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## Faculty of Science and Technology

### MASTER'S THESIS

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## Abstract

Oil and gas fields located in offshore are today being developed in even harsher and more challenging environments than anyone had thought of before. New designs, technologies, regulations and requirements have been developed and implemented along with these changes. As a result of these harsh conditions, the offshore structures will experience a lot of challenges in terms of design and maintenance integrity.

One of the most important concerns is the wave loadings which are critical on offshore structures in these environments due to their cyclic behaviour over time. The structures considered in this thesis are pipe lines, which are influenced by wave loadings. The wave loadings considered in this thesis are high cyclic loadings, which will accumulate damages on structures and then lead to fatigue failures. These failures are a result of a combination of the stress amplitudes and the number of cycles.

ASME B31.3 is the piping code that is utilized in design of most offshore process piping systems. But due to its lack of information about high cyclic fatigue failures, other codes need to be considered on this matter. There are different specifications which address fatigue failures, and the code used in this thesis is PD5500 British standard specification. This is used as a reference approach to estimate fatigue life. As an experiment there are two different other approaches discussed. One is covering the fatigue by calculating equivalent stress range and the other is covering the fatigue by assuming that a probability density function of stress range may be represented by a two parameter Weibull distribution.

Examples from the Eldfisk and the Snorre fields have been used for explanations of above given approaches. One has been the bridge piping on the Eldfisk field and the other one has been the flow line on the Snorre A field.

The approach given in the PD5500 specification and the equivalent stress range approach gives the same results for the fatigue life estimation, but the third approach which assumed a two parameter Weibull distribution of stress range, gives a different result than the other two. The equivalent stress range approach can be proven analytically, but hasn't proven earlier to be used with the two slopes SN curves. The thesis is discussed about the feasibility of using the equivalent stress range approach as another way of high cyclic fatigue assessment. This approach can be suitable to use in the industrial fatigue analyses but not the third approach.

Fatigue damages on offshore topside piping systems in the North Sea have been evaluated by using the above examples and it has identified that more than 80% of the fatigue damages happened at the wave heights between 2 m to 8 m.

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## Nomenclature

### Latin characters

$a$	speed of sound in the fluid (m/s)
$A$	wind exposed area ( $A = D$ ; for pipe) ( $m^2$ )
$A$	intercept of number of cycle axis
$c$	sum of mechanical allowances plus corrosion and erosion allowances
$C_A$	shape factor
$C_D$	drag coefficient
$C_D$	drag coefficient for blast
$D$	diameter of pipe (m)
$D$	pipe outside diameter
$d$	inside diameter of pipe
$D$	blast projected area = diameter of pipe (m)
$D$	cumulative fatigue damage ratio
$E$	quality factor
$E$	modulus of elasticity in $N/mm^2$ (MPa)
$f$	stress range factor
$f_m$	maximum value of stress range factor
$F$	pressure relief force (N)
$F_D$	drag force (N/m)
$F_D$	drag load (N/m)
$g$	gravity = $9.81 m/s^2$
$h$	shape parameter
$h$	wave height
$h_{max}$	maximum wave height
$k$	number of stress blocks
$k$	1.7 (for jacket structure)
$L$	length of the pipe (m)
$L_0$	time for the total number of stress cycles
$m$	inverse negative slope of the SN curve
$m_1, m_2$	inverse negative slope of the SN curve
$M_t$	torsion moment
$\dot{m}$	mass flow rate (kg/s)
$n$	number of stress cycles that exceeds $\Delta\sigma$
$n_i$	number of stress cycles in stress block $i$ with constant stress range $\Delta\sigma_i$
$n_0$	total number of stress cycles
$N$	equivalent number of full displaced cycles during expected service life of a piping system
$N_i$	number of cycles to failure at constant stress range $\Delta\sigma_i$
$N(\Delta\sigma)$	corresponding number of cycles to failure at a constant stress range $\Delta\sigma$
$P$	internal design gage pressure
$p$	dynamic drag pressure (Pa)
$P_1$	pressure of inlet (Pa)
$P_2$	pressure of outlet (Pa)
$q$	scale parameter
$r_2$	mean branch cross sectional radius
$S$	stress range obtained from appropriate design curve at the same life
$S$	allowable stress value



$S_c$	basic allowable stress at minimum metal temperature expected during the displacement cycle under analysis
$S_E$	computed displacement stress range
$S_h$	basic allowable stress at maximum metal temperature expected during the displacement cycle under analysis
$S_b$	resultant bending stress
$S_r$	allowable stress range for a particular life
$S_t$	torsion stress
$t$	time
$t$	pressure design thickness
$t$	thickness
$t_m$	minimum required thickness including mechanical, corrosion, and erosion allowances
$T_s$	effective branch wall thickness, lesser of $\bar{T}_h$ and $(i_i)(\bar{T}_b)$
$\bar{T}_b$	thickness of pipe matching branch
$\bar{T}_h$	thickness of pipe matching run of tee or header exclusive of reinforcing elements
$u$	wind speed (m/s)
$V_1$	fluid velocity of inlet (m/s)
$V_2$	fluid velocity of outlet (m/s)
$W$	weld joint reduction factor'
$Y$	coefficient provided in ASME B31.3 table 304.1.1 [1]
$Z$	sectional modulus of pipe/matching nominal pipe

### **Greek symbols**

$\nu$	Poisson's ratio
$\rho$	density of air ( $\text{kg/m}^3$ )
$\rho$	density of ice ( $\text{kg/m}^3$ )
$\rho$	fluid density ( $\text{kg/m}^3$ )
$\rho$	density of wind ( $\text{kg/m}^3$ )
$\alpha$	coefficient of linear thermal expansion ( $\text{m/m}^\circ\text{C}$ )
$\Sigma$	summation of
$\sigma_{1/2/3}$	Principle stresses
$\sigma_y$	Yield stress
$\tau$	shear stress
$\Delta$	displacement for the wave height
$\Delta L$	thermal expansion (m)
$\Delta P$	magnitude of pressure wave (Pa)
$\Delta T$	temperature change ( $^\circ\text{C}$ )
$\Delta V$	change in fluid velocity (m/s)
$\Delta\sigma$	stress range
$\Delta\sigma_0$	maximum stress range for a total of $n_0$ cycles
$\Delta_{\max}$	maximum displacement (515 mm/440 mm)

### **Abbreviations**

ASME	American Society of Mechanical Engineers
BS	British Standard
BSI	British Standard Institution
$^\circ\text{C}$	Celsius

DLF	Dynamic Load Factor
DNV	Det Norske Veritas
ESD	Emergency Shut Down
FEA	Finite Element Analysis
Hz	Hertz = cycles per second
kg	kilogram
lb	pound
m	meter
mm	millimeter
max	maximum
min	minimum
MPa	Mega Pascal
N	Newton
NDT	Non Destructive Test
Pa	Pascal
PED	Pressure Equipment Directive
PM	Pierson Moskowitz
psi	pound per square inch/pound-force per square inch
PSV	Pressure Safety Valve
s	second (time)
TLP	Tension Leg Platform
WAG	Water Alternative Gas

# 1 Introduction

## 1.1 Background

Offshore oil and gas industry has been using various design standards, codes and specifications through these years of success. Region to region, these may have differences due to its regulations and requirements. ASME B31.3 is one of the codes that used all over the world for process piping systems. In the Norwegian continental shelf most of offshore topside piping systems design and maintain by following this code. The background for this thesis is the desire to obtain better understanding about design part of the ASME B31.3 process piping code.

The ASME B31.3 provides set of rules to follow when design and maintain process piping systems. This may not address all the applications in the process piping systems. The main challenge of this piping code is to understand how the stresses in a pipe are treated and handled. This code has its own way of treating and handling these stresses.

As this code, piping systems can be imposed of various loadings. Due to cyclic loadings, piping systems can be failed even before stresses reaching the yield stresses of the pipe and this is called fatigue failure. Especially offshore piping systems which are subjected to high cyclic wave loadings can be critical on the fatigue failures. The ASME B31.3 piping code doesn't necessarily address these failures thoroughly. So there is a need to use other piping codes/specifications for the better understanding of these failures.

This thesis is defined in cooperation with the department of piping and layout in Aker Solutions MMO AS. The examples that used in this thesis for the explanation purposes are taken from ongoing projects in the North Sea with different Norwegian clients.

## 1.2 Purpose and Scope

The purpose of this study is to study design part of the ASME B31.3 piping code and also to understand detailed fatigue analysis of high cyclic failures according to the current standards. The thesis explains how to analyse high cyclic fatigue of wave affected piping systems and the background is limited to current standards and codes.

Scope of the thesis:

- Literature survey on the ASME B31.3 piping code and piping stress analysis
- Study static and dynamic pipe stress analysis
- Study detail fatigue analysis such as PD5500 specification
- Find maximum stress ranges of bridge piping and flow line examples using the CAESAR II stress analysis computer program
- Evaluation of fatigue analysis of the examples in different approaches
- Discussion
- Conclusions

## 1.3 Thesis organisation

Chapter 1 (Introduction) provides the background of the study, scope of the study and how the thesis is built up.

Chapter 2 (ASME B31.3 Process piping) describes about the ASME B31.3 code. This chapter is briefly pointing out most important equations used for the design conditions, allowable stresses and flexibility analysis. Also describe about the load requirements and stress limits.

Chapter 3 (Stress analysis of piping systems) discuss about the modes of failures, failure theories, loadings, piping stresses and stress analysis. Loadings to be considered in piping design and stress analysis and stresses in a pipe are briefly discussed in this chapter. Also about different stress analysis such as static and dynamic analyses are discussed.

Chapter 4 (Fatigue) describes about the fatigue analysis approaches. The equations that used for these different approaches are stated and derived in this chapter.

Chapter 5 (Bridge piping fatigue) gives detail fatigue analyse of bridge piping on the Eldfisk field. The analysis is done using different fatigue analysis approaches and obtained fatigue life. The PD5500 fatigue analysis approach is taken as a reference to compare with the other approaches.

Chapter 6 (Flow line fatigue) provides and analyses an example from the Snorre field flow line. As in the chapter 5, this example also analysed according to the approaches that discussed in the chapter 4.

Chapter 7 (Discussion) discuss about the results that obtained from the different approaches and gives possible explanations.

Chapter 8 (Conclusions) provides the conclusions and recommendation of all the approaches that used in this thesis.

## 2 ASME B31.3 Process piping

In worldwide, the ASME B31.3 design code is generally accepted standard for process piping such as piping for oil and gas, petro-chemical and chemical industries. Most of the North Sea topside piping systems are designed based on this code.

The code discussed about main three categories of fluid services in terms of possible degree of hazard. Those are category M, category D and normal.

Less hazardous fluid service can be called as category D and it includes fluids that are non toxic, non flammable, design gauge pressure less than 150 psi and design temperature is from  $-29^{\circ}\text{C}$  through  $186^{\circ}\text{C}$ .

A fluid service in which exposure to very small quantities of a toxic fluid can produce serious irreversible harm to persons on breathing or bodily contact, even when prompt restorative measures are taken can be considered as category M.

All fluid services can be considered normal unless the owner categorized them as category D or category M.

### 2.1 Design conditions

ASME B31.3 design conditions specifically intended for pressure design. There are two main design conditions discussed in the code. Those are design pressure and design temperature.

- Design pressure

When determining the design pressure it is required to consider all the possible conditions of internal pressure such as thermal expansion of trapped fluids, surge and failure of control devices. It is allowed to be used without protection of a pressure safety relief valve on a process piping system. The piping systems have to be designed to withstand the maximum pressure that can occur when none of the protections are provided and also it must be safe when all the protections are failed.

- Design temperature

The design temperature is mainly considered about the metal temperature of the pipe. There are several internal and external conditions can be involved with the design temperature such as the temperature of the process fluid, ambient cooling, ambient heating, solar radiation and maximum heat tracing temperature.

Minimum design temperature is the lowest temperature that a component can be expected while the system is in operation. This temperature is required to determine the design requirements and special material qualification requirements.

### 2.2 Pressure design

The required minimum pressure design thickness of a selected straight pipe, considering manufacturers minus tolerance, must be at least equal to  $t_m$ .

$$t_m = t + c \quad (\text{Eq 2.1}) [1]$$

Where;

$c$  = sum of mechanical allowances plus corrosion and erosion allowances

$t$  = pressure design thickness

$t_m$  = minimum required thickness including mechanical, corrosion, and erosion allowances

- For  $t < D/6$ , the pressure design thickness can be found from either of the following equations;

$$t = \frac{PD}{2(SE + PY)} \quad (\text{Eq 2.2}) [1]$$

$$t = \frac{P(d + 2c)}{2[SEW - P(1 - Y)]} \quad (\text{Eq 2.3}) [1]$$

Where;

D = pipe outside diameter

E = quality factor

P = internal design gage pressure

S = allowable stress value

Y = coefficient provided in ASME B31.3 table 304.1.1 [1]

W = weld joint reduction factor

d = inside diameter of pipe

- For  $t \geq D/6$  or for  $P/SE > 0.385$ , the calculation of pressure design thickness requires special consideration of factors such as thermal stresses, theory of failure and thermal stress.

The pressure design requirements for the other piping components such as pipe bends, elbows, branch connections, closures, flanges, blanks, reducers and non listed components have to be done as same as the above straight pipe pressure design.

## 2.3 Load requirements

It is necessary to consider different loadings when designing piping systems. The ASME B31.3 is discussed about the following loadings and shall be taken into account in the design of piping;

- Weight effect
  - Live loads
  - Dead loads
- Dynamic effects
  - Impact
  - Wind
  - Earthquake
  - Vibration
  - Discharge reactions
- Ambient effects
  - Cooling effects on pressure
  - Fluid expansion effects
  - Atmospheric icing
  - Low ambient temperature
- Thermal expansion and contraction effects
  - Thermal loads due to restraints
  - Loads due to temperature gradient

- Loads due to differences in expansion characteristic
- Effect of support, anchor and thermal movement
- Reduced ductility effect
- Cyclic effect
- Air condensation effects

## 2.4 Stress limits

The calculated stress limitations due to sustained loads and displaced strains can be described as follows;

- Internal pressure stresses

The stresses due to the internal pressure shall be considered safe when the wall thickness of the piping components satisfied with the above discussed pressure design requirement.

- External pressure stresses

The stresses due to external pressure shall be considered safe when the wall thickness of the piping components satisfied with the code.

- Stresses due to sustained loads,  $S_L$

All the longitudinal stresses due to sustained loads,  $S_L$ , (e.g. pressure and weight) of any component in a piping system shall not exceed  $S_h$ .

- Allowable displacement stress range,  $S_A$

The computed displacement stress range,  $S_E$  shall not exceed allowable displacement stress range,  $S_A$ .

$$S_E \leq S_A \quad (\text{Eq 2.4}) [1]$$

Where;

$$S_A = f(1.25S_c + 0.25S_h) \quad (\text{Eq 2.5}) [1]$$

When  $S_h - S_L$  is added to the term  $0.25S_h$  in the above Eq 2.5,  $S_A$  yields as follows;

$$S_A = f[1.25(S_c + S_h) - S_L] \quad (\text{Eq 2.6}) [1]$$

Where;

f = stress range factor

$$f = 6.0(N)^{-0.2} \leq f_m \quad (\text{Eq 2.7}) [1]$$

$f_m$  = maximum value of stress range factor

( $f_m = 1.2$  for ferrous materials which have minimum tensile strength  $\leq 517\text{MPa}$  and metal temperatures  $\leq 371^\circ\text{C}$  or  $f_m = 1.0$  for others)

N = equivalent number of full displaced cycles during expected service life of a piping system

$S_c$  = basic allowable stress at minimum metal temperature expected during the displacement cycle under analysis

$S_h$  = basic allowable stress at maximum metal temperature expected during the displacement cycle under analysis

- Weld joint strength reduction factor. W
- Unfinished weld strength reduction factors

The calculated stress limitations due to occasional loads can be described as follows;

- Operation

The longitudinal stresses,  $S_L$  due to sustained loads (e.g. pressure and weight) and occasional loads (e.g. wind, earthquake) shall be described as high as 1.33 times the allowable stress.

$$S_L \leq 1.33S_h \quad (\text{Eq 2.8})$$

There is no need to consider, the occasional forces (wind and the earthquake) will happen at the same time with sustained loads.

- Test

There is no requirement to consider, other occasional loads such as wind and earthquake will happen at the same time with the test loads.

## 2.5 Piping Flexibility analysis

When apply loads to a piping system, how the system responds to those is called flexibility analysis. Basically it is considered structural beam analysis model on pipe centre line. The fundamentals of the analysis can be described as follows [1], [2];

- Nominal dimension of the pipe will be considered to the analysis
- The piping flexibility and stress of piping components such as elbows and tees shall be calculated by inclusion of stress intensification factors and flexibility factors.
- Typically only moment and torsion will be considered for thermal stresses. The stresses due to axial and shear loads are not considered, since those are not significant in typical piping layout. However in special cases, it is necessary to include average stresses of those where they may be significant.
- Modulus of elasticity at 21 °C is normally used in the analysis.

### 2.5.1 Stress intensification factors

The stress intensification factor is telling about the severity of the stress in a component compared to the stress in nominal thickness straight pipe. This has developed from component fatigue testing.

The stress intensification factor and nominal stress in the component can be described as follows;

$$\text{Stress intensification factor} = \frac{\text{Nominal stress from the butt welded pipe fatigue curve at the number of cycles to failure in the component test}}{\text{Nominal stress in the component}} \quad (\text{Eq 2.9})$$

$$\text{Nominal stress in the component} = \frac{\text{Range of bending moment at the point of failure}}{\text{Section modulus of matching pipe with nominal wall thickness}} \quad (\text{Eq 2.10})$$



## 2.5.2 Flexibility analysis equations

The stresses due to thermal expansion loads in a piping system can be calculated with following standard equations;

$$S_E = \sqrt{S_b^2 + 4S_t^2} \quad (\text{Eq 2.11}) [1]$$

Where;

$S_E$  = computed displacement stress range

$S_b$  = resultant bending stress

$S_t$  = torsion stress

The torsion stress can be calculated as follows;

$$S_t = \frac{M_t}{2Z} \quad (\text{Eq 2.12}) [1]$$

Where;

$M_t$  = torsion moment

$Z$  = sectional modulus of pipe/matching nominal pipe

For full size outlet branch connections, the resultant bending stress  $S_b$  is calculated as follows;

$$S_b = \frac{\sqrt{(i_i M_i)^2 + (i_o M_o)^2}}{Z} \quad (\text{Eq 2.13}) [1]$$

Where;

$M_i$  = in plane bending moment

$M_o$  = out plane bending moment

$i_i$  = in plane stress intensification factor

$i_o$  = out plane stress intensification factor

For reducing branch connections, the resultant bending stress can be calculated in accordance to the following;

$$S_b = \frac{\sqrt{(i_i M_i)^2 + (i_o M_o)^2}}{Z_e} \quad (\text{Eq 2.14}) [1]$$

Where;

$$Z_e = \pi \cdot r_2^2 \cdot T_s \quad (\text{Eq 2.15}) [1]$$

Where;

$T_s$  = effective branch wall thickness, lesser of  $\bar{T}_h$  and  $(i_i)(\bar{T}_b)$

$\bar{T}_b$  = thickness of pipe matching branch

$\bar{T}_h$  = thickness of pipe matching run of tee or header exclusive of reinforcing elements

$r_2$  = mean branch cross sectional radius

The directions of in plane and out plane bending moments for full size outlets branch connections and reducing branch connections can be illustrated as in the following figures;

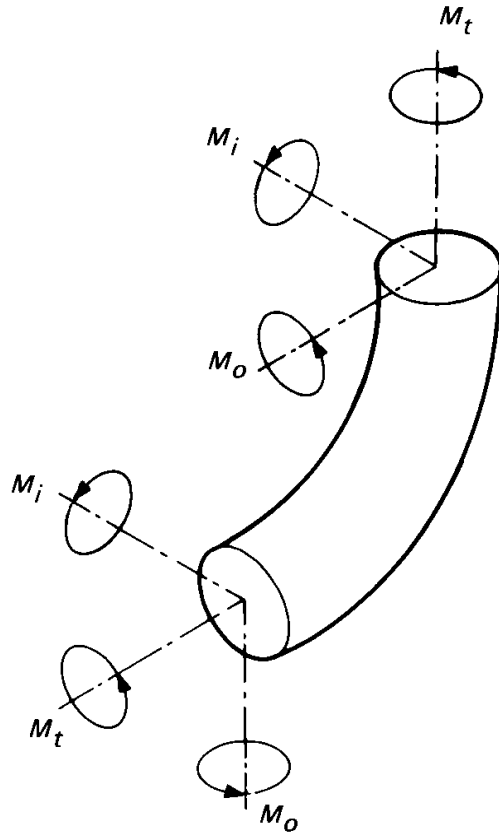


Figure 2.1 Moments in bend [1]

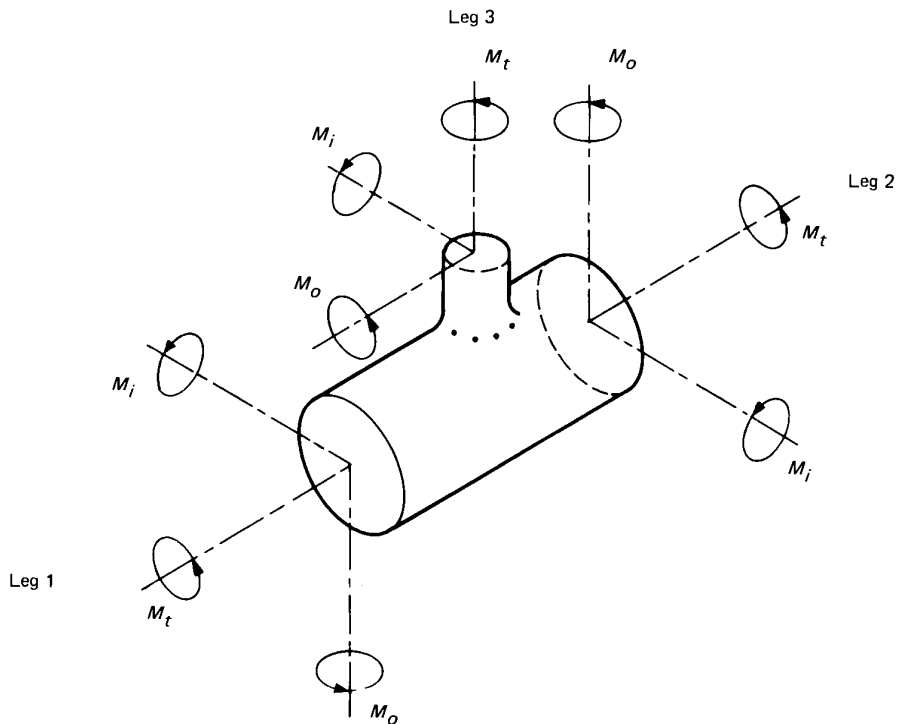


Figure 2.2 Moments in branch connection [1]

### 3 Stress analysis of piping systems

In the petroleum industry, transportation of the final product is the most important milestone of the total business. For this most efficient and common method is transporting through a piping system.

Either in onshore or offshore there can be seen very simple to most complex piping layouts. Some piping systems can be more critical and more difficult to design than others because of the temperature variations, vibrations, fatigue and connection to sensitive equipment such as turbines and compressors. Therefore it is important to do stress analysis for the piping systems. But most of the piping systems can be visually checked and see that the system is accepted. For others it is necessary to do a detailed stress analysis.

#### 3.1 Modes of failure

The main idea of piping stress analysis is to avoid the pipe failures. Therefore it is important to know about the different modes of failures. The pipes can be failed in different modes with many different mechanisms. Some of those are discussed as follows;

- Static stress rupture

When the stress reaches ultimate strength of the material, it will fail and it is called static stress rupture. There is no time involved in the static stress.

The static stress rupture can be further divided into two categories as ductile rupture and brittle rupture and can be illustrated as follows;

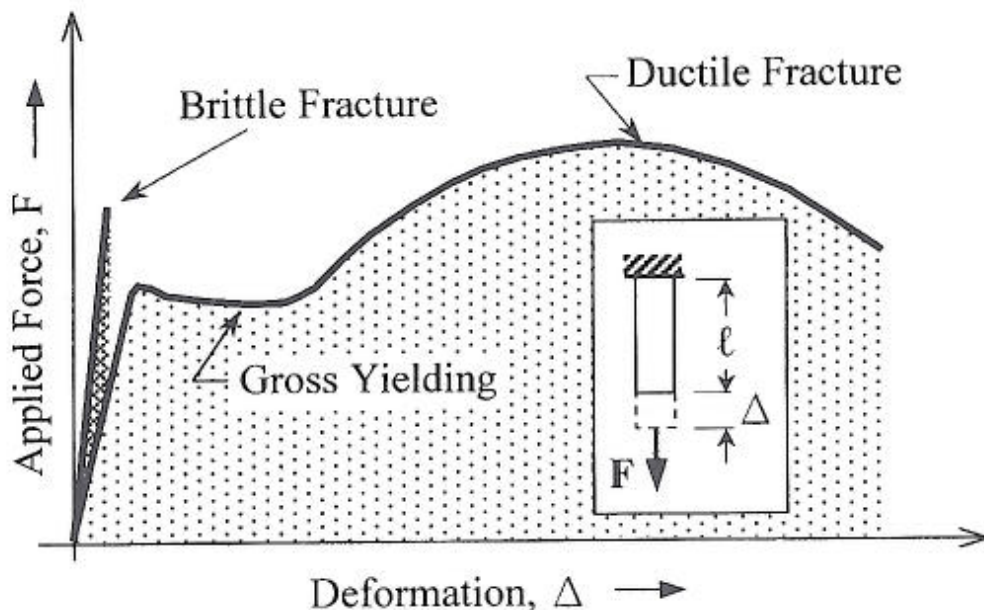


Figure 3.1 Static fracture [3]

- Ductile rupture

The material that fails with yielding is called ductile material and the pipe made out of these materials can yield producing a considerable plastic deformation. These materials can end up with 25% more or less elongation or contraction before the failure.

- Brittle rupture

The material does not yield or deform before it fail, can be called as brittle material. The brittle ruptures can be happened suddenly and unexpectedly. Most piping materials become

brittle as the temperature drops below a certain limit. Because of this it is important to identify the design conditions of the pipe and select specific materials for these situations.

- Fatigue failure

When the stress is cyclic, materials can be failed before the ultimate strength of the material. The failure due to cyclic loads is called fatigue failure. The fatigue failure is a result of a combination of the stress amplitude and the number of load cycles.

The fatigue failure can be divided into two with respect to number of cycles to failure.

- Low cyclic fatigue

Thermal expansion of a pipe can be produced low cyclic fatigue failures.

- High cyclic fatigue

Steady state vibration and rapid fluctuating thermal expansions can be attributed to high cyclic fatigue failure.

- Creep rupture

At high temperature environments, a pipe can be more vulnerable due to sustained stresses and reduction of the allowable stresses of the pipe material. The pipe will deform at higher temperature. Therefore the pipe can be failed after a certain time period even though the stress is much lower than the ultimate strength of the material. This phenomenon is called as creep and the failure is called as creep rupture.

The creep rupture can be illustrated as in the following curves and it can be categorized into three stages depend on the creep rate. The stage 3 has to be avoided in service due to failure region.

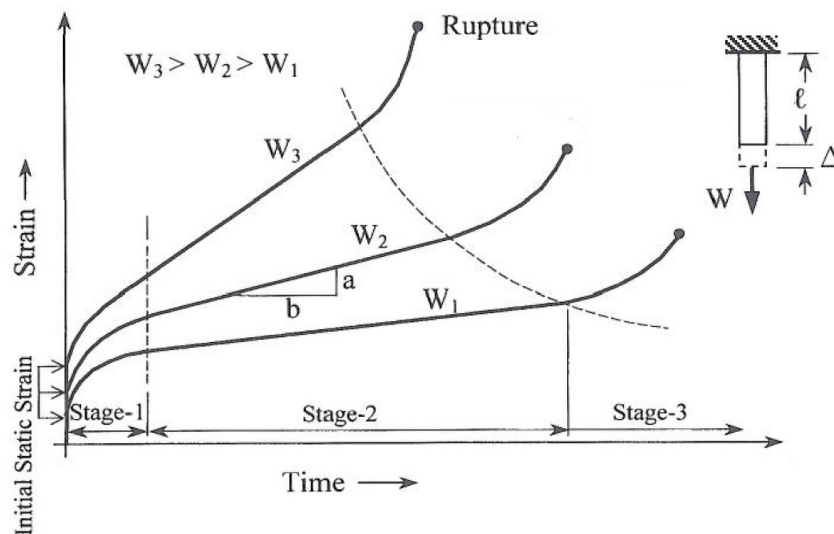


Figure 3.2 Creep rupture [3]

- Stability failure

Compressive stresses due to external loads of the pipe can be resulted in to stability failure. This is mainly can be seen on the large thin walled pipes. The pipe stability failure due to buckling may happen because of the external pressure and axial compressive stress. The buckling due to external pressure is attributed to different shapes such as two lobes, three lobes and four lobes.

The allowable stress can be expressed as follows for a long segment of pipe which produces two lobes buckling;

$$P_{allow} = \frac{2E}{3(1-\nu^2)} \cdot \left(\frac{t}{D}\right)^3 \quad (\text{Eq 3.1}) [3]$$

Where;

E = modulus of elasticity

t = thickness

D = outer diameter

$\nu$  = poisson's ratio

Wrinkling, square wave, column buckling, bending wrinkle and etc are different stability issues due to axial compressive stress buckling.

### 3.2 Theories of failure

There are several different theories of failure that used in the strength of materials basics as explained in the following;

- Maximum stress theory

The theory suggests that the material will yield when the absolute magnitude of any principle stresses reaches the yield strength of the material.

- Maximum strain theory

This predicts that the material will yield when the maximum strain ( $\epsilon_{max}$ ) reaches the yield strain.

$$\epsilon_{max} = \frac{\sigma_y}{E} \quad (\text{Eq 3.2}) [5]$$

- Maximum shear theory

The maximum shear theory stated that the material will yield when the maximum shear stress reaches the maximum shear stress at the yield point. The ASME is based on this theory for its piping and pressure vessel codes. The theory is also called as Tresca and can be formulated as follows;

When a force (F) applied to a rectangular prism, the shear stress ( $\tau$ ) can be written as follows;

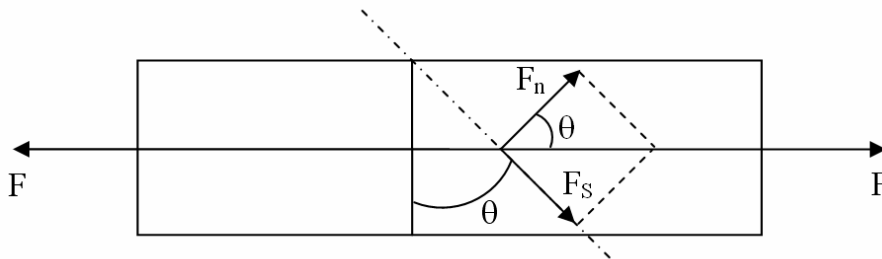


Figure 3.3 Stresses at skewed plane

$$\tau = \frac{F_s}{A_m} = \frac{F \sin \theta}{A / \cos \theta} = \sigma \sin \theta \cos \theta = \frac{\sigma}{2} \sin 2\theta \quad (\text{Eq 3.3}) [3]$$

$$\tau_{max} = \frac{\sigma_{max}}{2} \sin 2\theta_{max} = \frac{\sigma_y}{2} \cdot 1 = \frac{\sigma_y}{2} \quad \text{When; } \sin 2\theta = 1 = \sin 90^\circ \rightarrow \theta = 45^\circ \quad (\text{Eq 3.4})$$

According to the maximum shear stress theory (Tresca),

Tresca  $\rightarrow 2(\text{the maximum shear stress at the yield point}) = \text{tensile yield strength}$

Tresca  $\rightarrow 2(\tau_{\max}) = \sigma_y$

In two dimensional stress field  $\tau_{\max}$  becomes,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} \quad \text{Where; } \sigma_1 \geq \sigma_2 \text{ and } \sigma_1, \sigma_2 = \text{principle stresses} \quad (\text{Eq 3.5}) [5]$$

In three dimensional stress field  $\tau_{\max}$  becomes,

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \quad \text{Where; } \sigma_1 \geq \sigma_2 \geq \sigma_3 \text{ and } \sigma_1, \sigma_2, \sigma_3 = \text{principle stresses} \quad (\text{Eq 3.6}) [5]$$

Then the standardise formula for  $\tau_{\max}$ ;

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} \quad (\text{Eq 3.7})$$

$$\text{Tresca } \rightarrow \sigma_y = 2 \cdot \tau_{\max} = \sigma_{\max} - \sigma_{\min} \quad (\text{Eq 3.8})$$

- Maximum energy theory

In this theory, it predicts that the material will yield when the strain energy per unit volume in the material reaches the strain energy per unit volume at the yield point.

$$(\text{Strain energy/unit volume})_{\max} = \frac{\sigma_y^2}{2E} \quad (\text{Eq 3.9}) [5]$$

- Maximum distortion energy theory

The theory stated that the material will yield when the distortion energy per unit volume in the material reaches the distortion energy per unit volume at the yield point. This also called as von Mises theory.

$$(\text{Distortion energy/unit volume})_{\max} = \left[ \frac{(1+\nu)}{(3E)} \right] \cdot \sigma_y^2 \quad (\text{Eq 3.10}) [5]$$

### 3.3 Loadings to be considered in piping design/ stress analysis

A piping system is subjected to stresses and strains in different situations of its initial fabrication to service life of the system. When the piping system is in service, it is restrained by pipe supports and/or attached equipments. Mostly for the design purpose and the stress analysis purpose, it is only considered the loadings that are applied in the service life of the piping system.

Considered loadings are discussed as in the following.

#### 3.3.1 Dead weight

The dead weight load is the sum of weights from all the pipe and piping components such as flanges, bends, tees, bolts, valves, insulation, inside content and etc.

### 3.3.2 Internal pressure

The internal pressure load is the static end cap pressure load that act on the cross sectional area of the pipe caused by the internal pressure.

### 3.3.3 Sustained loads

The sustained loads are resulting to the primary stresses and those loads are not set limiting. Sum of the dead weight loads, axial loads caused by internal pressure and other axial loads that are not caused by the thermal expansion can be expressed as sustained loads. As these loads are acting, the longitudinal stresses will be resulted and all those stresses must not exceed the basic allowable stresses of the materials.

The pressure is normally considered as sustained load but there can be pressure cycles and pressure surges which are not considered as sustained loads. When there are pressure cycles, it has to take into consideration in fatigue analysis. For the stress analysis and the design purposes, it is required to use the design pressure not the operating pressure.

### 3.3.4 Occasional loads

Wind, earth quake, waves, snow and ice accumulation, dynamic loads such as pressure relief loads, fluid hammer, slug and etc are some examples of occasional loads.

In the North Sea installations design process, it may not a requirement to consider the earth quakes as a design load and therefore it is not discussed further in this thesis.

- Wind

Drag and lift forces can be induced on a pipe due to wind. The drag force can be found as follows and the lift force can be considered as negligible;

$$F_D = \frac{1}{2} \cdot \rho \cdot u^2 \cdot C_A \cdot A = \frac{1}{2} \cdot \rho \cdot u^2 \cdot C_D \cdot D \quad (\text{Eq 3.11})$$

Where

$F_D$  = drag force (N/m)

$\rho$  = density of air ( $\text{kg/m}^3$ )

$u$  = wind speed (m/s)

$C_A$  = shape factor

$C_D$  = drag coefficient

$A$  = wind exposed area ( $A = D$ ; for pipe) ( $\text{m}^2$ )

$D$  = diameter of pipe (m)

- Ice

Ice accumulation can be found as follows;

$$F_{ice} = \rho \cdot g \cdot A (N / m) \quad (\text{Eq 3.12})$$

Where;

$\rho$  = density of ice ( $\text{kg/m}^3$ )

$g$  = gravity =  $9.81 \text{ m/s}^2$

$$A = \frac{\pi}{4} (D_{ice}^2 - D_{pipe}^2) (\text{m}^2) \quad (\text{Eq 3.13})$$

- Pressure relief load

Pressure relief devices such as PSV are used in a piping system to safeguard the system by relieving excess pressure. There can be extra loads acting on the piping system because of the pressure relief process. As follows it is possible to derive an equation for the pressure relief load by applying the Bernoulli equation and the theory of momentum.

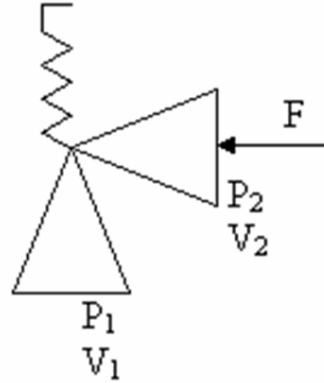


Figure 3.4 PSV

By applying the Bernoulli equation to the inlet and the outlet,

$$P_1 + \frac{1}{2} \cdot \rho \cdot V_1^2 = P_2 + \frac{1}{2} \cdot \rho \cdot V_2^2 \quad (\text{Eq 3.14})$$

$V_1 = 0$  (when PSV activates);

$$\Rightarrow P_1 = P_2 + \frac{1}{2} \cdot \rho \cdot V_2^2 \Rightarrow V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad (\text{Eq 3.15})$$

By applying the theory of momentum,

$$F = \dot{m} \cdot V_2 = \dot{m} \cdot \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad (\text{Eq 3.16})$$

Where;

$F$  = pressure relief force (N)

$V_1$  = fluid velocity of inlet (m/s)

$P_1$  = pressure of inlet (Pa)

$V_2$  = fluid velocity of outlet (m/s)

$P_2$  = pressure of outlet (Pa)

$\rho$  = fluid density ( $\text{kg/m}^3$ )

$\dot{m}$  = mass flow rate (kg/s)

- Fluid hammer

Another occasional load is fluid hammer and can be happened due to sudden change of direction or sudden motion stop of fluid. Pressure surge or wave is resulting due to this sudden reaction on the flow. The fluid hammer occurs commonly when a valve is closed suddenly and forces can act along the pipe on either direction of the valve.

When a valve closes instantaneously, the maximum fluid hammer can be calculated as follows and this is called the Joukowsky equation;



$$\Delta P = \rho \cdot a \cdot \Delta V \quad (\text{Eq 3.17}) [7]$$

Where;

$\Delta P$  = magnitude of pressure wave (Pa)

$\rho$  = density of fluid (kg/m<sup>3</sup>)

$a$  = speed of sound in the fluid (m/s)

$\Delta V$  = change in fluid velocity (m/s)

- Slug

In special circumstances of two phase gas liquid flow, there is a possibility of slugging in the flow. This is a special phenomenon that generates serious problems to the piping system such as unbalanced shaking load. The fluid slug in a straight pipe can be illustrated as in the following figure 3.5;

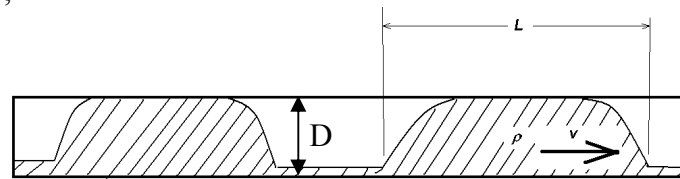


Figure 3.5 Fluid slug [8]

Fluid slug characteristics are;

- Slug length L (m)
- Slug speed V (m/s)
- Slug density  $\rho$  (kg/m<sup>3</sup>)
- Slug cylinder area  $A = \frac{\pi \cdot D^2}{4} (m^2)$

When the flow passes through a bend, it creates an impact force on the bend due to a change in the flow direction and thus a change in momentum. The slug force acting on a bend can be written as;

$$F = \rho \cdot A \cdot V^2 \quad (\text{Eq 3.18}) [8]$$

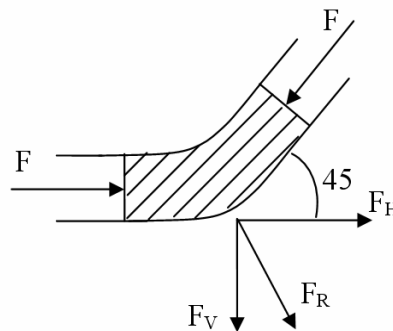


Figure 3.6 Slug bend 45deg

Horizontal and vertical force for a 45deg bend is as follows;

$$F_{H,45} = F - F \cos 45 = \rho \cdot A \cdot V^2 \left(1 - \frac{1}{\sqrt{2}}\right) = \rho \cdot A \cdot V^2 \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) \quad (\text{Eq 3.19})$$

$$F_{V,45} = F \cos 45 = \rho \cdot A \cdot V^2 \left(\frac{1}{\sqrt{2}}\right) \quad (\text{Eq 3.20})$$

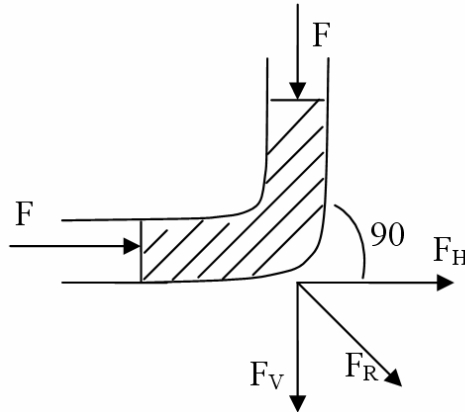


Figure 3.7 Slug bend 90deg

Horizontal and vertical force for a 90deg bend is as follows;

$$F_{H,90} = F_{V,90} = \rho \cdot A \cdot V^2 \quad (\text{Eq 3.21})$$

$$F_{R,90} = \sqrt{2} \cdot \rho \cdot A \cdot V^2 \quad (\text{Eq 3.22})$$

### 3.3.5 Environmental loads

Environmental loads are loads that caused by the nature such as wind, waves, earth quakes, snow and ice accumulation and etc. These environmental loads are considered either in the sustained or the occasional loads.

### 3.3.6 Live loads

Temporary deflection in the deck or supporting steel structure due to temporary loads can be considered as live loads. Filling or draining a column or pressure vessel, lifting or landing a load on to a deck that consists of sensitive equipments, deck deflection due to heavy crane operations and etc are typical examples of live loads that cause temporary deflections.

### 3.3.7 Displacement loads

There is significant displacement load caused by the thermal expansion and contraction loads. Due to its significance, the thermal expansion and contraction loads can result to damage the pipe itself, flanges, bolts, branches, pipe supports and also connected equipments such as pump and compressors.

The general equation for the thermal expansion can be given as follows;

$$\Delta L = \alpha \cdot L \cdot \Delta T \quad (\text{Eq 3.23}) [3]$$

Where;

$\Delta L$  = thermal expansion (m)

$\alpha$  = coefficient of linear thermal expansion (m/m°C)

L = length of the pipe (m)

$\Delta T$  = temperature change (°C)

The displacement load due to the pressure also has to consider, when talk about the total displacement load. On the other hand there are other displacements to consider such as displacements due to live loads, movements of the piping system and etc.

The equation for the longitudinal pressure expansion and strain for a pipe can be derived from the Hooke's law for linear, homogeneous and isotropic materials as follows;

$$\varepsilon_L = \frac{1}{E} [\sigma_L - \nu(\sigma_H + \sigma_r)] \quad (\text{Eq 3.24}) [5]$$

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E} [\sigma_L - \nu(\sigma_H + \sigma_r)] = \frac{1}{E} \left[ \frac{PD}{4t} - \nu \left( \frac{PD}{2t} + P \right) \right]$$

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E} \left[ \frac{PD}{4t} - \nu \frac{PD}{2t} \right] - \nu \frac{P}{E}$$

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E} \left[ \frac{PD}{4t} - \nu \frac{PD}{2t} \right] \ll \nu \frac{P}{E} \approx 0$$

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{PD}{4Et} [1 - 2\nu] = \frac{PD}{4Et} [1 - 2 \cdot 0.3] \ll (\nu = 0.3 \text{ for all temp \& all metals ASME B31.3})$$

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{PD}{4Et} [1 - 2\nu] = \frac{PD}{4Et} [0.4] = \frac{PD}{10Et} \left( \frac{m}{m} \right)$$

$$\varepsilon_L = \frac{\Delta L}{L} = 100 \frac{PD}{Et} \left( \frac{mm}{m} \right) \quad (\text{Eq 3.25})$$

### 3.3.8 Accidental loads

The code that discussed in this thesis, the ASME B31.3 doesn't address accidental loads or exceptional loads such as blast (explosion), fire and etc but European pressure equipment directive (PED) code EN 13480 does consider accidental loads.

It is required to perform a comprehensive stress analysis to evaluate the structural integrity of the piping and pipe supports during and after an accidental blast or exceptional event. If there is any load cases that is not covered by any codes, the stress engineer should agree on set of rules and limitations together with the safety engineer, the owner and the third party contractor.

The drag load from a blast (explosion) shall be calculated as following equation;

$$F_D = \frac{1}{2} \cdot \rho \cdot v^2 \cdot D \cdot C_D \cdot DLF \quad (\text{Eq 3.26}) [6]$$

$$F_D = p \cdot D \cdot C_D \cdot DLF \ll \left( p = \text{dynamic drag pressure} = \frac{1}{2} \cdot \rho \cdot v^2 \right) \quad (\text{Eq 3.27}) [6]$$

Where;

$F_D$  = drag load (N/m)

$\rho$  = density of wind ( $\text{kg/m}^3$ )

$D$  = blast projected area = diameter of pipe (m)

$C_D$  = drag coefficient for blast

$p$  = dynamic drag pressure (Pa)

$DLF$  = dynamic load factor

### 3.4 Piping stresses

Before look into the piping stress analysis, it is important to understand about the different piping stresses. Basically the stresses can be categorised into three such as primary stresses, secondary stresses and peak stresses.

- Primary stresses

The primary stresses are developed by the imposed loadings and are not self limiting. The sustained loads which are dead weight and internal pressure are examples of typical loadings that result to the primary stresses.

- Secondary stresses

When a piping system is limiting its free displacement, the stresses developed due to thermal loads or imposed displacements are called the secondary stresses. These stresses are self limiting.

- Peak stresses

The highest stresses in the considered stress region are called the peak stresses and always should take into consideration in fatigue and fracture mechanics calculations.

The stresses in a piping component can be categorised into two as follows;

- Stresses due to pressure, either internal or external
- Stresses due to forces and moments generated by weight, thermal expansion, wind, earth quake and etc

#### 3.4.1 Stresses due to internal pressure

The stress due to the internal pressure is the most common and the important stress at a component. When the pipe is pressurised, the pressure is acting on all direction of the pipe. The pressure force is acting normal to the surface of the pipe. Because of this pressure force, the pipe wall is stretched in all directions. There are three main stresses can be developed and those are longitudinal stress, hoop stress and radial stress.

##### 3.4.1.1 Longitudinal stress

It is assumed that the pipe is plugged both ends and then the longitudinal stress  $\sigma_L$  can be formulated as follows;

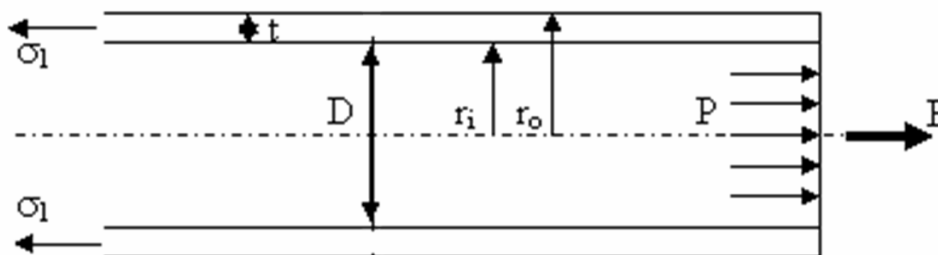


Figure 3.8 Longitudinal stress

The longitudinal stress is generally considered uniformly distributed across the wall thickness. Since the longitudinal stress is identical for both sides it is considered only one side of the pipe as in the above figure 3.8.

$$\text{Pressure force} = F = \pi \cdot r_i^2 \cdot P = \frac{\pi \cdot D^2}{4} \cdot P \quad (\text{Eq 3.28}) [3]$$

$$\text{Stress force} = \pi \cdot (r_o^2 - r_i^2) \cdot \sigma_L = \pi \cdot (r_o - r_i) \cdot (r_o + r_i) \cdot \sigma_L \approx \pi \cdot t \cdot D \cdot \sigma_L \quad (\text{Eq 3.29}) [3]$$

From the Eq 3.28 and Eq 3.29;

Pressure force = Stress force

$$\pi \cdot D \cdot t \cdot \sigma_L = \frac{\pi \cdot D^2}{4} \cdot P \Rightarrow \sigma_L = \frac{P \cdot D}{4t} \quad (\text{Eq 3.30}) [3]$$

### 3.4.1.2 Hoop stress

The hoop stress is not distributed uniformly in the wall thickness as in the longitudinal stress. In radial direction, the stress is higher at inner layer of the wall thickness and lower at the outer layer of the wall thickness. But for the following derivation it will be assumed that the stress is uniformly distributed across the pipe wall thickness. And also it is assumed that the length of the considered pipe is  $L$ m.

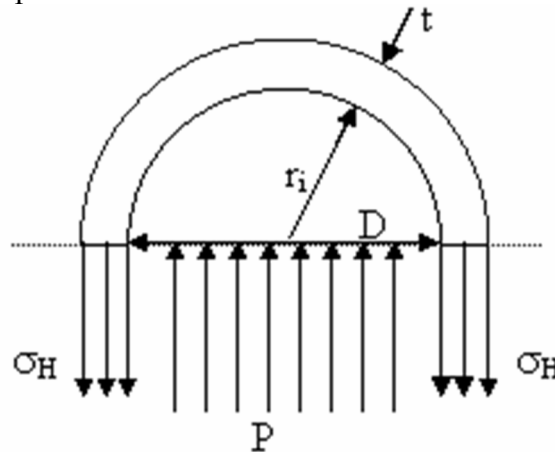


Figure 3.9 Hoop stress

$$\text{Pressure force} = 2 \cdot r_i \cdot L \cdot P = D \cdot L \cdot P \quad (\text{Eq 3.31}) [3]$$

$$\text{Stress force} = 2 \cdot L \cdot t \cdot \sigma_H \quad (\text{Eq 3.32}) [3]$$

From the Eq 3.31 and Eq 3.32;

Stress force = Pressure force

$$2 \cdot L \cdot t \cdot \sigma_H = 2 \cdot r_i \cdot L \cdot P = D \cdot L \cdot P \Rightarrow \sigma_H = \frac{P \cdot r_i}{t} = \frac{P \cdot D}{2t} \quad (\text{Eq 3.33}) [3]$$

From the Eq 3.30 and Eq 3.33, it is realised that the hoop stress is two times larger than the longitudinal stress of the pipe.

$$\sigma_H = 2 \cdot \sigma_L = \frac{P \cdot D}{2t} \quad (\text{Eq 3.34})$$

Since the hoop stress is not uniformly distributed across the pipe wall thickness, the hoop stress at any given  $r$  radius will be;

$$\sigma_H = \frac{P \cdot r_i^2}{(r_o^2 - r_i^2)} \cdot \left[ 1 + \frac{r_o^2}{r^2} \right]; \quad r_i \leq r \leq r_o \quad (\text{Eq 3.35}) [5]$$

### 3.4.1.3 Radial stress

The radial stress of inner layer of the pipe is equal to the inside pressure of the pipe and the radial stress at outer layer is equal to the outside pressure. Mostly in the offshore top piping the outside pressure considered as zero. So the radial stress at any given  $r$  radius point can be given as follows;

$$\sigma_r = \frac{P \cdot r_i^2}{(r_o^2 - r_i^2)} \cdot \left[ 1 - \frac{r_o^2}{r^2} \right]; \quad r_i \leq r \leq r_o \quad (\text{Eq 3.36}) [5]$$

### 3.4.2 Stresses due to forces

Same as the stresses due to the pressure, there are other stresses generated due to forces and moments by result of thermal expansion, weight, wind, earth quake and other internal and external loads.

The axial forces and shear forces acting on the pipe cross section can be illustrated as in the following figure 3.10;

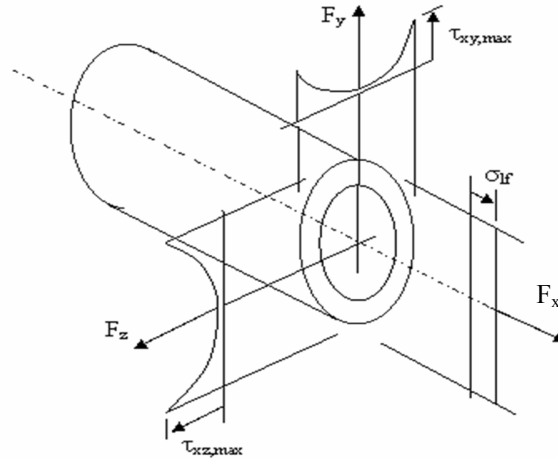


Figure 3.10 Stresses due to forces [3]

The shear stresses are not uniform across the cross section and maximum at the outer surface of the pipe. There is a factor introduced called shear distribution factor and gives the ratio between the maximum and the average shear stress. The factor is closed to 2.0 for most pipe cross sections. So the maximum shear stresses can be written as;

$$\tau_{xy,max} = \frac{2F_y}{A} \quad (\text{Eq 3.37}) [3]$$

$$\tau_{xz,max} = \frac{2F_z}{A} \quad (\text{Eq 3.38}) [3]$$

Where;

$$A = \pi(r_o^2 - r_i^2) \quad (\text{Eq 3.39}) [3]$$

The axial stresses are uniform across the cross section and can be written as;

$$\sigma_{lf} = \frac{F_x}{A} \quad (\text{Eq 3.40}) [3]$$

### 3.4.3 Stresses due to moments

Moment loads are divided into two categories such as bending moment and torsion moment. The stresses due to the moments can be illustrated as in the following figure 3.11;

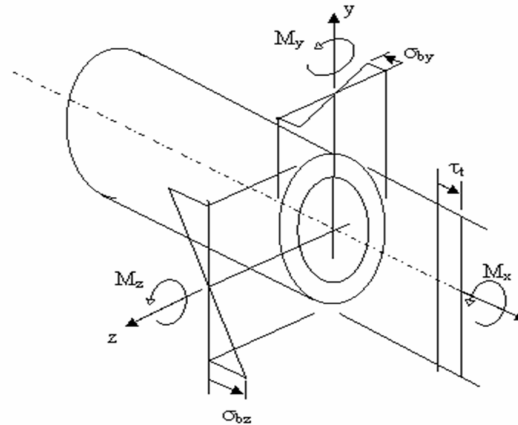


Figure 3.11 Stresses due to moments [3]

As in the illustrated figure 3.11 the stress distribution is linear and highest at the outer surface of the pipe. The highest stresses due to the bending can be written as;

$$\sigma_{by} = \frac{M_y}{Z} \quad (\text{Eq 3.41}) [3]$$

$$\sigma_{bz} = \frac{M_z}{Z} \quad (\text{Eq 3.42}) [3]$$

Where;

$$Z = \frac{\pi}{4r_o} (r_o^4 - r_i^4) \quad (\text{Eq 3.43}) [3]$$

The resultant bending stress will be;

$$\sigma_b = \sqrt{\sigma_{by}^2 + \sigma_{bz}^2} = \frac{1}{Z} \sqrt{M_y^2 + M_z^2} \quad (\text{Eq 3.44}) [3]$$

The bending stress due to the torsion moment  $M_x$  is uniformly distributed along the circumferential direction and maximum at the outer surface of the pipe. The maximum bending stress due to the torsion moment will be;

$$\tau_t = \frac{M_x}{2Z} \quad (\text{Eq 3.45}) [3]$$

### 3.5 Stress analysis

In the piping stress analysis, one of the main requirements is to check the flexibility of the piping system. The stress analysis also has to consider about the pipe wall thickness calculation with regard to the internal and external pressure, the required reinforcement calculation of the pipe and the piping components and the calculation of the maximum vertical deflection.

Most international design codes are more or less limited to static and dynamic stress analysis and it is basically used the allowable stress design methodology. But some piping codes are going further step and included also the load resistant factor design methodology.

Let's find more about the static and dynamic analysis.

### 3.5.1 Static analysis

When a piping system is subjected to internal static pressure, dead weight of the pipe and other sustained and displacement loads, the analysing or finding the sustained stresses, displacement stresses, pipe support loads and equipment loadings of the piping system is called the static analysis. When consider the dead weight of the pipe, it also has to consider the insulation, snow and ice accumulation, valve weights and etc.

### 3.5.2 Quasi static analysis

In quasi static analysis, it is considered the loads which are in dynamic nature such as earth quake, wind, explosion, slugs, water hammer, pressure surge and loads from pressure relieving devices.

### 3.5.3 Dynamic analysis

The dynamic analysis of a piping system is consisting of modal analysis, harmonic analysis, response spectrum analysis and time history analysis.

- Modal analysis

To find natural frequencies and mode shapes of the pipe, it is necessary to do a modal analysis. Various elastic piping components such as pipes, bends, tees, flanges and etc are part of a piping system and also the piping system is having uneven mass distribution because of size changes, different fittings and other various rigid components. Therefore once the system is displaced from static equilibrium, its components starts to oscillate at different mode shapes and starts to vibrate at associated frequencies.

It is vital to find the natural frequency of the piping system in order to determine the dynamic load factors (DLF) and also to determine the pipe support span to avoid harmful vibrations. It is necessary to do the modal analysis before the other dynamic analysis since these are using the natural frequencies of the system obtained from the modal analysis.

When a piping system is properly supported in according to the standards, the lowest natural frequency should not less than 4 or 5 Hz.

- Harmonic analysis

Steady state response of a piping system to applied loads which vary sinusoidal with time is determined by the harmonic analysis. The applied loads are modelled as displacements at a point or more points in the system. If the stress engineer identified multiple loads, those have to differentiate with phase angles.

- Respond spectrum analysis

The respond spectrum analysis is used to determine the respond of a piping system to a very exceptional load cases such as earth quakes and blasts.

- Time history analysis

The time history analysis is used to determine the system dynamic impact respond from time dependant loads such as activating a pressure relieving devices such as PSV, fast closing of emergency shut down (ESD) valve, uncontrolled start up, break down of a pump and etc.

## 3.6 Analysis tools

Normally the piping codes won't allow for simplified hand calculations except for specific piping systems. And also sometimes it can be allowed to design piping systems without extensive pipe stress calculations, if the system is a duplicate of existing with a known history of successful operation. But most piping codes deemed to analyse the new piping systems with thorough stress assessment.



To satisfy the piping code requirements and to do the analysis with reliable, practical and economical way, there is only one way and that is to get help from dedicated and commonly used pipe stress software based on the general purpose FEA programs with the piping code check module or the beam elementary theory. For extensive stress analysis there are many available types of software in the market and in this thesis there has been used software called CAESAR II 5.10 version.

The CAESAR II is complete pipe stress analysis software which used most widely in the world and this allow quick and accurate analysis of piping systems subjected to weight, pressure (sustained), thermal and other static and dynamic loads (operating). This includes a full range of latest international piping codes such as ASME B31.1 (power), B31.3 (process piping), B31.4 (offshore), B31.5 (refrigeration), British standards, TBK 5-6 Norwegian and much more.

This incorporates with the tables of the piping materials and the components plus expansion joints, spring hangers and the material properties including the allowable stresses. Because of this it can save considerable amount of time for searching those and also it ensure correct datasets are used for the each analysis.

When consider about the static analysis capabilities, the CAESAR II begins a static analysis by recommending load cases necessary to comply with the considered piping code stress requirements. The static analysis gives the piping stresses, displacements, moments and etc in each load cases and checks the piping code requirements are satisfied. This allows seeing clear results separately graphically or numerically within couple of seconds with respect to the each load cases.

Also the CAESAR II helps to identify the data needed for the dynamic analysis through specification. This allows doing the dynamic analysis such as the modal analysis for the natural frequency calculations, harmonic forces and displacement analysis, the model time history analysis, the dynamic response analysis and etc.

## 4 Fatigue

Fatigue is a localised and progressive failure that occurs when a material is subjected to cyclic loadings. Wave, current and wind loadings, vortex induced oscillation and etc are typical examples of loadings that may generate fatigue structural failures in offshore. The loading itself may not large enough to cause a sudden failure on a structure but the number of loading cycles may cause failure.

The fatigue failure goes through three stages;

1. Initiation (crack initiation)
2. Slow growth (crack propagation)
3. Onset of unstable fracture

When a loading is above a certain threshold, there will be initiation of microscopic cracks at the surface. After certain loading cycles during the service life these microscopic cracks will reach a critical size and then the structure will suddenly fail.

Fluctuation of stress due to cyclic loadings can be illustrated as follows;

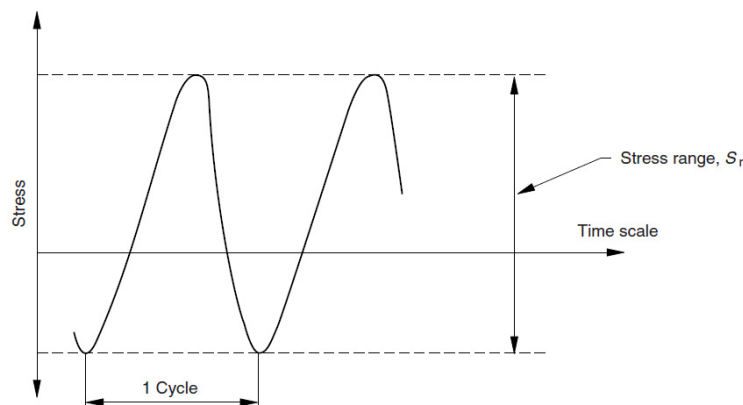


Figure 4.1 Stress variation [11]

The fatigue failure is caused by the cumulative effect of damage due to many load cycles. As in the above figure 4.1, there will be stress variations due to these load cycles and the most important component under this stress variation is the cyclic stress range or the cyclic stress amplitude. In the fatigue analysis the cyclic stress range is more important than the peak stress.

There are two types of fatigue such as high cyclic fatigue (low stress) and low cyclic fatigue (high stress) with respect to the stress levels and the number of cycles. Offshore structures in the North Sea are considered mainly the high cyclic fatigue due to the millions of cyclic loadings by waves, currents and etc.

Generally fatigue crack propagating direction is perpendicular to the maximum principal tensile stress.

Welded structures such as piping systems, the fatigue failure most often starts at a weld. When consider about a welded joint, the weld toe/root discontinuities are generally present and those are pre existing cracks. Consequently, the crack propagation stage represents the bulk of the total fatigue life of a welded joint. But for an un-welded component, the bulk of the total fatigue life can be attributed to crack initiation stage and that may exceed 95% of the fatigue life.

In this chapter the main focus goes to the fatigue life estimation or in other words fatigue design check and there can be different approaches for this. Those can be listed as follows with a different degree of refinement;

- Judgement by experience

If there is no significant amount of quantitative experience from similar cases, this approach has to be used with caution. It is advised not to go for this approach.

- Fracture mechanics

This approach is basically checking about how the structural endurance/tolerance of the design with a defect. The most important parameter or variable in fracture mechanics is the initial defect size. The system should be designed and inspected thoroughly so that the maximum initial defect size would not propagate in to a critical size during the service life or within inspection interval. The inspection interval is a critical parameter and it should be shorter than the duration for the crack to grow from a NDT detectable size to a critical crack size. By doing the crack propagation calculation, it is possible to determine the fatigue crack growth life.

When consider about a piping system, this fracture mechanics approach is not much practical and so that in this thesis it is focused only on the fatigue assessment based on the SN curve approach.

- SN curves

SN curves have been made on the basis of laboratory tests and those characterise the fatigue behaviour of materials. The SN curve expresses the number of cycles a material can withstand under repeated loading at a given stress level before the fatigue failure. There have been many fatigue SN curves developed during the time for different materials, systems, geometries, welded and un-welded components and etc.

When consider the fatigue assessment of a piping system which consist of welded joints, the weakest link is the weld. For this purpose, SN curves for different weld qualities have been developed by the BSI (PD5500) and these can be illustrated as in the following figure 4.2;

The SN curves have been developed mainly for structures in air and structures in seawater or cathodic protection. Since the most offshore topside piping is without the cathodic protection and not submerged in the seawater, the SN curves developed for the structures in air have used in the thesis.

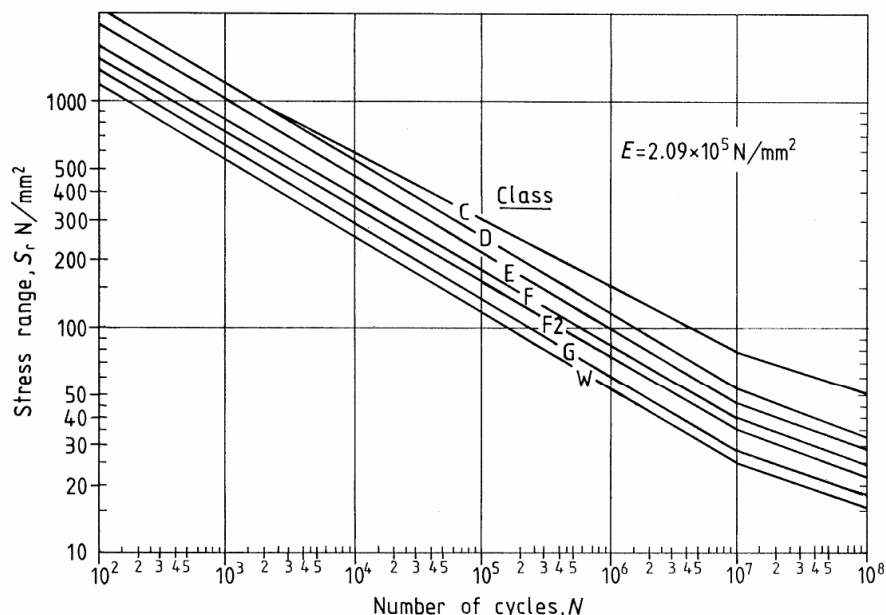


Figure C.3 — Fatigue design  $S-N$  curves for weld details applicable to ferritic steels up to and including  $350\text{ }^{\circ}\text{C}$ , austenitic stainless steels up to and including  $430\text{ }^{\circ}\text{C}$  [E], aluminium alloys up to and including  $100\text{ }^{\circ}\text{C}$ , nickel alloys up to and including  $450\text{ }^{\circ}\text{C}$  and titanium alloys up to and including  $150\text{ }^{\circ}\text{C}$  [A]

Figure 4.2 Two slopes SN curves for different weld qualities [11]

### 4.1 SN data and Miner-Palmgren approach

In 1945, A. Miner has developed a formula for the fatigue life estimation and it is called Miner-Palmgren formula and can be stated as follows;

$$D = \sum_{i=1}^k \frac{n_i}{N_i} \quad (\text{Eq 4.1}) [14]$$

Where;

D = cumulative fatigue damage ratio

k = number of stress blocks

$n_i$  = number of stress cycles in stress block i with constant stress range  $\Delta\sigma_i$

$N_i$  = number of cycles to failure at constant stress range  $\Delta\sigma_i$

The Miner-Palmgren formula basically finds the cumulative fatigue damage due to the stress cycles by the help of SN curves. This formula assumes when a fracture happens, the total damage ratio  $D = 1$ . At that time the calculated fatigue life can be given as follows;

$$L = \frac{L_0}{D} \quad (\text{Eq 4.2}) [14]$$

Where;  $L_0$  = time for the total number of stress cycles  $n_0 = \sum_{i=1}^k n_i$

The SN curves that are given in the above figure 4.2 have been developed by estimating the fatigue capacity of different welded joints subjected to constant amplitude uni-axial loading. So to find the corresponding number of cycles to failure for a given stress range  $\Delta\sigma$ , can be determined by going into the SN curve as illustrated in the following figure 4.3;

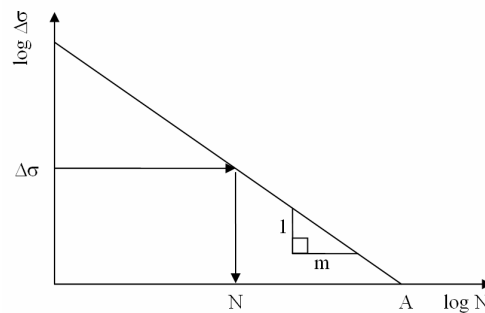


Figure 4.3 SN curve – relation

In the same time it is possible to develop an analytical expression for the SN curve to find the corresponding number of cycles to failure for the given stress range and vice versa. The relation between the two axes of the SN curve is logarithmic as follows;

$$\frac{1}{m} = \frac{\log \Delta\sigma}{\log A - \log N} \Rightarrow \log N = \log A - m \log \Delta\sigma \quad (\text{Eq 4.3}) [14]$$

Also this can be expressed as follows;

$$N = A \cdot \Delta\sigma^{-m} \quad (\text{Eq 4.4}) [14]$$

Where;

A = intercept of number of cycle axis

m = inverse negative slope of the SN curve

## 4.2 Closed form fatigue life equations

The stress cycles which considered are assumed to be randomly distributed with a probability density function  $f(\Delta\sigma)$ . This means that the number of cycles with stress range within  $\Delta\sigma$  and  $\Delta\sigma + d\Delta\sigma$  can be given as  $n_0 \cdot f(\Delta\sigma) \cdot d\Delta\sigma$ . Where  $n_0$  is the total number of stress cycles.

The cumulative damage ratio can be rewritten as;

$$D = \int_0^{\infty} \frac{n_0 \cdot f(\Delta\sigma)}{N(\Delta\sigma)} d\Delta\sigma \quad (\text{Eq 4.5}) [14]$$

Where;

$N(\Delta\sigma)$  = corresponding number of cycles to failure at a constant stress range  $\Delta\sigma$

By applying the Eq 4.4 to Eq 4.5;

$$D = \frac{n_0}{A} \int_0^{\infty} \Delta\sigma^m \cdot f(\Delta\sigma) \cdot d\Delta\sigma \quad (\text{Eq 4.6}) [14]$$

The probability density function of the stress range for offshore structures can be assumed to be represented by a two parameter Weibull distribution as follow;

$$f(\Delta\sigma) = \frac{h}{q} \cdot \left(\frac{\Delta\sigma}{q}\right)^{h-1} \cdot \exp\left(-\frac{\Delta\sigma}{q}\right)^h \quad (\text{Eq 4.7}) [14]$$

Where;

$h$  = shape parameter

$q$  = scale parameter

By applying the Eq 4.7 to Eq 4.6;

$$D = \frac{n_0}{A} \int_0^{\infty} \Delta\sigma^m \cdot \frac{h}{q} \cdot \left(\frac{\Delta\sigma}{q}\right)^{h-1} \cdot \exp\left(-\frac{\Delta\sigma}{q}\right)^h \cdot d\Delta\sigma \quad (\text{Eq 4.8}) [14]$$

Introducing;

$$t = \left(\frac{\Delta\sigma}{q}\right)^h \quad (\text{Eq 4.9}) [14]$$

$$\Delta\sigma = q \cdot t^{\frac{1}{h}} \quad (\text{Eq 4.10})$$

By derivation;

$$d\Delta\sigma = \frac{q}{h} \cdot t^{\frac{1}{h}-1} \cdot dt \quad (\text{Eq 4.11})$$

By applying the Eq 4.9, Eq 4.10 and Eq 4.11 to Eq 4.8;

$$D = \frac{n_0}{A} \int_0^{\infty} \left(q \cdot t^{\frac{1}{h}}\right)^m \cdot \frac{h}{q} \cdot (t) \cdot \left(\frac{q \cdot t^{\frac{1}{h}}}{q}\right)^{-1} \exp(-t) \cdot \frac{q}{h} \cdot t^{\frac{1}{h}-1} \cdot dt$$

$$D = \frac{n_0}{A} \cdot q^m \int_0^{\infty} t^{\frac{m}{h} - \frac{1}{h} + 1 + \frac{1}{h} - 1} \cdot \exp(-t) \cdot dt$$

$$D = \frac{n_0}{A} \cdot q^m \int_0^{\infty} t^{\left(\frac{m+1}{h}\right)-1} \cdot \exp(-t) \cdot dt \quad (\text{Eq 4.12}) [14]$$

The gamma function defined as;

$$\Gamma(n) = \int_0^{\infty} e^{-t} \cdot t^{n-1} \cdot dt \quad (\text{Eq 4.13}) [14]$$

By applying the Eq 4.13 to Eq 4.12 yield to;

$$D = \frac{n_0}{A} \cdot q^m \cdot \Gamma\left(\frac{m}{h} + 1\right) \quad (\text{Eq 4.14}) [14]$$

For the convenience let's eliminate the q by introducing the maximum stress range  $\Delta\sigma_0$  during the  $n_0$  number of cycles. The probability of exceedance of stress range  $\Delta\sigma$  is;

$$Q(\Delta\sigma) = 1 - \int_0^{\Delta\sigma} f(\Delta\sigma) \cdot d\Delta\sigma \quad (\text{Eq 4.15}) [14]$$

By applying the Eq 4.7 to Eq 4.15 and integrating, yield for the exceedance function;

$$Q(\Delta\sigma) = \exp\left[-\left(\frac{\Delta\sigma}{q}\right)^h\right] \quad (\text{Eq 4.16}) [14]$$

The probability that the maximum stress range  $\Delta\sigma_0$  is reached or exceeded for a total of  $n_0$  stress cycle is;

$$Q(\Delta\sigma_0) = \frac{1}{n_0} \quad (\text{Eq 4.17}) [14]$$

By combining the Eq 4.16 and Eq 4.17;

$$\begin{aligned} Q(\Delta\sigma_0) &= \frac{1}{n_0} = \exp\left[-\left(\frac{\Delta\sigma_0}{q}\right)^h\right] \\ \Rightarrow q &= \Delta\sigma_0 (\ln n_0)^{-\frac{1}{h}} \end{aligned} \quad (\text{Eq 4.18})$$

By applying the Eq 4.18 to Eq 4.16;

$$\begin{aligned} Q(\Delta\sigma) &= \exp\left[-\left(\frac{\Delta\sigma}{\Delta\sigma_0 (\ln n_0)^{-\frac{1}{h}}}\right)^h\right] \\ \Rightarrow Q(\Delta\sigma) &= \exp\left[-\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^h \cdot \ln n_0\right] \end{aligned} \quad (\text{Eq 4.19}) [14]$$

Since  $Q(\Delta\sigma)$  represents the probability of exceedance of  $\Delta\sigma$ , then the probability of exceedance of  $\Delta\sigma$  for n number of cycles can be given as;

$$Q(\Delta\sigma) = \frac{n}{n_0} = \exp\left[-\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^h \cdot \ln n_0\right] \quad (\text{Eq 4.20}) [14]$$

By rearranging;

$$\begin{aligned} \ln n - \ln n_0 &= -\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^h \cdot \ln n_0 \\ \ln n &= \left[1 - \left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^h\right] \cdot \ln n_0 \\ \left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^h &= 1 - \frac{\ln n}{\ln n_0} \\ \Rightarrow \Delta\sigma &= \Delta\sigma_0 \left(1 - \frac{\ln n}{\ln n_0}\right)^{1/h} \end{aligned} \quad (\text{Eq 4.21}) [14]$$

Where;

$n$  = number of stress cycles that exceeds  $\Delta\sigma$   
 $\Delta\sigma_0$  = maximum stress range for a total of  $n_0$  cycles  
 $n_0$  = total number of stress cycles

By applying the Eq 4.18 to Eq 4.14;

$$\begin{aligned} D &= \frac{n_0}{A} \cdot \left(\Delta\sigma_0 (\ln n_0)^{-\frac{1}{h}}\right)^m \cdot \Gamma\left(\frac{m}{h} + 1\right) \\ \Rightarrow D &= \frac{n_0}{A} \cdot \frac{\Delta\sigma_0^m}{(\ln n_0)^{\frac{m}{h}}} \cdot \Gamma\left(\frac{m}{h} + 1\right) \end{aligned} \quad (\text{Eq 4.22}) [14]$$

### 4.3 Equivalent stress range approach

By using the Eq 4.1 and Eq 4.4, the damage ratio for long term stress range distribution can be written as;

$$D = \sum_{i=1}^k \frac{n_i \cdot \Delta\sigma_i^m}{A} \quad (\text{Eq 4.23}) [14]$$

Let's find an equivalent stress range which is constant through the total number of cycles and gives the same damage as the above Eq 4.23;

$$D = \sum_{i=1}^k \frac{n_i \cdot \Delta\sigma_{eq}^m}{A} = \frac{\Delta\sigma_{eq}^m}{A} \cdot \sum_{i=1}^k n_i = \frac{\Delta\sigma_{eq}^m}{A} \cdot n_0 \quad (\text{Eq 4.24}) [14]$$

By considering the Eq 4.23 and Eq 4.24;

$$\begin{aligned} D &= \frac{\Delta\sigma_{eq}^m}{A} \cdot n_0 = \sum_{i=1}^k \frac{n_i \cdot \Delta\sigma_i^m}{A} \\ \Rightarrow \Delta\sigma_{eq} &= \left(\sum_{i=1}^k \frac{n_i \cdot \Delta\sigma_i^m}{n_0}\right)^{\frac{1}{m}} \end{aligned} \quad (\text{Eq 4.25}) [14]$$

By considering the Eq 4.22 and Eq 4.24, the equivalent stress range for Weibull distributed stress range can be given as follows;

$$D = \frac{\Delta\sigma_{eq}^m}{A} \cdot n_0 = \frac{n_0}{A} \cdot \frac{\Delta\sigma_0^m}{(\ln n_0)^{\frac{m}{h}}} \cdot \Gamma\left(\frac{m}{h} + 1\right)$$

$$\Rightarrow \Delta\sigma_{eq} = \frac{\Delta\sigma_0}{(\ln n_0)^{\frac{1}{h}}} \cdot \sqrt[m]{\Gamma\left(\frac{m}{h} + 1\right)} \quad (\text{Eq 4.26}) [14]$$

#### 4.4 Equivalent stress range approach for two slopes SN curves

The equivalent stress range approach can be further developed for the two slopes SN curves by considering the Miner's summation theory. Two groups can be considered such as stress ranges above the knee point (above  $\Delta\sigma$ ) of the two slopes SN curve and the stress ranges below the knee point (below  $\Delta\sigma$ ). The concept can be illustrated as in the following figure;

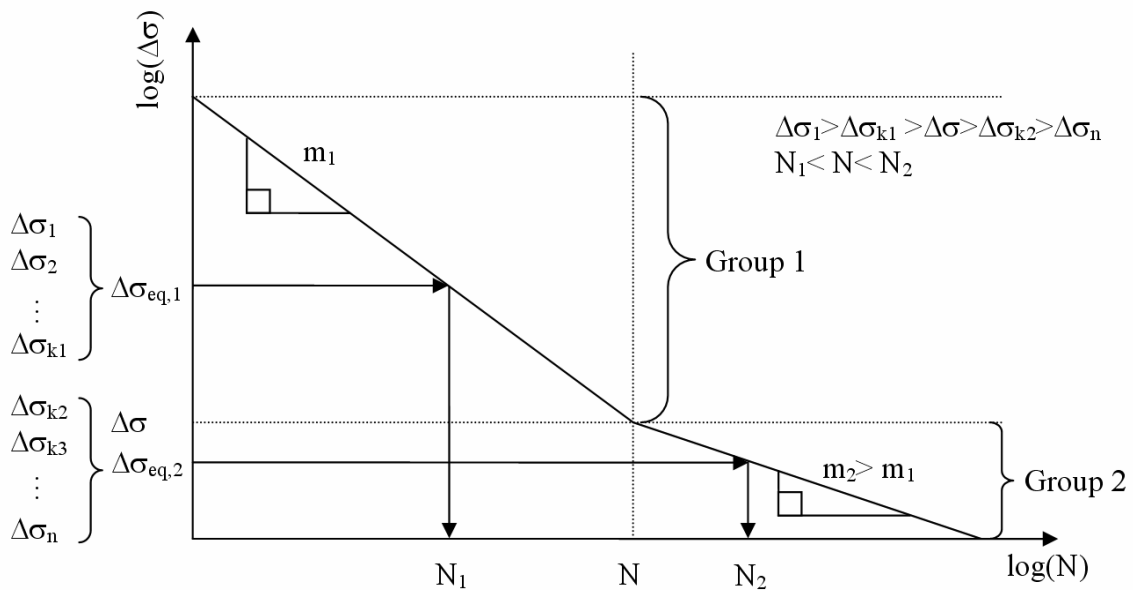


Figure 4.4 Equivalent stress range approach for two slopes SN curve

First equivalent number of cycles can be found for the each group as follows;

$$n_{0,i} = \sum_{i=1}^k n_i \quad (\text{Eq 4.27})$$

By using the Eq 4.25, it can be possible to find the equivalent stress range for the group 1 and 2. Once the equivalent stress ranges found, the corresponding number of cycles to failure can be found for the each group as illustrated in the above figure 4.4. Then the total fatigue damage can be calculated as follows;

$$D = \sum_{i=1}^2 \frac{n_{0,i}}{N_i} \quad (\text{Eq 4.28})$$

Where;  $N_i$  = number of cycles to failure

Since the same concept discussed in the above chapter 4.3 and 4.4, it can be easier to use the above derived Eq 4.24 to calculate the fatigue damage.



## 4.5 British standard PD5500

One of the main recognised standards for the estimation of fatigue life for offshore structures is the PD5500 standard published by the British standard institution. This has initially published as an enquiry to the BS code for unfired fusion welded pressure vessels and was named as enquiry BS5500/79.

The fatigue test data obtained from welded specimens that fabricated to normal standard of workmanship tested under load control or for applied strains exceeding yield under strain control, have been used to plot the design SN curves for the assessment of weld classes in the PD5500 standard. The design curves that used in this standard are two slopes SN curves in air and those are based on the following form of formula;

$$S_r^m N = A \quad (\text{Eq 4.29}) [11]$$

Where A and m are constants and have different values for different weld classes and can be given as the following table 4.1;

Different types of weld qualities/classes can be found in the appendix A.

**Table 4.1 Details about fatigue design curves for different weld classes [11]**

Class	Constants of S-N curve				Stress range at $N = 10^7$ cycles N/mm <sup>2</sup>
	for $N < 10^7$ cycles		for $N > 10^7$ cycles		
	$m$	$A^a$	$m$	$A^a$	
C <sup>b</sup>	3.5	$4.22 \times 10^{13}$	5.5	$2.55 \times 10^{17}$	78
D	3	$1.52 \times 10^{12}$	5	$4.18 \times 10^{15}$	53
E	3	$1.04 \times 10^{12}$	5	$2.29 \times 10^{15}$	47
F	3	$6.33 \times 10^{11}$	5	$1.02 \times 10^{15}$	40
F2	3	$4.31 \times 10^{11}$	5	$5.25 \times 10^{14}$	35
G	3	$2.50 \times 10^{11}$	5	$2.05 \times 10^{14}$	29
W	3	$1.58 \times 10^{11}$	5	$9.77 \times 10^{13}$	25

<sup>a</sup> For  $E = 2.09 \times 10^5$  N/mm<sup>2</sup>.  
<sup>b</sup> If  $S_r > 766$  N/mm<sup>2</sup> or  $N < 3$  380 cycles, use class D curve.

Since the SN design curves have two slopes, the constants A and m are changed at the  $10^7$  number of cycles.

In this standard there are two adjustments discussed to the general SN design curve formula due to the fact that the fatigue test data are obtained from a specific test specimen.

- Effect of material

For welded structures, the fatigue lives of welded details are independent of material yield strength and it is because the welded joints contain crack type defects and the fatigue life only depends on crack growth which independent of the yield strength of the material. Since that, for a given detail the same set of SN design curves can be used for all the steels and all aluminium, nickel and titanium alloys. The SN curves that are given in this standard are based to material which has modulus of elasticity of 209000 N/mm<sup>2</sup>. So when other materials and/or temperatures are being considered, there has to be considered adjustment to the stress range and can be given as follows;

$$S = S_r \cdot \left( \frac{209000}{E} \right) \quad (\text{Eq 4.30}) [11]$$

Where;

$S_r$  = allowable stress range for a particular life

$S$  = stress range obtained from appropriate design curve at the same life

$E$  = modulus of elasticity in  $\text{N/mm}^2$

- Effect of plate thickness

The SN design curves are applied for plate thickness up to 22mm. But due to fatigue strength decrease with increase in plate thickness, there has been introduction of adjustment for thickness greater than 22mm. The adjustment is the stress range obtained from appropriate design curves should be multiplied by factor  $(22/t)^{1/4}$ .

Taking the effect of the materials and the plate thicknesses into account, the Eq 4.29 can be rearranged as follows;

$$N = A \cdot \left( \frac{22}{t} \right)^{m/4} \cdot \left( \frac{S_r \cdot 209000}{E} \right)^{-m} \quad \Leftarrow \text{for } t > 22\text{mm} \quad (\text{Eq 4.31}) [11]$$

$$N = A \cdot \left( \frac{S_r \cdot 209000}{E} \right)^{-m} \quad \Leftarrow \text{for } t < 22\text{mm} \quad (\text{Eq 4.32}) [11]$$

## 5 Bridge piping fatigue

When consider about a piping system routed on a bridge which connected to offshore platforms, that piping system can be critical on fatigue design. For this specific fatigue assessment, this thesis is considered a bridge piping example on the Eldfisk field. Sketch of the example can be illustrated as the following figure 5.1;

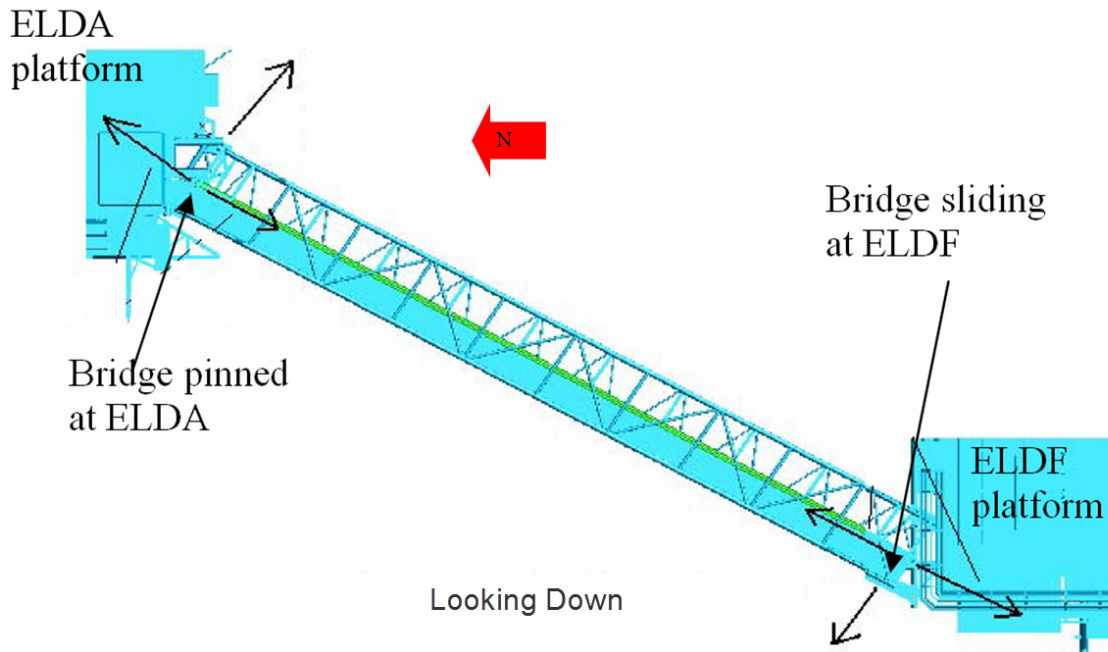


Figure 5.1 Sketch of bridge between ELDA and ELDF

The bridge is connected to two platforms and those are ELDA which is a production/wellhead jacket type platform and ELDF which is a processing jacket type platform. This bridge is pinned at the ELDA and guided and sliding on the ELDF.

Because of the wave forces, the platforms are relatively moving each other. The relative movements can be calculated by using different wave theories and hydrodynamic analyses. By calculating the stiffness of the platforms, it can be possible to calculate the maximum deflection of the platforms. These maximum deflections are given by the marine or the structural engineers to the piping stress engineers.

Wave conditions about wave heights, periods and frequencies can be obtained from the historical wave data collection in the Eldfisk area. These wave data can be statistically adjusted and scaled to required number of years.

### 5.1 System definition

This specific piping system is a system that consists of a 16" multi phase hydrocarbon production line which comes from the ELDA to the ELDE via ELDF on the bridges for the processing purposes. The bridge is 31.64 deg out of the global North direction as in the above figure 5.1. For this fatigue assessment, it has considered only one bridge as explained in the above section and that is the bridge piping in between the ELDA and the ELDF. To accommodate the relative movements of the platforms, the piping system has to consist of a piping expansion loop and in this specific example the loop is in the ELDF platform and it is because the bridge is sliding on the ELDF.

## 5.2 Design data

Most design data are mainly coming from the process engineer. The process engineer is responsible for issuing the line list which consists of all the process information relevant to the each line such as operating and design pressure and temperature, test pressure, flow rate, density of the fluid, insulation and etc. Then there can be a possibility to find the other relevant design data out of these line list data.

Relevant design data for the 16” production line can be given as follows;

### Line list and other data

Line number	06XXX-BD10-16”-PR
Max design temperature	100 °C
Min design temperature	-46 °C
Design pressure at low temp.	51.1 barg
Design pressure at high temp.	51.1 barg
Operating temperature max/min	80 °C/50 °C
Operating pressure	35.5 barg
Pipe internal diameter	393.7 mm
Insulation class and thickness	0/0 mm
Heat trace	No
Fluid density	867 kg/m <sup>3</sup>
Test pressure	77.75 barg
Pipe outer diameter	406.4 mm
Wall thickness	6.35 mm
Corrosion allowance	0.0 mm
Mill tolerance	12.5%
Pipe spec	BD10 (X company)
Pipe material	A 928 S32760 (25 Cr Duplex/super duplex)
Pressure class	300 lb
Elastic modulus	201200 N/mm <sup>2</sup>
Design code	ASME B31.3

Once all the design data are acquired, layout design has to be done according to the BD10 pipe spec. The pipe spec states the design details of all the pipes, fittings, bending, branches, flanges, valves and etc. For the layout design there has been used software called PDMS design and this is basically a three dimensional (3D) model as in the following figure 5.2.

Since this line is a critical line, there has to be done detailed stress analysis and for this it has been used the CAESAR II stress analysis software. The pipe support designer has to give an input to the stress engineer about the possible pipe support locations. Out of these data the stress analysis has to be done and pipe support loads and functions have to be given back to the pipe support designer. All those load cases and stress limits discussed in the above chapter two and three (ASME B31.3) have to be checked and satisfied during the stress analysis process.

In this thesis, the stress analysis is only limited to the fatigue analysis part. The fatigue assessment can be a huge challenge in this specific example due to high displacements and cyclic stresses. And other challenge is when it is allowed more flexibility on the piping system to accommodate the high displacements by using fewer guides in the pipe loop; there can be a problem with the low natural frequency of the system. As the same way when introduce more guides to increase the natural frequency, there can be a problem with the low fatigue life or failure due to the high stresses above the allowable. So these have to be done in a balance way.

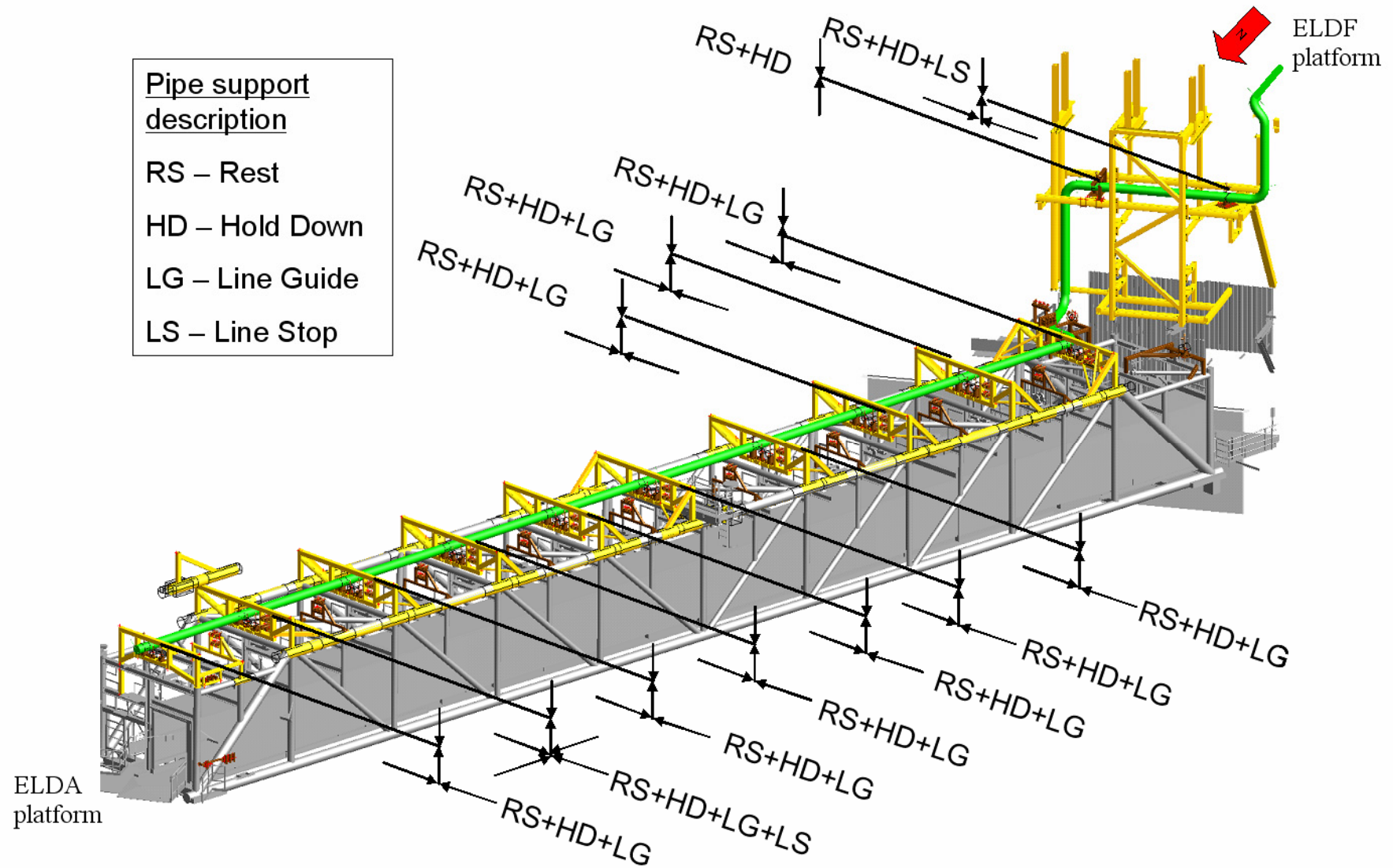


Figure 5.2 PDMS 3D model with pipe supports and bridge structure

### 5.3 Fatigue waves

Fatigue waves for the Eldfisk area have obtained from the historical wave data and given in the appendix B. In this appendix the wave data have scaled to thirty (30) years without any extrapolation to rare events and that means the wave data can be more or less considered as accurate as actual. These data contains different wave height blocks from 0.5 m to 19.5 m with median period and corresponding frequencies in different directions which divided into eight (8) sectors. For the fatigue life assessment, it has taken the OMNI directional wave heights as a conservative measure.

### 5.4 Platform relative displacements

The relative platform movements on the sliding side which is on the ELDF platform are taken from the data provided by the owner of platform. These displacements are calculated by means of the worst case scenario.

The maximum relative displacement of the ELDF platform moving towards the ELDA and the ELDF moving away from the ELDA are given as 515 mm and 440 mm.

Relative lateral displacements has considered as negligible.

These relative displacements have imposed on the line stop on the bridge at the node 30.

For the each fatigue waves defined in the above referred appendix B, the corresponding relative displacement have been calculated by using the relation between the maximum displacement and the maximum wave height and can be given as follows;

$$\Delta = \Delta_{\max} \cdot \left( \frac{h}{h_{\max}} \right)^k \quad (\text{Eq 5.1}) [14]$$

Where;

$\Delta$  = displacement for the wave height,  $\Delta_{\max}$  = maximum displacement (515 mm/440 mm)

$h$  = wave height,  $h_{\max}$  = maximum wave height (19.5 m),  $k = 1.7$  (for jacket structure)

### 5.5 Maximum stress range

As explained earlier, for the stress analysis the CAESAR II model has been developed and the calculated pipe support loads and displacements can be shown in a stress isometric drawing which has given in the following figure 5.3.

By imposing the displacements to the node 30, the maximum stress range can be found. The owner has decided that the wave heights above 10.5 m is a storm situation and so that the platforms will shutdown and out of operation. Due to this procedure the wave heights have divided into two groups such as wave heights from 0.5 m to 10.5 m and wave heights from 11.5 m to 19.5 m.

The first group which consider the wave heights from 0.5 m to 10.5 m can be analysed with the operating pressure and temperature and found the maximum stress range on each and every wave height block.

Since the platform will out of operation above the 10.5 m wave heights, the second group of wave heights can be analysed with the ambient pressure and temperature. Due to the pressure effect, the piping components become stiff. But above the 10.5 m waves, this piping system is without the pressure effect. Therefore the stress analysis has to be done without the pressure effect and it can be done by switching off the pressure stiffening option in the CAESAR II. When there is a piping system without the pressure effect, the system becomes more vulnerable at higher wave heights. Therefore the maximum stress range of the system will be higher without the pressure stiffening and ultimately this can be lead to lower fatigue life.

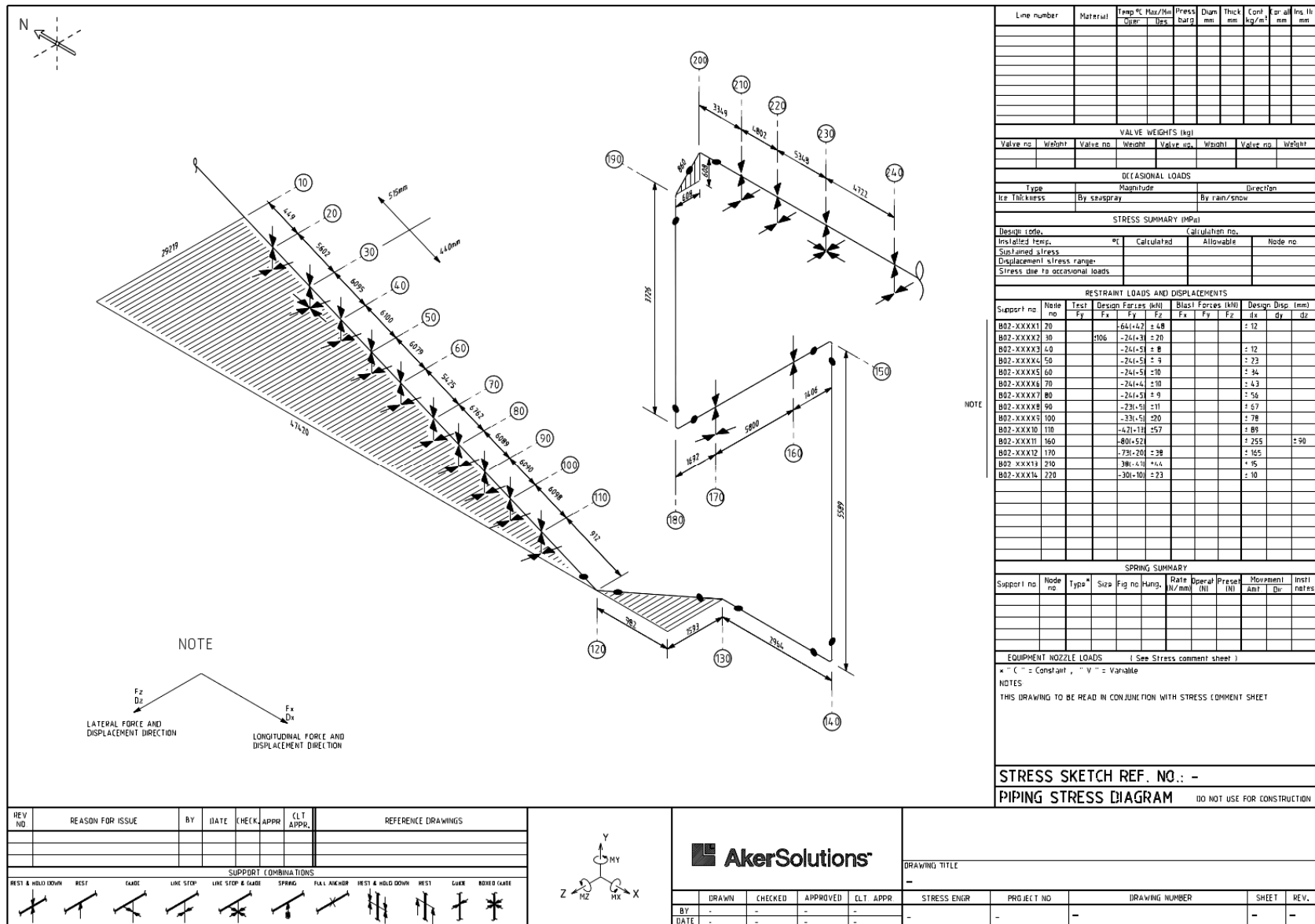


Figure 5.3 Stress iso – Bridge piping ELDA-ELDF

There are two CAESAR II models have developed for the group one and two because of the differences in design conditions and simplification of the calculation. These two models have named as ELDFELDA01 and ELDFELDA02.

Analysed load cases to find the maximum stress range for the fatigue analysis on each model/group can be given as follows;

**Table 5.1 Group 1 load combinations – bridge piping**

Group 1 (wave heights from 0.5 m to 10.5 m) operating condition, pressure stiffening ON			
Case No	Case description	Load case	Comment
L1	Weight + max operating pressure + max operating temperature	W+P1+T1	P1 = 35.5 barg, T1 = 80 °C
L2	Weight + max operating pressure + min operating temperature	W+P1+T2	P1 = 35.5 barg, T1 = -46 °C
L3	Weight + max operating pressure + max operating temperature + max bridge displacement towards ELDA	W+P1+T1+D1	P1 = 35.5 barg, T1 = 80 °C, D1 = 180 mm at 10.5 m wave height
L4	Weight + max operating pressure + max operating temperature + max bridge displacement away from ELDA	W+P1+T1+D2	P1 = 35.5 barg, T1 = 80 °C, D2 = 154 mm at 10.5 m wave height
L5	D1 = max bridge displacement towards ELDA	L3-L1	D1 = 180 mm at 10.5 m wave height
L6	D2 = max bridge displacement away ELDA	L4-L1	D2 = 154 mm at 10.5 m wave height
L7	Displacement range	L5+L6	Max displacement stress range

**Table 5.2 Group 2 load combinations – bridge piping**

Group 2 (wave heights from 11.5 m to 19.5 m) ambient condition, pressure stiffening OFF			
Case No	Case description	Load case	Comment
L1	Weight + ambient pressure + ambient temperature	W+P1+T1	P1 = 1 barg, T1 = 4 °C
L2	Weight + ambient pressure + ambient temperature	W+P1+T2	P1 = 1 barg, T1 = 4 °C
L3	Weight + ambient pressure + ambient temperature + max bridge displacement towards ELDA	W+P1+T1+D3	P1 = 1 barg, T1 = 4 °C, D3 = 515 mm at 19.5 m wave height
L4	Weight + ambient pressure + ambient temperature + max bridge displacement away from ELDA	W+P1+T1+D4	P1 = 1 barg, T1 = 4 °C, D4 = 440 mm at 19.5 m wave height
L5	D3 = max bridge displacement towards ELDA	L3-L1	D3 = 515 mm at 19.5 m wave height
L6	D4 = max bridge displacement away ELDA	L4-L1	D4 = 440 mm at 19.5 m wave height
L7	Displacement range	L5+L6	Max displacement stress range



The stress analysis has done by using the CAESAR II models and the maximum displacement stress ranges can be presented as follows;

**Table 5.3 Stress ranges calculated by CAESAR models and relation – bridge piping**

INDIVIDUAL LOADING EVENTS								
NO	WAVE HEIGHTS (m)	MAX DISP. TOWARDS ELDA (mm)	MAX DISP. FROM ELDA (mm)	NO OF CYCLES $n_i$ (30 years)	LOAD COM.	STRESS RANGE (MPa) CAESAR	PRES. STIFFENING ON/OFF	STRESS RANGE (MPa) RELATION
1	19.5	515	440	1	SA1	562.6	stiff OFF	562.6
2	18.5	471	402	23	SA2	514.3	stiff OFF	514.4
3	17.5	428	366	26	SA3	467.7	stiff OFF	468.1
4	16.5	388	331	82	SA4	423.5	stiff OFF	423.5
5	15.5	349	298	145	SA5	381.1	stiff OFF	380.8
6	14.5	311	266	429	SA6	339.9	stiff OFF	340.0
7	13.5	276	235	1161	SA7	301.0	stiff OFF	301.1
8	12.5	242	207	2066	SA8	264.4	stiff OFF	264.2
9	11.5	210	179	3916	SA9	229.2	stiff OFF	229.3
10	10.5	180	154	9224	SA10	158.7	stiff ON	158.7
11	9.5	152	130	20474	SA11	134.0	stiff ON	133.9
12	8.5	126	107	45249	SA12	110.7	stiff ON	110.8
13	7.5	101	87	111517	SA13	89.4	stiff ON	89.6
14	6.5	80	68	277285	SA14	70.3	stiff ON	70.2
15	5.5	60	51	717336	SA15	52.8	stiff ON	52.9
16	4.5	43	36	1933305	SA16	37.5	stiff ON	37.6
17	3.5	28	24	5424303	SA17	24.7	stiff ON	24.5
18	2.5	16	13	15808734	SA18	13.8	stiff ON	13.8
19	1.5	7	6	48670286	SA19	6.2	stiff ON	5.8
20	0.5	1	1	104473739	SA20	1.0	stiff ON	0.9

There has been found a relation between the displacement stress range, maximum displacement stress range and the wave heights and this can be given as follows;

$$S_r = S_{r,max} \cdot \left( \frac{h}{h_{max}} \right)^{1.7} \tag{Eq 5.2} [14]$$

Conservatively it may be enough to calculate the maximum displacement stress range for the maximum wave height by using the CAESAR II and then use the above relation to calculate the stress ranges for the lower state wave heights. By looking at the column seven (7) and nine (9), it is clearly shown that the stress range calculations by the above relation is no worst than the CAESAR II calculations.

### 5.6 Fatigue analysis based on PD5500

The fatigue curve E which is full penetrated butt weld has considered for the assessment. Since these fatigue curves contain two slopes which change at the  $10^7$  cycles, there are different A and m values identified as in the table 4.1 and can be given as follows;

Fatigue curve E

**Table 5.4 Weld class E details**

N<10 <sup>7</sup> /S <sub>r</sub> >47		N>10 <sup>7</sup> /S <sub>r</sub> <47		Stress range at 10 <sup>7</sup> N/mm <sup>2</sup>
m	A	m	A	
3	1.04x10 <sup>12</sup>	5	2.29x10 <sup>15</sup>	47

The fatigue analysis can be approached as follows;

To find the corresponding m and A constants for each wave block, the stress range has to be corrected/adjusted by considering the effect of the wall thickness and the materials and the adjusted stress range can be calculated as follows;

$$S_{r,adjusted} = S_r \cdot \left(\frac{t}{22}\right)^{1/4} \cdot \left(\frac{209000}{E}\right) \Leftarrow \text{for } t > 22mm \tag{Eq 5.3}$$

$$S_{r,adjusted} = S_r \cdot \left(\frac{209000}{E}\right) \Leftarrow \text{for } t < 22mm$$

Then m and A constants can be found by using the above table 5.4 and the adjusted stress range which is in the column three (3) of the table 5.5.

**Table 5.5 Fatigue life analysis PD5500 – bridge piping**

FATIGUE ANALYSIS PD5500									
FATIGUE CALCULATION FOR COMBINED LOADING EVENTS									
LOAD COMB.	STRESS RANGE CAESAR	ADJUSTED STRESS RANGE	m	A	n <sub>i</sub> (30 years)	N <sub>i</sub>	Damage D <sub>i</sub> =(n <sub>i</sub> /N <sub>i</sub> )	Damage percentage %	
SA1	562.6	584.4	3	1.04E+12	1	5210	0.000192	0.03 %	
SA2	514.3	534.2	3	1.04E+12	23	6821	0.003372	0.46 %	
SA3	467.7	485.8	3	1.04E+12	26	9069	0.002867	0.39 %	
SA4	423.5	439.9	3	1.04E+12	82	12216	0.006713	0.91 %	
SA5	381.1	395.9	3	1.04E+12	145	16763	0.008650	1.18 %	
SA6	339.9	353.1	3	1.04E+12	429	23628	0.018157	2.47 %	
SA7	301.0	312.7	3	1.04E+12	1161	34023	0.034123	4.64 %	
SA8	264.4	274.7	3	1.04E+12	2066	50199	0.041156	5.60 %	
SA9	229.2	238.1	3	1.04E+12	3916	77061	0.050817	6.91 %	
SA10	158.7	164.9	3	1.04E+12	9224	232139	0.039735	5.41 %	
SA11	134.0	139.2	3	1.04E+12	20474	385624	0.053093	7.22 %	
SA12	110.7	115.0	3	1.04E+12	45249	683968	0.066157	9.00 %	
SA13	89.4	92.9	3	1.04E+12	111517	1298571	0.085877	11.68 %	
SA14	70.3	73.0	3	1.04E+12	277285	2670622	0.103828	14.13 %	
SA15	52.8	54.8	3	1.04E+12	717336	6303426	0.113801	15.48 %	
SA16	37.5	39.0	5	2.29E+15	1933305	25532162	0.075720	10.30 %	
SA17	24.7	25.7	5	2.29E+15	5424303	205948753	0.026338	3.58 %	
SA18	13.8	14.3	5	2.29E+15	15808734	3783105927	0.004179	0.57 %	
SA19	6.2	6.4	5	2.29E+15	48670286	206673829280	0.000235	0.03 %	
SA20	1.0	1.0	5	2.29E+15	104473739	1893406805189770	0.000000	0.00 %	
							D	0.735009	100.00 %
							FATIGUE LIFE	40	years

The accumulated fatigue damage can be found according to the Miner-Palmgren approach and the damage for the load combination SA2 can be given as an example as follows;

The corresponding number of cycles to failure at 514.3 MPa stress range can be found by using the fatigue curve E or the analytical expression;

Since  $t < 22$  mm, the Eq 4.32 can be used;

$$N = A \cdot \left[ S_r \left( \frac{209000}{E} \right) \right]^{-m}$$

$$N_2 = (1.04E + 12) \cdot \left[ 514.3 \left( \frac{209000}{201200} \right) \right]^{-3} = 6821 \text{ cycles}$$

So the fatigue damage can be given as;

$$D_2 = \frac{n_2}{N_2} = \frac{23}{6821} = 0.003372$$

Likewise it can be possible to find accumulated fatigue damage as in the table 5.5.

$$\text{Accumulate total fatigue damage} = D = \sum_{i=1}^{20} \frac{n_i}{N_i} = 0.735009$$

$$\text{Fatigue life} = \frac{L_0}{D} = \frac{30}{0.735009} = 40.82 \approx 40 \text{ years}$$

The CAESAR II input listing for the two models that used for the analysis and the maximum displacement stress ranges can be found in the appendix C and D.

### 5.6.1 Modal analysis

Modal analysis is performed using the CAESAR II to analyse natural frequencies of the piping system and the results are given in the following table 5.6. The lowest natural frequency of the system can be given as 1.688 Hz, which can't be an acceptable frequency. The lowest natural frequency is calculated at the piping loop that has the nodes from 120 to 210, which are towards the ELDF platform. This section of the pipe is unable to provide enough line guides and line stops to increase the natural frequency, due to the high relative displacements. Due to the high relative displacements, even though if there are more guides and stops implemented, this section will be failed due to the high stresses. So this system has designed as it is now as a balanced solution.

When there is unacceptable low natural frequency, then the system has to go through a thorough fatigue assessment and it can be done as in the above section. The estimated total fatigue life for this piping system is 40 years and this can be considered as an acceptable fatigue design when compared with the design life time.

**Table 5.6 Natural frequencies – modal analysis**

Mode	Frequency (Hz)	Frequency (rad/s)	Period (s)
1	1.688	10.604	0.593
2	2.022	12.705	0.495
3	2.961	18.604	0.338
4	7.852	49.334	0.127
5	10.326	64.882	0.097

### 5.7 Fatigue analysis by calculating equivalent stress range

Let's take the same stress distribution as in the above section and find the new equivalent stress range which is constant through the total number of cycles and gives the same damage. The blocks are divided into two groups since the considered fatigue SN curves have two slopes. For the group one it has considered the load combinations from SA1 to SA15 and for the group two from SA16 to SA20. The fatigue assessment can be performed as in the table 5.7.

**Table 5.7 Fatigue life analysis by calculating equivalent stress range – bridge piping**

FATIGUE CALCULATION FOR COMBINED LOADING EVENTS									
LOAD COMB.	STRESS RANGE $\Delta\sigma_i$ (MPa)	$m_i$	$A_i$	$n_i$ (30 years)	$n_{0,i}=\sum n_i$ (30 years)	$\Delta\sigma_i^m n_i$	$\sum \Delta\sigma_i^m n_i$	Equi. Stress $\Delta\sigma_{eq,i}$ (MPa)	$D_i$
SA1	562.6	3	1.04E+12	1	1188934	199597047.4	653678123711.4	81.9	0.628537
SA2	514.3			23		3506972320.7			
SA3	467.7			26		2981469609.4			
SA4	423.5			82		6981183050.0			
SA5	381.1			145		8995799717.4			
SA6	339.9			429		18882770514.5			
SA7	301.0			1161		35488417662.4			
SA8	264.4			2066		42802545618.4			
SA9	229.2			3916		52849569281.0			
SA10	158.7			9224		41324252235.4			
SA11	134.0			20474		55216900317.9			
SA12	110.7			45249		68802905341.7			
SA13	89.4			111517		89311742610.7			
SA14	70.3			277285		107980984029.7			
SA15	52.8			717336		118353014354.8			
SA16	37.5	5	2.29E+15	1933305	176310367	173399667917072.0	243822754773917	16.9	0.106473
SA17	24.7			5424303		60314295029966.2			
SA18	13.8			15808734		9569385991239.6			
SA19	6.2			48670286		539279478819.6			
SA20	1.0			104473739		126356819.7			
D=D <sub>1</sub> +D <sub>2</sub>									0.735009
FATIGUE LIFE									40 years

As an example the equivalent stress range for the group one can be found as follows;

First the equivalent number of cycle  $n_{0,1}$  has to be found;

$$n_{0,1} = \sum_{i=1}^{15} n_i = 1188934$$

The material effect and the thickness effect have to take into consideration for the following calculation. If the wall thickness is greater than 22 mm,  $\Delta\sigma_i$  can be multiplied by  $(t/22)^{1/4}$  and otherwise no need to consider about the thickness effect. And also when consider the material effect,  $\Delta\sigma_i$  can be multiplied by  $(209000/E)$ . Where; t is the wall thickness and E is the elastic modulus of the considered material which is 201200 MPa in this example.

By considering the thickness effect and the material effect, the following Eq 5.4 and Eq 5.5 can be developed by modifying the Eq 4.25;

$$\Delta\sigma_{eq} = \left[ \sum_{i=1}^k \frac{n_i \cdot \left\{ \Delta\sigma_i \cdot \left( \frac{t}{22} \right)^{\frac{1}{4}} \cdot \left( \frac{209000}{E} \right) \right\}^m}{n_0} \right]^{\frac{1}{m}} \quad \Leftarrow \text{for } t > 22 \quad (\text{Eq 5.4})$$

$$\Delta\sigma_{eq} = \left[ \sum_{i=1}^k \frac{n_i \cdot \left\{ \Delta\sigma_i \cdot \left( \frac{209000}{E} \right) \right\}^m}{n_0} \right]^{\frac{1}{m}} \quad \Leftarrow \text{for } t \leq 22 \quad (\text{Eq 5.5})$$

Since the wall thickness is less than 22 mm, only the material effect needs to be considered and the equivalent stress range can be calculated as follows for the group one;

$$\Delta\sigma_{eq,1} = \left[ \sum_{i=1}^{15} \frac{n_i \cdot \left\{ \Delta\sigma_i \left( \frac{209000}{201200} \right) \right\}^{m_1}}{n_{0,1}} \right]^{\frac{1}{m_1}}$$

$$\Delta\sigma_{eq,1} = \left( \frac{653678123711.4}{1188934} \right)^{\frac{1}{3}} \approx 81.9 \text{ MPa}$$

Then the partial damage for the group one can be found by using the Eq 4.24.

$$D_1 = \frac{\Delta\sigma_{eq,1}^{m_1}}{A_1} \cdot n_{0,1}$$

$$D_1 = \frac{(81.9)^3 \cdot 1188934}{1.04 \cdot 10^{12}} \approx 0.628537$$

Once partial damage for the group two found as the same way, the total accumulated damage can be found by summing the partial damages;

$$D = D_1 + D_2 = 0.628537 + 0.106473 = 0.735009$$

The fatigue life can be found as follows;

$$\text{Fatigue life} = \frac{L_0}{D} = \frac{30}{0.735009} = 40.82 \approx 40 \text{ years}$$

## 5.8 Fatigue analysis by assuming Weibull distributed stress range

This analysis is performed by assuming that the stress range is Weibull distributed. So the above stress range is assumed as Weibull distributed and divided into two groups as using the two slopes fatigue curves.

Damage for the group one can be found by using the Eq 4.22 as follows;

$$D_1 = \frac{n_{0,1}}{A_1} \cdot \frac{\Delta\sigma_{0,1}^{m_1}}{(\ln n_{0,1})^{\frac{m_1}{h}}} \cdot \Gamma\left(\frac{m_1}{h} + 1\right)$$

$$\Delta\sigma_{0,1} = 562.6 \text{ MPa}$$

$$n_{0,1} = 1188934$$

$$m_1 = 3$$

$$A_1 = 1.04 \cdot 10^{12}$$

Weibull shape parameter for the stress range distribution assumed as;

$$h = 1.05$$

$$\Gamma\left(\frac{m_1}{h} + 1\right) = \Gamma\left(\frac{3}{1.05} + 1\right) = 5.0291$$

The material effect takes into consideration,  $\Delta\sigma_{0,i}$  can be multiplied by (209000/E).

$$D_1 = \frac{1188934}{1.04 \cdot 10^{12}} \cdot \frac{\left[562.6 \cdot \left(\frac{209000}{201200}\right)\right]^3}{[\ln(1188934)]^{\frac{3}{1.05}}} \cdot (5.0291) \approx 0.611132$$

The partial damage  $D_2$  for the group two can be found as the same way;

$$\Delta\sigma_{0,2} = 37.5 \text{ MPa}$$

$$n_{0,2} = 176310367$$

$$m_2 = 5$$

$$A_2 = 2.29 \cdot 10^{15}$$

$$D_2 = \frac{176310367}{2.29 \cdot 10^{15}} \cdot \frac{\left[37.5 \cdot \left(\frac{209000}{201200}\right)\right]^5}{[\ln(176310367)]^{\frac{5}{1.05}}} \cdot \Gamma\left(\frac{5}{1.05} + 1\right) \approx 0.000453$$

The total accumulated damage will be;

$$D = D_1 + D_2 = 0.611132 + 0.000453 = 0.611585$$

The fatigue life can be calculated as follows for the assumed Weibull shape parameter  $h = 1.05$ ;

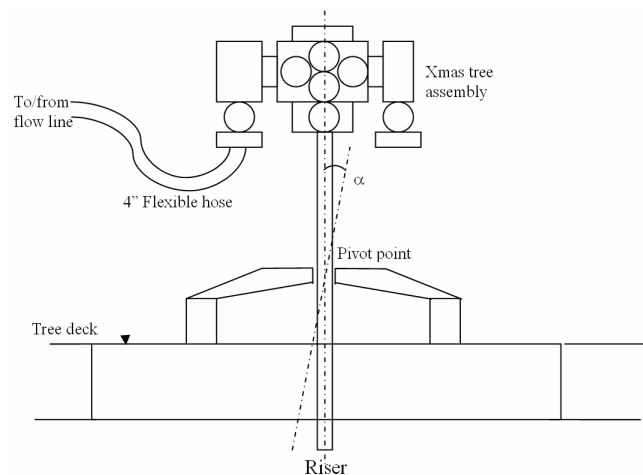
$$\text{Fatigue life} = \frac{L_0}{D} = \frac{30}{0.611585} = 49.05 \approx 49 \text{ years}$$

## 6 Flow line fatigue

Flow lines are designed to transport production hydrocarbon (liquid or gas) from a reservoir (production well) under pressure to process separator systems and as the same way these can be used to inject water or gas to the reservoir from water or gas injection manifolds. These lines are exposed to high cyclic loadings from Xmas tree movements and due to this, flow lines must be subjected to a comprehensive fatigue analysis. In this chapter as a fatigue analysis example, a flow line on the Snorre A field has considered.

The Snorre A platform is a tension leg platform (TLP) that has dry Xmas trees and those are connected to riser on top side of the platform. The riser is exposed to high cyclic wave loadings and because of this, the Xmas tree assembly moves relative to the platform at a pivot point as in the following figure 6.1. Since Snorre A is a TLP, it also moves relative to the wave loadings.

The riser and the Xmas tree assembly have the following configuration;



**Figure 6.1 Riser and Xmas tree assembly**

Even though the platform is moving relative to the waves, for the fatigue analysis it is only considered the movement of the Xmas tree assembly relative to the platform tree deck since the flow lines, production separators and manifolds are on the same platform tree deck. The Xmas tree is connected to the flow line through a 4" flexible hose. The flexible hose has been used to avoid direct transfer high relative displacements, forces and moments from the Xmas tree to the flow line.

### 6.1 System definition

The piping system considered in this chapter is connected to well slot P14. The well slot P14 is in production mode at this moment, but the client wanted to change it to water alternative gas (WAG) mode due to the changes in the well production quantity. For the fatigue analysis only the P14 gas injection flow line considered.

The gas injection flow lines in the Snorre A are belongs to the 2500 pound class rating due to the high pressure in the gas lines. The piping layout design has done using the PDMS design and this consists of 6" piping from the 8" manifold to the Xmas tree and also 2" blow down lines. The 6" flow line has designed using the compact flanges with the client approval. The compact flanges have been used to divide spools into the smaller spools and this has done due to installation difficulties in that area. These flanges can be considered as safe and strong as a weld with respect to the leakage of hydrocarbons and the strength.

The flow line 6"-PV-17DXX3-GC112 is used to transport gas from the 8"-PV-26AXX5-KC3 gas injection manifold to 6"-PV-17DXX2-GD254 converting spool. The converting spool is

designed to change the piping easily from the gas injection mode to the water injection mode when the client needed. From the converting spool to the 4" flexible hose, the line 6"-PV-17DXX1-GD254 is used.

The whole piping layout arrangement of the P14 flow line can be given as the following figure 6.2;

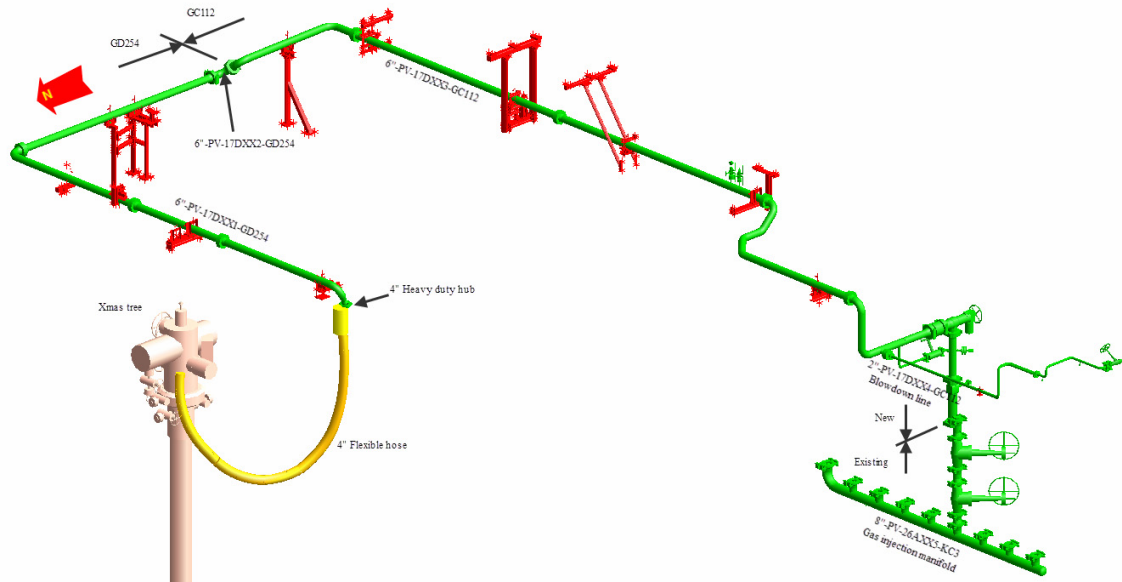


Figure 6.2 Piping and pipe support PDMS 3D layout

## 6.2 Design data

There are two different piping specifications used for the P14 flow line. Those are GC112 which is carbon steel 2500 pound pressure class and GD254 which is 25 Cr duplex (super duplex) 2500 pound pressure class.

The relevant design data can be given as follows;

### Line list data

Line number 1	6"-PV-17DXX1-GD254
Line number 2	6"-PV-17DXX2-GD254
Max design temperature	120 °C
Min design temperature	-46 °C
Design pressure at low/high temp.	370 barg/370 barg
Operating temperature max/min	95 °C/-8 °C
Operating pressure	281 barg
Pipe internal diameter	139.7 mm
Insulation class and thickness	0/0 mm
Heat trace	No
Test pressure	555 barg
Pipe spec	GD254 (Y company)
Pipe material	A790 S32760/A 928 S32760 (25 Cr super duplex)
Pipe outer diameter	168.28 mm
Wall thickness	14.27 mm
Pressure class	2500 lb
Elastic modulus	201200 N/mm <sup>2</sup>
Corrosion allowance	0.0 mm
Mill tolerance	12.5%
Design code	ASME B31.3



Line number 3	6"-PV-17DXX3-GC112
Max design temperature	120 °C
Min design temperature	-46 °C
Design pressure at low/high temp.	370 barg/370 barg
Operating temperature max/min	95 °C/-8 °C
Operating pressure	281 barg
Pipe internal diameter	139.7 mm
Insulation class and thickness	0/0 mm
Heat trace	No
Test pressure	555 barg
Pipe spec	GC112 (Y company)
Pipe material	5L X52 PSL2 S (carbon steel)
Pipe outer diameter	168.28 mm
Wall thickness	25 mm
Pressure class	2500 lb
Elastic modulus	204100 N/mm <sup>2</sup>
Corrosion allowance	3.0 mm
Mill tolerance	12.5%
Design code	ASME B31.3

Joint weights from the flanges, hubs, seal rings, clamps and valves have calculated as follows and those are modelled as rigid items in the CAESAR II;

#### Aker Solutions MMO

##### Calculation Sheet No.

Ref. Stress Sketch No. S1-AA-LAX-XXX1-001

##### Ref. Computer run No.

Calculated by:-

Date. 13.05.12

Checked by:-

DATA PT	LENGTH	FLANGES	SEAL RING	VALVE+ACT	SPECIAL	SPECIAL	CONTENT	TOTAL	NOTES
FROM TO	MM	RATING WEI. KG	WEI. KG	TYPE WEI. KG	TYPE WEI. KG	TYPE WEI. KG	N/M WEI. KG	WEIGHT N WEI. KG	
45 50	236	6"/52 HUB			6"/52 CLAMP			650 66	
60 70	236	6"/52 HUB			6"/52 CLAMP			650 66	
90 100	348	6"/52 HUB		HV143 115	6"/52 CLAMP 63,8			2097 214	
100 110	815	6"/52 HUB		CH762G 290	6"/52 CLAMP 31,9			3330 339	
160 170	240	6"COMPACT						673 69	
230 240	240	6"COMPACT						673 69	
270 280	240	6"COMPACT						673 69	
310 320	240	6"COMPACT						673 69	

360	117	6"/52 HUB			6"/52 CLAMP			<b>487</b>
370		17,1	0,6		31,9			50
370	117	6"/52 HUB						<b>168</b>
380		17,1						17
400	241	6"/54 HUB			6"/54 CLAMP			<b>567</b>
410		25,2	0,71		31,9			58
440	240	6"COMPACT						<b>673</b>
450		68	0,62					69
485	240	6"COMPACT						<b>673</b>
487		68	0,62					69
500	240	6"COMPACT						<b>673</b>
510		68	0,62					69
550	102	H4"/40 HUB			H4"/40 CLAMP			<b>281</b>
560		10	0,51		18,1			29
610	597	2"/14 HUB		BDV142	2"/20 CLAMP			<b>930</b>
620		3	0,36	85	6,4			95
700	273	2"2500#						<b>377</b>
710		38	0,42					38
760	144	2"COMPACT						<b>64</b>
770		6,4	0,08					6

6" TECH LOCK DATA: 6"/52 HUB: 16,9Kg -- 6"/52 CLAMP: 31,9Kg-- 6"/52SEALRING 0,60Kg

6" TECH LOCK DATA: 6"/54 HUB: 12,6Kg -- 6"/54 CLAMP: 31,9Kg -- 6"/54SEALRING 0,71Kg

DIMENSIONS: 6"/56/54/52 HUB: 117mm-- 6"/56/54/52 SEALRING: 7mm -- 6"/56/54/52 BLIND HUB: 73mm  
(GC112/WT25,GD254/120)



### 6.3 Maximum stress range

To find the maximum stress range of the piping system, parts of the existing piping arrangements and their supporting have been included into the CAESAR II model of the new piping to ensure that the correct boundary conditions are considered and modelled.

The client has provided the P14 riser behaviour and the number of cycles on each block to do the fatigue analysis and those data can be found in the appendix E.

Also the maximum end loadings imposed on the flow line by the flexible hose (due to the pressure expansion and displacement) have provided and those can be given as the below table 6.1;

**Table 6.1 Maximum end loads from flexible hose – P14**

Load case	Moment (kNm)	Tension (kN)	Shear (kN)
A (Sustained)	12.5	13.1	8
B (Design)	19.4	18.1	13.5
C (Accidental)	26.8	22.6	15.5

For the fatigue analysis it has only considered the load case A and B and the accidental load case has used to check the other piping code requirements.

Above loads are imposed at the 4" heavy duty hub which is the connection to the flexible hose (figure 6.2). Since the imposed loads are acting out of the global axes, these have corrected to the global axes and can be given as the following table 6.2;

**Table 6.2 Maximum end loads – global axes – P14**

	F1 (A)	F2 (B)	F3 (C)
FX	11352.5703	17464.3398	20820.1895
FY	-9160.4609	-11827.3799	-15227.7197
FZ	4776.5220	8060.3809	9254.5117
MX	-6764.0610	-10497.8193	-14502.1494
MY	-3154.1340	-4895.2158	-6762.4629
MZ	10027.4102	15562.5400	21498.7598

The F1, F2 and F3 forces and the moments have imposed at the node 560 in the CAESAR II model to do the stress analysis.

Once all the design inputs, loads, supports and etc are modelled completely in the CAESAR II, the required load combinations have defined. Those can be given as follows;

**Table 6.3 Load combinations P14 flow line**

Case No	Case description	Load case	Comment
L1	Weight + max design pressure + max design temperature + F1 (A)	W+P1+T1+F1	P1 = 370 barg, T1 = 120 °C
L2	Weight + max design pressure + min design temperature + F1 (A)	W+P1+T2+F1	P1 = 370 barg, T1 = -46 °C
L3	Weight + max design pressure + max design temperature + F2 (B)	W+P1+T1+F2	P1 = 370 barg, T1 = 120 °C
L4	Weight + max design pressure + max design temperature + F3 (C)	W+P1+T1+F3	P1 = 370 barg, T1 = 120 °C
L5	Maximum fatigue stress range F2 (B) – F1 (A)	L3-L1	Maximum fatigue stress range

Above load combinations have performed and the maximum stress range for the fatigue analysis calculated as 82.3 MPa at the node 540 which is the super duplex 6” pipe.

The CAESAR II input listing and the maximum stress range results can be found in the appendix F.

## 6.4 Fatigue analysis based on PD5500

The weld class F2 has been used for the fatigue analysis as a conservative measure. The corresponding constants for the SN curve can be given as follows;

**Table 6.4 Weld class F2 details**

N<10 <sup>7</sup> /S <sub>r</sub> >35		N>10 <sup>7</sup> /S <sub>r</sub> <35		Stress range at 10 <sup>7</sup> N/mm <sup>2</sup>
m	A	m	A	
3	4.31x10 <sup>11</sup>	5	5.25x10 <sup>14</sup>	35

The maximum stress range found above can be considered as a stress due to the flexible hose expansion. But by considering the envelop movement of the Xmas tree in the Snorre A, there has been a discussion and argument about the total maximum stress range and decided to multiply the above found stress range by two as a conservative measure. So the total calculated maximum stress range for the fatigue analysis can be given as 82.3x2 = 164.6 MPa. This has distributed along the blocks linearly as the below table 6.5;

The number of cycles to failure can be calculated at the each stress range and then the partial damage on each block can be found as in the following table 6.6;

Table 6.5 Stress distribution – P14

INDIVIDUAL LOADING EVENTS					
NO	WAVE HEIGHTS (m)	NO OF CYCLES $n_i$	LOAD COMB.	STRESS RANGE (MPa)	COMMENT (Max stress x stroke centre)
1	0.5	3826423	SA1	4.1	164.6x0.025
2	1.5	675935	SA2	12.3	164.6x0.075
3	2.5	165038	SA3	20.6	164.6x0.125
4	3.5	52568	SA4	28.8	164.6x0.175
5	4.5	19472	SA5	37.0	164.6x0.225
6	5.5	7942	SA6	45.3	164.6x0.275
7	6.5	3458	SA7	53.5	164.6x0.325
8	7.5	1583	SA8	61.7	164.5x0.375
9	8.5	754	SA9	70.0	164.5x0.425
10	9.5	371	SA10	78.2	164.5x0.475
11	10.5	187	SA11	86.4	164.5x0.525
12	11.5	97	SA12	94.6	164.5x0.575
13	12.5	51	SA13	102.9	164.5x0.625
14	13.5	28	SA14	111.1	164.5x0.675
15	14.5	15	SA15	119.3	164.5x0.725
16	15.5	9	SA16	127.6	164.5x0.775
17	16.5	5	SA17	135.8	160.5x0.825
18	17.5	3	SA18	144.0	160.5x0.875
19	18.5	2	SA19	152.3	160.5x0.925
20	19.5	1	SA20	160.5	160.5x0.975

Max stress range 164.6

Table 6.6 Fatigue life analysis PD5500 – P14

FATIGUE CALCULATION FOR COMBINED LOADING EVENTS									
LOAD COMB.	STRESS RANGE CAESAR	ADJUSTED STRESS RANGE	m	A	$n_i$ (1 years)	$N_i$	Damage $D_i=(n_i/N_i)$	Damage percentage %	
SA1	4.1	4.3	5	5.25E+14	3826423	367890486274	0.000010	0.08 %	
SA2	12.3	12.8	5	5.25E+14	675935	1513952618	0.000446	3.38 %	
SA3	20.6	21.4	5	5.25E+14	165038	117724956	0.001402	10.62 %	
SA4	28.8	29.9	5	5.25E+14	52568	21889123	0.002402	18.20 %	
SA5	37.0	38.5	3	4.31E+11	19472	7569815	0.002572	19.49 %	
SA6	45.3	47.0	3	4.31E+11	7942	4146052	0.001916	14.51 %	
SA7	53.5	55.6	3	4.31E+11	3458	2511787	0.001377	10.43 %	
SA8	61.7	64.1	3	4.31E+11	1583	1635080	0.000968	7.34 %	
SA9	70.0	72.7	3	4.31E+11	754	1123223	0.000671	5.09 %	
SA10	78.2	81.2	3	4.31E+11	371	804548	0.000461	3.49 %	
SA11	86.4	89.8	3	4.31E+11	187	595875	0.000314	2.38 %	
SA12	94.6	98.3	3	4.31E+11	97	453554	0.000214	1.62 %	
SA13	102.9	106.9	3	4.31E+11	51	353177	0.000144	1.09 %	
SA14	111.1	115.4	3	4.31E+11	28	280364	0.000100	0.76 %	
SA15	119.3	124.0	3	4.31E+11	15	226266	0.000066	0.50 %	
SA16	127.6	132.5	3	4.31E+11	9	185237	0.000049	0.37 %	
SA17	135.8	141.1	3	4.31E+11	5	153557	0.000033	0.25 %	
SA18	144.0	149.6	3	4.31E+11	3	128709	0.000023	0.18 %	
SA19	152.3	158.2	3	4.31E+11	2	108945	0.000018	0.14 %	
SA20	160.5	166.7	3	4.31E+11	1	93029	0.000011	0.08 %	
							$D=\sum D_i$	0.013197	100.00 %
							FATIGUE LIFE	75	Years

When estimating the fatigue life, the material effect has considered but not the thickness effect since the 6” super duplex pipe wall thickness 14.27 mm is less than 22 mm. Using the above table 6.6, the fatigue life estimation can be found as follows based on the PD5500 specification;

$$D = \sum_{i=1}^{20} \frac{n_i}{N_i} = \sum_{i=1}^{20} D_i = 0.013197$$

$$\text{Fatigue life} = \frac{L_0}{D} = \frac{1}{0.013197} = 75.77 \approx 75 \text{ years}$$

The modal analysis has performed using the CAESAR II and the lowest natural frequency calculated as 5.52 Hz, which is an acceptable frequency.

### 6.5 Fatigue analysis by calculating equivalent stress range

Two groups of blocks have defined such as group one from SA1 to SA4 and group two from SA5 to SA20. After finding the equivalent stress range which is constant through the total number of cycles and gives the same damage, then the partial damage can be found on each group of blocks.

The fatigue analysis can be approached as the below table 6.7;

**Table 6.7 Fatigue analysis by calculating equivalent stress range – P14**

FATIGUE CALCULATION FOR COMBINED LOADING EVENTS								
LOAD COMB.	STRESS RANGE ( $\Delta\sigma_i$ ) (MPa)	$m_i$	$A_i$	$n_i$ (1 year)	$n_{0,i} = \sum n_i$ (1 year)	$\Delta\sigma_i^m n_i$	Equivalent Stress ( $\Delta\sigma_{eq,i}$ ) (MPa)	$D_i$
SA1	4.1	5	5.25E+14	3826423	4719964	5460516512.3	13.7	0.004260
SA2	12.3			675935		234396949207.4		
SA3	20.6			165038		735994756190.8		
SA4	28.8			52568		1260818000753.6		
SA5	37.0	3	4.31E+11	19472	33978	1108670615.5	48.4	0.008937
SA6	45.3			7942		825605136.1		
SA7	53.5			3458		593361677.3		
SA8	61.7			1583		417271900.5		
SA9	70.0			754		289322732.3		
SA10	78.2			371		198746349.4		
SA11	86.4			187		135258304.3		
SA12	94.6			97		92176388.9		
SA13	102.9			51		62237859.8		
SA14	111.1			28		43044113.9		
SA15	119.3			15		28572596.0		
SA16	127.6			9		20940740.7		
SA17	135.8			5		14033832.0		
SA18	144.0			3		10045922.8		
SA19	152.3			2		7912243.0		
SA20	160.5			1		4632957.0		
							$D=D_1+D_2$	0.013197
							FATIGUE LIFE	75 years

The total damage and the fatigue life estimation can be calculated as follows in accordance with the above table 6.7;

First the equivalent number of cycle  $n_{0,1}$  can be found;

$$n_{0,1} = \sum_{i=1}^4 n_i = 4719964$$

Since the wall thickness is less than 22 mm, only the material effect needs to be considered and the equivalent stress range can be calculated by using the Eq 5.5 as follows for the group one;

$$\Delta\sigma_{eq,1} = \left[ \sum_{i=1}^4 \frac{n_i \cdot \left\{ \Delta\sigma_i \left( \frac{209000}{201200} \right) \right\}^{m_1}}{n_{0,1}} \right]^{\frac{1}{m_1}}$$

$$\Delta\sigma_{eq,1} = \left( \frac{2236670222664.1}{4719964} \right)^{\frac{1}{5}} \approx 13.7 \text{ MPa}$$

Then the partial damage for the group one can be found by using the Eq 4.24.

$$D_1 = \frac{\Delta\sigma_{eq,1}^{m_1}}{A_1} \cdot n_{0,1}$$

$$D_1 = \frac{(13.7)^5 \cdot 4719964}{5.25 \cdot 10^{14}} \approx 0.004260$$

Once partial damage for the group two found as the same way, the total accumulated damage can be found by summing the partial damages;

$$D = D_1 + D_2 = 0.004260 + 0.008937 = 0.013197$$

$$\text{Fatigue life} = \frac{L_0}{D} = \frac{1}{0.013197} = 75.77 \approx 75 \text{ years}$$

## 6.6 Fatigue analysis by assuming Weibull distributed stress range

The damage for the group one which consists of load combinations from SA1 to SA4 can be found by using the Eq 4.22 and assuming a Weibull distributed stress range as follows;

$$D_1 = \frac{n_{0,1}}{A_1} \cdot \frac{\Delta\sigma_{0,1}^{m_1}}{(\ln n_{0,1})^{\frac{m_1}{h}}} \cdot \Gamma\left(\frac{m_1}{h} + 1\right)$$

$$\Delta\sigma_{0,1} = 28.8 \text{ MPa}$$

$$n_{0,1} = 4719964$$

$$m_1 = 5$$

$$A_1 = 5.25 \cdot 10^{14}$$

Weibull shape parameter for the stress range distribution assumed as;

$$h = 1.1$$

$$\Gamma\left(\frac{m_1}{h} + 1\right) = \Gamma\left(\frac{5}{1.1} + 1\right) = 56.33133$$

The material effect takes into consideration,  $\Delta\sigma_{0,i}$  can be multiplied by (209000/E).

$$D_1 = \frac{4719964}{5.25 \cdot 10^{14}} \cdot \frac{\left[28.8 \cdot \left(\frac{209000}{201200}\right)\right]^5}{[\ln(4719964)]^{\frac{5}{1.1}}} \cdot (56.33133) = 0.000049$$

As the same way the partial damage  $D_2$  for the group two can be found;

$$\Delta\sigma_{0,2} = 160.5 \text{ MPa}$$

$$n_{0,2} = 33978$$

$$m_2 = 3$$

$$A_2 = 4.31 \cdot 10^{11}$$

$$D_2 = \frac{33978}{4.31 \cdot 10^{11}} \cdot \frac{\left[160.5 \cdot \left(\frac{209000}{201200}\right)\right]^3}{[\ln(33978)]^{\frac{3}{1.1}}} \cdot \Gamma\left(\frac{3}{1.1} + 1\right) = 0.002625$$

And the total accumulated damage will be;

$$D = D_1 + D_2 = 0.000049 + 0.002625 = 0.002674$$

The fatigue life can be calculated as follows for the assumed Weibull shape parameter  $h = 1.1$ ;

$$\text{Fatigue life} = \frac{L_0}{D} = \frac{1}{0.002674} = 373.97 \approx 373 \text{ years}$$

## 7 Discussion

Offshore topside piping systems consists of complicated systems which can be critical in fatigue design such as offshore bridge piping, flow lines, piping connected to rotary equipments and etc. Especially in offshore these systems can be more critical than in onshore, since more external loadings are imposed on these. In the above analysed examples, the main challenge and the difficulty is how to design these systems to withstand the high cyclic wave and other loadings. When there are critical systems, those have to go through extensive stress analysis processes. Fatigue analysis is one of the parts of this process.

In this thesis, the fatigue analyses have performed for two examples in three different approaches such as based on the PD5500 specification, by calculating the equivalent stress range and assuming a probability density function of stress range may be represented by a two parameter Weibull distribution. All these three approaches can be discussed as follows;

- PD5500 approach

Actually the PD5500 is a specification for pressure vessels. But the industry has considered the piping systems behave as pressure vessels and transformed the same fatigue analysis approach discussed in the PD5500 to the offshore topside piping systems.

In the bridge piping and the flow line fatigue examples, it has only considered the maximum stress range due to the platform movements and the riser movements. It hasn't considered the stresses due to pressure fluctuations, temperature variations, density fluctuations, slug loadings and etc. By the experience and the expert discussions, it has been identified that the damages imposed due to these stresses can be minimum compared to the fatigue damages due to the external loadings.

This can be possible to explain as these stresses and the corresponding number of cycles may not match to have greater damages. As an example, when the stress range is below 30 MPa for the weld class E, the number of cycles to failure becomes higher than  $10^8$  cycles. So even though the number of cycles at 30 MPa is  $10^5$  per year, the damage at this stress range is 0.001 which may be even less than 1% of the total damage. As the same way this can be explained for the higher stress ranges. Therefore the maximum stress range and the corresponding number of cycles both have to be a significant value to have critical fatigue damage.

But to have an accurate fatigue life estimation it may be better to have load combinations by considering the temperature variations, pressure fluctuations, density fluctuations and etc as in the PD5500 simplified fatigue analysis. However with or without considering these, shouldn't give much difference to the fatigue life estimation.

When considered about the above analysed two examples, those clearly shown that more than 80% of the total fatigue damage happens during the wave height region 2 m to 8 m (refer table 5.5 and table 6.6 damage percentages). This can be a common finding for most of the wave affected offshore piping fatigue analyses in the North Sea.

The fatigue estimation checks how long a system can be survived when the cyclic loadings imposed. So this fatigue estimation has to be checked against the design life time of the system. The flow lines are designed normally for 30 years unless otherwise specified by the owner. The P14 flow line example estimated fatigue life based on the PD5500 is 75 years and therefore this can be considered as acceptable fatigue design when compared with the design life time. By considering the same concept for the bridge piping example, the fatigue life estimation 40 years also can be considered as well acceptable design.



- Equivalent stress range approach

This method has derived from the Miner-Palmgren and the SN curve analytical expressions as in the chapter 4.3 and 4.4. But to find the fatigue damage, there need to be used data from the standard SN curves. The SN curves have been developed based on the laboratory tests by the different organisations such as BSI, DNV and etc. So when use these SN curves to this approach, the data need to be adjusted to the curve standards. As an example, when use the PD5500 SN curves, the data have to be adjusted by considering the material and the thickness effects. So the Eq 5.4 and Eq 5.5 have developed to calculate the equivalent stress range by considering the standards of the PD5500 specification.

And also the blocks have to be divided into two groups since the PD5500 has two slopes SN curves. Then the equivalent stress ranges have to be found for the each group and after that, the partial damages have to be calculated. By adding these partial damages together, the total fatigue damage can be found (refer table 5.7 and 6.7).

Even though this approach is proven analytically, this hasn't done using the two slopes SN curves by considering the two groups scenario as in the above analysed examples. This can be considered as same as the PD5500 specification but only difference is instead of calculating the number of cycles to failure for each block, this calculates the equivalent stress range for each group of blocks and then the partial damage on each group. The fatigue life calculations using the PD5500 and the equivalent stress range approaches give the same results for both the examples.

- Assuming Weibull distributed stress range approach

This is a simplified way of calculating the fatigue life of a system. The analytical expression (Eq 4.22) for this approach is derived by assuming the probability density function of the stress range may be represented by a two parameter Weibull distribution. This expression is completely based on full of assumptions. So this may be used with extremely care.

It can be obvious to see that the fatigue life estimation in this approach may not match with the other approaches due to these reasons.

The fatigue estimation results of the two examples can be presented as follows for all the analysed approaches;

**Table 7.1 Fatigue life estimations comparison**

Approach	Bridge piping (years)	Flow line (years)
PD5500	40	75
Equivalent stress range	40	75
Weibull stress range distribution	49	373
	$h=1.05,$ $n_0$ for 30 years	$h=1.1,$ $n_0$ for 1 year

The Weibull shape parameter is a sensitive parameter for the Eq 4.22. That mean when the shape parameter changes, the outcome deviates considerably. So it is advised to assume the shape parameter carefully and choice of this parameter should preferably be based on the results of a detailed analysis.

Other critical parameter is the number of years considered for the total number cycles. It has identified that the fatigue life estimation changes dramatically when the number of years considered changes. This can be explained as follows.

As an example let's consider the P14 flow line;

$$n_{0,1,1year} = 4719964$$

$$D_{1,1year} = 0.000049$$

$$n_{0,2,1year} = 33978$$

$$D_{2,1year} = 0.002625$$

$$L_{1year} = \frac{1}{D_1 + D_2} = \frac{1}{0.002674} = 373 \text{ years}$$

$$n_{0,1,30years} = 4719964 \cdot 30 = 141598920$$

$$D_{1,30years} = 0.000593$$

$$n_{0,2,30years} = 33978 \cdot 30 = 1019340$$

$$D_{2,30years} = 0.036479$$

$$L_{30years} = \frac{30}{D_1 + D_2} = \frac{30}{0.037072} = 809 \text{ years}$$

In the other two approaches when the number of years changed, the fatigue life estimations don't change with respect to that. But in this approach it does change dramatically as in the above explanation. This may be due to the assumptions that made such as the stress cycles are randomly distributed with a probability density function  $f(\Delta\sigma)$  and the probability density function of the stress range may be represented by a two parameter Weibull distribution. To find further about this, there needs to have an extensive investigation about the Eq 4.22 derivation. This can be done as a further study of the thesis.

Due to these complications, this approach should not be used in the industrial fatigue analyses unless otherwise have good understandings about the analytical expression and the assumptions.

Generally when talk about the fatigue analysis of a piping system, another important measure to consider is the lowest natural frequencies of the system. This can be done by performing a modal analysis. It is desirable and a common practice to keep the piping system's natural frequency above 4 Hz to mitigate fatigue induced failures by low frequencies of vibrations. But sometimes it can be hard to achieve this requirement, especially for the piping systems that has large expansion loops. As in the analysed bridge piping example, the lowest natural frequency calculated as 1.688 Hz and this is due to the large expansion loop that used to accommodate the high relative displacements of the platforms. Since there are not enough guides in the piping expansion loop, it is hard to have a higher natural frequency for this system. When there is a system like this, there needs to do extensive fatigue analysis and if the fatigue life estimation is satisfied compared to the designed life time, this system can be considered as acceptable design. So the analysed bridge piping example can be considered as acceptable design due to the 41 years of estimated fatigue life.

## 8 Conclusions

The fatigue analysis can be performed in various approaches and methods. But for the offshore piping systems and high cyclic stresses, few of these are used and most widely the PD5500 specification has been used in the today's industry.

In this thesis for the fatigue analysis, it is used the PD5500 specification as a reference to compare with the other approaches. The Miner-Palmgren summation and the SN data have been used in this approach. Since the PD5500 SN curves are developed by the laboratory testing of certain specific material ( $E = 209000$  MPa) and thickness (22 mm), there need to do adjustments when other than these specifics are used.

The examples analysed in this thesis are clearly shown that the PD5500 specification can be applied to applications much simpler way. When there are block data available or developed including the stress ranges and the number of stress cycles, all that need to calculate is the number of cycles to failure on each block and then the partial damage. This approach clearly shows the partial damage on each block and it gives good understanding about how the fatigue damage accumulates over the stress cycles. The thesis can be concluded that the PD5500 specification is one of the best and much simpler approach in high cyclic fatigue analyses with compared to other approaches analysed.

The equivalent stress range approach is used as an alternative method which also can be used to calculate the accumulated fatigue damage. The only difference from the PD5500 specification is that in this approach it calculates the equivalent stress range on each group of blocks instead of calculates the number of cycles to failure on each block. This approach gives the same fatigue life as the first when use the same two slopes PD5500 SN curves. To get the same results it is required to consider the material effect and the thickness effect since this approach uses the PD5500 SN curves. As a conclusion, this approach can be considered as another alternative to the high cyclic fatigue analysis of offshore piping systems.

The third approach which is assuming a Weibull distributed stress range can be considered as a simplified method. Due to the assumptions, the results calculated on the two examples didn't match with the other two approaches. Other issue in this approach is that when the number of years for the total number of cycles increased, the fatigue life estimation also increased. So it can be concluded that this approach has to be used carefully and can be possibly used only for an initial evaluation of the fatigue life. This approach should not be used for detail fatigue analysis.

More than 80% of the fatigue damage is accumulated by the wave heights from 2 m to 8 m. This can be the nature of the fatigue damages of wave affected offshore piping systems in the North Sea.

The lowest natural frequency above 4 Hz can be a challenge for bridge piping between two offshore platforms. The piping expansion loops implemented to accommodate the high relative displacements, may be an issue in these piping to have natural frequencies above the accepted limit. This is an unavoidable situation for the offshore bridge piping systems.

## 9 References

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# 10 Appendices

## 10.1 Appendix A – Weld classes

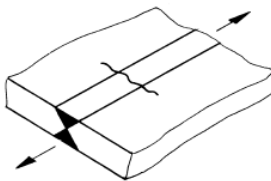
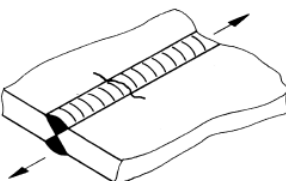
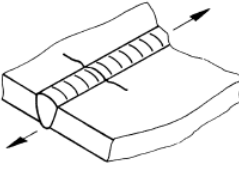

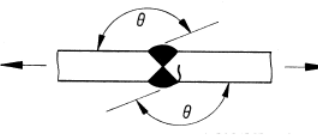

a) Seam welds			
<p>NOTE The highest fatigue strength transversely loaded seams are full penetration butt welds made from both sides or from one side using consumable inserts or a temporary non-fusible backing medium. Then, in the absence of significant defects, the fatigue strength of the joint depends on the overfill shape. In general, the overfill profile requirement for class D should be achieved with shop welds made in the flat position. However, special care may be needed in the case of submerged arc welding since it is known that very poor profiles can be obtained using this process.</p> <p>There is a reduction in the fatigue strength of transverse butt welds if they are made from one side only, unless a joint resembling one made from both sides can be achieved. This is possible using special consumable inserts or a temporary non-fusible backing medium. However, in all cases the weld should be inspected to ensure that full penetration and a satisfactory overfill shape have been achieved on the inside of the joint.</p> <p>As far as seam welds under longitudinal loading are concerned, there is an incentive to avoid the introduction of any discontinuous welds. In the absence of significant defects, their fatigue strengths are only reduced if they contain discontinuous welds.</p>			
Joint type	For stresses acting essentially along the weld		
	Sketch of detail	Class	Comments
Full penetration butt weld flush ground	 <p>Fatigue cracks usually initiate at weld flaws</p>	D	Weld shall be proved free from surface-breaking defects and significant sub-surface defects (see C.3.4.2) by non-destructive testing
Full penetration butt weld made from both sides or from one side on to consumable insert or temporary non-fusible backing		D	Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing
Full penetration butt welds made from one side without backing		D	Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing
Full penetration butt weld flush ground	 <p>Fatigue cracks usually initiate at weld flaws</p>	D	Weld shall be proved free from surface-breaking defects and significant sub-surface defects (see C.3.4.2) by non-destructive testing
Full penetration butt weld made from both sides or from one side on to consumable insert or temporary non-fusible backing		D E	Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing and, for welds made from one side, full penetration Overfill profile $\theta \geq 150^\circ$ Overfill profile $\theta < 150^\circ$
Full penetration butt welds made from one side without backing			Not recommended for fatigue loaded joints since fatigue life critically dependent on root condition. If full penetration can be assured, then class E. Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing

Figure 10.1 Relevant weld classes for piping fatigue [11]

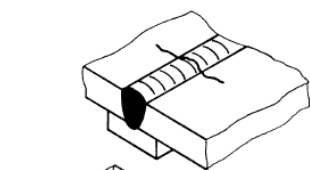
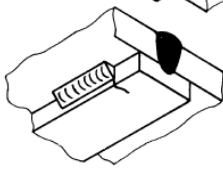
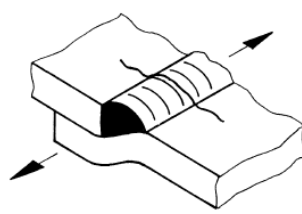
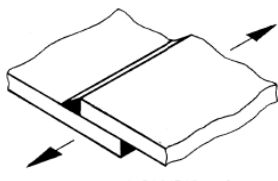
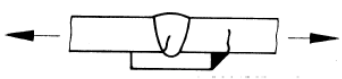
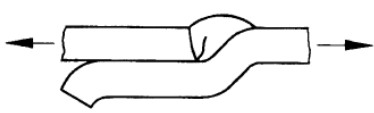
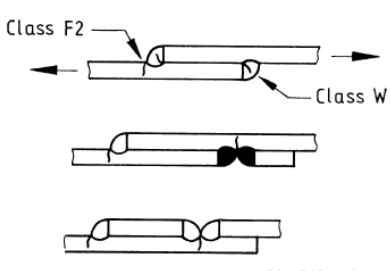
a) Seam welds (continued)			
Joint type	For stresses acting essentially along the weld		
	Sketch of detail	Class	Comments
Full penetration butt welds made from one side on to permanent backing		D	Backing strip shall be continuous and, if attached by welding, tack welds shall be ground out or buried in main butt weld, or continuous fillet welds shall be used
		E	Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing Backing strip attached with discontinuous fillet weld
		D	Joggle joint Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing
Fillet welded lap joint		D	Welds shall be continuous Based on stress range on cross-section of weld
a) Seam welds (continued)			
Joint type	For stresses acting essentially normal to the weld		
	Sketch of detail	Class	Comments
Full penetration butt welds made from one side on to permanent backing		F	Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing
		F	Weld shall be proved free from significant defects (see C.3.4.2) by non-destructive testing
Fillet welded lap joint		F2 W	Refers to fatigue failure in shell from weld toe Refers to fatigue failure in weld; based on stress range in weld throat

Figure 10.2 Relevant weld classes for piping fatigue cont... [11]

## 10.2 Appendix B – Fatigue waves

Table 10.1 Fatigue waves scaled to 30 years (without any extrapolation to rare events)

MID	MEDIAN	FREQUENCY								
		H(m)	T(s)	OMNI	N	NE	E	SE	S	SW
0.5	4.1	104473739	21586222	2549753	5098050	7721264	9110758	15234540	16342079	26831073
1.5	6.1	47601236	10056188	118782	2374985	3597039	4244351	7097185	7613144	12499562
2.5	7.2	15808734	3266379	385823	771426	1168364	1378620	2305257	2472847	4060018
3.5	7.9	5424303	1120762	132384	264692	400890	473033	790981	848485	1393076
4.5	8.5	1933305	399457	47184	94340	142883	168596	281918	302413	496514
5.5	9.0	717336	148215	17507	35004	53016	62556	104603	112208	184227
6.5	9.4	277285	57292	6767	13531	20493	24181	40434	43374	71213
7.5	9.8	111517	23041	2722	5442	8242	9725	16261	17444	28640
8.5	10.2	45249	9350	1104	2208	3344	3946	6598	7078	11621
9.5	10.5	20474	4230	500	999	1513	1785	2986	3203	5258
10.5	10.9	9224	1906	225	450	682	804	1345	1443	2369
11.5	11.2	3916	809	96	191	289	341	571	613	1006
12.5	11.5	2066	427	50	101	153	180	301	323	531
13.5	11.8	1161	240	28	57	86	101	169	182	298
14.5	12.0	429	89	10	21	32	37	63	67	110
15.5	12.3	145	30	4	7	11	13	21	22	37
16.5	12.6	82	17	2	4	6	7	12	13	21
17.5	12.8	26	5	1	1	2	2	4	4	7
18.5	13.1	23	5	1	1	2	2	3	3	6
19.5	13.3	1	1	0	0	0	0	0	0	0
<b>SUM</b>		<b>176430251</b>	<b>36674665</b>	<b>3262943</b>	<b>8661510</b>	<b>13118311</b>	<b>15479038</b>	<b>25883252</b>	<b>27764945</b>	<b>45585587</b>

### 10.3 Appendix C – Maximum stress ranges and CAESAR II input listing – ELDFELDA01 model

CAESAR II Ver.5.10.03, (Build 090206) Date: MAY 16, 2012 Time: 21:37  
 Job: H:\PERSONAL\THESIS\CAESAR\BRIDGE\ELDFELDA01  
 Licensed To: AKER OFFSHORE PARTNER -- ID #503579

Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 13 (EXP) L13=L11+L12

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 25.5 @Node 150  
 Code Stress: 158.7 Allowable: 622.4  
 Axial Stress: 1.5 @Node 179  
 Bending Stress: 158.7 @Node 150  
 Torsion Stress: 29.5 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 160.1 @Node 150

Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 16 (EXP) L16=L14+L15

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 21.5 @Node 150  
 Code Stress: 134.0 Allowable: 622.4  
 Axial Stress: 1.3 @Node 179  
 Bending Stress: 134.0 @Node 150  
 Torsion Stress: 24.9 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 135.1 @Node 150

Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 19 (EXP) L19=L17+L18

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 17.8 @Node 150  
 Code Stress: 110.7 Allowable: 622.4  
 Axial Stress: 1.0 @Node 179  
 Bending Stress: 110.7 @Node 150  
 Torsion Stress: 20.6 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 111.7 @Node 150

Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 22 (EXP) L22=L20+L21

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 14.4 @Node 150  
 Code Stress: 89.4 Allowable: 622.4  
 Axial Stress: 0.8 @Node 179  
 Bending Stress: 89.3 @Node 150  
 Torsion Stress: 16.6 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 90.1 @Node 150



INPUT LISTING

From 10 31.6391954 To 20 DX= 768.786 mm. DZ= 473.685 mm.  
 PIPE  
 Dia= 406.400 mm. Wall= 6.350 mm. Insul= .000 mm. Cor= .0000 mm.  
 GENERAL  
 T1= 80 C T2= -46 C P1= 35.5000 bars PHyd= 77.5500 bars  
 Mat= (400)B861 3 E= 106,866 N./sq.mm. EH1= 104,081 N./sq.mm.  
 EH2= 106,866 N./sq.mm. EH3= 106,866 N./sq.mm. EH4= 106,866 N./sq.mm.  
 EH5= 106,866 N./sq.mm. EH6= 106,866 N./sq.mm. EH7= 106,866 N./sq.mm.  
 EH8= 106,866 N./sq.mm. EH9= 106,866 N./sq.mm. v = .300  
 Density= 4.8440 kg./cu.dm. Insul= .0000 kg./cu.dm.  
 Fluid= 1.0000000 kg./cu.dm.

RESTRAINTS

Node 20 Y  
 Node 20 Z Dir Vec= -.5246 .0000 .8514

UNIFORM LOAD

UX1= .00 N./mm. UY1= .00 N./mm. UZ1= .00 N./mm. UX2= .00 N./mm.  
 UY2= -.22 N./mm. UZ2= .00 N./mm. UX3= .00 N./mm. UY3= -.51 N./mm.  
 UZ3= .00 N./mm.

WIND

Wind Shape= .650

ALLOWABLE STRESSES

B31.3 (2006)

-----  
 From 20 To 30 DX= 4,769.365 mm. DZ= 2,938.632 mm.

RESTRAINTS

Node 30 X Cnode 31 Dir Vec= .8514 .0000 .5246  
 Node 30 Y Cnode 31  
 Node 30 Z Cnode 31 Dir Vec= -.5246 .0000 .8514

DISPLACEMENTS

Node 31 DX1= -153.246 mm. DY1= .000 mm. DZ1= -94.422 mm. RX1= .000  
 RY1= .000 RZ1= .000 DX2= 131.111 mm. DY2= .000 mm. DZ2= 80.784 mm.  
 RX2= .000 RY2= .000 RZ2= .000 DX3= -129.408 mm. DY3= .000 mm.  
 DZ3= -79.734 mm. RX3= .000 RY3= .000 RZ3= .000 DX4= 110.678 mm.  
 DY4= .000 mm. DZ4= 68.194 mm. RX4= .000 RY4= .000 RZ4= .000  
 DX5= -107.272 mm. DY5= .000 mm. DZ5= -66.096 mm. DX6= 91.096 mm.  
 DY6= .000 mm. DZ6= 56.129 mm. DX7= -85.988 mm. DY7= .000 mm.  
 DZ7= -52.981 mm. DX8= 74.069 mm. DY8= .000 mm. DZ8= 45.637 mm.

-----  
 From 30 To 40 DX= 5,189.091 mm. DZ= 3,197.245 mm.

RESTRAINTS

Node 40 Y  
 Node 40 Z Dir Vec= -.5246 .0000 .8514

-----  
 From 40 To 50 DX= 5,193.347 mm. DZ= 3,199.867 mm.

RESTRAINTS

Node 50 Y  
 Node 50 Z Dir Vec= -.5246 .0000 .8514

-----  
 From 50 To 60 DX= 5,175.468 mm. DZ= 3,188.852 mm.

RESTRAINTS

Node 60 Y  
 Node 60 Z Dir Vec= -.5246 .0000 .8514

-----  
 From 60 To 70 DX= 4,618.673 mm. DZ= 2,845.784 mm.

RESTRAINTS

Node 70 Y  
 Node 70 Z Dir Vec= -.5246 .0000 .8514

```

From 70 To 80 DX= 5,756.953 mm. DZ= 3,547.132 mm.
RESTRAINTS
Node 80 Y
Node 80 Z Dir Vec= -.5246 .0000 .8514
-----
From 80 To 90 DX= 5,183.981 mm. DZ= 3,194.097 mm.
RESTRAINTS
Node 90 Y
Node 90 Z Dir Vec= -.5246 .0000 .8514
-----
From 90 To 100 DX= 5,184.834 mm. DZ= 3,194.623 mm.
RESTRAINTS
Node 100 Y
Node 100 Z Dir Vec= -.5246 .0000 .8514
-----
From 100 To 110 DX= 5,191.645 mm. DZ= 3,198.819 mm.
RESTRAINTS
Node 110 Y
Node 110 Z Dir Vec= -.5246 .0000 .8514
-----
From 110 To 120 DX= 776.448 mm. DZ= 478.406 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 89.988 Angle/Node @1= 44.99 119
Angle/Node @2= .00 118
-----
From 120 To 130 DX= 982.000 mm. DZ= -1,593.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 58.348 Angle/Node @1= 29.17 139
Angle/Node @2= .00 138
-----
From 130 To 140 DX= 2,964.000 mm. DZ= .000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 141
Angle/Node @2= .00 142
-----
From 140 To 150 DY= 5,589.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 149
Angle/Node @2= .00 148
-----
From 150 To 160 DX= .000 mm. DZ= 1,406.000 mm.
RESTRAINTS
Node 160 Y
-----
From 160 To 170 DX= .000 mm. DZ= 5,800.000 mm.
RESTRAINTS
Node 170 Y
Node 170 Z Dir Vec= -.5246 .0000 .8514
-----
From 170 To 180 DX= .000 mm. DZ= 1,692.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 179
Angle/Node @2= .00 178
-----
From 180 To 190 DY= 3,726.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 45.000 Angle/Node @1= 22.50 189
Angle/Node @2= .00 188
-----
From 190 To 200 DX= .000 mm. DY= 608.000 mm. DZ= -608.000 mm.
BEND at "TO" end

```

---

Radius= 609.600 mm. (LONG)    Bend Angle= 90.000    Angle/Node @1= 45.00 199

-----  
From 200 To 210 DX= 3,349.000 mm. DZ= .000 mm.

RESTRAINTS

Node 210 Y

Node 210 Z

-----  
From 210 To 220 DX= 4,802.000 mm. DZ= .000 mm.

RESTRAINTS

Node 220 Y

Node 220 Z

-----  
From 220 To 230 DX= 5,348.000 mm. DZ= .000 mm.

RESTRAINTS

Node 230 Y

Node 230 Z

Node 230 X

-----  
From 230 To 240 DX= 4,722.000 mm. DZ= .000 mm.

RESTRAINTS

Node 240 Y

Node 240 Z

## 10.4 Appendix D – Maximum stress ranges and CAESAR II input listing – ELDFELDA02 model

CAESAR II Ver.5.10.03, (Build 090206) Date: MAY 16, 2012 Time: 21:25  
 Job: H:\PERSONAL\THESIS\CAESAR\BRIDGE\ELDFELDA02  
 Licensed To: AKER OFFSHORE PARTNER -- ID #503579

Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 13 (EXP) L13=L11+L12

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 89.9 @Node 150  
 Code Stress: 562.6 Allowable: 625.9  
 Axial Stress: 3.0 @Node 40  
 Bending Stress: 562.4 @Node 150  
 Torsion Stress: 64.3 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 565.4 @Node 150

-----  
 Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 16 (EXP) L16=L14+L15

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 82.2 @Node 150  
 Code Stress: 514.3 Allowable: 625.9  
 Axial Stress: 2.8 @Node 40  
 Bending Stress: 514.2 @Node 150  
 Torsion Stress: 58.8 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 516.9 @Node 150

-----  
 Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 19 (EXP) L19=L17+L18

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 74.7 @Node 150  
 Code Stress: 467.7 Allowable: 625.9  
 Axial Stress: 2.5 @Node 40  
 Bending Stress: 467.6 @Node 150  
 Torsion Stress: 53.5 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 470.1 @Node 150

-----  
 Piping Code: B31.3 = B31.3 -2006, May 31, 2007

CODE STRESS CHECK PASSED : LOADCASE 22 (EXP) L22=L20+L21

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 67.7 @Node 150  
 Code Stress: 423.5 Allowable: 625.9  
 Axial Stress: 2.3 @Node 40  
 Bending Stress: 423.5 @Node 150  
 Torsion Stress: 48.4 @Node 190  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 425.7 @Node 150

INPUT LISTING

From 10 31.6391954 To 20 DX= 768.786 mm. DZ= 473.685 mm.  
 PIPE  
 Dia= 406.400 mm. Wall= 6.350 mm. Insul= .000 mm. Cor= .0000 mm.  
 GENERAL  
 T1= 4 C T2= -46 C P1= 1.0000 bars PHyd= 77.5500 bars  
 Mat= (400)B861 3 E= 106,866 N./sq.mm. EH1= 106,866 N./sq.mm.  
 EH2= 106,866 N./sq.mm. EH3= 106,866 N./sq.mm. EH4= 106,866 N./sq.mm.  
 EH5= 106,866 N./sq.mm. EH6= 106,866 N./sq.mm. EH7= 106,866 N./sq.mm.  
 EH8= 106,866 N./sq.mm. EH9= 106,866 N./sq.mm. v = .300  
 Density= 4.8440 kg./cu.dm. Insul= .0000 kg./cu.dm.  
 Fluid= .8670000 kg./cu.dm.

RESTRAINTS

Node 20 Y  
 Node 20 Z Dir Vec= -.5246 .0000 .8514

UNIFORM LOAD

UX1= .00 N./mm. UY1= .00 N./mm. UZ1= .00 N./mm. UX2= .00 N./mm.  
 UY2= -.22 N./mm. UZ2= .00 N./mm. UX3= .00 N./mm. UY3= -.51 N./mm.  
 UZ3= .00 N./mm.

WIND

Wind Shape= .650

ALLOWABLE STRESSES

B31.3 (2006)

-----  
 From 20 To 30 DX= 4,769.365 mm. DZ= 2,938.632 mm.

PIPE

Dia= 406.400 mm. Wall= 6.350 mm. Insul= .000 mm.

RESTRAINTS

Node 30 X Cnode 31 Dir Vec= .8514 .0000 .5246  
 Node 30 Y Cnode 31  
 Node 30 Z Cnode 31 Dir Vec= -.5246 .0000 .8514

DISPLACEMENTS

Node 31  
 DX1= -438.455 mm. DY1= .000 mm. DZ1= -270.153 mm. RX1= .000  
 RY1= .000 RZ1= .000 DX2= 374.602 mm. DY2= .000 mm. DZ2= 230.810 mm.  
 RX2= .000 RY2= .000 RZ2= .000 DX3= -400.994 mm. DY3= .000 mm.  
 DZ3= -247.072 mm. RX3= .000 RY3= .000 RZ3= .000 DX4= 342.250 mm.  
 DY4= .000 mm. DZ4= 210.877 mm. RX4= .000 RY4= .000 RZ4= .000  
 DX5= -364.386 mm. DY5= .000 mm. DZ5= -224.515 mm. DX6= 311.601 mm.  
 DY6= .000 mm. DZ6= 191.992 mm. DX7= -330.331 mm. DY7= .000 mm.  
 DZ7= -203.533 mm. DX8= 281.803 mm. DY8= .000 mm. DZ8= 173.632 mm.

-----  
 From 30 To 40 DX= 5,189.091 mm. DZ= 3,197.246 mm.

RESTRAINTS

Node 40 Y  
 Node 40 Z Dir Vec= -.5246 .0000 .8514

-----  
 From 40 To 50 DX= 5,193.347 mm. DZ= 3,199.867 mm.

RESTRAINTS

Node 50 Y  
 Node 50 Z Dir Vec= -.5246 .0000 .8514

-----  
 From 50 To 60 DX= 5,175.468 mm. DZ= 3,188.852 mm.

RESTRAINTS

Node 60 Y  
 Node 60 Z Dir Vec= -.5246 .0000 .8514

-----  
 From 60 To 70 DX= 4,618.673 mm. DZ= 2,845.784 mm.

RESTRAINTS

```

Node 70 Y
Node 70 Z Dir Vec= -.5246 .0000 .8514
-----
From 70 To 80 DX= 5,756.953 mm. DZ= 3,547.132 mm.
RESTRAINTS
Node 80 Y
Node 80 Z Dir Vec= -.5246 .0000 .8514
-----
From 80 To 90 DX= 5,183.981 mm. DZ= 3,194.097 mm.
RESTRAINTS
Node 90 Y
Node 90 Z Dir Vec= -.5246 .0000 .8514
-----
From 90 To 100 DX= 5,184.834 mm. DZ= 3,194.623 mm.
RESTRAINTS
Node 100 Y
Node 100 Z Dir Vec= -.5246 .0000 .8514
-----
From 100 To 110 DX= 5,191.645 mm. DZ= 3,198.819 mm.
RESTRAINTS
Node 110 Y
Node 110 Z Dir Vec= -.5246 .0000 .8514
-----
From 110 To 120 DX= 776.448 mm. DZ= 478.406 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 89.988 Angle/Node @1= 44.99 119
Angle/Node @2= .00 118
-----
From 120 To 130 DX= 982.000 mm. DZ= -1,593.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 58.348 Angle/Node @1= 29.17 139
Angle/Node @2= .00 138
-----
From 130 To 140 DX= 2,964.000 mm. DZ= .000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 141
Angle/Node @2= .00 142
-----
From 140 To 150 DY= 5,589.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 149
Angle/Node @2= .00 148
-----
From 150 To 160 DX= .000 mm. DZ= 1,406.000 mm.
RESTRAINTS
Node 160 Y
-----
From 160 To 170 DX= .000 mm. DZ= 5,800.000 mm.
RESTRAINTS
Node 170 Y
Node 170 Z Dir Vec= -.5246 .0000 .8514
-----
From 170 To 180 DX= .000 mm. DZ= 1,692.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 179
Angle/Node @2= .00 178
-----
From 180 To 190 DY= 3,726.000 mm.
BEND at "TO" end
Radius= 609.600 mm. (LONG) Bend Angle= 45.000 Angle/Node @1= 22.50 189
Angle/Node @2= .00 188

```

---

From 190 To 200 DX= .000 mm. DY= 608.000 mm. DZ= -608.000 mm.  
BEND at "TO" end  
Radius= 609.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 199

---

From 200 To 210 DX= 3,349.000 mm. DZ= .000 mm.  
RESTRAINTS  
Node 210 Y  
Node 210 Z

---

From 210 To 220 DX= 4,802.000 mm. DZ= .000 mm.  
RESTRAINTS  
Node 220 Y  
Node 220 Z

---

From 220 To 230 DX= 5,348.000 mm. DZ= .000 mm.  
RESTRAINTS  
Node 230 Y  
Node 230 Z  
Node 230 X

---

From 230 To 240 DX= 4,722.000 mm. DZ= .000 mm.  
RESTRAINTS  
Node 240 Y  
Node 240 Z

## 10.5 Appendix E – Snorre A wave blocks

Table 10.2 Snorre A wave blocks per year

Wave class no	stroke range		Number of cycles (per year)	Stroke mean	Riser angle
	Class range	Class centre			
1	0.00-0.05	0.025	3826423	-0.01	0.19
2	0.05-0.10	0.075	675935	-0.03	0.58
3	0.10-0.15	0.125	165038	-0.05	0.97
4	0.15-0.20	0.175	52568	-0.07	1.36
5	0.20-0.25	0.225	19472	-0.09	1.75
6	0.25-0.30	0.275	7942	-0.11	2.14
7	0.30-0.35	0.325	3458	-0.13	2.53
8	0.35-0.40	0.375	1583	-0.15	2.92
9	0.40-0.45	0.425	754	-0.17	3.31
10	0.45-0.50	0.475	371	-0.19	3.7
11	0.50-0.55	0.525	187	-0.22	4.09
12	0.55-0.60	0.575	97	-0.24	4.48
13	0.60-0.65	0.625	51	-0.26	4.87
14	0.65-0.70	0.675	28	-0.28	5.26
15	0.70-0.75	0.725	15	-0.3	5.64
16	0.75-0.80	0.775	9	-0.32	6.03
17	0.80-0.85	0.825	5	-0.34	6.42
18	0.85-0.90	0.875	3	-0.36	6.81
19	0.90-0.95	0.925	2	-0.38	7.2
20	0.95-1.00	0.975	1	-0.4	7.59



## 10.6 Appendix F – Maximum stress range results and CAESAR II input listing - P14

CAESAR II Ver.5.10.03, (Build 090206) Date: MAY 17, 2012 Time: 14:53  
 Job: H:\PERSONAL\THESIS\CAESAR\P14\CAESARII\P14\_FLOWLINE  
 Licensed To: AKER OFFSHORE PARTNER -- ID #503579

Piping Code: Multiple Codes  
 B31.3 = B31.3 -2006, May 31, 2007  
 TBK 5-6 = NORWEGIAN TBK 5-6 (1999)

CODE STRESS CHECK PASSED : LOADCASE 6 (OCC) L6=L3-L1

Highest Stresses: (N./sq.mm. )  
 CodeStress Ratio (%): 25.1 @Node 540  
 Code Stress: 82.3 Allowable: 327.5  
 Axial Stress: 1.2 @Node 550  
 Bending Stress: 81.2 @Node 540  
 Torsion Stress: 15.5 @Node 460  
 Hoop Stress: 0.0 @Node 20  
 3D Max Intensity: 82.3 @Node 540

### INPUT LISTING

From 10 To 20 DX= .000 mm. DY= .000 mm. DZ= 400.000 mm.  
 PIPE  
 Dia= 219.075 mm. Wall= 28.000 mm. Insul= .000 mm. Cor= 3.0000 mm.  
 GENERAL  
 T1= 120 C T2= -46 C P1= 370.0000 bars P2= .0000 bars  
 Mat= (331)API-5L X52 E= 204,099 N./sq.mm. EH1= 196,910 N./sq.mm.  
 EH2= 206,181 N./sq.mm. EH3= 204,099 N./sq.mm. EH4= 204,099 N./sq.mm.  
 EH5= 204,099 N./sq.mm. EH6= 204,099 N./sq.mm. EH7= 204,099 N./sq.mm.  
 EH8= 204,099 N./sq.mm. EH9= 204,099 N./sq.mm. v = .292  
 Density= 7.8334 kg./cu.dm. Fluid= 1.0000000 kg./cu.dm.  
 RESTRAINTS  
 Node 10 ANC  
 SIF's & TEE's  
 Node 20 Welding Tee  
 ALLOWABLE STRESSES  
 B31.3 (2006) Sc= 152 N./sq.mm. Sh1= 152 N./sq.mm.  
 Sh2= 152 N./sq.mm. Sh3= 152 N./sq.mm. Sh4= 152 N./sq.mm.  
 Sh5= 152 N./sq.mm. Sh6= 152 N./sq.mm. Sh7= 152 N./sq.mm.  
 Sh8= 152 N./sq.mm. Sh9= 152 N./sq.mm.

-----  
 From 20 To 30 DX= .000 mm. DY= .000 mm. DZ= 2,400.000 mm.  
 GENERAL  
 T1= 120 C  
 RESTRAINTS  
 Node 30 X Dir Vec= 1.0000 .0000 -.0001  
 Node 30 Y Dir Vec= .0000 1.0000 -.0000  
 ALLOWABLE STRESSES  
 B31.3 (2006) Sc= 152 N./sq.mm. Sh1= 152 N./sq.mm.  
 Sh2= 152 N./sq.mm. Sh3= 152 N./sq.mm. Sh4= 152 N./sq.mm.  
 Sh5= 152 N./sq.mm. Sh6= 152 N./sq.mm. Sh7= 152 N./sq.mm.  
 Sh8= 152 N./sq.mm. Sh9= 152 N./sq.mm.

-----  
 From 20 To 40 DX= .000 mm. DY= 178.000 mm. DZ= .000 mm.  
 PIPE  
 Dia= 168.275 mm. Wall= 25.000 mm. Insul= 30.000 mm.

GENERAL

Insul= .1500 kg./cu.dm.

-----  
 From 40 To 45 DX= .000 mm. DY= 2,428.000 mm. DZ= .000 mm.  
 RIGID Weight=19,620.00 N.

-----  
 From 45 To 50 DX= .000 mm. DY= 236.000 mm. DZ= .000 mm.  
 RIGID Weight= 650.00 N.

-----  
 From 50 To 60 DX= .000 mm. DY= 764.000 mm. DZ= .000 mm.

-----  
 From 60 To 70 DX= .000 mm. DY= 236.000 mm. DZ= .000 mm.  
 RIGID Weight= 650.00 N.

-----  
 From 70 To 80 DX= .000 mm. DY= 553.000 mm. DZ= .000 mm.  
 SIF's & TEE's  
 Node 80 Weldolet

-----  
 From 80 To 90 DX= .000 mm. DY= 295.000 mm. DZ= .000 mm.

-----  
 From 90 To 100 DX= .000 mm. DY= 348.000 mm. DZ= .000 mm.  
 RIGID Weight= 2,097.00 N.

-----  
 From 100 To 110 DX= -813.200 mm. DY= .000 mm. DZ= .000 mm.  
 PIPE  
 Dia= 168.275 mm. Wall= 25.000 mm. Insul= .000 mm.  
 RIGID Weight= 3,330.00 N.

-----  
 From 110 To 120 DX= -663.000 mm. DY= .000 mm. DZ= .000 mm.  
 SIF's & TEE's  
 Node 120 Weldolet

-----  
 From 120 To 130 DX= -734.000 mm. DY= .000 mm. DZ= .000 mm.  
 SIF's & TEE's  
 Node 130 Weldolet

-----  
 From 130 To 140 DX= -500.000 mm. DY= .000 mm. DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 228.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 139  
 Angle/Node @2= .00 138

-----  
 From 140 To 150 DX= .000 mm. DY= 1,267.000 mm. DZ= .000 mm.  
 SINGLE FLANGED BEND at "TO" end  
 Radius= 228.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 149  
 Angle/Node @2= .00 148

-----  
 From 150 To 160 DX= .000 mm. DY= .000 mm. DZ= -229.000 mm.

-----  
 From 160 To 170 DX= .000 mm. DY= .000 mm. DZ= -240.000 mm.  
 RIGID Weight= 673.00 N.

-----  
 From 170 To 180 DX= .000 mm. DY= .000 mm. DZ= -883.000 mm.  
 RESTRAINTS  
 Node 180 X Dir Vec= 1.0000 .0000 -.0001  
 Node 180 Y Dir Vec= .0000 1.0000 -.0000

-----  
 From 180 To 190 DX= .000 mm. DY= .000 mm. DZ= -2,326.000 mm.  
 BEND at "TO" end  
 Radius= 228.600 mm. (LONG) Bend Angle= 44.995 Angle/Node @1= 22.50 189  
 Angle/Node @2= .00 188

```

From 190 To 200 DX= 594.628 mm.  DY= .000 mm.  DZ= -594.726 mm.
BEND at "TO" end
Radius= 228.600 mm. (LONG)  Bend Angle= 45.005  Angle/Node @1= 22.50 199
Angle/Node @2= .00 198
-----
From 200 To 210 DX= 577.000 mm.  DY= .000 mm.  DZ= .000 mm.
BEND at "TO" end
Radius= 228.600 mm. (LONG)  Bend Angle= 45.000  Angle/Node @1= 22.50 209
Angle/Node @2= .00 208
-----
From 210 To 220 DX= 462.000 mm.  DY= 462.000 mm.  DZ= .000 mm.
SINGLE FLANGED BEND at "TO" end
Radius= 228.600 mm. (LONG)  Bend Angle= 90.000  Angle/Node @1= 45.00 219
Angle/Node @2= .00 218
-----
From 220 To 230 DX= .000 mm.  DY= .000 mm.  DZ= -229.000 mm.
-----
From 230 To 240 DX= .000 mm.  DY= .000 mm.  DZ= -240.000 mm.
RIGID Weight= 673.00 N.
-----
From 240 To 250 DX= .000 mm.  DY= .000 mm.  DZ= -154.000 mm.
RESTRAINTS
Node 250 X Dir Vec= 1.0000 .0000 -.0001
Node 250 Y Dir Vec= .0000 1.0000 -.0000
-----
From 250 To 260 DX= .000 mm.  DY= .000 mm.  DZ= -3,965.000 mm.
RESTRAINTS
Node 260 X Dir Vec= 1.0000 .0000 -.0001
Node 260 Y Dir Vec= .0000 1.0000 -.0000
Node 260 Z Dir Vec= .0001 .0000 1.0000
-----
From 260 To 270 DX= .000 mm.  DY= .000 mm.  DZ= -1,981.000 mm.
-----
From 270 To 280 DX= .000 mm.  DY= .000 mm.  DZ= -240.000 mm.
RIGID Weight= 673.00 N.
-----
From 280 To 290 DX= .000 mm.  DY= .000 mm.  DZ= -1,223.000 mm.
RESTRAINTS
Node 290 X Dir Vec= 1.0000 .0000 -.0001
Node 290 Y Dir Vec= .0000 1.0000 -.0000
-----
From 290 To 300 DX= .000 mm.  DY= .000 mm.  DZ= -4,501.999 mm.
RESTRAINTS
Node 300 Y Dir Vec= .0000 1.0000 -.0000
-----
From 300 To 310 DX= .000 mm.  DY= .000 mm.  DZ= -275.000 mm.
-----
From 310 To 320 DX= .000 mm.  DY= .000 mm.  DZ= -240.000 mm.
RIGID Weight= 673.00 N.
-----
From 320 To 330 DX= .000 mm.  DY= .000 mm.  DZ= -643.000 mm.
BEND at "TO" end
Radius= 228.600 mm. (LONG)  Bend Angle= 90.000  Angle/Node @1= 45.00 329
Angle/Node @2= .00 328
-----
From 330 To 340 DX= -1,305.000 mm.  DY= .000 mm.  DZ= .000 mm.
RESTRAINTS
Node 340 Y Dir Vec= .0000 1.0000 -.0000
-----
From 340 To 350 DX= -1,378.000 mm.  DY= .000 mm.  DZ= .000 mm.
SINGLE FLANGED BEND at "TO" end

```

Radius= 228.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 349  
 Angle/Node @2= .00 348

-----  
 From 350 To 360 DX= .000 mm. DY= -143.141 mm. DZ= -178.750 mm.  
 -----

From 360 To 370 DX= .000 mm. DY= -73.758 mm. DZ= -92.107 mm.  
 RIGID Weight= 293.00 N.

-----  
 From 370 To 380 DX= .000 mm. DY= -73.758 mm. DZ= -92.107 mm.  
 PIPE  
 Dia= 168.275 mm. Wall= 14.275 mm. Insul= .000 mm. Cor= .0000 mm.  
 GENERAL

T1= 100 C PHyd= 555.0000 bars Mat= (400)B861 3 E= 106,866 N./sq.mm.  
 EH1= 103,089 N./sq.mm. EH2= 106,866 N./sq.mm. EH3= 106,866 N./sq.mm.  
 EH4= 106,866 N./sq.mm. EH5= 106,866 N./sq.mm. EH6= 106,866 N./sq.mm.  
 EH7= 106,866 N./sq.mm. EH8= 106,866 N./sq.mm. EH9= 106,866 N./sq.mm.  
 v = .300 Density= 4.8440 kg./cu.dm. Fluid= .0000000 kg./cu.dm.  
 RIGID Weight= 293.00 N.

ALLOWABLE STRESSES  
 B31.3 (2006)

-----  
 From 380 To 390 DX= .000 mm. DY= -143.141 mm. DZ= -178.750 mm.

SINGLE FLANGED BEND at "TO" end  
 Radius= 228.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 389

-----  
 From 390 To 400 DX= -229.000 mm. DY= .000 mm. DZ= .000 mm.  
 -----

From 400 To 410 DX= -236.000 mm. DY= .000 mm. DZ= .000 mm.  
 RIGID Weight= 586.00 N.

-----  
 From 410 To 420 DX= -2,052.000 mm. DY= .000 mm. DZ= .000 mm.  
 RESTRAINTS

Node 420 Z Dir Vec= .0001 .0000 1.0000  
 Node 420 Y Dir Vec= .0000 1.0000 -.0000

-----  
 From 420 To 430 DX= -853.000 mm. DY= .000 mm. DZ= .000 mm.

RESTRAINTS  
 Node 430 X Dir Vec= 1.0000 .0000 -.0001

-----  
 From 430 To 440 DX= -2,530.000 mm. DY= .000 mm. DZ= .000 mm.  
 -----

From 440 To 450 DX= -240.000 mm. DY= .000 mm. DZ= .000 mm.  
 RIGID Weight= 673.00 N.

-----  
 From 450 To 460 DX= -229.000 mm. DY= .000 mm. DZ= .000 mm.  
 SINGLE FLANGED BEND at "TO" end  
 Radius= 228.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 459

-----  
 From 460 To 470 DX= .000 mm. DY= .000 mm. DZ= 1,873.000 mm.

RESTRAINTS  
 Node 470 +Y Dir Vec= .0000 1.0000 -.0000

-----  
 From 470 To 480 DX= .000 mm. DY= .000 mm. DZ= 1,498.000 mm.

RESTRAINTS  
 Node 480 X Dir Vec= 1.0000 .0000 -.0001  
 Node 480 Y Dir Vec= .0000 1.0000 -.0000  
 Node 480 Z Dir Vec= .0001 .0000 1.0000

-----  
 From 480 To 485 DX= .000 mm. DY= .000 mm. DZ= 350.000 mm.  
 -----

From 485 To 487 DX= .000 mm. DY= .000 mm. DZ= 240.000 mm.

RIGID Weight= 673.00 N.

-----  
 From 487 To 490 DX= .000 mm. DY= .000 mm. DZ= 2,274.000 mm.

RESTRAINTS

Node 490 X Dir Vec= 1.0000 .0000 -.0001  
 Node 490 Y Dir Vec= .0000 1.0000 -.0000

-----  
 From 490 To 500 DX= .000 mm. DY= .000 mm. DZ= 1,684.000 mm.

-----  
 From 500 To 510 DX= .000 mm. DY= .000 mm. DZ= 240.000 mm.

RIGID Weight= 673.00 N.

-----  
 From 510 To 520 DX= .000 mm. DY= .000 mm. DZ= 3,140.000 mm.

RESTRAINTS

Node 520 X Dir Vec= 1.0000 .0000 -.0001  
 Node 520 Y Dir Vec= .0000 1.0000 -.0000

-----  
 From 520 To 530 DX= .000 mm. DY= .000 mm. DZ= 504.000 mm.

BEND at "TO" end

Radius= 228.600 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 499  
 Angle/Node @2= .00 498

-----  
 From 530 To 540 DX= 126.648 mm. DY= -270.853 mm. DZ= .000 mm.

-----  
 From 540 To 550 DX= 29.650 mm. DY= -63.410 mm. DZ= .000 mm.

PIPE

Dia= 114.300 mm. Wall= 13.487 mm. Insul= .000 mm.

-----  
 From 550 To 560 DX= 43.204 mm. DY= -92.398 mm. DZ= .000 mm.

RIGID Weight= 179.00 N.

RESTRAINTS

Node 560 ANC Cnode 561

FORCES & MOMENTS

Node 561 FX1= 11,352.57 N. FY1= -9,160.46 N. FZ1= 4,776.52 N.  
 MX1= -6,764.06 N.m. MY1= -3,154.13 N.m. MZ1= 10,027.41 N.m.  
 FX2= 17,464.34 N. FY2= -11,827.38 N. FZ2= 8,060.38 N.  
 MX2= -10,497.82 N.m. MY2= -4,895.22 N.m. MZ2= 15,562.54 N.m.  
 FX3= 20,820.19 N. FY3= -15,227.72 N. FZ3= 9,254.51 N.  
 MX3= -14,502.15 N.m. MY3= -6,762.46 N.m. MZ3= 21,498.76 N.m.

-----  
 From 80 To 600 DX= -120.530 mm. DY= .000 mm. DZ= -120.530 mm.

PIPE

Dia= 60.325 mm. Wall= 12.000 mm. Insul= 30.000 mm. Cor= 3.0000 mm.

GENERAL

T1= 120 C T2= -45 C Mat= (177)A333 6 E= 204,099 N./sq.mm.  
 EH1= 196,910 N./sq.mm. EH2= 206,125 N./sq.mm. EH3= 204,099 N./sq.mm.  
 EH4= 204,099 N./sq.mm. EH5= 204,099 N./sq.mm. EH6= 204,099 N./sq.mm.  
 EH7= 204,099 N./sq.mm. EH8= 204,099 N./sq.mm. EH9= 204,099 N./sq.mm.  
 v = .292 Density= 7.8334 kg./cu.dm.

SINGLE FLANGED BEND at "TO" end

Radius= 76.200 mm. (LONG) Bend Angle= 45.000 Angle/Node @1= 22.50 599  
 Angle/Node @2= .00 598

ALLOWABLE STRESSES

B31.3 (2006) Sc= 138 N./sq.mm. Sh1= 138 N./sq.mm.  
 Sh2= 138 N./sq.mm. Sh3= 138 N./sq.mm. Sh4= 138 N./sq.mm.  
 Sh5= 138 N./sq.mm. Sh6= 138 N./sq.mm. Sh7= 138 N./sq.mm.  
 Sh8= 138 N./sq.mm. Sh9= 138 N./sq.mm. Sy= 241 N./sq.mm.

-----  
 From 600 To 610 DX= -35.000 mm. DY= .000 mm. DZ= .000 mm.

ALLOWABLE STRESSES

B31.3 (2006) Sc= 138 N./sq.mm. Sh1= 138 N./sq.mm.

Sh2= 138 N./sq.mm.    Sh3= 138 N./sq.mm.    Sh4= 138 N./sq.mm.  
 Sh5= 138 N./sq.mm.    Sh6= 138 N./sq.mm.    Sh7= 138 N./sq.mm.  
 Sh8= 138 N./sq.mm.    Sh9= 138 N./sq.mm.    Sy= 241 N./sq.mm.

-----  
 From 610 To 620 DX= -597.000 mm.    DY= .000 mm.    DZ= .000 mm.  
 RIGID Weight= 930.00 N.

-----  
 From 620 To 630 DX= -726.000 mm.    DY= .000 mm.    DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG)    Bend Angle= 90.000    Angle/Node @1= 45.00 629  
 Angle/Node @2= .00 628

-----  
 From 630 To 640 DX= .000 mm.    DY= 524.000 mm.    DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG)    Bend Angle= 44.998    Angle/Node @1= 22.50 639  
 Angle/Node @2= .00 638

-----  
 From 640 To 120 DX= .000 mm.    DY= 121.005 mm.    DZ= 120.995 mm.

-----  
 From 130 To 700 DX= .000 mm.    DY= .000 mm.    DZ= 144.000 mm.  
 GENERAL  
 T1= 100 C    Mat= (331)API-5L X52    E= 204,099 N./sq.mm.  
 EH1= 198,151 N./sq.mm.    EH2= 206,125 N./sq.mm.    EH3= 204,099 N./sq.mm.  
 EH4= 204,099 N./sq.mm.    EH5= 204,099 N./sq.mm.    EH6= 204,099 N./sq.mm.  
 EH7= 204,099 N./sq.mm.    EH8= 204,099 N./sq.mm.    EH9= 204,099 N./sq.mm.  
 v = .292    Density= 7.8334 kg./cu.dm.

ALLOWABLE STRESSES  
 B31.3 (2006)    Sc= 152 N./sq.mm.    Sh1= 152 N./sq.mm.  
 Sh2= 152 N./sq.mm.    Sh3= 152 N./sq.mm.    Sh4= 152 N./sq.mm.  
 Sh5= 152 N./sq.mm.    Sh6= 152 N./sq.mm.    Sh7= 152 N./sq.mm.  
 Sh8= 152 N./sq.mm.    Sh9= 152 N./sq.mm.

-----  
 From 700 To 710 DX= .000 mm.    DY= .000 mm.    DZ= 273.000 mm.  
 PIPE  
 Dia= 60.325 mm.    Wall= 12.000 mm.    Insul= 30.000 mm.  
 RIGID Weight= 377.00 N.

-----  
 From 710 To 720 DX= .000 mm.    DY= .000 mm.    DZ= 2,800.000 mm.  
 RESTRAINTS  
 Node 720 X    Dir Vec= 1.0000    .0000    -.0001  
 Node 720 Y    Dir Vec= .0000    1.0000    -.0000

-----  
 From 720 To 730 DX= .000 mm.    DY= .000 mm.    DZ= 605.000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG)    Bend Angle= 90.000    Angle/Node @1= 45.00 729  
 Angle/Node @2= .00 728

-----  
 From 730 To 740 DX= .000 mm.    DY= 697.000 mm.    DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG)    Bend Angle= 90.000    Angle/Node @1= 45.00 739  
 Angle/Node @2= .00 738

-----  
 From 740 To 750 DX= 278.000 mm.    DY= .000 mm.    DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG)    Bend Angle= 90.000    Angle/Node @1= 45.00 749  
 Angle/Node @2= .00 748

-----  
 From 750 To 760 DX= .000 mm.    DY= .000 mm.    DZ= 805.000 mm.

-----  
 From 760 To 770 DX= .000 mm.    DY= .000 mm.    DZ= 144.000 mm.  
 PIPE

Dia= 60.325 mm. Wall= 11.070 mm. Insul= 30.000 mm. Cor= 3.0000 mm.  
 Mill%(-)=12.50

GENERAL

T1= 120 C T2= -50 C P1= 370.0000 bars P2= .0000 bars  
 Mat= (331)API-5L X52 E= 204,099 N./sq.mm. EH1= 196,910 N./sq.mm.  
 EH2= 206,479 N./sq.mm. EH3= 204,099 N./sq.mm. EH4= 204,099 N./sq.mm.  
 EH5= 204,099 N./sq.mm. EH6= 204,099 N./sq.mm. EH7= 204,099 N./sq.mm.  
 EH8= 204,099 N./sq.mm. EH9= 204,099 N./sq.mm. v = .292  
 Density= 7.8334 kg./cu.dm. Insul= 1.0000 kg./cu.dm.  
 Fluid= .2840000 kg./cu.dm.

RIGID Weight= 64.00 N.

ALLOWABLE STRESSES

NORWEGIAN (1999) Sh1= 185 N./sq.mm. Fac= 460.0000

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 From 770 To 780 DX= .000 mm. DY= .000 mm. DZ= 90.000 mm.

RESTRAINTS

Node 780 X Dir Vec= 1.0000 .0000 -.0001  
 Node 780 Y Dir Vec= .0000 1.0000 -.0000

-----  
 From 780 To 790 DX= .000 mm. DY= .000 mm. DZ= 120.000 mm.

BEND at "TO" end

Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 789  
 Angle/Node @2= .00 788

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 From 790 To 800 DX= .000 mm. DY= 240.000 mm. DZ= .000 mm.

BEND at "TO" end

Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 799  
 Angle/Node @2= .00 798

-----  
 From 800 To 810 DX= 745.000 mm. DY= .000 mm. DZ= .000 mm.

BEND at "TO" end

Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 809  
 Angle/Node @2= .00 808

-----  
 From 810 To 820 DX= .000 mm. DY= .000 mm. DZ= 266.000 mm.

RESTRAINTS

Node 820 +Y Dir Vec= .0000 1.0000 -.0000

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 From 820 To 830 DX= .000 mm. DY= .000 mm. DZ= 560.000 mm.

BEND at "TO" end

Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 829  
 Angle/Node @2= .00 828

-----  
 From 830 To 840 DX= 76.000 mm. DY= .000 mm. DZ= .000 mm.

ALLOWABLE STRESSES

NORWEGIAN (1999)

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 From 840 To 850 DX= 720.000 mm. DY= .000 mm. DZ= .000 mm.

RIGID Weight= 1,348.00 N.

-----  
 From 850 To 860 DX= 76.000 mm. DY= .000 mm. DZ= .000 mm.

PIPE

Dia= 60.325 mm. Wall= 7.140 mm. Insul= 30.000 mm. Cor= .0000 mm.

GENERAL

Mat= (163)A312 TP316 E= 195,825 N./sq.mm. EH1= 188,306 N./sq.mm.  
 EH2= 198,206 N./sq.mm. EH3= 195,825 N./sq.mm. EH4= 195,825 N./sq.mm.  
 EH5= 195,825 N./sq.mm. EH6= 195,825 N./sq.mm. EH7= 195,825 N./sq.mm.  
 EH8= 195,825 N./sq.mm. EH9= 195,825 N./sq.mm. v = .292  
 Density= 8.0272 kg./cu.dm.

BEND at "TO" end

Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 859

RESTRAINTS

Node 859 Y Dir Vec= .0000 1.0000 -.0000  
 Node 859 Z Dir Vec= .0001 .0000 1.0000

ALLOWABLE STRESSES

NORWEGIAN (1999) Sc= 243 N./sq.mm. Sh1= 191 N./sq.mm.  
 Sh2= 243 N./sq.mm. Sh3= 243 N./sq.mm. Sh4= 243 N./sq.mm.  
 Sh5= 243 N./sq.mm. Sh6= 243 N./sq.mm. Sh7= 243 N./sq.mm.  
 Sh8= 243 N./sq.mm. Sh9= 243 N./sq.mm. Sy= 207 N./sq.mm. Fac= 650.0000

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 From 860 To 870 DX= .000 mm. DY= .000 mm. DZ= -152.000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 869

ALLOWABLE STRESSES

NORWEGIAN (1999) Sh1= 191 N./sq.mm. Sh2= 243 N./sq.mm.  
 Sy= 207 N./sq.mm.

-----  
 From 870 To 880 DX= -257.000 mm. DY= .000 mm. DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 879  
 Angle/Node @2= .00 878

ALLOWABLE STRESSES

NORWEGIAN (1999) Sh1= 191 N./sq.mm. Sh2= 243 N./sq.mm.  
 Sy= 207 N./sq.mm.

-----  
 From 880 To 890 DX= .000 mm. DY= .000 mm. DZ= -407.000 mm.

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 From 890 To 900 DX= .000 mm. DY= .000 mm. DZ= -141.000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 899  
 Angle/Node @2= .00 898

-----  
 From 900 To 910 DX= -279.000 mm. DY= .000 mm. DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 909  
 Angle/Node @2= .00 908

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 From 910 To 920 DX= .000 mm. DY= 74.992 mm. DZ= -212.003 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 919  
 Angle/Node @2= .00 918

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 From 920 To 930 DX= 147.000 mm. DY= .000 mm. DZ= .000 mm.

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 From 930 To 940 DX= 338.000 mm. DY= .000 mm. DZ= .000 mm.  
 RIGID Weight= 200.00 N.

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 From 940 To 950 DX= 76.000 mm. DY= .000 mm. DZ= .000 mm.  
 BEND at "TO" end  
 Radius= 76.200 mm. (LONG) Bend Angle= 90.000 Angle/Node @1= 45.00 949

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 From 950 To 960 DX= .000 mm. DY= .000 mm. DZ= -76.000 mm.

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 From 960 To 970 DX= .000 mm. DY= .000 mm. DZ= -587.000 mm.  
 PIPE  
 Dia= 60.325 mm. Wall= 2.800 mm. Insul= 30.000 mm.

GENERAL

T1= 50 C P1= 19.0000 bars  
 RIGID Weight= 940.00 N.

ALLOWABLE STRESSES

NORWEGIAN (1999) Sh1= 243 N./sq.mm. Sh2= 243 N./sq.mm.  
 Sy= 207 N./sq.mm.



-----  
 From 970 To 980 DX= .000 mm. DY= .000 mm. DZ= -51.000 mm.  
 PIPE  
 Dia= 60.325 mm. Wall= 2.800 mm. Insul= 30.000 mm.  
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From 980 To 990 DX= .000 mm. DY= .000 mm. DZ= -225.000 mm.  
 PIPE  
 Dia= 114.300 mm. Wall= 3.000 mm. Insul= 30.000 mm.

RESTRAINTS

Node 990 X Dir Vec= 1.0000 .0000 -.0001  
 Node 990 Y Dir Vec= .0000 1.0000 -.0000  
 Node 990 Z Dir Vec= .0001 .0000 1.0000  
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From 990 To 1000 DX= .000 mm. DY= .000 mm. DZ= -325.000 mm.  
 SIF's & TEE's  
 Node 1000 Welding Tee