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Hole Cleaning and Hydraulics

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Hole Cleaning and Hydraulics

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June 15, 2010.

Preface

The conclusion of this thesis brings to an end my master's program in the University of Stavanger. It leads to an MSc degree in Petroleum Engineering with specialization in Drilling Engineering.

I am highly indebted to my supervisor, Bernt Aadnøy Sigve for giving me the opportunity to work on this beautiful thesis work. His immense support and guidance throughout the thesis period is highly appreciated. Most importantly, I appreciate his open door policy toward me, I was welcomed at any time of the day and days of the week, a knock on the door was all I needed to have a chat with him and he was always happy and willing to give advice and tell me what to do next. Sincerely speaking, I wish this work continues in some sort of way because I have in the past six months become an expert in this part of drilling Engineering that is so important to the Petroleum industry.

I will not end this section without thanking my dearest and family for their support and encouragement.

And finally to all my friends and course mates in Petroleum Engineering, I wish to say a big thank you.

Contents

Nomenclature

A_S	*Surface area* (ft^2)
R_{Lam}	*Eccentric annulus laminar pressure ratio (dimensionless)*
e	*Eccentricity*
n	*Flow behavior index (Herschel-Bulkley fluids)*
d_p	*Pipe outside diameter (ft)*
dh	*Hole diameter or casing inside diameter*
R_{turb}	*Eccentric annulus turbulent pressure ratio*
d	*Diameter (ft)*
σ	*Shear stress* $(lb/100ft^2)$
σ_y	*Yield stress* $(lb/100ft^2)$
n_p	*Flow behavior index (power law fluids)*
y	*Shear rate* (s^{-1})
θ_{300}	*Viscosity reading at 300 Rpm (cp)*
θ_{600}	*Viscosity reading at 600 Rpm (cp)*
τ_y	*Suspension parameter*
K	*Consistency parameter*
K_p	*Consistency factor(power law)*
n_a	*Annular flow behavior index (H.B.F)*
Ka	*Annular consistency parameter*
μ	*Viscosity (cp)*
μ_p	*Plastic viscosity (cp)*
$\overline{V}$	*Average velocity (cp)*
Q	*Flow rate* $(ft^3/\min)$
Q_{Ver}	*Flow rate in vertical sections*
Q_{Dev}	*Flow rate in deviated sections*
d_2	*Outer diameter (ft)*
d_1	*Inner diameter (ft)*

N_{Re}	Renolds number
ρ	Density (lb / gal)
N_{Rec}	Critical Reynolds number
μ_a	Annular viscosity
V_f	Fluid average velocity
V_T	Transport velocity
G_p	Geometric shear-rate correction
α	Angle between centre point
B	Expansibility of conduit
V_{cb}	Critical velocity (Bingham Plastic Fluid)
V_{cp}	Critical velocity (power-law)
V_c	Critical velocity
R	Ratio yield stress
U_{LS}	Liquid superficial velocity
U_{gs}	Gas superficial velocity
Q_L	Liquid flow rate
ε	Fraction (liquid or gas)
u	phase velocity
u_s	Slip velocity
S	Slip ratio
X	gas fraction
C	Layer cutting concentration
U	Velocity of each layer
F_g	Force due to gravity
F_b	Buoyancy force
ρ_p	Particles density
g	Acceleration due to gravity
C_D	Drag Coefficient
C_L	Lift Coefficient
F_D	Drag force
F_L	Lift forces
F_{VAN}	Vander waals forces
A_H	Hamaker constant (N.m)
S	Particle separation distance (m)
Rep	Particle Reynolds number
R	Wellbore radius

U_{roll} *Critical velocity for rolling particle*

U_{lift} *Velocity (critical) for particle lift*

θ *Angle of repose/inclination*

A_T *Optimal nozzle area (in^2)*

P_s *Surface pressure*

P_{max} *Maximum pump pressure*

d(opt) *Optimal nozzle diameter*

Subscripts

b	*Cutting bed in transient segment*
hyd	*Hydraulic diameter*
l	*Liquid*
G	*Gas*
mb	*Moving bed layer*
mbsb	*interface between moving and stationary bed*
S	*Solid particle*
sd	*Solid dispersed fluid flow layer*
sb	*Stationary bed layer*
m	*Mixture*
f	*Fluid*
g	*gravitational*
D	*Drag*
L	*Lift*
c	*Critical*
Re	*Reynolds number*
Ver	*Vertical*
Dev	*Deviated*
a	*Annular*
c	*Cuttings*
i	*Inner*
o	*Outer*

Abbreviations

ECD	*Equivalent circulation density*
API	*American Petroleum Institute*
CCL	*Carrying capacity index*
TR	*Transport Ratio*
ROP	*Rate of penetration*
TI	*Transport index*
RF	*Rheology factor*
AF	*Angle factor*
PV	*Plastic viscosity (cp)*
YP	*Yield point* $(lb/100ft^2)$
Cc	*Cuttings concentration*
v	*Velocity*
A	*Area* (ft^2)
HHP	*Hydraulic horse power*
JIF	*Jet impact force*

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Abstract

Cuttings transport and efficient hole cleaning remains a vital challenge when planning and drilling vertical, deviated, high angle and extended reach wells. Optimization of cuttings transport depends on so many factors- hole angle, cutting size, drill string rotation, drill pipe eccentricity, optimization of bit hydraulics, etc.

As to date, most existing hole cleaning models and charts have been derived for wells with angles greater than 25 degrees, these models use charts developed from the original physical-based model. Hole cleaning in vertical holes have been evaluated by calculating the carrying capacity and most hole cleaning charts do not apply to near vertical and vertical wells.

This thesis will study the different factors that affect the efficient cleaning/transport of cuttings and bit hydraulics during drilling. Furthermore, a model is developed to bridge the gap between existing vertical and deviated models for predicting hole cleaning. A single model is proposed which can be used in predicting hole cleaning in both vertical and deviated wells. hole cleaning charts are also developed to be used in conjunction with existing hole cleaning charts in the field.

Also, the effect of washout is investigated in vertical wells. Corrections factors are proposed for correcting the critical flow rate. Finally, the predicted models are validated using field data and hole cleaning charts are produced.

Chapter 1: Introduction

This thesis studies the parameters affecting hole cleaning and hydraulics during drilling operation and development of new models and hole cleaning charts to predict hole cleaning in both vertical and deviated wells. Indebt study of models that have been put forward by different researchers in the past was conducted and new models derived for effective hole cleaning.

1.1 Background of thesis

As the world's energy demand continues to increase, more extended reach wells, ultra deep and highly deviated wells are being drilled to meet this energy demand. However, one of the biggest challenges in highly deviated and extended reach wells is the problem of hole cleaning during drilling. Poor hole cleaning can result to a number of drilling problems including: stuck pipe, possible hole pack-off, excessive ECD, formation break down and cuttings accumulation

The key to a successful hole cleaning relies upon integrating optimum drilling fluid properties with best drilling practices. In the fields, charts have been developed which can be used to predict hole cleaning in wells with deviation greater than 25 degrees, previously, hole cleaning in vertical and near vertical wells have been predicted by calculating the carrying capacity index (CCL), this thesis proposes new means of predicting hole cleaning in both vertical and highly deviated wells using the proposed model.

1.2 Study Objective

The objective of this work is to express and derive analytical formulations for predicting hole cleaning in both near vertical and highly deviated wells. The main objective is to breech the gap between the different formulas used in predicting hole cleaning in vertical and deviated wells by proposing a model that can be used to predict hole cleaning irrespective of the hole angle. Also, hole cleaning derivations and charts will be presented for the proposed model.

1.3 Report Structure

The theoretical bases for the study are the works by P.A. Bern and Yuejin Luo-[1, 2], M.S.Bizanti-[3] and API recommendation Practice-[4]. The review on bit hydraulics is based on the work of Bernt Aadnøy Sigve. Theoretical works by P.A Bern, Bernt Aadnøy and others are presented in chapter 2 and 3. Chapter 4 presents the derived models for efficient hole cleaning. Chapter 5 presents the verification of the derived models and suggests ways to applying them. Finally, conclusions are given based on the results in the work and the procedures for the derivation of the new models are given in the appendixes.

Chapter 2: Theory

2.1 Basic definitions of well and drilling fluid terminologies

Hole cleaning during drilling is a function of the well type, i.e. vertical, deviated or horizontal and also the type and properties of the drilling fluid. This section will present definition and description of well and drilling fluid terminologies relating to hole cleaning.

Viscosity Defined as the ratio of shear stress to shear rate, viscosity for most drilling fluids is not constant but varies with the shear rate. So we use the term effective viscosity to show that it was measured at a specific shear rate at an existing flow conditions in the wellbore.

Shear Stress This is the force needed to sustain a particular fluid flow rate, it is the ratio of force to the given area.

Shear rate This is the ratio of the velocity to the distance (velocity gradient),it can be seen as the rate at which one layer of fluid is moving past another layer. Shear rate is not constant across the flow channel and is highest next to the pipe wall.

Vertical well Any well with an inclination of 0 degrees, but most oil wells are drilled at an angle. So for the purpose of this work, we shall define a vertical well as one with angle less than 25 degrees.

High angle well This we will look at as the opposite of vertical wells. They are wells in which inclination is greater than 25 degrees.

Mud weight Also known as the fluid density, is the mass per unit volume of the fluid. It is a very important property of the drilling fluid.

Newtonian fluid This is any fluid that exhibits the following behaviors: (a)The only stress generated in simple shear flow is the shear stress, the two normal stress difference being zero, (b) shear viscosity does not vary with shear rate, (c) viscosity is constant.

Non Newtonian fluid Any fluid showing deviation from Newtonian behavior, most drilling fluids fall into this group, i.e. viscosity varies with shear rate.

Critical velocity Velocity at which cutting transport is optimized, where the Reynolds equal the critical Reynolds number.

Critical flow rate This is the flow rate at which the velocity equals the critical velocity.

Flow regime During fluid flow, the flow patterns and friction factors can be characterized by laminar, transitional and turbulent flow regimes.

Slip A term introduced when more than one phase is flowing down hole, i.e. slip velocity: the difference between the gas and liquid velocity.

2.2 Factors affecting hole cleaning

Optimal hole cleaning refers to the efficient removal of drill cuttings during drilling, for this condition to hold, many factors must be in place. To efficiently transport cuttings out of the hole, the transporting medium (drilling fluid) must be able to suspend the solid particles; also, there must be enough energy in the form of motion to push the solids out of the hole. Many researchers have been conducted to identify some of the factors affecting hole cleaning. The basic medium for cutting transport during drilling is via the circulation fluid better known as the drilling fluid. Cutting suspension and transport is one of the most important properties to be considered when selecting the drilling fluid. The movement of the drilling fluid in a vertical well is depicted below:

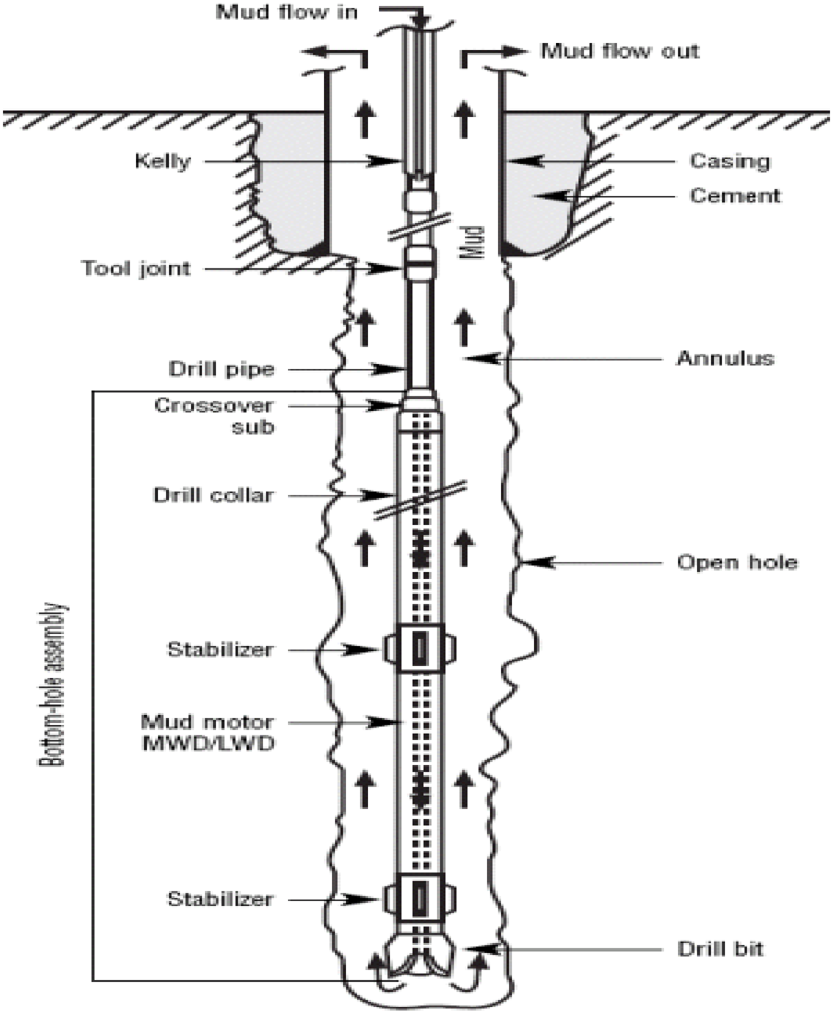

Figure2. 1: Drilling Fluid movement down hole-[E.B Nelson]

There are many factors that affects the ability of the drilling fluid to efficiently transport cuttings to the surface and provide optimal hole cleaning, some of such factors includes:

- Cutting size
- Drill pipe eccentricity
- Cutting density and mud weight
- Hole size and hole angle
- Rheology of circulation fluid
- Drill pipe rotation
- Multi-phase flow effect
- Hole cleaning pills
- Rate of penetration (ROP)
- Cuttings transport ratio
- Cuttings bed properties

These parameters all affect the removal of cuttings from the hole. Studies have been done to rank these parameters in order of importance to hole cleaning during drilling. In 2000, Rishi B, Stefan Miskan and Ergun Kuru came up with the figure below-[Adari, 5].

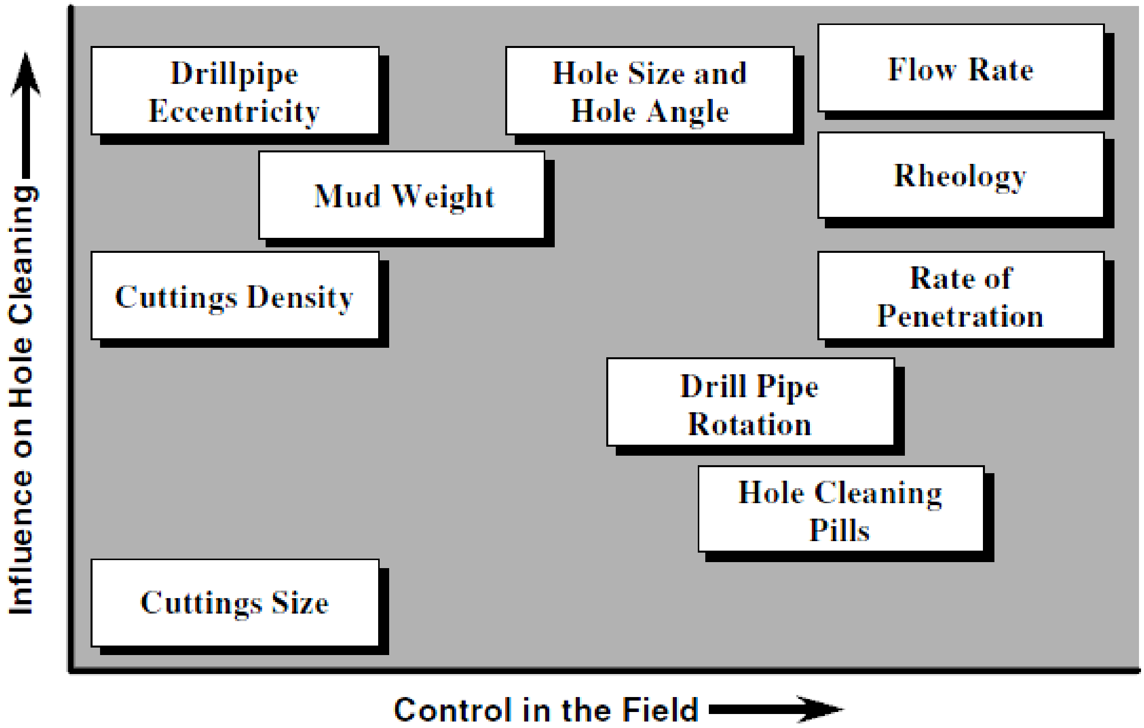

Figure2. 2: Key variables controlling cuttings transport-[Rishi B, etal]

2.2.1 Cutting/Particle size

Cutting characteristics such as shape, size and density are related to their dynamic behavior in a flowing media. The terminal velocity, drag force, buoyant forces and shear forces between cuttings are affected by both the characteristics of the cuttings and the properties of the circulation fluid. The sphericity of a cutting particle is the ration of the surface area of sphere of same volume to the surface area of the particle.

$$Sphericity = \frac{A_s(sphere)}{A_s(particle)} \qquad 2.1$$

The table below shows the sphericity for cutting particles of different shapes.

Table2. 1: Sphericity for different cuttings shapes

Shape	*Sphericity*
Sphere	1.0
Octahedron	0.85
Cube	0.81
Prism	0,77

According to investigation, there is a particle size that poses the most difficulty to clean out with water and from their study; it is of the order of 0.76mm diameter. They also concluded that smaller particles are harder to clean out than larger ones when the particle size is larger than 0.5mm, but for particles smaller than 0.5mm, the smaller particles are easier to clean out. The critical velocity needed to transport different sizes of particles is also dependent on the cutting concentration-[Walter, 6].

2.2.2 Drill Pipe Eccentricity

In vertical wells, it is easier to achieve a well centered drill string, but in deviated and high angle well, the drill string always tend to lie on the low side of the drilled well due to gravity. Experiments by-[Walker, 6] showed that solids are more difficult to be transported when the pipe is located near the bottom side of the hole. When this happens, the velocities in the narrow gaps close to the pipe are very low and this will cause solids to be deposited rapidly. This effect tends to be accentuated if the viscosity increases, as the drag forces on the liquid will reduce the velocity in the narrow gap the more.

An industrial method to estimate eccentricity involves multiplication of concentric-annulus pressure loss in each segment by the empirically derived ratio R_{lam} or R_{turb} depending on the flow regime:

$$R_{lam} = 1,0 - 0,072\frac{e}{n}\left[\frac{d_p}{dh}\right]^{0,8454} - \frac{3}{2}e^{2\sqrt{n}}\left[\frac{d_p}{d_h}\right]^{0,1852} + 0,96e^{3\sqrt{n}}\left[\frac{d_p}{d_h}\right]^{0,2527} \qquad 2.2$$

This equation is used to calculate eccentricity of the drill pipe in lamina flow. For calculation during turbulent flow conditions, we use the following:

$$R_{turb} = 1,0 - 0,048\frac{e}{n}[\frac{d_p}{dh}]^{0,8454} - \frac{2}{3}e^{2\sqrt{n}}[\frac{d_p}{d_h}]^{0,1852} + 0,285e^{3\sqrt{n}}[\frac{d_p}{d_h}]^{0,2527}$$

2. 3

In the diagram below, the effects of pipe eccentricity on fluid movement is shown. Cutting transport is most efficient in zone B where the pipe is well centered, while such is not the case in zones A and C where the pipe lies on one side

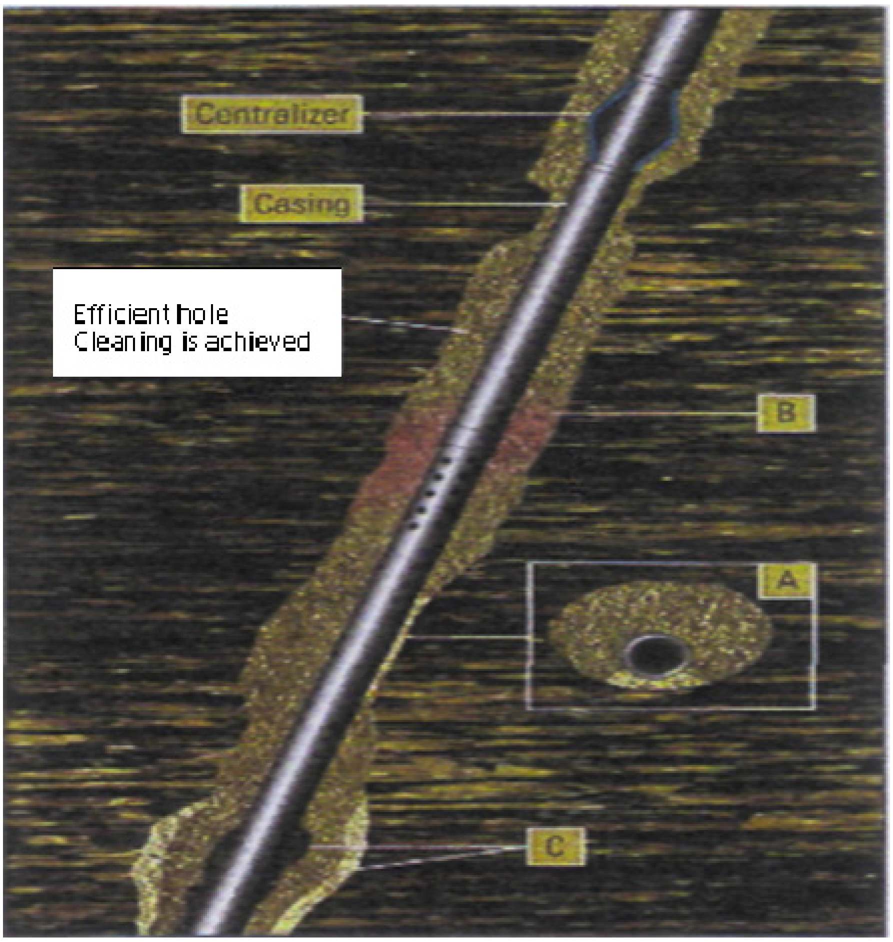

Figure2. 3: Depiction of pipe Eccentricity-[Nelson]

It is important to note that cutting transport is only affected slightly by the position of the pipe in the hole at low angle. As the inclination of the well increases towards the horizontal, the amount of fluid needed for proper hole cleaning increases-[6]. In conclusion, hole cleaning time is affected by the position of the pipe within the well bore. In order to optimize hole cleaning, reliable method to predict pipe eccentricity is needed.

2.2.3 Drill Pipe Rotation

In most cases during drilling using the rotary method, the drill pipe is always in rotation, except during making of connections or tripping.

It has been suggested that cutting transport is made easier in the presence of drill pipe rotation. Semi-consolidated beds can in some cases be removed because the drill string drags a large portion of the bed around from the bottom of the annulus to the top where a high flow rate is, the high flow rate can then disperse the removed cuttings to some degree and good hole cleaning may be achieved. This behavior is particularly a possibility for removing sand beds and other non-reactive cutting particles-[Saasen, 2007].

Figure2. 4: Pipe rotation helps fluid flow in the narrow side of an eccentric annulus-[N.Maroni]

Because of drill pipe rotation, fluid flow between a rotating pipe and the formation or a cased well is seldom stable. Pipe rotation tend to make flow turbulent and this turbulent like motion makes the frictional pressure loss to increase, causing an increased shear stress on the cutting bed surface. This increased shear stress will assist in cutting removal.

Pipe eccentricity is hardly achieved in most wells, in eccentric cases the pressure loss and thereby the ability to remove cuttings is increased because the effect of pipe rotation causes fast flowing fluid from the wide part of the hole down into narrow sections sandwiched between the formation and the drill pipe.

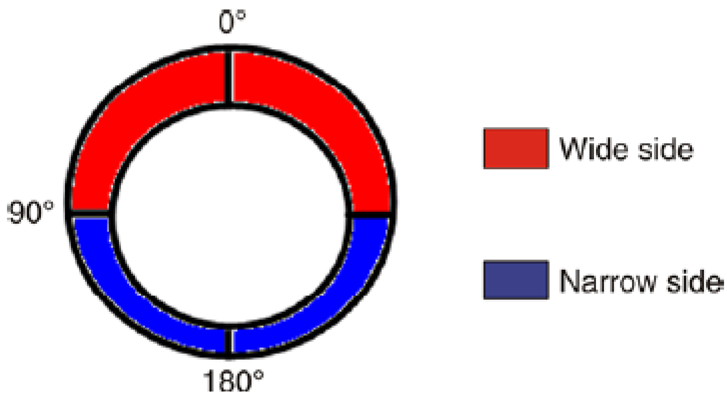

Figure2. 5: Illustration of wide and narrow sides in an eccentric annulus-[Maroni]

The fluids originally flowing through the narrow side is forced to move to the wide areas where the fluid velocity is higher. So the fluid in the narrow side is forced to accelerate. This alternating acceleration and retardation creates an increase in annular pressure losses. It may be okay to assume that the larger the rotation rate, the more turbulence like the motion becomes and the frictional pressure losses increases. Therefore for optimal hole cleaning, it is recommended to use as high drill pipe rotation as possible.

In a steady axial flow condition, the flow in the narrow and the wide remain constant along the length of the well bore.

When the pipe is rotated, it helps keep the cuttings in suspension as shown in 1, but when the pipe rotation is stopped, cuttings begin to form sediments in the narrow side-2, which may eventually lead to the pack-off of the well as shown in 3.

Adel Ali Bassal showed that drill string rotation has a moderate to significant effect on hole cleaning, and that this effect also depends on the hole angle and other cuttings properties. He found out that drill string rotation enhances hole cleaning more when the used mud has a higher viscosity with smaller cuttings sizes. He found that for hole angle at 65 degrees, and at horizontal, the effect drill string rotation caused an improvement in cuttings transport as shown in the plots below.

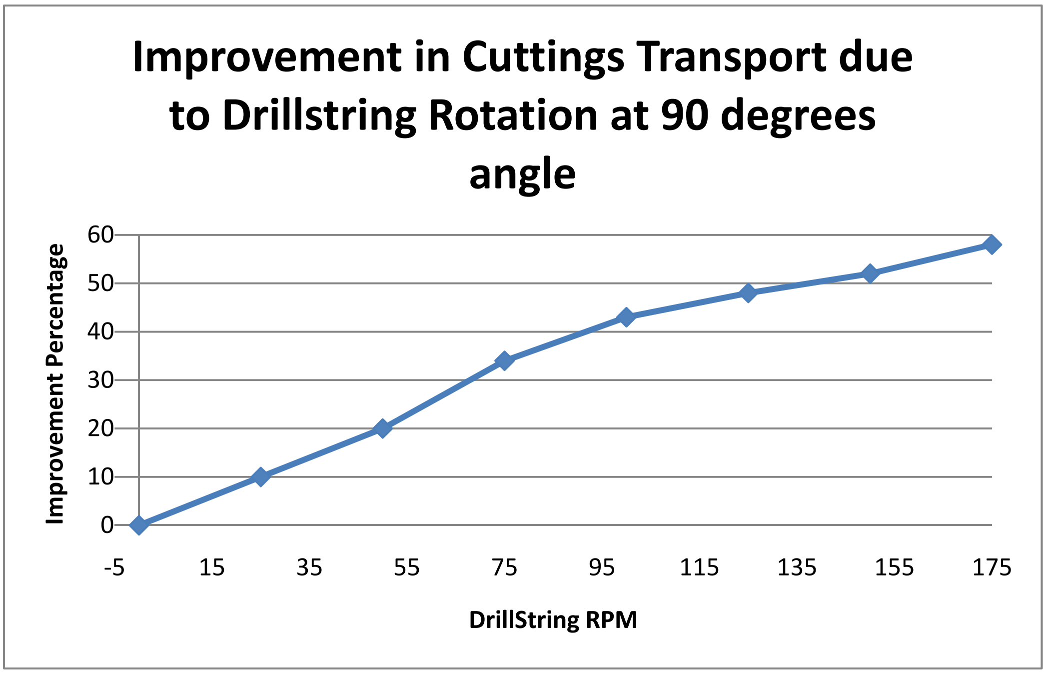

Figure2. 6: Effect of drill string rotation on cuttings transport in a horizontal wellbore-[8]

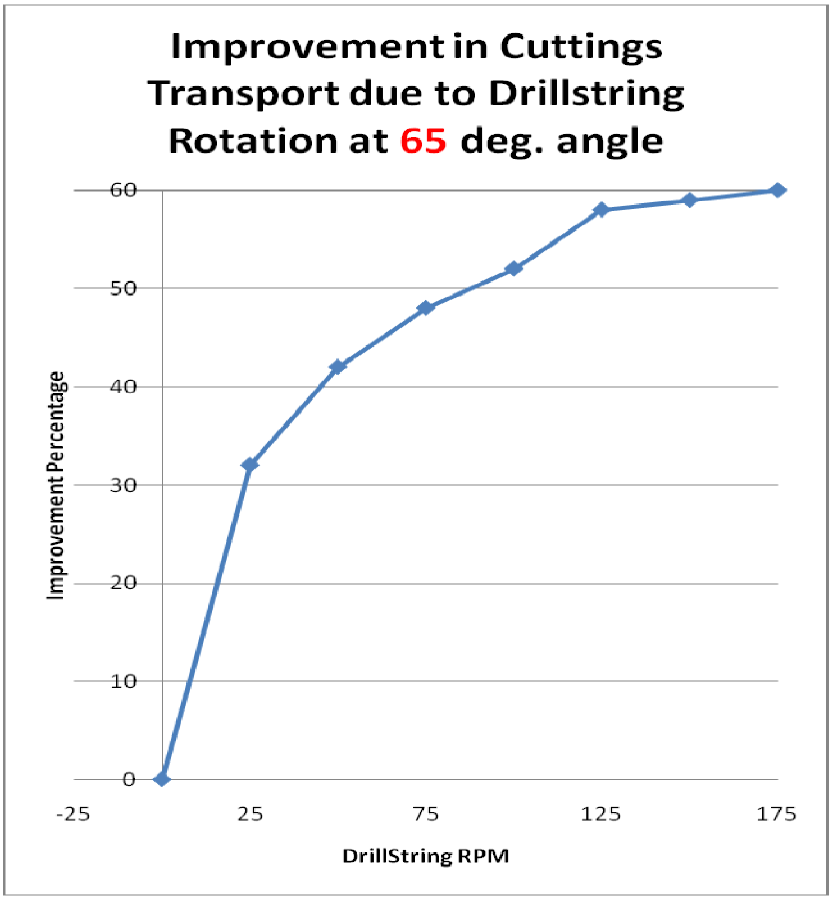

Figure2. 7: Effect of drill string rotation on cuttings transport in a wellbore at 65 degrees angle-[8]

When the pipe is not rotated, there is no fluid in the circumferential direction. Owing to the fundamental difference in the shear geometry between the narrow and wide sides of the annulus, the flow rates in the narrow side are always lower than those in the wide side, and this leads to difference in the top of fluids. When the pipe is rotated, tangential velocity in the fluid is initiated across the annulus gap, starting next to the casing. This leads to the transfer of fluid from the wide side to the narrow side, and vice versa-[Moroni, 9].

It should be noted that drill pipe rotation will affect the cuttings bed fraction differently depending on cutting bed rheology. For a bed formed in an oil based drilling fluid, there should be no gel structure that connect the cutting particles. Meaning that drill pipe rotation would primarily transport only the bed's surface particle into the annulus mainstream flow, but a water based drilling fluid with a lot of polymers could form a gel structure inter-linking the different cutting particles in the bed. Drill pipe rotation in this case can transport a larger volume of cutting to the annular mainstream. The lower the polymer content of the water based drilling fluid, the lesser the impact of drill pipe rotation on the cutting bed-[A.Saasen10].

2.2.4 Hole Size and Hole Angle

Studies have established the importance of the effect of inclination angle in cutting transport. J.Li showed that the toughest section for hole cleaning is the build section rather than the vertical or the horizontal section.

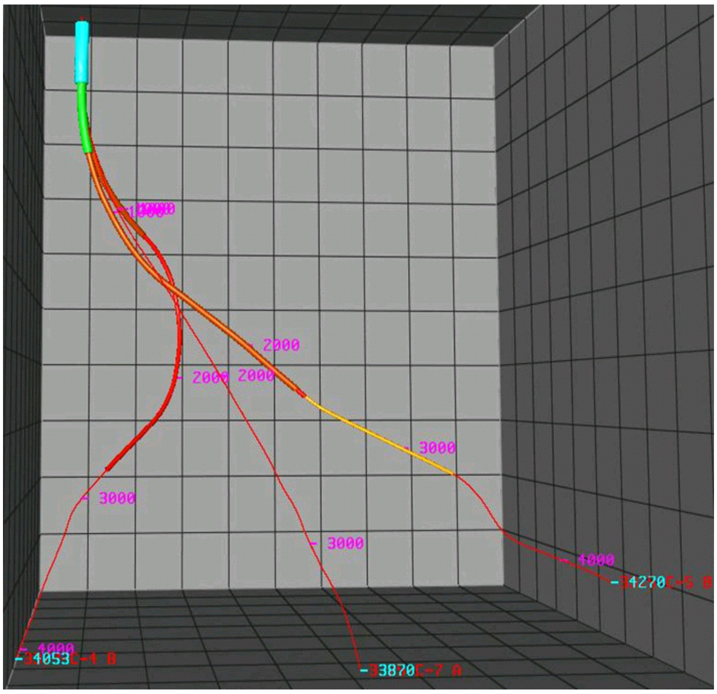

Figure2. 8: Effect of hole angle on particle sedimentation-[13]

- Vertical -< 20° - particles settle within the fluid, settling rates are generally low.
- Deviated – between 20° and 70° – particles settle out of the fluid, contact the borehole wall and slide downwards – Boycott settling.
- Horizontal – particles settle out of the fluid but do not move after this.

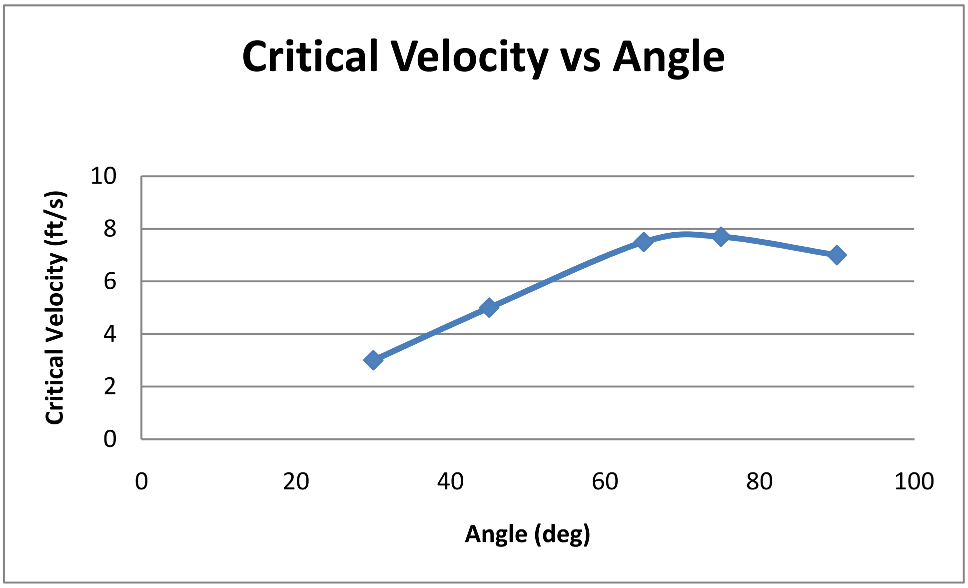

Figure2. 9: Effect of inclination angle on the critical velocity-[11]

Their studies showed that for different inclination angle, the minimum liquid velocity varies. The highest minimum in-situ liquid velocity is needed around 60 degrees. This is because cuttings tend to become unstable and to slide downward along the wellbore when angle increases above 60 degrees. Thus they concluded that hole cleaning is most difficult at close to 60 degrees. Their studies showed that increasing the inclination angle results in a higher cutting bed-[Li,S. Walker, 12].

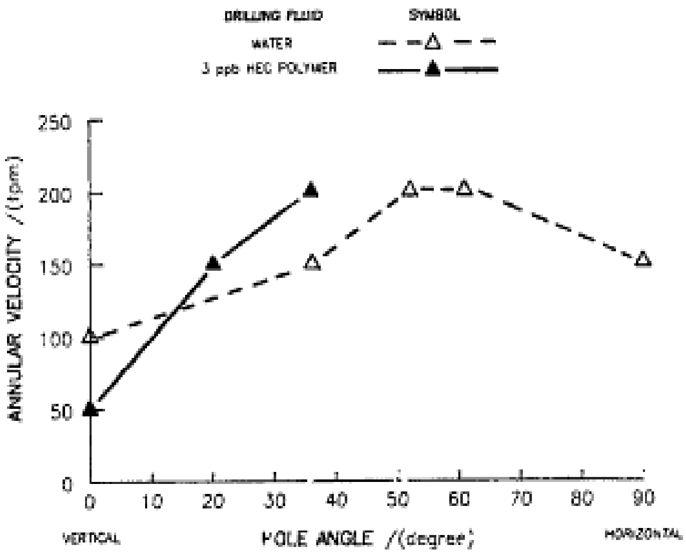

Figure2. 10: Annular velocity required to initiate the transport of 6mm beads with the drill pipe lying on the low-side of the hole-[N.P Brown]

In high angle hole intervals, the cuttings are no longer fully supported by fluid drag and it is inappropriate to make predictions of hole cleaning from techniques based on the fall velocity of the particles.

In high hole angles, the cuttings concentrate on the low side of the hole in the form of a bed. If the circulation rate is very low, cuttings are unlikely to be removed from the well bore. Upon increasing the flow rate, the bed becomes progressively eroded. The mobile cuttings on the interface saltate and form dunes or large ripples. The bed then starts to move and cuttings are cleaned from the well bore. This mechanism of bed movement is a more noticeable feature of hole cleaning with low viscosity fluids.-[Brown, 13]

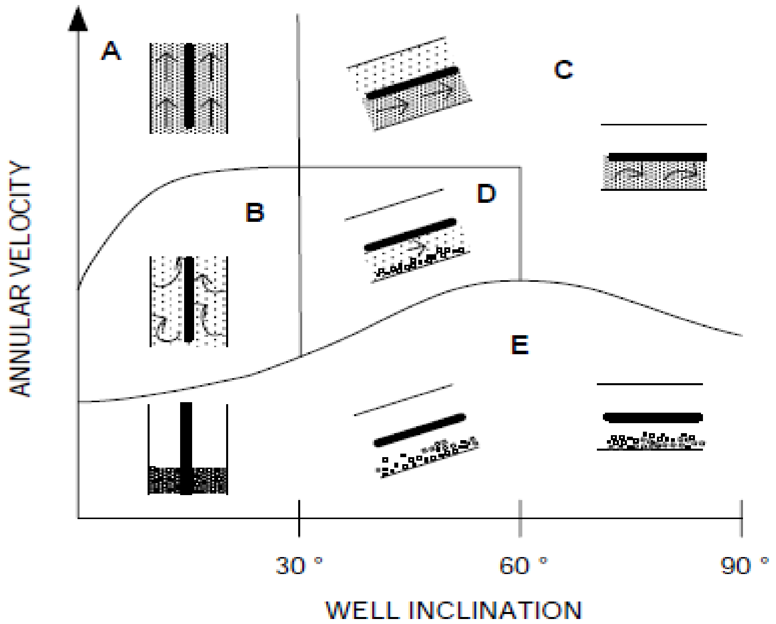

Figure2. 11: Cutting transport mechanism in vertical and deviated wells-[API Rec. 13D]

KEY

A	Zone 1 - Efficient hole cleaning
B	Zone 2 - Slow cuttings removal
C	Zone 3 - Good hole cleaning with moving cuttings bed
D	Zone 4 - Some hole cleaning – cuttings bed formed
E	Zone 5 - No hole cleaning

In holes inclined less than 30 degrees, the cuttings are effectively suspended by the fluid shear and beds do not form [Zone 1 &3], in such cases, conventional transport calculations based on vertical slip velocity are applicable. Beyond 30 degrees, the cuttings form beds on the low side of the hole which can slide back down the well, causing annular pack-off. Cuttings which form on the low side of the hole can either move en-masses as a sliding bed [Zone 4], or may be transported at the bed/drilling fluid interface as dunes or nipples [Zones 2]. The ideal zones for good hole cleaning are Zone 1 & 2.-[Bern,   14].

2.2.5 Rheology

Rheology can be defined as the science of deformation and flow, it refers to the different properties and characteristics of the drilling fluid. These properties of the circulation fluid have an effect on solids transport.

The shear stress at the bed interface for a near horizontal well bore plays the key role in solids transport. Eccentricity, flow regime and hole geometry also affect the rheological state of the liquid and have a significant impact on solids transport and removal.

A consistent conclusion indicate that for a horizontal or near horizontal well bore, hole cleaning is more efficient if a low viscosity fluid is pumped in a turbulent flow regime rather than a high viscosity fluid in a lamina regime-[6].

In their study, S.Walker, J.Li compared water, HEC and Xanvis polymers; they found out that the amount of solids that can be transported by a given volume of liquid is dependent on the rheological properties of the liquid. They found that Xanvis and HEC polymer based fluids are more effective than water in terms of carrying capacity but cannot erode a stationary bed. They also experimented with water and Xanvis, for the vertical well bore, hole cleaning is more efficient if a high viscosity fluid is pumped in a laminar flow regime rather than a low viscosity fluid in a turbulent flow-[6]

Different models have been propagated to provide assistance in characterizing fluid flow, but non of these models can completely describe rheological properties of drilling fluids over their entire shear-rate range.

The Bingham Plastic Model is used in fluids in which the shear stress to shear rate ratio is linear when a specific stress is exceeded. Mathematically, it is given as:

$$\sigma = \sigma_y + \eta_p \gamma \qquad 2.4$$

For such fluids, a specific yield stress must be exceeded for the fluid to flow. The model uses θ_{300} and θ_{600} to calculate the basic parameters PV and yield point YP.

$$PV = \theta_{600} - \theta_{300} \quad \text{(PV in cP)} \qquad 2.5$$

$$YP = \theta_{300} - PV \quad \text{(YP in } lb_f / 100 ft^2\text{)} \qquad 2.6$$

Also, the Herschel-Bulkley Model, also known as the modified power law model is used to describe flow of pseudo plastic drilling fluids which require a yield stress to flow. It is given mathematically as:

$$\tau = \tau_y + k\gamma^n \qquad 2.7$$

In the H-B model, the consistency parameter k can be considered as the PV or plastic viscosity term in the Bingham plastic model, also, the τ_y parameter describing the suspension characteristics of a drilling fluid can be seen as the Bingham plastic model Yield Point.

$$\tau_y = 2\theta_{300} - \theta_{600} \qquad 2.8$$

The fluid flow index is calculated using

$$n = 3,32 \log_{10}\left[\frac{(\theta_{600} - \tau_y)}{(\theta_{300} - \tau_y)}\right] \qquad 2.9$$

Then the consistency index is gotten via:

$$k = \frac{(\theta_{300} - \tau_y)}{511^n} \qquad 2.10$$

Finally, we have the Power Law model, which is used to describe the flow of shear thinning or pseudo plastic drilling fluid. A true power law does not exhibit a yield stress. This model uses two sets of viscometer dial readings to calculate index n and consistency index k for pipe flow and annular flow.

For pipe flow, we have:

$$n_p = 3,32 \log_{10}\left(\frac{\theta_{600}}{\theta_{300}}\right) \qquad 2.11$$

$$k_p = \frac{\theta_{300}}{511^{n_p}} \qquad 2.12$$

While for annular flow we have:

$$n_a = 0,657 \log_{10}\left[\frac{\theta_{100}}{\theta_{300}}\right] \qquad 2.13$$

$$k_a = \frac{\theta_{100}}{170,3^{n_a}} \qquad 2.14$$

The methods and formulas for determining basic variables using different Rheology models are summarized in the tables below

Table2. 2: Equations for determining flow behavior parameters

Rheological Model	*Flow Behavior Parameter*
Newtonian	$\mu = \theta_{300}$
Bingham Plastic	$\mu_p = \theta_{600} - \theta_{300}; \tau_y = \theta_{300} - \mu_p$
Power Law	$n = 3.32 \log \frac{\theta_{600}}{\theta_{300}}; k = \frac{510\theta_{300}}{511^n}$

Table2. 3: Equations for determining average velocity

Rheological Model	*Average Velocity*	*Average Velocity in Annulus*
Newtonian, Bingham Plastic, Power Law	$\bar{v} = \frac{q}{2.448 d^2}$	$\bar{v} = \frac{q}{2.448(d_2^2 - d_1^2)}$

The rheological properties of the mud will go a long way in determining its flow rate and suspension characteristics. According to Thor Inge F. Larsen, as the flow rate increases, the amount of cuttings in the annulus will decrease as shown below.

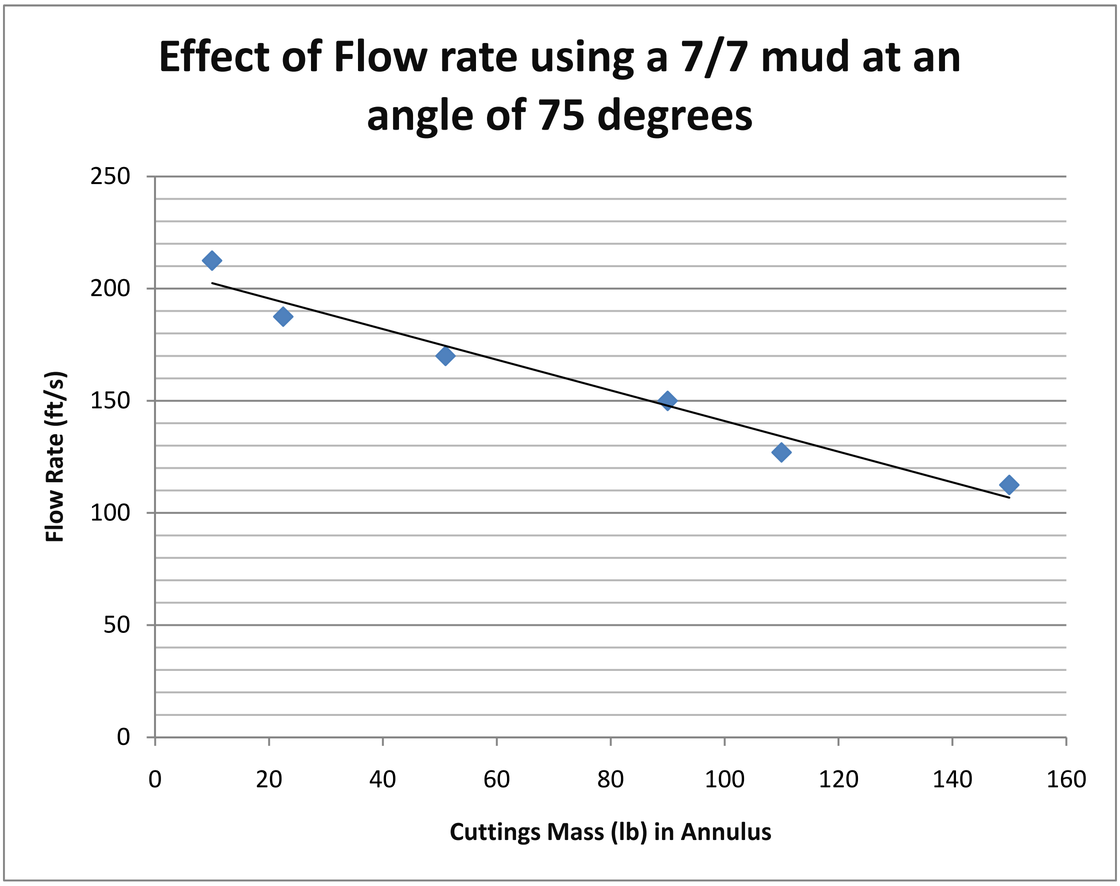

Figure2. 12: Effect of mud flow rate on annular cuttings mass concentration-[15]

2.2.6 Cutting Transport Ratio

This is the ratio of the cuttings concentration that is delivered or removed from the hole to the total cutting concentration in the well bore annular space. A higher cutting transport ratio means that a relatively lesser amount of solids stays in the well bore and a greater amount is transported with the carrying fluid.

Alternatively, cutting transport ratio can be seen as the transport velocity-V_T, divided by the fluid average annular velocity, V_f and is given by the equation:

$$T_r = \frac{V_T}{V_f} \qquad 2.15$$

Where the transport velocity is obtained using the following equation;

$$V_T = \frac{L}{t} \qquad 2.16$$

Where the length (distance), L, and time, t, are measured experimental data. The higher the transport ratio, the better the hole cleaning –[Belavadi, Chukwu, 16].

Researches have revealed that a critical flow rate exist at which cutting transport is optimized, and the well is efficiently cleaned. This critical flow velocity or flow rate have been modeled using different rheological models.

Using the power law model for fluids, we have:

$$V_{cp} = \left[\frac{28277(2,533 - n_p)k_p}{p} \left(\frac{1,6G_p}{dhyd} \right)^{n_p} \right]^{\frac{1}{2-n_p}} \qquad 2.17$$

Where

$$G_p = \left[\frac{(3-\alpha)n_p + 1}{(4-\alpha)n_p} \right] \left[1 + \frac{\alpha}{2} \right] \qquad 2.18$$

Using the Bingham-plastic model, we have

$$V_{cb} = \frac{67,86}{\rho} \left[B + \sqrt{B^2 + 9,42 \rho YP \left(\frac{4-\alpha}{3-\alpha} \right)} \right] \qquad 2.19$$

Where

$$B = \frac{PV\left(1 + \frac{\alpha}{2}\right)}{d_{hyd}} \qquad 2.20$$

A very close approximation can be achieved by an empirical combination of the critical velocity based on Power-law and that based on Bingham-plastic model. Using these we have:

$$V_c = V_{cp} + \left(V_{cb} - V_{cp}\right) R^{\sqrt{\frac{V_{cp}}{V_{cb}}}} \quad 2.21$$

Where

$$R = \frac{\tau_Y}{YP} \quad 2.22$$

2.2.7 Rate of Penetration

According to studies by Scott Walker, cutting bed is deeper for a higher ROP than it is for a lower ROP with the same circulation fluid rate. Also for a given ROP higher circulation fluid flow rate results in a lower bed height. When the ROP is constant, increasing the circulation flow rate results in a lower cutting concentration and a decreasing of the bed height. Also with a fixed circulation rate, increasing ROP results in a higher cutting concentration and a higher bed height.

According to Laxmikant Shrihari, the critical velocity increases with increase in the rate of penetration. For a 8 inch hole size, with a 4 inch pipe diameter and an eccentricity of 62% at an angle of 45 degrees, he got the following graph for a 25/25 mud.

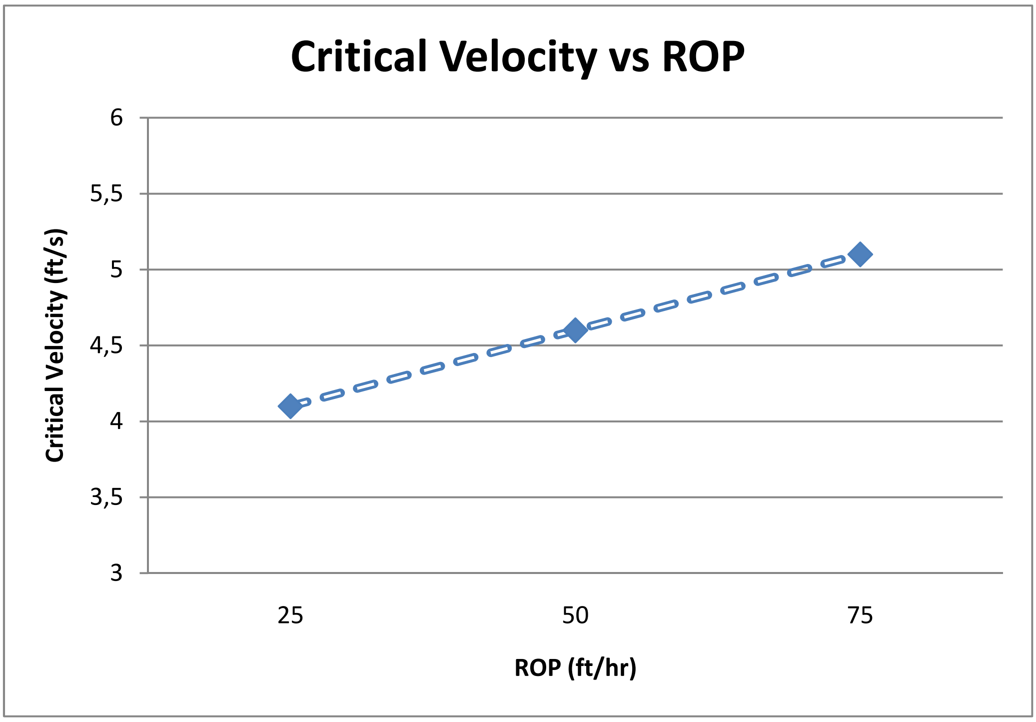

Figure2. 13: Effect of ROP on the critical velocity [15]

The cuttings velocity if a function of the rate of penetration as given in the equation below-[17].

$$V_{cut} = \frac{ROP}{36\left[1-\left(\frac{D_{pipe}}{D_{hole}}\right)^2\right]Cconc} \quad 2.23$$

Where according to Rudi,

$$Cconc = 0.01778ROP + 0.505 \quad 2.24$$

2.2.8 Multi-Phase Flow Effect

In most drilling operations, needs may arise that will warrant the addition of a gas phase to the circulation fluid. Also if the well is a gas well, or gas condensate, then there will be entrained gas in the drilling fluid. Situations that might warrant the addition of a gas phase to the drilling fluid includes drilling through a low pressure reservoirs or in under balanced conditions. It is important to note that the hole cleaning efficiency of the multi-phase system increases as the in-situ liquid velocity increases. The advantages of multi-phase flow are more pronounced at highly inclined angles up to 60 degrees with a Xanvis system. A multi-phase system will increase the efficiency of cutting transport but an important note according to studies is that increasing the gas volume fraction above 50% actually impedes the efficiency of cutting transport.

The general trend for a multi-phase system is an increase in solids transport and this in turn reduces the number of hole volume required to clean the hole-[6].

When there are more than one phase flowing in the drill pipe and annulus, challenges arise in determining the true flow velocity, fluid density, viscosity and other rheological parameters. To fully understand the conditions in the flow, we define U_{LS} and U_{GS} as the superficial velocity-assuming the phases are flowing alone.

$$U_{LS} = \frac{q_L}{A} \quad 2.25$$

$$U_{GS} = \frac{q_G}{A} \quad 2.26$$

While the phase velocity are defined thus

$$u_L = \frac{q_L}{A_L} \quad 2.27$$

$$u_G = \frac{q_G}{A_G} \quad 2.28$$

When there are more than one phase in the flow medium, the gas phase normally flows faster than the liquid phase, the relative phase velocity or the slip velocity is thus defined as:

$$u_S = |u_G - u_L| \quad 2.29$$

With the slip ratio being

$$S = \frac{u_G}{u_L} \quad 2.30$$

Many researches and models have been proposed for determining the effective density and viscosity for a two phase flow process. Provided the fluid fractions are known, using mixing rule, the density is given as.

$$\rho_m = \rho_L \varepsilon_L + \rho_G \varepsilon_G \quad 2.31$$

Unlike the density, viscosity for a given mixture is not a well defined quantity just in terms of fluid fraction and single phase velocities. The mixture velocity in fact depends strongly on dynamical processes as well including bubble size if gasses are present, flow regime etc-[multi phase hand out—include in end note]. It is therefore not unexpected that many different models exist for the determination of the mixture viscosity.

Cichitti: $$\mu_m = x\mu_G + (1-x)\mu_L$$

McAdams: $$\frac{1}{\mu_m} = \frac{x}{\mu_G} + \frac{(1-x)}{\mu_L}$$

Dukler: $$\mu_m = \varepsilon_G \mu_G + (1-\varepsilon_G)\mu_L$$

Geometrical average: $$\mu_m = \mu_G^{\varepsilon_G} . \mu_L^{(1-\varepsilon_G)}$$

For a given gas volume flow rate, increasing liquid flow rate results in a lower bed height, similarly for a given liquid flow rate, increasing gas volume flow rate results in a lower bed height. The range of liquid fraction may be limited due to the need to maintain under balanced conditions or due to pressure limitations of the drill pipe. Therefore a common job design requirement is to predict the critical gas volume fraction beyond which the hole cleaning may be significantly impaired-[12].

2.2.9 Effect of Cutting Bed Properties

The properties of the cutting bed have a major influence on hole cleaning, if the bed is loose or highly porous, then it may be necessary to remove single cutting particles that are not adhered to the bed. In which case removing the bed becomes easy. But if the cutting bed is highly consolidated with no cutting particle free to be removed alone from the bed by the flow, hole cleaning is then difficult-[7].

In their studies, they concluded that it is desirable to minimize the cutting bed consolidation as much as possible, fluids might migrate through a loose and porous cutting bed and help in keeping the cutting bed loose.

Laximikant Jalukar showed that under certain conditions, the cuttings bed height decreases as the flow rate increase, this is expected as the increased flow rate will increase cuttings removal, reducing the amount of cuttings in the annulus.

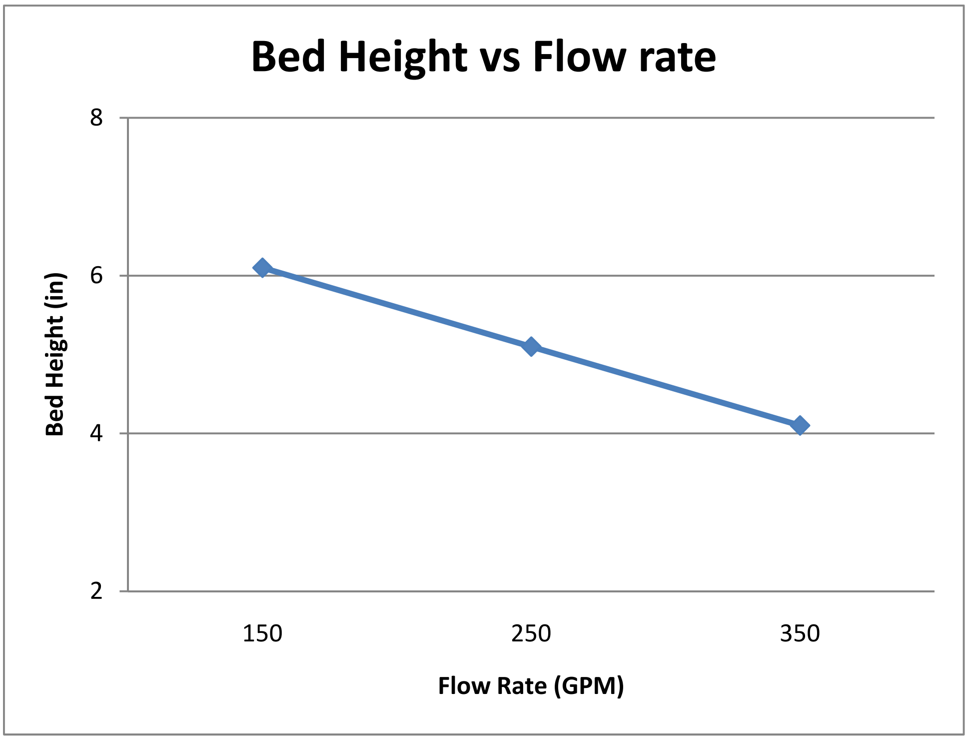

Figure2. 14: Effect of mud flow rate on cutting concentration and bed height-[11]

Arild Saasen, etal showed that the thickness of a cutting bed is affected by many different independent variables-[18]:

$$\frac{A_{bed}}{A_{wellbore}} = f\begin{bmatrix} ROP, Q, \Delta P, \theta, D_0, D_1, \\ \rho, \mu, rpm, \rho_c, d_c, g \end{bmatrix}$$ 2. 32

Using the PI theorem and experimental data, they presented cutting bed area as thus:

$$\frac{A_{bed}}{A_{wellbore}} = 0.7524\begin{bmatrix} \left(\frac{\rho v(D_0 - D_1)}{\mu}\right)^{-0.1023} & (\theta)^{0.0340} \\ \left(\frac{v^2}{D_0 - D_1}\right)^{-0.2933} & (Cc)^{0.2108} \end{bmatrix}$$ 2. 33

From there findings, one can solve this equation for a zero bed area and determine the critical fluid velocity, but because of the nature of the equation, they concluded that the left hand side cannot be zero, since that will give a trivial solution.

Chapter 3

Flow patterns and forces acting on a drill cuttings

3.1 Flow Patterns

Depending on the flow rate, conduit shape, fluid and solid properties and inclination, the liquid and solid phases may distribute in a number of different geometrical configurations during the flow of solid-liquid mixtures as is obtainable during drilling operations. It is possible to have a fully suspended symmetric flow pattern, asymmetric flow pattern or a moving bed flow pattern. Fully suspended symmetric flow pattern is found when the liquid velocity is very high, such that the solids are uniformly distributed in the liquid phase; it most observed when the solid particles are fairly fine-less than 1mm. As the liquid flow rate is reduced, there is a tendency for the solids to flow near the bottom of the pipe in highly inclined wells but still suspended, creating asymmetric solid concentration. This is known as asymmetric flow pattern. If the flow rate is further reduced, solids might deposit on the low side and bottom of the pipe in horizontal and highly inclined wells, forming a bed which will move in the direction of flow: this is the moving bed flow pattern-[19].

When velocity is further reduced, there will more deposition of solids, resulting in three layers, the top most consisting of a heterogeneous liquid, a moving solid bed and a stationary solid bed at the bottom. The diagram below depicts the different flow patterns-[19].

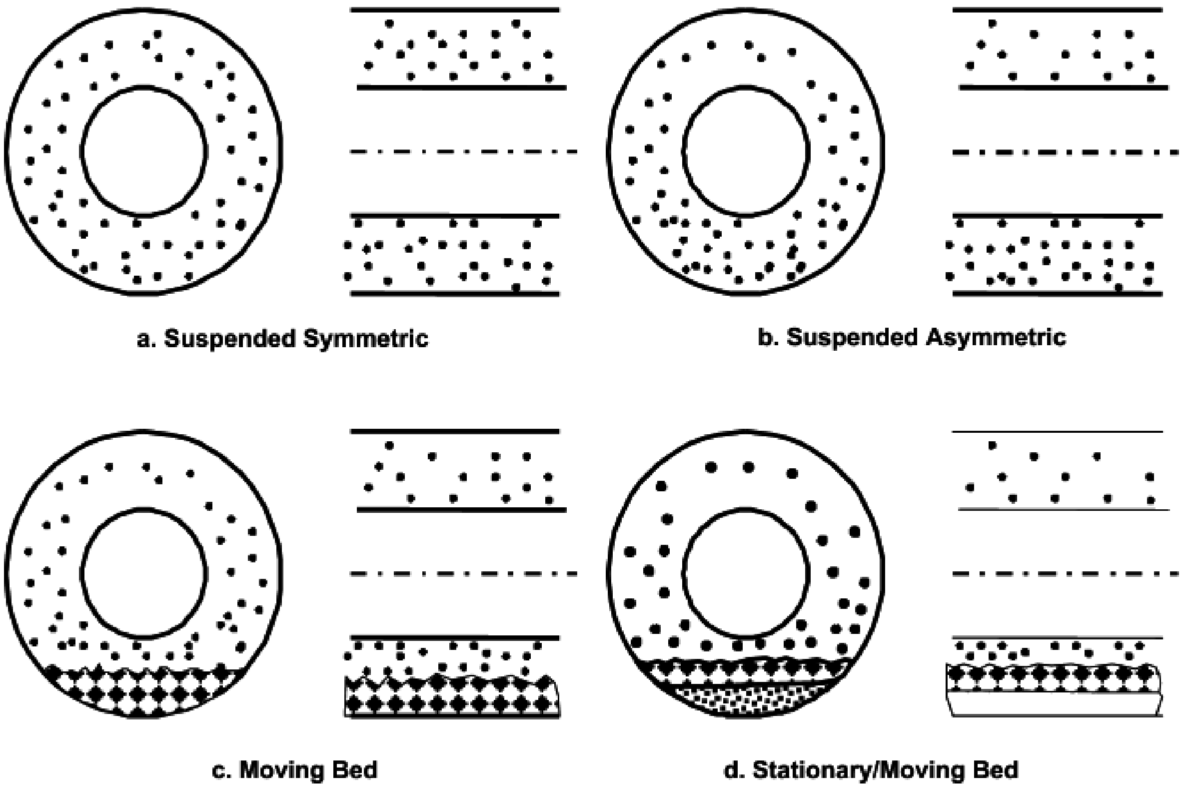

Figure3. 1: Flow patterns for solids/liquid flow in high angle and horizontal annulus-[Kelessidis &Bandelis]

According to Hyun Cho, Subhash Shah and Samuel Osisanya, by assuming no slip between the solid and liquid phase in the cutting bed, the continuity equation for the solid particles can be written as:

$$\frac{d}{dx} = (\rho_{sd} A_{sd} C_{sd} U_{sd} + \rho_{mb} A_{mb} C_{mb} U_{mb} + \rho_{sb} A_{sb} C_{sb} U_{SB}) = 0 \qquad 3.1$$

Here the subscripts sd, mb and sb refer to stationary bed layer, moving bed layer, and dispersed suspension layer respectively. With the density of each layer at any point in the system assumed to be a constant. The velocity of the stationary bed is almost zero since it is not moving, integrating the equation, we have:

$$A_{sd} C_{sd} U_{sd} + A_{mb} C_{mb} U_{mb} = A_a C_t U_t \qquad 3.2$$

The area relation can be described as follows:

$$A_{sd} + A_{mb} + A_{sb} = A_a \quad 3.3$$

They found the continuity equation for the liquid phase to be:

$$A_{sd}(1 - C_{sd})U_{sd} + A_{mb}(1 - C_{mb})U_{mb} = A_a(1 - C_t)U_t \quad 3.4$$

3.2 Forces Acting on a Suspended Drill Cutting

A drill cutting in suspension is acted upon by different forces; it is also affected by the effect of other drill cutting in contact with it. The shape of the cutting and the fluid properties are very important in determining which force is most active and dominates the system. The diagram below shows the fluid movement about a suspended and settling particle.

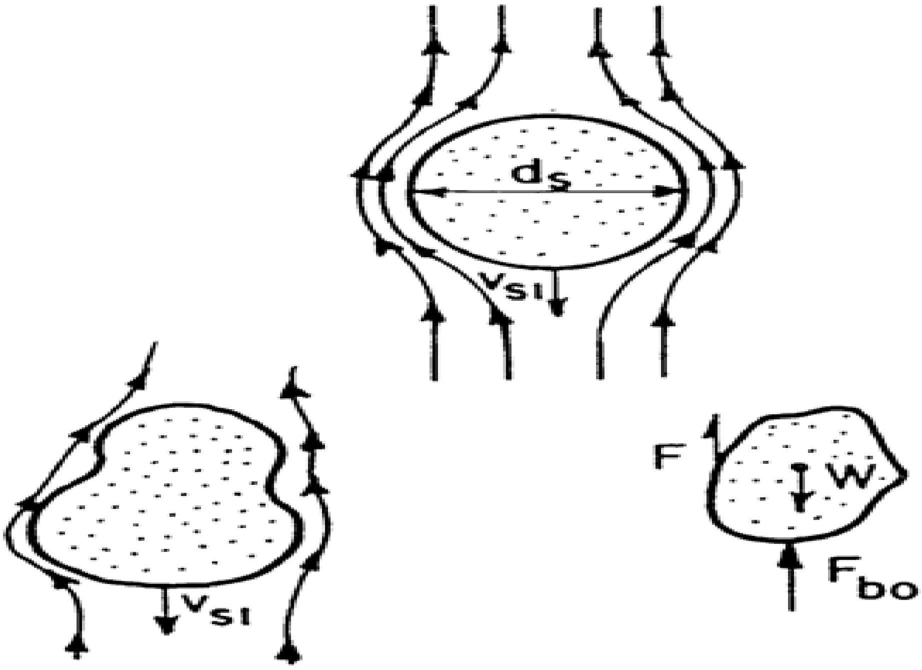

Figure3. 2: Streamline of fluid movement about a settling or suspended particle-[19]

For a particle in suspension the major forces acting on it includes the lift and drag forces, gravity and buoyancy forces, normal forces at point of contact and frictional forces. According to Mingin Duan, Stefan Miska, Mengjiao Yu, Nicolas Takach, and Ramadan Ahmed, these forces can be grouped into three groups. the static forces, hydrodynamic forces and the inter-particle forces. According to them, Gravity and buoyancy forces are static forces due to the properties of the particles and its surrounding fluid. Drag and lift are hydrodynamic forces incurred from the fluid flow. Van der Waals forces are inter-particle forces existing between any neighboring particles. They become dominant when the diameter of two closely neighbored particles are below 0.1mm-[20]

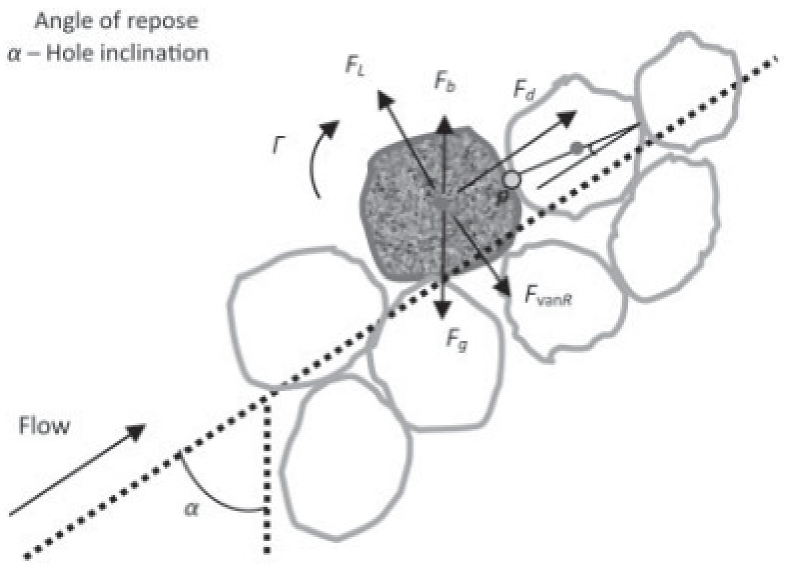

Figure3. 3: Forces applied to a particle on a solids bed-[Duan, Miska,etal]

Assuming the flow is steady and isothermal in a concentric annulus, the gas phase is free of cuttings, cutting particles are uniform and spherical, the effect of inner pipe rotation is not considered and cutting bed surface is uniform along the annulus, Mingqin Duan etal predicted:

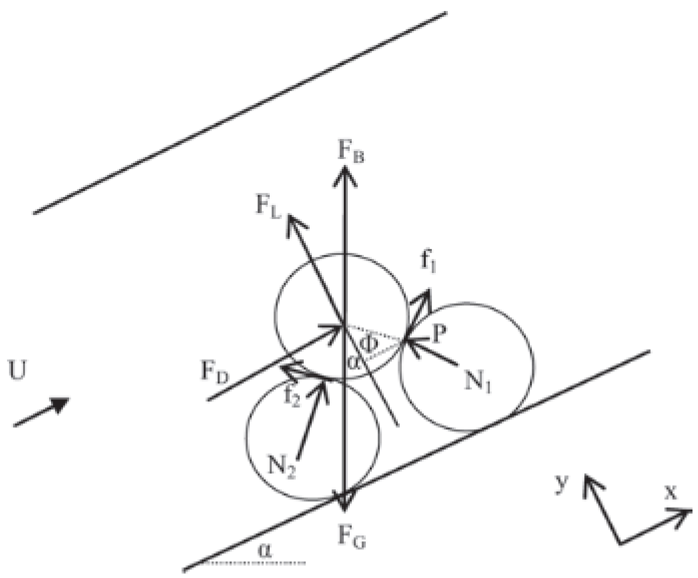

Figure3. 4: Forces acting on a single particle-[L.Zhou]

Static Forces, Gravity force as

$$F_g = \frac{\pi d_p^3}{6} \rho_p g \qquad 3.5$$

And the buoyancy forces as

$$F_b = \frac{\pi d_p^3}{6} \rho_f g \qquad 3.6$$

For the Hydrodynamic forces, they stated that a particle in a moving fluid experiences a force parallel to the direction of upstream velocity, drag and a force normal to the upstream velocity. They stated that these two forces can be obtained based on the definition of the drag coefficient, C_D, and the lift coefficient, C_L-[20].

$$C_D = \frac{F_D}{0.5 \rho_f u^2 A} \qquad 3.7$$

While the lift coefficient is defined as:

$$C_L = \frac{F_L}{0.5 \rho_f u^2 A} \qquad 3.8$$

The lift coefficient can also be found using the Saffman's lift, assuming a wall correction factor of 0.6, the lift coefficient for a small sphere in a slow shear flow is given by:

$$C_L = 2.47 \sqrt{\frac{d_p}{R_{ep} V_r} \frac{dV_r}{d_y}} \qquad 3.9$$

The Van der waals forces are evaluated because as a particle size decreases, its surface area per unit volume increases; therefore, surface forces begin to show an impact on the movement of the particle, making Van der waals forces an important factor to consider when the particles are closely connected.

This force is predicted using the equation:

$$F_{van} = -\frac{A_H d_p}{24 s^2} \qquad 3.10$$

Where according to Yu etal,

$$s = 1.78 \times 10^{-5} d_p^{0.77} \qquad 3.11$$

Table 3. 1: Formula for forces acting on bed cuttings

FORMULA FOR FORCES ACTING ON BED CUTTINGS	
FORCE	*EQUATION*
Gravity (F_G)	$F_G = \frac{1}{6} \rho_L g \pi d_p^3$
Buoyancy (F_B)	$F_B = \frac{1}{6} \rho_L g \pi d_p^3$
Lift (F_L)	$F_L = \frac{1}{8} C_L \rho_L u^2 d_p^2$
Drag (F_D)	$F_D = \frac{1}{8} C_D \rho_L u^2 d_p^2$
Van der waals (F_{van})	$F_{van} = -\frac{A_H d_p}{24 s^2}$

L.Zhou predicted that to initiate rolling of the bed particle, the moments of forces (F_B+F_L+F_D) at the contact point P which tend to cause downstream rotation must be greater than the moments of the force (F_G) that tend to prevent downstream rotation. He stated that the condition for initiation of particle rolling at the bed surface is given by:

$$F_L R \sin\theta + F_D R \cos\theta - (F_G - F_B) R \sin(\alpha + \theta) = 0 \qquad 3.12$$

Where α is the angle of inclination, and θ is the angle of repose. The angle of repose if defined as the maximum angle of slope measured from the horizontal plane at which cuttings come to rest on a pile. It is in the range of 33-38 degrees. Using these, Zhou predicted the critical rolling velocity as:

$$u_{roll} = \sqrt{\frac{4(\rho_C - \rho) g d_p \sin(\alpha + \theta)}{3\rho_L(\sin\theta C_L + C_D)}} \qquad 3.13$$

Also taking the normal and frictional forces at contact points as zero, when the particle lifting is about to occur, then the particle lifting condition at the bed surface is given by the following- [21].

$$F_L - (F_G - F_B)\cos\alpha = 0 \qquad 3.14$$

By substituting the force equations, Zhou found the critical velocity for particle lifting to be:

$$u_{lift} = \sqrt{\frac{4(\rho_C - \rho_L) g d_p \cos\alpha}{3\rho_L C_L}} \qquad 3.15$$

3.3 Initiation of Cuttings movement

P.A Bern etal stated that there are a total of seven independent variables that affect the critical conditions for the initiation of cutting transport and movement in a drilling annulus; however, they stated that the list is still not complete. Using the Buckingham PI theorem, they showed that the seven dimensional variables can yield four dimensional groups:

$$\pi_1 = \frac{d_s V_{cri} \rho_f}{\mu} \qquad 3.16$$

This group can be considered as the particle Reynolds number which is a representative of the ratio of the inertia force to the viscous force. They also stated that if the annular flow is turbulent, then Π1 can be viewed as representing the degree to which the cuttings are immersed into the viscous sub layer, or is projected into the turbulent core.

$$\pi_2 = \frac{V_{Cri}^2 \rho_f}{d_s g_F} \qquad 3.17$$

This is the ratio of the fluid dynamic forces to the effective gravitational forces. it can be considered as the modified particle Froude number and should have a significant effect as long as the gravitational force is important. They saw this parameter as the most important parameter in the initiation of cutting movement.

$$\pi_3 = \frac{d_s}{W} \qquad 3.18$$

This reflects the influence of cutting size relative to the size of the annular gap in cutting movement initiation. But if the cutting size is small compared to the size of the annular gap, then this effect should be neglected.

$$\pi_4 = \frac{\rho_f}{\rho_s} \qquad 3.19$$

This dimensional group shows the influence of the inertia force when the cuttings accelerate upon starting to move and the effect of this parameter are expected to be relatively small.

Based on the dimensionless groups, they stated that a general expression for describing the initiation of cuttings movement can be expressed thus:
(Π1, Π2, Π3, Π4) = 0
This model is valid for both Newtonian and non-Newtonian fluids which can be power-law, Bingham plastic fluids and any other fluid type.

3.4 Bit Hydraulics and Optimization

Cuttings transport efficiency is a function of the hydraulic parameters; it is a well known fact that the bit hydraulics affects frictional losses, which in turn affect the cuttings concentration in the annulus. To efficiently transport cuttings up the annulus, it is important to optimize bit hydraulics. The work done on hydraulics optimization by Aadnøy Bernt and others are reviewed below-[22-27].
During drilling, at the point of contact between the bit and the formation, as soon as the bit teeth fractures or cuts the rock, two things can take place:

1, The bit nozzles might provide enough jet impact force to transport the cuttings to the surface.
2, If the nozzles does not provide enough jet impact force, the cuttings will be regrind leading to decrease in the rate of penetration.
Bit hydraulics is related to parameters such as the nozzle size, the number of nozzles, jet velocity through the nozzles, and also the pressure loss at the bit.
According to Bernt Aadnøy, there are two basic criterion that are used in analyzing bit hydraulics for hole cleaning:
1, The Hydraulic Horsepower criteria (HHP)
2, The jet impact force criteria (JIF)

Using the optimum hydraulic horsepower criteria, the HHP at the bit can be determined from the relation below:

$$HHP = \frac{\Delta P_b Q}{1714}$$

Relating the HHP in terms of the circulation pressure P_e, we have:

$$HHP = \frac{Q(P_p - \Delta P_c)}{1714}$$

Lim presented a relationship between the pump pressure and the circulating pressure losses as:

$$P_p = (m+1)K^l Q^m = (m+1)\Delta P_c$$

Finding the optimal circulating pressure loss, we have:

$$\Delta P_{c(Opt)} = (\frac{1}{m+1})P_p$$

This corresponds to the model derived by Aadnøy for frictional losses in the hydraulic system, he defined the pressure drop equations as:

$$P_1 = P_2 + P_3$$

Where
P_1= Pump pressure
P_2= Pressure drop across nozzles
P_3= Parasitic pressure loss
From his correlations,

$$P_2 = Cq^m$$

Using the HHP criteria, he stated that:

$$P_3 = \frac{P_1}{m+1}$$

From the above, the optimum flow rate and nozzle sizes can be calculated as follows.

$$Q_{(Opt)} = [\frac{\Delta P_c}{K^l}]^{\frac{1}{m}}$$

And

$$d_{(Opt)} = (\frac{17.3\rho(Q_{opt})^2}{\Delta P_{b(opt)}})^{0.25}$$

Also, using the Jet Impact Force (JIF) criteria, Kien and Chukwu expressed the impact force developed by the bit as:

$$F_j = 0.01823 C_d Q \sqrt{\rho \Delta P_b}$$

From the above, the JIF is related to the flow rate and nozzle velocity as follows.

$$F_j = \frac{\rho Q V_n}{60g} = \frac{\rho Q V_n}{1932}$$

Relating the pump pressure to the circulating pressure loss, we have

$$2QP_p = (m+2)K^l Q^{m+1}$$

This gives

$$P_p = (\frac{m+2}{2})\Delta P_{c(opt)}$$

This gives the optimum pressure losses as:

$$\Delta P_c = (\frac{2}{m+2})P_p$$

This is also in agreement with the equation derived by Bernt Aadnøy using the jet impact force criteria, he stated that:

$$P_3 = \frac{2P_1}{m+2}$$

Aadnøy 1991 went further to derive a new set of hydraulic optimization criterion. These new criterion were field tested.

Criterion	Equation	Fraction parasitic pressure loss
Max. HHP	$P_2 q$	$\frac{1}{m+1}$
Max. JIF	$\sqrt{P_2} q$	$\frac{2}{m+2}$
New A	$\sqrt{P_2 q^{\frac{3}{2}}}$	$\frac{3}{m+3}$
New B	$\sqrt{P_2 q^2}$	$\frac{4}{m+4}$
New C	$\sqrt{P_2 q^{\frac{5}{2}}}$	$\frac{5}{m+5}$

Chapter 4: Model derivations

4.1 Derivation of models

The flow rate prediction equations and hole cleaning equations in deviated wells as presented in chapter two of this work and as derived by P.A.Bern and Yuejin Lou is valid only for angles greater than 30 degrees. However, it is possible to produce a model that can be valid for both vertical and deviated wells, by combining the models for the vertical and the deviated, such that the existing deviated charts can be combined with a new chart gotten from the derived model to predict hole cleaning in both near vertical and high angle wells. Hence this chapter will derive hole cleaning model, and will also consider the case of washout during drilling.

A model to find equivalent RF in a vertical section is derived, the angle factor is approximated for a vertical well and a correction factor for the effect of washout in a vertical well is modeled.

From appendix A, a model for approximating the rheological factor in vertical sections is derived:

$$RF = \frac{6k}{3585A_a}\left[\frac{TI}{Ccl}\right] \quad 4.1$$

In the derivation of the above equations, a constant flow rate was assumed in both deviated and vertical formulas, and the well was also assumed to be in gauge, thereby allowing for the use of a constant area. Finally, conditions in both cases were assumed the same.

The use of this equation is very important as will be shown in the next chapter, as it affords us the ability to go to the field with a single set of charts, which could be used for predicting hole cleaning in both vertical and deviated wells.

In the approximation of the angle factor in vertical section, a model is derived thus:

$$\frac{1}{AF}Q_{ver} = Q_{Deviated} \quad 4.2$$

In the above equation, Q_{VER} is the flow rate that the well will flow with if in a vertical well, while Q_{Dev} is the well's flow rate if in a deviated well, under the same conditions and using the same mud and fluids parameters.

Note also that in the above equation, both flow rates are calculated with different formulas, i.e.

$$Q_{vertical} = \frac{400,000A_a}{\rho k(0.13369)}$$, where the area is in ft^2 4. 3

while

$$Q_{deviated} = \frac{834.5TI}{\rho RF}$$ 4. 4

This is according to the procedure given in the API recommended practice.

Another model derived according to appendix C is the equation for predicting correction factors due to hole washout during drilling. According to appendix C, the model is derived thus:

$$Q_2 = \left[\frac{D_2}{D_1}\right]^2 Q_1$$ 4. 5

Where α is given by

$$\alpha = \left[\frac{D_2}{D_1}\right]^2$$ 4. 6

In the derivation of this equation, it is assumed that *Q1* is the initial critical flow rate when the well was in gauged, while *Q2* is the critical rates needed to clean the well after washout occurs and the well is enlarged.

Furthermore, to come up with a correction factor for the rates, to be able to predict appropriate critical flow rate that can efficiently clean the hole, we had to assume a constant flow velocity, such that a change in the well area affects the effective flow rate needed to maintain a constant velocity that will sufficiently clean the well.

Chapter 5: Practical Applications of Models

5.1 Application of equivalent Rheology factor Model in vertical sections

Finding equivalent RF in vertical sections, a model is derived thus:

$$RF = \frac{6k}{3585A_a}\left[\frac{TI}{Ccl}\right] \qquad 5.1$$

Applying data for a 8 ½ inch vertical hole size, using rheological data, such that the final plots gives same range as already existing hole cleaning charts-so they could be used together.

The charts below is to be used together with already existing hole cleaning charts as given in API 13D.

The procedure for determining the CFR or the maximum safe ROP using the charts is as follows:

1, With the PV and YP of the drilling fluid, read off the Rheology factor as given in the charts in Appendix D.

2, Get the angle factor AF from Table 5.9

3, Calculate the transport index, based on the RF, AF and MW.

4, For vertical well sections, read off TI/CCL values from Fig 5.1 and find your equivalent CCL value.

5, For deviated well section, using TI value and the angle factor, read off safe ROP from charts in appendix D, or with TI and desired ROP, read off CFR for proper hole cleaning, using charts in Appendix D.

6, If hole is washout, apply correction factor according to tables 5.10-12 for vertical sections.

Table 5. 1: Data set for verification of model 1

FOR 8 1/2 INCHES VERTICAL HOLE

TI/CCL	RF(K=25)	RF(K=50)	RF(K=75)	RF(K=100)	RF(K=125)	RF(K=150	RF(K=175)	RF(K=200)	RF(K=225)	RF(K=250)
0,5	0,02743	0,05487	0,0823	0,109733	0,137166	0,1646	0,192032	0,219465	0,246898	0,274331
1	0,05487	0,10973	0,1646	0,219465	0,274331	0,3292	0,384064	0,43893	0,493796	0,548663
1,5	0,0823	0,1646	0,2469	0,329198	0,411497	0,4938	0,576096	0,658395	0,740694	0,822994
2	0,10973	0,21947	0,3292	0,43893	0,548663	0,6584	0,768128	0,87786	0,987593	1,097325
2,5	0,13717	0,27433	0,4115	0,548663	0,685828	0,82299	0,960159	1,097325	1,234491	1,371656
3	0,1646	0,3292	0,4938	0,658395	0,822994	0,98759	1,152191	1,31679	1,481389	1,645988
3,5	0,19203	0,38406	0,5761	0,768128	0,960159	1,15219	1,344223	1,536255	1,728287	1,920319
4	0,21947	0,43893	0,6584	0,87786	1,097325	1,31679	1,536255	1,75572	1,975185	2,19465
4,5	0,2469	0,4938	0,74069	0,987593	1,234491	1,48139	1,728287	1,975185	2,222083	2,468981
5	0,27433	0,54866	0,82299	1,097325	1,371656	1,64599	1,920319	2,19465	2,468981	2,743313
5,5	0,30176	0,60353	0,90529	1,207058	1,508822	1,81059	2,112351	2,414115	2,71588	3,017644
6	0,3292	0,6584	0,98759	1,31679	1,645988	1,97519	2,304383	2,63358	2,962778	3,291975
6,5	0,35663	0,71326	1,06989	1,426523	1,783153	2,13978	2,496414	2,853045	3,209676	3,566306
7	0,38406	0,76813	1,15219	1,536255	1,920319	2,30438	2,688446	3,07251	3,456574	3,840638
7,5	0,4115	0,82299	1,23449	1,645988	2,057484	2,46898	2,880478	3,291975	3,703472	4,114969
8	0,43893	0,87786	1,31679	1,75572	2,19465	2,63358	3,07251	3,51144	3,95037	4,3893
8,5	0,46636	0,93273	1,39909	1,865453	2,331816	2,79818	3,264542	3,730905	4,197268	4,663631
9	0,4938	0,98759	1,48139	1,975185	2,468981	2,96278	3,456574	3,95037	4,444166	4,937963
9,5	0,52123	1,04246	1,56369	2,084918	2,606147	3,12738	3,648606	4,169835	4,691065	5,212294
10	0,54866	1,09733	1,64599	2,19465	2,743313	3,29198	3,840638	4,3893	4,937963	5,486625
10,5	0,5761	1,15219	1,72829	2,304383	2,880478	3,45657	4,03267	4,608765	5,184861	5,760957
11	0,60353	1,20706	1,81059	2,414115	3,017644	3,62117	4,224701	4,82823	5,431759	6,035288
11,5	0,63096	1,26192	1,89289	2,523848	3,15481	3,78577	4,416733	5,047695	5,678657	6,309619
12	0,6584	1,31679	1,97519	2,63358	3,291975	3,95037	4,608765	5,26716	5,925555	6,58395
12,5	0,68583	1,37166	2,05748	2,743313	3,429141	4,11497	4,800797	5,486625	6,172453	6,858282

TI/CCL	RF(K=25)	RF(K=50)	RF(K=75)	RF(K=100)	RF(K=125)	RF(K=150	RF(K=175)	RF(K=200)	RF(K=225)	RF(K=250)
13	0,71326	1,42652	2,13978	2,853045	3,566306	4,27957	4,992829	5,70609	6,419352	7,132613
13,5	0,74069	1,48139	2,22208	2,962778	3,703472	4,44417	5,184861	5,925555	6,66625	7,406944
14	0,76813	1,53626	2,30438	3,07251	3,840638	4,60877	5,376893	6,14502	6,913148	7,681275
14,5	0,79556	1,59112	2,38668	3,182243	3,977803	4,77336	5,568925	6,364485	7,160046	7,955607
15	0,82299	1,64599	2,46898	3,291975	4,114969	4,93796	5,760957	6,58395	7,406944	8,229938
15,5	0,85043	1,70085	2,55128	3,401708	4,252135	5,10256	5,952988	6,803415	7,653842	8,504269
16	0,87786	1,75572	2,63358	3,51144	4,3893	5,26716	6,14502	7,02288	7,90074	8,7786
16,5	0,90529	1,81059	2,71588	3,621173	4,526466	5,43176	6,337052	7,242345	8,147639	9,052932
17	0,93273	1,86545	2,79818	3,730905	4,663631	5,59636	6,529084	7,46181	8,394537	9,327263
17,5	0,96016	1,92032	2,88048	3,840638	4,800797	5,76096	6,721116	7,681275	8,641435	9,601594
18	0,98759	1,97519	2,96278	3,95037	4,937963	5,92556	6,913148	7,90074	8,888333	9,875925
18,5	1,01503	2,03005	3,04508	4,060103	5,075128	6,09015	7,10518	8,120205	9,135231	10,15026
19	1,04246	2,08492	3,12738	4,169835	5,212294	6,25475	7,297212	8,33967	9,382129	10,42459
19,5	1,06989	2,13978	3,20968	4,279568	5,34946	6,41935	7,489243	8,559135	9,629027	10,69892
20	1,09733	2,19465	3,29198	4,3893	5,486625	6,58395	7,681275	8,7786	9,875925	10,97325
20,5	1,12476	2,24952	3,37427	4,499033	5,623791	6,74855	7,873307	8,998065	10,12282	11,24758

Plotting the given data, we have the following hole cleaning chart:

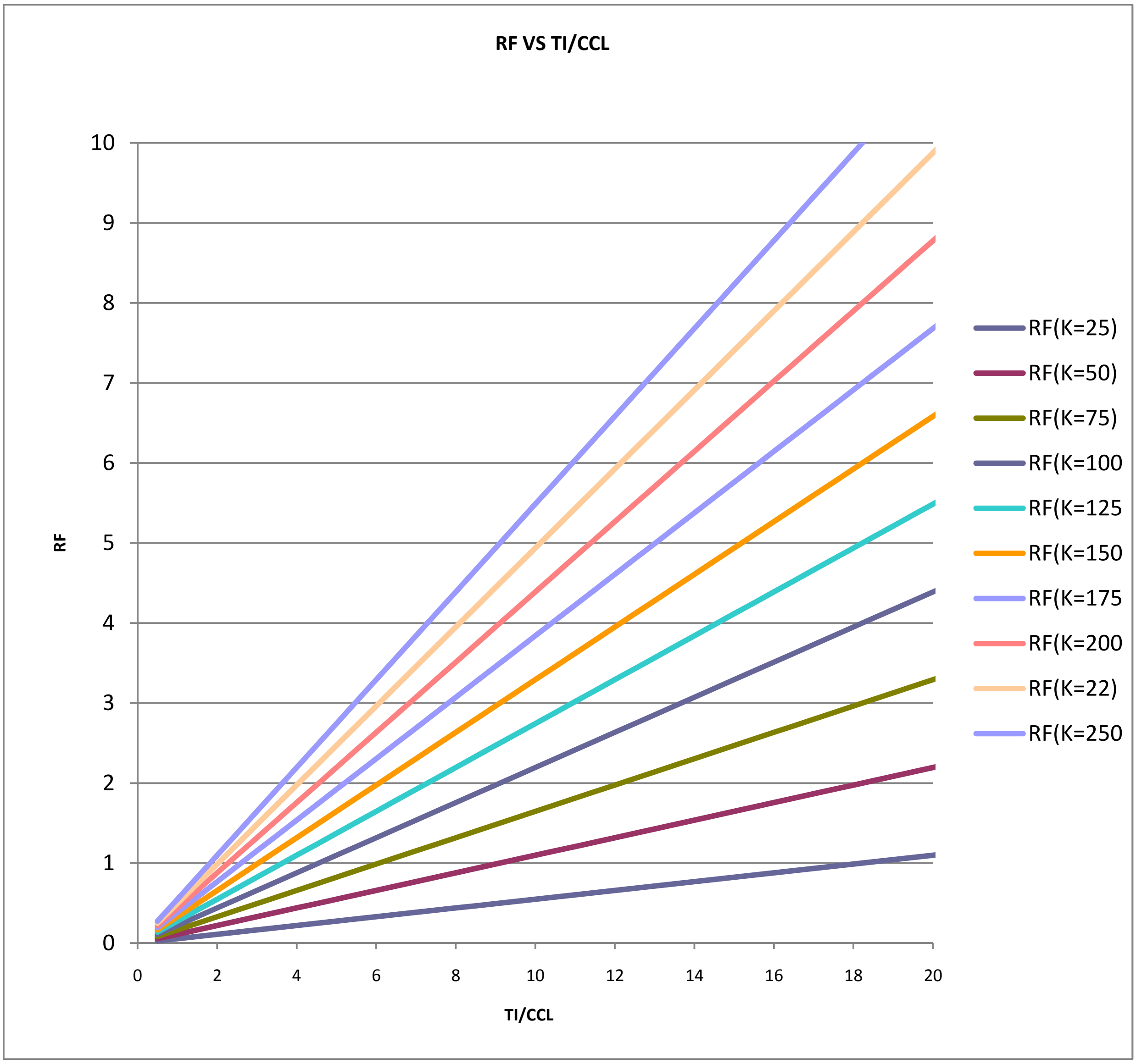

Figure 5. 1: Hole cleaning chart using the proposed model

Using the model, we can also predict the rheological properties i.e. the consistency index.

using the model, we can also predict the rheological properties i.e. the consistency index

Table 5. 2: Data set for predicting Rheological properties

K	RF(Ti/Ccl =4)	RF(Ti/Ccl =8)	RF(Ti/Ccl =12)	RF(Ti/Ccl =16)	RF(Ti/Ccl =20)	RF(Ti/Ccl =24)
0	0	0	0	0	0	0
25	0,21946501	0,43893002	0,65839503	0,877860041	1,097325051	1,316790061
50	0,43893002	0,877860041	1,316790061	1,755720081	2,194650101	2,633580122
75	0,65839503	1,316790061	1,975185091	2,633580122	3,291975152	3,950370183
100	0,87786004	1,755720081	2,633580122	3,511440162	4,389300203	5,267160243
125	1,09732505	2,194650101	3,291975152	4,389300203	5,486625254	6,583950304
150	1,31679006	2,633580122	3,950370183	5,267160243	6,583950304	7,900740365
175	1,53625507	3,072510142	4,608765213	6,145020284	7,681275355	9,217530426
200	1,75572008	3,511440162	5,267160243	7,022880325	8,778600406	10,53432049
250	2,1946501	4,389300203	6,583950304	8,778600406	10,97325051	13,16790061

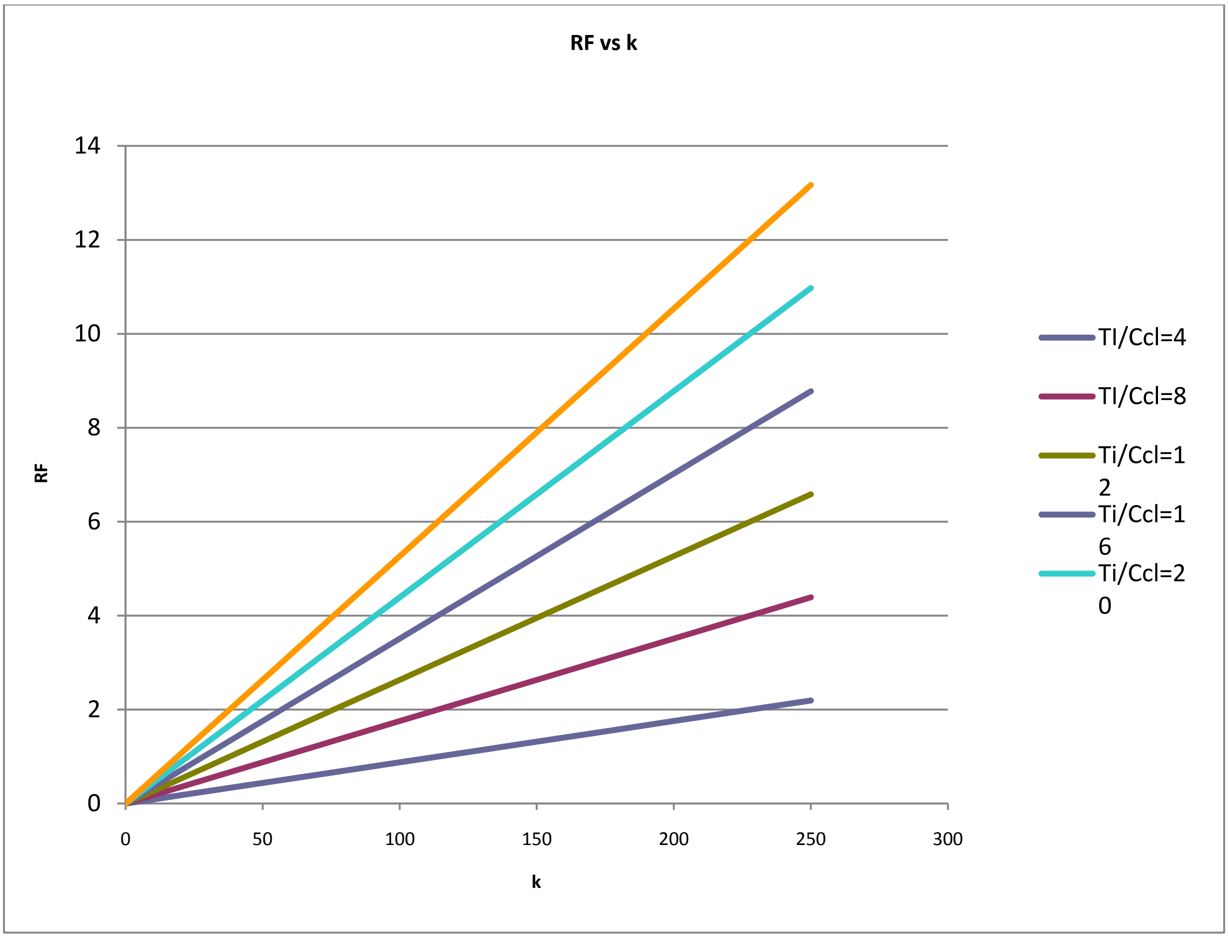

Figure 5. 2: Hole cleaning chart for predicting consistency factor

5.2 Evaluation of equivalent **AF in vertical sections-[3]**

Approximating the angle factor in vertical sections, using data from controlled experiments, we have the following calculations:

Data set 1

Table 5. 3: Data set 1 at an angle of about 30 degrees

Parameter	**Value**
PV	19cp
YP	$15 lb/100 ft^2$
ρ	$9.597\ ppg$
A_a	$0.28362\ ft^2$
Calculated k	321
Calculated RF	0.949
Calculated TI	1.506
Inclination	30.56

$$Q_{deviated} = \frac{834.5TI}{\rho RF} = \frac{1.506 \times 834.5}{9.597 \times 0.949} = 140 gal/\min$$

$$Q_{vertical} = \frac{400,000 A_a}{\rho k(0.13369)} = \frac{400,000 \times 0.28}{9.597 \times 321 \times 0.13369} = 275 gal/\min$$

$$\frac{1}{AF} Q_{ver} = Q_{Deviated} ------\Rightarrow \boxed{AF = 1.97}$$

Using data set two, we have:

Table 5. 4: Data set 2-at an angle of about 36 degrees

Parameter	**Value**
PV	19cp
YP	$15 lb/100 ft^2$
ρ	$9.597\ ppg$
A_a	$0.28362\ ft^2$
Calculated k	321
Calculated RF	0.949
Calculated TI	1.1.407
Inclination	36.13

$$Q_{deviated} = \frac{834.5TI}{\rho RF} = \frac{1.407 \times 834.5}{9.597 \times 0.949} = 129 gal / \min$$

$$Q_{vertical} = \frac{400,000 A_a}{\rho k (0.13369)} = \frac{400,000 \times 0.28}{9.597 \times 321 \times 0.13369} = 275 gal / \min$$

$$\frac{1}{AF} Q_{ver} = Q_{Deviated} ------\Rightarrow \boxed{AF = 2.1}$$

At an inclination of 45.31 degrees, we have:

Table 5. 5: Data set 3-at an angle of about 45 degrees

Parameter	**Value**
PV	19cp
YP	$17 lb / 100 ft^2$
ρ	$9.764 ppg$
A_a	$0.28362 ft^2$
Calculated k	407
Calculated RF	0.949
Calculated TI	1.421
Inclination	45.31

$$Q_{deviated} = \frac{834.5TI}{\rho RF} = \frac{1.421 \times 834.5}{9.674 \times 0.949} = 128 gal / \min$$

$$Q_{vertical} = \frac{400,000 A_a}{\rho k (0.13369)} = \frac{400,000 \times 0.28}{9.674 \times 407 \times 0.13369} = 214 gal / \min$$

$$\frac{1}{AF} Q_{ver} = Q_{Deviated} ------\Rightarrow \boxed{AF = 1.66}$$

Using data at an inclination of 54.88 degrees, we have:

Table 5. 6: Data set 4-at an angle of about 54 degrees

Parameter	**Value**
PV	19cp
YP	$17 lb/100 ft^2$
ρ	$9.764 ppg$
A_a	$0.28362 ft^2$
Calculated k	407
Calculated RF	0.949
Calculated TI	1.221
Inclination	54.88

$$Q_{deviated} = \frac{834.5 TI}{\rho RF} = \frac{1.221 \times 834.5}{9.674 \times 0.949} = 110 gal/\min$$

$$Q_{vertical} = \frac{400,000 A_a}{\rho k (0.13369)} = \frac{400,000 \times 0.28}{9.674 \times 407 \times 0.13369} = 214 gal/\min$$

$$\frac{1}{AF} Q_{ver} = Q_{Deviated} ------\Rightarrow \boxed{AF = 1.95}$$

Using data at an inclination of 60.44, we have

Table 5. 7: Data set 5- at an angle of about 60 degrees

Parameter	**Value**
PV	19cp
YP	$16 lb/100 ft^2$
ρ	$9.8474 ppg$
A_a	$0.28362 ft^2$
Calculated k	362.3
Calculated RF	0.949
Calculated TI	1.198
Inclination	60.44

$$Q_{deviated} = \frac{834.5 TI}{\rho RF} = \frac{1.198 \times 834.5}{9.8474 \times 0.949} = 107 gal/\min$$

$$Q_{vertical} = \frac{400,000A_a}{\rho k(0.13369)} = \frac{400,000 \times 0.28}{9.8474 \times 362.3 \times 0.13369} = 237 gal/\min$$

$$\frac{1}{AF} Q_{ver} = Q_{Deviated} ------\Rightarrow \boxed{AF = 2.2}$$

Finally, using data at an inclination of 71.75 degrees, we have:

Table 5. 8: Data set 6-at an angle of about 71 degrees

Parameter	**Value**
PV	19cp
YP	$16 lb/100 ft^2$
ρ	$9.8474 ppg$
A_a	$0.28362 ft^2$
Calculated k	362.3
Calculated RF	0.949
Calculated TI	1.142
Inclination	71.75

$$Q_{deviated} = \frac{834.5TI}{\rho RF} = \frac{1.142 \times 834.5}{9.8474 \times 0.949} = 102 gal/\min$$

$$Q_{vertical} = \frac{400,000A_a}{\rho k(0.13369)} = \frac{400,000 \times 0.28}{9.8474 \times 362.3 \times 0.13369} = 237 gal/\min$$

$$\frac{1}{AF} Q_{ver} = Q_{Deviated} ------\Rightarrow \boxed{AF = 2.32}$$

From the above calculations, we see that within the limits of experimental errors, we could take an average of the angle effect.

$$\frac{1.97 + 2.1 + 1.66 + 1.95 + 2.2 + 2.32}{6}$$

$$\Rightarrow \frac{12.2}{6} ---\Rightarrow 2.03$$

We see that the effect of angle in the vertical section approximates to two (2). Combining this with the angle factors derived by P.A Bern-[2], we have the following table:

Table 5. 9: Angle factor for different inclinations

Hole Angle	Angle Factor
0	2,03
25	1,51
30	1,39
35	1,31
40	1,24
45	1,18
50	1,14
55	1,10
60	1,07
65	1,05
70~80	1,02
80~90	1,0

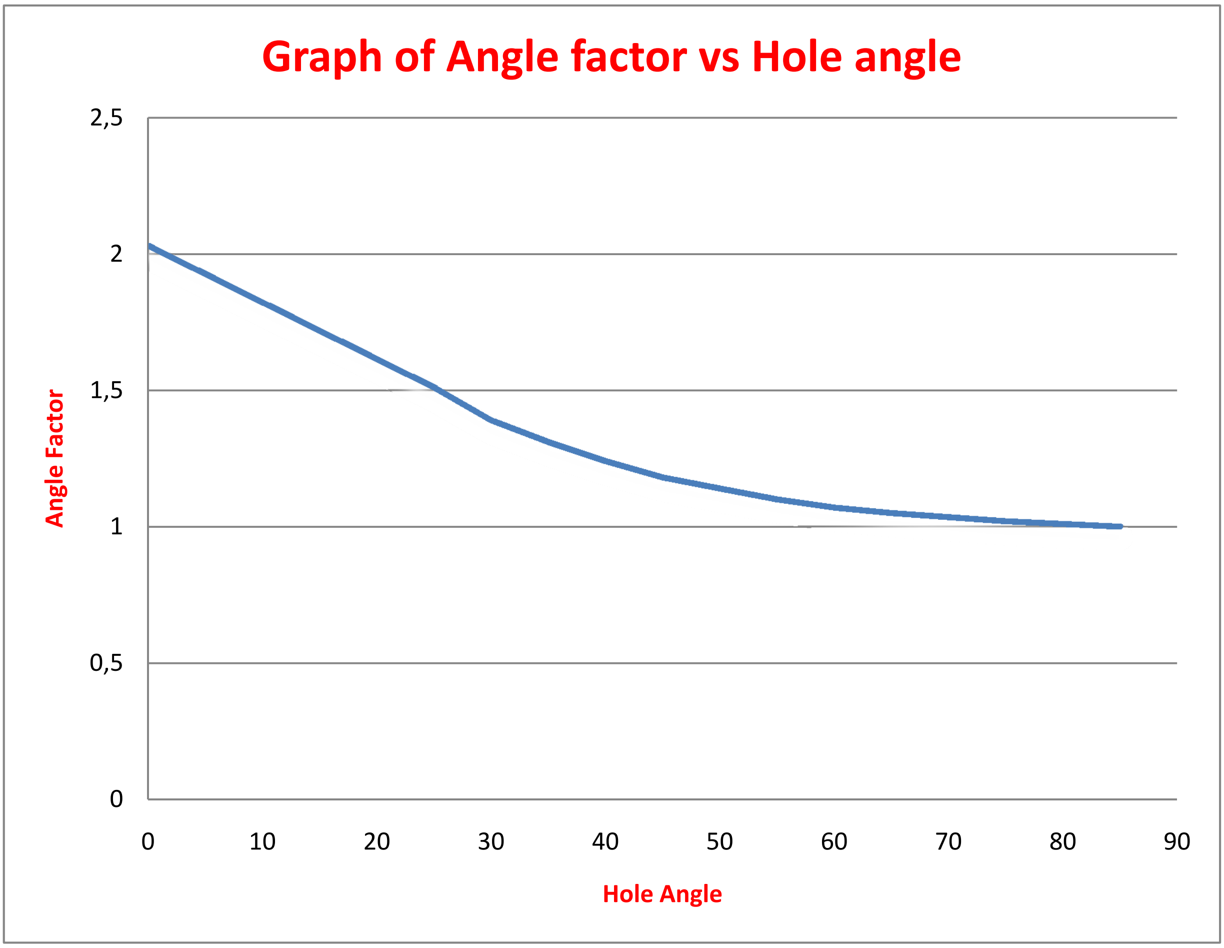

Figure 5. 3: Plot of angle factor versus the hole angle

5.3 Application of Vertical hole washout correction factor

It is known that washout in hole tend to reduce the velocity, resulting in poor hole cleaning. So the need arises to derive a correction factor to be used to correct for the effect of hole washout. This factor will help increase the rates such that efficient hole cleaning is achieved.

From appendix C, a correction factor is derived thus:

$$Q_2 = \left[\frac{D_2}{D_1}\right]^2 Q_1 \qquad 5.2$$

Where α is given by

$$\alpha = \left[\frac{D_2}{D_1}\right]^2 \qquad 5.3$$

Applying the derived formula to an 8 ½, 12 ¼, and 17 ½, for different hole washout sizes we have the following tables:

Applying to a 8 ½ inch hole,

Table 5. 10: Correction factors in 8 ½ wellbore

8 ½ inch Gauge hole size	
Washout size	α
9	1.12
10	1.38
11	1.67
12	1.99
13	2.34
14	2.71

Applying to a 12 ¼ inch hole:

Table 5. 11: Correction factors in 12 ¼ inch wellbore

12 ¼ inch gauge hole size	
Washout size	α
13	1.13
14	1.31
15	1.49
16	1.71
17	1.92
18	2.16

Finally, applying to 17 ½ inch hole size.

Table 5. 12: Correction factors in 17 ½ inch wellbore

17 ½ inch gauge hole size	
Washout size	α
18	1.06
19	1.18
20	1.31
21	1.44
22	1.58
23	1.73

Chapter 6: Conclusions & Recommendations

Hole cleaning and hydraulics play a critical role in the design and drilling of vertical, directional and horizontal wells, and can limit the reach of many wells especially for wells with long horizontal reach and angle greater than 60 degrees. This work concentrated on developing models for efficient hole in vertical and deviated wells.

From the extensive research and reviews done in chapters 2 and 3, the following conclusions have been drawn:

- Drill pipe rotation improves hole cleaning by keeping cuttings in suspension. This effect is also a function of the hole angle and cuttings properties.
- The minimum in-situ velocity needed for efficient hole cleaning increases with the hole angle and is highest at about 60 degrees. Making hole cleaning most difficult at angle close to 60 degrees.
- Annular cuttings concentration decreases as the flow rate increases, resulting in decrease in bed height as rate increases.
- An unbalanced hydraulic jet force improves hole cleaning, i.e. use of bit nozzles of different sizes or blanking off one nozzle in a three nozzle bit.

New models for the approximation of Rheological factor in vertical sections, approximation of angle factor in vertical wells and corrections for hole washout were derived in chapter 4. A hole cleaning chart was plotted in chapter 5 and the procedure for using the charts are given in chapter 5.

A mathematical solution for the approximation of the angle factor in vertical sections was done and the approximation gave a value of 2.03.

Finally, the model for the correction of hole washout in vertical sections was used to generate real data that can be applied in the field when the hole is suspected of being washed out.

I would recommend further studies to the models presented in this thesis, either as a thesis work or a further research work, with a flow loop being used to verify the models apart from the use of field data for verification which has been done in this work.

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Appendix

Appendix A: Derivation of A.8 Equation (Model 1)

Derivation of Model equation using simple relations and existing equations:

$$Q = VA \quad \Rightarrow \quad V_a = \frac{Q_a}{A_a} \quad \left[\frac{ft^3/gal}{ft^2}\right] \qquad \text{A. 1}$$

From API standards for hole cleaning in vertical wells, we have:

$$Ccl = \frac{\rho k V_a}{400,000} \quad (\text{For } \theta \leq 25) \qquad \text{A. 2}$$

Also from Bern's work or the API Standards for cleaning of deviated wells, we have:

$$TI = \frac{Q\rho RF}{834.5} \quad (\text{for } \theta \geq 26) \qquad \text{A. 3}$$

Assuming the flow rate to be constant, converting, we have:

$$Ccl = \frac{\rho k Q_a (0.13369)}{400,000 A_a} \qquad \text{A. 4}$$

$$Q_a = \frac{2991996.4 A_a Ccl}{\rho k} \qquad \text{A. 5}$$

From the deviated equation, we have that:

$$Q_a = \frac{834.5 TI}{\rho RF} ----------[2] \qquad \text{A. 6}$$

Equating, we have:

$$\frac{2991996.4 A_a Ccl}{\rho k} = \frac{834.5 TI}{\rho RF}$$

$$TI = \frac{400,000 A_a Ccl RF}{111.56 k} \qquad \text{A. 7}$$

$$RF = \frac{111.56 k}{400,000 A_a}\left[\frac{TI}{Ccl}\right]$$

$$\boxed{RF = \frac{k}{3585 A_a}\left[\frac{TI}{Ccl}\right]} -------- Model(1) \qquad \text{A. 8}$$

Where A_a is in ft^2 and k is the consistency factor.

Appendix B: Derivation of B.7 Equation (Peter Bern's Equations)

Using the factors affecting hole cleaning and the dimensional analysis.

From known equations:

$$g_{Force} = g_c(\rho_s - \rho_f)Sin\theta \qquad \text{B. 1}$$

$$V_{cri} = \left(\frac{\tau_{wc}}{\rho_f}\right)^{0.5}$$

Table B. 1: Variables affecting cuttings transport

VARIABLES	SYMBOL	UNIT
Apparent Viscosity (Pa.s)	μ	$ML^{-1}T^{-1}$
Diameter of cutting (mm)	d_s	L
Gravitational Force	g_F	$ML^{-2}T^{-2}$
Annular gap Width	W	L
Critical wall friction Velocity	V_{cri}	LT^{-1}
Density of cuttings	ρ_s	ML^{-3}
Density of fluid	ρ_f	ML^{-3}

MODELLING APPROACH

Table B. 2: Dimensionless variables

	μ	d_s	g_F	W	V_{cri}	ρ_s	ρ_f
L	**-1**	**1**	**-2**	**1**	**1**	**-3**	**-3**
T	**-1**	**0**	**-2**	**0**	**-1**	**0**	**0**
M	**-1**	**0**	**1**	**0**	**0**	**1**	**1**

Choosing three (3) independent variables and verifying using the determinant method, we have:

Checking for d_s , V_{cri} and ρ_f

Table B. 3: Three independent variables

	d_s	V_{cri}	ρ_f
L	1	1	-3
T	0	-1	0
M	0	0	1

$$\Rightarrow \begin{pmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \left|\begin{pmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right| \Rightarrow 1\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow 1 \times -1 = -1$$

With a determinant of -1, showing that the three variables are independent of each other, using the three pair to solve, we have:

Solving for μ

$$a\overrightarrow{d_s} + b\overrightarrow{V_{cri}} + c\overrightarrow{\rho_f} = \overrightarrow{\mu}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{pmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

a + b -3c = -1 ⇒ a=1

0 –b + 0 = -1 ⇒ b=1

0 + 0 + c=1 ⇒ c=1

$$\overrightarrow{d_s} + \overrightarrow{V_{cri}} + \overrightarrow{\rho_f} = \overrightarrow{\mu}$$

$$\boxed{\pi_1 = \frac{d_s V_{cri} \rho_f}{\mu}} \qquad \text{B. 2}$$

Solving for the gravitational force g_F, we have:

$$a\overrightarrow{d_s} + b\overrightarrow{V_{cri}} + c\overrightarrow{\rho_f} = \overrightarrow{g_F}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{pmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} -2 \\ -2 \\ +1 \end{bmatrix}$$

a + b -3c = -2 ⇒ a=-1

0 –b + 0 = -2 ⇒ b=2

0 + 0 + c=1 ⇒ c=1

$$\overrightarrow{-d_s} + 2\overrightarrow{V_{cri}} + \overrightarrow{\rho_f} = \overrightarrow{g_F}$$

Giving the following

$$\boxed{\pi_2 = \frac{V_{Cri}^2 \rho_f}{d_s g_F}} \qquad \text{B. 3}$$

Substitute for $g_{Force} = g_c(\rho_s - \rho_f)Sin\theta$ and $V_{cri} = \left(\frac{\tau_{wc}}{\rho_f}\right)^{0.5}$, we have

$$\boxed{\pi_2 = \frac{\tau_{wc}}{d_s g(\rho_s - \rho_f)\sin\theta}} \qquad \text{B. 4}$$

Solving for the annular gap width, we have:

$$a\overrightarrow{d_s} + b\overrightarrow{V_{cri}} + c\overrightarrow{\rho_f} = \overrightarrow{W}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{pmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

a + b -3c = 1 $\Rightarrow$ a=-1

0 –b + 0 = 0 $\Rightarrow$ b=0

0 + 0 + c=0 $\Rightarrow$ c=0

$$\overrightarrow{d_s} = \overrightarrow{W}$$

So we have:

$$\boxed{\pi_3 = \frac{d_s}{W}} \qquad \text{B. 5}$$

Finally, solving for the cuttings density, we have:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{pmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

Solving the equations, we have a=0, b=0 and c=1, giving:

$$\overrightarrow{\rho_f} = \overrightarrow{\rho_s} \quad \Rightarrow \quad \boxed{\pi_4 = \frac{\rho_f}{\rho_s}} \qquad \text{B. 6}$$

Using the pi theorem,

$$f(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

From experiments, it has been seen that π_3 and π_4 can be neglected, so we have the following:

$\pi_2 = A\pi_1^B$ Where A and B are constants, substituting for π_1 and π_2, we have

$$\boxed{\frac{\tau_{wc}}{d_s g(\rho_s - \rho_f)\sin\theta} = A\left[\frac{d_s V_c \rho_f}{\mu}\right]^B} \qquad \text{B. 7}$$

Appendix C: Derivation of C.5 Equation (Model)

Assuming flow rate in gauge hole = Q_{old}

Assuming flow rate in washout hole = Q_{new}

Knowing that:

$$Q = vA \quad \text{C. 1}$$

So we have

$$Q_1 = v_1 A_1$$

And

$$Q_2 = v_2 A_2$$

Now, a washout effect will reduce the equivalent velocity in the washed areas, so to find an appropriate correction factor, we need to maintain the same velocity to obtain good hole cleaning, so keeping the velocity constant, we have:

$$v_1 = v_2 = \frac{Q_1}{A_1} = \frac{Q_2}{A_2} \quad \text{C. 2}$$

Expressing in terms of the new required flow rate, we have:

$$Q_2 = Q_1 \frac{A_2}{A_1} \quad \text{C. 3}$$

But we know that-

$$A = \frac{\pi D^2}{4},$$

Substituting,

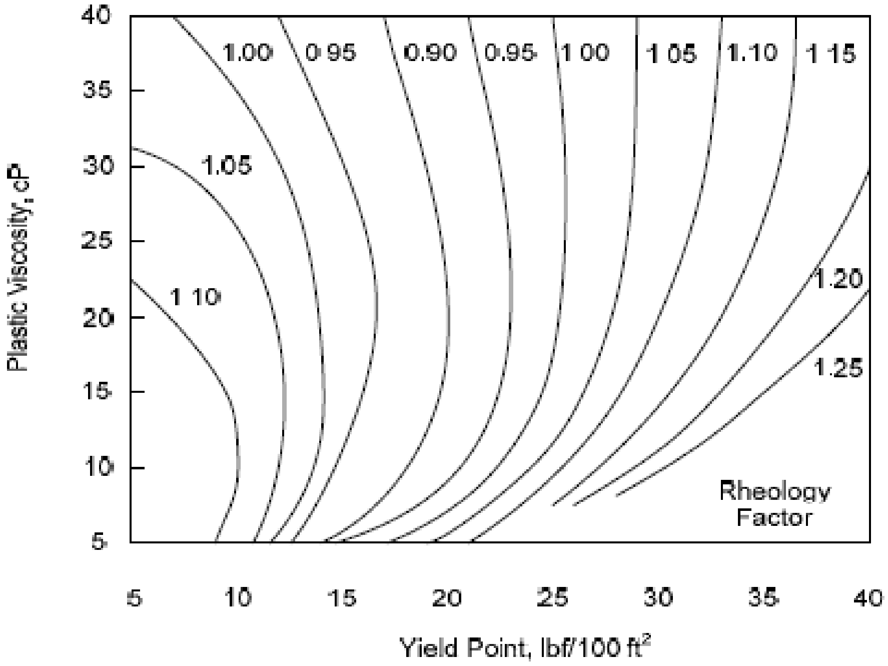
$$Q_2 = Q_1 \frac{4\pi D_2^2}{4\pi D_1^2}$$

$$Q_2 = Q_1 \frac{D_2^2}{D_1^2} \quad \text{C. 4}$$

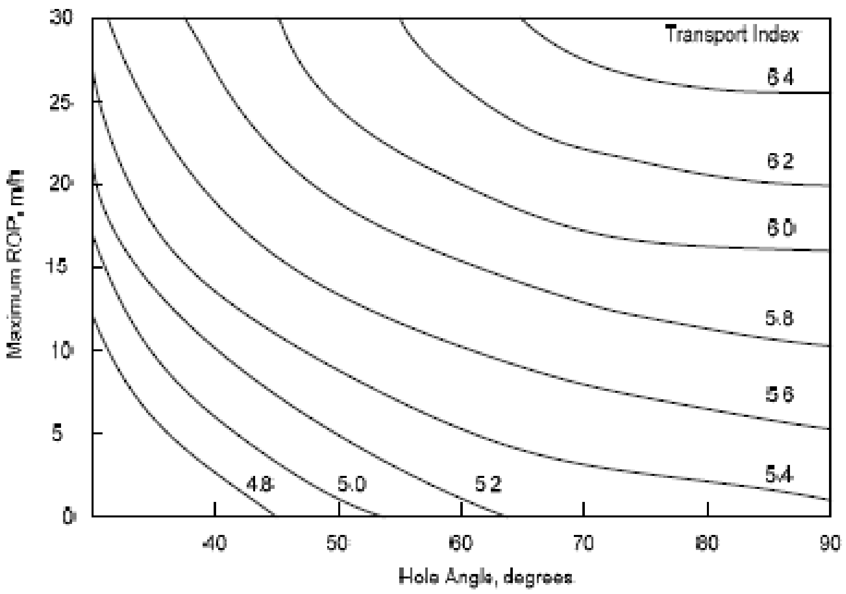
Where the correction factor α is given by:

$$\alpha = \frac{D_2^2}{D_1^2} = \left[\frac{D_2}{D_1}\right]^2 \quad \text{C. 5}$$

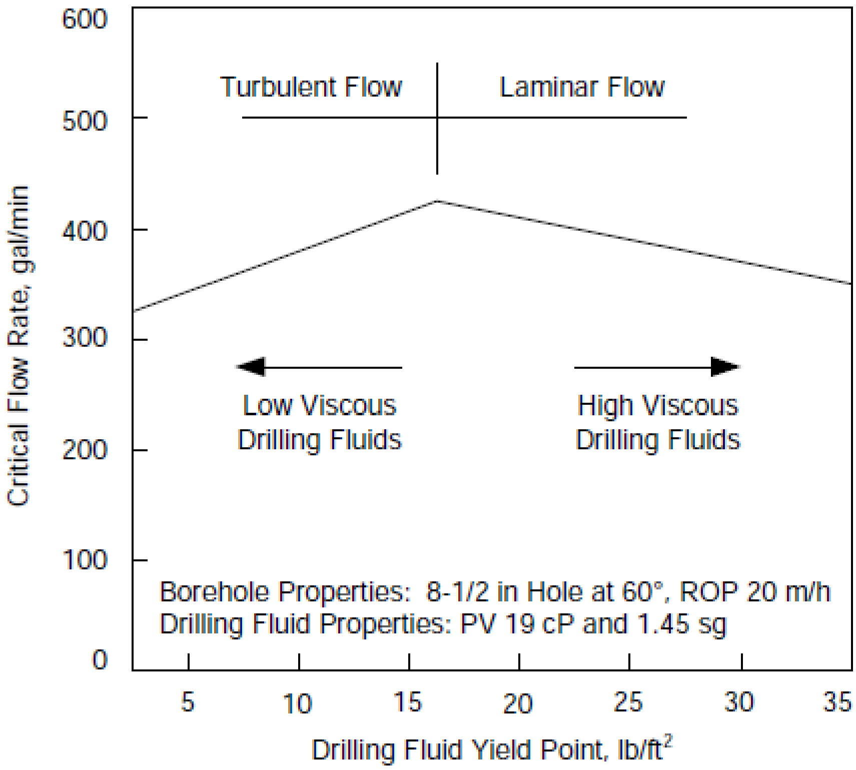
Appendix D: API Hole Cleaning charts



Rheology factor chart for 8-1/2 in. holes



Hole-Cleaning Chart for 8-1/2 in. Holes



Effect of yield point on critical flow rate