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Analysis of borehole failure related to bedding plane

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ABSTRACT

Borehole instabilities during drilling are more common in shale formations than in most other rock formations. Shale make up more than 80% of sediments and rocks in siliciclastic environments and about three quarters of borehole problems are caused by shale instability.
The assessment of in-situ stress and analysis of borehole failure due to instability and weak bedding plane represents one of the most critical factors when evaluating borehole stability that causes borehole failure. Significant amount of research have been done in this area which resulted in various mathematical models about the issue of borehole failure, stability and plane of weakness due to bedding. Especially Aadnoy, Chenevert , Jaeger and Zoback showed that at a certain angle rock failed at a very low load condition. Several material constitutive models have been considered for rock failure studies, including the Mohr-Coulomb, the Mogi-Coulomb, the Mohr-Coulomb elasto-plastic, Dracker-Prager, and the modified Lade models by all researchers. Shale instability is an extremely unpredictable and potentially costly problem in many foothill drilling operations still now. So far, unified decision about the plane of weakness and failure of borehole on shale is yet to be fully realized by the industry.

This paper analyzed the bedding plane failure and reproduced some of the results published in literature. This works studied based on the Aadnoy et.al. (2009) paper's field data and reproduced their combination of parameters that create bedding exposed positions. This thesis paper is based on a linear elastic and isotropic model for stresses around the wellbore, with the aim of trying to understand the general behavior of inclined boreholes due to anisotropy. It was found that borehole collapse was caused predominantly mainly by shear but also by tensile failure. The analysis remarkably found that for a laminated rock, a weakness of a plane may subject the well toward collapse for the hole angles between 10 to 40^0 (Aadnoy and Chenevert 1987).

This paper analyzed the 3D effect of attack angle with changing azimuth for a constant inclination on bedding plane. It is seen that bedding exposed is not only depends on inclination but also depends on dip of the formation, attack angle and azimuth. This paper also made a model which is enhanced Aadnoy et.al. (2009) model so that users can get the optimized well path and can make sure whether their well data has existed on the bedding exposed or protected positions. This thesis has tried to focus on mechanical wellbore stability and plane of weakness of shale formation and analyzed the Aadnoy et.al (2009) models to address the existing problem on this matter.

PREFACE

This thesis works have been prepared in order to partially fulfil the requirements for the degree of Master's Program in petroleum engineering. This thesis work has been carried out from January, 2011 to june, 2011 under the department of Petroleum Engineering, University of Stavanger, Norway. This thesis work was running under the BUET-NTNU linkage program (NOMA Scholarship). I am highly indebted to my main supervisor Dr. Eirik Karstad, Associate professor, Department of Petroleum engineering,University Stavanger, Norway and external Co-supervisor Dr. Mahbubur Rahman, Associate professor, department of PMRE, BUET, Bangladesh for giving me the opportunity to work on this topic. Their immense support and guidance throughout the thesis time is highly appreciated. Most importantly, I appreciate their open door policy toward me. I was welcomed at any time of the day and days of weeks. I needed to knock at the door frequently for discussing valuable issues, materials, suggestions for making the research paper comprehensive and valuable. they were very enthusiastic and willing to give advice. Sincerely speaking, I wish to continue this work in some sort of way to become an expert in this part of drilling engineering which is most important to the petroleum industry.

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I would like to acknowledge department of PMRE, BUET, Bangladesh and the UiS, Norway faculty for the fully funded scholarship by NOMA, Norway project. I greatly benefitted from them when studying on M.Sc. in Petroleum Engineering and working and in discussions on research related matters.

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Lastly and mostly, I thank to my wife, Habiba Sultana, for her patience during my thesis time and long study hours, making her distant. She helped me a lot about the technique of creating table of index, table of figures by MS word and MS excel. My sweet wife, missed me a lot as I would studied at late night, I am enormously indebted to my wife for understanding my hard work hours at the late Night. Habiba Sultana is not only a source of great inspiration but also the main source of pleasure and liveliness in my daily life. I shared my feelings with my parents, my brother, my sisters, close relatives and friends during this stressful time. Thanks to all of them for their mental support. I am also thankful to my parents for their support and care throughout my life. Their support and encouragement was, in the end, what made this thesis writing possible.

Md. Shamsuzzoha
Dhaka, June-2011, Bangladesh

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NOMENCLATURE

Symbol	Meaning & unit	Symbol	Meaning & unit
σ_v	Overburden In-situ stress, MPa	$\sigma_1,\sigma_2,\sigma_3$	Major, intermediate and minor Principal stress, MPa
σ_h	Minimum In-situ Horizontal stress, MPa	σ_θ, σ_r	Hoop or tangential and radial stress, MPa
σ_H	Maximum In-situ Horizontal stress, MPa	k	Effective Stress ratio
Pp	Pore Pressure, MPa	σ'	Effective Normal stress, MPa
Pwf	Fracture Pressure, MPa	$\sigma_{m,2}$	Mean effective stress, MPa
α_β	Biot Constant	$\sigma_{\theta z}$	Borehole shear stress, MPa
γ	Borehole deviation from vertical, Degrees	$\tau_{r\theta}$	Shear stress in radial direction, MPa
μ	Poisson's ratio	σ_x, σ_y	Virgin Formation stress at Borehole coordinate system (x-y axis)
a_z	Borehole azimuth angle from σ_H, degrees, or from North (Clockwise)	σ_{zz}	Virgin Formation stress at Borehole coordinate system (z-axis)
ϕ	Angle of internal Friction, degrees	σ_z	far field axial stress (Vertical)
Θ	Angular position around Borehole from x axis (Angle made by failure plane and minimum principal stress), degrees	FP	Fracture Pressure Gradient; Pa/m
β	Inclination of bedding to specimen axis, degrees or angle between applied force and bedding plane during tri-axial core testing, degrees	Aat	Attack angle, degrees
Ψ	Dilating angle, Degrees	β	Angle between applied force and normal to bedding plane (for **Chapter-2**)
A	Angle Between applied force and failure plane during triaxial core testing, degrees	g	Earth Gravity, m2/s
P_w	Well Pressure, Mpa		
P_{wc}	Critical Collapse Pressure, Mpa	LOP	Leak of Pressure, Mpa
Af	Angle of Failure Plane with wellbore	τ_o	Cohesive strength of material, Mpa
σ_t	Tensile strength of Rock of being fractured, Mpa	t	Plastic Thickness
P_{ep}	Elasto Plastic Barrier	a, φ	Well size, Porosity

ABBREVIATION

CPP	Collapse pressure prediction
Cp/CP	Collapse pressure
DEM	Discrete element modeling
DITF	Drill induced tensile failure
FEM	Finite element modeling
GMM	Geo-mechanical model
LOP	leak of Pressure
MPD	Managed pressure drilling
MW	Mud weight
MEM	Mechanical Earth Modeling
MWW	Mud weight window
M-C	Mohr-Coulomb
NF	Normal fault
OBD	Overbalanced drilling
OBM	Oil based mud
PP/Pf	Pore pressure
RF	Reverse-Fault
SS	Strike-Slip
Theta	Relative position of horizontal stresses
UBD	Underbalanced drilling
UCS	Uni-axial compressive strength
USM	Uniaxial strain Model
WMB	Water based mud

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CHAPTER 1 INTRODUCTION TO BEDDING PLANE

1 Introduction

The increasing demand for well bore stability analyses during the planning stage of a field arises from economic considerations and the escalating use of deviated, extended reach and horizontal wells. Well bore instability can result in lost circulation (**Figure l.6a**) where tensile failure has occurred, and spilling and/ or hole closure (**Figure1.6b**) in the case of compressive failure. The purpose of Wellbore Stability modeling is to create a safe operating window of annular pressures (mud pressures and mud weight) such that the designed fluid is high enough to ensure wellbore stability and low enough to ensure no loss of fluid. The drilling mud weight and the mud composition are two key operational parameters that can be adjusted to prevent hole-instability problems. Borehole instability is one of the largest sources of trouble and additional costs during drilling. Problems generally build up in time, starting with the fragmentation of the borehole wall, followed by transfer of the fragments to the annulus and finally-if hole cleaning is insufficient-culminating in such difficulties as a tight hole, packing off, filling of the hole, stuck pipe, etc. Drilling Problems are not often experienced in initial vertical exploration and appraisal wells. But drilling a highly deviated or even horizontal developments wells is prone to instability problems. The ultimate consequences may include losing the hole, having to side-track, an inability to log the well and poor cementations because of excessive washouts. New technologies, such as horizontal drilling and coiled tubing drilling, will not resolve borehole-instability problems; they too will suffer from borehole instability at least as much as conventional drilling.

To determine the safe mud window, The follwing data is used as a main input parameter for developing a model for ensuring against borehole collapse/fracture:

1. The insitu stress : overburden stress, maximum horizontal stress, Minimum horizonatal stress;
2. Pore pressure;
3. In-situ stress orientation;
4. Wellbore trajectory and bedding and weakplane directions;
5. Relevant rock stregth Data.

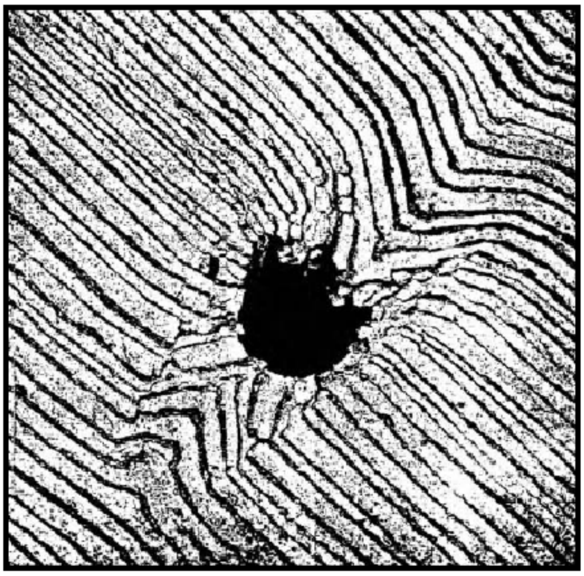
This thesis work is analyzed in details about the **number fourth point** above. Understanding the stress behavior of rocks is critical for drilling and completing vertical, horizontal and highly deviated wells. Rocks at a given depth in the earth's crust are exposed to compressive stresses of relatively large magnitude, vertically and horizontally, as well as to a pore pressure. These in-situ stresses are caused by the weight of the rock and by the confining lateral restraints. When a hole is drilled, the surrounding rock deforms slightly because of the stress relief induced by the cavity. For the rocks that behave linearly elastic, this leads to a stress concentration near the well. To balance the formation pore pressure and prevent rock failure, the well is usually filled with mud which offsets parts of the stress concentration. The Mud density can't, however, be increased by a large margin since this causes hydraulic fracturing of the formation, and a Potential lost circulation. According to A.A. Garrouch et. al., (2001), Addis et.al. (1990) the in-situ stress field in shales is different from that in adjacent sandstones or Carbonates. Many Shales are somewhat ductile, they tend to flow rather than fracture through bedding plane as a low load as the certain condition. As a consequence horizontal stresses become sometimes much closer in magnitude to the vertical stress than in Sandstones or Carbonates. This means horizontal stresses tend to be higher in the shale than in adjacent Sandstones and Carbonates. The two commonly rock failure criteria Such as Mohr-coloumb, Druker-Prager are used to wellbore stability computations, Maclean-Addis (1990). Mohr-coulomb criterion assumes that the intermediate principal stress has no effect on rock strength. On the other hand, Drucker-Prager criterion gives just as much as weight to the intermediate principal stress as it does to the major and minor principal stress.

Sedimentary rocks have a laminated structure, with directional elastic properties as well as directional shear and tensile strengths. To understand the fields situations better, a complete mathematical model have to be developed that takes into account all directional properties. Strength properties of bedded rocks have been known for some time. Anderson (1951) presented an early analysis of the phenomenon. Jaeger (1960) gives a thorough analysis of the various loading scenario that explain bedding failure. In particular, Jaeger includes friction in his analysis. A common way to model shear failure using Jaeger's approach is to use the Mohr Coulomb failure model, but vary the cohesive strength and the angle of internal friction, depending on the loading relative to bedding plane inclination.

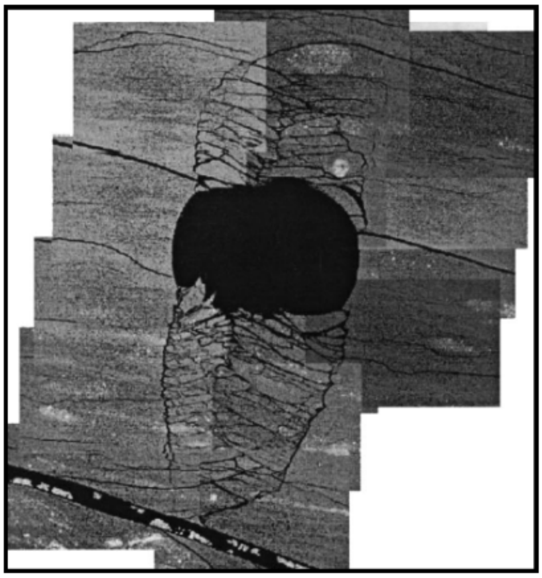
The **plane of weakness** was introduced in the oil industry by Aadnoy (1988). In modeling highly inclined boreholes, he investigated the effects of wellbore inclination, anisotropic elastic rock properties, anisotropic stresses, and anisotropic rock strength. It was shown that under certain conditions, the rock would fail along planes of weakness. Because of the geo-mechanical properties of shale (common high pore pressure, alignment of phyllosilicates due to overburden digenesis), slip surfaces may exhibit significantly more potential to fail as compared to stronger rock units, such as limestone and sandstone. For this reason, shale instability is an extremely important and potentially costly problem in many foothills drilling operations. From Aadnoy et.al (2009) paper, it is got, Layered rocks such as shale often exhibit different properties along or across bedding planes. Elastic properties like bulk modulus, Young's modulus and Poisson's ratio, show directional properties. The same can be concluded for compressive and tensile rock strength. From Literature, Rock strength is high when force vectors are applied at a high angle to bedding. At lower angles, on the order of 15° and 30°, stratal compressive strength is low. For this case, rock failure will occur along bedding planes. This type of rock behavior is often termed **"plane of weakness"**.

Wellbore instability is the primary cause of losses in boreholes and represents a serious challenge in the drilling industry. Instability is mechanical (Compressive) failure of formation surrounding the wellbore resulting wellbore enlargements which contribute to hole collapse, excess cuttings and hydraulic problems. Parameters affecting stability are optimum mud weight, Well trajectory for optimum mud weight, weak bedding Plane, time dependent- shale Fluid penetration, reactive Shale. This Paper is covered about the **weak bedding plane problem**. Drilling along bedding planes and in depleted reservoirs is risky, and when a well is drilled at shallow angles to thinly bedded shale, it is often highly unstable. Rock failure can occur as a result of large anisotropy in rock strength caused by bedding parallel weak planes. In these cases, an increased mud weight while drilling is required. However, when the reservoir immediately beneath the bedded shale is depleted, the increased mud weight can lead to lost circulation.

The following **Fig. 1.1 (a)** has shown wellbore buckling deformation and failure when penetrating horizontal or steeply dipping thinly-cycled beds which are shown by Bandis in 1987 and Barton in 2007. And wellbore failure obtained by a laboratory tests in shale with slightly dipping bedding is shown by **Fig 1.1 (b)** (okland and cook, 1998).



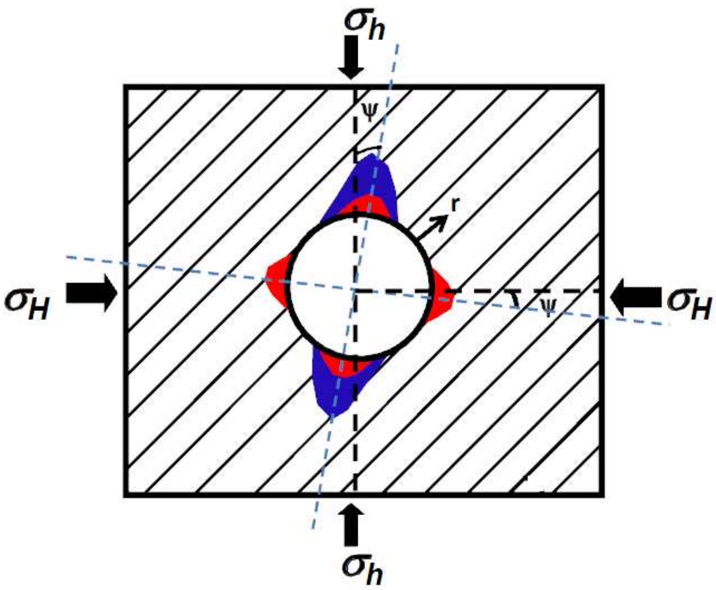
(a)



(b)

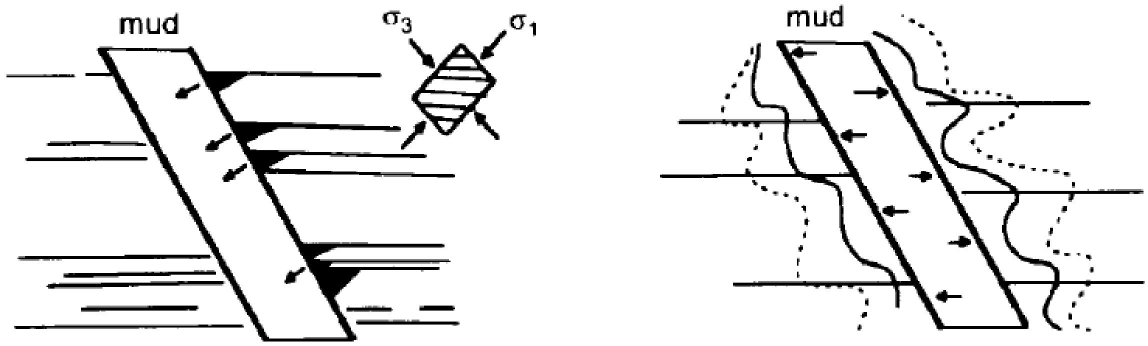
Figur 1-1 Well bore failure in formations with Bedding Plane (James lang et.al. 2011)

The following **figure 1.2** is shown that the maximum slip failure direction is not being no longer parallel to the horizontal stress direction, but with an angle of (ψ) to the minimum and maximum horizontal stress directions. The red area represents the failure caused by the slip failure in the **weak planes**. The blue area shows a schematic failure zone caused by both slip failure in weak planes and the shear failure of rock.



Figur 1-2 Wellbore shear failure and slip failure caused by the Weak planes

Fig. 1.3 shows the key destabilizing (due to weak bedding plane) mechanisms thought to be relevant to drilling deviated wells through fissile strata such as the Fernie. **Fig. 1.3a** shown fissile shale with weak bedding plane and **Fig.1.3b** shown pore pressure penetrates along bedding planes (i.e. overbalanced). One can see the following **Fig. 1.4**, is an example of directional stress wellbore failure cause weak bedding plane.



(a) **(b)**

Figur 1-3 Destabilizing mechanizm while drilling deviated well due to bedding

(P.J. Mclellan et .al. 1996)

Figur 1-4 Borehole Breakout due to a directional

There are several types of failure occurred due to bedding problem; these may be taken place along or across bedding plane either collapse or plastic/slip flow or king flow consists of a rotation of bedding plane. These phenomena shown in the **Fig. 1-5**

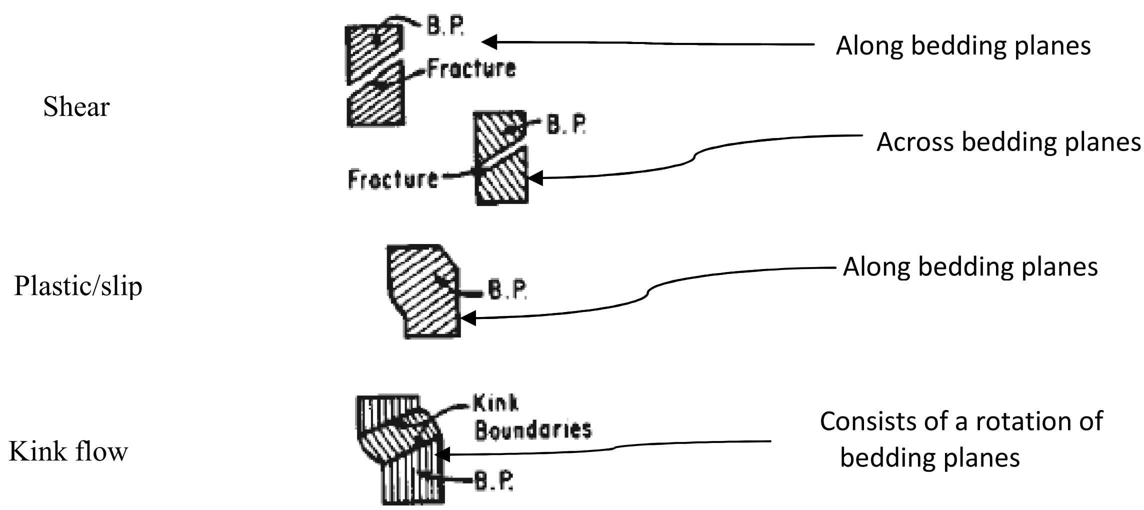


Figure 1 -5 types of failure (Mclamore et.al.1967)

The key solutions are to not only improve wellbore stability modeling associated with bedding planes, rock anisotropy, and pressure depletion, but also to account for their impact on horizontal stresses. This thesis work has developed models, are enabling calculations of wellbore failures along borehole trajectories with various drilling orientations versus bedding directions. The model has been verified by case studies as Aadnoy et.al (2009) Paper. The minimum stress and fracture gradient calculations are also considered for the rock anisotropy and the depletion. If shale is heterogeneous and in-situ stresses are anisotropic, borehole breakout could be more vicious and unpredictable. This is particular important when a well penetrates faults or **weak bedding planes**.

1.1 Objective of this Study

The overall objective of this thesis is to improve real-time down-hole knowledge of shale formation, achieve higher drilling efficiency through the use of geo-mechanical and borehole instability modeling. The focus of this study is about the weak bedding plane, borehole failure by bedding plane and analyzes these according to the paper of Aadnoy et.al. (1988, 2009) and Chenevert M.E. et.al. (1965, 1987). The aim of the thesis work is also to reproduce the result of their paper and give valuable comments at the end of this study. This paper focuses to enhance the Aadnoy et.al. (2009) model for users friendly and introduce the optimum well path so that any user can use during drilling as a quick reference. This paper discusses 3D effect of attack angle versus azimuth and clarify different variable (angle) about the Aadnoy et.al. Model so that users can understand easily. This works review different models considering the failure analysis by different Scientists and researchers and find out the challenge and applicability of their works.

1.2 Structure of the Study

This thesis works contain 7 chapters. Chapter 1 discussed about introduction of well bore stability and weak bedding plane, problem of weak bedding plane and different literature about bedding plane and borehole failure. Chapter 2 focused on the theory of Mohr-coulomb and plane of weakness and failure of bedding plane, anisotropy, minimum mud weight to face instability due to collapse. In Chapter 3 in-situ stresses, its measurements, its regime and fault of the formation were covered. Details available model and failure criterion of borehole, time delay failure and input source of geo-mechanical modeling were covered in Chapter 4. The literature of bedding plane and effect of bedding plane failure, Stress formation of a wellbore, direction and angle of different plane and the behavior of attack angle (3D effect) of different azimuth with constant inclination were discussed in Chapter 5. Also, the optimum well path and some parameters were talked about in this chapter. The Papers of Aadnoy et.al. (1988, 2009) and Chenevert (1965) were reviewed, analyzed and their findings were reproduced in Chapter 6. Finally in Chapter 7 conclusion and recommendation were specified based on the study.

1.3 Bedding Plane

Bedding Plane is a surface that separates one stratum, layer, or bed of stratified rock from another. A geological bed or stratification is a layer of sediment or volcanic material that is distinctly separate from other layers. Beds can vary in thickness from 1 cm thick to over 3 meters thick (from Internet) . Beds vary in texture and their resistance to weathering from one bed to another. The bedding plane separates beds and is a area easily fractured. Surface separating layers of sedimentary rocks. Each bedding plane marks termination of one deposit and beginning of another of different character, such as surface separating a sand bed from a shale layer. Bedding planes can be lines of weakness in that beds may slide over one another in a fold situation. This is greatly dependent on the types of beds involved. For example, a limestone may have joints from flexure, but it could slide laterally if sitting on shale. The bedding plane would be one line of weakness and the joints another. From the literature review observed that plane of weakness or Bedding plane range $10^0 < \gamma < 40^0$, is pronounced to collapse failure (Aadnoy 1988) along bedding plane.

1.4 Important of Shale research

Shale's make up over 75% of the drilled formations, and over 70% of the borehole problems are related to shale instability (Lal M., 1999). Shale is specifically mentioned in this setting, due to the fact that borehole instability is more pronounced in such formations than in any other formation (Horsrud et al., 2001; Aadnoy et al., 2004; Al-Ajmi et al., 2006; Fjær et al., 2008; Horsrud et al., 2001, Islam M.A. 2010). From field experience, it was found that shale (hard rock) make up of more than 80 % of the sediments and rocks in siliclastic environments and about three quarters of the borehole problems are caused by shale instability, leading up to troubles such as sloughing shale and stuck pipe. At best, an unstable wellbore would mean that drilling performance is impeded through lost time. At worst it could mean hole collapse and total loss of a well. All this means extra costs. A significant amount of lost time and extra cost about 2-5 billion USD/year (Aadnoy, 2009) is accounted to repair activity of shale related problems worldwide. It is believed that shear and radial tensile failure mechanisms are the two biggest concerns that can lead to mechanical instability when drilling in shale. Despite great efforts to reduce borehole instabilities, they remain a serious concern.

Stability issues in a depleted formation (inter-bedded shale-sand) are more difficult to handle due to a narrow drilling pressure window. When drilling in shale with heavy mud, using a conventional drilling approach can damage the sand formations and lead to lost circulation. However, attempting to drill in under-balance (UB) in the shale without collapsing the borehole may be one solution to obtain the necessary pressure in the sand formations. An alternative approach is to set the casing near the inter-phase region and continue "protected" drilling into the sand formation. All of these activities depend on the actual mechanical properties of the rock and on an accurate mud design programs.

1.5 Back ground of the study

The oil and gas industry still continues to fight borehole problems. The problems include hole collapse, tight hole, stuck pipe, poor hole cleaning, hole enlargement, plastic flow, fracturing, lost circulation, well control. Most of the drilling problems that drive up the drilling costs are related to wellbore stability. These problems are mainly caused by the imbalance created between the rock stress and strength when a hole is drilled. The stress-strength imbalance comes about as rock is removed from the hole, replaced with drilling fluid, and the drilled formations are exposed to drilling fluids (Lal M., 1999). Shale stability is affected by properties of both shale (e.g. mineralogy, porosity) and of the drilling fluid contacting it (e.g. wet ability, density, salinity and ionic concentration). The existence and creation of fissures, fractures and weak bedding planes can also destabilize shale as drilling fluid penetrates them. Drilling fluids can cause shale instability by altering pore pressure or effective stress-state and the shale strength through shale/fluid interaction. Shale stability is also a time-dependent problem in that changes in the stress-state and strength usually take place over a period of time (Horsrud et.el., 1994). This requires better understanding of the mechanisms causing shale instability to select proper drilling fluid and prevent shale instability. When drilled in Shale, native shale is exposed suddenly to the altered stress environment and foreign drilling fluid. The balance between the stress and shale strength is disturbed due to the following reasons:

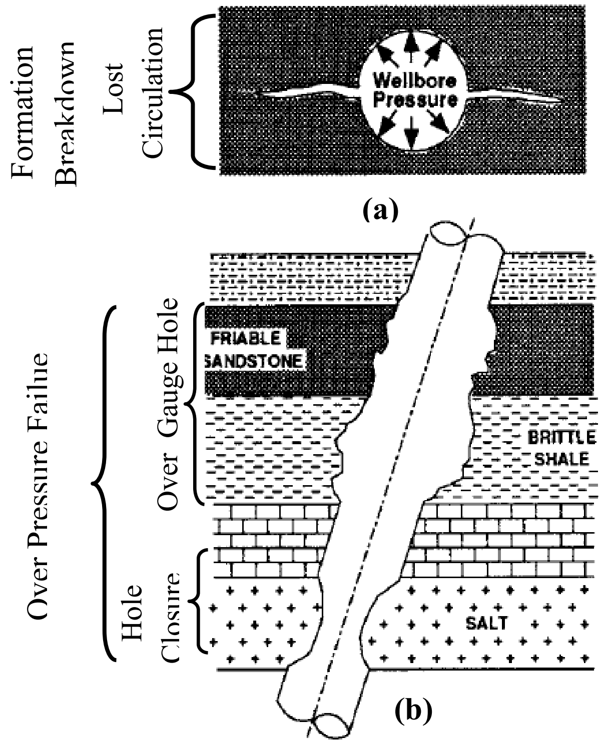
- Stresses are altered at and near the bore-hole walls as shale are replaced by the drilling fluid (of certain density) in the hole.
- Interaction of drilling fluid with shale alters its strength as well as pore pressure adjacent to the borehole wall. Shale strength normally decreases and pore pressure increases as fluid enters the shale.

When the altered stresses exceed the strength, shale becomes unstable, causing various stability related problems. To prevent shale instability, one needs to restore the balance between the new stress and strength environment. Factors that influence the effective stress at wellbore are wellbore pressure, shale pore pressure, far away in situ stresses, trajectory and hole angle etc.

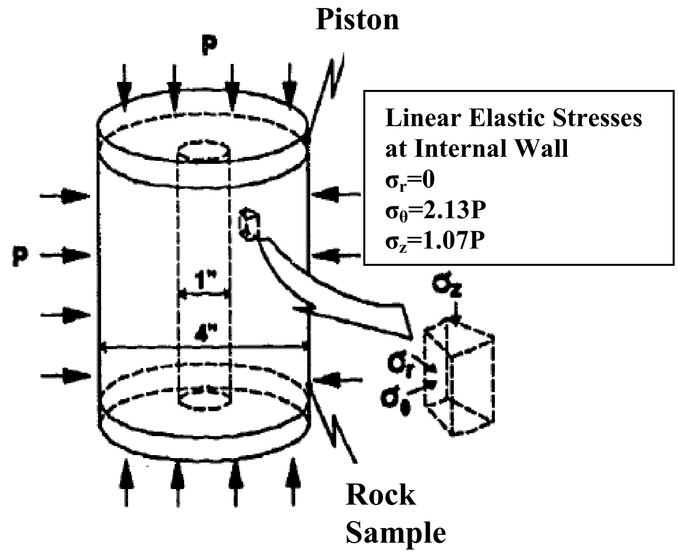
To prevent shear failure, the shear stress-state, obtained from the difference between the stress components (hoop stress, σ_θ -usually largest and radial stress, σ_r - smallest), should not go above the shear strength failure envelope. To prevent tensile failure causing fracturing, hoop stress should not decrease to the point that it becomes tensile and exceeds the tensile strength of the rock. The controllable parameters that influence the stress-state are drilling fluid, mud weight, well trajectory, and drilling/tripping practices. For example, radial stress (σ_r) increases with mud weight (wellbore pressure, P_w) and hoop stress (σ_θ) decreases with mud weight causing mechanical stability problem. The near wellbore pore pressure and strength are adversely affected by drilling fluid/shale interaction as shale is left exposed to drilling fluid (chemical stability problem).

Mechanical stability problem can be prevented by restoring the stress-strength balance through adjustment of mud weight and effective circulation density (ECD) through drilling/tripping practices, and trajectory control. The chemical stability problem, on the other hand, is time dependent (Horsrud et.el., 1994) unlike mechanical instability, which occurs as soon as we drill new formations. Chemical instability can be prevented through selection of proper drilling fluid, suitable mud additives to minimize/delay the fluid/shale interaction, and by reducing shale exposure time. Selection of proper mud with suitable additives can even generate fluid flow from shale into the wellbore, reducing near wellbore pore pressure and preventing shale strength reduction (Lal M., 1999).

Before a well is drilled, compressive stresses exist within the rock formations (**Figure 4.1a**). The stresses can be resolved in to a vertical or overburden stress, $\sigma_{v,}$ and two horizontal stresses, $\sigma_{H,}$ (the maximum horizontal stress), and σ_h (the minimum horizontal stress), which are generally unequal. When the well is drilled, the rock stresses in the vicinity of the wellbore are redistributed as the support originally offered by the drilled out rock is replaced by the hydraulic pressure of the mud. The redistributed stresses are normally referred to as the hoop stress, σ_θ which acts circumferentially around the wellbore wall, the radial stress, σ_r, and the axial stress, σ_z, which acts parallel to the wellbore axis (see **Figure 1.7**) for stress state within a hollow cylinder). In deviated wells an additional shear component,$\tau_{\theta z}$, (see **Fig 4.2**) is generated. If the redistributed stress state exceeds the rock strength, either in tension or compression, then instability may result (**Figure 1.6**). In order to evaluate the potential for wellbore stability a realistic constitutive model must be used to compute the stresses and/or strains around the wellbore. The computed stresses and strains must then be compared against a given failure Criterion.



Figur 1-6 Types of Stress Induced Wellbore Istability (Mclean and Addis 1990)



Figur 1-7 Typical loading adapted in Hollow (Mclean and Addis 1990,SPE-19941)

1.6 Problem Background and Challenge

The selection of a failure criterion for borehole stability analysis is a challenging task (Mclean et al., 1990). Proper selection of failure criteria for borehole stability analysis is therefore unclear to drilling engineers. Rock mechanic experts have applied several failure criteria in an attempt to relate rock strength measured in different simple tests to borehole stability.

The main challenge to reduce the wellbore failure is to need extensive experimental data realted to the characterization of shale anisotropy and heterogeneity. One of the crrucial challenges for evaluating and modeling the potential borehole stability problem in shale is the lack of relevant test data to desirable shale matrix anisotropy and heterogeneity. Shale anisotropy parametrs are evaluated from laboratory testa after a complex workflow from sampling to testing. Due to its heterogeneous nature, normally receive inconsistant tests results related to the directional properties of Shale. This variation creates confusion in numerical modeling, even when samples come from a similar core specimen, thus always has seen at calibration. During the stability analysis mud cooling have positive impact with respect to collapse , therefore mud Cooling effects should be accounted for in borehole Stability design, otherwise The risk of fracturing is underestimated (Islam 2010). Different fault has been given different failure stress of state and fault is a geological matter and this phenomenon is also time dependent. So It has to take careful measure which model is relevant to predict the stability of a borehole.

For the analyzing the borehole related failure has a lot of Challenges and difficulties. Different difficulties characterizing Shale and its elastic properties were noted (Islam 2010):

- Shale's isn't a reservoir Rocks, So it is not interesting in terms of Production;
- Difficulties regarding the collection and preservation of Shale ;
- Clay minerals are very sensitive to alteration of temperature, therefore it is difficult to obtain cores with preserved in-situ condition;
- Measuring technique is time consuming due to preservation, Physiochemical behavior and low permeability (barrier to obtain preferred saturation level) of shale;
- Massive time involvement in laboratory test due to low permeability;
- It is almost impossible to isolate the clay grains to an individual crystal to measure acoustic Properties;
- Structural and compositional complexity introduce difficulties in handling Shale;

It is clear that it my be impossible to consider all features of shale in one model.

The main consideration, when evaluating and modeling borehole stability problems in shale, is the lack of relevant test data to accurately describe shale directional properties .Coring a real shale specimen from an inter-bedded sand-shale layer is a challenging task. To characterize overburdened shale, an experimental setup is required. This is costly and time consuming, and therefore, it is well accepted to perform experimental investigations on outcropped shale and use this data to provide the necessary material data sets for the fundamental model. In practice, the fundamental models are calibrated against field cases and later readjusted. Neverthelss, wellbore stability assesment plays an important role in the design of drilling and production of oil and gas wells ; therefore, a methodology for arranging the data gap is needed

The drilling challenges of the 21st century, including greater depths, HPHT and often depleted reservoirs, demand that UBD/MPD and wellbore stability numerical techniques, together with real-time formation knowledge, are used to assist the driller in his or her daily business (SPE webside 2010)

CHAPTER 2 THEORY OF BEDDING PLANE AND ROCK STRENGTH

2.1 Strength Criterion for Anisotropic Rock

As of today, only four fracture criteria have been proposed for anisotropic rocks. In 1960, Jaeger proposed two fracture criteria for anisotropic rocks based on generalizations of the Mohr-Coulomb theory for isotropic rocks. The first theory, known as the "single plane of weakness" theory, considers an isotropic body that possesses a plane or parallel planes of weakness. The second theory proposed by Jaeger is called the "continuously variable shear strength" theory and assumes that the rock parameter 'τ_o, cohesive strength, is a function of the orientation of the anisotropic applied stress'. Consequently, when rocks fail in compression, They are actually failing in shear, as a result of inter-granular slip. Their resistance shear, i.e. shear strength, is due to a combination of Cohesion and friction between the rock grains. The third fracture criterion was proposed by Walsh and Brace (1964) and is an extension of the McClintock and Walsh (1963) modification of Griffith's (1924) tensile failure theory. It describes a material that possesses nonrandom oriented Griffith cracks that close under loading. The fourth fracture criterion, derived independently by Hoek (1964) is also a modification of Griffith theory and is essentially identical to the Walsh-Brace theory.

Here a brief summary of the first two theories considered in this paper follows. It will also be shown that the Walsh-Brace theory and Jaeger's single plane of weakness theory are identical in final form even though the fracture mechanisms involved are quite different in nature Mclean (1967).

2.2 Single plane of weakness Theory

As opposed to the Walsh-Brace theory which assumes failure occurs due to local tensile stress, the single plane of weakness theory, proposed by Jaeger (1960), assumes that the body fails in shear. This theory is a generalization of the well-known Mohr-Coulomb linear envelope failure theory and describes an isotropic body that contains a single plane or a system of parallel planes of weakness. The failure of the matrix material is given by

$$\tau = \tau_O + \sigma' Tan\phi \quad \ldots\ldots \quad (2.1)$$

Where, τ_o is the cohesive strength of the matrix material and tanα is the coefficient of friction. Failure along the plane of weakness is described by

$$\tau = \tau_W + \sigma' \mathrm{Tan}\,\phi_W \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(2.2)$$

Using the well-known Mohr circle relationship that relates τ and σ to σ_1 and σ_3, and the angle of internal friction, α, the final form of the single plane of weakness theory can be derived from equations (1) and (2).

This theory is evaluated by running tests at 0^0, 90^0 and 30^0 orientation for various confining pressure, plotting linear Mohr-Coulomb envelops and determining the value of the parameters ϕ, ϕ_w, τ_o and τ_w . Then the fracture strength of the material as a function of the orientation and the confining pressure. However, determing ϕ, and τ_o on a foot by foot basis presents more of a challenge. It clearly is not feasible to do this with laboratory strengths tests. As an alteranative, it is desirable to develop relationships for computing τ_o and α from wireline data. Therefore, rock strength correlation actually refers to relation with wire-lone log data for determing the Cohesive strength and friction angle (Lal M.et.al, 1996,1999). A more fundamental look at shale physics was taken to gain better insight into which factors need to be included in strength correlation. Several factors were considered clay mineralogy, clay content, compaction, water content, porosity, sonic velocity and Density.

The shale strength correlations, developed by Lal M. (1999), were tied only to compressional sonic velocity in shale. The relations were developed using an extensive shale data base. The following relations for friction angle, α (degrees) and cohesive strength, τ_o (MPa), were developed as a function of compressional sonic velocity V_p (km/sec):

$$\mathrm{Sin}\phi = \frac{V_p - 1}{V_p + 1} \dots\dots.(2.3) \quad \text{and} \quad \tau_o = \frac{5(V_p - 1)}{\sqrt{V_p}} = 10\mathrm{Tan}\phi \dots\dots\dots....(2.4)$$

The impact of clay mineralogy and contents on strength (and stability) can become quite significant while drilling, when a foreign drilling fluid contacts in situ smectitic shale and alters the salinity of native pore fluid through shale/fluid interaction. Smectitic shale has a lower tolerance to drilling fluid invasion, and will tend to fail easier than formations in which kaolinite and/or illite are the only clay types present. The effect of clay mineralogy on strength can be important if the drilling process severely disturbs a formation from its natural state. In those cases, as discussed below, smectitic formations will be more susceptible to failure. The strength of all geologic materials depends upon the effective confining. Therefore, if shale/drilling fluid interaction raise the pore pressure in the near wellbore region, the drop in effective confining pressure will make the hole more susceptible to failure.

Finally, even if we could design the best mud system for shale formations, continuous monitoring and control of drilling mud are critical elements for successful drilling. The mud composition continually changes as it circulates and interacts with formations and drilled solids. Unless concentrations of various mud additives are continually monitored (as opposed to the current practice of periodically monitoring just rheological and simple properties) and maintained, the desired results could not be achieved. The development and introduction of improved monitoring techniques for chemical measurements should proceed simultaneously with the development of more effective mud systems for shale stability, based on improved understanding of shale/fluid interaction (Lal M., 1999).

2.3 Continuously variable shear Strength Theory

The continuously variable shear strength theory was proposed by Jaeger (1960) and is based on the Mohr-Coulomb Theory (linear Mohr envelope). The theory assumes that the cohesive strength of the material is a continuous function of β and can be described by

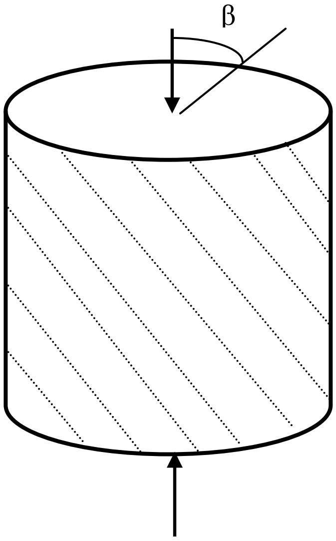
$$\tau_O = A - B\ \mathrm{Cos}\ 2(\gamma - \beta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

Where A and B are constants and γ is the orientation of β for which τ_o is a minimum. (As in the case of fracture strength, the minimum value of τ_o usually occurs at $\beta=\gamma=30^0$, Mclamore (1971). To evaluate the Continuously variable Shear strength theory, it is necessary to run a series of compression tests at orientation of 30^0 and 75^0 (assuming $\gamma=30^0$). Both of Chenevert (1965) and Aadnoy (1987,1988) did these works for various confining pressure, construct linear Mohr envelopes from the data, determine the value of τ_o and average $Tan\phi$ for the two orientations and then evaluate the constants A and B. Once A and B are known , the fracture strength of the material as a function of the orientation and the confining pressure.

2.4 Rock strength Anisotropy

Anisotropy of sedimentary rock is mainly due to the geometrical arrangement of particles that depends on the orientation of applied load respect to the bedding plane. More research on wellbore failure is needed (Zoback 2007) and points to a clear need for investigating the strength of a variety rocks (of dofferent stregth, Stiffiness, Permeability etc.) at range conditions (different loading rates, effective confining Pressures etc.) the presence of weak bedding plane in shally rocks (or finely laminated sanstones or foliation Planes in Metamorphic rocks) can sometimes have marked effect on rock strength. Discussed later on this report in details.

The influence of weak bedding planes on rock strength is referred to as strength anistropy. The improtrance of this depends both on the relative weakness of the bedding plane and the orientation of the plane with respect to the appled forces (Aadnoy, Chenevert 1987, Zoback 2007). This is illustrated the **figure 2.1**, for strength tests with bedding planes whose normal is at an angle β, to the appleid maximum stress. However when β ~60^0, slip on a weak



Figur 2-1 Angle between Normal to Bedding Plane and Maximum Principal stress

bedding pane would occur at a markedly lower stress level than that required to form a new fault (Zoback 2007). The intact rock would have its normal strength which would control

failure when slip on bedding planes did not occur and a lower strength , defined by the cohesion τ_w and internal frcition angle α_w of weak bedding planes which would apply. These parametrs only relevant, offcourse, when slip occurs along pre-existing planes of weakness and affects rock strength.

Mathmatically, it is possible to estimate the degree to which bedding planes lower rock strength using a theory developed by donath (1966) and Jaeger and Cook (1979). The maximum stress at which failure will occur, σ_1, will depend on σ_3, τ_o and α_w by:

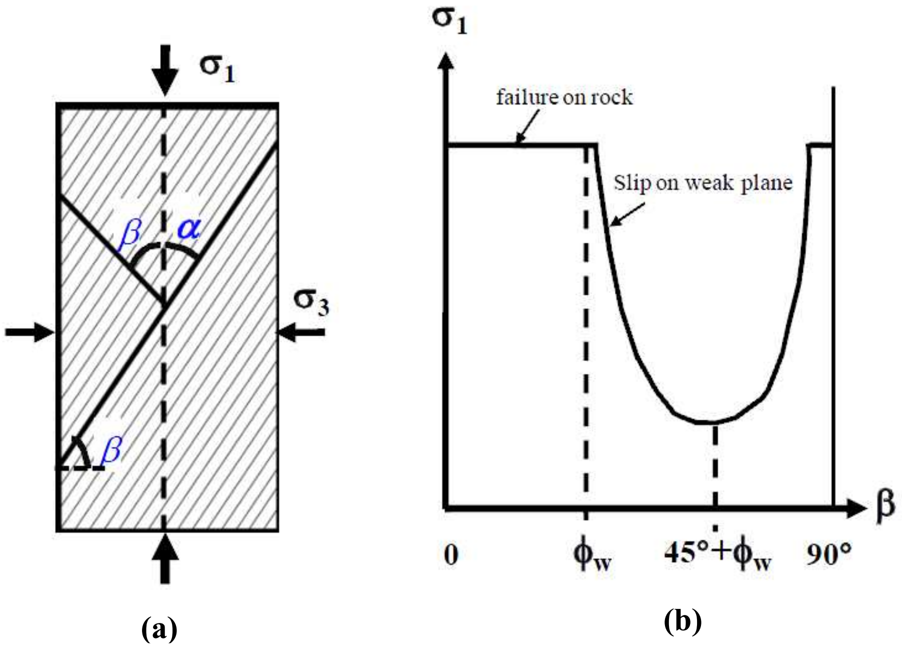
$$\sigma_1 = \sigma_3 \frac{2(\tau_W + \mathrm{Tan}\phi_W \sigma_3)}{(1 - \mathrm{Tan}\phi_W \mathrm{Cot}\beta_W)\mathrm{Sin}2\beta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

At high and low β, the intact rock strength is unaffected by the presence of the bedding planes. At β ~60^0 the strength is markedly lower using:

$$\mathrm{Tan}2\beta_W = 1/\mu_W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

It can be shown that the minimum strength is given by :

$$\sigma_1^{min} = \sigma_3 + 2(\tau_W + \mu_W \sigma_3)\ \left[(\mu_W{}^2)^{\frac{1}{2}} + \mu_W\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$



Figur 2-2 a) Transeversely Isotropic specimen with Bedding /weak planes in triaxial

Figur 2-2 b) Rock peak stregth variation with angle β, in the triaxial test at constant

For shear failure with consideration of weak planes in a vertical wellbore, Jang et.al. (2011) developed a equation to calculate the minimum mud pressure (P_{wd}) for preventing wellbore sliding (shear failure) in the weak planes. They call it minimum mud pressure to be "Slip failure pressure" and it's gradient is called "slip failure gradient":

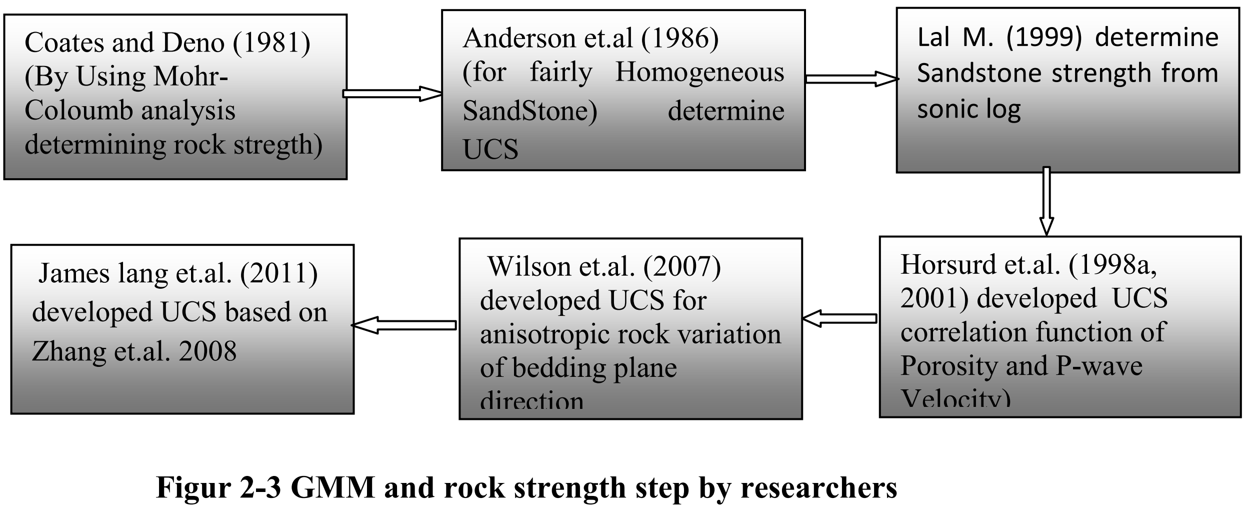
$$P_{wd} = \sigma_3 \frac{(\sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\mathrm{Cos}2\theta(1 - \mu_W \mathrm{Cot}\beta)\sin 2\beta - 2\tau_W + 2\mu_W P_p)}{2[\mu_W + (1 - \mu_W \mathrm{Cot}\beta)\mathrm{Sin}2\beta]} \ldots\ldots (2.9)$$

2.5 Rock Mechanical Testing

Rock mechanical Parameters are generally determined from two types of Test. First Tri-axial tests are performed to derive stiffness, Strength Characteristics and Input parameters for Numerical solution. Second, Thick wall Cylinder Test (TWC) **(Fig 1.6)** provided the 'TWC strength' used to calibrate the numerical Model and as a 'Quick look' experimental assessment of borehole stability.

TWC test is a routine and small scale borehole collapse test in which an external isotropic pressure is applied incrementally to the sample until failure which the linear hole maintained at atmospheric pressure. The failure pressure is called the TWC strength. Note that TWC strength may vary according to the TWC sample size and the Hole OD/ID ratio.

The tri-axial strength tests, each sample are loaded in the axial and radial directions. Shear failure is induced by increasing the axial stress after hydrostatic preloading. The tri-axial test is done at different confining pressures. Peak stress values shows the correspond failure of sample. From this data failure envelope and corresponding rock strength parameters (Peak friction angle α, and Cohesion Strength τ_o) are determined. A Schematic drawing is shown the following involved different researcher who are developed GMM and derive different equation of UCS/Strength of rock:



Figur 2-3 GMM and rock strength step by researchers

2.5.1 General Input Parameters of Geomechanical Model (GMM)

One challenge for constructing a geo-mechanical model is the generation of consistent input data. Many of the required parameters can be inferred from different sources, using some empirical correlations, theoretical expressions, or analogue data previously experienced. Both stress field and rock mechanical properties are part of the GMM. Various methods and techniques have been used to calculate necessary input to generate GMM. This study developed a standard GMM based on updated published work (Breckels et al., 1982; Tan et al., 1993; Aadnøy et al., 2005; Horsrud et al., 1998a). Details of present GMM along with data integration techniques are presented through Table 2.1.

Coates and Denoo (1981) shown the determination of Sand strength limits using Mohr-Coulomb analysis of Sand stability (Uniaxial Compressive Strength), which can be expreesed as follows:

$$\sigma_{UCS} = 0.0871\ 10^{-6} EKb\left(0.008V_{sh} + 0.0045(1 - V_{sh})\right) \ldots\ldots\ldots\ldots\ldots (2.10)$$

The cohesive strenth may be obtained from the following relation:

$$\tau_O = 2\sigma_{ucs}\, Tan\beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

Where β is the orientation Plane and defined as :

$$\beta = \frac{\pi}{4} + \frac{\phi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.12)$$

The incompressibilty Modulas (K_b) and Youngs Modulas (E) can be derived from Sonic Log Data (Compressional Travel Time, Δt_c and Compressional wave transit time Δt_s) shown in the following Figure-() contains corelatins for deriving the Sonig log data to K_b and E., **Table-2.1**(from Simangunsong et.al.,2006) shown also input Parametrs for mechanical wellbore stablity analysis.

2.5.2 Mechanical Input Parametrs and Correlation

Tabell 2.1 Data Input Source for Mechanical wellbore Stability Analysis

Parameters	Estimated From	Parameters	Estimated From
M	Seismic P-wave and S-wave	ρ_f	Kick influx into Borehole while Drilling
	Δt_s and Δt_c from sonic Log		Gas-cut mud while drilling
σ_H	Extended Leak off Test		Measurement while drilling
	Borehole Images, Best Gauge		Mud weight while drilling
σ_h	Δt_s and Δt_c from sonic Log		Equivalent depth density

	Micro Frac Test		D,Dc exponent drilling Parameter
	Mini Frac test		
	Leak off test	σ_v	ρb from density Log
	Massive Hydraulic Record	τ_o	Uni-axial or tri-axial core test
ρ_w	Mud weight required while Drilling, Drill stem tester	ϕ	Uni-axial or tri-axial core test
	Wire-line Formation Tester		
	Formation interval tester	(Source Simangunsong R.A. et. al. 2006)	

Rocks Mechanics Correlations

$$\text{Poisson's Ratio } \upsilon_{dyn} = \frac{\frac{1}{2}\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1}{\left(\frac{\Delta t_s}{\Delta t_c}\right)^2 - 1}$$

$$\upsilon_{sta} = 0.7\,\upsilon_{dyn}$$

$$G = \frac{\rho b}{\Delta {t_S}^2}$$

$$E = 2G(1 + \nu)$$

$$K = \rho_b\left(\frac{1}{\Delta {t_C}^2} - \frac{4}{3\Delta {t_S}^2}\right)$$

For fairly homegeneous Sandstone Anderson R., et al.(1986) simplified euation 2.9 the following way:

$$\sigma_{UCS} = E\big(0.008V_{sh} + 0.0045(1 - V_{sh})\big) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.13)$$

Borehole instability problem however are often oocured in Shaly zone, anlalyzing of shaly formations has proved to difficult, moreover Theses zones is considered unprofitable. Moreover Lower permeability nature of shales makes laboratory rock mechanics testing expensive and time consuming. This long-standing Problem investigated by Horsrud (2001) who observed an outcrop Clays database from a wide variety of sources in North Sea region. The data base Contains Triaxial test data of the Core samples taken from the outcrops.

Horsurded developed correlations to detect uinaxial Compressive Strength (UCS) as a function of Porosity or P-wave Velocity (V_p) from Sonig Log. The follwing relations are:

$$\sigma_{UCS} = 243.6\varphi^{-0.96} \ldots\ldots(2.14) \qquad \sigma_{UCS} = 0.77{V_p}^{2.93} \ldots\ldots\ldots..(2.15)$$

He also covered correlations to estimate failure angle (β) with respect the amount of Clay content, al follows:

$$\beta = 50.7 - 0.032C_{cl} + 0.24C_{kl} \;\ldots\ldots (2.16) \qquad \text{and} \qquad \phi = 2\beta - 90^0 \;\ldots. (2.17)$$

For Shale : $45^0<\beta<60^0$

The rock strengths are key input parameter in wellbore stability modeling. Rock strengths are preferably obtained from laboratory core tests and secondarily from compressional velocity correlations. Lal (1999) presented the following corelations for shale in gulf of Mexico:

$$\sigma_{UCS} = 10\left(\frac{304.8}{\Delta t_S}\right) \ldots\ldots\ldots..(2.18)$$

In the gulf of Mexico , some sandstones are weaker then shale, based on data Zhang et.al.2008 , James lang et.al. (2011) developed the following correlation:

$$\sigma_{UCS} = 0.68\left(\frac{304.8}{\Delta t_S}\right)^{2\cdot 5} \ldots\ldots\ldots..(2.19)$$

Where uniaxial compressive strength (σ_{UCS}) in MPA , and Δt_S is the sonic transit time in μS/ft.Wilson et.al. (2007) presented the following equation (rock strength in anisotropic formations) to calculate the rock uiniaxial compressive strength variation relative to bedding planes:

$$\sigma_{UCS\vartheta} = \sigma_{UCSmax}(Cos\vartheta + K_1 Sin\vartheta)(1 - \mathrm{Sin}\vartheta\mathrm{Cos}\vartheta)\left[1 - 2\mathrm{Sin}\vartheta\mathrm{Cos}\vartheta\left(1 - \frac{4k_2}{\sqrt{2}(1+K_1)}\right)\right] \ldots\ldots\ldots.(2.20)$$

Where, $\sigma_{UCS\vartheta}$ is the uniaxial comressive strength at ϑ with consideration of bedding effects; ϑ is the angle between the stress concentration orientation to the bedding, $\vartheta = 0$ represents loading perpendicular to bedding and $\vartheta = 90^0$ represents loading parallel to bedding; σ_{UCSmax} represents the maximum strength at any orientation; K_1 and K_2 are defined by the following equation:

$$K_1 = \frac{q_{para}}{q_{perp}}$$

$$K_2 = \frac{\sigma UCSmin}{\sigma UCSmax}$$

Where, q_{para} is the strength with bedding parallel to the sample axis, q_{perp} is the strength with bedding perpendicular to the sample axis.σ_{ucsmin} is the minimum strength at any direction.

2.5.3 Field Parameters

In addition to rock strength, Borehole stability is determined by the field values of the formation Pressure, Vertical stress (Overburden), Maximum and Minimum Horizontal stresses and Orientation of Borehole respected to In-situ stresses.

2.5.4 Formation pore pressure

Formation pressure is the presence of the fluids in the pore spaces of the rock matrix. Normal formation pressure is equal to the hydrostatic pressure of the native formation fluids. In most cases, the fluids vary from fresh water with a density of 8.33 Ib/gal (0.433 psi/ft) to salt water with a density of 9.0 lb/gal (0.465psi/ft). However, some field reports indicate instances when the normal formation fluid density was greater than 9.0 lb/gal. Regardless of the fluid density, the normal pressure formation can be considered as an open hydraulic system where pressure can easily be communicated throughout. The value of pore pressure at depth is usually desrcibed in relation to hydraustatic (or Normal) pressure, the pressure associated with a column of water from the surface to the depth of interest. Hydraustatic Pore pressure (P_p^{hydro}) increases with depth at the rate of 0.44 Psi/ft (Zoback, 2007 depending on salinity):

$$P_p{}^{hydro} \equiv \int_0^z \rho_w(z)\, gdz \approx \rho_W g z_W \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2.21)$$

A sealing mechanism must be present to trap the abnormal pressures in their environment. The most common sealing mechanism in continuous depositional basins is a low-permeability layer of rock, such as a clean shale section. The shale reduces normal fluid escape, causing under-compaction and abnormal fluid pressures. Formation pressures resulting from under-compaction often can be approximated with some simple calculations. If it is assumed that compaction does not occur below the barrier depth, the formation fluid below the barrier must support all overburden, rock matrix and formation fluids. The pressure can be calculated with Equation Following (Adams J.N. 1985):

$$P_p = 0.465\frac{Psi}{ft}.D_B + 1.00\frac{Psi}{ft}(D1 - D_B) \dots\dots\dots\dots\dots (2.22)$$

Where, D_1 depth of interest below barrier, ft., D_B depth of Barrier, i.e. low-Permeability section, ft., Pp formation Pressure at D_1, Psi.

Here the overburden pressure gradient is assumed to be 1.0 psi/ft and the normal formation fluid pressure gradient is 0.465 psi/ft.

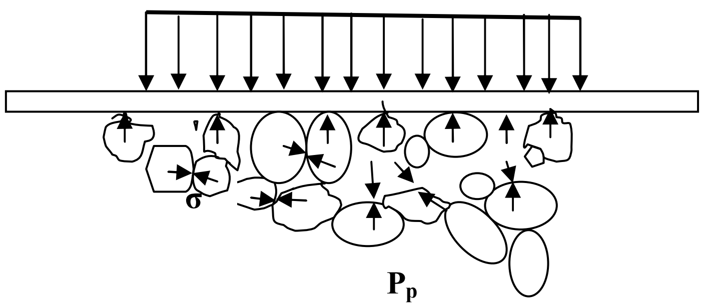
It is generally accepted that the upper limiting value for abnormal shale pore pressure is the minimum in-situ stress σ_h. The actual pore pressure values in shale formations, however, are one of the most difficult parameters to estimate quantitatively. Various methods are available for the estimation of abnormally high shale pore pressure. But they are empirical, and their reliability depends on field experience and the amount of data collected. Instead of carrying out an extensive analysis to quantify the abnormal pressures, various scenarios were assumed to see how sensitive our borehole stability analysis was with respect to such pressures. Sau-Wai Wong et. al (1993) represents two cases . The first case assumes a relatively low but constant abnormal pressure gradient of 0.48 psi/ft. The second case assumes that the abnormal pressure Pp increases with depth so that the effective stress ratio K' is constant. The effective stress ratio is given by:

$$K' = \frac{\sigma'h}{\sigma'v} = \frac{\sigma h - P_p}{\sigma v - P_p} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2.23)$$

$$P_p = \frac{(h - K'\sigma_v)}{(1 - K')} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2.24)$$

$$K\ of\ 0.55\ is\ assumed\ and\ then\ P_p\ can\ be\ calculated.$$

2.5.5 Effective stress



Figur 2-4 Stress and Pressure in a porous material (Aadnoy 2009)

Aadnoy (2009) covered that his Paper, Rocks are porous materials, which consist of a rock matrix and a fluid, which usually is under pressure. **Fig. 2.4** illustrates this. Assume a porous rock which is sealed by a plate. On the outside of the plate is a stress s acting. In order for equilibrium to exist, this stress must be balanced by stresses inside the rock on the other side of the plate.

Assume that the overburden stress, as an example, represent the total stress in **Fig. 2.4.** This represents the total stress, or the external loading. Inside the rock, this stress is partially taken up by the pore pressure inside the fluid and in the rock matrix. That is, the total stress is equal to the pore pressure plus the effective stress. Please note that this is an empirically defined principle, not an analytical model. In mathematical terms:

In mathematical terms:

$$\sigma = \sigma' + P_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.25)$$

Most of our analysis is related to failure of the rock matrix. Failure is in general governed by the effective stresses, which are given by:

$$\sigma' = \sigma - P_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.26)$$

A more general formulation of the effective stress principle includes a scaling factor in front of the pore Pp pressure term. This is called the Biot's constant, and looks as follows:

$$\sigma' = \sigma - \alpha_\beta P_p, \ldots.(2.27) \quad Where\ Biot\ constant, \alpha_\beta = \left(1 - \frac{E}{Ei}\frac{1-2\vartheta i}{1-2\vartheta}\right) \ldots..(2.28)$$

Here E is the Young's modulus, ν is the Poisson's ratio, the index i refers to the inter pore material, and the remaining terms to the bulk material. The Biots constant may have a value in the order of 0.8-1.0 for real rocks. This aspect of rocks is also called poroelasticity.

A fluid at rest cannot transmit shear stresses. This means that effective stresses are only valid for normal stresses. Shear stresses remain unchanged

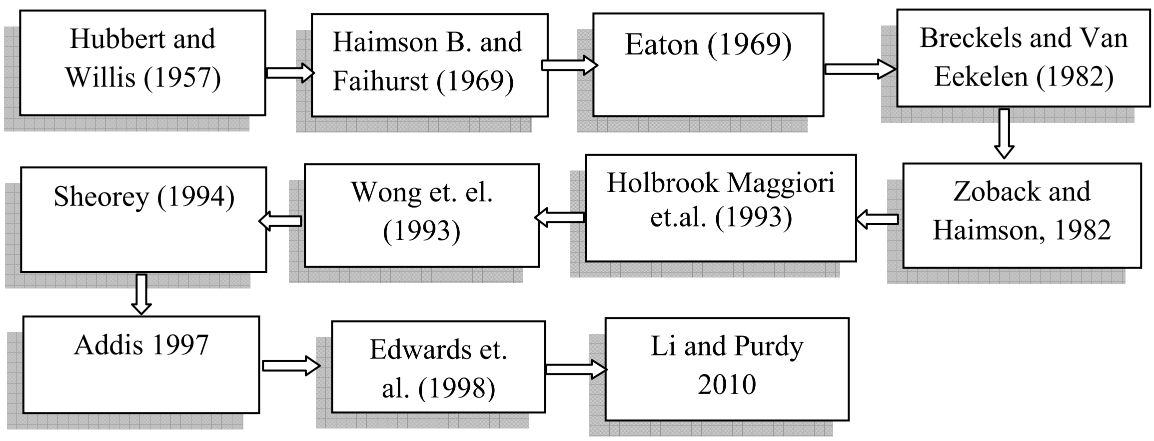
2.6 Operational aspects of shale drilling

An adequate understanding of the rock-mechanical problem posed by shale drilling does not necessarily imply that a solution can be easily formulated. Even in cases where it has been shown that borehole stability can be maintained through the use of suitable mud weights and mud type, it is still desirable to employ good drilling practices and operational procedures. Only then can an optimized drilling performance be achieved. All operational measures to reduce open hole time may be regarded as measures to combat shale problems. Additional measures, such as running pipe at lower velocity, may also be used to reduced hole instability. hole instability may be aggravated by swab/surge pressures. Therefore, proper mud conditioning is required to keep the gels and the plastic viscosity down to acceptable levels. This will generally lead to lower frictional pressure losses and smaller pressure shocks when one is circulating mud and pulling or running pipe. Hole cleaning (i.e. the efficient transport of cuttings to the surface) is of great importance for hole stability, especially in highly deviated holes. A massive amount of solids in the mud, combined with the inability to clean the hole efficiently, may increase the mud pressure significantly, causing existing fractures to re-open. It may even increase pore pressure penetration, thereby increasing the rate of borehole collapse and reducing the pressure required to cause formation breakdown. An excessive amount of colloidal shale in the mud will decrease the rate of penetration and may give rise to time-consuming differential-sticking problems in other parts of the hole. Proper solids removal and mud maintenance are therefore good hole-cleaning practices. In addition, the use of a top drive is desirable to ensure better mud circulation while tripping.

CHAPTER 3 INSITU STRESS AND ITS CONTITUENTS

3.1 In-situ stresses

In situ means in place, when something is "in situ," it is in its original location, means the rock stress acting at the undisturbed region on the underground/subsurface. Normally in the tectonically relaxed basin we consider three in-situ stress (1) vertical (overburden) stress, σ_V, (2) maximum horizontal stress, σ_H and (3) minimum horizontal stress, σ_h . Knowledge of the virgin stress field is very important in many problems dealing with rocks in Civil, Mining and Petroleum engineering as well as in Geology, Geophysics and Seismology. For soils, vertical stresses can be readily determined, while horizontal stresses are much more difficult to establish. Existing technique to estimate the horizontal stress magnitudes including borehole breakout analysis, in-situ estimates based on hydraulic fracturing and over coring. Extended leak-off test with flow back normally gives a reliable measure of the minimum horizontal stress. Maximum horizontal (for the magnitude and orientation) stress (σ_H) is estimated generally wellbore failure (breakout and drilling induced tensile failure) data. The following Schematic drawing has been shown in **Fig 3.1** about different researchers who are involved and give different theory to measure in-situ stresses:



Figur 3-1 Insitu stress estimation by different researchers

One of the biggest challenge to drill in the shale region due to instability, If we know the in-situ stress behavior we can predict proper well path and drilling would be done more economical way. The need for understanding in-situ stresses in rocks has been recognized by geologists and engineers for a long time and many methods to measure these stresses have been proposed since early 1930's. Probably the most accurate way of determining in- situ horizontal stress in deep boreholes is by conducting low-volume hydraulic fracturing experiments (Zoback and Haimson, 1982). Such measurements are expensive and can be done at only a few locations along the wellbore. New wire-line micro-fracturing tools (Thiercelin et al., 1993) may change this situation, but for now, well-logs are often used to provide stress estimates instead.

Borehole breakouts represent compressive-shear failure of borehole wall along the minimum horizontal direction where the maximum compressive hoop stress occurs. Generally, maximum horizontal stress must be determined from damage mechanics constraints based on borehole breakouts. However, estimation of Maximum horizontal stress magnitudes remains a challenge in the industry. In exploration wells, it is necessary first to drill a vertical pilot-hole. The zone of interest is cored, field tests are performed, laboratory testing is completed and an evaluation of the reservoir is made. With this information available, decisions can be made to optimize the borehole azimuth and well placement. A great deal of research, both theoretical and experimental, has been aimed toward a greater understanding of in-situ stresses. Much of this work has been a result of horizontal drilling, hydraulic fracturing and the need to examine more closely the state of stresses at depth for wellbore stability during prediction. Wellbores fail in a manner which is strongly controlled by the magnitude and orientation of the in-situ stress field.

Haimson B. and Faihurst (1969) consider an element of rock at a depth of 1,000 m below the surface. The weight of the vertical column of rock resting on this element is the product of the depth and the unit weight of the overlying rock mass (typically about 2.7 tones /m3 or 0.027 MN/m3). Hence the vertical stress on the element is 2,700 tones/m2 or 27 MPa. This stress is estimated from the simple relationship:

$$\sigma_v = \gamma z \qquad (3.1)$$

Where σ_v is the vertical stress, γ is the unit weight of the overlying rock and Z is the depth below surface. The horizontal stresses acting on an element of rock at a depth z below the surface are much more difficult to estimate than the vertical stresses. Normally, the ratio of the average horizontal stress to the vertical stress is denoted by the letter k such that:

$$\sigma_h = k\,\sigma_v = k\,\gamma z \qquad (3.2)$$

Terzaghi and Richart (1952) suggested that, for a gravitationally loaded rock mass in which no lateral strain was permitted during formation of the overlying strata, the value of k is independent of depth and is given by $k = \upsilon/1-\upsilon$, where υ is the Poisson's ratio of the rock mass. This relationship was widely used in the early days of rock mechanics but, as discussed below, it proved to be inaccurate and is seldom used today.

K tends to be high at shallow depth and decreases with increasing depth (Brown and Hoek, 1978, Herget, 1988). In order to understand the reason for these horizontal stress variations it is necessary to consider the problem on a much larger scale than that of a single site.

Sheorey (1994) developed an elastic-static thermal stress model of the earth (first order estimation) take into account tectonic forces. This model considers curvature of the crust and variation of elastic constants, density and thermal expansion coefficients through the crust and Mantle. A detailed discussion on Sheorey's model is beyond the scope of this report. He provides a simplified equation which can be used for estimating the horizontal to vertical stress ratio k. This equation is:

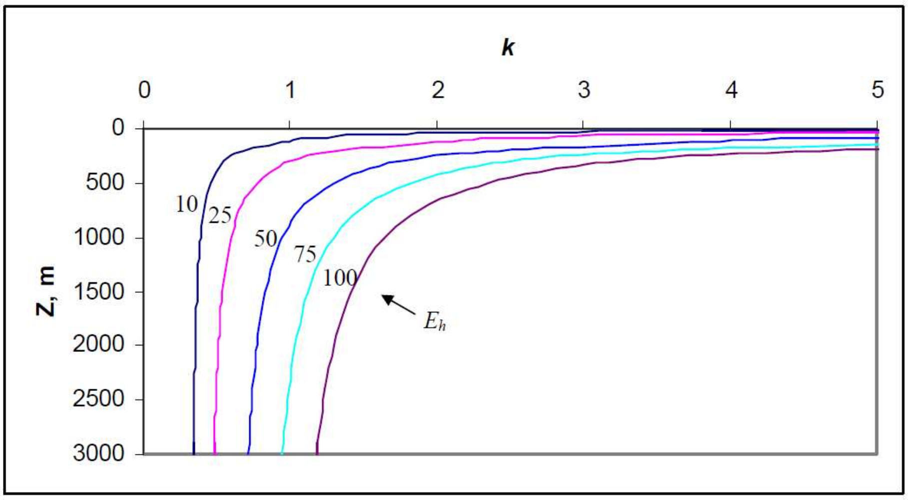
$$k = 0.25 + 7E_h\,(0.001 + 1/z) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

Where z (m) is the depth below surface and E_h (GPa) is the average deformation modulus of the upper part of the earth's crust measured in a horizontal direction. This direction of measurement is important particularly in layered sedimentary rocks, in which the deformation Modulus may be significantly different in different directions.

A plot of this equation is given in **Figure 3.2** for a range of deformation moduli. The curves relating k with depth below surface z are similar to those published by Brown and Hoek (1978), Herget (1988) and others for measured in situ stresses. Hence above equation is considered to provide a reasonable basis for estimating the value of k.

As pointed out by Sheorey, his work does not explain the occurrence of measured vertical stresses that are higher than the calculated overburden pressure, the presence of very high horizontal stresses at some locations or why the two horizontal stresses are seldom equal. These differences are probably due to local topographic and geological features that cannot be taken into account in a large scale model such as that proposed by Sheorey.

$$K = \frac{\text{Horizontal Stress}}{\text{Vertical Stress}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3.4)$$



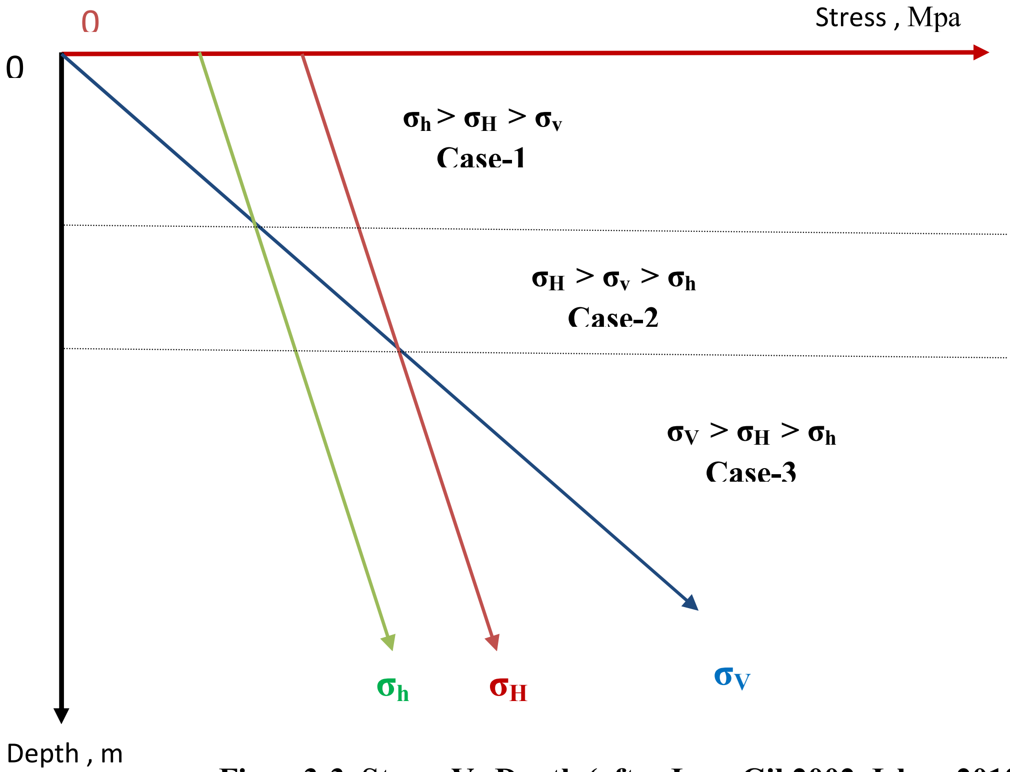
Figur 3-2 Ratio of horizontal to vertical stress from different deformation moduli based upon Sheorey's equation (after Shoerey's 1994)

3.2 Classification of In-situ stress and Fault

Based on in-situ stress magnitudes, Anderson (1951) classified three types of earth's in-situ stress states: extensional ($\sigma_V > \sigma_H > \sigma_h$), strike-slip ($\sigma_H > \sigma_V > \sigma_h$) and compression ($\sigma_H > \sigma_h > \sigma_V$). Borehole instability is in most of the cases, a direct reflection of these stress states. And in most of the cases wells are being drilled without the proper knowledge of stress pattern of the area. The situation becomes exceedingly critical if the drilling is being carried out in a tectonically active region involving multiple faults and variable degree of displacement of the adjoining structures. By considering a borehole stability incident from field reports it has been seen in many cases that it was not possible to maintain stable drilling.

until now, the drilling phase make use of stress direction mapping, relative in-situ stress magnitudes (i.e., $n_H = \sigma_H / \sigma_v$ and $n_h = \sigma_h / \sigma_v$), borehole breakout analysis and well path Optimization by inversion techniques. Breakout based determination of in-situ stress Orientation became an industry standard and has been discussed by many authors. Breakout is the zones that occur on the opposite side of the borehole due to spilling of the rock, especially when in-situ horizontal stress anisotropy exists in the region. However, in case of deviated wells the minimum stress direction cannot be estimated directly from the breakouts as its position changes in the borehole wall in relation to trajectory azimuth and to the in-situ stresses (Islam 2010, Jaeger et al., 1969; Mastin et al., 1988). Moreover, in some cases rock failure can happen due to orientation of the trajectory, drilling practices, improper mud property, and lower strength of the rock, to mention a few which are not directly related to the stress pattern of the area. A washout which is caused by natural weakness can easily be differentiated from breakouts, but shows no preferable orientation. Though, wellbore failures in the areas with very low horizontal stress anisotropy can confuse one in distinguishing breakouts from washouts. In areas with other parallel or complimentary information, like directionality obtained from sonic-shear-cross-dipole-anisotropy measurements, be considered (Dhruba et al., 2009).

A stress field model which varies with depth is therefore presented in **Fig. 3.3** (Islam 2010) this model represents wellbores drilled in shallow (case-I), medium-deep (case-II) and deep basins (case-III). These three cases are defined based on the in-situ stress magnitudes vs. depth of investigation. Inspection of these models revealed that the assessment of in-situ stresses is the focal weak side of in borehole instability analysis. A standard geo-mechanical model is essential for evaluating in- situ stresses. Estimated in-situ stresses may be used as input into shear failure models to evaluate different shear failure modes with reasonable accuracy. This paper presents a geo-mechanical model based on extensively used correlations for estimating rock strength, in-situ acting stresses and formation pore pressure. This current investigation will enhance the insight into borehole collapse risk.

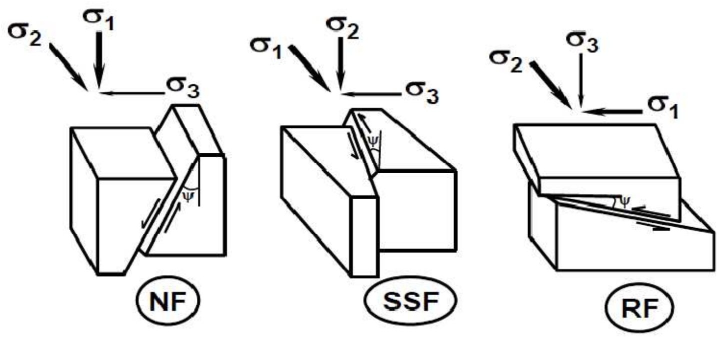


Figur 3-3 Stress Vs Depth (after Ivan Gil 2002, Islam 2010)

From Islam M.A (2010) paper said it is not known exactly how rock fails. The processes associated with failure are complex and not subjected to convenient characterization through simplified models. The Collapse criterion defines a state where the borehole is no longer stable, but becomes unstable to a degree where it is defined as collapsing. Many different arguments can be used to define the collapse criteria, e.g. scientific arguments based on mechanical criteria or operational argument based on practical limitations. Operational arguments are related to type and amount of caving or breakouts present in the drilling fluid, degree of wellbore instability with respect to section of angle & length, and the inclination of the borehole. Scientific arguments are fulfillment of a failure criterion, choice of failure criteria with respect to stress conditions, type of formation, and type of analysis method (analytical or numerical).

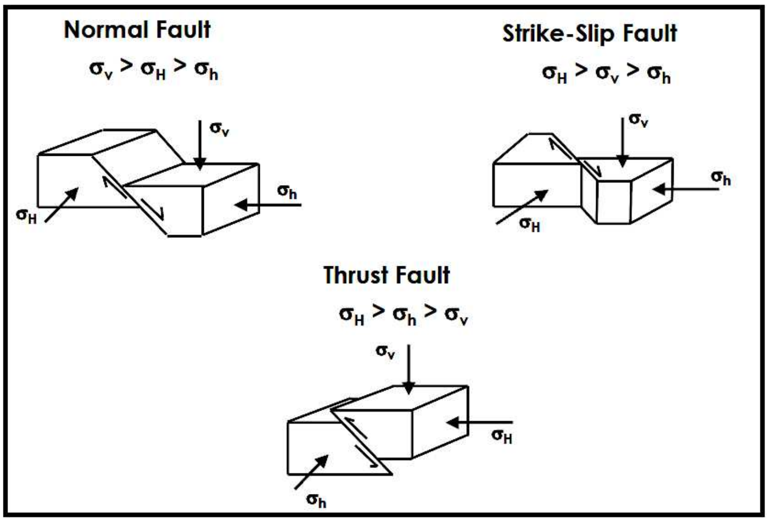
3.4 Fault Classification

Shale is heterogeneous and of laminated nature. Various types of fault have been found in shale and classified by Anderson (1951) and by Twiss and Moores (1992) as summarized in Fig. 3.4. As a rule of thumb for stress pattern with faulting, Anderson (1951) proposed a description for in-situ stress regimes. He suggested that normal or extensional faulting (NF) stress regimes are associated with $\sigma_v \geq \sigma_H \geq \sigma_h$, a reverse (RF) stress regime is associated with $\sigma_H \geq \sigma_h \geq \sigma_v$, and strike-slip (SS) setting occurs where the stress order is $\sigma_H \geq \sigma_v \geq \sigma_h$, particularly at shallow depths. Therefore, when σ_1 is vertical, normal faulting will occur, when σ_3 is vertical reverse faulting will occur, and strike-slip faulting occurs when σ_2 is vertical. Thus, the relative magnitudes of the in situ earth stresses control the tectonic pattern of the structure, and borehole stability analysis is dependent on stress pattern.



Figur 3-4 Stress axes and faults for relative stress magnitudes in normal NF), Reverse (RF) and strike-slip (SS) faulting regions (After Andersen 1951). the ψ is the angle between the maximum principal stress and the failure plane. For sand and sandstone values in the

Limited knowledge of in-situ stress often requires that data be estimated from the available correlations. In-situ stress consists of magnitude and orientation of principal stresses. These stresses magnitude depend on Tectonic activity and the presence of faults and folds. The following figure shows three types of fault and stress orientations by plate movement (**Figure 3.5**):



Figur 3-5 Fault Classification (SPE 99644)

According to Anderson's (1951) faulting mechanism, the principal stress ratios n_h defined as (σ_h/σ_v), and n_H defined as (σ_H/σ_v) uniquely define the stress regime. For instance, $n_h<n_H<1$ indicates extensional stress regime, while $n_h<1<n_H$, indicates strike-slip stress regime, and $1<n_h<n_H$ indicates compressional stress regime. In what follows most of the cases, only extensional and strike-slip stress regimes are considered because compressive stress regimes are not frequently encountered.

3.5 Estimated In-situ Stress

For a basin that is not tectonically active, the two horizontal stresses, Maximum (σ_H) and minimum (σ_h) can be assumed to be equal in magnitude (rare of occurrences of lateral strains during sediment burial). In a passive basin, Vertical stress (σ_v) is higher than the horizontal stresses. Hubbert and Willis (1957) proposed an empirical expression for the magnitude of the least principal stress as a function of depth in the Gulf of Mexico region:

$$\sigma_{h\ min} = 0.3(\sigma_V - P_p) + P_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.5)$$

Where the constant 0.3 was empirically determined from the analysis of hydraulic fracturing data. The scientific basis for this constant can be understood in terms of frictional faulting theory (Zoback and Healy 1984). Mathews and Kelly (1967) proposed a similar relation for the fracture pressure, or the magnitude of the pore pressure at which circulation is lost. As this requires propagation of a hydraulic fracture away from the wellbore, this value is essentially equivalent to the least principal stress. Thus they proposed:

$$\sigma_{h\ min} = K_i(\sigma_v - P_p) + P_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots.(3.6)$$

Where K_i is a function of z, using this relation, functions for the Louisiana Gulf coast and South Texas Gulf coast region were proposed that varied in a non-linear fashion from 0.4 and 0.48 at 2000ft to values exceeding 0.7 at depths greater than 10,000ft (see Mouchet and Mitchell 1989).

Eaton (1969) suggested a physically based technique for determination of least Horizontal stress can be estimated, using Poisson's ration the following correlation:

$$\sigma_H = \sigma_h = \left(\frac{\mu}{1-\mu}\right)\sigma_v + P_p\left(\frac{1-2\mu}{1-\mu}\right) \ldots\ldots..\ldots\ldots.\ldots.(3.7)$$

Another method to estimating minimum horizontal stress is reported by Breckels and Van Eekelen (1982), they used instantaneous shut in Pressure (ISIP), recorded from U.S. Gulf of Mexico, Venezuela, and Brunei, to estimate minimum horizontal stress and correlate the relation from the extensive amount of information. These relations are following by **Table-3.1**:

Tabell 3.1 Minimum Horizontal stress and Stress Depletion rate correlations in worldwide Basin

Region	Depth Range (ft)	σ_h (psi)	Stress Deplation ratio
US Gulf Coast	0 to 11,500	$0.197D^{1.145}$+0.46 (P_c-P_{cn})	0.46
	>11,500	1.167-4596+0.46 (P_c-P_{cn})	0.46
Venezuela	5,900 to 9,200	$0.21D^{1.145}$+0.56 (P_c-P_{cn})	0.56
Brunei	0 to 10,000	$0.21D^{1.145}$+0.56 (P_c-P_{cn})	0.49
(Source Simangunsong R.A. et.al. 2006)			

Edwards et. al. (1998) examined Leak off tests from the North Sea basin and concluded that the ration of Minimum horizontal stress to vertical stress, with respect to date is linearly dependent shown table 3.2 is following:

Table 3.2 Trend of σh/σv with Depth in the North Sea	
(Simangunsong et. al. 2006)	
Region	σ_h/σ_v
Northern North Sea	10^{-5} D + 0.7515
Central North sea	$2x10^{-5}$ D + 0.7439
Southern North sea	$3x10^{-6}$ D +0.8854

The in-situ principal stresses below the earth's surface are commonly assumed to lie in the vertical and horizontal directions. This is a good assumption for areas of little or no active tectonic activity, such as the North Sea. In such cases, the vertical total stress (σ_v) at a point in the formation is simply the gravitational weight of the vertical column of rock above that point, and it can be obtained by integration of the density log. The most reliable way of determining the minimum in-situ total stress (σ_h) is by a mini-frac or micro-frac test. Although less precise, σ_h can also be estimated indirectly from leak-off tests (LOTs). The leak-off point (LOP) in a LOT generally corresponds to the mud pressure at which the formation starts taking in mud fluid. Sau-Wai Wong et. al. (1993) examines The LOP values of about 170 LOTs performed in the northern North Sea. These values have not been corrected for the borehole deviations.

Wong et. el. (1993) collected data from 470 Leak off tests in the central Graven, North Sea, little Tectonic activity occurs in this area.

They established correlation for minimum Horizontal stress, Vertical stresses with respect to depth shown on table 3.3 is following:

Table 3.3 In-situ stress and Pore Pressure Correlations for Central Graben North Sea	
(Simangunsong et. al. 2006)	
Parameter	Correlations
Minimum Horizontal stress	σ_h= 31.5+0.472 x D +3.228 x 10^{-5}x D^2
Vertical (Overburden stress)	σ_v= 49 +0.747 x D +1.44 x 10^{-5}x D^2
Pore Pressure	P_p= (σ_h-0.55 σ_v) / 0.45

Finally, Holbrook, Maggiori et.al. (1993) proposed a porosity based technique for estimation of the least principal stress based on force balance concept:

$$\sigma_{h\,min} = (1-\varphi)(\sigma_v - P_p) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots.(3.8)$$

As porosity of over pressured shale is typically ~35%, it yields similar values to that predicted with Ki~65% in the Matthews and Kelly (1967) relation for over pressured shale at depth, but would seriously overestimate the least principal stress.

Li and Purdy (2010) proposed the following methods to calculate the upper bound of maximum horizontal stress (σ_H) based on generalized hook law with the equilibrium of stresses and pore pressure:

$$\sigma_H^{max} = \frac{(\sigma_h - \alpha_\beta P_p)}{\vartheta} - \sigma_v + 2\alpha_\beta P_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.9)$$

Li and Purdy (2010) presented an improved method to determine the maximum horizontal stress using observations of breakout width when the rock uni-axial compressive stress is known, that is:

$$\sigma_H \leq \frac{\sigma_{ucs} + (K+1)P_{mud} - \alpha_\beta(K-1)P_p - (1-2Cos2\beta_b)\sigma_h + \sigma^{\Delta t}}{1+2Cos2\beta_b} \ldots\ldots\ldots\ldots(3.10)$$

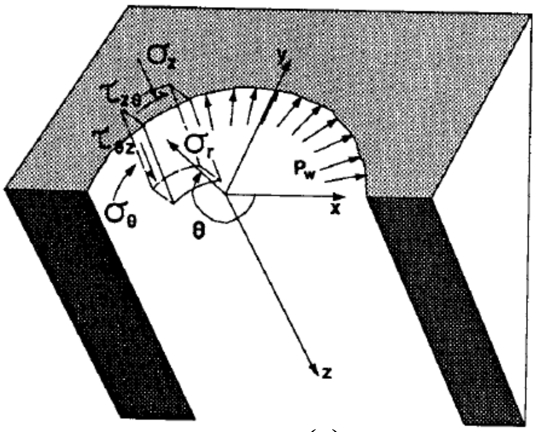
Where is $2\beta_b$ the wellbore breakout angle, P_{mud} is the mud pressure, K = $(1+Sin\phi_f)/(1-Sin\phi_f)$, ϕ_f is the angle of internal friction and $\sigma^{\Delta t}$ is the thermal effects stress. For some cases thermal effects are so small, can be avoided.

Simangunsong et.al (2006) suggests that Normal faulting Horizontal stresses are typically smaller between 25 and 50% than of the vertical stress. In the regions with folding or thrust faulting, the Horizontal stress is typically between 200 and 300% higher than vertical stress. Further more local structures can considerably alter the regional in-situ stress.

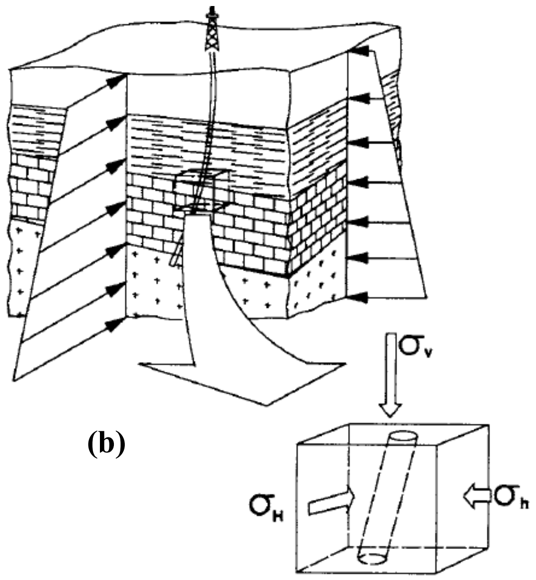
CHAPTER 4 FAILURE MODELS AND FAILURE CRITERIA

4.1 Back Ground of Wellbore Stability Modeling

Before driling a wellbore stres is copressive on the undergound with the exceptional structural complex area (near Saltdiapirs, tectonical causes etc). The In-situ stresses can be resoloved into a vertical (Overburden, σ_v) and two Horizontal stresses Such as Maximum stress (σ_H) and Minimum stress (σ_h) which are generally unequal. After Hole is drilled the stress is redistributed and hole is supported by the hydraulic pressure of Mud. The redistributed stress are generally designated by Hoop stress, σ_θ ,which act circumferenly around the wellbore wall, The radial stress σ_r and Axial stress σ_z which acts parallel to borehole axis. Incase of deivated wellbore another shear stress $\tau_{\theta z}$ come into account **Fig 4.1 a,b** (Addis et.al.,1991).



(a)



(b)

Figur 4-1 (a) Stress State at the Wall of a Deviated wellbore

Figur 4-1 (b) In-situ stress Field (Addis et.al. 1991)

4.2 Determination of Borehole stress State

The borehole stress is highly depend in stress-strain charcterictic from formation response to loading. Most of the cases assumed to be homogeneous,Isoropic and linear elastic (HILE) which is allowed to determine simple equations. Also more complex model (Modified lade criteion, Hoek-Brown Criterion etc.) frequently suffer from an exhaustive list of input parameters, Many of which can't be realistically determined in field cases, although sometimes they are created a accurate result compare to the HILE model. Appendix A has discussed in details about the stress of state.

4.3 Failure Criteion

To determine the stress along the borehole wall it is necessay to compare the formation strength of rock with borehole wall stress. At points where the stress states exceed the formation strength (either tension or compression) of rock failure is considered to have intitiated. Stress of wall converted to principal stresses (appendix A), Kirsch equation. One of the principal stress acts perpendicular to the wellbore and simply given by the wellbore Pressure ,P_w. The remaining two are found by transposing hoop stress σ_θ, the axial stress σ_z, and shear stress $\tau_{\theta z}$ (see figure-3), thus the three principal stresses can be expressed as

$$\sigma_1 = \frac{\sigma_\theta + \sigma_z}{2} + \sqrt{\left(\left(\frac{\sigma_\theta - \sigma_z}{2}\right)^2 + \tau_{\theta z}^2\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1)$$

$$\sigma_2 = \frac{\sigma_\theta + \sigma_z}{2} - \sqrt{\left(\left(\frac{\sigma_\theta - \sigma_z}{2}\right)^2 + \tau_{\theta z}^2\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.2)$$

$$\sigma_3 = P_W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.3)$$

However P_w may also be the intermediate or maximum principal stress depending on conditions.

4.3.1 Tensile Failure Criteria

This is discussed by Appendix B, Tensile failure criteria is simply determined by whether the minimum effective stress at the wall is less than the tensile strength of formation (assuming compression is positive), Thus the failure occurs when

$$\sigma'_3 \le -|\sigma_t| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4)$$

Where the σ_t is the tensile strength of rock and effective normal stress is given by Total normal stress minus pore pressure.

From Addis et. al. (1991), in certain instances the well pressure required to initiate the fracturing at the wellbore wall is lowered than the minimum Principal in-situ horizontal stress. In these cases the tensile fracture will only propagate a few radii from the wellbore resulting in only minor fluid losses, which is unlikely to constitute a problem. Thus when tensile failure is initiated we must also check to see if the fracture will propagate. Assuming the minimum Horizontal principal stress is less than to Vertical (overburden) stress, and then the propagation criterion can be writing on

$$P_w \geq \sigma_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.5)$$

Aadnoy (1988) discussed that effects of anisotropic elasticity parameters on the fracturing pressure are small. Therefore, with all possible sources of error involved in the interpretation of the fracturing pressure the anisotropy may be neglected. The anisotropy has a definite effect, however, on the position of the fracture around the hole wall. Permeability is an important parameter, such impermeable rocks as shale have high fracture pressures. The most important single factor in the fracturing analysis is probably the magnitude of in-situ stress field.

4.3.1.1 ElastoPlastic Fracture Model

Aadnoy et.al. (2007) developed an Elastic-plastic Fracture Model to improve the Predictions in deviated well. They analyzed that Kirsch (1898) equation under predicts the Fracture pressures, reason is that the Plasticity behaving mud cake is not properly accounted for. They developed a new elastic-plastic model consists of an anisotropic Part (in-situ stresses) and a Hydrostatic Part (The Plastic Mud cake). Kirsch equation works well for penetrating fluids without filtrate control, Therefore model with the kirsch equation simplest form may be written as:

$$P_w = \sigma_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.6)$$

It simply says that the borehole will fracture when the Minimum in-situ stress exceeded. The condition here is a Penetrating fluid such as water. But in drilling Processes, the fluids build a filter cake Barrier, Kirsch equation uses a non penetrating boundary condition. It assumes step function separating the borehole pressure and Pore pressure. The simple form is follows:

$$P_{wf} = 2\sigma_h - P_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.7)$$

Aadnoy et.al (2007) shown that above equation is general underestimates the fracture pressures. The problems is linear elasticity and a perfect (Zero filtrate Loss) Mud-cake.

Aadnoy and Belayneh (2004) shown their paper is that Nonpenetrating Boundary conditin actually consists of Several Parts, a bridge that yields during frcaturing and a crack in the rock that opens up with increasing borehole pressure, so inreality concentration factor 2 in above equation is not properly defining the frcature initiation Pressure because of an ill defined Boundary condition. Aadnoy et.al. (Feb.,2007) works drilling operation about this Issues. Aadnoy and Belayneh (2004) developed new elasto-Plastic model for the Non-Penetrating situation, and looks as simple form:

$$P_w = 2\sigma_h - P_p + \frac{2\sigma_y}{\sqrt{3}} \ln\left(1 + \frac{t}{a}\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (4.8)$$

The addiotional stregth obtained with elasto-Plastic Model is directly proportional with the yield stregth of the Particles form of Barrier. Aadnoy and Karstad (November,2007) shown the elastoplastic Barrier as follows:

$$P_{ep} = \frac{2\sigma_y}{\sqrt{3}} \ln\left(1 + \frac{t}{a}\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (4.9)$$

The general expression for the fracture equation becomes:

$$P_{wf} = P_{ep} + \sigma_x + \sigma_y - 2(\sigma_x - \sigma_y)\mathrm{Cos}2\theta - 4\tau_{xy}\mathrm{Sin}2\theta - \frac{\tau^2{}_{\theta z}}{\sigma_z - P_p} - P_p + \sigma_t \ldots (4.10)$$

The Postion of Fracture initiation is : $\mathrm{Tan}2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$……………………………..(4.11)

Above equation is the General form of frcature inintiation, if two horizointal stress is equal (relax depositional Basin), and normal fault is consudered, the facture equation can be simplified by:

$$P_{wf} = P_{ep} + \sigma_h(3 - \mathrm{Cos}^2\gamma) - \sigma_v\mathrm{Sin}^2\gamma - P_p + \sigma_t \ldots \ldots \ldots \ldots \ldots \ldots . . (4.12)$$

So the in-situ stress controls the the rock fractiuring and is an anisotropic process. The elasoplastic barrier, on the other hand ia an isotropic process, independent of wellbore inclination. From field experience (Aadnoy et.al.november 2007) conclude that A pure fluid with no filtrate control obeys the kirsch equation, Lab experimetns are required to determine the elastoplastic barrier for the mud used during drilling. They conclude that all method (such as LOT, XLOT, Correlation) have the drawback of estimating only one stress, i.e., minimum horizontal stress σ_h. Only the leakoff inversion method done by Adnoy (1990) determines the full 3-dimensional stress tensor and its direction. They also says always use the same model to estimate stress and derive predictions for new wells.

The new model (elastoplastic) model behave differently than Kirsch model for deviated well. The linear elastic part is sensitive to stress transformations and hence to wellbore orietation. On the otherhandThe elastoplastic barrier has no directional properties. Insome respect this shows that kirsch model overpredicts the directional effects.

From Adnoy et.al.(2007) Paper we got, there is no dependence of the azimuth for isotropic cases, only factor inclination. This example shows elastoplastic model offset the kirsch solution for inclinations, because of the elastoplastic barrier has no directional properties. The offset of Both models is constant regradless of inclination, only the rock frcatrure is sensitive to inclination, The realtive increase is 18.3% for a vertical well and 24.4% for a horizontal wellbore. In the early 1980's assumed that the oil could only drilled to certain inclinations, this assumption was based on Kirsch analysis, Field experience however showed that the most wells can be drilled to any inclination. The new elastoplastic model is therefore more in line with field experience. From the above discussion Adnoy et. al. (2007) conclude that Plastic behaviour of the filtercake is contributing significantly the fracture initiation pressure and the effect is isotropic. They concluded also that Kirsch model under predicts the fracture pressure in deviated wells and elastoplastic model is less conservative and should be used in highly deviated wells. But in mind that New model is valid only if Filtrate contro is applied with drilling muds.

4.3.2 Compressive Failure Criterion

The failure of rock in compression is a complex Process that involves microscopic failures manifest as the certain of small tensile cracks and frictional sliding on grain boundaries (Brace, Paulding et.al.1966). A main aspect of wellbore stability analysis is the selection of an appropriate rock failure criterion. Several linear elastic methods have been used to predict or describe at which stresses or stress conditions failure occur in a formation. The most popular models that have been used are Mohr-Coulomb, Mogi-Coulomb, modified Lade and Drucker–Prager. The principle used to predict borehole failures through those models are quite similar, but the involvement of principal stresses in the material failure process is different from model to model. For example, Mohr-Coulomb does not consider the effect of intermediate principal stress while Mogi-Coulomb and modified Lade do. Mclean-Addis (1990) are categorized different model in his study. They discussed two commonly failure criteria is reviewed for wellbore stability, Mohr-coulomb and Drucker-Prager (also known as extended Von-misses) criteria.

4.3.2.1 Mohr-coulomb criteria

The Mohr-coulomb criteria can be written on terms of Principal stress by:

$$\sigma_1 - P_p = \frac{1 + \text{Sin}\phi}{1 - \sin\phi}(\sigma_3 - P_p) + \frac{2\tau_0 \text{Cos}\phi}{1 - \text{Sin}\phi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

$$\tau_{max} = \tau_0 \text{Cos}\phi + \sigma_{m,2} \text{Sin}\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.14)$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \qquad \text{and} \qquad \sigma_{m,2}\ \frac{\sigma_1 + \sigma_3}{2}$$

Where τ_0 (rock cohesive strength) and α (friction angle) are material Parameters.

Cohesion (τ_0) is not a physically measurable parameter; it is more common to express rock strength rock strength in terms of C_o (UCS). The relationship (Zoback, 2007) between τ_o and C_o is:

$$C_0 = 2\tau_0\left[(1 + \text{Tan}^2\phi)^{\frac{1}{2}} + \text{Tan}\phi\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.15)$$

The criteria expressed above the equations will always first be satisfied on a plane that lies in the direction of σ_2; the value of σ_2 will not influence σ or This failure criterion implicitly assumes that σ_2 has no effect on failure, it only represents the failure situation in which $\sigma_2 = \sigma_3$ (Al-Ajmi et. al., 2005). The criteria expressed that the mean normal stress contributing to the creation of a failure plane is $\sigma_{m,2}$. At failure, a linear relationship is predicted between the maximum shear stress and the effective mean stresses.

According to A.A. Grouch (SPE, 2001) paper, Mohr-coulomb failure criterion is given by:

$$\sigma_1 - P_p \le 2\tau_0 \text{Tan}\left(\frac{\pi + 2\phi}{4}\right) + (\sigma_3 - P_p)\text{Tan}^2\left(\frac{\pi + 2\phi}{4}\right) \ldots\ldots (4.16)$$

If this condition is satisfied Failure will not occur.

4.3.2.2 The Drucker-Prager criterion

According to Addis et.al. (1990) paper, The Drucker-Prager criterion are expressed in terms of principal stresses as:

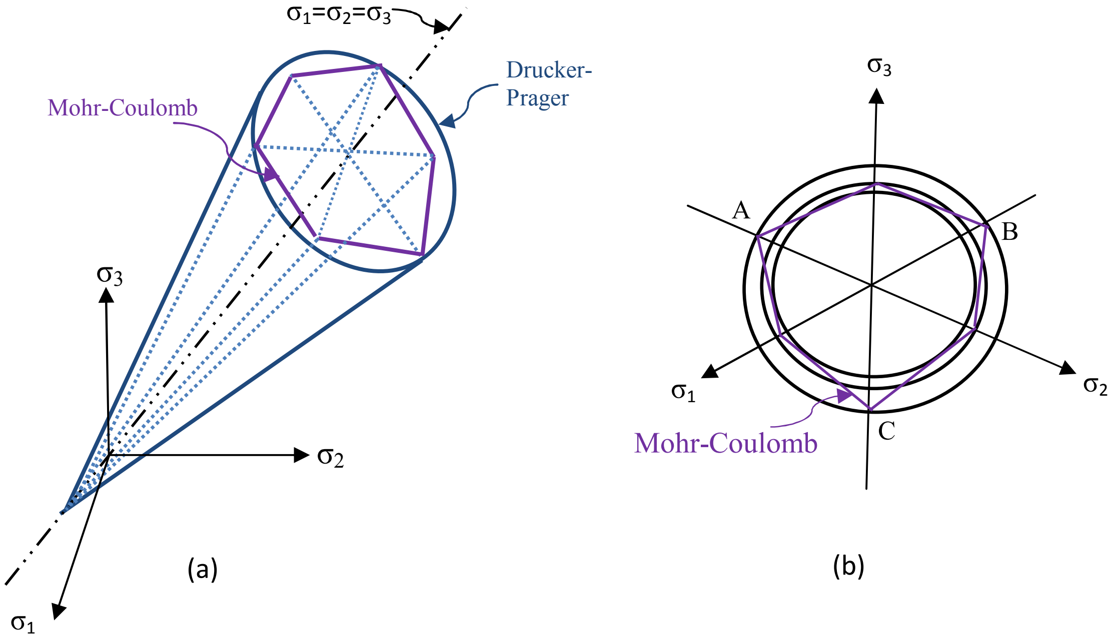
$$\tau_{oct} = C + m(\sigma_{oct} - P_p) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.17)$$

Where, $\tau_{oct} = \frac{1}{3}\sqrt{((\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}$

$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

C and m are materials Parameters.

Determination of rock strength in laboratory is frequently determined using standard tri-axial test equipment, if Core is available. It is straight forward to apply Mohr-Coulomb criterion for fitting the data (plot confining pressure x axis Vs axial stress on Y axis). According to Mclean and Addis (1990) paper, when using Drucker-Prager Criterion we are faced three choices when fitting the criterion to test the data. These choices have come through comparing with Drucker-Prager and Mohr-coulomb criterion. The projection of Mohr-coulomb and Drucker-Prager are plotted together shown in figure following (Fig 4.2 - a, b) and project at π-plane:



Figur 4-2 Projection in Failure criterion on Principal Stress Space and (Mclean and Addis 1990, SPE-20405)

A plane perpendicular to the line is defined by $\sigma_1=\sigma_2=\sigma_3$. The outer Drucker-Prager circle coincides with the outer apices of the Mohr-Coulomb hexagon and middle one with inner apices. Addis and Mclean (1990) fit the outer Drucker-Prager circle on their research. The other two are the result of trigonometric fitting exercises between the Mohr-coulomb and Drucker-Prager criterion. The relationships are following:

$$\text{Outer Circle: } m = \frac{2\sqrt{2}\text{Sin}\phi}{3-\text{Sin}\phi} \ldots (4.18) \qquad C = \frac{2\sqrt{2}\,\tau_0\text{Cos}\phi}{3-\text{Sin}\phi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.19)$$

$$\text{Middle Circle: } m = \frac{2\sqrt{2}\text{Sin}\phi}{3+\text{Sin}\phi} \ldots\ldots (4.20) \qquad C = \frac{2\sqrt{2}\,\tau_0\text{Cos}\phi}{3+\text{Sin}\phi} \ldots\ldots\ldots\ldots\ldots (4.21)$$

$$\text{Inner Circle: } m = \frac{\sqrt{6}\text{Sin}\phi}{\sqrt{9+3\text{Sin}^2\phi}} \ldots (4.22) \qquad C = \frac{\sqrt{6}\,\tau_0\text{Cos}\phi}{\sqrt{9+3\text{Sin}^2\phi}} \ldots\ldots\ldots\ldots\ldots (4.23)$$

Mclean and Addis proposed an improved mathematical model to evaluate the shear failure, using the drucker-Prager criterion in both underbalanced and overbalanced drilling conditions, This is one a powerful model, they concluded that Shear collapse can be estimated more realistically than with M-C and Von-Misses Criterion and using outer D-P Circle option.

4.3.2.3 Mogi-Coulomb Criterion

Several authors studied about well bore stability and discussed the performance of each constitutive model about their goodness and limitations (Takahashi et al., 1989; Horsrud et al., 1994; Haimson et al., 2000; Ewy, 1999; Chang et al., 2000, Al-Ajmi el al., 2005; Gil et al., 2002; Aadnøy et al., 2005; Islam et al., 2009c). It is obvious that intermediate strength effects cannot be ignored to estimate borehole collapse risk under strong anisotropic in–situ stress state. A 3D failure criterion related model is therefore required to account for poly-axial stress effects on collapse pressure prediction (CPP). The Mogi-Coulomb linear elastic model accounts for σ_2 effects on CPP. A closed form 3D elastic analytical solution (Ewy et al., 1998) worked well in conjunction with linear Mohr-Coulomb elastic model and modified Lade criterion. On the other hand, based on Mohr-Coulomb linear elastic theory, Islam et. al. (2010) developed a closed-form simplified analytical solution to estimate CP for complex well trajectory.

In general, 3D failure criteria that contain numerous parameters, or which require numerical evaluation, are difficult to apply in practice, particularly for wellbore stability problems. When it is intended to consider the influence of σ_2 on rock strength in wellbore stability analyses, the Drucker–Prager failure criterion is often used. This criterion is simple, since it contains only two fitting parameters. However, this failure criterion has been reported to overestimate (Islam, 2010) the intermediate principal stress effect, which may result in nonsensical stability predictions (Colmenares et al., 2002; Chang et al., 2000). As the Mohr-Coulomb criterion only represents rock failure under tri-axial stress states, it is expected to be conservative in predicting wellbore instability. To overcome the problem, Al Ajmi and Zimmerman (2005) introduced a new true tri-axial failure criterion called the Mogi-Coulomb criterion. This failure criterion is a linear failure envelope in the Mogi domain (τ_{oct} - $\sigma_{m,2}$ space), and the corresponding strength parameters were shown to be directly and simply related to Coulomb strength parameters, cohesion and friction angle. This linear failure criterion has been justified by experimental evidence from tri-axial tests as well as poly axial tests (Mogi, 1971b; Michelis et al., 1985 and 1987; Terzaghi et al., 1923; Takahashi et al., 1989; Haimson et al., 2000; Colmenares et al., 2002 and Al-Ajmi el al., 2005). The Mogi–Coulomb criterion neither ignores the strengthening effect of σ_2, as is done by the Mohr–Coulomb criterion, nor does it predict a strength as unrealistically high as does the Drucker–Prager criterion. Although both the Drucker–Prager and the Mogi–Coulomb failure criteria attempt to represent the failure surface of a material, they do so in different mathematical sub-spaces of the full three-dimensional space of principal stresses.

The solution of the Mogi-Coulomb CP model was derived in closed-form for vertical wellbores, for all stress regimes (Al-Ajmi et al., 2006b). For deviated or horizontal wellbores, a closed-form general solution could not be achieved and a numerical solution was needed. It is found that Mogi solutions have been used in several field cases and the model seems in each case to be consistent with field experience (Al-Ajmi et al., 2006a).

The most challenging part associated with the Mogi-Coulomb model is to estimate the model fitting parameters a and b (a is the intersection of the line with the (τ_{oct} - $\sigma_{m,2}$) axis, and b is its inclination from poly axial test data). Generally, poly axial test data are seldom available and in particular for shale. But the model fitting parameters can be estimated from tri-axial data (Al-Ajmi et al., 2005). Islam et.al. (2010) used tri-axial test data of Pierre-1 shale (relatively soft material) to estimate the model fitting parameters. They also developed and proposed a correlation which may estimate model fitting parameter from uniaxial compressive strength tests.

The Mogi-Coulomb criterion is a true-triaxial failure criterion. It is a natural extension of the classical Coulomb criterion into three dimensions defined by:

$$\tau_{oct} = a + b\sigma_{m,2} \text{—} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (4.24)$$

Where, the strength parameters a, b (Mogi-Coulomb strength parameters) and the octahedral shear stress are given by;

$$a = \frac{2\sqrt{2}}{3}\tau_0 \mathrm{Cos}\phi \ldots . (4.25) \quad b = \frac{2\sqrt{2}}{3}\mathrm{Sin}\phi \ldots \ldots \ldots \ldots \ldots \ldots \ldots . \ldots (4.26)$$

Where,

$$\tau_{oct} = \tfrac{1}{3}\sqrt{((\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}$$

4.3.2.4 Modified Lade criterion

The lade criterion (lade 1977) is a three dimensional failure criterion that was originally developed for frictional materials without effective Cohesion (Such as granular Soils). It was developed for soils with curved failure envelopes. This criterion is given by:

$$\left(\frac{I_1{}^3}{I_3} - 27\right)\left(\frac{I_1}{P_{atm}}\right)^{m'} = \eta_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . \ldots (4.27)$$

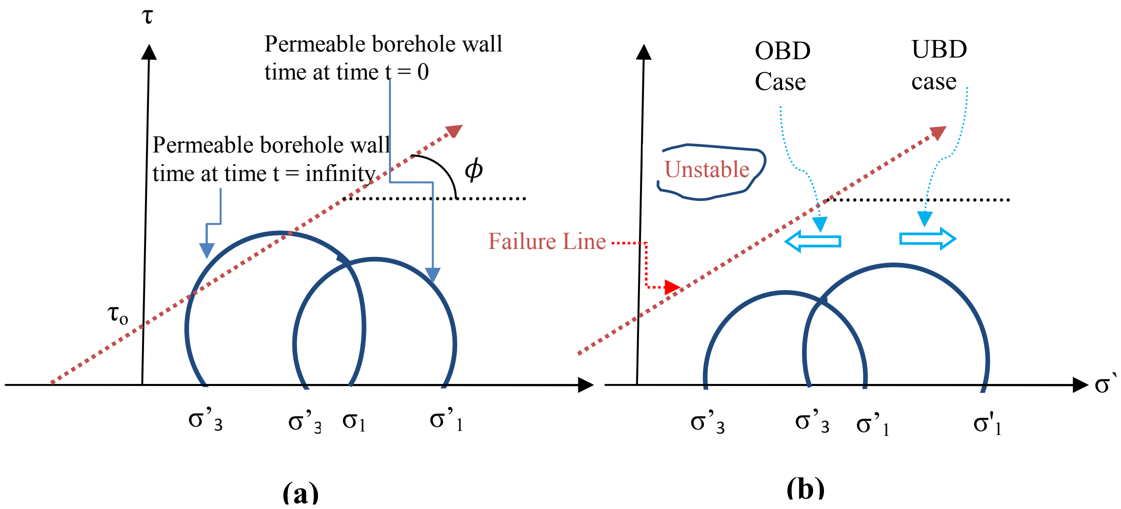
Where, $I_1=\sigma_1+\sigma_2+\sigma_3$ and $I_3= \sigma_1\sigma_2\sigma_3$ and m', η are material Constants. Modified Lade criterion developed by Ewy (1999) m' was set equal to Zero in order to obtain a criterion which is able to predict a linear shear strength increase with increasing mean stress ($I_1/3$). For considering material cohesion Ewy (1999) introduced Pore pressure as a necessary parameter and introduced S and η as material constants, determined from Mohr-Columb cohesion τ_o and internal friction angle α by:

$$S = \frac{\tau_o}{\mathrm{Tan}\phi} \ldots . (4.28) \qquad \eta = \frac{4(\mathrm{Tan}\phi)^2}{(1 - \mathrm{Sin}\phi)}(9 - 7\mathrm{Sin}\phi) \ldots\ldots\ldots\ldots ... (4.29)$$

ML criterion correctly describes the influence the intermediate Principal strength on rock strength. This corrective behavior results in a more realistic prediction compared to other criterion. For In details, Ewy (1998, 1999) provides the derivation of the Modified lade criterion.

4.4 Time delayed Failure

Normally, during borehole failure analysis based on Kirsch solution (1898 a), it is assumed that the borehole wall is impermeable and without mud cake. Shale has a very low but still a finite permeability (Nes, 2010), which means that impermeable wall conditions are valid only during drilling. After drilling, the pore pressure close to the borehole wall will gradually and eventually approach the well pressure depending on shale permeability (Aadnoy 2010). If the formation is exposed to the mud pressure for sufficiently long time, steady state pore pressure equilibrium can be reached. If the well is kept in overbalance (OBD), the pore pressure near the borehole wall will increase, leading to decreased effective stress, resulting in reduced hole stability (Rahman et al.,2000; Mengjiao et al., 2003). The opposite scenario can be seen for UBD. Based on an extension of the work of Terzaghi (1923), the solution is depicted graphically in Fig. 4.3b This model is valid for fully saturated rocks. Fig. 4.3a shows the displacement of the Mohr-Coulomb envelope in the direction of shear failure. In case of OBD, a pore pressure (P_p) consolidation effect has the adversed impact on time delayed borehole failure risk, since, under this operation P_p increases and the effective stress decreases. Conversely, in UBD, the instantaneous borehole failure risk is high; radial effective stress becomes negative and smaller than the tensile strength, tensile spalling will lead to stability problems. But with time, due to pore pressure equilibration, pore pressure reduces and the borehole becomes stable. According to the Mohr-Coulomb failure envelope, time delayed pore pressure effects on borehole stability issues is favorable in UBD, but the opposite is true in OBD.This paragraph is taken from PhD paper of Islam (2010), NTNU.



Figur 4-3 (a) Borehole condition for time delayed instability (Islam 2010)

Figur 4-3 (b) Mohr-Coulomb failure lines and time delayed pore pressure effects on borehole instability.

From the **fig. 4.3 (b),** during UBD, Pp decreases, stability increases and the failure circle moves towards the right and moves away from the failure line. In OBD, the opposite scenario takes place.

CHAPTER 5 ATTACK ANGLE, OPTIMUM WELL PATH AND DIFFERENT PARAMETRS RELATED TO BEDDING PLANE

5.1 literature review of bedding Plane and Single Plane of weakness

There are 3 fundamental processes which form rock, igneous, metamorphic, and sedimentary processes. Each of these basic rock types have inherent structural characteristics that define it strength, anisotrpy and durability, and hence, its usefulness in an engineering situation. Anisotropic rocks fail fail by (1) shear failure across the plane of anisotropy or shear faulting along the plane of anisotropy, (2) plastic flow or slip along the plane of anisotropy or (3) kinking. The exact nature of the failure is depend upon the confining pressure and the orientation of sample.

According to J.C. Jaeger (1960) the two-dimensional theory of two simple generalizations of the Coulomb-Navier criterion for shear failure is developed. The first of these refers to a material with a single plane of weakness which has a different shear strength and coefficient of internal friction from the remainder of the material. In this case it is shown that failure may take place, according to circumstances, either in the plane of weakness or in planes cutting across it. The second criterion refers to a layered material whose shear strength varies continuously from a maximum in one direction to a minimum in the perpendicular direction. In this case it appears that, instead of the two directions of failure possible for an isotropic material, there is only one possible plane of failure which lies between the plane of minimum shear strength and the nearest to it of the two Coulomb-Navier planes.

According to Aadnoy (1988, 2009) elastic properties like bulk modulas, Young's modulas and poisson's ratio, show directional properties. Rock strength is high when force vectors are applied at a high angle to bedding. At lower angles, on the order of 15^0 and $30^{0,}$ stratal compressive strength is low. For this case, rock failure will occur along bedding planes. This type of rock behavior is often termed **'Planes of weaknesses'**. Sedimentary rocks have a laminated structure, with directional elastic properties as well as directional shear and tensile strengths. This off course will affect the behavior of inclined boreholes. Aadnoy (1988) shows bedding planes of shale mainly affect high angle and horizontal wells drill close to the minimum horizontal stress direction. Shale is anisotropic due to their laminated structure as a consequence of depositional environment. Shale are almost non permeable, but porous and contain pore fluid at a given pressure. Shale can also be abnormally pressurized, pore pressure is also a very important parameter in shale because the fracture gradient is strongly sensitive to the magnitude of the pore pressure. From Aadnoy et.al. paper, two situations about Sand stone, It is permeable. First one: a perfect mud cake is assumed, we have analogous situation with shale; Inside the borehole contain the borehole pressure, transition being a step function. The second: assumes no Mud cake, therefore, fluid communication between the formation and the borehole are allowed; this means that during fracturing operations, the pore pressure immediately inside the porous rock wall is equal to the borehole pressure. Stress contributions caused by the flowing fluid are neglected. He concluded that due to Shale impermeable compare to sandstone, fracture gradient of Shale is higher than Sandstone.

Aadnoy addressed also, tensile strength decreases for increasing borehole angle. Isotropic and anisotropic solution gives error about 2%. Effects of fracture due to anisotropic elasticity parameters are small, the anisotropy has a definite effect, however on the position of the fracture gradient around the wall. The higher pore pressure, the more sensitive the borehole is toward Collapse, for such impermeable rocks as Shale, the pore pressure may be considered constant, regardless of loading.The deformation of borehole was completely independent of relative values of the in-situ stresses and the positions of it's maximum and minimum values were only function of the orientation of the bedding plane relative to the borehole. The fracture will orient itself normal to the least in-situ stress as it propagates away from the borehole. If the borehole pressure couldn't be transmitted through the mud cake or into an impermeable rock, a higher fracture pressure was observed. For relax depositional basins with equal horizontal in-situ stress, the borehole is sensitive to collapse for inclination between 10^0 and 35^0. This applies only to laminated rocks with a plane of weakness. If the horizontal in-situ stresses are different, a borehole very stable against collapse can be drilled by inclining the hole in direction of the least in-situ stress. If the least in situ stress is normal to the plane of the borehole axis and the axis is normal to the bedding plane, the directional shear strength properties come into account. 'Plane of weakness' theory to a deviated borehole two conditions determine whether the rock fails along a weakness plane---1) the relative magnitude of the two normal stresses 2) The angle between the borehole and the bedding plane.

Chenevert (1965) shown in case of bedded rock, young modulus was lower normal to bedding than along bedding. Orthotropic model (Nine constants) may be simplified to a laminate model (four constants) through the use of the strain energy density function and the assumption of horizontal isotropy. According to him, the tensile strength is 20 to 35% lower parallel to the bedding plane than perpendicular to it. Chenevert M.E. et. al. shows that Plastic properties depend on K_{ani}, is lithology dependent : Sandstone very anisotropic, Shales moderately anisotropic, Limestone isotropic.

Four failure criterio are reviewed to asses borehole stability and failure related to bedding palne, namely Mohr-coloumb, Drucker-Prager, Mogi-Coulomb and modified Lade criteria. These failure croteria are combined with linear and Isotropic rock mechanics behaviour. Results indicate that the modified Lade, Mogi-Columb criteria tend to be more realistic than the Mohr-coloumb and druck-Prager criteria for these evaluations.

Depending on the source of the problem, wellbore instability is classified (Islam, 2010) as either Mechanical or chemical. Chemical wellbore, instability often called Shale instability, is most commonly associated with water adsorption in shaly formations, where the water phase is present and can cause borehole collapse. In contrast Mechanical wellbore instability is caused by applying mud of insufficient weight, which will create greater hoop stresses around the borehole wall and excessive hoop stresses cause in rock failure. Critical parametrs (Aadnoyet.al. ,2009) are plane of weakness in the rock stregth, the relative normal stress values on the Borehole, and the relative angle between the Borehole and Bedding Plane.

According to Haimson and Herrik (1989) The major Breakout mechanism is apparently Tensile rupture along surfaces to the Borehole wall aided by shear failure in the redial direction. They also try to conclude that Collapse occurs at the position of the Borehole that corresponds to the direction of the least in-situ stress, Normal to the axis of the hole that is comply with the Aadnoy Paper (1988). According to Aadnoy (1988) increasing the depth of well doesn't affect the Shear stress of rock significantly and higher pore pressure the more sensitive the borehole is toward collapse. He also concluded that if the least in-situ stress is normal to the plane of the borehole axis and the axis is normal to the bedding plane is different, The directional shear strength come into play and potential collapse will occure $15^0<\beta<35^0$. He also took a decision If the least normal stress at the Borehole Wall is in the same plane as the Borehole axis and the normal axis to the bedding plane, one Mohr-coloumb envelope applies for all borehole angles.

5.2 Effect of Bedding Plane and Lamination

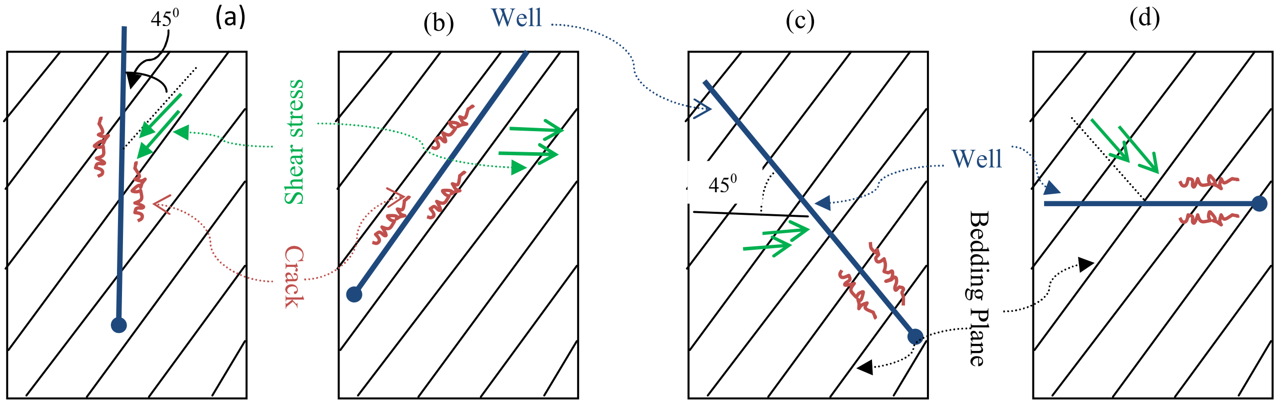
Shale has a laminated structure and contains numerous parallel bedded weakness planes. Two conditions determine whether the rock fails along a weakness plane or not; the relative magnitude of the two normal stresses and the angle between the borehole and the bedding plane. The plane of weakness was introduced in the oil industry by Aadnoy (1988), Chenevert (1965), Islam (2010). During modeling of highly inclined boreholes, they investigated the effects of wellbore inclination, anisotropic elastic rock properties, anisotropic stresses and anisotropic rock strength. It was shown that under certain conditions, the rock would fail along planes of weakness. Because of the geo-mechanical properties of shale (high P_p alignment of phyllosilicates due to overburden digenesis), slip surfaces may exhibit significantly higher potential to fails as compared to stronger rock units, such as limestone and sandstone.

From Islam (2010) PhD paper, illustrate physical models (**Fig. 5.1**) by representing different attacking angles between loading and weak bedding planes to diagnose instability and compare the physics of these models against performed experimental data.

Fig. 5.1a presents vertical wellbores drilled at 45^0 to the weak bedding plane in artificial shale. As per shear stress thumbs rule, the maximum shear stress direction will follow the bedding plane and the material is so weak in this direction that the evolved shear stress would be a potential challenge for material failure. Induced crack direction will be along the wellbore at 45^0 degree to the bedding plane, which may accelerate a material failure. Drilling a well in such a setting is considered to be the highest risk of mechanical borehole stability.

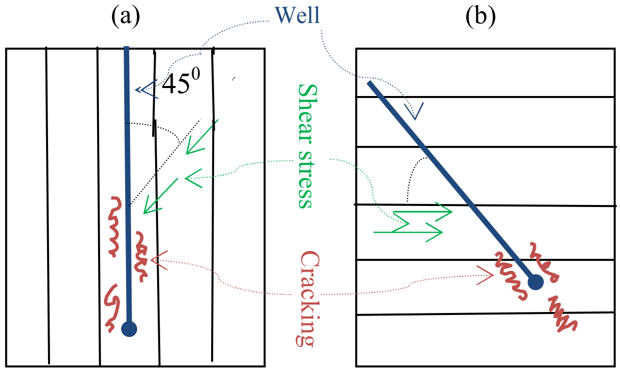
Fig. 5.1b shows deviated wellbores drilled parallel to the bedding plane. The maximum shear stress direction will be 45^0 to the weak plane and the material is relatively competent in this direction. Therefore, evolved shear stress would not be a challenge for the material. However, material failure may happen through induced cracking along the weak bedding plane. For UBD, induced cracks may help to raise ROP but it will be a key factor to initiate material failure. Drilling a well along the bedding plane is also considered to pose a high risk of mechanical borehole stability.

The remaining models shown in **Fig. 5.1c** deviated well at an angle ≥ 70^0 to the bedding plane and **Fig.5.1d** horizontal well are relatively less challenging with respect to material failure. Moreover, induced crack and shear stress direction are indicated in the model. Hypothetically, induced crack effects in this model will not be as critical as compared to previous model (Fig. 5.1a and Fig. 5.1b).



Figur 5-1 Wells drilled in different angles to the bedding plane

Fig. 5.2 presents two different sets of physical models, consisting of vertical (**Fig. 5.2a**) & horizontally (**Fig. 5.2b**) oriented bedding plane with different attacking angles. Vertical well with 0^0 attacking angle (**Fig.5.2a**) and vertical weak bedding plane, maximum shear stress acting at 45^0 to the borehole, induced cracking is the highest concern for instability. **Fig. 5.2b** Deviated well drilled at 45^0 to the horizontally oriented weak bedding plane. We have seen that in deviated trajectories the maximum shear stress is acting along the bedding plane. Shear failure is the highest concern in such conditions. But the material failure risk analysis will be the same as discussed in the aforesaid models. The most important features will be the attacking angle between the borehole and the weak bedding plane. In general, for any combination of weak bedding plane & hitting angle orientation, the evolved shear stress direction along the weak bedding plane pose a risk for initiating material failure (Islam et al., 2010c).



Figur 5-2 Wells drilled in different angles to the bedding plane

5.3 Stable Borehole Direction and Drilling along Principal stress axis

It is well accepted from different researchers that the minimum differential stress indicates minimum stress anisotropy in the plane perpendicular to the borehole. This stress condition (Islam 2010) determines stable borehole conditions under the in-situ stress state. For instance, in case of NF stress state, wellbores parallel to σ_h means a smaller stress difference ($3\sigma_v$ -σ_H) while wellbores parallel to σ_H means larger stress difference ($3\sigma_v$ - σ_h). The generalized form of maximum and minimum stresses is therefore:

$$\sigma_{\theta\ max} = 3\sigma_{max} - \sigma_{min} = \text{Initiation of Collapse}$$
$$\sigma_{\theta\ min} = 3\sigma_{min} - \sigma_{max} = \text{Initiation of fracture}$$

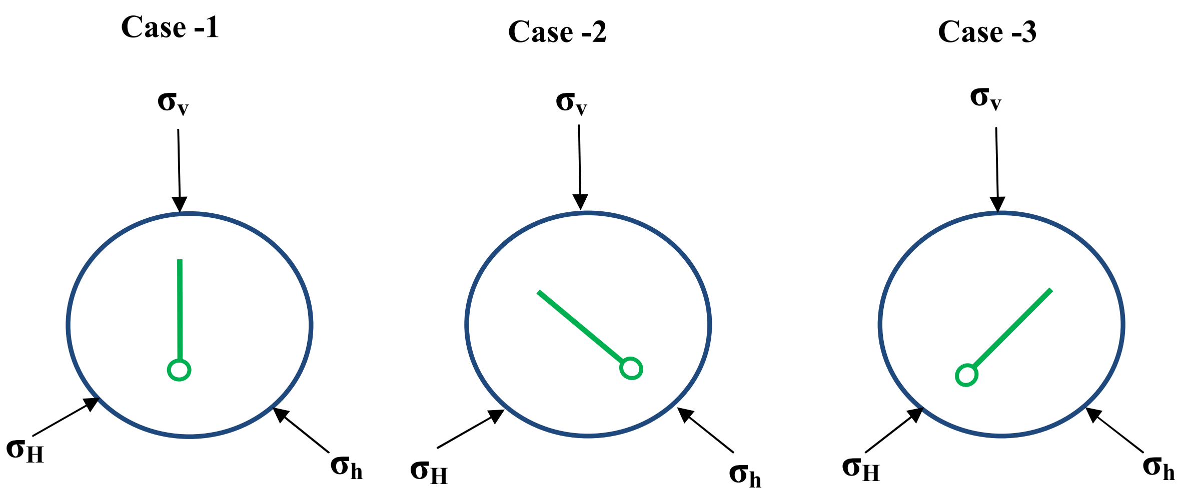
Therefore, minimum stress differences imply stable boreholes. From literature review (Islam 2010) we got the following general results:

- Normal fault: Least stable well direction is horizontal along σ_H.
- Strike- slip: Least stable well direction is vertical.
- Reverse fault: Least stable well direction is horizontal along σ_h.

Example from Islam (2010):

In NF stress regimes; if σv =1 Kg/l and σH=0.8 kg/l and σh=0.5 Kg/l, which direction would give the most Stable wellbore? The answer is given below and summarized in Table 5.1

- Case 1 : Drilling parallel to largest in-situ stress (σ_V)
- Case 2 : Drilling parallel to Smallest Horizontal in-situ stress (σ_h)
- Case 3 : Drilling parallel to largest Horizontal in-situ stress (σ_H)

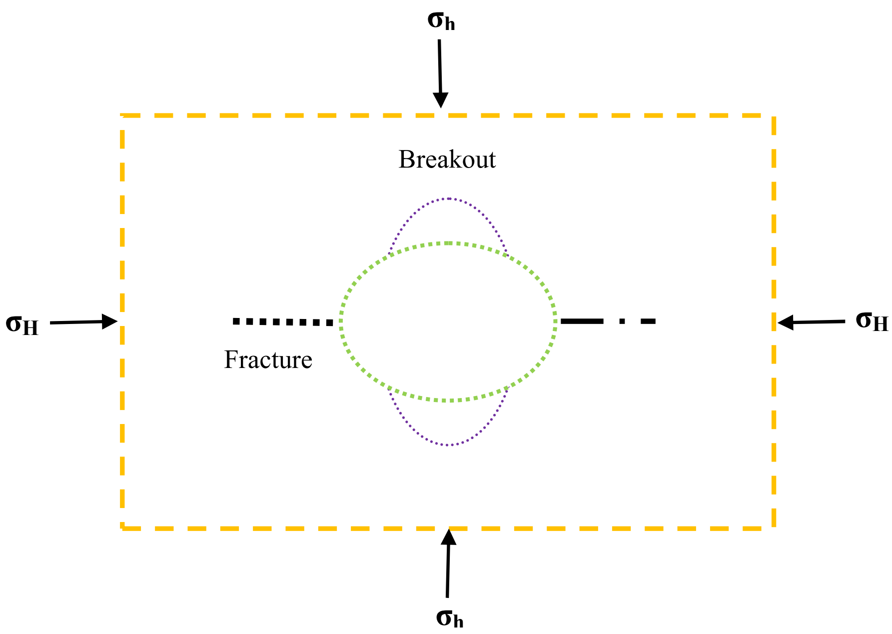


Figur 5-3 Insitu stress and Borehole direction as estimated in Table

Table 5.1 Determination of potential Boreholes Problems based on differential hoop stress Model		
Case1	Case2	Case3
$\sigma_{\theta\ max} = 3\sigma_H - \sigma_h = 1.9$ $\sigma_{\theta\ min} = 3\sigma_h - \sigma_H = 0.7$ $\Delta\sigma_{\theta 1} = 1.2$ (Alternative)	$\sigma_{\theta\ max} = 3\sigma_v - \sigma_H = 2.2$ $\sigma_{\theta\ min} = 3\sigma_H - \sigma_v = 1.4$ $\Delta\sigma_{\theta 2} = 0.8$ (Stable)	$\sigma_{\theta\ max} = 3\sigma_v - \sigma_h = 2.5$ $\sigma_{\theta\ min} = 3\sigma_h - \sigma_v = 0.5$ $\Delta\sigma_{\theta 3} = 2.0$ (Unstable)
(Source Islam 2010)		

It is generally accepted that drilling against the smallest differential stress is good for stable drilling. Such a condition is satisfied by $\Delta\sigma_2<\Delta\sigma_1<\Delta\sigma_3$, case 2 from the exampled above will be the best option for obtaining a stable well. However, case1 is also an option if it is considered the lowest tangential stress at the borehole wall. Recently Islam (2010), Dhruba et al. (2009), Al-Ajmi et al. (2006), worked on simulations of collapse pressures and optimized drilling direction to minimize borehole stability problems by using a linear elastic model. From their work it was found that the most stable horizontal hole will be along the direction of minimum horizontal stress (same conclusion as from the above example) which minimizes the stress anisotropy in the plane perpendicular to the borehole. In a similar way, a stable drilling direction can be addressed when the well goes through the principal stress axis, which is illustrated in Appendix A Fig. A1-a.

A common believe in the petroleum industry is that fracture and collapse occur in the direction of the maximum and minimum principle in-situ stress, respectively. This is true for vertical boreholes, but Islam 2010 paper showed, by using stereographic projections on numerical examples, that this is not the general case for deviated wells. In-situ stress direction and the well trajectory are equally important to assess the fracture and collapse initiation position. He also discussed that failure azimuth is close to 90^0 for low borehole inclinations, but varied more at high inclinations. This is the general solution for all stress regimes. However, for Strike-Slip stress regimes, this increase initiates at a much higher inclination.



Figur 5-4 Failure directions when drilling in Principal in-situ stress direction

(Source Islam 2010)

Normally A wellbore will fracture in the direction of the largest horizontal in-situ stress, and break out in the direction of the smallest horizontal in-situ stress as illustrated in Fig. 5.4. This is well known and it is valid only for vertical boreholes. From literature review we have known that the fracture and break out position is altered due to the effects of well trajectory. It is a common belief that the fracture angle and collapse angle differ by 90^0. Kårstad and Aadnøy (2005) showed by numerical examples that this is not always the case, and that the fracture and collapse angle might even be the same in some situation. The consequence and analyses of **Fig. 5.4** are presented in **Table 5.2**. 'θ' is the relative position to the direction of the major horizontal stress (σ_H). This angle indicates the orientation of the stresses around the wellbore circumference.

Table 5.2 Condition for maximum and minimum magnitudes of Tangential stress in Vertical wells and its consequences (Islam 2010)

Stress	Magnitude	θ Degree	Analytical solution	Consequences
σ_θ	Maximum	±π/2	$\sigma_{\theta\ max} = 3\sigma_{max} - \sigma_{min} - P_w$	- Causes Shear Failure - Controls the orientation of fractures which occurs in the direction of σ_h
σ_θ	Minimum	0, 180	$\sigma_{\theta\ min} = 3\sigma_{min} - \sigma_{max} - P_w$	- Causes Tensille Failure - Fracture initiates and occurs in the direction of σ_H

Fig. A1 (a) on Appendix A illustrates wellbore orientation along the principle stress direction. Aadnøy et al. (2005) worked on this model and it was found that three possible principle stress orientations for borehole geometry can be obtained. These are parallel to the vertical stress (bound I), parallel to the minimum horizontal stress (bound II) and finally parallel to the intermediate horizontal stress (bound III). A critical stable condition can be determined if the tectonic stress regime is known (i.e., the orientation and magnitudes of σ_H and σ_h).

- NF - the sequential stable borehole condition will be stress bound (II > I >III), however, the best option will be bound II when $\sigma_H > \sigma_h$, but for $\sigma_H = \sigma_h$, at bound I will be preferred. Bedding planes with intermediate and high stress anisotropy have a considerable effect on wellbore stability. The impact is larger on wellbores orientated in directions close to that of σ_h (Chen et al., 1999, Islam 2010).

- SS- the chronological stable borehole condition will be stress bound (III > II >I), but the best option will be bound III (horizontal boreholes); Although bedding planes also have larger influence on wellbore orientated in directions close to that of σ_h, they have significant effects even with low strength anisotropy (Chen et al.,1999, Islam 2010).

- RF - stable borehole condition will be stress bound (I > III > II), but the best option will be bound I; when $\sigma_H = \sigma_h$. However, non vertical wells with the borehole along σ_H (bound III) can be a potential solution. Bedding planes with low strength anisotropy also have a major effect on wellbore stability, but with a larger influence on wellbore orientated in directions close to that of σ_H (Chen et al., 1999, Islam 2010).

5.4 Evaluate optimum well path arbitrary stress axis

The optimum well path for increased mechanical borehole stability is the borehole orientation that maximizes the difference between the fracture and the collapse pressure. Based on hoop stress analysis, According to Islam (2010) Al-Ajmi et al. (2006) developed a mathematical expression to optimize well path under in-situ stress state. The optimum well path (γ_{opt}) deviates from the maximum principal in-situ in the $\sigma_{highest}$-$\sigma_{minimum}$ plane by this amount:

$$\gamma_{opt} = \mathrm{Sin}^{-1}\sqrt{\frac{\sigma_H - \sigma_h}{\sigma_v - \sigma_h}} = \mathrm{Sin}^{-1}\sqrt{M} \quad \ldots\ldots \quad (5.1)$$

Thus in general, according to Islam (2010) the maximum fracture pressure and minimum Collapse pressure always occur when the borehole is directed from $\sigma_{highest}$ towards $\sigma_{minimum}$ with an angle γ_{opt} . M is anisotropic function defined by:

$$M = \frac{\sigma_H - \sigma_h}{\sigma_v - \sigma_h} \quad \ldots\ldots \quad (5.2)$$

According to stress regime above equation become:

$$M_{NF} = \frac{r_H - r_h}{1 - r_h} \quad \ldots\ldots \quad (5.3)$$

$$M_{SS} = \frac{1 - r_h}{r_H - r_h} \quad \ldots\ldots \quad (5.4)$$

$$M_{RF} = \frac{r_h - 1}{r_H - 1} \quad \ldots\ldots \quad (5.5)$$

Where, $r_H = \sigma_H/\sigma_v$, $r_h = \sigma_h/\sigma_v$, from Islam (2010) paper as per Anderson (1975) , $1 > r_H > r_h$ indicates normal fault stress regime, $r_H > 1 > r_h$ indicates strike slip stress regime and $r_H > r_h > 1$ indicates a reverse stress regime. In general, equation Eq.5.1 should be used as a quick, rough guideline to design the most favorable drilling trajectory with regards to wellbore stability (Al-Ajmi et.al.2006, Islam 2010).

Islam (2010) is addressed his paper by:

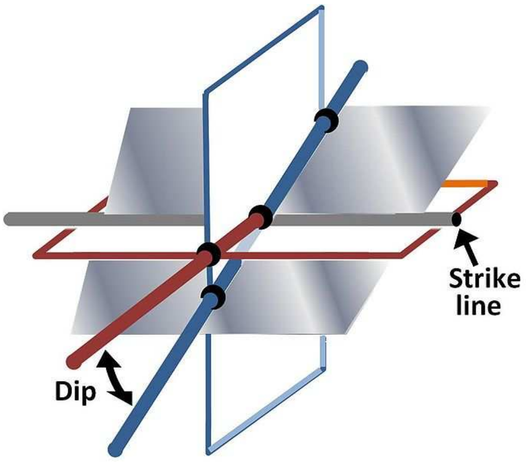
- The trend of optimum well path for NF and SS regimes are strongly non-linear, as where 'M' is relatively small (i.e., M = 0 to 0.2). However, this optimal well path becomes reached at linear trend for 'M' greater than 0.2 until it reached at 0.9. For, M=1, this can be obtained in an isotropic stress field. Otherwise, intermediate horizontal stress determinate the optimum well path.
- When $\sigma_H = \sigma_h$; value of 'M' = 0; 'γ_{opt}' = 0 degree; this means the 'γ_{opt}' is parallel to the maximum principal in-situ stress (vertical well). This can be seen in NF and in SS-RF stress systems. But when $\sigma_H \neq \sigma_h$;the optimum drilling trajectory deviates from the vertical by inclination = optimum well path in a direction parallel to the minimum principal in-situ stress. Optimum well path is reached at horizontal well when 'M' = 1 and $\sigma_{highest} = \sigma_{intermediate}$ ($\gamma_{opt} = 90^0$). This will take place in RF (with $\sigma_H = \sigma_h$) and in NF-SS stress regimes.
- In SS stress regimes, the most stable borehole is horizontal with a drilling direction (azimuth) = optimum well path.
- In RF stress regimes, the wellbore should be drilled in the direction of the σ_H (azimuth = 0) with a drilling inclination of (90- γ_{opt}) to minimize borehole instability.

5.5 Relation between borehole direction and borehole failure

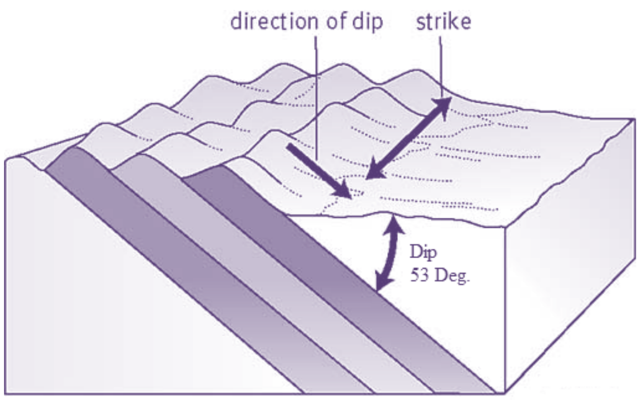
The **strike line** of a bed, fault, or other planar feature is a line representing the intersection of that feature with a horizontal plane. It is also called Slip Fault that primarily displays horizontal displacement.

An angle giving the orientation of a planar feature such as bedding or a fault plane; it is the acute angle measured between the planar feature and the horizontal is called **dip**. It is measure perpendicular to the strike direction. Dip is also called inclination angle of the formation as measured at right angles to strike. **Dip and strike** are a method of describing the orientation of a plane in three dimensional space. think of the direction of dip as the direction that a ball would roll if placed on the surface. The angle of dip is measured in degrees. The following **Figures (5.5, 5.6, 5.7, 5.8)** show dip, strike and attack angle.

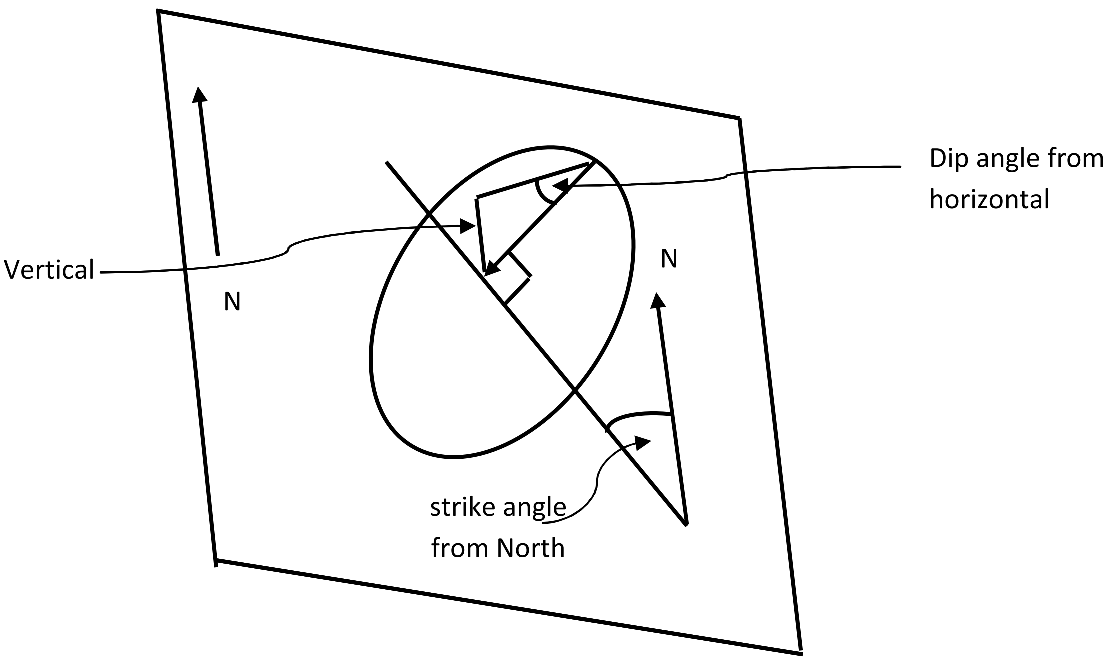
Bedding Plane is a surface that separates one stratum, layer, or bed of stratified rock from another. A geological bed or stratification is a layer of sediment or volcanic material that is distinctly separate from other layers.



Figur 5-5 Relation with Dip and Strike(Internet)

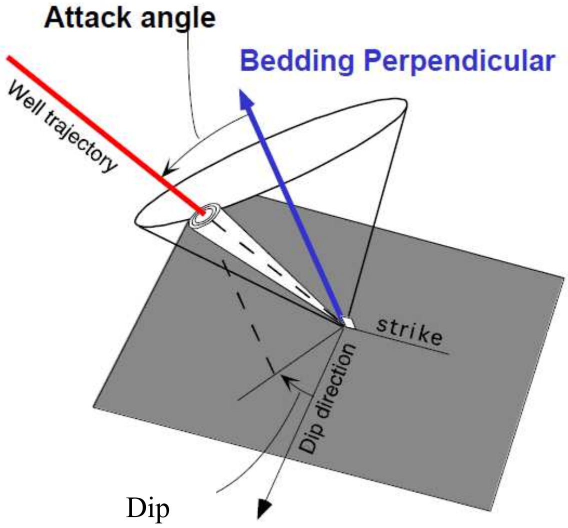


Figur 5-6 3D view of Dip and Strike(Internet)



Figur 5-7 Measuring Dip and Strike

Attack angle (a_{at}) is the angle between the wellbore and the bedding plane, it's normally taken as acute angle. Attack angle 0^0 means the Wellbore is parallel to the bedding plane and 90^0 means wellbore is perpendicular to bedding plane. Attack angle is extremely important because, if favourable conditions exist (10^0 < Attack < 30^0) plane of weakness may occur at tremendous low load condition.



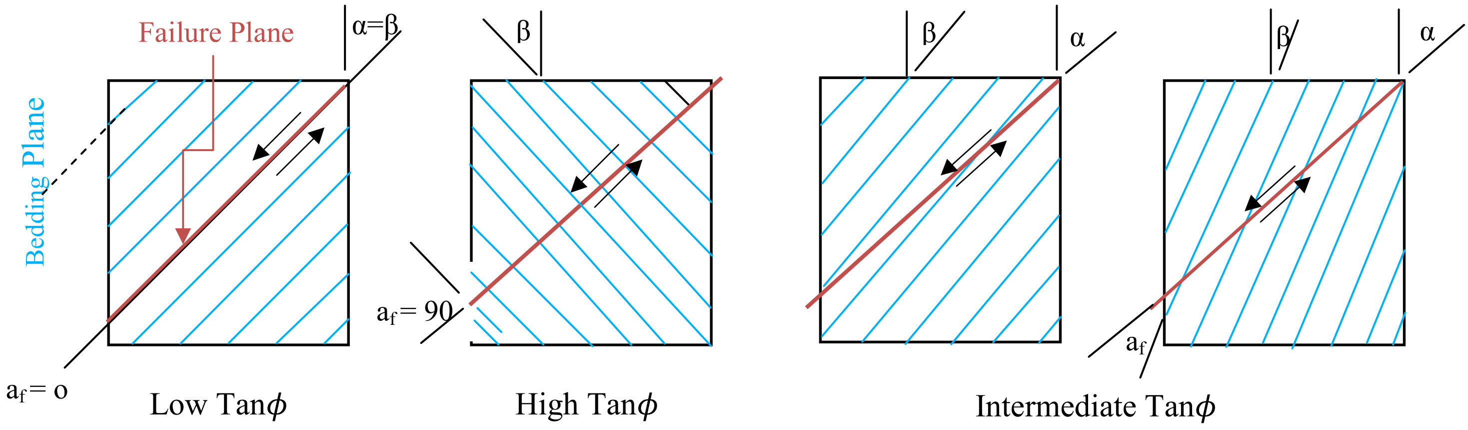
Figur 5-8 Measuring attack angle and dip

Failure plane means in what plane the wellbore/speciman will fail. One can analyze failure plane by Mohr-columb and tri-axial test (under different load condition) and can be determined angle of fracture (α) from a speciman. This may be considered a complex matter when one think on underground condition, because of complex insitu stresses and pore pressure are acting and changing that matter due to deplation of the reservoir.

Our work has given clear idea to remove the confusion of the different angle such that reader should get fruitful idea about the relation of different angle, shown on the following **Fig 5.9**. Here a_f is the angle between the bedding Plane and failure planes varied with the Tanϕ (Internal friction coefficent). Their relation is following:

$$a_f = \beta - \alpha = \beta - \frac{\frac{\pi}{2} - \phi}{2} \quad \ldots\ldots\ldots (5.6)$$

Here α is the angle between applied force and failure plane during triaxial core-testing and β is the angle between applied force and bedding plane during triaxial core testing. One thing for reader don't confused about γ and β, γ is related for wellbore inclination from vertical, on the other hand β is related to Core-plug. If you want to compare attack angle and β, They are eqiuvalent, But they are considered in different positions.

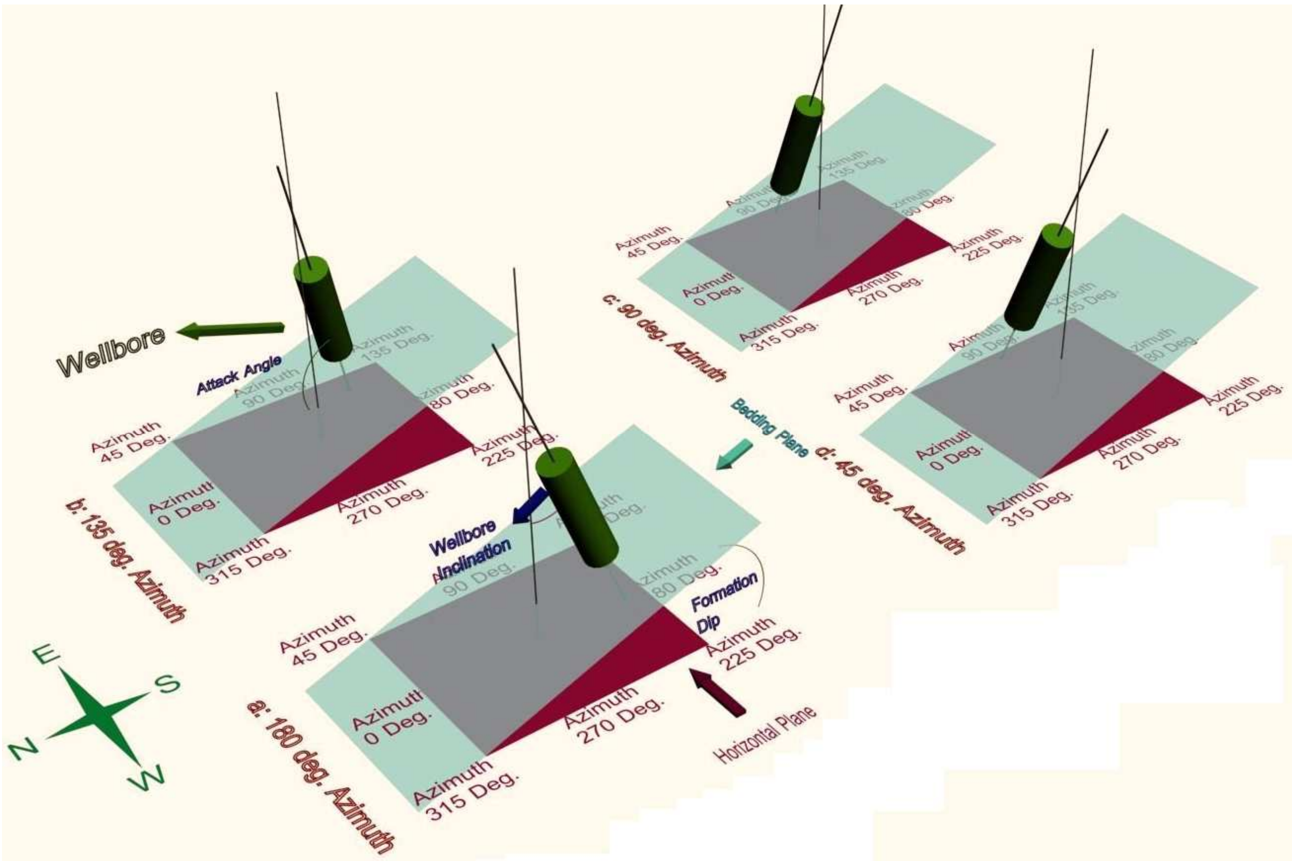


Figur 5-9 Failure Plane Vs Bedding Plane with variations in the angle between the failure plane and bedding plane(a_f) , (Chenevert et.al.1965, Aadnoy 1988)

So the array of bedding plane, borehole position and the dip/strike angle of formation are important parameters if any one want to apply the in-stu stresses equations (**Appendix-A**) for determining failure criterion, rock stregth and want to develop models of wellbore failure. Especially, any researcher can apply the triaxial / poly-axial test from the core, he can know the plane of weakness, that is extremely important for smooth drilling of a well to avoid the wellbore instability problem.

5.6 Relation with Attack angle (3D effect) and different Azimuth with constant Inclination

Azimuth is the angle of well direction from True North (or Sometimes taken from σ_H) and taken positive Clockwise from North normally. The following figures (**Fig. 5.10 and Fig. 5.11**) shown the effect of attack angle with changing the azimuth. We certainly found that Although inclination doesn't change, But attack angle changes with the azimuth. So it's extremely important for testing the Bedding exposed with different Azimuth with constant inclination. We also found that attack angle is the lowest value on the downdip position and the highest value on the updip Position. From the another picture above (**Fig 5.9**) we also found that Dip is a formation preperties with relatate to srike direction, so attack angle is a function of Dip and strike also, Altough both are geolgical propetries. Dip and strike give true picture of the underground with 3D view of a well.



Figur 5-10 Attack Angle Vs Azimuth with constant inclination on a bedding Plane (3D) effect

Aadnoy et.al. (2009) paper did not address about the effect of attack angle and azimuth. This Paper and our research confirmed that attack angle is affected the azimuth angle definitely which ultimately will affect the result of bedding exposed position. From **figure 5.10** above one also see the minimum attack angle will create at the down dip position and maximum on the up dip position. Some of the positions are analogous, such as 90^0 and 270^0, 45^0 and 315^0 etc.

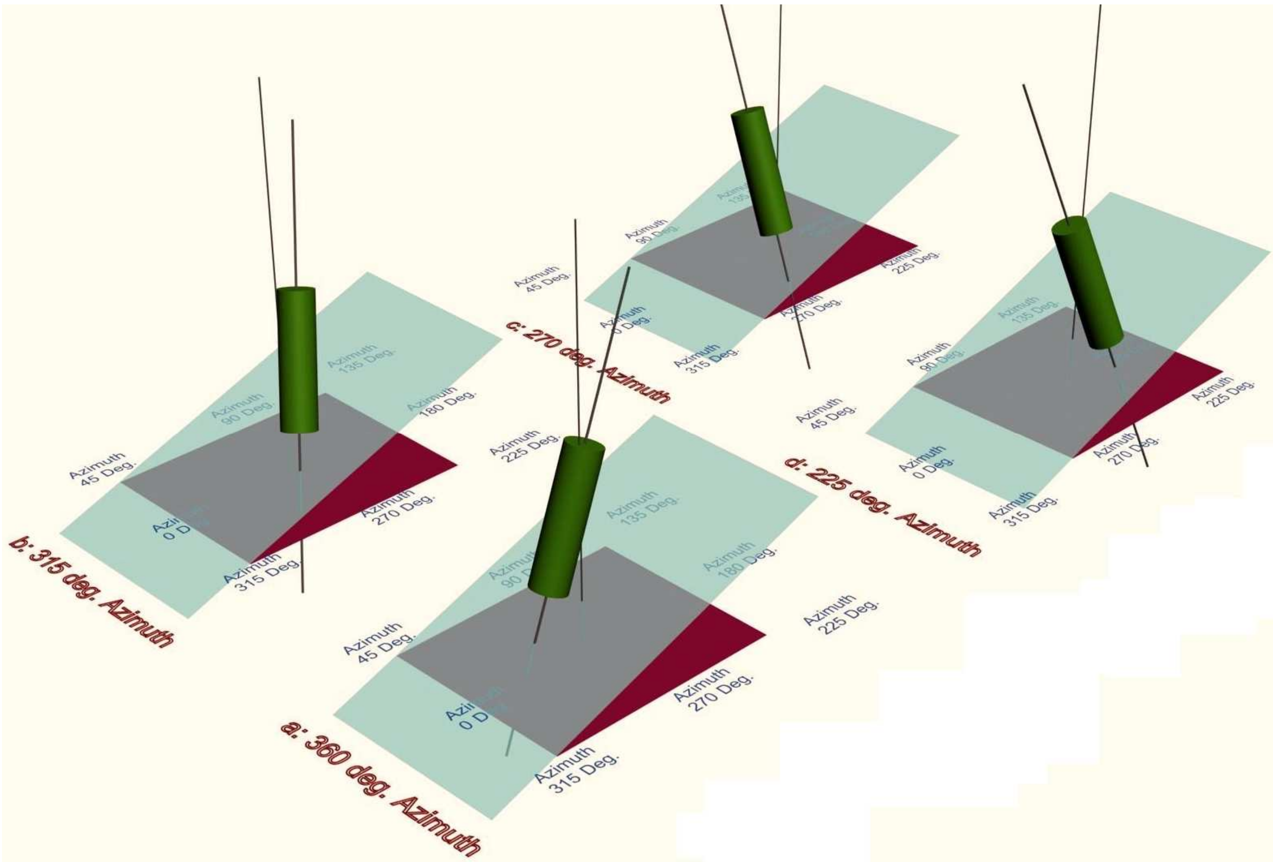


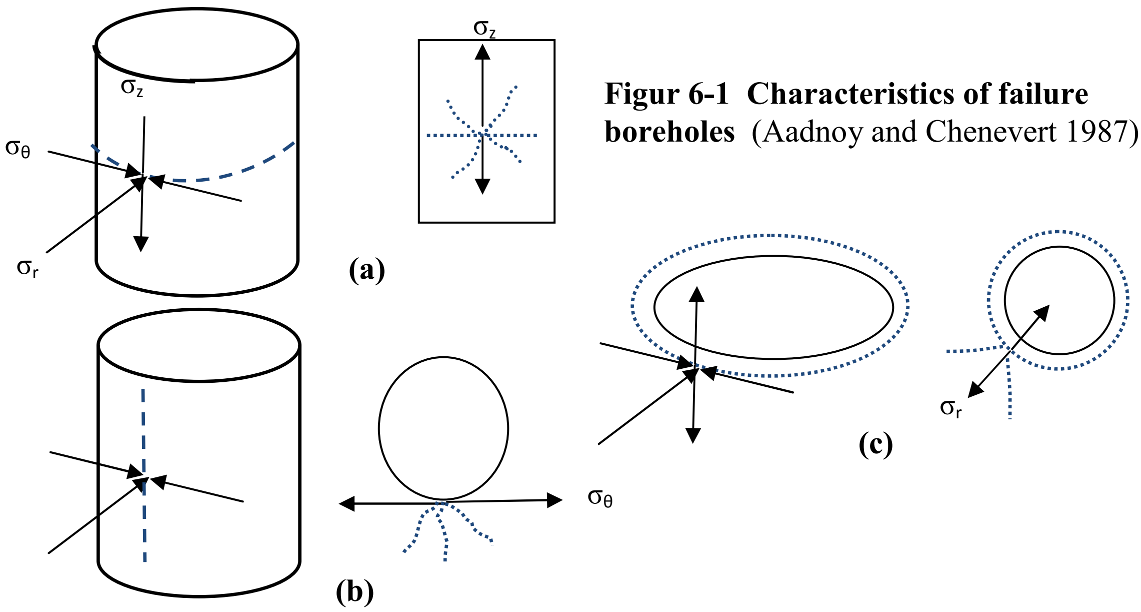
Figure 5.11 Attack Angle Vs Azimuth with constant Inclination on a bedding Plane (3D) effect

CHAPTER 6 ANALYZED OF AADNOY ET.AL (2009) PAPER

6.1 Review Aadnoy et.al. (1988, 2009) and Chenevert (1965) Paper

Layered rocks such as shales often exhibit different properties along or across bedding planes. Elastic properties like bulk modulus, Young's modulus and Poisson's ratio, show directional properties. The same can be concluded for compressive and tensile rock strength. According to Aadnoy et.al. paper (2009) rock strength is high when force vectors are applied at a high angle to bedding. At lower angles, on the order of 15° and 30°, stratal compressive strength is low. For this case, rock failure will occur along bedding planes. This type of rock behavior is often termed "plane of weakness" .An increased angle, however, brings about new problems. Cuttings transport,Casing setting and cementing and drillstring frictions are examples of difficulties encountered in highly deviated boreholes. That also decreased fracturing gradient , with an increased application of oil based muds the prediction of fracturing gradients becomes more important than ever. This works assumed for developing model that linear-isoropic plane strain conditions, All in-situ stresses are principal and directed horizontally and vertically. The key in this analysis is that when a well is drilled , the rock surrounding the hole must take the load that was previously taken by the removed rock. As a result increases stress concentration around the wall. If the rock is not strong enough, the borehole will fail. According to their paper Borehole collapse and fracture occures at different depth and condition is shown in **fig. 6.1**. A typical fracturing (horizontal fracture) of the wellbore in shallow well shown in **fig. 6.1a**, where the overburden is being lifted .The axial stress σ_z , goes tensile , while radial and tengential stress remain in a compressive stress. Shear effects occur between (σ_θ , σ_z), (σ_θ, σ_r), and (σ_r, σ_z) because of large stress differences. This shear stress will merely aid the fracturing process caused by axial stress going tensile.No rock peices will be released because of both tensile and shear stresses cause fracturing act radially outward from the borehole. **Fig. 6.1b** illustrates the fracturing of deeper well, where vertical fracturing is occurred. Here radial and axial stresses are compressive and circumperentail or hoop stress is tensile in nature. A Borehole collapse is described in Fig.6.3c ,This a typical drawdown problem, Here both axial and tengentila stress goes compressive and radial effective stress goes in tension . For linear elastic theory, Failure should occur in the wellbore wall. It is visualized that wellbore sometimes fails in tension around a circumference shown in **fig.6.1c**. this case presence of radial failure aids the shear stresses in releasing piece of rocks from wellbore.If the wellbore pressure is lower than the formation pressure, formation fluid flow into the wellbore will wash the released pieces of rock from the wellbore.

Chenevert et.al. (1965) provided a number of shear measurements on coreplugs as a function of bedding plane orientation. **Fig 6.2 and 6.3** illustrades a borehole in laminated rock formation, with borehole inclination γ, in the x-z plane. Two infinitesimally small pieces of rocks are shown on the Boreholewall.The Borehole typically fail at $\theta=0^0$ (Case A) or 90^0 (Case B). If the applied stress in the x direction is the smallest, the Borehole will fail as in case B and the y direction is the smallest direction the Borehole will fail in Case B. For a typical collapase, the radial stress is the smallest, according to Mohr-coloumb criterion, We can avoid axial stress of laboratory data (as an intermediate pressure). The radial stress is the minor principal stress and the tangential stress (hoop stress) is the major principal stress. The redial stress is always in a principal stress direction, tangential stress is not exactly in principal stress direction because some shear stress components change the direction slightly. The equivalent (as well) core plug shown in **Fig. 6.3**, for case A the tangential stress acts parallel to the bedding plane, Therefore $\beta=0^0$, for this case regardless of inclination between borehole and bedding plane, one shear data set are applied for all borehole angles. In case B, tangential stress is applies at an angle with respect to the bedding plane and values now β=γ, For this case directional shear stress come into account with respect to bedding plane. Although core plug applies confined pressure (two equal stress) and one major axial stress, But accuracy of applying core-plug data to real boreholes is not known. Only case B bedding exposed a certain Borehole inclination and for case A no bedding exposed shown from their paper (Aadnoy et.al. 1987, 2009).

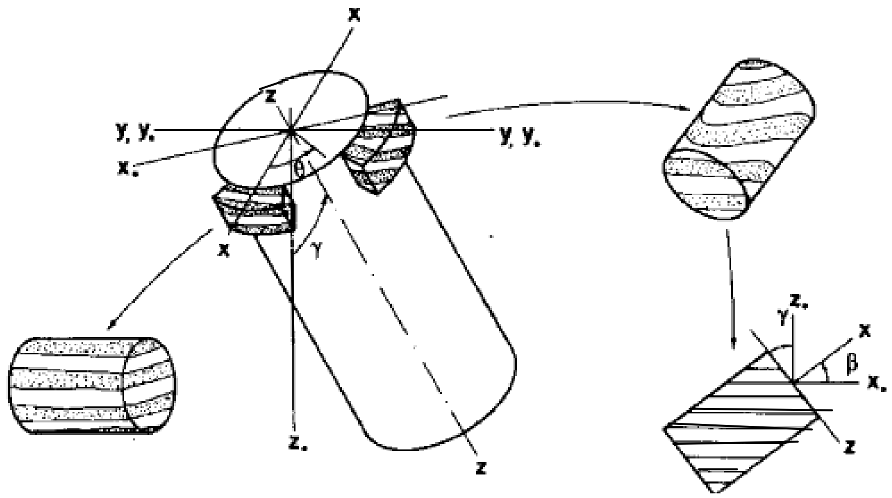


Figur 6-1 Characteristics of failure boreholes (Aadnoy and Chenevert 1987)

6.2 Conditions where the 'Plane of weakness' control well bore Failure

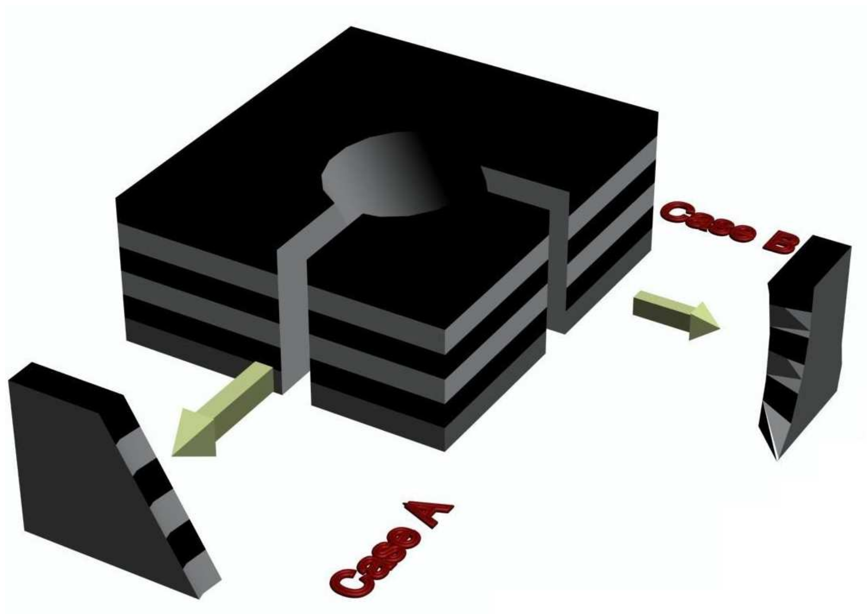
Wellbore failure is controlled by:

- Borehole orientation versus in situ stress orientation
- The magnitude of the in situ stresses
- The failure position on the borehole wall versus the bedding plane orientation



CasA at θ=0^0 **CasB** at θ=90^0

Figur 6-2 Test Plug Bedding Plane at related to Wellbore Position (Adanoy et. al.2009)



Figur 6-3 Test Plug Bedding Plane as related to Wellbore Position 3D view

Fig. 6.1 and 6.2 shows a deviated borehole in bedded rock. The layered rock at the borehole wall is shown as a core plug. Shaded and un-shaded regions in schematic plugs represent inter-bedded shale units that have the same bedding orientation. The borehole may fail at position A (high or low side of wellbore) or at position B (borehole sides). The stress conditions that cause failure are related to the failure positions as follows:

- If $\sigma_x < \sigma_y$, borehole fails at position A
- If $\sigma_y < \sigma_x$, borehole fails at position B

For case A, the weakness plane is not exposed and a stable borehole exists. For case B, the plane of weakness is exposed for certain wellbore/bedding plane inclinations, leading to an unstable borehole.

The stress conditions above are found from the stress transformation equations between the in situ stress tensor and the borehole direction. They are:

$$\sigma_X = (\sigma_H \mathrm{Cos}^2 a_Z + \sigma_h \mathrm{Sin}^2 a_Z)\cos^2\gamma\,,\quad \sigma_y = (\sigma_H \mathrm{Sin}^2 a_Z + \sigma_h \mathrm{Cos}^2 a_Z) \ldots\ldots..(6.1)$$

For in case-B, $\sigma_y < \sigma_x$ putting this value in to equation (6.1), the directions that plane of weakness exposed are given by:

$$\sigma_H(\mathrm{Sin}^2 a_Z - \mathrm{Cos}^2 a_Z.\mathrm{Cos}^2\gamma) + \sigma_h(\mathrm{Cos}^2 a_Z - \mathrm{Sin}^2 a_{Z.}\mathrm{Cos}^2\gamma) < \sigma_V \mathrm{Sin}^2\gamma)..(6.2)$$

For in case-A, $\sigma_x < \sigma_y$ putting this value in to equation (6.1), the follwing equation we

$$\sigma_H(\mathrm{Sin}^2 a_Z - \mathrm{Cos}^2 a_Z.\mathrm{Cos}^2\gamma) + \sigma_h(\mathrm{Cos}^2 a_Z - \mathrm{Sin}^2 a_{Z.}\mathrm{Cos}^2\gamma) > \sigma_V \mathrm{Sin}^2\gamma)..(6.3)$$

This condition applies only in the bedding inclination range determined by the compressive strength data. According to Adnoy et.al. (2009) Paper Plane of weakness is shown in the interval 10°-30°. Note in Fig. 6.1 that the bedding inclination of the core plug (β) is equivalent to the borehole versus bedding inclination for the actual well (a_{at}).

If the in-situ stress tensor is aligned with the bedding plane, the inclination γ also applies to the bedding plane. If there is a dipping bedding plane, the relative orientation between borehole and bedding plane is: $\gamma - k_{dip}$, where k_{dip} is the formation dip.

If there is a strong stress contrast between σ_x and σ_y, the above analysis typically holds true. However, for a small stress contrast and within the sensitive borehole/bedding orientation, other failures may occur. It depends on the degree of planes weakness. The borehole strength modeling should therefore always test and compare failures at both positions A and B.Table 6.1 below defines six different stress states. We will investigate to what extent these influence bedding plane failure.

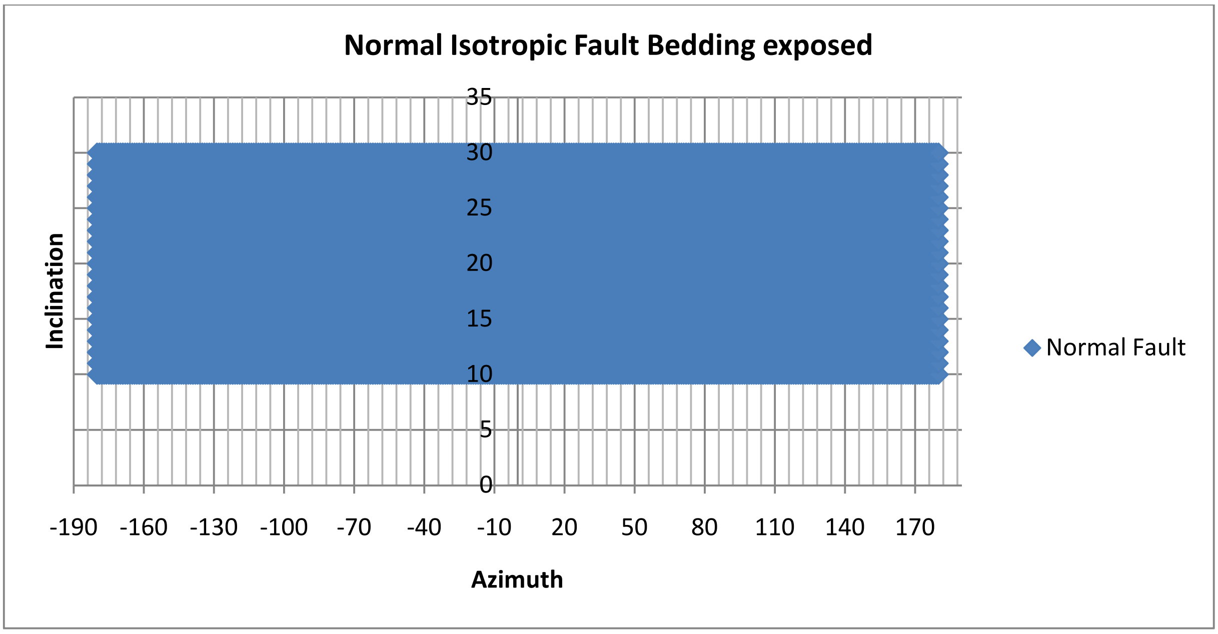
Table 6.1 Stress States (Aadnoy et.al. 2009)

Stress States	Normal Fault	Strike/Slip Fault	Reverse Fault
$\sigma_v, \sigma_H, \sigma_h$	1, 0.8, 0.8	0.8, 1, 0.8	0.8, 1, 1
$\sigma_v, \sigma_H, \sigma_h$	1, 0.9, 0.8	0.9 ,1 , 0.8	0.8,1, 0.9

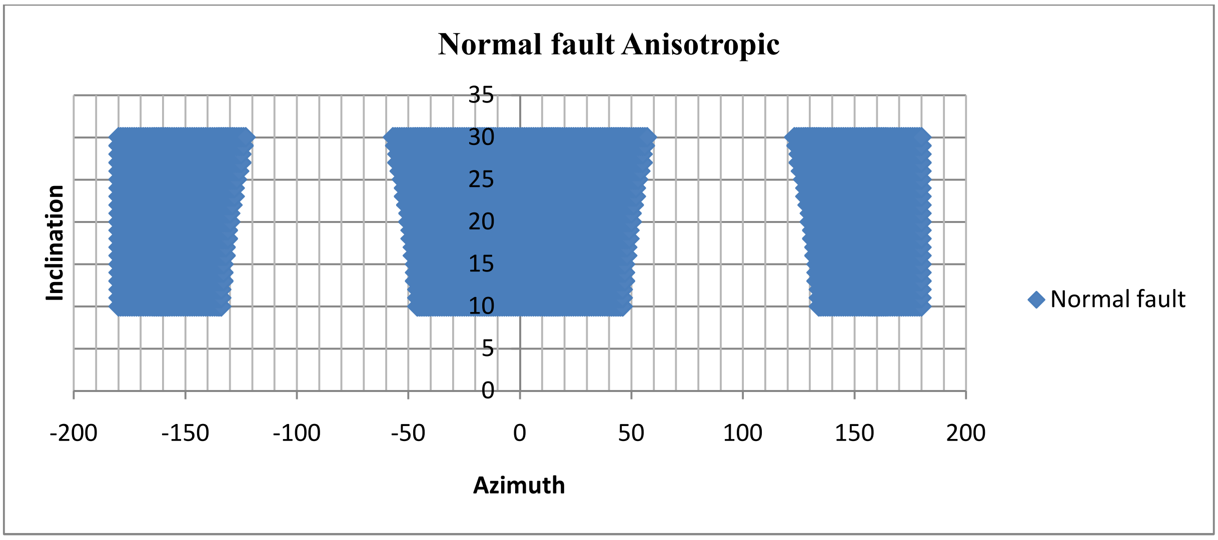
Table 6.2 BC well Data (Aadnoy et. al. 2009)

Parameter	Magnitude	Source
Azimuth	N 60^0 E	planned
Inclination	30 Deg.	planned
Porepressure, Pp	1125 Kg/m^3	Drilling record
Overburden Pressure Gradient , σ_v	25 Kpa/m (1.1 psi/ft)	Density Log
Maximum Horizontal Pressure Gradient , σ_H	29 Kpa/m (1.3 psi/ft)	Aadnoy et.al.2009
Minimum Horizontal Pressure Gradient , σ_h	20 Kpa/m (0.94 psi/ft)	Aadnoy et.al.2009
Maximum Horizontal Stress Orientation	N 32^0 E	Borehole Breakout analysis
Strike Dip, S_p	S 50^0 E	Image Log
Dip, K_{dip}	53^0 SW	Image Log

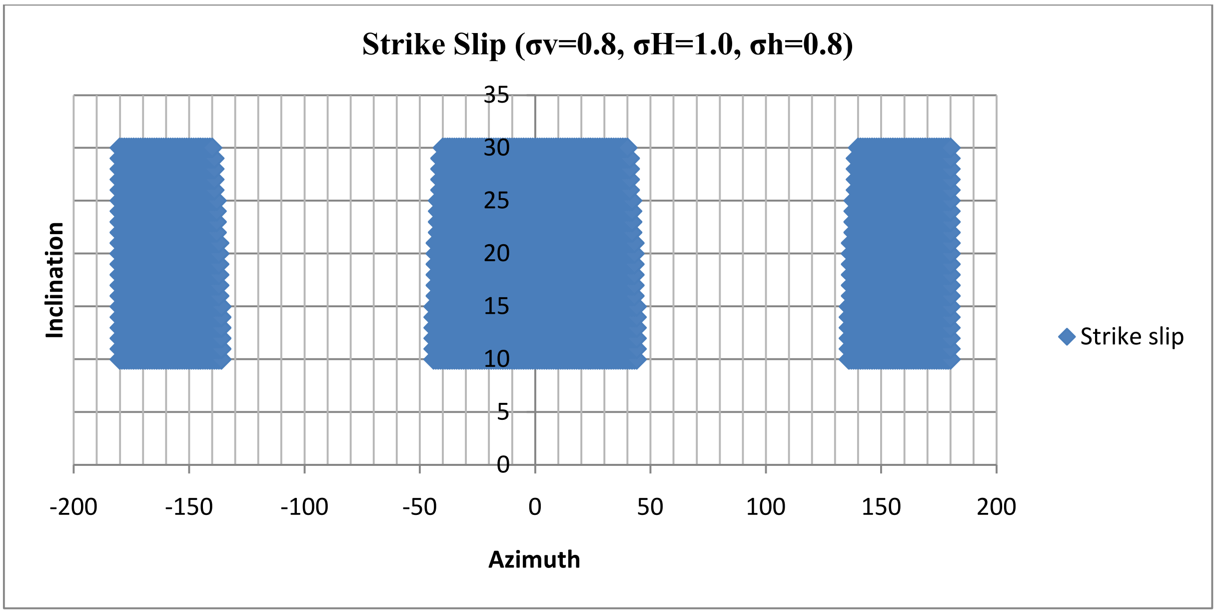
By applying the **equation 6. 2** and from **table 6.1** the range of wellbore inclination interval 10°-30°, bedding planes of weakness exposed (**Blue Color Shaded area**) results are reproduced by the following our results:



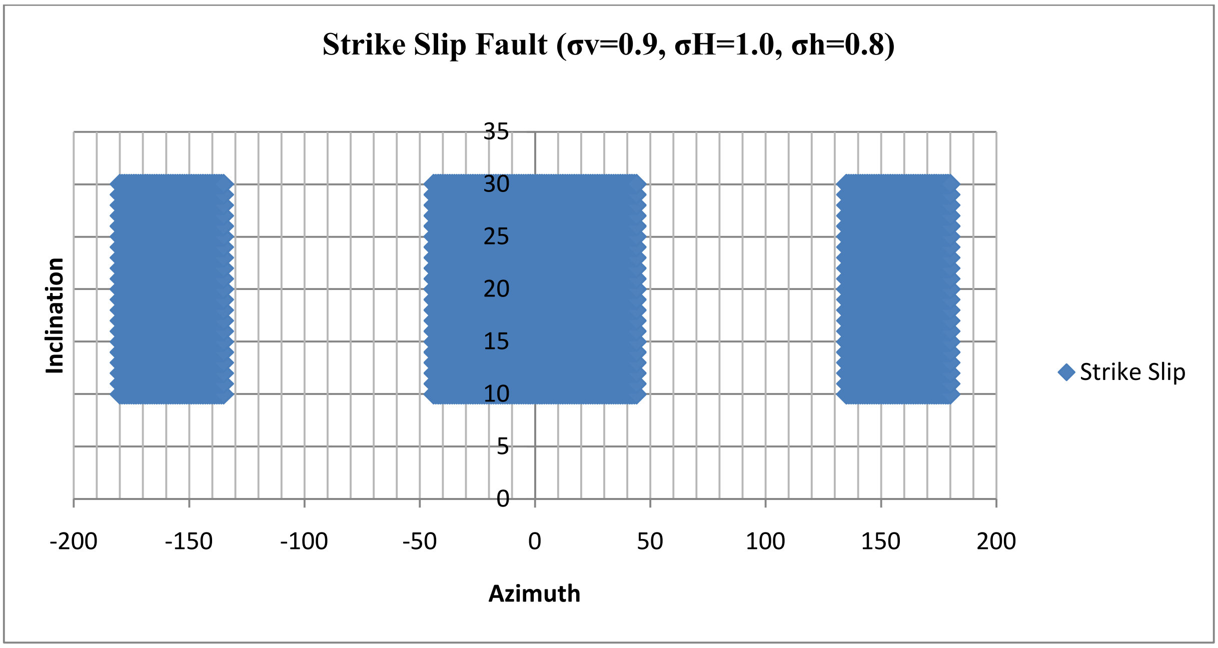
Figur 6-4 Normal Fault Isotropic Bedding Exposed



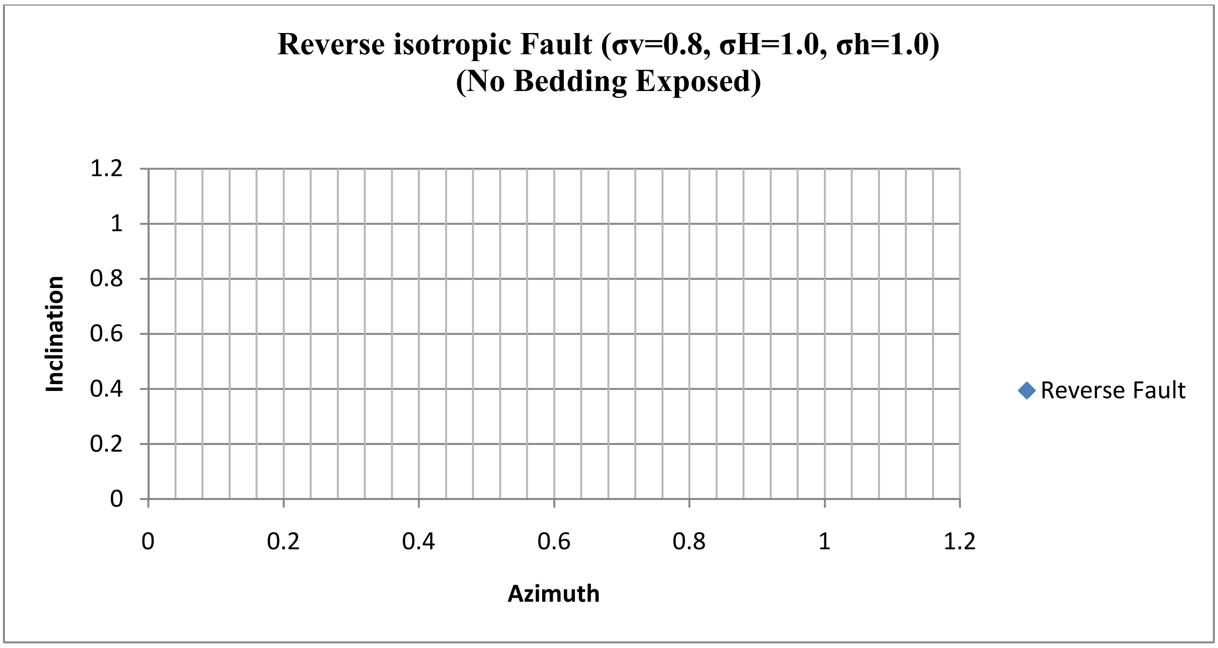
Figur 6-5 Normal Fault Anisotropic Bedding Exposed



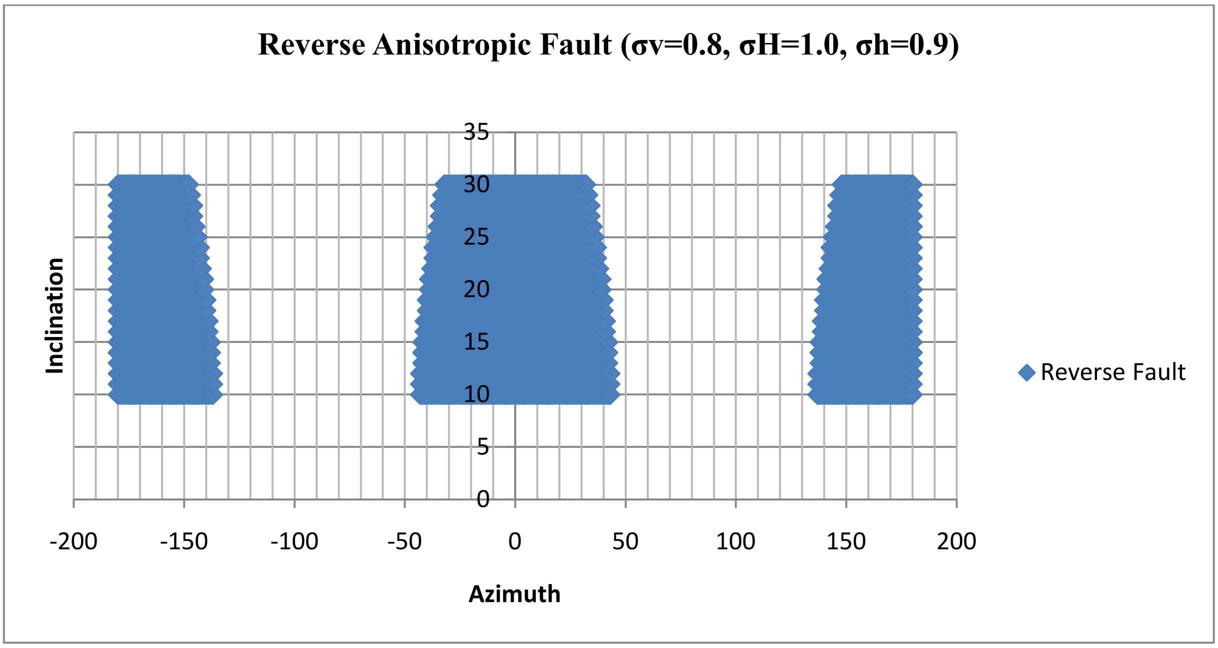
Figur 6-6 Strike Slip Bedding Exposed



Figur 6-7 Strike Slip Fault Bedding Exposed



Figur 6-8 Reverse Fault Isotropic Bedding Exposed

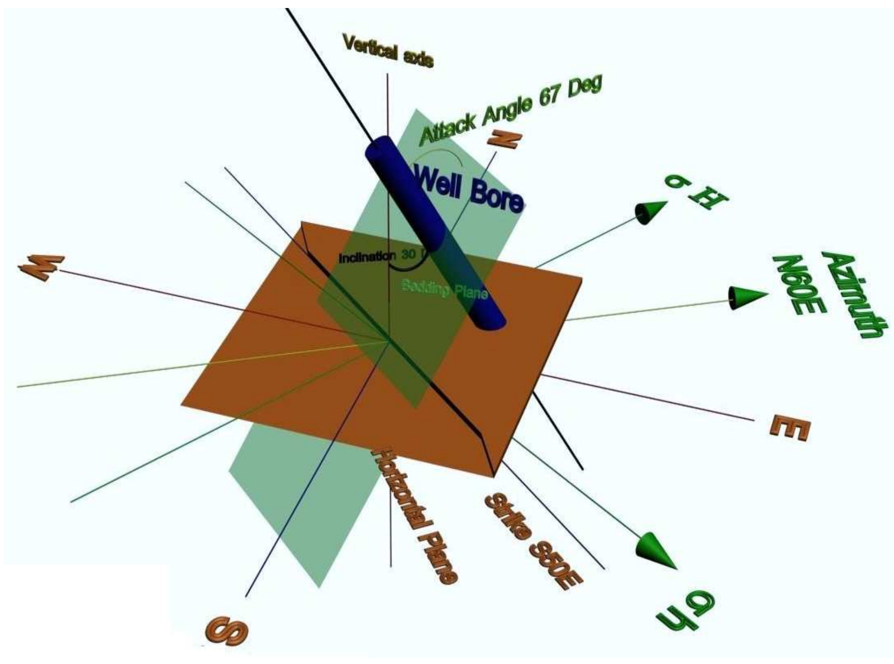


Figur 6-9 Reverse Fault Anisotropic Bedding Exposed

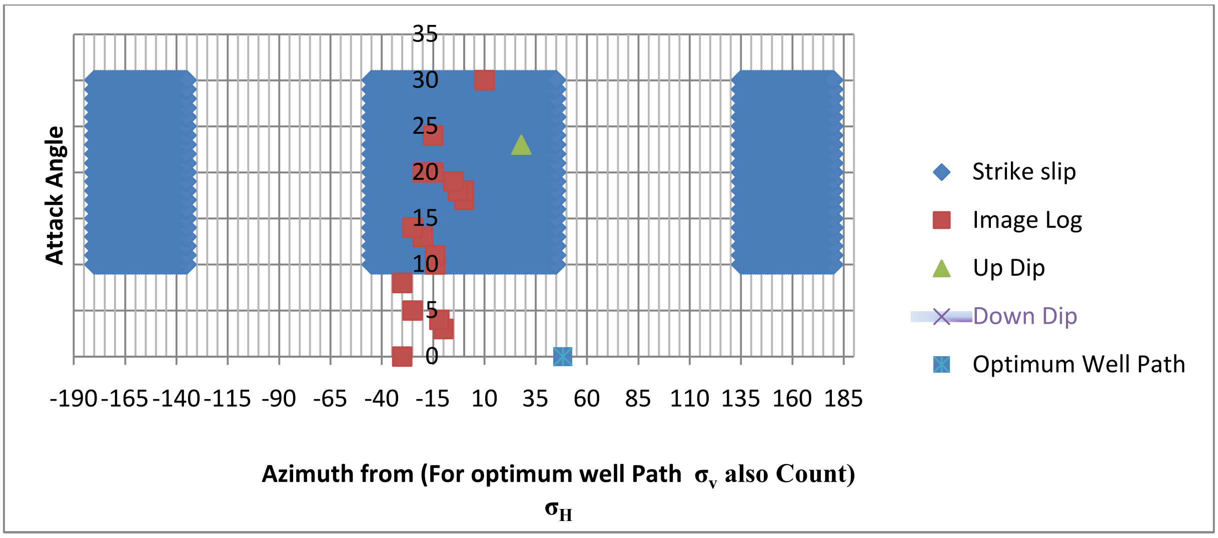
From the above spread (**Fig. from 6.4 to Fig. 6.9**) sheet this reasearch paper got same result compare to Aadnoy et.al.(2009) paper. Incase of normal fault isotropic case, plane of weakness esposed for all azimuths as shown in **Fig. 6.4**. Incase of isotropic reverse fault it is seen that plane of weakness is not been formed, shown in **Fig. 6.8**. Rest of the other cases, model found at a certain combination of parameters plane of weakness (bedding exposed) has been formed. This research works have run the model with formation dip, results shown that formation dip will be affected the bedding exposed area. One should consider dip angle if one want to get good result. Main limitation of our model, assumed linear elastic rock, took only integer of inclination angle and took 0.5*interger of azimuth. Our model will run another minimum value if Computer has capacity to run this model, Otherwise you have to take time to obtain the result.

6.3 Field case Data

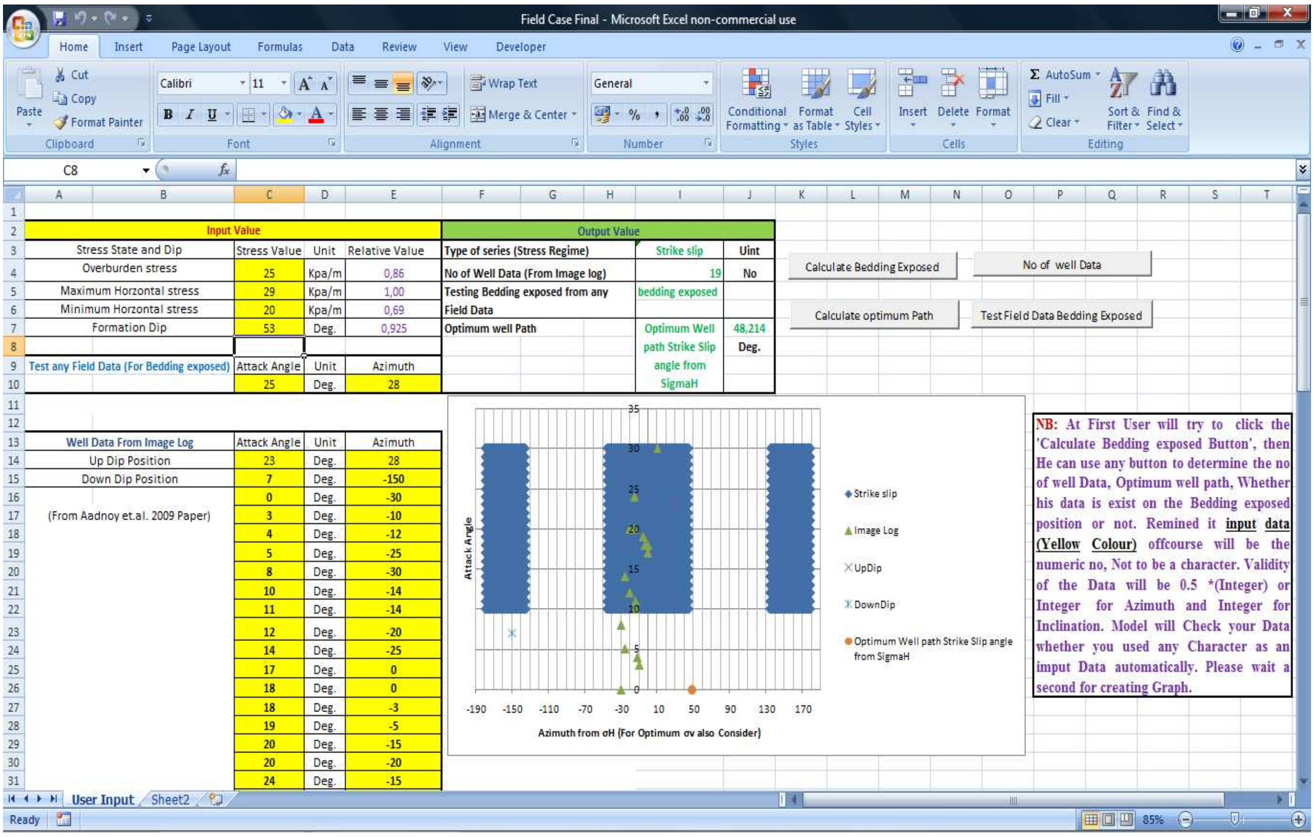
From **table 6.2** field data of Aadnoy et. al. (2009) paper, this paper reproduced the following spread sheet **Fig 6.11** and introduced new matter which was optimum well path. This work has obtained the angle of **optimum well path was about 48.2^0** from maximum principal stress (σ_H) by applying Alajmi (2006) – Islam (2010) equations. This result was close to the Aadnoy et.al. (2009) paper result, that was 45^0 from the maximum horizontal stress. Our works faced difficulty to deterimine the attack angle. We got attack angle 67^0 (shown in 3D **Fig. 6.10)** that was definitley differed from Aadnoy et.al. result. Their well direction datas were azimuth N60^{0}E, well inclination 30^0, strike S50^{0}E and dip 53^{0}SW. So it has be needed further research about the conflict of Aadnoy et.al. and our findings. Our model view and result has been shown in the **Fig. 6.11** and **Fig. 6.12**. Any user can introduce field data to obtain their different values such as type of fault, bedding exposed and safe positions, optimum well path, no of well data by applying our enhanced model .



Figur 6-10 3D view of Field Data (From Aadnoy et.al 2009 Paper)



Figur 6-11 field data Bedding Exposed (Aadnoy et.al. 2009 Paper)



Figur 6-12 Model View and User input Sheet

They deal with that the critical parameters are planes of weakness for rock strength, the relative normal stress values on the borehole and the relative angle between the borehole and bedding plane. According to study, this paper found that the critical angle between borehole and bedding would 10 to 30 degrees for many bedded rocks. If the angle between borehole and bedding is zero or 90 degrees, then the wells would be more stable. Under appropriate down-hole conditions, any borehole may fail in an isotropic horizontal stress field except for reverse fault. In anisotropic stress field, wells may be stable for some azimuths, but fail under another drilling direction. The study provided invaluable pre-drill wellbore stability analysis of a complex geological structure. This study shows that planes of weakness in bedded rocks may lead to severe borehole collapse problems. However, in the three dimensional space there are combinations of wellbore inclinations and azimuth where the weak planes are not exposed to failure. At these directions a very stable borehole may result.

6.4 Results and Discussion

From the Aadnoy (1987,1988,2009) paper it is found that in cases with high tectonic in-situ stress in one direction, the borehole may be made very stable toward collapse by inclining it in the direction of the least in-situ stress. Hence one failure envelopes for all inclination as shown in case A of **Fig. 6.1, 6.2** and plane of weakness does not come into play at all. On the other hand inclining the borehole in the direction of maximum horizontal in-situ stress stress gives the conditions for the weakness plane to apply, with resultatnt collapse problems between 10 and 40^0 inclination. Generally deeper the well , the morelikely the borehole is to become sensitive towards collapse. Altough Von-Mises (Bradley 1979) found increasing the inclination didn't increase the sensitivity toward collapase, According to M-C shear failure theory and Jaeger's (1960) weakness- plane theory found that more incilnation is sensitive for collapse. Aadnoy et. al. found that higher inclination more sensitive towards collapse the isotropic rocks become. For weak Chacks, insence, may become a serious problem. For laminated rocks the weakness plane makes the rock stongly sensitive towards collapse in the range of 10 and 40^0 inclination for relax depositional basine. Fracturing of the borehole mainly a tensile failure, In general the fracturing gradient decreased with increased borehole inlination. Aadnoy and chenevert (1987) developed and estimated a simple formula to derive fracturing gradient at any borehole angle provided that farcuring gradient of a vertical hole and pore pressure are known. To arrive this results, the horizontal in-situ stress had to be adjusted with a correlation coefficient.

Our thesis works found from 3D view of **Fig. 5.10 and Fig. 5.11**, attack angle had been changed with different azimuth although inclination was same. Attack angle depends on the relative position of bedding plane and a plane that contain horizontal stress. So it has to be confirmed what are the dip angle and azimuth before taking the drilling action in to a formation. So one should take clear idea about the above planes before running our model.

After introducing the field data into the model, this thesis got the new thing that enhance model now can be determined the optimum well path and know whether the well data is secured or exist on the bedding exposed position. Our model got the optimum **well path 48.2^0** from maximum horzontal stress. This thesis works also found the change of attack angle (3D effect) with different azimuth. Our works also drew the different azimuth of wellbore position shown in the 3D view of **Fig. 5.10 and 5.11** such as 0^0, 45^0, 90^0, 135^0, 180^0, 225^0, 270^0, 315^0. The most important matter has been discussed at chapter2, such as one can determine minimum mud weight that prevent borehole collpase/fracture due to plane of weakness by applying the equation based on chapter 2.

CHAPTER 7 CONCLUSION AND RECOMMENDATION

The failure behavior of anisotropic laminated rocks may be desribed by the Single plane of weakness theorey of Jaeger, or by variable coefficient approach depending on rock behavoir (Cohesive strength and Internal friction). According to the literature review, the tensile strength of rocks is the most important factor for rock fracturing. In case of relaxed depositional basins, the borehole is sensitive to collapse/fracture for a range of 10 to 35^0 of inclination. This may be applicable only to laminated rocks and the phenomenon of plane of weakness. If the horizontal in-situ stresses are different, a borehole very stable against collapse can be drilled by inclining the hole in direction of the least in-situ stress. The critical parameters are planes of weakness in rock strength, the relative normal stress values on the borehole, and the relative angle between the borehole and bedding plane. The wells, which are drilled into 0 or 90 degrees of attack angle, are more stable. Under appropriate down-hole conditions, any borehole may fail in an isotropic horizontal stress field except for reverse fault. It is also found from literature review that relative position of wellbore and bedding plane is more important compared to the rock anisotropy. One can determine minimum mud weight that prevent borehole collpase/fracture due to plane of weakness by applying the equation discussed in this thesis paper.

According to the research regarding 3D is that the attack angle changes with changing azimuth having the inclination unchanged. One model cannot be sufficient to address all type of rocks. Before conclusion, one should correlate the model (even our model) results with the laboratory results. This research has dealt with the relation of different angles clearly and discussed available model related to the wellbore failure. In this study, theory and field cases of Aadnoy et. el.(2009) have been reproduced, enhanced their model and introduced some parameters. The following matters can be justified by our model.

- The user may apply their field data whether their applied field data is on the bedding exposed or safe positions.
- They may get quick result of optimum well path.
- They can test how many well data they have.
- Attack angle with Borehole inclination and azimuth relation can be addressed clearly.
- The difference of this research finding regarding up-dip and down-dip positions from those of Aadnoy et. al. (2009) field data can be further analyzed and justified by means of further study.

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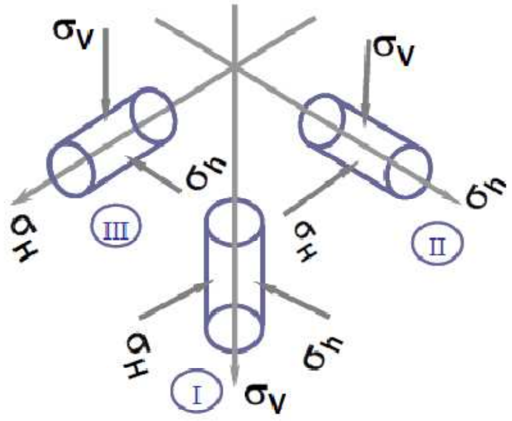
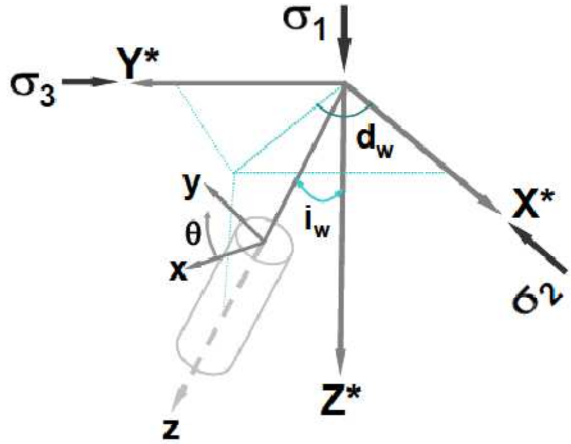
APPENDIX

Appendix A: State of stress and transformation





(a) **(b)**

Figur A-1 Wellbore orientation vs. principle stress direction(a) and vs. an arbitrary stress axis (b) (after Aadnøy et al., 2005)

For an arbitrarily oriented borehole the rotation of the stress tensor from the global in-situ stress coordinate system (x*,y*, z*) to a local borehole coordinate system (x,y,z) were applied as illustrated in Fig. A1. Assume that the in-situ principal stresses (σ_1, σ_2, σ_3) are associated with the co-ordinate system (x*, y*, z*). These virgin formation stresses should be transformed to another co-ordinate system (x, y, z) to conveniently determine the stress distribution around a borehole where the z-axis is parallel to the borehole axis, the x-axis is parallel to the lower most radial direction of the borehole, and the y-axis is horizontal. This transformation can be obtained by a rotation 'a_z' around the 'x'*-axis, and then a rotation 'γ' around the z*-axis. Using the stress transformation equation given by (Jaeger et al., 1969), the virgin formation stresses expressed in the (x, y, z) coordinate system is presented in Eq. (A1) From Eq.(A6), the angle 'a_z' corresponds to the deviation (inclination) of the borehole from σ_H, and the angle 'γ' represents the deviation of the borehole from σ_V.

Far-field stresses in a coordinate system referred to the borehole. Since the different stress regimes differ only by a transformation of coordinate axis, the locations and values of the extreme failure pressures for a general stress regime are obtained from the normal faulting case by letting $\sigma_v \rightarrow \sigma_1$, $\sigma_H \rightarrow \sigma_2$, $\sigma_h \rightarrow \sigma_3$, where σ_1, σ_2 and σ_3 are the ordered principal in-situ stresses.

It is common in the oil industry to assume three principal in-situ stresses, the vertical or overburden stress σ_v, and the maximum and minimum horizontal stress, σ_H and σ_h. We will here use these definitions, but please observe that the three principal stresses may not always assume a horizontal/vertical orientation. When analyzing image logs, deviations may occur. In these cases one could replace σ_v, σ_H, σ_h to $\sigma_{11,}$ $\sigma_{12,}$ σ_{13} to avoid confusion.

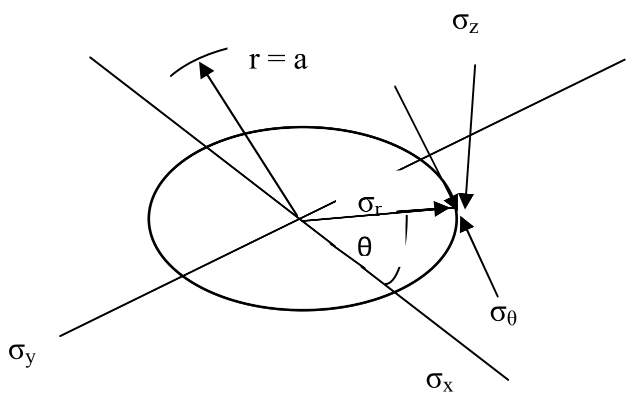
Figure A1 shows the most important stresses. The input stresses are the in-situ stresses σ_v, σ_h and σ_H. Since the borehole may assume any orientation, these stresses must be transformed to a new coordinate system x, y, z where we observe stresses as σ_x, σ_y, σ_z. The directions of the new stress components are given by the borehole inclination from vertical,γ, the geographical azimuth,a_z and the borehole position from the x-axis, θ. One of the properties of this transformation is that the y-axis is always parallel to the plane formed by σ_H and σ_h.

The following equations define all transformed stress components as shown in Fig. A1

$$\sigma_X = (\sigma_H \mathrm{Cos}^2 a_z + \sigma_h \mathrm{Sin}^2 a_z)\mathrm{Cos}^2\gamma + \sigma_v \mathrm{Sin}^2\gamma \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (A1)$$

$$\sigma_Y = (\sigma_H \mathrm{Cos}^2 a_z + \sigma_h \mathrm{Sin}^2 a_z) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (A2)$$

$$\sigma_{ZZ} = (\sigma_H \mathrm{Cos}^2 a_z + \sigma_h \mathrm{Sin}^2 a_z)\mathrm{Sin}^2\gamma + \sigma_v \mathrm{Cos}^2\gamma \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (A3)$$



Figur A-2 Stress acting at the Borehole

$$\tau_{yz} = \frac{1}{2}(\sigma_H - \sigma_h)\mathrm{Sin}(2a_z)\mathrm{Sin}\gamma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A4)}$$

$$\tau_{xz} = \frac{1}{2}(\sigma_H \mathrm{Cos}^2 a_z + \sigma_h \mathrm{Sin}^2 a_z - \sigma_v)\ \mathrm{Sin}(2\gamma) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A5)}$$

$$\tau_{xy} = \frac{1}{2}(\sigma_H - \sigma_h)\mathrm{Sin}(2a_z)\mathrm{Cos}\gamma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A6)}$$

Chenevert and Aadnoy Combined the total stress tensors will the borehole inner system to take into account the stress around a borehole as a function of radial distance from the centre of borehole, expressed as follows:

$$\sigma_r = \frac{1}{2}(\sigma_x + \sigma_y)\left(1 - \frac{a^2}{r^2}\right) - \frac{1}{2}(\sigma_x + \sigma_y)\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\mathrm{Cos}2\theta$$
$$- \tau_{xy}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\mathrm{Sin}2\theta + \frac{a^2}{r^2}P_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A7)}$$

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y)\left(1 + \frac{a^2}{r^2}\right) - \frac{1}{2}(\sigma_x - \sigma_y)\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\mathrm{Cos}2\theta$$
$$- \tau_{xy}\left(1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right)\mathrm{Sin}2\theta + \frac{a^2}{r^2}P_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A8)}$$

$$\sigma_z = \sigma_{zz} - 2\mu(\sigma_x + \sigma_y)\left(\frac{a^2}{r^2}\right)\mathrm{Cos}2\theta - 4\mu\tau_{xy}\left(\frac{a^2}{r^2}\right)\mathrm{Sin}2\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A9)}$$

$$\tau_{r\theta} = \frac{1}{2}(\sigma_x + \sigma_y)\mathrm{Sin}2\theta + \tau_{xy}\left(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}\right)\mathrm{Cos}2\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A10)}$$

$$\tau_{rz} = (\tau_{xz}\mathrm{Cos}\theta + \tau_{yz}\mathrm{Sin}\theta)\left(1 - \frac{a^2}{r^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A11)}$$

$$\tau_{\theta z} = (-\tau_{xz}\mathrm{Sin}\theta + \tau_{yz}\mathrm{Cos}\theta)\left(1 - \frac{a^2}{r^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A12)}$$

At the Bore hole at r = a, equation from A7-A12 reduces to

Radial stress: $\sigma_r = P_W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A13)}$

Tangential Stress: $\sigma_\theta = (\sigma_x + \sigma_y - P_W) - 2\mu(\sigma_v - \sigma_y)\mathrm{Cos}2\theta - 4\tau_{xy}\mathrm{Sin}(2\theta) \ldots\ldots \text{(A14)}$

Axial Stress , Plain strain: $\sigma_Z = \sigma_{zz} - 2\mu(\sigma_v - \sigma_y)\mathrm{Cos}2\theta - 4\mu\tau_{xy}\mathrm{Sin}(2\theta) \ldots\ldots\ldots \text{(A15)}$

Axial Stress , Plain stress: $\sigma_Z = \sigma_{zz} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A16)}$

Shear Stress: $\tau_{\theta z} = 2(\tau_{yz}\mathrm{Cos}\theta - \tau_{xz}\mathrm{Sin}\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A17)}$

Shear Stress: $\tau_{rz} = (\tau_{r\theta}) = (0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A18)}$

As failure is governed by the principal stresses, the following matrix defines planes of principal stress:

$$\begin{bmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_\theta & \tau_{\theta z} \\ 0 & \tau_{\theta z} & \sigma_z \end{bmatrix} \equiv \begin{bmatrix} \sigma_i & 0 & 0 \\ 0 & \sigma_j & 0 \\ 0 & 0 & \sigma_k \end{bmatrix}$$

Taking the determinant of the above matrix, the principal stresses are given by the following eigenvalue equation:

$$(\sigma_r - \sigma)\{(\sigma_\theta - \sigma)(\sigma_z - \sigma) - \tau^2{}_{\theta z}\} = 0$$

The solutions are:

$$\sigma_i = P_W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A19)}$$

$$\sigma_j = \frac{1}{2}(\sigma_\theta + \sigma_z) + \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z) + 4\tau_{\theta z}{}^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(A20)}$$

$$\sigma_k = \frac{1}{2}(\sigma_\theta + \sigma_z) - \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z) + 4\tau_{\theta z}{}^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(A21)}$$

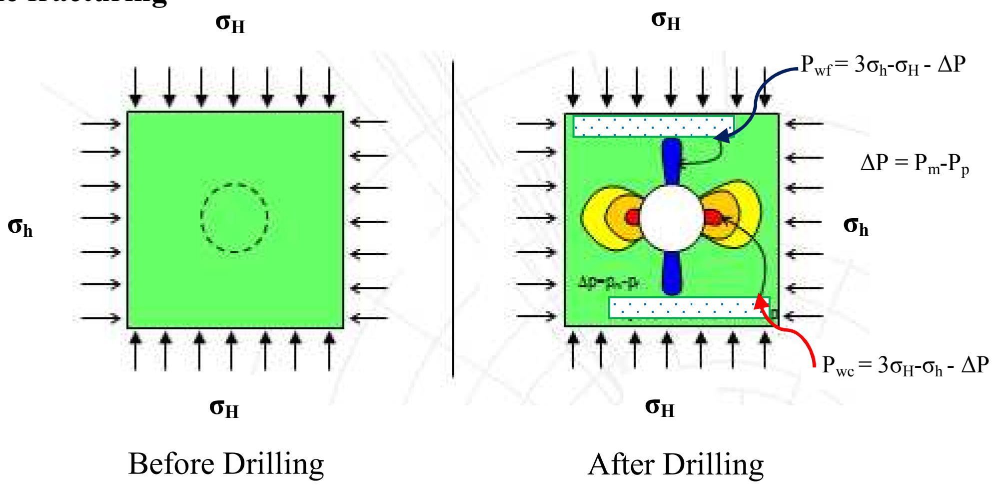
After each computation the indices are rearranged according to the convention, i, j, k assumes values: 1 is largest, 3 is the least principal stress.

Table A1 Stress state of Vertical Hole $\gamma = 0$

Transformed In-situ Stress (A1-A12)	Borehole Stresses (A13-A18)	Principal Stresses (A19-A21)
$\sigma_x = \sigma_H \mathrm{Cos}^2 a_z + \sigma_h \mathrm{Sin}^2 a_z$ $\sigma_y = \sigma_H \mathrm{Sin}^2 a_z + \sigma_h \mathrm{Sin}^2 a_z$ $\sigma_z = \sigma_v$ $\tau_{xy} = 1/2\ \mathrm{Sin}2a_z(\sigma_h - \sigma_H)$ $\tau_{xz} = \tau_{yz} = 0$	$\sigma_r = P_w$ $\sigma_\theta = \sigma_H + \sigma_h - P_w - 2(\sigma_H - \sigma_h)\ \mathrm{Cos}2(a_z + \theta)$ $\sigma_z = \sigma_v$ $\tau_{\theta z} = 0$	$\sigma_i = \sigma_r$ $\sigma_j = \sigma_\theta$ $\sigma_k = \sigma_z$

Appendix B: Borehole Failure criteria

Borehole fracturing



Figur B-1 Borehole Affect In-situ stress Field Before and After drilling (Eirik)

Fracturing condition for the Borehole is defined as follows:

$$\sigma'_3 = \sigma_3 - P_0 = -\sigma_t \dots\dots\dots\dots\dots (B1)$$

The minimum principal stress equal to the tensile rock strength defines this condition. Often the rock tensile strength is neglected assuming natural cracks and fissures.

Appendix C Collapse and Mohr-Columb Model

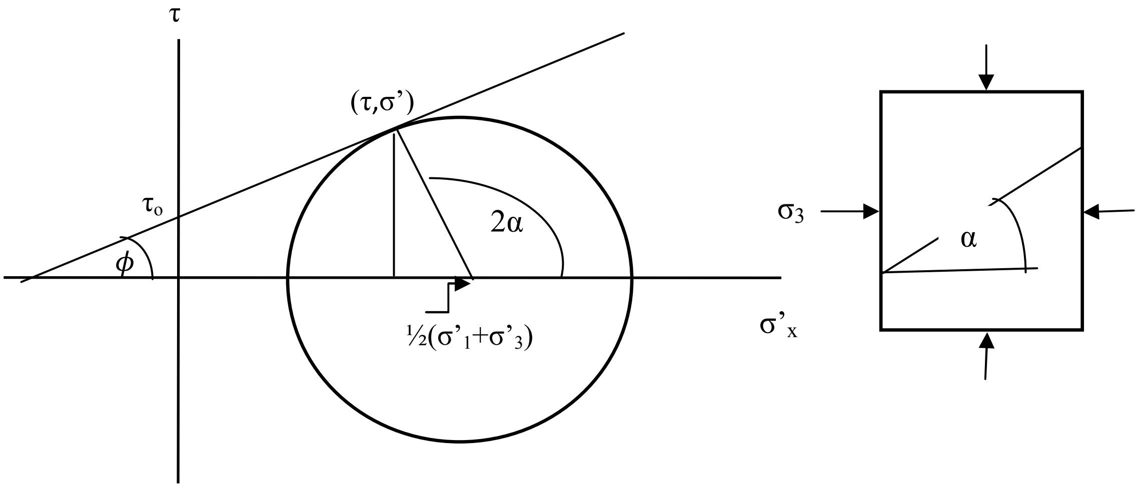
Borehole Collapse Mohr Coulomb failure Model

The Mohr-Coulomb failure equations are used to understand the stress conditions at which the rock sample fails. The equations describe a circular locus of paired values (σ_n, τ) of the normal and shear stresses that operate on any and all orientations within a given body that has been subjected to known values of σ_1, and σ_3. Using the Mohr-Coulomb failure diagram it is possible to identify a plane of any orientation relative to σ_1 and to read the values of normal stress, σ_n and shear stress, τ, acting on the plane (Figure C1). The failure envelope is a collection of Mohr circles which is developed experimentally by subjecting a suite of samples to successively higher confining stresses, σ_3, and determining the resultant value for failure, σ_1 . A minimum of four plug samples are used to construct the envelope. One sample is run under uniaxial (unconfined) compression and another is subjected to the in-situ effective confining stress, σ'_c. Two additional samples are run at conditions proportionally above and below, σ'_c to complete the envelope.

Coulomb theorized that fracturing occurs when the shear stress on a plane exceeds both the cohesion of the material and the friction developed by stress normal to the plane. Thus, failure occurs outside of the envelope and the area within the envelope is referred to as the region of stability. The relationship between the magnitude of the shear stress, τ and normal stress, σ_n , is:

$$\tau = \tau_o + \sigma_n \mathrm{Tan}\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(C1)}$$

where, τ_o, is the cohesive or shear strength of the rock under zero normal stress and, ϕ, is defined as the angle of internal friction. Both of these variables are critical to the understanding of how rocks fail and the reader is referred to Jaeger and Cook for a complete discussion on rock mechanics. The tensile strength of the rock, σ_t, is measured indirectly by the Brazilian disk method and is used in the evaluation of rock failure mechanisms. For practical applications, it could be useful to derive expressions for the particular stress states . This will be performed in the following. Figure C1 shows the stresses at failure:



Figur C-1 Stresses at failure for the Mohr-Coulomb Failure Model

In the figure above we use effective stresses. Inspection of the figure reveals that the coordinates (σ, τ') at failure is defined by the following equations:

$$\tau = \frac{1}{2}(\sigma'_1 - \sigma'_3)\mathrm{Cos}\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C2)$$

$$\sigma' = \frac{1}{2}(\sigma'_1 + \sigma'_3) - \frac{1}{2}(\sigma'_1 - \sigma'_3)\ \mathrm{Sin}\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C3)$$

For applications of the model, Equation B.2-3 should be inserted into Eqn.(B.1) The resulting equation defines the stress state at failure.

$$(\sigma'_1 - \sigma'_3) - (\sigma'_1 + \sigma'_3)\ \mathrm{Sin}\phi = 2\tau_0\mathrm{Cos}\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C4)$$

It should be emphasized that shear strength is an experimentally determined material property. There is often little physical arguments for particular models. Rather, empirical models are developed which fits the data. The Mohr-Coulomb model describes a few material properties. The angle α is defined as the angle of friction. Sandstone, for example, will exhibit friction along a shear plane as the grains will restrict motion. This is irrespectively if the sand grain are cemented or not. The cohesive strength τ_o, on the other hand, reflects the degree of cementation of the material. The fracture angle (α) on the plug specimen shown in Fig. 5.6 can be determined from the following expression:

$$\alpha = 45^0 + \frac{\phi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C5)$$

For borehole collapse we assume mohr-coulomb failure model. This is directed by the maximum and minimum principal stresses.

For borehole collapse at low borehole pressure, σ_j will be the maximum principal stress σ_1 whereas σ_i will be the least principal stress σ_3 (Appendix A). Angle ϕ defined as the Angle of friction. Sand stone, for example, will exhibit friction along a shear plane as the grains will restrict motion. This is irrespectively if the sand grain cemented or not. The cohesive strength τ_o, on the other hand, reflects the degree of cementation of the material.

Appendix D: Principal stress, average and deviatroic stress

The stress components can be written in a tensorial form as follows:

$$\begin{bmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{bmatrix} \equiv \begin{bmatrix} \sigma_x & \tau_{xy} & \sigma_z \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \ldots\ldots\ldots..(D.1)$$

A tensor in space has 3^n components, where n is the order or the rank. Examples are given below:

Tensor:	Rank:	No. of components:	Example:
Scalar	0	$3^0 = 1$	Temperature, mass
Vector	1	$3^1 = 3$	Velocity, force
Tensor	2	$3^2 = 9$	Stresses, deformation
	3	$3^3 = 27$	Anisotropic stresses, material properties
	4	$3^4 = 81$	

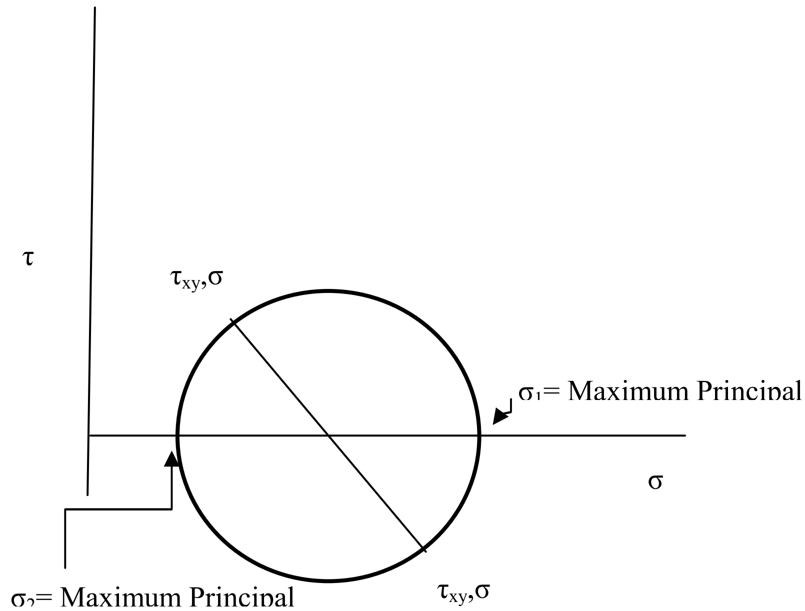
In applied rock mechanics, the material properties E and ν are considered scalar by assuming isotropic properties. This means that the properties are equal in all directions. However, real rock is often anisotropic with directional properties defined as for example E_x, E_y and E_y. The reason for neglecting this is often that we do not know the real properties, and that we assume that the effect of anisotropy is negligible. The reader should be aware of these simplifications.

Principal stress

The general definition of the stress state is reproduced below:

$$[\sigma]=\begin{bmatrix} \sigma_x & \tau_{xy} & \sigma_z \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots.(D2)$$

Imagine that a given stress state is defined, that is; each of positions in the matrix above is given a number. By transforming these stresses in space, all of these stress components will change according to the transformation laws. This complicates the exact definition of stress, as to stress matrices may look quite different, but may describe the same stress state if transformed to another orientation. This problem is avoided by introducing principal stresses. If we rotate our coordinate system to an orientation where all shear stresses vanishes, the normal stresses are defined as principal stresses. This is illustrated in two dimensions by Mohr's circle (**Fig. D1**):



Figur D-1 Mohr's circle for a two dimensional

The general definition of the principal stresses is as follows:

$$[\sigma]=\begin{bmatrix}\sigma_x & \tau_{xy} & \sigma_z\\ \tau_{xy} & \sigma_y & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & \sigma_z\end{bmatrix}=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(D3)$$

Equation D.3 actually is a set of homogeneous linear equations. By moving the right hand matrix over to the left and taking the determinant, a solution for the principal stresses is:

$$\begin{bmatrix}(\sigma_x-\sigma) & \tau_{xy} & \tau_{xz}\\ \tau_{xy} & (\sigma_y-\sigma) & \tau_{yz}\\ \tau_{xz} & \tau_{yz} & (\sigma_z-\sigma)\end{bmatrix}=|0|$$

To determine the principal stresses, σ, the determinant of the equation above must be calculated. The result is:

$$\sigma^3\text{-}I_1\sigma^2\text{-}I_2\sigma\text{-}I_3 = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(D4)$$

Where:

$$I_1=\sigma_x+\sigma_y+\sigma_z$$

$$I_2=\tau_{xy}^2+\tau_{xz}^2+\tau_{yz}^2-\sigma_x\sigma_y-\sigma_x\sigma_z-\sigma_y\sigma_z$$

$$I_3=\sigma_x(\sigma_y\sigma_z-\tau_{yz}^2)-\tau_{xy}(\tau_{xy}\sigma_z-\tau_{xz}\tau_{yz})+\tau_{xz}(\tau_{xy}\tau_{yz}-\tau_{xz}\sigma_y)$$

I_1, I_2, I_3 are called invariants, as they remain invariant for a given stress state regardless of the orientation of the coordinate system. Equation D.4 always has three real roots. These roots are called principal stresses, where: $\sigma_1 > \sigma_2 > \sigma_3$ actually they are the eigen values of the matrix.

Average and deviatoric stresses

First we will define an average stress as:

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

We will decompose Eqn. D.2 by defining the total stress as the sum of the average stress and the deviatoric stress.

$$[\sigma] = \begin{bmatrix} (\sigma_m) & 0 & 0 \\ 0 & (\sigma_m) & 0 \\ 0 & 0 & (\sigma_m) \end{bmatrix} + \begin{bmatrix} (\sigma_x - \sigma_m) & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & (\sigma_y - \sigma_m) & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - \sigma_m) \end{bmatrix} \dots\dots (D5)$$

The total stress is equal to the average stress plus the deviatoric stress. The reason for separating the stress into two components is that many failure mechanisms are governed by the deviatoric stress because it actually reflects the shear stress level. To determine the principal deviatoric stresses, The deviatoric invariants then become:

$$J_1 = 0 \dots\dots (D6)$$

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2] \dots\dots (D7)$$

$$J_3 = I_3 + \frac{1}{3}I_1 I_2 + \frac{2}{27}I_1^3 \dots\dots (D8)$$

A physical definition may be as follows. Any stress state can be decomposed into a hydrostatic and an deviatoric stress component. The hydrostatic component may cause volume change in the body, but no shape change. The deviatoric component causes shape change, and give therefor rise to shear stresses. The equation for J_2 is often used in calculations of shear strength of materials. It is often called for the von Mises theory of failure or flow criterion.

Two-dimensional stresses

We will now consider a two-dimensional loading case, where there is no stress along the z-axis. For this case, the stresses $\sigma_z = \tau_{xz} = \tau_{YZ} = 0$, and Eqn. D2 reduces to:

$$[\sigma] = \begin{bmatrix} (\sigma_x) & \tau_{xy} & 0 \\ \tau_{xy} & (\sigma_y) & 0 \\ 0 & 0 & (0) \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \dots\dots (D9)$$

The equation for the principal stress becomes:

$$\sigma\left(\sigma^2-\left(\sigma_x+\sigma_y\right)\sigma-\left(\tau_{xy}^2-\sigma_x\sigma_y\right)\right)=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D10)}$$

The roots of this equation is:

$$\sigma_{1,2}=\frac{1}{2}\left(\sigma_x+\sigma_y\right)\mp\sqrt{\left(\frac{1}{2}\left(\left(\sigma_x-\sigma_y\right)\right)\right)^2+\tau_{xy}^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(D11)}$$

Analysis is based on: linear elasticity that is non-penetrating kirsch solution for fracturing and Mohr-coulomb shear-failure model for borehole collapse. If more complex material models are used, such as nonlinear elasticity or elasto-plasticity, deformation and failure characteristics may change as given by the constitutive rock behavior.