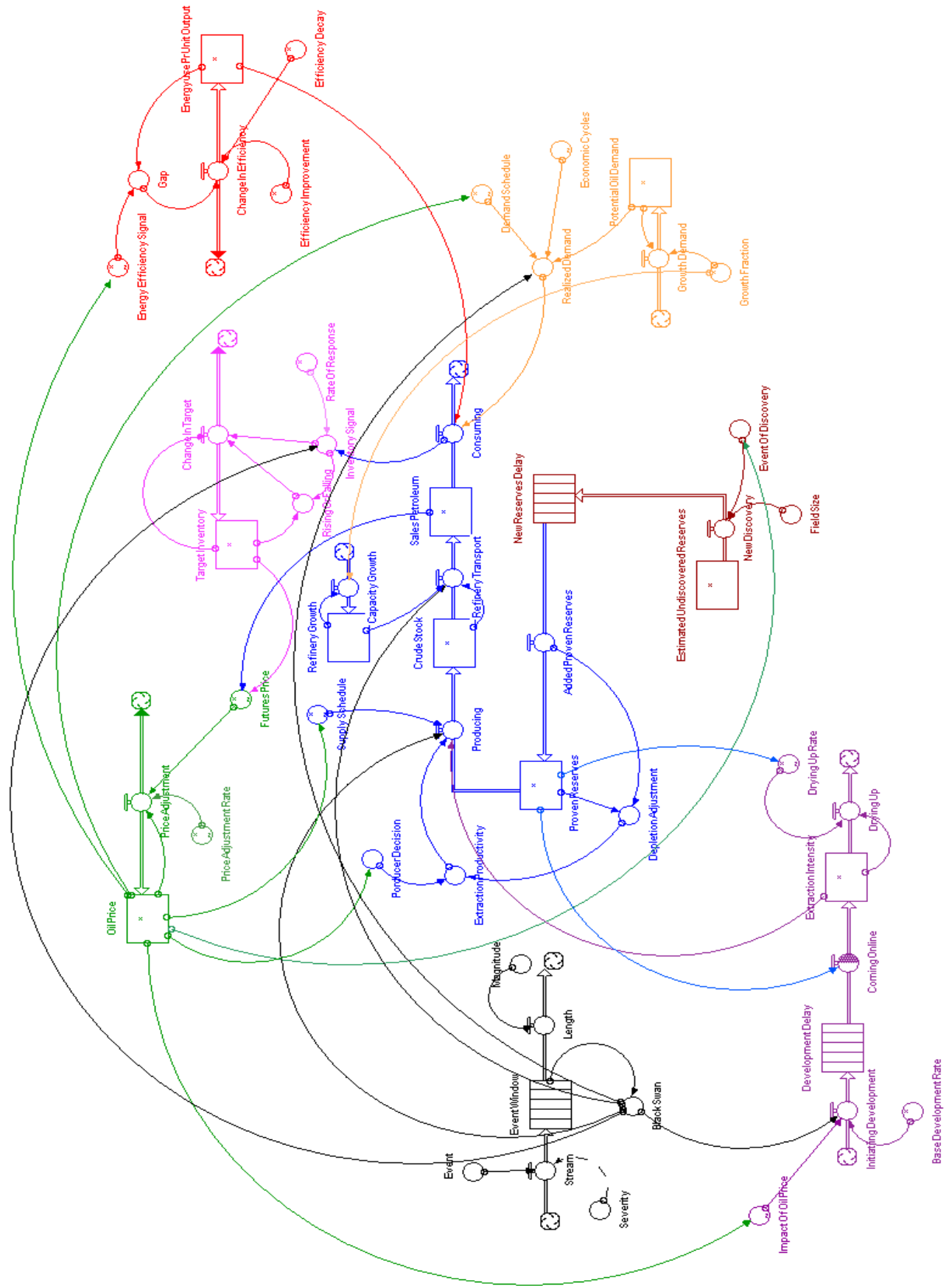


Figure 32 Oil price model as constructed in Stella

### 6.3.1.2 Undiscovered Oil

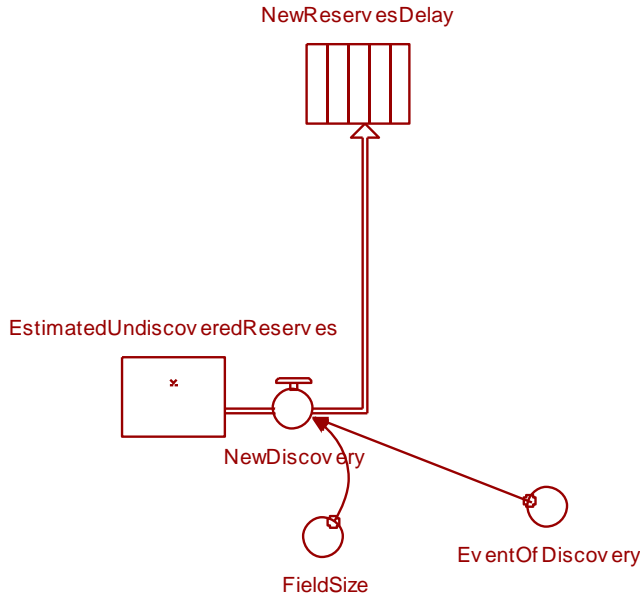


Figure 33 Model of Undiscovered oil

From a discovery receives a status as proven reserves it will require time to test and examine the prospect, depending on location, size and accessibility this can vary among the different discoveries. This sub-model only comprises the transition from discovery to proven reserves. Development of a project is covered in sub-model 6.3.7 Development and extraction model.

**EstimatedUndiscoveredReserves** An estimate of probable reserves not yet discovered. Average of estimates made by

$$= N \sim (\mu, \sigma) \quad [10^6 \text{ bbl}]$$

where  $\mu$  is the mean of the 2 triangulated distributions below and  $\sigma$  its associated standard deviation.

$$1 \text{ (USGS)} = \text{RiskTriang}(394.381; 724.228; 1.202.168)$$

$$2 \text{ (SPE report)} = \text{RiskTriang}(1.800.000; 2.900.000; 4.400.000) - \text{Proven reserves}$$

The distribution of output values are shown in Figure 34<sup>5</sup>.

<sup>5</sup> Negative values are not imported into Stella. In the case of negative values, 0 is imported.

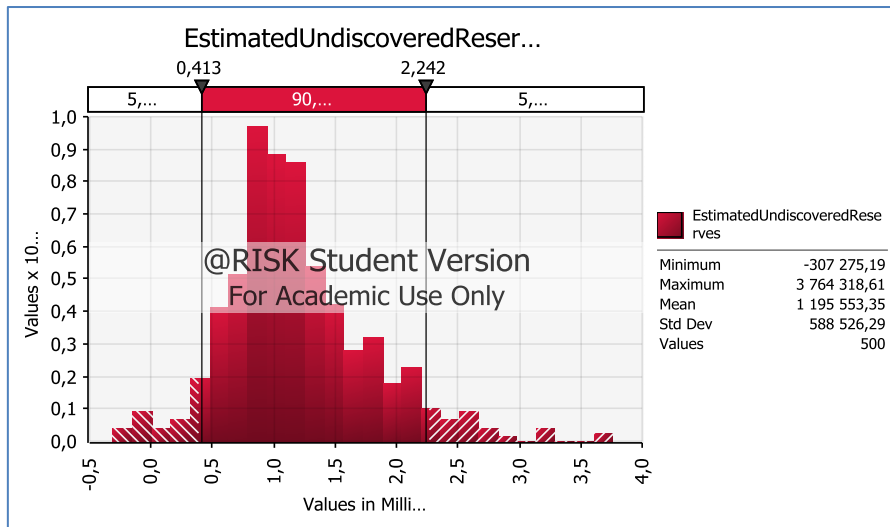


Figure 34 Distribution of EstimatedUndiscoveredReserves

**FieldSize** Estimate on size of total monthly discoveries. Distribution of output values are shown in Figure 35.

$Field\ Size = N\sim(2000,500)$  [10<sup>6</sup> bbl]

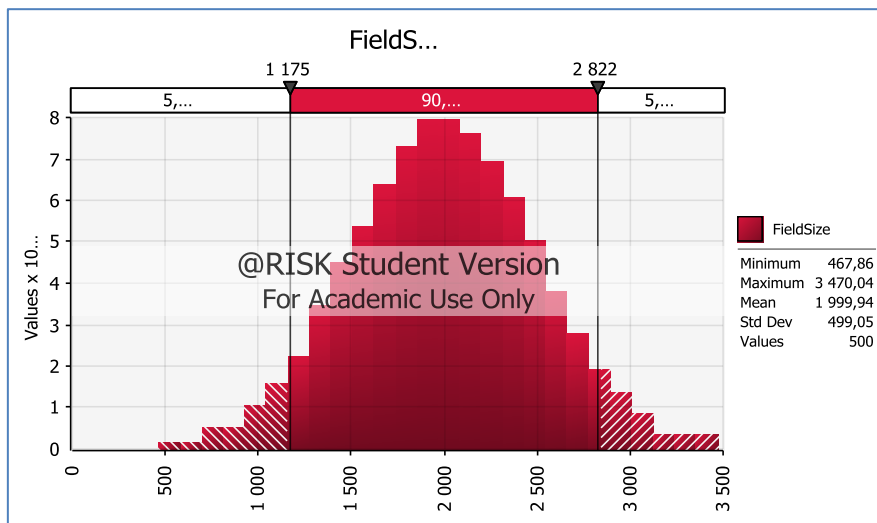


Figure 35 Distribution of total monthly discoveries

**EventOfDiscovery** Random event of discovery of a new field or new reserves. Chance of a discovery is set to 65% each month and where an increase in price will increase it to 80%, because of higher expected exploration or recovery activities.

$EventOfDiscovery = IF\ OilPrice > delay(OilPrice, 3)\ THEN\ Montecarlo(80)\ ELSE\ Montecarlo(65)$   
[0,1]

**NewDiscovery** Flow of new discoveries. The data used in FieldSize and EventOfDiscovery are set in order to match the assumptions

made by ASPO on future discoveries. The cumulative output from NewDiscovery will approximate the ASPO data.

$$\text{NewDiscovery} = \text{FieldSize} * \text{EventOfDiscovery} \quad [10^6 \text{ bbl}]$$

**NewReservesDelay**

Stocks up new discoveries for a normal distributed amount time before making them available as proven reserves. This relates to drilling an appraisal well to verify the characteristics and size of a discovery.

$$\text{INIT}(\text{New ReservesDelay}) = 6 \quad [\text{months}]$$

### 6.3.1.3 Base oil flow

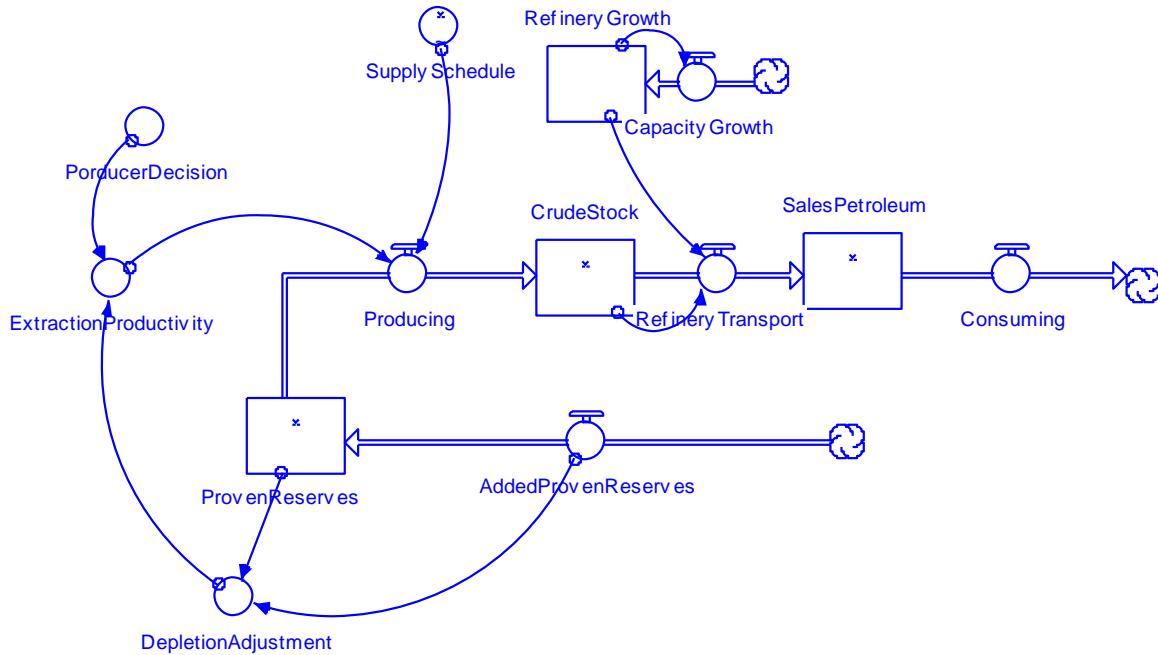


Figure 36 Model of Base oil flow

This model represents the up-, mid- and downstream of the petroleum market. The upstream part is represented from the AddedProvenReserves to RefineryCapacity. Reserves are added a time after a discovery, which is set to be  $N-(24,6)$ , to replicate the time from a discovery to a receiving field status. Development will require more time and this is handled in the sub-model Development & Extraction. Production rates are based on available data and adjusted continuously by several factors from data and within the total model. The CrudeStock and Transport parts represent mid-stream and exerts a bottleneck function to the flow, as the refinery capacity in the world is limited, the same limit is applied here. RefineryGrowth represents the growth in refinery capacity; the growth factor is set identical to the growth factor for increased demand for oil. The downstream is represented with the stock SalesPetroleum and the flow Consuming. SalesPetroleum will represent the inventory levels of petroleum products available for consumption. The SupplySchedule represents the elasticity of supply towards the oil price. Higher prices will trigger SupplySchedule to send a signal to increase the production in the model. The ProducerDecision node will provide a signal to ExtractionProductivity to cut-off 20% in Producing if there is negative trend in the price movements over a minimum of 5 months. All other price movements will keep ExtractionProductivity equal to the input of DepletionAdjustment. DepletionAdjustment will provide an output based on the replenishment of ProvenReserves. If ProvenReserves are produced faster than AddedProvenReserves can replenish the stock, the productivity will decrease consequently lowering the production.

**AddedProvenReserves**

Flow of proven reserves from NewReservesDelay to ProvenReserves. This node represents the transition time from a discovery becomes proven reserves. The distribution of transit times is shown in Figure 37.

$AddedProvenReserves = N\sim(24,6)$

[months]

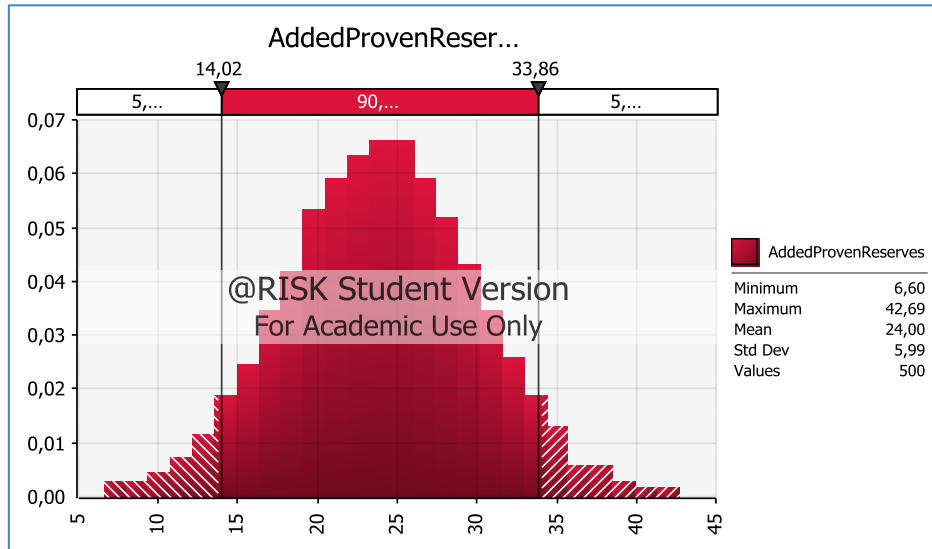


Figure 37 Distribution of transit times for AddedProvenReserves

**ProvenReserves**

An estimate of proven reserves in the world. The distribution of the initial values is shown in Figure 38.

$Initial\ value = N\sim(1.406.800,69\ 600)$

[10<sup>6</sup> bbl]

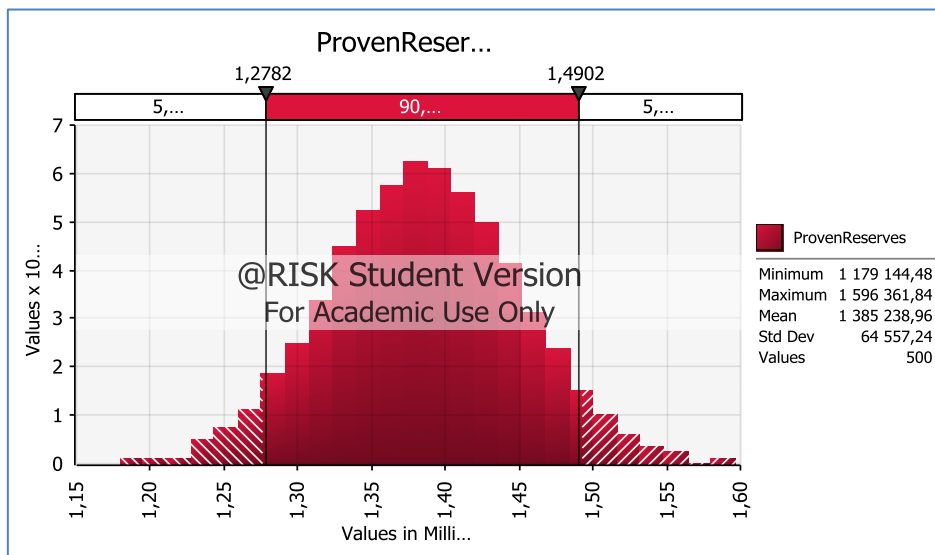


Figure 38 Distribution of ProvenReserves

**DepletionAdjustment**

Factor of rate of depletion of proven reserves from initial value of proven reserves and added reserves.

$$DepletionAdjustment = \frac{ProvenReserves}{INIT(ProvenReserves)+AddedProvenReserves} \quad []$$

**ExtractionProductivity** Factor of productivity based on ProducerDecision and DepletionAdjustment. Producer Decision is governed by the OilPrice and a lowering of the price over a given time, 5 months, will result in ProducerDecision turning to 1, thus lowering the production with 20%.

$$ExtractionProductivity = IF\ ProducerDecision = 1, 0,8 * DepletionAdjustment, 1 * DepletionAdjustment$$

**SupplySchedule** Elasticity of supply towards the oil price. Governed by graphical input (Appendix G)

**Producing** Flow toggle based on SupplySchedule, ExtractionProductivity, ExtractionIntensity and BlackSwan. This node reflects the monthly production of oil in the world.

$$Producing = ExtractionIntensity * ExtractionProductivity * SupplySchedule * BlackSwan \quad [10^6\ bbl/month]$$

**CrudeStock** Stock of produced resources, outflow from the stock is limited to refinery capacity in the world. Distribution of initial values is shown in Figure 39.

$$CrudeStock = Initial\ value = N\sim(2.669,51) \quad [10^6\ bbl]$$

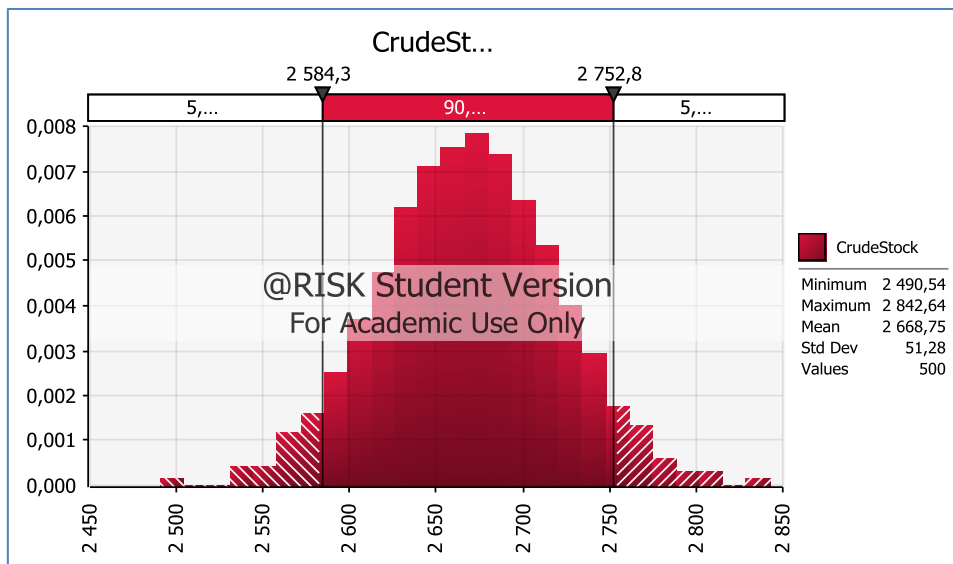


Figure 39 Distribution of initial values in CrudeStock

**RefineryTransport** Transport from refineries to markets. Outflow in this node is limited by the adjusted refinery capacity and the occurrence of BlackSwan events.



$$\text{Transport} = \text{BlackSwan} * \text{MIN}(\text{CrudeStock}, \text{RefineryGrowth} * \text{INIT}(\text{CrudeStock}))$$

[10<sup>6</sup> bbl/month]

**RefineryGrowth** Factor for increased refinery capacity during a simulation.

$$\text{RefineryGrowth} = \text{Initial Value} = 1 \quad []$$

**CapacityGrowth** Inflow of increased capacity. Growth factor is set identical to the growth factor in the demand for oil. It is assumed that a rise in demand will trigger a similar increase in the capacity of refineries. Typically this node generates values within the range of 0.05% – 0.5%. The growth factor is kept identical during each simulation. Distribution of growth factors is shown in Figure 40.

$$\text{CapacityGrowth} = \text{RefineryGrowth} * \text{GrowthFraction} \quad []$$

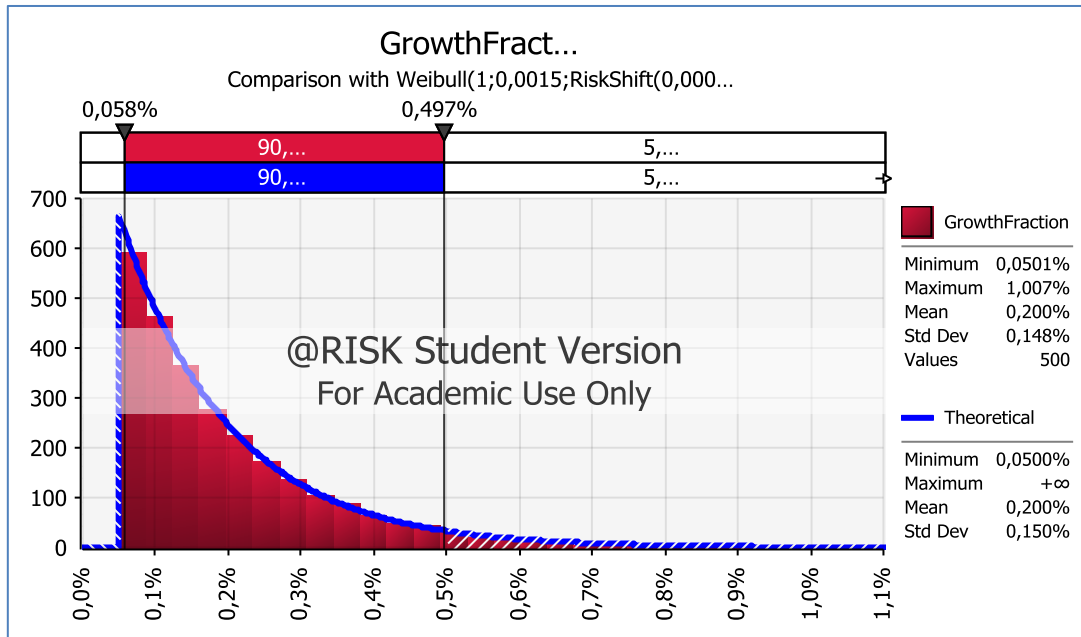


Figure 40 Distribution of GrowthFraction for RefineryGrowth

**SalesPetroleum**

Produced crude and refined petroleum products ready for consumption. Initial value assumed a triangle distribution of available data. The distribution of the values is shown in Figure 41.

$$SalesPetroleum = Initial\ value = RiskTriang(3.780; 3.950; 4.200) \quad [10^6\ bbl]$$

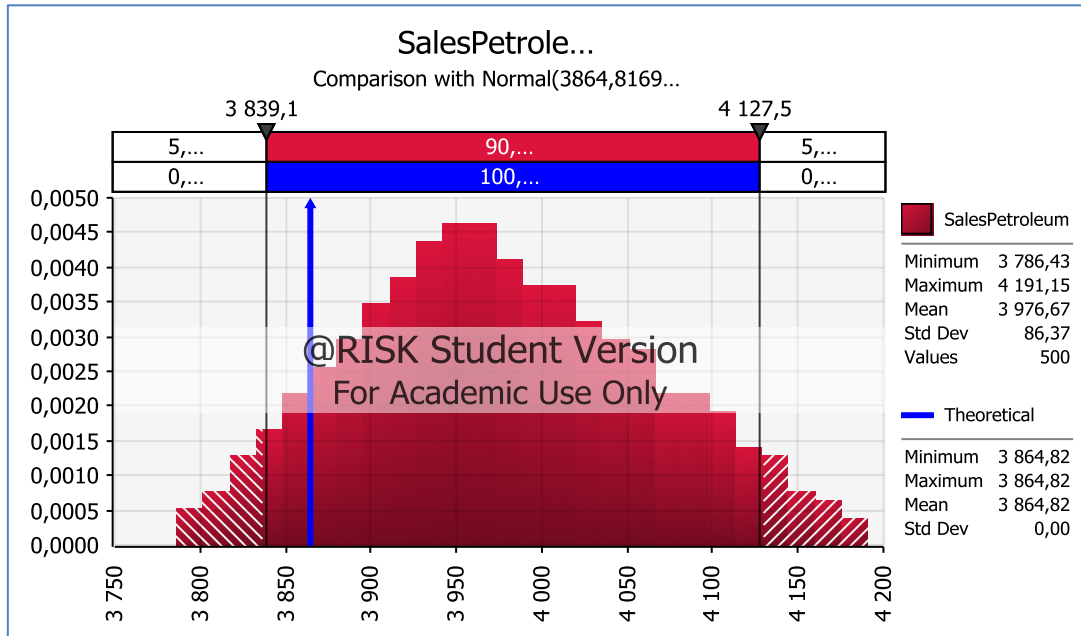


Figure 41 Distribution of initial values of SalesPetroleum

**Consuming**

Consumed petroleum in the world based on RealizedDemand and EnergyusePrUnitOutput.

$$Consuming = RealizedDemand * EnergyusePrUnitOutput \quad [10^6\ bbl/month]$$

### 6.3.1.4 Demand

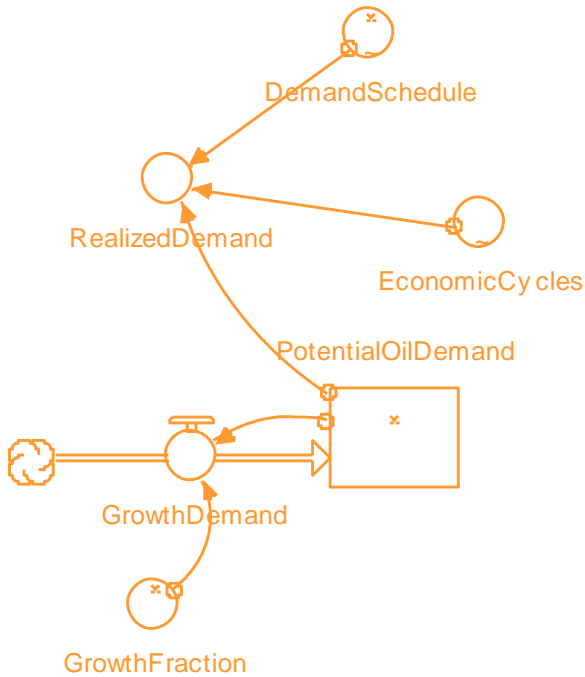


Figure 42 Model of demand

Demand is driven upwards by the GrowthDemand flow at the given rate in GrowthFraction. DemandSchedule will adjust the demand according to the demand-elasticity towards the price. EconomicCycles adjust for the demand cycles experienced in oil trade over a year.

#### GrowthFraction

Estimated monthly growth in demand of oil. Typically generates values within the range of 0.05% – 0.5%. The growth factor is kept identical during each price path simulation. The mean growth rate from this distribution is 0,2% per year, which after 20 years is equivalent to 60% total growth. This is the same total growth rate for demand of oil estimated by EIA. The distribution of GrowthFraction is shown in Figure 43below.

= *RiskWeibull*(2; 0,005; *RiskShift*(0,0005))

[%]

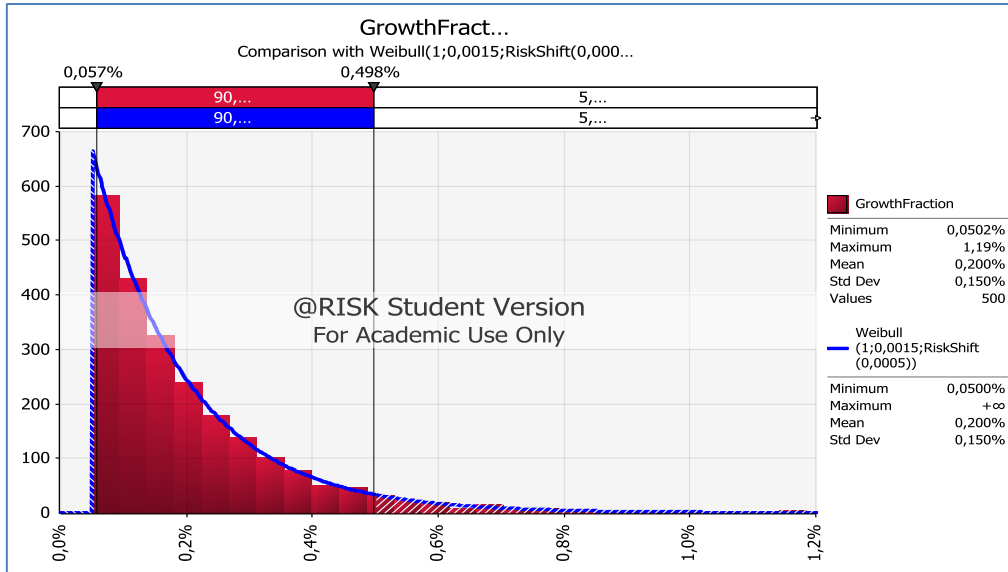


Figure 43 Distribution of GrowthFraction used in System Thinking simulations

**GrowthDemand**

Represents growth in demand for oil. Uni-flow for the growth of demand based on percentage of the potential oil demand. The growth in PotentialOilDemand will be at the fixed growth rate during each simulation.

$$= GrowthFraction * PotentialOilDemand \quad [10^6 \text{ bbl}]$$

**PotentialOilDemand**

An estimate of demand of oil in the world. Distribution of the initial values is shown In Figure 44.

$$Initial \ Value = N \sim (2.473,95) \quad [10^6 \text{ bbl}]$$

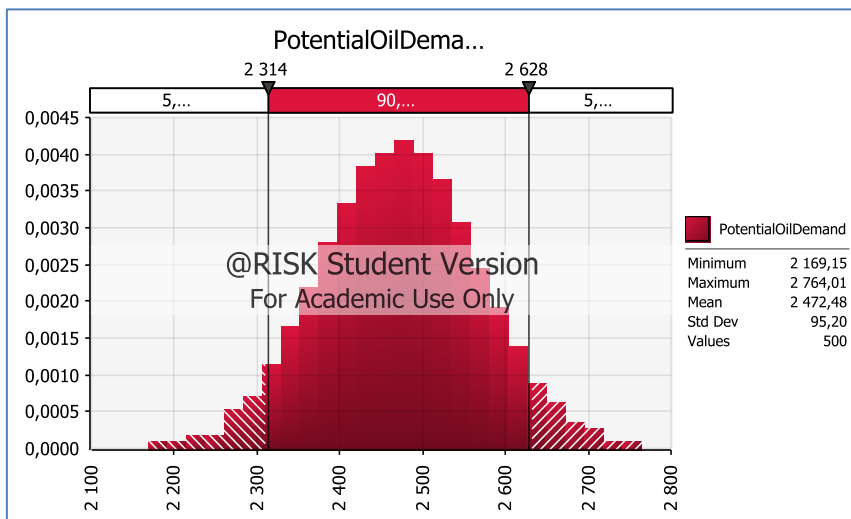


Figure 44 Distribution of initial values of PotentialOilDemand

<b>EconomicCycles</b>	A factor seasonally distributed between 0.9975 and 1.0025, reflecting the difference of demand during a year. This is replicated in the model by a curve with a sine structure.
<b>DemandSchedule</b>	Demand correlated to the oil price governed by a variable graphical output based on historical data. It shows the elasticity of demand towards the oil price. (Appendix X).
<b>RealizedDemand</b>	Demand adjusted for impact of price, unforeseen events and seasonal adjustments. A BlackSwan event is considered to generate an increase in demand.

$$\text{RealizedDemand} = \text{PotentialOilDemand} * \text{DemandSchedule} * \text{EconomicCycles} * (2 - \text{BlackSwan}) \quad [10^6 \text{ bbl}]$$

### 6.3.1.5 Inventory

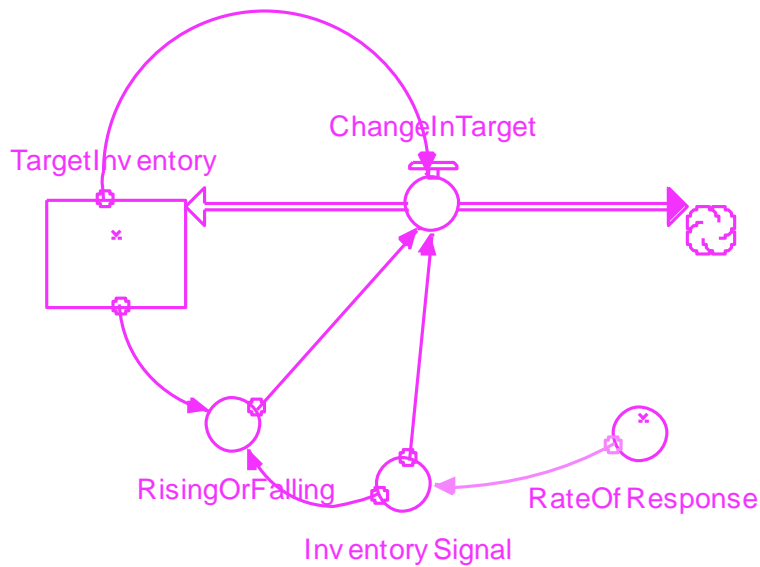


Figure 45 Model of Inventory

TargetInventory is the level of crude stock which is considered to be the appropriate amount based on the given consumption rate.

A major and rough assumption to this model is that the mechanism to hold a 90 day petroleum supply is assumed for the global consumption.

#### InventorySignal

Signal produced to be used in adapting the target inventory to present consumption levels multiplied with the given response factor. BlackSwan will send a signal to increase stock level if an event occurs.

$$InventorySignal = Consuming * RateOfResponse * (2 - BlackSwan) \quad [10^6 \text{ bbl}]$$

#### RateOfResponse

Yields a response-factor to which a target inventory level should be according to consumption levels. The factor RateOfResponse is kept identical for each simulation. The distribution of the factor is shown in Figure 46.

$$RateOfResponse = N\sim(3, 0, 3) \quad []$$

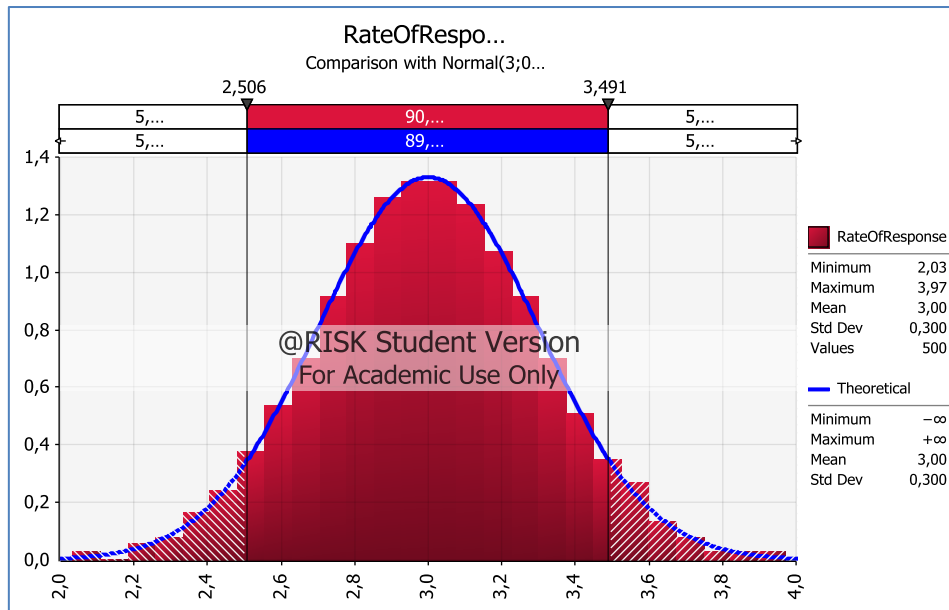


Figure 46 Distribution of RateOfResponse factor.

**RisingOrFalling** Difference between real- and target-levels yields a signal to change the target accordingly. If target is lower than the signal created from the consumption a greater reaction is created than if target is higher than the signal created from the consumption.

$$RisingOrFalling = IF TargetInventory > InventorySignal THEN N\sim(0.03,0.004) ELSE N\sim(0.5,0.1)[10^6 \text{ bbl}]$$

**ChangeInTarget** Changes the target according to the signal and equation below. This flow regulator is bi-flow able to adjust the target up or down.

$$ChangeInTarget = (InventorySignal - TargetInventory) * RisingOrFalling \quad [10^6 \text{ bbl}]$$

**TargetInventory** Target set by consumption levels and the corresponding appropriate levels. Initial value generated in Excel. Distribution of initial values is shown in Figure 47.

$$TargetInventory = RateOfResponse * N\sim(2473,95) \quad [10^6 \text{ bbl}]$$

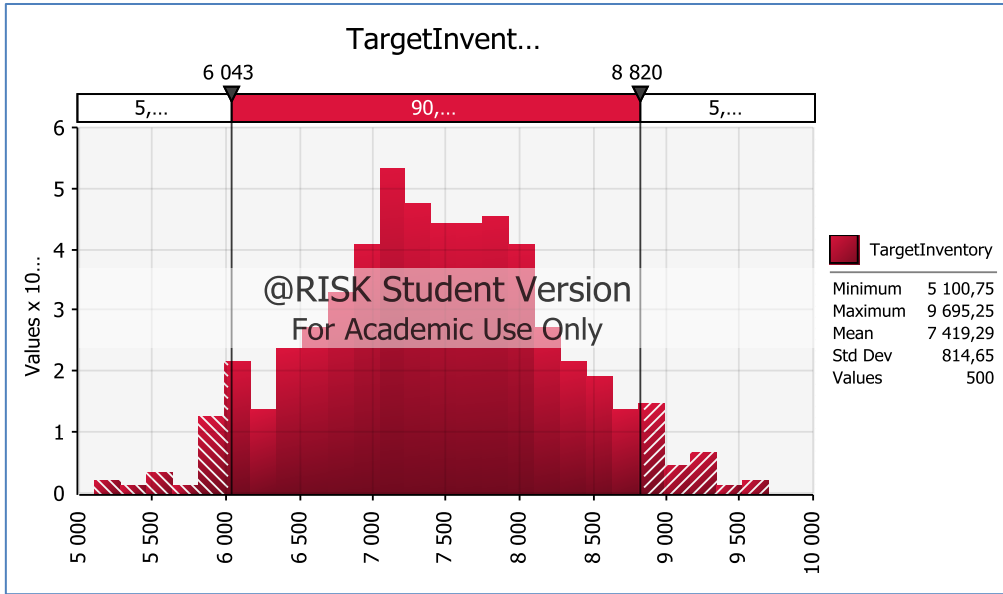


Figure 47 Distribution of initial values of TargetInventory



### 6.3.1.6 Efficiency

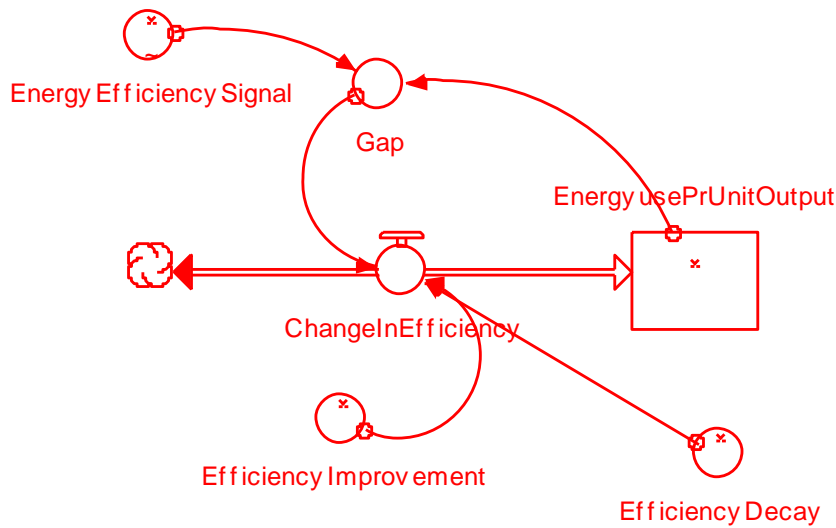


Figure 48 Model of Efficiency

A high oil price will give incentives to become more energy efficient and trigger transition to other sources of energy or substitutes. The end product of this model is a factor between 0 and 1, where 0 relates to the state of total transition to other sources of energy and 1 relates to a price level which there is no incentives to be efficient. It is assumed that the change in efficiency revert faster to less efficient due to a fall in prices than to more efficient due to higher prices.

**EnergyEfficiencySignal** Signal based on OilPrice divided by the initial OilPrice. The elasticity between prices and efficiency is governed by a variable graphical input (Appendix I)

$$EnergyEfficiencySignal = \frac{OilPrice}{INIT(OilPrice)} \quad []$$

**Gap** Difference between current efficiency level and signal created by the simulated OilPrice

$$Gap = EnergyEfficiencySignal - EnergyusePrUnitOutput \quad []$$

**EnergyusePrUnitOutput** How efficient the use of petroleum is at current price level. Initial value set to 0.8.

$$EnergyUsePrUnitOutput = 0.8 \quad []$$

**ChangeInEfficiency** Change in energy efficiency dependent on the price of oil.

$$ChangeInEfficiency = IF\ Gap > 0\ THEN\ EfficiencyImprovement * Gap\ ELSE\ EfficiencyDecay * Gap$$

**EfficiencyImprovement**

Factor for improvement in efficiency (due to a rise in oil price). Typical values generated between 1 and 5,5. The factor remains identical through each simulation. The distribution of the factor is shown in Figure 49.

$$EfficiencyImprovement = RiskExpon(1,9; RiskShift(1); RiskTruncate(0; 6)) \quad []$$

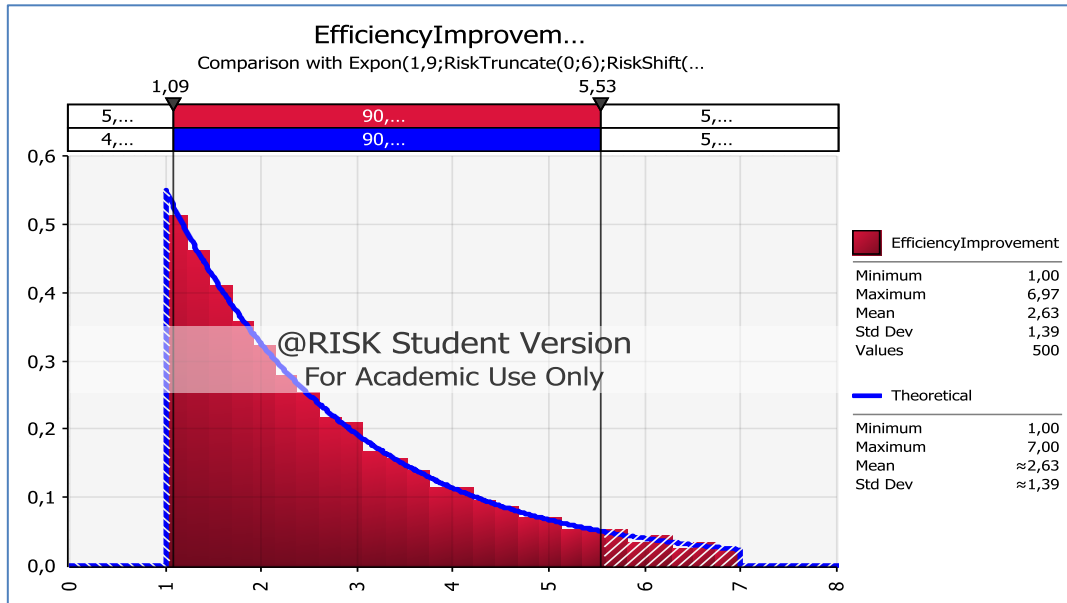


Figure 49 Distribution of EfficiencyImprovement factor

**EfficiencyDecay**

Factor for worsening of efficiency (due to a fall in oil price). Typical values generated between 10-15. The factor remains identical through each simulation. The distribution of the factor is shown in Figure 50.

$$EfficiencyDecay = RiskErlang(2; 1,25; RiskShift(9,5)) \quad []$$

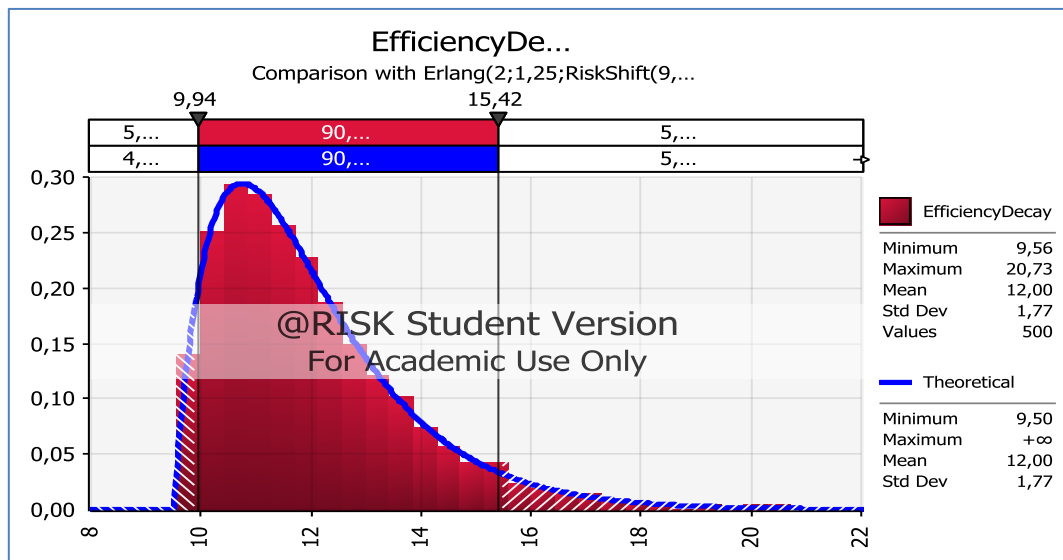


Figure 50 Distribution of EfficiencyDecay factor

### 6.3.1.7 Price

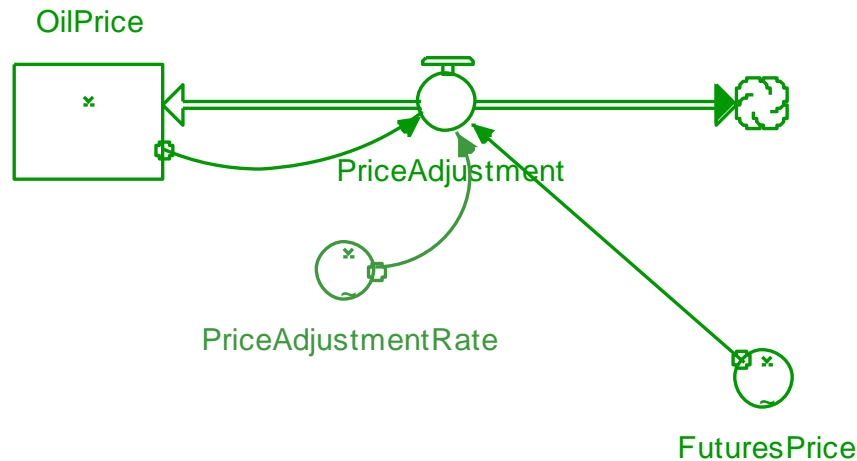


Figure 51 Model of Price

It is in this sub-model the oil price is simulated. The FuturesPrice is the main input to regulation of the oil price. This node will be sending a signal to adjust the OilPrice towards its own movement. The rate at which it adjusts is controlled by the PriceAdjustmentRate. FuturesPrice is regulated by the elasticity between the price and the real inventory level compared to the target inventory level. There are several methods to extract prices from these inventories. The elasticity or correlation between inventories used in the model in this thesis is based on historical data which tracks future price responses to reported inventory levels provided by Simmons & Company. (Appendix H)

#### FuturesPrice

The futures price is based on input from the inventory levels from TargetInventory and SalesPetroleum and governed by a graphical input based on historical data replicating the coherence between the two (Appendix H).

$$FuturesPrice = \frac{SalesPetroleum}{TargetInventory} \quad [\$]$$

#### PriceAdjustmentRate

The rate at which the price will adjust itself. This input is governed by graphical input which is made up by 11 independent triangular distributed factors towards time (Appendix K)

$$PriceAdjustmentRate = f(RiskTriang(2,3.5,8)) \quad []$$

#### PriceAdjustment

Flow of price adjustments based on input factors.

$$PriceAdjustment = \frac{(FuturesPrice - OilPrice)}{PriceAdjustmentRate} \quad [\$]$$

#### OilPrice

The current simulated price of oil. Initial value set by historical data. WTI @ December 2009

$$OilPrice = 74.48 \quad [\$]$$

6.3.1.8 Development and extraction

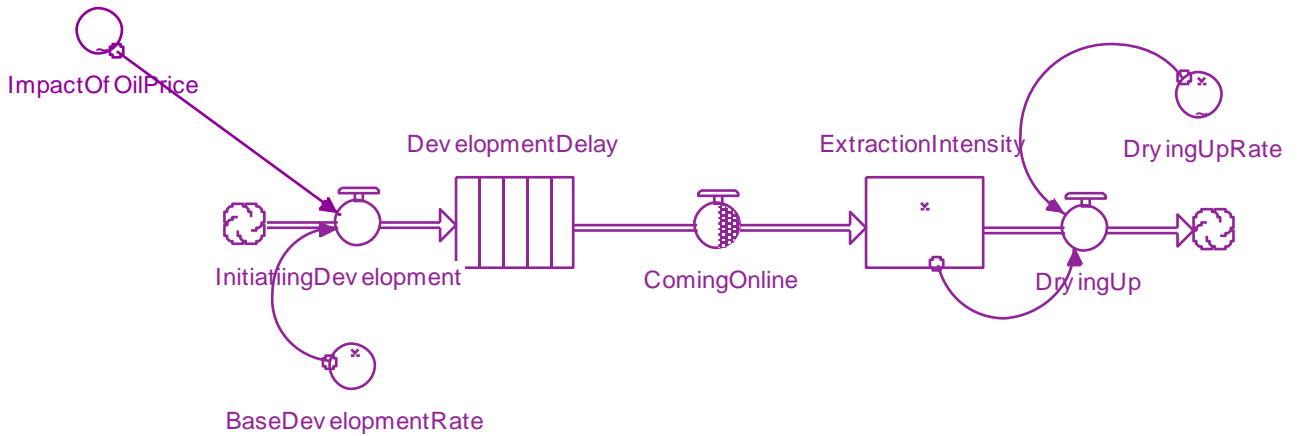


Figure 52 Model of Development and extraction

Initiating development of projects will increase with higher oil prices and will also be affected by BlackSwans, however there will be a development delay for the project. ExtractionIntensity will reflect the production rate by the state of proven reserves, thus a maximum of extraction intensity will indicate a peak oil of total world oil production. Drying up rate will reflect the loss of production in matured fields.

**ImpactOfOilPrice** Signal to increase development governed by graphical input related to the oil price. Produces values in the range of 0 - 2

$$ImpactOfOilPrice = \frac{OilPrice}{INIT(OilPrice)} \quad []$$

**BaseDevelopmentRate** The normal rates at which projects are developed. Distribution of values is shown in Figure 53.

$$BaseDevelopmentRate = N\sim(1000,50) \quad [10^6 \text{ bbl/month}]$$

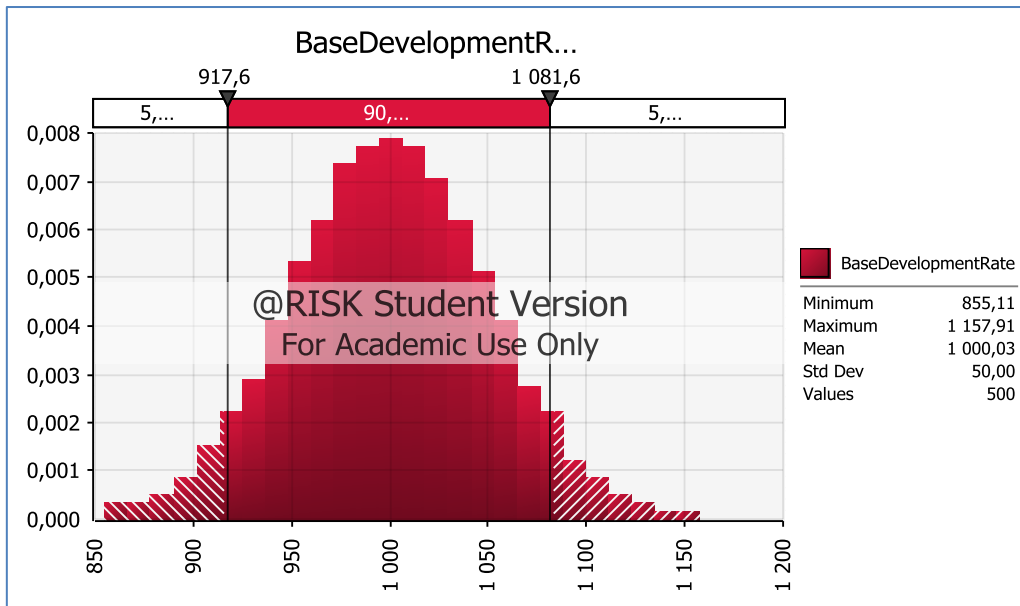


Figure 53 Distribution of values for BaseDevelopmentRate

### InitiatingDevelopment

Flow of new development. It is assumed that unforeseeable events will create an increase in new project development

$$\text{InitiatingDevelopment} = \text{BaseDevelopmentRate} * \text{ImpactOfOilPrice} * (2 - \text{BlackSwan})$$

[10<sup>6</sup> bbl/month]

### DevelopmentDelay

Conveyor of development of new fields. This conveyor will hold the initiated development until the flow toggle ComingOnline channels it through. Initial value set at 10000\*10<sup>6</sup> bbl. This value may seem high, but was chosen because of the delay built into the model regarding the addition of proven reserves. With a lower value there would be a gap that would not support the exploration and development 0-3 years before simulation-start.

$$\text{DevelopmentDelay} = 10000 \quad [10^6 \text{ bbl}]$$

### ComingOnline

This flow toggle is uni-flow and increases the extraction intensity based on a relationship between the actual proven reserves and the initial value of proven reserves. The flow is multiplied for correct unit conversion.

$$\text{ComingOnline} = 0.8 * \left( \frac{\text{INIT}(\text{ProvenReserves})}{\text{ProvenReserves}} \right) \quad [10^6 \text{ bbl}]$$

### ExtractionIntensity

Represents the intensity which proven reserves are produced. This can also be related to term peak oil, where the maximum number achieved during a simulation will reflect the peak production of oil. Initial value is based on data of end 2009. The distribution of initial values is shown in Figure 54

$$ExtractionIntensity = N \sim (2340, 200)$$

[10<sup>6</sup> bbl/month]

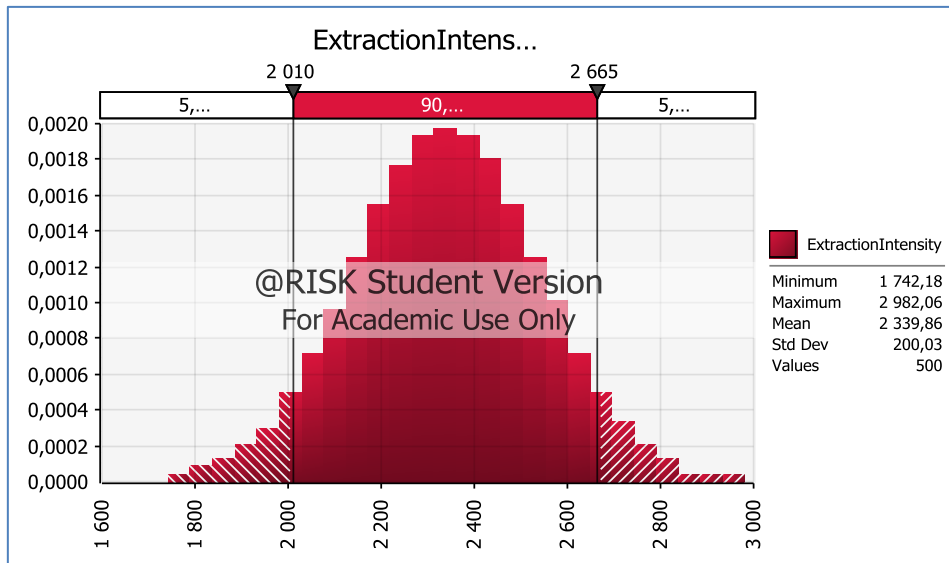


Figure 54 Distribution of initial values in ExtractionIntensity

**DryingUp**

Flow of lost production from matured fields or abandonment of fields from extraction.

$$DryingUp = ExtractionIntensity * DryingUpRate$$

[]

**DryingUpRate**

The rate at which proven reserves dry up or abandonment of producing fields. The node is governed by a variable graphical input. (Appendix X).

$$DryingUpRate = Time * \frac{1000000}{INIT(ProvenReserves)}$$

[month/10<sup>6</sup> bbl]

### 6.3.1.9 Black Swans

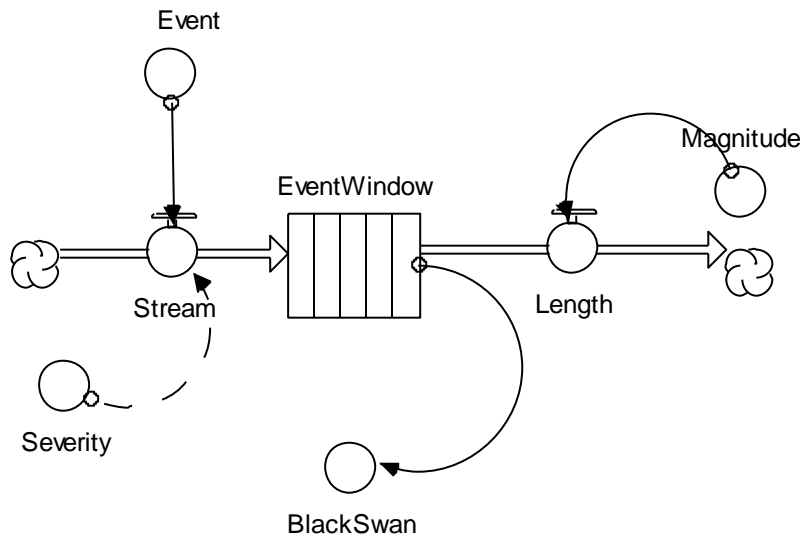


Figure 55 Model of BlackSwan

BlackSwan is in this model not limited to events according to the definition by Taleb, but will also include events which have an impact on the oil price, but are not embraced properly by the sub-models. Even though some of these events are not unpredictable and the impact uncertain, they should still be modelled since they have such large impact. As an event will vary its impact during the time it is active, replicating this in the model has been done by creating shorter interval impacts with large severity. These events will create jumps and the model itself will by these jumps use some time to recover to its normal movement.

The node BlackSwan will receive a number between 0 and 1 from the EventWindow, where 0 is high impact and 1 is no impact. BlackSwan is then assumed to impact the listed nodes in the total model the following way:

- Producing                      BlackSwan will influence production negatively.
- InitiatingDevelopment      BlackSwan will increase further development and exploration.
- Transport                      BlackSwan will disrupt the mid and down-stream supply chain and refinery capacity negatively.
- RealizedDemand              BlackSwan will trigger an increase in demand for oil.
- InventorySignal              BlackSwan will trigger an increased target for the inventory levels.

The input data of BlackSwan are set in order to replicate impact patterns experienced in the past.

**Event**                              The probability for an event to happen, where the number 1 is generated by the Montecarlo command at a given normal distribution. The distribution of probability is shown in Figure 56.

$Event = Montecarlo(N \sim (10,5))$

[]

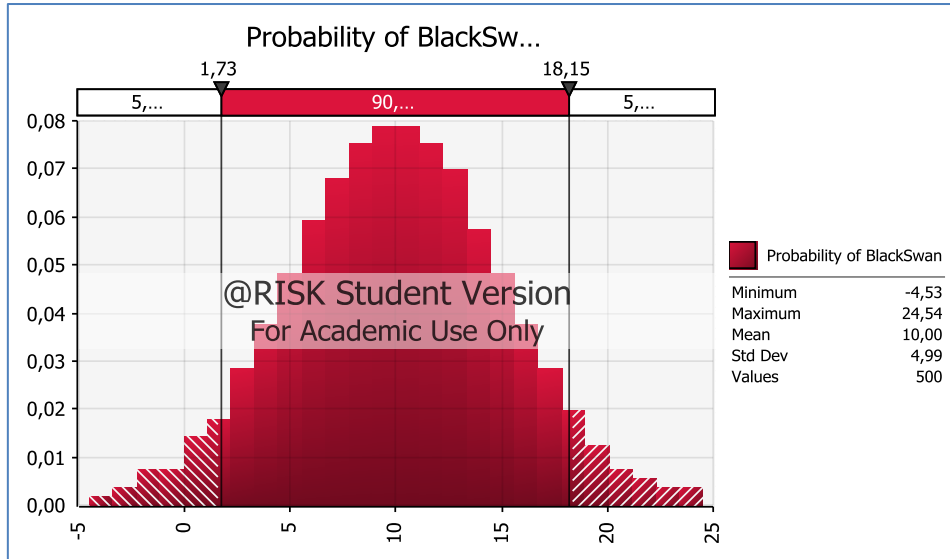


Figure 56 Probability distribution of BlackSwan

**Severity**

The severity from an event where 0 is no impact and 1 is huge impact. This is set to be a continuous normal distribution during the simulation. The distribution of values is shown in Figure 57.

$Severity = N\sim(0.45,0.15)$

[]

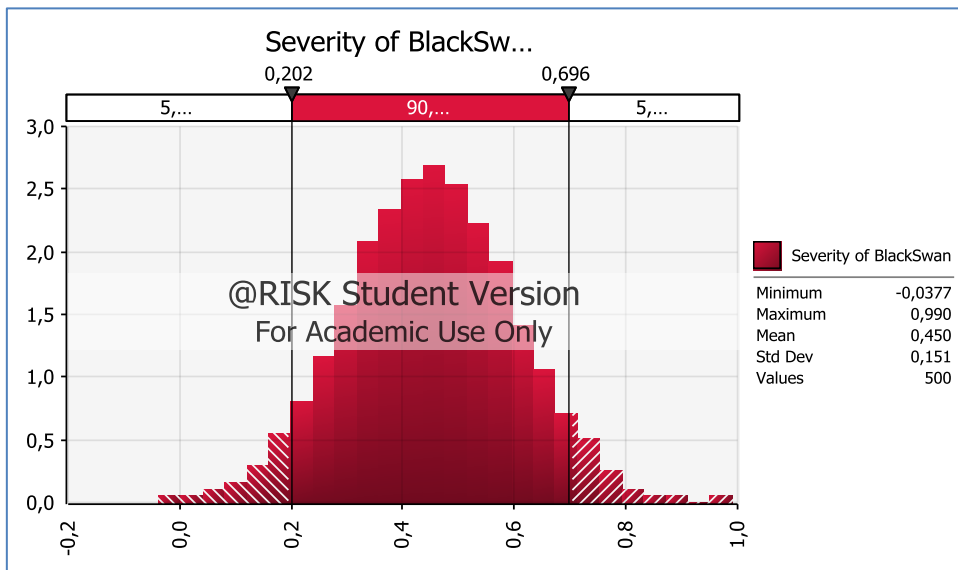


Figure 57 Distributon of Severity of BlackSwan

**Stream**

Flow of events and their severities into the conveyor.

$Stream = Event * Severity$

[]

**EventWindow**

Stock of events with a given severity. Initial value set to 0.

$EventWindow = 0$

[]



**Magnitude** The length at which an event affects the model in the EventWindow.  
Distribution of transit times is shown in Figure 58

$Magnitude = N \sim (4,2)$  [months]

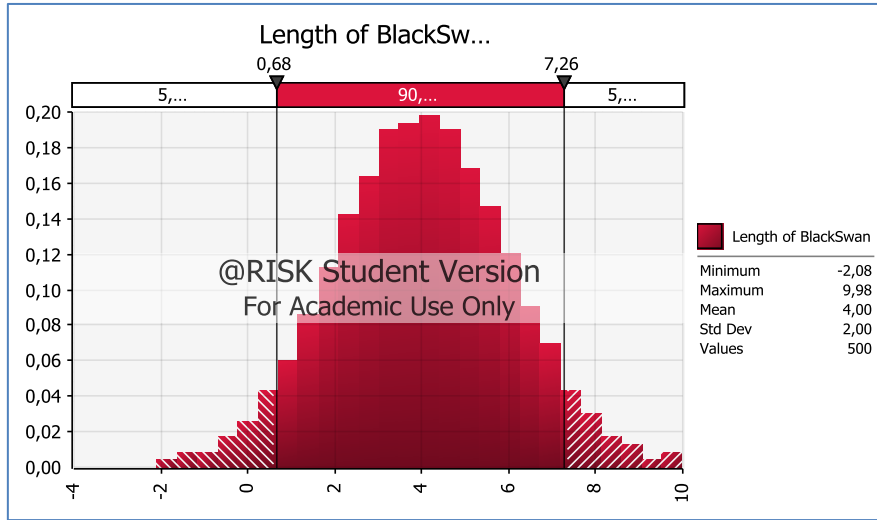


Figure 58 Distribution of impact lengths of BlackSwan, numbers in months

**Length** Flow regulator, will flow out an event at the given time produced by Magnitude

$Length = Magnitude$  [months]

**BlackSwan** Represents the event generated by the model.

$BlackSwan = 1 - EventWindow$  []

## 7 Results

*"However beautiful the strategy, you should occasionally look at the results"*  
*Winston Churchill*

This chapter will display the results from the simulations of the price models and their impact in the selected projects. The price models will first be shown in comparison to each other, before they are presented separately in their own section. Special interest has been taken in studying the output from the system thinking model. The ST-model made for this thesis requires a high number of input parameters and assumptions. The Stella-software has the possibility to monitor and export values of all key parameters. This allows for several outputs, not only the price simulation itself. By inspecting how the key input parameters behave during a simulation, it is possible to validate the input data or assumptions that have been made. In the same way it offers the chance to refine the model for further improvements.

All four models provide different output and carries different attributes, such as volatility and long term trend. The graphic output and statistical analyses from all the models are based on 500 sample runs. A sample run is a single simulated run for twenty years based on the input parameters and their associated uncertainty. The P10, P50, P90 and Mean are analysed over the total of 500 sample-runs. The following page containing Figure 59 displays the output from all four models in a descending order of complexity. All models are presented in identical windows for easier comparison.

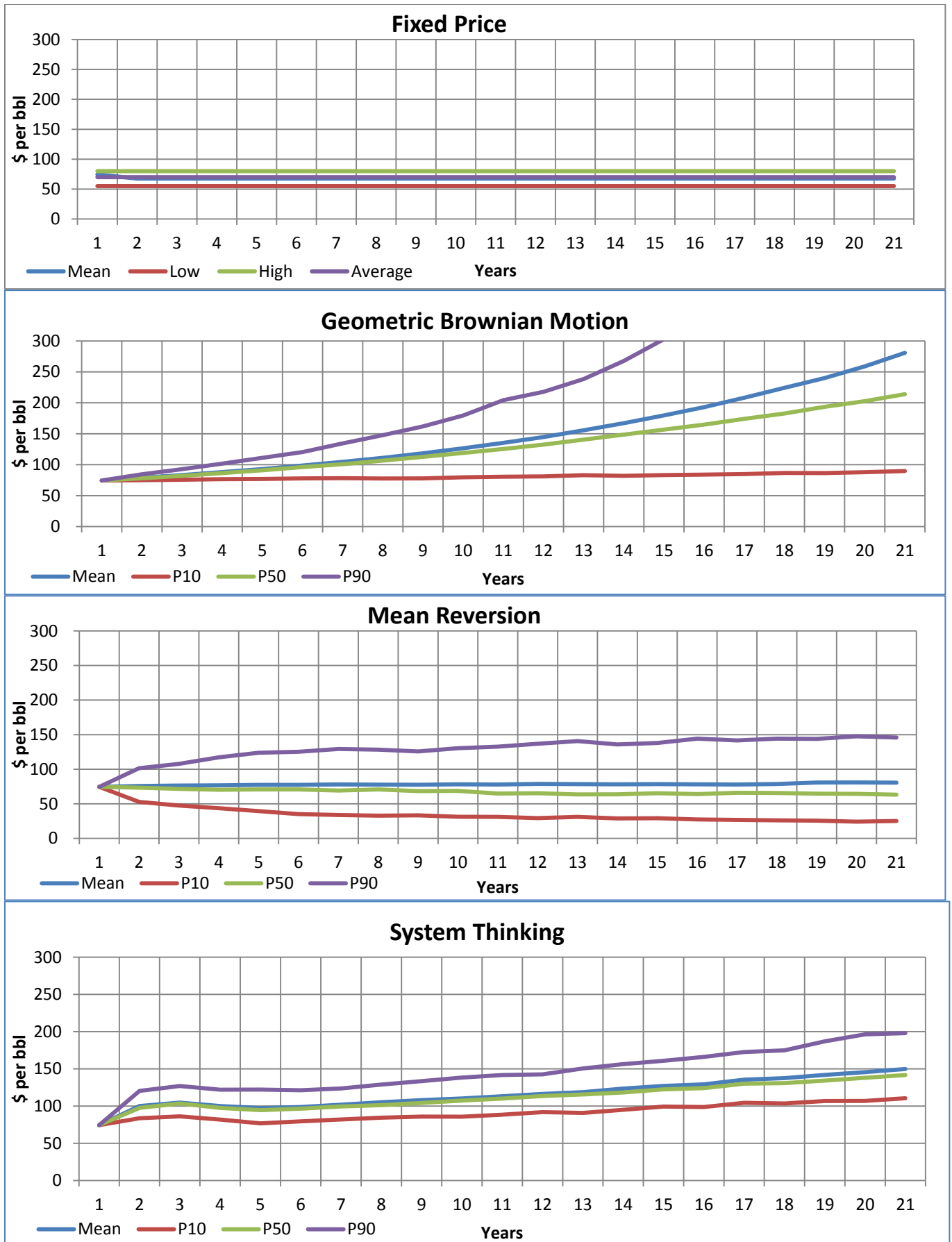


Figure 59 Graphic display of simulation output from all four price models

## 7.1 Single Price model

### 7.1.1 FP model

The fixed price model shows a flat projection of the oil price in Figure 60. Even though it will most likely be updated at a yearly basis, that update will only be for future projections, thus the output would look similar with a different price level. The shift in “Mean” in year one is the move from starting point spot price (Dec 2009 – 74,48 \$/bbl) to the future mean estimated in chapter 6.1.



Figure 60 Fixed Price model output

The “High” and “Low” are the highest and lowest values obtained from the Pareto survey<sup>24</sup>. These values are not to be perceived as uncertainty; they are neither used in further simulation, as the fixed price model will only use a fixed price.

## 7.2 Stochastic price models

### 7.2.1 GBM model

The GBM results shown in Figure 61, display a large space between the upper and lower values simulated and also a considerable growth in the “Mean” price path, which resembles an almost exponential growth. This can be traced to the large uncertainty in the drift factor. The volatility factor also holds an uncertainty to it, but the relatively larger size of the drift factor compared to the volatility factor dominates the simulation runs. The large width experienced in the estimated drift factors yields a large uncertainty in the overall simulation. However it is important to note that the width does not become fairly large until 8 years into the simulation.

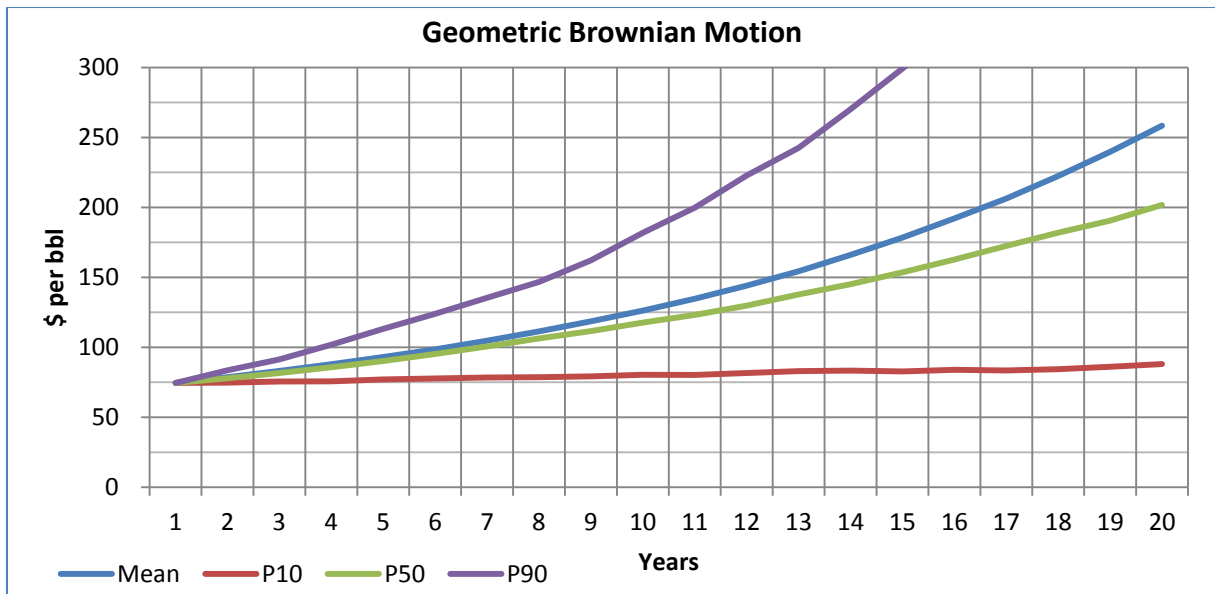


Figure 61 Geometric Brownian Motion model output

Figure 62 shows the total extent of the total model output.

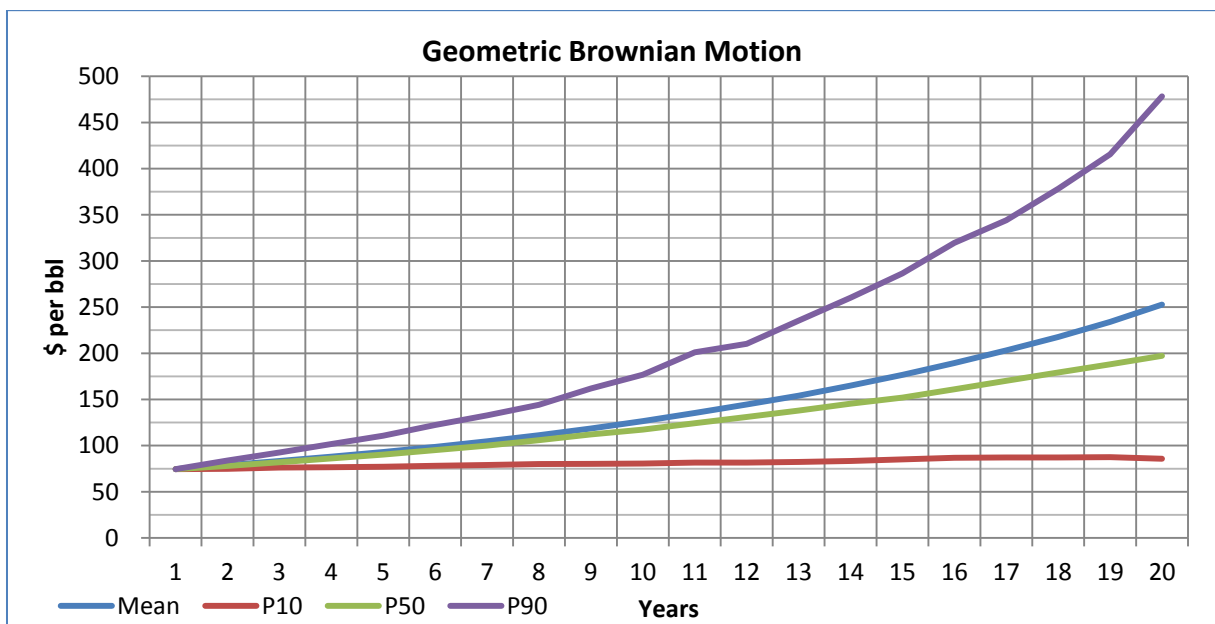


Figure 62 Geometric Brownian Motion model output with increased y-axis value

In Figure 63, the sample runs show that the main influence of the model is the drift factor, as the volatility factor is relatively small compared to the drift in the price paths. There is however an abrupt movement which can be related to the volatility factor.

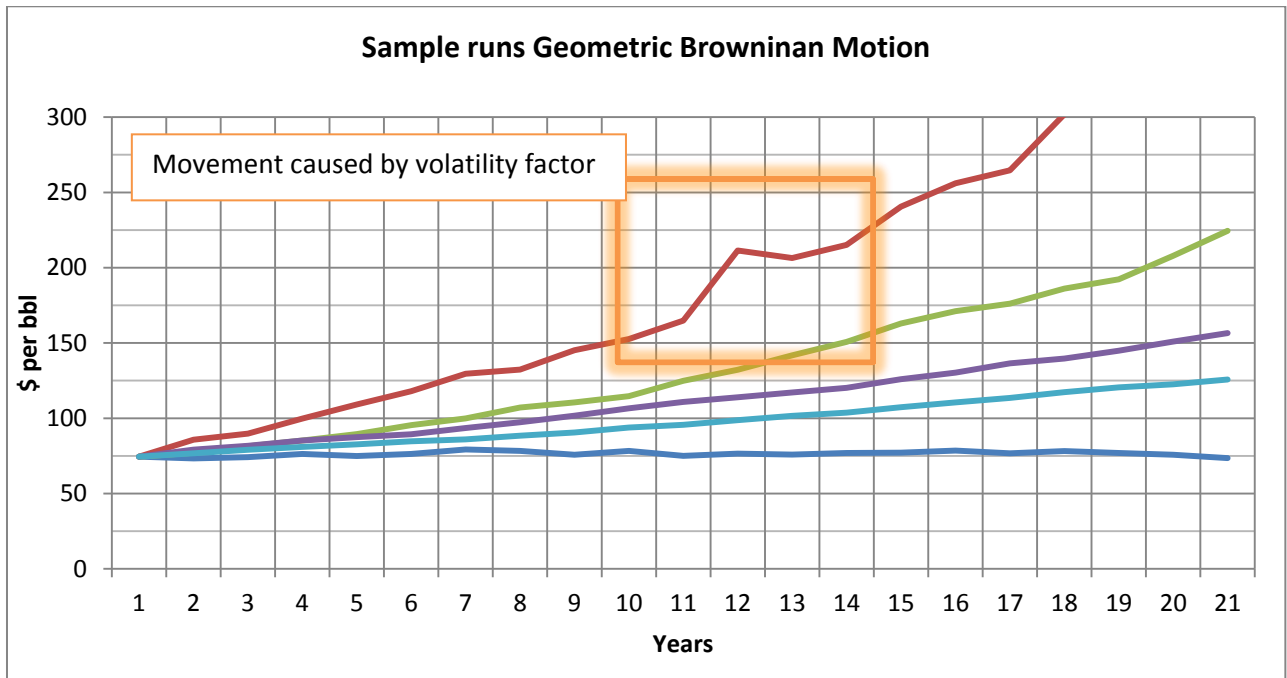


Figure 63 Geometric Brownian Motion sample runs.

### 7.2.2 MR model

The results from the Mean Reversion model show almost a symmetrical behaviour of uncertainty factors around the long term mean price. It also displays a larger upside than downside in the uncertainty. An important feature to notice in the MR model is how it is able to divert at a quite early stage and creates a wide probability distribution. This is shown in the output from the simulations in Figure 64.

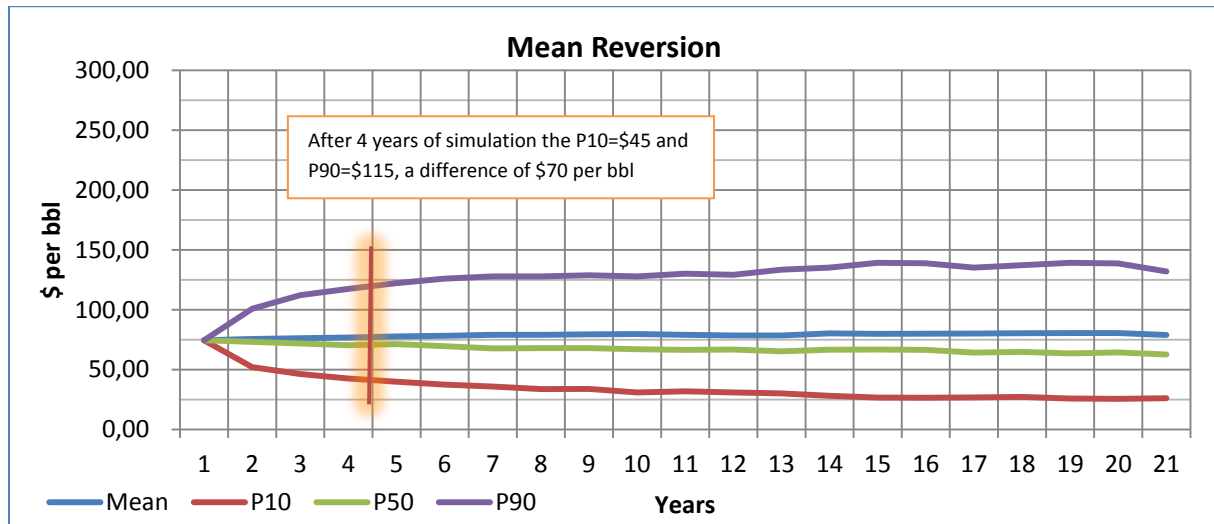


Figure 64 Mean reversion model output

For the sample runs it is evident that there is a mean reversion effect in the model as all the sample runs fluctuate around a mean trend. This is shown in Figure 65 with trend-lines added for the purple and red sample run. The values in the graph are adapted to better show the mean reversion effect.

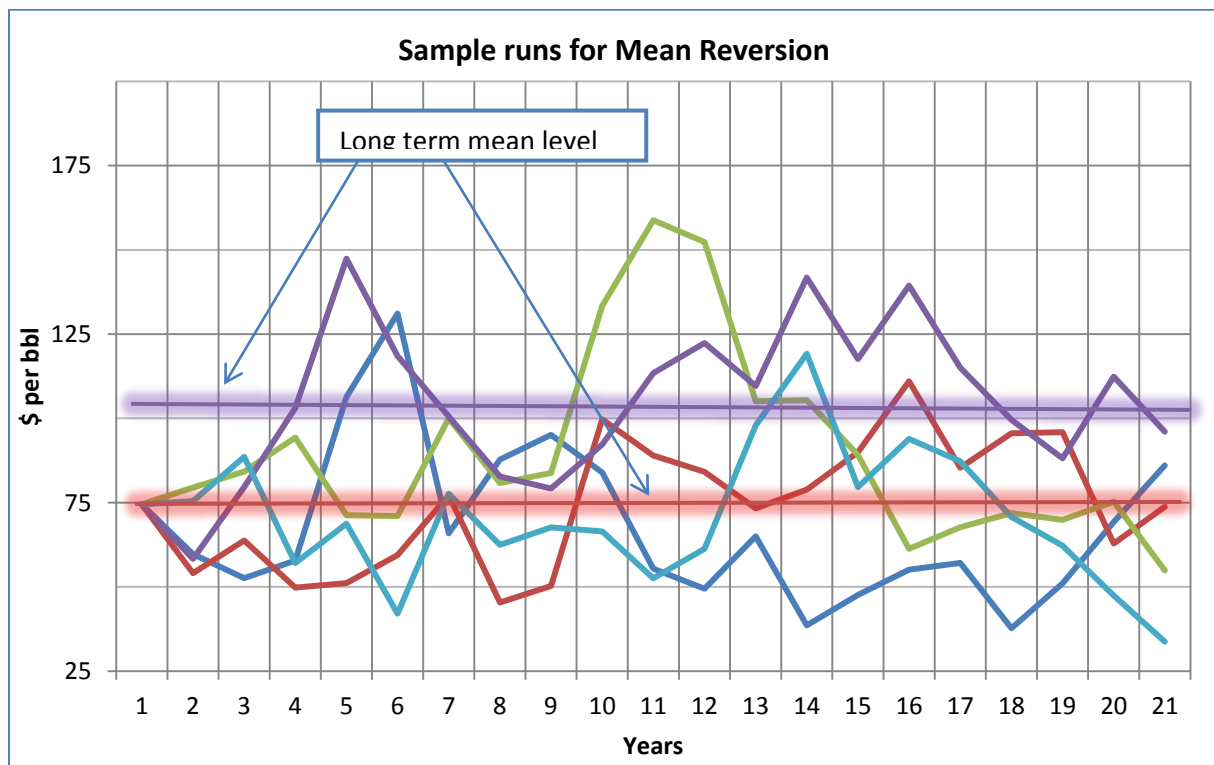


Figure 65 Sample runs for Mean reversion model

## 7.3 Stochastic- and system thinking approach price-models

### 7.3.1 ST model

Each run was performed in Stella with stochastic variables, variable input data and variable elasticity generated by @Risk in MS Excel for each run. A total of 500 runs were then exported to MS Excel. As simulations in Stella were performed for 240 months, all runs were averaged at a yearly basis. The results from sensitivity analysis are shown in Figure 66. Selections of yearly sample runs are displayed in Figure 67 and monthly sample runs are shown in Figure 68.

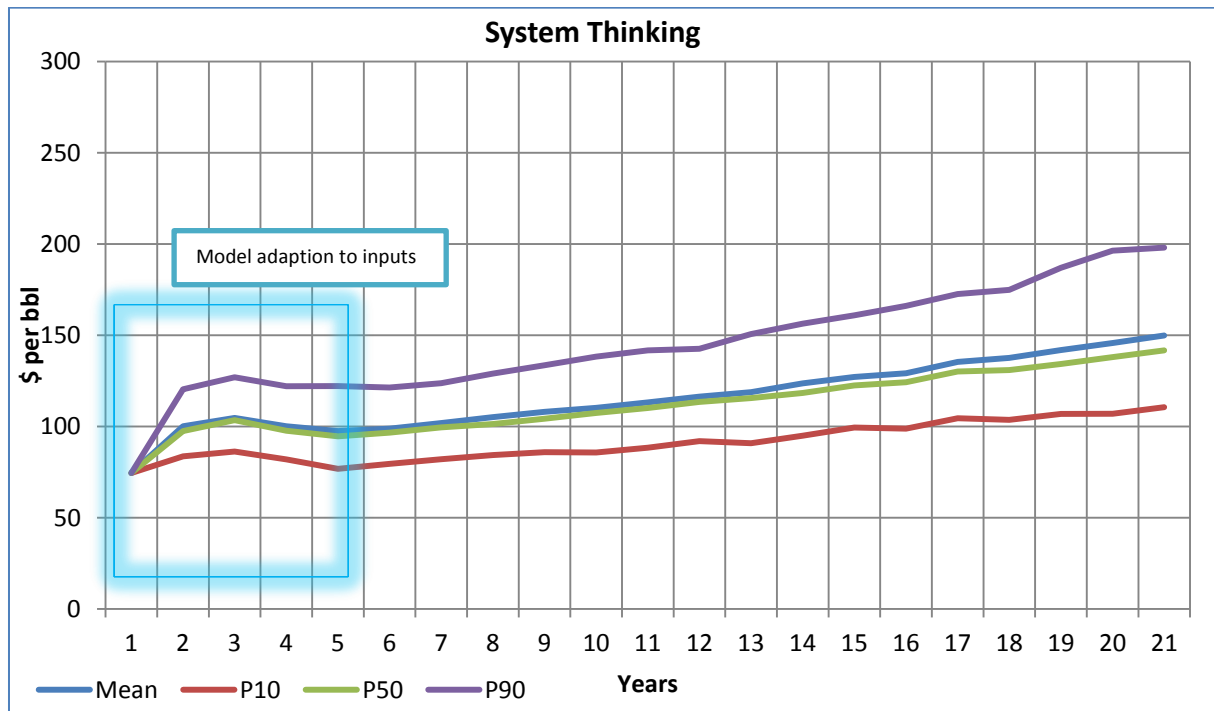


Figure 66 Graph of sensitivity analysis from System Thinking model.

The graph in Figure 66 shows an increasing trend in the price path for the whole simulation period. An interesting part of the output is the start of the simulation; there is a small peak for over a period of four years. This may be result of the initial values of the input parameters adapting themselves to all the interdependencies in the model. The graph also shows the increasing probability distribution experienced through the simulation period. This gives evidence to an increasing volatility experienced in the model.



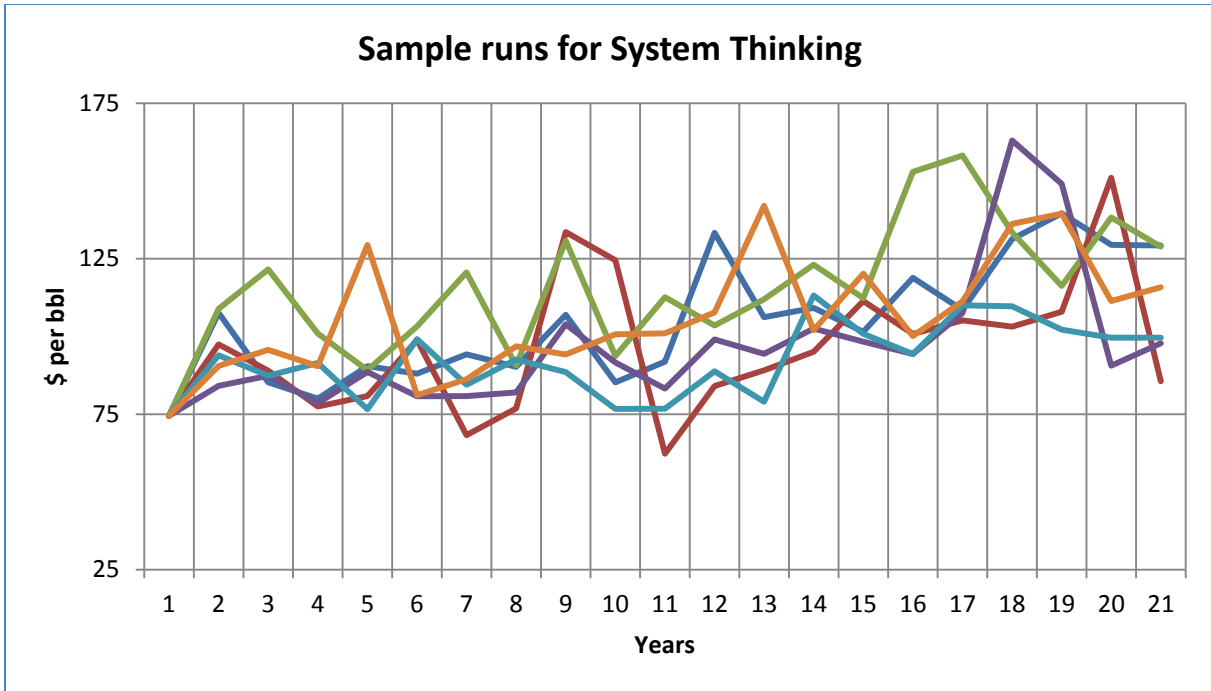


Figure 67 Graph of sample runs for System Thinking

The sample runs in Figure 67 show an increasing mean trend for all runs, but quite volatile movements around it. This shows the model’s ability to balance price movement. As these sample runs are yearly averaged, a better understanding to the model and the peaks is obtained by inspecting the monthly outputs from the model in Figure 68.

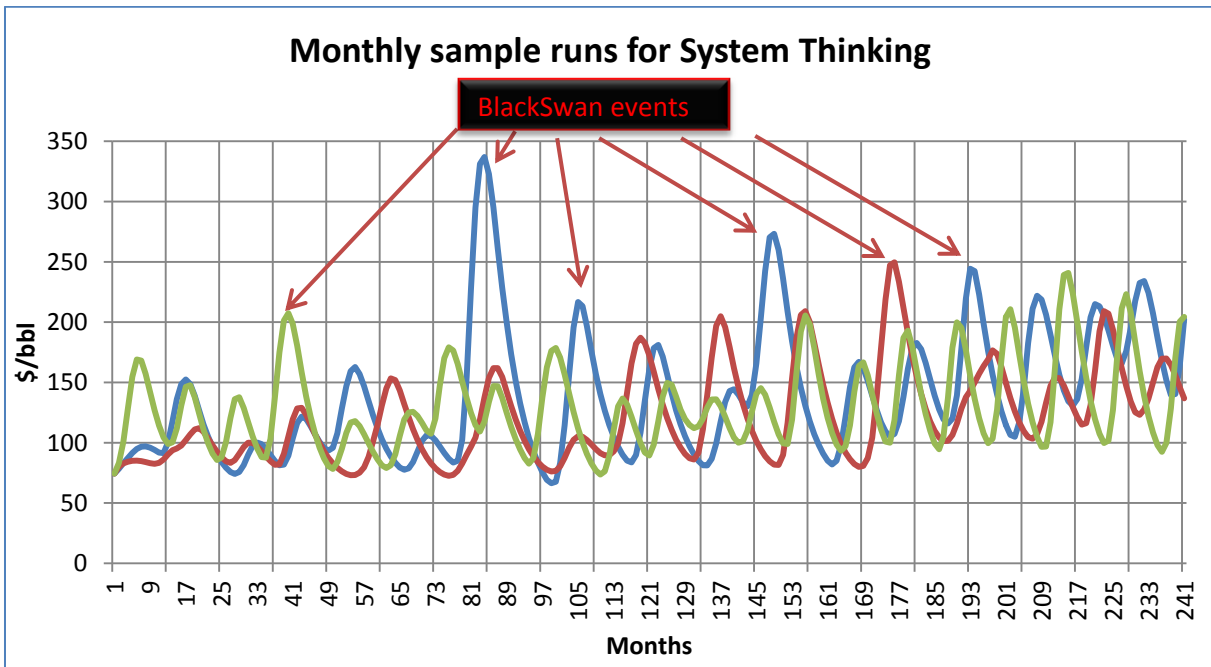


Figure 68 Graph of monthly sample runs for System Thinking

Figure 68 shows the large volatility experienced from the simulations in the model. The larger spikes in the graph are caused primarily by BlackSwan events (red arrows). The dips occurrences after the peaks are a trait in the model to balance out the peaks. The fluctuations seen through the overall

simulation time are related to overall uncertainty and volatility from all the inputs in the entire model.

The System thinking method together with the Stella software enables outputs from every node in the entire model. This makes it possible to validate the data and the assumptions made to inputs in the model. It is then possible to see how other parameters other than the oil price behave during simulations. This can give validity to the model as a whole if data behaves as expected or to give evidence on where to improve input parameters. Some of these outputs are investigated and for a better overview they are separated into the same sub-models from section 5.4.1.

- **Undiscovered oil**
- **Base oil flow**
- **Demand**
- **Inventory**
- **Efficiency**
- **Price**
- **Development and extraction**
- **Black swans**

### 7.3.1.1 Undiscovered oil

From this section the total amount of undiscovered resources are depleted at an estimated discovery rate and size and then through time added to the proven reserves stock in the Base oil flow -section.

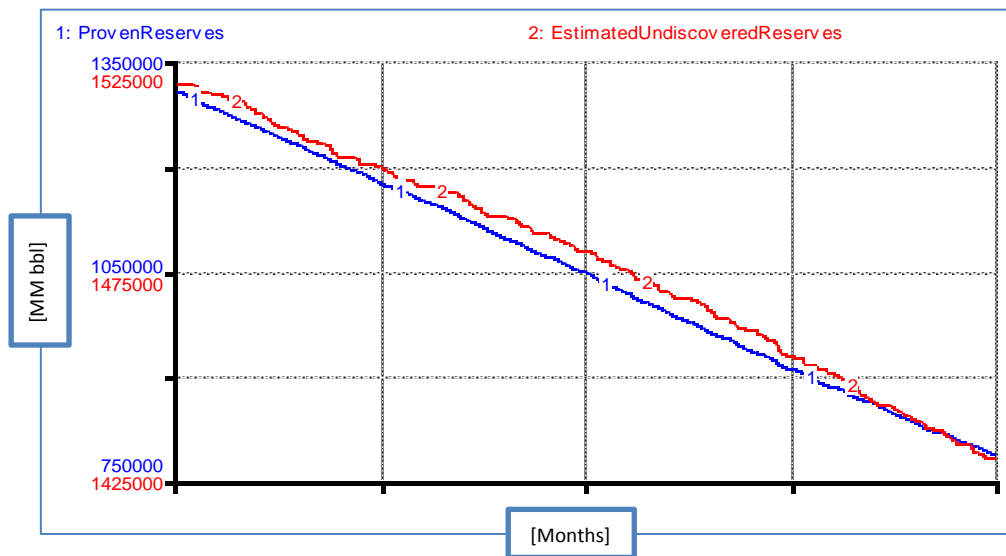


Figure 69 Sample-run showing reserves quantity during simulation.

The sample run in Figure 59 shows the depletion of ProvenReserves is almost at a total of 50% for a simulation. The EstimatedUndiscoveredReserves is depleted based on the DiscoveryRate and DiscoverySize giving the outflow of AddedProvenReserves. For this particular run, a total of almost 100.000 MM bbl are discovered and added to ProvenReserves over 20 years of simulation. This discovery amount is about 6,56% of EstimatedUndiscoveredReserves for this run. The addition of discoveries gives a replacement rate of 16,67%. In other words, only 16,67% of the total petroleum production was replaced .In Figure 70, a sample run shows cumulative discoveries add up to almost 60.000 MM bbl. The columns show the additions of discoveries based on DiscoveryRate and DiscoverySize.

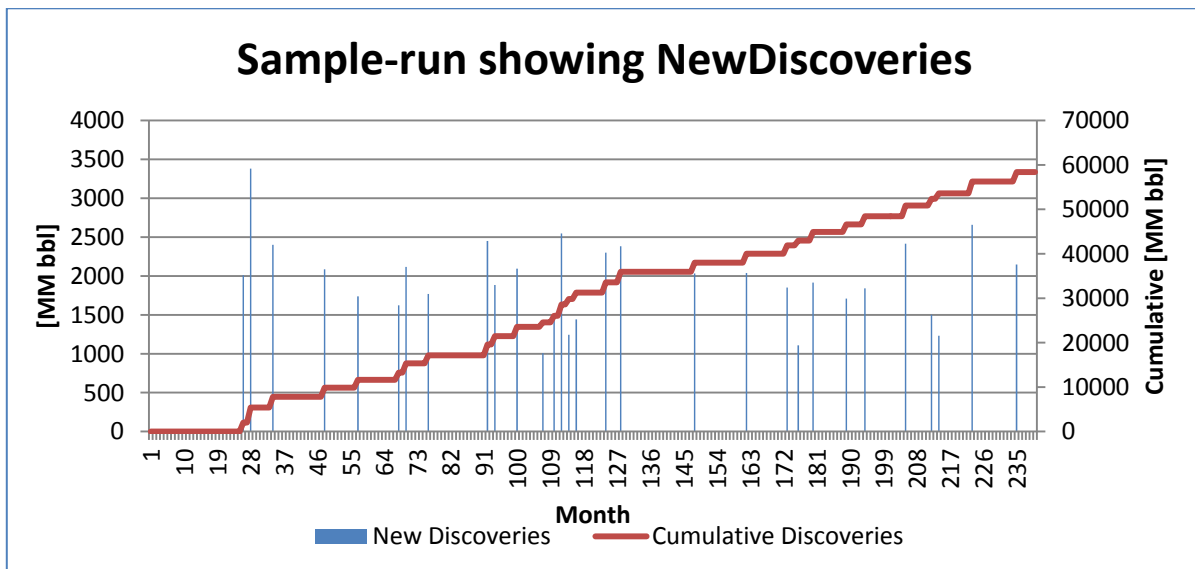


Figure 70 Sample run showing discoveries and cumulative discovery volume

### 7.3.1.2 Base oil flow

From the Base Oil Flow section the following outputs may prove valuable to investigate.

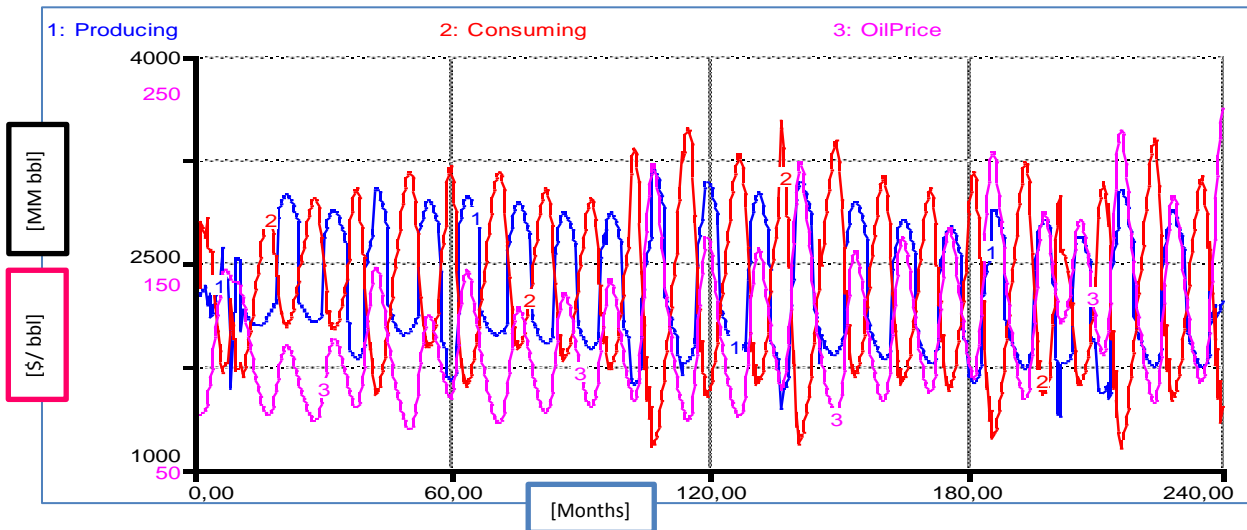


Figure 71 Sample-run showing Producing, Consuming and OilPrice.

Figure 71 shows a sample-run of the Producing and Consuming flows together with the OilPrice. This shows a balancing feature between oil price and consumption, where the production is following the oil price. It would also be valuable to inspect the elasticity in the supply versus the oil price and its effect on the production pattern.

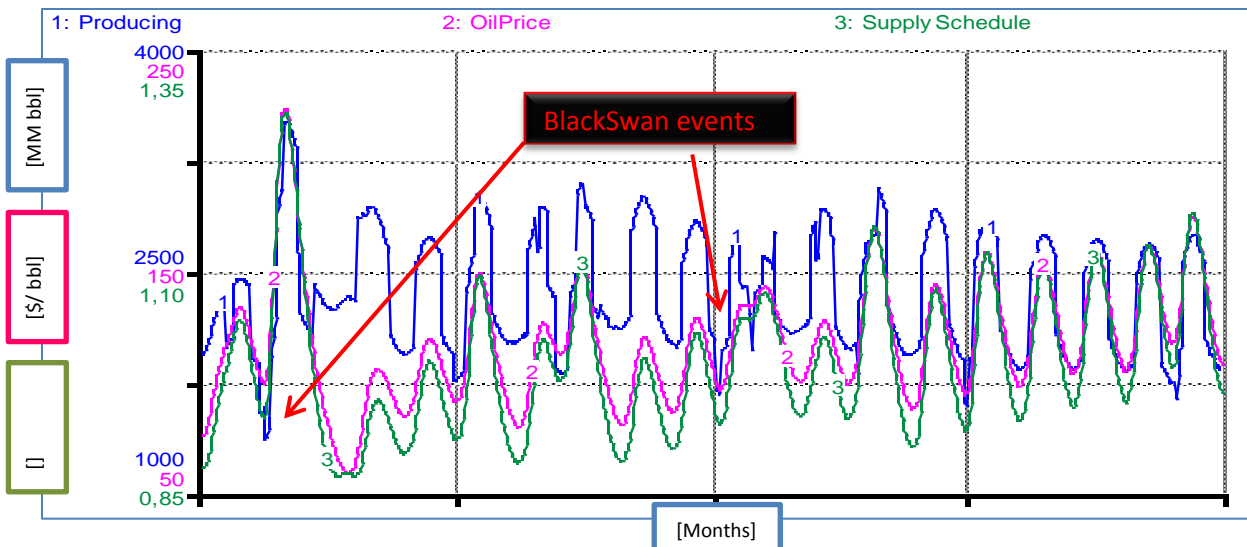


Figure 72 Sample-run showing Producing, OilPrice and SupplySchedule.

Figure 72 shows how SupplySchedule is influenced by the OilPrice and subsequent adjusts Producing thereafter. In this sample-run, SupplySchedule will at a price level of 150 \$/bbl increase production with 10%, and for a price level at 60 \$/bbl decrease the production with 15%. Two interesting points from the graph are at around 30 and 120 months (marked with red arrows). These dips are caused by BlackSwan events. At both point there is a dip in Producing. The first point creates a large peak in price and subsequently the production. The second point however, does not cause the same reaction. Both events were severe and can be seen in Figure 74 as the graph is from the same run.

The reason for this may be difference in inventory levels, such as the second point having a sufficient inventory level to withstand a BlackSwan impact.

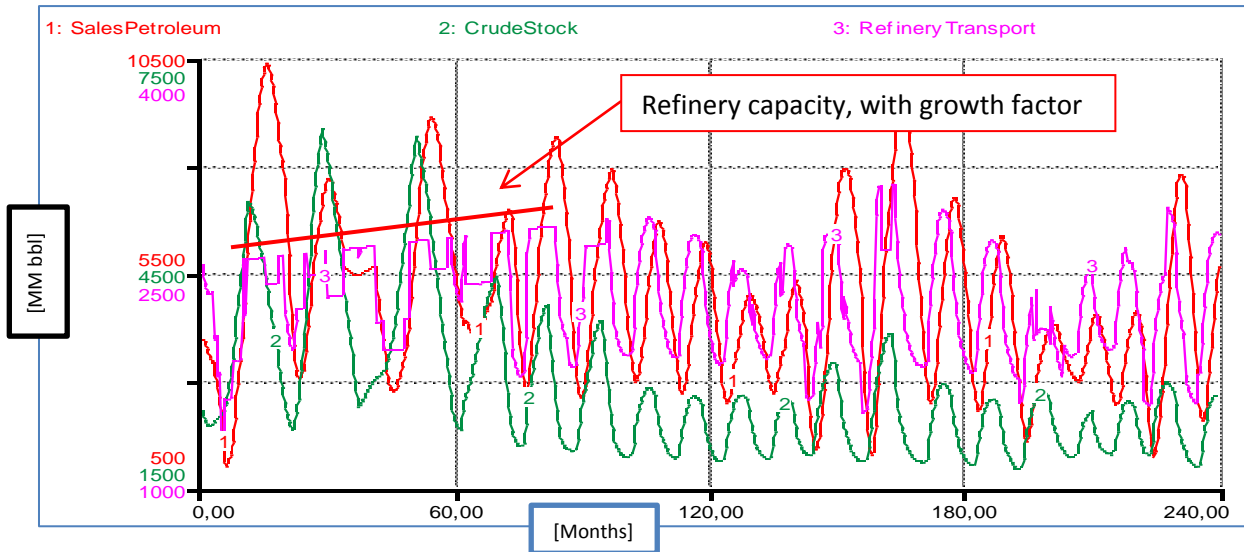


Figure 73 Sample-run showing SalesPetroleum, CrudeStock and RefineryTransport.

In Figure 73, three important nodes are displayed. SalesPetroleum refers to the inventory levels which trigger futures price movements. CrudeStock is produced crude oil which is not yet refined. RefineryTransport refers to the output from refineries. RefineryTransport is limited by the refinery capacity, which can be seen in the graph in the area from 0 – 60 months. The limit is reached several times (red line). The limit also shows a growth, as there is built in a growth factor to refinery capacity.

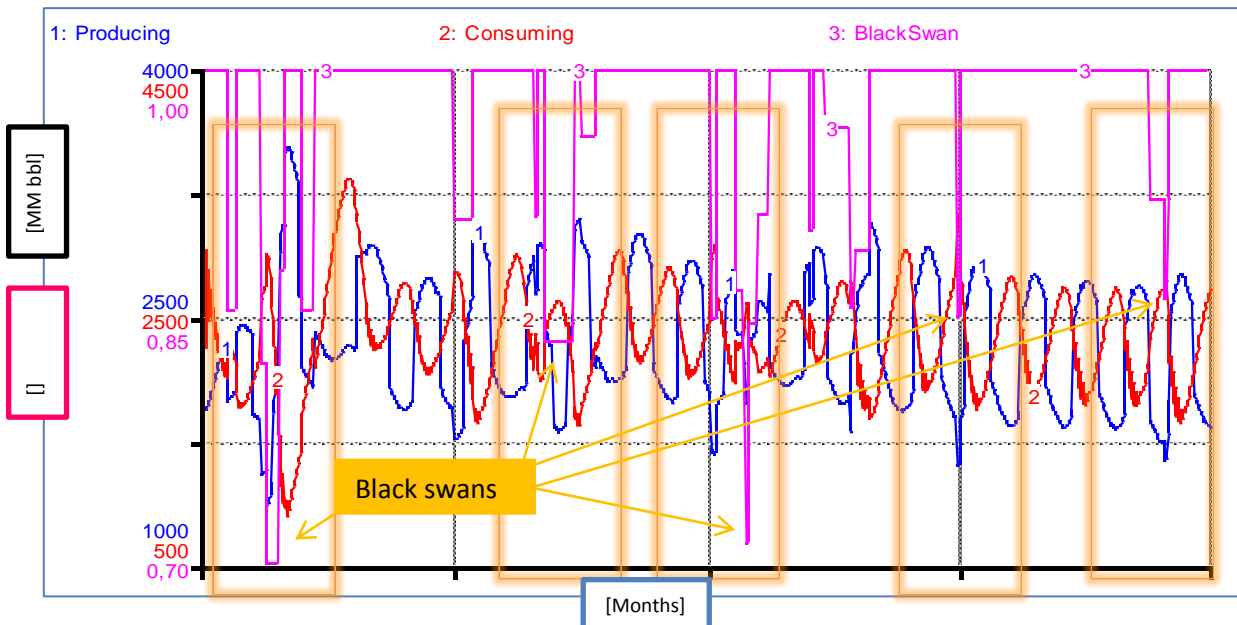


Figure 74 Sample-run showing Producing, Consuming and BlackSwan

Figure 74 shows the BlackSwan events and how it affects Producing directly and Consuming indirectly through increased demand. As seen in the graph and also in Figure 71 (the same run) it is the first event which creates the most disturbances in the model. The other events are also quite severe,

creating 15% - 25% decrease in production and increase in demand, but inventory levels are large enough to handle change. The events are shown with orange frames in Figure 74.

### 7.3.1.3 Demand

PotentialOilDemand is the initial demand at start of a simulation and is set with an initial value generated from available datasets. This demand will grow depending on the growth factor. RealizedDemand is the PotentialOilDemand adjusted for reactions to price levels and general cycles in the demand structure. Figure 65 show both potential and realized demand for a sample run.

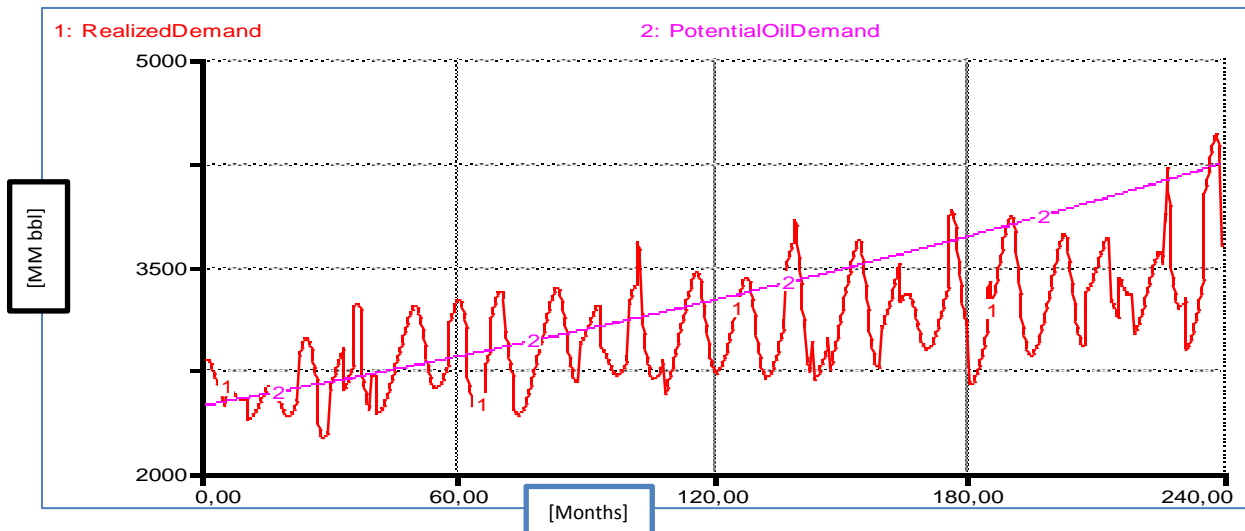


Figure 75 Sample-run showing demand of oil.

As the graph in Figure 75 shows, the potential demand for oil grows almost 75% over 20 years. This particular sample run was set with a monthly growth factor of 0,22%, the lower values experienced in RealizedDemand versus PotentialOilDemand relates to the elasticity of demand towards the oil price and fluctuation arisen from economic cycles and BlackSwan events. Because of these inputs the realized demand does not increase by the same factor as the potential oil demand.

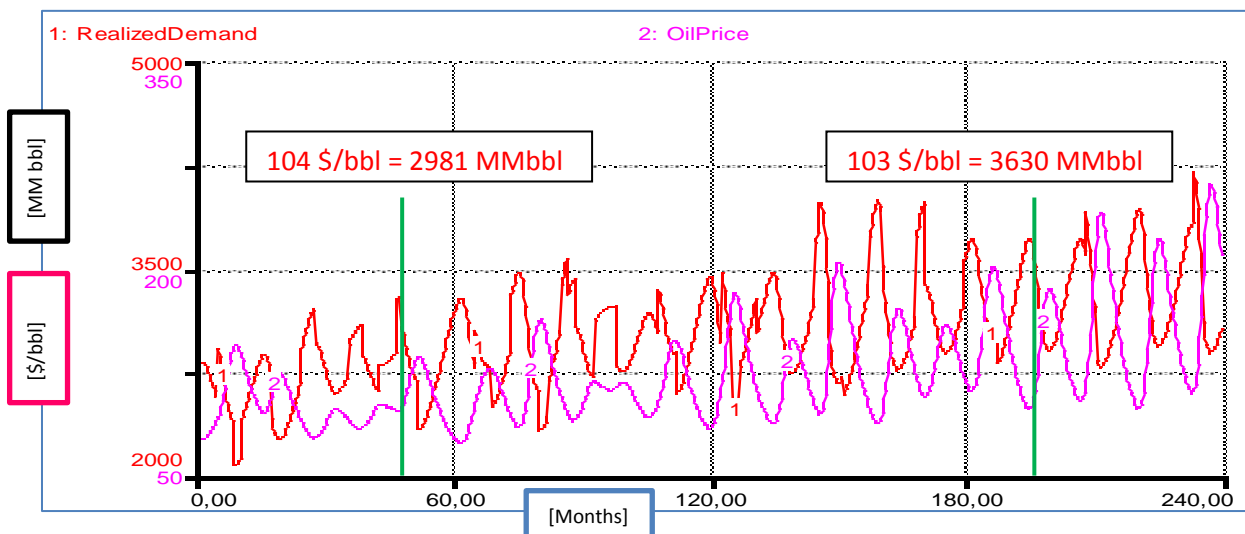


Figure 76 Sample-run showing RealizedDemand and OilPrice.

Figure 76 show the elasticity of RealizedDemand towards the OilPrice. Taking two points from the graph (marked with green lines) at the same price level but different times, shows the following relation: 4 years out gives a price of 104 \$/bbl and demand of 2.981 MM bbl. 16,5 years out gives a price of 103 \$/bbl and demand of 3630 MM bbl. This shows and adaption for demand to higher price levels in the model.

### 7.3.1.4 Inventory

Inventory levels relate s to the general inventory worldwide which contributes to drive the futures price in this model. The inventory target is set by the consumption level and a response factor.

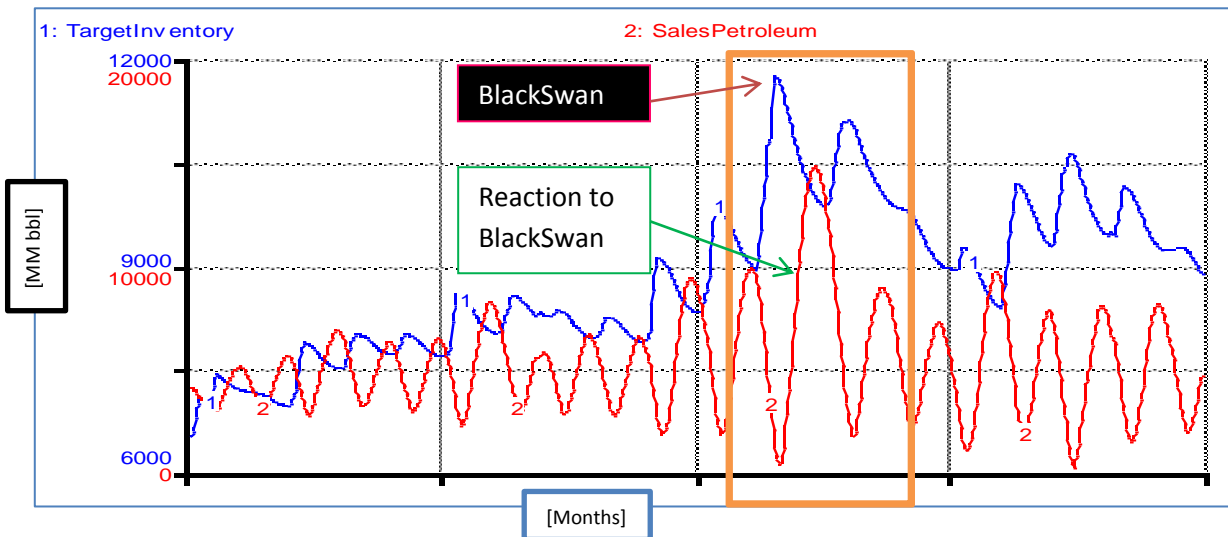


Figure 77 Sample-run showing the TargetInventory and the levels in SalesPetroleum.

Figure 77 shows the TargetInventory in relation to SalesPetroleum. Dividing SalesPetroleum by TargetInventory yields a FuturesPrice level from a elasticity graph (Appendix H). Thus a low level in SalesPetroleum and a high level in TargetInventory will reflect scarcity in the market and the FuturesPrice will increase according to the given elasticity. The large spikes (marked by orange frame) show the effect of a BlackSwan event. It also shows the reaction by the model to stabilize the event.

### 7.3.1.5 Efficiency

The efficiency section is directly related to the oil price and provides input to the consumption in the model. It relates to how the price of oil will force an improvement or decay into energy efficiency. High prices will readily force consumption to become more efficient and a low price will not give any incentives to do so. The energy efficiency is governed by elasticity towards the oil price (Appendix I).

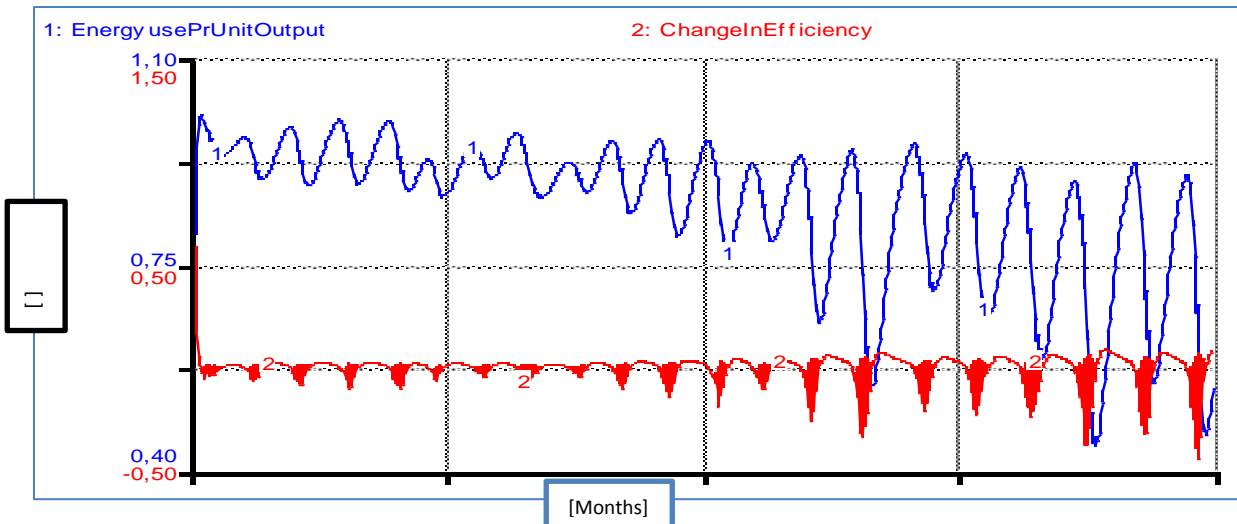


Figure 78 Sample-run showing Energy use Pr Unit Output and Change In Efficiency

Figure 78 shows that during simulation there is a declining trend showing increasing effectiveness as the price increases. The larger spikes are showing a reaction to high prices, causing a sharp increase in the energy efficiency to the consumption of oil. The Change In Efficiency shows how efficiency patterns change according to the price levels experienced. It also shows a swifter reaction to low prices than adapting to high prices.

### 7.3.1.6 Price

The oil price is governed by the FuturesPrice input and is adjusted at a given rate from PriceAdjustmentRate (Appendix K).

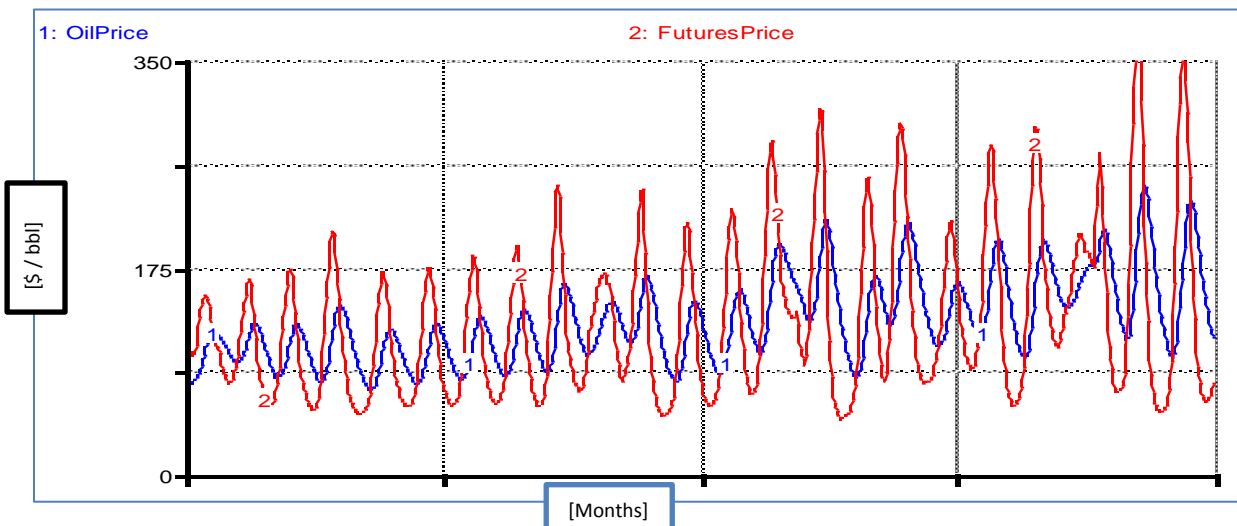


Figure 79 Sample-run showing FuturesPrice and OilPrice.

The graph in Figure 79 shows the relation between the price paths. The OilPrice is chasing the movement of FuturesPrice, replicating both normal backwardation and contango markets. An interesting point would be to see the impact of BlackSwan events to the OilPrice. This is shown in Figure 80.



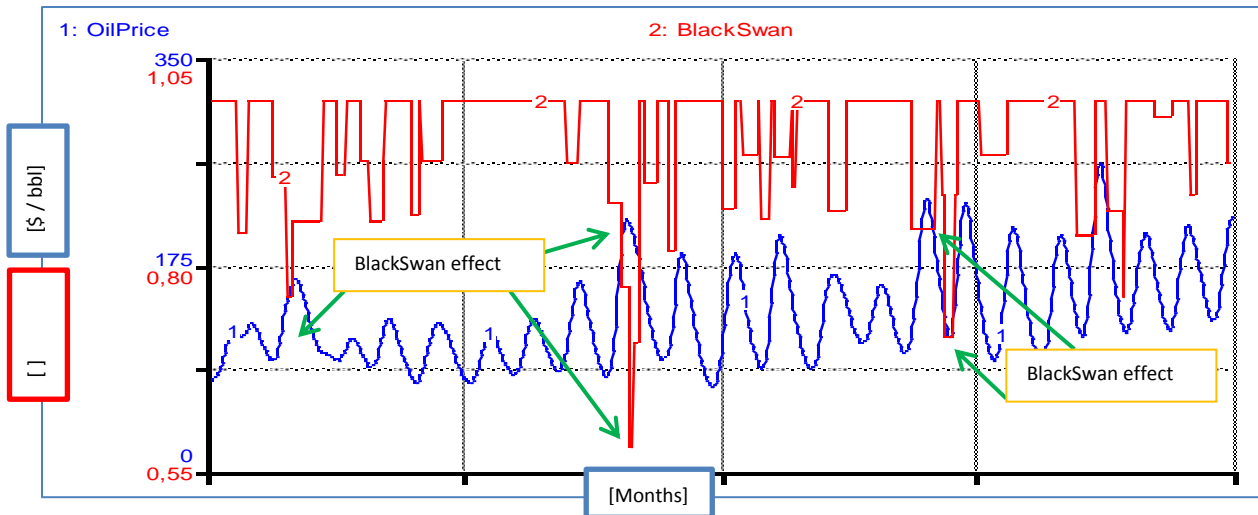


Figure 80 Sample-run showing BlackSwan and OilPrice.

The graph clearly indicates that more severe events influence the price path greatly, while smaller events do cause any major interference. This will be shown in more detail in section 7.3.1.8.

### 7.3.1.7 Development and extraction

This section replicates the functions of development and extraction into the full model. There will be initiated new development dependent on the oil price, higher price equals more development. As new development comes on-line, there will also be resources from mature fields drying up during simulations.

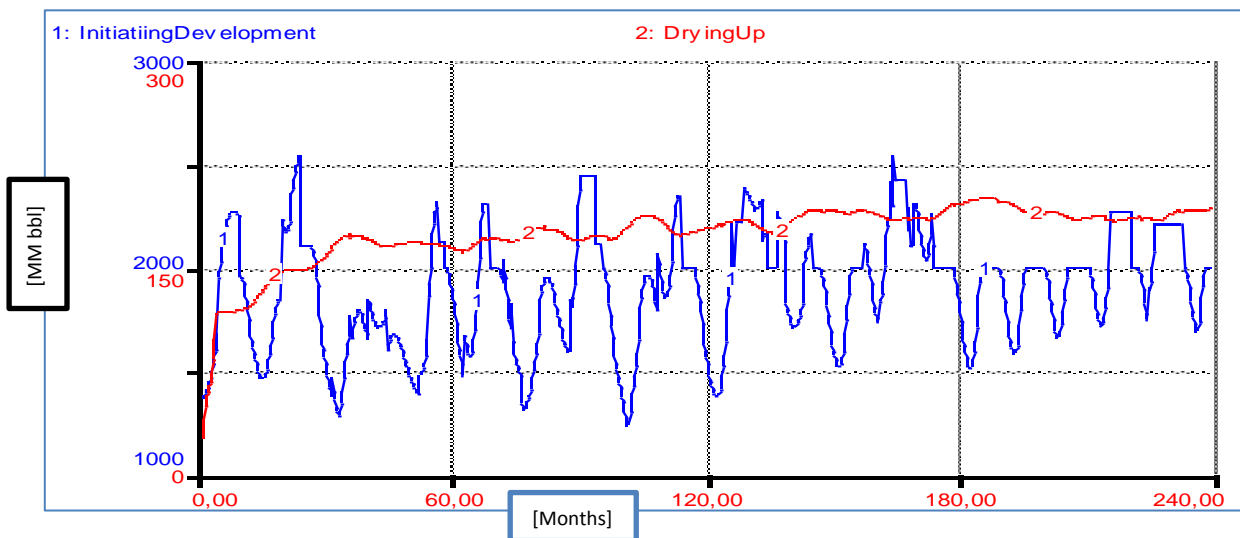


Figure 81 Sample-run showing the rate of new development and rate of reserves drying up.

Figure 81 shows the rate of new development initiated and the rate of reserves drying up. The fluctuations in InitiatingDevelopment are a combination of reactions to the OilPrice and BlackSwan. A BlackSwan event will trigger an increase the rate of development. Drying up is related to ExtractionIntensity shows the relation between extractions versus proven reserves and governs the production in the base oil flow section. ExtractionIntensity can thus be seen as an indicator of the much debated theme, peak oil. Figure 82 shows a sample run of ExtractionIntensity.

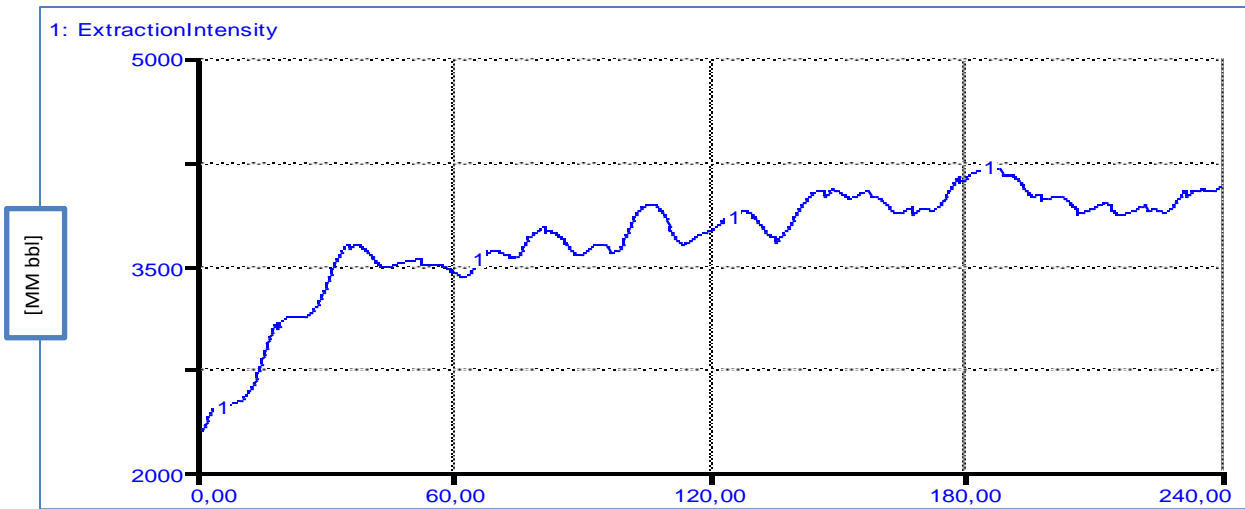


Figure 82 Sample-run showing ExtractionIntensity.

In Figure 82 there is no clear indication of a peak, more of a plateau. In Figure 83, sensitivity analysis of 500 samples is done in order to find a trend.

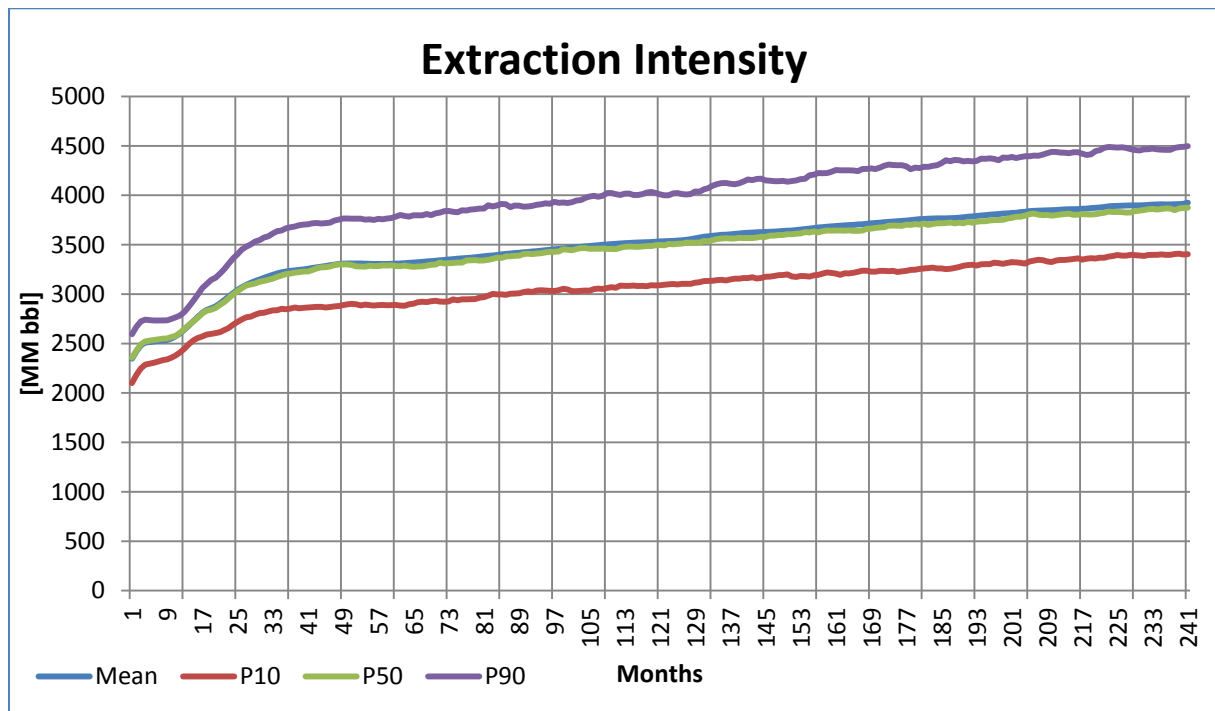


Figure 83 Sensitivity analysis on ExtractionIntensity

This graph indicates no imminent peak oil from a collection of 500 samples.

### 7.3.1.8 Black Swan

BlackSwan provides upsets within the model and can be seen as the main initiator of the most volatile fluctuations which are evident in almost all of the sample-runs as large peaks and dips. BlackSwan makes impact in the model at different magnitudes and time periods.

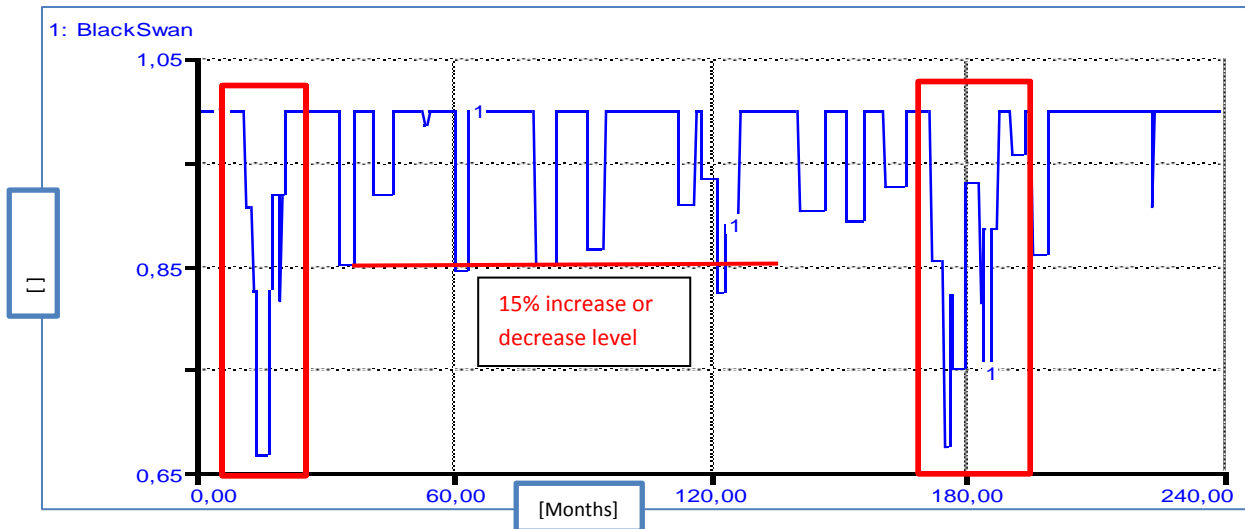


Figure 84 Sample-run showing the output of BlackSwan.

In Figure 84 the graph shows two large incidents (red boxes) and some smaller events. The smaller events do not impact the model with more than around 15% (red level-bar). The events last for different periods of time and will influence the model in different ways. The output from BlackSwan is between 0 and 1 where 1 is a normal state and 0 is absolute blocking of the system. For areas where there is an increasing effect of BlackSwan, the factor will be multiplied with: 2-BlackSwan. And for a decreasing effect, the node will just be multiplied with BlackSwan directly. BlackSwan will affect the listed nodes in the following ways:

- InitiatingDevelopment; it will increase development of new fields.
- Production; it will decrease the production of oil.
- Transport; it will decrease the output from refineries.
- InventorySignal; it will increase the target levels for inventory.
- RealizedDemand: it will increase the demand for oil.

Since the primary model output is the oil price, Figure 85 will show the OilPrice with BlackSwan operating at normal state, and Figure 86 will show the OilPrice with no BlackSwan in the model. Both runs are with practically the same input variables, except for the Monte Carlo simulations and variables which are run directly in Stella.

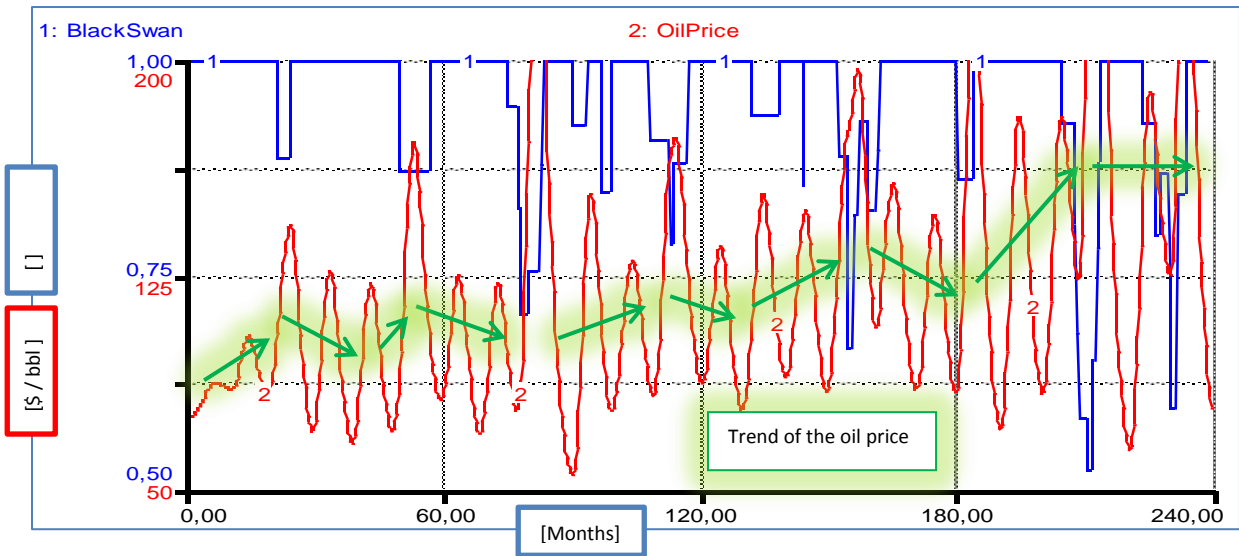


Figure 85 Sample-run showing OilPrice and BlackSwan.

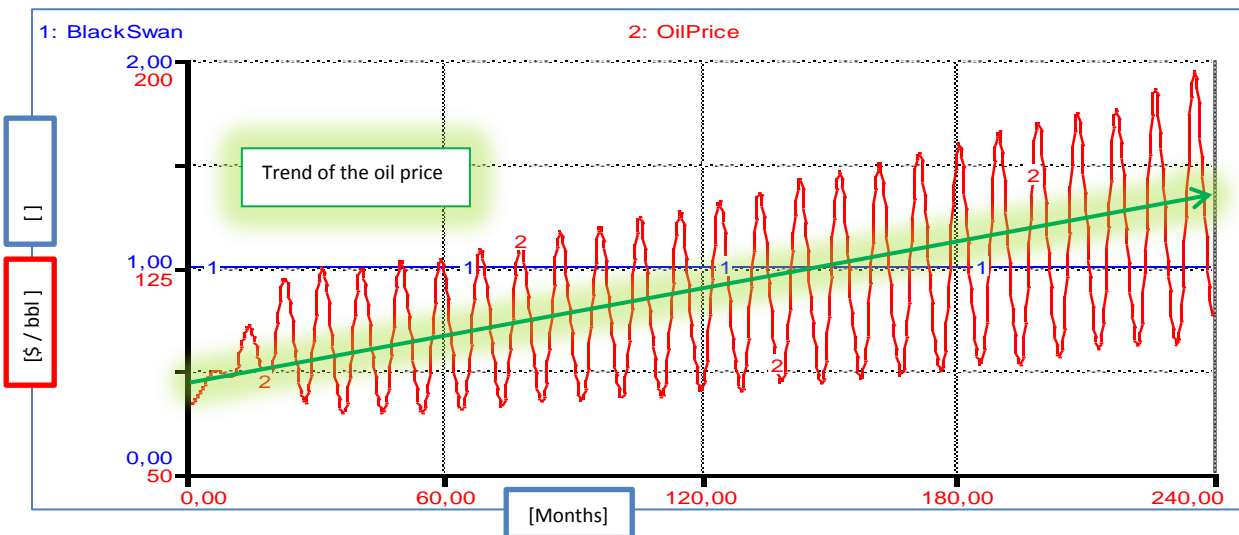


Figure 86 Sample.run showing OilPrice and a non operating BlackSwan

Comparing Figure 85 and Figure 86, it is possible to see how the model tries to stabilize itself after a shock. This is shown with trend arrows in both graphs (green arrows).

An important issue is to look at is the total occurrences of BlackSwan. In Figure 87 a frequency distribution is shown.

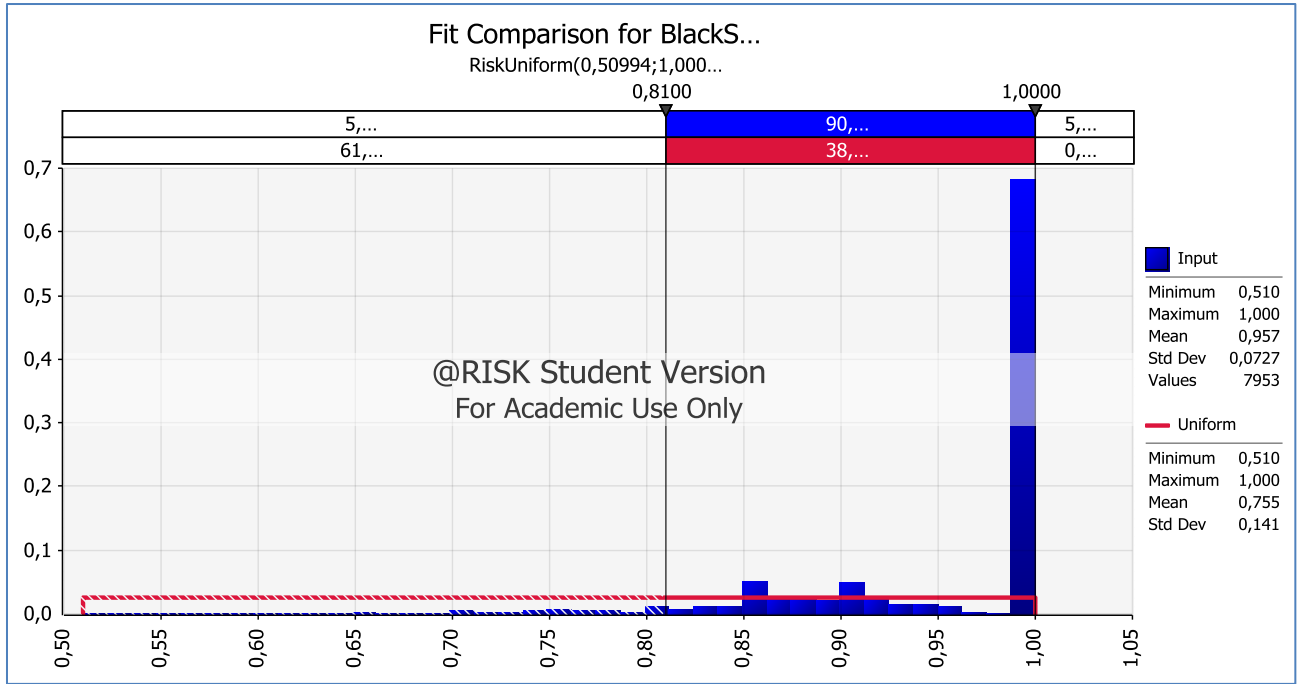


Figure 87 Relative Frequency distribution of BlackSwan events.

The graph in Figure 87 shows the relative frequency of BlackSwan events in the simulations. A value of 1 accounts for almost 70% of the outputs, meaning normal state. If there is a BlackSwan event happening, these usually takes on values between 0,85 and 0,925. Which means that the effect on subjected nodes will be in the range of 7,5% - 15% increase or decrease of normal state.

### 7.3.1.9 Summary points

Stella provides a valuable tool for the modelling, but it can also produce outputs from all nodes in the model. This can be used for validation for the assumptions made to inputs and to inspect how single input and outputs relates in the model. In Figure 88 and Figure 89 both the OilPrice and ExtractionIntensity are displayed at different growth rates. Fixing many of the key parameters in the model a display of the effect on yearly growth factors can be investigated. The growth rate is the main reason for increased demand and will therefore indirectly affect the long term trend in the oil price.

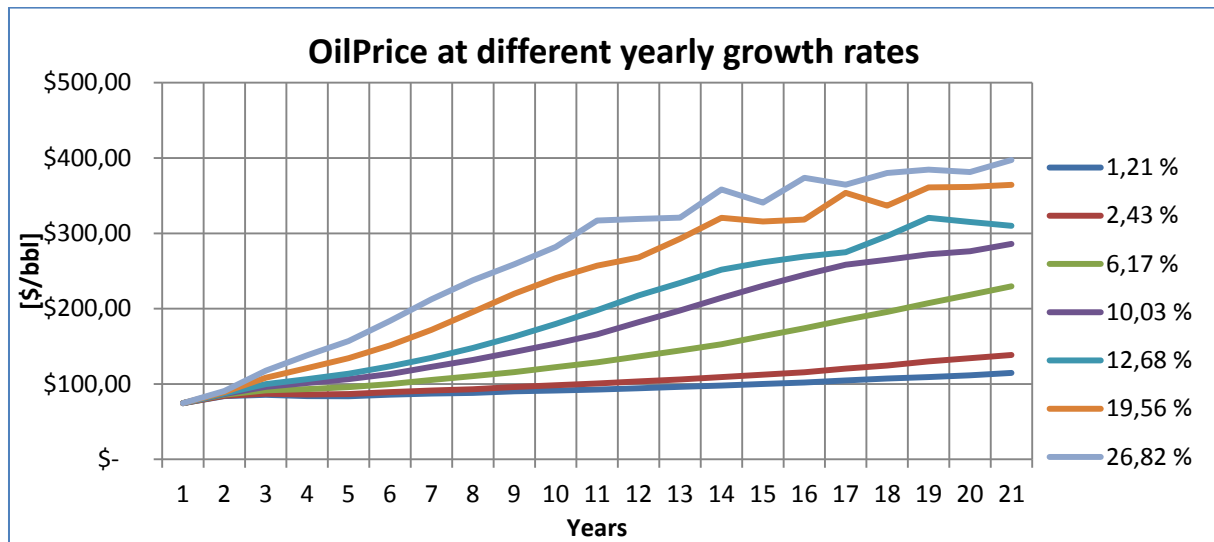


Figure 88 OilPrice at different yearly growth rates.

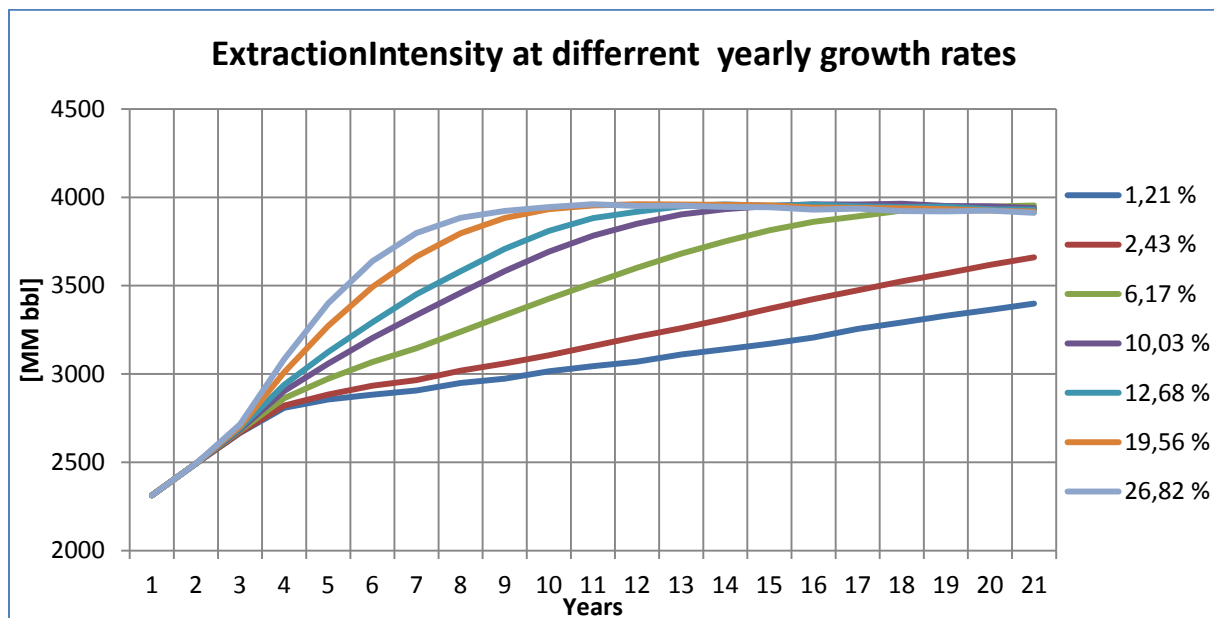


Figure 89 ExtractionIntensity at different yearly growth rates.

Figure 88 shows the effect on the oil price at different yearly growth rates in the demand for oil. As the demand factor grows, so does the price and also the volatility in the price paths. In Figure 89, a higher demand increases the extraction intensity, or production, at an earlier stage. Here it is possible to see a plateau for the larger growth rates and a possible decline, which can indicate peak oil in about 20 years at these extremely high growth rates.

The data which are frequently published and receives attention from the industry players are typically *Production*, *Consumption*, *Demand* and *Proven Reserves*. In Figure 89 these are all displayed in one graph. The outputs are from a single run.

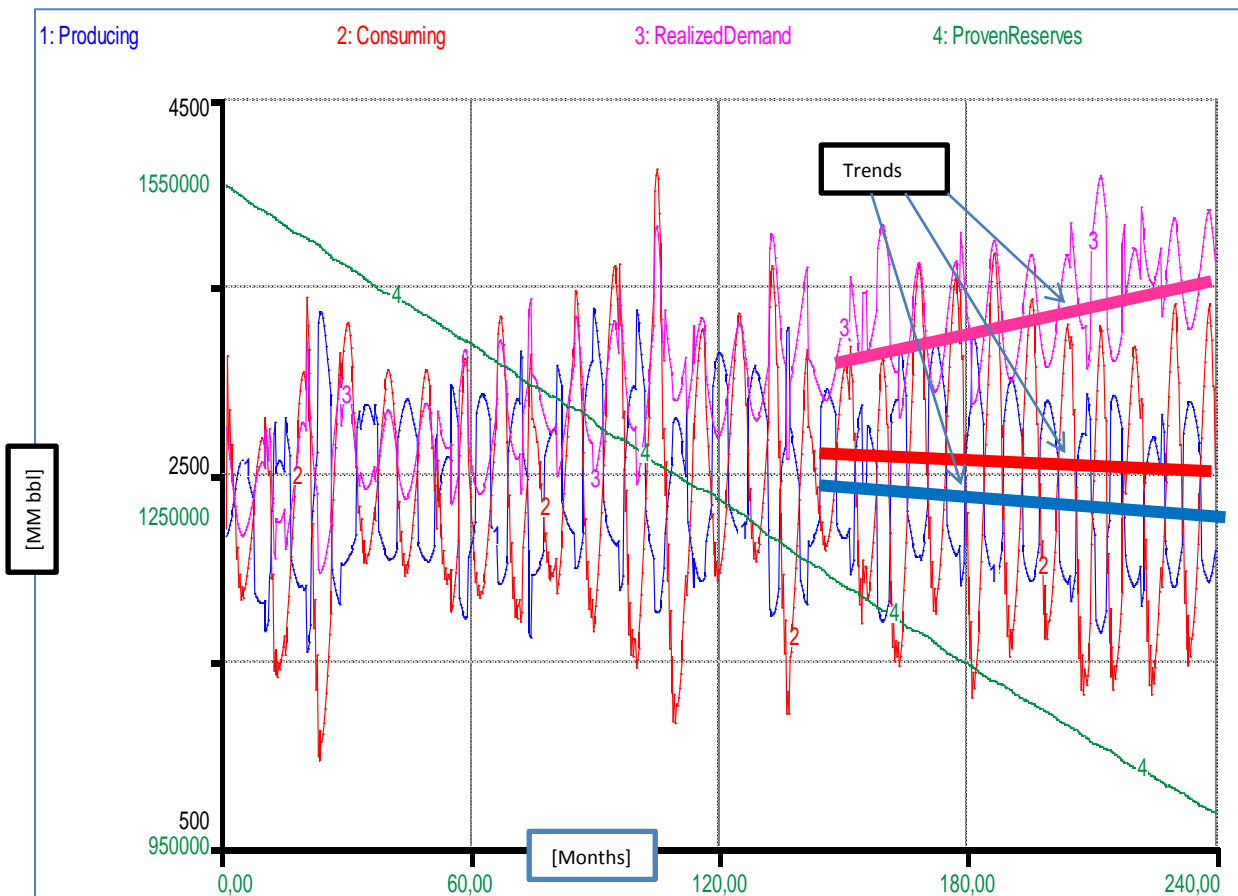


Figure 90 Sample-run showing Producing, Consuming, RealizedDemand and ProvenReserve. Y-axis [MM bbl], X-axis [months]

An interesting point in Figure 90 is the increasing trend in RealizedDemand especially from 140 months into the simulation. As the demand increases, Production and Consumption both have a decreasing trend. This can be related to a higher oil price which changes the efficiency level in the model. In this sample-run a total of 600.000 MM bbl of oil are produced from the initial proven reserves. This is equivalent to an average of:

30.000 MM bbl per year, 2.500 MM bbl per month and 83,33 MM bbl per day.

## 7.4 Price model impact in field economics

The price models were run at the exact same conditions in each field examples. The values from both production and economic variables were generated similarly for all four price models, such as the price models comparison are for the exact same conditions for each field. The price models were compared in two scenarios; one, with open parameters where production and economic inputs are variables. Second, with fixed parameters where all inputs except for the oil price is set at best estimate or most likely outcome. All project sheets technical and economic data can be found in Appendix L-N.

Firstly the difference between the two scenarios will be shown in each project with the use of two different metrics, NPV and IRR, and all the price models. The comparisons are done by looking at the standard deviation from the probability distribution of the two metrics. A summary of the findings will be presented afterwards.

Then, a presentation of each project and impact of the price models in the specific projects at *fixed* parameters. For each projects the best estimate of the production profile will be presented as this will provide insight into different results from the price models. For the graphic presentation, the values P10, P50 and P90 will be presented in a modified boxplot as seen in the example in Figure 91. Thereafter a general comparison of the models in the all three projects will be performed.

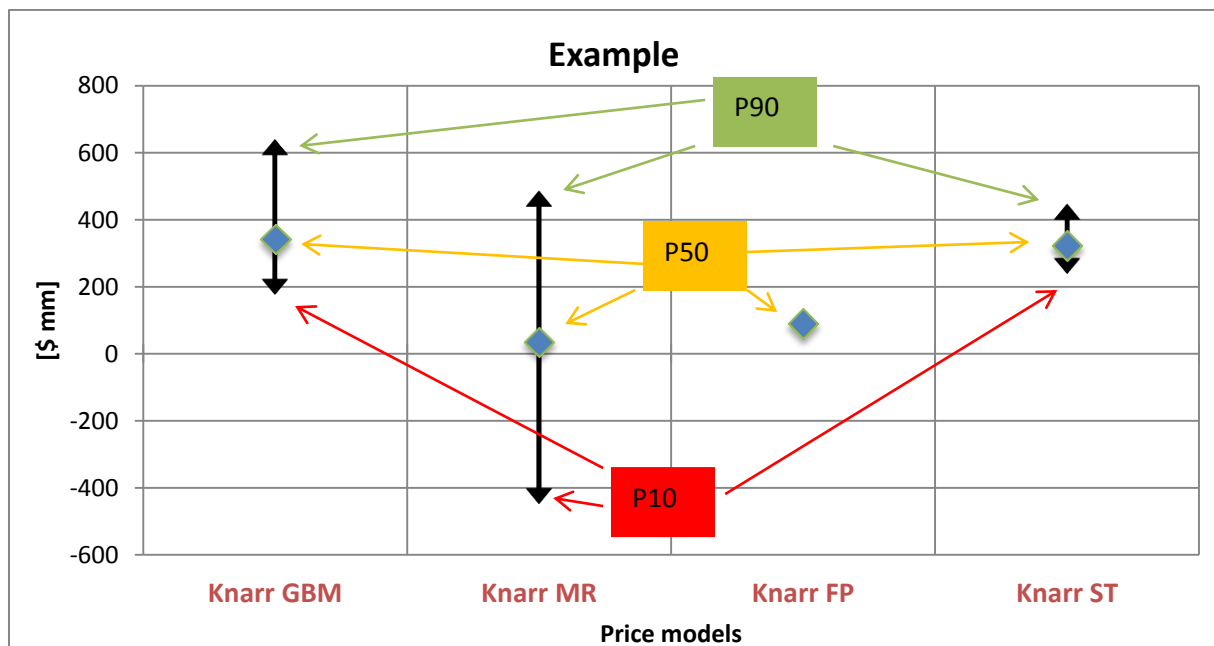


Figure 91 Example of display method for price model impact in field.



### 7.4.1 Scenario comparisons. Open and Fixed parameters

To compare the two different scenarios the simulations for both the fixed and open parameters were run with the exact same price paths. The total comparisons sheets can be reviewed in Appendix O-T. As an example the NPV calculation for Tawke field with respectively GBM and ST models is shown in Figure 92

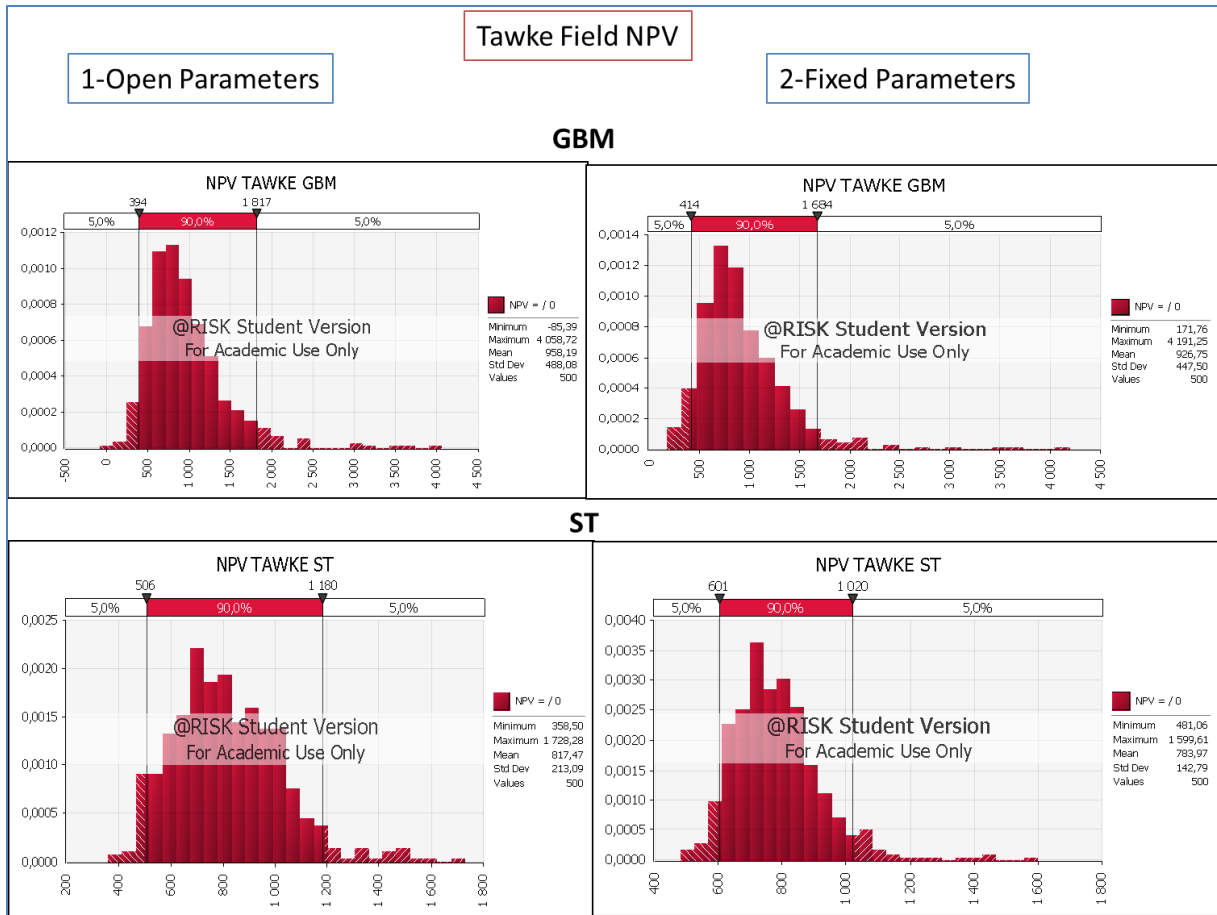


Figure 92 Tawke field, scenario comparison.

The main factor to focus on in this comparison will be the standard deviation. This factor will be able to indicate how much of the difference in value of the total project that can be related to the oil price. In Table 13, the difference in standard deviation from a project evaluation at open versus fixed parameters is listed for the different price models. A lower percentage will indicate that most of the difference in project value is reflected by the oil price. A high percentage will indicate that none of the difference in project value is caused by the oil price.

Scenario comparisons output		Scenario comparisons output	
%-change in std.dev with NPV		%-change in std.dev with IRR	
Price model	FP	Price model	FP
Knarr	-100 %	Knarr	-100 %
Tawke	-100 %	Tawke	-100 %
Tiber	100 %	Tiber	100 %
Price model	GBM	Price model	GBM
Knarr	5 %	Knarr	-9 %
Tawke	-8 %	Tawke	-33 %
Tiber	-14 %	Tiber	-14 %
Price model	MR	Price model	MR
Knarr	-7 %	Knarr	-8 %
Tawke	-14 %	Tawke	-20 %
Tiber	-13 %	Tiber	-13 %
Price model	ST	Price model	ST
Knarr	-29 %	Knarr	-46 %
Tawke	-33 %	Tawke	-64 %
Tiber	-49 %	Tiber	-49 %

Table 13 Change in Standard deviation from open to fixed parameter simulations

From Table 13 the increased or decreased relative difference in the standard deviation is displayed. It is shown for both the NPV and IRR metric. As expected the FP model does not reflect any of the uncertainty of a project value. Both MR and GBM have similar values for two of the projects, Knarr and Tiber, while there is some difference in the Tawke project. The ST model does not provide the amount of uncertainty as the two stochastic models, but it still accounts for at least 50% total project uncertainty for all projects and metrics, except for IRR at the Tawke project. As seen in Figure 93 , the part which the price models accounts for on a projects total uncertainty in value is substantial.

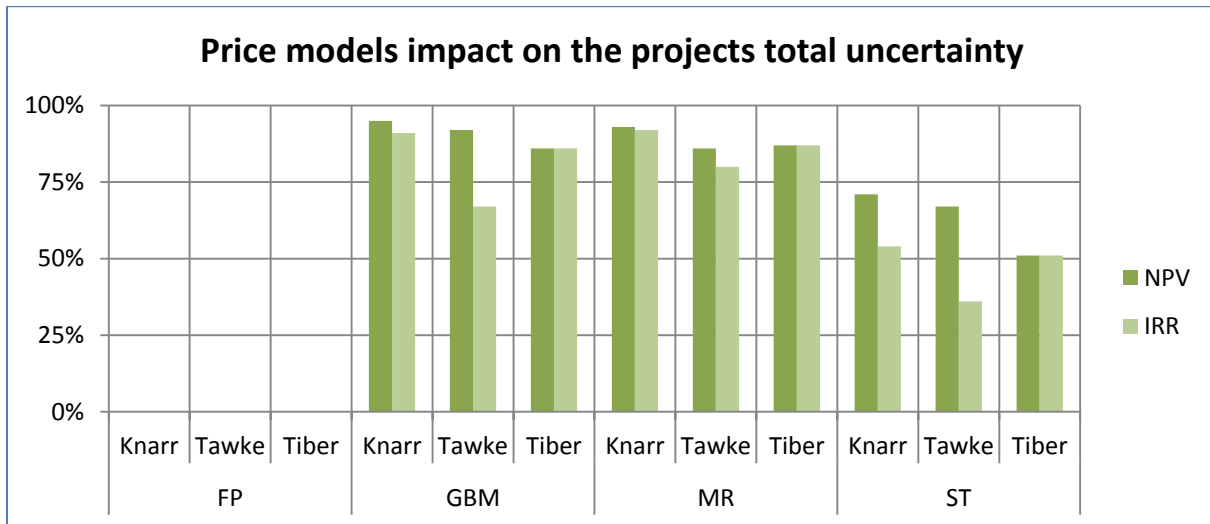


Figure 93 Part of price models impact on the projects total uncertainty

### 7.4.2 Knarr results

Below in Figure 94 is the best estimate of the production profile for Knarr.

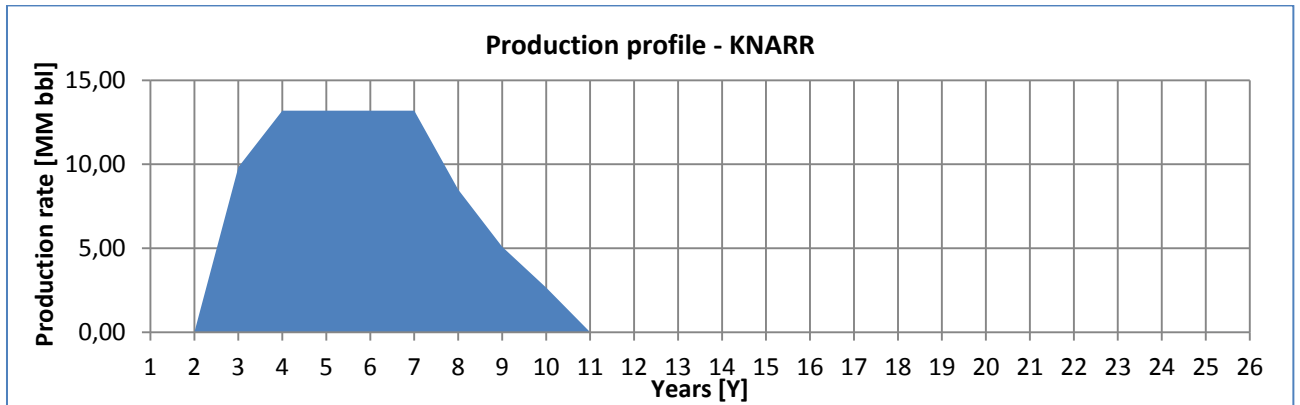


Figure 94 Production profile for Knarr, based on best estimate values.

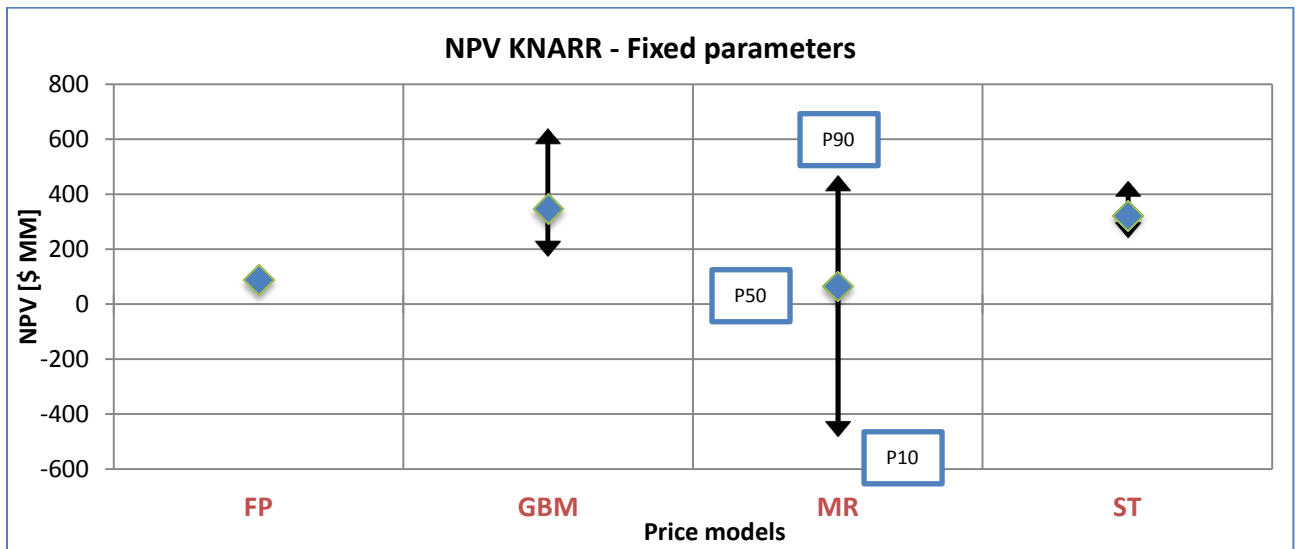


Figure 95 NPV Knarr results for all price models

In Figure 95 the NPV calculation for all four price models in Knarr is displayed. The blue marker indicates the P50 and the arrows indicate the P90 and P10 values. For Knarr the largest uncertainty is found with MR model. Focusing on the P50, the FP and MR models does not give a high value for this project. While the GBM and ST models rate it fairly similar. The GBM and ST models give the NPV substantially higher value than the FP and MR, almost 4 times more. However the uncertainty in the MR model encompasses both the P50 in GBM and the whole range from the ST model. The ST and GBM model provides a higher mean price path than the MR and FP at the time when plateau production is reached. This therefore yields a higher NPV value from these two models.

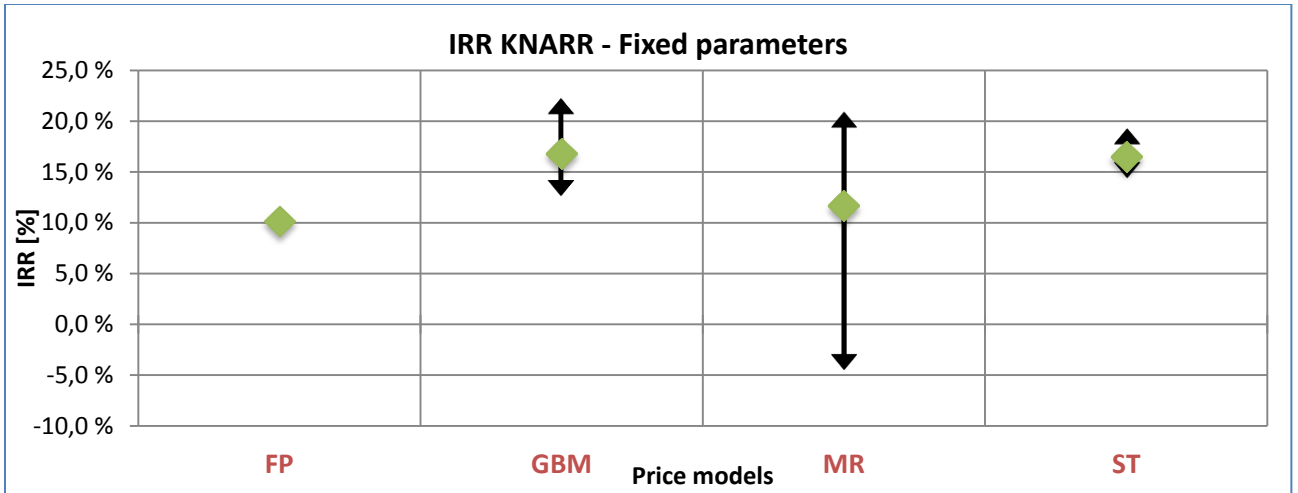


Figure 96 IRR Knarr results for all price models

By inspecting Figure 96 the same tendency as in Figure 95 can be seen. However the IRR calculation provides a better value for the project with FP and MR than the NPV calculation.

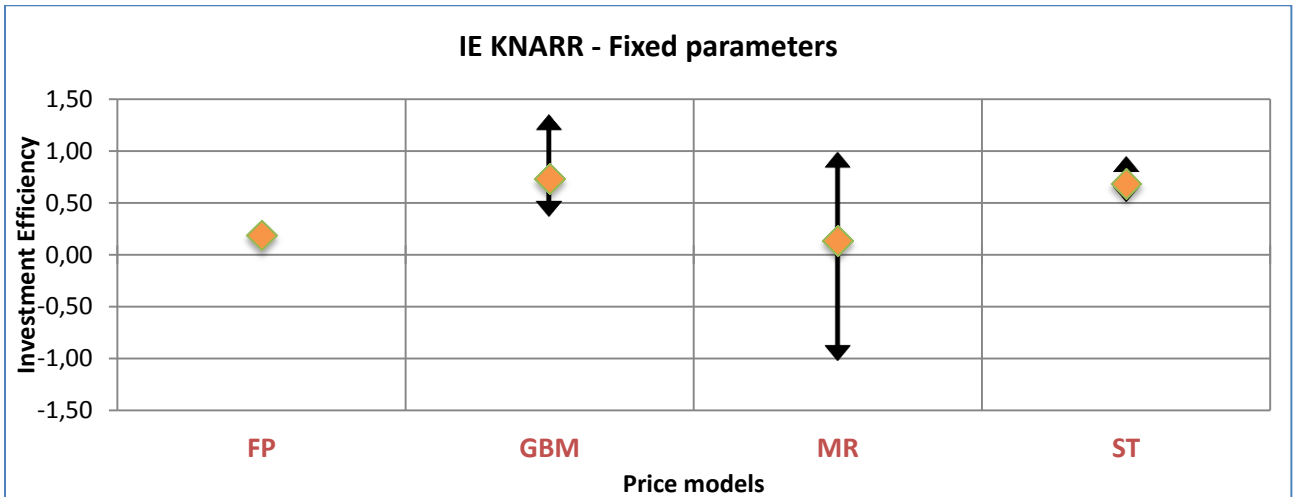


Figure 97 IE Knarr results for all price models

As the metric used in Figure 97 is based on the NPV, it shows more or less the same behaviour as Figure 95. The best estimate for Capex for Knarr used in the calculation is \$ 1.050.000.000. Knarr is a fairly capital intensive projects regarding its recoverable reserves, thus the general low investment efficiency.

### 7.4.3 Tawke results

Below in Figure 98 is the best estimate of the production profile in Tawke.

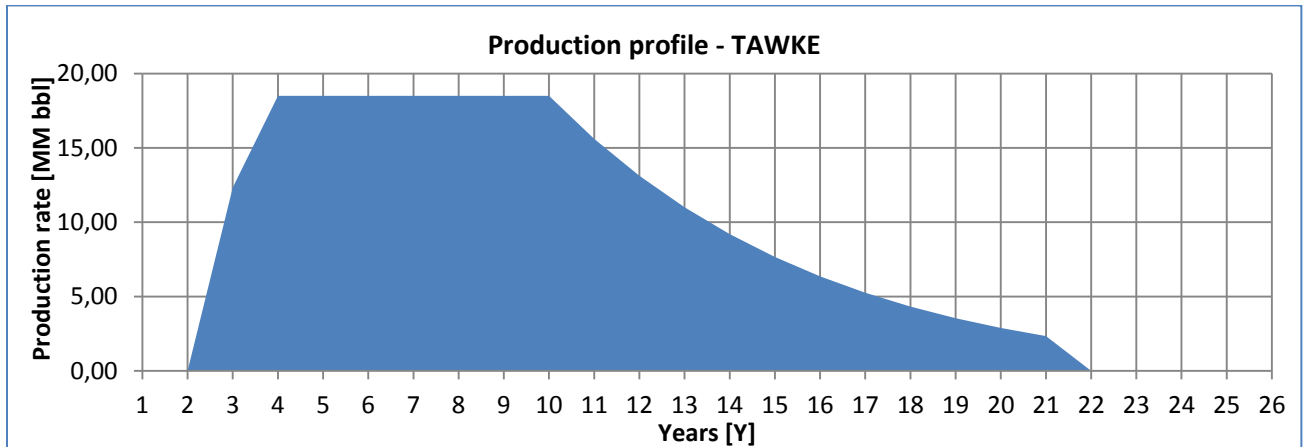


Figure 98 Production profile for Tawke, based on best estimate values.

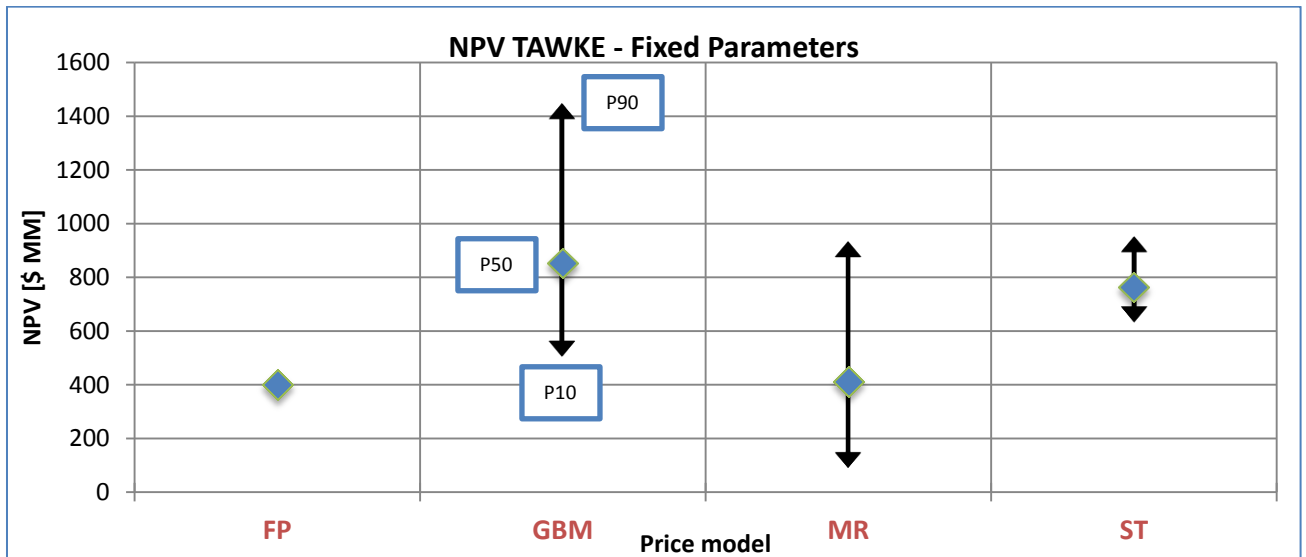


Figure 99 NPV Tawke results for all price models

For the Tawke NPV values seen in Figure 99, the same tendencies as before are seen. Both GBM and ST provide a higher value than MR and FP models. The Tawke field is reviewed to be a positive venture for all price models. This can be traced to its onshore location, making it a less expensive field to operate. It has also a long plateau rate and large reserves. From the production profile in Figure 98, the lifetime of the field is estimated to be 22 years. This means that the increased value in ST and both the increased value and range of the GBM reach their full potential at this field. The GBM and ST models P50-values are at around two times that of the FP and MR models. An interesting point is that both the GBM and MR show approximately the same range between their P10 and P90 estimates, respectively \$ 945 MM and \$ 832 MM.

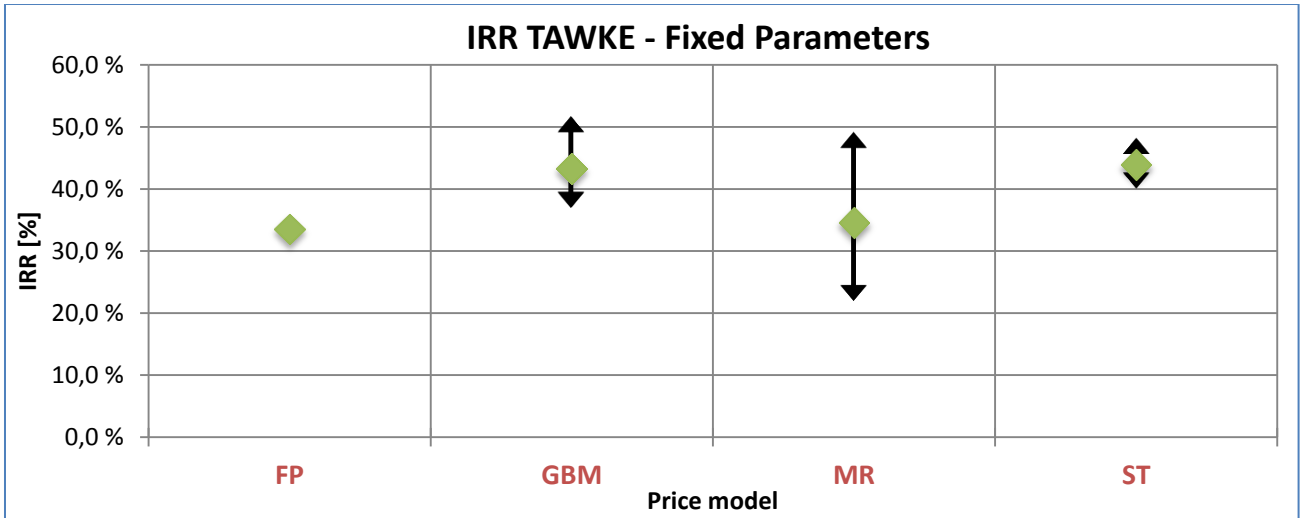


Figure 100 IRR Tawke results for all price models

Figure 100 shows that for all price models the IRR value is very large. The differences between the models are the same as in Figure 99, but not to the same extent.

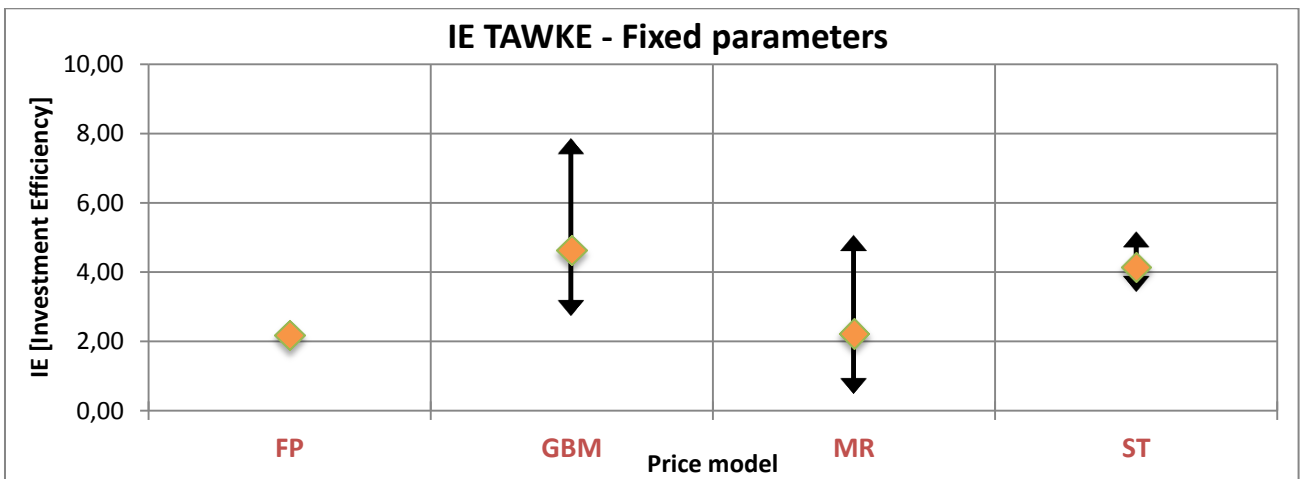


Figure 101 IE Tawke results for all price models

The investment efficiency metric gives a high value in all the price models. This is due to the high NPV experienced and the low Capex, which is estimated at \$ 335 MM, for Tawke. In Figure 101, the same uncertainty range from the GBM and MR model is experienced here as in the NPV metric.

### 7.4.4 Tiber results

Below in Figure 102, is the best estimate for the production profile in Tiber.

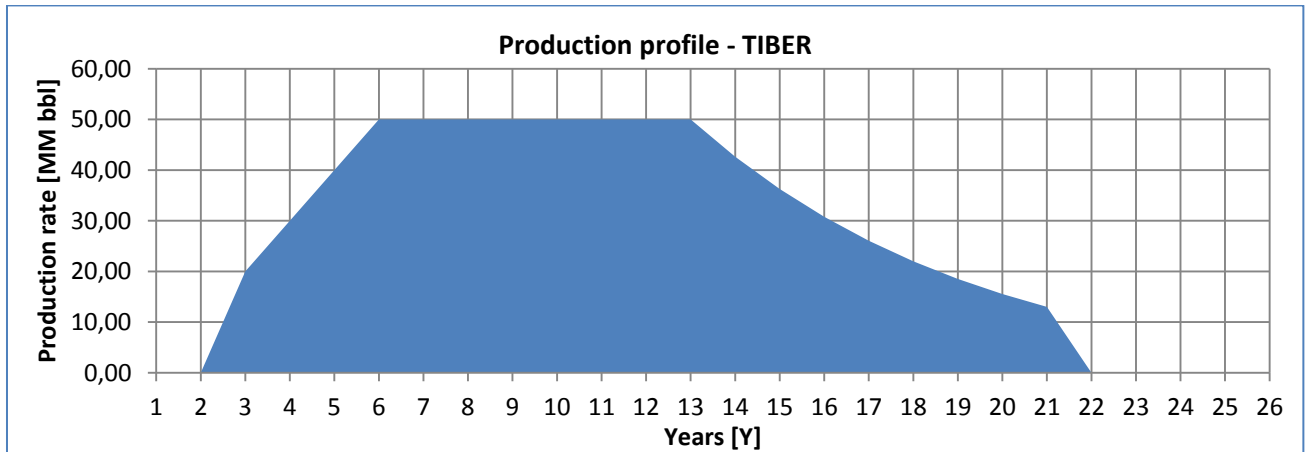


Figure 102 Production profile for Tiber, based on best estimate values

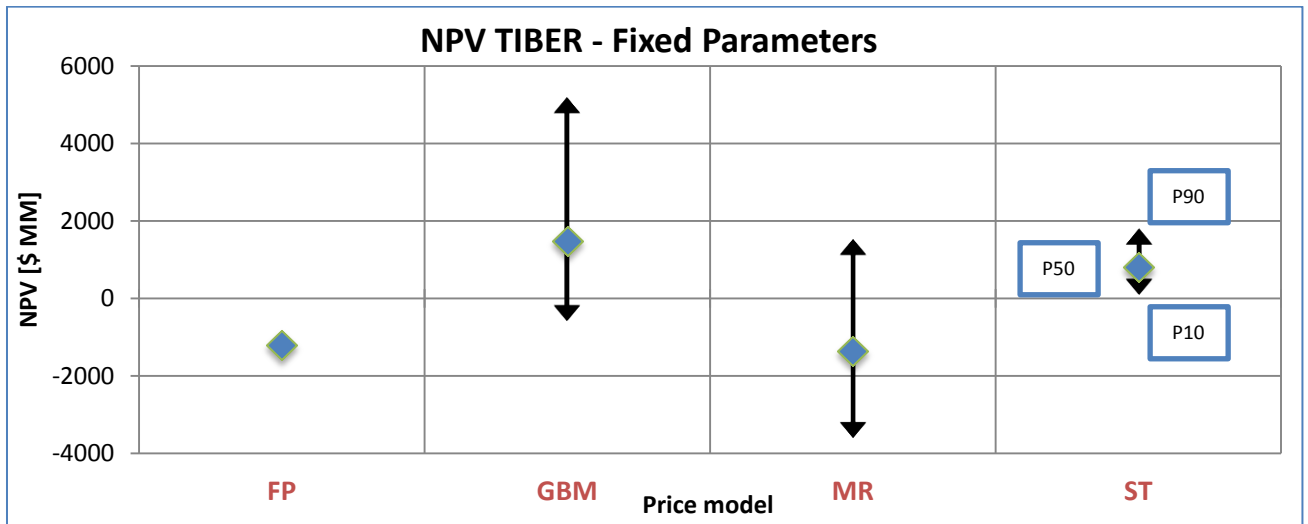


Figure 103 NPV Tiber results for all price models

In Figure 103 the same valuation from the models are seen as in the previous fields. The long lifetime as seen in Figure 102 favours the increased price paths of GBM and ST. Because of the high capital intensity of Tiber in the early years and long time to reach full production the NPV does not support the project at low price paths. The uncertainty range seen in GBM and MR in previous examples show the same behaviour here, however because of the increased volume from Tiber, the upside of GBM model becomes larger. An interesting point is the FP and MR models negative P50-value for this field which is considered to be a giant oil field.

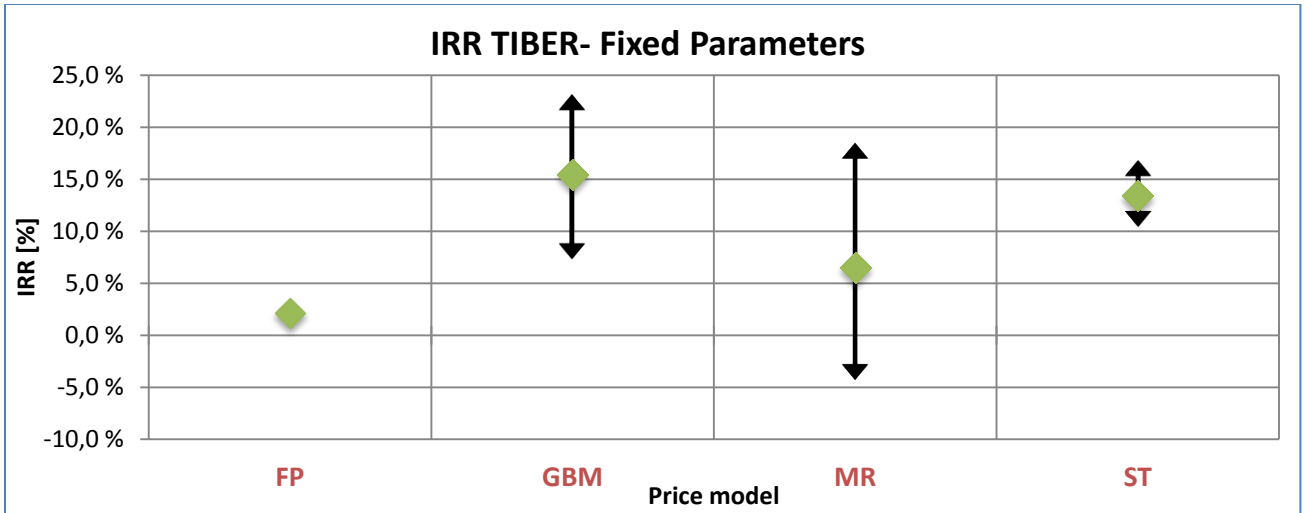


Figure 104 IRR Tiber results for all price models

Figure 104 shows the same for Tiber as it did for the Knarr example. Although the NPV is not favourable for all models, the IRR is. However the same tendency between models is seen here; High values from GBM and ST, lower from FP and MR.

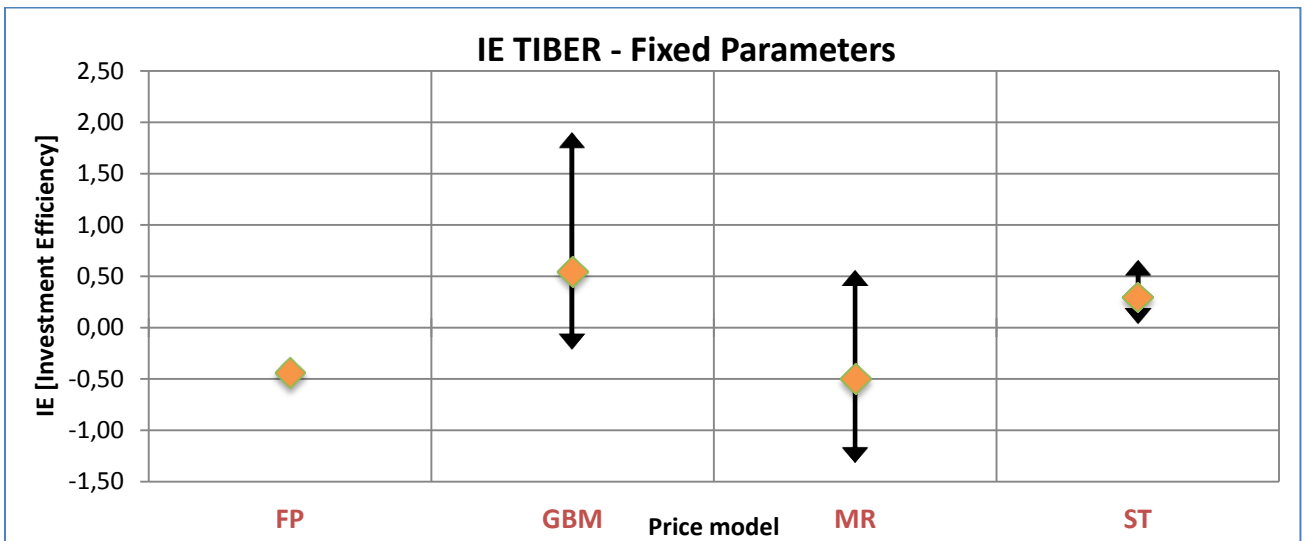


Figure 105 IE Tiber results for all price models

The investment efficiency metric does not give clear incentives to invest in this project. Mainly due to its large Capex and long start up time (causing a low NPV). The best estimate of the Capex is \$ 4.125 MM. In Figure 105, the same large uncertainty range provided by the MR and GB, is seen here, as in Tawke. The 20 year lifetime of the field captures the full uncertainty in the price projections.



### 7.4.5 Hurdle rate

By extracting the mean price path for all the price models (Figure 106) and comparing them to the given hurdle rates obtained from the Pareto report, there are some interesting points to be made.

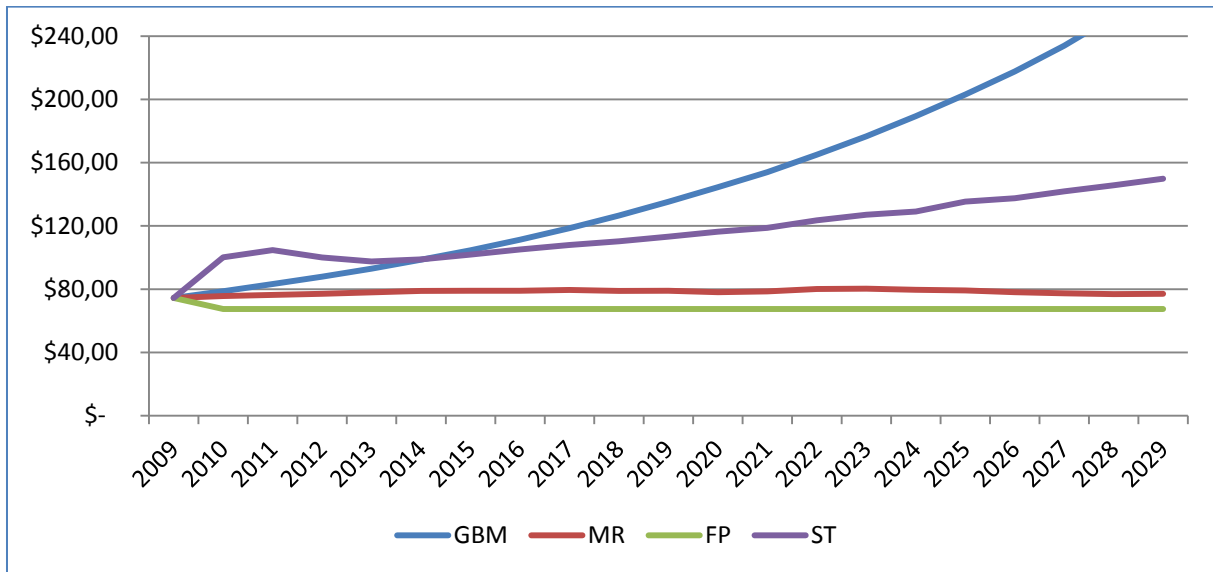


Figure 106 Mean price paths for all price models for 20 years.

Inserting the price paths from Figure 106 into the hurdle rate diagram, a comparison can be seen in Figure 107. The most expensive type of projects (Deep Water, Heavy oil, Arctic oil and oil shales) will need maturity for some years at a high price levels before an investment can be done with an FP and MR as these models does not support these investment at the price level in the model, while ST and GBM gives better incentives to invest in high cost projects at an earlier stage.

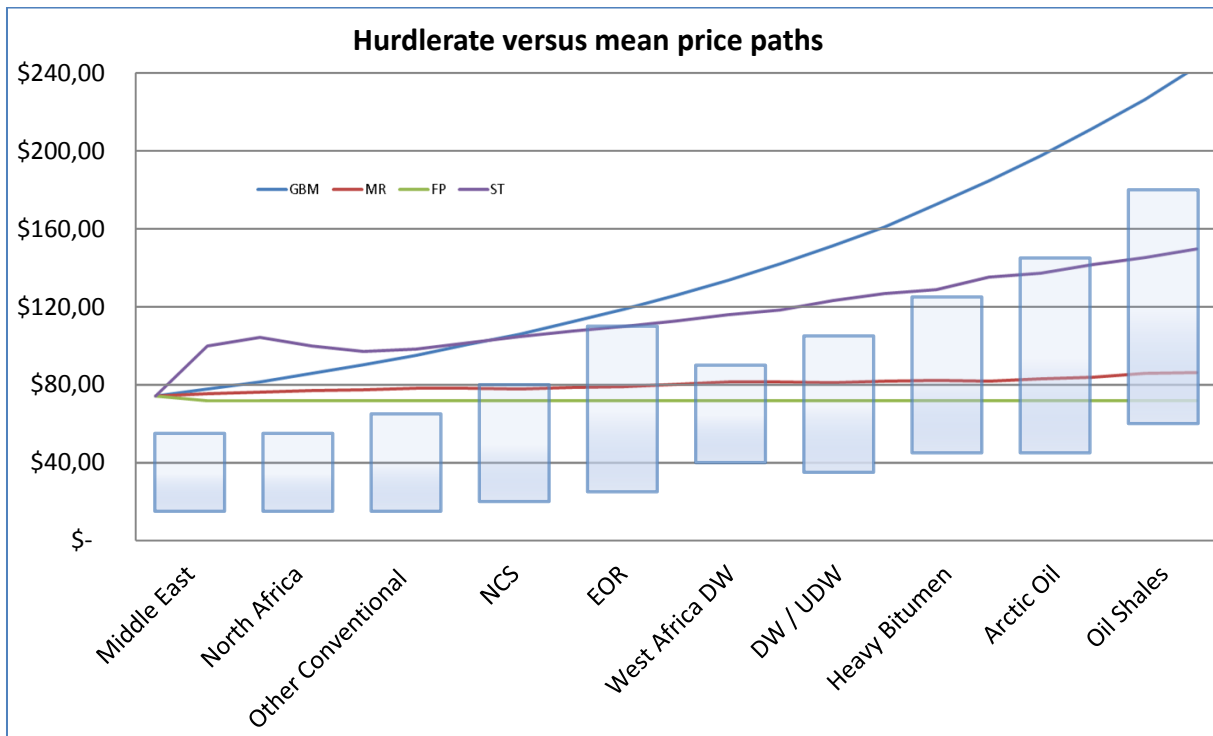


Figure 107 Hurdle rates versus mean price paths

### 7.5 Project comparisons by price models

For all comparison of the projects by the different metrics, the mean value from the simulations is used.

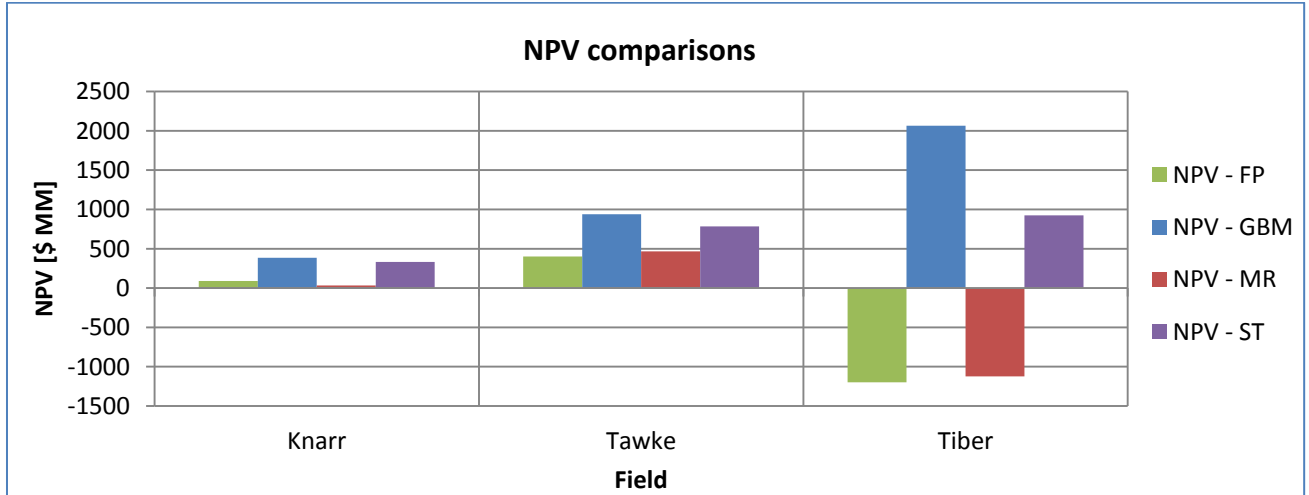


Figure 108 NPV comparison between selected fields

The NPV calculations in Figure 108 show some different results for the different price models. The GBM and ST favours all the projects and in the same order. While the MR and FP models do not favour all projects, they both rank similarly. The largest inconsistency between the models is in the Tiber project. The FP and MR models fail to recognize the potential in the field by using NPV.

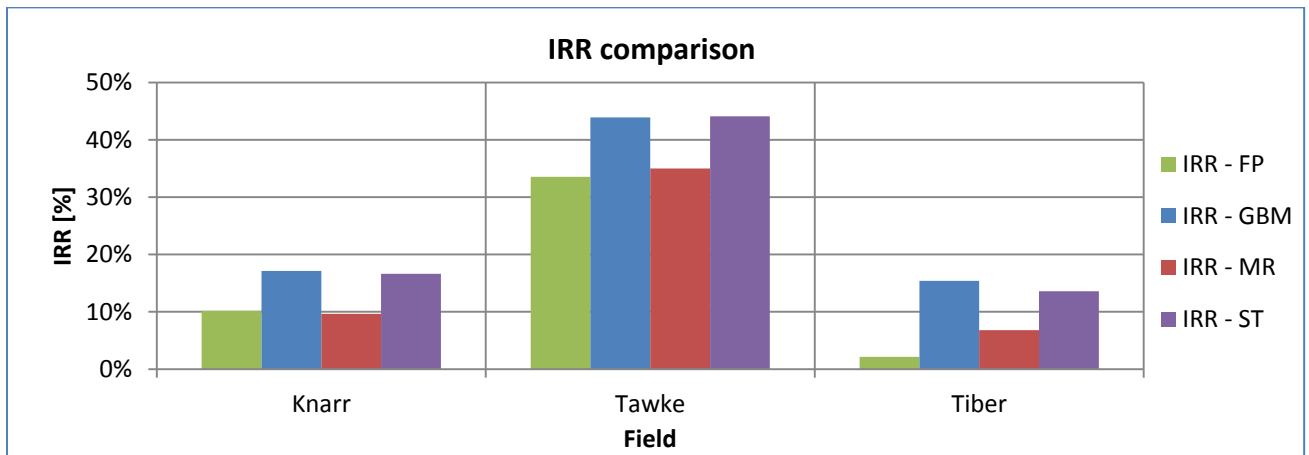


Figure 109 IRR comparison between selected fields

For the IRR comparisons in Figure 109, all price models favour the Tawke field. This is caused by the IRR being biased towards projects with a low initial investment and early cash-flow. The ranking in total for IRR is identical for all price models in IRR.

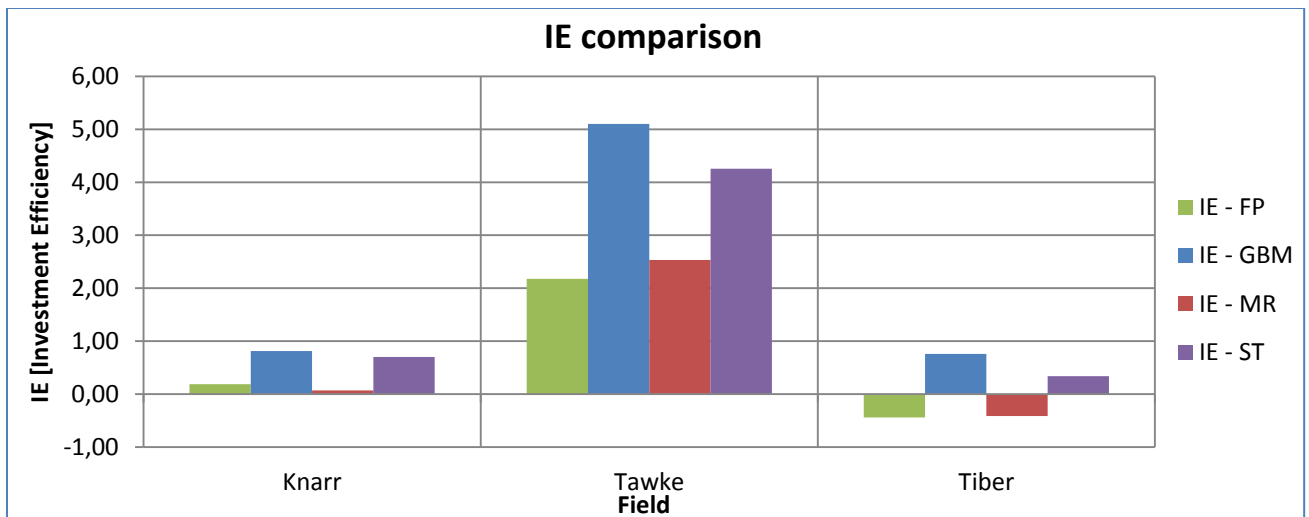


Figure 110 IE comparison between selected fields

As displayed in Figure 110 The investment efficiency metric shows to be favourable towards the Tawke field. The IE ranks the projects in the same manner as the IRR. The Tiber field is also here not recognized by the FP and MR. Also for the Knarr field, FP and MR do not give a clear incentive to invest.

## 8 Main results & discussion

*“The sooner we admit how poor we are at predicting the future, the better we will become at (incrementally) predicting the future”*

*Stephen J. Dubner*

Firstly the models will be discussed; the ST-model will be discussed particularly regarding its structure and inputs. Then, impacts of the price models on projects, before a general summary

### 8.1 Model discussions

The four price models show all different behaviour and trends. All models are commented separately in the following sections, but presented first are some key points about each model:

- Fixed Price:
  - Single price reflects no uncertainty.
  - Will favour low cost projects, with high volumes.
- Geometric Brownian Motion:
  - Exponential growth in uncertainty and mean price.
  - Will favour projects with high production at a later stage
- Mean Reversion
  - Long term fixed price and large uncertainty around the mean.
  - Will reflect wide uncertainty in every stage of a project production time
- System thinking
  - Increasing trend in the price, low uncertainty range
  - Able to favour project with a short production time

Note that all models are based on the WTI oil price and simulated from December 2009. The price is simulated in 2009 US dollars. This means future values of US dollars or exchange factors are not accounted for in respect to other currencies or commodities and the future price paths which are simulated here should be calculated as having a US dollar value as of December 2009.

#### 8.1.1 Fixed price model

The FP model may be the simplest, but as the other models this model is also not independent. Setting a fixed price or a planning price for a project is based on assumptions on market development and historical prices. If the last 2 years shows an average price of \$50 per barrel, a price would probably not be set at \$100 per barrel. The FP model shows therefore a dependency on previous price level and then adjusted for future expectations for the price level. However, the FP fails to display any volatility which is one of the main characteristic of the oil price. Thus providing an upside and downside to a project is not an attribute of this model. By fixing all other variable in a projects production and economic input, a FP model will only provide a single estimate. This estimate will then not reflect any exogenous uncertainty; rather just confirm a projects inherent uncertainty from the production and economic variables. The FP model when used, are also often chosen to be conservative. This will dampen the interest for high cost projects and ventures which threaten the economic comfort zone. As a consequence FP-models will limit the investment grade.

### 8.1.2 Geometric Brownian Motion model

The GBM model provides the widest range of possible price paths of all the price models investigated here. It behaves as predicted according to the parameters used. The parameter estimation gave a large uncertainty, especially in the drift factor, creating almost exponential price paths. As there is no constraint in the GBM in order to revert to a long time trend, it continues in its path, only dependent on its last value. There is however a price floor experienced in the simulations as the P10 is almost constant at the start value, just transiting slowly towards \$100. As a consequence GBM fails to identify lower price levels than the initial value of the simulation. The exponential price path experienced in the P90, P50 and mean values, surely gives the model a range of uncertainty, but the question arises as if the whole range should be shifted downwards and compressed severely. The behaviour or price level seen in the GBM-model may not be unlikely, but it fails to embrace the characteristics evident in the oil price, such as returning to a long term trend. And the huge range of uncertainty at the latter part of the projection may show a too large upside for a project.

### 8.1.3 Mean Reversion

The MR-model embraces almost all of the characteristics of the oil price and behaves as anticipated in the simulations; reverting to a long term mean. It starts to fluctuate to its full range early in the simulations and reaches its full uncertainty range after 7-8 years. The model carries a wide range of uncertainty, so wide that the impact that arises from its use might not be desired. The uncertainty range is about \$100 bbl from 7-8 years out and grows slightly to the end of the projections. The MR model is the only model that recognizes a price level underneath \$70 per bbl. But it can be debated if this low level is still valid today, when assessing the industry and markets. However this model may give a better incentive to employ by companies, as it gives a mean long term price which may replicate a typical planning price and at the same time can add uncertainty.

### 8.1.4 System Thinking

The ST-model shows a different total behaviour than the other models, but it can be seen as a combination of both the MR and GBM with a lower uncertainty range. Even though at a monthly basis the ST model is very volatile, it does not reflect it at the average yearly price path used in the simulations. The large peaks experienced in the monthly simulations are often balanced out with dips right after. Consequently an averaging of prices over a year, consumes these fluctuations. This can be seen as hedging, as a company will never be fully exposed to the larger peaks and dips in the market when selling oil. Thus this becomes a realistic effect. But it might not reflect the true uncertainty in the price model. Averaging has its flaws; just as men have drowned crossing a stream with an average depth of six inches. As for uncertainty, the ST-model shows a range of \$50 between the P10 and P90 from 4 years out and up to a range of \$75 from 14 years out. As the MR model the ST revert around a mean trend, but the trend in the ST-model have more similarities to the trend in GBM-model. In summary the ST-model proves to include the characteristics found in the oil price. But it might be flawed to not reflect a low price level. Since the ST model is modelled by months, a comparison of the monthly output from December 2009 versus the real WTI price is done. This is shown in Figure 111.

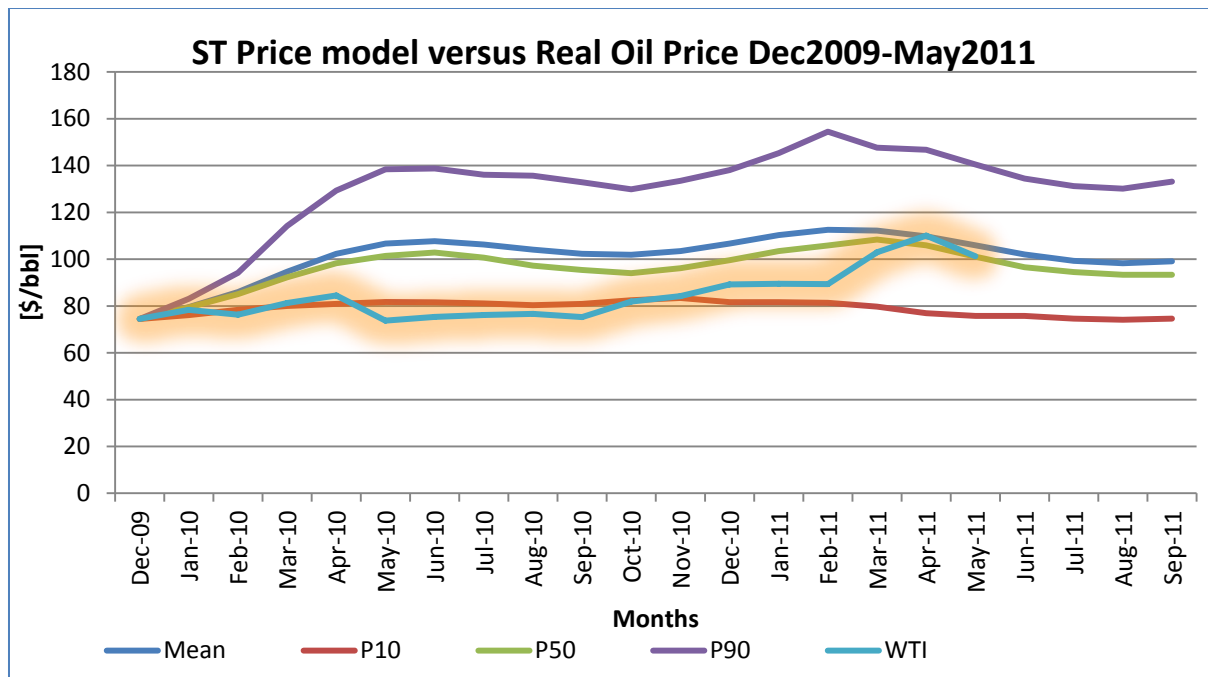


Figure 111 Graph of ST price model versus the real oil price, from December 2009 until May 2011, including projections.

The real price is not encompassed by the model in the first year; it even drops below the P10. A remarkable thing is however that the trajectories have some similar movement pattern. This gives a slight evidence for a true behaviour of the price model, but the uncertainty does not embrace the lower values to fully grasp the real price level.

#### 8.1.4.1 System Thinking, review of model outputs

Regarding the modelling, there are some important factors to review. The ST-model used in this thesis is extensive and complex; it will not be advisable to detail into further steps as it is volatile at this stage already. By past experiences, regional modelling has been futile, thus this model is kept for a global environment. The model requires data for many important inputs and some of the data is under considerable amount of debate. Much of the debated themes are about peak oil, black swans, production and inventory numbers, growth in China and India, exchange rates versus the oil price and climate incentives. All of which are difficult to include in a model, but have to some degree been addressed in the model and will be discussed here.

How much oil is left? By using the best data available at the time for estimated undiscovered conventional oil, and the anticipated discovery rate set by ASPO, there should be no reason to panic. There is large uncertainty to the value of undiscovered reserves in the model; 400 billion bbl and 2.200 billion bbl. This however, does not constrain the model in any way as the discovery in the model generates cumulative values around 50.000 to 100.000 MM bbl per simulation. These values coincide with the future discovery projections of ASPO. Giving truth to these data, it suggests that there still is an abundance of conventional oil to explore for and that there should be volumetric incentives to do so. By also looking at the proven reserves over the course of a simulation shows a relatively low but still good comparison to recent data: In a sample-run a total of 600.000 MM bbl of oil are produced from the initial proven reserves. This is equivalent to an average of:

*30.000 MM bbl per year, 2.500 MM bbl per month and 83,33 MM bbl per day.*

As a comparison the average production in 2010 worldwide was:

*29.556 MM bbl per year, 2.463 MM bbl per month and 82,1 MM bbl per day<sup>42</sup>.*

However a curious point from the updated BP Statistical Review(2011) is the increase of proven reserves from 2009 till 2010 by a number 10.000 MM bbl. Reasons for this may be an update of reserves estimates of larger fields not yet started production, increased recovery factors on mature fields or new substantial discoveries. But the main point is that there has been an increase in the proven reserves. As seen in the results in section 7, the model does not prove any peak oil in the near time at reasonable growth rates in demand.

Efficiency levels seem to be a weak point in the model. Even though it shows correct behaviour in the elasticity towards the oil price, there is no solid base or research for it to make proper decisions on. It also fails to take include irreversibility of energy efficiency into account, such as changing the infrastructure of heating from heating oil to natural gas and from petrol engine to hybrids are not reflected in this model.

WTI price behaviour from inventory levels show good historic correlations, but will it continue to do so in the future? A weak point from the model is the assumption made that all global consumption should be stored for approximately 90 days. This generally does only apply for OECD countries, although it can be assumed that other consuming countries keep a certain amount of stock of their own. There is tendencies in the market of a growing gap between the US and Europe/Asia. China and India are now respectively the second and fourth largest consumer of oil in the world (BP Stat. Review, 2011). In May 2011 Russia surpassed Saudi Arabia as the largest producer of petroleum (EIA). Russia has for longer periods asked to settle their sales in different currency than USD, as the dollar is not favourable to them. As the largest producer in the world, they have more power to do so. All of these factors, may obsolete the price structure in the model for the WTI oil price. As for the futures market, this could have a serious impact on oil the oil trade.

The system thinking approach would nonetheless give a possibility to try and implement most of these changes.

## 8.2 Project

Addressing the impact the different price models contribute to a project, can be done with expecting the table in section 7.1. Here the amount of the total uncertainty related to the price models was investigated and the results were conclusive.

From the table and by its nature it is clear that a fixed price model does not contribute to any variation of the project value. The GBM and MR models do not provide any significantly difference between themselves, but both models contribute highly to the variation of the project value. This can be related to the nature of the price paths. From the results in section 7.1 both GBM and MR have larger price variations during simulation, where GBM has an increasing large span and MR a large variation over its long term price. A large variation in price will affect project economics greatly. For the ST model the variation in project value is not as large as MR or GBM, but for almost all of the metrics and fields it still accounts for over 50% of the difference in total project value.

This gives evidence to the great impact the price models have on the uncertainty in a project. All but the FP model contribute to well over 50% of the uncertainty of a projects value

The nature of the price paths and their uncertainty makes them favour different projects.

It is clear that the FP- model favour project with low unit cost and high volumes. For almost all the metrics, the FP-model was conclusive to invest in one field, Tawke.

GBM, by its nature will favour projects with a long production time and a sizeable volume at tail production. GBM came to be conclusive to invest in all projects, but showed high values in the metric for especially Tawke and Tiber. It also shows a huge upside in the projects and in comparisons to the other models.

MR, shows the widest range of possible values for all of the projects in all the metrics. The MR-model yielded negative mean NPV-values for both Knarr and Tiber. But with its extensive range of uncertainty it also managed to show an upside to both.

The ST model shows a positive value towards all three projects. Compared to the GBM and MR model it does not offer the same uncertainty in the results which may prove to be the biggest drawback of the model.

### **8.3 Impact by price models**

The different behaviour of the price models will, by using them in a project, give incentives for different production schemes according to the expected range of a price level. The ST model and especially the GBM model will favour a high production volume at the later stages of a production. The MR and FP model are no neutral in this respect. By example, having a conservative planning price and in the event of a price above the planning price; it may trigger full production and selling at a price which is relatively high according to the planning price. This might however be relatively low, looking at an expected future price range from a more realistic price model. Although there is a profit above what was initially expected, a large upside has been foregone by a short term decisions. Having a realistic price model with an uncertainty range will also give possibility to further develop a field and employ IOR/EOR methods. Inspecting different hurdle rates with a realistic model will most probable produce more upside potential and trigger more investments into new areas and also mature areas making smaller fields economical viable projects. The choice of a more realistic model or a model with uncertainty may realize better utilizations of field as to change abandonment options. A price path similar to the ST-model would give clear incentives to prolong the life of a field, either by choking production or implement measures to further extract resources.



## 9 Conclusions

*“A holistic view brings realistic action”*

*Dalai Lama*

In this thesis a comparisons of four different price models were undertaken to investigate their impact in project economics. Particular effort was taken to refine a price model based on system thinking which has not yet been popularized.

It is clear through the results that uncertainty in a price model brings better basis for decision making in a project as it can display upside or downside to a project at different price scenarios. The selected price models, all but one, contributes to over 50% of the total uncertainty in a projects value.

A price model used in project economics should therefore have uncertainty associated to it in order to reflect the possible values different price scenarios could impose in a project.

Two of the models include the key characteristics which are inherent in the real oil price and are chosen as recommended models from this thesis work; The Mean Reversion (MR) model and the System Thinking (ST) approach. The Mean Reversion model used here offers a larger uncertainty range, but fails to embrace an increasing trend in the price. The System Thinking Model shows an increasing trend and has a reasonable uncertainty range; however it fails to embrace lower price levels.

Their impact in projects shows however a large difference in the employed metrics. Both models bring the same characteristic, but very different results.

The MR model has the largest uncertainty range, but in the present world and for future predictions reviewed in this thesis it would seem highly unlikely to experience a price level in the lower uncertainty ranges of the MR model. More likely, is an increasing trend in the price. It is therefore recommended to use the MR-model in a more holistic manner as to not solely base the model on historic data, but take into consideration future expectations and trends.

The System Thinking approach will through its nature seem to be the most realistic model, but the use of this model is recommended under further refinement of its properties. It may show the true characteristic of the oil price, but the low level of uncertainty relative to the MR-model might limit its ability to truly reflect especially the downside to a project. However the ST-model is more adaptable to model a dynamic petroleum industry and as it is based on both historic data and present interdependencies and influence patterns, it has the capability to adjust for a changing world.

To conclude; the uncertainty in price models are highly influential to the valuation of projects. It is therefore vital to implement a realistic price model with uncertainty when assessing projects. The choice of model should be approached with respect to historic data and attuned for present and future outlook.

## 10 Nomenclature

BBL	Barrel
CAPEX	Capital expenditure
CCF	Capital cash flow
D	Depreciation
DCF	Discounted cash flow
DR	Discount rate
DW	Deep water
EIA	US Energy Information Administration
EOR	Enhanced oil recovery
E&P	Exploration and Production
FC	Fixed cost
FP	Fixed Price
GBM	Geometric Brownian Motion
GOM	Gulf of Mexico
GR	Gross revenue
GRR	Gross revenue return
IE	Investment efficiency
IEA	International Energy Association
IOR	Increased Oil Recovery
IRR	Internal rate of return
MR	Mean Reversion
MM	Million
MM BBL	Million barrels
NCF	Net cash flow
NCS	Norwegian continental shelf
NGL	Natural gas liquids
NPV	Net present value
NYMEX	New York Mercantile Exchange
OECD	Organization of Economic Co-operation and Development
OOIP	Original oil in place
OPEC	Organization of Petroleum Exporting Countries
OPEX	Operating expenses
P	Price
P10	Covers 10% of outcomes
P50	Covers 50% of outcomes
P90	Covers 90% of outcomes
Q	Production
SPR	Strategic Petroleum Reserve
ST	System Thinking
TI	Taxable income
TR	Tax rate
UDW	Ultra deep water
VC	Variable cost
WTI	West Texas Intermediate

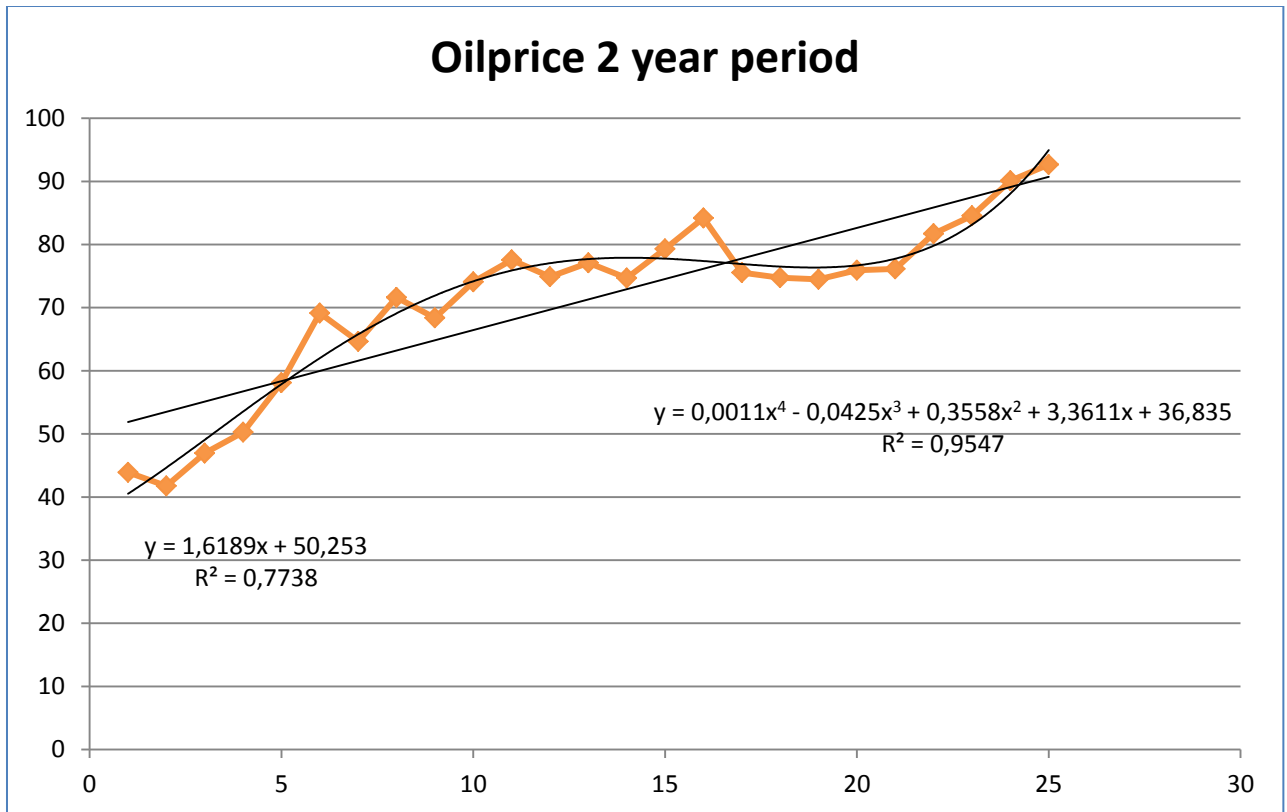
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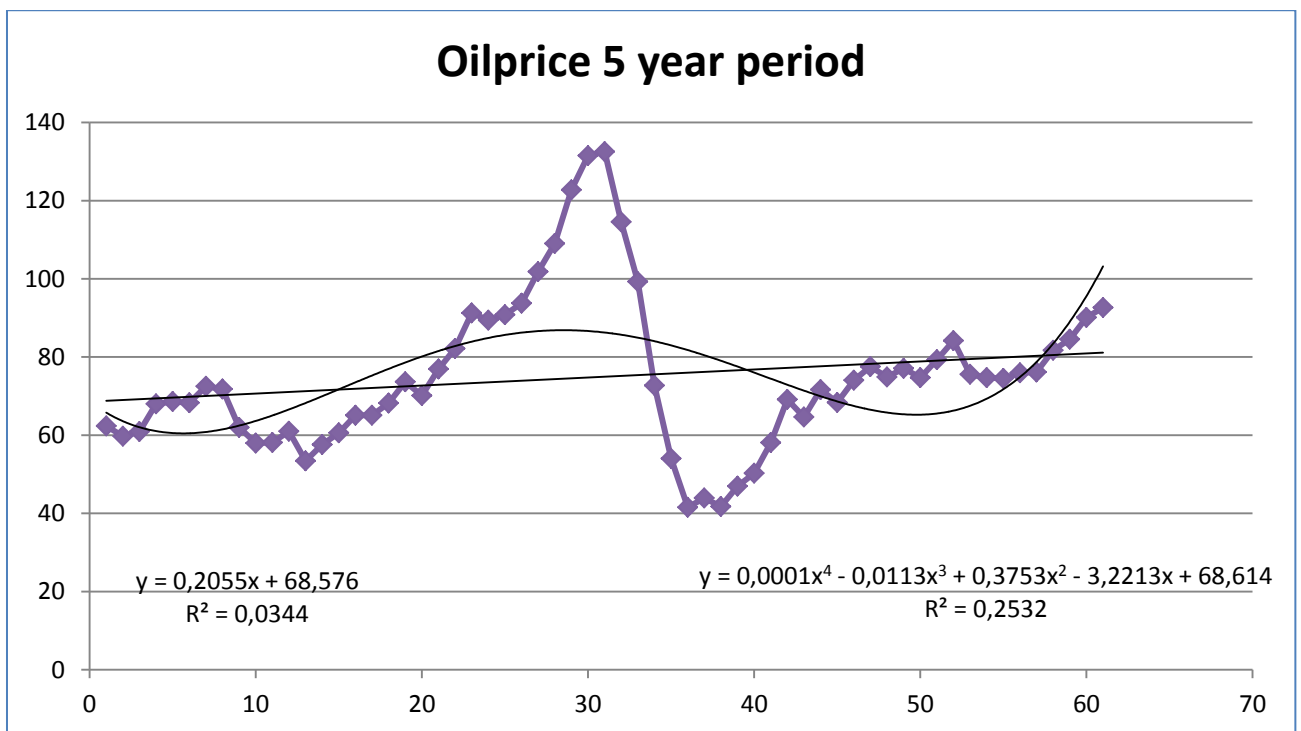
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<sup>42</sup> BP statistical review 2011.

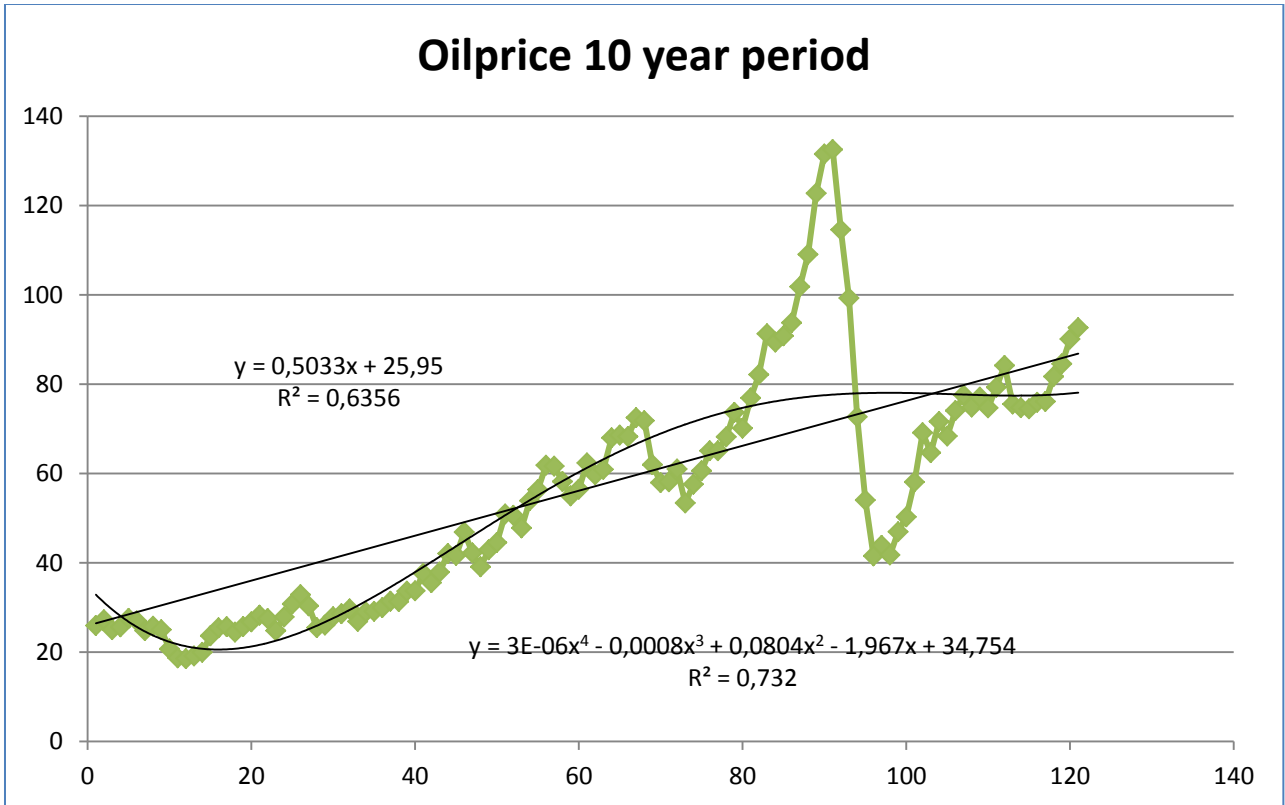
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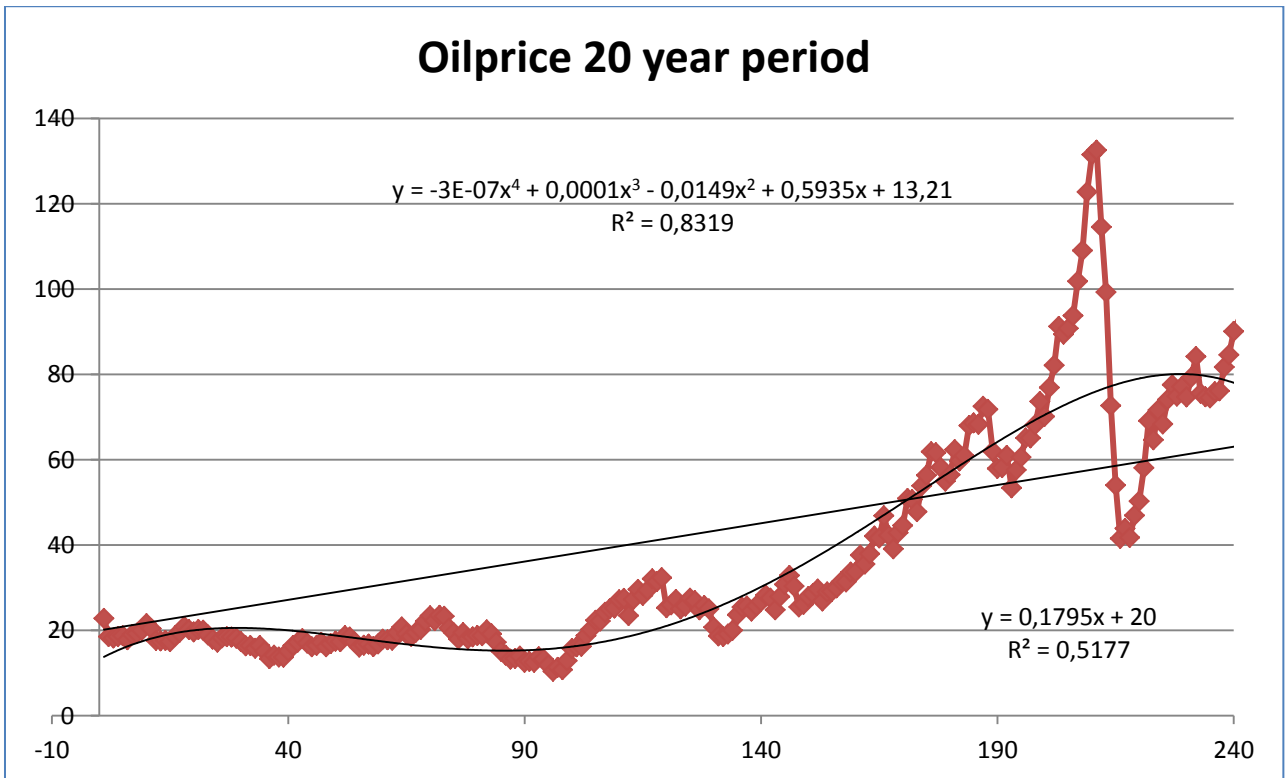
Appendix A: Oil price movements from January 2009 – January 2011, monthly average price



Appendix B: Oil price movements from January 2006 – January 2011, monthly average price



Appendix C: Oil price movements from January 2001 to January 2011, monthly average price.

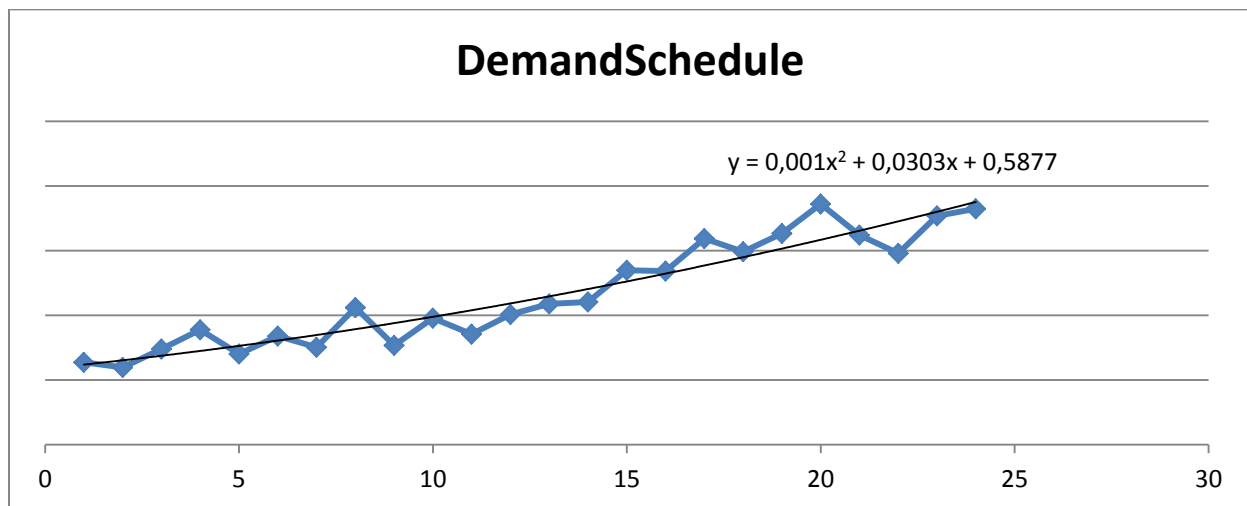


Appendix D: Oil price movements from January 1991 to January 2011, monthly average price.

## E

Year	Money of the day	2009 equivalent	Year	Money of the day	2009 equivalent	Year	Money of the day	2009 equivalent	Year	Money of the day	2009 equivalent
1861	\$ 0,49	\$ 11,65	1898	\$ 0,91	\$ 23,37	1935	\$ 0,97	\$ 15,15	1972	\$ 2,48	\$ 12,72
1862	\$ 1,05	\$ 22,47	1899	\$ 1,29	\$ 33,13	1936	\$ 1,09	\$ 16,87	1973	\$ 3,29	\$ 15,89
1863	\$ 3,15	\$ 54,66	1900	\$ 1,19	\$ 30,56	1937	\$ 1,18	\$ 17,62	1974	\$ 11,58	\$ 50,41
1864	\$ 8,06	\$ 110,11	1901	\$ 0,96	\$ 24,66	1938	\$ 1,13	\$ 17,20	1975	\$ 11,53	\$ 45,98
1865	\$ 6,59	\$ 91,99	1902	\$ 0,80	\$ 20,55	1939	\$ 1,02	\$ 15,74	1976	\$ 12,80	\$ 48,25
1866	\$ 3,74	\$ 54,58	1903	\$ 0,94	\$ 23,21	1940	\$ 1,02	\$ 15,59	1977	\$ 13,92	\$ 49,24
1867	\$ 2,41	\$ 36,84	1904	\$ 0,86	\$ 20,45	1941	\$ 1,14	\$ 16,60	1978	\$ 14,02	\$ 46,13
1868	\$ 3,63	\$ 58,27	1905	\$ 0,62	\$ 14,74	1942	\$ 1,19	\$ 15,65	1979	\$ 31,61	\$ 93,41
1869	\$ 3,64	\$ 58,43	1906	\$ 0,73	\$ 17,36	1943	\$ 1,20	\$ 14,88	1980	\$ 36,83	\$ 95,89
1870	\$ 3,86	\$ 65,22	1907	\$ 0,72	\$ 17,12	1944	\$ 1,21	\$ 14,74	1981	\$ 35,93	\$ 84,80
1871	\$ 4,34	\$ 77,41	1908	\$ 0,72	\$ 16,51	1945	\$ 1,05	\$ 12,51	1982	\$ 32,97	\$ 73,30
1872	\$ 3,64	\$ 64,92	1909	\$ 0,70	\$ 16,65	1946	\$ 1,12	\$ 12,29	1983	\$ 29,55	\$ 63,65
1873	\$ 1,83	\$ 32,64	1910	\$ 0,61	\$ 13,99	1947	\$ 1,90	\$ 18,24	1984	\$ 28,78	\$ 59,43
1874	\$ 1,17	\$ 22,10	1911	\$ 0,61	\$ 13,99	1948	\$ 1,99	\$ 17,72	1985	\$ 27,56	\$ 54,95
1875	\$ 1,35	\$ 26,27	1912	\$ 0,74	\$ 16,38	1949	\$ 1,78	\$ 16,01	1986	\$ 14,43	\$ 28,25
1876	\$ 2,56	\$ 51,37	1913	\$ 0,95	\$ 20,54	1950	\$ 1,71	\$ 15,23	1987	\$ 18,44	\$ 34,82
1877	\$ 2,42	\$ 48,56	1914	\$ 0,81	\$ 17,28	1951	\$ 1,71	\$ 14,11	1988	\$ 14,92	\$ 27,06
1878	\$ 1,19	\$ 26,35	1915	\$ 0,64	\$ 13,51	1952	\$ 1,71	\$ 13,81	1989	\$ 18,23	\$ 31,53
1879	\$ 0,86	\$ 19,72	1916	\$ 1,10	\$ 21,60	1953	\$ 1,93	\$ 15,47	1990	\$ 23,73	\$ 38,94
1880	\$ 0,95	\$ 21,03	1917	\$ 1,56	\$ 26,08	1954	\$ 1,93	\$ 15,39	1991	\$ 20,00	\$ 31,51
1881	\$ 0,86	\$ 19,04	1918	\$ 1,98	\$ 28,18	1955	\$ 1,93	\$ 15,45	1992	\$ 19,32	\$ 29,54
1882	\$ 0,78	\$ 17,27	1919	\$ 2,01	\$ 24,92	1956	\$ 1,93	\$ 15,23	1993	\$ 16,97	\$ 25,20
1883	\$ 1,00	\$ 22,93	1920	\$ 3,07	\$ 32,86	1957	\$ 1,90	\$ 14,47	1994	\$ 15,82	\$ 22,90
1884	\$ 0,84	\$ 19,98	1921	\$ 1,73	\$ 20,73	1958	\$ 2,08	\$ 15,43	1995	\$ 17,02	\$ 23,95
1885	\$ 0,88	\$ 20,93	1922	\$ 1,61	\$ 20,59	1959	\$ 2,08	\$ 15,29	1996	\$ 20,67	\$ 28,26
1886	\$ 0,71	\$ 16,88	1923	\$ 1,34	\$ 16,84	1960	\$ 1,90	\$ 13,75	1997	\$ 19,09	\$ 25,52
1887	\$ 0,67	\$ 15,93	1924	\$ 1,43	\$ 17,93	1961	\$ 1,80	\$ 12,90	1998	\$ 12,72	\$ 16,74
1888	\$ 0,88	\$ 20,93	1925	\$ 1,68	\$ 20,55	1962	\$ 1,80	\$ 12,76	1999	\$ 17,97	\$ 23,14
1889	\$ 0,94	\$ 22,35	1926	\$ 1,88	\$ 22,78	1963	\$ 1,80	\$ 12,61	2000	\$ 28,50	\$ 35,50
1890	\$ 0,87	\$ 20,69	1927	\$ 1,30	\$ 16,05	1964	\$ 1,80	\$ 12,43	2001	\$ 24,44	\$ 29,61
1891	\$ 0,67	\$ 15,93	1928	\$ 1,17	\$ 14,64	1965	\$ 1,80	\$ 12,23	2002	\$ 25,02	\$ 29,84
1892	\$ 0,56	\$ 13,32	1929	\$ 1,27	\$ 15,89	1966	\$ 1,80	\$ 11,89	2003	\$ 28,83	\$ 33,62
1893	\$ 0,64	\$ 15,22	1930	\$ 1,19	\$ 15,28	1967	\$ 1,80	\$ 11,56	2004	\$ 38,27	\$ 43,46
1894	\$ 0,84	\$ 20,74	1931	\$ 0,65	\$ 9,15	1968	\$ 1,80	\$ 11,09	2005	\$ 54,52	\$ 59,89
1895	\$ 1,36	\$ 34,93	1932	\$ 0,87	\$ 13,66	1969	\$ 1,80	\$ 10,53	2006	\$ 65,14	\$ 69,32
1896	\$ 1,18	\$ 30,31	1933	\$ 0,67	\$ 11,09	1970	\$ 1,80	\$ 9,94	2007	\$ 72,39	\$ 74,90
1897	\$ 0,79	\$ 20,29	1934	\$ 1,00	\$ 16,01	1971	\$ 2,24	\$ 11,85	2008	\$ 97,26	\$ 96,91
									2009	\$ 61,67	\$ 61,67

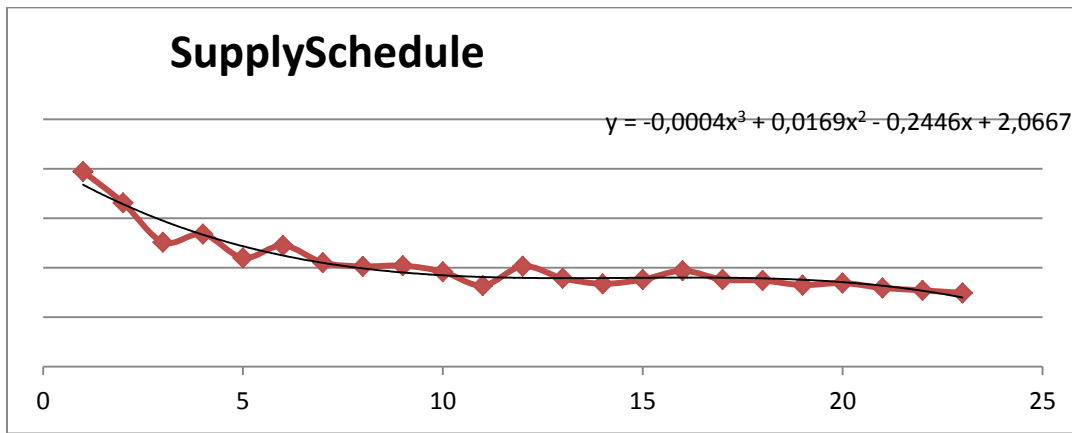
## F



	Demand Basis	Demand Shift	stdev	raw demand	smoothed demand
1	0,63	-0,060914	0,12	0,569086	0,681548127
2	0,644	0,116816	0,1235	0,760816	0,699378732
3	0,679	0,025271	0,1270	0,704271	0,720850641
4	0,715	0,269418	0,1304	0,984418	0,745963852
5	0,775	-0,196945	0,1339	0,578055	0,774718367
6	0,829	-0,116572	0,1374	0,712428	0,807114185
7	0,871	-0,015926	0,1409	0,855074	0,843151306
8	0,9	0,232053	0,1443	1,132053	0,88282973
9	0,931	0,178018	0,1478	1,109018	0,926149457
10	0,968	-0,138949	0,1513	0,829051	0,973110488
11	0,992	0,114774	0,1548	1,106774	1,023712822
12	1,016	-0,229328	0,1583	0,786672	1,077956459
13	1,076	0,042938	0,1617	1,118938	1,135841399
14	1,142	-0,051572	0,1652	1,090428	1,197367642
15	1,226	0,087238	0,1687	1,313238	1,262535189
16	1,31	-0,218893	0,1722	1,091107	1,331344038
17	1,418	0,024775	0,1757	1,442775	1,403794191
18	1,496	0,278810	0,1791	1,774810	1,479885647
19	1,587	0,178470	0,1826	1,765470	1,559618407
20	1,671	-0,140981	0,1861	1,530019	1,642992469
21	1,755	-0,113997	0,1896	1,641003	1,730007835
22	1,833	-0,046586	0,1930	1,786414	1,820664504
23	1,893	0,212809	0,1965	2,105809	1,914962476
24	1,98	-0,119328	0,2	1,860672	2,012901751

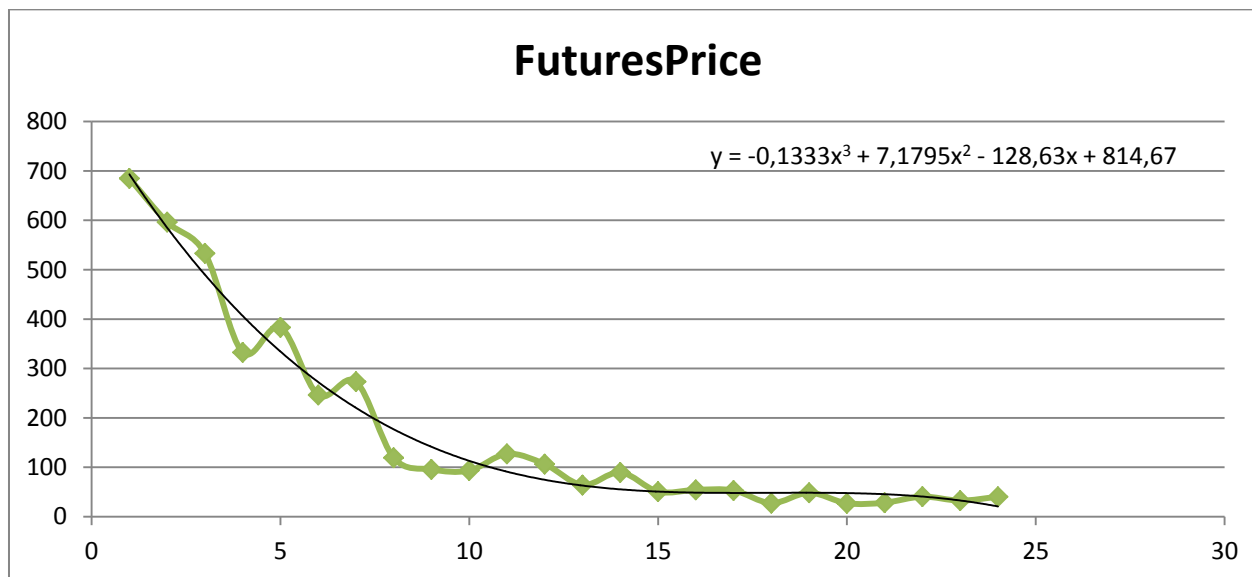


G



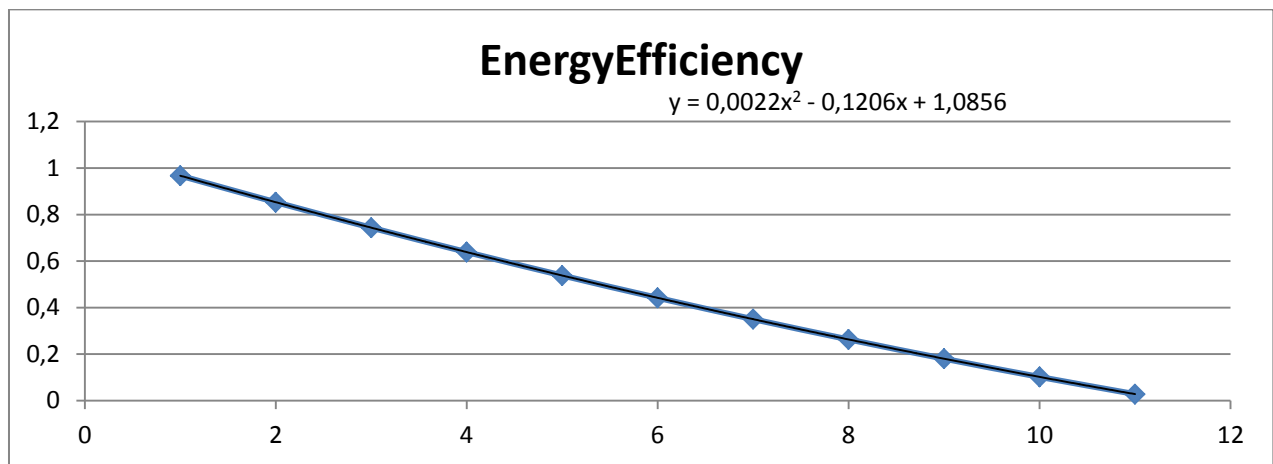
	Supply Basis	Demand Shift	stdev	raw demand	smoothed demand
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2	1,667	0,006356	0,115217391	1,673356	1,683082432
3	1,386	0,031867	0,110434783	1,417867	1,511386391
4	1,245	0,050719	0,105652174	1,295719	1,36732175
5	1,152	0,146205	0,100869565	1,298205	1,248492996
6	1,086	-0,007087	0,096086957	1,078913	1,152504616
7	1,058	-0,093442	0,091304348	0,964558	1,076961099
8	1,03	0,063678	0,086521739	1,093678	1,019466931
9	1,011	-0,108563	0,08173913	0,902437	0,9776266
10	0,993	0,126173	0,076956522	1,119173	0,949044593
11	0,974	0,009653	0,072173913	0,983653	0,931325398
12	0,955	0,053696	0,067391304	1,008696	0,922073502
13	0,936	0,034161	0,062608696	0,970161	0,918893391
14	0,918	-0,082584	0,057826087	0,835416	0,919389554
15	0,9	0,021496	0,053043478	0,921496	0,921166478
16	0,88	0,021070	0,04826087	0,901070	0,921828651
17	0,861	0,028042	0,043478261	0,889042	0,918980559
18	0,852	0,068477	0,038695652	0,920477	0,910226689
19	0,824	0,020757	0,033913043	0,844757	0,893171531
20	0,815	0,018274	0,029130435	0,833274	0,865419569
21	0,787	-0,001914	0,024347826	0,785086	0,824575293
22	0,777	-0,000533	0,019565217	0,776467	0,768243189
23	0,758	0,004989	0,014782609	0,762989	0,694027745

## H



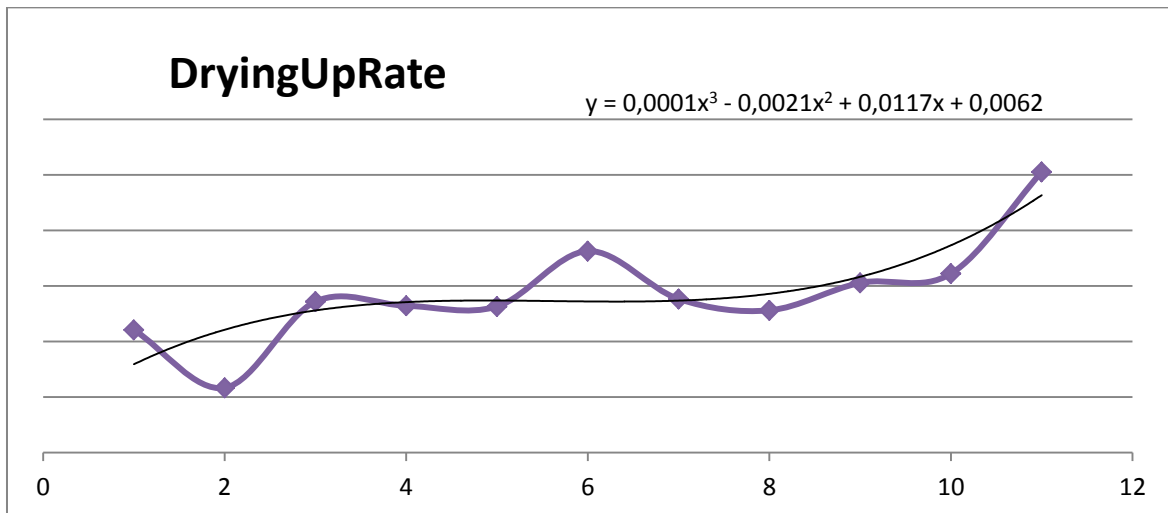
	futures price base			raw	smoothed
1	617	-44,097358	50	572,902642	646,5577614
2	533	64,894491	47,95652174	597,894491	553,3688466
3	452	74,699927	45,91304348	526,699927	470,6718087
4	392	-4,410723	43,86956522	387,589277	397,8302581
5	347	-19,315077	41,82608696	327,684923	334,2078048
6	287	43,483408	39,7826087	330,483408	279,1680592
7	233	59,872577	37,73913043	292,872577	232,0746314
8	170	-31,616044	35,69565217	138,383956	192,2911315
9	130	-77,093115	33,65217391	52,906885	159,1811699
10	110	-10,485796	31,60869565	99,514204	132,1083567
11	98	42,317896	29,56521739	140,317896	110,436302
12	92	-10,312057	27,52173913	81,687943	93,52861615
13	85	19,727122	25,47826087	104,727122	80,74890929
14	76	-0,252521	23,43478261	75,747479	71,46079161
15	68	-1,891704	21,39130435	66,108296	65,02787332
16	61	21,330665	19,34782609	82,330665	60,81376462
17	55	37,378116	17,30434783	92,378116	58,1820757
18	50	-8,647277	15,26086957	41,352723	56,49641676
19	46	4,274409	13,2173913	50,274409	55,12039801
20	43	9,000719	11,17391304	52,000719	53,41762963
21	41	5,075294	9,130434783	46,075294	50,75172183
22	39,5	3,240348	7,086956522	42,740348	46,48628481
23	38,5	-8,091963	5,043478261	30,408037	39,98492876
24	37,6	-0,154531	3	37,445469	30,61126388

I

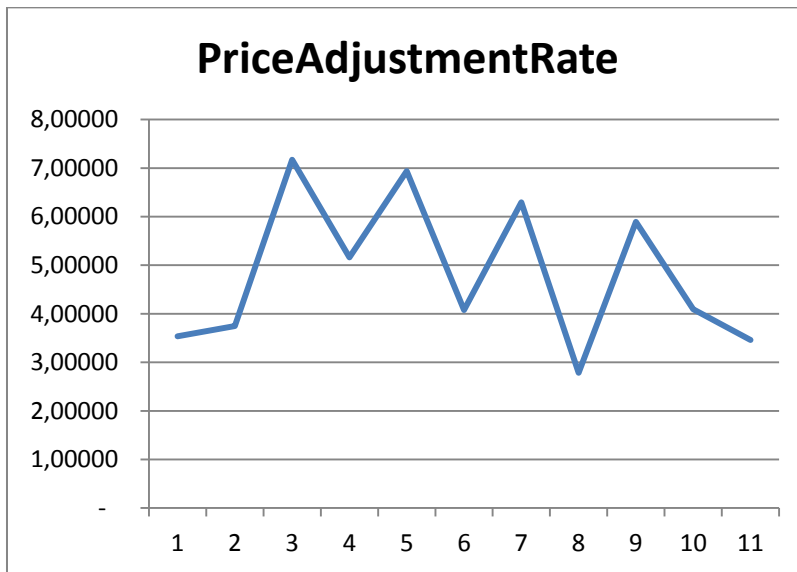


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1	0,994	0,248119	0,12	1,2421	1,148260246
2	0,89	0,041960	0,108	0,9320	0,979666293
3	0,79	-0,073067	0,096	0,7169	0,824139538
4	0,675	0,016388	0,084	0,6914	0,68167998
5	0,565	-0,023991	0,072	0,5410	0,55228762
6	0,48	-0,012618	0,06	0,4674	0,435962457
7	0,405	-0,081222	0,048	0,3238	0,332704492
8	0,285	0,021480	0,036	0,3065	0,242513725
9	0,215	-0,041274	0,024	0,1737	0,165390155
10	0,1	0,019508	0,012	0,1195	0,101333783
11	0	0,000000	0	0,0000	0,050344608

J



	drying up rate bse			ras	smoothed
1	0,021	0,003674	0,005	0,024674	0,024712014
2	0,021	0,005029	0,005	0,026029	0,024650443
3	0,022	0,000686	0,005	0,022686	0,024491861
4	0,023	-0,001710	0,005	0,021290	0,02443657
5	0,025	0,004604	0,005	0,029604	0,02468487
6	0,026	0,001579	0,005	0,027579	0,025437062
7	0,028	-0,006724	0,005	0,021276	0,026893449
8	0,031	0,000874	0,005	0,031874	0,029254331
9	0,034	-0,001953	0,005	0,032047	0,03272001
10	0,038	-0,000344	0,005	0,037656	0,037490786
11	0,049	-0,005179	0,005	0,043821	0,043766961

**K**

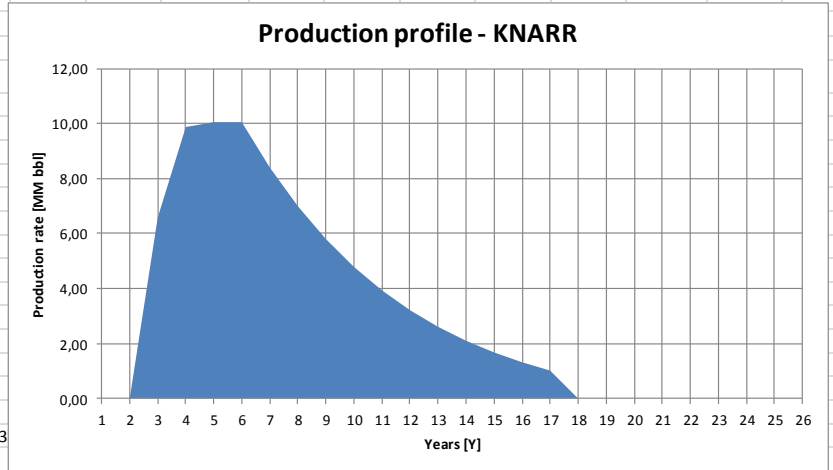
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4,09443
3,46129

L

		Input PDFs					Type \ Classification		
		Real.	Min	ML	Max	Corr	Low	Medium	High
$N_p$	Reserves, MMbbl	69,0000	52,836	69	100	Indep			
$y_R$	Length of Ramp Up (to plateau), yrs	3,0000	2	3	4	0,00			
$q_p$	Yearly Plateau Rate, MMbbl/yr	11,0000	7,00	11,00	22,00	0,00			
$P$	Fraction reserves produced at end plateau	0,6000	0,40	0,60	0,70	0,00			
$q_L$	Field Economic Rate Limit, MMbbls/yr	0,6900	0,48	0,69	0,90	0,00			

Year	Prod rate	Cum Prod	Decline rate
$Y$	$q_t$	$N_t$	$\alpha$
0	0,000	0,000	
1	0,000	0,000	
2	0,000	0,000	
3	11,000	11,000	
4	11,000	22,000	
5	11,000	33,000	
6	11,000	44,000	0,286389
7	8,261	52,261	0,302827
8	6,102	58,363	0,323333
9	4,416	62,780	0,350335
10	3,111	65,891	0,389234
11	2,108	67,999	0,456085
12	1,336	69,335	0,645266
13	0,000	69,335	2,060597
14	0,000	69,335	2,060597
15	0,000	69,335	2,060597
16	0,000	69,335	2,060597
17	0,000	69,335	2,060597
18	0,000	69,335	2,060597
19	0,000	69,335	2,060597
20	0,000	69,335	2,060597
21	0,000	69,335	2,060597
22	0,000	69,335	2,060597
23	0,000	69,335	2,060597
24	0,000	69,335	2,060597
25	0,000	69,335	2,060597
26	0,000	69,335	2,060597
27	0,000	69,335	2,060597
28	0,000	69,335	2,060597
29	0,000	69,335	2,060597
30	0,000	69,335	2,060597
31	0,000	69,335	2,060597
32	0,000	69,335	2,060597
33	0,000	69,335	2,060597
34	0,000	69,335	2,060597
35	0,000	69,335	2,060597
36	0,000	69,335	2,060597
37	0,000	69,335	2,060597
38	0,000	69,335	2,060597
39	0,000	69,335	2,060597
40	0,000	69,335	2,060597
Total Produced		69,335	
% Produced		100,5%	



Field Lifetime	13	Mean	13,00
		Stddev	0,00



M

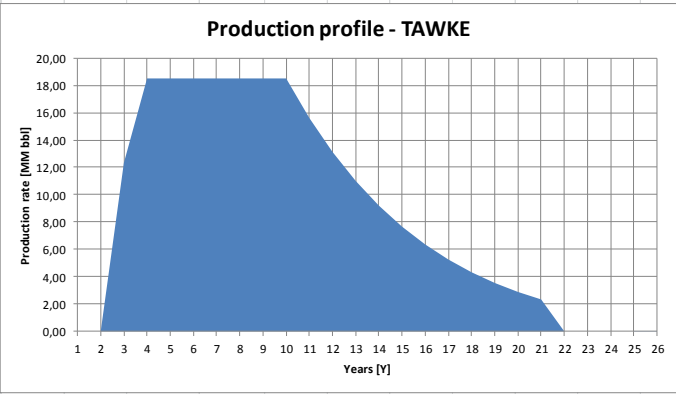
		Input PDFs					Type \ Classification			
		Real.	Min	ML	Max	Corr	Low	Medium	High	
$N_p$	Reserves, MMbbl	230,0000	150	230	370	Indep				
$y_R$	Length of Ramp Up (to plateau), yrs	3,0000	2	3	4	0,00				
$q_P$	Yearly Plateau Rate, MMbbl/yr	18,5000	12,00	18,50	33,00	0,00				
$P$	Fraction reserves produced at end plateau	0,6000	0,40	0,60	0,70	0,00				
$q_L$	Field Economic Rate Limit, MMbbls/yr	0,3450	0,23	0,35	0,46	0,00				

Year	Prod rate	Cum Prod	Decline rate
$Y$	$q_t$	$N_t$	$\alpha$
0	0,000	0,000	
1	0,000	0,000	
2	12,333	12,333	
3	18,500	30,833	
4	18,500	49,333	
5	18,500	67,833	
6	18,500	86,333	
7	18,500	104,833	
8	18,500	123,333	
9	18,500	141,833	0,170203
10	15,605	157,438	0,173077
11	13,125	170,563	0,17612
12	11,005	181,568	0,179352
13	9,198	190,766	0,182798
14	7,662	198,428	0,186487
15	6,358	204,786	0,190456
16	5,256	210,041	0,194752
17	4,325	214,367	0,199437
18	3,543	217,910	0,204591
19	2,888	220,798	0,210328
20	2,340	223,138	0,216807
21	0,000	223,138	-0,05028
22	0,000	223,138	-0,05028
23	0,000	223,138	-0,05028
24	0,000	223,138	-0,05028
25	0,000	223,138	-0,05028
26	0,000	223,138	-0,05028
27	0,000	223,138	-0,05028
28	0,000	223,138	-0,05028
29	0,000	223,138	-0,05028
30	0,000	223,138	-0,05028
31	0,000	223,138	-0,05028
32	0,000	223,138	-0,05028
33	0,000	223,138	-0,05028
34	0,000	223,138	-0,05028
35	0,000	223,138	-0,05028
36	0,000	223,138	-0,05028
37	0,000	223,138	-0,05028
38	0,000	223,138	-0,05028
39	0,000	223,138	-0,05028
40	0,000	223,138	-0,05028
	0,000	223,138	
	% Produced	97,0%	

### Production profile - TAWKE



Field Lifetime	21	Mean	21,00
		Stddev	0,00





N

		Input PDFs					Type \ Classification		
		Real.	Min	ML	Max	Corr	Low	Medium	High
$N_p$	Reserves, MMbbl	750,000	600	750	900	Indep			
$y_R$	Length of Ramp Up (to plateau), yrs	5,0000	4	5	7	0,00			
$q_P$	Yearly Plateau Rate, MMbbl/yr	50,0000	20,00	50,00	100,00	0,00			
$P$	Fraction reserves produced at end plateau	0,6000	0,40	0,60	0,70	0,00			
$q_L$	Field Economic Rate Limit, MMbbls/yr	0,6000	0,38	0,60	0,75	0,00			

Year	Prod rate	Cum Prod	Decline rate
$Y$	$q_t$	$N_t$	$\alpha$
0	0,000	0,000	
1	0,000	0,000	
2	20,000	20,000	
3	30,000	50,000	
4	40,000	90,000	
5	50,000	140,000	
6	50,000	190,000	
7	50,000	240,000	
8	50,000	290,000	
9	50,000	340,000	
10	50,000	390,000	
11	50,000	440,000	
12	50,000	490,000	0,1593548
13	42,635	532,635	0,1616719
14	36,270	568,905	0,1641025
15	30,781	599,686	0,1666576
16	26,056	625,741	0,1693498
17	21,997	647,738	0,1721939
18	18,517	666,255	0,1752074
19	15,541	681,796	0,1784114
20	13,002	694,798	0,1818315
21	0,000	694,798	-0,010869
22	0,000	694,798	-0,010869
23	0,000	694,798	-0,010869
24	0,000	694,798	-0,010869
25	0,000	694,798	-0,010869
26	0,000	694,798	-0,010869
27	0,000	694,798	-0,010869
28	0,000	694,798	-0,010869
29	0,000	694,798	-0,010869
30	0,000	694,798	-0,010869
31	0,000	694,798	-0,010869
32	0,000	694,798	-0,010869
33	0,000	694,798	-0,010869
34	0,000	694,798	-0,010869
35	0,000	694,798	-0,010869
36	0,000	694,798	-0,010869
37	0,000	694,798	-0,010869
38	0,000	694,798	-0,010869
39	0,000	694,798	-0,010869
40	0,000	694,798	-0,010869
% Produced		92,6%	

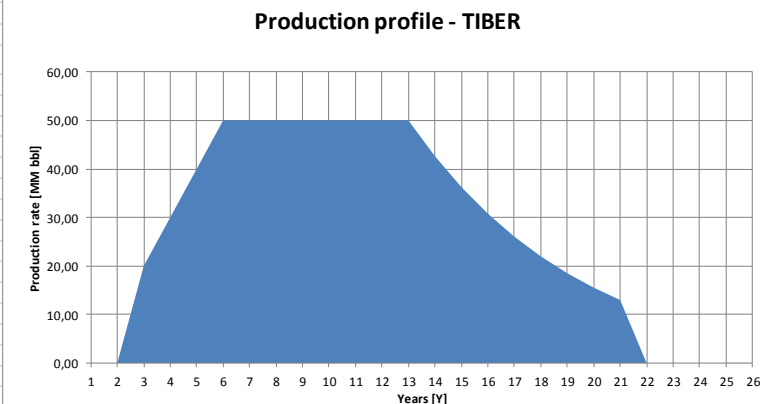
  

Field Lifetime	21
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Mean	21,00
Stddev	0,00

### Production profile - TIBER



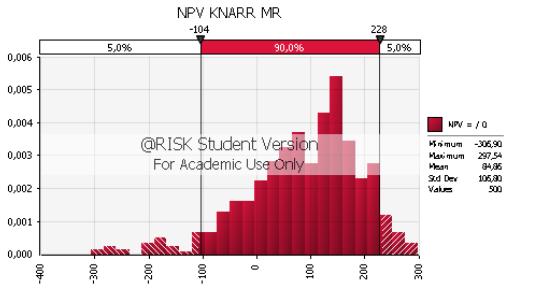


0

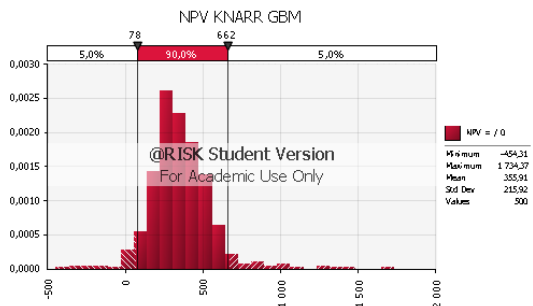
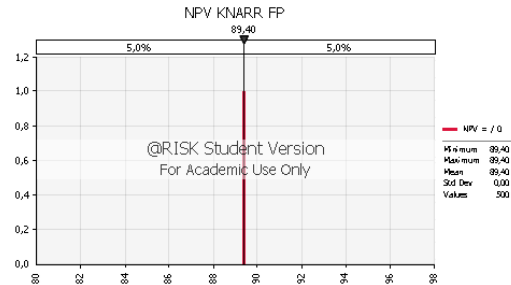
## Knarr Field NPV

### 1-Open Parameters

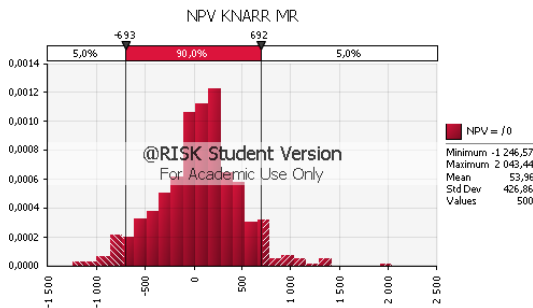
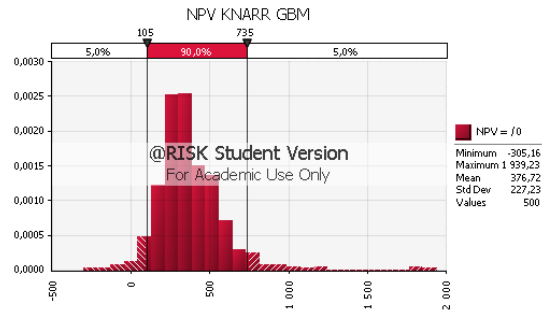
### 2-Fixed Parameters



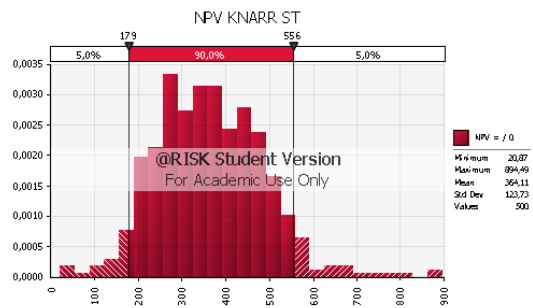
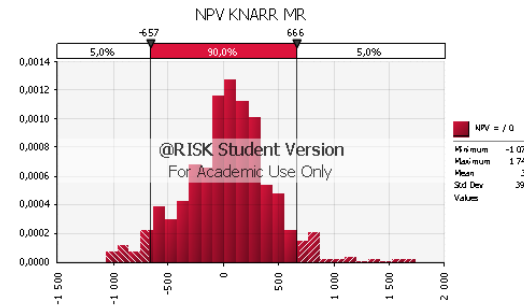
**FP**



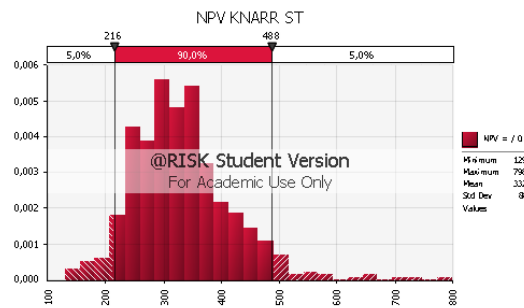
**GBM**



**MR**



**ST**

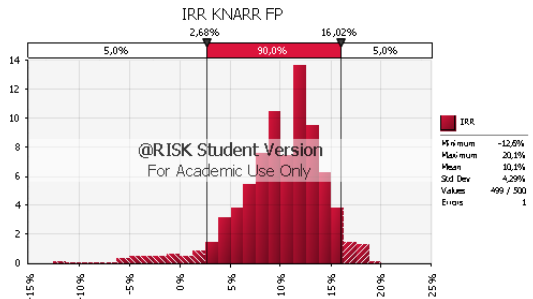


P

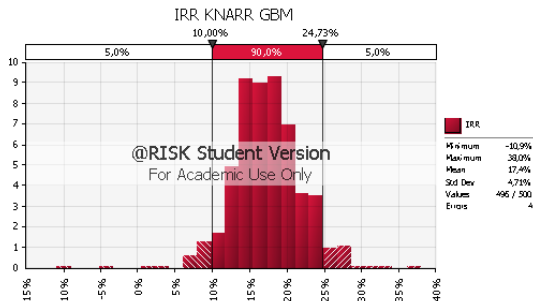
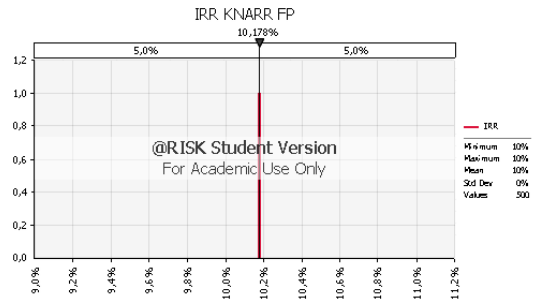
Knarr Field IRR

1-Open Parameters

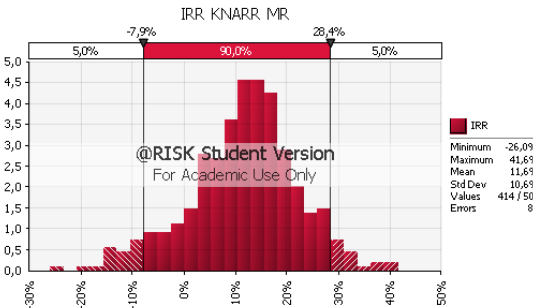
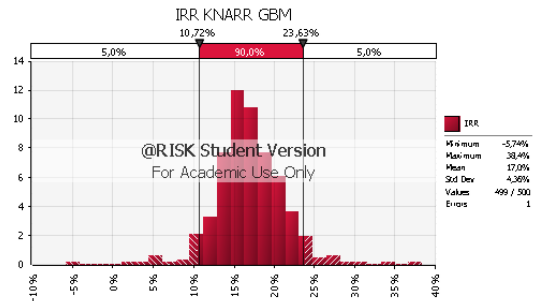
2-Fixed Parameters



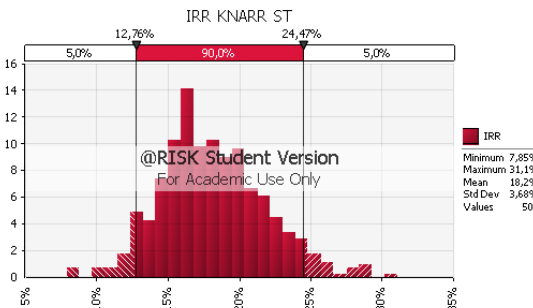
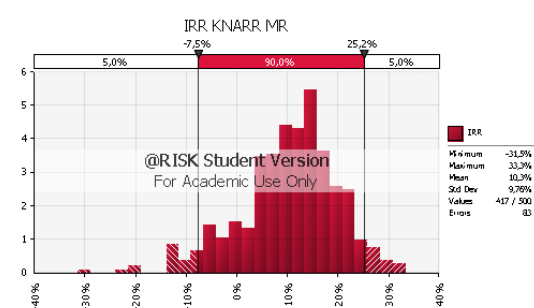
FP



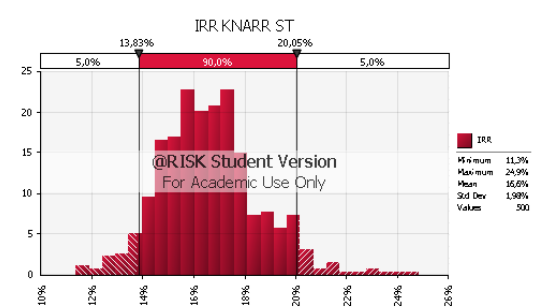
GBM



MR



ST

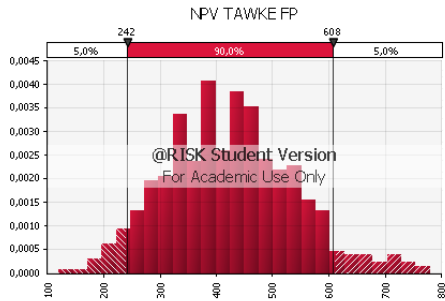


Q

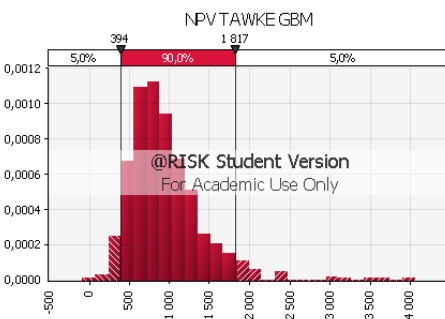
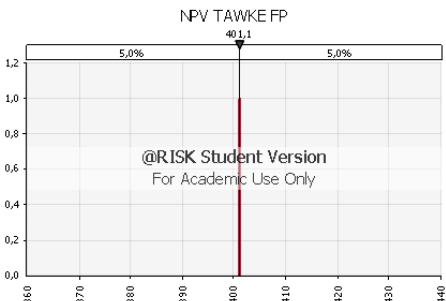
Tawke Field NPV

1-Open Parameters

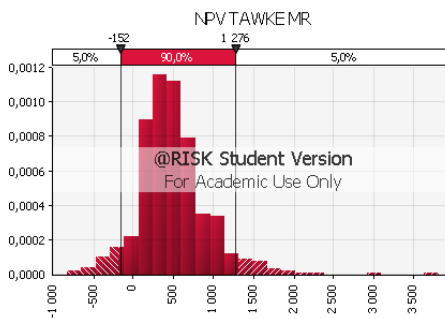
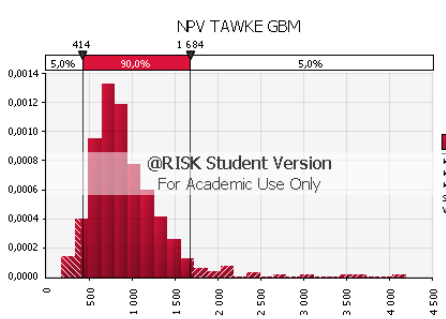
2-Fixed Parameters



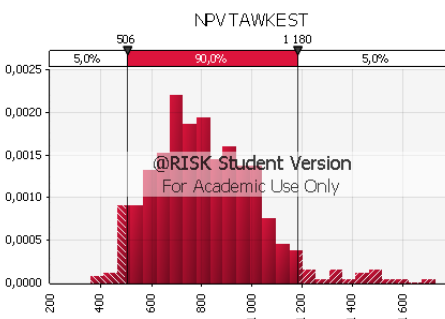
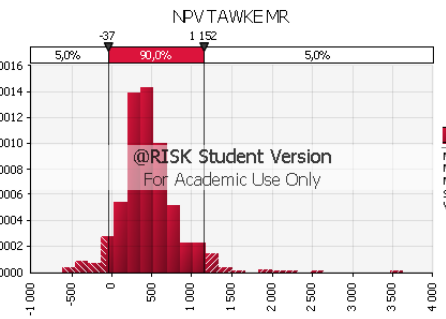
FP



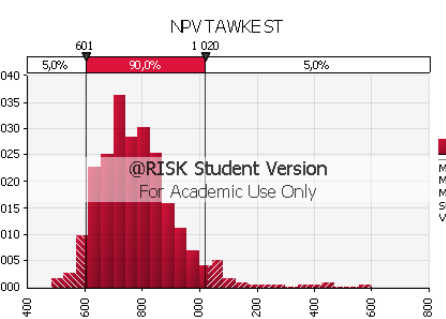
GBM



MR



ST

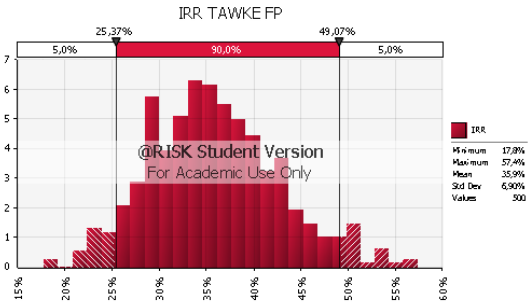


R

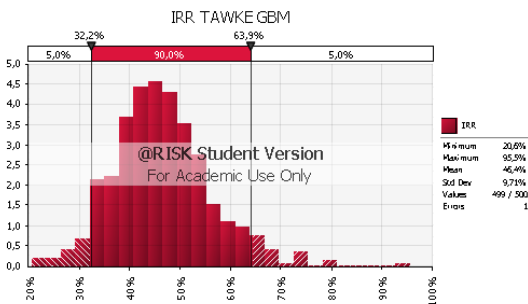
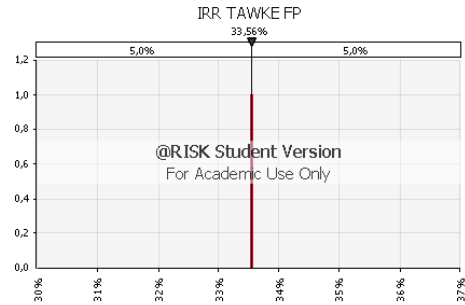
Tawke field IRR

1-Open Parameters

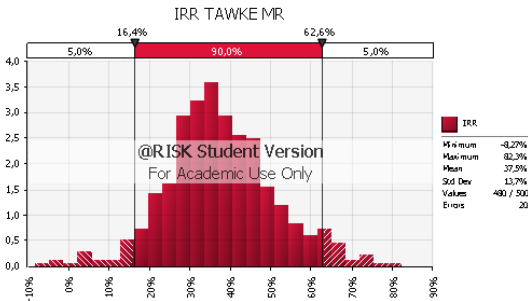
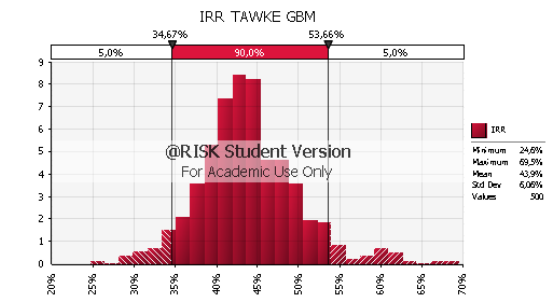
2-Fixed Parameters



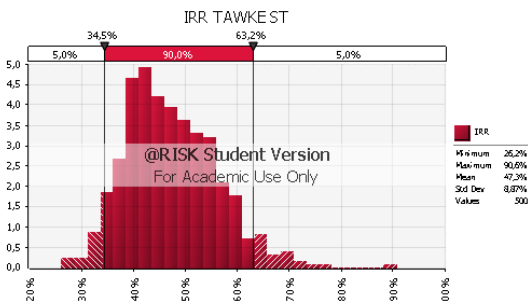
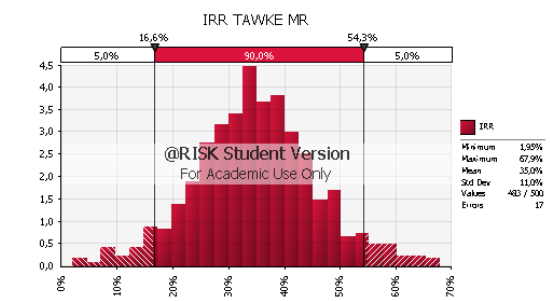
FP



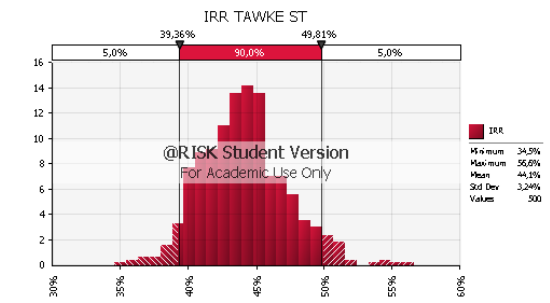
GBM



MR



ST

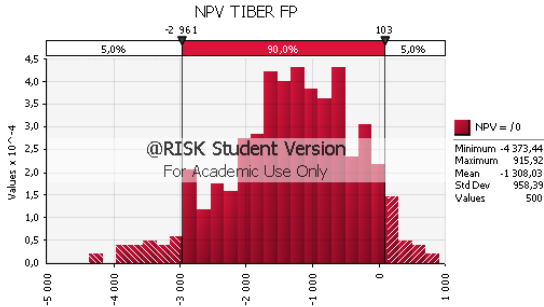


S

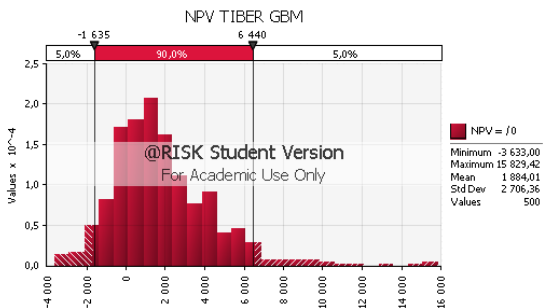
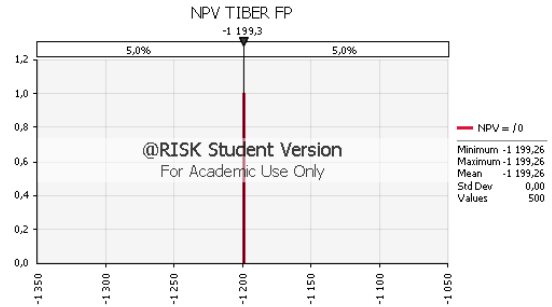
### Tiber Field NPV

#### 1-Open Parameters

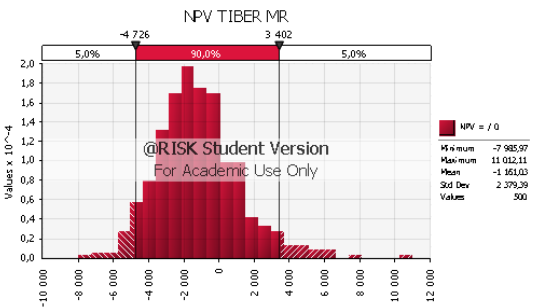
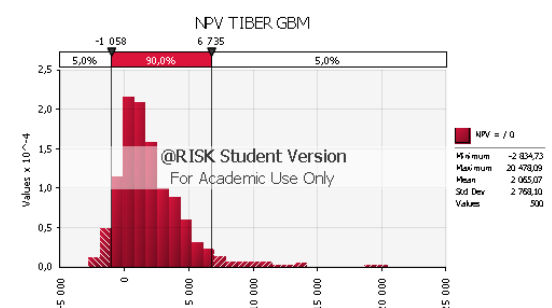
#### 2-Fixed Parameters



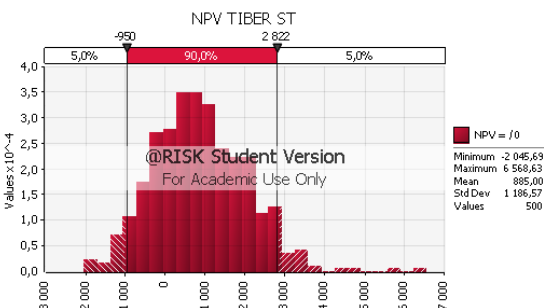
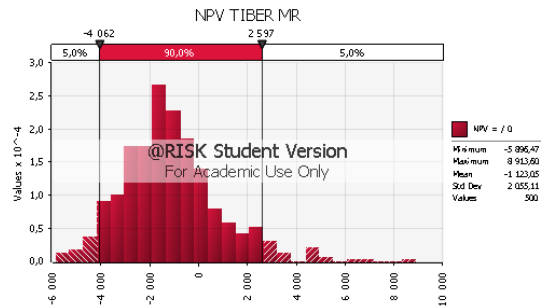
FP



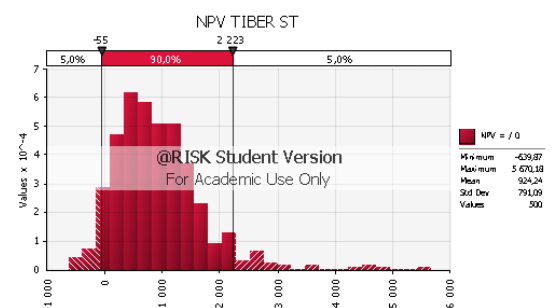
GBM



MR



ST



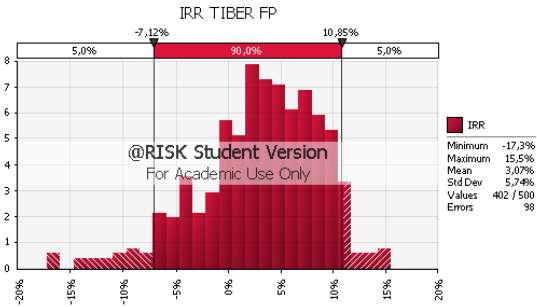


T

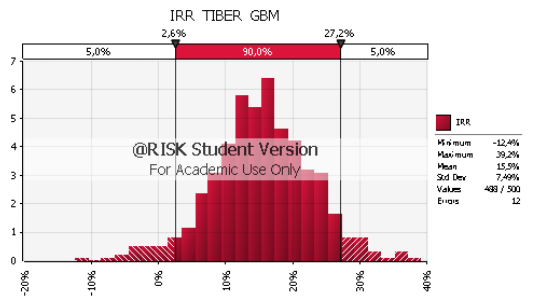
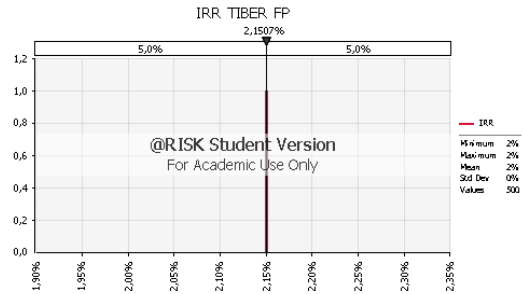
Tiber Field IRR

1-Open Parameters

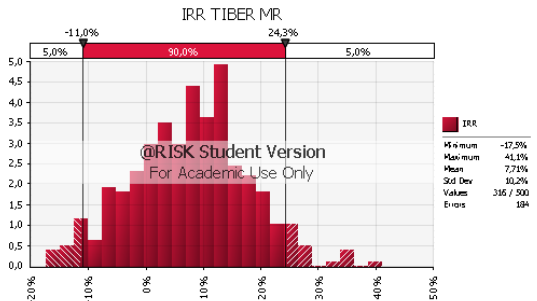
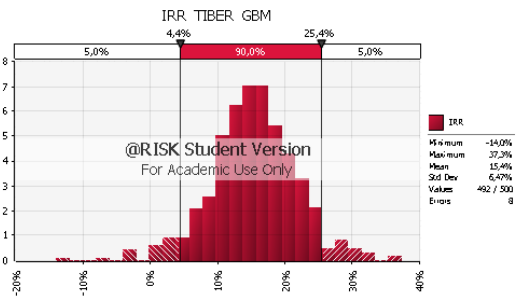
2-Fixed Parameters



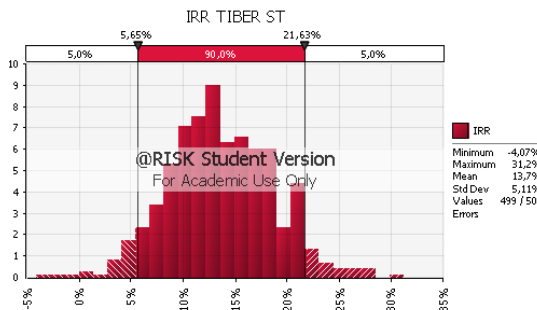
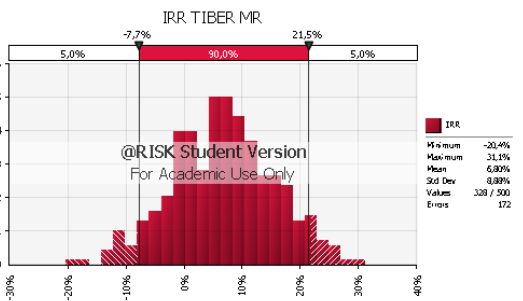
FP



GBM



MR



ST

