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## **Abstract**

This master thesis presents the influence of different fiber in high-performance lightweight concrete and the ductility capacity of reinforced lightweight concrete beam. Twelve beams with length of 2.2m and reinforcement ratio 0.24 have been tested under 4 point bending, three of them were made by normal density aggregates as references beams. The target concrete compressive strength for all beams were 50MPa.

Three different types of fibers such as steel fiber, Polypropylene macrofiber and Polypropylene microfiber (PP-fiber) with two different fraction volume has been used to study the influence of fibers on high-performance concrete.

Two types end-hooked steel fibers with a length/diameter 50mm/1mm (N50) and 35mm/0.55mm (N35) has been evaluated. The behavior of plastic-fiber with a length/diameter 50mm/1mm and Polypropylene fiber with length/diameter 12mm/0.22mm have been also investigated. Steel-fibers and plastic-fiber were studied by two volume fraction 1 and 2%. The volume fraction of PP-fibers were decided to be 0.4 and 0.6% because of low volume density.

Test results showed that use of plastic-fiber and PP-fiber has no influence on crack-width development. Beams with plastic-fiber showed the same crack-width comparing to concrete beams without fibers. PP-fiber give negative results specially by increasing the volume fraction from 0.4% to 0.6%. End-hooked steel fibers decreased the crack opening under loading comparing to references beams. Test result showed that use of N50 in a volume fraction greater than 1% will decrease the crack-width at beam under same loading conditions comparing to beam with 1% volume fraction.

All beams failed due flexural failure under loading. The lightweight concrete showed brittle failure and sudden fracture. Beams with end-hooked steel fibers had a ductile behavior during failure, increasing in volume fraction of steel fiber from 1 to 2% increased the flexural moment capacity of beam under loading. Both type of steel-fiber with a 1 and 2% volume fraction showed the highest increasing in ductility and moment capacity during loading to failure.

Plastic-fiber resulted in to increasing ductility of beams, volume fraction greater than 1% had better influence on beam ductility and moment capacity of beam. The plastic-fiber beam with volume fraction 2% failed at the same load as normal density concrete beam but with higher ductility capacity.

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PP-fiber with higher volume fraction than 0.4% caused segregation by swallow cement pasta, and caused bleeding by releasing the water after casting of concrete. Bleeding and segregation caused low concrete compressive strength and low moment capacity for beam during loading.

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### Ductility in Lightweight Concrete with Fiber

#### I. Symbols

##### Latin upper case letters

- $A_c$  Cross sectional area of concrete  
 $A_s$  Cross sectional area of tensile reinforcement  
 $A_{s,min}$  Minimum cross sectional area of reinforcement  
 $E_c$  Modulus of elasticity of normal weight concrete  
 $E_{cm}$  Secant Modulus of elasticity of concrete  
 $E_f$  Young's modulus of the fibers  
 $E_s$  Young's modulus of reinforced bar  
 $l_f$  Length of fiber  
 $M_{Ed}$  Design value of the applied internal bending moment  
 $V_{Ed}$  Design value of the applied shear force  
 $V_f$  Fiber volume fraction  
 $w_k$  Crack width

##### Latin lower case letters

- $d$  Effective depth of a cross-section  
 $f_c$  Compressive strength of concrete  
 $f_{cd}$  Design value of concrete compressive strength  
 $f_{ck}$  Characteristic compressive cylinder strength of concrete at 28 days  
 $f_{cm}$  Mean value of concrete cylinder compressive strength  
 $f_{ctk}$  Characteristic axial tensile strength of concrete  
 $f_{ctm}$  Mean value of axial tensile strength of concrete  
 $f_y$  Yield strength of reinforcement  
 $f_{yd}$  Design yield strength of reinforcement  
 $f_{tk,res}$  The residual stress of fiber reinforced concrete  
 $\gamma$  Partial factor  
 $\gamma_C$  Partial factor for concrete  
 $\gamma_S$  Partial factor for steel reinforcement  
 $\gamma_m$  Partial factor for fiber property  
 $\varepsilon_c$  Compressive strain in the concrete  
 $\varepsilon_{cu}$  Ultimate compressive strain in the concrete  
 $\varepsilon_{sm}$  The main strain in the reinforcement

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- $\varepsilon_{cm}$  The main strain in the concrete  
 $\varepsilon_{mu}$  First crack strain of plain concrete  
 $\varepsilon_u$  Strain of reinforcement  
 $\sigma_c$  Compressive stress in the concrete  
 $\sigma_{cu}$  Compressive stress in the concrete at  $\varepsilon_{cu}$   
 $\sigma_{fk,max}$  Maximum stress of fiber  
 $\sigma_{fk,mid}$  Average stress in all fibers bridging a crack

#### Abbreviations

- ACI American concrete institute  
EC2 Eurocode 2  
FA Fly ash  
FRC Fiber reinforced concrete  
LWC Lightweight concrete  
M12 Fiber with length 12 mm  
NDC Normal density concrete  
SCC Self-compacting concrete  
SFRC Steel fiber reinforced concrete  
VCC Vibrator-compacting concrete  
N50 End-hooked steel fiber with  $(l_f / d) 50mm / 1mm$   
N35 End-hooked steel fiber with  $(l_f / d) 35mm / 0.55mm$   
M50 Polypropylene plastic fiber with  $(l_f / d) 50mm / 1mm$   
M12 Polypropylene microfiber with  $(l_f / d) 12mm / 22\mu m$

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#### Abbreviations of beams and concrete ID

Ref_SCC_MF	Referance beam with self-compacting concrete with 2% N50 steel fiber
Ref_VCC_MF	Referance beam with vibrator-compacting concrete with 2% N50 steel fiber
Ref_SCC_UF	Referance beam with self-compacting concrete without fiber
Ref_LWC_UF	Referance beam with lightweight concrete without fiber
LWC_N50_2%	Lightweight concrete beam with 2% N50 steel fiber
LWC_N50_1%	Lightweight concrete beam with 1% N50 steel fiber
LWC_N35_2%	Lightweight concrete beam with 2% N35 steel fiber
LWC_N35_1%	Lightweight concrete beam with 1% N50 steel fiber
LWC_PF_2%	Lightweight concrete beam with 2% M50 polypropylene plastic fiber
LWC_PF_1%	Lightweight concrete beam with 1% M50 polypropylene plastic fiber
LWC_PPF_2%	Lightweight concrete beam with 2% M12 polypropylene microfiber
LWC_PPF_1%	Lightweight concrete beam with 1% M12 polypropylene microfiber



## **II. Preface**

The Popularity of high rise buildings with great design has explored in the last two decades. The title of highest building does not last more than few years, since 1996 which Willis tower was highest building with 442m height, the title of highest building have been beaten 6 times. Today's record holder is Burj Khalifa (828m tall). Use of normal concrete in such a high building will give a very high dead load which is a big challenge for design engineering. A secondary use of lightweight concrete which is 25 % lighter than normal concrete can reduce use of reinforcement bars and support work which leads to considerable cost saving and less dead load design.

Use of lightweight concrete specially high-performance lightweight concrete has been founded challenges in seismic zones because of low ductility capacity. The ductility has to be increased to satisfy the minimum seismic design requirements. Use of fiber-reinforced concrete can be one of solutions for improved ductility.

The purpose of this thesis is to study the influence of fibers on ductility behaviors in high-performance lightweight concrete. The funds for this thesis provided by Mechanical and Structural Engineering Institute of University of Stavanger. The author would like to extend his gratitude to his supervisor PhD. Kjell Tore Fosså for his time dedication and his support and guidance in the process of testing and writing this thesis.

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Stavnager, June 15th 2011,

Milad Ahmadyar

## **Chapter 1**

### **1.1 Introduction to concrete**

Use of cement and concrete went back to 2000 year B.C. Roman Empire used quicklime, pozzolana and aggregate of pumice to build pantheon and Baths of Caracalla. The name “concrete” comes from Latin concretus, which means to grow together.

First modern concrete was made in 1756 by British engineer John Smeaton. He defined his concrete as mixture of hydraulic cement and pebbles as a coarse aggregate. The most important year for concrete industry is 1824 when Joseph Aspdin invented Portland Cement. He burned ground limestone and clay together. He knew that he can change the chemical properties of the ground limestone by burning it, the process create a stronger cement.

Today is concrete most used material in construction business. It is because of properties such as high compressive strength, durability, flexibility of shape and fire resistance. In 2009 produced worldwide cement factories about 2,800,000,000,000 tons Portland cement ref.[1].

Concretes strength mostly controls by w/c ratio, low w/c ratio give higher strength and lower permeability. Normally w/c ratio starts from 0.4 up to 0.8.

Hydration process starts between water and cement and result of this reaction is cement paste. Cement paste glues the aggregate together. Aggregate has normally higher strength compare cement paste but it depends on strength of concrete. In normal strength concrete will cement paste fail due compressive pressure. Hydration process is a critical phase for strength of concrete.

Concrete weakness is in tension, cement pasta will crack fast when concrete is under tension. To solve these problems used Joseph Monier in 1849 steel reinforcing bars. Today they use steel fibers, glass fiber, or plastic fiber to increase strength in tension.

### **1.2 Types of concrete**

Concrete is divided into three categories based on unit weight. Normal weight concrete which contains natural sand and gravel or crushed rock aggregates, has a density around 2400 kg/m<sup>3</sup>. Lightweight concrete(LWC) which contains lightweight aggregates with lower bulk density, has a density around 1800 kg/m<sup>3</sup>. The third type of concrete is heavyweight concrete with a density higher than 2400 kg/m<sup>3</sup>.

**1.3 Strength classes of concrete**

Strength of concrete is classified into three strength classes. Strength lower than 20MPa knows as low-strength concrete. It is used in structures member such as floor-system, facade etc. Main reason of using low strength concrete is saving in project cost.

Moderate-strength concrete has a strength between 20 to 40MPa. It is also referred to as normal concrete and most used in structural work ref.[2].

Concrete with a strength higher than 40MPa is called high strength concrete or high-performance concrete. It is used for structures, offshore, oil and gas industry. In case of saving in material cost, it is possible to replace normal concrete by high-performance concrete to reduce the size of structural member and the use of reinforcement.

*Table 1-1: Strength classes for concrete [Table 3.1 En1992-1-1:2004E]*

**Strength classes for concrete(MPa)**

$f_{ck}$	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{ck, cube}$	15	20	25	30	37	45	50	55	60	67	75	85	95	105
$f_{cm}$	20	24	28	33	38	43	48	53	58	63	68	78	88	98
$f_{ctm}$	1'6	1'9	2'2	2'6	2'9	3'2	3'5	3'8	4'1	4'2	4'4	4'6	4'8	5'0

**1.4 Aggregate**

Aggregate occupies between 65 and 75% of the concrete volume. Manufacturers produce usually aggregates with grading 0-8, 8-16 or 8-22. Optimized packing and minimal voids in concrete depends on how well aggregates are graded.

There are two types aggregates, coarse aggregate and fine aggregate.

Coarse aggregate are such as Natural crushed stone which is produced by crushing natural stone, natural gravel, Artificial coarse aggregates and Heavyweight aggregates.

Artificial coarse are mainly slag and expanded shale and it can be used for lightweight concrete.

Heavyweight aggregates produce from barites, limonites and magnetites.

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The size of coarse aggregate is larger than 5mm. Fine aggregate is smaller than 5mm and made of sand and it helps to get better well graded aggregates and reduce use of the cement paste. Adding adequate fine material in a concrete mixture will avoid segregation and bleeding. Addition more than necessary fine material will make concrete sticky and unworkable.

Bulk density of aggregates determine the weight and category of concrete whether it is lightweight, normal or heavyweight concrete.

### 1.5 Cement

For describe cement in general sense, can say that it is a binder which used for adhesive other material together. Revolution in cement industry start in early years of industry age.

Cement divides in two categories, Hydraulic cement and non-hydraulic cement.

Hydraulic cement is also called water resistance cement or Portland cement. Non-hydraulic cements are gypsum and lime cement.

Portland cement is the most used cement in structural concrete. The main Portland clinker is limestone.

The limestone is mixed with bauxite, quartz and gypsum to provide the correct composition of oxides.

The production of Portland cement is depend on the resource and local availabilities to materials.

Limestone is a calcium silicates product and can be replace by chalk, marl and sea shell. They are all common material sources of calcium silicate. The Portland cement clinker also contains iron, aluminates, and aluminoferrites of calcium.

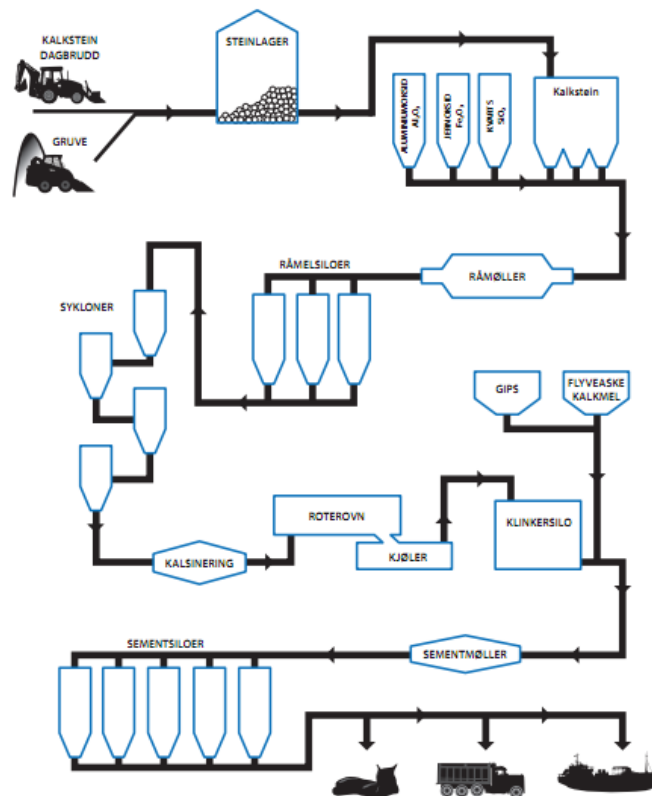
*Table 1-2: Oxides and clinker compound of Portland cement ref[2]*

Abbrevation	Oxid	Name	Compound	Abbrevaition
C	CaO	Calcium oxide, or lime	3 CaO • SiO <sub>2</sub>	C3S
S	SiO <sub>2</sub>	Silicon dioxide, or silica	2 CaO • SiO <sub>2</sub>	C2S
A	Al <sub>2</sub> O <sub>3</sub>	Aluminium oxide, or alumina	3 CaO • Al <sub>2</sub> O <sub>3</sub>	C3A
F	Fe <sub>2</sub> O <sub>3</sub>	Iron oxide	4 CaO • Al <sub>2</sub> O <sub>3</sub> • Fe <sub>2</sub> O <sub>3</sub>	C4AF
M	MgO	Magnesium oxide	4 CaO • 3Al <sub>2</sub> O <sub>3</sub> • SO <sub>3</sub>	C4A3S

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*Table 1-3: Notation and mechanical properties of different cement ref[3]*

Product Name	Norcem Standard	Norcem Standard FA	Norcem Industri	Norcem Anlegg	Norcem Anleggsement FA
Nation according NS-EN197-1	CEM I 42,5 R	CEM II/A-V 42,5 R	CEM I 42,5 R	CEM I 52,5 N	CEM II/A-V 42,5 N
Compressive strength Mpa					
Setting time(min)	130(B)/125(K)	130	100	120	165
1day	21	21	32	18	13
2days	32	31	40	30	22
7days	42	40	49	46	35
28 days	52	52	57	60	53



*Figure 1-1: overview over the cement production process ref[4].*

### **1.6 Chemical Admixtures**

Admixtures are used as ingredients of concrete and are added to the batch immediately before or during the mix operation. The purpose of using admixtures is for change the properties of concrete such as increasing the strength, workability, plasticity, durability of concrete. Admixture can be used for reducing water demand and accelerating the cement hydration reaction.

Types of admixtures can be summarized such as:

- Accelerating admixtures
- Air-entraining admixtures
- Water-reducing admixtures
- Superplasticizers
- Polymers and latexes
- Antifreeze admixtures
- Anti-washout admixtures for underwater concreteing
- Anti-shrinkage admixtures

### **1.7 Plasticizers/Superplasticizers**

Superplasticizers admixture is most used admixture in Norway, approximately 95% of all admixtures sold in Norway[7]. This admixture has ability to reduce three to four times the mixing water in given concrete mixture compared to normal water-reducing admixtures[2]. Workability and slump retention would be approved by adding superplasticizers admixture at batching plants without any refreshing of the concrete at work site. Adding of superplasticizers requires experience, by overdosing will concrete start segregation and can be described as like as soup.



*Figure 1-2: overdosing of superplasticizers admixture*

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#### **1.8 Pozzolan**

Pozzolan is type of active mineral additions which can be added to cement in manufacturing or during concrete mixing. The important effect of pozzolan is increasing the long-term strength of concrete.

There are two types of pozzolans used in Norway, Fly Ash and Silica Fume.

##### **1.8.1 Fly Ash**

Fly Ash is added directly under cement production, Norcem produces two types FA cement in Norway, NORCEM STD-FA with 20% FA and ANLEGG FA with 30%FA.

Particle size of FA is around 10-20 $\mu$ m which is the same size as Portland cement.

FA in Portland cement will increase the long term strength of concrete and reduce cement consumption. Anlegg-FA cement is usually used for producing high strength concrete. The benefits of Anlegg-FA can be named such as:

- Reducing alkali contents in concrete
- Low temperature development
- Excellent workability and permeability
- reducing risk of bleeding

Compressive strength of concrete with FA will be low at early age (3-7days) because of low temperature development.

FA is an industrial byproduct from Coal industry and since there is no

##### **1.8.2 Silica Fume**

Silica Fume has a particle size of 0,1 $\mu$ m and it is a byproduct of smelting process of silicon metal (SiO) and ferrosilicon alloys. SF has become a very efficient filler because of the small size and it effect directly on fresh concrete properties. The benefits of using SF in concrete is depend on volume percentage of cement replacing which is normally up to 20%. ref[5]

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Benefits can be summarized such as:

- increasing compressive Strength
- increasing workability
- reducing permeability
- reducing reinforcement corrosion
- more coherent
- eliminate bleeding
- homogenous concrete
- increasing frost resistance
- finer pore structures

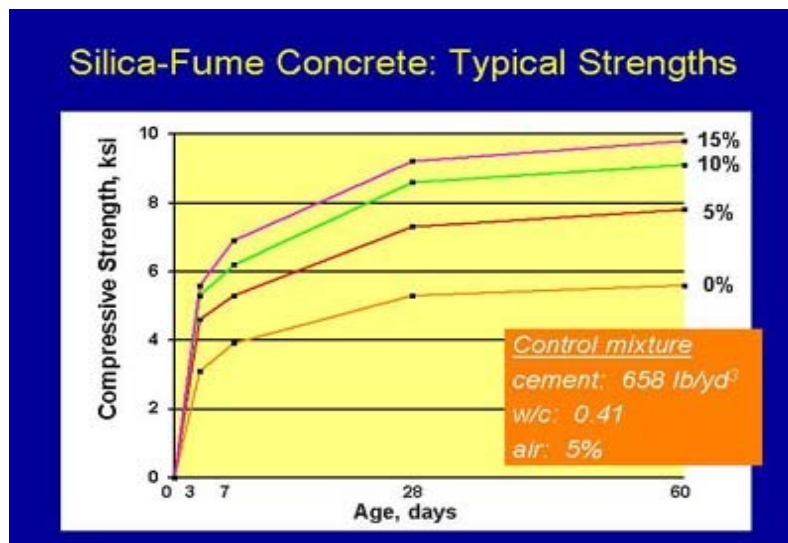


Figure 1-3: Effects of increasing silica fume.ref[6]



### **1.9 Self-compacting concrete (SCC)**

Durable concrete structure requires professional compaction by skilled workers. The reduction in the number of skilled workers and increasing the cost of construction's site in Japan, made Japanese engineers to find a new concreting method in early 1980s. The research led them to self-compacting concrete (SCC) which is a durable concrete and can reach every corner of a form-work without vibrating compacting. SCC is made of exactly same material as normal concrete with different in matrix volume, often higher than 340 l/m<sup>3</sup>. SCC is a matrix dominated concrete.

SCC must have high deformability and good resistance to segregation between coarse aggregate and mortar when the concrete flows through the confined zone of reinforcing.

NS-EN 206-9:2010 is the European standard for SCC and requirement for slump-flow has been given in 3 classes which shows in table 1-4.

*Table 1-4: Slump-flow classes [table 1:NS-EN 206-9:2010]*

Class	Slump-flow in mm
SF1	550 to 650
SF2	660 to 750
SF3	760 to 850

Water/binder (w/b) ratio in SCC is normally lower than 0.45 which gives a matrix dominated concrete. With higher w/b ratio than 0.45 is necessary to change the concrete composition for increasing the matrix volume. Increasing in matrix volume can cause separation because of low viscosity in matrix, it need more attention to keep matrix viscosity high enough to avoid separation.

Norwegian experience recommended to keep w/b ratio lower than 0.45 and higher than 0.3. Low w/b ratio can cause poor stability, more plastic shrinkage and drying shrinkage and gives higher strength than was planned. With w/ b ratio lower than 0.3 will concrete become very tough and flows heavily because of high matrix volume. [7]

Superplasticizers admixture is a important aids for given better flow and slump result in low water/cement ratio. Using of air-entrained admixture will increase the level of cohesion and stability of the final product and it will protect SCC against freezing.

SCC may show more plastic shrinkage and creep than normal concrete because of higher cement pasta

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to aggregate ratio. Adding silica fume in SCC gives a better stability but it is depend on the amount of addition.

Benefits of using SCC can be summarized such as:

- Decrease construction's time
- Improved durability
- Better surface finish
- Reduction of site manpower
- Better safety working environment

## Chapter 2: Ductility of High Performance Lightweight Concrete

### 2.1 Introduction to lightweight concrete

The density of normal concrete is  $2400 \text{ kg/m}^3$ , with replacing normal aggregate with lightweight aggregate will density of concrete decrease to  $1750 \text{ kg/m}^3$ . Reducing of dead load up to 35%, will allowing greater design flexibility, reducing size of structure, longer spans and decrease reinforcing steel bars. The density class of LWC is showed in table 2-5.

Table 2-5: Density class for LWC

Density class	1	1.2	1.4	1.6	1.8	2
Density( $\text{kg/m}^3$ )	801-1000	1001-1200	1201-1400	1401-1600	1601-1800	1801-2000

There are two types lightweight aggregate, manufacture which are produced by expanding products such as Vermiculite or perlite, calcining, blast-fumace slag, clay, fly ash, shale or slate. Second type is from natural materials, such as pumice, tuff or scoria.

The shape of particles can be rounded, angular or irregular. It is depend on source and production process. The lightweight aggregate which are produced in Norway have rounded shape and called Leca. Lightweight aggregates is barley used in structural concrete in Norway. The production of Leca is depending of each projects.

The southwestern Bell Telephone office building [8]in Kansas City is a great example of how effective lightweight concrete can be. Building was built originally 14 storeys structure but in 1928 the owner wished to add more storeys on the same foundation. After study the building they found out that foundation would support eight more storey with using normal density concrete. But if they replace concrete with lightweight concrete would be able to add 14 storeys on the same foundation. That was start of using lightweight concrete. The first structural lightweight concrete was built in 1929 in St. Louis, they designed both frame and floor systems by lightweight concrete.

## **2.2 High performance lightweight aggregates concrete**

The proportioning of high performance lightweight concrete is based on cementitious material content such as cement type, chemical admixture and pozzolanic. The recommended slump is between 76 to 100 mm for workability to be proved ref.[18].

The water/cementitious ratio is essential for the compressive strength and need extra attention. Because of larger cellular structure of lightweight coarse aggregate, it is important to remember that it has a high degree of water adsorption (up to 20%, depend on aggregates type). In case of Leca it is 10% water absorption in first 24 hours and up to 25% after 28 days ref[appendix 1a].

## **2.3 Lightweight self-compacting concrete**

The trial batch tests by Ben C. Gerwick in California [9] showed that lightweight concrete with a slump flow below 430 mm is not likely to have full compacting performance. Lightweight concrete with a slump flow over 660 mm often leads to severe segregation. Slump flow in the range of 460 mm to 560 mm provided the required workability[9]. European standard for SCC [NS-EN 206-9:2010] does not apply to lightweight SCC because of limited experience. However, it requires a slump flow between 550 to 650 mm for class SF1 as it showed in table 2-5 ref [NS-EN 206-9:2010].

The most challenge to produce self-compacting lightweight concrete is the workability, low density of lightweight aggregates make flowability and cohesion low. For concrete become self compacting it needs to be able to flow under the own weight without any need for vibration or additional compaction. Separation of concrete depends on stability of concrete. Stability increases with high flow resistance of the matrix. Higher workability decrease flow resistance, which results decreasing in the stability and risk for separation.

Flow resistance of the matrix  $\lambda_Q$  determined by FlowCyl-tests, experience shows when flow resistance become larger than approximately 0.75 may concrete become too viscous or tough which will cause low flowability. Flow resistance lower than approximately 0.5 may cause concrete bleeding and segregating. Ref.[7]

The shape and size of lightweight aggregate affects the workability and strength of concrete. Leca which is manufactured in Norway, have rounded particle shape and the density is lower than angular aggregate which results lower strength but better workability.

The total cementitious materials content should be approximately 550 kg/m<sup>3</sup> based on researches has been done on self-compacting lightweight concrete. ref [9]

## 2.4 Mechanical properties of LWC

### 2.4.1 Compressive strength

Normal aggregates has a higher strength and stiffness than cement pasta in normal concrete, which means that aggregate carry most part of tension in concrete. That will cause a tensile stress between mortar and aggregate which will occur micro-crack after a while. When aggregates capacity exceeded will tension in micro-cracks become larger and cracks grow. In that phase concrete capacity is exceeded and crack go through cement pasta. ref[10]

However, lightweight aggregate and mortar has almost the same stiffness, and produce a higher elastic compatibility leading to reduces stress concentrations in the mortar-aggregate interface. Low w/c ratio and using of admixtures will make sure a good mortar-aggregate interface which can also prevent bleeding. When tensile stress in LWC become larger, lightweight aggregate can not handle the stress and it will cause a suddenly crack and fracture which will go through mortar and aggregates. That make compressive strength of LWC a function of lightweight aggregate and cement paste strength and stiffness.

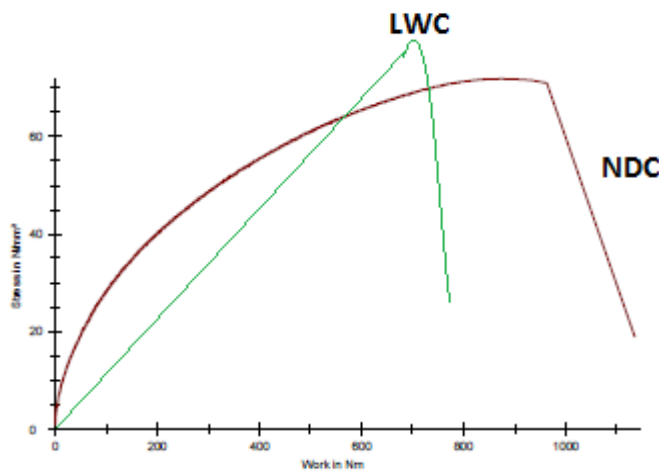
When strength of concrete become larger than lightweight aggregates strength, mortar has to carry most part of concrete tensile stress. For high-performance LWC increases the mortar stiffness and strength.

Table 2-6: Strength classes for LWC table 11.3.1 EC2

f <sub>lck</sub> (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80
f <sub>lck,cube</sub> (MPa)	13	18	22	28	33	38	44	50	55	60	66	77	88
f <sub>lcm</sub>	17	22	28	33	38	43	48	53	58	63	68	78	88

### 2.4.2 E-modulus

LWC has a linear loading curve and faster fall curve compare to NDC. LWC give more strain in maximum stress than NWC which cause lower E-modulus compare to NDC. Low E-modulus and higher strain in LWC will prevent shrinkage. ref[10]



E-module for LB50

$$f_1 = (\rho / \rho_1)^2 \quad \rho_1 = 2200 \text{ kg/m}^3$$

$$\rho = 1932 \text{ kg/m}^3$$

$$f_1 = 0.77$$

E-module for lightweight concrete

$$E_{LWC} = f_1 \cdot E_{NWC}$$

$$E_{NWC,B50} = 37 \text{ GPa} \quad \text{table 3.1 EC2}$$

$$E_{LWC} = 0.77 \cdot 37 = 28.5 \text{ GPa}$$

Figure 2-4: Stress-strain diagram LWC & NDC

### 2.4.3 Cracking and shrinkage

The most critical time for concrete is when concrete starts hardening. Most properties of concrete develops during this phase. Hardening process is between 1 to 2 weeks and it start between 6 to 12 hours after casting.

C-S-H adsorbs amount of water, when a hydrated cement paste is under sustained stress will C-S-H loss a most part of physical adsorbed water and it cause a creep strain in cement paste.

Reducing of cement paste volume during hardening of concrete will cause cracking and shrinkage in concrete ref.[5].

High strength concrete has a tendency to develop early age cracking. Early ages defines as minutes after casting when the concrete is in plastic age and some hours after casting when concrete is in early hardening age. The typical cracking problem will be caused of plastic settlement and plastic shrinkage.

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It has been assumed that LWC crack development behaves the same as NWC in early ages. It should be noted that Leca which has a particle density of 1300 kg/m<sup>3</sup> have a water absorption about 40 liter per m<sup>3</sup> concrete, which corresponds to a volumetric shrinkage of 40 000 μstrain. ref[11]

The creep coefficient  $\phi$  for LWC is equal to creep coefficient  $\phi$  of normal density concrete (NDC) multiplied by  $(\rho/2200)^2$ .

The creep strains is the same as NDC by strength classes higher than LC16/18, but the final drying shrinkage value for LWC increases 20% compare to NDC.[ref.EC2 11.3.3]

$\eta_3$  is a coefficient for determining creep coefficient

$$\eta_3 = \begin{cases} 1.5 & \text{for } f_{lck} \leq LC16/18 \\ 1.2 & \text{for } f_{lck} \geq LC16/18 \end{cases}$$

In case of relative humidity 40%:

The drying shrinkage values for NDC  $\epsilon_{cd,0} = 0.46\text{‰}$

The drying shrinkage values for LWC  $\epsilon_{cd,0} = 0.46 \times 1.2 = 0.552\text{‰}$

Type of shrinkage can summarized as:

- Plastic shrinkage: It occurs in fresh phase of concrete. The cause is water evaporating from the surface of the concrete. Plastic shrinkage can be expected in hot summer days and by strong wind.
- Plastic settlement: when concrete start bleeding in plastic phase, the solid particles move downward and water stays on top and cover the surface. The mechanism is as same as plastic shrinkage but in higher degree and depth.



Figure 2-5: Plastic shrinkage,ref[12]

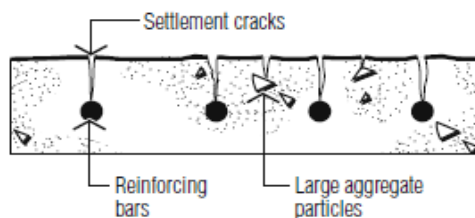


Figure 2-6: Plastic settlement ref[13]

- Autogenous shrinkage: Hydration process starts when cement react with water. The absolute volume will decrease when cement paste is in a liquid state. The macroscopic volume change occurring with no moisture transferred to the exterior surrounding environment cause Autogenous shrinkage which is an external volume change ref[24].
- Chemical shrinkage: It starts at same time with autogenous shrinkage. Pores in the binder phase start growing because of volume loss, which also increases air volume in the capillary pores. That means that shrinkage turns into air voids within hardened cement paste. Chemical shrinkage is an interval volume reduction ref[24].
- Drying shrinkage: The mechanism is the same as plastic shrinkage but it expected in a stiff concrete. The shrinkage cracks on surface start after evaporation of surface's water and the pressure of negative pore develops crack and shrinkage on surface of concrete.



Figure 2-8: Drying shrinkage ref[15]

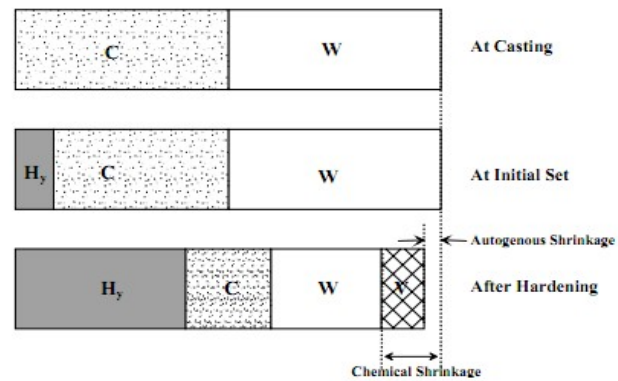


Illustration 2-7: Reactions causing autogenous and chemical shrinkage. [24] C = unhydrated cement, W = unhydrated water, Hy = hydration products, and V = voids generated by hydration.



## **2.5 Non-steel reinforcement**

Non-steel reinforcement can be categorized as fiber reinforced polymer (FRP) which can also be called fiber reinforced plastic (FRP) and Glass reinforced plastic (GRP).

Plastic fiber was used since the 1960s in some structural elements but not as main reinforcement bars. The important development of plastic fiber started in the 1980s. The new development of medical equipment such as magnetic resonance imaging (MRI) became the main reason for focus on use of plastic fiber as main reinforcement bars in concrete. MRI cannot tolerate the presence of any steel reinforcement. Engineers found a solution which was using glass fiber reinforcement for surrounding and supporting MRI.

Environmental and chemical attacks became new considerations for using glass fiber as composite reinforcement in construction such as sea walls, industrial roof decks, chemical industry floor slabs and reactor equipment.

The major consideration in using plastic reinforcement as main reinforcement is the bond between fiber rods and concrete. The transformation of stress from the reinforcement to concrete depends mainly on the bond interaction between those two materials.

Four types of bond failure are identified ref[18]:

1. Shearing off failure of the rod windings
2. Local mechanical frictional pull-out (the rods are ribbed, and frictional resistance covers only a part of the surface)
3. Full frictional pull-out where failure resistance is along the entire rod length
4. Plain frictional resistance in isolated strand rods

These four types of bond failure show that the bond strength depends on rod surface configuration and deformation.

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Benefits of using PP fiber:

- Increasing fire resistance
- Increasing concrete density and durability
- Reducing amount of cracks from plastic shrinkage forces
- Reducing risk of separation/bleeding
- Faster and better construction

Type of non-steel reinforcement:

- Polyolefin
- Polypropylene (twisted, fibrilated and monofilament)
- PVA
- Nylon
- Ar-glass
- Wollastonite

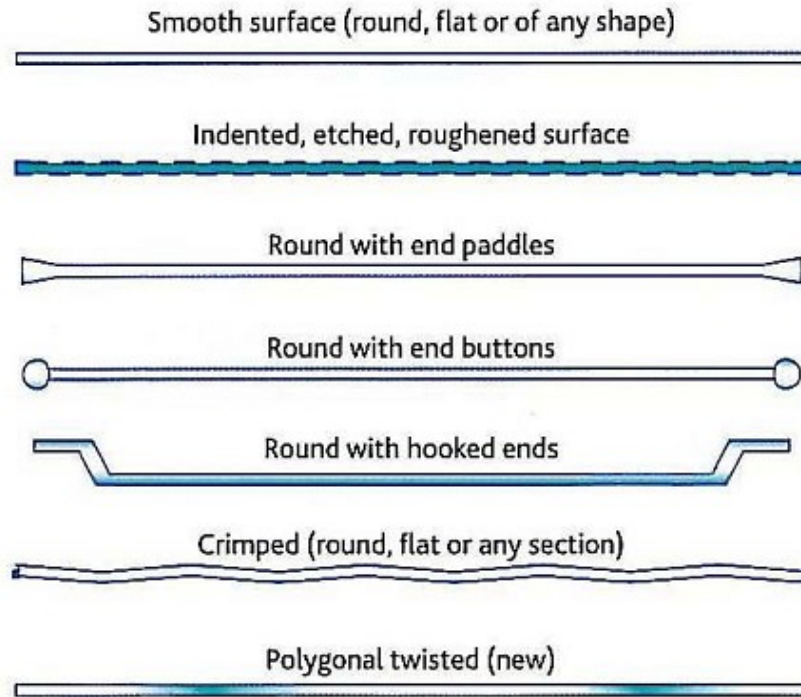
## **2.6 Steel fiber concrete**

Concrete has high compressive strength but very low tensile strength capacity. Steel reinforcing bar carry the tensile strength in concrete and it prevent development of cracks under loading. Concrete structural members under loading develop crack before reinforcement take over the tensile forces. The cracks occur because of low tensile strain capacity of concrete. Cracks will reduce service life of concrete by reducing durability of concrete. Low durability will reduce the ability of concrete to resist weathering action and chemical attack. Development of cracks could be limited by using of steel-fiber. Steel-fiber has high tensile strength and would take over the concrete tensile stress at early crack development phase. Fibers over the crack in concrete will decrease moment, shear and punching resistance. When a crack start growing, the fibers will transmit tensile force over the crack into the surrounding concrete ref.[17].

Steel bar reinforcement can be replaced by steel fiber and be added direct into the concrete at batching plant. Steel fiber-reinforced concrete(SFRC) is most used for slabs on grade, it reduce work time on construction and it reduce cracks compare to steel bar reinforcement.

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The quality of SFRC is depend on how good fibers are distribute in concrete.  
Self compacting concrete improve the homogenous distribution of fiber in concrete. Vibrator compacted concrete (VCC) may cause poor distribution of fiber and give critical and weak points in concrete.



*Figure 2-9: Type of steel fiber ref[17]*

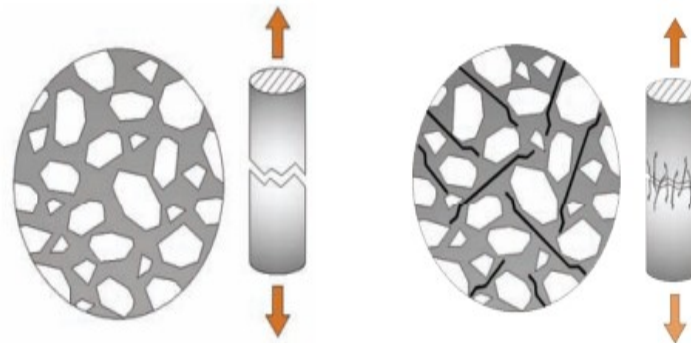
The volume fraction of fibers added to a concrete mix is measured as a percentage of the total volume of the concrete and fibers. The respect ratio of fiber is measured by dividing fiber length by fiber diameter.

Volume percentage of steel fiber is usual between 0.2%(20kg/m<sup>3</sup>) to 2%(157kg/m<sup>3</sup>). By adding more than 2% volume fraction, may cause poor concrete workability and reducing fiber dispersion.

### 2.6.1 Mechanical properties of steel-fiber concrete

The steel-fibers have little or no effect on compressive strength and E-modulus of concrete, in some cases it may increase the compressive strength but it is not mentionable. The E-modulus and compressive strength should be determined experimentally when volume fraction of steel-fiber is greater than 1%.

Tensile strength of concrete increase by adding steel-fiber ref[16]. The effect of fiber on direct tensile strength is depend on the direction and distribution of fibers. Measurement of the direct tensile strength of SFRC is not given in any standard and it has no agreement how it can be tested by experiment. Earlier researches where distribution of fiber were randomly, showed that the tensile strength increase up to 60%, but it is depend on how homogenous is the distribution ref[16].



*Figure 2-10: The influence of steel fiber content on tensile strength*

Steel-fibers have most effect on the flexural strength of concrete, increasing up to 100% has been reported by earlier experiments ref[16]. Aspect ratio of the fibers and the fraction volume of fibers are important parameters for increasing the flexural strength.

The post-crack tensile strength of SFRC is depend on the distribution and orientation of fibers, as earlier been mentioned the casting process, workability and the size of specimen are important parameters.

## **2.7 Fiber bridging**

The main reason of using steel fiber is the increasing the toughness and energy absorption capacity of concrete which can lead to a greater ductility. Fibers bridge across crack and prevent it from growing and slow down the development of cracks opening in the concrete. Fracture or pull out of the fibers are depend on bond strength between concrete and fibers. For concrete become more ductile is best that fibers pull out, that means enough bond strength is important for ductility in concrete. The bond between fibers and concrete is primary decided from friction between the fibers and concrete. The shape of fiber is important parameter for the bond strength and amount of energy observation. For example end-hooked fibers give better friction and bond strength because of deformed ends. The end hooked needs to bend and yield before it can be pulled out of concrete. For pulling out a hooked steel fiber requires a work that is four times greater than for a smooth steel fiber ref[21].

For avoid fiber rupture the yield capacity of the fiber has to be sufficient. Pull out failure need lot of energy to take place which result a higher ductility ref[21].

Neutral axis in concrete beam section will move toward the compressive face when crack in concrete start growing. This movement of neutral axis become less and equilibrium by fibers pullout resistance and fibers bridging the cracks ref[22].

## **2.8 Material qualities for steel fiber reinforced concrete (SFRC)**

Norwegian preliminary guidelines (NPG) describe the residual stress as:

“The residual stress of SFRC based on assumption that steel fibers crossing a concrete crack contributes to the tension capacity of the reinforces concrete in the same way as for reinforcement bars. There will be no main change of direction of the fibers direction at the crack, the another assumption is that maximum force in a steel fiber at a crack is defined by the fiber's anchorage capacity” The residual stress of SRFC is showed in figure 2-11 and it is given by NPG:

$$f_{tk, res} = \eta_0 v_f \eta_1 \sigma_{fk, max} = \eta_0 v_f \sigma_{fk, mid}$$

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where

- $v_f$  Fiber volume ratio (fiber volume)/(concrete volume)
- $\sigma_{fk,max}$  Maximum stress of fiber with anchorage length  $l_b = l_f/2$  at a crack, decided by bond and upper yield limit
- $\sigma_{fk,mid}$  Average stress in all fibers bridging a crack, with random embedded length and orientation, based on experience and experimental, SINTF recommended 500MPa for steel fiber and 250MPa for PP fiber
- $\eta_1$  Aspect ratio,  $\sigma_{fk,mid} / \sigma_{fk,max}$
- $\eta_0$  approximate 1/3 for randomly 3D distribution and orientation of fibers, 1/2 for fibers in planes parallel to tension direction

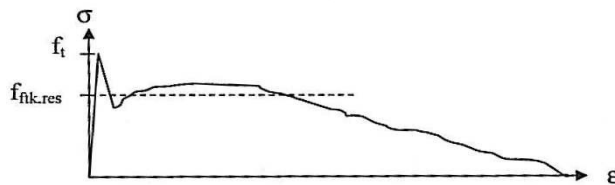


Figure 2-11: Typical tension behavior for SFRC ref. [17]

**2.8.1 Calculation of the residual strength in fiber reinforced concrete(FRC)**

Residual strength of steel-fiber and PP-fiber in case of FRC has been calculated based on NPG formulation, this calculation is for steel-fiber with 2 % volume fraction ref[17]:

$$f_{tk,res} = \eta_0 v_f \sigma_{fk,mid}$$

$$v_f = V_f / V_c = 0.02$$

$$\sigma_{fk,mid} = 500$$

$\sigma_{fk,mid}$  This value is given by SINTEF, it is based on experimental result.

$$f_{tk,res} = 1/3 \times 0.02 \times 500 = 3.33 MPa$$

$$f_{fd,res} = f_{tk} / \gamma_m = 3.33 / 1.55 = \underline{\underline{2.15 MPa}}$$

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The residual strength for steel-fiber will be 2.15MPa. The residual strength in PP-fiber decrease approximate half part residual strength than steel-fiber, calculation shows residual strength for plastic fiber with 2% volume fraction:

$$f_{tk,res} = \eta_0 v_f \sigma_{fk,mid}$$

$$v_f = V_f / V_c = 0.02$$

$$\sigma_{fk,mid} = 250MPa$$

$$f_{tk,res} = 1/3 \times 0.02 \times 250 = 1.67MPa$$

$$f_{fid,res} = f_{tk} / \gamma_m = 3.33 / 1.55 = \underline{\underline{1.1MPa}}$$

Strain and Stress distribution over a SFRC cross-section has been described in (NPG) as showed in figure2-11.

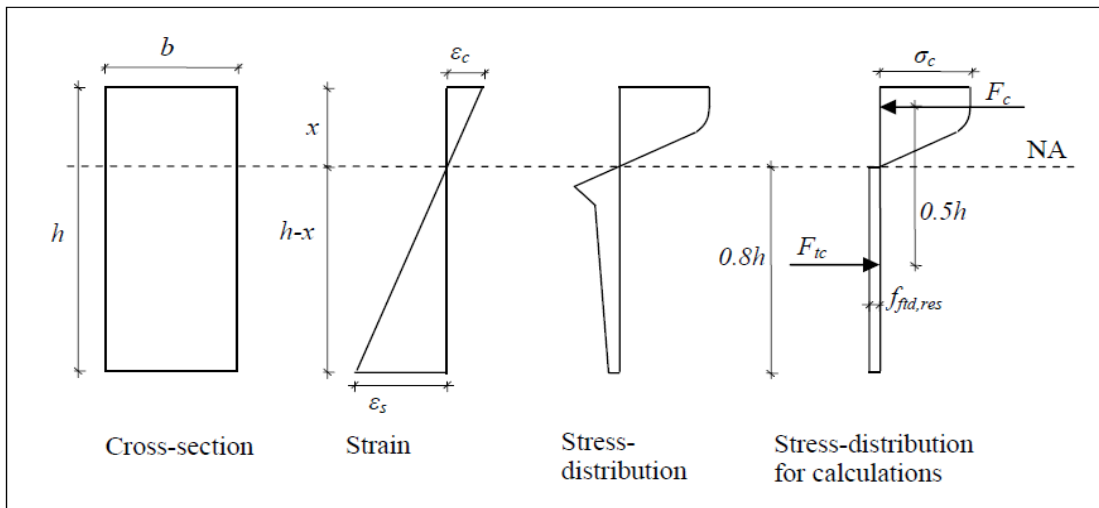


Figure 2-12: Strain and Stress distribution over a SFRC cross-section ref[17]

The stress and strain illustration is only for concrete with fiber reinforcement FRC. It has to be noted that when residual stress  $f_{tk,res}$  become larger than 2.5 N/mm<sup>2</sup>, the cross-section's compression zone height has to be determined.

The moment capacity of SFRC can be determined by this equation:

$$M_{fid,res} = 0,4 f_{fid,res} \times p \times e \times b \times h^2$$

$$f_{fid,res} = f_{tk,res} / \gamma_m \quad \gamma_m = 1.55$$

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Here  $p$  presents scale factor for load carrying capacity of SFRC which is depend on dimension of beams and  $e$  is yield factor for residual strength for self-compacting concrete (SCC).

$$p=1,1-0,7h>0,75$$

$$e=0,9 \text{ in upper parts of the element for SCC}$$

$$e=1,2 \text{ in lower parts of the element for SCC}$$

$$e=1 \text{ when the residual strength is determined by bending tests of beams made of SCC}$$

## 2.9 Concrete Reinforced composites

Definition of a composite material has been defined in ASTM 3878-95c as:

“Composite Material. A substance consisting of two or more materials, insoluble in on other, which are combined to form a useful engineering material possessing certain properties not possessed by the constituents.”

The difference between fiber reinforced concrete (FRC) and concrete reinforced composite is that FRC has no steel bar reinforcement in concrete.

NPG requires that all structures with safety level 2 or higher has to used reinforced composites, to make sure that conventional bars transfer all external forces in addition to the fibers. Because of using conventional bars all safety factor for fiber can be changed to  $\gamma_m=1$ .

The requirement is because of limited experience with fiber reinforced concrete.

Strain and stress distribution over a cross-section reinforced with both reinforced bar and steel fibers is showed in figure 2-13.

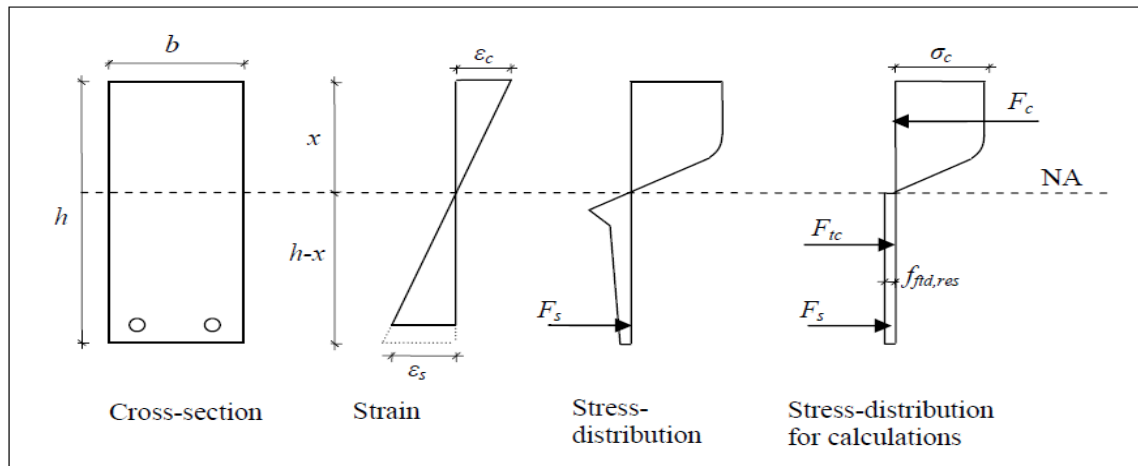


Figure 2-13: Strain and stress of concrete reinforced composite ref.(17)



By removing the safety factor, will the residual strength of concrete reinforced composite for steel fiber with 2% volume fraction be 3.33Mpa and for plastic fiber with 2% volume fraction be 1.67MPa.

$$f_{fd, res} = f_{tk, res}$$

**2.10 Shear in concrete reinforced composites**

Shear strength increase with using steel fibers in concrete. Calculation of shear strength in SFRC can be determined by:

$$V_{fd} = 0,8 f_{fd, res} \times p \times b \times d$$

Shear strength because of conventional reinforcement can be determined by EURO CODE2:

$$V_{Rd,s} = \frac{A_{sw}}{s} \times z \times f_{yw} \times \cot \theta$$

Total concrete's shear strength :

$$V_d = V_{cd} + V_{Rd,s} + V_{fd}$$

Shear capacity of beam with SFRC by volume fraction 2% will increase 34% compare to a beam without fiber. [apandix2]

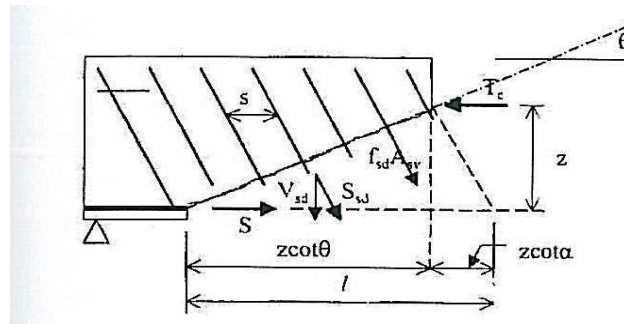


Figure 2-14: Shear forces on beam ref[17]

Diagonal cracks can be reduced and controlled by adding stirrup reinforcement in beam. Reason of using stirrup reinforcement is to carry the redistributed shear stresses, mainly through tension after the formation of diagonal cracks. Tension will be transferred back to the concrete. [21] This will cause more diagonal cracks, and cracks opening will grows slowly compare with no stirrup reinforcement. Addition of stirrup reinforcement will prevent the shear failure in beam. The steel-fibers increase shear capacity and by calculation can reduce use of stirrup.

## 2.11 Crack Width Analysis for normal reinforced concrete

Theoretical analysis of crack width of beam with a bar reinforcement is given in EC2 7.3.4 as:

$$W_k = s_{r,max} (\varepsilon_{sm} - \varepsilon_{cm})$$

$s_{r,max}$  maximum crack spacing

$\varepsilon_{sm}$  the mean strain in the reinforcement

$\varepsilon_{cm}$  the mean strain in the concrete

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0,6 \frac{\sigma_s}{E_s}$$

$\sigma_s$  the stress in the tension reinforcement assuming a cracked section

$\alpha_e$  is the ratio  $E / E_{cm}$

$$\rho_{p,eff} = \frac{A_s + \xi_1^2 A_p'}{A_{c,eff}}$$

$$A_{c,eff} = b \times h_{c,ef}; \quad h_{c,ef} = \min \{ 2,5(h-d); (h-\alpha d)/3; h/2 \}$$

$$k_t = \begin{cases} 0,6 & \text{for short term loading} \\ 0,4 & \text{for long term loading} \end{cases}$$

$s_{r,max}$  present the average final crack spacing.

where

$$s_{r,max} = k_3 \times c + k_1 k_2 k_4 \times \phi / \rho_{p,eff}$$

$\phi$  is bar diameter

$c$  is the cover to the longitudinal reinforcement

$$k_1 = \begin{cases} 0,8 & \text{for high bond bars} \\ 1,6 & \text{for bars with an effectively plain surface} \end{cases}$$

$$k_2 = \begin{cases} 0,5 & \text{for bending} \\ 1,0 & \text{for pure tension} \end{cases}$$

$$k_3 = 3,4 \quad ; \quad k_4 = 0,425$$

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Table 2-7 shows the maximal crack width in all exposure class.

Table 2-7: Values of  $w_{max}(mm)$ (table 7.1N EC2)

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4 <sup>1</sup>	0,2
XC2, XC3, XC4	0,3	0,2 <sup>2</sup>
XD1, XD2, XS1, XS2, XS3		Decompression
<p><b>Note 1:</b> For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.</p> <p><b>Note 2:</b> For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.</p>		

It is important to be noted that crack spacing in SFRC is depend on bond between concrete and steel fibers. If steel fiber's bond with the concrete is stronger than permitted will it cause brittle crack on steel fibers and if bond is poor, fiber will have no effect.

**2.12 Crack width analysis for fiber and conventional reinforcing**

Analysis of mechanical behavior of concrete become complicated by adding fiber to concrete mix. There are parameters such as fiber substrate, fiber geometry, fiber volume concentration, type of fiber, the distribution and orientation of fibers which are important to be in consideration in analysis of concrete members. The problem is that before adding fibers, a concrete structure member has already lot of design parameters such as stiffness, concrete strength, conventional reinforcement type, structural geometry and reinforcement type which it can be difficult to connect all those design parameters. The orientation and distribution of fibers are depend on workability and type of concrete such as self compacting concrete or vibrate compacting concrete. Use of vibrator can have negative effect on distribution of fibers. The each point where vibrator has been pocked can be empty of fibers and make it the weakest point in concrete. However, it has been done lot of studies on how crack width can be decrease by adding fiber to concrete mix.

To show how fiber affect crack width can use stress-crack width model. To calculate it theoretical it

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need to drive a micromechanical to find the stress-crack width relationship. [23]

There are few guidelines for crack width analysis for concrete reinforced composite, RILEM TC 1622-TDF is one of few guidelines for determine the crack width. The problem with this formulation is that it does not take the effect of fibers volume fraction and rotation in consideration.

$$S_{r,max} = \left( 50 + 0,25k_1 \times k_2 \frac{\phi}{\rho_{eff}} \right)^{\frac{1}{3}} \times \left( \frac{50}{L_f / \phi_f} \right)^{\frac{1}{3}}$$

$k_1$  is coefficient which takes account of the bond proerties of the bars

$k_2$  is coefficient which takes account of the form of the strain distribution

$\phi$  is bar size

$\rho_{eff}$  is effective reinforcement ratio  $\frac{A_s}{A_{c,eff}}$

$A_s$  is the area of the reinforcement contained

$A_{c,eff}$  is the effective tension area

$L_f$  is the fiber length

$\phi_f$  is the fiber diameter

As it showed in formula  $S_{r,max}$  multiplied by new parameter which take fiber length and diameter in perspective not volume fraction. The new parameter has to be equal or less than 1, this is depend on the aspect ratio of fiber. This parameter shows that use of fiber will decrease the crack spacing which will cause increasing the number of cracks and decreasing of crack opening.

To be noted that steel fiber with aspect ratio 50 will have no effect on crack spacing based on this parameter. Polypropylene microfiber (PP-fiber) have a high aspect ratio which cause error in this calculation.

COIN-rapport (may 2011) has present new guidelines for calculation of crack width for fiber-reinforced concrete. Formulation is principle identical to EC2 7.3.4 (section 2.11) except addition of a factor  $k_5$  to calculation of maximum crack spacing. This factor takes the fiber additive into the consideration.

$$w_k = s_{r,max} (\epsilon_{sm} - \epsilon_{cm})$$

$$s_{r,max} = k_3 \times c + k_1 \times k_2 \times k_4 \times k_5 \times \frac{\phi}{\rho_{s,eff}}$$

$$k_5 = (1 - f_{ftk,res2,5} / f_{ctm})$$

Calculation of the residual strength in fiber reinforced concrete is showed in section 2.8.1.

### 2.13 Ductility of concrete

Ductility is an important property in reinforced structure in seismic zones.

It allowed the structure to deform when the maximum bearing capacity is exceeded and response to in-elasticity in server earthquake. Ductility may be defined as the ability to undergo deformation without a substantial reduction in the flexural capacity of the member (Park & Ruitong 1988). High ductility will cause high deflection when structural member were loaded until failure which will be a warning before total collapse.

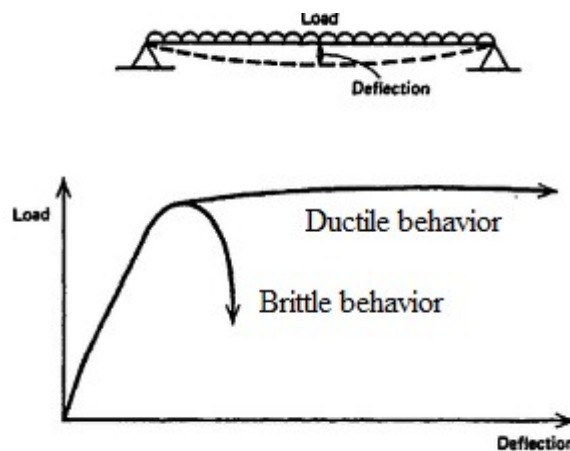


Figure 2-15: Load-deflection behavior of a flexural member

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The ductile failure is initiated by yielding of steel reinforcement, concrete will crush when steel reinforcement begins to plastic deformation and loss the carrying properties.

Ductility is measured in terms of strain, displacement and rotation. [ref.18].

**Strain ductility:**

$$\mu_e = \frac{\varepsilon}{\varepsilon_y}$$

$\varepsilon$  is maximum sustainable strain

$\varepsilon_y$  is the yield strain ductility

**Curvature ductility:**

$$\mu_\phi = \frac{\phi_m}{\phi_y}$$

$\phi_m$  is the maximum sustainable curvature

$\phi_y$  is the yield curvature

**Displacement ductility**

$$\mu_\Delta = \frac{\Delta}{\Delta_y}$$

$\Delta$  is the sum of yield displacement and plastic displacement

For inelastic behavior to be sustainable it should all these ductility factor be grater than 1,0.

Concrete is known as a brittle material with low tensile strain capacity and poor fracture toughness.

With increasing concrete strength expecting more brittleness and decreasing in the concrete strain,  $\varepsilon_{cu}$ .

In this study will get a concrete strain:

$$f_{cm} = 58$$

$$\varepsilon_{cu} = 2.8 + 27[(98 - f_{cm})/100]^4 = 0,0035$$

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In case of concrete reinforced with both steel fiber and reinforcement bars the maximum strain of steel can not exceed 2.5‰(0.0025) ref[17]. This limitation give a higher compression zone and lower internal torque arm. By increasing the maximum strain in reinforcement will capacity increase too.

Brittle behavior of high-performance concrete make lot of engineers skeptical to use it in seismic region. It should be mentioned that ductility is not only depend on strength of concrete. Most part of the ductility performance dependent on reinforcement details and other parameters such as the material characteristics of the concrete, the geometry of structure, the material characteristics of reinforcement and the amount of longitudinal compressive reinforcement.

High strength concrete improved the bond between the reinforcement and concrete which can limited the deformation capacity of reinforcement and decrease ductility.

Ductility can increase by adding compression reinforcement bar in beams. In the case of seismic zones, concretes member should have better ductility performance than normal ductility. It is required more analysis on each structural member geometry and reinforcement design specially when high strength concrete is used.

For seismic zones is important that failure in structural member occurs by ductile flexural failure, instead of shear brittle failure. For avoid shear brittle failure it need to increase shear resistance of structural members by adding secondary shear steel reinforcements bar.

Ductility has relationship with crack of concrete, when cracks occur in concrete because of overloading will fibers bridge the cracks, in that way fibers decrease crack width and increase ductility of concrete .

#### **2.14 Curvature ductility**

Curvature ductility ( $\mu_\phi$ ) describes the ductility of reinforced concrete section and it is a ratio between curvature of concrete strain when it reach the ultimate strain and curvature of yield strength of tension reinforcement at start point of yielding. The moment and curvature at first reinforcement yield can be determined by assuming an under-reinforced section which has been suggest by Park and Paulay (1975). ref[20]

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$$M_y = A_s f_y d''$$

$$\phi_y = \frac{\varepsilon_{sy}}{(1-k)d} = \frac{f_y}{E_s(1-k)d}$$

where

$d''$  is the distance from centroid of compressive forces  
in the steel and concrete to the centroid of tensile

$$k = \sqrt{(\rho + \rho')^2 n^2 + 2(\rho + \rho') \times d / d_1 n} - (\rho + \rho') n$$

$\rho$  is the tensile reinforcement ratio  $A_s / bd$

$\rho'$  is the compression ratio  $A'_s / bd$

The curvature at ultimate of reinforced concrete section can be calculated by finding the maximum value of concrete strain at the compressive fiber ref[20].

$$M_u = 0.85 f_c \times ab(d - \frac{a}{2}) + A'_s f_y (d - d')$$

$$\phi_u = \frac{\varepsilon_c}{c} = \frac{\varepsilon_c \times \beta_1}{a}$$

where

$$a = \frac{A_s f_y - A'_s f_y}{0.85 f_c \times b}$$

$\beta_1$  is the depth of the equivalent rectangular stress block

The value of concrete strain for normal concrete and lightweight concrete has been given in EC2 which is 0.0035.



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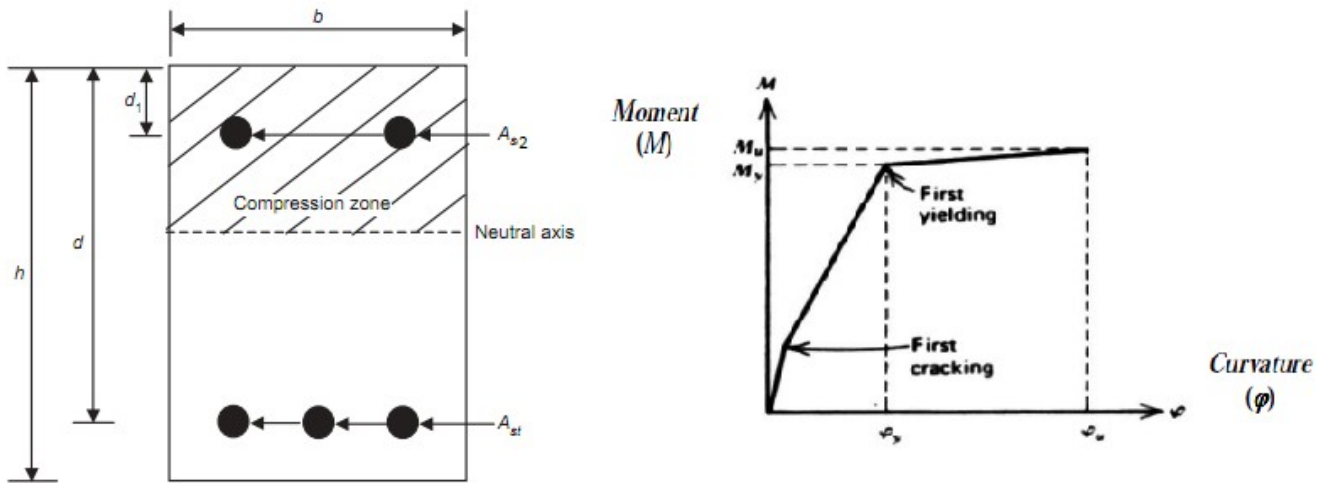


Figure 2-16: Beam section analysis and moment-curvature diagram

When the steel tension increase, will both  $k$  and  $a$  increase which will result a higher  $\phi_y$  and a lower  $\phi_u$ . That means decreasing the ductility. According to curvature formulation will ductility increase by increasing the concrete strength because  $k$  and  $a$  will decrease.

Study of lightweight concrete curvature ductility by adding fiber become more complicated.

Lightweight concrete has lower E-modulus compare to normal density concrete and by adding fiber to the concrete it will influence on the E-modulus.

### **2.15 Concrete fracture mechanical**

Ductility can be studied from fracture mechanical parameters such as fracture energy, brittleness number and characteristics length. By study the relationship between the accumulated elastic energy and the fracture energy in the structure can define brittleness number B (bache 1989).

$$\text{Elastic energy} = \frac{\sigma_0^2}{E} \alpha \beta L^3$$

$$\text{Fracture energy} = G_f \alpha \beta L^2$$

$$\text{Brittleness number } B = \frac{\text{Elastic energy}}{\text{Fracture energy}} = \frac{L\sigma_0^2}{G_f E}$$

Where:

$\sigma_0$  is maximum stress

L is beam length

$G_f$  is fracture energy

E is modulus of elasticity

By study Brittleness number formula can understand that ductility of beam is depend on length of beam and maximum stress, when a length or maximum stress increase will get higher brittleness number which means ductility decrease. For increasing ductility in a specific beam length and maximum stress must increase fracture energy and the elastic modulus.

The maximum stress can be replaced by concrete tensile strength to find the brittleness number and by rewriting formula can get the characteristics length too, in that case can brittleness by length.

$$B = \frac{Lf_t^2}{G_f E} \quad l_{ch} = \frac{EG_f}{f_t^2}$$

**2.16 Fracture energy in concrete**

Besides tensile strength and modulus of elasticity, fracture energy should also be in consideration in ductility analysis. The fracture energy can be measured by three point beam bending test.

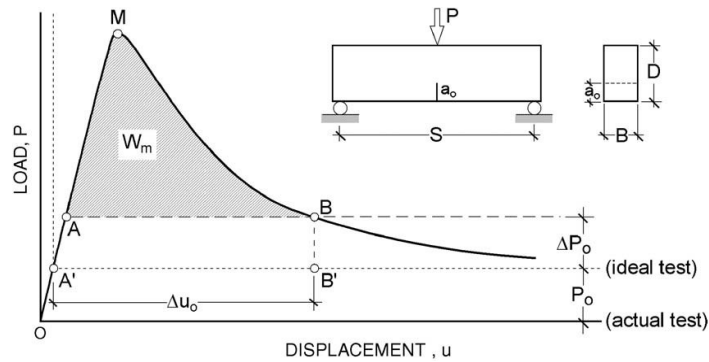


Figure 2-17: load displacement curve ref.[22]

Fracture energy of concrete can be measured by:

$$G_f = \frac{W_F}{B(D - a_0)}$$

$B$  is the thickness of beam

$D - a_0$  is the ligament length

$$W_F = W_m + 2 \times \Delta u_0 \Delta P$$

Aggregates quality (size, shape and hardness) and interface between aggregate and mortar are two important parameters which influence the fracture energy of concrete. A research by RILM-TC50 concluded that when the size of aggregate become larger, will fracture energy increase. ref[22] while the interface between aggregates and mortar has direct effect on the tensile properties of concrete. Strong matrix-aggregates interface increase the area of broken particles, but it is depend on how weak or strong the aggregates are. By having a strong aggregates will the particles just debonded not broken. In theory can say that because of strong bond and strong aggregates in concrete will crack path wander around aggregates which will result increasing the fracture area and fracture energy of concrete. In case of using weak aggregates such as lightweight aggregate which has almost same stiffness as mortar, will crack path go through mortar and aggregates. It will cause a sudden fracture. The breaking

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aggregates take place when the interface is strong enough, and by having a weak matrix-aggregates interface will aggregates just be debonded not broken, that will result a lower fracture energy.

Study on strong and weak aggregates shows that fracture energy is first of all depend on how strong is matrix-aggregates interface and than how strong or weak is the aggregates. A various of researches shows that fracture energy of weak aggregates with strong aggregates bond will give lower fracture energy than a strong aggregates with strong aggregate bond ref.[22].

Also for increasing the ductility of lightweight concrete, it should increase the strength of the matrix-aggregates interface which will give increasing in fracture energy in concrete. For reaching the highest value of fracture energy in lightweight concrete, the percentage of broken lightweight aggregates has to be 100%.

## **Chapter 3 : EXPERIMENTAL PROGRAMME**

### **3.1. Mix Proportions**

Three type concrete mixes have been evaluated in the tests, normal density self compacting concrete, normal density vibrator compacting concrete and lightweight vibrator compacting concrete mix for all lightweight beams. The Concrete is a high strength concrete with target strength of 50MPa. The w/b ratio is constant for all LWC mixes.

In this experiment test for LWC has been used Anlegg-FA cement which contain 30% fly ash. The cement has been replaced by 6% Elkem silica fume. For normal density concrete has been used Anlegg cement with 8% silica fume replacement.

*Table 3-8: SCC mix proportions (kg/m<sup>3</sup>)*

Cement	Silica fume	Water	w/b ratio	Sand	Coarse aggregate	Superplasticizer
387,6	31	188,8	0,46	853,3	908,8	7,8

*Table 3-9: Normal concrete mix proportions (kg/m<sup>3</sup>)*

Cement	Silica fume	Water	w/b ratio	Sand	Coarse aggregate	Superplasticizer
311	24,9	165,9	0,46	894	952	4,7

*Table 3-10: LWC mix proportions (kg/m<sup>3</sup>)*

Cement	Silica fume	Water	w/b ratio	Sand	Leca	Superplasticizer
395,5	23,7	186	0,42	826,6	453,2	5,9

Workability of the mix has been kept constant. Because of use of different fiber types, amount of superplasticizer become adjusted.

Moisture of Leca and sand has been measured before each mix and since they stored outside of building, they gave variable value which has been noted and changed in proportion.

**3.2 Fiber types**

There were made one pair beams of each type of fibers. Beams from each pair were identical except the volume fraction of fibers. These fibers were manufactured by Resconmapei. In this study steel fibers, plastic fibers and polypropylene microfiber has been used.

Steel fibers are end hooked at their ends and has been used 2 type of them, DE 50/1N (N50) and DE 35/0,55N (N35). Steel fibers are made of normal strength wire with a tensile strength of 1100MPa ref. [appendix 1]



*Figure 3-18:* DE 35/0,55N                      DE 50/1N

Two single reference beams which contains 2% fiber of type DE 50/1N and are NDC, were cast by self-compacting and vibrator-compacting method. While two LWC consist of fiber with volume fraction 1 and 2%.

*Table 3-11:* Properties of hooked steel fibers

Fiber type	Material	Length mm	Nominal diameter	Aspect ratio	Tensile strength(MPa)
DE 50/1N	Steel	50	1	50	1100
DE 35/0,55N	Steel	35	0,55	63,5	1100

Plastic fiber is a macrofiber which is made of plastic polypropylene. The properties of this fiber helps to control shrinkage and crack in concrete ref.[appendix 1]. Two beams were cast with 1% and 2% volume fraction.

PP-fiber M12 is a multifilament Polypropylene fiber. It were cast two beams with M12-fiber with volume fraction 0,4% and 0,6%. Normal dosage of PP-fiber is 1 kg/m<sup>3</sup>(0,1%), and for a fire resistance

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concrete is 2 kg/m<sup>3</sup>(0,2%) ref.[appendix 1]. It has been noted that PP-fiber in contrast to another type of fibers attract water, and the real challenge is to keep sufficient workability in the fresh concrete. It has been tried to add 1% PP-fiber to concrete mix but it made the concrete real dry and separation of leca was the result. Adding extra water and superplastcizer admixture did not help either. It has been observed that after a while that PP-fiber M12 release the water (cement paste) and rat bleeding.



*Figure 3-19: PP-fiber M12 Plasticfiber M50*

*Table 3-12 : Properties of PP-fiber M12 and Plastic-fiber M50*

Fiber type	Material	Length mm	Nominal diameter	Aspect ratio	Tensile strength(MPa)
Polypropylene	Microfiber	12	0,22	54.5	400
Plasticfiber	Macrofiber	50	1	50	250

### 3.3 Specimen details

12 beams with and without fiber has been made in this experiment program. Three beams are casted by normal density concrete(NDC) and 9 beams with LWC. Fibers type which has been used in this experimental program is listed in table 3-13. Two beams of NDC are self compacting concrete (Ref\_SCC\_MF and Ref\_SCC\_UF). The last NDC beam is vibrator-compacting concrete with 2% steel fibers (Ref\_VCC\_MF). All LWC beam specimens are VCC. The Beam (Ref\_LWC\_UF) has no fiber content.

*Table 3-13: Overview of beams and the volume fraction of fibers*

Beam ID	Type of Fiber	Fiber volume fraction
Ref_SCC_MF	DE 50/1 N	2.00%
Ref_VCC_MF	DE 50/1 N	2.00%
Ref_SCC_UF	None	0
Ref_LWC_UF	None	0
LWC_N50_2%	DE 50/1 N	2.00%
LWC_N50_1%	DE 50/1 N	1.00%
LWC_N35_2%	DE 35/0,55N	2.00%
LWC_N35_1%	DE 35/0,55N	1.00%
LWC_PF2%	M50	2.00%
LWC_PF1%	M50	1.00%
LWC_PPF0.6%	M12	0.60%
LWC_PPF0.4%	M12	0.40%

All beams have same dimension of 2,2m long and with cross section of 0,3m height and 0,25m width.

All beams has been design based on normal density concrete.

Calculation of reinforced bars from EC2:(complete calculation appendix 2)

$$z = 0,84 \times 259 = 217,5$$

$$A_s = \frac{M_{Ed} \times 10^6}{f_{yd} \times z} = \frac{63 \times 10^6}{435 \times 217,5} = 666 \text{ mm}^2$$

Use:

$$3 \text{ } \phi 20 \text{ } A_s = \underline{942 \text{ mm}^2}$$

Detail of reinforcement of beam is given in figure 3-20 and 3-21. The beam was reinforced with three 20 mm diameter bars in yield strength section and two 12mm bar in the compression section. The stirrup reinforcement were 8 mm diameter with c/c 130 mm.



Calculation of stirrups based on EC2:(complete calculation appendix 2)

$$\frac{A_{sw}}{s} \geq \frac{V_{Ed}}{z \times f_{ywd} \times \cot \theta} = \frac{90000}{233 \times 434 \times 2,5} = 0,356 \text{ mm}^2 / \text{mm}$$

$$\left(\frac{A_{sw}}{s}\right)^{\min} = 0,1 \times b_w \times \frac{\sqrt{f_{ck}}}{f_{yk}}$$

$$S_{\max} = 0,6 \times h'$$

$$\left(\frac{A_{sw}}{s}\right)^{\min} = 0,1 \times 250 \times \frac{\sqrt{28}}{500} = 0,265 \text{ mm}^2 / \text{mm}$$

$$s = 190 \text{ mm}$$

$$h' = 220$$

$$s_{\max} = 0,6 \times 220 = 130 \text{ mm}$$

$$s_{\max} < s \text{ so use } s_{\max}$$

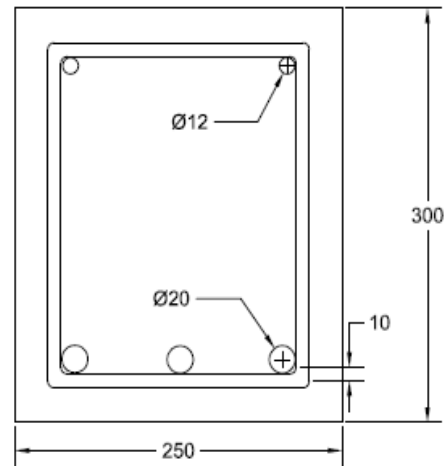


Figure 3-20: Reinforcement detail

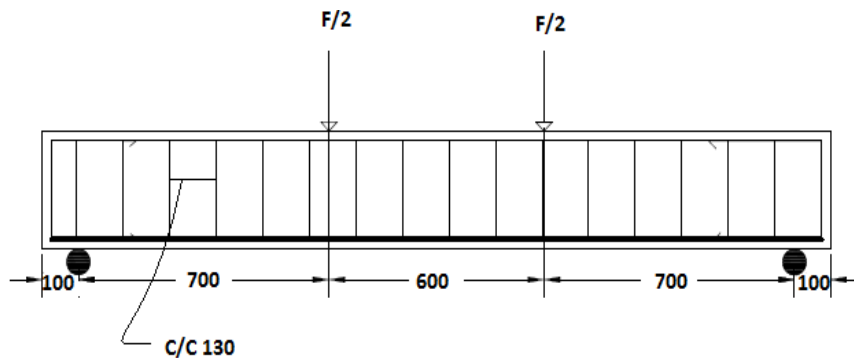


Figure 3-21: Reinforcement detail and test setup

For finding the influence of each fibers on beams ductility, it was decided to use the same type of reinforcement and stirrups for all beams in order to get less parameters.

### 3.4 Mixing and casting

Concrete mixing took place at the Engineering laboratory of University of Stavanger. The mixer drum has a capacity of 110 liter concrete mix. Each beam including cylinders and cubes needed 230 liter fresh concrete. The mixing was divided in 3 mixes, one 70 liters and two 80 liters concrete mix. Cement, sand and Leca were mixed in approximate 3 min and than the water was added in to the

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mixing drum. This way mixed together in one minute before adding superplasticizer admixture. Fibers were added after admixtures because of the high volume fraction. The reason that fibers were added after admixture is that rotating drum hadn't enough power to mix the fibers in high cohesive and dry concrete.

Beam specimens were cast by two layers, each layer had approximately 150 mm thickness and was compacted by vibrator. It has been noted that use of vibrator can give weak point and less fiber in those area vibrator was used.

The fresh concrete test process followed NS-EN 12350-2:2009. For each beam specimen concrete mix were cast 8 cylinders with a 150 mm diameter and 300 mm height and 6 cubes with dimension 100x100x100. Cylinders and cubes were sampled following NS-EN 12390-1:2009. Compacting and curing of specimens followed NS-EN 12390-2:2009. Cubes were covered with plastic sheet and cylinders were covered by lid for 24 hours and than moved to a tank of water where the temperature were kept constant 20 degree. Specimens were pulled out from water 30 min before testing.

### 3.5 Beam testing

All beams were tested by 4 point bending test. The beams were supported on two points. The length between support points were 2 meters. Beams were loaded with two concentrated loads with arm length of 0.7m from support points. The test machine is a type of Toni Technick with a maximum 400 kN pressure force. Each beam has been present by a load-deflection curves. Elasticity, plasticity and ductility has been highlighted in each curve. Concrete beams are under-reinforced by design which means steel reinforcement would yield before failure. This area is highlighted as plasticity area.

Concrete crushing will initiate after yielding and beams will loss carrying load capacity A brittle failure would give a low deflection after yielding right before reaching the ultimate failure. The deflection area after initiation of concrete crushing were highlighted as ductility area.

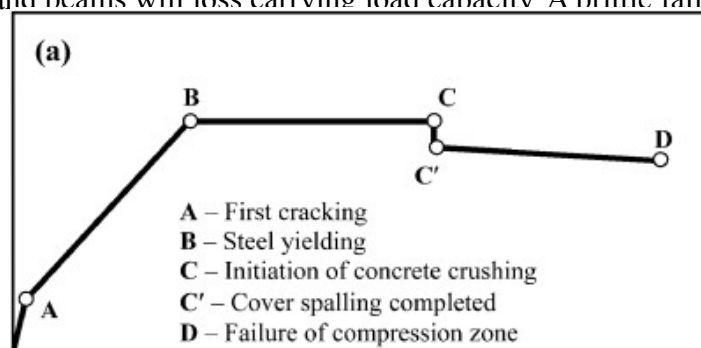


Figure 3-22: Concrete reinforced behavior under loading ref.[22]

## Chapter 4: Experimental Result

### 4.1 Compressive strength

The strength class of concrete for each beam has been decided based on the obtained compressive strength tests of 6 cubes (3 at age 7 and 3 at age 28days) and one cylinder specimen. The compressive test is performed according to NS-EN 12390-3-2009. The strength classes in EC2 which is showed in table 1-1 and table 2-2 are based on the characteristic cylinder strength  $f_{ck}$ . Strength class for each concrete is decided from characteristic compressive cube since the value is more precise than the characteristic compressive cylinder strength.

Each specimen has been evaluated after failure to satisfy the requirement according NS-EN 12390-3-2009. The volume density of each cube has been determined by following EN-12390-7 and the average is showed in table 4-15. Calculation of density of cube specimens follows this formula:

$$V = \frac{m_a - m_w}{\rho_w}$$

$$D = \frac{m}{V}$$

$V$  is the volume of the specimen

$m_a$  is the mass of the specimen in air

$m_w$  is the apparent mass of the immersed specimen

$\rho_w$  is the density of water

The characteristic cube strength is calculated based on the values  $f_{cm}$  and the standard deviation  $s$ . The standard value is assumed to be 4. Calculation is followed by NS-EN 206-1:2000+NA:2007.

Table 4-14: Criteria for compressive strength (NS-EN 206-1)

Production	Number of tests	Criterion 1	Criterion 2
		Average of test results $f_{cm}$	All individual test results
Preliminary	3	$\geq f_{ck} + 4$	$\geq f_{ck} - 4$
Continuous	more than 15	$\geq f_{ck} + 1.48\sigma$	$\geq f_{ck} + 4$

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**Test result** from compressive strength test is present in table 4-15 and strength classed are chosen from NS-EN 206-1.

*Table 4-15: Strength class*

Beam ID	density(kg/l)	Mean value $f_{cm}$ (MPa)	std.division	$f_{ck,cube}$ (MPa)	$f_{ck,cylinder}$	Strength class
Ref_SCC_MF	2.48	78	4.0	72	61	C55/67
Ref_VCC_MF	2.54	94	4.0	88	82	C70/85
Ref_SCC_UF	2.38	80	4.0	74	65	C60/75
Ref_LWC_UF	1.96	74	4.0	68	65	LC60/66
LWC_N50_2%	1.85	71	4.0	65	57	LC60/66
LWC_N50_1%	1.92	49	4.0	43	46	LC40/44
LWC_N35_2%	1.89	66	4.0	60	55	LC55/60
LWC_N35_1%	1.94	68	4.0	62	59	LC55/60
LWC_PF2%	2	72	4.0	66	57	LC60/66
LWC_PF1%	2.04	56	4.0	50	43	LC45/50
LWC_PPF0.6%	1.99	31	4.0	25	21	LC20/22
LWC_PPF0.4%	2.01	53	4.0	47	49	LC40/44

#### 4.1.1 Discussion

By studying the experimental results, it can be concluded that compressive strength of steel fiber-reinforced concrete and plastic fiber-reinforced concrete have not been increased by presence of fiber in the concrete. The only change that has been noted, is that the cubic specimens did not crush in failure because of the fibers bridging which change failure mode from fragile to ductile.

Results shows that lightweight PP-fiber reinforced concrete with 0.6% volume fraction (LWC\_PPF0.6%) give half strength class compared to lightweight concrete without fiber (LWC\_UF). By considering that all lightweight concrete has been made of the same receipt with constant w/c ratio the deviation of LWC\_PPF\_0.6% is considerable large. The strength decreasing is most likely because of high volume addition of polypropylene microfiber (PP-fiber). The manufacture description has recommended the maximum addition of 0.2% in case of fire resistance concrete. During the casting of LWC\_PPF\_0.6% it has been noted a very low workability and disappearing of cement paste in to the PP-fiber which cause a separation of Leca and cement paste. It has also been noted that PP-fiber release amount of water after 30 minutes which cause bleeding in concrete. All these incidents seems to cause reduction in quality and strength of concrete.

#### 4.2 Tensile strength

The splitting tensile strength is measured as the average of 2 cylinder by split cylinder test  $f_{ct,sp}$ .

According to EC2 the axial tensile strength ( $f_{ct}$ ) is equal to 90% of splitting tensile strength ( $f_{ct,sp}$ ).

Test method and calculating of tensile strength followed NS-EN 12390-6:2009.

The mean value of axial tensile strength of concrete has been given in table 1-1 for normal density concrete.

For lightweight concrete has EC2 given the formula  $f_{lcm} = f_{cm} \times \eta_1$  where  $\eta_1 = 0.45 + 0.6\rho / 2200$

$$f_{ct,sp} = \frac{2 \times F}{l \times b \times \pi}$$
$$f_{ct} = f_{ct,sp} \times 0.9$$

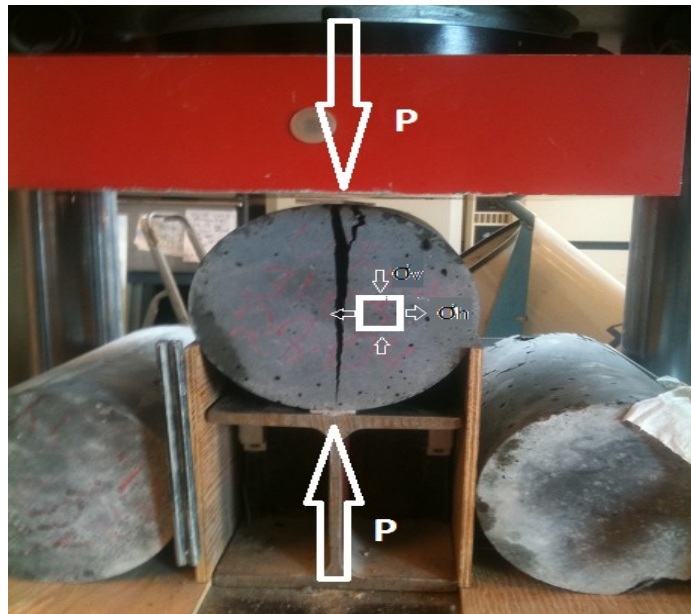
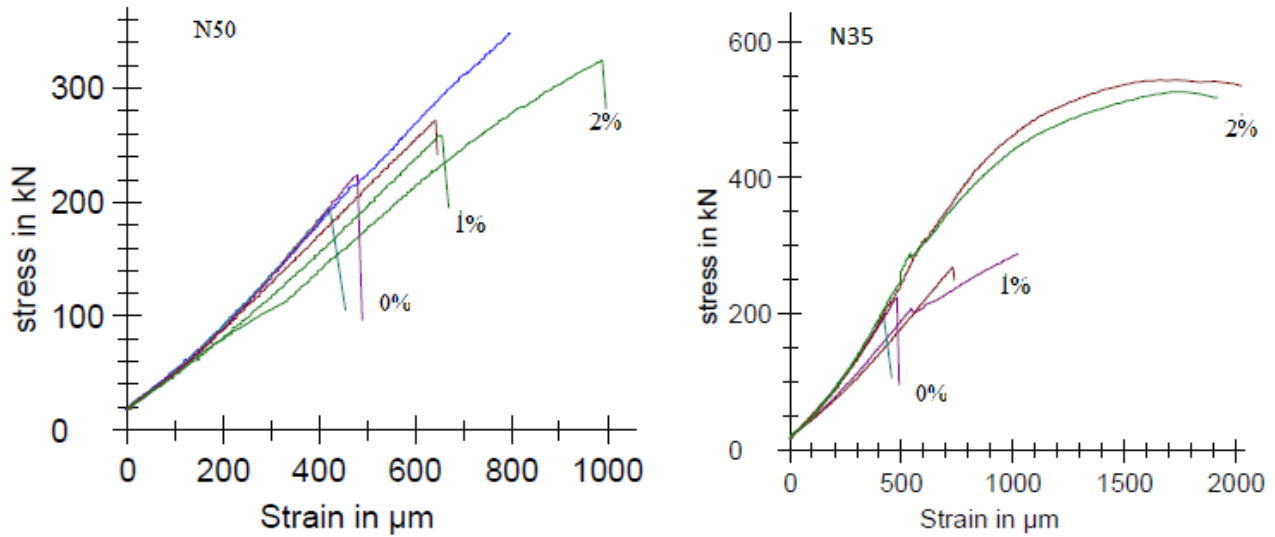


Figure 4-23: Split test

**Test results of split test** is reported in table 4-16 and diagram of each fiber has been present in figure 4-24, 4-25.

*Table 3-16: split test result by cylinder test*

Cylinder ID	Split test		
	F(kN)	f <sub>ct,sp</sub> (MPa)	f <sub>ct</sub> (MPa)
Ref_VCC_MF	724	10.24	9.22
Ref.SCC.UF	314	4.44	4.00
Ref.LWC_UF	224	3.17	2.85
LWC_N50_2%	348	4.92	4.43
LWC_N50_1%	271	3.83	3.45
LWC_N35_2%	540	7.64	6.88
LWC_N35_1%	288	4.07	3.67
LWC_PF2%	209	2.96	2.66
LWC_PF1%	224	3.17	2.85
LWC_PPF0.6%	148	2.09	1.88
LWC_PPF0.4%	240	3.40	3.06



*Figure 4-24 Split test steel fiber*

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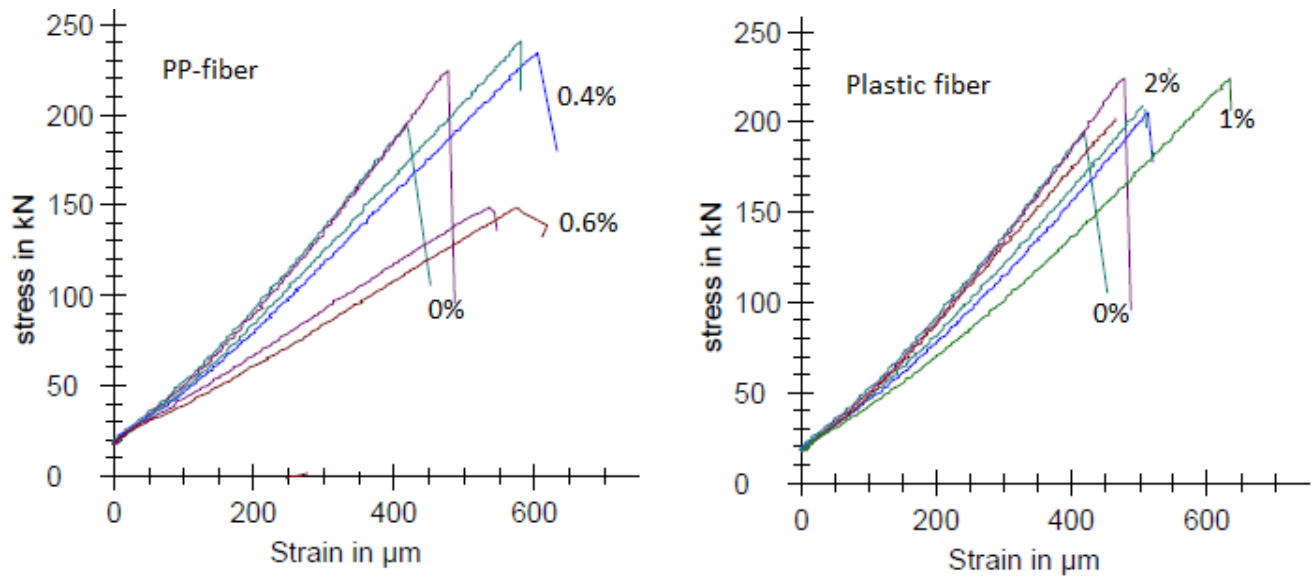


Figure 4-25 Split test PP- and Plastic-fiber

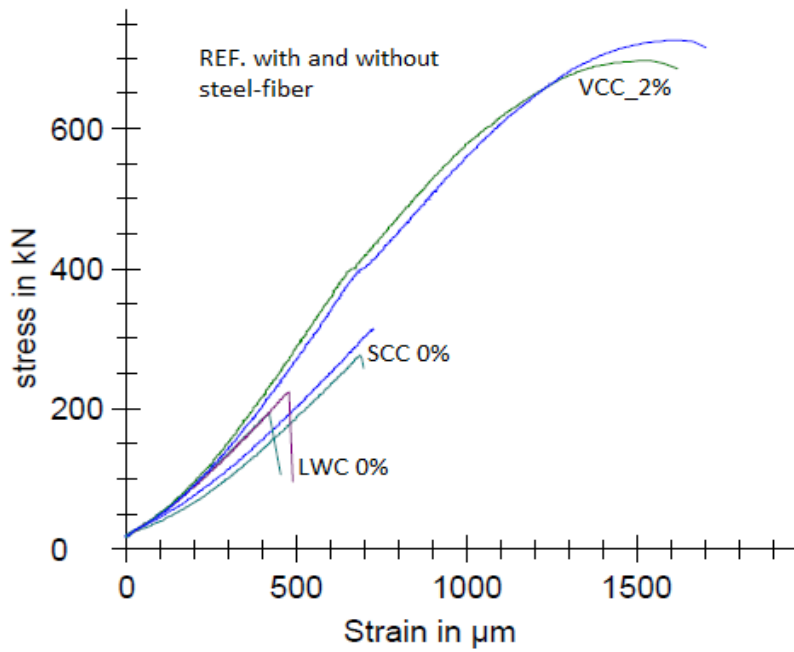


Figure 4-26: Split test references with normal and lightweight concrete

4.2.1 Discussion of split test result

Table 4-17 shows the result from split test and characteristic axial tensile strength for each strength class. The result of tensile test is compared with characteristic axial tensile strength which is given in EC2.

Table 4-17: Tensile strength (result from split test and EC2)

Cylinder ID	Split test			fct/fcm	EC2		
	F(kN)	fct,sp(MPa)	fct(MPa)		strength class	fctm	fck0.05
Ref_VCC_MF	724	10.24	9.22	11.24%	C70/85	4.60	3.22
Ref_SCC_UF	314	4.44	4.00	6.83%	C60/75	4.08	3.22
Ref_LWC_UF	224	3.17	2.85	4.88%	LC60/66	4.08	2.86
LWC_N50_2%	348	4.92	4.43	8.63%	LC60/66	4.08	2.86
LWC_N50_1%	271	3.83	3.45	8.33%	LC40/44	3.24	2.86
LWC_N35_2%	540	7.64	6.88	12.51%	LC55/60	3.89	2.27
LWC_N35_1%	288	4.07	3.67	6.90%	LC55/60	3.89	2.72
LWC_PF2%	209	2.96	2.66	5.19%	LC60/66	4.08	2.72
LWC_PF1%	224	3.17	2.85	7.37%	LC45/50	3.52	2.86
LWC_PPF0.6%	148	2.09	1.88	9.95%	LC20/22	2.04	2.46
LWC_PPF0.4%	240	3.40	3.06	7.01%	LC40/44	3.24	1.43

Results of two split test has been suspected to be somehow wrong and incorrect. By comparing the results of Ref\_VBB\_MF and LWC\_N35\_2% by others split test results will see the difference between them are unrealistic high. Test method of those 2 cylinder have been followed correctly as it has been described earlier, but the cause of this deviation can be that the test machine could not record the first crack because of high presents of steel fiber. The diagram which is shown in figure 4-26 shows that it is a double strain compare to another results. From diagram can find the real value of axial tensile strength since the correct value is in the end of the elastic curve. New tensile strength is suggest in table 4-18 by analysis the diagram from earlier result.

Table 4-18: New value of tensile strength

Cylinder ID	Split test			EC2			Percentage increasing
	F(kN)	fct,sp(MPa)	fct(MPa)	strength class	fctm	fck0.05	
Ref_VCC_MF	400	5.66	5.09	C70/85	4.60	3.22	58
LWC_N35_2%	340	4.81	4.33	LC55/60	3.89	2.72	59

Improvement of the tensile strength compare to given tensile strength value in EC2, has been noted in all type of fibers reinforced concrete except in one case which is plastic-fiber 2%. Lightweight concrete without any fiber has the same tensile strength as expected in EC2. The test results shows that tensile



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strength in lightweight concrete is increased by using steel-fiber. PP-fiber with 0.4 % volume fraction shows improvement in tensile strength.

The result from LWC\_PP\_0.6% gave the lowest tensile strength in this experiment, as it is already discussed earlier, adding more than 0.4% fiber will cause bleeding and segregation under casting.

The tensile strength of normal density concrete is 4.0MPa, the table 4-19 shows the percentage of increasing/decreasing of each tensile strength compare to SCC\_UF.

*Table 4-19: comparing tensile strength of SCC\_UF with LWC*

Cylinder ID	Split test			Percentage VS Ref.SCC.UF
	F(kN)	fct,sp(MPa)	fct(MPa)	
Ref_VCC_MF	400	5.66	5.09	27.4%
Ref_SCC_UF	314	4.44	4.00	0.0%
Ref_LWC_UF	224	3.17	2.85	-28.7%
LWC_N50_2%	348	4.92	4.43	10.8%
LWC_N50_1%	271	3.83	3.45	-13.7%
LWC_N35_2%	340	4.81	4.33	8.3%
LWC_N35_1%	288	4.07	3.67	-8.3%
LWC_PF2%	209	2.96	2.66	-33.4%
LWC_PF1%	224	3.17	2.85	-28.7%
LWC_PPF0.6%	148	2.09	1.88	-52.9%
LWC_PPF0.4%	240	3.40	3.06	-23.6%

the impact of replacing normal density concrete with lightweight concrete, is that the tensile strength must increase by adding steel fiber. According to the result of this experiment it should not be less than 2% volume percent. Both type of steel fiber with N35 and N50 with 2% fraction volume give a positive increasing compare to normal density concrete.

### 4.3 E-modulus

The E-modulus of each concrete has been identified by pressure testing of two cylinders. The test method and calculation is performed according NS-3676. Strain in concrete is measured by using 2 strain sensors which was installed on a stand. The equipment is special made for E-modulus test which give a precise strain on each test. The figure 4-27 shows a cylinder during testing by cycles pressure system according to NS-3676.

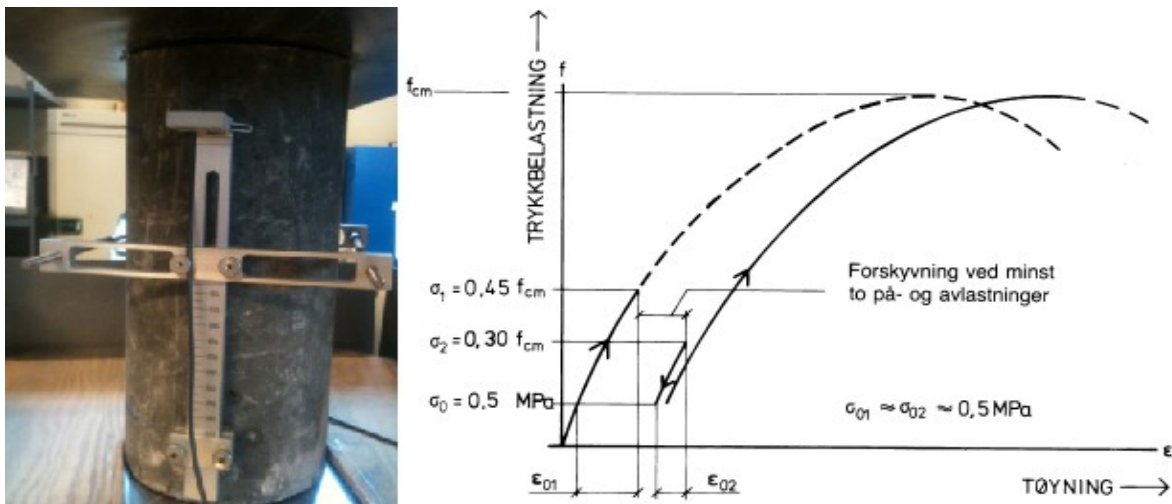


Figure 4-27: E-modulus testing

The mean value of cylinder compressive strength  $f_{cm}$  is given in table 4-15 and  $\sigma_1, \sigma_2$  is calculated for each cylinder base on earlier results. Strain at  $\sigma_1, \sigma_2$  and  $\sigma_0$  from digital measurement was noted and the result of E-modulus was calculated by:

$$E_o = \frac{(\sigma_1 - \sigma_0) \times L_1}{(\varepsilon_1 - \varepsilon_0)} \quad E_c = \frac{(\sigma_2 - \sigma_0) \times L_1}{(\varepsilon_2 - \varepsilon_0)}$$

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Results of E-modulus test is present in table 4-20. The result is the average of two cylinder test.

*Table 4-20:Result of E-modulus test*

Concrete ID	Eo(GPa)	Ec(GPa)
<b>SCC_UF</b>	<b>32.1</b>	<b>32.0</b>
<b>VCC_2%</b>	<b>33.0</b>	<b>33.5</b>
LWC_UF	22.6	22.6
LWC_N35_1%	19.3	19.4
LWC_N35_2%	21.5	22.0
LWC_N50_1%	22.9	23.0
LWC_N50_2%	24.0	24.0
LWC_PF1%	21.0	21.4
LWC_PF_2%	22.4	22.1
LWC_PP_0.4%	17.6	17.6
LWC_PP_0.6%	13.5	13.3

The result shows that E-modulus of LWC is lower than normal density concrete which has been expected. E-modulus for each concrete strength class is given in EC2. Table 4-21 compare the experimental result by theoretical standard value.

*Table 4-21:E-modulus base on strength class*

Concrete ID	Eo(GPa)	Strength Class	EC2 (Gpa)
<b>SCC_UF</b>	<b>32.1</b>	<b>C60/75</b>	<b>39</b>
<b>VCC_2%</b>	<b>33.0</b>	<b>C70/85</b>	<b>41</b>
LWC_UF	22.6	LC60/66	30.03
LWC_N35_1%	19.3	LC60/66	30.03
LWC_N35_2%	21.5	LC55/60	29.26
LWC_N50_1%	22.9	LC40/44	26.95
LWC_N50_2%	24.0	LC55/60	29.26
LWC_PF1%	21.0	LC45/50	27.77
LWC_PF_2%	22.4	LC60/66	30.03
LWC_PP_0.4%	17.6	LC40/44	29.95
LWC_PP_0.6%	13.5	LC20/22	23.1

The result from experimental E-modulus shows a approximate 10GPa difference between E-modulus which is given in EC2 and the test results. LWC\_N50 with 1 and 2% volume fraction are the only concrete that have a higher E-modulus compare to LWC\_UF.

**4.4 Stress-strain diagram**

Three cylinders have been tested for stress-strain diagram for each beam specimen. The test has been followed by SINTEF quality control for stress-strain diagram by pressure load ref[19]. The pressure load controlled by deformation speed 0.3 % per min. The pressure on cylinders continues until that specimens collapse. Studying stress-strain diagram can give a basic understanding of the ductility ability of concrete. A ductile concrete would not have a brittle fracture and it can be decided when it have plastic behavior. Brittle concrete will break at rather low strain.

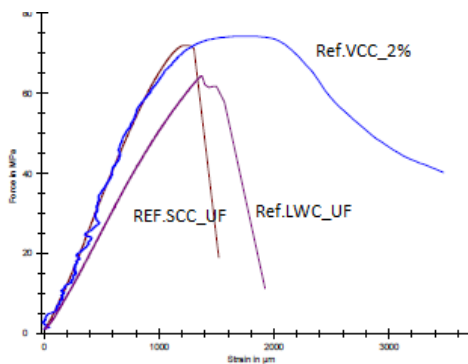


Figure 4-28: Stress-strain diagram for ref. beams

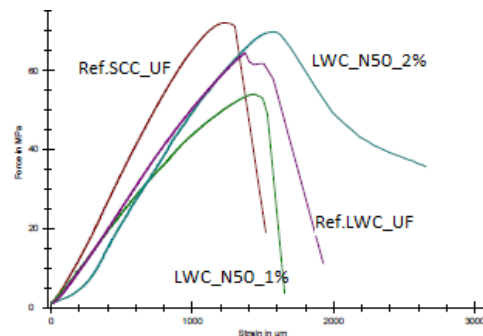


Figure 4-29: Stress-strain diagram for N50

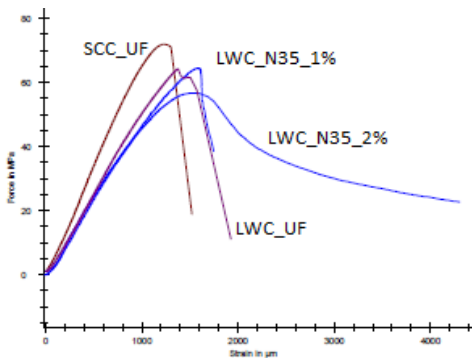


Figure 4-30: Stress-strain diagram for N35

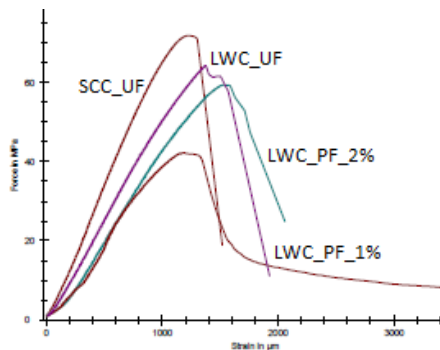


Figure 4-31: Stress-Strain diagram for Plastic fiber

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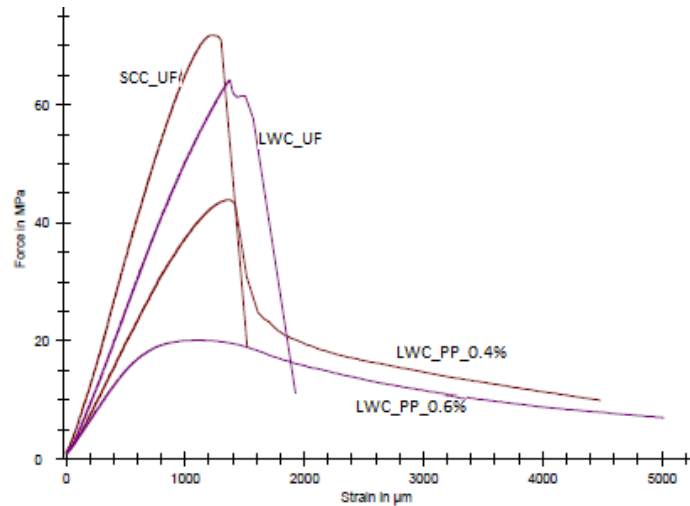


Figure 4-32: Stress-Strain diagram for PP-fiber

SCC\_UF and LWC\_UF are references concrete without any fibers and figure 4-28 shows that they got suddenly fall after the strength capacity is reached. The behavior in plasticity area shows the ductility capacity for each specimen.

The strain ductility coefficient  $\mu_e$  can be identified as maximum sustainable strain ( $\epsilon_y$ ) divided by the yield strain ( $\epsilon$ ) ref.[19]

Table 4-22: Ductility coefficient for stress-strain diagram

Cylinder ID	strain( $\mu\text{m}$ )		Ductility coeff. $\mu_y$
	$\epsilon_y$	$\epsilon$	
Ref_VCC_MF	1200	3500	2.92
Ref_SCC_UF	1200	1600	1.33
Ref_LWC_UF	1400	1900	1.36
LWC_N50_2%	1600	2750	1.72
LWC_N50_1%	1200	1620	1.35
LWC_N35_2%	1400	4200	3.00
LWC_N35_1%	1600	1800	1.13
LWC_PF2%	1450	2100	1.45
LWC_PF1%	1200	3400	2.83
LWC_PP_0.6%	1400	4600	3.29
LWC_PP_0.4%	800	5000	6.25

LWC\_PP\_0.4% and 0.6% which contain polypropylene microfiber show high ductility capacity but lower ultimate stress compare to SCC\_UF and LWC\_UF.

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LWC\_N50 and LWC\_N35 cylinders with a 2% volume fraction have a higher ductility coefficient compare to the references concrete. Steel-fiber in normal density concrete VCC (2%) shows increasing in ductility ratio compare to normal density concrete without fiber (SCC\_UF) which will confirm that addition of steel fiber improves the ductile behavior. Concrete reinforced by plastic-fiber M50 also improves the ductility coefficient compare to LWC\_UF but it has lower ultimate stress compare to SCC\_UF.

#### 4.5 Behavior of beam specimens

All beam specimens were covered by plastic sheets right after casting. Each beam specimen was stripped and stored outside the laboratory for 3 days after casting for 28 days. No type of shrinkage or crack was observed on specimens during this period. The weather condition was humid climate. For better observation of crack were all beams painted before testing at 28 days age. The testing method was 4 point bending. The speed of load decided to be 100 N/s. The maximum allowed crack width decided to be 0.4mm which is shown in table 2-7. The load on beam specimens continued until fracture stage. Each diagram is the elasticity, plasticity and ductility highlighted.

In this section result and discussion of each beam will be present in a single section.

A general result of relation between Crack width, force and deflection for each beam specimen is presented in table 4-23 and result from calculation of ductility coefficient ( $\mu$ ) which is based on deflection parameters ( $\Delta$ ) from laboratory test is presented in table 4-24.

$$\mu = \frac{\Delta_{uf}}{\Delta}$$

$\Delta_{uf}$  is the deflection of beam at ultimate fracture load

$\Delta$  is the deflection of beam when yield of reinforcement start

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*Table 4-23: Overview of results from beam specimens*

Beam ID	Force(kN)		Deflection(mm)	
	Crack width (0,4mm)	Ultimate Load	Crack width (0.4 mm)	Ultimate failure (mm)
Ref.SCC.MF	265	459	6.9	70
Ref.VBB.MF	280	464.7	7.93	74
Ref.SCC.UF	220	392.31	6.84	35
Ref.LWC_UF	220	384.31	7.72	25.8
LWC_N50_2%	260	414.68	8.25	60.2
LWC_N50_1%	230	385.89	8	54.2
LWC_N35_2%	255	414.44	8.8	76
LWC_N35_1%	260	406.55	7.8	63.5
LWC_PF2%	235	392.24	8.83	66
LWC_PF1%	225	381.65	7.5	55
LWC_PPF0.6%	195	315.15	10.77	43
LWC_PPF0.4%	230	342.97	10	45

*Table 3-24: Ductility ratio  $\mu$  regarding deflection*

Beam ID	Deflection(mm)		Ductility coefficient $\mu$
	$\Delta$	$\Delta_{uf}$	
Ref.SCC.MF	15	70	4.67
Ref.VBB.MF	10	74	7.40
Ref.SCC.UF	15	35	2.33
Ref.LWC_UF	13	25.8	1.98
LWC_N50_2%	14	60.2	4.30
LWC_N50_1%	14	54.2	3.87
LWC_N35_2%	14.5	76	5.24
LWC_N35_1%	14.5	63.5	4.38
LWC_PF2%	13.5	66	4.89
LWC_PF1%	13	55	4.23
LWC_PPF0.6%	21	43	2.05
LWC_PPF0.4%	18	45	2.50

**4.5.1 Lightweight concrete beam without fiber (LWC\_UF)**

The specimen LWC\_UF is a reference beam specimen without fiber. Figure 4-34 and figure 4-35 show crack width development versus load and the load versus deflection of beam specimen.

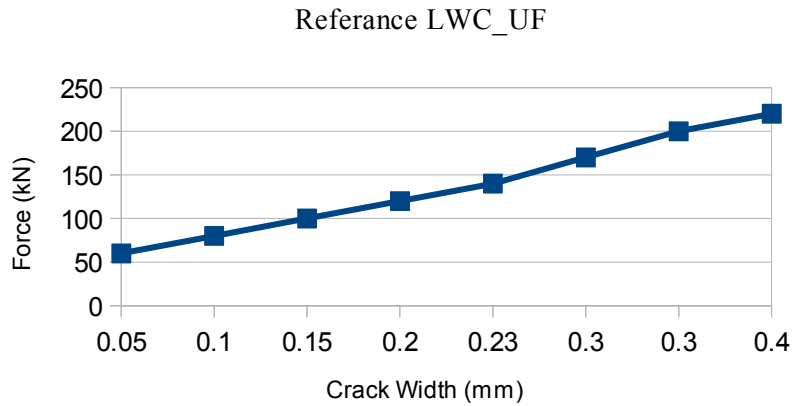


Figure 4-33: Crack width development for LWC\_UF

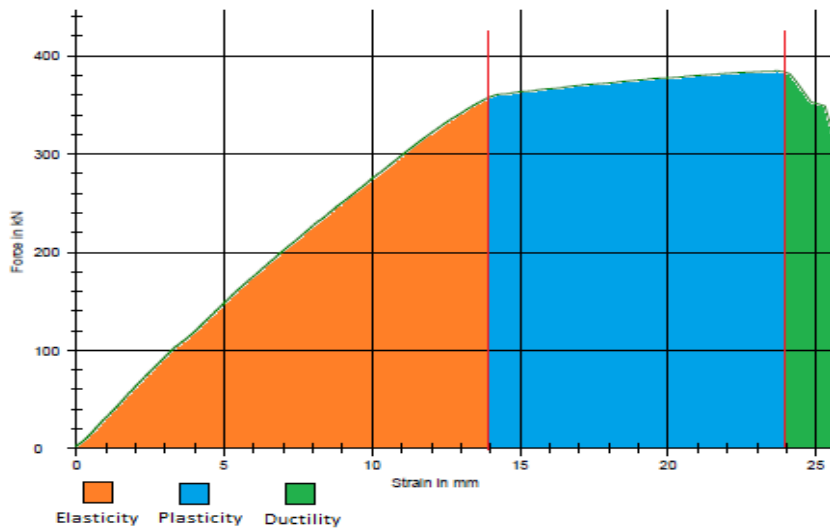


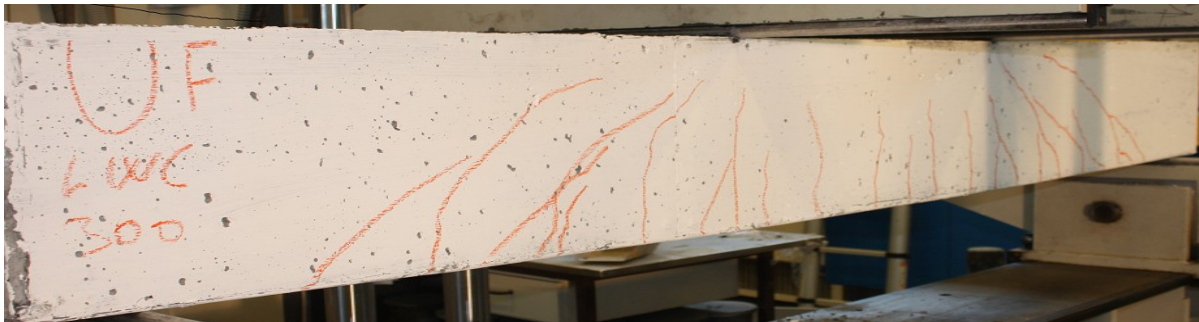
Figure 4-34: Load-deflection diagram for LWC\_UF

First crack on beam become visible at 60kN and the flexural cracks developed in region of maximum moment from bottom of surface. The crack width grow faster in felt of maximum moment which caused at end a flexural failure. As it was expected the high strength lightweight concrete had a brittle



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failure. LWC\_UF had a strength class LC60/66 and it was designed for a maximum load 180kN but as result show the failure took place at load 384kN. The ductility was very low because of high brittleness and maximum deflection was registers at 26 mm right before flexural failure.



*Figure 4-35: Crack pattern before failure LWC\_UF*



*Figure 4-36: Brittle flexural failure LWC\_UF*

**4.5.2 Reference beam self-compacting concrete without fiber (SCC\_UF)**

The beam specimen SCC\_UF had a concrete strength class C60/70, the first crack was observed at load 60kN. Figure 4-37 and fig. 4-38 show crack width development versus load and the load versus deflection of the specimen by comparing with LWC\_UF.

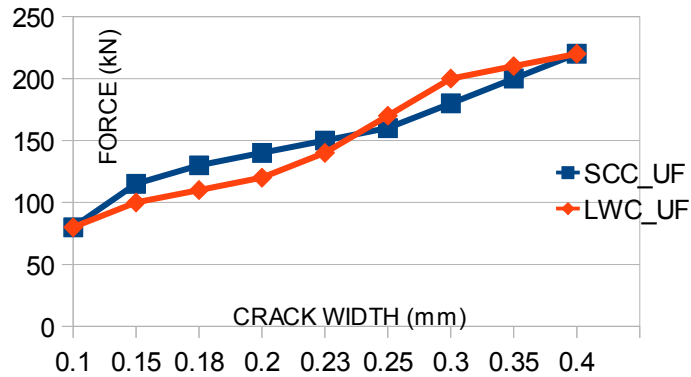


Figure 4-37: Development of crack width for SCC\_UF

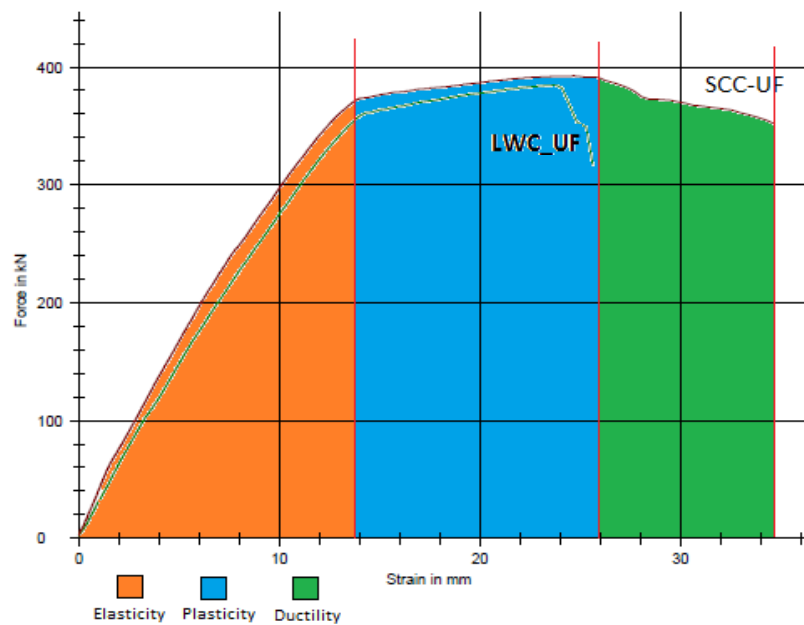
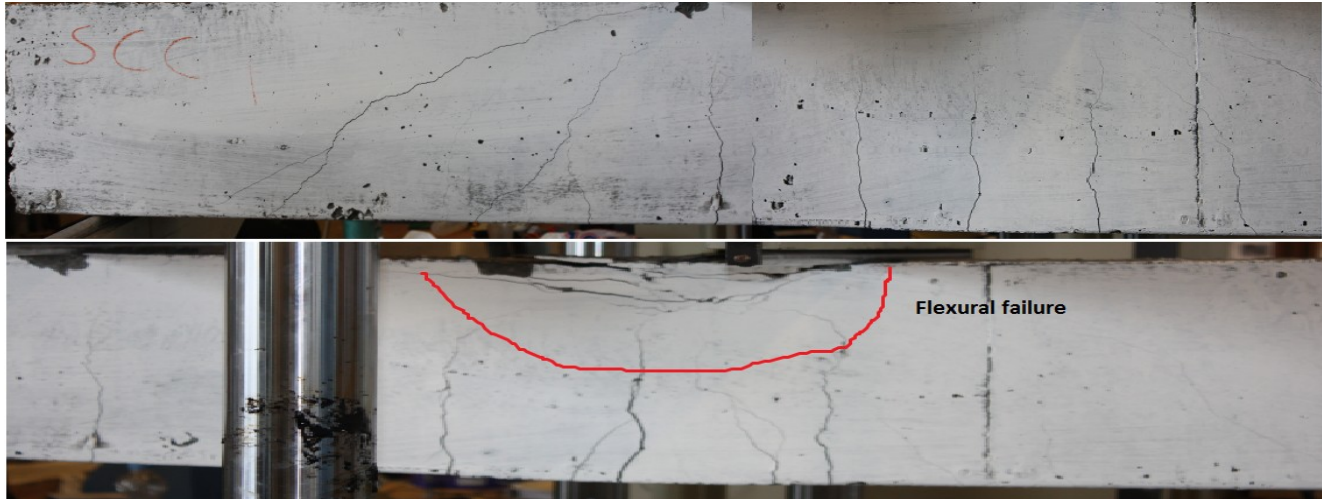


Figure 4-38: load versus deflection for SCC\_UF and LWC\_UF

The deflection of beam specimen at crash point was 35 mm which is 10 mm higher than LWC\_UF. LWC\_UF and SCC\_UF have same concrete strength class and they are designed for same load. Figure 4-38 shows that the lightweight concrete become more brittle and less ductile compare to normal

density concrete. The beam specimen crashed due flexural failure. The crack width development was similar to LWC\_UF. The horizontal cracks grows faster than diagonal cracks and it is the horizontal cracks which cause failure in maximum moment zone. Figure 4-39 shows the cracks pattern and the failure mode.

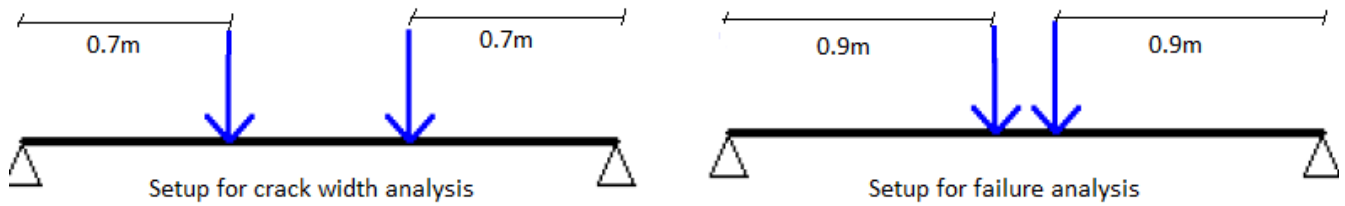


*Figure 4-39: Crack pattern before and after failure SCC\_UF*

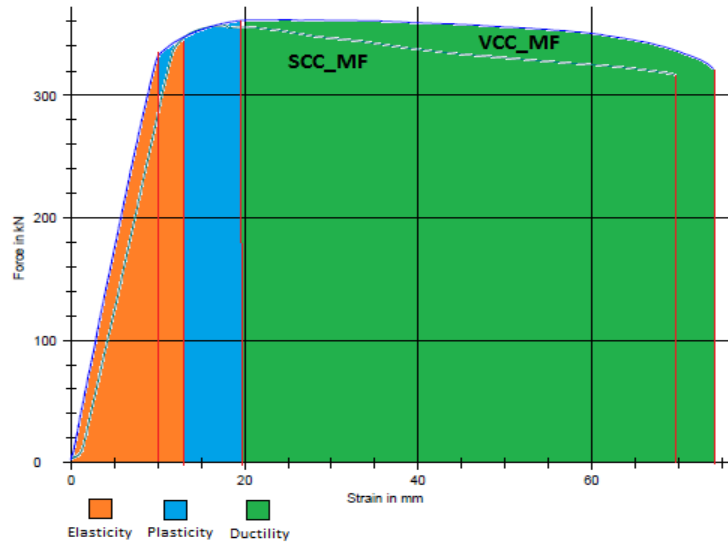
#### **4.5.3 Reference beams self-compacting and vibrator-compacting concrete (SCC\_MF, VBB\_MF)**

Testing of these two beam specimens were a challenge. The hydraulic of the laboratory machine had a maximum load pressure up to 400kN. These two beam specimens were tested after lightweight concrete beam specimens and by study results from lightweight beams could see that the failure load is will be higher than 400kN. Since the failure was expected to be a flexural failure, it was decided to increase the torque arm from 0.7m to 0.9m which increase the maximum moment. Figure 4-40 shows the beam test setup for both beam specimens. In order to study the relation between load and crack width, it was decided to run the test as similar as the other beam specimens and they were loaded under identical conditions until the crack width of 0.4mm was detected.

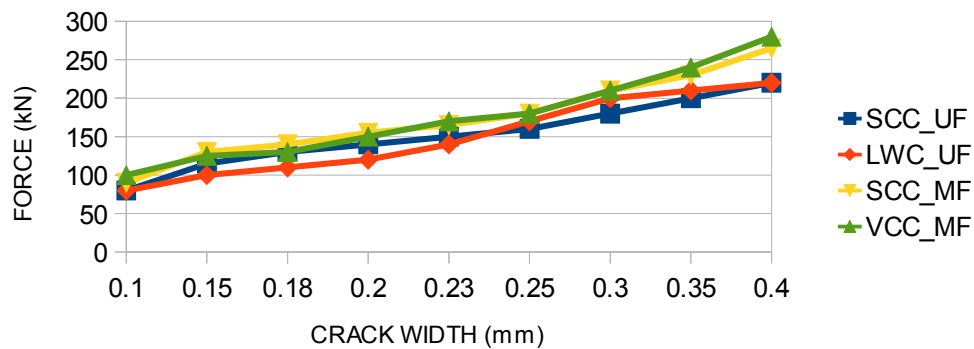
**Result** of these two beams are present in the same diagram. Concrete strength class of SCC\_MF is C50/60 and VBB\_MF is C70/85. Figure 4-41 shows the load versus deflection relationship and figure 4-41 shows the development of crack width versus load.



*Figure 4-40: beam setup for SCC\_MF and VCC\_MF*



*Figure 4-41: Load -deflection relation*



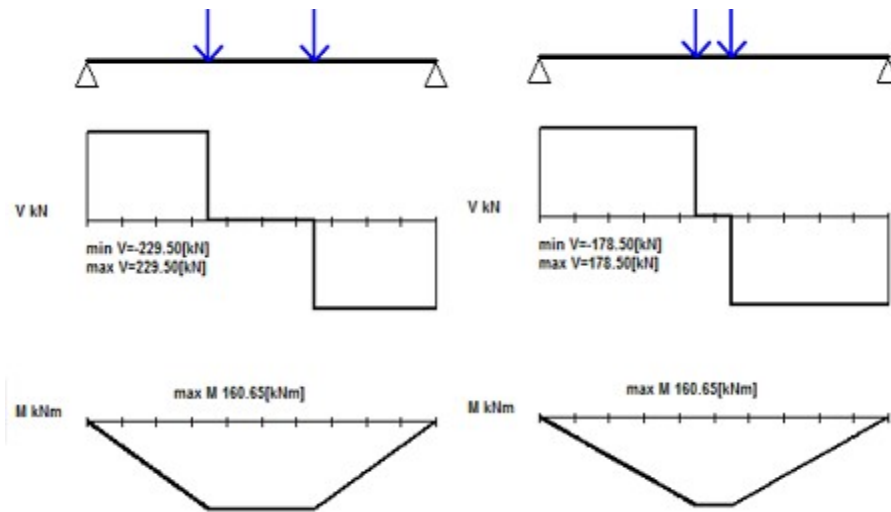
*Figure 4-42: Crack development for VCC\_MF and SCC\_MF*

VCC\_MF beam specimen had a higher strength class compare to SCC\_MF beam specimen. Figure 4.41 shows that the normal density concrete VCC\_MF and SCC\_MF which both have 2% volume fraction end-hooked steel fiber, have approximated same ductility range. Difference between those two

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beams are the compacting method, one self-compacting (SCC) and the other vibrator-compacting(VCC).

The failure load has been calculated by moment calculation in order to approximate value corresponding to test setup when running as similar as another beam specimens. The beam specimen VCC\_MF failed at a maximum moment of 162.65kNm with 0.9 torque arm. By assuming that the beam specimen would have a flexural failure of approximate same moment of a 0.7 torque arm, then the load will be 459kN. Note that maximum moment area decrease in 0.9m torque arm, which will cause a earlier failure and more concentrated load at a smaller area. Shear failure is not in consideration even it will be decreased by increasing the torque arm. Figure 4-43 shows the difference between moment and shear diagram by 0.7 and 0.9 torque arm.



*Figure 4-43: Moment and shear diagram for SCC-MF*

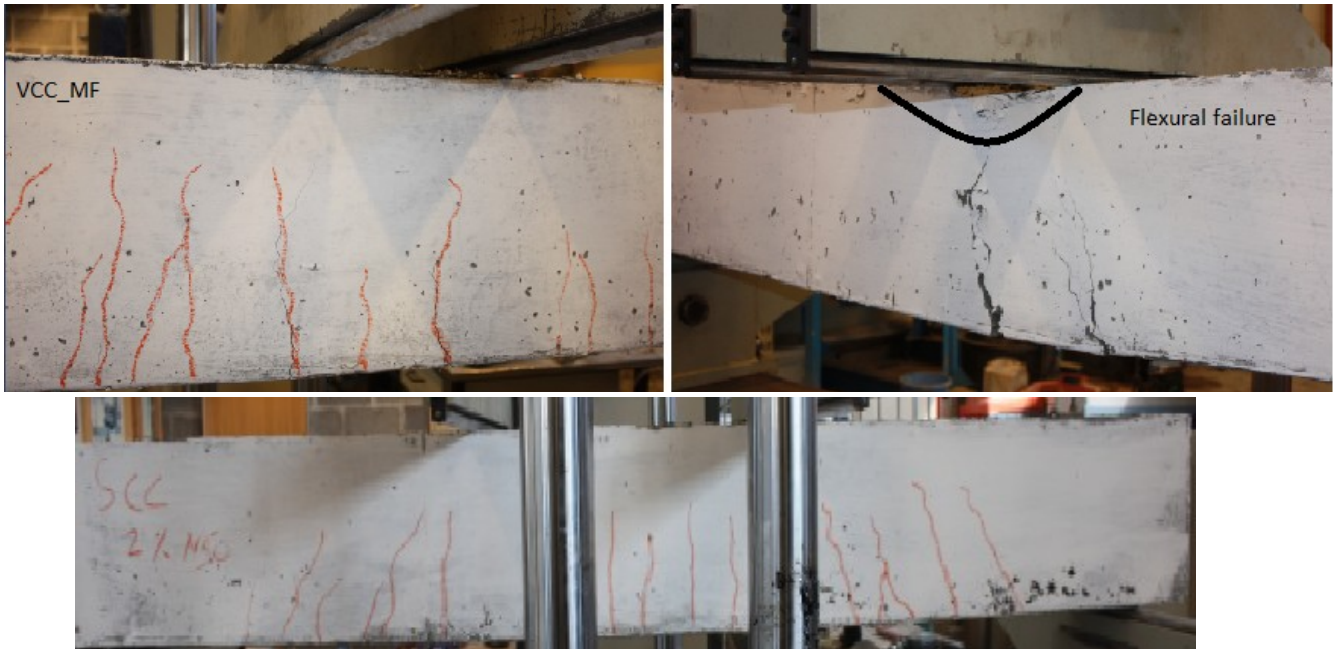


Figure 4-44: Crack pattern for VCC\_MF and SCC\_MF

Figure 4-45 shows the behavior of steel-fibers inside the cracks after failure. A few fracture failure in the steel fibers was observed but the most part of fibers failed by pull-out failure. In pull-out failure which is desirable for the test results, occur after yielding the end of steel fibers. Note that the steel fibers in references beam is of type end-hooked N50 with aspect ratio 50 which is given in table 3-13.

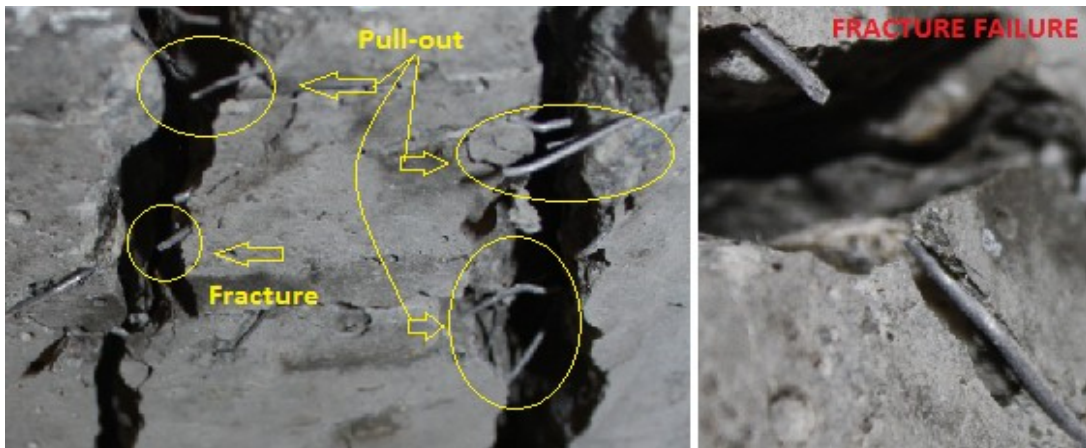


Figure 4-45: Bottom of VCC\_MF beam, pull-out and fracture of steel-fiber

4.5.4 Beam specimens with steel-fiber N50 (LWC\_N50) and N35 (LWC\_N35)

Four lightweight beam specimens with 1% and 2% steel-fiber of type N50 and N35 were loaded until the ultimate failure occurred. The figure 4-46 shows load and crack width relation of LWC\_N50 and LWC\_N35 beams including LWC\_UF and SCC\_UF, which are the reference beams. Figure 4-47 shows load-deflection diagram for beams with N50 and N35 fibers.

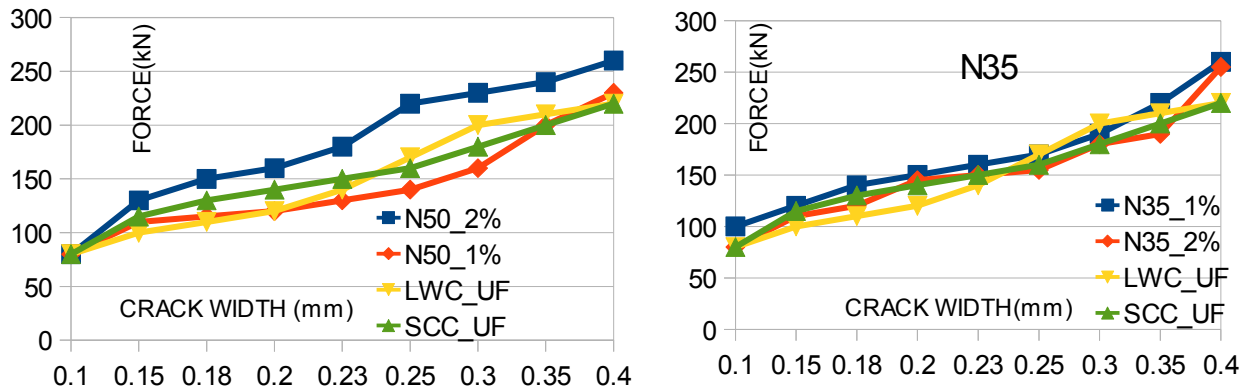


Figure 4-46: load-crack width diagram for N50 and N35

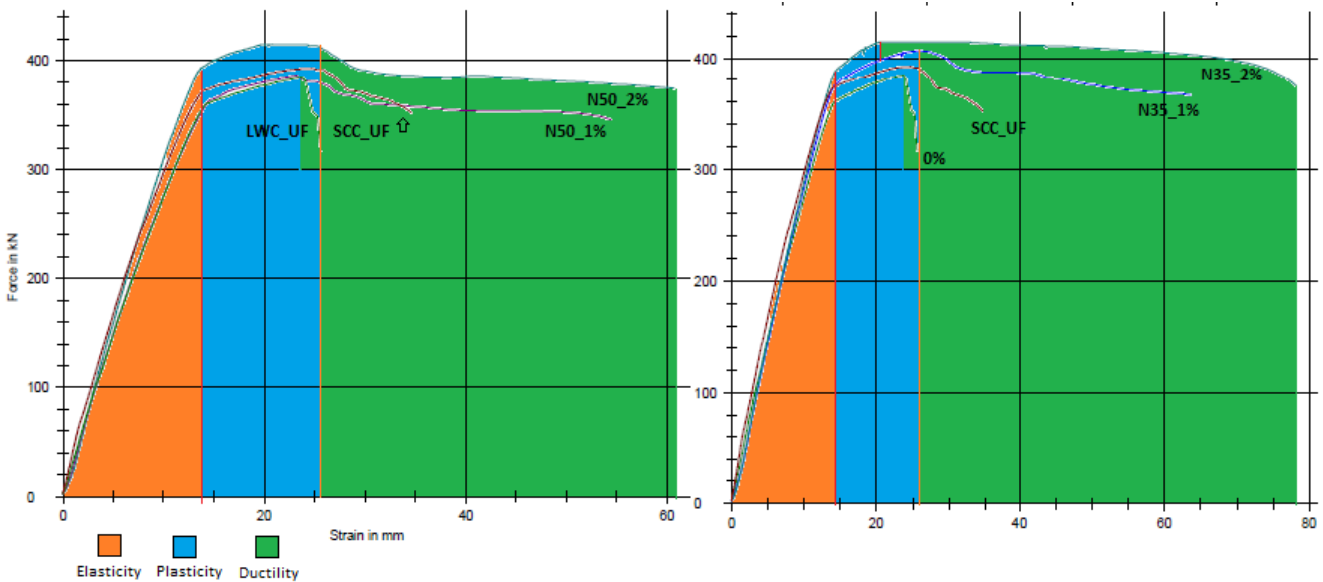


Figure 4-47: Load-deflection diagram of beams LWC\_N50

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The ductility behavior of beam specimens LWC\_N50 and N35 began approximated at 26mm deflection where the reference lightweight concrete beam had a flexural failure. LWC\_UF, LWC\_N50\_2% and LWC\_N35\_1% have identical concrete strength class (LC60/66) which give a better understanding of steel-fiber's influence on ductility.

Crack width of 0.4mm on LWC beams with 2% N50 and N35(1% and 2%) were discovered when load was 260kN. The result shows 18% load increasing compare to both reference beams and 13% compare to presents of 1% N50steel-fiber. It shows that presents of fiber controlled the crack opening up to 18% load increasing.

Deflection at crash point increases by 77% for LWC\_N50\_2% and 117% for LWC\_N35\_2% comparing to SCC concrete without fiber.

Figure 4-48 shows crack pattern at load 300kN. Fiber failure on crack opening were smiler to SCC\_MF and VCC\_MF. Pull-out failure was the failure mode for the most part of fibers. Diagonal cracks opening grew slower compared to cracks in middle of beam which shows increasing in shear capacity in beam specimen because of presents of steel fiber.



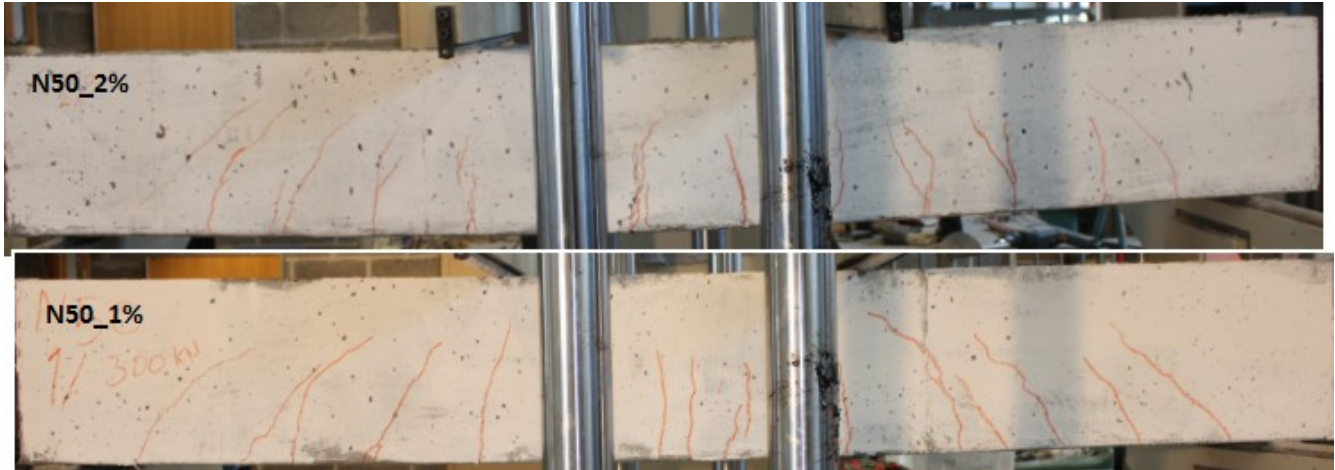


Figure 4-48: Crack pattern beam LWC\_N50 (2% and 1%).



Figure 4-49: Flexural failure in beam LWC\_N50\_2%

**4.5.5 Beam specimens with polypropylene microfiber M12(PP-fiber) and Plastic-fiber M50 (PF)**

Result of microfiber M12 (PP-fiber) and plastic-fiber beam specimens is showed by figure 4-50 and 4-51.

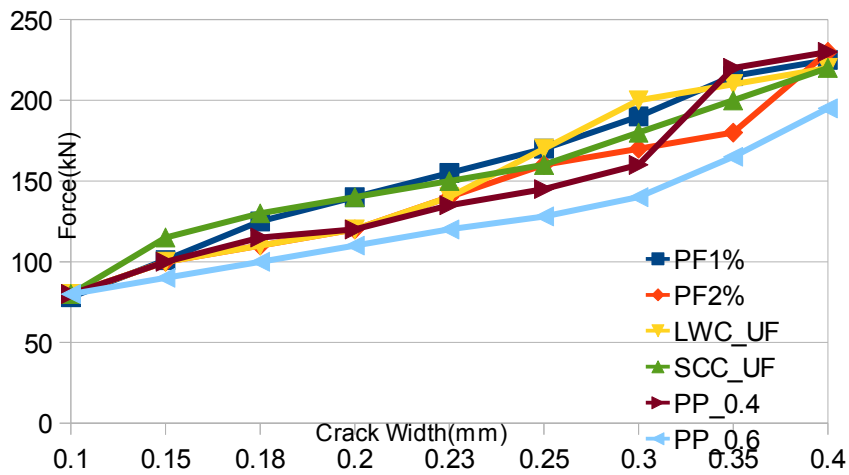


Figure 4-50: Load - crack width for PP- and plastic-fiber

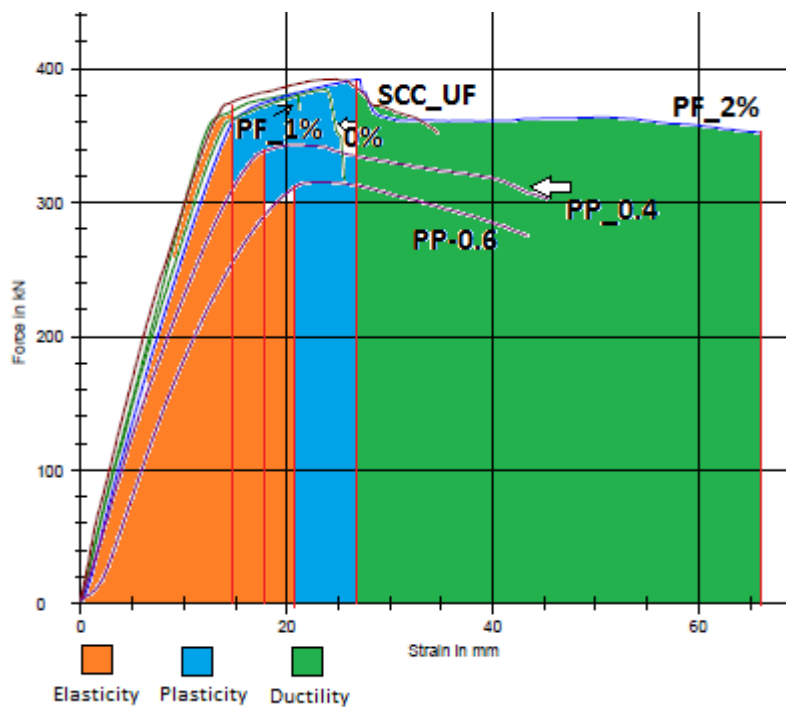


Figure 4-51: load-deflection relation for PP- and Plastic-fiber beams

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The beam specimen LWC\_PF\_1% was the first beam specimen which tested. There were some confusing with wizard and programming of the test setup which cause that test stopped right after plasticity zone was over. The ductility behavior is not registered.

The crack width development were almost similar in all beam specimens comparing with reference beam except beam PP\_0.6%.

PF\_2% beam specimen start the plasticity behavior at almost same deflection point as SCC\_UF beam specimen but the deflection at flexural failure had 88% increasing.

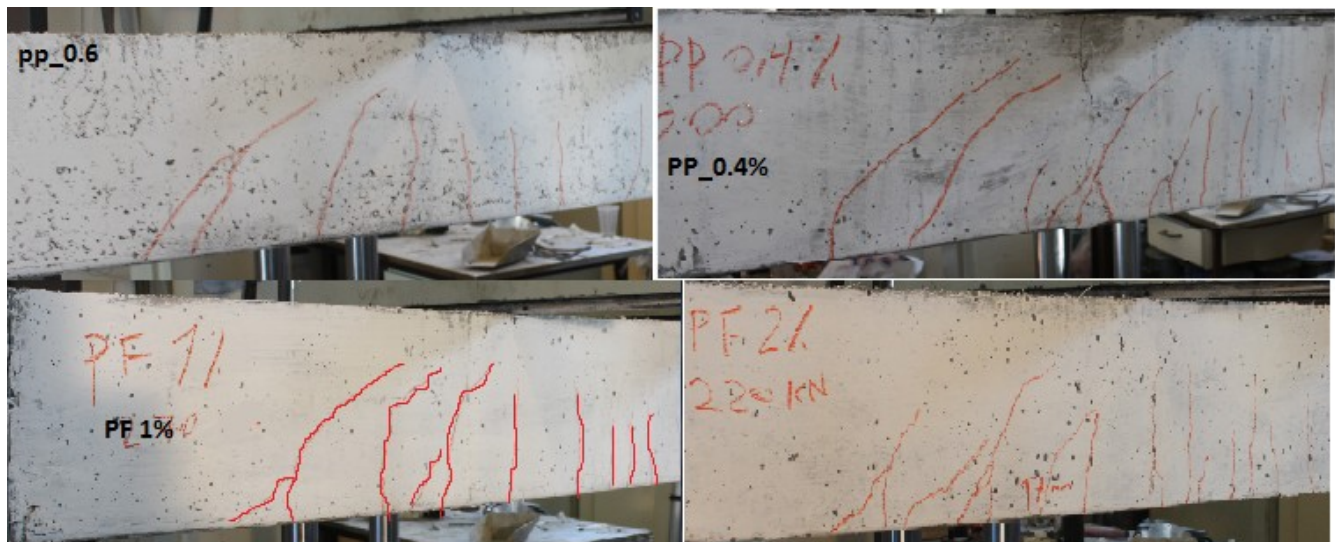


Figure 4-52: crack pattern on PP- and Plastic-fiber on load 300kN

#### 4.6 Analysis of results

Failure mode of beams were flexural failure, cracks were mainly vertical in middle of the beam span. In brittle concrete this failure is very unfortunate, the collapse of beam will be suddenly without any warning. This brittle failure has been observed in lightweight concrete without fiber-reinforced. The failure occurred as like as a explosive force.

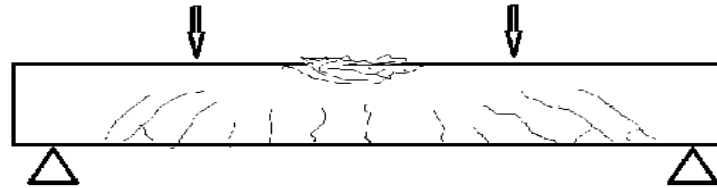


Figure 4-53: Flexural failure described in ACI

In general, all steel fiber reinforced beams exhibited elastic behavior through approximately 129 kN-m of applied moment beside PP-fiber reinforced beams which maximum reached 105 kn-m. The crack was discovered on all beam specimens approximated on load 60kN, and a few very fine crack started to develop in the midspan area of beam where the bending moment was maximum. The load and crack width on each beam specimens has been recorded. Table 4-25 shows the load on beam specimens when the crack width of 0.4mm were observed.

Table 4-25: Load on crack width 0.4mm

CRACK WIDTH 0.4	PF1%	PF2%	N50_2%	N50_1%	LWC_UF	N35_1%	N35_2%	PP_0.4	SCC_UF	SCC_N50	PP_0.6	VCC_N50_2%
Load(kN)	225	230	260	230	220	260	255	230	220	265	195	280
Comparing beams to LWC_UF & SCC_UF	2.3%	4.5%	18.2%	4.5%	0.0%	18.2%	15.9%	4.5%	0.0%	20.5%	-11.4%	27.3%

The result which is present in table 4-25 shows that steel-fiber has most effect on decreasing of crack width during loading of the beams. The difference between N50 1% and N50 2% steel-fiber content is significant regarding to control of crack width due stress on cracks. In beam specimens LWC\_N35\_1% and 2% has been noted that 1% had better result compare to N35\_2%. The result from table 4-25 is the crack width from the critical and the weakest point which reach the width of 0.4 mm first. The weakness in beam N35\_2% can be explain by bad distribution of fibers and using of vibrator which can make a weak point with no fiber or the bond between fiber and concrete is not strong enough to get a maximum utilization at early stage of crack development. Polypropylene microfiber M12 (PP-fiber) has showed no reduction on crack width during loading.

Plastic-fiber has not much contributions to decrease crack-width at early age but in ultimate load it control the crack opening which cause the ductile behavior.

#### 4.6.1 Proposal for crack width analysis

Determining of crack width for concrete reinforced composites is given in RILEM TC 1622-TDF, as it mentioned in previous section, this formulation does not take the volume fraction and direction of fibers in concrete. The main change in calculation is that the crack spacing become added a factor which will decrease the crack spacing.

Theoretical crack width calculation in this thesis are based on formulation from EC2.

$$W_k = s_{r,max} (\varepsilon_{sm} - \varepsilon_{cm})$$

$$s_{r,max} = k_3 \times c + k_1 k_2 k_4 \times \phi / \rho_{p,eff}$$

In EC2 is given that  $\varepsilon_{cm}$  is the mean strain in the concrete between cracks, and  $\varepsilon_{sm}$  is the mean strain in the reinforcement under loading. By assuming that the mean strain in the concrete is equal to strain in first crack, it is possible to calculate a new strain which take the volume fraction of fiber and random rotation in consideration. Crack strain can be identified by calculate the crack tensile strength and elastic modulus of fiber reinforced concrete ref[21].

Composite tensial strength at first crack

$$\sigma_{cc} = \eta_1 \eta_2 \sigma_f V_f + \sigma_{mu} (1 - V_f)$$

Stiffness:

$$E_c = \eta_1 \eta_2 E_f V_f + E_m (1 - V_f)$$

$$\varepsilon_c = \frac{\sigma_{cc}}{E_c}$$

where :

$\sigma_f$  is the tensial strength of fiber

$V_f$  is the volume variation of fiber

$\sigma_{mu}$  is the the matrix tensial strength at first crack

$E_f$  is the fiber modulus

$E_m$  is the matrix modulus

$\eta_1$  is fiber length factor

$\eta_2$  is the fiber orientation factor(1/5 for 3D)

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The mean strain in the reinforcement under loading is assumed to be equal to strain in the tension reinforcement on crack section.

$$\epsilon_s = \frac{\sigma_s}{E_s}$$

$\sigma_s$  is stress in tension reinforcement based on a crack section

The theoretical crack width is calculated based on this formulation. The formula is similar to EC2 crack width expression by difference in calculation of strain in concrete and reinforcement.

$$W_k = s_{r,max} (\epsilon_s - \epsilon_c)$$

The result from calculation of crack width showed in table 4-26 and calculation sheet is presented in appendix 2. The value of concrete E-modulus and load on each beam in this calculation were used the value from the earlier experiment tests.

*Table 4-26: theoretical and experimental result of crack width*

beam ID	load(kN)	Crack width (exp) mm	Crack width(mm) (proposed formula)	Crack width(mm) (COIN,2011)
Ref.SCC_MF	285	0.4	0.37	0.32
Ref.VBB_MF	280	0.4	0.39	0.32
Ref.SCC_UF	220	0.4	0.33	0.33
Ref.LWC_UF	220	0.4	0.29	0.29
LWC_N50_2%	260	0.4	0.38	0.29
LWC_N50_1%	230	0.4	0.33	0.29
LWC_N35_2%	255	0.4	0.35	0.28
LWC_N35_1%	260	0.4	0.36	0.32
LWC_PF2%	230	0.4	0.34	0.26
LWC_PF1%	225	0.4	0.34	0.29
LWC_PPF0.6%	195	0.4	0.28	0.25
LWC_PPF0.4%	230	0.4	0.34	0.3

The proposed approach of crack-width analysis has been checked by experiment test result. Table 4-26 and table 4-27 verify that result obtained with the proposed model are compared with crack-width observed in the test.

*Table 4-27: Crack width for beam LWC\_N50\_2%*

LWC_N50_2%		
Load on beam (kN)	Crack width (observation)mm	Crack width Porposed Theo (mm)
80	0.1	0.06
130	0.15	0.14
160	0.2	0.19
200	0.25	0.259
230	0.3	0.308
240	0.35	0.324
260	0.4	0.36

Theoretical Crack-width calculation for LWC\_N50\_2% which is showed in table 4-27, shows that proposed crack width calculation give approximate same value as experimental crack-width during loading.

#### 4.6.2 Deflection of beam specimens

Deflection of beam should be limited, it is given in EC2 that deflection in beam may not excesses span/250 or in some sensitive situation should be consider a limit value span/500. The stiffness and deflection of each beam specimens on service load 200kN has been calculated and compared to the deflection result from experiment. It has to be mentioned that the data such as E-modulus and strength class which are used in calculation of beam deflection are based on cylinder test result. Deflection of beam depend of beam stiffness and the concrete E-modulus, by increasing the E-modulus will result in decreased deflection.

$$\alpha = \rho \cdot \eta \cdot \left(1 + \frac{\rho'}{\rho}\right) \left[ \sqrt{1 + \frac{2(1 + \frac{\rho' \cdot d'}{\rho \cdot d})}{\rho \cdot \eta \cdot (1 + \frac{\rho'}{\rho})^2}} - 1 \right]$$

$$I_c = \frac{1}{2} \cdot \alpha^2 \cdot \left(1 - \frac{\alpha}{3}\right) \cdot b \cdot d^3$$

$$EI = E_c \cdot I_c$$

$$\delta_{\max} = \frac{a \cdot F}{2 \cdot 24 \cdot EI} (3L^2 - 4 \cdot a^2)$$

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*Table 4-28: result from beam test*

Beam ID	Deflection(mm)		Curvature	Ductility coeff.	% VS	%VS
	Theoretical(200kN)	Exp.test(200kN)	Ductility coeff	Deflection $\mu$	SCC	LWC_UF
Ref.SCC.MF	4.23	6.9	3	4.67	100.0%	135.7%
Ref.VBB.MF	4.22	7	3.05	7.40	217.6%	273.7%
Ref.SCC.UF	4.26	7	3.01	2.33	0.0%	17.8%
Ref.LWC_UF	4.8	6	1.82	1.98	-18.9%	0.0%
LWC_SF2%_N50	4.67	6.5	2.6	4.30	84.5%	117.2%
LWC_SF1%_N50	4.75	7.5	2.54	3.87	66.1%	95.5%
LWC_SF2%_N35	4.85	6.8	2.46	5.24	124.9%	164.7%
LWC_SF1%_N35	5.05	7.5	2.33	4.38	88.0%	121.2%
LWC_PF2%	4.78	7.5	2.49	4.89	110.0%	146.9%
LWC_PF1%	4.89	7.6	2.43	4.23	81.5%	113.7%
LWC_PPF0.6%	5.8	10.5	1.9	2.05	-12.0%	3.4%
LWC_PPF0.4%	5.22	9	2.2	2.50	7.3%	26.3%

Table 4-28 shows the result of deflection and curvature ductility coefficient calculation which are based on theoretical calculation. Curvature ductility coefficient do not take consideration the influence of fibers directly. The deflection ductility coefficient give more accurate results compared to curvature ductility coefficient as it is presented in table 4-28. Deflection and curvature ductility calculation sheet is presented in appendix 2d.

By comparing the deflection ductility coefficient of each beam to SCC-UF and LWC\_UF will shows that ductility has been increased in all type fiber reinforced concrete except Polypropylene microfiber M12 (PP-fiber).



## **5. Conclusion**

Concrete become more brittle and less ductile by increasing the compressive strength of concrete. This become more obvious with lightweight concrete since the strength of cement paste is similar than lightweight aggregates. This combination will cause a sudden fracture through aggregates and cement paste.

Fiber-reinforced concrete has been tested for compressive strength, E-modulus, tensile strength and beams cracks behavior during loading. The following conclusions can be made from this thesis:

1. The use of end hooked steel fibers and plastic fibers in volume fraction 1 and 2% did not influenced the compressive strength of lightweight concrete. Polypropylene microfiber (PP-fiber) decreased the compressive strength of lightweight concrete because of high volume fraction which caused segregation and low workability.
2. The tensile strength of LWC and NDC were improved by using steel-fiber. Increasing of volume fraction of steel-fiber caused increasing in tensile strength. Polypropylene microfiber M12 (PP-fiber) and plastic macrofiber M50 showed no effect on tensile strength.
3. The lightweight concrete with 2% end-hook steel fiber was the only SFRC which showed improvement of ductility with high ultimate stress under stress-strain diagram test. All SFRC had improvement in ductility but with low ultimate stress compare to SCC without fiber(SCC\_UF).
4. Steel-fibers showed high resistance to crack development, end-hooked N50 by 2% volume fraction showed better ability to arrest cracks compare to 1% volume fraction. Plastic-fiber and microfiber showed similar crack-width development as references beam. Polypropylene microfiber M12 (PP-fiber) with volume fraction greater than 0.4% caused segregation and bleeding in fresh concrete which resulted lower resistance to crack development in beam with 0.6% volume fraction PP-fiber.
5. The result showed that all type of fibers improved the ductility behavior for concrete beam specimens compare to beam specimens without fiber. Increasing in volume fraction of end hock steel-fiber and plastic-fiber increased the flexurale moment capacity and ductility of beam

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### Ductility in Lightweight Concrete with Fiber

specimens. Polypropylene microfiber M12 (PP-fiber) result higher deflection on beam and better ductility but the failure moment capacity decreased compared to others beam specimen.

6. The results from beam specimens under loading showed that brittleness of a high performance lightweight concrete can be modified by using additional reinforcement such as fibers. Following fibers with given volume fraction would improve the ductility behavior of beams compare to a NDC beam without fiber and could be considered as an option to replace NDC in seismic zones: lightweight concrete reinforced by end-hooked steel fibers (1 and 2% volume fractions) and Lightweight concrete reinforced by polypropylene plastic-fiber with 2% volume fraction.

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### Ductility in Lightweight Concrete with Fiber

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## Appendix 1

1.a Leca 700 4-8mm

1.b Leca 3-6mm

1.c Plastic fiber M50

1.d PP-fiber M12

1.e Steel-fiber N35

1.f Steel-fiber N50

## Appendix 2

2.a Beam design

2.b Calculation of crack width based on COIN 2011

2.c Calculation of crack width based on proposed formula

2.d Calculation of curvature ductility coefficient



 04 1111-CPD-0003	 maxit Group Postboks 216 Alnabru N-0614 Oslo
	<b>EN 13055-1</b> <b>Lette tilslag til betong, mørtel og injeksjonsmørtel</b>

**Produktnavn:** Leca 700, 4-8 mm

**Varenummer:** 1029,1

Egenskap	Analysemetode	Deklarert verdi	
Kornform		Rund	
Kornstørrelsesfordeling	EN 933-1	4-8 mm Overkorn $\leq 10\%$ , Underkorn $\leq 15\%$	%
Finstoff (<63 $\mu\text{m}$ )	EN 933-1	$\leq 2$	%
Bulkdensitet	EN 1097-3	$700^{2)} \pm 5 \%$	$\text{kg/m}^3$
Andel knuste korn		NPD <sup>1)</sup>	%
Renhet		NPD <sup>1)</sup>	
Egenfasthet	EN 13055-1, annex A, proc. 1 Vibrasjonstid: 20 sek	Minimumsverdi $\geq 7,0$	$\text{N/mm}^2$
Klorid	EN 1744 -1	$\leq 0,04$	% CL-
Syreoppløselig sulfat	EN 1744 -1	NPD	% $\text{SO}_3$
Totalt svovelinnhold	EN 1744 -1	Uttrykt som S: Gjennomsnittsverdi 0,05, maksimumsverdi $\leq 0,32$ . Uttrykt som $\text{SO}_3$ : Gjennomsnittsverdi 0,15, maksimumsverdi $\leq 0,80$ .	% S
Volumstabilitet	EN 13055-1, annex B	Holdbar ifølge lang tids erfaring	%
Vannabsorpsjon	EN 1097-6, Annex C	< 10% etter 1 time < 20% etter 24 timer For bruk i lettbetong bør eksakt 1 timers verdi måles på hvert enkelt parti.	%
Emisjon av radioaktivitet		NPD <sup>1)</sup>	
Utlekking av tungmetaller		NPD <sup>1)</sup>	
Utlekking av polyaromatiske karboner		NPD <sup>1)</sup>	
Utlekking av andre farlige stoffer		NPD <sup>1)</sup>	
Holdbarhet overfor frysing / tining.	EN 13055-1, annex C	Holdbar ifølge lang tids erfaring	%
Holdbarhet over for alkalireaksjoner		Holdbar ifølge lang tids erfaring	

<sup>1)</sup> NPD: No Performed Data, Ikke relevant informasjon, <sup>2)</sup> Ordreproduksjon, densitet fastsettes sammen med kunde, kan avvike fra deklart verdi.

**Supplerende deklarasjon (utenfor CE - merkning):**

Egenskap	Analysemetode	Deklarert verdi	
Partikkeldensitet	EN 1097-6, annex C	$1200^{2)} \pm 10 \%$	$\text{Kg/m}^3$

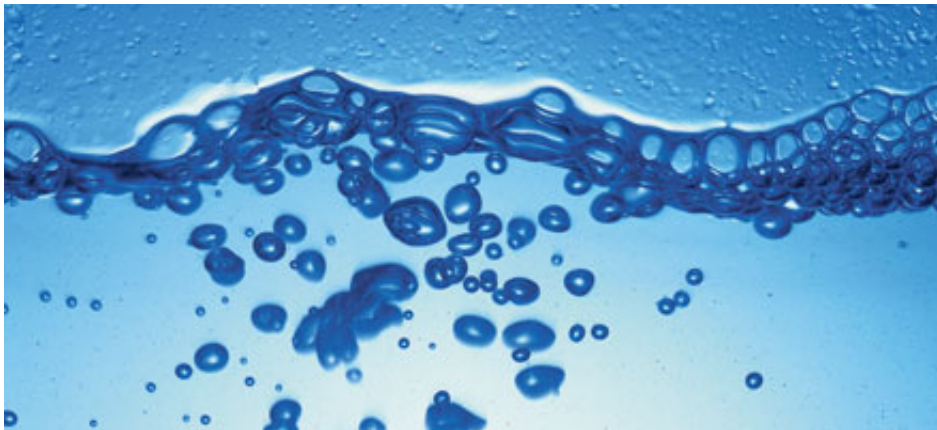


Hjem > Filtralite > Filtralite > FILTRALITE HR 3-6 mm

## FILTRALITEHR 3-6 mm

Filtralite egner seg godt til vannrensing, enten det er drikkevann, avløpsvann eller forbehandling før avsaltning. Høy densitet og runde leireaggrater.

- Enkel å installere i filtre
- Stor overflate for bakterier å vokse på
- Effektiv partikkelfiltrering



[Åpne / Lukke](#)

### FILTRALITE HR 3-6 MM

Commercial name	FILTRALITE HR 3-6 mm
Density	Bulk density: 825 kg/m <sup>3</sup> particle density: 1450 kg/m <sup>3</sup>
Type of material	Expanded clay
Appearance	Round particles, smooth surface structure
Manufactured by	Weber Leca Rælingen, Norway

### SIZE AND WEIGHT

Size/weight	Value	Deviation	Comments
Effective size	3,5 mm	± 0,3 mm	d10
Particle size range	3-6 mm	< 2,5 mm max 3 % + Δ < 0,125 mm > 6,0 mm max. 5 %	
Coefficient of uniformity	< 1,5		d60 / d10
Bulk density, dry	825 kg/m <sup>3</sup>	± 75 kg/m <sup>3</sup>	EN 1097-3
Particle density, dry (PDD)	1450 kg/m <sup>3</sup>	± 100 kg/m <sup>3</sup>	Exclay Norm

### OTHER PROPERTIES

Other properties	Value	Comments
Floating particles	< 2%	Maximum volume floating particles after 2 days in water.



# Plasticfibre M50

## PRODUKTBEKRIVELSE

**Plasticfibre M50** er en makrofiber laget i plastmaterialet polypropylen.

**Plasticfibre M50** er en strukturell fiber for armering av betong. Fiberen har gode mekaniske og kjemiske egenskaper for bruk i betong.

## KVALITETER

- Lav egenvekt og dermed lavere dosering sammenlignet med stålfiber.
- Kjemisk inaktiv.
- Godt egnet for bruk i korrosivt miljø.
- Kjemikaliebestandig, angripes ikke av syrer eller løsemidler.
- Alkali-resistent.
- Ingen kjent helsesisiko.
- Leveres ferdig buntet i bunter av 250 g, som løses opp ved innblanding i betong

## BRUKSOMRÅDE

**Plasticfibre M50** er godt egnet til armering av betongdekker på grunn for å hindre svinnriss og oppsprekking av betong. Vær klar over at betong som støpes på dekker bør beskyttes mot uttørking uavhengig av fibertype og dosering. Dette kan være påføring av membranherder, ettervanning og tildekking.

**Plasticfibre M50** er meget godt egnet i dekker, og konstruksjoner der man har stadig varierende laster som må tåle støtbelastning men ikke har moment-belastning over lang tid tid:

- Verkstedgulv / biloppstillingsplasser
- Kaier, anker
- Stableblokker
- Betongbrystinger og støpte sikkerhetsgjerd.

## INNBLANDING OG DOSERING

Det beste er å tilsette fiber i betongen i blanderen. Alternativt kan den tilsettes i betongbilen. **Plasticfibre M50** leveres i

bunter på 250 gram. Fibrene entrer derfor betongen orientert parallelt med hverandre, og dette hindrer risikoen for dannelse av fiberballer. Rundt hver bunt med 250 gram fiber er det er plastbånd som er vannløselig, og som løses opp ved innblanding i betong.

Ved tilsetning av fiber i betongbil, sørg for at trommel går på maksimal hastighet under dosering, og la trommel gå på maksimal hastighet i ytterligere 5 minutter etter at siste fiber er tilsatt.

For bruk i betongdekker på grunn anbefales det å dosere 3,5 – 5,0 kg/m<sup>3</sup> betong.

## LAGRING OG HOLDBARHET

Må lagres tørt og beskyttet mot direkte sollys.

## VERNETILTAK

For helse-, miljø- og sikkerhetsinformasjon - se eget sikkerhetsdatablad. Sikkerhetsdatabladene finnes på [www.resconmapei.com](http://www.resconmapei.com).

## MERK

*De tekniske anbefalinger og detaljer som fremkommer i denne produktbeskrivelse representerer vår nåværende kunnskap og erfaring om produktene.*

*All ovenstående informasjon må likevel betraktes som retningsgivende og gjenstand for vurdering.*

*Enhver som benytter produktet må på forhånd forsikre seg om at produktet er egnet for tilsiktet anvendelse.*

*Brukeren står selv ansvarlig dersom produktet blir benyttet til andre formål enn anbefalt eller ved feilaktig utførelse.*

*Alle leveranser fra Rescon Mapei AS skjer i henhold til de til enhver tid gjeldende salgs- og leveringsbetingelser, som anses akseptert ved bestilling.*



## TEKNISKE SPESIFIKASJONER

Material:	Polypropylen
Fiberlengde, mm:	50 (± 10 %)
Fiberbredde/tykkelse mm:	1,28/0,81 (gjennomsnitt: 1,1) (± 10 %)
Materialdensitet, g/cm <sup>3</sup> :	0,91
E-modul, N/mm <sup>2</sup> :	754
Smeltepunkt, °C:	160
Strekkefasthet, N/mm <sup>2</sup> :	250
Forpakning:	Bunter á 250 gram, 12 kg pr. eske, 81 esker pr. pall.

## Produsent:

Rescon Mapei AS  
Vallsetvegen 6, 2120 Sagstua, Norway  
Tlf: +47 62 97 20 00 Fax: +47 62 97 20 99  
post@resconmapei.no  
www.resconmapei.com



# PP-fibre M12

## Polypropylen mikrofiber

### PRODUKTBSKRIVELSE

**PP-fiber M12** er en multifillament-propylenfiber med optimale mekaniske, fysiske og kjemiske egenskaper for bruk i mørtel og betong.

### KVALITETER

- Stor overflate pr mengde fiber
- Høy bindings-/ heftevne
- Kjemisk inaktiv
- Problemfri innblanding
- Lett å pumpe, ingen slitasje på rør og slanger
- 100 % alkali-resistent
- Ingen kjent helsesisiko
- Angripes ikke av syrer eller løsemidler

### Spesielt effektiv i de første kritiske timene i størkning og herdefasen

Under den første tiden i størknings- og herdefasen utvikles betongens styrke langsommere enn sammentrekkingskreftene. Dette plastiske svinnet kommer primært av avdamping fra overflaten, samt den kjemiske reaksjonen mellom sement og vann. Som en konsekvens av plastisk svinn vil betongen risse opp. Ved å tilføre **PP-fiber M12** i fersk betong, vil betongens mulighet til å ta opp strekk- kreftene som oppstår i de første kritiske timene etter utstøping øke betydelig.

### Betong med PP-fiber

- Økt brannmotstand
- Reduserer forekomsten av riss pga av plastisk svinn
- Reduserer risikoen for "bleeding"
- Øker betongens tetthet
- Øker holdbarheten ved temperatur- endringer
- Forbedrer betongens motstand mot slag, forhindrer skader på hjørner og kanter
- Reduserer skader ved avforming og under transport av elementer
- Raskere og bedre konstruksjon pga økt kohesjon (sammenholdning) i betongen

### Produsent:

Rescon Mapei AS  
Vallsetvegen 6, 2120 Sagstua, Norway  
Tlf: +47 62 97 20 00 Fax: +47 62 97 20 99  
post@resconmapei.no  
www.resconmapei.com

### Tilsetning

- **PP-fiber M12** kan tilsettes i tørre materialer eller i fersk betong i blanderen, eller den kan tilsettes direkte i automikseren
- Betong med **PP-fiber M12** kan pumpes og sprøytes
- Betong med **PP-fiber M12** reduserer betongens synk. Kombineres derfor med bruk av superplastiserende stoffer (**Dynamon**-produkter).
- Blandingstid:
  - I betongblanderen: normal blandetid er tilstrekkelig
  - I betongbil: blandingstiden økes med 1 minutt pr m<sup>3</sup> med høy hastighet på trommelen.

### Dosering

Normaldosering 1 pose a 910 g pr m<sup>3</sup>  
Antall poser pr kartong: 20  
Antall kartonger pr pall: 30  
**PP-fiber M12** er pakket i vannløselige poser

### LAGRING OG HOLDBARHET

Må lagres tørt og beskyttet mot direkte sollys.

### VERNETILTAK

For helse-, miljø- og sikkerhets- informasjon - se eget sikkerhetsdatablad. Sikkerhetsdatabladene finnes på [www.resconmapei.com](http://www.resconmapei.com).

### MERK

*De tekniske anbefalinger og detaljer som fremkommer i denne produktbeskrivelse representerer vår nåværende kunnskap og erfaring om produktet. All ovenstående informasjon må likevel bli betraktet som retningsgivende og gjenstand for vurdering. Enhver som benytter produktet må på forhånd forsikre seg om at produktet er egnet for tilsiktet anvendelse. Brukeren står selv ansvarlig dersom produktet blir benyttet til andre formål enn anbefalt, eller ved feilaktig utførelse.*

*Alle leveranser fra Rescon Mapei AS skjer i henhold til de til enhver tid gjeldende salgs- og leveringsbetingelser som anses akseptert ved bestilling.*



### TEKNISKE SPESIFIKASJONER

Materialtype:	Polypropylen
Vekt pr pose (kg):	0,91
Lengde (mm):	12 ± 2
Diameter (µm):	22 ± 3
Densitet (g/dm <sup>3</sup> ):	0,91
Antall fibre (mill/kg):	224
E-modul (N/mm <sup>2</sup> ):	3500 - 3900
Strekkstyrke (N/mm <sup>2</sup> ):	400
Kjemisk reaksjon:	Inert
Farge:	Hvit/gjennomsiktig
Emballasje:	Poser à 910 g, 20 poser per kartong. Vannløselige
Lagring:	Lagres tørt



# Steelfibre DE 35/0,55 N

## PRODUKTBEKRIVELSE

**Steelfibre DE 35/0,55 N** er en høy-pres-terende stålfiber til armering av betong. Fiberlengden er 35 mm og er derfor meget godt egnet for tynnere betong-skikt og sprøytebetong. **Steelfibre DE 35/0,55 N** leveres magnetisk orientert i 20 kg papkartonger. Magnetisk orientering sikrer effektiv innblanding i betongen. Fiberen er trådbasert og har endeforankring i begge ender.

**Steelfibre DE 35/0,55 N** har strekkfasthet på 1100 ( $\pm 15\%$ ) MPa.

## ANBEFALINGER

Det anbefales alltid å tilsette fiber på betongfabrikk.

I betongblander:

- Tilsett aldri fiber som første komponent. Tilsett fiber under omrøring i tilslag eller i ferdig blandet betong.
- Bland betongen til fibreene er homogent fordelt.

I betongbil:

- Tilsett fiber under maksimal omdreining på trommel (12 – 18 rpm).
- Betongens synk skal være minst 120 mm.
- Ikke tilsett mer enn 60 kg/min.
- Etter endt tilsats, la trommel gå på maksimal hastighet i 5 min.

## DOSERING I SPRØYTEBETONG

Dosering er avhengig av ønsket energi-absorpsjonsklasse. Målinger er utført av SINTEF Byggforsk etter NB Publikasjon nr. 7.

## LAGRING OG HOLDBARHET

Ved tørr lagring i uåpnet original emballasje, beskyttet mot fukt, er holdbarheten minst ett år. Fuktskadet produkt med begynnende korrosjon kan ikke lenger påregnes å oppfylle de tekniske spesifikasjonene.

## VERNETILTAK

For helse-, miljø- og sikkerhetsinformasjon - se eget sikkerhetsdatablad. Sikkerhetsdatabladene finnes på

[www.resconmapei.com](http://www.resconmapei.com)

## MERK

*De tekniske anbefalinger og detaljer som fremkommer i denne produktbeskrivelse representerer vår nåværende kunnskap og erfaring om produktene.*

*All ovenstående informasjon må likevel betraktes som retningsgivende og gjenstand for vurdering.*

*Enhver som benytter produktet må på forhånd forsikre seg om at produktet er egnet for tilsiktet anvendelse. Brukeren står selv ansvarlig dersom produktet blir benyttet til andre formål enn anbefalt eller ved feilaktig utførelse.*

*Alle leveranser fra Rescon Mapei AS skjer i henhold til de til enhver tid gjeldende salgs- og leveringsbetingelser, som anses akseptert ved bestilling.*

### TEKNISKE SPESIFIKASJONER

Energiabsorpsjonsklasse:	E700	E1000
Fiberdosering (kg/m <sup>3</sup> ):	20	Dokumenteres i prosjekt

### Produktspesifikasjoner:

Lengde:	35 mm ( $\pm 10\%$ )
Diameter:	0,55 mm ( $\pm 10\%$ )
Strekkfasthet:	1100 MPa ( $\pm 15\%$ )
Slankhetstall (l/d):	63,5



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# Steelfibre DE 50/1,0 N

## PRODUKTBEKRIVELSE

**Steelfibre DE 50/1,0 N** er en høy-presterende stålfiber til armering av betong. Fiberlengden er 50 mm.

**Steelfibre DE 50/1,0 N** leveres som løs fiber i 20 kg pappkartonger. Fiberen blir orientert ved hjelp av magnet før pakking, så alle fibre ligger parallelt i pappkartongen. Dette gjør tilsetning og innblanding i betong enklere. Fiberen er trådbasert og har endeforankring i begge ender.

**Steelfibre DE 50/1,0 N** har strekkfasthet på 1100 ( $\pm 15\%$ ) MPa.

## ANBEFALINGER

Det anbefales alltid å tilsette fiber på betongfabrikk.

I betongblander:

- Tilsett aldri fiber som første komponent. Tilsett fiber under omrøring i tilslag eller i ferdig blandet betong.
- Bland betongen til alle fibre er homogent fordelt.

I betongbil:

- Tilsett fiber under maksimal omdreining på trommel (12 – 18 rpm).
- Betongens synk skal være minst 120 mm.
- Ikke tilsett mer enn 60 kg/min.
- Etter endt tilsats, la trommel gå på maksimal hastighet i 5 min.

## LAGRING OG HOLDBARHET

Ved tørr lagring i uåpnet original emballasje, beskyttet mot fukt, er holdbarheten minst ett år. Fuktskadet produkt med begynnende korrosjon kan ikke lenger påregnes å oppfylle de tekniske spesifikasjonene.

## VERNILTAK

For helse-, miljø- og sikkerhetsinformasjon - se eget sikkerhetsdatablad. Sikkerhetsdatabladene finnes på [www.resconmapei.com](http://www.resconmapei.com)

## MERK

De tekniske anbefalinger og detaljer som fremkommer i denne produktbeskrivelse representerer vår nåværende kunnskap og erfaring om produktene.

All ovenstående informasjon må likevel betraktes som retningsgivende og gjenstand for vurdering.

Enhver som benytter produktet må på forhånd forsikre seg om at produktet er egnet for tilsiktet anvendelse.

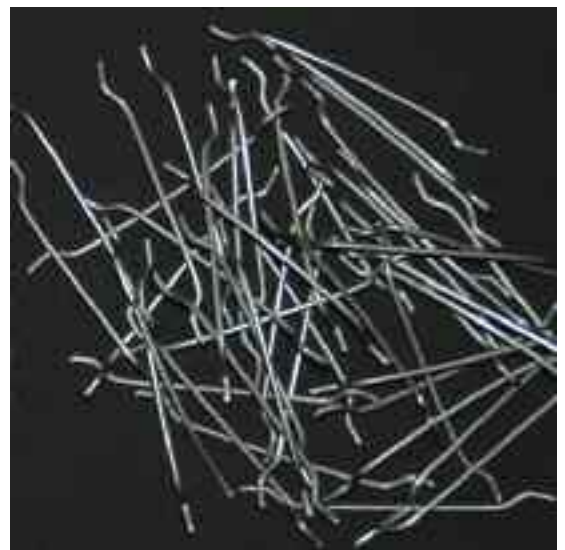
Brukeren står selv ansvarlig dersom produktet blir benyttet til andre formål enn anbefalt eller ved feilaktig utførelse.

Alle leveranser fra Rescon Mapei AS skjer i henhold til de til enhver tid gjeldende salgs- og leveringsbetingelser, som anses akseptert ved bestilling.

## TEKNISKE SPESIFIKASJONER

### Produktspesifikasjoner:

Lengde:	50 mm ( $\pm 10\%$ )
Diameter:	1,0 mm ( $\pm 10\%$ )
Strekkfasthet:	1100 MPa ( $\pm 15\%$ )
Slankhetstall (l/d):	50



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*Design of beam**Section properties :*beam width/heights  $b/h = 250/300\text{mm}$ 

nom\_c = 25mm

*design values*

$$V_{ED} = 90\text{kN} \quad M_{ED} = 63\text{kNm}$$

*Materials and characteristic strengths :*

concrete: C50/60

$$f_{ck} = 50\text{N/mm}^2 \quad f_{ctm} = 4,1\text{N/mm}^2 \quad f_{yk} = 500\text{N/mm}^2$$

$$E_s = 2 \times 10^5 \text{N/mm}^2$$

*Partial safety factor*

$$\gamma_c = 1,5 \quad \gamma_s = 1,15 \quad \gamma = 1$$

$$\alpha_{cc} = 0,85$$

$$\left( \begin{array}{l} \lambda = 0,8 \\ \eta = 1 \end{array} \right) \text{ for strength class } \leq \text{B50}$$

*Characteristic strengths :*

$$f_{yd} = f_{yk} / \gamma_s = 435\text{N/mm}^2 \quad f_{cd} = \alpha_{cc} \times f_{ck} / \gamma_c = 28,3\text{N/mm}^2$$

$$\varepsilon_{yd} = f_{yd} / E_s = 0,00217$$

$$d = 300 - 25 - 8 - 16 \times 0,5 = 259\text{mm}$$

*Analysis*

$$M_{Rd} = 0,275 f_{cd} b d^2 = 130\text{kNm}$$

$$\alpha_b = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = \frac{0,0035}{0,0035 + 0,00217} = 0,617$$

$$A_{s,b} = \frac{\lambda \times \eta \times f_{cd} \times b \times d \times \alpha_b}{f_{sd}} = \frac{0,8 \times 1 \times 28,3 \times 250 \times 259 \times 0,617}{435} = 2079\text{mm}^2$$

 $A_s < A_{s,b}$  reinforcement yield before concrete crush

$$z = 0,84 \times 259 = 217,5$$

$$A_s = \frac{M_{Ed} \times 10^6}{f_{yd} \times z} = \frac{63 \times 10^6}{435 \times 217,5} = 666\text{mm}^2$$

*Use :*

$$3 \text{ } \phi 20 \quad A_s = 942\text{mm}^2$$

Control of minimum reinforcement

$$A_{s,\min} = \frac{0,26 \times f_{ctm} \times b_t \times d}{f_{yk}} = \frac{0,26 \times 2,8 \times 250 \times 259}{500} = 94\text{mm}^2$$

$$A_{s,\min 2} = 0,0013 \times b_t \times d = 84,17\text{mm}^2$$

$$A_{s,\min} = \text{MAX}(A_{s,\min} ; A_{s,\min 2}) = 94 < 666 \text{ O.K.}$$

(2b)

### Calculation of the residual strength in fiber reinforced concrete(FRC)

$\sigma_{fk} := 500\text{MPa}$  This value has been suggested by SINTEF for steel fiber

$$\eta_0 := \frac{1}{3}$$

$v_f := 0.02$  Volume fraction of fibers

$$\gamma_m := 1.55$$

$$f_{tk.res} := \eta_0 \cdot v_f \cdot \sigma_{fk} = 3.333\text{MPa}$$

$$f_{ftd.res} := \frac{f_{tk.res}}{\gamma_m} = 2.151\text{MPa}$$

### Calculation of crack width for lightweight concrete beam with steel-fiber (LWC\_N50\_2%)

This calculation is based on EC2 and result of beam each is showed in table 4-26

Max force on beams  $F_{max} := 260\text{kN}$   $l_t := 2\text{m}$   $p := \frac{F_{max}}{2}$   
 $l_m := 0.7\text{m}$

Max Moment on beam  $M_{max} := \frac{F_{max} \cdot l_m}{2}$

$$M_{max} = 91\text{m}\cdot\text{kN}$$

Tensile strength of concrete  $f_{ctm} := 4.43\text{MPa}$

Material data:

$E_s := 200\text{GPa}$   $b := 250\text{mm}$

$k_1 := 0.8$   $d := 257\text{mm}$

$k_2 := 0.5$

$k_3 := 3.4$

$k_4 := 0.425$

$$k_5 := \left( 1 - \frac{f_{ftd.res}}{f_{ctm}} \right) = 0.515$$

$\phi := 20\text{mm}$

$A_s := 942\text{mm}^2$

$d_1 := 28\text{mm} + 10\text{mm} + 6\text{mm}$



$$\rho_{p,eff} := 0.051$$

$$A_{s1} := 226 \text{ mm}^2$$

$$\phi_b := 8 \text{ mm}$$

$$c_m := 25 \text{ mm} + \phi_b = 33 \text{ mm}$$

Crack spacing

$$S_{rmax} := \left( k_3 \cdot c_m + k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot \frac{\phi}{\rho_{p,eff}} \right) = 146.503 \cdot \text{mm}$$

For finding strain in reinforcement need to calculate the stiffness of beam

**E concrete**

$$E_{cm} := 24 \text{ GPa}$$

(E – modulus) for steel

$$E_s = 200 \cdot \text{GPa}$$

material stiffness:

$$\eta := \frac{E_s}{E_{cm}} = 8.333$$

reinforced ratio

$$\rho := \frac{A_s}{b \cdot d} = 0.015$$

$$\rho_1 := \frac{A_{s1}}{b \cdot d} = 3.518 \times 10^{-3}$$

$$\eta \rho := \eta \cdot \rho = 0.122$$

For finding EI

$$I_c := 0.5 \cdot \alpha^2 \cdot \left( 1 - \frac{\alpha}{3} \right) \cdot b \cdot d^3$$

$$\alpha := \rho \cdot \eta \cdot \left( 1 + \frac{\rho_1}{\rho} \right) \left[ \sqrt{1 + \frac{2 \left( 1 + \frac{\rho_1 \cdot d_1}{\rho \cdot d} \right)}{\rho \cdot \eta \cdot \left( 1 + \frac{\rho_1}{\rho} \right)^2}} - 1 \right] = 0.375$$

$$EI := E_{cm} \cdot I_c$$

reinforcement stress:

$$\sigma_s := E_s \cdot M_{max} \cdot (1 - \alpha) \cdot \frac{d}{EI} = 466.106 \cdot \text{MPa}$$

$$f_{ct,eff} := 4.4 \frac{\text{N}}{\text{mm}^2}$$

$$k_t := 0.6$$

$$\Delta \varepsilon := \frac{\sigma_s - k_t \cdot \frac{f_{ct,eff} \cdot (1 + \eta \cdot \rho_{p,eff})}{\rho_{p,eff}}}{E_s}$$

$$\Delta \varepsilon = 1.962 \times 10^{-3}$$

$$S_{rmax} = 146.503 \cdot \text{mm}$$

Crack width:

$$w_k := S_{rmax} \cdot \Delta \varepsilon = 0.287 \cdot \text{mm}$$

## Crack width analysis for concrete reinforced composites steel fiber N35 2% volume fraction

**2.W**

Max force on beams  $F_{\max} := 260\text{kN}$   $l_t := 2\text{m}$   $p := \frac{F_{\max}}{2}$   
 $l_m := 0.7\text{m}$

Max Moment on beam  $M_{\max} := \frac{F_{\max} l_m}{2}$   $M_{\max} = 91\text{ m}\cdot\text{kN}$

Matrial data:

$E_s := 200\text{GPa}$   $b := 250\text{mm}$  distance from compression reinforcement  
 $E_c := 24\text{GPa}$   $d := 259\text{mm}$   $d_1 := 28\text{mm} + 10\text{mm} + 6\text{mm}$   
 E-modulus of concrete

$k_1 := 0.8$   $k_2 := 0.5$   $k_3 := 3.4$   $k_4 := 0.425$

$\phi := 20\text{mm}$  Reinforcement diameter

$A_s := 942\text{mm}^2$   $A_{s1} := 226\text{mm}^2$

$\rho_{p,\text{eff}} := 0.051$

$c_m := 35\text{mm}$

fiber lengde

$l_f := 50\text{mm}$

$d_f := 1\text{mm}$

Crack spacing  $S_{r\max} := \left( k_3 \cdot c_m + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi}{\rho_{p,\text{eff}}} \right)$

$S_{r\max} = 185.667 \cdot \text{mm}$

Composite tensile strength at first crack

$V_f := 0.02$	volume of fiber	
$\sigma_{mu} := 5\text{MPa}$	for high strength concrete ref.[21]	
$\sigma_f := 1100\text{MPa}$	Tensile stress of fiber	
$\eta_2 := \frac{(1)}{5}$	Value from earlier reaserch ref[21]	
$\eta_1 := 1$		
$E_f := 200\text{GPa}$	(E – modulus)of fiber	
$\sigma_{cc} := \eta_1 \cdot \eta_2 \cdot \sigma_f \cdot V_f + \sigma_{mu} \cdot (1 - V_f)$		$\sigma_{cc} = 9.3 \cdot \text{MPa}$
$E_{cm} := \eta_1 \cdot \eta_2 \cdot E_f \cdot V_f + E_c \cdot (1 - V_f)$		$E_{cm} = 2.432 \times 10^4 \cdot \text{MPa}$

tensile strain at first crack in concrete will be

$$\epsilon_{cm} := \frac{\sigma_{cc}}{E_{cm}} = 3.824 \times 10^{-4}$$

For findind strain in reinforcement need to find stifness

(E – modulus)for steel	$E_s = 200 \cdot \text{GPa}$
------------------------	------------------------------

matrial stifness:	$\eta := \frac{E_s}{E_{cm}} = 8.224$
-------------------	--------------------------------------

reinforcement ratio	$\rho := \frac{A_s}{b \cdot d} = 0.015$	$\rho_1 := \frac{A_{s1}}{b \cdot d} = 3.49 \times 10^{-3}$
---------------------	---	--

$$\alpha := \rho \cdot \eta \cdot \left( 1 + \frac{\rho_1}{\rho} \right) \left[ \sqrt{1 + \frac{2 \left( 1 + \frac{\rho_1 \cdot d_1}{\rho \cdot d} \right)}{\rho \cdot \eta \cdot \left( 1 + \frac{\rho_1}{\rho} \right)^2}} - 1 \right] = 0.372$$

For beam by compression reinforcement can find the  $\alpha$  by this formula

For finding EI

$$I_c := 0.5 \cdot \alpha^2 \cdot \left( 1 - \frac{\alpha}{3} \right) \cdot b \cdot d^3$$

$$EI := E_{cm} \cdot I_c$$

$$\sigma_s := E_s \cdot M_{max} \cdot (1 - \alpha) \cdot \frac{d}{EI}$$

$$EI = 6.411 \times 10^6 \text{ m}^2 \cdot \text{N}$$

reinforcement stress:

$$\sigma_s = 461.524 \cdot \text{MPa}$$

The main strain in reinforcement:

$$\epsilon_{sm} := \frac{\sigma_s}{E_s} = 2.308 \times 10^{-3}$$

$$\epsilon_{cm} = 3.824 \times 10^{-4}$$

crack width:

$$w_k := S_{rmax} \cdot (\epsilon_{sm} - \epsilon_{cm})$$

$$w_k = 0.357 \cdot \text{mm}$$

This calculation is runned for all beams and results is showed in table 4-26

**Deflection of beam is calculated on load 200 kN and result is showed in table 4-28**

$$a := 0.7 \text{ m}$$

$$P := 200 \text{ kN}$$

$$L_1 := 2 \text{ m}$$

$$\delta_{maks} := \frac{a \cdot P \cdot (3L_1^2 - 4 \cdot a^2)}{2 \cdot 24 \cdot EI} = 4.567 \cdot \text{mm}$$

## 2.X

### Calculation of curvature ductility coefficient

This is an example of curvature ductility calculation for beam SCC\_UF. Result of calculation for each beam is given in table 4-28.

Reinforcement and beam data:

$$A_s := 942 \text{mm}^2$$

$$A_{s1} := 226 \text{mm}^2$$

$$b := 250 \text{mm}$$

$$h := 300 \text{mm}$$

$$d := 257 \text{mm}$$

$$d_1 := 28 \text{mm} + 10 \text{mm} + 6 \text{mm}$$

$$f_{yd} := 434 \frac{\text{N}}{\text{mm}^2}$$

Concrete data:  $\frac{\text{C60}}{75}$

$$f_{ck} := 60 \frac{\text{N}}{\text{mm}^2}$$

$$\epsilon_{cm} := 0.0035$$

$$E_c := 32 \text{GPa}$$

$$E_s := 200 \text{GPa}$$

$$f_{cd} := f_{ck} \cdot \frac{0.85}{1.5} = 34 \cdot \text{MPa}$$

$$\epsilon_{yd} := \frac{f_{yd}}{200 \text{GPa}} = 2.17 \times 10^{-3}$$

$$\rho := \frac{A_s}{b \cdot d} = 0.015$$

$$\rho_1 := \frac{A_{s1}}{b \cdot d} = 3.518 \times 10^{-3}$$

$$n := \frac{E_s}{E_c} = 6.25$$

For beam by compression reinforcement can find the  $\alpha$  that way:

$$\alpha := \rho \cdot n \cdot \left(1 + \frac{\rho_1}{\rho}\right) \left[ \sqrt{1 + \frac{2 \left(1 + \frac{\rho_1 \cdot d_1}{\rho \cdot d}\right)}{\rho \cdot n \cdot \left(1 + \frac{\rho_1}{\rho}\right)^2}} - 1 \right] = 0.338$$

$$x_1 := \alpha \cdot d = 86.794 \cdot \text{mm}$$

$$p := \rho + \rho_1 = 0.018$$

$$k := \sqrt{p^2 \cdot n^2 + 2 \cdot \left( \rho + \frac{\rho_1 \cdot d_1}{d} \right) \cdot n} = 0.451$$

$$a := \frac{A_s \cdot f_{yd} - A_{s1} \cdot f_{yd}}{0.85 f_{ck} \cdot b} = 0.024 \text{ m}$$

$$Q := \left( \frac{\rho_1 \cdot \epsilon_{cm} \cdot E_s - \rho \cdot f_{yd}}{1.7 \cdot f_{ck}} \right) = -0.038$$

$$\beta_1 := \frac{a}{x_1} = 280.805 \text{ m}^{-1} \cdot \text{mm}$$

$$Q_1 := \frac{\rho_1 \cdot \epsilon_{cm} \cdot E_s \cdot \beta_1 \cdot d_1}{0.85 f_{ck} \cdot d} = 2.321 \times 10^{-3}$$

Curvature ductility coeff.

$$\phi_u := \frac{\beta_1 \cdot E_s \cdot \epsilon_{cm}}{f_{yd}} \cdot \frac{1 + p \cdot n - k}{\sqrt{Q^2 + Q_1 - Q}} = 3.007$$