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Abstract

Deciding whether or not to stop producing oil and start producing gas is a difficult decision. This due to the decision's irreversible nature and dependency on the many uncertain factors. With one of the main uncertainties being the oil price this thesis evaluate its effect on the optimal timing of transition from oil to gas production. To do this a Real Options model using Monte Carlo simulation was made in Excel. The model was built and fitted for a fictive case which was used as a basis for the evaluation. To model the oil price a Mean reverting Ornstein-Uhlenbeck-processes was chosen for its ability to include the main characteristics of the oil price.
The analysis showed that the optimal timing was dependent on the oil price and its inherent uncertainty, and varying in terms with the nature of the oil price model.

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1 Introduction

Producing an oil and gas field is in many ways a complicated manner. Depending on the field, this may vary from planning the wells, oil and gas separation, infrastructure to reservoir mechanics. However, in many cases one of the most difficult issues is the timing of the transition from oil to gas production, also known blow down. This is due to the fact when producing the late oil reserves some of the pressure driving the oil up the well is exerted by the overlaying gas and if the gas then is produced the pressure will drop so that the reservoir will unable to produce more oil. This is thus an important and irreversible decision which will be highly dependent on several uncertain factors such as the reserves, future production and maybe the most important ones, oil and gas prices[16].
When today's managers evaluate this option, the decision is generally based on Net Present Value calculations where the blow down is postponed x number of years. By doing it this way the economical value of future decisions does not seem to be considered thus the project is undervalued. E.g. should the future oil price be highly under estimated the decision to carry out blown down would be a big financial loss. A method that is better suited for this evaluation is Real Options valuation[6]. This is a method using different techniques to take uncertainty and flexibility in to consideration. Although the real options thinking have been shown a high degree of interest and acceptance in the petroleum industry, it has still not come to widespread use[14]. It has been written numerous of papers about the different approaches, about the effects of the oil price etc., but to the knowledge of the author, little has been written about its application to the blow down optimization problem.

The purpose of this thesis is thus; *to analyze the effect of the oil price' uncertainty on the optimal timing of transition from oil to gas production.*

To achieve this it was first undertaken an extensive literature review. The review aimed to give an overall understanding of the real options scheme and the conceptual underpinnings of different approaches. Further, a valuation model in Excel was chosen to study the concept closer. The effect of the oil price was then analyzed based on a fictive case where the parameters of both the field and the model were varied.

The remainder of this thesis is organized as follows. Chapter 2 reviewing real options theory and oil price modelling. Chapter 3 presenting the chosen procedure and example case. Chapter 4 presenting the analysis done and a discussion around the findings. And chapter 5 presenting a summary and conclusions.

2 Theory survey

This chapter consists of two main parts. The first part will present the history of real options, some methods and their main features. The second part will give a brief overview of some oil price models and further evaluation of two of these.

2.1 Real options theory

The importance of project valuation and value creation has always been a focus of academic and managerial interest, and throughout the time there has been a variety of approaches attempting to determine which decision that will maximize asset value. Starting in the 1950`s with the traditional Discounted cash flow method it became possible to predict the stream of the cash flow using a present value table. The net present value of a project is calculated by discounting the future expected cash flows at a discount rate that takes the risk of the project and time value of money into account. With these premises, a project with a positive NPV is assumed profitable.
However, the simplicity of this method has several shortcomings and one of the biggest critiques of the DCF is that it fails to include managerial flexibility[8]. In many situations it will be possible for the management to make decisions that will affect the value of the project while the project is under progress. These choices will be very crucial for the project's success and has to be taken into consideration.
As the shortcomings were recognized it was needed to alter the approach and solutions to this were the financial markets Option pricing method, and decision analysis methods. These methods added the value of managerial flexibility and were brought out due to the uncertainty and complexity of real projects, which the traditional DCF method was not able to value correctly. Originating from the financial options and taken into the real world valuing

real investments under uncertainty, the option valuation methods were named real options[7]. RO valuation is in many ways the extension and application of Option pricing methods that where developed for the 1970's finance industry.[4,13,14]

2.1.1 Discounted Cash Flow Method

The DCF method values the project by discounting the future expected cash flows at a single discount rate compensating for the time value of money and risk. When operating in complete markets (i.e. that there is one or more securities whose payoff(s) can replicate the projects payoffs in all states and periods), the discount rate can be derived from the market price of a portfolio replicating these expected cash flows. However, dealing with incomplete markets there will always be a tracking error due to the difference between the cash flows of the replicating portfolio and those of the project[7]. Thus for these reasons most investment projects are valued using the DCF approach with a discount rate set to the firms "weighted average cost of capital", WACC. The WACC is the minimum return that the creditors, owners and other capital providers demand for the investment, and is determined using the capital asset pricing model (CAPM)[7]. For further description of the WACC, see Grinblatt and Titman (1998, Chapters 10 and 12).

The DCF approach implicitly assumes that cash-flow uncertainty grows at a constant rate over the life of a project and that the project's outcome will be unaffected by future decisions of the firm. DCF analysis is based on static budget where the management is seen as passive pieces, and typically presumes one line of action from the beginning, thus not taking the value created by flexibility into account. Doing this in a project with a high level of cash flow uncertainty makes it impossible to generate an appropriate value of the investment, making the DCF inadequate for real investments[18].

2.1.2 Real Options Analysis (ROA) and Decision analysis (DA)

The core of the real option theory is to try to ensure the most efficient use of assets. It is an optimization problem of how to maximize the NPV, given risk factors in the market, technical risks and the relevant options i.e. oil price, production volume, and flexibility respectively. A real option is a right, but not the obligation to exercise an action for a predetermined cost. This may be the right to expand or close down existing operations, to contract new rigs, etc. I.e. one has the option to act in the future, and this flexibility has a value that must be taken in to account. The value of these options can only be determined through option pricing or decision analysis methods[7]. At its most basic, RO is by Yao et. al (2006) said to comprise the creation, valuation and exercise of flexible responses to manage the impacts of uncertainty: where

> -by flexibility*: "is meant the ability (or right), to take an action (make a decision) after some uncertain event has been resolved."*
>
> -by valuation: *"the calculation of the economic value of an investment opportunity that includes such flexibility, plus any associated costs."*

Both DA and ROA acknowledge upfront that the future is uncertain. However, they differ in the way they approach it. In short DA divides the problems into smaller parts and effectively accounts for the value of managerial flexibility. By finding the key uncertainties and subjectively assessing these, the NPV can be found by typically constructing a binomial lattice, decision tree or simulations models of the project cash flows. Using risk-neutral techniques and stochastic processes they derive the option value. The major weakness of this approach is its biased focus on the decision maker's subjective beliefs and ignorance of the markets and their affect on projects[4]. Although DA and

ROA serve the same goal, they are founded on different foundations. While DA uses subjective beliefs and preferences, ROA seeks objective market information to determine the project and real option value. The approach has its roots in the financial industry and the concepts developed by Black and Scholes (1973) and Mereton (1973) for valuing financial options. However, applications of real option methods have been restricted by the mathematical complexity. This is because general problems require a probabilistic solution to the company's optimal investment policy at all instances in time up to the maturity of its options and not only at present. To solve this, the evolution of uncertainty in the value of the real asset over time is first modeled as a stochastic process. The value of the optimal policy can then be determined by Bellman`s principle of optimality. (For further description see Bellman (1957))[1]. Comparing the two methods they both show different strengths and weaknesses. Although, DA suits handling subjective uncertainties, it poorly takes into account the markets effect on projects[18]. Contrary, as option pricing works well when it exist a complete market, it fails in absence of such. To be able to deal with these incomplete markets Smith and Nau (1995) proposed a method that integrates the two approaches. By distinguishing between market risks (e.g. oil price) which can be hedged by trading securities and private uncertainties (e.g. reserves) which are project-specific risks, these can be valued using option pricing theory and decision analysis techniques respectively[4].

The integrated RO approach is very useful when valuing projects where both market and project-specific components exist. This distinction often appears natural in oil and gas E&P projects, (e.g. with oil price as market risk and reserves a project-specific risk), thus making it equally popular in the industry[4,8].

Another method which compensates for the DCF methods flaw of discounting the whole cash flow at one single discount rate is the risk neutral approach. This is a method where risk premiums are incorporated by risk-adjusting probabilities rather than risk-adjusting with discount rates. The adjusted probabilities can be derived from market and technical information. E.g. for oil prices the forward price presented in the market is the expected value of one barrel of oil discounted for risk but not time. This yields the risk neutral expected values and to achieve time equivalence, the cash flows are discounted at a risk free rate[13].

As the years have passed the continuing evolution of computational tools and some progress of the underlying theory has created a widespread selection of valuation and analysis techniques[7]. However, summing up, the different approaches are based on the same methodologies either its Lattices, Trees, simulation or a combination. Depending on scenario and the project uncertainties, one may be better fitted than the other. E.g. projects having complex cash flow structures which are affected by multiple uncertainties may not be as easily valued by lattice or trees and analysts then may have to resort to methods such as Monte Carlo simulation[8,13].

2.1.3 Stochastic processes

A stochastic process is the counterpart to a deterministic process and is by Sheldon. M. Ross (2007)[11] described as: {X(t), t ∈ T }, a collection of random variables. Meaning, for each t ∈ T, X(t) is a random variable. The set T is called the index set of the process, and if it is countable, the process is said to be a discrete-time process. On the other hand if the set T is not countable, e.g. and interval, the stochastic process is said to be a continuous-time process. Further, the index t represents the time, thus making X(t) the state of the process at time t. The array of all potential values that the random variables X(t) can assume is defined as the *state space.* Thus, a stochastic process is a system of random variables that describes the evolution of some process through time[11,24].

A well explanatory description of a stochastic process is: " *a process which describes the uncertainty in how a variable changes over time. The values that the variable can take on are defined by an appropriate probability distribution for each time. These distributions are a function of a set of parameters such as the current price, standard deviation of annual fluctuations (volatility) and expected trends."* Begg and Smit (2007)

2.2 Modeling oil price

It is well known that the uncertainty in the oil price is an important factor for the commercial success of offshore E&P projects[9]. It is therefore important choosing the best fit-for-purpose price model describing its characteristics as high volatility, abnormal jumps and tendency to revert to the long term mean[15]. A key component when choosing oil price model for this thesis is thus the characteristics of the way it fluctuates through time.

A model that is widely used for both modeling stock and commodity prices is the continuous-time stochastic Geometric Brownian Motion (GBM).[15,18] This model implies that returns have a log-normal distribution, meaning that the logarithmic returns, which are simply continuously compounded returns, follow a normal distribution. Another popular model for both stock and oil price is the Ornstein-Uhlenbeck Mean Reverting (MR) model. This model describes that the oil price has a tendency to revert back to some long-term mean over time. The further the value is from the long-term mean, the stronger is its attraction to it. Another edition of the MR model is the Mean Reversion with Jumps. Copying the features of the MR and adding a random jump this model tries to mimic some of the rare, but large "jumps" which often the oil price tend to have. As for the RO approach, research has produced new and more sophisticated methods. For example Schwartz and Smith (2000)[5] simulate a drift in the long term mean and suffice a 2-factor model where the long term mean price follows a GBM whereas short-term deviations from the mean follow a MR process.[15,18]

2.2.1 Geometric Brownian Motion

A Geometric Brownian Motion is a continuous-time stochastic process which implies that returns have a log-normal distribution, meaning that the logarithmic returns follow the normal distribution. The variance of the price grows proportionally with time, and due to the nature of a log-normal distribution the price will never fall below zero[25].

Describing the change in price ∂P, the process can be written:

$$\partial P = \alpha P \partial t + \sigma P \varepsilon \sqrt{\partial t} \tag{2.1}$$

Where α is the percentage drift, σ percentage volatility, ε a standard normal [0,1] random variable and ∂t, the change in time.

To be able to simulate the future price following the GBM the expected volatility and the drift has to be estimated. This can be done several ways, some use estimates based on market option prices, while others prefer using historical spot prices. Using historical data the volatility can be estimated by computing the standard deviation of
($\ln P_t - \ln P_{t-1}$), the drift by averaging the values of ($\ln P_t - \ln P_{t-1}$), while the spot price, P, is directly observed in the market.[13,15,22,25]

2.2.2 Mean Revering Process

The Mean Reverting Process is a log-normal diffusion process. The model concept is that the high and low values of the price are interim and that the value will tend to move towards the average, long term mean over time. The further the value is from the long-term mean, the stronger its attraction is to it. This response is explained as the market's reaction to change and captures one of the main features of energy prices. Unlike the Geometric Brownian Motion, the MR's variance does not grow proportionally to the time interval, but increases in the beginning and after some time stabilizes on a given value.[15,23]

In financial economics literature, it appears several different ways to model the mean-reversion process. The format presented here in Equation 2.2 was studied by Dixit & Pindyck (1994), and is also known as the *Geometric Ornstein-Uhlenbeck* model.[23]

"The model assumes that the relative change over a single period is due to the combined effect of the mean reversion term, $\eta(P - P^{*})\partial t$, and the probabilistic fluctuation component, $\sigma\varepsilon\sqrt{\partial t}$. " Begg and Smit (2007)

$$\partial P/P = \eta(P - P^{*})\partial t + \sigma\varepsilon\sqrt{\partial t} \tag{2.2}$$

Where η is the mean reversion rate, σ the volatility, P* the long term mean value, and ε a standard normal distribution. As for the GBM these parameters can be extracted from current forward/futures prices and historical spot price series. However, due to the comprehensiveness of some of these parameters they will not be reviewed further. For additional reading, see Blanco and Soronow (2001).[23]

The price in the MR is still following a stochastic process like the GBM with an expected trend and a random component. However, it is now not only dependent of the time, but also the present value. As mentioned above, the MR models drift will tend to move towards the long term mean. Depending on if the current value is high or low compared to the mean, the drift will change sign. If the current value is beneath the long term mean, the drift is positive. And if the value is above, the drift is negative[15,23,26]

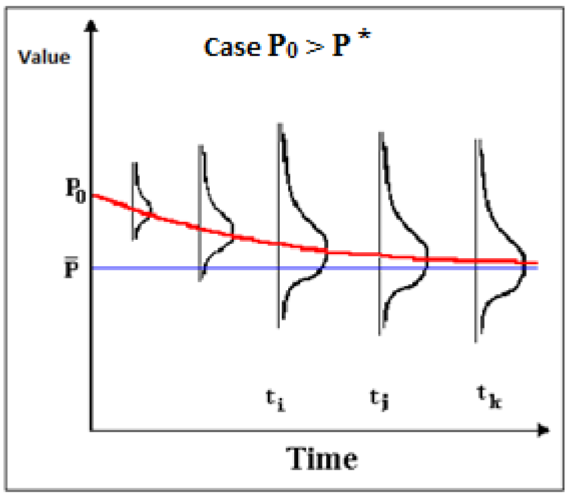

Figure 2.1 - Mean Reversion high price case

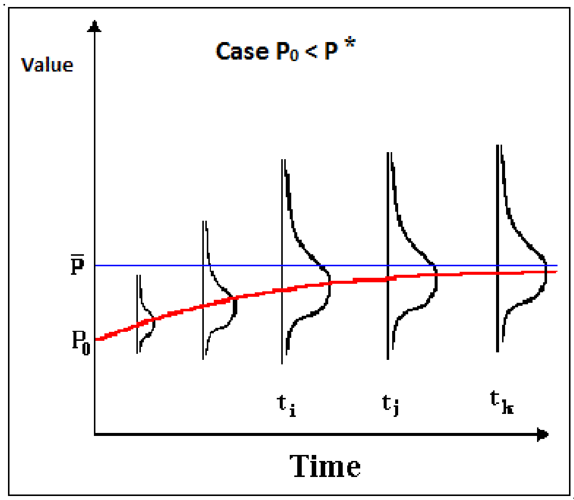

Figure 2.2 - Mean Reversion low price case

Comparing the two models they show a wide difference in both an operational and computational sense. While one of the greater advantages of the GBM model is the ease of estimating the input parameters, this is a more complex and comprehensive task with the MR.[15] Having a quick look at the GBM it is seen that it may not be the best fitted process for modeling oil price. As mentioned earlier, one of the most important features of an oil price model is to describe the characteristics of the price. The GBM fail to capture some of these characteristics, and maybe the main characteristic for oil prices; its tendency to revert back to some long-term mean over time.[23] It assumes that price changes are independent of each other and consequently there is no factor that causes it to tend back towards the mean when deviating. This memoryless characteristic don't tend to be supported by historical prices and therefore favoring the MR model which recognizes this. Also questionable, is whether future price behavior will conform to past behavior, although this also concerns the MR model.[15] The advantages of the MR model have become more and more popular the past decade and managers have been able to assign greater accuracy to their models, or at a least to their model assumptions[23].

2.2.3 Monte Carlo simulation

Monte Carlo simulation (MSC) is a powerful tool for uncertainty and risk analysis, and is used by professionals in such widely disparate fields as finance, manufacturing, research and development, insurance, oil & gas and the environment. It is a computerized mathematical technique that produces probability distributions of possible outcome values. This is done by computing a model containing different input variables and inherent probability distributions. MSC selects one set input variables which generates the outputs. This procedure is repeated numerous of times and each round with new random sampled inputs.[19]

"The input variable distributions do not need to be approximated, because the technique is not limited to the use of theoretical probability distributions or to discrete approximations of continuous distributions. This is important, because there is generally no "right" probability distribution for any variable – we are using probability to quantify our degree of belief in what the actual outcome will be." (Bratvold and Begg, 2010, p. 95)

Modeling of correlation and dependencies is also easily done with MCS software and the most common approach is the rank-order correlation. This technique only requires choosing the two distributions to be correlated and a correlation coefficient between -1 and 1 for modeling of the linear dependency between the ranks of the variables. Two distributions with coefficient 1 are perfectly correlated and will move synchronous. The opposite movement will occur with a coefficient of -1 and if the coefficient is 0 there will be no dependency, and the distributions are considered uncorrelated.[17,20]

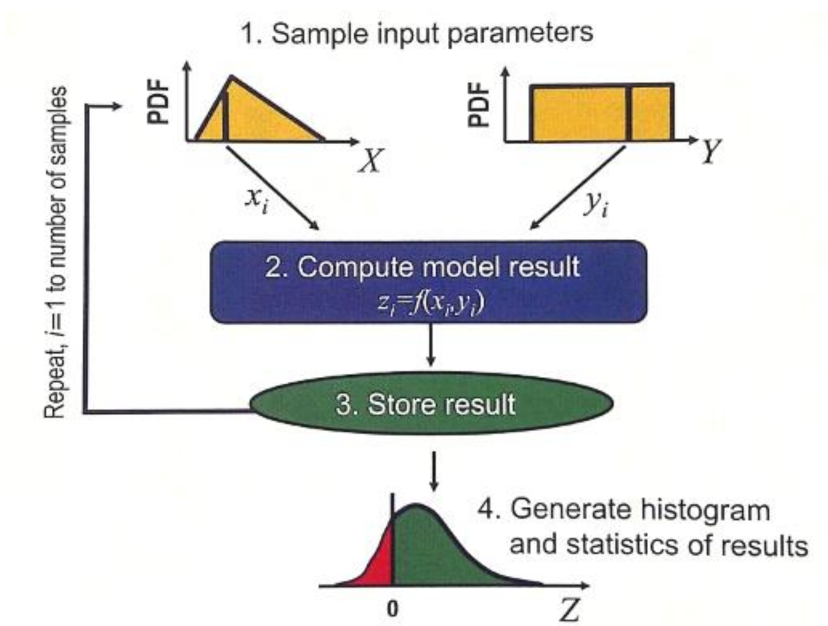

Figure 2.3 - Schematic of the Monte Carlo simulation procedure.
(Bratvold and Begg, 2010, p. 95)

3 Methodology and procedure

To evaluate the theory of this thesis a valuation model in excel has been made. The model is built and fitted for a fictive case and will be the tool used in the analysis of the oil price 'uncertainties effect on the optimal time to carry out blow down(henceforth OT). In the next sections the base case which is used through the analysis is presented before presenting the procedure for analyzing the effect of the oil price. Subsequently the modeling uncertainties and assumptions are explained before finishing with troubleshooting in Excel.

3.1 Example case

The following case presented is a modified study of the numerical example earlier discussed by Brando et al.(2005), Smith (2005) and Willigers and Bratvold (2009). In contrast to these examples it is here investigated an oil field also containing a gas cap. The oil field has estimated reserves of 60 million bbl left. It is at its ending life and has a water cut of 90% increasing by a 0,4% per year. Confining the pressure is a gas cap with estimated reserves of 25 billion Sm^3. The oil and gas price are $90 per barrel and $0.21 per Sm^3. The decision makers are facing a choice either to start blow down, meaning starting to produce gas and ending oil production, or continue to produce oil for one additional year before having the to make the decision over again. Upon the start of gas production a cost of $70 million is assumed for well and platform modifications as perforating, completion, compressors etc. A production stop for 3 months is thus anticipated. In addition there are also variable cost related to the operational oil and gas production of $40 per barrel of oil and $0.21 per Sm^3 of gas, and a $50 million per year fixed cost. Ending the production after x additional years of oil production, and 7 years of gas production requires an additional abandonment cost of $200 million.

3.2 Procedure

When developing a model for decision valuation and analysis it is important to realize what type of options one is to be faced with and the key uncertainties which will have major impact on the outcome of the project. A valuation model should also reflect that projects are subject to uncertainty that resolve over time as well as capture the relationship between the project uncertainties and the markets.[13] As mentioned, to solve the emphasis of this thesis a valuation model in excel using Monte Carlo simulation has been made. The main uncertainties taken into consideration were the production profiles, oil and gas prices, variable operational costs and oil reserves. These are treated as either public or private uncertainty in a risk-neutral framework. The simulation method was chosen for its practicality for modeling point decisions and suitable ability to handle the many underlying uncertainties.[12]

The option to either start blow down and producing gas or to postpone and continue producing oil until the next decision point is a repeating option if it is not exercised. For the purpose of this analysis the option is seen as a possibility over a 6 year period. This is done by composing 6 cash flow cases representing the 6 decision points, where the only difference from one to the other is the change in the gas production profile and one less or extra year of oil production. I.e. cash flow 0(henceforth CF#0) would represent the value of starting to produce gas from the beginning, while CF#1 would represent the value of producing one extra year with oil and then starting to produce oil. Assuming gas production for 7 years, the different cash flow streams would then span over a time period from 7 to 12 years, with CF#0 over a 7 years period of only gas production, CF#1 a 8 years period where producing oil for the first year and gas the next 7 and CF#2 for a 9 years period where producing oil the first two years and gas the next 7, etc. When blow down is undertaken it is assumed that oil production stops immediately due to the pressure drop.

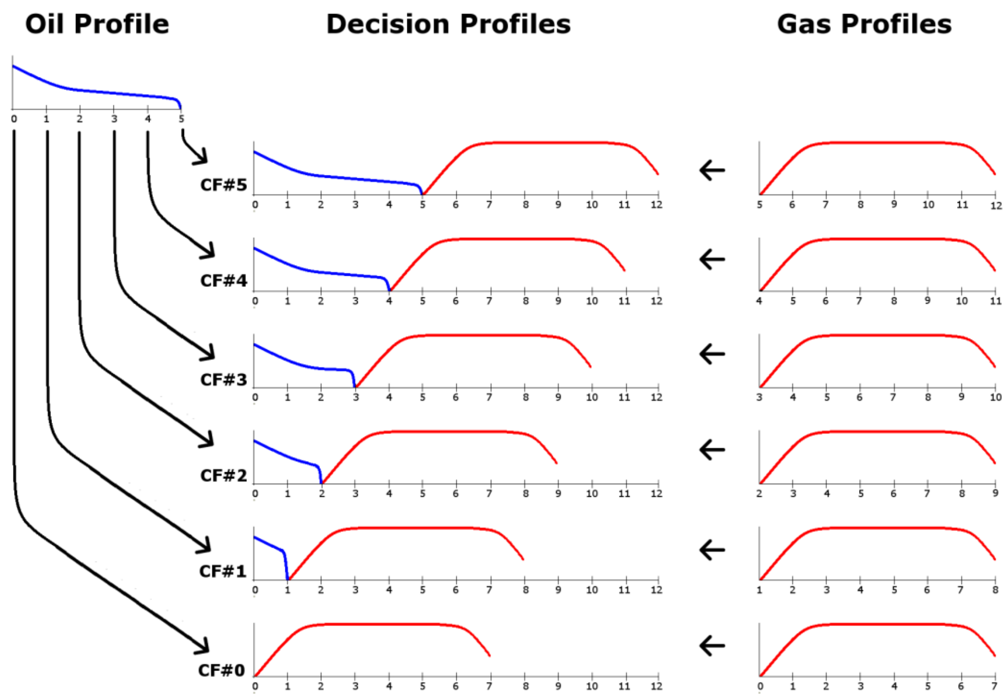

Figure 3.1- Generation of decision profiles for one iteration.

Summarized, one simulation path would generate one oil production profile over a 5 year period where the two first production years would be used for CF#2 etc. 6 gas production profiles; one for each CF, one oil reserve estimation used for all the cash flows cases, and the variable operational costs; one profile for the oil production and one for the gas, each used for all the cash flows. The cash flow streams of each case are then discounted for time and their NPV calculated. The optimal timing for this iteration is then the flexibility having the highest NPV, and by repeating this process for 5000 iterations using the simulation tool @risk, the optimal decision will then be the CF case with the highest excepted NPV. For further analysis of the different parameters effect on the optimal transition point, the model parameters is changed and compared to the each other after 5000 new iterations.

3.3 Model uncertainties and assumptions

In this section a description of the different uncertainties and the underlying assumptions is reviewed. The uncertainties are categorized as market and private uncertainties. Using a risk neutral framework this requires the composition of risk-neutral stochastic processes for the public uncertainties oil and gas price. However, for the remaining private risks which all are assumed uncorrelated to the markets, these do not require risk-neutral probabilities and can be set by an experts ‘judgment. [13]

Oil and gas price

The oil and gas price are clearly market uncertainties and is therefore modeled as risk-neutral Mean reverting Ornstein-Uhlenbeck-processes. This model is chosen for its ability to include the main characteristics of the oil and gas price, following a mean reverting stochastic process[5]. A two-factor model with a drift in the long term mean and a Mean Reversion with Jumps model were also consider appropriate, but not used due to their inconvenient mathematical properties.

To develop a risk-neutral MR process for the oil and gas prices, a risk premium, $\mu - r$, is subtracted from the real drift of the process, $\eta(P - P^*)$. Where, μ is the risk-adjusted discount rate and r the risk free interest rate. This yields the equation shown beneath. For a detailed derivation of the equation, see Dias, M.[26]

$$\partial P = (\eta(P - P^*) - \mu + r)\partial t + \sigma\varepsilon\sqrt{\partial t}. \tag{3.1}$$

The different oil and gas price parameters are listed in table 3.1. These are collected from earlier work done by Willigers and Bratvold (2008,2010)[12,21] Trying to reflect the real world the two commodities were also correlated using @risk and a correlation factor of 0.85.

Table 3.1 – Price model parameters		
Denomination	**Oil**	**Gas**
P -Price (t_0)	\$90 per barrel	\$0,21 per sm^3
P*-Long term mean	\$90 per barrel	\$0,21 per sm^3
μ -Risk-adjusted discount rate	8%	8%
r - Risk free interest rate	5%	5%
σ – Volatility	20%	46,64%
η -Reversion rate [Annual]	34,66%	22%

Variable oil and gas operating costs

The variable gas operational cost is modeled as a standard GBM process, while the variable operational oil cost is modeled as a modified version of the GBM made to compensate for the increasing water production. The modification implies that the annual drift, α, starts at 3% and increases annually 0.5%. The costs are based on the paper of Willigers and Bratvold[12], and the first quarterly rapport of 2011 from Statoil.[27] Differing here is the start costs, which are set $5 per barrel and $0,02 per Sm^3 higher compensating for the late phase of production and the more complicated circumstances.

Table 3.2 – Cost parameters		
Denomination	**Oil**	**Gas**
α - Annual drift	3%	3%
σ - Volatility	10%	10%
Δ α- Drift increase	0.5%	0
P- Start cost	$40 per barrel	$0,1 per Sm^3

Oil production and reserves

The recoverable oil reserves is assigned a PERT distribution with min, most likely and max values of 30, 60, and 90 million barrels and probabilities of 10%, 50% and 90% respectively. This is in line with operators in the industry which often use minimum, expected and maximum values for their estimates.

When simulating one path a possible outcome of the reserves is estimated. This figure is then used to generate a base profile of the oil production. This base production profile stretches over 5 years with an initial production of 10% of the expected reserves and a decline rate of 10% per year. The final production profile of one iteration is then estimated using the normal distribution with the base profile as the expected values and standard deviations of 10% of the base case. So, one iteration generates an estimation of the reserves and a base profile which then generates the end simulation sample path. Table 3.3 and figure 3.3 shows how one simulation path could turn out.

Table 3.3 - Sample path - Oil profile						
Year	**0**	**1**	**2**	**3**	**4**	**5**
Reserves (PERT) [mill. barrels]	50	-	-	-	-	-
Reserves left [mill. barrels]	50	45	40,5	36	32,35	29,07
Generated base profile [mill. barrels]	-	5	4,5	4,05	3,65	3,28
Standard deviation [mill. barrels]	-	0,05	0,045	0,0405	0,0365	0,0328
Simulated profile path	N(μ;σ)	N(5;0,05)	N(4,5;0,045)	N(4,015;0,0405)	N(3,65;0,0365)	N(3,28;0,0328)

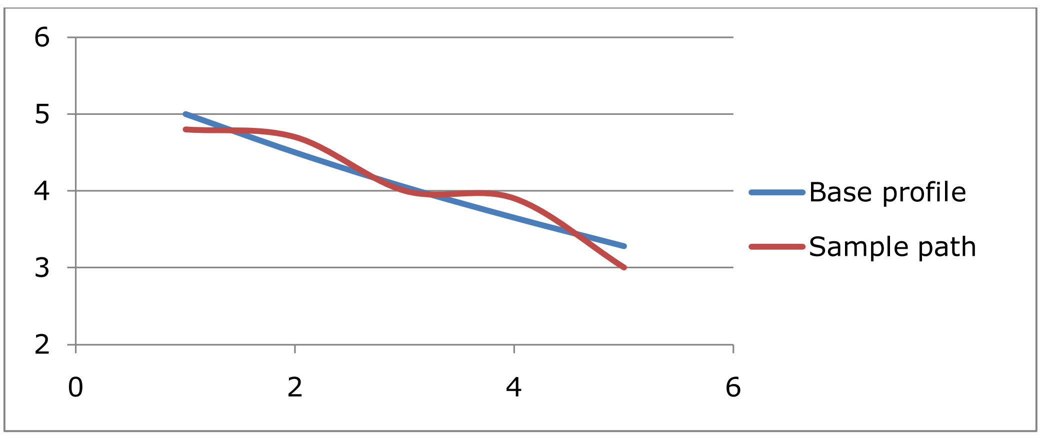

Figure 3.2 - Example oil production profile

Gas production

The gas production profile is built up after the "standard profile" for gas production with a build up rate at the start, going over to a more stable plateau rate for some years and finishing in fast decline. With basis in a production of 2.625, 3.5, 3.5, 3.5, 3.5, 3.5, and 2.5 Sm^3 for the seven years of production, the expected production for the 6 cash flow streams is estimated using a normal distribution with this base as the expected values and a standard deviation of 0.3 Sm^3. In short, one iteration will generate six different profiles using a normal distribution with the basis production as expected value and standard deviation of 0.3Sm^3.

It should be mentioned that the somewhat long build up in the start is set to reflect the 3 months of production stop due to recompletion of the wells. The buildup, normalization and end production for both the base and a sample case can be seen in figure 3.1.

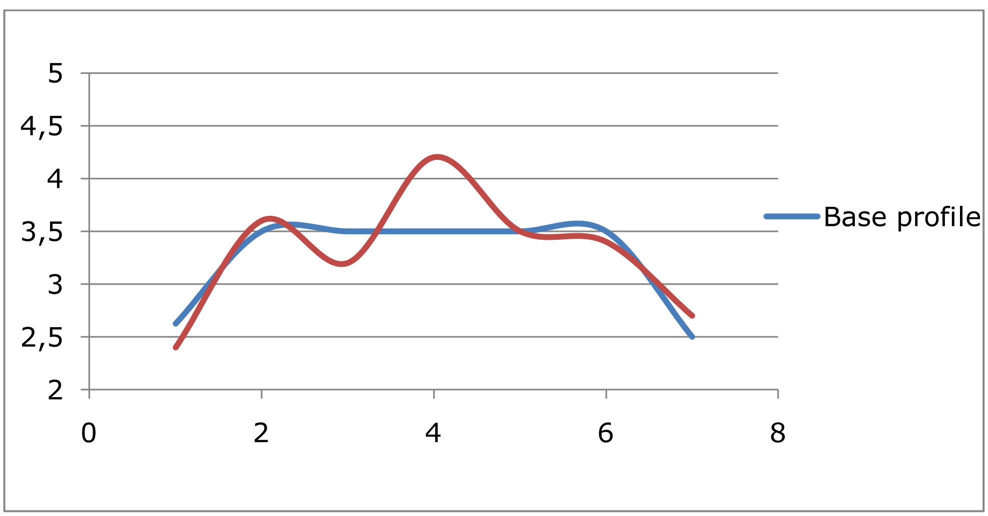

Figure 3.3 - Example path - yearly gas production.

3.4 Troubleshooting in Excel

Ever so careful a built spreadsheet often contains errors. These may come from default references, wrong copying, lack of parameterization etc., and can lead to major faults in the end results. To prevent this different precaution and tests are accomplished. By separating the model and testing extreme values e.g. by setting the oil reserves to zero, the oil production should be the same. Also the built-in Error Checking function and auditing tools for Excel has been used. The Trace Precedents auditing option gives a good visual overview presented by arrows showing the different cells and their dependencies. An error in the model will often appear as an irregularity in the pattern formed.

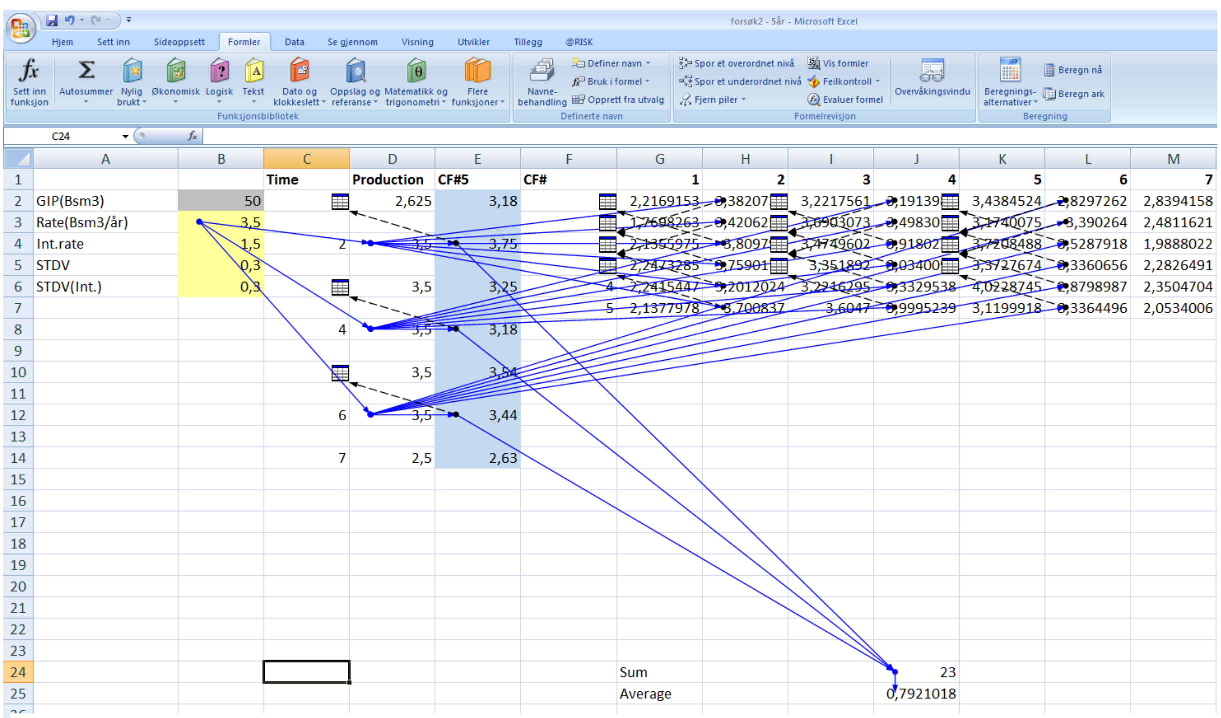

Figure 3.4 – Spreadsheet showing regularity.

4 Results and discussion

To be able to do a thorough analysis of the oil price's influence on the transition from oil to gas production it is critical not only to define the key uncertainties and parameters, but also to analyze their influence on each other and the final outcome. This has been done by using the figures presented in chapter 3 as a standard base and changing the different parameters and comparing the results. All the different parameter combinations have also been simulated with a fixed oil price of $90 per barrel to better view the interactions with the oil price uncertainty. This standard case which from now on will be referred to as the base case is presented with its main model figures in table 4.1. Also, the results which are highlighted with red text and in parenthesis in the tables below, display the OT and Standard deviation respectively.

Table 4.1 – Base case model figures		
Oil and gas price simulation		
Denomination	**Oil**	**Gas**
P -Price (t_0)	$90 per barrel	$0,21 per sm^3
P*-Long term mean	$90 per barrel	$0,21 per sm^3
μ -Risk-adjusted discount rate	8%	8%
r - Risk free interest rate	5%	5%
σ – Volatility	20%	46,64%
η -Reversion rate [Annual]	0,3466	0,22
Variable operating costs		
Denomination	**Oil**	**Gas**
α - Annual drift	3%	3%
σ - Volatility	10%	10%
Δ α- Drift increase	0.5%	0
P- Start cost	$50 per barrel	$0,1 per Sm^3
Other parameters		
Reserves	PERT distributed, 30;60;90 million barrels at 10,50,90%	Fixed 25 billion Sm^3

4.1 Parameter analysis

4.1.1 Oil reserves

With the uncertainty regarding how the recoverable reserves would influence the OT and appurtenant NPV's, this parameter was first analyzed to verify its function and validity.To do this the PERT distribution describing the reserves in the base case was altered to minimum, most likely, and maximum values of 30, 50, 70 and 40, 70, 100 million barrels with the same probabilities of 10/50/90% as for the base case. Further the PERT distribution was substituted with fixed reserves of 60 million barrels. Table 4.2 gives a summary of the generated NPV's.

Table 4.2 – Oil reserves valuation						
Oil reserves – Sim#2.1.2	**CF#5**	**CF#4**	**CF#3**	**CF#2**	**CF#1**	**CF#0**
PERT – 30, 50, 70	1032	1086	1127	1150	**1159**	1146
PERT – 30, 60, 90/Base case	1212	1246	**1260**	1250	1220	1152
PERT – 40, 70, 100	1353	**1369**	1360	1323	1260	1150
Fixed reserves – Sim#1.1.2						
Fixed 60MM barrels	1172	1205	**1225**	1221	1195	1132

Comparing the fixed reserves case with the base case the OT does not differ and is at CF#3 for both simulations. However, comparing the NPV's of the two simulations the base case yields higher NPV's thus showing the importance of catching the upside of the uncertainty in the oil reserves. Also when comparing the three PERT distributions with the increasing oil reserves, as one would expect, it is seen that the OT moves towards more oil production and changes from CF#1 to CF#4. The same changes were also done for other Long term mean and Spot prices, they also supporting these same modes. (See Sim#1.1 and 2.1 in Appendix)

4.1.2 Oil price volatility

To evaluate the effects of the volatility in the oil price several simulations have been run varying both the volatility and the Long term mean and Spot price (See Sim#4 in Appendix). The volatility for the oil price model was also investigated with three Uniform distributions of 10-20%, 10-30% and 10-40%. In general the results do not show any clear consistency and can be summarized by the base case with the different oil price volatilities presented in table 4.3

Table 4.3 – Volatility valuation						
Volatility- Sim#3.2	**CF#5**	**CF#4**	**CF#3**	**CF#2**	**CF#1**	**CF#0**
10% volatility	1200 (1967)	1239 (2005)	**1256** (2048)	1251 (2080)	1216 (2061)	1150 (1979)
30% volatility	1216 (2121)	1251 (2145)	**1263** (2165)	1256 (2150)	1219 (2090)	1151 (1978)
50% volatility	1204 (2479)	1230 (2448)	**1246** (2385)	1239 (2293)	1210 (2162)	1151 (1982)
Base case -Uniform distribution						
Volatility – Sim#4.2	**CF#5**	**CF#4**	**CF#3**	**CF#2**	**CF#1**	**CF#0**
10-20%	1198	1227	**1242**	1235	1206	1142
20-40%	1210	1244	**1261**	1250	1217	1152
30-60%	1203	1237	**1254**	1249	1218	1155

Simulating the base case with a volatility of 10% yielded an OT at CF#3 with the NPV of $1256 million. Increasing the volatility to 30% and 50% further gave the OT of CF#3 for both cases and yielding NPV's of $1263 and $1246 million respectively. For the uniform distributions the volatility was first set to a minimum of 10% and maximum 20%. This setup gave the OT for CF#3 and a NPV of $1242 million. Also in this case when increasing the uniform distribution to volatilities 20-40% and 30-60% CF#3 yields the OT, this time with NPV's of $1261 and $1254 million. Summarized, the six different volatilities did not change the OT compared to the base case nor did they show any consistency in terms of the NPV's.
Comparing the standard deviations show increasing values with increasing volatility for all the outcomes. This relation has been recognized before and can seen as a result of the mere use of a price model that embodies the fluctuations of historical data.[15] However, comparing the cases of 10% and

50% volatility it is seen that for the 10% case the standard deviation tend to increase when going from CF#5 to CF#2 and, opposite decreasing from CF#5 to CF#1 with 50%. This may be interpreted as lower volatility favoring longer oil production in terms of risk and the other way around, high volatility earlier gas production.

4.1.3 Oil price

Analyzing the base case which has a Long term mean and Spot price of $90, one would expect the same case only with a $90 fixed oil price to give a similar answer. However, the results in table 4.4 show that the OT shifts from producing oil for three years to four years and that the fixed price case yields a considerable higher NPV than the base case. This margin is seen repeated for all the fixed oil price simulations (see Sim#3, 4 and 5 in Appendix) and can be described as the impact of the risk premium incorporated in the risk-neutral oil price model yielding lower profits for the oil production and thus causing the shift in OT. Comparing the standard deviations of the two cases it is also seen that this margin is consistent with the base case yielding a little higher values.

Table 4.4 – Fixed oil reserves

Base case	**CF#5**	**CF#4**	**CF#3**	**CF#2**	**CF#1**	**CF#0**
Base case – Sim#3.2	1212 (2000)	1246 (2051)	**1260** (2082)	1250 (2089)	1220 (2072)	1152 (1971)
With fixed $90 oil price – Sim#2.2.2	1316 (1953)	**1328** (1992)	1317 (2041)	1289 (2054)	1237 (2032)	1155 (1944)

Depending on the oil price at the time a decision is made or what it is expected to be in the future it will have large effect on the outcome of the decision. As an example, for the base case the OT is identified for CF#3 generating a NPV of $1260 million, while changing the Long term mean to $100 and Spot price to $90 the OT changes to CF#4 with a NPV of $1324 million. In addition to how we set and estimate the Long term mean and

Spot price, the reversion speed of the oil price to the Long term mean will be decisive. To evaluate these effects on the OT the base case has been altered for the different combinations of the Long term mean and Spot prices of 80, 90 and $100 as well as for the three annual reversion speeds of 20, 35 and 50%. The OT, NPV's and belonging standard deviation for the different combinations and cases are shown in table 4.5

Table 4.5 – Changing oil price and Reversion speed						
Reversion speed, η, 20, 35, 50						
LTM 90-SP 80-Sim#5.1	**CF#5**	**CF#4**	**CF#3**	**CF#2**	**CF#1**	**CF#0**
η -20%	1058 (2006)	1110 (2049)	1143 (2073)	1164 (2086)	**1169** (2060)	1150 (1966)
η -35%	1111 (1977)	1145 (2022)	1166 (2049)	**1172** (2084)	1170 (2055)	1150 (1964)
η -50%	1165 (1971)	1193 (2016)	**1206** (2043)	1204 (2077)	1183 (2067)	1152 (1969)
LTM 90 –SP 90- Sim#5.2						
η -20%	1192 (2031)	1227 (2069)	**1246** (2098)	1239 (2087)	1211 (2054)	1145 (1942)
η -35%	1212 (1992)	1246 (2042)	**1260** (2065)	1250 (2074)	1220 (2045)	1152 (1945)
η -50%	1244 (1959)	1264 (2004)	**1267** (2050)	1252 (2075)	1214 (2049)	1151 (1956)
LTM 90 –SP 100- Sim#5.3						
η -20%	1314 (2093)	1342 (2127)	**1348** (2146)	1320 (2136)	1261 (2083)	1153 (1980)
η -35%	1295 (2046)	1313 (2095)	**1320** (2120)	1306 (2128)	1253 (2082)	1150 (1976)
η -50%	1293 (2029)	1313 (2085)	**1317** (2101)	1300 (2118)	1247 (2077)	1148 (1976)
LTM 80-SP 90-Sim#6.1	**CF#5**	**CF#4**	**CF#3**	**CF#2**	**CF#1**	**CF#0**
η -20%	1118 (2136)	1172 (2176)	1207 (2208)	**1218** (2203)	1200 (2165)	1145 (2037)
η -35%	1068 (2113)	1128 (2143)	1174 (2180)	**1199** (2185)	1194 (2141)	1151 (2037)
η -50%	1086 (2089)	1142 (2128)	1179 (2170)	**1195** (2191)	1189 (2142)	1145 (2028)
LTM 90 –SP 90- Sim#5.2						
η -20%	1192 (2031)	1227 (2069)	**1246** (2098)	1239 (2087)	1211 (2054)	1145 (1942)
η -35%	1212 (1992)	1246 (2042)	**1260** (2065)	1250 (2074)	1220 (2045)	1152 (1945)
η -50%	1244 (1959)	1264 (2004)	**1267** (2050)	1252 (2075)	1214 (2049)	1151 (1956)
LTM 100 –SP 90- Sim#6.3						
η -20%	1263 (2062)	**1278** (2117)	1275 (2169)	1249 (2180)	1206 (2131)	1131 (2039)
η -35%	1323 (2042)	**1324** (2106)	1310 (2164)	1282 (2185)	1226 (2145)	1148 (2050)
η -50%	**1367** (2015)	1361 (2082)	1344 (2145)	1298 (2173)	1237 (2144)	1149 (2031)

Looking at all the simulations at a 35% reversion speed the changes in the Long term mean price,(presented in the lower part of table 4.5), seem to have larger effect on the OT compared to the cases with the changing Spot prices (presented in the upper part). While the OT changes from CF#2 to CF#3 to CF#4 for the Long term means of 80, 90 and $100, the OT only changes from CF#2 to CF#3 for the increment in the Spot price. Further, comparing the case with the Long term mean of $90 and Spot price of $80 against the Long term mean of $80 and a Spot price of $90 both at 35% reversion speed, the former yields the highest NPV's for CF# 5 and 4 with $1111 and $1145 million, while the latter yields the highest NPV's for CF#3-1 with $1174, $1199 and $1194 million respectively. For CF#0 the NPV's are almost equal around $1150 million. These variations can be seen in relation with the MR model's attraction to the Long term mean, and the equally large CF#0's as a result of no oil production. In the same way comparing the CF#'s for the Long term mean $100 and $90 Spot price case against the Long term mean of $90 and Spot price of $100 at 35% reversion speed, the former generates the highest NPV's of $1323 and $1324 million for CF#5 and 4, while the latter price parameters generates the highest NPV's of $1320, $1306 and $1253 million for CF#3, 2 and 1 respectively. However, this attraction to the Long term mean is governed by the reversion speed of the price model and looking at the two cases with reversion speeds of 20 and 50% the effect is clearer. With 20% reversion speed the Long term mean of $90 and Spot price of $100 case yield the highest NPV's for all the CF#'s. In the case using the 50% reversion speed, the Long term mean of $100 and Spot price of $90 case yields the highest NPV's for CF#5, 4 and 3, while the Long term mean $90 and Spot price $100 case yields the highest NPV's for CF#2 and 1. This show large dependency of these three parameters on the outcome of the NPV's.

Looking over the standard deviations of the six simulations it's seen that by increasing the reversion speed the standard deviation of the separate CF#'s tend to decrease. Also, one of the biggest differences between two CF#'s at the same reversion speed is only about 5%. This is between CF#5 and CF#1 for the Long term mean $90 and Spot price $80 case yielding NPV's and standard deviations of $1111 ($1977) and $1170 ($2055) respectively. It should also be registered that for almost every case of the six simulations the standard deviation is at its minimum at CF#5 and increases for each CF before topping around CF#2.

4.2 Summary

Overall it is seen a general dependency between the OT and the changes in the oil price, and with the OT varying in terms with the nature of the oil price model. Changes in the volatility did not show any consistency to the NPV's nor change the OT. However, as one could expect increased volatility generated larger standard deviations for the NPV's. Showing larger effects for the OT was changes in the Long term mean price, however little was detected for changes in the Spot price. Also the reversion speed showed significance in changing the OT, but only when the value of the Long term mean was set higher than the Spot price. Increments in the reversion speed were also recognized for decreasing the standard deviations of the NPV's.

5 Conclusion and Recommendations

It has been created a valuation model in excel using Monte Carlo simulation to analyze the oil
price' uncertainties effect on the optimal timing for transition from oil to gas production. The model has been used for analyzing one specific case and the results presented should therefore be seen as guidelines. Nevertheless, the changes in the oil price show that the optimal timing is dependent on the oil price and its inherent uncertainty.

For further research an interesting extension to this thesis would be to include a more accurately approach to the reserves and production profiles and to examine the approach with a case study.

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Appendix

Sim#1.1				**Fixed Reserves 60MM bbl Oil price 80;90;100**					
Sim#1.1.1									
L_Mean_G	0,21		Oil reserves	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	80								
P_G	0,21								
P_O	80		60	979	1034	1076	1116	1133	1126
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Sim#1.1.2									
L_Mean_G	0,21		Oil reserves	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	90								
P_G	0,21								
P_O	90		60	1172	1205	1225	1221	1195	1132
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Sim#1.1.3									
L_Mean_G	0,21		Oil reserves	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	100								
P_G	0,21								
P_O	100		60	1408	1407,002	1393	1345	1262	1148
V.oil.prod.cost	40								
V gas.prod.cost	0,1								

Sim#1.1 – Analysis of fixed oil reserves with the Long term mean set equal to the Spot price's of 80, 90 and $100.

Sim#1.2		**Fixed oil reserves and oil price**							
L_Mean_G	0,21		Fixed oil price	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	N/A								
P_G	0,21								
P_O	N/A		80	1042	1091	1124	1145	1148	1139
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Reserves Fixed	60MM bbl								
L_Mean_G	0,21		Fixed oil price	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	N/A								
P_G	0,21								
P_O	N/A		90	1291	1300	1291	1256	1203	1127
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Reserves Fixed	60MM bbl								
L_Mean_G	0,21		Fixed oil price	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	N/A								
P_G	0,21								
P_O	N/A		100	1511	1483	1444	1374	1265	1136
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Reserves Fixed	60MM bbl								

Sim#1.2 – Fixed reserves of 60MM bbl with fixed oil price’s of 80, 90 and $100.

Sim#2.1			**Reserves Riskpertalt distrid. 10/50/90% Oil price 80;90;100**						
Sim#2.1.1									
L_Mean_G	0,21		Res. Risk.p.alt	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	80		30-50-70						
P_G	0,21		30-60-90	976	1040	1089	1127	1144	1142,426
P_O	80		40-70-100						
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Sim#2.1.2									
L_Mean_G	0,21		Res. Risk.p.alt						
L_Mean_O	90		30-50-70	1032	1086	1127	1150	1159	1146
P_G	0,21		30-60-90	1212	1246	1260	1250	1220	1152
P_O	90		40-70-100	1353	1369	1364	1323	1260	1150
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Sim#2.1.3									
L_Mean_G	0,21		Res. Risk.p.alt						
L_Mean_O	100		30-50-70						
P_G	0,21		30-60-90	1428	1423	1402	1355	1275	1150
P_O	100		40-70-100						
V.oil.prod.cost	40								
V gas.prod.cost	0,1								

Sim#2.1 - Testing the PERT distributions functioning by varying the distribution parameters and oil price.

Sim#2.2									
Sim2.2.1		**Fixed oil price**							
L_Mean_G	0,21		Fixed oil price	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	N/A		80	1090	1133	1158	1178	1177	1157
P_G	0,21		STDV	1944	1992	2033	2043	2019	1940
P_O	N/A								
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Reserves riskpert 10/50/90%	30-60-90								
Sim2.2.2									
L_Mean_G	0,21		Fixed oil price						
L_Mean_O	N/A		90	1316	1328	1317	1289	1237	1155
P_G	0,21		STDV	1953	1992	2041	2054	2032	1944
P_O	N/A								
V.oil.prod.cost	40			600	622	677	731	772	794
V gas.prod.cost	0,1								
Reserves riskpert 10/50/90%	30-60-90								
sim2.2.3									
L_Mean_G	0,21		Fixed oil price						
L_Mean_O	N/A		100	1544	1520	1476	1401	1293	1156
P_G	0,21		STDV	1975	2013	2045	2055	2028	1944
P_O	N/A								
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Reserves riskpert 10/50/90%	30-60-90								

Sim#2.2 – Evaluating the base case with fixed oil price.

Sim#3									
			Reserves Riskpertdistrid. 10/50/90% Oil price 80;90;100						
Sim#3.1									
L_Mean_G	0,21		Volatility oil	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	80		10	979	1036	1084	1122	1139	1140
P_G	0,21		20	976	1034	1083	1118	1139	1134,978
P_O	80		30	982	1041	1090	1123	1143	1135
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Res. riskpert 10/50/90%	30-60-90								
Sim#3.2			Volatility oil						
L_Mean_G	0,21		10	1200	1239	1256	1251	1216	1150
L_Mean_O	90		STDV	1967	2005	2048	2080	2061	1979
P_G	0,21		20	1212	1246	1260	1250	1220	1152
P_O	90		STDV	2000	2051	2082	2089	2072	1971
V.oil.prod.cost	40		30	1216	1251	1263	1256	1219	1151
V gas.prod.cost	0,1		STDV	2121	2145	2165	2150	2090	1978
Res. riskpert 10/50/90%	30-60-90		50	1204	1230	1246	1239	1210	1151
			STDV	2479	2448	2385	2293	2162	1982
Sim#3.3									
L_Mean_G	0,21		Volatility oil						
L_Mean_O	100		10	1409	1407	1383	1337	1262	1141
P_G	0,21		20	1418	1414,701	1387	1342	1259	1142
P_O	100		30	1404	1403,637	1381	1333	1259	1138
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Res. riskpert 10/50/90%	30-60-90								

Sim#3 – Changing volatilities- presented in the results.

Sim#4		Oil price 80;90;100, Uniform oil Volatility						
Sim#4.1								
L_Mean_G	0,21	Unifiorm Volatility	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	80	10-20%	979	1035	1077	1117	1137	1135,304
P_G	0,21	20-40%	986	1040	1084	1123	1136,647	1139
P_O	80	30-60%	978	1033	1081	1120	1139	1139
V.oil.prod.cost	40							
V gas.prod.cost	0,1							
Res. riskpert 10/50/90%	30-60-90							
Sim#4.2								
L_Mean_G	0,21	Volatilitet olje						
L_Mean_O	90	10-20%	1198	1227	1242	1235	1206	1142
P_G	0,21	20-40%	1210	1244	1261	1250	1217	1152
P_O	90	30-60%	1203	1237	1254	1249	1218	1155
V.oil.prod.cost	40							
V gas.prod.cost	0,1							
Res. riskpert 10/50/90%	30-60-90							
Sim#4.3								
L_Mean_G	0,21	Volatilitet olje						
L_Mean_O	100	10-20%	1398	1397,323	1381	1338	1258	1136
P_G	0,21	20-40%	1390,258	1397	1381	1329	1256	1136
P_O	100	30-60%	1395	1394,349	1378	1333	1257	1135
V.oil.prod.cost	40							
V gas.prod.cost	0,1							
Res. riskpert 10/50/90%	30-60-90							

Sim#4 – Evaluating volatilities in terms of oil price changes.

Sim#5								
Sim#5.1			**Oil price L_Mean_O;90, P_O = 70;90;110**					
L_Mean_G	0,21	Reversion Speed	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	90	20 %	1058	1110	1143	1164	1169	1150
P_G	0,21	STDV	2006	2049	2073	2086	2060	1966
P_O	80	35 %	1111	1145	1166	1172	1170	1150
V.oil.prod.cost	40	STDV	1977	2022	2049	2084	2055	1964
V gas.prod.cost	0,1	50 %	1165	1193	1206	1204	1183	1152
Res. riskpert 10/50/90%	30-60-90	STDV	1971	2016	2043	2077	2067	1969
Sim#5.2								
L_Mean_G	0,21	Reversion speed						
L_Mean_O	90	20 %	1192	1227	1246	1239	1211	1145
P_G	0,21	STDV	2031	2069	2098	2087	2054	1942
P_O	90	35 %	1212	1246	1260	1250	1220	1152
V.oil.prod.cost	40	STDV	1992	2042	2065	2074	2045	1945
V gas.prod.cost	0,1	50 %	1244	1264	1267	1252	1214	1151
Res. riskpert 10/50/90%	30-60-90	STDV	1959	2004	2050	2075	2049	1956
Sim#5.3								
L_Mean_G	0,21	Reversion speed						
L_Mean_O	90	20 %	1314	1342	1348	1320	1261	1153
P_G	0,21	STDV	2093	2127	2146	2136	2083	1980
P_O	100	35 %	1295	1313	1320	1306	1253	1150
V.oil.prod.cost	40	STDV	2046	2095	2120	2128	2082	1976
V gas.prod.cost	0,1	50 %	1293	1313	1317	1300	1247	1148
Res. riskpert 10/50/90%	30-60-90	STDV	2029	2085	2101	2118	2077	1976

Sim#5 – Altering Spot price and reversion speed

Sim#6									
Sim#6.1				**Oil price L_Mean_O 80;90;100 P_O = 90**					
L_Mean_G	0,21		Reversion speed	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	80		20 %	1118	1172	1207	1218	1200	1145
P_G	0,21		STDV	2136	2167	2208	2203	2165	2037
P_O	90		35 %	1068	1128	1174	1199	1194	1151
V.oil.prod.cost	40		STDV	2113	2143	2180	2185	2141	2037
V gas.prod.cost	0,1		50 %	1086	1142	1179	1195	1189	1145
Res. riskpert 10/50/90%	30-60-90		STDV	2089	2128	2170	2191	2142	2028
Sim#6.2									
L_Mean_G	0,21		Reversion speed						
L_Mean_O	90		20 %	1192	1227	1246	1239	1211	1145
P_G	0,21		35 %	1212	1246	1260	1250	1220	1152
P_O	90		50 %	1244	1264	1267	1252	1214	1151
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Res. riskpert 10/50/90%	30-60-90			51	65	73	71	55	23
				-28	-11	10	24	27	2
Sim#6.3									
L_Mean_G	0,21		Reversion speed						
L_Mean_O	100		20 %	1263	1278	1275	1249	1206	1131
P_G	0,21		STDV	2062	2117	2169	2180	2131	2039
P_O	90		35 %	1323	1324	1310	1282	1226	1148
V.oil.prod.cost	40		STDV	2042	2106	2164	2185	2145	2050
V gas.prod.cost	0,1		50 %	1367	1361	1344	1298	1237	1149
Res. riskpert 10/50/90%	30-60-90		STDV	2015	2082	2145	2173	2144	2031

Sim#6 – Altering the long term mean and reversion speed

Sim#7				**Gas price L_Mean_G = 0,21 P_G = 0,18;0,21;0,24**					
Sim#7.1									
L_Mean_G	0,21		Gas volatility	CF#5	CF#4	CF#3	CF#2	CF#1	CF#0
L_Mean_O	90								
P_G	0,18		46,64 %	1141	1148	1133	1084	1000	867
P_O	90								
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Res. riskpert 10/50/90%	30-60-90								
Sim#7.2									
L_Mean_G	0,21		Gas volatility						
L_Mean_O	90		20,00 %	1205,51	1233,919	1245,846	1238,935	1205,88	1140,112
P_G	0,21		46,64 %	1208,103	1236,141	1240,027	1229,602	1195,026	1131,817
P_O	90		60,00 %	1208,181	1233,476	1234,777	1225,406	1186,427	1127,517
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Res. riskpert 10/50/90%	30-60-90								
Sim#7.3									
L_Mean_G	0,21		Gas volatility						
L_Mean_O	90								
P_G	0,24		46,64 %	1256	1303	1337	1361	1375	1371
P_O	90								
V.oil.prod.cost	40								
V gas.prod.cost	0,1								
Res. riskpert 10/50/90%	30-60-90								

Sim#7 – Testing the gas price volatility