An Analysis of Commodity Price Dynamics with Focus on the Price of Salmon

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Preface

This thesis is submitted in partial fulfilment of the requirements for the degree of *Philosophiae Doctor (PhD)* at the University of Stavanger, Faculty of Science and Technology, Norway. The research has been carried out at the University of Stavanger from January 2007 to December 2009, except from August 2009 to December 2009 where I spent a semester at Duke University, Nicholas School of the Environment.

The main contributors to this thesis have been my supervisors Frank Asche and Ragnar Tevterås, in addition to my college, fellow PhD student and co-author on one paper, Marius Sikveland. I wish to extend a special thank to these very resourceful academics, colleges and friends. For my supervisors, Frank Asche and Ragnar Tveterås I am especially grateful for their patience, allowing me to focus on the fields of economics closest to my interest, and always with critical, insightful and friendly advice guiding this project to its completion. I would also like to thank my colleagues at the University of Stavanger for providing a memorable and enjoyable three years. For my semester at Duke University I would like to thank Professor Marty Smith and his PhD students at the Nicholas School of the Environment for taking an interest in my research and providing the resources and insights to further improve my research.

Contents

Summary

This thesis is concerned with studying the short run dynamics of commodity prices. The industry of interest and primary study is Norwegian Aquaculture, with the price of farmed salmon as the main data-set. Even though most of the cases studied are related to the salmon market, it is my hope that some of the insights and results can be applied to a more general set of agricultural or conventional commodities.

This thesis falls in line with a large collection of research papers and thesis' on the Norwegian aquaculture industry. Motivated by dissecting what has largely been a highly successful growth industry, coupled with availability of detailed high quality data, a great deal of economic research on the industry has been conducted. Much of this research is related to long run supply side effects. A large low frequency panel data set has laid the ground for successful economic research into amongst other productivity effects in the industry. Due to a lack of high frequency data, the short run effects have been less studied. The only reliable high frequency data available is price data. This thesis contributes to the body of research on the industry by focusing on the short run price dynamics of the commodity. In addition to studying short run effects, the thesis introduces tools originally used in finance to study the price data. Incorporating both traditional economic analysis and finance is relevant when doing short run price analysis, and provides an alternative angle for looking at the commodity market.

Due to the lack of detailed high frequency data on state variables other than price, the thesis applies non-structural time series analysis as the method for empirical analysis. This necessarily restricts direct inference of causality relationships. However, non-structural time series analysis provides a large battery of models to reliably and thoroughly describe the dynamics of the series studied. The detailed output from time series models are used in combination with knowledge of predictable relative changes in underlying state variables to understand the market dynamics.

1. Introduction

The variations of prices around their long-run trends define their volatility. These short-run fluctuations are generally dominated by noise. Noise in the observed data can enter from exogenous sources, such as weather/climatic effects, or from measurement errors. Despite noise being a major part of short run fluctuations, we often observe persistent and deterministic patterns. In economic theory, such patterns are predicted to arise from the speculative property of the commodity. The ability to store and speculate on the future value of the commodity allows economic cost variables to enter the short-run dynamics of prices.

The general economic theory proposing to explain short-run dynamics of commodity prices is the theory of storage (Kaldor, 1939; Working, 1948, 1949; Brennan, 1958). As stated by Fama & French (1987), the theory is not controversial. Its initial strength is its simplicity and use of tangible economic variables as explanation variables. The theory of storage states that in order to carry a commodity forward, the carrier's needs to be compensated for the cost of storing and the alternative cost of capital. This compensation comes in the form of expected price changes. In its general form, the theory gives an intuitive explanation to observed patterns in short run dynamics.

Commodity markets are heterogonous. In its general form the theory of storage is to general to provide detailed predictions on all commodity markets. Being the main economic variable, the cost of storage needs to be specified for the particular commodity market analysed. Assuming, for example, a constant positive cost of storing fails to explain certain properties observed in commodity prices, such as backwardation in futures markets (Deaton & Laroque, 1996). Each commodity market has a specific cost of storing function. The perishability of the commodity, the flexibility of changing production scale, the exogenous shocks to stocks, the use of the commodity (input value, multiple outputs) and seasonal effects all contribute to the cost of storing function. This necessitates a unique specification of the function for each commodity market analysed. These specifications allow use of the theory of storage as an analytical tool in interpreting output from empirical models where statistical causality inference is hard or impossible, such as non-structural time series models.

This thesis investigates the dynamics of commodity prices with main focus being on the price of Norwegian farmed salmon. The theoretical background for my analysis is the theory of storage. I use the theory of storage and its market specific modifications as an analytical framework to interpret output from the empirical analysis.

My method of empirical analysis is non-structural time series analysis. I focus not only on statistical fitting, but also on how empirical models needs the flexibility to accompany the dynamics predicted by economic theory. Modern time series analysis provides a battery of flexible empirical models. Many of these models originate from empirical work in finance. Since commodity price dynamics in the short-run, to some degree, emulate the price dynamics of financial assets, the application of financial asset models in commodity market analysis seems valid. The main difference, in model application, is that commodity models need to accompany the effect of non-exogenous changes in stocks.

Non-structural time series analysis using prices alone disallows direct causation inference between price and other state variables. To arrive at statistical satisfactory models, where direct inference is possible, we need a detailed data-set of state variables. For many commodity markets, such data-requirements are unfeasible. In addition, a clear definition of the representation of state variables is not always apparent. If we, for example, define an inventory as a reserve of market ready stocks, it is not straight forward how this state variable should be constructed. Storage in agriculture, for example, can be done by delaying harvest. If we ignore direct inference of causation relationships, modern time-series analysis provides a detailed descriptive representation of commodity prices.

The descriptive representation of prices provides a strong indirect inference framework for evaluating predictions made by the theory of storage. One might argue that the lack of direct inference is compensated by the greater availability of price data. In order to analyse the output of non-structural time series models in the light of economic theory, the models need to be flexible enough to accommodate the dynamics predicted by theory. A consistent price series model should therefore be sensitive to changes in state variables.

Even though exact quantitative representations of state variables are unavailable, we often have information on relative structural changes in state variables. For example, regular demand shifts or growth patterns in stock variables provide information on predictable changes in state variables. If the price model is sensitive to such state variable changes, the model can be used to derive indirect inference of predictions from theory. Combining time series analysis and knowledge of changes in market fundamentals provides an indirect means to gain insight into the driving processes' of price determination. I focus on using time series analysis in combination with knowledge of market fundamentals to analyse the dynamics of commodity prices.

1.1 ESSAYS OF THE THESIS

In the thesis' first paper, *The Behaviour of Salmon Price Volatility,* written in collaboration with Marius Sikveland, we provide an analysis of salmon price volatility. Salmon prices exhibit substantial volatility. An understanding of the structure of volatility is of great interest since this is a major contributor to economic risk in the salmon industry. The volatility process in salmon prices was analysed based on weekly price data from 1995 to 2007. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model was used to test for volatility clustering and persistence for prices. We find evidence for and discuss the degree of persistence and reversion in salmon price volatility. Further, we find that volatility is larger in periods of high prices. For the industry this means that larger expected profits more often than not comes at a trade-off of larger price risk.

In the second paper, *Supply Side Explanations for Volatility Spill-Over in Primary Commodity Prices*, I seek to use storage theory to explain volatility spill-over patterns in commodity prices. I suggest that correlations in inventory stocks and differences in inventory flexibility, as well as common price deterministic factors such as input factor usage and substitutability between goods, allow non sporadic volatility spill-over to persist in primary commodity markets. To investigate these claims empirically, I apply a volatility spill-over test to a basket of goods from the Norwegian aquaculture industry, where differences in characteristics of goods allow examination of the economic significance of volatility spill-over.

In the paper *Regime Shifts in Commodity Prices with Application to the Price of Salmon* I apply a flexible regime shifting model to the price of salmon in order to accommodate the non-linear dynamics predicted by the Theory of storage. Both commodity price theory and statistical evidence suggests that the underlying price processes of commodity prices are non-linear in dynamics. By allowing both mean and variance parameters to change between states we find evidence that underlying skewness and kurtosis in residuals arising in linear models disappear. I further argue, based on the theory of storage that a two-state regime shifting model is a suitable price model for commodity prices. Theory predicts that as underlying fundamentals such as stock sizes change, the persistence and volatility of prices will change. Using the regime shifting model we are able to indirectly test predictions made by the theory of storage. The state probabilities emerging as a product of regime shifting models can provide a basis for examining theoretical predictions on commodity markets. For the case of salmon, I find that there are seasonal patterns in volatility in addition to industry profitability conditions affecting the emergence of volatility regimes.

In the final paper *Stochastic Long Run Cycles in Agricultural Commodity Prices,* written in collaboration with Frank Asche, we analyse the degree of stochasticity of long run cycles in commodities using an expansion of the HEGY unit root test. In a rational speculative agricultural market long run predictable cycles not founded in cost shifting factors should not be allowed to persist. None the less arbitrage possibilities have documented in long run cycles of for example hogs, broiler and cattle. If cycles are sufficiently stochastic the cost of identifying and reacting to the perceived cycles will increase to the degree of allowing some residual cyclicality in the markets. As such realised cycles and arbitrage will only be reasonably identified following the cycle completion. During the cycle movement persistence is allowed due to its stochasticity. In this paper we propose two approaches to examining the stochasticity of long run cycles in agriculture. The first approach depends on the notion of duration dependence in cycles and is a non-parametric index to test for convergence in cycles. The second approach is based on the realisation that most commodity prices are non-stationary and near-unit or unit root. We expand on the classical seasonal unit root test to test for existence of unit roots outside the seasonal frequencies.

My main data-set in this thesis will be weekly observations on the price of Norwegian farmed salmon from 1990 to 2007. I also use monthly and annual observations on frozen salmon, hogs, wheat, corn, poultry, eggs oil and gold. Since my main data-set is the price of Norwegian farmed salmon I will give a summary of the Norwegian Aquaculture industry. In the summary my main focus will be on discussing state variables which determine the price of salmon. Following this, I move on to a general illustration of the economic theory of commodity price dynamics before I move on to the main time-series methods used in this dissertation.

2. THE NORWEGIAN SALMON FARMING INDUSTRY

In 2008 Norway accounted for 51% of farmed Atlantic salmon production. Total production in 2008 was approximately 754 000 tonnes at a value of 18 billion NOK. Norway is today the largest exporter of farmed Atlantic salmon, and it is one of Norway's biggest exporting industries. The main product form is fresh whole salmon accounting for 77% of the industries revenues in 2008 (figure 1).

The trend in the industry has been one of fewer and larger companies (Guttormsen, 2002; Tveteras, 1999; Tveteras, 2000). In 2008, 186 companies and 929 production licenses where registered, the same statistics for 1999 on the other hand show 467 companies and 799 licenses.

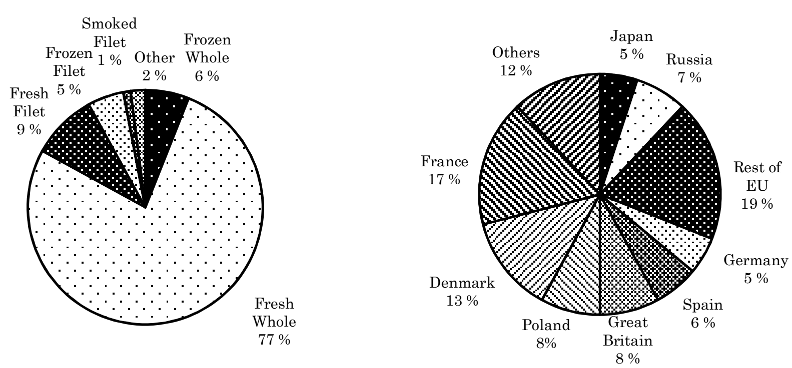

FIGURE 1. Market shares for different types of salmon products and main export markets, 2007. Source: Norwegian Seafood Export Council.

Despite the consolidation of the salmon industry it is still a fairly competitive industry, with many medium and small companies representing a large share of the production.

The long Norwegian coastline offers an array of potential farm locations, providing farmers with potential hedging of production risk as correlation of farm specific risks, such as disease outbreaks and temperature fluctuations, decreases over geographical distance (Oglend & Tvetras, 2009). At present salmon farms are located along most of the Norwegian coastline.

The main export market for Norwegian salmon is the European Union, with France and Poland as the two biggest EU markets. Besides EU, Russia has emerged as an important market. Followed by Japan these two markets remain the two largest outside of the EU (figure 1).

2.1 SALMON AQUACULTURE AND THE FISHERY SECTOR

Currently salmon farming is the most valuable single sector of Norwegian food production. The industry remains profitable and competitive in international markets without subsidies and regulatory assistance from the central government. From its beginnings in the 1970's, the industry has evolved positively both in terms of production output and market value. Figure 2 illustrates how production output and market value has increased steadily over the last twenty years.

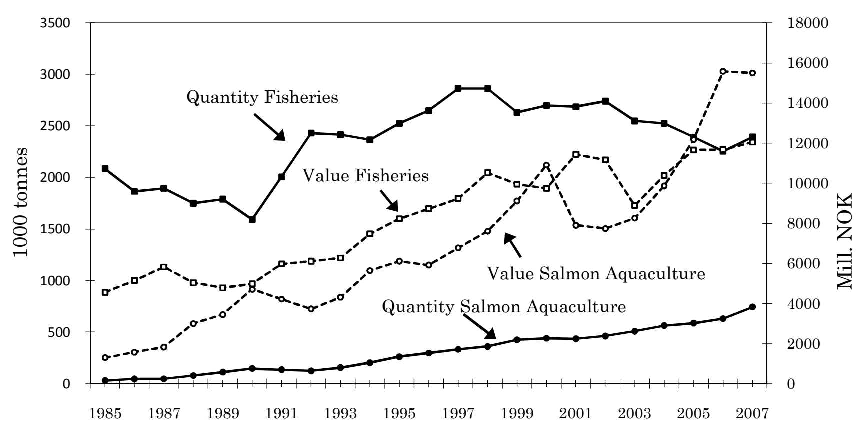

FIGURE 2. Developments in quantity harvested and value of Norwegian fisheries and Norwegian salmon aquaculture. Source: SSB.

By comparing aquaculture to the traditional fishery sector we observe that fisheries output has remained relatively stable, while salmon production has experienced a consistent positive growth. Despite salmon having a considerably lower output quantity, the sales value surpassed the fishery sector in 2005, and has remained considerably above it ever since.

From figure 2 we can observe another important difference between the sectors. In comparison to the fishery sector, the fluctuation in value relative to production output is considerably higher in aquaculture. This suggests that price has been a major source for fluctuations in industry value; an issue which will be discussed further in the thesis and remain an underlying motivation for my work.

What separates aquaculture production from fisheries is the degree of control over production and output (Anderson, 2002). The intensive production process characterising Norwegian salmon farming allows a closed production cycle with a controlled feeding process. This high degree of control enables the aquaculture industry to provide a stable supply of fish. A stable supply has provided the aquaculture industry with a major advantage in expanding markets to include super-market outlets, and has allowed a steady supply and innovation of processed salmon products (Bjørndal, 1990; Tveteras, 2000; Guttormsen, 2002; Vassdal 2006). In an industry with substantial productivity growth this has provided an outlet for market growth, where positive shifts in both the demand and the supply curve has allowed persistently decreasing prices and on average positive, yet very volatile, returns (Asche, 1997).

2.2 THE SALMON PRODUCTION CYCLE

The strategy for successful aquaculture production lies in first replicating the species life in nature, and then improving conditions such that both productivity and stability of output can be increased (Asche, 2008). In salmon farming there are two major production steps; the production of smolt and the cultivation of smolt to harvest ready fish.

At the first stage in the production chain, brood stock is kept in order to produce offspring. When the salmon offspring hatches they are raised on land in fresh water tanks. The fry are kept in tanks and fed for 12 to 18 months until they reach the smolt stage. At this stage the fish is ready to be transferred to sea.

In the final stage the fish are kept in salt water pens where they are fed until reaching harvest ready weight, this process usually takes from 12 to 24 months. At this stage that most of the growth of the salmon occurs. The smolt transfer to sea is, due to biological factors, done in the months of March to October. One "generation" of fish is transferred either in the spring or the fall segment of the transfer window.

Salmon growth is dependent on the sea temperature. Growth is highest during mid to late summer. Since only the spring transfer will benefit from the increased growth period during its first months in sea, most small fish is available during the spring to early summer, before the major growth period. For the larger fish most harvesting is done during the summer, due to larger fish having an increased probability of reaching sexual maturity in late summer. When the salmon reaches sexual maturity the quality of fish severely decline (Asche & Guttormsen, 2001).

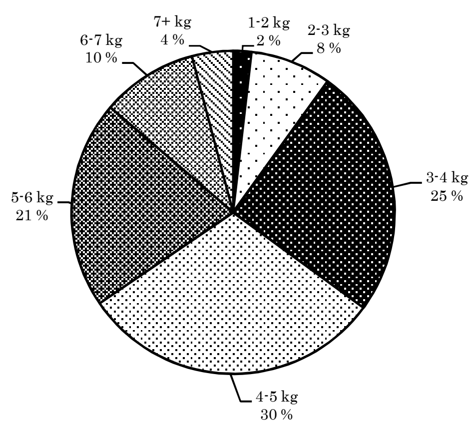

FIGURE 3. Average harvest weight for salmon, 2007. Source: Norwegian Seafood Export Council.

From figure 3 we observe that in 2006, 76% of the harvested fish was in the 3-6 kg range. This implies that the aggregate stock of sellable fish is highest during the fall to late winter, where both the latest spring transfer and second to last fall transfer reaches this weight class. This is also supported by the facts that demand for salmon increases at Christmas and Easter, leaving less available fish for the early summer months.

Another way of looking at this is to think of the potential growth of salmon as an alternative cost of harvesting. By harvesting in the spring the immediate growth period is sacrificed. This suggests that the convenience yield for keeping stocks in the pens is greater in the spring. Knowing the seasonal dynamics of stock development is important in accounting for the short term dynamics of price. This subject will be more thoroughly discussed later and emerges throughout the thesis.

2.3 INDUSTRY TRENDS

Since its start in the 1970's, the Norwegian aquaculture industry has experienced a persistent growth in productivity, as can be seen in the real production cost per kg/Salmon (figure 4). These improvements in productivity have consistently created incentives for new companies to enter the market, in addition to increased production amongst incumbent companies (Asche, 1997). This has contributed to increases in aggregate supply and declining prices. We observe a clear long-run downward trend in prices explained by a persistent decline in production costs (figure 4) until 2000.

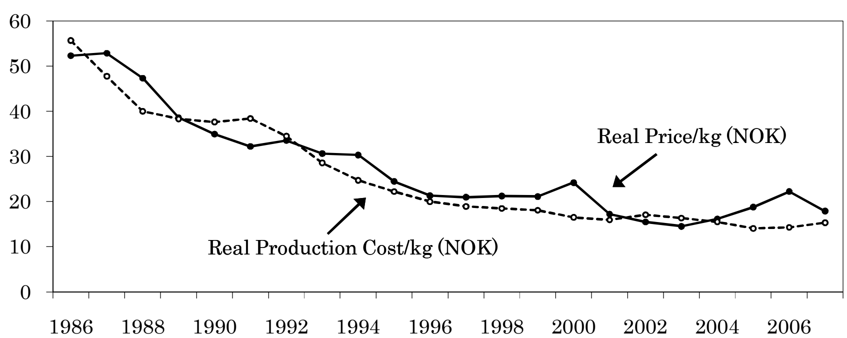

FIGURE 4. Developments in real price and production cost per kg. Salmon in Norwegian Aquaculture. Source: Norwegian Seafood Export Council.

The total effect of increased productivity can be explained by several factors. One important reason for increased productivity is the consistent improvements in fish health; allowing lower losses due to fish death (Asche, 1999). Related to improved fish health are improvements to feed and feed-technology (Asche et al., 1999; Guttormsen, 2002). The feed-factor (the relationship between feed usage and salmon growth) has decreased from about 3 during the early nineties to just above one. Despite feed costs accounting for close to 60% of production costs today (20 % in 1985) the cost of feed per kg/fish has been reduced by 29% since 1985. Another main contributor to declining costs is the decline in labour related costs. Inflation adjusted labour costs declined from 9.55 NOK/kg. in 1986 to 1.33 NOK/kg in 2007. Further the cost share of labour dropped from 15% in 1986 to 9% in 2007. A final reason for decreased production costs is an increase in capacity utilization. As a proxy for better capacity utilization the inflation adjusted depreciation cost of capital per kg. salmon produced has dropped from 2.59 NOK in 1986 to 0.75 NOK in 2005 (Tveteras & Guttormsen, 2007).

2.4 STOCK AND PRICE DYNAMICS

The main interest in this thesis is the price dynamics of commodities. It is therefore of interest to examine how important state variables, such as live fish stock size and consumer demand, have affected both industry returns and the price of salmon. In this introductory analysis we use graphical representations of state variables.

For Norwegian salmon aquaculture we notice that despite a persistent decline in production costs per kg/salmon, the return per kg fish is very volatile. The declining production costs can account for the long run decreasing trend in the price for salmon.

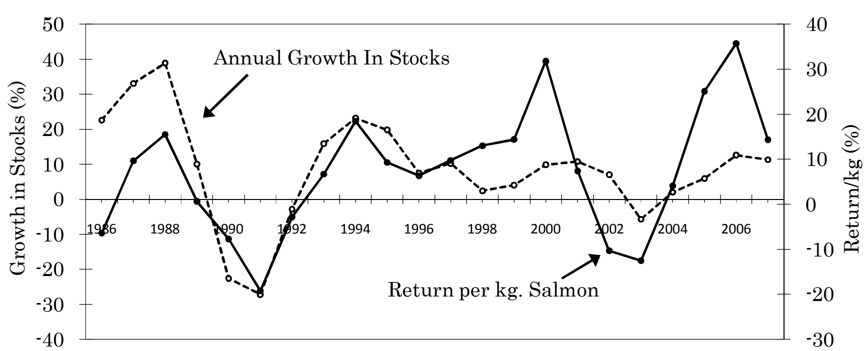

FIGURE 5. Development in Real Return per kg. salmon and the Annual change in Stock Size as measured by outstanding fish. Source: SSB.

However the price volatility; accounting for much of the fluctuations in returns, is not explained by variations in production costs.

In figure 5 we observe both the annual change in fish stock, as measured by quantity of fish, and the return per kg. salmon. As the figure illustrates, returns fluctuate quite drastically throughout the period. The mean return in this period is approximately 7.4%. Looking at annual changes in stock size, this variable is constructed as:

$$\frac{stock(t+1)-stock(t)}{stock(t+1)}=\frac{newfish(t)-sales(t)-loss(t)}{stock(t+1)}$$

The stock size is measured as quantity of fish, and as such does not account for outstanding biomass. However, the stock variable contains information on future availability of sellable fish. In figure 5 we see that stock growth follows return in an apparently pro-cyclical manner. In a non-stochastic production environment this relationship could be explained by an inverse relationship between stock and sales growth, generating a pro-cyclical relationship between output price and stock-growth (growth in terms of number of fish). In a stochastic production environment, such as aquaculture, this relationship does not hold absolutely. Stock increases can be the result of favourable output developments. In such cases sales do not have to decrease to increase stocks. The stochastic output also affects returns through effective production costs.

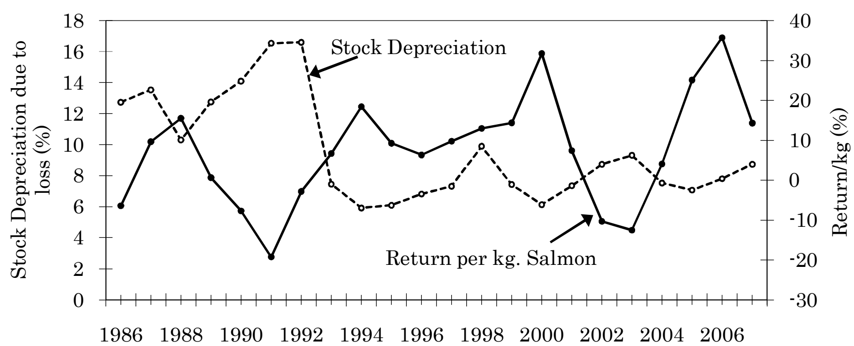

FIGURE 6. The development in real return per kg. salmon and annual stock depreciation by loss of fish. Source: SSB.

In example, the large negative return in 1991 can be explained by a large loss of fish (figure 6), and as such a severe reduction in stock size.

The stock loss caused effective production costs per kg salmon to increase and returns to be negative. Following the large reduction in stock size, price increased (figure 7) and return peaked again in 1994.

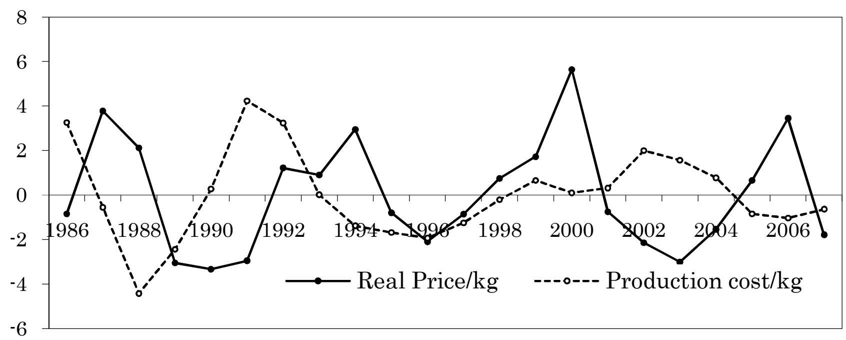

FIGURE 7. Real Price and Production cost per kg Salmon adjusted for linear and quadratic trend. Source: SSB.

In an industry where time to replace a lost stock is high, it is natural that high prices are allowed to persist as stocks are being rebuilt. The entire process of producing a salmon of sellable weight takes from 2 to 3.5 years. This allows long-run transfer effects from stock to prices.

In 2000 and 2006 peaks on returns emerged explainable by cyclical movements in price. Again, this cyclicality can be traced back to movements in stock size, and thus appear to have a purely stochastic causation. Both two years prior and two and three years after the peak in price and returns, a positive shock to loss occurred (figure 6). As stated, these losses are likely to have had lagged effects on price movements as losses have persistent effects on stock sizes. The impression that this cyclical movement is deterministic due to its apparent regularity in occurrences seems to be faulty; they seem merely to be the result of a coincidentally spaced loss in stocks.

For short-run dynamics stock developments are important in accounting for movements in price and return. In terms of control of production output it is likely that price and return volatility will decrease as stock stability increases. Further, how producers react to loss in production is fundamental in how prices move. In figure 8 we see how the ratio of new stock to sales moves with stock loss.

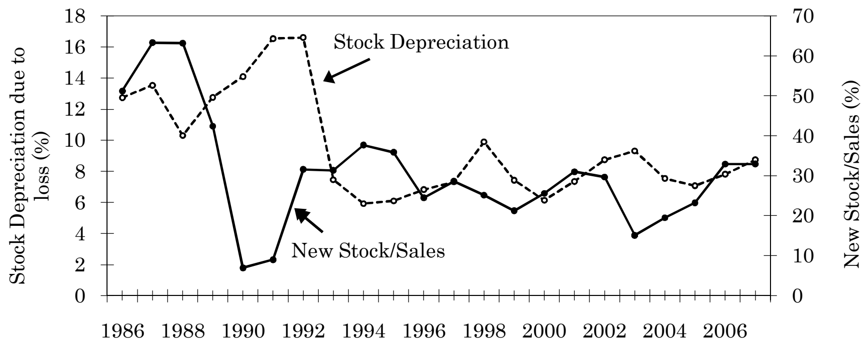

FIGURE 8. Developments in Stock Depreciation and percentage of annual sale relative to new stock. Source: SSB.

A low value of new stock to sales suggests that stocks outside what is lost are being reduced to keep up supply. The immediate effect of this is to counter spiking in spot prices. As stocks are reduced, expected prices will increase as stocks are not immediately replaced. A high loss and a low new stock to sales value is an indicator that prices will increase. The stock to sales ratio is a control variable for producers, equivalent to the storage variable.

The flexibility in this control works as an immediate opposing force to stock loss on price effects. In a forward looking market the management of new stock to sales can be used to buffer price movements, and to exploit arbitrage possibilities If this variable was held constant, short term price movements would be directed purely by the loss variable under constant structural demand. In this case an improvement in technology to reduce loss would have the effect of reducing spot price volatility.

In the industry today there are indications that the flexibility in the new stock to sales control variable is reduced. The reason for this is the markets demand for a stable fish supply. Large super-market chains require a predictable and stable supply of fish. As such the flexibility of producers in changing short-run supply to counter short-run effects are reduced. In terms of price volatility there are

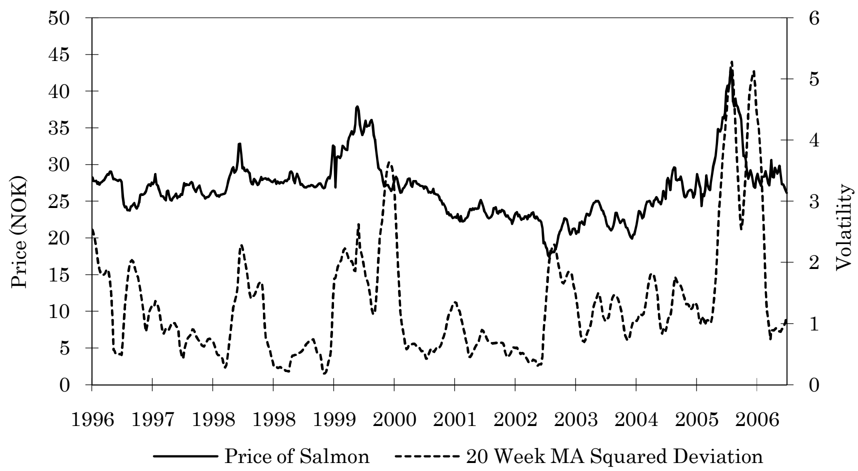

FIGURE 9. Weekly observations of salmon price and a 20 week moving average squared deviation of prices.

opposing forces on the determination of future spot price volatility. In figure 9 I have plotted the weekly price of salmon and a measure of price volatility. For the volatility measure we observe that spot price volatility move in clusters with periods of high volatility. In terms of evolution of volatility there is no indication that volatility has been reduced over the years.

2.5 SEASONAL EFFECTS

The quantity of salmon sold (figure 10) has been steadily increasing over the years. Atlantic salmon has moved from a niche luxury commodity to a bread and butter fish available in major super market outlets. Figure 10 illustrates the seasonal patterns in sales. Traditionally, a major increase in sales occurs at Christmas when demand increases.

A lesser demand surge also occurs at Easter. Following the Easter season, sales usually flattens out in the early summer and starts picking up again in the fall, before peaking at Christmas. The major demand shifts are predictable through-out the season, and should have little effect on price movements outside of cost changes related to shipping and changing stock sizes.

In summer months the temperature in the North-Atlantic increases, providing better conditions for the salmon to grow. The availability of fish prior to the growth period depends on the preceding seasonal demand, fish health and status of production facilities.

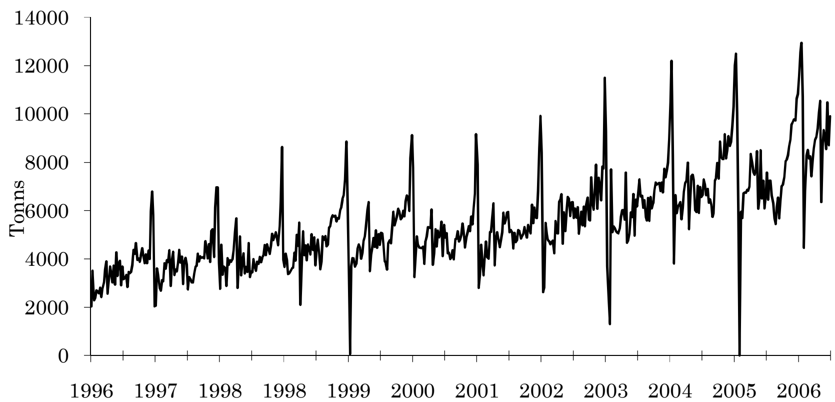

FIGURE 10. Development in quantity of salmon sold.

Prior to the summer growth the stock is comparatively dominated by smaller fish. Having a sizable stock in this period is valuable. Reducing the stock by harvesting comes at a cost of sacrificing the benefit of the higher growth to come. This higher alternative cost of harvesting generates a convenience yield on remaining stocks. A higher convenience yield necessitates that price must increase for slaughtering to occur. The producer needs to be compensated for the sacrificed growth period. In figure 11 I averaged the price and quantity sold over weekly observations to more easily illustrate the seasonal movements in price and quantity sold.

We observe that prices on average are highest from April to early June, prior to the major growth period when the stock of weight classes where most fish is sold is lowest. In this period, the value of remaining stock is high. As fall approaches and growth declines, the biomass increases and sales increase. As sales increase, prices decline before it spikes up at the Christmas demand surge.

The reason that the spring/summer seasonal pattern persist is most likely due to biological restrictions. One might argue that if regular patterns emerged that were not founded on biological restrictions they would be eliminated by changes to production plan. The price increase in Christmas is also accompanied by an increase in sales, suggesting that price increase is the result of greater demand. For the higher prices in early summer we observe no such co-movement in price and sales. This further lends support to the claim that higher prices in this period are the result of changes in supply side fundamentals, necessitating higher prices to keep up supply. As stated, this is likely due to the increased alternative cost of harvesting in this period.

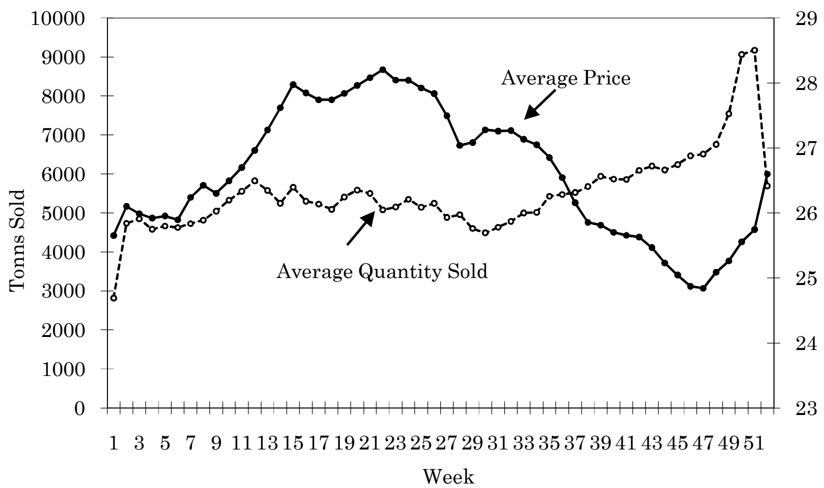

FIGURE 11. Seasonality in Price and Quantity Sold. Averages are taken from 1995-2008.

2.6 CONCLUDING REMARKS

The Norwegian salmon farming industry has experienced a consistent growth both in productivity and value since its beginnings in the 1970's. The increase in productivity has allowed the price of salmon to follow a long run declining trend. Despite a steady increase in the control of production output, industry profits are still sensitive to shocks to stocks through disease, escape of fish, algae influx or temperature changes. The nature of the biophysical restriction leads such effects to have persistent effects on fish stock, return and price dynamics.

We observe how industry profits are volatile, a volatility that has been relatively stable throughout the sample period. Cost factors are unlikely to account for the variations in profitability. The main

cause for volatility appears to be the stochastic output, both through its direct effect on sellable stock and the short term spot price dynamics. The stochastic production and the producers' reaction to windfall loss or gain to output remains crucial in explaining short-run price and profit dynamics. In terms of price volatility, we do not observe a decline in magnitude of the years. Volatility is itself stochastic and varies regularly throughout the data-period. One would perhaps expect volatility to decline as the industry attains better control over production. However, short term supply flexibility has most likely been reduced in later years, owing to salmon becoming a main provider of "bread and butter" fish, necessitating a stable supply of salmon to processors and outlet chains. A lower flexibility in changing short term supply to counter loss or gain to output will lead prices to correct more heavily and as such to increased volatility.

In the next section I look more generally at price characteristics of commodities. I discuss the economic theory underlying the short run dynamics of prices and the econometric theory used for modelling price dynamics in this thesis.

3. SHORT-RUN COMMODITY PRICE DYNAMICS

In the short run the commodity behaves similarly to a financial asset. This similarity arises because storage allows speculation on the future value of the commodity. When storage is allowed, the set of consumers and producers are no longer disjoint. Producers can now enter on the demand side of the market. The flexibility and cost of storing places each commodity market on a continuum between a classical economic market of disjoint producers and consumers and a modern financial asset market.

If the set of consumers and producers are disjoint, the price prevailing at each instant in time will (under the assumption of stationary demand) be the outcome of a stationary stochastic process having expected value at the competitive equilibrium value – the marginal cost of production. In this traditional case, price reflects the intersection between marginal consumption and production value.

For financial assets, the set of "consumers" and "producers" are the same. Financial assets cannot be consumed, they are held solely for their ability to generate monetary value over time. Ignoring emissions, the total stock of financial assets are at all times constant. The financial asset is valued by its inter-temporal ability to generate a dividend to the holder. For holders of financial assets, it is the return process that determines its value. Asset prices are statistically well approximated by a random walk, a statistically divergent process. That financial asset prices might diverge does not conflict with financial theory since returns, not levels, is the determining factor in the assets valuation.

Modern commodity markets exists on a continuum between the traditional economic commodity market of independent consumers

and producers, and the joint set characterizing financial asset markets. In commodity markets, agents can turn speculators or hedgers by holding commodities over time. Storage opens an alternative to selling or consuming the commodity immediately. Based on expectations on the future, agents might find it optimal to sit on the commodity. When this happens, the commodity takes on an additional speculative property, and the commodity more strongly emulates a financial asset.

When analysing the short run dynamics of commodity prices, we apply ideas from both finance and conventional producer and consumer theory. It is natural that in the short run, where producers are allowed some speculative freedom, ideas from finance enter into the modelling of commodity prices.

3.1 THE THEORY OF STORAGE

When modelling price spreads (the difference between spot and expected future prices) there exists both a financial and economic explanation model. In finance the spread is explained in part by a predictable element, the expected price, and a risk premium accruing to the holder of a futures contract on the commodity (Cootner, 1960; Dusak, 1974; Breeden, 1980; Hazuka, 1984). In economic theory, the spread is explained by the alternative cost of capital and the cost of storage. The economic model was in its initial form developed by Kaldor (1939), Working (1948, 1949) and Brennan (1958).

3.1.1 THEORY AND DATA

The theory of storage remains uncontroversial (Fama & French, 1987). However, in its simplest form (constant costs of storing) the theory fails to explain certain characteristics of commodity prices. Also, since the theory only accounts for short run dynamics, any permanent structural changes is not accounted for by the theory. To make the theory explain certain characteristics of modern commodity prices, several modifications to the net cost of storage variable has been introduced in more recent works. Below I list three price characteristics which the general theory of storage has proved unable to explain. I also explain some of the remedies to the theory proposed in order to explain the data.

1. High, near unit root autocorrelation. In general, commodity prices have an autocorrelation much higher than what can be accounted for by speculative storing alone. This discrepancy has been pointed out by several authors (Deaton & Laroque 1992, 1996; Chamber & Bailey, 1996; Pindyck, 2001; Williams & Wright, 1991).

Collecting monthly data on a selection of commodity prices from the IMF website, I provide in table 1 some descriptive statistics. All prices are corrected for inflation. I calculate three month autocorrelations, normalised standard deviations in addition to skewness and kurtosis measures on price levels. I also perform an Augmented Dickey Fuller test for unit roots in prices. The unit root test results are not shown in the table.

As seen from table 1, half of the commodities have a first order autocorrelation greater than 0.95. Even for the third month autocorrelation, the average value is 0.81. This high autocorrelation often makes commodity prices fail standard unit root tests such as the Augmented Dickey Fuller test (Wang & Tomek; 2007, Tomek; 2000). Of the 36 commodities in table 3 only 6 of the commodities rejects a unit root using the Augmented Dickey Fuller test with a constant and trend. In order to accommodate the problems associated with empirical analysis using non-stationary data (Granger & Newbold, 1974), prices are normally put through a first-difference filter.

TABLE 1. Price characteristics of monthly commodity prices (1983-2008).

Category	Commodity	AC(1)	AC(2)	AC(3)	Std. Dev.	Skewness	Kurtosis
Beverages	Cocoa beans	0.95	0.89	0.84	27.66	0.58	0.15
	Coffee	0.96	0.96	0.83	36.31	0.85	0.55
	Tea	0.84	0.66	0.54	21.55	0.80	0.61
	Sugar	0.96	0.90	0.85	32.34	0.24	-0.28
Cereals	Barley	0.95	0.90	0.85	36.36	1.95	5.17
	Maize	0.91	0.83	0.75	26.71	2.34	7.64
	Rice	0.88	0.68	0.50	33.55	4.61	31.95
	Wheat	0.95	0.89	0.83	31.57	2.79	10.11
Fruits	Bananas	0.75	0.56	0.38	28.34	0.90	0.96
	Oranges	0.91	0.80	0.71	38.32	1.42	1.82
Meat	Beef	0.96	0.92	0.89	14.52	-0.19	-1.29
	Poultry	0.98	0.95	0.92	21.41	0.14	-0.74
	Lamb	0.97	0.93	0.89	21.30	0.27	-0.91
	Swine	0.93	0.85	0.77	35.30	1.23	1.48
Seafood	Fish (salmon)	0.98	0.95	0.91	28.79	0.66	-0.16
	Shrimp	0.89	0.80	0.74	18.71	0.27	-0.72
Vegetable Oils and Protein Meal	Fishmeal	0.97	0.94	0.91	31.36	1.56	2.48
	Olive Oil	0.99	0.96	0.94	34.64	0.64	-0.62
	Palm oil	0.95	0.89	0.84	40.83	1.89	5.16
	Groundnuts (peanuts)	0.92	0.81	0.70	27.47	2.19	5.34
	Rapeseed Oil	0.94	0.89	0.83	38.94	2.28	7.21
	Soybeans	0.92	0.84	0.76	25.26	2.34	7.57
Agricultural - Raw Materials	Wool	0.97	0.93	0.89	28.92	1.38	2.01
	Cotton	0.97	0.91	0.85	29.40	0.49	0.20
	Hard Logs	0.97	0.92	0.86	21.65	1.19	3.21
	Hides	0.93	0.83	0.74	34.33	-0.17	-0.06
	Rubber	0.96	0.91	0.87	16.32	1.65	2.51
Metals	Aluminum	0.95	0.91	0.87	29.72	1.23	0.99
	Copper	0.97	0.93	0.89	63.64	2.16	3.81
	Iron Ore	0.95	0.89	0.84	57.88	2.85	8.63
	Lead	0.98	0.94	0.89	73.73	3.20	10.70
	Nickel	0.98	0.94	0.90	79.22	2.52	7.02
	Tin	0.94	0.87	0.80	44.30	2.55	7.77
Energy	Coal	0.89	0.81	0.75	46.08	3.81	18.75
	Crude Oil (petroleum)	0.94	0.89	0.85	68.98	2.33	5.89
	Natural Gas	0.96	0.93	0.89	59.39	1.99	3.49

Note: AC(1) denotes first order autocorrelation, AC(2) second order and so on. Standard deviation is normalized by the sample mean price.

Unit roots in commodity prices, however, are not supported by the theory of storage. None the less, for suitably long time series' non-stationarity is likely to emerge.

The structure of demand and technology is expected to change in time. Under this explanation for non-stationarity it is reasonable that the theory of storage cannot account for non-stationarity. The theory of storage is a short-run model. The data we collect on commodity prices is the sum of both short-run and long-run structural effects. To generate an autocorrelation in line with what the data shows, Deaton & Laroque (1996) and Chamber & Bailey (1996) suggests adding an autoregressive element to the exogenous harvest component. The remedy itself is straightforward, however the evidence that harvests are autocorrelated is still lacking.

2. Price Backwardation. A constant positive cost to storing cannot account for the fact that producers tend to hold stocks even when prices are expected to fall. To account for this, the cost of storing must be allowed to take negative values. There must be a real positive dividend to holding a commodity. As such, a less tangible economic variable, the convenience yield, is introduced to the theory. The convenience yield is defined as the marginal benefit of sitting on a commodity. Several proposed intuitive explanations for the convenience yield have been put forward. For example, stocks are held as an insurance against stock-outs (Deaton & Laroque 1992, 1996), taking on a real option value (Heaney, 2002; Litzenberger & Rabinowitz, 1995; Routledge, Duane & Chester, 2000) as it becomes increasingly valuable to sit on a commodity as it becomes more scarce. A further proposition to why producers tend to sit on commodities when prices are expected to fall is found in the benefit of being able to produce when stocks are low and/or avoid costly changes in production (Considine & Larson, 2001; Litzenberger & Rabinowitz, 1995). That convenience yields are linked to some economic dividend on the stock is further supported by Milonas & Henker (2001) who find that convenience yields in oil are strongly seasonal and inversely related to stocks.

3. Mean Reversion in Price Spreads. Backwardation also implies that commodity price returns display consistent mean reversion. The theory of storage, with constant cost of storing, does not give rise to this mean reversion pattern. To get a price spread consistent with the mean reversion observed in prices, the net cost of storage is modelled as stochastic, being the sum of both a positive cost of

storing and stochastic convenience yield component (Gibson & Schwartz, 1989; Miltersen & Schwartz, 1998). By allowing the convenience yield to be mean reverting, the net cost of storing will be mean reverting and the price spread will mean revert.
Outside the high autocorrelation observed in prices, the theory of storage modified by a flexible cost of storing function can explain many of the characteristics observed in commodity prices today. The cost of this theoretical consistency is the introduction of the less tangible convenience yield variable. The lesson from these necessary modifications is that one general theory to account for the dynamics of all commodity prices is unfeasible. Within the theory of storage we need to specify a specific cost of storage function for each commodity market analysed.

Despite the need to take account for the heterogeneity in each commodity market, there are some characteristics of commodity prices the general theory can account for. Theory predicts that the price distribution is right skewed. The positive probability of a stock-out generates positive price spikes, leading to right skewed price distributions. Looking at table 1, we observe that most commodity prices have right skewed distributions relative to the normal distribution.

Further, the asymmetric price process generates a non-constant conditional variance of prices. Theory predicts that when stocks are low – the convenience yields are high, and price volatility increases. These predictions are shown to hold in several studies (Ng & Pirrong, 1994; Ng & Ruge-Murcia, 2000 and Nilesen & Schwartz, 2004) – illustrating that implications from the theory of storage are consistent with patterns in observed volatility. Further Fama & French (1987) test both the economic and financial model of storage for a set of 21 commodities. They test the economic and financial model of storage for a set of 21 commodities. The authors find that the theory of storage provides a better explanation for the observed price characteristics than the financial model. However the authors point out that the financial and economic theory for commodity price spreads are not mutually excluding.

3.2 OPTIMAL STORAGE AND PRICE SPREAD DYNAMICS

We now turn to providing a more rigorous explanation for the theory of storage. As is conventional we assume that producers exhibit Rational Expectations (Lucas, 1978). The producers' beliefs concerning the future are assumed to coincide with the true probability distribution underlying state realisations. We further

assume that a continuous, real-valued and strictly decreasing demand function $D(p)$ exist where $D(p)$ tends to $+\infty$ as price tends to a lower bound $\underline{p}$. The inverse demand function $P(x)$ thus exists and is strictly converging down to a lower bound on amount at hand $\underline{x}$. We also assume that demand and technology is structurally constant. Also, production scale and intensity is assumed fixed in the short run.

Denoting stock levels at time t as X_t , stocks are assumed to follow the discrete time process:

$$X_t = X_{t-1} + I_{t-1} + v_t . \qquad (1)$$

In equation (1), the variable v_t accounts for exogenous additions to stocks from planned production. In the theory of storage this is the source for variations in prices. In its simplest interpretation, variations enter due to weather effects affecting realized output. However, since temporary changes in demand has an equivalent but inverse effect on stocks, the stochastic component can also be seen as an exogenous excess demand variable (Deaton & Laroque, 1996).

I_{t-1} is the amount stored in the previous period. Amount stored is the control variable available to the producer in short run. Storage is restricted to take non-negative values; it is not possible to carry over stocks from tomorrow to today. This restriction is crucial to the dynamics of commodity prices. It implies that price dynamics becomes asymmetrical as the pricing function changes regime when a stock-out occurs. When no storage is done, the stock dynamics is fully dependent on the exogenous addition to stocks v_t. Note that the storage variable I_{t-1} can also be defined as $I_{t-1} = \phi_{t-1} X_{t-1}$, $\phi_{t-1} = [0,1]$; ,a ratio of stocks carried over to the next period. Looking

at storage in this way makes it more general. In agriculture, storage can hence be viewed as the option not to harvest. Storing is done through further cultivation.

We now introduce the cost of storing variable. On the margin this variable can be defined as:

$$\chi(I_t) = c(I_t) + \delta(I_t). \tag{2}$$

The marginal cost of storing $\chi(I_t)$ is decomposed into a non-negative cost factor $c(I_t)$ and a non-positive convenience yield factor $\delta(I_t)$. Under competitive storage it is assumed that cost of storing depends on amount stored. In the Kaldor-Woking hypothesis it is assumed that $D_I\chi > 0$, an increasing marginal cost of storing. For higher storage levels, the cost effect is expected to dominate the convenience yield effect. The marginal benefit to increasing storage is generally hypothesized to be decreasing.

The theory of storage states that in order to store a commodity, the producer must be compensated on the margin for the cost of storing. This restriction generates the Euler condition for optimal storage:

$$\beta E(p_{t+1} \mid I_t) - P(X_t - I_t) = c(I_t) + \delta(I_t) \quad . \tag{3}$$

Here β is a discount factor accounting for the alternative cost of capital. $E(p_{t+1} \mid I_t)$ is the expected price tomorrow with storage I_t today. Further, $P(X_t - I_t)$ is the realized spot price. In equilibrium, the discounted expected price spread $\beta E(p_{t+1} \mid I_t) - P(X_t - I_t)$ must cover the marginal cost of storing $c(I_t) + \delta(I_t)$. When condition (3) is satisfied, any arbitrage from storing has been eliminated.

The price spread $\beta E(p_{t+1} \mid I_t) - P(X_t - I_t)$ decreases in storage. This implies that the maximum price spread is achieved when storage is zero. We define the expected maximum price spread $E\eta_{t+1}^{Max}$ as:

$$E\eta_{t+1}^{Max} = \beta E(p_{t+1} \mid 0) - P(X_t) \tag{4}$$

Following equation (4), the market stocks-out ($I_t = 0$) when $E\eta_{t+1}^{Max} < c(I_t) + \delta(I_t)$. In this case the optimal storage level is

negative. However, due to the non-negativity constraint on storage the storage, the closest point to optimality is zero.

Combining equation (3) and (4) the expected discounted price spread $E\eta_{t+1}$ can be expressed by the functional:

$$E\eta_{t+1} = \min\{c(I_t) + \delta(I_t), \beta E(p_{t+1} \mid 0) - P(X_t)\} \tag{5}$$

This is the price spread process predicted by the theory of storage. The left term on the right side of equation (5) is the expected price spread under storage, while the right term is the stock-out spread. Note how equation (5) illustrates how the control variable I_t determines the spread under storage, while the spread under stock-out is determined by the exogenous addition to stocks today. Equation (5) also illustrates the importance of a convenience yield. Since the convenience yield is a negative cost component it reduces left side expression, increasing the probability that storage is the prevailing regime.

We observe how the spread process is non-linear, moving between the two regimes dependent on stock availability today X_t. The probability of stocking-out tomorrow can be expressed as:

$$\begin{aligned} &prob(\beta E(p_{t+1} \mid 0) - P(X_{t-1} + I_{t-1} + v_t) < c(I_t) + \delta(I_t)) \Leftrightarrow \\ &prob(P(X_{t-1} + I_{t-1} + v_t) + c(I_t) + \delta(I_t) > \beta E(p_{t+1} \mid 0)) \end{aligned} \tag{6}$$

Under the assumption that the harvest is normally distributed $v \sim N(\mu, \sigma)$, the price process moves according to a two state Markov chain having non-constant transition probabilities given by:

$$P_t = \begin{bmatrix} \rho_t \rho_{t-1} & \rho_t (1-\rho_{t-1}) \\ (1-\rho_t)\rho_{t-1} & (1-\rho_t)(1-\rho_{t-1}) \end{bmatrix} \tag{7}$$

Where $\rho_t = \Phi_{\mu_{s,t1}, \sigma^2_{s,t}}(\beta E(p_{t+1} \mid 0))$ is the probability of storing today. The probability distribution is conditional on stock availability today. Its mean and variance can be approximated by:

$$\mu_{s,t} = P(X_t) + c(I_t) + \delta(I_t) \tag{8}$$

$$\sigma^2_{s,t} \approx \left(\frac{\delta P(X_t)}{\delta(X_t)} + \frac{\delta c(I_t)}{\delta(X_t)} + \frac{\delta \delta(I_t)}{\delta(X_t)} \right)^2 \sigma^2$$

This derivation implies that the more that was stored yesterday, the higher is expected stocks today, and the higher is the probability of not stocking out today. Hence the higher stocks are the greater is the likelihood of staying in the storage state. In essence the spread process under optimal storage follows a non-linear process – a two state time dependent Markov chain determines the likelihood of changing price regimes. Note also that since price spreads can change regime the volatility process becomes non-constant.

3.3 THE ROLE OF RATIONAL EXPECTATIONS

Storage creates a short run price dynamic different from a white noise process. The drift in the process arises due to a compensation for the cost of storing. If fundamentals such as planned production output and demand follow non white-noise processes, storage decisions must anticipate the signals in these variables to the extent they are predictable. The knowledge that productivity increases in a certain period must lead expected prices period to decrease for this period as storage accommodates this information.

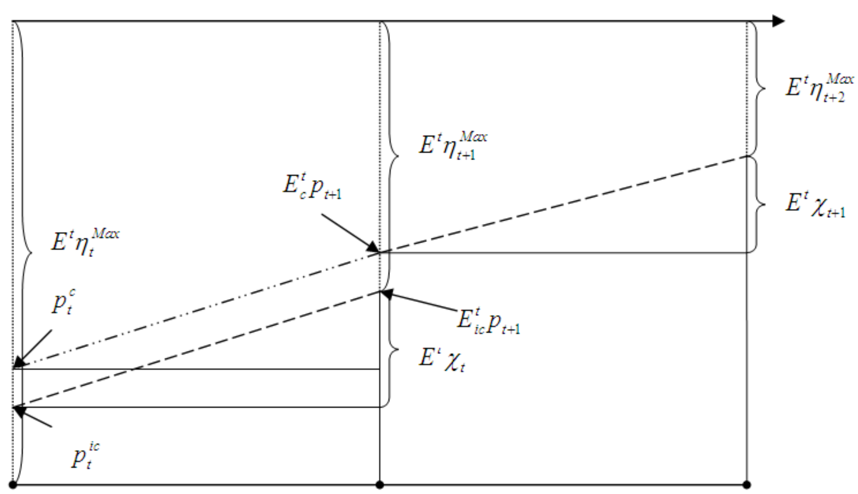

FIGURE 12. Rational Expectations under Storage

It is not rational behaviour to store as under normal conditions when information is in the market that a demand surge will occur in the next period. As the market absorbs all available information, expectations becomes rational to the degree that any divergence from expectations arise purely due to unpredictable shocks.

Assume for example that producers only look one period ahead. In this case the expected price will be equal to the price achieved when selling the stock tomorrow (Expected price with subscript **ic** in

Figure 12). This expected price can be consistent with the condition that expected price spread covers the cost of storage, and will indeed be correct if stocks tomorrow are insufficient to initiate storage tomorrow. However, once storage today is so high that storage is likely done tomorrow as well, this expectation becomes inconsistent with rational expectations. As storage will be done tomorrow, the spot price tomorrow diverges from what was expected today, and the divergence arises from a predictable reason. Price will then jump from $E_{ic}^{t} p_{t+1}$ to $E_{c}^{t} p_{t+1}$. A producer knowing that the market only looks one period ahead can exploit this information to earn a marginal profit of $E_{c}^{t} p_{t+1} - E_{ic}^{t} p_{t+1}$. Under rational expectations, divergence due to available information at time *t* will not occur, only divergence in price spread due to information arriving at *t+1* will move future spot price away from what is expected at time *t.*

Another way of stating this is that expected prices cannot be used as information to earn abnormal profits in the future:

$$
\begin{aligned}
&E(p_{t+n} \mid p_t) - Ep_t = \\
&(E(p_{t+1} \mid p_t) - Ep_t) + (E(p_{t+2} \mid E(p_{t+1} \mid p_t)) - E(p_{t+1} \mid p_t)) \\
&+ \ldots + (E(p_{t+n} \mid E(p_{t+n-1} \mid p_t)) - E(p_{t+n-1} \mid p_t))
\end{aligned}
\tag{9}
$$

In effect, the expected price *n* periods ahead is equal to the sum of expected prices up to *n,* conditional on price at time *t.* The assumption of rational expectations is a fundamental axiom in the theory of storage.

3.4 THE THEORY OF STORAGE AND AGRICULTURE

When it comes to agriculture in general, there are certain characteristics which influence the theory of storage. Agriculture deals with production of "living" commodities. This implies that the flexibility involved in storing is limited by the perishability natural to agricultural commodities. The value of living commodities depreciates under storage, suggesting that the cost of storing in time increases by the rate of decay. Depreciation due to perishability can be remedied by deferring from harvesting. Delaying harvest is equivalent to storing the commodity. Storing by further cultivation suggests a stochastic cost of storing contributing to the volatility of price spreads. Contrary to keeping stocks warehouses, storing by cultivation implies that stocks are subject to the ordinary weather effects of agricultural production.

Agricultural commodities are by nature used as inputs in own production. Its input value suggests that as stock levels decline, their marginal productive value increases, generating a positive convenience yield on remaining stocks. As is shown by bio-economic models, growth rates of agricultural commodities vary with stock size. If stocks are too high or too low, this can significantly punish growth rates. The convenience yield from storing by cultivation can be seen as the alternative cost of harvesting. The expected growth-rate of the stock becomes a component of the convenience yield. Hence we should observe seasonal variations in convenience yields for agricultural commodities. Growth rates vary predictably throughout the season and contribute to the alternative cost of harvesting at any time.

Many agricultural commodities can be sold in separate markets conditional on its property at the time of harvest and/or some transformation applied by producers. Certain fish species can be sold at different weight-classes, meat can be sold fresh, frozen or conserved etc. This implies that stocks of commodities are related, and that theory of storage could be used to, not only explain the single commodity price dynamic, but also relative price dynamics across related markets. The stock of one commodity can have positive or negative productivity effects on the stock of other commodities. The theory of storage provides not only an inter-temporal explanation for price dynamics, but also a spatial explanation for common patterns in mean-reversion and/or volatility.

Aquaculture is a subset of agriculture dealing with production of farm bred marine species. For salmon aquaculture, markets exist for frozen, fresh and conserved salmon. Fish is also sold as whole or filets, in addition to different weight-classes. This means that stocks are highly correlated; a fixed amount of fish stock is distributed over each product form. Further, salmon can be stored by delaying harvest or by harvesting and storing. When the fish is harvested, it can be stored fresh or in some conserved form, such as frozen. An array of varying degrees of flexible storing options is available to salmon producers. Further, growth rates of salmon vary predictably across seasons, with higher growth rates during

summer when water temperatures rise. The convenience yield from storing by cultivation increases during early summer since the alternative cost of slaughtering prior to a high growth period is greater. For producers of salmon, potential arbitrage opportunities exist not only inter-temporally but also spatially across different product forms. The theory of storage provides a suitable framework to analyse the price relationships both inter-temporally and spatially in the short run. The theory suggests that inter-temporal and spatial arbitrage needs to be evaluated simultaneously in light of a commodities connection to a common stock starting point. In effect the theory of market integration becomes equivalent to the theory storage where the cost of storing is replaced by the cost of transformation between markets.

4. EMPIRICAL METHODS

In order to model dynamics of prices empirically we need to establish a suitable statistical representation of our data. We would also prefer that, in addition to an appropriate statistical representation, the empirical model should encompass the dynamics demanded by the structural model. We have seen how the dynamics of commodity prices move dependent on state-variables in a constant feedback relationship. None of the state variables, except additions to stocks from production, can be reasonably assumed exogenous. Analysing price series in the light of the theory of storage has in general taken three approaches.

The first approach I call *Direct Inference Using Inventory/Stock Data.* The benefit to this approach is it allows direct inference of predictions made by theory. In this approach causality relationships are empirically possible to achieve. Due to the low availability of good, high frequency, inventory/stock and cost of

storing data this approach is less frequently used in the literature. Examples of this approach are Rucker, Burt & LaFrance (1984), Krane (1993), Pindyck (1993) and Considine & Larson (2001).

The second approach I call *Indirect Inference Using Price Data.* This approach uses price data to test predictions on price characteristics from the theory of storage. Due to the higher availability of high frequency price data, this approach is predominant in the literature. The drawback of this approach is naturally the inability to draw direct inference on the relationships between inventory/stocks, cost of storing and price. Using only price data prevents identification of parameters. The modelling framework for this approach is non-structural time series analysis. One of the benefits to this approach is accessibility to a large arsenal of modern time series tools. Examples of this approach are Fama 6 French (1987); Bessembinde & Sequin (1993); Ng & Pirrong (1994); Litzenberger & Rabinowitz (1995); Kim & Rui (1999); Routledge, Duane & Chester (2000); Heaney (2002); Cassaus & Collin-Dufresne (2005)).

A third approach uses different variations of simulated methods of moments to directly estimate parameters of the structural model (Michaelides & Ng. 2000). This approach has emerged with the availability of greater computing power. The approach uses simulated methods of moments from finance to estimate the parameters of the theoretical storage model.

In this thesis I use the *Indirect Inference Using Price Data* approach for the empirical analysis. I focus not only on statistical fitting of empirical models but also on how empirical models need the flexibility to accompany the dynamics predicted by economic theory.

4.1 NON-STRUCTURAL TIME SERIES ANALYSIS

With a non-structural time series representation of prices we can achieve a well defined statistical representation of dynamics. The cost of this is a sacrifice of direct causality inference between state variables. However, since theory suggests that state variables exist in a constant feedback relationship, the sacrifice of direct causation inference seems less grave. Even though direct inference is sacrificed, this does not imply that hypothesis testing is impossible, or that empirical representation of data is disjoint from economic theory. The theory produces several predictions on price characteristics in which results can be evaluated.

In addition to focusing on empirical models consistent with the flexibility required by theory, we can also use knowledge of relative changes in underlying state variables to evaluate model output. Even if exact quantitative representation of state variables is missing, we often have available information relating to predictable structural changes. Such information can come from seasonal patterns, as in growth rates of stocks, or predictable changes in demand. This knowledge can be used, in combination with the theory of storage, to interpret the output of a consistent empirical model.

Developments in modern time series analysis have provided many useful models to represent prices in a manner consistent with the flexibility demanded by theory. Some of these models include the (G)ARCH models of Engle(1982) and Bollerslev(1986) in addition to the different modifications to the model such as the EGARCH model of Nelson(1991), the GJR-GARCH model of Glosten, Jagannathan & Runkle(1993) or the GARCH-M model. Various non-linear models such as the regime shifting model of Hamilton(1986), structural models (Harvey, 1991) and smooth transmission models (Chan & Tong, 1986) provide a great degree of flexibility when modeling the mean. In addition, long memory models such as the fractionally integrated models of Granger & Joyeux (1980) and seasonal unit root models (Hylleberg, Engle, Granger & Yoo, 1990) provide flexibility in modelling longer run effects in prices.

4.2 STATIONARITY AND DATA PREPARATION

Before we derive the general time series model of prices used in this thesis, it is worth mentioning a fundamental and important issue when doing time-series analysis. The issue is that of data stationarity. We define a data-set as a discrete $n \times 1$ vector Y, where a single element is identified by y_t in the time subscript *t.* In our data-set *n* represent the number of observations available and Δt denotes the time interval between observations, usually a day, week or month in commodity price analysis. It is assumed that $Y = \{y_t\}_{t=t_0}^{t=t_0+n}$ is a single outcome of the underlying stochastic process that generated our data set. The underlying stochastic process is

called the data generating process (DGP). Our entire data set Y is assumed a single draw from the DGP. The objective of time series analysis, like any econometrical analysis, is to find a statistically suitable approximation of the DGP. The DGP is said to be stationary if:

$$E(y_t) = \mu \qquad \forall t$$

$$E(y_t - \mu)(y_{t-j} - \mu) = \gamma_j \qquad \forall t, \ \forall j$$

That Y is stationary implies that that the mean μ and autocovariance γ_j of the series is independent of the time of observation *t.* Working with a stationary data set is important both in terms of statistical validity of coefficient estimates, and economic validity of output analysis. Granger & Newbold (1974) illustrated how a non-stationary process, specifically a process with a unit root, generates spurious regression effects which severely bias' statistical inference. In addition, if the non-stationarity is not accounted for, the output of our model is only relevant for the specific time period of our data set. By not accounting for non-stationary components, the future underlying DGP will change in time while our model approximation will not.

That the data is stationary leads to a fundamental theorem in time series analysis: the Wold decomposition theorem. The theorem states that given a stationary data-set, the DGP can be written as an infinite moving average process:

$$y_t = \mu_t + \sum_{i=1}^{\infty} b_i \varepsilon_{t-i} + \varepsilon_t \tag{10}$$

Where ε_t is an uncorrelated innovation sequence, the b's are the moving average coefficients, assumed absolutely summable, and μ_t is the deterministic component. Thus we are able to decompose the price series as the sum of a deterministic component μ_t and a random component $\sum_{i=1}^{\infty} b_i \varepsilon_{t-i} + \varepsilon_t$. Since the process is dependent on an infinite innovation sequence it is standard to express the time series as an autoregressive model, or a mixed autoregressive moving average model:

$$y_t = \mu_t + \sum_{j=1}^{q} a_j y_{t-j} + \sum_{i=1}^{p} \beta_i \varepsilon_{t-i} + \varepsilon_t \ . \tag{11}$$

This is equivalent to the Wold representation. The Wold decomposition theorem allows the dynamics of the series to be represented by a linear model. This way of representing the dynamics of a model is fundamental in time series analysis. The primary criterion to allow this representation is that the data is stationary. In most economic data, the raw data does not allow this property. To make the data stationary it is common to apply a filter to the data, removing the non-stationary component. What filter one applies is naturally dependent on the form non-stationarity. Deterministic or seasonal trends can be removed by applying polynomial or trigonometric filters. Stochastic trends can be removed by difference filters. A difference filter takes the form $(1 - L^n)$, where L is the backshift operator and n is some positive integer. The process might also be fractionally integrated such that a filter $(1 - L)^d$, where d can be a real number, is necessary to achieve stationarity. To analyse returns for example, the filter

$(1-L)$ is applied. This filter removes the zero frequency unit root and allows stationary modelling of zero frequency unit root data.

4.3 THE EMPIRICAL PRICING FUNCTION

To derive the time series representation of prices, we start from commodity price theory. Supply today S_t determines spot price as:

$$p_t = P(S_t) \qquad\qquad S_t = X_t - I_t \,. \tag{12}$$

The supply S_t is further the result of previous period realisation of sales through stock X_t and storage I_t as discussed in the previous section. Since state variables exist in a constant feedback relationship, the pricing function does not generally allow a closed form solution. By focusing specifically on the price variable at time *t,* we can construct an approximate price model as:

$$\hat{p}_t \approx P(E(S_t \mid S_{t-1})) \qquad \Leftrightarrow \qquad \hat{p}_t \approx P(E(S_t \mid P^{-1}(S_{t-1}))) \tag{13}$$

Since the demand function is assumed invertible, a one-to-one mapping exists between the previous period price and sale. Using

previous period price, an estimate of current sales and price can be achieved. We define the approximate price today as:

$$\hat{p}_t = f(p_{t-1}) + \varepsilon_t \tag{14}$$

Here ε_t is the error in the approximation model. Due to the lack of a one-to-one mapping, direct inference of state variables is not possible. Even though a one-to-one mapping between sales and price exists, a one-to-one mapping between sales and underlying state variables such as stock and storage costs does not exist. Low supply might suggest low stock levels. However, low supply might also be the result of a high convenience yield. This implies that direct inference on state variables from price alone is not possible. Denoting $\bar{p}$ as the long run equilibrium price; the price when storage is at its steady state, the price process (14) can be approximated by a Taylor series expansion as:

$$p_t = \bar{p}(1+r) + f^{(1)}(\bar{p})(p_{t-1} - \bar{p}) + \varepsilon_t$$
$$\varepsilon_t = \sum_{n=2}^{\infty} f^{(n)}(\bar{p})(p_{t-1} - \bar{p})^n \tag{15}$$

This is the fundamental non-structural time series representation of prices from the theory of storage. The linear term measures the speed of convergence to the long run equilibrium, $\bar{p}$. This is the mean reversion term. The term ε_t measures the error of applying the linear model. If $E(\varepsilon_t) = 0$ and $E(\varepsilon_t \varepsilon_{t+j}) = 0$ is equal to a constant for $j = 0$ and zero for $j \neq 0$ the linear model is a valid representation. If this is the case the model is said to be stationary and the Wold decomposition theorem applies.
From the theory discussion above we do not expect linearity to be the case. The derivatives $f^{(n)}$ measure how the market reacts to changes in the state variables outside of equilibrium. If linearity is a valid representation, we need costs of storage to be constant; we also need storage to be a constant ratio, planned production to be constant and demand stationary and linear. Assuming all this seems unreasonable. Also, looking at the table of commodity price characteristics these restrictions generally do not appear to be satisfied. For example, symmetric mean reversion of prices around the long run price is unreasonable. The possibility of stock-outs and

non-linear storage costs makes mean reversion asymmetric. Note that if $f^{(1)} = 1$ the price dynamics collapses to a random walk process where prices appreciate at $\bar{p}r$. Further, when the market stocks out, mean reversion generally vanishes such that $f^{(1)} = 0$. In reality $f^{(1)}$ will vary in state variables leading to a non-linear price process.

How the price process diverges from the linear model represents important information on price dynamics. Moreover, it allows the use of output from time-series models to indirectly test hypothesis from theory. What is not picked up by the linear model remains in the error term $\varepsilon_t = \sum_{n=2}^{\infty} f^{(n)}(\bar{p})(p_{t-1} - \bar{p})^n$. Analysing both the structure of this term and the invariance of this term by changing the linear model remains crucial in understanding commodity price dynamics. Since price itself does not provide a one-to-one mapping to other state-variables, we apply knowledge of exogenous factors in specific markets to evaluate the empirical output of the model against theory.

The dynamics of $\varepsilon_t = \sum_{n=2}^{\infty} f^{(n)}(\bar{p})(p_{t-1} - \bar{p})^n$ contains information, on amongst others, the conditional variance, skewness, kurtosis and serial-correlation in prices. Several models are available to examine the structure of ε_t. The (G)ARCH model focuses mainly on the second order derivative $f^{(2)}$. Further, regime shifting models focus on the asymmetry of convergence, picked up amongst others by the third order derivative $f^{(3)}$. This can account for the skewenss in distributions. By including several lags in the linear model we can also gain information on the kurtosis and form of mean reversion in prices. The eigenvalues and modulus of the mean reversion coefficient vector provides information on how prices converge to the long run mean. Real eigenvalues higher than unity in absolute values suggests diverging prices, while complex eigenvalues lower than unity generates oscillating reversion patterns. Improved understanding of commodity prices can be achieved by time-series modelling when we analyse linear model output against the theoretical pricing model. Since commodity price theory predicts that errors from linear models should emerge we can analyse the empirical error by time series methods and knowledge of exogenous state variables in specific markets.

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The Behaviour of Salmon Price Volatility[1]

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2008

Abstract Salmon prices exhibit substantial volatility. An understanding of the structure of volatility is of great interest since this is a major contributor to economic risk in the salmon industry. The volatility process in salmon prices was analysed based on weekly price data from 1995 to 2007. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model was used to test for volatility clustering and persistence for prices. We find evidence for and discuss the degree of persistence and reversion in salmon price volatility. Further, we find that volatility is larger in periods of high prices. For the industry this means that larger expected profits more often than not comes at a trade-off of larger price risk.

Introduction

In general, producers face two main types of risk, production risk, which influences how much is produced with a given input factor combination, and price risk, which influences the revenue one will obtain from the quantity produced (Just & Pope 1978; Sandmo 1971). A number of studies have recognized that salmon farming is risky (Asche & Tveteras, 1999; Tveteras, 1999; 2000, Kumbhakar, 2002 & Kumbhakar and Tveteras, 2003). Production risk is the main focus of these studies. Despite that price volatility seems to be one of the main sources for cycles in profitability, price risk in salmon aquaculture has received little focus. In this paper we investigate the price volatility for Norwegian salmon, and thereby obtain information with respect to the nature of the price risk that salmon farmers are facing.

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To put the salmon industry into a broader perspective, we can compare it with meat producing sectors in agriculture. From 1995 to 2007 the standard deviation of monthly salmon prices around their linear trend was 14.9%. For US beef and pork the standard deviation in the same time period was 11.9% and 24.9%, respectively. One particular distinguishing factor with salmon is that as it approaches harvest-ready sizes it also approaches sexual maturation, which causes a significant decline in quality and growth. Salmon farmers will often have a relatively short time window for harvesting, and will consequently be concerned about week-to-week variation price dynamics during that time window.

For the salmon industry, providing information on the volatility of prices is potentially valuable. There is substantial variability in industry profit levels (Tveteras, 1999), and an important part of this variability is due to fluctuating prices.

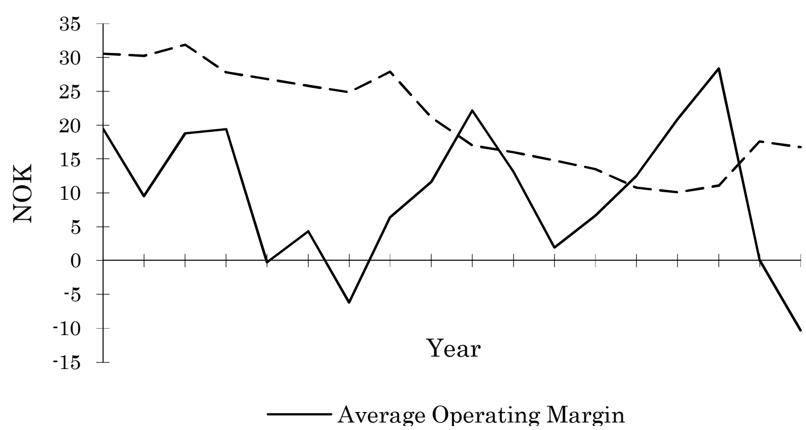

FIGURE 1. Average and standard deviation of the export margin for Norwegian salmon exports 1985-2002. Source: The Norwegian Directorate of Fisheries.

Not only first hand sellers experience the economic costs of highly fluctuating prices. The costs of price volatility are transferred to the entire value chain. Retailers and consumers increasingly demand stability of price and supply, and often have little understanding for biological and other mechanisms driving the formation of prices in the market. Modern value chains for food products are organized and have capital-intensive technologies that are geared towards predictability and stability of supplies and prices. From the fluctuating first-hand prices to the relative stable retail prices many intermediary agents in the value chain, such as fish processors, can experience substantial variability of capacity utilization and profits as prices fluctuate.

Revealing information on the volatility term contributes to the literature on price processes in aquaculture. Studies of price forecasting (Guttormsen, 1999; Gu and Anderson, 1995; Vukina & Anderson, 1994) rely on precise knowledge of the noise generating part of prices. The question of how precise we can expect price forecasts to be is highly related to the volatility term. Also, studies of market integration (Asche, Bremnes, & Wessells, 1999; Asche, Gordon, & Hannesson, 2004) rely on knowledge of the volatility term. If markets for comparable goods are integrated, which imply that we can describe them by a single price measure, this should also include the integration of the volatility processes of the comparable goods.

In addition, volatility of prices is important in establishing the value of contingent claims. Forward and futures market for salmon is now under establishment in Norway and Switzerland, although they have not been successfully established on a large scale. This is due to many factors outside the scope of this paper, but since the value of a contingent claim is dependent on the underlying asset (in this case the salmon price) it is important to establish the true properties of the volatility generating part of the price process. Simply assuming an independent zero-mean normally distributed term for describing volatility can be costly if the price process contains properties and connections diverging from a random walk. For instance, assuming normality if the distribution displays fat tails can lead to underestimation of extreme events and consequently to severe losses, as many speculators and investors on the world's stock markets have experienced. For example the probability of a trading loss as that incurred from the Black Monday stock market crash has been estimated, using a normal distribution, to occur with a probability of 1 in 10^{157} per day (James & Zetie, 2002).

Previous research on salmon prices has been predominantly concerned with issues such as price forecasting and market integration, and as such has for the most part focused on the price levels and the drift term of the price process. As far as we know, little work has been done on examining the volatility process of salmon prices. Thus this paper contributes to the study of salmon prices by analytically and descriptively investigating the volatility term of the price process. In essence we will look for indications that the volatility term cannot be described by a, generally assumed, independent zero-mean normally distributed random variable. We do this econometrically by applying the GARCH model (Bollerslev 1986) to our price time-series. The GARCH model allows us to model the variance term of the price process as a regression equation dependent on some explanatory variables, where the lagged variance and squared error term of the price process is assumed as default variables. This in essence allows us to empirically model any heteroskedasticity in the process. The result from the analysis of this process will reveal information on the volatility term in the form of bringing to light attributes such as volatility clustering[2] and the persistence of volatility shocks. This again allows us to discuss how volatility reverts after a shock, and as such reveal the predictive powers of volatility. The persistence of any volatility shock will also provide an indicator on the level of efficiency in the market; how fast prices revert to a conceived equilibrium following a shock. In addition we will investigate the distributional properties of the error term in the price structure in order to reveal non-normality attributes such as leptokurtosis and skewness. When deriving the distributional form of the error term we apply the kernel density estimation method.

The paper starts by giving a short overview of the aquaculture industry and some of the processes generating price risk. After this we start our analysis by descriptively trying to analyse the behaviour of price volatility. We apply some measures of volatility to our time series in order to apprehend indications of the properties of volatility that will in turn direct our further analysis. Following the descriptive analysis we apply the GARCH model to our time series so as to more rigorously investigate the properties suggested by the descriptive analysis. Our results reveal that the volatility term is not independent and that persistence and clustering is present in the short term dynamics of prices. As such the investigation provides valuable information on the salmon price path for any risk averse market participant.

[2] Volatility clustering is the property that prices are correlated in higher powers.

AQUACULTURE PRODUCTION AND RISK

The salmon farming industry is an industry in rapid growth; from 1996 to 2006 the volume of salmon sold from Norwegian farms more than doubled (from 298 to 626 thousand metric tonnes). This development has transformed what was once a relative small scale periphery of the biological production sector into a multi-billion dollar industry. For the biological production sector the breeding and cultivation of salmon has been one of the most commercially successful endeavours. Today Norway is the leading producer of salmon accounting for around 40% of the world production. Most of the industry growth comes from a substantial productivity growth which over time has substantially reduced unit costs. The reduction of production costs is due to two main factors. Firstly, fish farmers are able to produce more with a given amount of inputs, and secondly, improved input factors has made the production process cheaper (for example the development of better feed and feeding technology). The reduction of unit costs has lead the price of salmon to decrease over time (figure 2), providing a long term trend for the general direction of the salmon equilibrium price. In Norway most of the salmon farms where established during the 1980s. The long Norwegian coastline provides a large array of potential farm locations, and provides the farmer with potential hedging of production risk as correlation of farm specific risks such as disease outbreaks and temperature fluctuations decreases over geographical distance. At present salmon farms are located along most of the Norwegian coastline.

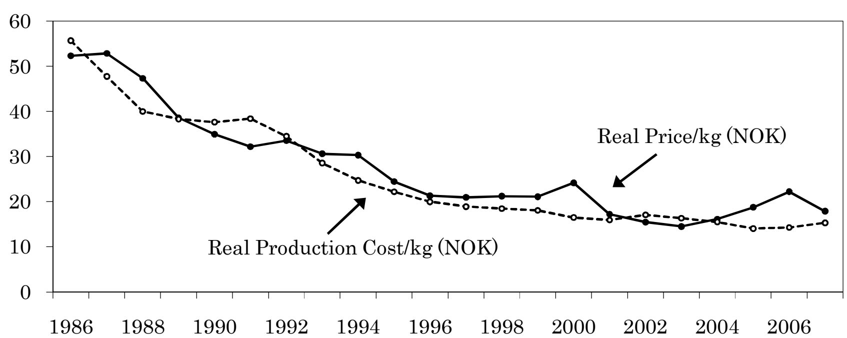

FIGURE 2. Cost of production and price per kilo fish for the Norwegian salmon farming industry 1986-2005. Source: The Norwegian Directorate of Fisheries.

We can define volatility as the fluctuations of prices above and below some pre-conceived long term trend or equilibrium. These price movements are for the most part risky as the direction and force of the motions are largely unknown on the short term basis. For the salmon industry the level of prices functions as the target for which production is evaluated. When prices increase the farmers seek to increase profits by increasing the amount of salmon produced and sold, when prices decline the farms might choose to reduce the intensity of production and the amount of fish sold in an attempt to wait for prices to increase. The biological nature of production implies that desired production output does not always meet its target. Disease, escape of fish and water temperature conditions are important factors that determine the final stock of fish. As such the possibility at any time to clear the market is not at unity. There will be periods of over and under supply which will cause prices to fluctuate. In particular the market for fresh fish will be subject to volatility as inventory keeping is limited, although some flexibility is allowed through the stocking of fish in pens. This inventorying is limited because the fish eventually reaches sexual maturity and when it does its quality deteriorates rapidly. Salmon in Norway have the largest probability of reaching sexual maturity during August-September. Thus one would expect seasonal differences in the flexibility available for the farmers in exploiting profit probabilities. Further at the demand side, factors such as seasonality and changes in preferences (e.g. caused by information on animal diseases and potentially harmful and beneficial substances) and changing exchange rates in different markets will also contribute to the volatility of prices.[3] So, if salmon farmers are risk averse they will use the volatility of prices, in addition to the level, as a target to evaluate the amount of salmon produced and sold at the market at a given time. Information on the short term dynamics of volatility can thus provide valuable information since on the short term basis farmers have some level of flexibility in realising an optimal utility of profits.

We now start our analysis of the salmon price volatility. We do this with the assumption that the volatility term is approximated by a random process, an assumption that seems reasonable in light of the large degree of uncertainty inherit in the market. As we will see

[3] Kinnucan & Myrland (2002, 2001) provide a discussion of the impact of exchange rates.

this assumption will soon break down; the analysis will show that the volatility term itself contains valuable information.

THE SHORT RUN DYNAMICS OF SALMON PRICES

Our data set is provided by the Norwegian Seafood Export Council and consists of 650 weekly observations of salmon prices in Norwegian Kroners from the start of 1995 to week 21 in 2007. One observation of price at time t will be denoted as X_t. As a starting point we decompose the price path as such.

$$dX_t = \mu X_t + \sigma X_t dB_t \tag{1}$$

The above Stochastic Differential Equation breaks the price movement down in two parts. One predictable, or trend part μX_t, and one noise part, $\sigma X_t dB_t$ accounting for the uncertainty of price movements. The uncertainty of price movements σ is driven by the Brownian motion B_t, which in its increments is normally distributed with mean zero and variance equal to the size of the time increment. Note that the price decomposition contains two information terms, namely the drift term and a constant volatility term. The Brownian motion is pure noise and contains no information.

This basic way of modelling price movements is much applied in financial economics. We will argue that the price process in the salmon industry may be described by the same process. The selling and buying of salmon is motivated by the same incentive for utility maximization as any financial asset investment. The sale of salmon does not have to occur at the exact moment the fish reaches sellable size; the profit maximizing policy of sellers is a dynamic problem, they might hold the salmon and wait for price to change or sell it immediately. This strengthens the speculative forces underlining the price of salmon.

Since uncertainty is a fundamental attribute of the salmon production process we know that the price of salmon is volatile. A hypothesis concerning salmon prices is therefore that the price process is very much explained by the Brownian motion, and that long term predictability is limited. In our time series the long term predictability, or drift term, is linked to any trend observed in the given time domain.

The relative difference in price levels, or return, from week to week is denoted as $R = X_t / X_{t-1}$. To account for proportional changes in returns we apply a logarithmic transformation of the price difference such that $Y_t = \ln X_t - \ln X_{t-1}$. The logarithmic transformation is also applied to the price process; transforming both the variables and the shape and moments of the probability distribution $dY_t = (\mu - \tfrac{1}{2}\sigma^2)dt + \sigma dB_t$. The log return Y_t is normally distributed with mean $(\mu - \tfrac{1}{2}\sigma^2)\Delta t$ and variance $\sigma^2 \Delta t$. This simple model, in the case of zero drift, assumes that log returns are independent. For the Black-Scholes option pricing formula, for example, the pricing equation does not contain a local mean rate of return. Generally this seems like a strict assumption, and as such the seminal work done by Black & Scholes has been criticized for this independence assumption. In fact, empirical analysis of stock returns indicates that non-linear functions of returns are autocorrelated (Jones, 2003). The non-zero correlation between different powers of return gives rise to volatility clustering. Thus log-returns, at least for stocks, often seem to be connected not only through a drift term but also through a non-zero conditional variance.

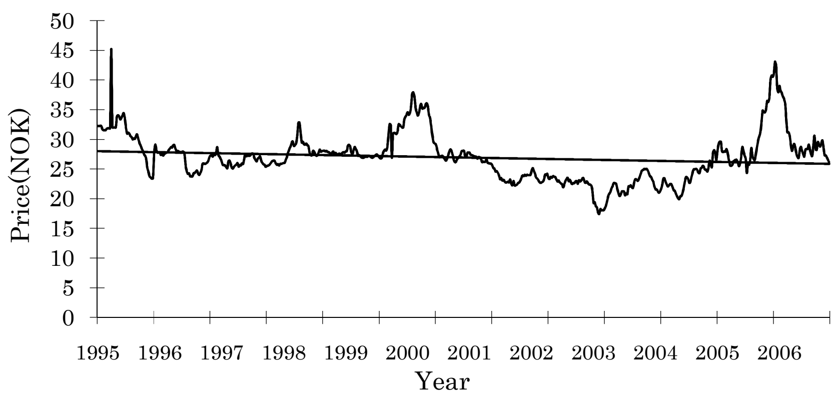

FIGURE 3. Weekly salmon prices from 1995 to 2007 with fitted trend line.

If the noise term σ is equal to zero, the price movement is completely predictable and described by the linear relationship $Y_0 + \mu t$. Thus we see that volatility is the term describing the divergence of prices from its predictable level. In relation to salmon prices we might expect that the price will often diverge from any assumed predictable level. From 1996 to 2007 we observe that the trend line in prices (figure 3) is weakly declining. Increasing industry productivity subsequently explains the decline in prices over time.[4] In our figure prices are nominal so that the downward effect from increased productivity on prices is counteracted by inflation.

If the market for salmon is completely efficient, meaning that all relevant information concerning the future value of salmon is incorporated in its price, the predictable part of the price movement approximates to zero; more precisely, any trend observed in the price in the case of an efficient market is due to inflation. Thus the change in price from week to week should be completely described by the noise term $\sigma X_t dB_t$. The parameter σ in the price process is the fundamental measure of volatility, and is in this simple description assumed to be constant. From figure 3 it is hard to argue that the predictable factor μ is very dominant, there seems to be little drift in the price process and the dominant part of the given price movement seems to be given by the Brownian motion. If this holds then no patterns in prices can be found, and thus for the market participants they would be unable to acquire any information on the future price movements. The best prediction on future prices would simply be today's price levels, where the volatility term would be a simple white noise term.

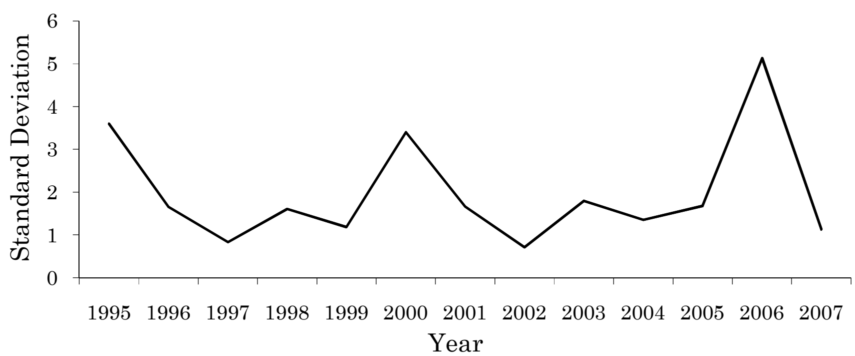

FIGURE 4. Annual average of weekly standard deviation of salmon prices.

[4] See e.g. Asche (1997), Asche & Tveteras (2002) .

In order to examine the noise term of the production process, we now apply two empirical measures of volatility on the salmon price series, a standard deviation measure and a historical rolling volatility measure. The standard deviation measure in Figure 4 gives us the annual average variation of prices from its mean. This simple measure gives us our first indication that volatility itself fluctuates. The annual standard deviation only gives one observation per year and hence it does not contain much information.

To give a more detailed picture of volatility we expand on the annual standard deviation measure by using a rolling measure in which we measure the divergence of prices from a 20 week moving average.

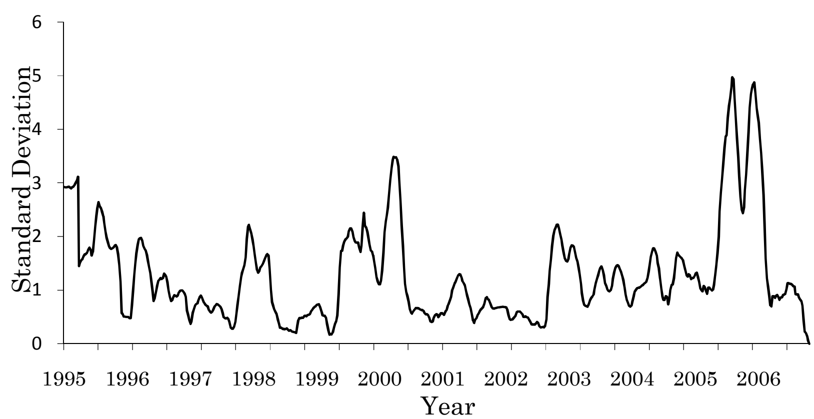

FIGURE 5. Twenty weeks moving average of salmon price volatility.

As indicated in Figure 4 and 5, the volatility is displaying significant variation over time. In addition, volatility seems to "spike" in certain time intervals. There appear to be significant positive shifts in the volatility process. This suggests that the volatility σ in our price process is stochastic, and that the assumption that volatility σ is constant seem insufficient in describing the price process. When modelling stochastic volatility to incorporate spikes, the Ornstein-Uhlenbeck process for volatility has been applied (Zerili, 2005). The Ornstein-Uhlenbeck process allows for autocorrelation in volatility.

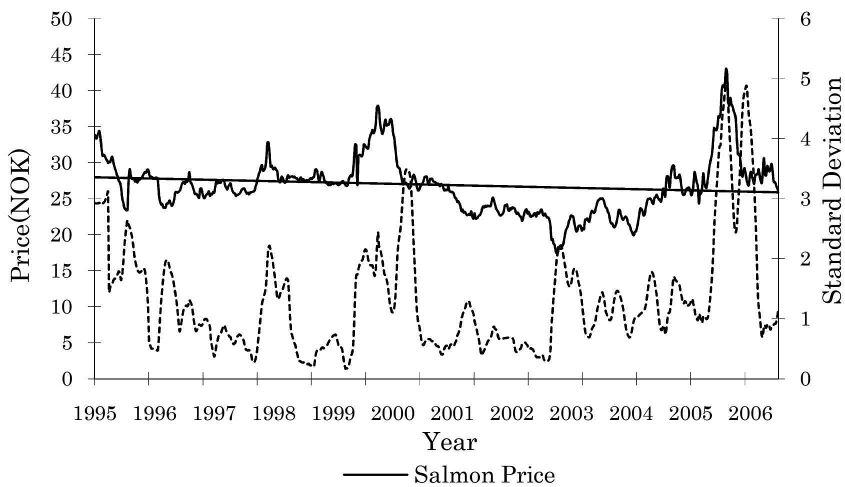

FIGURE 6. Salmon Price and Volatility.

The discrete time counterpart to the Ornstein-Uhlenbeck process can be implemented by the GARCH model. The indication that volatility is a stochastic process opens up the possibility that volatility is connected across time, and that a GARCH model is suitable to describe the volatility process for the discrete time approach.

By examining Figure 6 another pattern in the volatility process seems to emerge. The figure suggests that volatility is greater in periods of relative high prices; that a positive correlation between price and volatility exists. In the theory of storage it has been conjectured that such a relationship should exist (Deaton & Laroque, 1992; Chambers & Bailey, 1996). In periods with scarce availability of goods, for example due to a streak of bad harvests, the price is allowed to persist above the long run equilibrium level. As inventories are emptied the producers reach a state where excess demand cannot be satisfied. This gives rise to the characteristic price spikes observed in commodity markets; and as such larger than average volatility. In order to examine this property we divide our data-set in two; one set where price is below the trend and one where it is above. Thus this functions as a proxy for a high and low price data set. Further we test, using both the Levene (1960) and Brown & Forsythe (1974) test, whether the standard-deviation of the two price sets are significantly different, as shown in Table 1. We note that the standard deviation of the "high price" and "low price" series are 3.47 and 2.27, respectively.

TABLE 1. Levene/Brown and Forsythe test for equality of variance.

Dummy	Mean	St.Dev.	Freq.
Low price	24.33	2.27	360
High price	30.19	3.47	290
Total	26.95	4.08	650

w0 = 40.14	df(1,648)	Pr > F = 0.0000000
w50 = 13.26	df(1,648)	Pr > F = 0.0002914
w10 = 24.15	df(1,648)	Pr > F = 0.0000011

*The term w0 reports Levene's statistic, and w50(median) and w10(10 percent trimmed mean) replaces the mean with the two alternative location estimators as proposed by Browne and Forsythe.

Both the Levene's and the Brown & Forsythe test indicate that the standard deviations are different. As such this approach supports the suspicion that volatility is larger in periods of high prices. For market participants this means that larger expected profits generally come at a trade-off of larger price risk.

Next we move to the log-space, where we apply our measures of volatility to the log-return of prices. By examining returns instead of levels we are able to say something about the short term dynamics of price movements; that is the corrective movements in prices. The return movement indicates how the market price converges to the equilibrium price. If the equilibrium price level is constantly changing, as we would assume in a market with much uncertainty, this would lead to large volatility in returns as prices constantly "catches up" to the equilibrium price.

If drift is absent from the return process we should observe that the log returns are independent and (in the case of a constant volatility term) fluctuate unsystematically around zero as drift-less Brownian motion increments (the Brownian motion is as stated independent and normally distributed in its increments).

Figure 7 shows the movements of log-returns. The mean of log returns is estimated to -0.00032. Thus this simple description seems to indicate that log-returns are reasonably approximated by Brownian motion increments. But as we will see later this simple analysis is incomplete as it cannot isolate what part of volatility is random and independent and what is correlated.

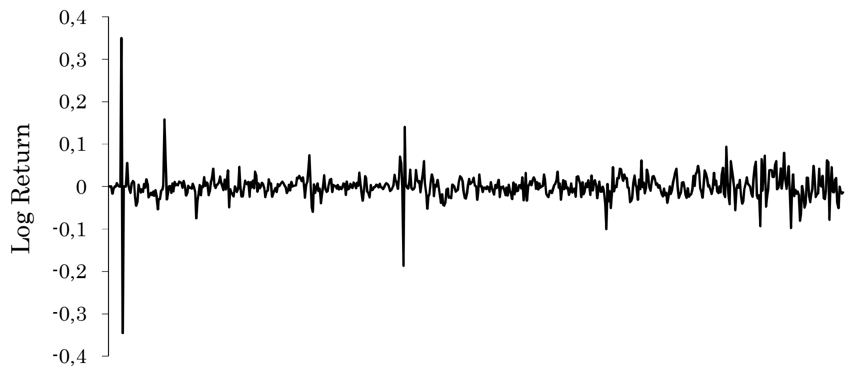

FIGURE 7. Log-Return of salmon prices.

If we were to assume that log-returns are in fact normally distributed and follow Brownian motion increments we can reach an estimate on annual standard deviation of log returns based on the expression:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y}_t)^2}\sqrt{\Delta T}$$

Where $\overline{Y}_t$ is the mean annual log-return, ΔT is the number of periods, here 52 weeks, and n is number of observations per year, also 52.

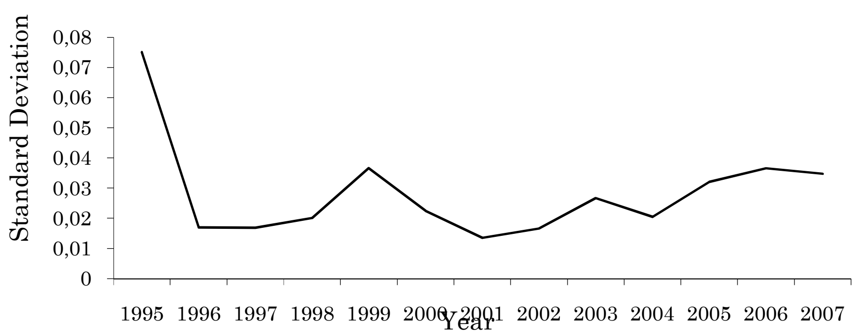

FIGURE 8. Estimation of annual volatility of log returns based on assumption of log-normal returns.

Now, Figure 8, as well as previous figures, suggests that the variance of salmon price is itself volatile, such that the volatility term σ becomes stochastic. Thus the simple estimate on annual standard deviation becomes unreasonable, since it assumes that returns are drawn from a fixed log-normal distribution in the sample interval.

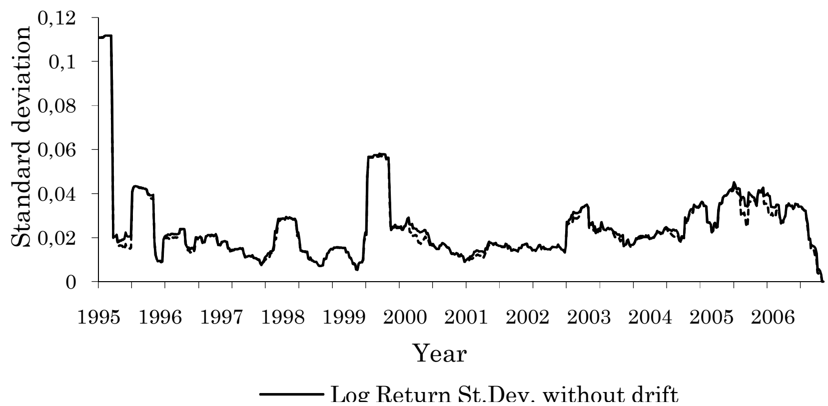

FIGURE 9. Twenty weeks moving average of log returns with and without drift.

To obtain a more complete picture of log-return volatility we apply the dynamic moving average measure. Figure 9 depicts the moving average with and without drift. The figure supports the hypothesis that drift is largely absent in the salmon return process. There seems to be little divergence between a drift and a zero drift process. The difference between the two moving average measures is a mean adjustment term to the log-returns in the case of the drift measure. If there were significant drift in the price process this would lead to a notable difference between the two measures since log-returns would over time diverge from zero.

This figure also suggests that volatility displays clustering. The indication of volatility clustering further strengthens our suspicion that the volatility term of the price process is stochastic. It appears to depend on previous week(s) volatility.

It is also necessary to determine the time series properties of the variables in order to avoid the problem of non-stationarity. We test for non-stationarity using the Augmented Dickey-Fuller (ADF) test. We included a constant in all the variables that do not appear to be trending, and a trend in the ADF test on the highly trending volume traded variable. The results are shown in table 2. Lag length was chosen to minimize the Akaike Information Criterion. The most important tests are the tests on log returns and log volume change (log-diff.-volume). The ADF tests reject the null of non-stationarity on both of these variables at the five percent level.

TABLE 2. Augmented Dickey Fuller unit root test.

Series	t-adf	Lag length	Options included
Salmon price	-2.748	2	Constant
Log-Return	-26.84**	0	Constant
Volume	-12.10**	1	Constant and trend
Log.-diff.-volume	-10.75**	14	Constant

We also tested for "ARCH effects" (Engle 1982) on both log return and log-diff.-volume We regressed the dependent variable (log return and log-diff.-volume sequentially) on a constant, and saved the residuals, squared them, and regressed them on five own lags to test for ARCH of order 5. We obtained R^2 and multiplied with the number of observations. This test statistic is distributed as Chi-square. The test statistic (table 3) for both log return and log-diff.-volume, shows that the series show evidence of ARCH effects. A test for autocorrelation in the data was also performed. The Ljung-Box test suggests that autocorrelation is present in all series except log returns.

TABLE 3. Autocorrelation and ARCH tests.

Price Series	Autocorrelation	ARCH
	Ljung-Box (25)	Chi ^{2}
Salmon Price	8405**	
Log-Return	24	220**
Volume	6096**	
Log-diff.-volume	114**	265**

The analysis so far suggests that long run predictability is severely limited; a drift in the price process is largely absent in our time frame, and such that the volatility movements is important in describing the price process. Further, the existence of spikes and clustering of volatility suggests that volatility is described by a stochastic process and that it is not independent across time.

Despite a lack of predictability arising from an approximately zero drift term, the log returns still might display correlations arising from a non zero conditional volatility. Thus the natural extension of the analysis is to apply the GARCH model to our price process.

ECONOMETRIC APPROACH

If we simulate an ARCH(1) series, we can see that the ARCH(1) error term u_t has clusters of extreme values. This is a consequence of the autoregressive structure of the conditional variance. That the variance is dependent on the squared variance of the previous period leads to the possibility of higher power correlations between log-returns. If the realized value of u_{t-1} is far from zero, h_t (the conditional variance of u_t) will typically be large. Therefore, extreme values of u_t are followed by other extreme values, and thus we observe the clustering seen in financial market returns.

There have been some difficulties implementing the ARCH model. A problem is that often a large number of lagged squared error terms in the equation for the conditional variance are found to be significant on the basis of pre-testing. In addition, there are problems associated with a negative conditional variance, and it is necessary to impose restrictions on the parameters in the model. Consequently in practice the estimation of ARCH models is not always straight forward. Bollerslev (1986) extended the ARCH model and allowed for a more flexible lag structure. He introduced a conditional heteroskedasticity model that includes lags of the conditional variance (h_{t-1}, h_{t-2},…, h_{t-p}) as regressors in the model for the conditional variance in addition to lags of the squared error term $(u_{t-1}^2, u_{t-2}^2, ..., u_{t-q}^2)$, which leads to the generalized ARCH (GARCH) model. In a GARCH(p,q) model, u_t is defined as:

$$u_t = \varepsilon_t (\alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j})^{1/2}$$

where ε_t ~NID(0, 1); $p \geq 0, q \geq 0; a_0 > 0, a_i \geq 0, i = 1,...,q$ and $\beta \geq 0$, j = 1, 2,…, p. It follows from manipulation of the above equation that h_t (the conditional variance of u_t) is a function of lagged values of u_t^2 and lagged values of h_t:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \ .$$

Earlier in the paper we noted that volatility is larger in periods of higher prices, as such it seems reasonable that the volatility process is asymmetric and positively skewed. In order to incorporate asymmetric volatility it is normal to apply the EGARCH (exponential GARCH) rather than GARCH model. In describing our price series we have not found this to be a suitable approach. Under leptokurtic distributions such as the Student-t distribution, the unconditional variance does not exist for EGARCH. The exponential growth of the conditional variance changes with the level of shocks, this leads to the explosion of the unconditional variance when extreme shocks are likely to occur. In empirical studies it has been found that EGARCH often overweighs the effects of larger shocks on volatility and thus results in poorer fits than standard GARCH models[5].

ECONOMETRIC RESULTS AND DISCUSSION

A normality test (Doornik & Hansen, 1994), presented in Table 4, indicate non-normality, which is not surprising considering many large residuals. Non-normality is an inherent feature of the errors from regression models of financial data, and hence robust standard errors are calculated. Further, the price level series displays kurtosis (1.6361) and skewness (0.8663). Concerning log returns, the distribution displays excess kurtosis (45.324) but as opposed to the price level series skewness (0.094122) is to a large degree eliminated. Furthermore, the large kurtosis in log returns means that more of its variance is explained by infrequent extreme deviations from its mean. This illustrates the uncertainty and risk underlying the return process in the industry.

TABLE 4. Descriptive Stastistics

Series	Mean	Std.Dev.	Skewness	Kurtosis	Normality Chi^2
Salmon Price	26.946	4.0835	0.8663	1.6361	67.858**
Log-Return	-0.00032	0.0318	0.0941	45.324	3607**
Volume	5305.9	1954.6	0.8459	1.0401	81.885**
Log-diff.-volume	0.002309	0.4935	0.0300	129.11	9449.3**

Corresponding results for both volume and log-diff.-volume can be seen in table 4 below. Applying kernel density estimation with a

[5] See the empirical study of Engle and Ng (1993).

Gaussian distribution term we can estimate the distribution of the price series and log-returns.

As figure 10 shows the skewness is to a large degree eliminated when looking at log-returns. The low level of skewness suggests that in the short term there is little possibility of any reliable excess return. In addition the high kurtosis in log returns means that more of its variance is explained by infrequent extreme deviations from its mean. This would suggest that large returns are generated by unpredictable shocks. The distributional analysis indicates that assuming a normally distributed error term in the price structure of salmon is non-trivial, and that any research on salmon prices should account for the distributional form of the price series in their time domain.

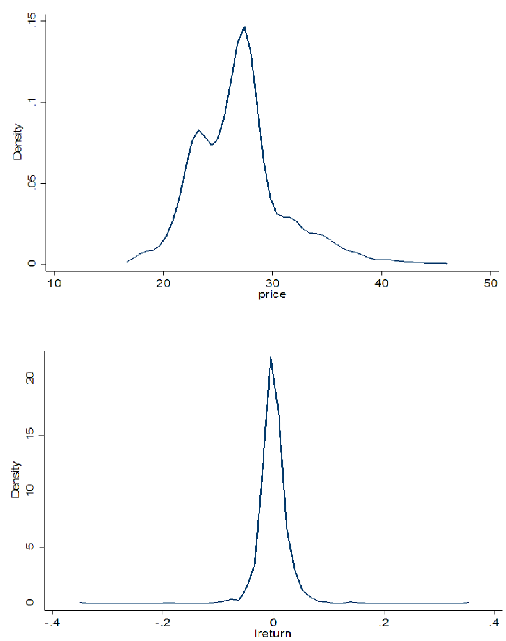

FIGURE 10. Kernel density estimates of price level and log return.

In the volatility equation we include the stationary time series of log volume traded differences. The reason for including volume can be found in the relationship between storage and short term price dynamics in commodity prices (Deaton & Laroque, 1992; Chambers & Bailey, 1996). The theory states that storage allows the smoothing of short term price fluctuations. In production of goods with limited durability; such as fresh salmon, the possibility for inventorying is limited. One might conjecture that the only possibility for storage of fresh fish in aquaculture is through a continuation of cultivation. As such there exists an inverse relationship between the growth in volume sold and the availability of inventories to smooth future prices; or alternatively that the growth in volume indicates the utilization of inventories. The relationship between volatility and volume is also investigated in financial markets (c.f. Bessembinder & Seguin 1993).

We estimate the GARCH model with Student-t distributed errors, as proposed by Bollerslev (1987)[6]. The distribution tends to the standard normal when degrees of freedom go to infinity. From table 5 below we observe that the optimal number of lags in our model is five.

The model is estimated with a five week lag in the price equation and a one week lag in the GARCH and ARCH terms.

$$y_t = \mu + \sum_{i=1}^{5} \eta_i y_{t-i} + u_t \tag{2}$$

$$h_t = \alpha_0 + \gamma \Delta Volume + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

Here $\Delta Volume$ is along with return defined on log form. The model (2) – (3) was estimated sequentially using maximum likelihood[7].

[6] Likelihood Equation evaluated

$$\ell(\Phi) = T \log\left\{ \frac{\Gamma(\upsilon+1)/2}{\pi^{\frac{1}{2}}\Gamma(\upsilon/2)} (\upsilon-2)^{-\frac{1}{2}} \right\} - \left(\tfrac{1}{2}\right)\sum_{t=1}^{T} \log(h_t) - [(\upsilon+1)/2]\sum_{t=1}^{T} \log\left[1 + \frac{\left(y_t - \mu - \sum_{i=1}^{5} \eta_i y_{t-1}\right)^2}{h_t(v-2)} \right]$$

Where Φ represents the parameters space; υ the degrees of freedom and Γ the gamma function. See Hamilton (1994) pp 662 for further detail on GARCH estimation with t-distributed errors.

[7] Akaike Information Criterion also confirms that log-diff-volume in the variance equation should be included. AIC with volume included is -4.88, and is -4.87 without log-diff-volume in the estimation.

TABLE 5. Akaike Information Criteria (AIC) and Bayesian Information Critaria (BIC).

GARCH(1,1)*		
	AIC	BIC
AR(1)	-3139.51	-3133.84
AR(2)	-3139.13	-3132.66
AR(3)	-3139.46	-3132.17
AR(4)	-3140.02	-3131.92
AR(5)	-3146.11	-3137.21
AR(6)	-3137.3	-3127.58
AR(7)	-3132.59	-3122.06
AR(8)	-3123.85	-3112.52
AR(9)	-3116.76	-3104.62
AR(10)	-3108.86	-3095.91

*Extending the GARCH terms to GARCH(2,1), GARCH(1,2) or GARCH(2,2) does not improve the fit over the GARCH(1,1) alternative

From table 6 we observe that both previous period variance and error term is significant on the 5% level on today's variance of price. Thus the large spiking and clustering in volatility indicated earlier can be explained by the conditional variance term. Intuitively the lag 1 structure of variance suggest that if price was very volatile last week it is more likely than not to be very volatile this week as well. After a period with high volatility, one can expect that the volatility reverts to a more stable level. For aquaculture firms this means that volatility last week has some predictive power concerning this week's volatility, and as such can offer information to a risk averse firm who values information on price volatility.

In the variance equation, we see that $\Delta Volume$ is negative and significant on the five percent level: the conditional variance of salmon prices is negatively (positively) related to positive (negative) changes in traded volume. Following the reasoning for including volume movements in the volatility equation, the results state that as the utilization of inventories increase volatility decreases.

TABLE 6. AR(5)-GARCH(1,1) Estimation Results.

Parameter			
Mean Function	Coefficient	Robust Std.Dev.	t-values
μ	-0.00024	0.00068	-0.358
η_1	0.35227**	0.04683	7.52
η_2	-0.02208	0.04079	-0.541
η_3	-0.06444	0.04129	-1.56
η_4	0.02923	0.03537	0.827
η_5	0.08648**	0.03061	2.83
Variance- Function			
α_0	0.00018**	0.000003	2.81
γ	-0.00035*	0.00016	2.13
α_1	0.44230**	0.1259	3.51
β_1	0.3694**	0.1214	3.04
Log likelihood	1581.8		

** implies significance on the one percent level, * implies significance on the five percent level

This supports the relationship that the availability of inventories helps smooth prices. However the utilization of inventories today comes at a trade-off of lower inventories tomorrow such that the option of smoothing prices in the future has decreased. We should note that although the difference in volume traded is statistical significant, it is less likely to be economically significant due to a low coefficient value. Figure 11 shows the relationship between log return standard deviation and changes in volume traded. In table 6 we observe how the conditional mean (return) is related to its previous values. Particularly, lag 1 and lag 5 are significant and positive. The return in week *t* depends on the return last week and return five weeks ago.

Next we perform misspecification tests on the model residuals. The Portmanteau statistic for the scaled residuals returns a Chi square value of 15.453 (a *p*-value of 0.75). The Portmanteau statistic for squared residuals results in a Chi square value of 0.31328 (a *p*-value of 1). Hence, the Portmanteau tests reject autocorrelation in the residuals. We test for error ARCH from lags 1 to 2. With a *p*-value of 0.97 we reject ARCH 1-2 in the residuals. Lastly, a normality test is performed. A *p*-value of 0.00 implies serious non-normality. With regressions from speculative prices, we do not get normally distributed errors. We therefore report robust standard errors.

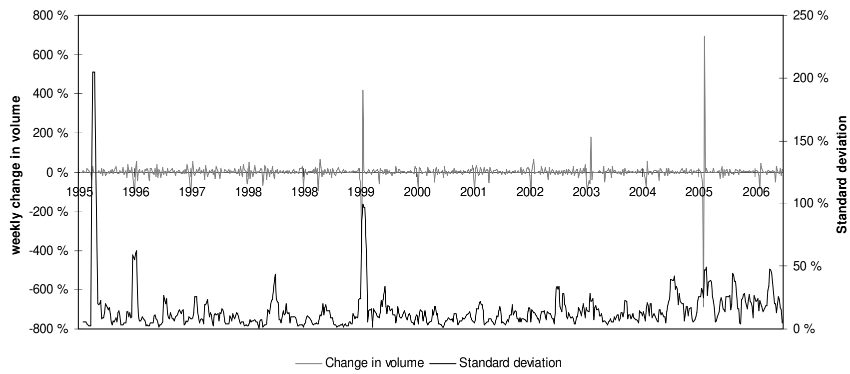

FIGURE 11. Change in volume traded and standard deviation of log-returns.

In a GARCH(1,1) model, the sum $(\alpha_1 + \beta_1)$ measures the degree of volatility persistence in the market; the speed at which the market dissipates a shock. What this means is that the larger the persistence is the lower is the speed of reversion in the market. We note that the value of volatility persistence in our model is estimated to 0.81. To put this in context we note that Buguk, Hudson & Hanson (2003) found persistence values for catfish, corn, soybeans and menhaden equal to 0.98, 0.94, 0.88 and 0.38, respectively. Moreover, this suggests that the market for salmon displays a lower degree of volatility persistence than catfish, corn and soybeans products, but greater than menhaden.

Furthermore, we might use the degree of volatility persistence in the market to estimate the half life of a volatility shock. The half life estimate measures the time it takes for a shock to fall to half of its initial value and is determined by (Pindyck, 2004):

$$\text{Half-life time} = \log(.5)/\log(\alpha_1 + \beta_1)$$

We calculate a half life time of 3.3 weeks. Recent literature on volatility persistence suggests that the persistence in the conditional variance may be generated by an exogenous driving variable that is itself serially correlated. Hence the inclusion of such an exogenous variable in the conditional variance equation would reduce the observed volatility persistence (see Lamourex & Lastrapes, 1990; Kalev et al., 2004). We find that excluding the exogenous variable results in a half life time of 4.4 weeks.

CONCLUDING REMARKS

While production risk in salmon aquaculture has received substantial attention, little focus has been given to price risk. It is important to understand price risk as is seems to be a main factor driving the profitability cycles the industry is experiencing. Our results indicate that the assumption of an independent zero mean normally distributed error term is not trivial when modelling salmon prices. We find that the salmon prices and log-returns are non-normal, and display skewness and kurtosis for the former and kurtosis for the latter case. Assuming normality when modelling salmon prices is not supported by our study. Moreover, we find that a AR(5)-GARCH(1,1) process describes the salmon price process. Thus academic research applying salmon prices should account for the fact that there is persistence of volatility itself on the short-term dynamics. For studies of price forecasting, for example, this means that in periods of large shocks we cannot expect as precise forecasts, even in periods following the shock since volatility will generally persist for some time as the market corrects. Also for studies of market integration we note that if comparable salmon goods are to be aggregated they should also display some of the same volatility patterns, we should observe some volatility spill over effects between comparable goods. For the relevant market participants the fact that volatility clustering is existent offers some predictive information on the price fluctuations in the market. More specifically we find that previous week's volatility offers some indication of next week's volatility. This provides information to a market chronically missing stability and predictability. Risk averse market participants can avoid trading next week if they observe that volatility is large this week. This gives the market participants an additional hedging possibility; there is clear evidence that volatility reverts following a shock. We also find support for the hypothesis that volatility is larger in periods of high prices. For the industry this means that larger expected profits more often than not comes at a trade-off of larger price risk.

Our results also indicate that the degree of volatility persistence in the market for salmon aligns itself with a small sample of other agricultural goods. We also note that following a shock, the volatility half's in an estimated 3.3 weeks, which offers some planning information for the market participants. Concerning the predictability of prices we find that today's log-returns are dependent on a 1 and a 5 week lag of log-returns. This means that there is some level of short term predictability present in the market. To some degree this supports studies that claim to offer some level of short term predictions of salmon prices. Concerning long term predictions on price levels we find that the long term trend in prices is weakly declining. The decline is mostly due to increasing industry productivity. As such, any prediction on future price levels can, at least in the long run, be found in the continuation of the productivity increase. Short term price correlations offer no predictive powers on any long term price levels. The focus of this paper has been on understanding price risk in salmon prices. Future research can be conducted on evaluating forecasting performance of different volatility models.

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SUPPLY-SIDE EXPLANATIONS FOR VOLATILITY SPILLOVER IN PRIMARY COMMODITY PRICES

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Abstract In this paper, I investigate supply-side explanations for nonsporadic volatility spillover patterns in perishable commodities. I suggest that correlations in inventory stocks and differences in inventory flexibility, as well as common price deterministic factors such as input factor usage and substitutability between goods, allow nonsporadic volatility spillover to persist in primary commodity markets. To investigate these claims empirically, I apply a volatility spillover test to a basket of goods from the Norwegian aquaculture industry, where differences in characteristics of goods allow examination of the economic significance of volatility spillover. I find that volatility spillover is present in the salmon market and that the less perishable frozen commodity takes a leading role in spillover causation.

INTRODUCTION

In the theory of storage (Kaldor, 1939; Working, 1948,1949; Deaton & Laroque, 1992; Pindyck, 2001; Chambers & Bailey, 1996) the persistence of short-run price movements is largely explained by suppliers' ability to use inventories to smooth price variations. Inventories enable producers and/or speculators to link the spot price to the expected future price, and under a rational expectations regime, dampen price movements by satisfying short run excess demand. By linking spot price to expected future prices, a positive low-order autocorrelation in prices is generated. Deaton & Laroque(1996) find that speculative behaviour alone is not sufficient to explain the large autocorrelations displayed by commodity prices. They conclude that most likely underlying autocorrelations in fundamental supply and demand processes add to the high low order autocorrelations observed in commodity prices. Moreover, higher price volatility generally leads to an increase in the demand for inventories as the value of having inventories increase. In commodity markets, high-volatility regimes are often identified by periods of exhausting inventories where price smoothing is less efficient. Since inventories are so pivotal in accounting for short-run commodity price dynamics, it seems reasonable that inventory dynamics might also account for short-run co-movements in related commodity prices. In this paper, I investigate the significance of inventories in explaining volatility spillover between commodity prices when inventories are correlated. By this I provide supply-side explanations for the existence of non-sporadic price volatility spillover. To test for volatility spillover we apply the bi-variate causation in variance test of Cheung & Ng (1996).

One motivation for keeping inventories is so to quickly react to emerging profit opportunities. Different commodity markets have different possibilities for storage, and storage can take different forms. Producers of fresh agricultural goods, for example, will have inferior storage flexibility in the traditional sense of keeping goods at warehouses. On the other hand, for these producers, storing can be done by delaying harvest, in effect storing by further cultivation. The decision to reduce inventories can have effects outside immediate stocks effect. A commodity stock is often not independent of the stock of other commodities. An example of this is use of commodities as inputs in the production of processed goods. Certain commodities are also inputs in own production. Live animal stocks are used for the breeding of future stocks. These dynamics in commodity stocks are conjectured to generate long run price cycles. The classic example is the hog, cattle and broiler cycles (Hayes & Schmitz, 1993; Shonkwilder & Spreen, 1986; Harlow, 1960; Rosen, Murphy & Scheinkman, 1994; Rausser & Cargill, 1970).

In this paper we argue that inventory dynamics, through correlations in stocks levels, can generate price volatility spillover effects. When commodities enter high-volatility regimes, it is often due to increased scarcity of the good. Goods become unique in the sense that arbitrage is allowed to persist as prices are allowed to spike to abnormal levels. If different commodity stocks are correlated, this also suggests that related commodities will enter high-volatility regimes, in effect generating volatility spillover. Furthermore, differences in inventory flexibility between related goods will generate a leading, causal relationship in volatility spillover. Goods better suited for storage will appear to cause volatility spillover into markets with less flexible storage. This happens because goods better suited for storage will buffer price movements with greater ease than less flexible goods.

In a rational expectation market the speed of volatility transmission does not have to follow the speed of inventory correlations. Stock levels in one market can provide information on the future stock levels in related markets, leading prices to move as the market anticipates future stock changes. This correlation in stocks provides a supply-side explanation for the existence of volatility spillover in commodity markets. If the speed of reacting to anticipated inventory correlation effects is sufficiently great, this effect will however not be reasonably isolated from market integration effects. In this case causation inference is impossible due to common price driving factors. Common stochastic trends can gives rise to apparent volatility spillover effects at lower lag orders.

In this paper I test for volatility spillover in the Norwegian salmon aquaculture industry. Farm bred salmon is sold in several different product forms, where the stocks of each product is related to a common stock of fish. We test for volatility spill-over between the markets for different weight-classes of fresh salmon in addition to frozen salmon. This provides a spectre of stock correlations and storage flexibility in commodities relevant to investigate to supply side explanations for volatility spillover.

A common approach to volatility spillover analysis is to specify a multivariate GARCH model, where conditional variances are regressed on own past conditional variance and squared residuals, in addition to other lagged variables of related goods. Cheung & Ng (1996) provide an alternative test for volatility spillover, where the sample cross-correlations between standardized squared residuals are used to determine any significant volatility spillover. This simplifies the analysis by evaluating all prices against each other one at a time. One can then avoid the potentially complex equations involved in fitting a multivariate GARCH model. The approach of Cheung & Ng will be the method of analysis in this paper.

In the literature on volatility spillover, most interest has been directed at analyzing traditional financial assets such as exchange or interest rates (Apergis & Rezitis, 2003; Reyes, 2001; Hong, 2001; Kanas & Kouretas, 2001; Kim & Rui, 1999; Tse, 1999; Gallagher & Twomey, 1998; Byars & Peel, 1995). In addition, it seems that a multivariate GARCH model has been the preferred tool for analysis, at least prior to the article by Cheung & Ng (1996). In primary commodity markets, the study of volatility spillover has received less attention. Buguk, Hudson & Hanson (2002) examine volatility spillover effects in the US catfish markets. They subsequently detected strong price spillover from feeding materials to catfish feed and output prices. Apergis & Rezitis (2006) study spillover effects in Greek agricultural products and find that disequilibrium at the input level is transmitted to the retail level, and vice versa. Furthermore, Yang, Zhang & Leatham (2003) study the volatility spillover in international wheat futures markets.

The paper is organized as follows. We start by positing the supply-side explanations for volatility spillover emerging from the theory of storage. Following this, we provide a short summary of Norwegian salmon aquaculture, deriving the predictions made on volatility spillover from our hypothesis. We then derive the empirical test for volatility spillover. We investigate the time-series properties of our data and derive specifications of the price processes driving the data generation. We then perform the empirical test and analyse these results in light of the postulated supply-side explanations. Our analysis shows that volatility spillover effects exist in the markets for farm bred salmon. Strong zero lag effects suggest that common price driving factor connect volatility. In addition, we find that volatility transmission effects are stronger from lower weight classes of fish to higher, consistent with correlation patterns in stocks.

INVENTORY CORRELATIONS AND VOLATILITY SPILLOVER

We might define storage as the action of keeping a stock of goods readily available for shipping to the market. In this sense, storage and its cost can take several forms. For agricultural commodities storage can for example be done by delaying harvest. From this definition we treat storage as the carrying forward of market ready stocks. In this sense, storage can be applied to a broad spectre of agricultural commodities where perishability inhibits traditional storage.

The theory of storage states that availability of stocks determines the market's ability to buffer price movements. Arbitrage opportunities due to positive price spiking are less likely to persist in markets where inventories can be emptied quickly to increase supply. Following the assumption that lower stock levels generate higher price volatility, we might state some implications regarding volatility spillover.

Implication 1: A significant cross-correlation between commodity stocks will generate price volatility spillover between markets.

To illustrate the intuition behind this implication, imagine that the price of a certain commodity increases. The price increase reduces the stocks of that good as producers capture the profits associated with higher prices. If a positive correlation in stocks exists between this commodity and another, the future stock of the related commodity will decrease. The lower future stock of the related commodity will increase the price volatility in the market. If stocks are negatively correlated, the opposite effect will occur. The speed at which this transmission effect occurs depends on the degree which the effect is anticipated by market participants.

Implication 2: The commodity with superior inventory possibility will take a leading role in causing volatility spillover between related goods.

As long as the cost and flexibility of storage varies across commodities, the superior storage commodity will be better suited to buffer price movements. This will generate a leading role in volatility spillover. An example of this is the relationship between fresh and preserved commodities. A preserved commodity will generally have superior storage opportunity and thus be better dampening effects of shocks from the fresh market.

Another issue related to volatility spillover worth mentioning is market integration. Some price determining factors can, to a varying degree, be common across commodities. These common factors generate a common trend and restrict the allowable divergence in relative prices. When investigating the market for fresh salmon, Asche & Guttormsen (2001) find that markets are highly correlated and that a common stochastic trend exists. This implies that fresh salmon prices are strongly linked at the zero frequency. The existence of common stochastic trends thus suggests that some volatility spillover exists in the markets outside of stock correlation effects. Market integration suggests a strong zero-lag volatility transmission. In our analysis, we are not able to differentiate between volatility transmissions that are a result of common factor shocks and those generated by heterogeneous shocks. In addition, if the co-integration relationships display persistence—for example, following a stationary autoregressive process—this translates to patterns in volatility spillover that give the impression of direct volatility causation. Failing to account for market integration relationships might bias the economic interpretation of causation relationships, and could be wrongly ascribed to properties such as the market's efficiency in substituting between goods. In the test by Cheung & Ng (1996), volatility spillover effects can exist even if no causality in the mean can be found, but the analysis does not distinguish between heterogeneous and common factor shocks. Our analysis thus reveals volatility spillover effects, if they exist, but does not account for the qualitative difference and causation of shocks. However, a strong zero-frequency volatility transmission indicates market integration.

EMPIRICAL APPLICATION

In order to examine volatility spillover, we use a basket of goods conjectured to be connected in stocks, but with somewhat different inventory correlations and flexibility. Specifically, the dataset consists of time series of fresh salmon of 2–3 kg, 3–4 kg, 4–5 kg, 5–6 kg and 6–7 kg in addition to frozen salmon.

As with agriculture in general, there are considerable biological dependencies in aquaculture production. In the common stock of farm bred salmon the growth of the fish provides the mechanism for correlation between stocks of different weight classes. Growth generates a positive correlation in stock sizes across different weight cohorts. This correlation is further asymmetrical as larger stocks of lower weight-classes generate larger stocks of higher weight-classes, but not the other way around. After harvesting the fish remains fresh for only a couple of weeks. Hence the option of storing fresh commodities outside of delaying harvest is limited. However, the farmer has the option to freeze the fish and considerably increase the storage opportunity. The more flexible nature of storage in the frozen fish suggests that the stock is less dependent on the stock of the fresh fish in the short run. The option of freezing the fish will, in general, decrease its market value, as a premium exists on the willingness to pay for fresh fish.

A preliminary indication of the effect of stock correlations on prices can be found by investigating relative prices and volume traded. We generate a series of volume traded and price of fresh salmon of 3–4 kg and 5–6 kg weight cohorts relative to the 4–5 kg cohorts. The correlations in stocks suggest that increasing the supply of 3–4 kg salmon should reduce the availability of 4–5 kg salmon. The sale of 5–6 kg however should not affect the availability of 4–5 kg salmon. In Figure 1, we plot the relative values of volume traded and prices from 1999:01 to 2006:11.

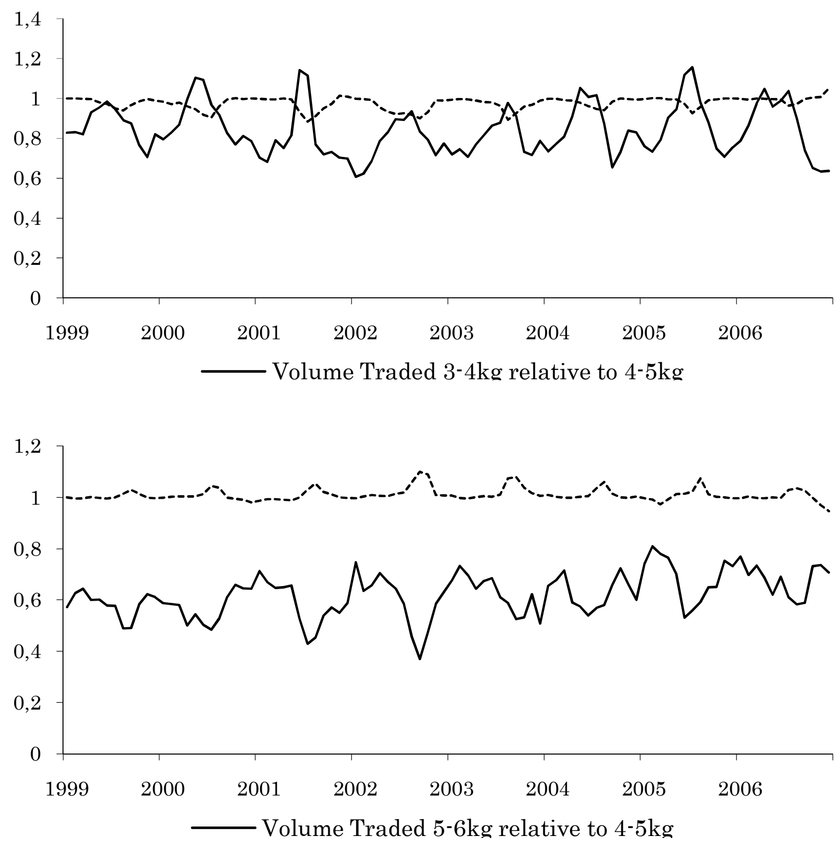

FIGURE 1. Relative price and volume traded

The relevant patterns reveal themselves by examining the peaks in relative prices with respect to movements in volume traded. In figure 1 we observe how an increase in sales of 3–4 kg relative to 4–5 kg is accompanied by a negative peak in the price of 3–4 kg salmon relative to 4–5 kg salmon. The same negative peak is not observed for 5–6 kg salmon relative to 4–5 kg salmon because 5–6 kg salmon fetches a higher price than 4–5 kg salmon. However, in this figure, the price peaks upward as supply of 4–5 kg salmon increases relative to 5–6 kg salmon.

These differences in price cycles suggest that it is the lowest weight class in both cases that are detrimental for relative price movements. The explanation for this in our model is simply that inventories, which are negatively related to sales, are positively correlated. If market participants observe an increase in the sales of a certain weight class of fish, in effect reducing its inventory, they know that this leads to a reduction in the future availability of fish of a higher weight class. The expected price of fish of higher weight classes will increase, thus increasing the spot price as arbitrage equate the current price to the appropriately discounted expected future price. Therefore, one expects a positive volatility spillover between fish of different weight classes.

With regard to our analysis we expect to observe some specific patterns in the analysis of volatility spillover in Norwegian aquaculture. In relation to market integration and common equilibrium factors, previous research has shown that the markets for different sizes of fresh salmon are integrated. Hence we expect to observe a high volatility spillover at the zero frequency. Because substitutability in the sales of fish of different weight classes exists, we also expect that volatility spillover is larger for goods closer in weight. Concerning storage flexibility we expect that frozen salmon will take a leading role in volatility causation due to the superior conditions for buffering shocks from fresh fish

In regards to correlations in inventories, movements in prices are not generally bounded by movements in physical stocks. As long as expectations are assumed rational, prices adjust to changes in expected stock levels. Thus, correlations in inventories are assumed to contribute to a strong zero-lag volatility spillover. For volatility spillover effects outside of the zero frequency, if they exist, we expect a stronger volatility transmission from lower to higher weight classes due to the nature of stock correlations.

EMPIRICAL METHODOLOGY

In order to estimate volatility spillover effects we apply the method first proposed by Cheung & Ng (1996). We derive the cross-correlation function (CCF) of squared standardized residuals. These series' are then used to test for Granger causality in variance. Under the null hypothesis of no spillover effects, the CCF will be asymptotically normally distributed with unit variance and zero mean.

The notation of the following derivation follows Cheung & Ng (1996). Assume that the data generating process of two time series can be specified as:

$$X_{1t} = \mu_{1t} + h_{1t}^{0.5}\varepsilon_t \qquad\qquad X_{2t} = \mu_{2t} + h_{2t}^{0.5}\xi_t,$$

where ε_t and ξ_t are two i.i.d. normally distributed stochastic processes with zero mean and unit variance. The conditional variance $h_{it}^{0.5}, i = 1,2$ is assumed to exist. The conditional mean and variance can be modelled by commonly used models such as ARMA or (G)ARCH. Next, we construct the squared standardized residuals as:

$$U_t = \frac{(X_{1t} - \mu_{1t})^2}{h_{1t}} = \varepsilon_t^2, \qquad V_t = \frac{(X_{2t} - \mu_{2t})^2}{h_{2t}} = \xi_t^2.$$

Furthermore, the CCF at lag k is defined as:

$$r_{uv}(k) = \frac{c_{uv}(k)}{\sqrt{c_{uu}(0)c_{vv}(0)}},$$

Where $c_{uv}(k)$ is the cross-covariance at lag k between the squared standardized residuals U_t and V_t. Furthermore, $c_{uu}(0)$ and $c_{vv}(0)$ are sample variances. By definition, $E(c_{uu}(0))$ and $E(c_{vv}(0))$ is equal to one. It follows that if no cross-correlation exists between U_t and V_t, $E(c_{uv}(k))$ is zero, and the CCF will tend to zero. Given a correctly specified sample counterpart of the CCF, $\hat{r}_{uv}(k)$, it follows that $\sqrt{T\hat{r}_{uv}(k)}$ converges to $N(0,1)$ (Theorem 1, Cheung and Ng, 1996). Thus, in order to test for causation in variance we compare

$\sqrt{T\hat{r}_{uv}(k)}$ to the standard normal distribution. The null hypothesis of no causation in variance between two time series can then be stated as:

$$H_0 : \sqrt{T\hat{r}_{uv}(k)} \sim N(x,1), x = 0\ ,$$

$$H_1 : \sqrt{T\hat{r}_{uv}(k)} \sim N(x,1), x \neq 0\ .$$

At this point, it is necessary to note that the assumption of a strict causal relationship between variances hinges on our definition of the information sets generating price movements in each market. Only under the strict assumption of two disjoint sets of information, each affecting the relevant time series, can we with certainty say that a causal relationship exists. If co-integration vectors exist, for example, we know that an additional information set can be isolated that affects both prices through a common equilibrium relationship. As stated above, what appear to be causal relationships can then simply be the result of common information entering the market. Based on this we will use the term "volatility correlation" rather than "causation" from now on.

DATA

The dataset used for the formal analysis is provided by the Norwegian Export Council and consists of seven time series of prices (salmon 2–3 kg, salmon 3–4 kg, salmon 4–5 kg, salmon 5–6 kg, salmon 6–7 kg, and frozen salmon) observed monthly from January 1990 to July 2007. This provides 211*6 (1266) unique observations. These goods are chosen because they display somewhat different fundamental properties and relations to each other relevant to the posited explanations for volatility spillover. Specifically, the flexibility available for storage is different between fresh and frozen commodities. Further, the growth of fish ensures that one way correlations in stocks exist upward in weight classes.

Table 1 provides some summary statistics of our dataset. Both the normal and asymptotic normal distribution can be rejected for fresh salmon, albeit with the lower weight classes of salmon being closest to reaching normality. For frozen salmon we cannot reject the hypothesis of asymptotic normality.

TABLE 1. Summary Statistics - monthly observations January 1990 - July 2007

	2-3kg	3-4kg	4-5kg	5-6kg	6-7kg	Sal. Frozen
Mean	26.215	28.199	28.971	29.251	29.457	31.103
Maximum	42.625	48.625	50.875	52.375	54.75	44
Minimum	13.643	16.078	16.303	16.25	16.293	18.08
Std. Dev.	6.5006	7.2086	7.574	7.8189	7.9106	5.8953
Skewness	0.34183	0.59169	0.6478	0.7157	0.78518	-0.015499
Kurtosis	-0.72959	-0.58373	-0.58673	-0.51451	-0.31628	-0.78116
Normality	16.386**	38.63**	48.976**	58.969**	62.09**	6.385*
Asy. Norm.	8.789**	15.308**	17.784**	20.34**	22.56**	5.3733
Obs.	211	211	211	211	211	211

[1] All prices are denoted in NOK.

[2] The measure of kurtosis is corrected to make the kurtosis of the normal distribution standard with value zero.

[3] The normality test is based on Doornik and Hansen (1994) where the null hypothesis is that the series is normally distributed. Values reported are the chi-squared test statistic.

[4] The asymptotic normality test is derived from the normality test but applies transformed skewness and kurtosis measures that create statistics that are closer to the standard normal distribution.

All distributions except for frozen salmon right skewed, having larger probabilities than average of values above the mean. Right skewness in commodity prices is common, arising due to positive price spiking. The lack of significant right skewness in frozen salmon can be explained by superior storage flexibility.

TABLE 2. Augmented Dickey Fuller unit root tests.

Series $H_0 : \beta = 1$	Lag	Level β	Lag	First difference β
Salmon 2–3 kg	6	0.958	2	0.058**
Salmon 3–4 kg	6	0.951	5	–0.261**
Salmon 4–5 kg	6	0.950	5	–0.250**
Salmon 5–6 kg	6	0.948	5	–0.328**
Salmon 6–7 kg	6	0.943	8	–0.599**
Frozen salmon	7	0.930	4	–0.252**

Note: ** 5% significance

None of the individual price series are leptokurtic, but they display lower probability than the normal distribution for values near the mean.

We apply the Augmented Dickey–Fuller test to test for unit roots at the zero frequency. We test this using a constant and a maximum lag level of 14 months, derived from the formula $L^{\max} = \text{int}[12[T/100]^{0.25}$ as suggested by Schwert(2002). To trace out the specific lag length from these 14 possibilities, we use the Akaike Information Criteria (AIC). The result from this test is reported in Table 3. For all fresh salmon goods, the AIC suggests a lag length of six months, where the hypothesis of a unit root cannot be rejected at the 95% level. For frozen salmon, the AIC suggests a seven-month lag, where again the hypothesis of a unit root cannot be rejected. To ensure stationarity, we apply the logarithmic first-difference filter to each series. As Table 2 shows, applying the first-difference eliminates the unit root.

PRICE PROCESS SPECIFICATION

In order to derive statistical valid cross correlation functions (CCF) we need consistent estimates of residuals and conditional variances. To ensure that any correlations revealed by the CCF represent volatility transmission effects between goods, we focus our model selection procedure on purging the error terms of serial correlation. We apply a dynamic autoregressive model on both the return and variance processes. For the conditional variance, we apply the general autoregressive conditional heteroskedasticity model (GARCH) of Bollerslev (1986). We also allow two extensions of the model; namely, the exponential GARCH (EGARCH) of Nelson(1991) and the GJR-GARCH of Glosten, Jagannathan & Runkle(1993). The GJR-GARCH model allows a leverage effect to account for asymmetry of residuals. This is done to allow for possible asymmetries in the price process, with the traditional price spikes of commodity prices in mind.

Each volatility model is allowed one or two lags to the lagged squared residual and variance. The lag length of the mean model is chosen according to the Akaike information criteria. Model selection is done by evaluating serial correlation in standardized residuals (Ljung Box Q test (Box, Jenkins & Reinsel,1994)) and normality of the residual distribution (Chi^2 test and Kolmogorov

TABLE 3. Price model estimation results for each commodity.

	2-3kg	3-4kg	4-5kg	5-6kg	6-7kg	Frozen
Model:	AR(4)-GJR(1,2)	AR(6) GJR(1,2)	AR(6)-GJR(1,2)	AR(6)-GJR(1,2)	AR(6) - GARCH(21)	AR(14)-GJR(1,2)
Mean Model:						
Constant	.00 (.00)	-.01 (.01)	-.01 (.01)	-.01 (.01)	-.01 (.01)	.00 (.01)
AR-L(1)	.23 (.08)	.17 (.09)	.24 (.09)	.28 (.08)	.37 (.08)	.06 (.09)
AR-L(2)	-.09 (.09)	-.01 (.09)	.00 (.08)	-.01 (.09)	-.04 (.07)	.17 (.08)
AR-L(3)	-.09 (.08)	-.17 (.08)	-.18 (.08)	-.22 (.07)	-.28 (.07)	-.12 (.09)
AR-L(4)	.13 (.07)	-.13 (.09)	-.14 (.09)	-.13 (.09)	-.10 (.09)	-.26 (.08)
AR-L(5)	- (-)	.02 (.09)	.01 (.09)	.02 (.08)	.07 (.08)	.00 (.09)
AR-L(6)	- (-)	-.15 (.09)	-.14 (.09)	-.21 (.08)	-.19 (.07)	.08 (.08)
AR-L(7)	- (-)	- (-)	- (-)	- (-)	- (-)	.00 (.08)
AR-L(8)	- (-)	- (-)	- (-)	- (-)	- (-)	-.16 (.07)
AR-L(9)	- (-)	- (-)	- (-)	- (-)	- (-)	.04 (.07)
AR-L(10)	- (-)	- (-)	- (-)	- (-)	- (-)	-.09 (.07)
AR-L(11)	- (-)	- (-)	- (-)	- (-)	- (-)	-.05 (.07)
AR-L(12)	- (-)	- (-)	- (-)	- (-)	- (-)	-.10 (.07)
AR-L(13)	- (-)	- (-)	- (-)	- (-)	- (-)	-.11 (.06)
AR-L(14)	- (-)	- (-)	- (-)	- (-)	- (-)	-.10 (.06)
Variance Model:						
Constant	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)	.00 (.00)
ARCH-L(1)	.13 (.14)	.03 (.09)	.06 (.40)	.01 (.10)	.39 (.16)	.13 (.19)
ARCH-L(2)	.56 (.27)	.04 (.11)	.03 (.12)	.00 (.10)	- (-)	.52 (.21)
GARCH-L(1)	.05 (.16)	.57 (.48)	.59 (.14)	.61 (.46)	.00 (.15)	.39 (.16)
GARCH-L(2)	- (-)	- (-)	- (-)	- (-)	.44 (.22)	- (-)
Leverage-L(1)	.36 (.30)	-.03 (.19)	-.06 (.18)	-.01 (.14)	- (-)	.21 (.28)
Leverage-L(2)	.05 (.34)	.14 (.22)	.18 (.20)	.20 (.23)	- (-)	-.49 (.23)
Ljung-Box	18.87 (.53)	10.05 (.97)	12.16 (.91)	13.42 (.86)	10.78 (.95)	27.19 (.13)
Chi^2	2.20 (.70)	2.44 (.66)	2.80 (.91)	.93 (.92)	1.96 (.86)	9.60 (.09)
KS test	.04 (.87)	6.35 (.10)	.04 (.90)	.04 (.90)	.04 (.83)	.07 (.30)
LL	487.03 (.00)	467.76 (.00)	468.83 (.00)	460.47 (.00)	456.55 (.00)	508.19 (.00)

Smirnov(KS) test (Masseym,1951)). We also select by model fitting (Log likelihood values). The normality test of residuals is preferred because valid inference of the CCF is based on asymptotic normality. The Portmanteau Q statistics are used to avoid biased measures in the CCF as related to autocorrelation in residuals. The full statistics for each model is reported in the appendix.

In table 3 the output for the selected models for each commodity is shown. For each model selected, the residuals satisfy the serial correlation and normality tests. The standard deviation for each estimate is reported in parenthesis. For the test statistics, the parenthesis contains p-values. Having satisfactory purged the residuals we are now ready to derive the cross correlation functions.

VOLATILITY SPILLOVER ANALYSIS

From the price process specifications above, we estimate each model and store residuals and conditional variances. These series are then used to generate the CCFs derived above. In the volatility spillover analysis, every price is evaluated against every other price, and this is done for up to up to ten-month lags. Each cross-correlation is evaluated against the null hypothesis of zero volatility spillover, in which the cross-correlation is asymptotically normally distributed with zero mean and unit variance. Full cross-correlations are reported in table 4, p-values are reported in parenthesis. The bolded correlations are significant at the 5% or lower level.

Our analysis focuses on evaluating the postulated supply-side explanations for volatility spillover derived above. Firstly, we note that all significant volatility spillover effects are positive in sign. This is expected from a positive correlation in stock sizes. The most easily noticeable pattern is the strong zero-lag volatility spillover between fresh salmon commodities. It appears that within a month, much of the volatility has already been absorbed.

As conjectured, the market integration relationship leads to correlations in corrections to shocks as prices share common equilibrium factors. A common equilibrium factor in prices moves all prices that are connected and results in the error term of each price being non-independent; this has the effect linking volatility at the zero frequency. No zero-lag volatility spillover appears to exist between fresh and frozen salmon prices.

TABLE 4a. Volatility spillover analysis results.

	2-3kg vs. 3-4kg.	2-3kg vs. 4-5kg.	2-3kg vs. 5-6kg.	2-3kg vs. 6-7kg.	2-3kg vs. Frozen
LAG					
-10	-.09 (.19)	-.04 (.53)	-.01 (.87)	-.01 (.91)	.05 (.50)
-9	.08 (.23)	.09 (.19)	.16 (.03)	.08 (.26)	.02 (.81)
-8	-.05 (.52)	-.06 (.41)	-.04 (.53)	-.05 (.48)	-.01 (.90)
-7	.00 (.95)	.07 (.34)	.03 (.70)	.00 (.97)	**.15 (.03)**
-6	-.06 (.42)	-.07 (.36)	-.08 (.24)	-.03 (.62)	.05 (.48)
-5	-.06 (.40)	-.05 (.46)	-.02 (.76)	-.01 (.92)	**.29 (.00)**
-4	-.04 (.54)	-.03 (.66)	-.05 (.51)	-.03 (.71)	.03 (.64)
-3	.12 (.10)	.10 (.15)	.07 (.30)	.00 (.97)	.11 (.13)
-2	-.11 (.14)	-.11 (.11)	-.07 (.30)	-.06 (.42)	.03 (.64)
-1	-.07 (.31)	-.04 (.58)	-.03 (.69)	.07 (.33)	-.07 (.34)
0	**.76 (.00)**	**.70 (.00)**	**.60 (.00)**	**.42 (.00)**	.07 (.35)
1	.00 (.96)	-.01 (.91)	.00 (1.00)	-.06 (.37)	.10 (.16)
2	.06 (.39)	.06 (.37)	.07 (.33)	.07 (.36)	.06 (.40)
3	**.16 (.03)**	**.14 (.05)**	**.16 (.02)**	.11 (.11)	.00 (.99)
4	.01 (.85)	.03 (.71)	.00 (.97)	.00 (.97)	.00 (.95)
5	-.07 (.31)	-.08 (.29)	-.06 (.42)	-.02 (.76)	-.05 (.49)
6	.04 (.56)	.02 (.75)	.02 (.82)	.05 (.46)	-.02 (.79)
7	-.05 (.47)	-.02 (.77)	-.06 (.42)	-.02 (.79)	.00 (.98)
8	-.06 (.36)	-.07 (.29)	-.05 (.45)	-.09 (.20)	.00 (.96)
9	-.05 (.52)	-.08 (.27)	-.03 (.65)	-.01 (.94)	.00 (.95)
10	.04 (.60)	.03 (.72)	.02 (.77)	.03 (.72)	.02 (.79)

The stock of frozen commodities is less dependent on immediate effect of live stock effects. The improved storage flexibility in keeping stocks of frozen fish provides a buffer for stochastic movements in the live stock. The volatility spill-over in fresh fish appear to decrease as the weight difference increases. Fish that are closer in weight are more strongly linked in correlations of stocks and effects of single-good inventory decisions. For fresh salmon production, this is stated in the fact that the time to transform one salmon, as indexed by weight, to another is lower for salmon in which the target weight is closer to the original weight.

TABLE 4b. Volatility spillover analysis results.

LAG	3-4kg vs. 4-5kg.	3-4kg vs. 5-6kg.	3-4kg vs. 6-7kg.	3-4kg vs. Frozen	4-5kg vs. 5-6kg.
-10	.03 (.62)	.01 (.90)	-.03 (.66)	.05 (.45)	-.02 (.73)
-9	.01 (.86)	.04 (.58)	.04 (.58)	.00 (.96)	.01 (.93)
-8	-.06 (.44)	-.05 (.47)	-.02 (.83)	.05 (.45)	-.07 (.36)
-7	.01 (.91)	-.04 (.61)	-.07 (.33)	.12 (.09)	-.02 (.74)
-6	.03 (.72)	-.01 (.86)	.00 (.97)	.09 (.18)	.00 (.99)
-5	-.06 (.41)	.00 (.98)	.05 (.52)	**.25 (.00)**	-.01 (.85)
-4	-.02 (.73)	-.04 (.58)	-.03 (.63)	.10 (.16)	-.04 (.57)
-3	**.19 (.01)**	**.16 (.02)**	.10 (.18)	.06 (.42)	**.15 (.03)**
-2	-.08 (.27)	-.09 (.22)	-.10 (.15)	.02 (.74)	-.06 (.43)
-1	.00 (.97)	.04 (.54)	**.16 (.03)**	-.07 (.29)	.00 (.97)
0	**.93 (.00)**	**.85 (.00)**	**.61 (.00)**	.09 (.18)	**.89 (.00)**
1	-.07 (.35)	-.08 (.25)	**-.16 (.03)**	.07 (.31)	-.04 (.55)
2	-.03 (.71)	-.03 (.64)	.02 (.74)	.03 (.70)	-.04 (.60)
3	**.18 (.01)**	**.25 (.00)**	**.20 (.01)**	-.07 (.31)	**.24 (.00)**
4	-.01 (.85)	.00 (.97)	-.07 (.32)	-.02 (.77)	-.01 (.89)
5	-.08 (.24)	-.08 (.27)	-.06 (.37)	-.05 (.51)	-.08 (.28)
6	.02 (.77)	.02 (.80)	.03 (.69)	-.08 (.28)	.03 (.71)
7	-.06 (.41)	-.07 (.33)	-.03 (.69)	.01 (.89)	.00 (.96)
8	-.07 (.34)	-.06 (.41)	-.09 (.20)	-.01 (.85)	-.05 (.44)
9	-.06 (.41)	-.03 (.63)	-.01 (.92)	-.07 (.33)	.00 (.95)
10	-.02 (.74)	-.02 (.75)	-.04 (.55)	.02 (.83)	.04 (.59)

We observe that a pattern emerges in how a shock to fresh salmon goods transfers to the frozen salmon market. Within three to five months of a shock in the frozen salmon market, there are significant volatility transmission effects on the fresh market. On the other hand a shock in the fresh market has no significant effects on the frozen fish volatility. This supports the hypothesis that storage possibilities in commodity prices helps to buffer price shocks. As stated above, the possibility for inventory keeping is greater for frozen than fresh fish.

TABLE 4c. Volatility spillover analysis results.

	4-5kg vs. 6-7kg.	4-5kg vs. Frozen	5-6kg vs. 6-7kg.	5-6kg vs. Frozen	6-7kg vs. Frozen
LAG					
-10	-.03 (.69)	.04 (.53)	.00 (.99)	.04 (.53)	.02 (.77)
-9	.01 (.90)	-.01 (.89)	.04 (.59)	-.01 (.87)	.04 (.61)
-8	-.03 (.70)	.05 (.49)	-.03 (.71)	.01 (.93)	-.03 (.69)
-7	-.06 (.42)	**.15 (.03)**	-.06 (.40)	**.15 (.03)**	.14 (.06)
-6	.02 (.81)	.09 (.22)	.01 (.90)	.05 (.51)	.05 (.48)
-5	.02 (.82)	**.25 (.00)**	.02 (.76)	**.24 (.00)**	**.29 (.00)**
-4	-.02 (.74)	.06 (.38)	.02 (.79)	.04 (.58)	.02 (.82)
-3	.09 (.21)	.03 (.67)	.11 (.13)	.05 (.45)	.05 (.50)
-2	-.08 (.28)	.03 (.67)	-.11 (.12)	.01 (.88)	.01 (.85)
-1	.12 (.10)	-.07 (.36)	.13 (.07)	-.04 (.62)	.05 (.50)
0	**.67 (.00)**	.09 (.18)	**.77 (.00)**	.07 (.30)	.01 (.84)
1	-.13 (.07)	.12 (.09)	**-.16 (.02)**	.06 (.38)	.04 (.56)
2	.03 (.66)	.01 (.84)	.00 (.96)	-.02 (.78)	-.06 (.43)
3	**.20 (.01)**	-.06 (.40)	**.14 (.05)**	-.05 (.48)	-.01 (.93)
4	-.09 (.22)	-.02 (.82)	-.09 (.22)	.01 (.87)	-.08 (.29)
5	-.07 (.34)	-.01 (.84)	-.04 (.59)	.01 (.93)	.08 (.25)
6	.04 (.54)	-.09 (.19)	.02 (.80)	-.07 (.32)	-.09 (.23)
7	.03 (.69)	-.02 (.81)	-.02 (.76)	-.02 (.73)	.00 (.96)
8	-.10 (.18)	-.03 (.69)	-.07 (.31)	-.04 (.53)	-.09 (.21)
9	.02 (.80)	-.04 (.57)	.01 (.89)	.02 (.79)	.03 (.70)
10	-.03 (.70)	.05 (.52)	-.05 (.47)	.04 (.57)	-.01 (.89)

In volatility spillover, this effect emerges as a leading relationship in volatility transference for frozen fish. It is likely that frozen salmon volatility transmission is a result of direct causation effects, because of the relative long time period of transference and the strong correlation relationship. The low flexibility in fresh fish inventories makes these goods less elastic in reacting to shocks from the frozen market. The market returns to equilibrium either through changing price or supply. In a competitive market, the change in supply is expected to dominate the change in price. However, if supply is inflexible, as with lower inventory possibilities, the market is more likely to tend to equilibrium through changes in prices. This would explain the stronger correlation relationship in volatility from frozen to fresh fish. In addition, the correlation from frozen salmon is larger for fresh salmon of larger weight. The stronger effect for higher weight classes can be explained by the lower flexibility in inventorying available to the farmer as the fish increases in weight, making the alternative of freezing the fish more desirable.

As postulated the mechanism that correlates fresh stocks are the growth of fish. This suggests that shocks to lower weight-class stocks transmit to higher weight-classes, but not the other way around. From the assumption that lower stocks generate higher price volatility we would thus expect that price volatility is transmitted in the same manner. Looking at the volatility spill-over results it thus appears that there are stronger transmission effects from lower weight to higher weight classes. This is consistent over all weight-classes. From these indirect results we can explain some of the patterns emerging in price volatility transmission by the fundamental correlation existing in stocks of the commodity.

CONCLUSION

Our analysis of volatility spillover illustrated how fundamental properties of markets, common equilibrium factors and correlation and flexibility in inventories can carry over to how shocks in prices are transmitted across markets. The theory of commodity prices states the importance of inventories in accounting for the volatility of prices. This analysis also suggested that correlations in inventories affect transmission in volatility. The analysis in this paper found evidence supporting this hypothesis. Therefore, the existence of volatility spillover can be partly explained by these correlations in inventories. In addition, we provided evidence that common equilibrium factors generating a common stochastic trend further lead to a strong zero-lag volatility spillover. The analysis provides justification for the existence of nonsporadic volatility spillover effects outside of what can be explained by non-stationary demand-side effects. We found that the application of volatility spillover analysis to commodity prices can yield patterns explainable by economic theory even when applied to monthly observations. This paper suggested that systematic volatility spillover in primary commodities can occur and are explainable by economic theory; specifically, common equilibrium factors can account for a strong zero-lag spillover, while correlations and differing degrees of flexibility in inventorying can explain greater-than-zero-lag correlations.

The volatility spillover test of Cheung & Ng (1996) was applied to a basket of Norwegian aquaculture goods. We found a strong zero lag, in addition to higher-level lag effects between fresh salmon prices, but with the effects decreasing as the difference in weight increases. We found that frozen salmon takes a leading role in volatility spillover, explained by the more flexible inventory possibilities available for frozen fish. We further found evidence that the volatility of fresh salmon of lower weight classes is more closely related to trout, while the volatility of higher weight classes is more closely related to frozen salmon. In light of the analytical results on volatility spillover, these volatility spillover effects are believed to be generated by the differences in inventorying flexibility and common equilibrium factors such as prices on primary input factors.

Asche & Guttormsen (2001) also show that representing salmon prices through one unified price is valid. Our analysis follows this and suggests that the volatility in salmon prices can be represented by a unified salmon volatility measure. In addition, any precision of price forecasting is dependent on the level of volatility in the given market, where the study of Oglend & Sikveland (2009) already shows that salmon price volatility is stochastic with clustering properties such that the precision of forecasts is dependent on its previous volatility patterns. Our analysis shows that the precision of forecasts in one market is not independent of volatility patterns in other markets. In addition, our analysis suggests that a vector autoregressive representation of salmon prices should include a multivariate representation of variance with relevant cross-correlation terms. This is especially relevant in representations where a correctly specified conditional variance is important; for example, the emerging market for Norwegian aquaculture derivatives.

APPENDIX

Results from model selection procedure.

			Salmon 2-3kg.			
Model		LAG	Ljung-Box Q	Chi ^{2}	KS	LL
GARCH	(1,1)	1	15.59 (.742)	2.44 (.656)	.05 (.639)	480.8
	(1,2)	7	18.62 (.546)	2.91 (.714)	.05 (.591)	486.0
	(2,1)	3	15.59 (.742)	2.44 (.656)	.05 (.639)	480.8
	(2,2)	7	18.62 (.546)	2.91 (.715)	.05 (.591)	486.0
EGARCH	(1,1)	7	20.08 (.453)	3.68 (.451)	.05 (.639)	484.4
	(1,2)	1	22.95 (.291)	2.75 (.739)	.03 (.981)	491.8
	(2,1)	9	25.91 (.169)	2.96 (.565)	.07 (.279)	490.6
	(2,2)	7	23.71 (.255)	20.12 (.003)	.09 (.057)	511.0
GJR	(1,1)	8	11.87 (.920)	2.89 (.576)	.06 (.446)	482.9
	(1,2)	8	18.87 (.531)	2.20 (.699)	.04 (.865)	487.0
	(2,1)	8	11.87 (.920)	3.36 (.499)	.06 (.446)	482.9
	(2,2)	8	18.87 (.531)	2.20 (.699)	.04 (.865)	487.0

			Salmon 3-4kg.			
Model		LAG	Ljung-Box Q	Chi ^{2}	KS	LL
GARCH	(1,1)	1	9.97 (.969)	5.54 (.236)	.04 (.815)	465.9
	(1,2)	7	10.25 (.963)	3.93 (.415)	.05 (.768)	467.3
	(2,1)	3	9.95 (.969)	1.72 (.632)	.05 (.659)	466.5
	(2,2)	7	10.25 (.963)	3.93 (.415)	.05 (.768)	467.3
EGARCH	(1,1)	7	12.43 (.901)	7.16 (.209)	.05 (.640)	475.7
	(1,2)	1	10.17 (.965)	3.32 (.506)	.03 (.986)	470.3
	(2,1)	9	16.74 (.670)	7.80 (.167)	.09 (.061)	478.1
	(2,2)	7	13.57 (.852)	10.32 (.171)	.07 (.288)	492.1
GJR	(1,1)	8	20.88 (.404)	10.63 (.031)	.06 (.475)	461.8
	(1,2)	8	10.05 (.967)	2.44 (.656)	.04 (.868)	467.8
	(2,1)	8	21.19 (.386)	6.35 (.096)	.07 (.283)	462.5
	(2,2)	8	10.05 (.967)	2.44 (.656)	.04 (.868)	467.8

Model		LAG	Salmon 4-5kg. Ljung-Box Q	Chi ^{2}	KS	LL
GARCH	(1,1)	1	12.77 (.887)	5.91 (.206)	.05 (.567)	466.8
	(1,2)	7	13.06 (.875)	2.91 (.714)	.04 (.879)	468.0
	(2,1)	3	12.34 (.904)	13.09 (.023)	.06 (.521)	467.1
	(2,2)	7	13.06 (.875)	2.91 (.714)	.04 (.879)	468.0
EGARCH	(1,1)	7	12.84 (.884)	7.53 (.110)	.04 (.786)	466.3
	(1,2)	1	15.04 (.774)	2.99 (.560)	.04 (.932)	467.8
	(2,1)	9	14.91 (.781)	4.53 (.605)	.05 (.707)	481.0
	(2,2)	7	24.27 (.231)	6.15 (.407)	.07 (.263)	483.2
GJR	(1,1)	8	12.49 (.898)	9.94 (.077)	.05 (.766)	467.0
	(1,2)	8	12.16 (.911)	2.80 (.911)	.04 (.898)	468.8
	(2,1)	8	11.50 (.932)	2.99 (.701)	.05 (.751)	468.2
	(2,2)	8	12.16 (.911)	2.79 (.732)	.04 (.898)	468.8

Model		LAG	Salmon 5-6kg. Ljung-Box Q	Chi ^{2}	KS	LL
GARCH	(1,1)	1	13.96 (.833)	4.40 (.494)	.05 (.669)	458.6
	(1,2)	7	13.83 (.839)	10.29 (.067)	.05 (.588)	459.0
	(2,1)	3	12.34 (.904)	8.10 (.151)	.05 (.604)	459.8
	(2,2)	7	13.82 (.839)	10.29 (.067)	.05 (.587)	459.0
EGARCH	(1,1)	7	13.33 (.863)	5.00 (.416)	.05 (.660)	459.3
	(1,2)	1	13.91 (.835)	1.83 (.767)	.04 (.818)	460.1
	(2,1)	9	22.28 (.326)	9.31 (.157)	.04 (.948)	472.6
	(2,2)	7	16.75 (.669)	8.38 (.212)	.04 (.932)	475.0
GJR	(1,1)	8	13.50 (.855)	5.59 (.348)	.05 (.692)	459.4
	(1,2)	8	13.42 (.859)	.93 (.920)	.04 (.901)	460.5
	(2,1)	8	11.96 (.918)	5.31 (.379)	.04 (.794)	460.7
	(2,2)	8	13.42 (.859)	.93 (.920)	.04 (.901)	460.5

Model		LAG	Salmon 6-7kg. Ljung-Box Q	Chi ^{2}	KS	LL
GARCH	(1,1)	1	13.19 (.869)	5.88 (.208)	.03 (.973)	456.0
	(1,2)	7	13.31 (.864)	5.40 (.249)	.03 (.968)	456.0
	(2,1)	3	10.78 (.952)	1.96 (.855)	.04 (.829)	456.5
	(2,2)	7	10.96 (.947)	2.79 (.732)	.04 (.775)	456.6
EGARCH	(1,1)	7	13.48 (.856)	4.14 (.387)	.04 (.928)	458.1
	(1,2)	1	15.30 (.759)	2.14 (.709)	.03 (.998)	459.9
	(2,1)	9	16.53 (.683)	8.73 (.189)	.06 (.408)	484.8
	(2,2)	7	16.55 (.682)	1.88 (.866)	.06 (.483)	487.5
GJR	(1,1)	8	12.16 (.910)	4.44 (.350)	.03 (.966)	457.3
	(1,2)	8	13.11 (.873)	4.74 (.315)	.03 (.965)	458.8
	(2,1)	8	12.16 (.910)	4.44 (.350)	.03 (.966)	457.3
	(2,2)	8	13.11 (.873)	4.74 (.315)	.03 (.965)	458.8

Model		LAG	Frozen Salmon Ljung-Box Q	Chi ^{2}	KS	LL
GARCH	(1,1)	1	30.84 (.057)	13.45 (.009)	.08 (.111)	504.0
	(1,2)	7	33.50 (.030)	12.99 (.023)	.07 (.218)	506.1
	(2,1)	3	30.84 (.057)	13.45 (.009)	.08 (.110)	504.0
	(2,2)	7	31.38 (.050)	10.99 (.052)	.08 (.121)	506.2
EGARCH	(1,1)	7	31.27 (.052)	14.24 (.007)	.09 (.077)	504.2
	(1,2)	1	41.40 (.003)	16.01 (.025)	.05 (.556)	525.7
	(2,1)	9	27.92 (.111)	9.52 (.090)	.07 (.299)	510.9
	(2,2)	7	24.83 (.208)	11.10 (.049)	.07 (.208)	519.2
GJR	(1,1)	8	28.19 (.105)	10.08 (.039)	.07 (.254)	504.9
	(1,2)	8	27.19 (.130)	9.60 (.087)	.07 (.300)	508.2
	(2,1)	8	28.19 (.105)	10.08 (.039)	.07 (.253)	504.9
	(2,2)	8	27.19 (.130)	9.60 (.087)	.07 (.300)	508.2

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Regime Shifts in the Structure of Commodity Price Dynamics with Application to the Price of Salmon

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Abstract Both commodity price theory and statistical evidence suggests that the underlying price processes of commodity prices are non-linear in dynamics. By allowing both mean and variance parameters to change between states we find evidence that underlying skewness and kurtosis in residuals arising in linear models disappear. I further argue, based on the theory of storage that a two-state regime shifting model is a suitable price model for commodity prices. Theory predicts that as underlying fundamentals such as stock sizes change, the persistence and volatility of prices will change. Using the regime shifting model we are able to indirectly test predictions made by the theory of storage. The state probabilities emerging as a product of regime shifting models can provide a basis for examining theoretical predictions on commodity markets. For the case of salmon, I find that there are seasonal patterns in volatility in addition to industry profitability conditions affecting the emergence of volatility regimes.

Introduction

When analysing time series of commodity prices, two common statistical characteristics can often be detected. Firstly, the distributions of price levels are often right skewed, arising from the characteristically positive price spikes. Secondly, low order autocorrelations are high, often to the degree of failing to reject standard unit root tests (Deaton & Laroque 1992, 1996; Chamber & Bailey, 1996; Pindyck, 2004; Tomek, 2000). The primary economic explanation model for the short run dynamics of commodity prices is the theory of storage (Kaldor ,1939; Working , 1948, 1949 and Brennan, 1958). In the theory of storage, shocks to price levels are allowed to persist in order to cover the discounted cost of storage. Since stocks cannot be carried from the future to the present, a non-negativity constraint is imposed on the storage control variable. This non-negativity constraint generates non-linear price dynamics as the price process jumps from regimes of speculative storage to stock out periods. Such periods of scarcity allows the commodity some uniqueness, divorcing its value from marginal costs.

In the theory of storage inventory levels and cost of storage are important state variables accounting for non white noise patterns in short-run price movements. Neither inventory levels nor net costs of storage are exogenous in the theory. A feedback relationship exists between state variables. In general, a closed form solution to the pricing function from the theory of storage cannot be derived. When constructing an empirical commodity price model consistent with dynamics predicted by theory of storage, we should reasonably allow price dynamics to change states. The theory of storage suggests two fundamental states: *the storage regime*, where speculative storage is done to eliminate arbitrage. Prices are here smoothed so that the spread recoups the net cost of storage and alternative cost of capital. In the other state, *the scarcity regime*, no speculative storage is done and prices are allowed to persist at abnormal positive levels. In regards to Granger and Swanson (1997), the first regime will be a convergent regime, where storage prohibits divergence of prices. The second regime will, on the other hand, be a regime of mildly explosive prices and no directed mean reversion driven by economic incentives.

To account for this conjectured change in price dynamics, we propose a model for price dynamics where parameters of a linear model are allowed to change between two states. This non-structural model does not allow mapping of regime shifts to specific changes in state variables. We have no direct way to infer if states produced by the empirical model are consistent with regime changes in the theoretical model. However, using output from the empirical model and knowledge of structural changes in state variables (specifically the time series of state probabilities, the coefficient values in each state, and knowledge of predictable changes in state variables), we are able to indirectly infer if our empirical model produces dynamics consistent with economic theory. In example, knowledge of seasonal effects in stock levels or demand surges can be used to evaluate if the empirical model produces dynamics consistent with what would be expected from the theory of storage. The theory, for example, predicts that when stocks are low the market is more likely to enter a scarcity period where price volatility increases and mean reversion decreases. Using knowledge of the market in addition to model output, such hypothesis can indirectly be examined.

Our econometric approach is Hamilton's regime switching model (Hamilton , 1989). By defining a probability measure as a stochastic Markov process we allow state dependent parameters to be weighted according to probability measures. The probability measure generates a time dependant convex combination of two sets of parameters. Examining the probability measures reveals information on which regime at any specific time best represents the data generating process. We apply the empirical model on a data-set of weekly observations of the price of fresh salmon. The price of salmon is a non-stationary stochastic process dominated by what appears to be relative large shifts in a stochastic variance parameter Oglend & Sikveland (2009). Modelling prices as the outcome of an unconditional distribution will generate residuals diverging from normality. By allowing the distribution to change in time, such non-normality issues can be properly addressed.

An important advantage of regime switching models is that parameter values are allowed to shift independently of lagged variables. We draw information from the errors of linear models to infer statistical properties of the time series. This is beneficial in cases where we believe that non-stationarity is due to non-observable state variables not readily available to the econometrician. When analysing short run commodity price dynamics, the full range of relevant state variables are often not available. Relying on inference from non-observable state variables is then often necessary. This paper investigates whether a two state regime switching is a consistent and suitable non-structural time series representation of prices. We further investigate how the output from such a model can be used to evaluate predictions made by the theory of storage.

Our empirical results for the price of salmon suggest that the non-linear regime shifting model is able to produce approximately normally distributed residuals. It appears that the leptokurtic properties observed in the data can be addressed parametrically by the model. Furthermore, using the model we find indirect support for the theory of storage. We find that under the normal regime dynamic multipliers are stable and converging, while under the high volatility regime the multipliers a far more unstable with oscillating patterns. Furthermore, as prices increase the probability of entering the high volatility regime increases.

BACKGROUND

The modern formulation of the theory of storage is found in the rational expectation competitive storage theory, derived amongst other in Deaton & Laroque (1992, 1996) and Chambers &Bailey (1996). A derivation of the theoretical pricing function illustrates how commodity price dynamics is the result of a two state process. As is standard for competitive storage models, an equilibrium storage level is reached when price spreads (the difference between expected price and spot price) are equal to the net cost of storage and alternative cost of capital. The decision variable facing the inventory manager is the level of inventories.

Denoting the stock levels at time t as X_t , stocks follows the discrete time process:

$$X_t = X_{t-1} + I_{t-1} + \nu_t . \tag{1}$$

In equation (1), the variable ν_t accounts for exogenous additions to stocks from planned production. In the theory of storage this is the source for variations in prices. In its simplest interpretation, variations enter due to weather effects affecting realized output. However, since temporary changes in demand has an equivalent but inverse effect on stocks, the stochastic component can also be seen as an exogenous excess demand variable (Deaton & Laroque, 1996).

I_{t-1} is the amount stored in the previous period. Amount stored is the control variable available to the producer in short run. Storage is restricted to take non-negative values; it is not possible to carry over stocks from tomorrow to today. This restriction is crucial to the dynamics of commodity prices. It implies that price dynamics becomes asymmetrical as the pricing function changes regime when a stock-out occurs. When no storage is done, the stock dynamics is fully dependent on the exogenous addition to stocks ν_t. Note that the storage variable I_{t-1} can also be defined as $I_{t-1} = \phi_{t-1} X_{t-1}$, $\phi_{t-1} = [0,1]$; ,a ratio of stocks carried over to the next period. Looking at storage in this way makes it more general. In agriculture, storage can hence be viewed as the option not to harvest. Storing is done through further cultivation.

We now introduce the cost of storing variable. On the margin this variable can be defined as:

$$\chi(I_t) = c(I_t) + \delta(I_t). \tag{2}$$

The marginal cost of storing $\chi(I_t)$ is decomposed into a non-negative cost factor $c(I_t)$ and a non-positive convenience yield factor $\delta(I_t)$. Under competitive storage it is assumed that cost of storing depends on amount stored. In the Kaldor-Woking hypothesis it is assumed that $D_I\chi > 0$, an increasing marginal cost of storing. For higher storage levels, the cost effect is expected to dominate the convenience yield effect. The marginal benefit to increasing storage is generally hypothesized to be decreasing.

The theory of storage states that in order to store a commodity, the producer must be compensated on the margin for the cost of storing. This restriction generates the Euler condition for optimal storage:

$$\beta E(p_{t+1} \mid I_t) - P(X_t - I_t) = c(I_t) + \delta(I_t) \ . \tag{3}$$

Here β is a discount factor accounting for the alternative cost of capital. $E(p_{t+1} \mid I_t)$ is the expected price tomorrow with storage I_t today. Further, $P(X_t - I_t)$ is the realized spot price. In equilibrium, the discounted expected price spread $\beta E(p_{t+1} \mid I_t) - P(X_t - I_t)$ must cover the marginal cost of storing $c(I_t) + \delta(I_t)$. When condition (3) is satisfied, any arbitrage from storing has been eliminated.

The price spread $\beta E(p_{t+1} \mid I_t) - P(X_t - I_t)$ decreases in storage. This implies that the maximum price spread is achieved when storage is zero. We define the expected maximum price spread $E\eta_{t+1}^{Max}$ as:

$$E\eta_{t+1}^{Max} = \beta E(p_{t+1} \mid 0) - P(X_t) \tag{4}$$

Following equation (4), the market stocks-out ($I_t = 0$) when $E\eta_{t+1}^{Max} < c(I_t) + \delta(I_t)$. In this case the optimal storage level is negative. However, due to the non-negativity constraint on storage the storage, the closest point to optimality is zero.

Combining equation (3) and (4) the expected discounted price spread $E\eta_{t+1}$ can be expressed by the functional:

$$E\eta_{t+1} = \min\{c(I_t) + \delta(I_t), \beta E(p_{t+1} \mid 0) - P(X_t)\} \tag{5}$$

This is the price spread process predicted by the theory of storage. The left term on the right side of equation (5) is the expected price spread under storage, while the right term is the stock-out spread. Note how equation (5) illustrates how the control variable I_t determines the spread under storage, while the spread under stock-out is determined by the exogenous addition to stocks today. Equation (5) also illustrates the importance of a convenience yield. Since the convenience yield is a negative cost component it reduces left side expression, increasing the probability that storage is the prevailing regime.

We observe how the spread process is non-linear, moving between the two regimes dependent on stock availability today X_t. The probability of stocking-out tomorrow can be expressed as:

$$\begin{aligned} &prob(\beta E(p_{t+1} \mid 0) - P(X_{t-1} + I_{t-1} + v_t) < c(I_t) + \delta(I_t)) \Leftrightarrow \\ &prob(P(X_{t-1} + I_{t-1} + v_t) + c(I_t) + \delta(I_t) > \beta E(p_{t+1} \mid 0)) \end{aligned} \tag{6}$$

Under the assumption that the harvest is normally distributed $v \sim N(\mu, \sigma)$, the price process moves according to a two state Markov chain having non-constant transition probabilities given by:

$$P_t = \begin{bmatrix} \rho_t \rho_{t-1} & \rho_t (1 - \rho_{t-1}) \\ (1 - \rho_t)\rho_{t-1} & (1 - \rho_t)(1 - \rho_{t-1}) \end{bmatrix} \tag{7}$$

Where $\rho_t = \Phi_{\mu_{s,t1}, \sigma^2_{s,t}}(\beta E(p_{t+1} \mid 0))$ is the probability of storing today. The probability distribution is conditional on stock availability today. Its mean and variance can be approximated by:

$$\mu_{s,t} = P(X_t) + c(I_t) + \delta(I_t) \tag{8}$$

$$\sigma^2_{s,t} \approx \left(\frac{\delta P(X_t)}{\delta(X_t)} + \frac{\delta c(I_t)}{\delta(X_t)} + \frac{\delta\delta(I_t)}{\delta(X_t)} \right)^2 \sigma^2$$

This derivation implies that the more that was stored yesterday, the higher is expected stocks today, and the higher is the probability of not stocking out today. Hence the higher stocks are the greater is the likelihood of staying in the storage state. In essence the spread process under optimal storage follows a non-linear process – a two state time dependent Markov chain determines the likelihood of changing price regimes. Note also that since price spreads can change regime the volatility process becomes non-constant.

We observe that the price process moves between the two regimes. The two fundamental state variables determining if regime shifts occur in the following period is the stock level and cost of storage. Quantitative representation of these state variables does not exist at the desired frequency for the commodity market analysed in this paper. We therefore treat state transition probabilities as time invariant parameters to be estimated. In essence, the error of applying the linear model functions as a state variable in the empirical model. This necessary disallows direct in inference. The important feature is that the empirical model allows price dynamics to changes regimes, as is the implication on price dynamics as predicted by theory.

EMPIRICAL MODEL

In order to draw valid statistical inference from our model we need to derive a model which satisfactory can encompasses the dynamics of the underlying data generating process. The previous analysis suggests that price follows an asymmetrical two regime process. To model these asymmetries empirically we apply a two state regime shifting model, as first derived in Hamilton (1989). Regime switching models have become popular, specifically in application to macroeconomic and financial issues. However, less application of the model can be found in commodity price theory.

The regime switching model was first developed by Hamilton (1989), where regime shifts are applied to the mean term of the model. This was initially done to identify business cycles in macroeconomic data. The model has later been expanded to allow regime shifts in variance parameters (Hamilton & Susmel, 1994) or in relation to co-integration (Krolzig, 1996). The regime switching model has been applied to explain problems in macroeconomics such as the business cycle (Bansal, Tauchen and Zhou, 2004; Hamilton, 1989) or interest rate movements (Garcia & Perron, 1996; Gray, 1999; Ang & Bekaert 2002; Engstad & Nyholm, 2000). In addition, the model has been applied to model exchange rate movements (Sachs, 1996) and financial asset returns (Guidolin & Timmermann (2007); Ang & Bekaert (2000)). Furthermore, different estimation approaches has been proposed such as the EMM estimator, where a simulation approach is evaluated against the real time series (Bansal et al. (1995) and Gallant and Tauchen (1996). Concerning commodity prices, application has been found in regards to the study of macroeconomic regimes (Tomek, 1997; Cuddington & Liang, 1999).

In our approach we are interested in the shift of the price process, specifically in mean reversion and volatility movements. We initially model the price as an autoregressive model with regime dependant autoregressive parameters. To allow stochastic volatility shifts we model the constant term in an ARCH representation to be regime dependent. The regime probabilities evolve according to a two state Markov process, where we label the two states as "normal" and "extreme" regime.

We start by assuming that the data generating process can be represented by the autoregressive switching ARCH model. The model allows regime dependent parameters both in the mean and variance equation:

$$y_t = \mu + \sum_{i=1}^{q} \beta_i(S_t)\Delta y_{t-1} + \sigma_t^2(S_t)\varepsilon_t . \tag{9}$$

$$\sigma_t^2(S_t) = \alpha_0\theta(S_t) + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 . \tag{10}$$

Equation (9) describes the development of prices, where y_t denotes the price observed today, μ is the intercept and $\beta_i(S_t)$ is the autoregressive parameters dependent on the state at time *t*. Furthermore, $\sigma_{s,t}^2$ describes the conditional variance. To allow shifts in volatility, the conditional variance intercept α_0 is multiplied by a state dependent scale factor $\theta(S_t)$. The parameter $\theta(S_t)$ takes on a unit value during the normal regime, while be being scales by θ_s in the extreme regime. Note that we do not impose any value on the scaling factor; it is purely selected by data fitting.

In order to make the model tractable we specify the state determining variable S_t as a stochastic variable. We follow Hamilton (1989) and model S_t as the result of a discrete time, discrete state first order Markov process. We define S_t as a 2×1 vector $[S_{1t}, S_{2t}]$ where element j of S_t takes value one if state j is realised and zero otherwise. Furthermore, we define the 2×2 matrix P as the transition probabilities between states[8]:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

Hence, the state process S_t evolves as the result of a first order autoregressive process where transition probabilities functions as parameters:

$$S_t = PS_{t-1} + v_{t+1} \quad . \tag{11}$$

The outcome of equation (11) can be interpreted as the implied probability of the system in being in state one or two at time *t*. Using this equation (9) and (10) can be rewritten as:

$$y_t = \mu + \sum_{i=1}^{q} \left(\varphi_0 + \varphi_{i,S_t=1} S_{1t} + \varphi_{i,S_t=2} S_{2t}\right) y_{t-i} + \left(\alpha_0 \left(S_{1t} + \theta_s S_{2t}\right) + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2\right) \varepsilon_t^2$$

Note that for variance, the scaling coefficient θ_s is normalized for state one. This implies that during the *a priori* extreme state, the variance is scaled by the parameter θ_s, presumably different from unity, but not necessarily larger.

The problem we face is to reach an optimal inference on the process S_t determining the weights in the linear combination of parameters. The solution to this problem is the non-linear filter proposed by Hamilton. If the two eigenvalues of the transition

[8] We must assume that the rows of the transition probabilities sum to unity so as to represent them as probability measures.

probabilities P are respectively one and less than one, the Markov chain is said to be ergodic. Ergodicity implies that the eigenvector associated with the unit eigenvalue, $P\pi = \pi$, produces the unconditional probability of being in state *j* at any point in time, $P(s_t = j;\Phi) = \pi_i$. Here Φ is the vector of model parameters. The vector π can be interpreted as the limit of the Markov process $\lim_{m\to\infty} P^m S_t = \pi$, or the stable probability vectors independent of time. The density of the observed point y_t conditional on the state *j* and parameters θ is:

$$f(y_t \mid s_t = j;\Phi) \qquad \text{for j=1,2.}$$

The probability of observing state *j* and value y_t is:

$$P(y_t, s_t = j;\Phi) = f(y_t \mid s_t = j;\Phi) \times P(s_t = j;\Phi)\,. \tag{12}$$

Summing this over all states generates the unconditional probability of observing the data point at time t:

$$f(y_t;\Phi) = \sum_{j=1}^{2} f(y_t \mid s_t = j;\Phi) \times P(s_t = j;\Phi)\,. \tag{13}$$

In order to derive the probability weights that produce our parameters at any given time, we need to produce the probabilities that the unobserved regime responsible for observation t was regime *j*. Using Bays' rule this can be stated as:

$$P(s_t = j \mid y_t;\Phi) = \frac{P(y_t, s_t = j;\Phi)}{f(y_t;\Phi)} = \pi_i \left(\frac{f(y_t \mid s_t = j;\Phi)}{f(y_t;\Phi)} \right) \tag{14}$$

This is just state *j*'s part of the total unconditional probability of observing the data. If the distribution implied by state *j* totally describes the data observed at time *t*, the probability of observing *j* is equal to the unconditional probability of observing state *j* at any given time. This gives us an intuitive interpretation of the weights as being proportional to how much of the observed data each regime can account for. Further, note that this is the sample or post-data counter part of the Markov process $S_t = PS_{t-1} + v_{t+1}$ defined above.

Given a set of parameters Φ, Hamilton shows how optimal inference about the regimes can be ensured by iterating equation (12) and (13), written in its composite form as:

$$\bar{S}_{t|t} = \frac{(\bar{S}_{t|t-1} \otimes \eta_t)}{1'(\bar{S}_{t|t-1} \otimes \eta_t)} \qquad \bar{S}_{t+1|t} = P \times \bar{S}_{t|t} . \tag{15}$$

Where η_t is the (2×1) vector of conditional densities and P is the probability transition matrix. As a by-product of this iteration the likelihood function $\ell(\Phi) = \sum_{t=1}^{T} (f(y_t; \Phi)) = \sum_{t=1}^{T} 1'(\bar{S}_{t|t-1} \otimes \eta_t)$ is produced. The value of the parameters Φ that maximizes the log likelihood function can then be found by conventional maximum likelihood methods. In order to start the procedure we need a starting value for the probability weights $\bar{S}_{1|0}$. For this we use the unconditional or ergodic probabilities π. Thus for the two state approach two likelihood functions are calculated, one for each state. These are combined according to weights $\bar{S}$ to construct a composite distribution to represent the data generating process. The weight ascribed to each parameter value is, as stated, the result of a first order Markov process where the parameters of the Markov process is the state transition probabilities. By the ergodicity of this process the weights can be interpreted as probabilities that the system is in a given state. Further by maximum likelihood we find the optimal parameters, including the state transition probabilities so as to best combine the state dependant distributions. These parameters are derived in combination with equation (15) to produce our desired probability weights. Since the probability weights $S_{t|t}$ are only evaluated at up to the information available at time t we perform a sweep through the data at the end of the estimation so as to use the full sample to evaluate each time dependant probability (See Hamilton (1994) pp. 694).

MODEL ESTIMATION

The composition of the salmon price used in the empirical evaluation is achieved by averaging the market price for fresh salmon of different weight classes. This is assumed a valid representation of price of salmon (Asche & Guttormsen, 2001). Prices are observed weekly from the end of 1992 to the summer of 2007, totalling 844 observations.

In order to draw valid statistical inference from the empirical analysis we need to ensure stationarity. We detect only weak linear trends in our time series, unlikely to account for all non-stationarity. As is common, a stochastic trend component is likely to exist in our data. To avoid the possibility of spurious regression effects we test for unit roots at the zero frequency using the Augmented Dickey Fuller test. Failing to detect a unit root suggests no stochastic trend in our data. We test using a constant and a maximum lag level of 20 weeks, as suggested by the formula $L^{max} = \text{int}[12[T/100]^{0.25}$ (Schwert ,1989).

TABLE 1. Augmented Dickey Fuller Unit Root Test.

	Constant			Constant + Trend		
Series	*Lag*	y_{t-1} coef.	t	*Lag*	y_{t-1} coef.	t
Price level	17	0.989	-2.123	17	0.9802	-2.716
1. difference	16	-0.15311	-9.083**	16	-0.15461	-9.083**

** Significance at the 1% level

To trace out the specific lag length from these 20 possibilities, we use Akaike's Information Criteria (AIC). The AIC suggests an evaluation at a lag of 17 months. The test fails to reject a unit root (see Table 1). To eliminate non stationary components we apply a first difference filter. The series in first differences rejects unit roots at all lag levels.

TABLE 2. Descriptive Statistics.

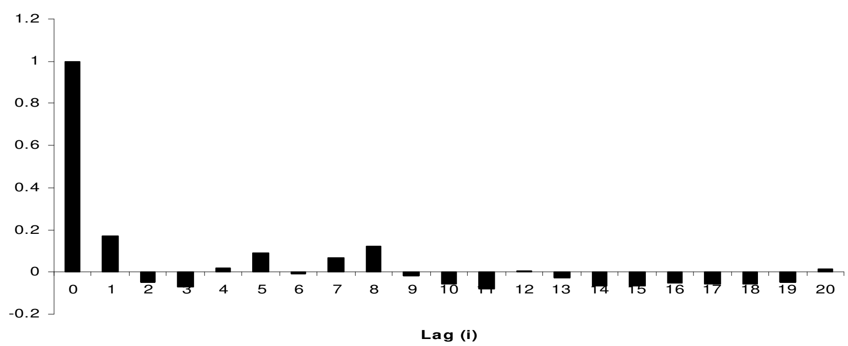
	Price Level	First Difference
Mean	26.41	-0.021941
Standard Deviation	6.125	1.023
Skewness	0.787	0.44105
Excess Kurtosis	0.384	12.775
Minimum	14.508	-6.8333
Maximum	49.167	8.7083
Normality Test	129.39**	1079.1**

Note: The normality test is the Jarque Bera test, where ** indicate rejection of the null hypothesis of normality on the 95% significance level. Excess kurtosis is kurtosis outside of the normal distribution.

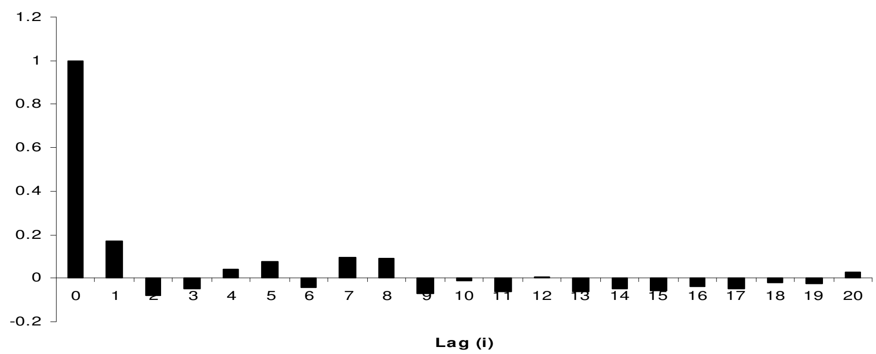
As such we will in our further analysis use the first differenced price series.

Table 2 provides some summary statistics for both the level and first difference of prices. We note that normality is rejected for both levels and returns using the Jarque Bera test. In both levels and returns the distributions display positive skewness and excess kurtosis. The positive skewness implies a longer tail to the right of the mean; arising due to the characteristic positive price spikes. For the first difference some skewness has been transferred to excess kurtosis.

A large portion of the return seems to be concentrated around the mean, but with occurring periods of large corrections. Below we plot autocorrelations and partial autocorrelations for the first differenced series. The plot suggests that autocorrelations move in a weak oscillating pattern, with waves above and below zero respectively.



Sample autocorrelations
Confidence interval: $\left[-2/\sqrt{T}, +2/\sqrt{T}\right] \approx [-0.07, 0.07]$



Sample partial autocorrelations

FIGURE 2. Sample Autocorrelations and Partial Autocorrelations

The effect seems to vanish for higher order lags, indicating an oscillation decreasing in magnitude. Oscillating correlations with decaying amplitude is a property of systems with complex eigenvalues of modulus less than one. If we represent the price process of salmon through a second order autoregressive model, the eigenvalues of the system is approximately $[0.09 \pm 0.342i]$ which has modulus 0.361.

These eigenvalues are converging dynamic multipliers of oscillating pattern with peaks appearing approximately monthly. Evaluating the likelihood values for lag length of three weeks does not significantly improve the representation of data. Increasing lag length beyond three lags does provide a better fitting to the data, as would be expected, but at the cost of possibly over specifying the model. An AIC analysis on a linear autoregressive model suggests the application of a twenty week lag. However for a non-linear model, this suggestion becomes invalid. By allowing non-linear parameters the need for such long lag lengths disappears. By constructing parameters as a flexible convex combination of linear parameters we can replicate the complex eigenvalues arising in the linear model. Indeed, as we will see in the next chapter, a two week lag for the non linear model performs well under relevant sensitivity analysis.

SENSITIVITY ANALYSIS

In order to verify the adequacy of the models fit to data, we now apply some sensitivity analysis to the basic model. The model we use is a second order autoregressive model for the development of the mean and a first order autoregressive conditional heteroskedasticity model for the conditional variance. The model applies regime dependant parameters for the autoregressive coefficients in the mean, and for the constant term of the conditional variance:

$$y_t = \mu + \sum_{i=1}^{2} \beta_i (S_t) \Delta y_{t-1} + \sigma_t^2 (S_t) \varepsilon_t \,.$$

$$\sigma_t^2 (S_t) = \alpha_0 \theta(S_t) + \alpha_i \varepsilon_{t-1}^2 \,.$$

We evaluate the model by comparing the non-linear models to that its linear counterpart (the same model with only one regime). When evaluating a linear versus non linear model, we must confront a problem arising due to lack of equivalent parameters under the linear model. For example, transition probabilities do not exist for the linear counterpart. Further, as pointed out by Hansen(1996), when evaluating a linear autoregressive model against a regime switching model, some key assumptions about the asymptotic distributional theory of non-linear models are compromised. In order for the non-linear models to produce true parameter estimates, we need to assume a locally quadratic likelihood surface; where locally is defined as a region where both the null and global optimum lie. In addition, we need to guarantee that the scores have positive variance, allowing the application of a central limit theorem stating the existence of a multivariate normal distribution. In evaluating Hamilton's switching model, both these assumptions are violated because of the existence of the nuisance parameters. The null of a linear model arrives at a local optimum, and higher order derivatives also appear to vanish. As such the standard Likelihood Ratio test used to evaluate such models will reject a linear model too often. Hansen (1992) & Garcia (1989 improve around these difficulties by deriving bounds for the asymptotic distribution valid under the existence of nuisance parameters. Hamilton (1996) further created a set of score functions based on the derivatives of the model's log likelihoods. From these score functions one can test the existence of autocorrelations, ARCH effects or the validity of the Markov assumptions. Hamilton's testing approach has not been applied on a large scale in the literature, most likely due to the complexities of the switching models fitted to data (Breunig, Najarian & Pagan, 2003). In order to evaluate the validity of applying a non-linear model, we apply a modified Likelihood Ratio test from Tillman(2003). We adjust the LR test statistics to a $\chi^2(r+n)$ distribution where r is the degrees of freedom and n is number of nuisance parameters. Being aware of the existence of nuisance parameters, we take a conservative approach to the likelihood ratio test:

$$LR = 2\left(L(\theta \mid Y_T) - L(\theta^{rest} \mid Y_T)\right) .$$

Where θ^{rest} is the parameters of our restricted (linear ARCH) model. Evaluating this model against the equivalent two state switching model produces a likelihood ratio of 102 (Table 3). The relevant critical value is 20.5 for the 1% interval. The test suggests that introducing non-linear parameters strongly improves the models fit to data. In addition, the non linear model performs better than the linear model even with a twenty week lag. Testing for more than two states is more problematic, and an optimum of the likelihood surface is hard to find. As stated, the asymptotic properties of this test is not technically satisfactory because of the nuisance parameters, however such a high rejection rate evaluated against a conservative critical value provides strong evidence that introducing the non-linear model is justifiable.

In order to validate our choice of model further, we test the normality of the standardized residuals in the linear and non-linear model using the Jarque Bera test (Table 3). The Jaque Bera test for normality of residuals for the linear model produces a statistics of 152.71, strongly rejecting the hypothesis of normality. For the non-linear model, we note that the distribution of the final model is multivariate, with the final distribution generated by a linear combination of the regime dependent distributions. For the normality of residuals the test value is 5.94, satisfying normality on the five percent level. Introducing multivariate distributions through non-linear modelling allows the representation of normal residuals, and more importantly able us to model the non-normality properties of the data.

TABLE 3. Likelihood and Normality Test.

	Value	95% Critical Value
Likelihood Ratio	102	20.5
JB for linear model	152,71	5.99
JB for non-linear model	5.94	5.99

Linear model:

$\Delta y_t = \delta + \beta_1 \Delta y_{t-1} + \sigma_t^2 \varepsilon_t$

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$

Non-Linear model:

$\Delta y_t = \delta_s + \beta_1 \Delta y_{t-1} + \sigma_{s,t}^2 \varepsilon_t$

$\sigma_{s,t}^2 = \alpha_0 \theta_s + \alpha_1 \varepsilon_{t-1}^2$

Where the subscript S denotes regime dependant variables

To further facilitate our choice of model we will test some predefined moment functions on the residuals of the model. Formally we define three such moment functions:

$$m_{1,t} = \sum_{t=1}^{T} \left(y_t - y_t(\hat{\theta}) \right). \tag{16}$$

$$m_{2,t} = \frac{1}{T} \sum_{t=2}^{T} \left(y_t - y_t(\hat{\theta}) \right) \left(y_{t-1} - y_{t-1}(\hat{\theta}) \right). \tag{17}$$

$$m_{3,t} = \frac{1}{T} \sum_{t=3}^{T} \left(y_t - y_t(\hat{\theta}) \right) \left(y_{t-2} - y_{t-2}(\hat{\theta}) \right). \tag{18}$$

Equation (16) measures the residual as implied by the model, and we should find under proper specification that $E(m_1)$ converges to zero. Further equation (17) and (18) tests the autocorrelations of residuals. Thus under the expectation that all relevant information is included in the model no serial correlation should exist. In order to estimate equation (16), (17) and (18) consistently we regress $m_{1,t}$, $m_{2,t}$ and $m_{3,t}$ individually against intercepts and scores and use robust standard errors when forming t-statistics. The scores are defined as the derivatives of the log likelihood with regards to the parameters. We include the scores in the estimation to correct the variance of the moment functions for effects generated by the nuisance parameters Tauchen(1985). Specifically it can be shown that the asymptotic variance of the moment functions converges to the sample variance and a term consisting of the score and information matrix of the likelihoods. Failing to account for this extra term will hence overestimate the sample variance, leading to under rejection of the hypothesis of zero expected moments (See Breunigi, Najarin & Pagan (2003), pp.709).

TABLE 4. Consistency test for residuals.

	Intercept	t-value	Rob. std.errors
m1	0.0105961	0.38	0.0277576
m2	0.0046653	0.13	0.0359982
m3	-0.007281	-0.16	0.0453619

We evaluate the t-values of the intercept terms in the moment regressions to assess the value of our three moment functions. As reported in table 4 no significant autocorrelation in residuals can be found. It does appear that our moment functions are satisfied, strengthening our specification of the data generating process.

ESTIMATION RESULTS

We now apply our model to the estimation procedure derived above. The estimation is performed using the maximum likelihood approach of Broyden, Fletcher, Goldfarb, Shannon (BFGS). Initial values are traced using the Simplex method. Table 5 reports the estimation results. All parameter values, except the mean, are significant at the 95% level. We note that the normal state displays greatest stability, where the probability of remaining in the state following a normal state is 97.8%, while changing state occurs only with a 9% probability. During what we define as the normal state, the autoregressive parameters of the mean model are for week one and week two lag respectively 0.499 and 0.28. This indicates mean reversion in the normal state. For the high volatility regime, the autoregressive parameters are negative, with value -0.20 and -0.232 for lag one and two respectively. The negative parameter values during the high volatility regime illustrate the fluctuating nature of excess return during these periods. Turning to the competitive storage theory this fluctuating reversion patterns can be explained by a lack of speculative stocks to smooth prices and generate positive autocorrelations.

Concerning the conditional variance of returns, we observe that mean reversion exist in the autoregressive parameter. Persistence in volatility exists, giving rise to volatility clustering.

TABLE 5. Estimation Results.

Parameter	Coefficient	Std.Dev.	t-value
P_{11}	0.9781	0.0072	135.8088
P_{12}	0.09	0.0258	3.4816
μ	-0.0261	0.0225	-1.157
$\beta_{t-1} \mid S=1$	0.49968	0.043	11.6148
$\beta_{t-1} \mid S=2$	-0.2	0.0366	-5.4587
$\beta_{t-2} \mid S=1$	0.2801	0.1066	2.6266
$\beta_{t-2} \mid S=2$	-0.2318	0.0902	-2.568
α_0	0.2732	0.0254	10.724
α_1	0.2446	0.072	3.3937
$\theta \mid S=2$	8.8609	0.8617	10.282
Obs. 844		LL	-939.3436

For the high volatility regime the conditional variance is multiplied by a scalar approximately equal to 8.86. As such, during the high volatility regime the variance of the distribution increases almost nine fold relative to the normal state.

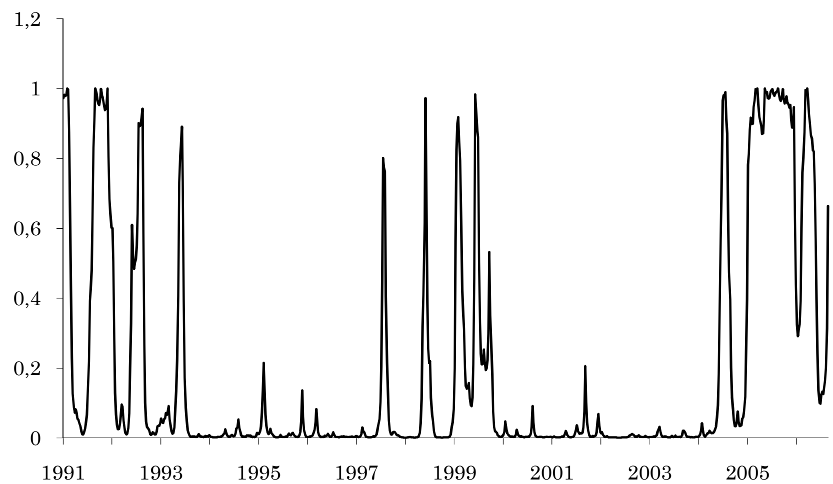


FIGURE 3. Probability of being in high volatility regime

In figure 3 we plot the smoothed unconditional probabilities of existing in the high volatility state. We observe that some years are relative "quiet". Specifically 1994 to 1997 and 2000 to 2004 are relative quiet periods. Following 2004 up to 2007 the market again enters a high volatility regime. From the estimation results we now compare how the implied price dynamics compare to knowledge of market fundamentals and the implications from theory of storage.

DISCUSSION

Regarding the mean reversion parameters, we note that the eigenvalues for the normal state $[0.2 \pm 0.585]$ are real and converging, providing a state where excess returns are absorbed to an unconditional mean value. For the "high" volatility regime the eigenvalues $[-0.1 \pm 0.47i]$ are complex, with modulus 0.48. Thus the dynamic multipliers in the high volatility regime are oscillating but converging. Hence the mean reversion in the price process moves between states of stable convergence of returns to fluctuating convergence. Under speculative storage the autocorrelations are believed to be relatively high as current price is equated to the discounted expected future price. As such, inventories are used to absorb excess returns and mean reversion is expected. In periods of low stocks the price movements are to a greater degree dominated by the stochasticity of demand and supply, and excess returns are allowed to persist as stocks are insufficient to eliminate arbitrage.

In the salmon market demand is not stationary across the season. Periods exist, for example during Christmas and Easter, where demand increases. Further, because of the productive nature of stocks, the total biomass of stocks varies across the season, following seasonal growth and maturity patterns. This provides a mean to examine indirectly the effects predictable shifts in state variables on the price process. In order to isolate specific seasonal patterns in supply we fit some cyclical patterns to a time series of quantity of salmon sold from 1999 to 2007. From our preliminary knowledge we know that during summer periods, growth of fish increases while at Christmas and Easter a surge in demand occurs. In the weekly quantum traded data we can pick up these effects by modelling the trend by trigonometric sinusoid and cosine functions. We allow for one, two and four cycles a year in addition to a quadratic and linear time trend. The trend is modelled as:

$$\mu_t = \mu_q + \alpha_q t + \alpha_{qq} t^2 + \sum_{n=1}^{4}\left[\varphi_n \sin\left(2\pi t / 13n\right) + \omega_n \cos\left(2\pi t / 13n\right)\right].$$

Figure 4 displays the trends fit to data. The major peak in the seasonal trend is the Christmas surge in demand, accounting for the largest seasonal effect. The smaller peaks following Christmas is the Easter surge, while the larger peak following this is the summer seasonal effect. We note that the minor peaks fit the data poorer in later observations.

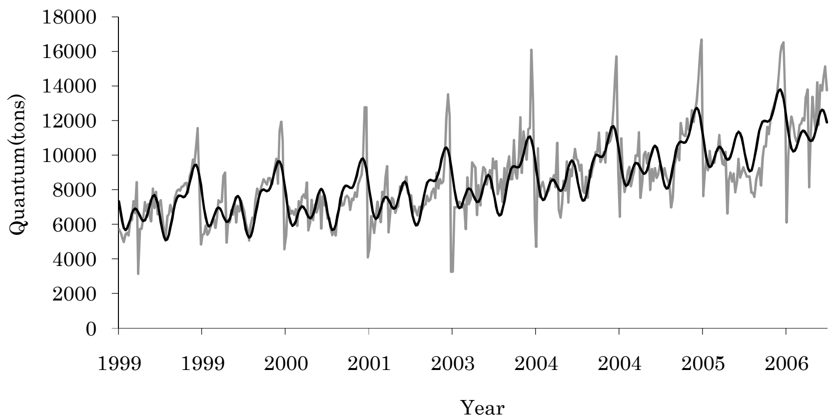


FIGURE 4. Quantum Traded and fitted trend

This representation of the trend is not in any way complete but is sufficient to illustrate and identify the summer, Christmas and Easter seasonal effects.

To identify the seasonal effects we generate a series of the weekly averages of the probability of being in a high volatility state from 1992 to 2007. From 1992 to 2007 there are 15 data points. To evaluate the statistical significance of the seasonal patterns in volatility we apply the bootstrap method. From the entire series of probabilities we draw 15 observations randomly, representing a random seasonal observation. From these observations an average is created. This is done for 10 000 draws, allowing the formation of a distribution of averages. From the distribution the 1%, 5%, 95% and 99% percentiles are generated. In figure 5 these percentiles in addition to the mean and seasonal series is displayed. We observe from figure 5 that the seasonal pattern lays within the 5% and 95%

confidence interval, however with the extremes of the series being close to the intervals bounds.

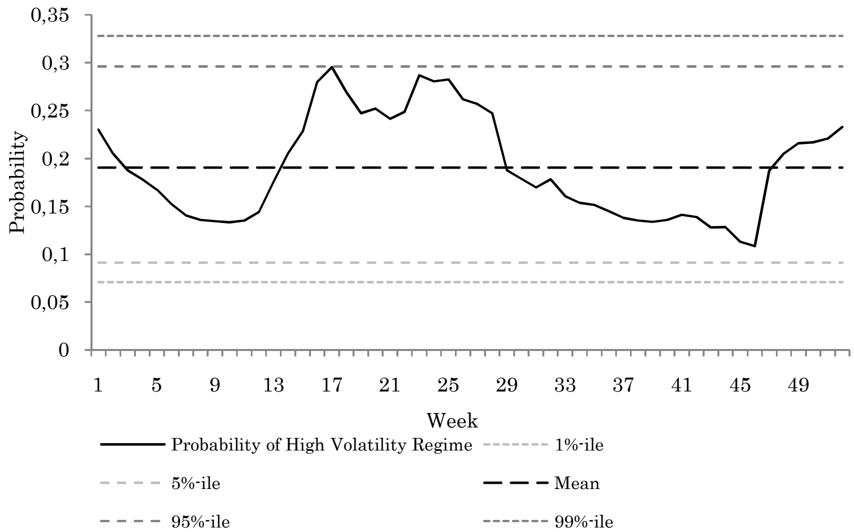


FIGURE 5. Weekly probability of high volatility state.

The probable high volatility weeks in figure 5 coincide with the seasonality patterns conjectured above. Specifically, we find that the demand surge during Christmas times increases the probability of the market in entering a high volatility regime. The demand surge effectively empties the inventory of available fish for sale, reducing the markets flexibility in satisfying future excess demand. The market is more likely to be equated by price movements than flexibility in supply. Concerning the early summer period seasonal effects, the convenience yield of stocks increases as the alternative cost of harvesting fish increases.

This is due to the fact that harvesting prior to a growth period sacrifices the immediate high growth period. As the convenience yield increases more stocks are withheld and the price necessary to initiate harvesting increases. In figure 6 we plot the seasonal patterns in volatility in addition to the seasonal price level patterns. In the figure we observe how the probable high volatility regimes coincide with the seasonal patterns in prices. This result is in line with the theory of storage, where high prices are accompanied with lower stocks and higher price volatility.

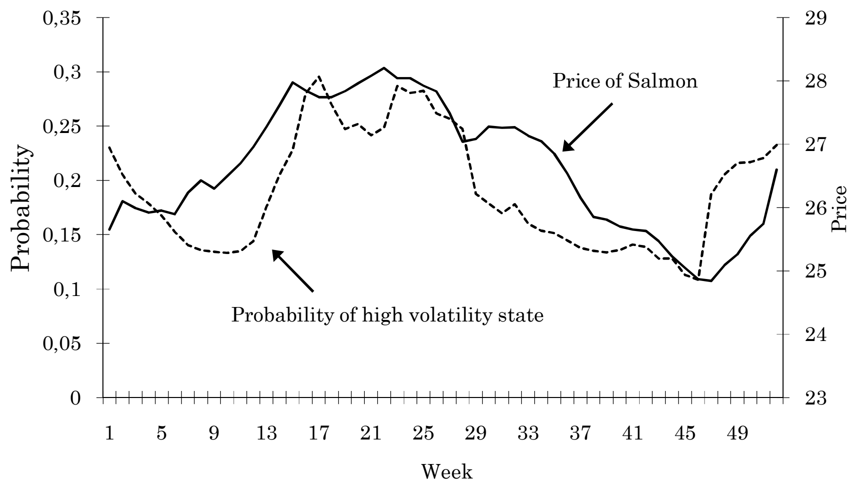


FIGURE 6. Seasonal pattern in volatility and price levels.

Turning back to the full probability of high volatility series, we can also compare the periods of probable high volatility with the series of annual industry return, that is the difference between price and production cost per kilogram fish.

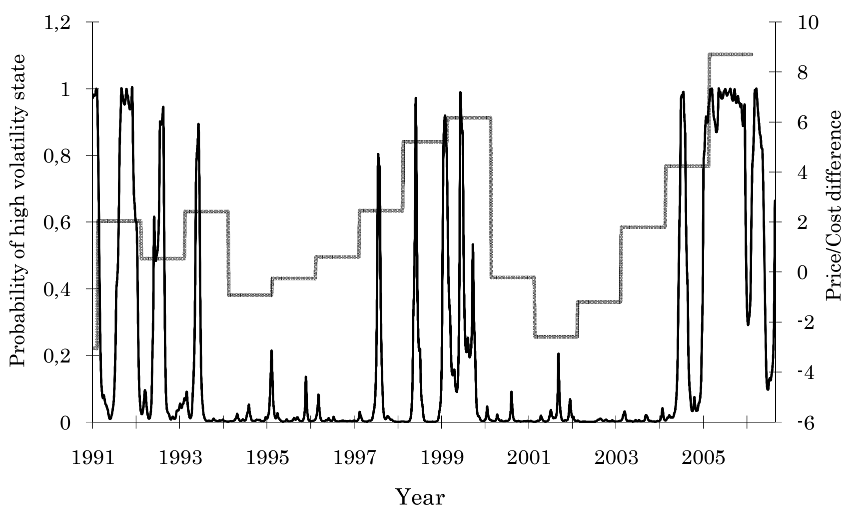


FIGURE 7. Volatility and Profitability.

From figure 7 we see that periods of high profitability; measured by the price/cost difference, is closely followed by an increasing likelihood of existing in a high volatility regime. When profitability is large stocks are exhausted leaving less flexibility in smoothing future price movements.

The results indicated above suggest that the two state regime shifting model can provide a successful basis in modelling commodity prices. We observe the movements in fundamentals such as stock levels are important in accounting for changes in the price process in the short run and that a model with shifting parameters can pick up such changes.

CONCLUSION

In this paper I have argued that a two state Markov switching model is a consistent price model for commodity prices. The price model allows the flexibility in dynamics demanded by the theory of storage. The output of the model are furthermore useful in making indirect inference of predictions made by storage theory. We applied the regime shifting model to a time series of salmon prices in order to examine predictions made by commodity price theory. The competitive storage theory states that price follows an asymmetrical process identified by speculative storage. By modelling mean and variance as the result of a two state regime swicthing model, the fit to data is improved over its linear counterpart. We defined the two states as a normal volatility regime and a high volatility regime. The econometric analysis show that during the normal regime, returns behave according to a traditional mean reverting process. In the high volatility regime, volatility is scaled almost nine fold relative to the normal regime, in addition to generating negative signs on the mean reversion coefficients of returns.

From the empirical analysis we also find that during early to middle summer and around Christmas there exist a greater probability of existing in a high volatility state. These periods coincide with seasonal patterns observed in the market. Specifically the summer seasonality is associated with improved growth of salmon. This improved growth period generates a convenience yield on sitting on stocks, leaving less stocks available for speculative storage. A higher price is necessary to initiate harvesting. The Christmas seasonality is associated with a surge in demand, effectively emptying stocks as the temporal excess demand is satisfied. We also find that higher prices and improved profitability is associated with a higher likelihood of existing in a high volatility regime. This is consistent with the theory of storage, where high prices are characterised by a scarcity of goods. The high prices lead farmers to empty current stock, lowering the flexibility in reacting to future changes in excess demand. The feedback effect between high prices and low inventories is expected to generate a high volatility regime

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STOCHASTIC LONG RUN CYCLES IN COMMODITY PRICES

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Abstract In a rational expectations market, long run predictable cycles not founded in cost shifting factors should not persist. Still, arbitrage possibilities have been documented in long run cycles of for example hogs, broiler and cattle. If cycles are sufficiently stochastic the cost of identifying and reacting to the perceived cycles will increase to the degree of allowing some residual cyclicality in the markets. As such, realised cycles and arbitrage will only be precisely identified following the cycle completion. In this paper, we follow two approaches to examining the stochasticity of long run cycles in commodity prices. The first approach based on duration dependence in cycles and is a non-parametric index to test for convergence in cycles. The second approach is based on the realisation that most commodity prices are non-stationary and near-unit or unit root. We expand on the classical seasonal unit root test to test for existence of unit roots outside the seasonal frequencies.

INTRODUCTION

The existence of long run predictable cycles in commodity prices have been thoroughly examined, studied, and documented, where the classical studies are those of the hog cycle (Hayes & Schmitz, 1993; Shonkwilder & Spreen, 1986; Harlow, 1960; Dean & Heady 1958; Chavas & Holt, 1991) in addition to cyclicality in cattle prices (Rosen, Murphy & Scheinkman, 1994) and broiler prices (Rausser and Cargill, 1970). The out of seasonal cycles are believed to emerge due to a counter cyclical reaction in supply and the lag from production decision to realised output. A positive price shock will increase supply as profits are captured, leading spot prices to decline. The increased supply will further reduce the availability of future stocks, leading prices to increase again until a new stock emerges allowing prices to be corrected. Earlier studies founded on the Cobweb theorem (Ezekiel, 1938) argued that these cycles persist due to producers ignoring public information and forming price expectations on some model of historical price realisations (Breymeyer, 1959; Larie, 1947; Harlow, 1960; Larson, 1950; Rucker, Burt & LaFrance, 1984). Based on the cobweb model, it was predicted that a four year cycle in hog prices should exist. However, cycles both shorter and longer than this has been found in empirical studies. This discrepancy motivated the development of multi-frequency cobweb explanations, allowing for several cyclical patterns to exist (Talpaz, 1974).

Following the cobweb analysis a second school of thought emerged, basing its analysis on the efficient market hypothesis of Fama (1970), Muth (1961) and Samuelson (1965). These studies argued that in an efficient market, predictable cycles will be eliminated due to arbitrage opportunities. Under this regime, a cobweb style cycle should not persist since it generates a profitable trading strategy. The cobweb model allows identification of emerging cycles since expected prices are based on historical data. The elimination of arbitrage, and hence cyclicality, in the efficient market framework hinges on two criteria: (1) the ability to identify and predict the cyclical patterns and (2) the marginal gain in changing trading strategies overshadowing marginal costs. The argument that long-run cycles will be eliminated by arbitrage thus hinges on the assumption of a predictable cycle.

Shonkwilder & Spreen (1986) examined the statistical stability of hog cycles, and found evidence that cycles undergo a complicated change based on cycle length. Further studies (Holt & Craig, 2006; Chavas & Holt, 1991) based on a non-linear and chaotic approach to cycles conclude that cycles follow what appear to be non-linear dynamics, and that hog cycle is a non-stationary process. It appears that earlier attempts at modelling cycles as deterministic changes in well behaved stationary markets is unsatisfactory in explaining the persistence of cycles. However, if cycles emerge from a non-stationary process, their persistence is not necessarily in contradiction with the efficient market hypothesis. Identification and profitable trading strategies can be derived after the cycle unfolding, in a suitably long time series. The cost and risk in reacting to a perceived cycle might outweigh the expected benefits during the cycle unfolding.

The existence of unit roots in commodity prices is debated (Tomek, 2000; Tomek & Wang, 2007). Economic theory suggests that commodity prices should be convergent, yet empirical tests often find that unit roots cannot be rejected. High autocorrelation in low order lags are often detected in commodity prices. Near unit root, or stochastic root, processes are further difficult to reject using traditional unit root tests. Hence, it is necessary to maintain a scepticism vis-à-vis equating commodity prices in general to unit-root processes. However, statistical evidence that commodity prices contain unit roots should not simply be discarded as the result of potentially inferior statistical methods. Existence of non-stationary components in commodity prices can provide an economic explanation for the existence of persistent long run cycles in commodity prices. If one assumes that slowly evolving unit-root processes are significant components in commodity prices, this provides an explanation within the efficient market hypothesis for the existence of long-run cycles in commodity prices.

There are several economic rationales for unit roots outside of the zero frequency. For instance, if producer's price by marginal cost, unit roots outside of the zero frequency can be detected if marginal costs follow a sufficiently variable non-zero frequency unit root process. Implementation of new technology is generally not immediate, taking some time to be functional. Stock levels might also move in a near unit root process dependant on stocks at some non-immediate previous time period. In a market where the correlation between stocks and sales are high this will transmit to stochastic cyclicality in prices. Further, in pricing regimes where producers base planned production output on spot prices, cyclicality in can occur. Some time generally passes before changes in planned production takes effect. That many commodity prices appear non-stationary opens for the possibility that cycles in prices are generated by out of seasonal unit roots. This paper contributes to the literature of long run cycles in commodity prices by examining for the existence of stochastic cycles generated by out of seasonal unit root processes.

To test the stochasticity of observed cycles we take two approaches. Our first approach is non-parametric and consists of evaluating a constructed index that allows the examination of duration dependence in cycles. This method is based on the approach used by Ohn, Taylor & Pagan (2004) for identifying business cycles. By simulation we infer the statistical properties of the index under the null of no duration dependence. From this we test the sampled index values. Failing to reject duration dependence implies non-converging and random cycle evolution. The second approach is a parametrical econometric test, expanding on the classical seasonal unit root test of Hylleberg, Engle, Granger & Yoo (1990) paper (HEGY). In the parametric test we explicitly define the relevant cyclical frequencies. The HEGY test initially tested for seasonal unit roots in quarterly data, but was later expanded to monthly data by Beaulieu & Miron (1993) and Franses (1991). The expanded test allowed the existence of twelve seasonal unit roots in addition to the conventional zero frequency root. In the seasonal unit root literature the economic incentive for the existence of these roots are not thoroughly discussed. It is generally argued that some seasonal fluctuations may be caused by the behaviour of economic agents and may therefore not be constant (Hylleberg, Jørgensen & Sørensen, 1993). Concerning cyclical patterns outside of the seasonal span, the most well know cycle is the business cycle. The business cycle can be generated by a random walk process, and it is not possible to separate the stochastic trend from the cycle (Harding & Pagan, 2002). We use this approach to test for unit roots at frequencies that can account for out of seasonal cycle lengths.

We start the analysis by investigating an index which enables us to test the degree of convergence and predictability in cycles. We evaluate the index for monthly observations of the spot price of some selected commodities. Following the index test we continue to the parametric unit root test, where we apply the test to the same monthly observations in addition to annual observations of different commodities. Our analysis suggests that cycles are non-deterministic and stochastic with out of seasonal frequencies being significant in determining cyclical movements. We find support for the hypothesis that long run cycles are allowed to persist due to their stochastic nature and that reliable identification can only be done after the cycle unfolding.

CYCLE PHASE CORRELATION

Samuleson (1965) demonstrated that if all available information is incorporated in the probability distribution of price movements, prices will fluctuate randomly. One model for this process is the random walk model, a zero frequency unit root process where returns are generated from an unconditional stable distribution. The random walk process contains the Markov property that current prices moves randomly conditional on previous period information; the price history outside of the immediate previous period has no predictive power on future prices. If prices display cyclicality it appears that the Markov property is violated as the price history can predict future price movements. This suggests that if cycles not founded in cost shifting factors persist in the efficient market paradigm, they must behave like a unit root process. Cycle length and amplitude cannot have predictive power on future movements. Profitable trading strategies for such cycles can only be found after their unfolding, much like novel chart analysis show how to buy low and sell high in a historical chart.

A property of unpredictable cycles is a lack of duration dependence. A positive (negative) duration dependence implies that the likelihood of an expansion or contraction phase to revert is positively (negatively) proportional to the time spent in the cycle phase. In a data generating process with no duration dependence, the duration of cyclical phases are independent of the history of the phases. A unit root process will not display duration dependence as the Markov property implies that expected future price is equal to the spot price.

A starting point to examine the stochasticity of cycles is to examine whether duration dependence is present in the series. In this section we apply an index to measure the tendency of the series to be in the same cycle phase at different lags. Our null hypothesis will be no duration dependence. No duration dependence is a characteristic of series where no economic forces correct price movements; the random walk model for example contains no duration dependence. We start by transforming a time series of price observations $\{y_t\}_{t=0}^{T}$ into a series of expansion and contraction phases by the mapping:

$$\begin{aligned} s_t &= 1 \quad \text{if} \quad (1-L)y_t \geq 0 \\ s_t &= 0 \quad \text{if} \quad (1-L)y_t < 0, \end{aligned}$$

where L is the backshift operator. A string of ones in $\{s_t\}_{t=0}^{T}$ for example indicates the existence of a cycle expansion phase. The event of being in a certain phase and the duration of each phase is assumed drawn from some probability distributions. Under no duration dependence, the event of being in a phase is independent of previous realisations and as such is drawn from the Poisson distribution. The duration of each phase will be drawn from the exponential distribution, or the geometrical distribution in discrete time (Ohn, Taylor & Pagan, 2005). In a series with convergent cyclical behaviour the independence assumption of each phase will be broken as the duration of each phase will be dependent on the duration of previous phases. Evaluating the duration dependence of cycles allows evaluation of the stochasticity of cycles. The index, when evaluated against the null hypothesis of no-duration dependence provides a measure of the stochasticity in the observed cyclical behaviour.

The index is created from the binary sequence $\{s_t\}_{t=1}^{T}$ by measuring the regularity in occurrence of phases at different lag levels:

$$\hat{I}_{t,t-k} = \left(\sum_{t=k}^{T} s_t s_{t-k}\right)T^{-1} + \left(\sum_{t=k}^{T}(1-s_t)(1-s_{t-k})\right)T^{-1} \tag{1}$$

The index measures the rate at which the phases at time t and t-k are equal. This approach is equivalent to the Harding & Pagan (2002) measure of concordance between different cycles. We do not impose any restrictions on the minimum duration of each phase as we at this point will not discriminate between cycle lengths. An advantage of the non-parametric approach is that non-stationarity of the original series is not an issue. The expected value of the index is equal to:

$$E(I_{t,t-k}) = E(s_t)E(s_{t-k}) + E(1-s_t)E(1-s_{t-k}) + 2\operatorname{cov}(s_t, s_{t-k}).$$

For no cyclical regularity the covariance term vanishes. In a pure random walk process the probability of moving up or down conditional on current price is equal to one half, and the expected value of the index will be equal to one half for all lag choices. Under the null hypothesis the covariance term $2\operatorname{cov}(s_t, s_{t-k})$ vanishes for all lag values k. Under stationarity conditions of $\{s_t\}_{t=1}^{T}$, the ergodic theorem imply that:

$$\lim_{T\to\infty} \hat{I}_{t,t-k} = \left(\sum_{t=k}^{T} s_t s_{t-k}\right)T^{-1} + \left(\sum_{t=k}^{T}(1-s_t)(1-s_{t-k})\right)T^{-1} = E(I_{t,t-k}).$$

The geometric distribution associated with no duration dependence is defined as $prob(X=k) = (1-p)^k p$ where $prob(X=k)$ describes the probability of observing a cycle with length k. The probability p

denotes the probability of a phase collapse. The expected cycle length will be equal to $E[X] = p^{-1}$. In the case of a drift-less random walk, $p = 0.5$. Note that for a non-zero frequency unit root processes such as $y_t = y_{t-l} + \varepsilon_t$, the probability of a phase collapse p is also one half when $y_0 = 0$ because:

$$
\begin{aligned}
&p = prob((1-L)y_t > (<)0) = \\
&prob(\varepsilon_t + \varepsilon_{t-l} + \ldots + \varepsilon_{t-nl} > (<)\varepsilon_{t-1} + \varepsilon_{t-l-1} + \ldots + \varepsilon_{t-nl-1}) = 0.5
\end{aligned}
$$

If the process contains a drift, the probability of an expansion and contraction phase collapse is not identical. To eliminate drift we de-trend the original series before applying the index.

When evaluating the empirical index we compare our derived index values against the implied distribution of the index under the null hypothesis. Our procedure to derive the sample distributions under the null is as follows: (1) we de-trend the original price series and generate the sequence $\{s_t\}_{t=1}^{T}$. Following this we derive the implied probabilities of existing in an expansion or contraction phase. The probabilities are derived as the rate at which the series $\{s_t\}_{t=1}^{T}$ is in an expansion (contraction) phase. Under stationarity conditions of the sequence $\{s_t\}_{t=0}^{T-1}$ this provides an unbiased measure of the probability of existing in given a cycle phase. (2) Applying the derived probabilities we generate the distribution of the index under the null hypothesis by simulating series $\{\hat{s}_t\}_{t=1}^{T}$, where phase lengths are drawn from the geometric distribution. This is done for each series that we evaluate. From 24 000 simulations of the series $\{\hat{s}_t\}_{t=1}^{T}$ we derive critical values of the simulated distribution as relevant fractiles of the statistics:

$$
cv_k = \frac{\left(n^{-1}\left\{\sum_{t=1+k}^{T} \hat{s}_t \hat{s}_{t-k} + (1-\hat{s}_t)(1-\hat{s}_{t-k})\right\} - 0.5\right)\sqrt{T}}{std\left(n^{-1}\left\{\sum_{t=1+k}^{T} \hat{s}_t \hat{s}_{t-k} + (1-\hat{s}_t)(1-\hat{s}_{t-k})\right\}\right)} .
$$

The critical values are generated for each series and each lag value k of interest. (3) If the critical values of our empirical index fall outside of the relevant percentile areas of the simulated critical values we reject the hypothesis that the associated cyclical movement is generated with no-duration dependence.

TABLE 1. Cycle Phase Correlation Index for a selection of commodities.

Lag	Wheat	Beef	Corn	Hog	Poultry	Gold	Oil
1	0.57	0.58	0.62	0.58	0.64**	0.52	0.53
2	0.54	0.51	0.55	0.51	0.57*	0.46	0.46
3	0.51	0.48	0.52	0.48	0.51	0.53	0.52
4	0.48	0.49	0.48	0.51	0.41**	0.54	0.45
5	0.49	0.48	0.49	0.49	0.42**	0.53	0.52
6	0.45	0.49	0.48	0.51	0.42**	0.49	0.49
7	0.44*	0.46	0.50	0.48	0.41**	0.56	0.49
8	0.48	0.54	0.51	0.46	0.46	0.56	0.48
9	0.51	0.56*	0.49	0.49	0.49	0.48	0.50
10	0.54	0.51	0.54	0.51	0.54	0.49	0.56
11	0.57*	0.48	0.53	0.54	0.57*	0.55	0.52
12	0.53	0.49	0.53	0.62**	0.63**	0.52	0.49
13	0.57*	0.43*	0.50	0.53	0.58*	0.48	0.49
14	0.49	0.42*	0.53	0.43*	0.53	0.47	0.44
15	0.44	0.43*	0.51	0.43*	0.43*	0.51	0.45
16	0.48	0.47	0.48	0.47	0.42**	0.54	0.43*
17	0.44	0.46	0.49	0.44*	0.37**	0.47	0.55
18	0.47	0.52	0.49	0.45	0.37**	0.52	0.51
19	0.51	0.49	0.49	0.48	0.40**	0.51	0.50
20	0.48	0.52	0.51	0.41**	0.41**	0.52	0.60**
21	0.52	0.52	0.52	0.45	0.51	0.49	0.52
22	0.50	0.50	0.60**	0.49	0.54	0.47	0.46
23	0.56*	0.49	0.54	0.51	0.53	0.55	0.48
24	0.55	0.50	0.50	0.52	0.61**	0.56*	0.46
25	0.46	0.51	0.52	0.53	0.55	0.52	0.53
26	0.49	0.56*	0.48	0.46	0.51	0.46	0.45
27	0.46	0.54	0.49	0.44*	0.46	0.50	0.45
28	0.46	0.52	0.45	0.47	0.44*	0.55	0.52
29	0.50	0.48	0.48	0.51	0.44*	0.50	0.50
30	0.41*	0.50	0.51	0.48	0.48	0.48	0.57*
31	0.50	0.51	0.47	0.48	0.47	0.55	0.52
32	0.51	0.48	0.49	0.45	0.48	0.54	0.48
33	0.48	0.49	0.50	0.49	0.51	0.48	0.51
34	0.51	0.52	0.50	0.54	0.57*	0.48	0.44*
35	0.53	0.50	0.52	0.60**	0.59**	0.48	0.52
36	0.53	0.53	0.53	0.61**	0.64**	0.47	0.52

Note: * 5% significance level, ** 1% significance level
For details on data-sources see Appendix A.

This will be evidence for the existence of economic forces underlying the cyclical movements.

The appendix reports the full critical value tables for up to 36 lags. For monthly observations of Wheat, Beef, Corn, Hogs, Poultry, Gold and Oil from 1983:08 to 2008:06 we calculate the index and critical values. The results are reported in table 1 above.

The series with most consistent cyclicality is poultry. For poultry we observe a strong annual phase correlation where the probability of being in the same state of the cycle is significantly higher at the annual lag order. The production cycle for poultry is kept within one year, making seasonal patterns dominate any long run out of seasonal effects. The effect further persists back to the three year lag. For hogs the same consistent annual cycle vanishes. We have a clear 12 month correlation but this effect does not persist at the 24 month lag but does emerge again at the 36 month lag. There appear to be both intra seasonal and out of seasonal effects in hogs, with a significant three year cycle. For beef, wheat and corn in addition to gold and oil the persistence in cycles seem weaker. The lack of a regular pattern in the emergence of cycle correlation, such as a clear correlation in 12, 24 and 36 months suggests that cycles are stochastic. A unit root process outside of the zero frequency will give rise to such inconsistencies in patterns, where integer multiples of significant lags are not significant themselves. As such hogs and poultry appear to have less stochastic cycles than for example beef, wheat and corn. Thus the index gives a preliminary indication to which commodities have the most deterministic cycles.

OUT OF SEASONAL UNIT ROOTS

We now turn to testing parametrically for out of seasonal unit roots. The original HEGY test explains how to test for seasonal unit roots in quarterly data. Franses (1991) and Beaulieu & Miron (1993) expanded this test to allow for testing within monthly observations. Our goal in this section is to further generalize this approach to allow for cycle lengths outside of the seasonal span. In effect, we treat the notion of a season as longer than a year, thereby allowing us to test for unit roots outside of the traditional seasonal span.

We define our data as the sequence $\{y_t\}_{t=0}^{T}$. The data is assumed generated by the autoregressive process:

$$\varphi(L)y_t = \varepsilon_t \ . \tag{2}$$

Here $\varphi(L)$ is a polynomial in the backshift operator L and ε_t a white noise process. We assume that deterministic trends are absent from equation (2). Equation (2) allows up to n unit roots where some or all might be complex. A process with n unit roots can further be written $(1 - L^n)y_t = \varepsilon_t$, that is with n unit roots. If in monthly data we are interested in stochastic cycles up to three years, the maximum amount of unit roots n would be equal to 36+1, where the one is due to the possible unit root at the zero frequency. In the complex plane a particular root can be expressed by the complex polar representation as:

$$e^{\alpha i} = \cos(\alpha) \pm i\sin(\alpha). \tag{3}$$

The minus sign accounts for polar opposites, which is related to the aliasing of the series. A specific root can further be identified by frequency $\alpha = 2\pi j / n, \quad j = 1, \ldots, n-1$. Our interest is to examine whether the series y_t has unit roots at any of the zero or lower frequencies, that is weather applying a filter assuming unit roots at the specific frequencies is a valid representation of the data generating process. HEGY achieves this by linearizing the polynomial $\varphi(L)$ so to isolate specific frequencies, allowing testing of each complex root. Further a unit root process generated by $(1 - L^n)y_t = \varepsilon_t$ can be factored as:

$$\left[\prod_{j=1}^{n-1}(1 + 2\cos\left(\frac{2\pi j}{n}\right)L + L^2)\right]y_t = \varepsilon_t \ . \tag{4}$$

We wish to examine whether the data y_t has unit roots at any of the zero or lower frequencies. For n unit roots the specific frequencies of the roots are:

$$\left[\begin{matrix}\frac{0\times 2\pi}{n}, \frac{1\times 2\pi}{n}, \frac{2\times 2\pi}{n}, \ldots, \frac{(n-1)\times 2\pi}{n}, \pi, \frac{(n+1)\times 2\pi}{n}, \ldots, \\ \frac{(2n-2)\times 2\pi}{n}, \frac{(2n-1)\times 2\pi}{n}\end{matrix}\right].$$

Where frequencies correspond to cycles in the specifically defined seasonal span *n* as:

$$\left[0, n, \frac{n}{2}, ..., \frac{(n-1)}{n}, 1, \frac{n}{(n+1)}, ..., \frac{n}{2n-2}, \frac{n}{2n-1}\right]$$

If the ordinary seasonal span is equal to *s* (for example *s=12* for monthly observations) we are interested in unit roots generating cycles longer than *s.* Note that due to the aliasing problem; that a cyclical pattern can be fitted by more than one unique cycle, we can only test the presence of unit roots in frequencies by pair wise comparison (other than for the unique frequencies at zero and π). From the polar representations $e^{\alpha i} = \cos(\alpha) \pm i\sin(\alpha)$ a cycle with frequency $\pi/6$ can be fitted by its polar opposite $11\pi/6$ since $\cos(\pi/6) = \cos(11\pi/6)$. This will enable us to specifically distinguish out of seasonal from inter seasonal unit roots. As such the rejection of the null hypothesis of no unit root at the frequency indicates no out of seasonal unit roots, however the non-rejection suggests the possibility of out of seasonal unit roots at the frequency.

Equation (3) shows how each unit root is associated with a specific cyclical frequency. Beaulieu & Miron (1993) further show how a filter in the backshift operator can be applied to the series so as to isolate the specific cyclical frequencies. Applying the fact that each frequency has a corresponding polar opposite the specific filters can, for *n* seasonal unit roots written, be written as:

$$y_{nAt} = \sum_{j=1}^{N} \cos\left(\frac{2\pi \times n}{N}\right) L^{j-1} \qquad y_{nBt} = \sum_{j=1}^{N} \sin\left(\frac{2\pi \times n}{N}\right) L^{j-1} \quad \text{for} \quad \frac{2\pi \times n}{N} < \frac{\pi}{2}$$

$$y_{nAt} = \sum_{j=1}^{N} \cos\left(\frac{2\pi \times n}{N}\right) L^{j-1} \qquad y_{nBt} = -\sum_{j=1}^{N} \sin\left(\frac{2\pi \times n}{N}\right) L^{j-1} \quad \text{for} \quad \frac{2\pi \times n}{N} > \frac{\pi}{2}$$

$n = 1,..,N/2$ and N the maximum unit roots. The filter associated with the B subscript corresponds to the polar opposite of the specific frequency. We use the negative sign to account for the negative sign emerging when the frequency starts in the negative quadrant of the unit circle, where the cosine value is negative. The equation to be estimated can then be written as:

$$\varphi(L)^* y_{N+1,t} = \sum_{n=1}^{N/2} (\pi_{nA} y_{nAt} + \pi_{nB} y_{nBt}) + \mu + \alpha t + \varepsilon_t \tag{5}$$

The polynomial $\varphi(L)^*$ is a remainder with roots outside of the unit circle. The parts following the unit root filters is a part accounting for intercept and trend, note that this part also allows for seasonal dummy effects. Equation (5) is further estimated by Ordinary Least Squares. For frequency zero and π we compare $\pi_{nA} = 0$ against $\pi_{nA} < 0$ since the polar opposites vanish for these values. For the other frequencies we test $\pi_{nA} = 0$ with a two sided test. If we cannot reject $\pi_{nA} = 0$ we need to test $\pi_{nB} = 0$ versus $\pi_{nB} < 0$. Under unit-root the coefficient values are zero. Further if $\pi_{nA} \neq 0$ for frequency π and for at least one of the sets $\{\pi_{nA}, \pi_{nB}\}$, no unit roots outside of the zero frequency unit root exists. Note also that in testing a specific filter, for example the $(1-L^{36})$ filter, we are able to test filters which are factors of $(1-L^{36})$. In this specific case we are also able to test the validity of $(1-L)$, $(1-L^2)$, $(1-L^3)$, $(1-L^4)$, $(1-L^6)$, $(1-L^9)$, $(1-L^{12})$ and $(1-L^{18})$ as the frequencies associated with these filters are subsets of the higher order filter $(1-L^{36})$. We thus note that the seasonal filter from Beaulieu & Miron (1993), namely $(1-L^{12})$, becomes a special case of the general test. Increasing the number of possible unit roots would increase the number of possible subset filters however at the cost of reducing degrees of freedom.

The distribution of the t-statistics for frequencies zero and π is equal to the Dickey & Fuller(1979) distribution. Further Beaulieu & Miron(1993) proves that the t-distribution for π_{nA} and π_{nB} is equal to that in Hylleberg, Engle, Granger & Yoo(1990) and as such is assumed indifferent of initially amount of unit roots. For the testing procedure we simulated critical values in order to replicate the critical values in Beaulieu & Miron (1993) and Hylleberg, Engle, Granger & Yoo (1990). This is done for six unit roots in a series of annual observations and 36 unit roots in a series of monthly observations. These values complement the tables generated by Beaulieu & Miron (1993) and Hylleberg, Engle, Granger & Yoo (1990) and the values appear consistent with the traditional seasonal test values.

We will apply the above approach to a maximum of 6 unit roots (a maximum six year cycle) for annual observations of corn wheat, beef, corn, hogs, eggs, gold and oil. We also apply a 36 unit roots (a maximum three year cycle) for monthly observations of wheat, beef, corn, hog, poultry, gold and oil.

UNIT ROOT ESTIMATION RESULTS AND DISCUSSION

Before we continue to the unit root estimation we provide some descriptive statistics of our data (table 2). We also perform Augmented Dickey Fuller tests with intercept and trend for the commodities and series used.

TABLE 2. Descriptive Statistics and Augmented Dickey Fuller test for Unit Root at Zero Frequency.

	Annual						
	Wheat	Beef	Corn	Hogs	Eggs	Gold	Oil
Data Range	1909: 2007	1910: 2007	1909: 2008	1910: 2008	1909: 2006	1901: 2007	1901: 2007
Mean	2.10	32.745	1.51	24.08	0.42	127.76	9.1346
% Dev. From	54.63	81.50	55.60	66.17	41.23	152.22	125.20
Skewness	0.44	0.70117	0.49	0.51	0.24	1.2633	2.2178
Kurtosis	-0.90	-0.97448	-0.72	-1.23	-1.07	0.20652	5.4761
Δy_t Lag Order :	3	2	3	2	2	4	7
βy_{t-1} : ADF Constant	0.96	1.00	0.94	0.99	0.95	0.99	1.04
Δy_t Lag Order	2	1	1	2	10	4	5
βy_{t-1} : ADF Constant + Trend	0.72*	0.88	0.69*	0.82	0.54*	0.89	0.90
	Monthly						
	Wheat	Beef	Corn	Hogs	Poultry	Gold	Oil
Data Range	83:01-08:06	83:01-08:06	83:01-08:06	83:01-08:06	83:01-08:06	83:01-08:06	83:01-08:06
Mean	155.03	102.81	113.03	74.50	56.93	394.07	28.496
% Dev. From	31.57	14.52	26.71	35.30	21.41	68.98	21.41
Skewness	2.88	-0.22	2.1251	1.01	0.15	2.344	2.3968
Kurtosis	10.93	-1.21	6.7115	0.65	-0.83	6.3864	6.4079
Δy_t Lag Order :	7	2	8	12	7	5	11
βy_{t-1} : ADF Constant	1.00	0.97	1.02	0.93	1.00	1.02	1.03
Δy_t Lag Order	7	2	8	12	12	5	2
βy_{t-1} : ADF Constant + Trend	0.99	0.97	1.01	0.89*	0.89**	1.02	1.03

Note: * Reject unit root at 5% significance level, ** Reject unit root at 1% significance level. For details on data-sources we refer to Appendix A.

In determining lag length for the zero frequency unit root test apply the Akaike Information Criteria.

For the annual data the ADF test with a constant fails to reject the null of a unit root for all six series. However, when including a trend the null of a unit root is rejected for three of the series at a 5% level. It appears that the drift term can explain some of the high autocorrelation for the annual data, and wheat, corn and eggs appear trend stationary in annual observations. Turning to the monthly data we cannot reject a unit root at the 5% significance level for any series using only a constant. The drift term in the monthly data are less able to explain the high autocorrelation and including a trend a unit root at the zero frequency can only be rejected for hogs and poultry.

We now turn to the application of the unit root test for annual data. We allow up to six unit roots outside of the zero frequency, where these unit roots relate to cycles of length 2 years, 6 years, 1.2 years, 3 years and 1.5 years). The test results are reported in table 3 and models include both a constant and a trend term. Lags are again chosen from the Akaike Information Criteria. As with the ADF tests, these tests are also us unable to reject the zero frequency unit root π_1 for all commodities with only a constant. For the trend case the critical values are closer to rejection but we can still not reject unit root at the zero frequency. Moving to the two year long cycle π_2 we can reject a unit root for all series except beef and corn, and this result is carried over for the trend model.

Further for wheat and eggs we can reject at least one of the F-tests such as no unit roots appear present in these series outside of the zero frequency. For hogs we cannot reject the null of unit roots associated with cycles longer than two years. For beef we reject all frequencies except the zero and Nyquist frequency.

For corn the frequency associated with a 6 and 1.2 year cycle cannot reject a unit root. Gold and oil appear dominated by the zero frequency, an intuitive result since these commodities are dominated by speculative behaviour forcing the spectrum of the series to fall steeply after the zero frequency.

As stated above, we can test downwards for the appropriate differencing filter. The six year test allows us to test for the filters

$(1-L)$, $(1-L^2)$ and $(1-L^3)$. We do this by testing the unit root at the shortest cycle and moving upwards if we cannot reject unit roots, we stop if all cycles of a given length cannot be reject while all above can be rejected. Doing this gold and oil are characterised by the $(1-L)$ filter, beef by the $(1-L^2)$ filter, allowing up to a two year cycle as generated by a unit root process, and eggs by the $(1-L^3)$ filter, associated with a longest unit root cycle of 3 years. For wheat, corn and hogs the appropriate the filter cannot be found from this specific test. We can reject the zero frequency unit roots for wheat, but not for corn and hogs.

TABLE 3. Results from Unit root test with six unit roots in Annual Data.

	Constant									
			0	π	$\frac{\pi}{3}$		$\frac{2\pi}{3}$		$\frac{\pi}{3}$	$\frac{2\pi}{3}$
	Period	Lags	π_1	π_2 (2y)	π_3 (6y)	π_4 (1,2y)	π_5 (3y)	π_6 (1.5y)	$F_{3,4}$	$F_{5,6}$
Wheat	1909:2007	8	0.6	-2.41*	-0.54	-2.13*	-0.97	2.98**	2.43	4.94**
Beef	1909:2008	6	0.67	-0.53	-1.71	-2.8*	-1.95*	2.43**	5.73**	5.21**
Corn	1909:2008	8	0.09	-1.3	-1	-1.72*	-2.94**	2.71**	2	7.65**
Hogs	1910:2008	12	-0.01	-3.38**	-2.08*	-0.93	-0.98	1.15	2.71	1.17
Eggs	1909:2006	12	-0.55	-2.42*	-1.43	-0.88	-2.8**	1.63*	1.44	5.63**
Gold	1901:2007	8	0.63	-2.03*	-0.35	-3.57**	-2.04**	2.43**	6.42**	5.04**
Oil	1901:2007	9	1.9	-2.81**	-3.49**	-1.98	-1.69	1.69	8.82**	3.01**

	Constant and Trend									
			0	π	$\frac{\pi}{3}$		$\frac{2\pi}{3}$		$\frac{\pi}{3}$	$\frac{2\pi}{3}$
	Period	Lags	π_1	π_2 (2y)	π_3 (6y)	π_4 (1,2y)	π_5 (3y)	π_6 (1.5y)	$F_{3,4}$	$F_{5,6}$
Wheat	1909:2007	8	-2.2	-2.48**	-0.53	-1.5	-0.99	3.05**	1.28	5.17**
Beef	1909:2008	6	-1.64	-0.53	-1.96*	-1.96*	-2.06*	2.45**	6.05**	5.51**
Corn	1909:2008	8	-2.61	-1.38	-0.91	-1.02	-2.79**	2.86**	0.94	7.64**
Hogs	1910:2008	12	-2.5	-3.52**	-2.39**	-0.87	-1.09	1.11	3.36	1.24
Eggs	1909:2006	12	-2.45	-2.48**	-1.63	-0.86	-2.97**	1.58*	1.72	6.06**
Gold	1901:2007	8	-1.21	-2.03*	-0.43	-3.43**	-2.04*	2.42**	5.96**	5.01**
Oil	1901:2007	9	0.33	-2.79**	-3.42**	-1.91	-1.73	1.7	8.33**	3.11**

Note: Significance at 5% level, **Significance at 1% level

Having examined annual data we move to the monthly test where we allow for thirty six unit roots. This is applied to observations of wheat, beef, corn, hog, poultry, gold and oil from 1983:08 to 2008:06. Due to space restraints we only report the two first unique frequencies along with the F-tests in the table below, the complete statistics is reported in the appendix. The models include both a constant, constant and trend and seasonal dummy. In the analysis we will refer to the F-test for all frequencies except the zero and Nyquist frequency as reported in the table below.

What primarily distinguishes the monthly data from the annual data is the tendency to reject unit roots at the zero frequency for all series. For wheat we reject unit root at the 5% for the zero frequency, except for the trend case. The results here are ambiguous as we are close to the rejection values. For the Nyquist frequency, the frequency associated with a 2 month cycle, we reject unit root only with the seasonal dummy. Including deterministic dummies have an effect, but make rejection less likely and thus support that both deterministic and stochastic cyclical effects are present.

Further the intra-seasonal frequencies appear important in wheat as we cannot reject unit root for any of the models, except for the seasonal dummy effect appearing to account for the annual cycle. Further the frequency related to the three year cycle cannot be rejected. Beef appear to be the commodity with most stochastic effects in cycles. We fail to reject both the zero and Nyquist frequency in addition to the frequency associated with the three year cycle. Again unit roots at the seasonal frequencies seem more relevant, however not to the extent of wheat. For beef the seasonal associated with a three month cycle can be explained by the seasonal deterministic dummy. For corn the results are more ambiguous, with a rejection of the zero frequency for all cases except the trend case.

The Nyquist frequency is rejected at a 5% significance level in all cases with the exception of the seasonal dummy case. The seasonal dummies appear less powerful to explain intra seasonal cycles in corn in respect to wheat; however the three year cycle frequency is again significant. Still, the rejection of the Nyquist frequency cast doubt to the existence of lower frequency unit roots. Hogs have the strongest rejection of a unit root at the zero frequency, while no rejection can be found at the Nyquist frequency.

TABLE 4. Results from Unit Root Test with 36 possible Unit Toots in Monthly Data.

	Frequency	0	π (s)	$\frac{\pi}{2}$ (s)	$\frac{5\pi}{9}$	$\frac{4\pi}{9}$	$\frac{11\pi}{8}$	$\frac{7\pi}{18}$	$\frac{2\pi}{3}$ (s)	$\frac{\pi}{3}$ (s)	$\frac{13\pi}{18}$	$\frac{5\pi}{18}$	$\frac{7\pi}{9}$	$\frac{2\pi}{9}$	$\frac{5\pi}{6}$ (s)	$\frac{\pi}{6}$ (s)	$\frac{8\pi}{9}$	$\frac{\pi}{9}$	$\frac{17\pi}{18}$	$\frac{\pi}{18}$
	Cycle Length			4m 1.33m	3.6m 1.38m	4.5m 1.28m	3.27m 1.44m	5.14m 1.24m	3m 1.5m	6m 1.2m	2.77m 1.56m	7.2m 1.16m	2.57m 1.63m	9m 1.125m	2.4m 1.71m	12m 1.1m	2.25m 1.8m	18m 1.06m	2.11m 1.9m	36m 1.03m
		π_1	π_2	$F_{3,4}$	$F_{5,6}$	$F_{7,8}$	$F_{9,10}$	$F_{11,12}$	$F_{13,14}$	$F_{15,16}$	$F_{17,18}$	$F_{19,20}$	$F_{21,22}$	$F_{23,24}$	$F_{25,26}$	$F_{27,28}$	$F_{29,30}$	$F_{31,32}$	$F_{33,34}$	$F_{35,36}$
Wheat	C	-2.73**	_-2.15*_	3.01*	5.58**	6.67**	3.29*	0.34	_1.42_	_1.99_	1.05	4.83**	6.92**	7**	_0.75_	_2.47_	6.1**	4.54**	4.51**	1.98
	C+T	-2.91	_-2.14*_	2.99*	5.56**	6.64**	3.27*	0.34	_1.42_	_1.98_	1.05	4.79**	6.91**	6.93**	_0.75_	_2.48_	6.09	4.47**	4.5**	1.64
	C + SD	-2.64	_-1.96_	3.42	5.82*	6.1*	3.53	0.28	_1.33_	_1.32_	1.06	4.71*	6.26*	7.25**	_1.76_	_5.01*_	5.62*	4.54*	4.23*	2.1
Beef	C	-1.76	_-1.86*_	2.17	1.53	3.41*	3.86*	2.32	_1.07_	_2.98*_	4.45**	5.07**	4.47**	1.87	_6.26**_	_4.24**_	5.92**	10.04**	6.81**	0.72
	C+T	-1.73	_-1.85*_	2.16	1.52	3.4*	3.84*	2.32	_1.07_	_2.98*_	4.42**	5.05**	4.44**	1.87	_6.21**_	_4.19*_	5.89**	9.98**	6.77**	0.73
	C + SD	-1.71	_-2.15_	2.47	1.45	3.08	4.23	2.43	_4.51*_	_2.94_	3.74	4.93*	4.38	1.83	_7.13**_	_4.33*_	5.61*	9.12**	6.64**	0.67
Corn	C	-2.99**	_-2.36*_	2.69	7.88**	5.16**	0.53	5.29**	_5.26**_	_5.24**_	2.64	7.26**	2.7	7.66**	_3.2*_	_4.67**_	5.91**	7.26**	6.31**	1.89
	C+T	-3.06	_-2.35*_	2.69	7.85**	5.14**	0.53	5.26**	_5.23**_	_5.19**_	2.62	7.22**	2.67	7.6**	_3.17*_	_4.72**_	5.86**	7.19**	6.25**	1.73
	C + SD	-2.84*	_-2.53_	1.11	8.57**	2.93	1.07	3.67	_3.66_	_2.81_	1.92	5.11*	1.97	5.23*	_4.28_	_5.01*_	7.06**	9.11**	5.88*	2.44
Hog	C	-4.13**	_-1.67_	7.43**	6.78**	13.23**	8.63**	7.51**	_2.73_	_11.06**_	5.12**	17.91**	5.9**	6.61**	_1.17_	_5.03**_	4.84**	15.14**	6.68**	3.07*
	C+T	-3.52*	_-1.66_	7.41**	6.79**	13.13**	8.65**	7.49**	_2.73_	_10.9**_	5.13**	17.55**	5.89**	6.51**	_1.17_	_4.98**_	4.81**	14.97**	6.62**	2.62
	C + SD	-3.92**	_-1.56_	8.87**	6.21*	14.23**	8.22**	8.17**	_4.51*_	_13.62**_	4.38*	20.45**	5.23*	8.45**	_1.4_	_12.13**_	4.43*	18.25**	6.59*	3
Poultry	C	-2.9**	_-0.97_	6.41**	7.36**	11.06**	6.07**	4.24*	_3.94*_	_1.91_	7.68**	16.74**	7.22**	8.67**	_6.62**_	_2.48_	7.41**	8.11**	5.93**	1.6
	C+T	-3.92**	_-0.99_	6.56**	7.23**	11**	6.16**	4.15*	_4.06*_	_1.9_	7.78**	16.26**	7.43**	7.92**	_6.75**_	_2.61_	7.55**	7.72**	6.06**	1.29
	C + SD	-3.06*	_-1.93_	10.73**	7.67**	13.82**	7.17**	4.93*	_8.87**_	_7.63**_	7.28**	16.62**	8.03**	9.49**	_7.2**_	_1.76_	7.53**	9.05**	6.76**	1.97
Gold	C	0.38	_-0.47_	9.11**	4.15**	6.35**	7.88**	2.1	_11.03**_	_5.49**_	6.69**	5.71**	3.58*	6.54**	_7.02**_	_2.5_	0.83	7.27**	7.6**	10.47**
	C+T	1.02	_-0.37_	9.62**	4.52**	6.64**	8.11**	2.24	_11.08**_	_5.9**_	6.66**	6.25**	3.49*	7.16**	_6.84**_	_2.78_	0.85	7.99**	7.36**	12.77**
	C + SD	0.36	_-0.62_	9.05**	3.89	6.27**	7.41**	2.09	_13.06**_	_5.55*_	7.06**	5.59*	3.63	6.68**	_7.49**_	_3.3_	0.6	7.03**	7.01**	9.84**
Oil	C	1.11	_-2.37*_	5.16**	1.57	10.2**	7.19**	3.93*	_5.27**_	_5.24**_	3.83*	7.43**	11.46**	3.93*	_6.2**_	_2.89*_	8.69**	10.18**	12.52**	2.01
	C+T	0.98	_-2.37_	5.27**	1.6	10.32**	7.22**	4*	_5.31**_	_5.22**_	3.86*	7.4**	11.45**	3.99*	_6.23*_	_2.7_	8.71	10.22**	12.67**	1.84
	C + SD	0.91	_-2.64_	5.75**	1.81	10.61**	7.05**	4.32	_5.63**_	_6.39**_	3.75	8.06**	11.66**	4.27	_7**_	_4.02_	7.34**	10.95**	11.52**	2.18

Note: C : Constant, T : Trend, SD: Seasonal Dummy. The parts marked in italics are seasonal unit roots where cycles are completed within seasonal span.

For lower frequencies, we find that most frequencies can be rejected except for the three year frequency and the seasonal frequency associated with the 2.4 month seasonal cycle. Note that the power to reject seasonal frequencies increase significantly for the model including seasonal effects.

For poultry we reject the zero frequency and not the Nyquist frequency. Lower order frequencies mostly rejects unit roots and all seasonal cycles expect for the frequency associated with the twelve month cycle appear explainable by the seasonal dummies, this result is in line with the results for the index where clear intra-seasonal cycles appears. The frequencies associated with the twelve month and three year cycle lack rejection of the unit root hypothesis. Gold and oil are the commodities less likely to reject the unit root hypothesis at the zero frequency. However, for gold we cannot reject the Nyquist frequency. For seasonal frequencies the only one we cannot reject unit roots is for the 12 month cycle. Gold is further the only commodity we can reject the frequency associated with the three year cycle. It appears that the seasonal frequencies dominate for most goods but that the unit root associated with frequency of a 3 year cycle cannot be rejected, except for gold. The goods where stochastic cycles appear most predominant are for beef, hog and poultry. This result is in line with the hypothesis that the dynamics in animal production, the usage of outputs as inputs in a stochastic production process can generate cyclical patterns in prices. These dynamics often span outside of the traditional seasonal lengths associated with for example plant production and appear highly stochastic as we cannot reject unit roots outside of the zero frequency.

We also turn to testing downwards in order to find an appropriate filter associated with the factors of the 36 unit root test. For corn, hog and poultry we reject unit root at the zero frequency, the lowest order simple filter $(1-L)$, thus no simple filter fits these goods since the zero frequency is the common denominator for the all higher order filters. Further for the $(1-L^2)$ filter only the zero and Nyquist frequency must be unit root, however this is not satisfied in any of the remaining series such that from our testing procedure no lower order appropriate filter can be found. Oil is the closest to be described by a $(1-L)$ filter while gold is closest to the $(1-L^2)$ filter. Other filters which are not a factor of 36 could be relevant but this has not been pursued here. For example, for all integer filters inside the seasonal span one could create enough high order filters so that all factors is an integer between zero and twelve. For our test beef appear the series with most frequencies not rejecting unit root, but also for soybeans, poultry and hogs we cannot reject unit root at other than the zero frequency although no general lower order filter seem appropriate. Also the seasonal frequencies seem important for most series. The failure to reject a unit root at other frequencies, and in particular the frequencies associated with a three year cycle, implies that we cannot reject the hypothesis that highly stochastic long run dynamics exist. These dynamics can account for stochastic cyclical patterns.

Looking at the cycle phase correlation index together with the unit root tests, the results indicate that the cycle phases are not independent of their duration, that some force exist which reverts cycles in a consistent pattern, and further that seasonal patterns are important. However, looking at the three commodities with the lowest rejections of unit roots outside of the zero frequency, beef, hog and poultry, we observe that these are the commodities with the most significant cycle phase correlation. Furthermore, in the unit root test, beef appear to be the commodity with the most stochastic cyclical behaviour. This is in line with the cycle phase correlation index where little consistency in regular repetition of significant correlations in cycle phases indicates a stochastic

cyclical pattern. Poultry, in the cycle correlation index, appear to have a regular repetition in emergence of correlations and further have a high degree of unit root rejection for frequencies other than the Nyquist frequency. As such for poultry deterministic cyclical behaviour inside the seasonal spans seem more important than for beef where longer run stochastic cycles appear more prevalent. The unit root tests illustrate the importance of the seasonal patterns and deterministic mean shifting effects at seasonal frequencies. Despite the results from the cycle correlation index that economic forces seem fundamental in determining cyclical behaviour, a lack of regularity in emergence of correlations suggests that previous cyclical behaviour cannot predict future cyclical behaviour, and as such, that cycles are highly stochastic, especially for the prices of animal products. Hence, cycles can be identified after their duration but complex long run dynamics appear to exist such that the prediction of cycle lengths and amplitudes during its unfolding seems harder. This supports the hypothesis that long run cycles can persist due to their inherent randomness. Further, as the oil and gold commodities illustrate, strong speculative forces are important in tying prices together over time and as such forcing the mass of the spectrum to the zero frequency. In addition, the underlying mechanics of the production process where not only current production decisions but also sales or harvesting decisions affect future stocks is important in generating potentially long run dynamics in price connections. If, in addition, production is stochastic and storage possibilities limited, the cycle will appears highly stochastic and can explain the emergence and persistence of apparently predictable and profitable long run cycles in prices.

CONCLUSION

In this paper we pursue the hypothesis that long run cycles in commodities can persist due to their inherent randomness leading to a high cost of identifying cycles and low precision in forecasting future cycle lengths and amplitudes. We examine the stochasticity of the long run dynamics by examining the most stochastic cycle, the ones generated by unit root processes. Two test approaches are derived. First, we derived a non-parametric index which tests for no-duration dependence in the cycles. No duration dependence is associated with cycle phases independent of phase length. This is consistent with cycles generated by unit root processes where the Markov property dictates that price moves only according to present level. We generate this index for monthly observations of wheat, beef, corn, poultry, hog, gold and oil to test the hypothesis. Our tests suggests that we can reject no-duration dependence for all commodities, that there apparently is some force dictating the cycle properties but that the regularity in emergence of cycle phase correlation is weak, except for the case of poultry. The index helps our analysis by in essence telling us that no, cycles are generally not completely independent of their history, but this dependence is irregular and that during a time series the cycle properties might change.

Our second approach is parametric and consists of formally stating cycle frequencies which we test for unit roots. The test is an expansion of the classical seasonal unit root test where we allow for cycle lengths greater than the seasonal span. From the generalisation we derive two specific tests, one for six unit roots and one for thirty six unit roots. The six unit root test is applied to annual observations of wheat, beef, corn, eggs, hogs, gold and oil allowing for a cycle length of maximum six years. We find that gold and oil are characterised by the $(1-L)$ filter, beef by the $(1-L^2)$

filter, allowing up to a two year cycle as generated by a unit root process, and eggs by the $(1-L^3)$ filter, associated with a longest unit root cycle of 3 years. For wheat, corn and hogs the appropriate the filter cannot be found from this specific test. We can reject the zero frequency unit roots for wheat, but not for corn and hogs.

The thirty six unit root test is applied to monthly observations of wheat, beef, corn, poultry, hog, gold and oil allowing a maximum of a three year cycle length. We find that for oil and gold the zero frequency unit root is most predominant. This is consistent with the knowledge that these commodity markets are more dominated by speculative forces and less restricted by seasonal productive restrictions. Seasonal frequencies dominate for most goods, but the unit root associated with the frequency of a 3 year cycle cannot be rejected, except for gold. The goods where stochastic cycles appear most predominant are for beef, hog and poultry. This result is in line with the hypothesis that the dynamics in animal production, the usage of outputs as inputs in a stochastic production process can generate long cyclical patterns in prices. For poultry the stochastic effects seem less dominant and intra-seasonal deterministic regular cycles seem important. Beef appear to be the commodity with most stochastic cycles. This result is also in line with the cycle phase correlation index where poultry has a clearer regularity in emergence of cycle phase correlation, indicating more

deterministic cycle behaviour. We conclude that fundamental production properties, such as the impact of future production on current sales decisions are important in generating cycles and coupled with at stochastic production process and low inventory flexibility can generate cycles which are severely stochastic and as such that cycles are allowed to persist due to their randomness and are only clearly identifiable after their unfolding.

Appendix A. Data description:

Monthly Series

	Source	Description
Wheat	www.indexmundi.com	No.1 Hard Red Winter, FOB Gulf of Mexico, US$ per metric tonnes.
Beef	www.indexmundi.com	Australian and New Zealand 85% lean fores, FOB U.S. import price, US cents per pound.
Corn	www.indexmundi.com	U.S. No.2 Yellow, FOB Gulf of Mexico, U.S. price, US$ per metric tonnes.
Hog	www.indexmundi.com	Swine (pork), 51-52% lean Hogs, U.S. price, US cents per pound.
Poultry	www.indexmundi.com	Poultry (chicken), Whole bird spot price, Georgia docks, US cents per pound.
Gold	Global Insight	Per Ounce, US$
Oil	www.indexmundi.com	Crude Oil (petroleum); Dated Brent, US$ per barrel.

Annual Series

Wheat	USDA-NASS	USD/bu
Beef	USDA-NASS	Prices Received By Farmers, US$ per Cwt.
Corn	USDA-NASS	USD/bu
Hogs	Historic and Quick Stats	
Eggs	Historic and Quick Stats	
Gold	Global Insight	Per Ounce, US$
Oil	Historic and Quick Stats	US$ per barrel

APPENDIX B.

TABLE B1. Critical values from Distributions of Test Statistics for 6 Unit Roots in Annual Data, 24000 simulations, data generating process $\Delta_6 y = \varepsilon_t \quad nid(0,1)$.

Auxilliary Regressions	T	Fractiles $t:\pi_1$ 0.01	0.05	0.1	$t:\pi_2$ 0.01	0.05	0.1	$t:\pi_{ODD}$ 0.01	0.05	0.1
No intercept	50	-2.50	-1.84	-1.51	-2.50	-1.84	-1.52	-2.49	-1.83	-1.49
No trend	100	-2.54	-1.88	-1.56	-2.57	-1.92	-1.58	-2.52	-1.88	-1.54
	1k	-2.54	-1.92	-1.59	-2.54	-1.92	-1.59	-2.54	-1.89	-1.55
Intercept	50	-3.38	-2.74	-2.44	-2.49	-1.84	-1.51	-2.47	-1.81	-1.48
No trend	100	-3.39	-2.81	-2.50	-2.52	-1.90	-1.57	-2.53	-1.87	-1.53
	1k	-3.45	-2.85	-2.54	-2.53	-1.90	-1.59	-2.52	-1.90	-1.56
Intercept	50	-3.95	-3.28	-2.97	-2.47	-1.83	-1.51	-2.47	-1.81	-1.48
Trend	100	-3.89	-3.33	-3.05	-2.51	-1.89	-1.56	-2.50	-1.87	-1.55
	1k	-3.95	-3.38	-3.08	-2.55	-1.92	-1.58	-2.52	-1.89	-1.55

Auxilliary Regressions	T	Fractiles $t:\pi_{EVEN}$ 0.01	0.05	0.10	0.90	0.95	0.99	$F:\pi_{ODD,EVEN}$ 0.90	0.95	0.99
No intercept	50	-2.30	-1.60	-1.24	1.24	1.62	2.31	2.33	3.03	4.77
No trend	100	-2.35	-1.62	-1.25	1.25	1.61	2.31	2.37	3.09	4.76
	1k	-2.30	-1.64	-1.28	1.27	1.63	2.31	2.36	3.08	4.81
Intercept	50	-2.26	-1.59	-1.23	1.23	1.59	2.29	2.30	3.02	4.71
No trend	100	-2.32	-1.62	-1.26	1.25	1.62	2.28	2.34	3.06	4.70
	1k	-2.30	-1.63	-1.27	1.25	1.60	2.28	2.36	3.04	4.65
Intercept	50	-2.24	-1.58	-1.22	1.19	1.54	2.23	2.19	2.89	4.55
Trend	100	-2.26	-1.59	-1.24	1.23	1.59	2.28	2.31	2.97	4.58
	1k	-2.29	-1.62	-1.26	1.24	1.59	2.26	2.32	3.03	4.71

TABLE B2. Coefficient Standard Deviations from simulation.

Auxilliary Regressions	T	y1	y2	y3	y4	y5	y6
No intercept	50	0.045	0.044	0.045	0.036	0.044	0.035
No trend	100	0.025	0.026	0.027	0.021	0.027	0.021
	1000	0.014	0.014	0.015	0.012	0.015	0.012
Intercept	50	0.080	0.044	0.044	0.035	0.044	0.035
No trend	100	0.043	0.025	0.026	0.020	0.026	0.021
	1000	0.022	0.014	0.015	0.011	0.014	0.011
Intercept	50	0.103	0.042	0.043	0.034	0.042	0.033
Trend	100	0.055	0.024	0.025	0.020	0.026	0.020
	1000	0.029	0.014	0.015	0.011	0.014	0.011

TABLE B3. Critical values from Distributions of Test Statistics for 36 Unit Roots in Monthly Data, 24000 simulations, data generating process $\Delta_{36} y = \varepsilon_t \quad nid(0,1)$.

Auxilliary Regressions	T	$t:\pi_1$ 0.01	0.05	0.1	$t:\pi_2$ 0.01	0.05	0.1	$t:\pi_{ODD}$ 0.01	0.05	0.1
No Intercept	200	-2.32	-1.74	-1.44	-2.33	-1.75	-1.46	-2.29	-1.69	-1.39
No Seasonal	400	-2.43	-1.84	-1.51	-2.41	-1.84	-1.53	-2.42	-1.81	-1.49
No Trend	1k	-2.48	-1.86	-1.55	-2.49	-1.88	-1.57	-2.46	-1.85	-1.53
Intercept	200	-3.16	-2.62	-2.35	-2.33	-1.75	-1.45	-2.31	-1.73	-1.42
No Seasonal	400	-3.27	-2.74	-2.44	-2.45	-1.84	-1.52	-2.42	-1.81	-1.50
No Trend	1k	-3.35	-2.80	-2.51	-2.49	-1.89	-1.56	-2.49	-1.87	-1.55
Intercept	200	-3.10	-2.56	-2.29	-3.06	-2.54	-2.26	-3.14	-2.50	-2.13
Seasonal	400	-3.23	-2.70	-2.42	-3.24	-2.69	-2.42	-3.32	-2.65	-2.28
No Trend	1k	-3.36	-2.81	-2.52	-3.34	-2.82	-2.52	-3.41	-2.73	-2.34
Intercept	200	-3.67	-3.14	-2.87	-2.30	-1.75	-1.44	-2.31	-1.72	-1.41
No Seasonal	400	-3.78	-3.23	-2.96	-2.43	-1.84	-1.53	-2.42	-1.82	-1.50
Trend	1k	-3.89	-3.33	-3.05	-2.53	-1.90	-1.59	-2.50	-1.87	-1.55
Intercept	200	-3.09	-2.54	-2.28	-3.13	-2.57	-2.28	-3.13	-2.50	-2.13
Seasonal	400	-3.70	-2.94	-2.51	-3.24	-2.70	-2.42	-3.29	-2.63	-2.25
Trend	1k	-3.88	-3.33	-3.06	-3.35	-2.79	-2.50	-3.41	-2.72	-2.33

Auxilliary Regression	T	$F:\pi_{ODD,EVEN}$ 0.01	0.05	0.10	0.90	0.95	0.99	$t:\pi_{EVEN}$ 0.90	0.95	0.99
No Intercept	200	-2.17	-1.52	-1.17	1.17	1.51	2.17	2.08	2.68	4.13
No Seasonal	400	-2.25	-1.59	-1.24	1.23	1.59	2.26	2.23	2.88	4.40
No Trend	1k	-2.28	-1.61	-1.25	1.25	1.60	2.26	2.29	2.95	4.50
Intercept	200	-2.19	-1.55	-1.20	1.19	1.53	2.17	2.06	2.67	4.10
No Seasonal	400	-2.25	-1.59	-1.24	1.23	1.58	2.24	2.22	2.87	4.39
No Trend	1k	-2.29	-1.62	-1.26	1.26	1.62	2.29	2.34	3.02	4.61
Intercept	200	-2.25	-1.58	-1.23	1.21	1.56	2.24	3.11	4.00	5.93
Seasonal	400	-2.36	-1.66	-1.29	1.27	1.64	2.34	3.47	4.45	6.54
No Trend	1k	-2.42	-1.70	-1.32	1.32	1.69	2.40	3.73	4.78	6.98
Intercept	200	-2.19	-1.55	-1.21	1.17	1.51	2.15	2.04	2.65	4.09
No Seasonal	400	-2.25	-1.59	-1.24	1.22	1.57	2.23	2.22	2.86	4.38
Trend	1k	-2.29	-1.62	-1.26	1.25	1.61	2.28	2.33	3.00	4.59
Intercept	200	-2.26	-1.59	-1.24	1.21	1.57	2.24	3.11	4.01	5.91
Seasonal	400	-2.34	-1.65	-1.28	1.27	1.64	2.33	3.47	4.46	6.55
Trend	1k	-2.43	-1.70	-1.33	1.31	1.69	2.40	3.72	4.77	7.00

TABLE B4. Results from Unit Root test with 36 possible Unit Roots in Monthly Data, Constant Included.

			0	π (s)	$\frac{\pi}{2}$ (s)		$\frac{5\pi}{9}$		$\frac{4\pi}{9}$	
	Period	Lags	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8
Wheat	83:08-08:06	2	-2.91	-2.14	0.04	-2.45	-1	3.09	-0.99	-3.49
Beef	83:08-08:06	2	-1.73	-1.85	-0.53	-2.01	-0.93	1.4	-2.44	-0.96
Corn	83:08-08:06	2	-2.99	-2.36	-2.12	-0.84	-1.24	3.64	-2.27	-2.25
Hog	83:08-08:06	3	-3.52	-1.66	-3.27	-2.06	-1.77	3.08	-4.16	-2.92
Poultry	83:08-08:06	2	-3.92	-0.99	0.02	-3.62	-2.14	2.92	-0.35	-4.67
Gold	83:08-08:06	2	0.38	-0.47	-4.07	-1.16	-2.88	-0.11	-3.34	-1.22
Oil	83:08-08:06	1	1.11	-2.37	-2.77	-1.56	-0.75	1.59	-2.57	-3.75

	$\frac{11\pi}{8}$		$\frac{7\pi}{18}$		$\frac{2\pi}{3}$ (s)		$\frac{\pi}{3}$ (s)		$\frac{13\pi}{18}$	
	π_9	π_{10}	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}	π_{16}	π_{17}	π_{18}
Wheat	-2.29	1.12	0.07	-0.82	-1.64	0.42	-1.99	-0.04	0.25	1.43
Beef	-2.21	1.64	-0.53	-2.08	-1.16	0.89	-1.72	-1.7	-2.89	0.73
Corn	-0.54	0.88	0.22	-3.25	-0.95	3.1	0.42	-3.21	-1.82	1.41
Hog	-3.6	2.02	-1.73	-3.43	-1.52	1.74	-4.09	-2.13	-3.17	0.48
Poultry	-1.84	3	0.49	-2.85	-0.69	2.76	0.01	-1.95	-2.71	2.87
Gold	-3.95	0.47	-1.4	-1.5	-3.98	2.47	-1.04	-3.14	-1.03	3.52
Oil	-1.89	3.3	0.29	-2.79	-2.69	1.82	-1.69	-2.74	-2.42	1.33

	$\frac{5\pi}{18}$		$\frac{7\pi}{9}$		$\frac{2\pi}{9}$		$\frac{5\pi}{6}$ (s)		$\frac{\pi}{6}$ (s)	
	π_{19}	π_{20}	π_{21}	π_{22}	π_{23}	π_{24}	π_{25}	π_{26}	π_{27}	π_{28}
Wheat	-0.32	-3.08	-1.35	3.43	1.76	-3.16	-1.06	0.65	-1.24	-1.76
Beef	-1.26	-2.95	-2.71	1.3	-0.45	-1.91	-3.22	1.52	-2.59	-0.99
Corn	0.77	-3.71	-2.32	0.06	2.25	-3.02	-2.05	1.49	-1.41	-2.61
Hog	-4.66	-3.6	-2.81	2	0.37	-3.58	-1.51	0.24	-2.7	-1.41
Poultry	1.24	-5.52	-2.29	3.07	1.78	-3.36	-3.12	1.9	-2.24	0.46
Gold	-1.52	-3.04	-2.05	1.71	0.37	-3.57	-3.7	-0.41	-1.52	-1.67
Oil	-1.88	-3.36	-4.72	-0.61	0.5	-2.73	-3.15	1.49	-1.56	-1.69

	$\frac{8\pi}{9}$		$\frac{\pi}{9}$		$\frac{17\pi}{18}$		$\frac{\pi}{18}$	
	π_{29}	π_{30}	π_{31}	π_{32}	π_{33}	π_{34}	π_{35}	π_{36}
Wheat	-3.25	1.34	1.04	-2.83	-2.94	0.53	1.59	-0.31
Beef	-3.41	-0.21	-0.4	-4.45	-3.51	0.97	-0.15	-1.16
Corn	-2.79	2.01	0.83	-3.78	-3.19	1.44	1.91	0.43
Hog	-3.1	0.05	-2.61	-4.61	-3.6	0.55	1.67	-1.02
Poultry	-3.32	1.96	-0.54	-3.8	-3.48	0	1.29	1.56
Gold	-0.87	-0.95	1.31	-3.58	-3.81	0.88	-1.23	-4.57
Oil	-3.94	-1.41	-0.51	-4.47	-3.88	2.44	1.49	-1.24

TABLE B5. Results from Unit Root test with 36 possible Unit Roots in Monthly Data, constant and Trend Included.

			0	π (s)	$\frac{\pi}{2}$ (s)		$\frac{5\pi}{9}$		$\frac{4\pi}{9}$	
	Period	Lags	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8
Wheat	83:08-08:06	2	-2.91	-2.14	0.04	-2.45	-1	3.09	-0.99	-3.49
Beef	83:08-08:06	2	-1.73	-1.85	-0.53	-2.01	-0.93	1.4	-2.44	-0.96
Corn	83:08-08:06	2	-3.06	-2.35	-2.12	-0.83	-1.25	3.63	-2.27	-2.24
Hog	83:08-08:06	3	-3.52	-1.66	-3.27	-2.06	-1.77	3.08	-4.16	-2.92
Poultry	83:08-08:06	2	-3.92	-0.99	0.02	-3.62	-2.14	2.92	-0.35	-4.67
Gold	83:08-08:06	2	1.02	-0.37	-4.21	-1.1	-3.01	-0.19	-3.42	-1.23
Oil	83:08-08:06	1	0.98	-2.37	-2.8	-1.6	-0.72	1.62	-2.58	-3.77

	$\frac{11\pi}{8}$		$\frac{7\pi}{18}$		$\frac{2\pi}{3}$ (s)		$\frac{\pi}{3}$ (s)		$\frac{13\pi}{18}$	
	π_9	π_{10}	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}	π_{16}	π_{17}	π_{18}
Wheat	-2.29	1.12	0.07	-0.82	-1.64	0.42	-1.99	-0.04	0.25	1.43
Beef	-2.21	1.64	-0.53	-2.08	-1.16	0.89	-1.72	-1.7	-2.89	0.73
Corn	-0.54	0.88	0.21	-3.24	-0.96	3.09	0.4	-3.2	-1.82	1.4
Hog	-3.6	2.02	-1.73	-3.43	-1.52	1.74	-4.09	-2.13	-3.17	0.48
Poultry	-1.84	3	0.49	-2.85	-0.69	2.76	0.01	-1.95	-2.71	2.87
Gold	-4.02	0.33	-1.51	-1.48	-4.09	2.31	-1.21	-3.21	-1.12	3.49
Oil	-1.9	3.3	0.29	-2.81	-2.71	1.82	-1.69	-2.74	-2.42	1.35

	$\frac{5\pi}{18}$		$\frac{7\pi}{9}$		$\frac{2\pi}{9}$		$\frac{5\pi}{6}$ (s)		$\frac{\pi}{6}$ (s)	
	π_{19}	π_{20}	π_{21}	π_{22}	π_{23}	π_{24}	π_{25}	π_{26}	π_{27}	π_{28}
Wheat	-0.32	-3.08	-1.35	3.43	1.76	-3.16	-1.06	0.65	-1.24	-1.76
Beef	-1.26	-2.95	-2.71	1.3	-0.45	-1.91	-3.22	1.52	-2.59	-0.99
Corn	0.74	-3.71	-2.31	0.06	2.21	-3.03	-2.04	1.48	-1.46	-2.6
Hog	-4.66	-3.6	-2.81	2	0.37	-3.58	-1.51	0.24	-2.7	-1.41
Poultry	1.24	-5.52	-2.29	3.07	1.78	-3.36	-3.12	1.9	-2.24	0.46
Gold	-1.72	-3.11	-2.04	1.68	0.25	-3.76	-3.63	-0.52	-1.6	-1.77
Oil	-1.88	-3.36	-4.72	-0.59	0.48	-2.76	-3.16	1.48	-1.52	-1.62

	$\frac{8\pi}{9}$		$\frac{\pi}{9}$		$\frac{17\pi}{18}$		$\frac{\pi}{18}$	
	π_{29}	π_{30}	π_{31}	π_{32}	π_{33}	π_{34}	π_{35}	π_{36}
Wheat	-3.25	1.34	1.04	-2.83	-2.94	0.53	1.59	-0.31
Beef	-3.41	-0.21	-0.4	-4.45	-3.51	0.97	-0.15	-1.16
Corn	-2.78	1.99	0.8	-3.77	-3.17	1.43	1.82	0.44
Hog	-3.1	0.05	-2.61	-4.61	-3.6	0.55	1.67	-1.02
Poultry	-3.32	1.96	-0.54	-3.8	-3.48	0	1.29	1.56
Gold	-0.83	-1.01	1.3	-3.78	-3.78	0.89	-1.4	-5.05
Oil	-3.94	-1.42	-0.47	-4.49	-3.91	2.44	1.52	-1.06

TABLE B6. Results from Unit Root test with 36 possible Unit Roots in Monthly Data, constant and Seasonal dummy Included.

			0	π(s)	$\frac{\pi}{2}$ (s)		$\frac{5\pi}{9}$		$\frac{4\pi}{9}$	
	Period	Lags	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8
Wheat	83:08-08:06	2	-2.64	-1.96	-0.53	-2.53	-1.12	3.12	-1.02	-3.33
Beef	83:08-08:06	2	-1.71	-2.15	-0.48	-2.16	-0.91	1.37	-2.18	-1.2
Corn	83:08-08:06	2	-2.64	-1.96	-0.53	-2.53	-1.12	3.12	-1.02	-3.33
Hog	83:08-08:06	3	-3.92	-1.56	-3.7	-2.01	-1.87	2.82	-4.54	-2.7
Poultry	83:08-08:06	2	-3.06	-1.93	-0.56	-4.59	-2.1	3.08	-0.28	-5.24
Gold	83:08-08:06	2	0.36	-0.62	-4.05	-1.17	-2.79	-0.09	-3.3	-1.25
Oil	83:08-08:06	1	0.91	-2.64	-2.84	-1.81	-0.55	1.81	-2.65	-3.8

	$\frac{11\pi}{8}$		$\frac{7\pi}{18}$		$\frac{2\pi}{3}$ (s)		$\frac{\pi}{3}$ (s)		$\frac{13\pi}{18}$	
	π_9	π_{10}	π_{11}	π_{12}	π_{13}	π_{14}	π_{15}	π_{16}	π_{17}	π_{18}
Wheat	-2.4	1.12	0.01	-0.75	-1.61	0.28	-1.61	-0.21	0.13	1.45
Beef	-2.14	1.93	-0.56	-2.12	-1.51	2.59	-1.51	-1.86	-2.68	0.58
Corn	-2.4	1.12	0.01	-0.75	-1.61	0.28	-1.61	-0.21	0.13	1.45
Hog	-3.49	2.02	-2.29	-3.27	-2.25	1.96	-4.57	-2.37	-2.96	0.15
Poultry	-2.04	3.21	0.61	-3.09	-1.86	3.77	-0.04	-3.91	-2.64	2.76
Gold	-3.85	0.31	-1.42	-1.47	-4.35	2.62	-1.1	-3.14	-1.21	3.57
Oil	-1.86	3.28	0.34	-2.92	-2.7	2	-2.01	-2.94	-2.39	1.33

	$\frac{5\pi}{18}$		$\frac{7\pi}{9}$		$\frac{2\pi}{9}$		$\frac{5\pi}{6}$ (s)		$\frac{\pi}{6}$ (s)	
	π_{19}	π_{20}	π_{21}	π_{22}	π_{23}	π_{24}	π_{25}	π_{26}	π_{27}	π_{28}
Wheat	-0.43	-3.04	-1.29	3.27	1.5	-3.38	-1.57	1.07	-0.96	-3
Beef	-1.21	-2.93	-2.69	1.32	-0.42	-1.9	-3.47	1.59	-2.49	-1.31
Corn	-0.43	-3.04	-1.29	3.27	1.5	-3.38	-1.57	1.07	-0.96	-3
Hog	-5.11	-3.72	-2.73	1.77	-0.46	-4.1	-1.53	0.7	-3.01	-3.93
Poultry	1.04	-5.64	-2.53	3.07	1.87	-3.7	-3.26	1.91	-0.86	-1.52
Gold	-1.55	-2.98	-2.06	1.73	0.21	-3.63	-3.85	-0.19	-2	-1.65
Oil	-1.93	-3.53	-4.79	-0.46	0.54	-2.85	-3.3	1.66	-1.28	-2.41

	$\frac{8\pi}{9}$		$\frac{\pi}{9}$		$\frac{17\pi}{18}$		$\frac{\pi}{18}$	
	π_{29}	π_{30}	π_{31}	π_{32}	π_{33}	π_{34}	π_{35}	π_{36}
Wheat	-3.15	1.22	0.78	-2.93	-2.86	0.47	1.75	-0.44
Beef	-3.33	-0.16	-0.27	-4.26	-3.46	1.01	-0.13	-1.1
Corn	-3.15	1.22	0.78	-2.93	-2.86	0.47	1.75	-0.44
Hog	-2.97	0.01	-2.7	-5.26	-3.59	0.56	1.55	-1.32
Poultry	-3.38	1.85	0.13	-4.24	-3.68	0.07	1.94	1.61
Gold	-0.72	-0.84	1.38	-3.49	-3.65	0.88	-1.2	-4.43
Oil	-3.59	-1.37	-0.58	-4.63	-3.84	2.2	1.64	-1.16

TABLE B7. Critical Values for Distribution of Test Statistics for Index Cycle Correlation, 24000 simulations, T=600.

	$prob_f = 0.25$				$prob_f = 0.256$				$prob_f = 0.263$				$prob_f = 0.27$			
LAG	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01
1	16.0	15.3	11.2	10.5	15.6	14.7	10.9	10.1	15.1	14.3	10.5	9.71	14.6	13.8	10.1	9.31
2	9.02	8.16	4.13	3.27	8.63	7.78	3.79	3.02	8.25	7.50	3.44	2.68	7.87	7.04	3.19	2.43
3	5.96	5.08	0.92	0.08	5.69	4.82	0.75	-0.08	5.52	4.56	0.50	0.34	5.17	4.39	0.34	-0.50
4	4.56	3.61	-0.59	-1.43	4.34	3.44	-0.76	-1.60	4.21	3.27	-0.84	1.60	3.96	3.11	-0.92	-1.77
5	3.79	2.85	-1.35	-2.28	3.61	2.77	-1.43	-2.28	3.53	2.68	-1.52	2.28	3.44	2.51	-1.52	-2.37
6	3.36	2.51	-1.77	-2.63	3.27	2.43	-1.77	-2.63	3.23	2.35	-1.77	2.71	3.19	2.35	-1.86	-2.63
7	3.19	2.35	-1.94	-2.88	3.11	2.26	-1.94	-2.80	3.11	2.26	-1.94	2.80	3.02	2.18	-1.94	-2.80
8	3.11	2.18	-2.03	-2.97	3.02	2.18	-2.03	-2.97	3.02	2.18	-2.03	2.88	3.02	2.09	-2.03	-2.88
9	3.11	2.18	-2.11	-2.97	2.94	2.09	-2.11	-3.06	3.02	2.09	-2.11	2.88	2.94	2.09	-2.03	-2.88
10	3.02	2.18	-2.11	-2.97	2.94	2.09	-2.11	-3.06	3.02	2.09	-2.11	2.97	2.94	2.01	-2.03	-2.88
11	3.02	2.18	-2.11	-3.01	3.02	2.09	-2.11	-2.97	3.02	2.09	-2.03	2.88	2.94	2.09	-2.03	-2.88
12	3.02	2.18	-2.11	-3.06	2.94	2.09	-2.11	-3.06	3.02	2.09	-2.03	2.88	2.94	2.01	-2.11	-2.88
13	3.02	2.09	-2.11	-2.97	2.94	2.09	-2.20	-3.06	2.94	2.09	-2.03	2.97	2.85	2.01	-2.03	-2.97
14	3.02	2.09	-2.11	-3.06	2.94	2.09	-2.11	-3.06	2.94	2.09	-2.03	2.97	2.94	2.01	-2.11	-2.88
15	3.02	2.09	-2.11	-2.97	2.85	2.09	-2.20	-3.06	2.94	2.09	-2.11	2.97	2.85	2.01	-2.03	-2.88
16	3.02	2.09	-2.11	-3.06	2.94	2.09	-2.20	-3.06	2.94	2.09	-2.11	2.97	2.85	2.01	-2.03	-2.97
17	3.02	2.09	-2.11	-3.06	2.94	2.09	-2.11	-2.97	3.02	2.09	-2.11	2.97	2.85	2.09	-2.11	-2.97
18	3.02	2.09	-2.11	-3.06	3.02	2.09	-2.11	-3.06	3.02	2.09	-2.03	2.97	2.94	2.01	-2.11	-2.97
19	3.02	2.18	-2.11	-2.97	2.94	2.09	-2.11	-2.97	2.94	2.09	-2.03	2.97	2.85	2.01	-2.03	-2.97
20	3.02	2.18	-2.11	-2.97	3.02	2.09	-2.11	-3.01	2.94	2.09	-2.03	2.88	2.85	2.01	-2.03	-2.97
21	3.02	2.18	-2.11	-3.06	3.02	2.09	-2.11	-2.97	2.94	2.09	-2.11	2.97	2.85	2.01	-2.03	-2.88
22	3.11	2.09	-2.11	-3.06	2.94	2.09	-2.11	-2.97	2.85	2.01	-2.11	2.97	2.85	2.01	-2.03	-2.88
23	3.02	2.09	-2.11	-3.06	2.94	2.09	-2.11	-3.06	2.94	2.01	-2.11	2.97	2.94	2.01	-2.03	-2.88
24	3.02	2.09	-2.11	-3.06	3.02	2.09	-2.11	-2.97	2.94	2.09	-2.11	2.97	2.94	2.09	-2.03	-2.97

	$prob_f = 0.277$				$prob_f = 0.285$				$prob_f = 0.294$				$prob_f = 0.303$			
LAG	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01
1	14.18	13.3	9.61	8.82	13.7	12.8	9.12	8.44	13.1	12.3	8.63	7.87	12.6	11.7	8.16	7.50
2	7.59	6.77	2.85	2.09	7.13	6.32	2.51	1.76	6.77	5.96	2.18	1.42	6.41	5.61	1.84	1.17
3	4.95	4.13	0.17	-0.67	4.65	3.87	-0.08	-0.92	4.47	3.61	-0.25	1.09	4.21	3.36	-0.42	-1.18
4	3.79	2.94	-1.01	-1.86	3.61	2.77	-1.18	-1.94	3.44	2.60	-1.26	2.03	3.27	2.51	-1.35	-2.11
5	3.36	2.43	-1.60	-2.45	3.19	2.35	-1.60	-2.45	3.11	2.26	-1.69	2.45	2.94	2.14	-1.69	-2.54
6	3.11	2.18	-1.77	-2.67	2.94	2.18	-1.86	-2.63	2.94	2.09	-1.86	2.63	2.85	2.01	-1.86	-2.63
7	2.94	2.09	-1.94	-2.80	2.85	2.01	-1.94	-2.71	2.85	2.01	-1.86	2.71	2.77	2.01	-1.94	-2.71
8	2.94	2.09	-1.98	-2.88	2.85	2.01	-1.94	-2.80	2.77	1.93	-1.94	2.71	2.68	1.93	-1.94	-2.80
9	2.94	2.01	-2.03	-2.97	2.77	2.01	-2.03	-2.80	2.77	1.93	-1.94	2.80	2.68	1.93	-1.94	-2.71
10	2.94	2.01	-2.03	-2.88	2.85	1.93	-2.03	-2.80	2.77	2.01	-1.94	2.71	2.68	1.93	-1.94	-2.71
11	2.85	2.01	-2.03	-2.88	2.77	1.93	-1.94	-2.80	2.77	1.93	-1.94	2.80	2.77	1.93	-1.94	-2.80
12	2.85	2.01	-2.03	-2.88	2.77	1.93	-2.03	-2.88	2.77	1.93	-1.94	2.71	2.77	1.93	-1.94	-2.71
13	2.85	2.01	-2.03	-2.88	2.77	1.93	-2.03	-2.80	2.77	1.93	-1.94	2.76	2.77	1.93	-1.94	-2.71
14	2.85	2.01	-2.03	-2.88	2.77	2.01	-1.94	-2.80	2.77	1.93	-1.94	2.71	2.77	1.93	-1.94	-2.80
15	2.94	2.01	-2.03	-2.80	2.85	2.01	-2.03	-2.80	2.77	1.93	-1.94	2.80	2.77	1.93	-1.94	-2.71
16	2.85	2.01	-2.03	-2.88	2.81	2.01	-2.03	-2.80	2.77	1.93	-1.94	2.80	2.72	1.93	-1.94	-2.80
17	2.85	2.01	-2.03	-2.88	2.85	2.01	-2.03	-2.80	2.68	1.93	-1.94	2.80	2.68	1.93	-1.94	-2.80

18	2.85	2.01	-2.03	-2.88	2.77	1.93	-2.03	-2.80	2.77	1.93	-1.94	2.80	2.68	1.93	-1.94	-2.80
19	2.85	2.01	-2.03	-2.88	2.77	1.93	-2.03	-2.80	2.77	1.93	-1.94	2.71	2.68	1.93	-1.94	-2.71
20	2.85	2.01	-2.03	-2.88	2.77	2.01	-1.94	-2.84	2.68	1.93	-1.94	2.71	2.77	1.93	-1.94	-2.71
21	2.85	2.01	-2.03	-2.88	2.77	2.01	-1.94	-2.80	2.77	1.93	-1.94	2.80	2.85	1.93	-1.94	-2.71
22	2.85	2.01	-2.03	-2.84	2.77	1.93	-1.94	-2.88	2.77	1.93	-1.94	2.80	2.68	1.93	-1.94	-2.80
23	2.85	2.01	-2.03	-2.80	2.85	2.01	-2.03	-2.88	2.77	2.01	-1.94	2.76	2.77	1.93	-1.94	-2.80
24	2.85	2.01	-2.03	-2.88	2.85	1.93	-2.03	-2.88	2.85	1.93	-1.94	2.80	2.68	1.93	-1.94	-2.71

	$prob_f = 0.3125$				$prob_f = 0.3225$				$prob_f = 0.333$				$prob_f = 0.345$			
LAG	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01
1	12.05	11.2	7.69	6.86	11.4	10.7	7.13	6.41	10.8	10.0	6.50	5.87	10.2	9.41	5.87	5.25
2	6.05	5.25	1.59	0.75	5.69	4.82	1.17	0.42	5.25	4.47	0.84	0.08	4.82	4.13	0.50	-0.25
3	3.96	3.19	-0.59	-1.43	3.70	2.94	-0.76	-1.56	3.44	2.68	-0.92	1.77	3.27	2.51	-1.09	-1.86
4	3.19	2.35	-1.43	-2.20	3.11	2.26	-1.52	-2.20	2.85	2.09	-1.52	2.37	2.77	2.01	-1.60	-2.37
5	2.85	2.09	-1.77	-2.54	2.77	2.01	-1.77	-2.54	2.68	1.93	-1.77	2.54	2.60	1.93	-1.77	-2.54
6	2.77	1.93	-1.86	-2.63	2.68	1.93	-1.86	-2.63	2.68	1.93	-1.86	2.54	2.60	1.84	-1.77	-2.54
7	2.72	1.93	-1.86	-2.63	2.60	1.84	-1.86	-2.63	2.68	1.84	-1.86	2.63	2.60	1.84	-1.77	-2.54
8	2.68	1.93	-1.94	-2.71	2.60	1.84	-1.86	-2.71	2.60	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.54
9	2.68	1.84	-1.94	-2.63	2.60	1.84	-1.86	-2.63	2.68	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.54
10	2.68	1.93	-1.94	-2.63	2.60	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.60	1.84	-1.77	-2.54
11	2.68	1.93	-1.94	-2.71	2.60	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.51	1.84	-1.77	-2.54
12	2.68	1.93	-1.94	-2.71	2.60	1.93	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.63
13	2.68	1.93	-1.94	-2.71	2.68	1.84	-1.86	-2.71	2.60	1.84	-1.86	2.63	2.51	1.76	-1.86	-2.63
14	2.68	1.93	-1.94	-2.71	2.68	1.93	-1.86	-2.63	2.68	1.84	-1.86	2.63	2.51	1.84	-1.86	-2.63
15	2.68	1.84	-1.86	-2.71	2.68	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.51	1.84	-1.86	-2.54
16	2.68	1.93	-1.94	-2.63	2.68	1.84	-1.86	-2.71	2.60	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.63
17	2.68	1.93	-1.86	-2.71	2.60	1.84	-1.86	-2.71	2.60	1.84	-1.77	2.63	2.60	1.84	-1.86	-2.63
18	2.68	1.84	-1.86	-2.67	2.68	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.63
19	2.68	1.93	-1.86	-2.63	2.68	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.51	1.76	-1.86	-2.63
20	2.60	1.93	-1.86	-2.63	2.60	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.56	1.84	-1.86	-2.63
21	2.64	1.84	-1.86	-2.71	2.60	1.84	-1.86	-2.63	2.51	1.84	-1.86	2.63	2.60	1.76	-1.77	-2.54
22	2.68	1.88	-1.86	-2.71	2.68	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.51	1.84	-1.86	-2.54
23	2.68	1.93	-1.86	-2.71	2.60	1.84	-1.86	-2.71	2.60	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.54
24	2.68	1.93	-1.94	-2.71	2.60	1.84	-1.86	-2.63	2.60	1.84	-1.86	2.63	2.60	1.84	-1.86	-2.63

	$prob_f = 0.357$				$prob_f = 0.3703$				$prob_f = 0.384$				$prob_f = 0.4$			
LAG	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01	0.99	0.95	0.05	0.01
1	9.51	8.73	5.34	4.65	8.73	8.06	4.65	3.96	8.06	7.32	3.91	3.19	7.22	6.50	3.19	2.51
2	4.56	3.79	0.25	-0.50	4.13	3.36	-0.13	-0.92	3.79	3.02	-0.50	-1.18	3.36	2.68	-0.76	-1.47
3	3.11	2.35	-1.18	-1.94	2.94	2.18	-1.35	-2.11	2.77	2.01	-1.43	-2.20	2.60	1.93	-1.52	-2.28
4	2.68	1.93	-1.60	-2.37	2.60	1.84	-1.60	-2.37	2.51	1.84	-1.69	-2.37	2.43	1.76	-1.69	-2.37
5	2.60	1.84	-1.77	-2.45	2.51	1.76	-1.77	-2.45	2.43	1.76	-1.77	-2.54	2.43	1.67	-1.69	-2.45
6	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.43	1.67	-1.77	-2.45
7	2.51	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.45	2.35	1.67	-1.77	-2.45
8	2.51	1.84	-1.77	-2.45	2.51	1.76	-1.77	-2.45	2.51	1.76	-1.69	-2.45	2.43	1.67	-1.69	-2.45
9	2.51	1.84	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.43	1.76	-1.69	-2.45	2.43	1.76	-1.69	-2.45
10	2.51	1.84	-1.77	-2.45	2.43	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45
11	2.51	1.84	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45
12	2.51	1.84	-1.77	-2.54	2.43	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.54	2.35	1.67	-1.69	-2.45
13	2.51	1.76	-1.77	-2.63	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45

14	2.51	1.76	-1.86	-2.54	2.51	1.76	-1.77	-2.45	2.43	1.76	-1.77	-2.45	2.43	1.76	-1.69	-2.45
15	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.45	2.39	1.67	-1.77	-2.45
16	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.45	2.51	1.76	-1.77	-2.54	2.43	1.67	-1.77	-2.45
17	2.51	1.76	-1.86	-2.54	2.51	1.76	-1.77	-2.45	2.51	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45
18	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45
19	2.60	1.84	-1.77	-2.54	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45
20	2.60	1.84	-1.77	-2.54	2.43	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.45	2.43	1.76	-1.77	-2.45
21	2.51	1.76	-1.86	-2.54	2.51	1.76	-1.77	-2.45	2.43	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.37
22	2.51	1.76	-1.86	-2.54	2.51	1.76	-1.77	-2.54	2.43	1.76	-1.69	-2.45	2.43	1.67	-1.77	-2.45
23	2.51	1.76	-1.77	-2.63	2.51	1.76	-1.77	-2.54	2.51	1.76	-1.77	-2.45	2.43	1.67	-1.77	-2.45
24	2.51	1.76	-1.86	-2.54	2.43	1.76	-1.77	-2.54	2.43	1.76	-1.77	-2.45	2.43	1.67	-1.69	-2.45

	$prob_f = 0.416$				$prob_f = 0.4347$				$prob_f = 0.4545$				$prob_f = 0.48$			
LAG	**0.99**	**0.95**	**0.05**	**0.01**	**0.99**	**0.95**	**0.05**	**0.01**	**0.99**	**0.95**	**0.05**	**0.01**	**0.99**	**0.95**	**0.05**	**0.01**
1	6.41	5.69	2.35	1.67	5.52	4.82	1.42	0.84	4.56	3.83	0.50	-0.17	3.44	2.77	-0.50	-1.26
2	3.02	2.35	-1.01	-1.77	2.77	2.09	-1.26	-1.94	2.60	1.84	-1.43	-2.20	2.35	1.67	-1.60	-2.37
3	2.51	1.84	-1.60	-2.28	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.60	-2.37	2.26	1.59	-1.69	-2.37
4	2.43	1.67	-1.69	-2.37	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
5	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.33
6	2.43	1.67	-1.69	-2.37	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
7	2.43	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.26	1.67	-1.69	-2.37
8	2.43	1.67	-1.69	-2.37	2.43	1.67	-1.69	-2.41	2.35	1.67	-1.69	-2.37	2.35	1.59	-1.69	-2.37
9	2.43	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.60	-2.37
10	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
11	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.26	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
12	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.41	2.35	1.67	-1.69	-2.37	2.26	1.67	-1.69	-2.37
13	2.35	1.67	-1.69	-2.37	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
14	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.60	-2.37
15	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.41	2.35	1.67	-1.69	-2.37	2.35	1.59	-1.69	-2.37
16	2.43	1.67	-1.69	-2.37	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.33
17	2.43	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37
18	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
19	2.43	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.60	-2.37	2.35	1.67	-1.69	-2.37
20	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
21	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
22	2.43	1.67	-1.69	-2.45	2.26	1.67	-1.69	-2.45	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37
23	2.35	1.67	-1.69	-2.37	2.43	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.60	-2.37
24	2.43	1.67	-1.69	-2.45	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37	2.35	1.67	-1.69	-2.37

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