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Writer: Marco Antonio Céspedes Guzmán	
Faculty supervisor: Prof. Erik Skaugen – University of Stavanger External supervisor(s): None	
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Abstract

During the make-up and break-out of the connections, the drill string is wedged in the rotary table and the top of the drill string follows the platform heave movement as the bit is off bottom. This oscillation may be transmitted down the string and produce pressure fluctuations around the string, especially at the bottom of the string due to the small clearance between BHA and borehole. This can cause high surge and swab pressures that may lead to undesired loss of circulation or influx of formation fluid.

This thesis presents a numerical method developed at the University of Stavanger (programmed in MatLab), for calculating the behavior of elastic drill strings when exposed to the forced movement at the top (platform heave movement). The method is restricted to treating only one-dimensional waves, travelling along the string axis. It considers all the elastic effects and some of the effects of friction, such as contact friction and viscous friction inside the drill string. The down hole oscillations of the drill string drill bit and the pressure fluctuations below the bit were calculated, assuming that the pressure is generated by forcing the mud to flow up and down the annulus as the bit moves down and up, respectively.

The basic well case consists of a “Build and Hold” directional well, where the well is drilled vertically 500 m to the KOP, where the well is smoothly deflected until a maximum desired deviation. The established deviation is maintained while drilling to the target depth. The drill string has a 5” DP section, a 200 m 8” BHA section, and a 12” drill bit. The wellbore is partly cased, and the mud properties are considered as standard. Several scenarios were simulated in order to evaluate the effects of the drill string length, deviation, wave conditions, amount of numerical segments, etc. upon the pressure fluctuations and bit movement. As expected the pressure fluctuations and bit movement increases when the wave conditions are harsher, more accuracy is obtained when the amount of numerical segments is increased, and longer wells with high deviation damp down the bit movement due to the increased contact friction, thus also the pressure fluctuations.

The results obtained by the method can be regarded as conservative, since the actual pressure fluctuations probably will be less than the ones calculated here. Thus, they still can be used for better planning of drilling operations, and a better understanding of the drill string oscillations, giving an enhanced wellbore pressure profile management, and a strong indication that in very sensitive reservoirs a heave compensated drill floor could be used to drill more efficiently and safely.

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1. Introduction

1.1 Background

The search for offshore oil and gas, or at least the interest for such ventures, started almost simultaneously with the modern oil industry (approx. 10 years after the Drake well). Initially, offshore drilling was an extension of onshore activities; consequently, the first offshore wells were drilled near shore and in shallow waters. By 1947, the first well “out of sight of land” was drilled 14.5 km offshore in the Gulf of Mexico – Ship Shoal coast of Louisiana; this discovery not only provided the first major discovery in the Gulf of Mexico but also represented the starting point for the modern offshore industry (1).

As the use of floating drilling vessels began, due to the interest in deep waters; problems related to heave motion appeared which adversely affect drill string operations. Some of these were: sub optimal drilling operations (bit on bottom) due to fluctuation of WOB, reaming operations which require very low WOB and a smooth, uniform rate of lowering, setting packers or seal assemblies, directional surveys which require the drill string to be held stationary, and severe vessel heave causing drilling operations to stop altogether as safety becomes compromised.

Therefore, in the late fifties, telescopic joints known as bumper subs were developed (2), to decouple the lower part of the drill string from the vessel heave. Afterwards, in 1970 a motion compensator was designed by E. Larralde and the prototype built by Vetco Offshore Inc. (3). This device was found in almost all floating drilling units, certainly in all of those that worked in harsh environments. In 1974, it was reported in (4) that the introduction of Drill String Compensators (Passive compensators) to the North Sea Operations had increased productive time by 16 days per unit per day. Later, active compensators systems were introduced to proactively hold the suspended load motionless or hybrid systems were used as well. This was a major advance in the offshore industry, as weather down time is reduced, optimum bit loads can be achieved which allows better ROP's and increased bit life; as well as safer operations.

However, at present, it appears we have drilled most of the easy prospects. Those remaining require more *precision in managing the wellbore pressure profile, to drill safely and efficiently* (5) (6). Therefore, wells are getting more difficult to drill, specially due to small margins between the pore and fracture gradient; usually present when drilling depleted reservoirs (7), pressurized cap rock due to water injection (Gullfaks), HPHT wells (8) and deep water drilling (1). Consequently, kick – loss scenarios are very frequent which results in much Non-productive time (NPT) dealing with these troublesome zones.

Moreover, now, the industry is learning that precise monitoring of wellbore hydraulics and pressure, when the bit is not on bottom, is often as important to drilling safety and efficiency as when drilling ahead (5). This issue is more related to this thesis in that maintaining the bottom hole pressure within acceptable limits when drilling from a mobile offshore drilling unit (MODU) is more complicated due to the motion of the MODU (heave specially). The heave compensator controls the position of the drill string in drilling mode and also in tripping mode. However during make-up and break-out of the connections, the drill string is suspended by the slips in the rotary table and the top of the drill string moves up and down in conjunction with the heave of the MODU (9). This heave oscillation may be transmitted down the string and give pressure fluctuations around the string; especially at the bottom end, where due to the small clearance between the BHA and drill bit with the borehole, act as leaky pistons. This can in turn cause high surge and swab pressures that may lead to lost circulation or influx of formation fluid, which may limit the drilling length of the borehole section and may also result in well control challenges, especially when drilling prospects specified in the previous paragraph, such as narrow drilling windows.

The thesis presents a numerical method developed at the University of Stavanger, for calculating the behavior of elastic drill strings when exposed to the forced movement at the top (MODU's heave). The method is restricted to treating only one-dimensional waves, travelling along the string axis; these waves are pure stress waves (10). It considers all the elastic effects and some of the effects of friction, which will be further developed in the next chapter, so the down hole oscillations of the drill string drilling bit and the actual pressure fluctuations at the bottom (below the bit) will be calculated, considering that the pressure is generated by forcing the mud to flow up and down the annulus as the bit moves down and up, respectively.

1.2 Motivation

As explained before, drilling wells have become more difficult and challenging when drilling in “trouble zones”, which are a major contributor to economically un-drillable prospects. Such drilling related situations are loss circulation, differential stuck pipe, well control situations, kick-loss situations; that result in NPT that may exceed the authorized expenditure for the well's drilling program (11).

All drilling related challenges mentioned, have one thing in common: they indicate a requirement for more precise wellbore pressure management, containment and control with fewer interruptions when drilling.

Therefore, several technological advancements have aided in this pursuit of not only analyzing the pressures when drilling, but also when the bit is off bottom, as when using Managed Pressured Drilling (MPD). In (7) is stated that the Kristin development wells may be the first

wells globally to use MPD techniques in a harsh weather offshore environment on a floating drilling installation; and it was identified as a challenge that while the use of a closed loop drilling system eliminates the slip joint volume changes that occur with heave on a conventional set-up, it introduces another variable termed “Closed System Heave”. This is the change in volume that occurs as the drill pipe cycles in and out of the closed system, as will happen during rig heave when the bit is off bottom. These effects are in the order of 5 to 10 bar for the volume of a typical Kristin case. It is clearly seen that the swab-surge effects due to heave were identified in this HPHT field development.

Furthermore, in (9) are presented magnitudes of surge and swab pressures that can occur in typical drilling operations, but with focus on Trough Tubing Rotary Drilling (TTRD). As this type of drilling allows smaller clearances between the hole and the drill string, and as most of the current methods for MPD from MODUs do not have a functionality to compensate for both surge and swab pressures when the drill string is wedged to the rotary table, it represents a huge problem.

Finally, recently in March 2012, at the SPE/IADC Managed Pressure Drilling and Underbalanced Operations Conference & Exhibition (12), a presentation was given of how heave motion of floating rigs complicates the control of pressure in MPD. As said before, during connections, the drill string moves with the heaving rig causing down-hole pressure fluctuations. Therefore, as a step forward designing control schemes to actively attenuate the fluctuations, a fit-for-purpose mathematical model of well hydraulics was derived based on a finite volumes discretization. The model was validated using experimental data from UIIRig – A full-scale experimental drilling facility.

Therefore, based on the motivation to contribute to a more precise wellbore pressure management, this thesis is a contribution in evaluating the magnitude of pressure fluctuations (surge and swab) below the bit when the drill string is wedged to the drill floor, and how the rig heave affects these pressures. As the numerical method is programmed in MatLab, it can consider several scenarios with different waves, heaves, hydraulic considerations, friction, drilling fluids, drill string length, well deviation, etc.; and how these will affect the Surge and Swab pressure variations which are very important information when drilling, specially, in narrow window prospects to avoid NPT, waiting on weather, losses, excessive well control situations or kick-loss scenarios.

1.3 Scope of study

This thesis investigates how the heave movement of a floating drilling vessel affects the swab and surge pressures when the drill string is wedged to the rotary table every time the drill string length need to be increased or decreased by adding or removing a stand; as the heave

compensator cannot alleviate the movement when the drill string is not hanging from the travelling block. These pressure fluctuations were calculated by using a numerical Method developed at UiS, which calculates the behavior of strings (tubes, rods, etc.) when exposed to changing forces.

The method uses the one dimensional, second order wave equation which describes the actual motion of the material in the string; and the numerical solution calculates the movement of a drill string at a number of equidistant points along this string, and only at equidistant points in time. This is based upon a numeric, finite element method (FEM), specially designed to handle problems with long strings exposed to different external forces and/or forced movements of parts of the string. This method includes full calculations of the string elasticity in the axial direction, and allows inclusion of linear viscous friction, and contact friction between the drill string and the walls of the well. The numeric calculation was programmed in MatLab.

Research on wave theory and significant wave heights, periods and other statistical data related to the North Sea harsh environment was done. So that the calculations consider different wave scenarios (heave, period) and their effects on the surge and swab pressures. It is worthwhile to mention that MatLab allows the easiness of changing different parameters, like drilling mud properties, drill string spec's, contraflow considerations, number of numerical segments, length between drill string and bottom-hole, deviation of the well and specially the number of segments discretized for the calculations; which let us evaluate their effects on the pressure fluctuations.

Therefore, this thesis will show that the calculation of the surge and swab pressures due to the heaving rig, when the heave compensator is not used, is possible by using the numerical method presented here. So, different parameters can be changed to evaluate their degree of affection upon the pressure fluctuations. The practical approach that has been taken here, while perhaps not definitive, at the very least gives proof of concept and provides pretty accurate pressure calculations within the inherent limitations.

1.4 Outline

Chapter one – *Introduction* is as its name says an introductory section outlining the framework of the study. It introduces the motivation behind the investigation within the context of the historical background, and sketches the scope of the study and its contributions. At last it describes the structure of the present written work.

Chapter two – *Literature Review* begins by introducing drilling and how problems had evolved in order to appreciate, more than in the past, the evaluation of swab and surge pressures. Then it presents the motions of the rig, with special focus on heave, North Sea waves and its theory.

Finally it describes the UiS numerical method in detail, definitions, equations and limitations; for its further modeling in MatLab.

Chapter three – *Numerical Calculations and Programming* explains how the whole new Numerical Method program is applied for a well case scenario in this thesis. The logic of the program is illustrated in a Flow Diagram, where dependencies between different parts of the program are shown. Moreover, this chapter presents equations, explaining their purpose and meaning, also presents the required inputs and all calculations done to achieve the objective of the program, calculate the pressure fluctuations below bit and bit movement. Finally, it shows how the results from the programmed method are presented.

Chapter four – *Results Analysis and Discussion*, this chapter addresses the results obtained from the application of the Numerical Method, explained in Chapter 3; in order to calculate the pressure fluctuations below bit and bit movement when the drill string is attached to the drill floor and its top part is forced to follow the rig heave movement. Results are based on a well case scenario (“build and hold well path”), and different parameters introduced by the user (Table 4.1) were evaluated on how critical they are for making a more realistic calculation. Moreover, the drill string length and deviation of the well were also changed in order to evaluate their effect on the bit movement and pressure fluctuations. The results are explained with Graphs and Tables for a better understanding.

Chapter five – *Conclusions and Recommendations* summarizes and discuss important parts of the preceding chapters of the thesis and draws conclusions from the results of the investigation. A review of the goals of the thesis is contrasted with the results obtained from the study. Finally recommendations are given regarding future improvements and application of this method.

Chapter six – *References* presents all the sources and bibliography used for this thesis.

Chapter seven – *Nomenclature* summarizes all the abbreviations meaning and the name, sign and units for all the variables used both in this thesis and in the program developed in MatLab.

Chapter eight – *Appendixes* has five sections; in the first, the wave equation is derived and its solution; second, the Standard Numerical Equation is proven using the wave equation, and third, the physical ball spring model is introduced to explain how to find the numerical equations for different space boundaries. Finally, the MatLab program code is presented with all the commentaries for a better understanding, and at last the intermediate results and variables’ values obtained from running the program in MatLab for a given scenario.

2. Literature Review

2.1 Offshore Drilling

As stated in (13), offshore drilling refers to a mechanical process where a wellbore is drilled through the seabed. It is carried out to explore and subsequently produce hydrocarbons which lie in rock formations beneath the seabed. Hydrocarbons production is more challenging than land-based installations due to the remote and harsher environment, plus the need to provide very large production facilities. However, the trend today is to conduct more of the operations subsea, with no installations visible above the sea. Moreover, these operations present logistics and human resources challenges, where efforts such as integrated operations are being applied.

There are two basic types of drilling rigs – fixed platforms rigs and mobile rigs. Fixed platform rigs are installed on large offshore platforms and remain in place for many years. Most of the large fields in the North Sea were developed using this kind of rigs (14). Mobile rigs comprise two types: jack-up rigs used in shallow waters less than 100 meter deep and semi-submersible rigs used in deeper waters down to 1000 meters or more. These two can be seen in Figure 2.1. In very deep waters, drilling ships are used. This thesis is focused on rigs which are floating at all times, but obviously when in position for drilling are anchored and ballasted.



FIGURE 2.1 - SEMI SUBMERSIBLE AND JACK UP RIGS (14)

In drilling operations, the drilling fluid is pumped down the drill string and flows through the drill bit in the bottom of the well. Then it flows up the well annulus carrying cuttings out of the well. It is also used to keep the pressure in the annulus at a desired level. This pressure control is crucial in all drilling operations, as the pressure has to be within certain boundaries. Specifically, it has to be above the pore pressure to prevent unwanted inflow from the surrounding formations into the well, and below the fracture pressure of the surrounding formations to prevent the well from fracturing (12).

2.1.1 Drill String

The drill string is the combination of the drill pipe, the bottom hole assembly (BHA) and any other tools used to make the drill bit turn at the bottom of the wellbore (15). The drill string is hollow so that drilling fluid can be pumped down through it and circulated back up the annulus.

The drill string is typically made up of three sections. The BHA, which is the lower portion of the drill string, consisting of the bit, bit sub, a mud motor (some cases), stabilizers, drill collars, jarring devices and crossovers for various thread forms. This section must provide axial force for the bit to break the rock (WOB), survive a hostile mechanical environment and provide the driller with directional control of the well. The assembly often includes a mud motor, directional drilling and measuring equipment, measurements while drilling (MWD) tools, logging while drilling (LWD) tools and other specialized devices. Second, there is a transition pipe, which is a type of drill pipe whose walls are thicker and collars are longer; usually called Heave Weight Drill Pipe (HWDP). It tends to be stronger and has higher tensile strength than common drill pipe. Finally, drill pipe, which by far is the largest part of the string. Its purpose is to support the bit and the BHA, and to provide means to pull the bit out of hole. It also provides means to rotate the bit or act as supporting means for a down hole motor to rotate the bit.

The selection process for the drill pipe consist of, first, strength considerations, then size, and finally cost. Strength refers to several properties, including the pipe's ability to pull the string out of hole (tension capacity) and to transmit torque to the bit (torque capacity). There are other strength considerations such as internal pressure from the drilling fluid, bending in directional wells, fatigue, external pressure, compressive load and buckling. Pipe size and tool joint size are determined by hydraulics, fishability, and elevator hoisting capacity; other considerations for the size include buckling strength, fatigue resistance, external pressure, and bending stress. Finally, items affecting the cost are pipe availability, drill pipe features to enhance the pipe's performance, and features that enhance the pipe's usable life (1).

The present thesis' calculations are performed for a homogeneous string, where the forces are acting along its axis. In general, a sequence of strings of possibly different diameter and material can be considered; but for simpler calculations, the present investigation considers a long drill string which mainly consists of a long section of drill pipes at the top, and a shorter section of heavier drill collars at the bottom, both made of the same material but of different dimensions. The simplified drill string can be seen in the following Figure:

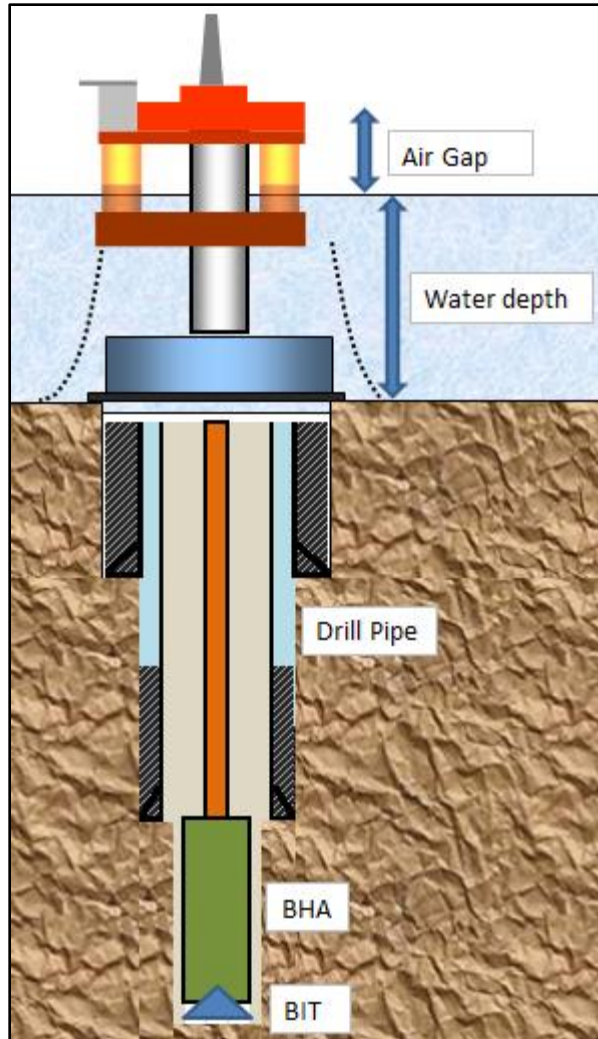


FIGURE 2.2 - SIMPLIFIED EXAMPLE OF DRILL STRING ON AN OFFSHORE DRILLING OPERATION

The physical drill string parameters required for calculation are, from (10):

- String length: measured and true vertical depths needed when a deviated well is considered.
- Material cross section and weight: is always needed when external forces are acting on the string, heave. The most important parameters are outer and inner diameters and weight per unit of length (nominal). The drill pipe and drill collars specifications have standard sizes, weights, grades, tensile strength, etc. For the calculations, one can choose any, depending of the section that is being drilled. However, for drill pipes there are nine commonly used sizes ranging from 2 3/8 to 6 5/8 in.
- Any two of the three string material parameters: density ρ_{steel} , speed of sound in the material c_{steel} , and the modulus of elasticity for the solid string E_{steel} . As these parameter are connected by the equation:

$$c_{steel}^2 = E_{steel} / \rho_{steel} \quad (2.1)$$

Regarding these properties, they can vary significantly according to the grade of steel used, alloys needed, etc. For calculations it was assumed to have a steel density of 7850 kg/m³ (16), modulus of elasticity of approx. 30000000 psi (17) and the speed of sound considering longitudinal waves travelling in long rods is 5172 m/s.

- Coefficients of friction: for calculating the contact friction (solids sliding against each other), these coefficients must be known specially for friction of drill string against casing and drill string against formation in open hole. Standard values of 0.23 (steel – steel) and 0.3(steel-rock) were adopted.
- The liquid friction will be defined as a function of string speed relative to the liquid.
- Finally, the string axis deviation from the vertical (angle of deviation) must be known, which can change along the string. This is used to calculate any force against any support (normal force) for finding contact friction, and to find the component of gravity acting along the string axis.

The well path used in the thesis will be the type “Build and hold”. The well is drilled vertically from surface to the kickoff point (KOP), where the well is steadily and smoothly deflected until a maximum angle and desired direction is achieved. The established angle and direction are maintained while drilling to the target depth.

2.1.2 Hydraulics and drilling fluids

As explained before, during drilling of oil and gas wells, drilling fluid is circulated from surface to the bottom hole through the drill string, the bit nozzles, and returns to surface in the annular region between the borehole and the drill string. The drilling fluid has a number of important functions, it removes cuttings from the bottom hole, holds cuttings and weight material in suspension when circulation is interrupted, controls subsurface pressure, and transmit hydraulic horsepower.

For conventional drilling applications, we use water-based mud (WBM), oil-based mud (OBM), and synthetic-based mud (SBM). In addition, aerated fluids and foam are frequently used for underbalanced drilling (UBD) applications. For the purpose of hydraulics analysis, drilling fluids are generally classified as Newtonian and non-Newtonian fluids. Newtonian fluids such as water and mineral oil exhibit a direct proportionality between shear stress and shear rate in laminar flow, where the constant of proportionality is the viscosity of the fluid and is independent of the shear rate.

The relationship between the shear stress and the shear rate is used as the main rheological classification method for drilling fluids. Accordingly, different rheological models (constitutive equations) such as Bingham plastic, power-law and Herschel-Bulkley (Yield Power law) have been developed to represent correctly this relationship and perform wellbore hydraulics analysis. Figure 2.3 show a plot of shear stress vs. shear rate for Newtonian fluids, power-law fluids and Bingham plastic fluids. The vast majority of drilling muds do not exhibit pure Newtonian, power-law nor Bingham plastic behavior, it is more likely the dashed line in the Figure.

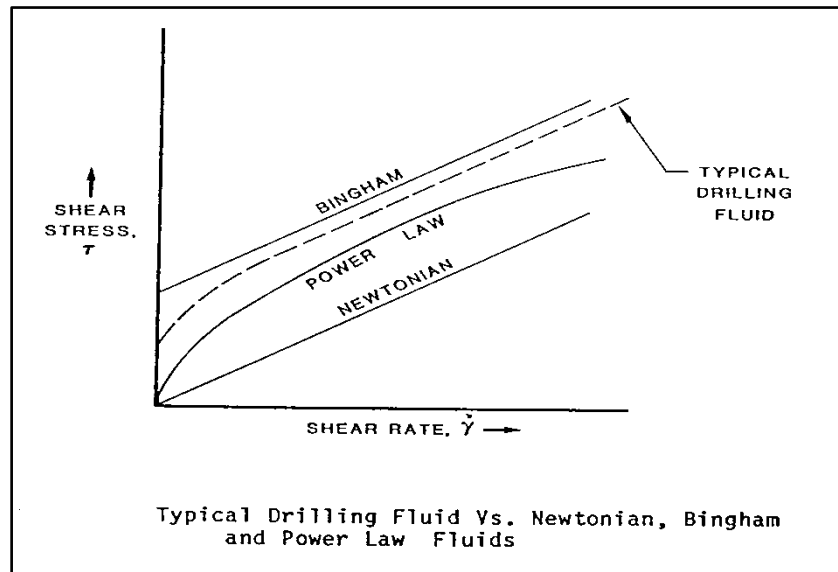


FIGURE 2.3 - COMPARISON OF FLUID BEHAVIORS (18).

Wellbore hydraulic calculations require rheological parameters of the fluid. These parameters normally are obtained using viscometric measurements that present shear stress and shear rate at the same known points in the viscometer. Different types of viscometers have been developed to determine rheological properties of fluids, such as (1):

- Rotational viscometer: Couette viscometers (concentric-cylinder rotational viscometers) became the most popular due to their operational simplicity and mechanical reliability.
- Pipe viscometer: Often these show better reliability and accuracy than rotational viscometers. However, pipe viscometers are relatively expensive and not convenient for field applications. As a result, they are commonly used for research purposes.

Furthermore, regarding fluid flow in pipes (19), all fluids lose part of its energy when flowing; this is absorbed by dissipation in friction forces: Internal friction due to its viscosity, and external friction due to pipe roughness. This loss of energy is called the friction pressure drop or loss, and is expressed by the difference in the pressure of the fluid between two points of a horizontal pipe. For example, a circulating drilling mud has an initial energy represented by the pump discharge pressure. This energy is totally lost in the mud circuit because the mud pressure is zero when it

returns to the pits. In this case the pump discharge pressure represents the total pressure losses in the mud circuit.

The pressure losses occur in the surface equipment, inside the drill pipes and drill collars, through the bit, and in the annulus between the wellbore and the drill string. The pressure loss equations are a function of the rheology of the fluid, type of flow (laminar or turbulent), and the pipe and hole geometry.

In this thesis, for calculating the pressure loss in the drill string and in the annulus, a Bingham Plastic fluid was considered. Using units and equations specified in the Drilling Data Handbook. The pressure loss due to friction are calculated with: (units and variables are defined in nomenclature)

TABLE 2.1- PRESSURE LOSS EQUATIONS FOR BINGHAM FLUID (19)

	IN DRILL STRING	IN ANNULUS
LAMINAR	$P = \frac{l * q * \mu_p}{612.95 * D^4} + \frac{\tau_o * L}{13.26 * D}$	$P = \frac{L * Q * \mu_p}{408.63 * (D_o + D_i) * (D_o - D_i)^3} + \frac{\tau_o * L}{13.26 * (D_o - D_i)}$
TURBULENT	$P = \frac{L * d^{0.8} * Q^{1.8} * \mu_p^{0.2}}{901.63 * D^{4.8}}$	$P = \frac{L * d^{0.8} * Q^{1.8} * \mu_p^{0.2}}{706.96 * (D_o + D_i)^{1.8} * (D_o - D_i)^3}$

2.1.3 Drilling window

From the very beginnings of the oil industry, the mechanical behavior of the formation has played an important role. After all, to drill a well, a large volume of rock has to be broken up and removed. Now, after roughly 30 years of development, geo-mechanics plays an accepted role in the oil and gas industry. Many companies now require screening or auditing of all proposed projects to examine the potential for costly drilling or production problems arising from the response of the rock (1).

Drillers must, as far as possible, avoid kicks, wellbore instability, and loss of circulation, usually by selecting an appropriate mud weight. Knowledge of formation pore pressure and fracture gradient is essential for selection of a safe range of mud weights. If the mud pressure falls below the local pore pressure in highly permeable formation, then a kick is taken; if this happens in a soft but essentially impermeable formation, the well may collapse. This consideration provides a lower limit on mud weight in terms of safety, although in many cases drillers will drill underbalanced to increase the ROP. On the other hand, if the mud pressure exceeds the local tensile breakdown pressure for the formation, a fracture is formed. With loss of circulation, the fracture propagates if the mud pressure exceeds the minimum horizontal earth stress (more

accurately, the least principal stress). This provides the upper limit on the mud weight (20). These limits are called the drilling window, as seen in the following Figure:

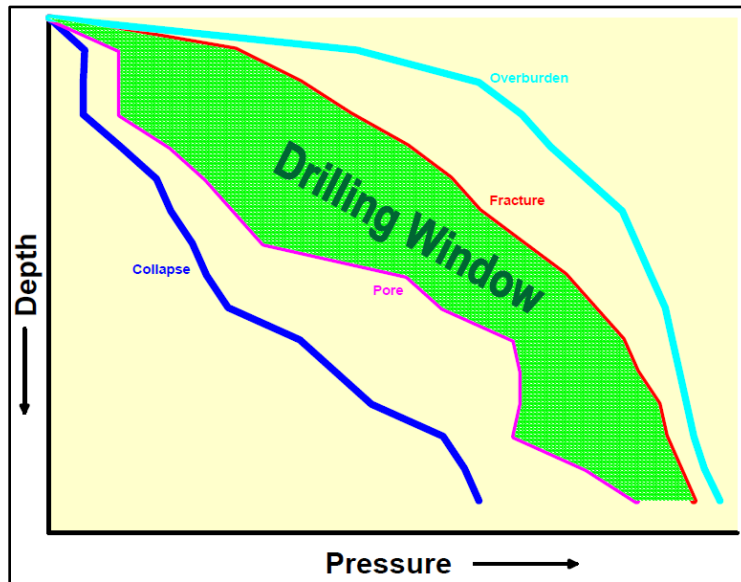


FIGURE 2.4 - DRILLING WINDOW WITH PORE AND FRACTURE GRADIENTS AS LIMITS (6).

In the example illustrated in Figure 2.5, when the drilling window is sufficiently large to avoid kick & losses and to drill deep open holes for each casing size, conventional wells drilled overbalanced and in an open vessel environment are most often used. In this case the annular pressure management is primarily controlled by the mud density and mud pump flow rates (annular friction pressure).

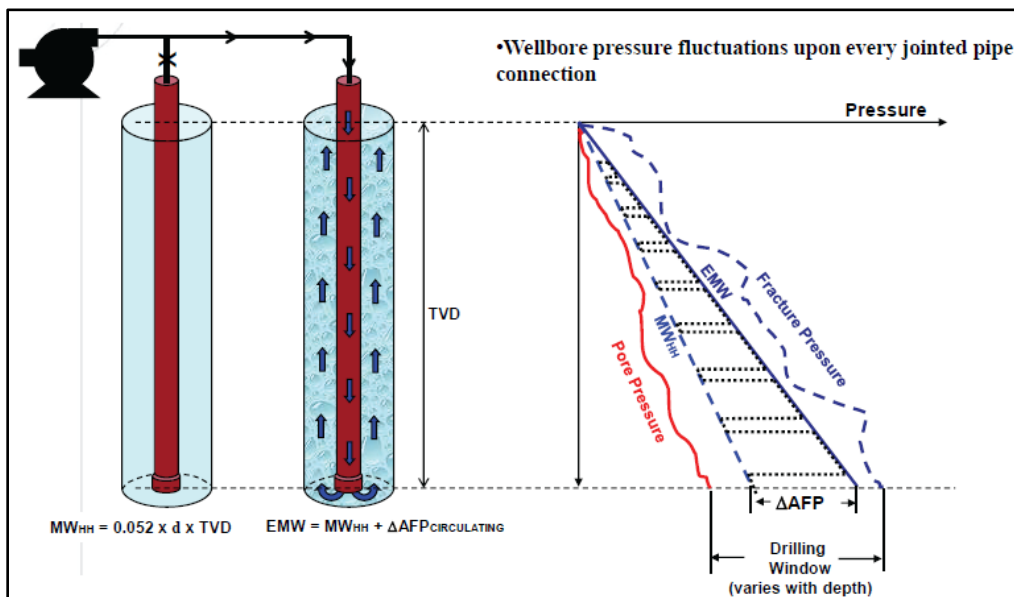


FIGURE 2.5 - HYDRAULICS AND DRILLING WINDOW FOR CONVENTIONAL DRILLING (21)

Many thousands of wells have been drilled safely and effectively with this technology and it has served the industry for over a century. However, fewer prospects exist today that have such wide drilling windows (5). Furthermore, in an open-vessel environment, drilling operations are often subjected to kick-stuck-kick-stuck scenarios that significantly contribute to NPT. Because the vessel is open, increased flow, not pressure, from the wellbore is often an indicator of an imminent well control incident. Often the inner bushings are pulled to check for flow. In that short span of time, a tiny influx has the potential to grow into a large volume kick. Pressures cannot be adequately monitored until the well is shut-in and becomes a closed vessel.

2.1.4 Challenging prospects

Present prospects, with challenging drilling environments, where the wellbore stability pressure and pore pressure may be in close proximity to one another, or even in some wells these might cross each other; a precise control of the annular pressure profile is critical to simultaneous well control and wellbore stability. Being a major cause of loss of time and equipment during drilling, estimates of its total cost to the industry vary but figures of USD 2 to 5 billion per year are widely quoted (1).

Examples of such difficult prospects are:

- HPHT wells: Most challenging to drill safely and efficiently due to the nature of their drilling hazards and elevated consequences if not mitigated with appropriate risk management and the best available technology. Several of the more predominant drilling hazards are kicks as a result of encountering unexpected formation pore pressure, swabbing effects when tripping out that invite an influx of reservoir fluids, and kick-loss scenarios common to drilling in narrow or relatively unknown pressure environments (5).
- Depleted reservoirs: Where the main concern is uncertainty in the reservoir pressures in development areas of the producing intervals. As depletion reduces pore pressure due to production, and the upper boundary of the fluid density (fracture, loss); this implies that the drilling window changes with depletion. Such variation can be seen in the following Figure, where the fracture and pore pressures change with time, indicating the drillable margins in the Kristin Field (7).

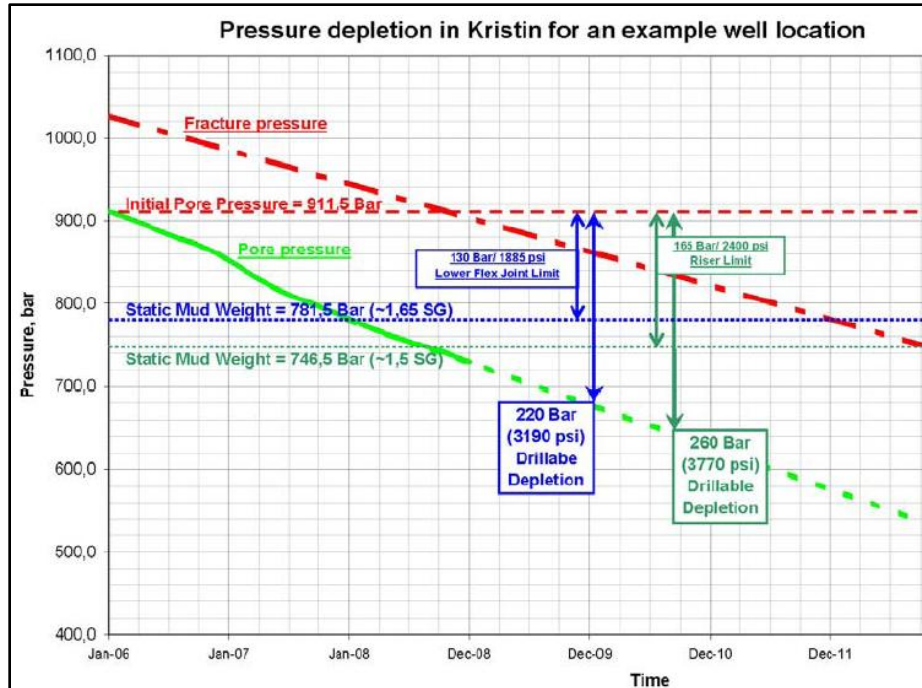


FIGURE 2.6 - PRESSURE DEPLETION IN KRISTIN FIELD

- Over-pressured cap rock due to water injection: One example is the Gullfaks field, where the abnormal drilling window is presented in the following Figure (SPE Bergen – 2005, Gullfaks):

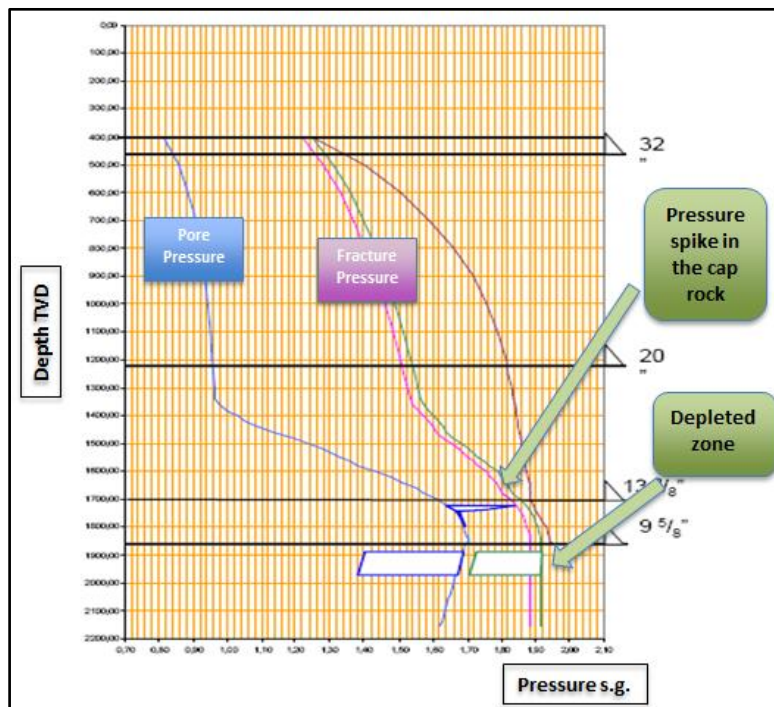


FIGURE 2.7 - PRESSURE PROGNOSIS GULLFAKS FIELD

- Deep water drilling: The substantially increased water depth has impacted the subsurface geo pressure profile and consequently all aspects of exploration drilling. The behavior of the subsurface geo pressure profile is driven by subsea water depth and stresses created by sedimentation rate, lithology, and structural setting. Stress reduction due to subsea water depth is the main reason of narrowing the safe drilling window between the pore and the fracture pressures (22).

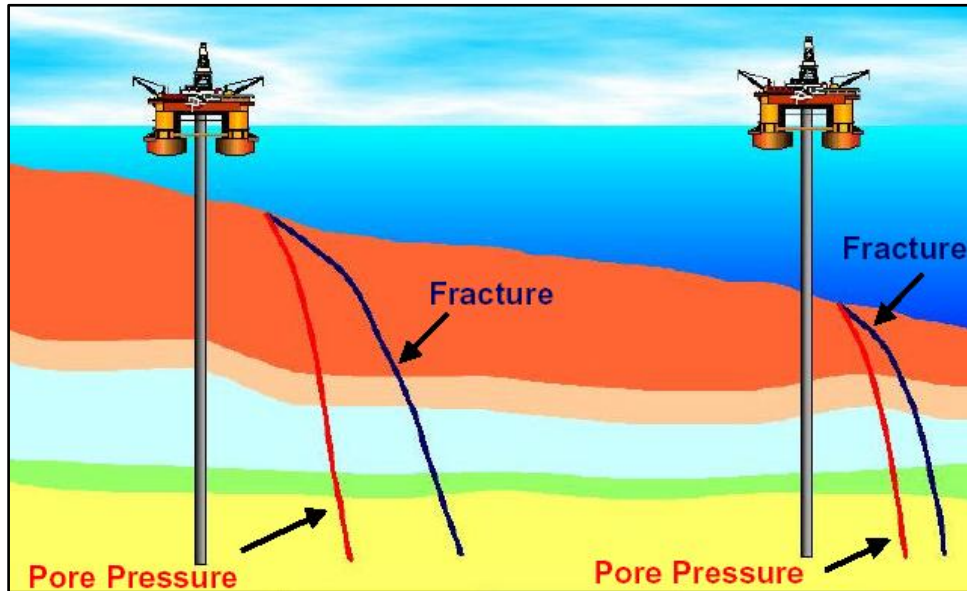


FIGURE 2.8 - DRILLING OPERATIONAL WINDOWS FOR SHALLOW AND DEEP WATER DRILLING

Due to the drilling challenges explained above, new drilling methods had to be used, such methods are referred to as MPO (Managed Pressure Operations, earlier MPO), DMG (Dual Mud Gradient), UBD (Underbalanced Drilling). These allow to drill more efficiently, faster, safer, and also to drill wells that would not be possible to drill with conventional methods. As seen in Figure 2.10. MPO, UBD and partly DMG rely upon some special equipment, where the most important is the RBOP (Rotating BOP), which is a device that allows rotation of the drill string and drilling while the top of the well annulus is sealed; making it possible to control the annulus top pressure (23).

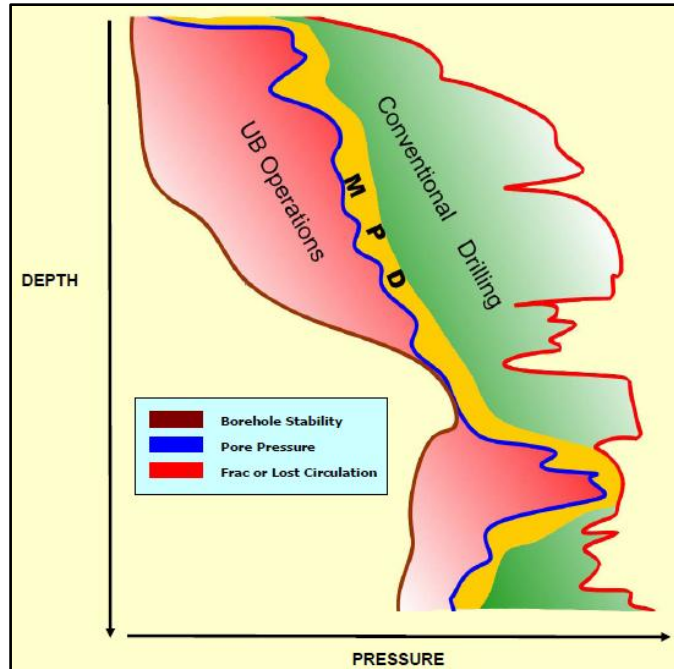


FIGURE 2.9 - DRILLING WINDOWS FOR CONVENTIONAL DRILLING OPERATIONS, MANAGED PRESSURE DRILLING OPERATIONS, AND UNDERBALANCED DRILLING OPERATIONS

As stated in (11), Managed Pressure Drilling (MPD) is relatively new offshore technology that addresses a litany of issues associated with drilling into “trouble zones”; encountering issues such as: excessive mud cost, differential stuck pipe, well control situations associated with loss circulation issues, wellbore instability, kick-loss scenarios when drilling into narrow down hole pressure environments. To address these issues, various techniques can be employed to keep the well pressure constant, especially in drilling mode, connections and tripping.

The detailed explanation of the MPD methods, equipment and operation is beyond the scope of this thesis. However, the intent of all the previous discussion regarding MPD methods related to difficult prospects (especially narrow drilling windows), is to show towards which direction the industry has been moving when encountering such challenging drilling prospects. Moreover, as the present investigation is more focused on swab and surge pressures produced when the drill string is wedged to the heaving drill floor on floating rigs, which can lead to kick or loss scenarios when drilling with small pore and fracture margins; it represents a different step forward to a better management of the wellbore pressure.

2.1.5 Surge and Swab pressures

Surge and swab friction pressures take place due to the displacement of fluid caused by drill string movement (piston effect) in a fluid-filled borehole. When the pipe moves up, pressure is reduced (swab effect); and when pipe moves down hole, pressure will increase (surge effect).

The string movement, in this case, will be the heaving rig, as the drill string is held by the slips during connection, see Figure 2.11 (9).

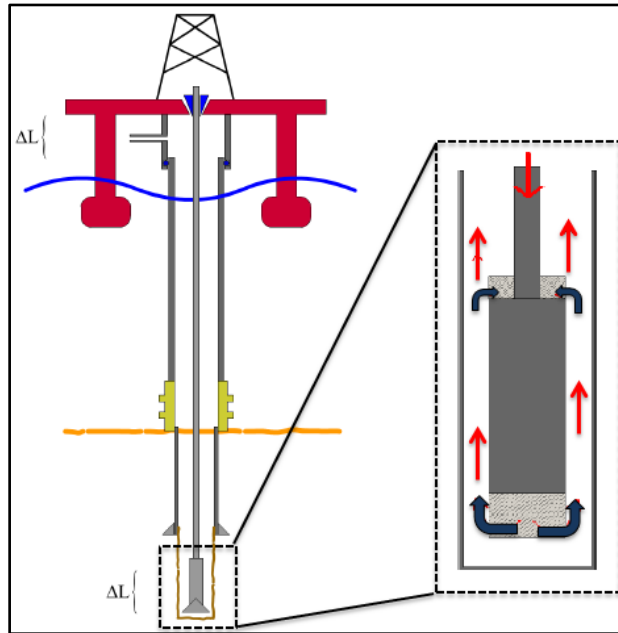


FIGURE 2.10 - DRILL STRING MOVEMENT DURING CONNECTIONS (OLVE SUNDE RASMUSSEN, NTNU, 2008, REF (9))

As no control of surge and swab pressure during drill string connection is possible, due to the MODU's heave motion; the paper referenced in (9) presents magnitudes of surge and swab pressures that can occur in typical drilling operations, but with special focus on TTRD operations, as most of the current methods for MPD from MODUs do not have a functionality to compensate for both surge and swab pressures. For the calculations in this paper, a closed pipe model is used and drill pipe stretch is not accounted for (Figure 2.11). These calculations are regarded as conservative as the wellbore /casing expansion and the drill pipe cross section area reduction due to increased borehole pressure are not taken into account. Table 2.1 shows surge and swab pressures generated by heave motion of the MODU for a range of heave scenarios.

TABLE 2.2 - SURGE AND SWAB PRESSURES DUE TO HEAVE MOTION (9)

Amplitude ± (m)	Period (sec)	Max Velocity ± (m/s)	Surge/swab pressures ± (bar)	
			TTRD	Conventional
0.5	11	0.29	10.18	11.74
1.0	14	0.45	14.00	15.23
1.5	11	0.86	24.32	22.13
3.0	13	1.45	55.63	38.69

As clearly observed in the previous Table, the conventional well case provides only a slight reduction in pressure variation compared to the TTRD case. In a normal drilling operation, a float sub will prevent flow into the drill string from below. However, when the pipe is moved out from the borehole, some of the content of the drill string is likely to flow into the annulus. This gives slightly less swab pressures compared to a closed pipe.

In long, low annular clearance, directional wells, mechanical friction and fluid drag will reduce and slow the motion of the lower end of the drill string, and, as a result, the surge and swab pressures will be smaller than seen in shorter, vertical wells.

Moreover in (7), when using MPD issuing depletion problems in HPHT Kristin wells, a “closed system heave” was defined as the change in volume that occurs as the drill pipe cycles in and out the closed system during the rig heave when the bit is off bottom. These effects were in the order of 5 – 10 bar for a typical Kristin case, therefore not much trouble was encountered because of it, but the problem was present and known.

Furthermore in (12), to actively attenuate the down hole pressure fluctuations, during the connection when the drill string is detached from the draw-works and moving with the heaving rig; a mathematical model of well hydraulics is derived for alleviating the complication of controlling the pressure in MPD on heaving floating rigs. The dynamical model that describes the distribution of flow and pressure in a drilling fluid shows that the friction model based on standard, Newtonian friction factor correlations gives a simple and accurate way of describing the pressure losses during flow, with the viscosity of the fluid as a tuning parameter.

Nowadays, as mostly only difficult prospects are left to be discovered, a precise control of wellbore pressure is required for drilling operations, tripping, connections, and, with more emphasis when narrow windows present, when the drill string is wedged to the heaving drill floor. As stated in (9), in some cases, surge and swab pressures caused by heave motion may be higher than the annular pressure loss experienced during drilling; therefore when large heave motions are present, it may be difficult to achieve a complete compensation. However, if these pressure fluctuations can be calculated, they will represent very valuable information for avoiding kicks and losses scenarios; or, furthermore, to justify the use of a heave-compensated drill floor which is not commercially available to the industry.

Helping in that objective, the thesis contributes to a better pressure management by introducing a numerical method that calculates the pressure fluctuations below the bit, considering the elasticity of string, contact friction against formation and casing, buoyancy, viscous friction and harsh environment in North Sea.

2.2 Ship motion

Since about 1950, the industrial and commercial interest for the behavior of ships and other floating structures in sea has been increased significantly. Nowadays, it becomes more and more usual to judge a ship or other offshore designs on its seakeeping performance. The motion of a ship or offshore structure in waves is important from a safety point of view and economic as well. Because of its drilling capability, the design of a floating offshore drilling structure is generally such that its vertical motion response to waves is small; as it must be designed for a maximum workability in the environment (24).

As the ships motion is a complicated interaction between the air, sea and ship's hull, which is beyond the scope of this thesis; this part of the chapter introduces some of the basic concepts regarding waves, the modeling of them and the characteristics of the North Sea waves. Furthermore, some basic theory of ship statics and motions with emphasis on the vertical motion – known as heave, is presented.

2.2.1 Waves

In its simplest scientific form, a wave is an expression of the movement or progression of energy through a medium. Such waves are often called progressive waves, a category that includes seismic waves, sound waves, light waves and ocean waves. As energy is transmitted through a fluid, the particles in the fluid may move up and down and back and forth in a kind of orbital motion as a sinusoidal line, i.e., as a sine wave. From experience observing waves in nature, it is known that real waves are much more complex (25).

Ocean surface waves are generally distinguished in two states: sea or wind waves, when the waves are being worked on by the wind that raised them; and swell waves, when they have escaped the influence of the generating wind. Sea is usually of shorter period (higher frequency) than swell. Sea is shorter in length, steeper, more rugged and more confused than swell. Since wind-generated waves have their origin in the wind they are changeable, varying both seasonally and regionally. Wind waves, specially, are short crested and very irregular. Even so, they can be seen as a superposition of many simple, regular harmonic wave components, each with its own amplitude, length (or period or frequency) and direction of propagation. Such consideration can be very handy, as it allows one to predict very complex irregular behavior in terms of the much simpler theory of regular waves. This method of “superposition” will be further explained and it is the one used for predicting the movement of the string when wedged in the slips.

Ocean waves, when under the influence of the wind, have a shape closely related to trochoidal waves. This wave shape in mathematically defines as the continuous line formed by the path of a fixed point within a circle as that circle is rolled along a straight line. As the wave moves out from under the wind's influence, the wave height diminishes and the wave shape decays to a

more sinusoidal shape. The differences between the two wave shapes can be seen in Figure 2.12. The trochoidal wave shape is used for some fundamental predictions but, for deterministic studies, is usually replaced by the mathematically simpler sinusoidal shape (Blagoveshchensky, 1962).

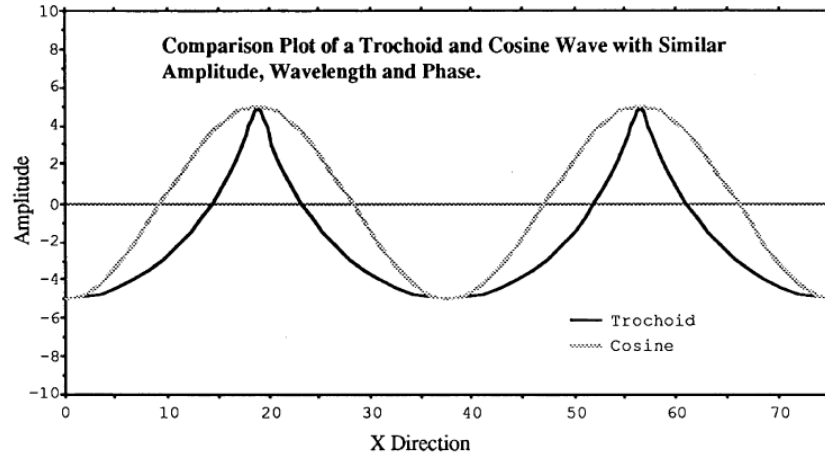


FIGURE 2.11 - TROCHOIDAL VS SINE WAVE (26)

Regular Waves. Figure 2.13 shows a harmonic wave ζ , from two different perspectives. “a” shows what one would observe in a snapshot photo made looking at the side of a (transparent) wave flume; the wave profile (with wave amplitude ζ_a and wave length λ) is shown as a function of distance “x” along the flume at a fixed instant in time:

$$\zeta = \zeta_a * \cos \left(2\pi * \frac{x}{\lambda} \right) \quad (2.2)$$

Figure 2.13 – b, is a time record of the wave profile (with wave amplitude ζ_a and wave frequency ω) observed at one location along the flume; it looks similar in many ways to the first figure, but the angle $2\pi x / \lambda$ has been replaced by ωt :

$$\zeta = \zeta_a * \cos (\omega * t) \quad (2.3)$$

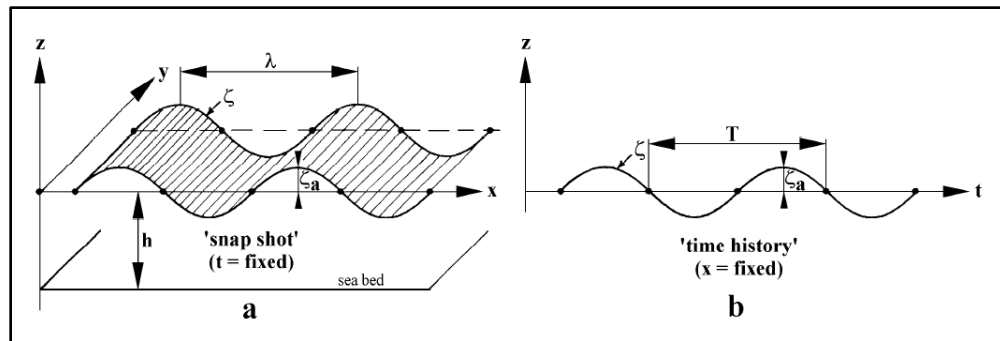


FIGURE 2.12 - HARMONIC WAVE PARAMETERS (24)

Other parameters that help define the waves are (previous Figure):

- Still Water Level: is the average water level or the level of the water if no waves were present. This level is the origin of the coordinate system, with positive z-axis upward and x-axis positive in the direction of the wave propagation.
- Water depth (h): is a positive value and is measured between the sea bed and the still water level.
- Crest: the highest part of the wave above the still water level.
- Trough: the lowest part of the wave below the still water level.
- Amplitude (ζ_a): If the wave is described by a sine wave, then its amplitude is the distance from the still water level to the crest, or to the trough.
- Wave height: is two times the wave amplitude and is measured vertically from the wave trough level to the wave crest level.
- Wave length (λ): is the horizontal distance (measured in the direction of wave propagation) between any two successive wave crests or troughs.
- Wave period (T): is the time it takes for two successive waves to pass a particular point. It can be said that the period is the same distance as the wave length but along the time axis.

Since the distance between any two corresponding points in successive sine waves is the same, wave lengths and periods are usually measured between two consecutive upward (or downward) crossings of the still water level. Such points are also called zero-crossings, and are easier to detect in a wave record.

- Wave frequency (f): is the number of waves that pass a particular point in a given time period ($f = 1/T$)
- Significant wave height: is the average wave height of the highest 1/3 of the waves present and is good indicator of potential for wave damage.

Since sine or cosine waves are expressed in terms of angular arguments, the wave length and period are converted to angles using:

$$k * \lambda = 2\pi \text{ or } k = \frac{2\pi}{\lambda} \quad (2.4)$$

$$\omega * t = 2\pi \text{ or } \omega = \frac{2\pi}{T} \quad (2.5)$$

Where, k is the wave number rad/m and ω is the circular wave frequency rad/s. Obviously the wave form moves one wave length during one period so that its propagation speed or phase velocity, c , is given by:

$$c = \frac{\lambda}{T} = \frac{\omega}{k} \quad (2.6)$$

Fortunately, the water particles themselves do not move with this speed, only the wave form (wave crests or troughs) moves with this phase velocity.

Irregular Waves. As explained before, ocean surface waves can be classified into two basic categories (24):

- Sea: is a train of waves driven by the prevailing local wind field. The waves are short-crested with the lengths of the crests only a few (2-3) times the apparent wave length. Also, these are very irregular, high waves are unpredictably followed by low waves and vice versa. Individual wave crests seem to propagate in different directions with tens of degrees deviation from the mean direction. The crests are fairly sharp and sometimes even small waves can be observed on these crests or there are dents in the larger wave crests or troughs (Figure 2.14). The apparent or virtual wave period, T' , varies continuously, as well as the virtual or apparent wave length, λ' .
- Swell: are waves which have propagated out of the area and local wind in which they were generated. They are no longer dependent upon the wind and can even propagate for hundreds of kilometers. Individual waves are more regular and the crests are more rounded than those of a sea. The lengths of the crests are longer, now several (6-7) times the virtual wave length (Figure 2.14). The wave height is more predictable too.

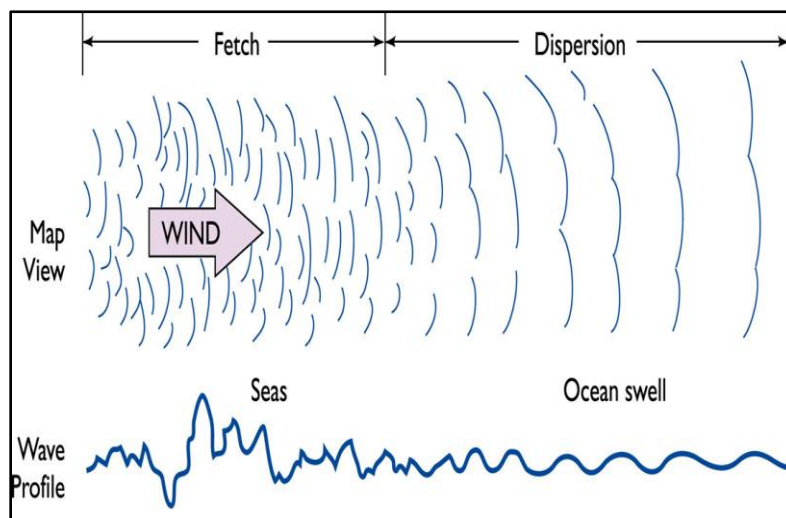


FIGURE 2.13 - IRREGULAR WAVES

In the general case, the total irregular waves are a superposition of the sea and the swell at that location; they can be added as is shown in the following section.

Superposition. Wind waves, especially, are very irregular. If a cross section through a real sea would be taken, it shall look like Figure 2.15-b. These can be seen as a superposition of many simple, regular harmonic wave components, each with its own amplitude, length, period or frequency and direction of propagation. Therefore, the real seas can be thought to be the sum of many individual regular waves as seen in Figure 2.15-a.

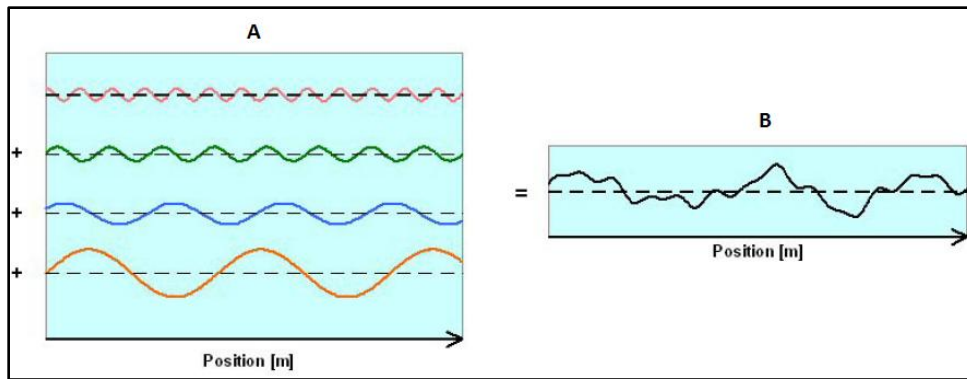


FIGURE 2.14 - A: SEVERAL SINUSOIDAL WAVE COMPONENTS WITH DIFFERENT PARAMETERS. B: RESULT OF SUM OF COMPONENTS

This consideration allows predicting very complex irregular behavior in terms of much simpler theory of regular waves. This so-called superposition principle was first introduced in hydrodynamics by St. Denis and Pierson, 1953; and will be used for modeling the movement of the top of the drill string when it is held by the slips and follows the rig heave movement. Note that due to the lack of information the rig heave movement is assumed to follow the wave amplitude, and the wave components must have whole numbers relations between their frequencies.

2.2.2 North Sea environment

The severity and variability of North Sea weather are well known to those engaged in North Sea Oil Exploration and Production. The environment, in one way or another, influences virtually every aspect of offshore development, be it at the planning, design or field operational stage. There is a definite need for the offshore industry to increase its knowledge and understanding of the North Sea environment, and to quantify the range and kind of conditions likely to be encountered (27).

As the first semi-submersibles began operating in North Sea in mid-60's (28), it was generally assumed that the area was fairly sheltered and that conditions did not vary significantly North to South. Experience in the field soon showed that these assumptions were wrong, and that the severity and variability of the weather presented formidable challenges to those engaged in offshore development. To meet these challenges, detailed measured information on the North Sea environment was needed, which was not available. Even though, visual estimates of wind speeds and wave heights were logged by most ships and trawlers working in the area over a considerable number of years, they were inconsistent and inaccurate when compared to actual measurements. Therefore, reliable measurements were needed.

Figure 2.16 shows the average wave conditions occurring throughout the period 1952-72. Wave heights given include a value for swell based on the seasonal average; it must be noted that a

long swell of period 10 to 12 seconds has a greater effect on the rig's movements than does a larger wave of lower period. These long swells are generally less than 14 ft.

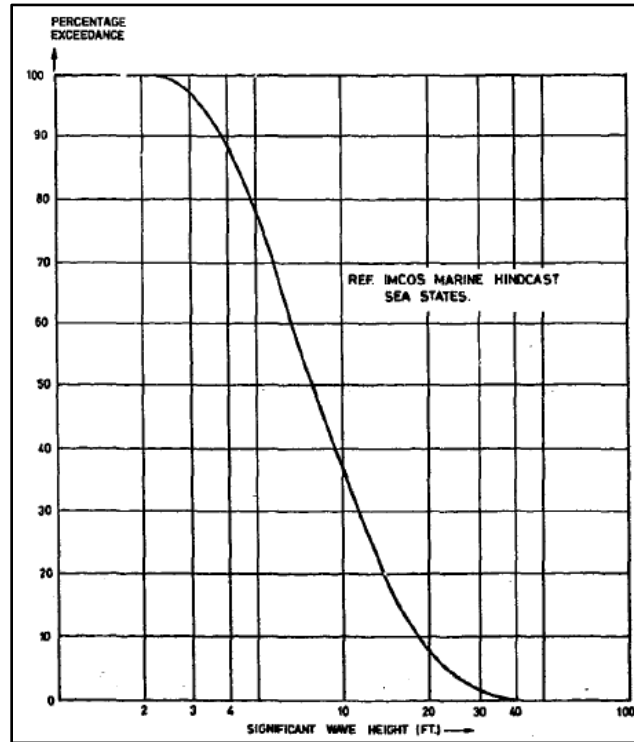


FIGURE 2.15 - PERCENTAGE-EXCEEDANCE CURVE OF AVERAGE SIGNIFICANT WAVE HEIGHT FOR THE NORTHERNS NORTH SEA (LAT. 61°) (4)

Later, in 1968, a group of six oil companies and two associated British Government bodies formed the North Sea Environmental Study Group (NSES) who had a measuring program of three years where data was collected from a number of fixed platforms in the Southern North Sea, and from semisubmersible rigs in the Northern North Sea (29). When activities were completed, knowledge of environment certainly improved, but it was neither extensive nor reliable enough to generate long term design statistics by themselves. Then oil companies continued to measure the environment from their own installations and exploration rigs, but further important oil discoveries and plans to go into full-scale production demanded a more comprehensive and reliable knowledge of the North Sea.

This need led to the formation in 1972 of the North Sea Oceanographic Study Group (NSOSG), made of fifteen oil companies operating in the North Sea and the British Government. This reliable information helped construct desired statistics to obtain design information, which will aid not only in the short term (operational decisions) but also in long term (design, construction) (27).

In (30), 1979, it is stated that as semisubmersibles spend all their life in conditions of marine operations, so their entire behavior is connected to their motion under the action of environmental forces (wave forces). Therefore, both wave height and period were considered as very important environmental parameters. For the wave height, data corresponds to sea conditions around 60°N in North Sea, no error data gathering was assumed and a Weibull distribution of significant heights was established, then using Monte-Carlo techniques a distribution of design (100 years) wave height was obtained. As for the wave period, a scatter diagram of significant amplitude and period was recorded in the North Sea Area. Both results, for wave height and period are shown in the following Table.

TABLE 2.3 - WAVE PARAMETERS

Exceedence Probability	Wave Height m	Wave Period s
0,1	26,6	12
Most probable	29,4	14
0,9	32,2	16

For the design and certification of offshore structures, it is essential to have probability estimates of the most extreme wave conditions expected during the lifetime of the structure (20 years typically). However as a minimum, it is required 10 years of data to extrapolate to 25 years or more (31). As data, more of the times, is intermittent and cover few years; a very useful method, in order to construct wave statistics is based on a wave hindcast approach using a data base of historical wind fields. These models are beyond the scope of this thesis; therefore, the previous discussion was established in order to explain how North Sea environment information had been gathered, analyzed and used for designing structures or planning operations; moreover, data regarding wave height and periods were shown.

However, the most recently information found regarding the Norwegian Sea wind and waves (period and wave heights) conditions was found through the Norwegian Petroleum Directorate (NPD), in the *Scientific Basis for a Management Plan for the North Sea and Skagerrak: Area Report* (Faglig grunnlag for en forvaltningsplan for Nordsjøen og Skagerrak: Arealrapport), made by the Institute of Marine Research (Havforsknings Instituttet, 2010), reference (32).

This report states that in winter there is a large low-pressure activity over the North Sea and thus a rough wave climate in general. Area 58-62°, North Sea, shows a wave climate of great variations throughout the year. The largest waves occur most frequently in fall season. In spring and summer season wave climate is calmer. The dominant wind direction is from south and southwest in winter, with increasing proportion of northerly winds in summer. Area 56-58°, North Sea, the dominant wind direction during the year are from southwest, with a stronger element of western and northerly winds in summer. An important map of the North Sea is presented, where the maximum wave height and period for 50 years are shown (Figure 2.17).

This information will give the appropriate inputs for calculating the swab and surge pressures when the drill string is moving due to the heave of the rig. The pressures will be calculated for different weather conditions which are the roughest in North Sea, and their effects on the pressure fluctuations will be established.

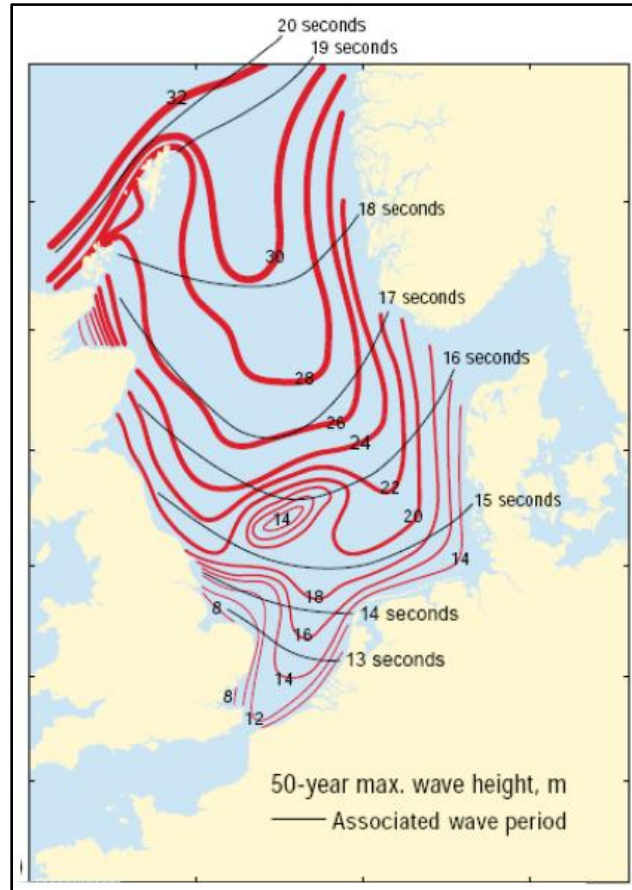


FIGURE 2.16 - 50 YEARS WAVE HEIGHT AND PERIOD (32)

2.2.3 Ship movement

A rig can be considered a rigid body floating upon still or disturbed water. The motions are described using a three dimensional reference frame where six quantities can be measured: rotations about each of the axes and translations in the direction of each of the axes. These motions are coupled and therefore not easily interpreted when looking for insight into the behavior of the ship. The common terminology and definitions are presented below and are with respect to the equilibrium position (Blagoveshchensky, 1962).

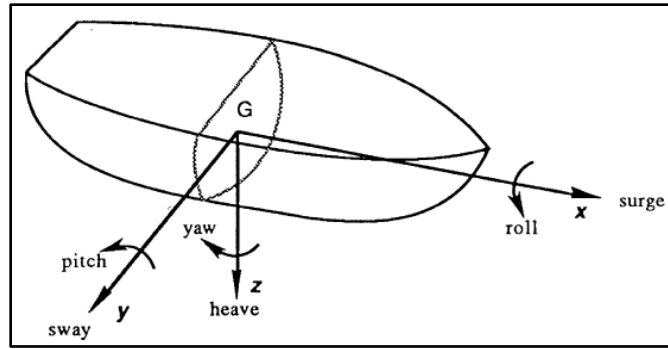


FIGURE 2.17 - SHIP MOTIONS (26)

- Roll – Rotation about the longitudinal axis. Positive if the starboard (right) side is low.
- Pitch – Rotation about the transverse axis. Positive if the bow is raised.
- Yaw – Rotation about the vertical center axis. Positive if the bow is to starboard.
- Heave – Translation in the vertical direction. Positive when the ship moves downward.
- Surge – Translation in the longitudinal direction. Positive in the forward direction.
- Sway – Translation in the transverse direction. Positive in the starboard direction.

Roll and pitch limit the ability to handle pipe on the rig floor and hence, may be considered to hamper well control operations. However, heave is of more concern in well control operations since this is transferred to the drill string; and is the movement that more focus will be placed on in this thesis.

2.2.4 Heave

The vertical heave motion a vessel imparts to the drill string is influenced by the frequency an ocean wave encounters the vessel. Then, the amplitude of heave which is transferred to the drill string is a function of the wave height and the natural heave period of the vessel. It may be graphically determined from a plot from the heave response operator, which describes the percentage of heave to wave height versus the period of the wave. The curve indicates that for a short period wave, one less than the distance between the vessel's outermost columns, no appreciable heave effect is noted. As the period of the wave approaches the natural heave period of the vessel the heave increases rapidly. In between these two points the magnitude of the heave response operator is of interest to a drilling supervisor, as this defines the amplitude of the motion imparted to the drill string (33).

At some point drilling must be suspended due to heave. Each operator has to establish his own limits of heave depending on the capability of the equipment and environment (length, height and period of wave) where the operations are carried on.

To absorb the heave motion from a floating unit various compensators have been manufactured. Initially bumper subs, or concentric sliding sleeves, generally with a five foot stroke were made part of the BHA. But as offshore exploration moved into deeper and rougher waters, failures achieved new dimensions. Therefore, in late 60's, Vetco Offshore Inc, introduced the concept of hydraulic and pneumatic heave compensator, which was proved viable and became a standard equipment aboard offshore drilling vessels. Its objective is to hold a load suspended from the vessel while simultaneously reducing the oscillating movement of that load due to vessel motion (34).

As minimum down time is of prime importance, the heave compensators aided in several issues, such as faster operations, improved control over down hole equipment position, enhanced load and motion control, and weight and position control. This has given faster drilling, less damage to down hole connectors, easier setting of seals and hangers, and made all operation easier and safer allowing operations in rougher weather conditions, savings in bit life, weather down time, shocks on BHA minimized, less hook load variations, and less BOP stack oscillation during installation, among others.

However, as prospects are harder to drill, as explained in previous sections, small variations in the bottom hole pressure can lead to unwanted kick-loss scenarios, which is considered as down time because drilling is suspended. For example, just the annular friction pressure that is lost whenever the mud pumps are off due to connections is enough to allow unwanted influxes getting into the well. Therefore the vessel's heave that is transferred to the drill string when it is wedged to drill floor may be enough to fracture the formation or create kick scenarios, especially in narrow drilling window prospects. This issue is the one addressed in this thesis.

2.3 UiS Numerical Model to calculate swab and surge pressures due to heave movement

The UiS Numerical Model was first introduced by the Prof. Erik Skaugen in the subject "Advanced Drilling Technology" at the University of Stavanger. Therefore, all the equations, derivations and theory described in this section are from the compendiums, summaries and lectures elaborated by him, reference (10).

2.3.1 Introduction

The numerical method presented is for calculating the behavior of strings (rod, tubes or other similar structures, and possibly with different diameter) when exposed to changing forces. These forces may be due to connections to other objects or strings, to gravity and/or to friction. Changing of forces might be due to breaking of connections, collisions, braking due to friction,

etc. This will set up stress waves in the string, sometimes so strong that the material might yield or break. Even though these equations do not include yielding as they assume perfect linearity in the material, they will indicate the yielding the first time it happens.

The numerical method is restricted to only one-dimensional waves, travelling along the string axis. These waves are pure stress waves, giving the local stress along the string. In reality, if the load change is along the string as assumed here, some waves across the string might be induced but will be neglected as they are considerably weaker than the waves along the string. Shear waves are also neglected, even those with shear forces along the string, as they require at least two-dimensional description.

The theoretical background is the one-dimensional, second order wave equation which describes the actual motion of the material in an infinitely long, straight string of constant cross section, when the string material is not stressed beyond its linear elastic limit (yield limit). The derivation of this equation is presented in the Appendix 8.1, as well as its general solution.

For the numerical calculation the whole string is divided along its axis into finite elements or numerical segments (purely fictive, for numerical calculations only), each with length Δz (space step length), and the displacements of these segments from their original positions are calculated at certain points in time. The time interval between these points in time is constant and equal to the time step Δt . The numerical calculation will thus give no information of what is happening between these points in time. The basic unit for calculating the displacements is Δx , which can be freely chosen to give integer values for easy handling.

The numerical calculations give as a result the actual movement of the numerical segment “mid points”. This movement is the actual movement of the string material at the segment midpoint, as the string is divided in numerical segments and each segment has its own midpoint, the calculations give the observed movements of these midpoints.

2.3.2 Division of string into numerical segments

The whole string must be divided into a number of whole segments, except at the ends, where half segments can be used. Their length Δz can be freely chosen, subjected only to the restriction that the string must be divided into n or $n+0.5$ segments, n being a whole number. As a whole segment has a length of Δz , a half segment has a length of $\Delta z/2$, and is counted as 0.5 segments. Examples of dividing the string can be seen in the Figure 2.19, where N is the number of segments. Between the end segments there must always be whole segments, so a segment is defined as being inside the string if its midpoint is a distance Δz or more away from the ends of the string.

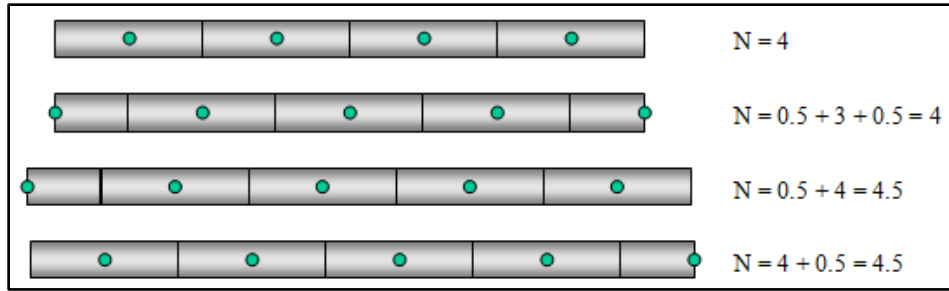


FIGURE 2.18 – EXAMPLE OF DIVIDING STRINGS CORRECTLY INTO NUMERICAL SEMENTS. FOR EACH STRING THE NUMBER N OF SEGMENTS IS SHOWN. THE MIDPOINTS ARE SHOWN AS SMALL CIRCLES.

For strings of changing diameter, segments of corresponding diameters must be used. The requirement for a segment diameter is that each half of a whole segment must have a constant diameter. Therefore, a whole segment is allowed to have two different diameters, as a half segment can have only one diameter. Possible ways of representing a change of diameter are shown in the following Figure, where in the lower case one segment with two different diameters has been used. Note that in both cases the number of segments is 4, but the number of midpoints is 4 and 5.

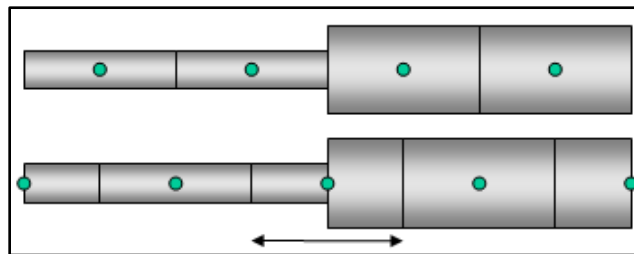


FIGURE 2.19 - TWO DIFFERENT WAYS OF DIVIDING A STRING WITH CHANGING DIAMETERS.

According to previous rules, only strings with abrupt changes of diameter can be modeled exactly. As real strings may have gradual changes of diameter, these can only be approximated by introducing several shorter numerical segments with gradually increasing constant diameters.

Finally, the segment mid points should be numbered in the positive direction, from 1 and upward, from left to right in a horizontal string. If gravity is included, the positive direction is downward along the string, and then the gravity force acting along the string is positive as well.

2.3.3 Displacements

The displacement of a numerical segment is given by the displacement of its midpoint. This displacement is the actual physical movement of the segment midpoint away from its original position, where the string is assumed to be completely relaxed (no tension, no stress in the

string). There will be no information of displacement values of the string between these midpoints in space. The notation and calculation procedure are as follows:

- The displacement at present time t of segment midpoint number j is X_j .
- The displacement at one time step Δt before present time (at time $t-\Delta t$) of segment midpoint number j is XG_j (the old displacement – “gammel” in Norwegian).
- When these displacements are known for all segment midpoints the new displacements XN_j at one step into the future (at time $t+\Delta t$) can be calculated from the numerical equations which will be explained in the following sections.
- After all new displacements are calculated, the time $t+\Delta t$ is taken as the present time t , and all XN_j becomes X_j , and all former X_j becomes XG_j . This shift of values has to be done for each time step.
- In order to start the calculation, one complete set of X_j and XG_j must be known, for all values of j . This is the initial condition or the boundary condition in time. These initial displacements must be found from the actual condition of the string.

The calculation is best done in a table format, as shown Table 2.4. It is recommended to include the segment number at the top of the table. The actual time at the start of the calculation can be freely chosen, but it is usually equal to zero. The positive direction is from left to right. For this example, the initial conditions chosen are those of a string with constant cross section, grabbed at both ends and stretched. It is assumed that the left end is staying in place, while the right end is pulled in the positive direction due to the stretch. The string material will then move to the right all along the string (except at the left end), and increasingly more towards the right end. This is shown by the increasing displacements in the positive direction, in Figure 2.21.

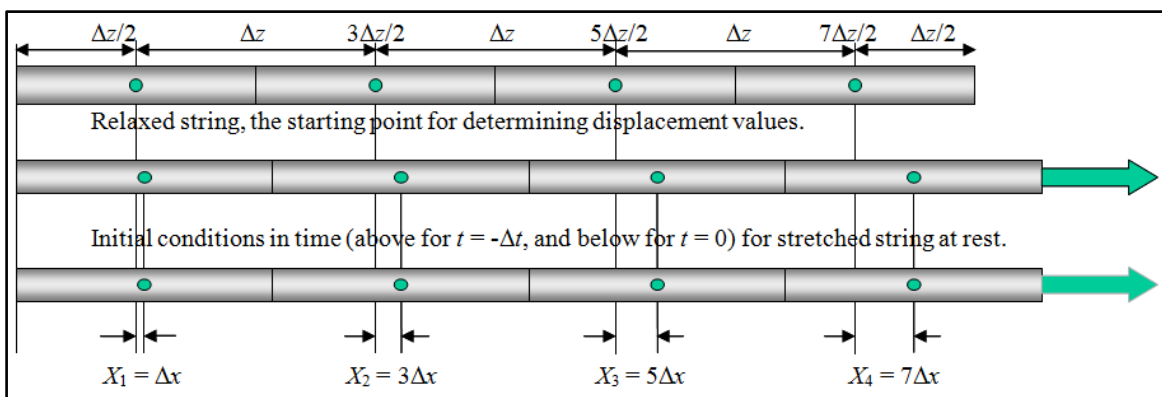


FIGURE 2.20 - CONSTANT DIAMETER STRING IS STRETCHED RESULTING IN DISPLACEMENTS OF ITS 4 SEGMENT MIDPOINTS (FROM LEFT TO RIGHT) 1, 3, 5 AND 7, MEASURED IN UNITS OF Δx .

TABLE 2.4 - CALCULATION TABLE FOR EXAMPLE FROM FIGURE 2.21

Distance form left end:	$\Delta z/2$	$3*\Delta z/2$	$5*\Delta z/2$	$7*\Delta z/2$	TIME	COMMENTS
Segment Number:	1	2	3	4		
Initial displacement (XG) at $t=-\Delta t$:	1	3	5	7	$t=-\Delta t$	Initial conditions
Initial displacement (X) at $t=0$:	1	3	5	7	$t=0$	in time

As seen above, the reason to choose the present time as $t=0$ and $t=-\Delta t$ for one step time before, is that now the numerical calculations can begin for the future time (XN). Besides starting at $t=0$ is convenient, as usually something happens at this time, when calculations start; for instance the stretched string in Figure 2.21 can be released from its right end.

For convenience of calculations, it is often chosen a special unit for measuring the displacements (Δx), which can be freely chosen, but is better to choose one so that all displacements are integers. As seen in Figure 2.21, the actual positions of the segment midpoints, in the relaxed string, are measured as the distance from the left end of the string in terms of the space step unit Δz . Then the actual positions of the segment midpoints in the stretched string are found by adding the displacement to the position in the relaxed string.

It is very important to be certain that at each time step the displacements show the actual positions of the string material at the midpoints in the relaxed string. This displacement is the distance the string material has moved away from its position in the relaxed string. Note that “relaxed string” means the string in the original position used for position reference. The string might be relaxed in a later time or different position, as long as all its segment midpoints are displaced an equal amount, then the distances between the midpoints have not changed from their values in the original reference position.

As seen in Table 2.4, the displacements do not include the units of the parameters used; then if a displacement has a value of 7, this represents 7 times the length unit, which is Δx . Therefore, it is useful to determine Δx from forces acting upon the string (i.e. gravity), or from the initial speed of the string or part of it.

2.3.4 Boundary conditions in space

When the calculations start, the present time is increased by one time step Δt so when all the future displacements (XN) are calculated, these become the present values as described earlier. The initial conditions in time are left behind, as after two time steps of calculations they will have no influence on further calculations. But there are other conditions which will keep influencing the calculations; these are called the boundary conditions in space because they are found at certain positions along the string. In general all deviations from a continuous string of constant cross section, freely moving, can be considered as boundary conditions in space. The most common are:

- String terminated with a free end: This end can move freely, and no forces are acting on it; for example, a string that ends in air with nothing connected to this end.
- String terminated with fixed end: This end has a fixed position, and no movement of this end is possible; for example, a string welded to a big, heavy block of steel.
- Any part of the string fixed to something heavy or to the ground, preventing the string to move at this point: This is effectively dividing the string, making two independent strings, each with a fixed end at this point.
- Any part of the string (including ends) fixed to a piece of equipment forcing the string to move in a predetermined manner at this point.
- Change of cross section of the string.
- A piece of equipment clamped to the string and moving freely with the string.
- Etc.

General forces, like gravity and friction along the string, are not usually called boundary conditions, as they affect every part of the string.

For other boundary conditions, like different endings, diameter changes, masses clamped to the string, etc., they are included in the numerical calculations by special numerical equations for those midpoints affected. These equations will be explained in the following sections. Is very important to notice that any segment where its midpoint is not closer than Δz to the point where special conditions are present, will not be directly affected by those conditions; therefore its displacement will be calculated by the standard numerical equation.

On the other hand, as there are two ways of ending a string in terms of segments, either ending with a whole segment or with a half segment; two different equations are needed as they represent different space boundary conditions. In the following section, this and some other useful boundary conditions are presented with their equations and derivations.

2.3.5 Numerical equations

The numerical equations are used to calculate the new displacements in the future time (XN) of a segment midpoint. They can vary according to the different space conditions, but the equations do not change in time. The most important equations are explained below.

STANDARD EQUATION

This equation is used to calculate the new displacement (XN) for a segment midpoint, when this is **inside the string with constant cross section**. The actual cross section of the string does not enter in the equation in this case. For the segment number j the equation is:

$$XN_j = X_{j-1} + X_{j+1} - XG_j \quad (2.9)$$

This equation applies to all the segments of constant cross section where at least half of the neighboring segments, closest to the segment in question, have the same cross sections. The following Figure shows examples of this case.

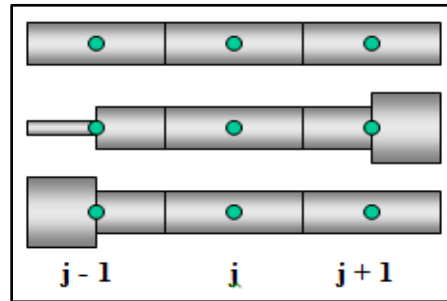


FIGURE 2.21 - EXAMPLE OF SEGMENTS WHERE THE STANDARD EQUATION CAN BE APPLIED TO SEGMENT J (AND ONLY TO SEGMENT J IN THE CASES SHOWN)

The proof of the above equation can be found in Appendix 8.2, using the one-dimensional, second order wave equation. Moreover, its derivation can be done using the physical ball-spring model explained in Appendix 8.3. This model gives the displacements as lengths. The standard length unit may not be so convenient for the calculation; therefore, eq. 2.10 is divided by the unit length chosen (Δx):

$$\frac{XN_j}{\Delta x} = \frac{X_{j-1}}{\Delta x} + \frac{X_{j+1}}{\Delta x} - \frac{XG_j}{\Delta x} \quad (2.10)$$

Then, since Δx also has a length unit, the units are divided away. This is actually the equation used in numerical calculations; therefore, for any result found, for instance, $XN_j=9$, the real physical length is $9*\Delta x$. As it is a bit inconvenient to use the fraction $(X/\Delta x)$ all the time, the numerical equations use XN , instead. But, it is very important to remember that this is a relative displacement and actually means $XN_j=9\Delta x$, because the numerical equations show the displacement as pure numbers.

The general rule here is that during one time step (Δt), the midpoint displacement of any segment cannot be influenced in any way by the situation beyond the midpoints of the neighboring segments. This is because the information travels with the speed of sound in materials, and during one time step any information about cross section, forces acting, and so on, cannot travel further than $c*\Delta t=\Delta z$, which is the distance from one midpoint to another.

END BOUNDARY NUMERICAL EQUATIONS

As explained before, the end of a string can be free or fixed. First, the free end is discussed here. When the **end is free**, not fixed to anything, the number of the segment closest to the string end is usually given as either 1 or n . Besides, the string can end with a whole segment or a half segment, giving different equations. The equations for these cases correspond to the Figure 2.23.

- A) String beginning with whole segment: $XN_1 = X_2 + X_1 - XG_1$ (2.11)
- B) String ending with whole segment: $XN_n = X_{n-1} + X_n - XG_n$ (2.12)
- C) String beginning with half segment: $XN_1 = 2X_2 - XG_1$ (2.13)
- D) String ending with half segment: $XN_n = 2X_{n-1} - XG_n$ (2.14)

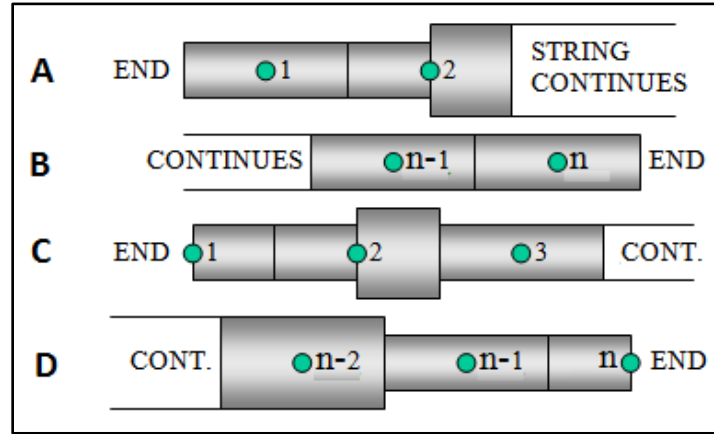


FIGURE 2.22 - POSSIBLE FREE ENDS, AT THE BEGINING AND END OF THE STRING, AND WITH WHOLE AND HALF SEGMENTS

Note that the free end equations shown, here assumes that the free end segment midpoints will not be affected by any deviation (i.e. change in cross section) unless they are closer than Δz to the segment midpoint in question. Observe that in Figure 2.23, segment midpoints 2 in A, 2 in C and $n-2$, $n-1$ in D, will have equations different from the standard equations, as the change in cross section affect them.

When the **end is fixed**, it is kept completely at rest with a constant and fixed displacement (to the left and to the right, whether j is 1 or n , respectively). Similarly, the number of the segment closest to the string end should be either 1 or n ; and the string can end with a whole or half segment as well. The equations for these cases correspond to the Figure 2.24.

- A) String beginning with whole segment: $XN_1 = X_2 - X_1 - XG_1 + 2X_{LE}$ (2.15)
- B) String ending with whole segment: $XN_n = X_{n-1} - X_n - XG_n + 2X_{RE}$ (2.16)
- C) String beginning with half segment: $XN_1 = X_{LE}$ (2.17)
- D) String ending with half segment: $XN_n = X_{RE}$ (2.18)

Where X_{LE} is the constant, fixed displacement of the left end, and X_{RE} is the constant, fixed displacement of the right end. These constants values can have any value, including zero; that is when, for instance, the string end is fixed when the string is in its relaxed reference position where all displacements are zero.

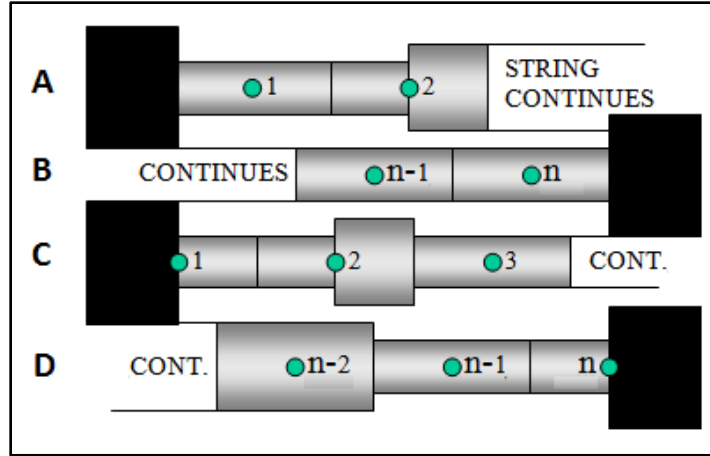


FIGURE 2.23 - POSSIBLE FIXED ENDS, AT THE BEGINNING AND AT THE END OF THE STRING; AND WITH WHOLE OR HALF SEGMENTS

CHANGE OF CROSS SECTION

Here is considered a simple cross section, which means that there are not several changes in close succession, the next change must be at least a distance Δz away. Any change of cross section is allowed at a segment border, or at a segment midpoint. If the cross section changes from A_1 to A_2 , in the positive direction, the numerical equations are presented below, which correspond to the Figure 2.25.

- A) Change of cross section at the segment midpoint: Only the segment midpoint j is affected by the change in cross section. As for the neighboring midpoints, the change is Δz away from them; therefore they are not affected and will be calculated by the standard equation.

$$XN_j = \frac{2A_1}{A_1+A_2} X_{j-1} + \frac{2A_2}{A_1+A_2} X_{j+1} - XG_j \quad (2.19)$$

- B) Change of cross section at the segment border: as the change is in the border between j and $j+1$, both segments midpoints are at a distance $\Delta z/2$ from the change, therefore both are affected by the change:

$$XN_j = X_{j-1} - \frac{A_2-A_1}{A_1+A_2} X_j + \frac{2A_2}{A_1+A_2} X_{j+1} - XG_j \quad (2.20)$$

$$XN_{j+1} = \frac{2A_1}{A_1+A_2} X_j + \frac{A_2-A_1}{A_1+A_2} X_{j+1} + X_{j+2} - XG_{j+1} \quad (2.21)$$

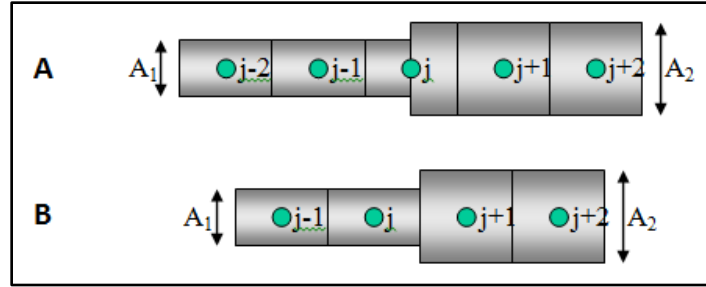


FIGURE 2.24 - A SINGLE CHANGE OF THE STRING MATERIAL CROSS SECTION AT A SEGMENT MIDPOINT AND AT THE SEGMENT BORDER (SEGMENT “J” ONLY)

Note that there is no need to know the actual cross sections, only their relative size, for instance, that A_2 is twice A_1 . Moreover, the cross sections can have any values, including zero and infinity. If A_1 is zero it means that there is no string to the left and then the equation turns into the equation of a free end.

The equations presented above are sufficient for the drill string calculation within this thesis; however, there are other possibilities for cross sections changes which are shown in (35).

FORCED POINT

A forced point can be defined as an external force that affects the string. These external forces will be further explained in section 2.3.6. For now we are interested in the movement when the drill string is attached to the drill floor (forced point) and transfers the heave movement directly to the string. That means that a forced point moves any part of the string in a predetermined way, and being sufficiently powerful to do this without being influenced by the strings own movement or oscillations. Any movement of a forced point will, however, influence the parts of the string connected to it.

The displacement for this kind of point is given by an equation that is independent of the string movement, but is a function of time. An example is:

$$X_p = 5 * \sin (\pi * t) \quad (2.22)$$

Where X_p is the displacement of the forced point in the string and t is time. This forced point must be represented by a segment midpoint in the string, as stated before. If so, X_p equation is used directly in the numerical calculations, which gives the displacement of that midpoint. This gives simpler calculations, for that reason, one should try to choose a segment division of the string in such a way that the forced point is placed at the segment midpoint. Moreover, the function describing the forced point movement will be structured according to wave superposition theory and considering the North Sea environment.

2.3.6 External forces

Any part of the string can be attacked by external forces, that is, forces not generated within the string by its own movements (stretching and compressing the string material). These forces may be due to force fields, like gravity, contact of surrounding materials or fluids giving friction, masses attached to the string giving inertial forces, springs, etc. When a force attacks the string in any position, it is highly recommended to divide the string into numerical segments in such a way that a segment midpoint can be placed there. It is, however, also possible to place the border between two segments there, but this is more complicated to handle and may give numerical oscillations (see section 2.3.8).

If an actual force is not acting along the string, the force component along the string axis must be used, since the string model used here assumes movement along the string axis only. As examples, the gravity vertical component acting on a drill string in a deviated well, or the contact force from the well wall acting on the string which will give friction against the movement along the string axis.

The main divisions of this kind of forces are:

- Fixed and forced points: Forces that move any part of the string in a predetermined way; called fixed when part of the string is forcedly kept at rest; or forced point when actually moving, this has been discussed before.
- Point force: A force attacking the string over a length small compared to the numerical segment length Δz . The best is to have a segment midpoint at this position; but if it is not the case, one can assume that the force is acting on the closest midpoint, where the inaccuracy will be at most $\Delta z/2$.
- Distributed force: A force attacking the string over a length equal or longer than, the numerical segment Δz . Examples of these forces are gravity and friction, which will be further explained in the next section. For each segment the total force is calculated and assumed to act at the segment midpoint.
- Inertial force: This force appears when a mass is attached to a string and it is forced to move because the string is moving, and the mass resists this. This mass is assigned to the closest midpoint and added to the mass of the segment; alternatively, it can be distributed between two segment midpoints.

In the general expression, for one segment midpoint acceleration, the sum of all forces acting on the segment can be divided into two sums as follows:

$$m_s a = m_s \frac{XN_j - 2X_j + XG_j}{\Delta t^2} = \sum \text{Internal forces} + \sum \text{External forces} \quad (2.23)$$

If the previous equation is divided by the segment mass m_s , and multiply with Δt^2 , the standard numerical equation is found when no external forces are acting and for a string with constant cross section. The above equation then becomes:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + \frac{\Delta t^2}{m_s} \sum \text{External forces} \quad (2.24)$$

As the previous equation is applicable for a segment inside the string and with constant cross section; if the segment in question is an end segment, or a segment sufficiently close to a cross section change to become affected by it, then the appropriate equation for this case must be used.

2.3.7 Gravity

It is defined as an external force which is a distributed force, as it affects the string over a length equal or longer than a numerical segment length. This force is assumed to act at the segment midpoint. As an example, consider a string hanging vertically (in air), with its positive direction chosen to be downward. From the physical ball model (Appendix 8.3) it is found that, as each segment in the string experience the force of gravity, which is acting in the positive direction (downward), the standard numerical equation (away from ends or change of cross sections) is given by:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + g\Delta t^2 \quad (2.25)$$

Where g is the acceleration of gravity, 9.81 m/s^2 . Note that this equation is correct in terms of units, all the displacements are lengths and the term $g\Delta t^2$ is also a length. Therefore, this equation may be called a physical numerical equation for the new displacement of the segment midpoint, as it is an equation between physical quantities.

If now, the displacement unit Δx is chosen to be equal to $g\Delta t^2$, the physical numerical equation can be written:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + \Delta x \quad , \text{ where } \Delta x = g\Delta t^2 \quad (2.26)$$

The previous is still a physical equation, but if divided by Δx , the numerical equation used for calculations is defined; where any relative displacement $X/\Delta x$ is still written as X , and this is even simpler:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + 1 \quad (2.27)$$

This equation describes the standard numerical equation (inside the string with constant cross section) when gravity is considered. Similarly, the other numerical equations such as string ends, changes in cross sections, have the same consideration regarding gravity.

In a well there is liquid giving buoyancy. This is a force acting upwards, only at the bottom of the string (if it has a constant cross section), and when the well is deviated. This force can be calculated as the weight of the liquid displaced by the steel volume V_s of the string:

$$F = V_s \rho_L g = \frac{M_s}{\rho} \rho_L g = \frac{Nm}{\rho} \rho_L g = \frac{\rho_L}{\rho} Nm g \quad (2.28)$$

Where N is the number of whole segments, m is the mass of one whole segment, while ρ_L and ρ are densities of liquid and steel, respectively.

Assuming a drill string hanging vertically, where the positive direction is downward, and the last segment of the string (bottom of string) is a free end ending with a half segment (Eq. 2.14). The equation for the last segment midpoint is found by introducing gravity and buoyancy into Eq. 2.24, considering Eq. 2.14:

$$XN_n = 2X_{n-1} - XG_n + g\Delta t^2 - \frac{\Delta t^2}{m_s} \left(\frac{\rho_L}{\rho} Nm g \right) \quad (2.29)$$

The reason for the negative sign is that this force of buoyancy is upwards, in the negative direction. Is important to remember that this equation was found for the bottom of the string; however, buoyancy also will affect the string in the non-vertical segments in a deviated well.

2.3.8 Friction

Friction as explained before is a special case of external forces acting on the string and can be treated in the same way. The main difference to an external force, in general, is that friction, like gravity, usually is acting on every segment in the string. But its action is much more complex than gravity, because it is always acting in the opposite direction of the segment velocity. If the segment velocity is positive then friction is negative, and vice versa.

The simplest case is **linear friction**, where the force of friction is proportional to the segment velocity. We accordingly need a numerical expression of the velocity. The velocity of a segment mid point at any time t can be estimated by taking the average of the average speeds in the former and the following (future) time interval Δt :

$$v_j = \frac{v_j(t-\Delta t, t) + v_j(t+\Delta t, t)}{2} = \frac{X_j - XG_j}{2\Delta t} + \frac{XN_j - X_j}{2\Delta t} = \frac{XN_j - XG_j}{2\Delta t} \quad (2.30)$$

If the previous equation is introduced in Eq. 2.24, for a string with constant cross section:

$$\begin{aligned} XN_j &= X_{j-1} + X_{j+1} - XG_j - \frac{\Delta t^2}{m} \left(C * \frac{XN_j - XG_j}{2\Delta t} \right) \\ &= X_{j-1} + X_{j+1} - XG_j - C \frac{\Delta t}{2m} (XN_j - XG_j) \end{aligned} \quad (2.31)$$

Where m is the mass of a segment on the string and C is a constant of proportionality between the speed of a segment midpoint and the force of friction acting on this segment. The negative sign, in the external forces term, shows that the force of friction acts in the opposite direction of the direction of the movement. Solving the equation in respect to the unknown XN_j :

$$XN_j = \frac{1}{1+\alpha}X_{j-1} + \frac{1}{1+\alpha}X_{j+1} - \frac{1-\alpha}{1+\alpha}XG_j \quad , \text{ where } \alpha = C \frac{\Delta t}{2m} \geq 0 \quad (2.32)$$

Rewriting previous equation in a more convenient manner:

$$XN_j = (1 - \varepsilon)X_{j-1} + (1 - \varepsilon)X_{j+1} - (1 - 2\varepsilon)XG_j \quad , \text{ where } \varepsilon = \frac{\alpha}{1+\alpha} = \frac{C\Delta t}{2m+C\Delta t} \geq 0 \quad (2.33)$$

Note that α and ε are pure numbers. The constant C has the unit Ns/m and multiplied by Δt s , results $\text{Ns}^2/\text{m} = \text{kg}$ which is the unit of the segment mass m . Note that Eq. 2.33 gives actual displacements (physical lengths), not the numeric displacements that are pure numbers; however, Eq, 2.33 can be easily divided by Δx , giving the numerical equation for calculations.

This numeric equation will not give exact results, but will be more accurate if the value of ε is decreased. For this purpose, as the constant C of friction is proportional to the segment length $\Delta z=c\Delta t$, C can be written as $C=C_o\Delta t$, where C_o is a constant independent of the chosen step length. The segment mass m is also proportional to the chosen step length: $m=A\Delta z\rho=Ac\rho\Delta t$. These give:

$$\varepsilon = \frac{\alpha}{1+\alpha} = \frac{C\Delta t}{2m+C\Delta t} = \frac{(C_o\Delta t)\Delta t}{2Ac\rho\Delta t+(C_o\Delta t)\Delta t} = \frac{C_o\Delta t}{2Ac\rho+C_o\Delta t} \quad (2.34)$$

In this equation all parameter except Δt are constants, independent of the chosen step length (Δz). If a smaller value of Δt is chosen (by choosing a smaller value of Δz), the constant ε (or α) will decrease, giving more accurate calculations of the friction effects.

In the industry, with relatively large pipe diameters, liquid flow against equipment is in most cases at least partly turbulent. The friction is then not linear, and more complex equations must be used. But even if the flow of liquid against the string is laminar (not turbulent), the friction against a string changing its speed will not be given by the above equations (this is usually the case when it is dynamically loaded). This is because linear friction assumes steady state conditions, for instance that the string is moving at a constant speed. But, the whole point of the numerical calculations developed here is to treat strings that are not moving at a constant speed, on the contrary vibrating, suddenly accelerating, and so on. Moreover, even if the flow is laminar the friction against the string will not only depend upon the speed, but also the acceleration. And it will not depend only upon present values, but also upon the near history of string movement. Then, the assumption of linear friction in this case, as for a vibrating drill string, is completely wrong.

- The (+) is used X *IS CONSISTENT IF* \Rightarrow $XN < X$
- IS NOT CONSISTENT IF* \Rightarrow $XN > X$

So, if it is inconsistent again, it is a self-contradictory case. If this situation appears, an approximation is to assume that the segment has stopped due to friction, and simply set $XN=X$. If this rule is adhered, the string will not start moving again until:

- $XN > X$, when the (-) sign was used.
- $XN < X$, when the (+) sign was used.

The calculation with contact friction are exact except when this stopping situation takes place, or if the velocity of the segment turn from positive to negative even without stopping. The reason is because the exact time when the segment had stopped cannot be determined. It can only be found to lie between two following times in the calculation table, giving a maximum uncertainty of $\Delta t/2$. Therefore by using smaller time steps Δt (or Δz) the error can be minimized; furthermore, the position where the segment stopped can be found by a 3 points parabolic approximation, instead of just assuming $XN=X$. This more complicated approximated will be further explained in the next chapter.

2.3.9 Numerical Oscillations

Numerical oscillations have been mentioned earlier as a problem. In some cases there will appear oscillations in the displacements with a space wavelength of $2 \Delta z$ and a time wavelength of $2\Delta t$. This indicates the presence of numerical oscillations. To be absolutely certain about this, one must redo the calculations with different values of Δz and Δt . Then, if it was a numerical oscillation it will either disappear because this change has changed some of the conditions giving these oscillations, or they will appear again but now with wavelengths two times the new values of Δz and Δt . If it had been real oscillations, actually present in the system, their wavelength cannot be proportional to an arbitrarily chosen numerical parameter. The only effect on real oscillations would be a small shift of amplitudes and-or wavelengths due to a change in accuracy.

For travelling waves and constant forces no numerical oscillations will be initiated, except when a constant force is turned on or off during the calculations.

Numerical oscillations will typically appear when any external force acting on the string changes its value, for instance a force suddenly turned on or turned off. But not always, if a force, for instance, is turned on and lasts for an even number of time steps before it is turned off, there will be no numerical oscillations. However, there will be an error in the displacements of the segments as it will change in steps of two time units instead of smoothly.

An example to show this oscillation is presented. Assume that in a long string of constant cross section a force F acts on segment midpoint number 5; this force starts acting at time zero and ends at time T . Then the numerical equations are:

- For all segments but 5 ($j \neq 5$), standard equation applies Eq. 2.9 for all times:

$$XN_j = X_{j-1} + X_{j+1} - XG_j$$

- For $j = 5$, (external force) apply Eq. 2.24:

$$XN_5 = X_4 + X_6 - XG_5 + \alpha \frac{F\Delta t^2}{m_s} \quad , \text{ where } \alpha = 1 \text{ for } 0 \leq t \leq T \text{ else } \alpha = 0$$

By choosing the displacement unit $\Delta x = \frac{F\Delta t^2}{2m_s}$, and dropping the unit:

$$XN_5 = X_4 + X_6 - XG_5 + 2 \frac{F\Delta t^2}{2m_s} = X_4 + X_6 - XG_5 + 2\Delta x = X_4 + X_6 - XG_5 + 2 \quad (2.39)$$

Then the calculation table is, for $T=3\Delta t$:

TABLE 2.5 - CALCULATION TABLE FOR $T=3\Delta t$.

Segment Number:	1	2	3	4	5	6	7	8	9	10	COMMENTS
TIME	DISPLACEMENTS										
$t=-\Delta t$	0	0	0	0	0	0	0	0	0	0	
$t=0$	0	0	0	0	0	0	0	0	0	0	Force begins to act
$t=\Delta t$	0	0	0	0	2	0	0	0	0	0	Eq. 2.39 for $j=5$
$t=2\Delta t$	0	0	0	2	2	2	0	0	0	0	Eq. 2.39 for $j=5$
$t=3\Delta t$	0	0	2	2	4	2	2	0	0	0	Force stops acting
$t=4\Delta t$	0	2	2	4	2	4	2	2	0	0	Standard Eq. for $j=5$
$t=5\Delta t$	2	2	4	2	4	2	4	2	2	0	Standard Eq. for $j=5$
$t=6\Delta t$	2	4	2	4	2	4	2	4	2	2	Standard Eq. for $j=5$

In the previous Table, it can be seen the beginning of a numerical oscillation of constant amplitude, which is spreading out over the whole string, in time. Here the force was active for three time steps. Now, if the force is active for four time steps ($T=4\Delta t$), it can be seen that no oscillations are present but the displacements do not increase smoothly (Table 2.6).

TABLE 2.6 - CALCULATION TABLE FOR $T=4\Delta t$

Segment Number:	1	2	3	4	5	6	7	8	9	10	COMMENTS
TIME	DISPLACEMENTS										
$t=-\Delta t$	0	0	0	0	0	0	0	0	0	0	
$t=0$	0	0	0	0	0	0	0	0	0	0	Force begins to act
$t=\Delta t$	0	0	0	0	2	0	0	0	0	0	Eq. 2.39 for $j=5$
$t=2\Delta t$	0	0	0	2	2	2	0	0	0	0	Eq. 2.39 for $j=5$
$t=3\Delta t$	0	0	2	2	4	2	2	0	0	0	Eq. 2.39 for $j=6$
$t=4\Delta t$	0	2	2	4	4	4	2	2	0	0	Force stops acting
$t=5\Delta t$	2	2	4	4	4	4	4	2	2	0	Standard Eq. for $j=5$
$t=6\Delta t$	2	4	4	4	4	4	4	4	2	2	Standard Eq. for $j=5$

A quite simple way to remove the numerical oscillations when external forces act on a relaxed string, is to take the average value of the row $n-1$ and row n and substitute this values in the row n (row is referred to the time step lines). Applying this on Table 2.5 (for $T=3\Delta t$):

TABLE 2.7 - CALCULATION TABLE FOR $T=3\Delta t$ (OSCILLATIONS REMOVED BY AVERAGING IN TIME TECHNIQUE)

Segment Number:	1	2	3	4	5	6	7	8	9	10	COMMENTS
TIME	DISPLACEMENTS										
$t=-\Delta t$	0	0	0	0	0	0	0	0	0	0	
$t=0$	0	0	0	0	0	0	0	0	0	0	Force begins to act
$t=\Delta t$	0	0	0	0	1	0	0	0	0	0	Eq. 2.39 for $j=5$
$t=2\Delta t$	0	0	0	1	2	1	0	0	0	0	Eq. 2.39 for $j=5$
$t=3\Delta t$	0	0	1	2	3	2	1	0	0	0	Force stops acting
$t=4\Delta t$	0	1	2	3	3	3	2	1	0	0	Standard Eq. for $j=5$
$t=5\Delta t$	1	2	3	3	3	3	3	2	1	0	Standard Eq. for $j=5$
$t=6\Delta t$	2	3	3	3	3	3	3	3	2	1	Standard Eq. for $j=5$

Note that this way of removing oscillations, by averaging, does not give quite correct wave amplitudes when waves are also travelling along the string.

For accurate calculations of complex systems, the best strategy is to choose the time step so small that changes in forces for each time step are small compared to peak values. Also, in realistic systems there is almost always friction, and including this in the calculations will gradually reduce all amplitudes, also those of numerical oscillations.

However, for waves traveling along the string this is never a problem, all types of end segments and cross section changes give correct numerical solutions, without any numeric oscillations appearing.

2.4 MatLab Program for the UiS Numerical Model

MATLAB (Matrix Laboratory) is a numerical computing environment and fourth generation programming language. It allows matrix manipulations, plotting of functions and data,

implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

Why MatLab is used for this kind of numerical calculations? Because its easiness to handle matrixes and numerical loops overcome several calculation limitations usually present in programs like Excel. For instance, in MatLab one can divide the string in any desired number of segments, while in Excel this should be done manually, which is not so practical.

Moreover, the flexibility it offers, for changing any desired parameter from the drill string, well path, time steps, etc. can be done fast and the sensibility of the results can be readily seen. MatLab results can be easily exported to Excel where further operations can be done to the Calculation Tables.

3. Numerical Calculations and Program (MatLab)

As discussed in the previous section, MatLab was chosen to make the numerical calculations more flexible and to change easier parameters in order to evaluate their effects on the pressure fluctuations below the drill bit.

As discussed earlier, the problem addressed by the thesis is based on the context when during drilling the drill string has to be wedged to the drill floor each time the drill string length needs to be increased or decreased by adding or removing stands. Therefore, when drilling for a floating rig the top part of the drill string will follow the heave movement of the rig, as no heave compensation takes place (drill string not hanging from travelling block). This forced heave oscillation of the top of the drill string may be transmitted down the string and give pressure fluctuations around the string, especially at the bottom end where the BHA and the drill bit act as leaky pistons (swab and surge pressures). However, as the drill string is elastic, the bottom movement of the drill string may be quite different from the forced movement at the top. Friction forces along the drill string will reduce the amplitudes of the forced oscillations, but also depends on elastic effects and wave interference phenomena. All the elastic effects and some of the effects of friction are calculated in this program. The down hole oscillations of the drill string drilling bit are also found, and the pressure fluctuations at the bottom were calculated.

The calculations are based on an example well case study, which is a “build and hold” type of well, with 5” DP, 8” BHA and 12” wellbore, and a standard mud rheology. The main parameters that will be evaluated are the ones entered by the user, such as contraflow percentage, mud escape area, degree of mud moving with string, number of numeric segments in the drill string, and wave parameters. Moreover the drill string length and well deviation will be changed as well. All in order to evaluate their effects on the swab and surge pressure fluctuations and bit movement given by the well case example.

The flow diagram for the whole Numerical Method is presented in Figure 3.1, and the MatLab program code can be found in Appendix 8.4.

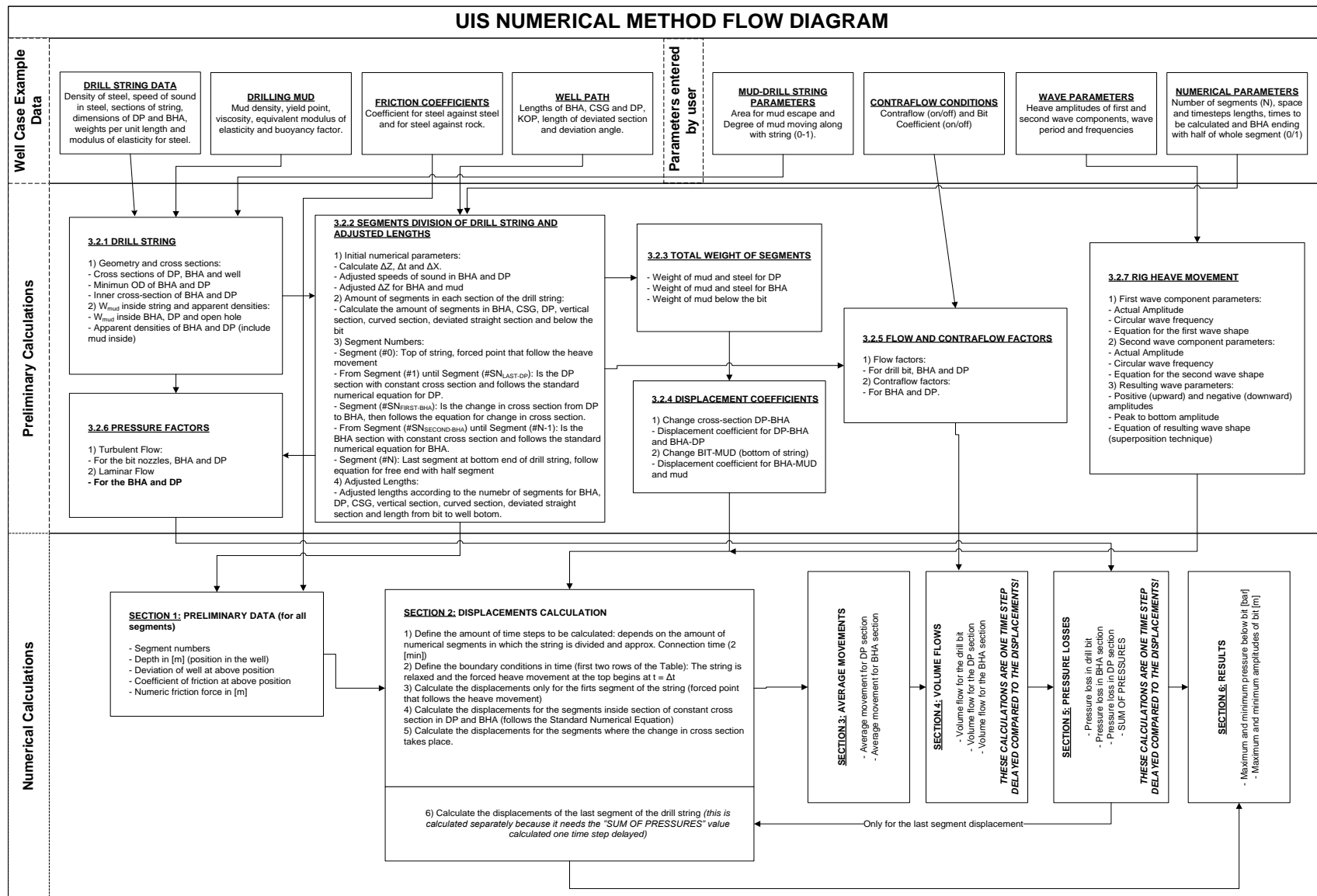


FIGURE 3.1 – FLOW DIAGRAM FOR THE UIS NUMERICAL METHOD

The following sections represent the steps of the UiS Numerical Method (which can be seen in the flow chart as well) in order to calculate the pressure fluctuations and bit movement. Note that the same reference numbers for the sections are used for both the Flow Diagram and this thesis. The sections explain the theory behind the equations, how they were calculated, and with what purposes.

3.1 Inputs for the program

The inputs for the numerical calculation are many, and can be divided into two groups. The first, comprises the physical, material and geometry parameters of the well, drill string and drilling fluid; which are limited by the type of well that is being under analysis. However, the drill string length and the deviation will be evaluated for different calculation scenarios. The second, comprises parameters that are assigned by the user himself which are not limited by the type of well under consideration and their effects on the pressure fluctuations is emphasized.

Note that all the variable definitions and units are presented in Chapter 7; and their corresponding variable names in MatLab can be found there as well.

3.1.1 Well case example data

As explained before, this data is referred to a specific well case example. For this thesis, it is a “Build and hold” well, where the well is drilled vertically from surface to the KOP, where the well is steadily and smoothly deflected until a maximum angle and desired direction is achieved. The established angle and direction are maintained while drilling to the target depth. The kick off section must have a constant radius of curvature. The shallower part of the well is cased, while the rest is open hole.

The drill string is assumed to be simply made of drill pipes, a BHA section made of drill collars and a bit at the end.

Drill String:

- Density of steel (ρ_{steel}) is 7850 kg/m³.
- Speed of sound in steel (c_{steel}) is 5172 m/s.
- Modulus of elasticity for steel: $E_{steel} = \frac{\rho_{steel} * c_{steel}^2}{100000} = bar$ (3.1)
- Minimum inner diameter of string (MIN_{ID}) is 0.06 m.
- Outer Diameter of DP (OD_{DP}) is 5 in.
- Outer Diameter of BHA (OD_{BHA}) is 8 in.
- Outer Diameter of Bit (OD_{BIT}) is 12 in.
- Weight of DP (W_{DP}) is 30 kg/m.

- Weight of BHA (W_{BHA}) is 200 kg/m.
- Depth from bit to well bottom, when the string is attached on slips is ($D_{Bit-bottom}$) is 20 m.

Drilling fluid:

- Density (ρ_{MUD}) is 1200 kg/m³.
- Viscosity (μ_{MUD}) is 15 Cp.
- Yield point (YP_{MUD}) is 14 lb/100ft².
- Speed of sound (c_{MUD}) is 1000 m/s.
- Equivalent modulus of Elasticity for mud: $E_{MUD} = \frac{\rho_{MUD} * c_{MUD}^2}{100000} = \text{bar}$ (3.2)
- Buoyancy factor (F_b): $F_b = 1 - \frac{\rho_{MUD}}{\rho_{steel}} = \text{number}$

Friction Coefficients:

- Coefficient of friction Steel-Rock ($f_{steel-rock}$) is 0.3 adim
- Coefficient of friction Steel-Steel ($f_{steel-steel}$) is 0.23 adim

Well path:

- Length of BHA section (L_{BHA}) is 200 m.
- Length of Cased section (L_{CSG}) is 1200 m.
- Total Measured Depth of well (MD_{total}) is 3000 m.
- Depth to kick off point (KOP) is 500 m.
- Length of curved section (L_{curved}) is 500 m.
- Deviation after curved section (Dev) is 70 deg..
- Length of straight deviated section (L_{dev}) is 2000 m.

As explained before, the majority of these parameters will be kept constant through all the calculations, as they represent the base case for this study. Only the length of the drill pipe and the deviation of the well after the KOP will be changed for making different calculation examples and evaluate their effects onto the pressure fluctuations and bit movement. Other parameters which will be evaluated are explained in the next section.

3.1.2 Parameters entered by user

These parameters are chosen freely by the user in order to change specific conditions in the calculations, however these have limitations. It is very important to evaluate which are the most relevant to the pressure fluctuations below the bit and on the accuracy of calculations of the numerical method.

Mud-drill string parameters:

- Area for the bit mud escape section ($A_{MUD-ESCAPE}$): This area is illustrated in the Figure 3.2, by the red colored area between the well bore and the bit.

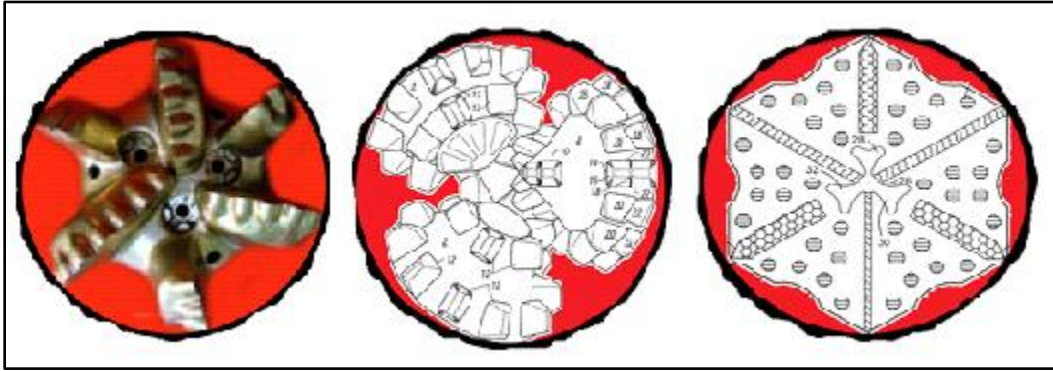


FIGURE 3.2 - DRILL BITS CROSS SECTION IN THE WELLBORE (REF. WWW.CHINA-OGPE.COM)

As seen before, this area depend greatly on the type of bit used, if the wellbore is in gauge, hole cleaning, etc. As the bit is pulled approx. 20 m from the well bottom when making the connections, it is assumed that only drilling mud is around the bit, which means that cuttings or rock debris is not plugging this area. Therefore, as this area can vary, the user may choose between two areas, for the calculations:

- 1) An absolute bit-mud escape cross section area, for instance $20/10000 \text{ m}^2$.
- 2) A relative bit-mud escape cross section area: $A_{MUD-ESCAPE} = 0.1 * A_{BIT} \text{ m}^2$
(3.3)

As this area for mud escape is used to calculate the flow and pressure loss factors for the drill bit, this parameter affects directly the friction of the string against the upward mud flow when the bit moves down and vice versa, increasing or decreasing the pressure below the bit. It will be evaluated how these pressures below the bit affects the amplitudes of the bit movements; therefore it can be calculated as a non-leaky piston and analyze the amplitudes of the bit movements and the pressure variations below the bit.

- Degree of mud moving with string ($De_{MUD-MOVE}$) is a factor between 0 and 1, which can be chosen by the user in order to define what percentage of the drilling mud, inside a drill string segment, that is moving along with the drill string segment itself. It is a parameter which helps making the calculation more accurate, regarding the friction inside the drill string. As it is known, the mud to string friction inside the drill string is larger than on the outside, both because of the smaller space, and also because all the friction from the mud is only acting on the string inside wall; while on the outside the total mud friction is distributed between the walls of both the string and the well or casing.

To check if this friction has an important effect on the calculations, an extreme form of inside friction will be tried where the mud is forced to follow the movement of the string ($Deg_{MUD-MOVE} = 1$). This will only affect the mass of the string segments and the apparent densities of DP and BHA, which are possible to calculate exactly, as shown later.

Contraflow conditions:

- Contraflow addition to volume flow (*Contraflow*): Is a factor that turn on or off the contraflow addition to volume. When it has a value equal to 1 the contraflow is on, and when 0 the contraflow is off. This parameter affects directly the “Contraflow Factors” of the DP and BHA, so when this factor is zero there will not be any “Contraflow Factors”. Therefore when calculating the volume flow for each time step, the contraflows can increase or decrease the volume flow in the DP and BHA sections, as they are added directly to the “Flow Factors”.
- Bit Coefficient (BIT_{coeff}): Similarly to the *Contraflow*, it is a factor that turns on or off the bit coefficient. It affects directly the “Flow Factor” for the drill bit, when it is on the Mud Escape Area will be reduced, giving a larger flow through the bit; and when it is off the Mud Escape Area will remain and the flow through the bit will be smaller.

Wave (heave) parameters:

As explained in the Section 2.2, regular waves with their own specifications plus the superposition theory will be used to model the waves in North Sea, which will transmit the heave to the top of the drill string. It is assumed that the platform heave movement follows the wave amplitude. Moreover by using the information from Figure 2.17 (32), the wave parameters will be changed according to that graph in order to get more realistic wave environments. These parameters are:

- Heave amplitude of the first wave (A_{h1}): Is the first wave heave amplitude which is larger than the second wave and affects greatly the heave movement of the platform. This heave variation is very important to check how the bit displacement and pressures below bit are affected by it, verifying the rig heave limitations and possible kick and loss scenarios due to a narrow drilling window.
- Heave amplitude of the second wave component (A_{h2}): It is the second wave heave amplitude which helps model the sea waves in a more realistic wave. As seen in Figure 2.12, the sea wave most of the times have a trochoidal shape; therefore, this second wave aids in this objective and has a maximum value of $A_{h1}/4$. This limitation is included in order to avoid a possible peak at the trough of the first wave component; which would not make a complete flat trough as desired. Moreover the frequencies of the wave components must have whole numbers relationships.

- Heave period (T_h): is the period of the wave which is very important in order to calculate the frequencies of the wave components and this will vary according to Figure 2.17 for the North Sea.

The pressure fluctuations and bit displacements generated by different waves, and whose parameters are changed by the user, will be analyzed. This is of prime importance for the thesis findings.

Numerical parameters:

- Number of numerical segments (N): This is the number of segments in which the user wants the drill string to be divided into. It affects directly the length Δz (space step length), and consequently the time step Δt . These parameters are very important in order to define the number of segments and calculations accuracy. As the program was done in MatLab, this parameter can be easily changed; for instance, if N is increased, then the string will be divided into more segments (Δz and Δt smaller), and the resulting calculation table will increase in size. The effects on the pressure fluctuations below the bit and bit displacements will be evaluated due to the change of this variable.
- Amount of time steps to be calculated (*times*): is directly chosen by the user and should not be confused with the time step Δt . This amount just refers to how many time steps one wants to calculate. One must be careful not to choose a very small value, because the oscillations at the top of the string might not have enough time to travel down to the bottom of the drill string; and not too large because this will make the calculations slow and impractical.
- BHA type of end (BHA_{end}): As explained in section 2.3.5, a drill string can have two types of ends, half segment end or whole segment end. If $BHA_{end}=0$ then the drill string end with a half segment, if $BHA_{end}=1$ then the end is with a whole segment but coupled to the mud. As the objective is to calculate the pressure fluctuations and bit displacement, it is better to end the string with a half segment as the calculations for that last midpoint will be more accurate and will represent directly the point of interest for the thesis.

3.2 Preliminary calculations

This section shows all the calculations, equations and some results; however the next chapter will discuss all the results in detail. As all these preliminary calculations are made before beginning with the UiS Numerical Method calculation itself, they are very important for the overall results as must be done as accurate and simple as possible. Therefore, these calculations are divided into groups which are easier to follow in the Flow Diagram of the whole Program.

3.2.1 Drill string

Geometry and cross sections of drill string:

- Material cross section of DP (CS_{DP}):

$$CS_{DP} = \frac{W_{DP}}{\rho_{steel}} = m^2 \quad (3.4)$$

- Material cross section of BHA (CS_{BHA}):

$$CS_{BHA} = \frac{W_{BHA}}{\rho_{steel}} = m^2 \quad (3.5)$$

- Cross section of well (A_{well}): is simply the area of the well using the outer diameter of the bit in m^2 .
- Minimum outer diameter of DP, depending on its own weight and the drill string minimum inner diameter (MIN_{OD-DP}):

$$MIN_{OD-DP} = \sqrt{\frac{4*W_{DP}}{\pi*\rho_{steel}} + MIN_{ID}^2} = m \quad (3.6)$$

- Minimum outer diameter of BHA, depending on its own weight and the drill string minimum inner diameter (MIN_{OD-BHA}):

$$MIN_{OD-BHA} = \sqrt{\frac{4*W_{BHA}}{\pi*\rho_{steel}} + MIN_{ID}^2} = m \quad (3.7)$$

- Inner cross section of DP (ICS_{DP}), is the rest between the maximum outer area, considering the specified outer diameter from data and the calculated minimum outer diameter, of the DP minus the material cross section:

$$ICS_{DP} = \pi \frac{(\max(OD_{DP}, MIN_{OD-DP}))^2}{4} - \frac{W_{DP}}{\rho_{steel}} = m^2 \quad (3.8)$$

- Inner cross section of BHA (ICS_{BHA}), is the rest between the maximum outer area, considering the specified outer diameter from data and the calculated minimum outer diameter, of the BHA minus the material cross section:

$$ICS_{BHA} = \pi \frac{(\max(OD_{BHA}, MIN_{OD-BHA}))^2}{4} - \frac{W_{BHA}}{\rho_{steel}} = m^2 \quad (3.9)$$

Weight of mud inside drill string and apparent densities:

- Weight of mud in DP (W_{DP}) is the weight of the mud inside the DP per length unit of DP:

$$WM_{DP} = ICS_{DP} * \rho_{mud} = \text{kg/m} \quad (3.10)$$

- Weight of mud in BHA (W_{BHA}) is the weight of the mud inside the BHA per length unit of BHA:

$$WM_{BHA} = ICS_{BHA} * \rho_{mud} = \text{kg/m} \quad (3.11)$$

- Weight of mud in Open Hole (W_{OH}) is the weight of the mud in open hole per meter:

$$WM_{OH} = A_{well} * \rho_{mud} = \text{kg/m} \quad (3.12)$$

- Apparent density of DP material including the mud ($A\rho_{DP}$) is basically the density of the DP steel which is added by the mud density which is inside the DP according to the “Degree of mud moving with string”, defining how much mud inside the DP is moving along with the DP:

$$A\rho_{DP} = \rho_{steel} + Deg_{MUD-MOVE} * \frac{WM_{DP}}{CS_{DP}} = \text{kg/m}^3 \quad (3.13)$$

- Apparent density of BHA material including the mud ($A\rho_{BHA}$) is analogue to the DP value:

$$A\rho_{BHA} = \rho_{steel} + Deg_{MUD-MOVE} * \frac{WM_{BHA}}{CS_{BHA}} = \text{kg/m}^3 \quad (3.14)$$

3.2.2 Segments division of drill string and adjusted lengths

As explained in Section 2.3.5, the drill string has to be divided into segments in a certain manner in order to place the segment midpoints in parts of the string that are of interest. For this thesis example, as the drill string is made of DP (5”), BHA (8”) and Bit (12”), there are two changes in cross section; however, only the change in cross section from DP to BHA will be considered, as the length of the Bit is not significant compared to the DP or BHA length. Therefore one segment midpoint should be placed in this change of diameter connection; but as the string can be divided into any desired amount of segments (N), sometimes a segment midpoint will not be able to be placed exactly in this position, so an approximation is made by choosing the closest segment midpoint to the change of cross section.

Another two important midpoints are: the top of the drill string which will move according to the platform heave movement, and the bottom of the drill string where the bit displacement and pressure fluctuations are calculated. For the first, it is assumed that the first segment midpoint of the whole string is the one that is attached to the slips in the drill floor, so as it is a forced point it will transmit the heave forces/oscillations to the string. For the second, it is extremely important to have a segment midpoint at the bottom end of the drill string (free end with a half segment), so that the bit displacement and pressure will be accurately calculated for that exact position of the drill string.

Initial numerical parameters:

For making these calculations, the only input needed is the desired amount of numerical segments that the user wants to divide the string into (N), so it is assumed to be known.

- Space step length (Δz) is the length of the numerical segment.

$$\Delta z = \frac{MD_{total}}{N} = m \quad (3.15)$$

- Adjusted speed of sound in DP (c_{adj-DP}) is the new speed of sound for the DP considering the mud that is inside itself, by introducing the apparent density calculated in the previous section. This will affect the time step later.

$$c_{adj-DP} = \rho_{steel} * \sqrt{\frac{\rho_{steel}}{A\rho_{DP}}} = m/s \quad (3.16)$$

- Time step length (Δt) is the time lag between each calculation, as the displacements for all the segment midpoints are calculated in time intervals of Δt .

$$\Delta t = \frac{\Delta z * 1000}{c_{adj-DP}} = msec. \quad (3.17)$$

- Displacement unit (Δx) is the unit chosen for converting the numerical displacements no units into physical displacements m , for instance. As gravity is present, it is convenient to be calculated as follows:

$$\Delta x = g * \left(\frac{\Delta t}{1000}\right)^2 = m \quad (3.18)$$

This unit of displacement will be helpful in order to calculate the “Numeric Friction force in meters”, explained later.

- Adjusted speed of sound in BHA ($c_{adj-BHA}$) is the new speed of sound for the BHA considering the mud that is inside itself, by introducing the apparent density calculated in the previous section. This will affect the space step length for the BHA, which will be adjusted by this new speed of sound.

$$c_{adj-BHA} = \rho_{steel} * \sqrt{\frac{\rho_{steel}}{A\rho_{BHA}}} = m/s \quad (3.19)$$

- Adjusted space step length for BHA (Δz_{BHA}) is adjusted due to the adjusted speed of sound in the BHA and is used to calculate the final length of BHA after placing the segments and adjusting the lengths.

$$\Delta z_{BHA} = \frac{\Delta t * c_{adj-BHA}}{1000} = m \quad (3.20)$$

- Equivalent space step length for the drilling mud (Δz_{MUD}) is used to calculate the segments below the bit, between the drill string and the well bottom.

$$\Delta z_{MUD} = \frac{\Delta z * C_{MUD}}{c_{steel}} = m \quad (3.21)$$

Amount of Segments in each section:

These are basically found by rounding up the number of segments that would fit in the lengths of DP, BHA, etc. defined by the well path. It is important to remember that the number of segment is an integer number; that is why they are rounded for each section of the drill string.

- Number of segments in the BHA section ($N_{SEG-BHA}$):

$$N_{SEG-BHA} = round\left(\frac{L_{BHA}}{\Delta z}\right) = \text{integer number} \quad (3.22)$$

- Number of segments in the cased section of the well ($N_{SEG-CSG}$):

$$N_{SEG-CSG} = round\left(\frac{L_{CSG}}{\Delta z}\right) = \text{integer number} \quad (3.23)$$

- Number of segments in the DP section (N_{SEG-DP}):

$$N_{SEG-DP} = round\left(\frac{MD_{total} - L_{BHA}}{\Delta z}\right) = \text{integer number} \quad (3.24)$$

- Number of segments in the vertical section before the KOP ($N_{SEG-VERT}$):

$$N_{SEG-VERT} = round\left(\frac{KOP}{\Delta z}\right) = \text{integer number} \quad (3.25)$$

- Number of segments in the curved section ($N_{SEG-CURV}$):

$$N_{SEG-CURV} = round\left(\frac{KOP + L_{CURVED}}{\Delta z}\right) - N_{SEG-VERT} = \text{integer number} \quad (3.26)$$

- Number of segments in the deviated straight section ($N_{SEG-DEV}$):

$$N_{SEG-DEV} = N - N_{SEG-VERT} - N_{SEG-CURV} = \text{integer number} \quad (3.27)$$

- Number of segments below the bit ($N_{SEG-MUD}$):

$$N_{SEG-MUD} = round\left(\frac{D_{bit-bottom}}{\Delta z_{mud}}\right) = \text{integer number} \quad (3.28)$$

After finding the amount of segments that each section have, one must calculate the segment numbers of the different sections in order to later apply the corresponding numerical equation to each segment number. These are referenced in Figure 3.3 and their corresponding segment number and numerical equation are also presented.

Segment numbers:

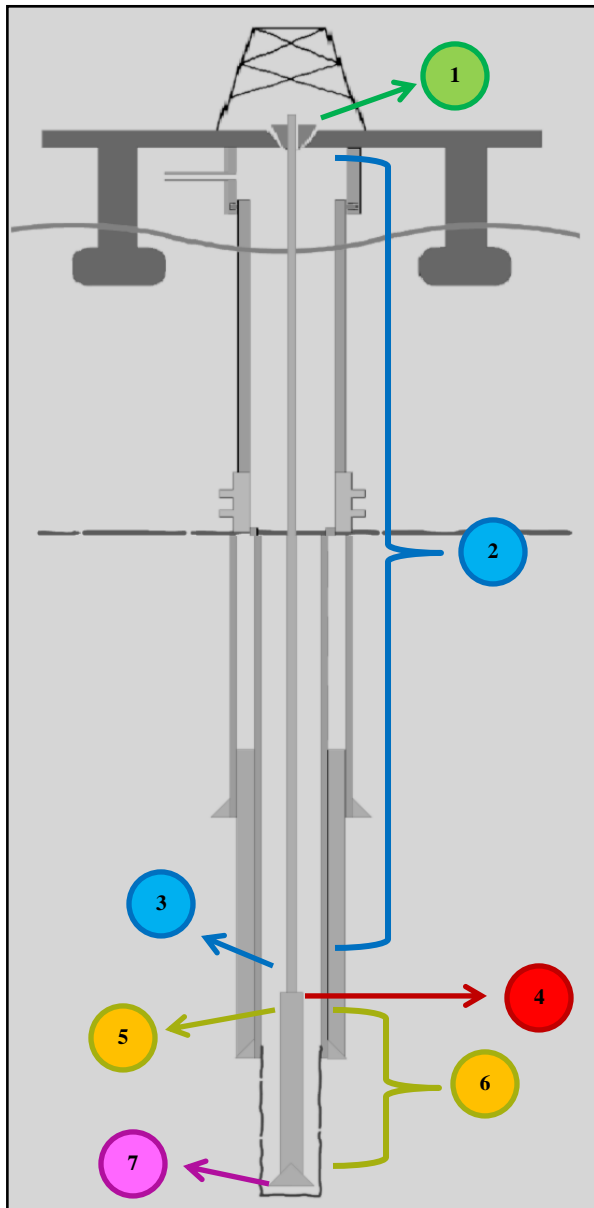


FIGURE 3.3 - IMPORTANT NUMBER OF SEGMENTS ALONG THE DRILL STRING

At (1), segment #0: The top part of the string, which is wedged to the drill floor, will follow the rig heave. This midpoint will be calculated by the numerical equation for forced points Eq. 3.87 (modeled heave).

In (2) are all the segments in the DP section that will follow the Standard Numerical equation for a constant cross section string given by Eq. 2.9, until the segment number is equal to $\#SN_{LAST-DP}$, explained below.

In (3) is the last segment of the DP section ($\#SN_{LAST-DP}$), which will follow the Standard Numerical equation as explained in the previous paragraph.

$$\#SN_{LAST-DP} = \#SN_{FIRST-BHA} - 1 \quad (3.29)$$

In (4) is the first segment of BHA ($\#SN_{FIRST-BHA}$), which will follow the Numerical Equation for change in cross section (Eq. 2.19).

$$\#SN_{FIRST-BHA} = N - N_{SEG-BHA} \quad (3.30)$$

In (5) is the segment of BHA ($\#SN_{SECOND-BHA}$), where the Standard Numerical equation begins to be used again, as in all segments in (6).

$$\#SN_{SECOND-BHA} = \#SN_{FIRST-BHA} + 1 \quad (3.31)$$

Finally, in (7) is the last segment number (N), which is a free end half segment and follows the numerical equation Eq. 2.14.

Adjusted Lengths:

Due to the fact that the user can define the amount of segments in which the drill string is going to be divided into (N); the lengths of the sections that at the beginning were defined by the well path now have to be adjusted according to the number of “entire” segments of each section.

- Adjusted length of the BHA considering its segments. Note that for the BHA, two adjustments are done. The first, due to the number of segments and second due to the adjusted step length for the BHA which considers the apparent density of BHA (including mud):

$$ADJL_{BHA} = \Delta z * N_{SEG-BHA} = m \quad (3.32)$$

$$ADJL_{BHA} = \Delta z_{BHA} * N_{SEG-BHA} = m \quad (3.33)$$

- Adjusted length of the cased section considering its segments:

$$ADJL_{CSG} = \Delta z * N_{SEG-CSG} = m \quad (3.34)$$

- Adjusted length of the DP section considering its segments:

$$ADJL_{DP} = \Delta z * N_{SEG-DP} = m \quad (3.35)$$

- Adjusted length of vertical section before the KOP considering its segments:

$$ADJL_{VERT} = \Delta z * N_{SEG-VERT} = m \quad (3.36)$$

- Adjusted length of the curved section considering its segments:

$$ADJL_{CURV} = \Delta z * N_{SEG-CURV} = m \quad (3.37)$$

- Adjusted length of the deviated straight section considering its segments:

$$ADJL_{DEV} = MD_{total} - ADJL_{VERT} - ADJL_{CURV} = m \quad (3.38)$$

- Adjusted length of BHA considering its segments:

$$ADJL_{BHA} = \Delta z * N_{SEG-BHA} = m \quad (3.39)$$

- Adjusted measured depth at the end of the curved section:

$$ADJMD_{END-CURVED} = ADL_{VERT} + ADJL_{CURVE} = m \quad (3.40)$$

- Adjusted depth from bit to well bottom, considering the space step in mud:

$$ADJD_{BIT-BOTTOM} = \Delta z_{MUD} * N_{SEG-MUD} = m \quad (3.41)$$

These correctly adjusted lengths are very important in order to calculate the loss of pressure due to friction in the annulus (pressure factors), explained later.

3.2.3 Total weight of segments

These parameters define the weight of one space step length (one whole segment) of drill string plus the mud contained inside according to the “Degree of mud moving with the string”, which is the percentage that helps modeling the friction of the mud inside the string (Section 3.1.2). These weights are used to calculate the Displacement coefficients in the next section.

- Weight of mud and steel for the DP section ($W_{MUD-steel DP}$) is the weight of the steel plus the mud contained in one space step length Δz :

$$W_{MUD-steel DP} = (W_{DP} + Deg_{MUD-MOVE} * WM_{DP}) * \Delta z = kg \quad (3.42)$$

- Weight of mud and steel for the BHA section ($W_{MUD-steel BHA}$) is the weight of the steel plus the mud contained in one space step length Δz :

$$W_{MUD-steel BHA} = (W_{BHA} + Deg_{MUD-MOVE} * WM_{BHA}) * \Delta z = kg \quad (3.43)$$

- Weight of mud below drill bit ($W_{MUD-below bit}$) is only the weight of mud in one space step length equivalent for the mud Δz_{MUD} :

$$W_{MUD-below bit} = A_{well} * \Delta z_{MUD} * \rho_{MUD} = kg \quad (3.44)$$

3.2.4 Displacement coefficients

These parameters are used along with the numerical equations to calculate new displacements; they are especially helpful when calculating special midpoints displacements, such as change in cross section (DP-BHA) and bottom end of string (BHA-MUD). Is important to know in which number of segments, (presented in Fig. 3.3), these coefficients must be applied.

Change in cross section (DP-BHA):

- Displacement coefficient for DP-BHA (DC_{DP-BHA}):

$$DC_{DP-BHA} = \frac{2 * W_{MUD-STEEL DP}}{W_{MUD-STEEL DP} + W_{MUD-STEEL BHA}} = \text{number} \quad (3.45)$$

- Displacement coefficient for BHA-DP (DC_{BHA-DP}):

$$DC_{BHA-DP} = \frac{2 * W_{MUD-STEEL BHA}}{W_{MUD-STEEL BHA} + W_{MUD-STEEL DP}} = \text{number} \quad (3.46)$$

Change BIT-MUD (bottom of string):

- Displacement coefficient for BHA-MUD ($DC_{BHA-MUD}$):

$$DC_{BHA-MUD} = \frac{2 * W_{MUD-STEEL BHA}}{W_{MUD-STEEL BHA} + W_{MUD-below bit}} = \text{number} \quad (3.47)$$

- Displacement coefficient for mud (DC_{MUD}):

$$DC_{MUD} = \frac{W_{MUD-below\ bit}}{W_{MUD-below\ bit} + W_{MUD-STEEL\ BHA}} = \text{number} \quad (3.48)$$

3.2.5 Flow and Contraflow Factors

The name Factor means that when multiplied by the displacements, calculated with the numerical equations, the results will have a predetermined meaning and units. For instance, if volume factors are multiplied to the displacements, the results are volumes with volume units.

Now, the flow factors, when multiplied by the displacement of the last midpoint (at the bottom of the string) represents the volume that flows through the drill string at any specific time step Δt , for any given section, such as DP, BHA or bit. To calculate the total volume flow, it must be added the contraflow factors that when multiplied by the “Average movement” of the whole drill string section in question, will increase or decrease the volume flow. However, the total volume flow calculation for each section will be explained in Section 3.3.4.

Flow Factors:

- For drill bit (FF_{bit}):

$$FF_{bit} = \frac{A_{well} - A_{MUD-ESCAPE} + 0.5 * BIT_{coef} * A_{MUD-ESCAPE} * 60000}{\Delta t / 1000} = \frac{L}{min.m} \quad (3.49)$$

- For BHA (FF_{BHA}):

$$FF_{BHA} = \frac{\pi * (\max(OD_{BHA}, MIN_{OD-BHA}))^2 * 60000}{4\Delta t / 1000} = \frac{L}{min.m} \quad (3.50)$$

- For DP (FF_{DP}):

$$FF_{DP} = \frac{\pi * (\max(OD_{DP}, MIN_{OD-DP}))^2 * 60000}{4\Delta t / 1000} = \frac{L}{min.m} \quad (3.51)$$

Contraflow Factors:

- For BHA (CFF_{BHA}):

$$CFF_{BHA} = \frac{\max(OD_{BHA}, MIN_{OD-BHA})}{\max(OD_{BHA}, MIN_{OD-BHA}) + OD_{BIT}} * Contraflow * \frac{(A_{well} - \pi * (\max(OD_{BHA}, MIN_{OD-BHA}))^2) * 60000}{4\Delta t / 1000} = \frac{L}{min.m} \quad (3.52)$$

- For DP (CFF_{DP}):

$$CFF_{DP} = \frac{\max(OD_{DP}, MIN_{OD-DP})}{\max(OD_{DP}, MIN_{OD-DP}) + OD_{BIT}} * Contraflow * \frac{(A_{well} - \pi * (\max(OD_{DP}, MIN_{OD-DP}))^2) * 60000}{4\Delta t / 1000} = \frac{L}{min.m} \quad (3.53)$$

3.2.6 Pressure Factors

Even though, these are factors, they will not be directly multiplied by the displacements calculated with the numerical equations. On the other hand, these are built in such way to be multiplied by the “Volume Flows” of each section at any time step; so that pressures are obtained in bar. These pressure factors are obtained for each section of the drill string using the equations specified in the Table 2.1, for laminar and turbulent flow.

It is very hard to calculate exactly which flow regime is present at any given time step or section of the string, because the time steps are very small thus is not practical. However, as the objective is to calculate the surge and swab pressures below the bit, these are going to be greater when the friction pressure loss in the annulus have larger values. Therefore, for making the calculation even more severe, when calculating the pressures, only the maximum value between turbulent and laminar flow will be considered.

Turbulent Flow:

- Pressure factor for the drill bit nozzle (PF_{BIT}): As seen in its units, it must be multiplied by the squared volume flow in the drill bit, so bar is obtained.

$$PF_{BIT} = \frac{\rho_{MUD}}{200000 * (60000 * A_{MUD-ESCAPE})^2} = bar \left(\frac{min}{L} \right)^2 \quad (3.54)$$

- Pressure factor for the BHA ($PF_{BHA-TUR}$): As seen in its units, it must be multiplied by the volume flow in the BHA with exponent 1.8, so bar is obtained.

$$PF_{BHA-TUR} = \frac{\left(\frac{\rho_{MUD}}{1000} \right)^{0.8} * \mu_{MUD}^{0.2} * ADJL_{BHA}}{70696 * (OD_{BIT} + \max(OD_{BHA}, MIN_{OD-BHA}))^{1.8} * (OD_{BIT} - \max(OD_{BHA}, MIN_{OD-BHA}))^3} = bar \left(\frac{min}{L} \right)^{1.8} \quad (3.55)$$

- Pressure factor for the DP (PF_{DP-TUR}): As seen in its units, it must be multiplied by the volume flow in the DP with exponent 1.8, so bar is obtained.

$$PF_{DP-TUR} = \frac{\left(\frac{\rho_{MUD}}{1000}\right)^{0.8} * \mu_{MUD}^{0.2} * (MD_{total} - ADJL_{BHA})}{70696 * (OD_{BIT} + \max(OD_{DP}, MIN_{OD-DP}))^{1.8} * (OD_{BIT} - \max(OD_{DP}, MIN_{OD-DP}))^3} = bar \left(\frac{min}{L}\right)^{1.8} \quad (3.56)$$

Laminar Flow:

As seen in Table 2.1, the pressure loss equations for laminar flow have two terms, the first depending on the “volume flow” and the second independent of it: $P=a*Q + b$. This means that they cannot be multiplied by the volume flows directly, as the turbulent pressure factors. Therefore, when applying the laminar flow pressure factors, the pressure must be calculated by 2 operations: First, multiply the volume flow with the laminar pressure factor (a), and second add the independent term (b).

- For the BHA, the multiplying factor is:

$$PF_{BHA-LAM} = \frac{\mu_{MUD} * ADJL_{BHA}}{100 * 408.63 * (OD_{BIT} + \max(OD_{BHA}, MIN_{OD-BHA})) * (OD_{BIT} - \max(OD_{BHA}, MIN_{OD-BHA}))^3} = bar \left(\frac{min}{L}\right) \quad (3.57)$$

And the adding term is:

$$ADD_{BHA-LAM} = \frac{YP_{MUD} * ADJL_{BHA}}{100 * 13.26 * (OD_{BIT} - \max(OD_{BHA}, MIN_{OD-BHA}))} = bar \quad (3.58)$$

Then for calculating the pressure in one time step, in the BHA section:

$$P = PF_{BHA-LAM} * (volume\ flow) + ADD_{BHA-LAM} = bar \quad (3.59)$$

- For the DP, the multiplying factor is:

$$PF_{DP-LAM} = \frac{\mu_{MUD} * (MD_{total} - ADJL_{BHA})}{100 * 408.63 * (OD_{BIT} + \max(OD_{DP}, MIN_{OD-DP})) * (OD_{BIT} - \max(OD_{DP}, MIN_{OD-DP}))^3} = bar \left(\frac{min}{L}\right) \quad (3.60)$$

And the adding term is:

$$ADD_{DP-LAM} = \frac{YP_{MUD} * (MD_{total} - ADJL_{BHA})}{100 * 13.26 * (OD_{BIT} - \max(OD_{DP}, MIN_{OD-DP}))} = bar \quad (3.61)$$

Then for calculating the pressure in one time step, in the DP section:

$$P = PF_{DP-LAM} * (volume\ flow) + ADD_{DP-LAM} = bar \quad (3.62)$$

The “volume flow” will be calculated in the next section.

Note that when the “volume flow” is zero, if one applies the previous equations directly for laminar flow, one would still experience a pressure loss due to the adding term which is independent of the “volume flow”. This leads to unrealistic results. Then, in order to avoid this problem, a conditional operator should be used carefully when programming.

Another important factor is the pressure to force factor (PPF_{DS}) against the drill string. As its name states, it is the factor that converts the sum of pressures losses into a force with upward direction (-) which acts on the drill bit area (borehole area minus mud escape area) at the bottom of the string. This factor is calculated as follows:

$$PPF_{DS} = \frac{200000*(A_{well}-A_{MUD_{ESCAPE}})*(\Delta t/1000)^2}{W_{BHA}*\Delta z_{BHA}} = \frac{m}{bar} \quad (3.63)$$

As this factor unit is m/bar , when multiplied by the sum of pressures losses in DP, BHA and drill bit in each time step, it gives a displacement in m that is used to calculate the last segment midpoint displacement (at the very bottom of the string) by introducing it into its numerical equation. This calculation is explained in the Section 3.3.3.

3.2.7 Rig heave movement

The intent of this part is to describe the platform heave movement as realistic as possible, but simple as well. As the sea waves do not follow a sinusoidal movement, the waves must be modeled using the superposition theory in order to add up simpler regular waves to get a realistic sea wave. Therefore, two waves with different parameters will be considered for this. But the wave components must have whole numbers relations between their frequencies.

First some preliminary parameters are calculated, and at the end the full wave equation is presented, which will only depend on time.

- Adjusted amplitude of the second wave component ($ADJA_{h2}$): The second wave amplitude must be adjusted in case its input is larger than $A_{h1}/4$, in order to represent correctly the physical wave shape. This must be done in order to avoid the possible formation of peaks at the trough of the first wave component.

First wave component parameters:

- Actual amplitude of the first wave component (AA_{h1}):

$$AA_{h1} = \frac{A_{h1}^2}{A_{h1}+ADJA_{h2}} = [m] \quad (3.64)$$

- Circular wave frequency of the first wave component (f_{h1}):

$$f_{h1} = \frac{4\pi}{T_h * 1000} = \frac{rad}{msec} \quad (3.65)$$

- Wave shape:

$$A1 = AA_{h1} * \sin \left(2 \frac{2\pi}{T_h} t \right) = m \quad (3.66)$$

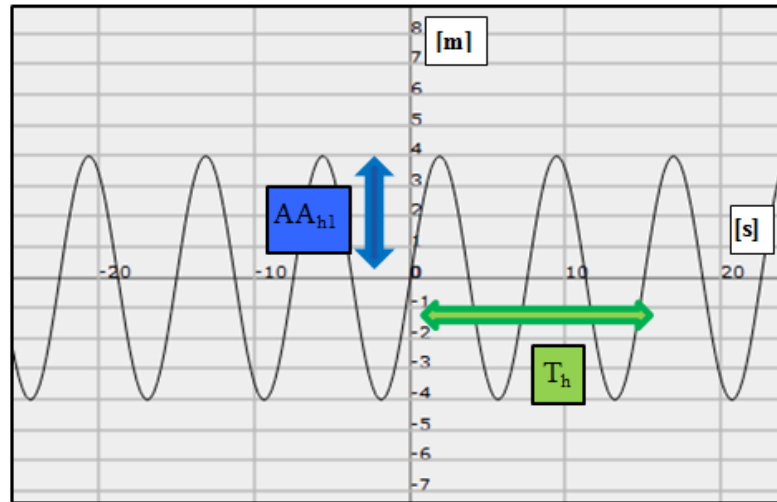


FIGURE 3.4 - SHAPE OF THE FIRST WAVE COMPONENT (GRAPH MADE IN WWW.FOOPLOT.COM)

Second wave component parameters: Is with a phase of $\pi/2$ ahead from the first wave and has double the circular wave frequency (whole number relation).

- Actual amplitude of the second wave component (AA_{h2}):

$$AA_{h2} = A_{h1} - AA_{h1} = m \quad (3.67)$$

- Circular wave frequency of the second wave component (f_{h2}):

$$f_{h2} = \frac{8\pi}{T_h * 1000} = \frac{rad}{msec} \quad (3.68)$$

- Wave shape:

$$A2 = AA_{h2} * \sin \left(4 \frac{2\pi}{T_h} t - \pi/2 \right) = m \quad (3.69)$$

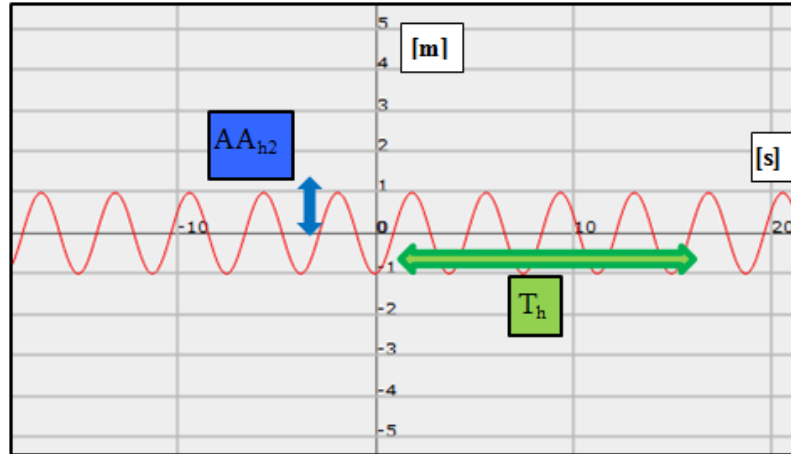


FIGURE 3.5 - SHAPE OF THE SECOND WAVE COMPONENT (GRAPH MADE IN WWW.FOOPLOT.COM)

Resulting wave parameters:

- Positive Upward amplitude (UA): $UA = AA_{h1} + AA_{h2} = m$ (3.70)

- Negative Downward amplitude (DA): $DA = AA_{h1} - AA_{h2} = m$ (3.71)

- Peak to bottom amplitude (PBA): $PBA = 2 * AA_{h1} = m$ (3.72)

- Wave shape:

$$A12 = AA_{h1} * \sin\left(2\frac{2\pi}{T_h}t\right) + AA_{h2} * \sin\left(4\frac{2\pi}{T_h}t - \pi/2\right) = m \quad (3.73)$$

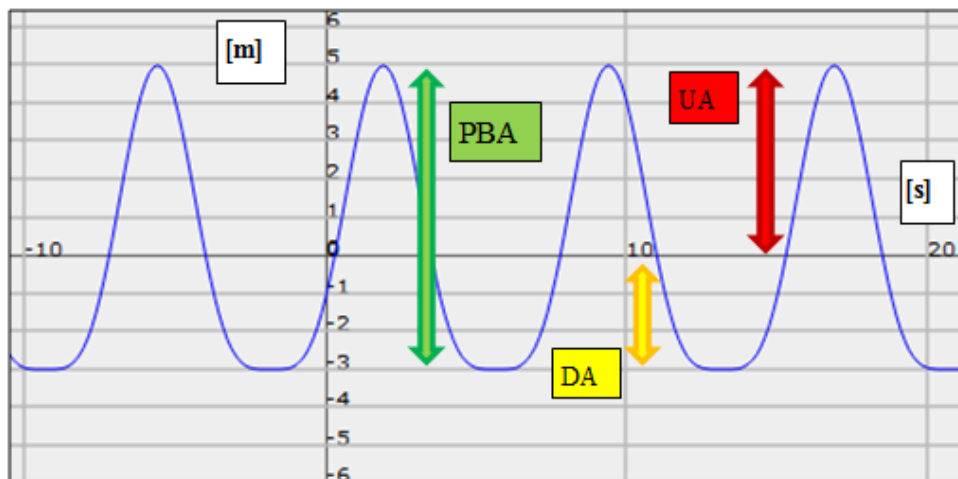


FIGURE 3.6 - RESULTING WAVE SHAPE (GRAPH MADE IN WWW.FOOPLOT.COM)

Due to the lack of information, it is assumed that the platform heave movement will follow the wave amplitude described above. Note that more complex wave models can be used in order to better represent the North Sea waves. However, it is more important to analyze the effect that different average platform heave movements have over the pressure fluctuations below the bit, and how harmful they can be to the drilling operations. As explained in Section 2.2.2,

representative wave parameters will be used in order to better represent the harsh North Sea environment.

3.3 Numerical Calculations

These so called “Numerical Calculations” represent how the different numerical equations, developed in the UiS Numerical Method, are applied to the drill string whose top part follow the rig movement. It is explained how these numerical equations represent different characteristics of the drill string and its oscillation movement. Then, it is important to discuss how the friction, hydraulics and buoyancy are considered within the numerical method, and what limitations they represent.

However, the calculation does not finish by only calculating the displacements using the numerical equations; but the bit movement and pressure fluctuations below the bit must be calculated as well, which are the prime objectives of the thesis. For this end, another calculation Table is required, next to the Numerical Calculation Table, where the volume flows, pressures and movements are calculated using the displacements calculated in the previous Numerical Calculation Table. Both Tables will be explained in detail later.

As MatLab has the capability to export data to Excel sheets, it will be explained what data is exported to Excel and how, in order to easily analyze which variables affect the pressure fluctuations and bit movement, and build figures and tables for a better appreciation.

3.3.1 Friction

The forced heave oscillation of the top of the drill string may be transmitted down the string and give pressure fluctuations around the string, especially at the bottom end where the BHA and the drill bit act as leaky pistons. However, as the drill string is elastic the bottom movement of the drill string may be quite different from the forced movement at the top. Friction forces along the drill string will reduce the amplitudes of the forced oscillations, but depends also on elastic effects and wave interference phenomena. All the elastic effects and some of the effects of friction are calculated here.

The numerical method calculates the movements of a drill string at a number of equidistant points along the string, and only at equidistant points in time. This method includes full calculations of the string elasticity in the axial direction, and allows the inclusion of the linear viscous friction (inside and outside the string) and contact friction between the drill string and walls of the well. These frictions can be seen in the following Figure:

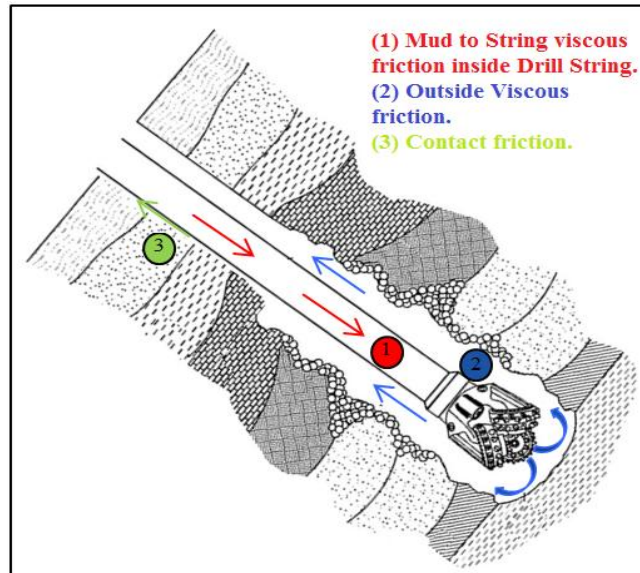


FIGURE 3.7 - TYPES OF FRICTION PRESENT IN A DEVIATED WELL (REF. [HTTP://WWW.FREEPATENTSONLINE.COM/6806232.HTML](http://www.freepatentsonline.com/6806232.html))

(1) *Mud to String viscous friction inside Drill String:*

This friction is the result of the drilling fluid flowing inside the drill string. It is certainly larger than the fluid friction on the outside, because of the smaller space inside the string than in the annulus; and also because all the friction from the mud is on the inside wall, while on the outside the total mud friction is distributed between the walls of both the string and the well.

This inside drill string friction will be represented by the “Degree of mud moving with the string”, which will define what percentage of mud is moving along with the string. To see if this friction is important, an extreme form of inside mud friction will be tried, were the mud will be forced to follow the movement of the string ($Deg_{MUD-MOVE}=1$). This will only add mass to the string, and is possible to calculate exactly.

(2) *Outside viscous friction:*

The increasing pressure below the bit due to friction against the mud flow upward when the bit moves down, and decreasing pressure when the bit moves up, strongly reduces the amplitudes of the bit movements. In addition it is very complicated to calculate friction forces against the drill string when the string surface velocity keeps changing due to oscillations in the axial direction. For these calculations, liquid friction was therefore included only as steady state viscous friction, giving smaller viscous friction forces than expected. However, it is very important, first, to find these friction forces against the drill string due to only its axial vibrations; then prove that its effect, compared to the contact friction in deviated wells, is much less.

Using only a simple but not very accurate method is to use the equation for pressure loss due to friction in annulus, from the Drilling Data Handbook, presented in Table 2.1, turbulent flow. Then the pressure gradient is (all units in SI system):

$$\frac{\Delta P_f}{\Delta L} = \Delta p_f = \frac{\rho^{0.8} \mu^{0.2} Q^{1.8}}{5.07599(D+d)^{1.8}(D-d)^3} = \frac{Pa}{m} \quad (3.74)$$

In order to generate the pressure gradient, a force gradient is needed which is equal to the pressure gradient, times the cross section area of the annulus: $A_A = (\pi/4)(D^2 - d^2)$. This force gradient is balanced with the total friction force against the walls of the well in the annulus (Newton's third Law).

The total friction force against the walls of the annulus is assumed to be evenly distributed between the outer surface of the drill string and the inner surface of the wellbore. This is exact for a very narrow and parallel annulus; however for a real well, the friction on the string will be somewhat less than on the walls of the well. Therefore, by making this overestimation of the viscous friction against the drill string will not matter much, as the purpose is to prove that the contact friction is much larger than the viscous friction over the drill string. So, if it is proven that the contact friction is larger than the overestimated viscous friction over the string; then the contact friction will certainly be greater than the actual value of viscous friction. The friction force per unit length along the drill string is the less than:

$$\frac{F_s}{\Delta L} = \frac{1}{2} \Delta p_f A_A = \frac{1}{2} \frac{\rho^{0.8} \mu^{0.2} Q^{1.8}}{5.07599(D+d)^{1.8}(D-d)^3} \frac{\pi}{4} (D^2 - d^2) = \frac{\rho^{0.8} \mu^{0.2} Q^{1.8}}{12.9259(D+d)^{1.8}(D-d)^2} = \frac{N}{m} \quad (3.75)$$

A constant friction against the drill string cannot influence axial drill string vibrations; it is only a change of axial friction due to vibration that can reduce the amplitude of these vibrations. Then, in order to evaluate this, the variation of the axial velocity due to axial vibrations must be calculated. A simple example of generation of axial vibrations is the formation of a lobe pattern (series of radial ridges) in the well bottom when drilling. As the parts of the drill bit, in contact with the formation, move around it is forced to move up and down over the lobe pattern ridges. The best known example is when drilling with a tri-cone bit, when a stable tri-lobe patten often is generated. This forces the bit to move up and down three times for each revolution of the bit. The axial movement of the drill bit, for a smooth n-lobe patten where the cross sections of the lobes have a sinusoidal shape, is given by:

$$a = A * \text{sine} (2\pi n f t) = m \quad (3.76)$$

Where, A is the lobe pattern amplitude, f is the RPM of the bit, t is time, $n=3$ is the contact points for a tri-cone.

The axial velocity of the bit is found by differentiating the amplitude with respect to time; and the maximum velocity is found by inserting the maximum value of the time function on the differentiated equation ($\cos(2\pi nft)=1$), as shown below:

$$v = \frac{da}{dt} = \frac{d A \cdot \sin(2\pi n f t)}{dt} = 2\pi n f A \cdot \cos(2\pi n f t) = \frac{m}{s} \Rightarrow v_{max} = 2\pi n f A = \frac{m}{s} \quad (3.77)$$

In a hypothetical situation where the drill string has a constant axial velocity (v_{max}), the change of mud friction force against the drill string will be approximately equal to the change found by changing the mud flow velocity by this amount. The reason it is not exactly equal to this, is that then the wall of the well must also move with this velocity. The difference in velocity between the well and the drill string will slightly skew the velocity distribution of the mud in the annulus, and this will slightly affect the velocity of the mud against the drill string. This effect is, however, very small when the string axial velocity is much smaller than the mudflow velocity.

In order to verify that the string axial velocity is indeed smaller than the mud flow velocity, it can be assumed some reasonable values for calculating them. A bit with a pattern amplitude of $A=3 \text{ mm}$, and $f=120\text{RPM}=2 \text{ s}^{-1}$, results in $v_{max}=2\pi*3*2*0.003=0.1131 \text{ m/s}$. This velocity is considerably larger than the ROP, which for a soft rock is around $30 \text{ m/hr}=0.0083 \text{ m/s}$. On the other hand, the average flow velocity of mud in the annulus, corresponding to 3000 lpm , $5'' \text{ DP}$ and $8'' \text{ ID}_{well}$, is $v_{mud}=Q/A_A=5.06 \text{ m/s}$. As seen here, the axial vibration velocity is considerably smaller than this average flow velocity of mud; moreover as ROP is even smaller than the axial vibration velocity, the ROP is neglected.

Then, by changing the average mudflow velocity by v_{max} , which is equivalent to change the mudflow volume flow rate by $v_{max} * A_A$, a change in friction force is given by:

$$\frac{F_s + \Delta F}{\Delta L} = \frac{\rho^{0.8} \mu^{0.2} (Q + v_{max} A_A)^{1.8}}{12.9259 (D+d)^{1.8} (D-d)^2} = \frac{\rho^{0.8} \mu^{0.2} (Q + 2\pi n f A (0.25\pi (D^2 - d^2)))^{1.8}}{12.9259 (D+d)^{1.8} (D-d)^2} = \frac{N}{m} \quad (3.78)$$

$$\frac{\Delta F}{L} = \frac{F_s + \Delta F}{\Delta L} - \frac{F_s}{\Delta L} = \frac{\rho^{0.8} \mu^{0.2} \left(Q + 2\pi n f A (0.25\pi (D^2 - d^2)) \right)^{1.8}}{12.9259 (D+d)^{1.8} (D-d)^2} - \frac{\rho^{0.8} \mu^{0.2} Q^{1.8}}{12.9259 (D+d)^{1.8} (D-d)^2} = \frac{N}{m} \quad (3.79)$$

Where ΔF is the friction force acting on the drill string due to its vibrations.

Given that $v_{mud}=Q/A_{ann}=Q/(0.25\pi(D^2-d^2))$, Eq. 3.79 can be rearranged:

$$\frac{\Delta F}{\Delta L} = \frac{\rho^{0.8} \mu^{0.2} Q^{1.8}}{12.9259 (D+d)^{1.8} (D-d)^2} \left[\left(1 + \frac{2\pi n f A (0.25\pi (D^2 - d^2))}{Q} \right)^{1.8} - 1 \right] = \frac{F_s}{\Delta L} \left[\left(1 + \frac{2\pi n f A}{v_{mud}} \right)^{1.8} - 1 \right] \quad (3.80)$$

Then by using the fact that the axial vibrating velocity (v_{max}) of the string is small compared to the mud flow speed (v_{mud}), this can be approximated by: $(v_{max} \ll v_{mud}) = (y \ll x)$

$$(x + y)^n = x^n(1 + y/x)^n = x^n \left(1 + n y/x + 0.5n(n-1)(y/x)^2 + \dots \right) = x^n(1 + n y/x) + e \quad (3.81)$$

Where e =error terms, with its first error term equal to $0.5x^n n(n-1)(y/x)^2$.

Finally, the friction force acting on the string due to vibrations is:

$$\frac{\Delta F}{\Delta L} = \frac{F_s}{\Delta L} \left[1.8 \frac{2\pi n f A}{v_{mud}} \right] = \frac{N}{m} \quad (3.82)$$

With a maximum error of:
$$e = \frac{1.8(1.8-1)}{2} \left(\frac{2\pi n f A}{v_{mud}} \right)^2 = 0.72 \left(\frac{2\pi n f A}{v_{mud}} \right)^2 \quad (3.83)$$

If these both last equations are evaluated for any reasonable values, one can see that the absolute maximum error, when making this simplification, is less than 1% of $\Delta F/\Delta L$ calculated with Eq. 3.82. Therefore, it can be concluded that the error due to the simplification of the friction calculation is quite small. However, as it is very complicated to calculate friction forces against the drill string when the string surface velocity keeps changing due to oscillations in axial direction, the error done by assuming steady state flow condition in deriving the equation for the changing friction due to the string axial vibrations is not small. Unfortunately, it is very difficult to calculate the correct friction for the non-steady state flow actually taking place around the vibrating string.

For steady state laminar flow it is possible to calculate the correct friction force, up to the point where the vibrations are sufficient violent to induce turbulent flow. For the calculations done here, it is assumed turbulent steady flow, as used in the beginning in Eq. 3.74; so the friction against the vibrations cannot increase due to inducing turbulent flow as for laminar steady state flow. Nevertheless, the mud friction against the vibrating string is probably considerably larger than the one calculated here. But as shown later, this outside viscous friction is very small compared to contact friction; therefore it is reasonable to assume that for deviated wells the contact friction dominates.

(3) Contact friction:

As discussed in Section 2.3.8, as linear friction assumes steady state conditions (string moving with constant speed), which contradicts completely what the numerical method developed here is trying to do: analyze strings that are not moving with constant speed, that are vibrating, suddenly accelerating, and so on. Therefore, contact friction is the dominating friction against the string, and the one that will be treated here.

The contact friction was calculated for a no tension string. But this was only done to get the contact friction forces as a function of depth. This was accurately calculated considering the friction coefficients between steel to casing, and steel to rock.

To show the difference between frictions damping of the drill string vibrations due to outside mud flow and due to contact friction, the “contact friction force gradient” ($F_c/\Delta L$) is calculated and compared to the mud friction force along the drill string ($\Delta F/\Delta L$) due to vibrations, calculated already in the previous section. The contact friction for a deviated well is shown in the following Figure, as well as some basic equations for the weight of the string and its components.

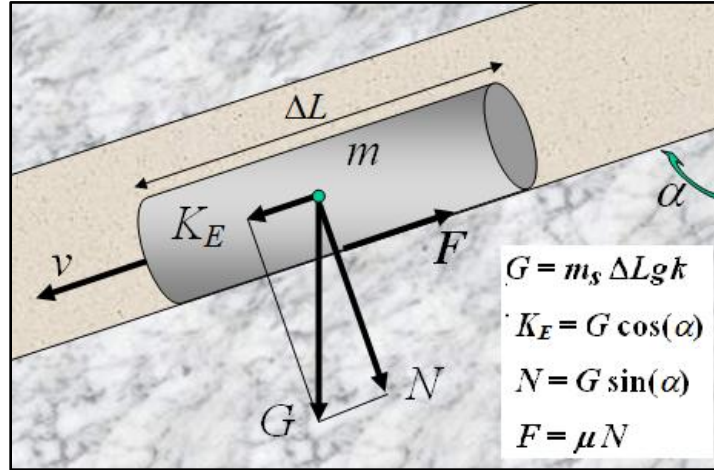


FIGURE 3.8 - FORCES ACTING ON A DRILL STRING SEGMENT IN A DEVIATED WELL

Then the contact friction gradient is given by, according to the previous Figure:

$$\frac{F_c}{\Delta L} = \mu * m_s * g * k * \sin(\alpha) = \frac{N}{m} \quad (3.84)$$

Where μ is the contact friction coefficient, k is the buoyancy factor, m_s is the drill string mass per unit length and α is the deviation angle.

Therefore to make the final comparison between the outside viscous friction (Eq. 3.82) and the contact friction (Eq. 3.84), one can assume some reasonable values. For the first equation, an average flow velocity of mud in the annulus, corresponding to 3000 lpm, 5" DP and 8" ID_{well}, is $v_{mud} = Q/A_A = 5.06 \text{ m/s}$, a bit with a pattern amplitude of $A = 3 \text{ mm}$, $f = 120 \text{ RPM} = 2 \text{ s}^{-1}$, and $n = 3$ for a tri-cone. And for the second $k = 0.85$, $m_s = 40 \text{ kg/m}$, $\alpha = 30^\circ$, and $\mu = 0.2$.

$$\frac{\Delta F}{\Delta L} = \frac{F_s}{\Delta L} \left[1.8 \frac{2\pi n f A}{v_{mud}} \right] = \frac{\rho^{0.8} \mu^{0.2} Q^{1.8}}{12.9259(D+d)^{1.8}(D-d)^2} \left[1.8 \frac{2\pi n f A}{v_{mud}} \right] = 0.373 \frac{N}{m}, \text{ with an error } \approx 1\%$$

$$\frac{F_c}{\Delta L} = \mu * m_s * g * k * \sin(\alpha) = 0.2 * 40 * 9.81 * 0.85 * \sin(30) = 33.354 \frac{N}{m}$$

In this case the contact friction is almost 90 times larger than the friction against the mud. Even if the actual mud friction may be considerably larger than calculated here; due to the more or less impossibility to avoid small bends and deviations from a perfect well path, the contact friction

will also be larger than the calculated here because the drill string will then on the average press harder against the walls of the well. Therefore the contact friction is generally considered to be much larger than the friction against the mud in a deviated well, so it is concluded that more accurate and much more complicated calculations of this outside viscous friction will not change the results much.

On the other hand, as contact friction certainly influences the results, effort must be placed in trying to calculate this as accurate as possible. Then for contact friction there are no numeric errors except for the inaccuracy in determining the moment when the drill string changes the direction it slides. It is explained in Section 2.3.8, as a problem to determine the plus or minus sign of the friction term in the numerical Eq. 2.38, when inconsistencies appear and make the concept self-contradictory. This situation was explained by the fact that the segment has stopped due to friction, or if the velocity of the segment turn from positive to negative even without stopping. The reason is because the exact time when the segment stopped cannot be determined, and then by using smaller time steps the error can be minimized. However, there are 2 approximations that can be done in order address this issue.

The first, and the easiest, is to assume that the segment has stopped and simply set $XN=X$. If this rule is adhered, the string will not start moving again until:

- $XN > X$, when the (-) sign was used.
- $XN < X$, when the (+) sign was used.

This first solution can be easily adhered to the numerical equations when calculating the displacements. For example:

- String with constant cross section (Standard numerical equation, Eq. 2.9)
- Assumed that the segment is moving in positive direction ($X > XG$) then (-) sign for the friction shall be used, for a start.
- An “if” statement can be build using Eq. 2.37 to represent this situation according to the Figure 3.9 below:

$$XN_j = X_{j-1} + X_{j+1} - XG_j \mp XF_j = XA_j \mp XF_j , \text{ where: } XF_j = \mu \Delta t^2 g * \sin(\alpha) \left[1 - \frac{\rho_{mud}}{\rho_{steel}} \right]$$

Case 1: If $(XA - XF) > X$ then $XN = XA - XF$ **Consistent!**
 else Case 2 happens **Inconsistent!**

Case 2: Applies when $(XA - XF) < X$, this indicates that (+) sign must be used, which lead to Case 3.

Case 3: If $(XA + XF) < X$ then $XN = XA + XF$ **Consistent!**

else Case 4 happens **Inconsistent!**

Case 4: Applies when $(XA + XF) > X$, which indicates that (-) sign must be used, which leads to Case 1 again, which is inconsistent and self-contradictory! Therefore, in this situation it is assumed that the segment stopped and $XN = X$.

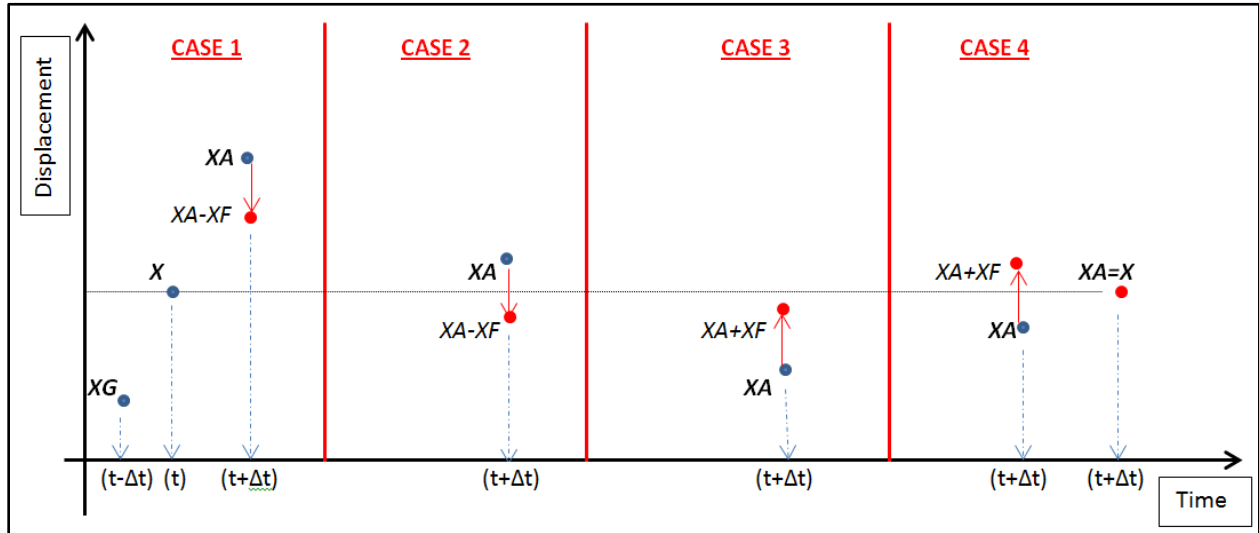


FIGURE 3.9 - FIRST SOLUTION FOR INCONSISTENCY IN THE FRICTION SIGN

The second approximation, quite more complex but more accurate, is to calculate the position where the segment has stopped by a 3 points parabolic approximation, rather than just assume the same position ($XN=X$). This approximation is calculated as follows:

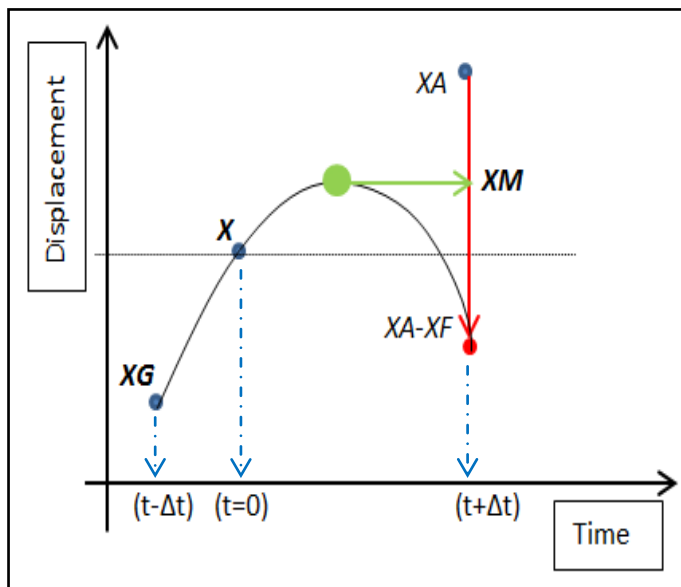


FIGURE 3.10 - PARABOLA APPROXIMATION FOR FINDING THE POSITION WHERE THE SEGMENT HAS STOPPED.

Parabola Eq.: $X = At^2 + Bt + C$

Replacing data points from Figure 3.11:

@ $t = -\Delta t$: $XG = A(-\Delta t)^2 + B(-\Delta t) + C$

@ $t = 0$: $X = A(0)^2 + B(0) + C$

@ $t = +\Delta t$: $XA - XF = A(\Delta t)^2 + B(\Delta t) + C$

Finding the variables A , B and C :

$$A = \frac{XA - XF + XG - 2X}{2\Delta t^2}$$

$$B = \frac{XA - XF - XG}{2\Delta t}$$

$$C = X$$

Therefore the parabolic equation is:

$$X = \left[\frac{XA - XF + XG - 2X}{2\Delta t^2} \right] t^2 + \left[\frac{XA - XF - XG}{2\Delta t} \right] t + C \quad (3.85)$$

Finally, by applying the first derivative on Eq. 3.85, and equalizing to zero, one can readily find the maximum point of the parabola; therefore, this point will be considered as the position where the segment stopped ($XN=XM$).

All these considerations and limitations regarding the viscous friction inside the string, outside the string and contact friction are considered when making the program in MatLab. Therefore it was very important to explain their limitations, how they are calculated and what approximations can be done in order to address some issues encountered, as accurate as possible.

3.3.2 Numerical Calculation Tables

This section intent is to show and explain the structure of the numerical calculations, which calculations are done earlier, in which order calculations must be done, what does the resulting table show, how is the calculation Table divided, etc. It is basically the infrastructure of the program results, showing what the numbers mean and how they are displayed for a better appreciation.

The Calculation Table follow the same structure as Table 2.5 explained in Section 2.3.9, but it is a little more complicated since more information has to be shown and more calculations are done. Moreover, some preliminary calculations will be shown as well, needed for the displacements calculations explained in the next section.

The whole results Table is divided into 2 main Tables, the first is “Displacement Calculations Table”, and the second “Bit Movement and Pressure Calculations Table”; both can be observed below and each section of them will be explained.

TABLE 3.1 - SECTIONS OF THE UIS NUMERICAL METHOD CALCULATION TABLE

DISPLACEMENTS CALCULATIONS TABLE							BIT MOVEMENT AND PRESSURE CALCULATIONS TABLE																
Segment Number:	0	1	N-1	N	Average movement of DP in one time step [m]	Average movement of BHA in one time step [m]	Volume Flow Drill Bit [lpm]	Volume Flow BHA [lpm]	Volume Flow DP [lpm]	Pressure Drill Bit [bar]	Pressure BHA [bar]	Pressure Dpt [bar]	SUM OF PRESSURES [bar]	Maximum Pressure below bit [bar]	Minimum Pressure below bit [bar]	Maximum Amplitude of Bit [m]	Minimum Amplitude of Bit [m]	Maximum Pressure below bit [numeric]	Minimum Pressure below bit [numeric]		
Depth [m] (position in the well)	-						3	4				5						6					
Deviation of the well at above position	-	1																					
Coeff. Of friction at above position	-																						
Numerical Friction Force in [m]	-																						
TIME		DISPLACEMENTS																					
t=-Δt	0	0	0	0	0	0																	
t=0	0	0	0	0	0	0																	
t=Δt	2																						
t=2Δt																							
t=3Δt																							
.																							
.																							
t=nΔt																							

Displacement Calculations Table:

Section 1: It basically comprises all the information and preliminary calculations before beginning with the displacement calculations.

The first row “Segment Number”, it is an informative row that represents all numerical segments in which the drill string was divided (*N, introduced by the user*). It is very important for checking if the right numerical equation was used for the right segment number and for calculating the position in the well (next row).

Second row “Depth m (position in the well)”, represents the position of the segment in meters, is calculated by multiplying the segment number by the space step length (*Seg.Number*Δz*).

Third row “Deviation of the well at above position”, represents the deviation in degrees at the position calculated in the second row. Therefore, according to the position that the segment is in the well and the well path, this value is calculated as shown below for each segment:

If (*Position in the well < Depth of KOP*)

Then $Dev = 0^\circ$

Else if (*Position in the well < Depth of KOP + Length of curved section*)

$$\text{Then } \left(Dev = \frac{Dev.after\ curved\ section * (Position\ in\ the\ well - Depth\ of\ KOP)}{Length\ of\ curved\ section} \right) \tag{3.86}$$

Else (*Dev=Dev. after curved section*)

Fourth row “Coefficient of friction at above position”, it simply shows the value of the coefficient of friction according to the segment position in the well, considering the well path. If the segment position is inside the cased section or open wellbore section, it will show different values.

Fifth row “Numerical Friction Force in m ”, is perhaps the most important row in this section, because it represents the contact friction that will be considered when calculating the displacements in the next section. It is calculated according to the Eq. 3.84, using the parameters specified in the fourth and third row, for each segment number:

$$F_c = \Delta x * \mu_{friction} * F_{buoyancy} * \sin(Deviation) = m$$

Section 2: Is basically the part where the displacements are calculated for each and every segment of the string, at every time step. As seen in Table 3.1, this section is divided into two main columns, one is the time steps (“TIME”) and the other the displacements for each segment (“DISPLACEMENTS”).

Note that there is a more yellowish section (times: $t=-\Delta t$ and $t=0$), where the boundary conditions in time must be specified. These initial displacements must be found from the actual condition of the string, which in this case are all zero as the string is relaxed, and the heave movement transferred to the string will begin at time $t= \Delta t$.

Therefore, the numerical calculations begin, indeed, at the row where time $t=\Delta t$, from segment zero until segment N . The displacement calculation must begin from left to right, considering any boundary conditions in space for the different segments. These calculations will be explained in Section 3.3.3.

Bit Movement and Pressure Calculations Table:

This Table is complementary to the Displacements calculation Table, as it has two main purposes only.

The first, which comprises **sections 3, 4 and 5**, its main objective is to calculate the last column of section 5, which is “Sum of pressures bar ”. The detailed calculation of each column will be explained in Section 3.3.4; however a brief explanation of them is given here.

To begin with it is very important to explain that the calculations for these columns is delayed one time step compared to the “Displacement Calculations Table”, because the displacement of the last segment (N) is calculated using the “Sum of pressures” result from one time step before. This means that, for example, if the displacements are being calculated for $t=\Delta t$ for the segments 1 to $N-1$, then for calculating the displacement for the segment N , the “Sum of pressures” must be already calculated for one time step before $t = \Delta t - \Delta t$.

Section 3 calculates the average movement of the whole DP section and BHA section, considering the displacements of all the segments corresponding to each section and the amount of segments that each section has. Section 4 calculates the “Volume Flow lpm ” of each section, using the flow factors (Section 3.2.5) and the average movements of each section. Finally Section 5 calculates the “Pressures bar ” for each section, considering the outside viscous friction calculated using the pressure factors explained in section 3.2.6. Then “Sum of pressures” is calculated just by adding up the pressures calculated for each section; therefore, when multiplied by the “pressure to force factor” it gives the negative force acting at the very bottom of the string (last segment number); which is used to calculate the displacement of the last segment (N).

The second purpose of this Table, **section 6**, is to calculate and show the results basically from the whole program, which are the bit displacement “Maximum and minimum amplitude of bit m ” and the pressure fluctuations below the bit “Maximum and minimum pressure below bit bar ”. These two columns are the most important results from the Numerical Method, and will be explained and discussed in the Section 3.3.4.

3.3.3 “Displacement Calculations” Table

This section focuses only on the displacements calculations, and it will describe how the calculations were done, how the equations were used, how the boundary conditions in space were applied, which logic the program follows, etc. It is important to note, that the order in which the several calculation are explained here is the correct order in which they must be programmed for easiness and practicality.

As explained in the previous section, this Table is divided into two sections as seen in Table 3.2.; the first (green color), preliminary calculations which was explained before, and the second (yellowish color), will be explained in here. Therefore it is assumed that the Section 1 of this Table is already calculated and known.

TABLE 3.2 - SECTIONS OF THE DISPLACEMENTS CALCULATIONS TABLE

DISPLACEMENTS CALCULATIONS TABLE						
Segment Number:	0	1	N-1	N
Depth [m] (position in the well)	-					
Deviation of the well at above position	-					
Coeff. Of friction at above position	-					
Numerical Friction Force in [m]	-					
TIME	DISPLACEMENTS					
t=-Δt	0	0	0	0	0	0
t=0	0	0	0	0	0	0
t=Δt						
t=2Δt						
t=3Δt						
.						
.						
.						
t=nΔt						

This whole Table is considered as a matrix in MatLab, which is calculated from left to right and from up to down. So, this matrix has to be populated (filled up) with the required displacements in order to make the calculations.

The first operation, before calculating the displacements, is to define the time steps which are the first column in the Section 2 (Table 3.2.). As the amount of time steps, to be calculated, are directly entered by the user (*times*) is fairly simple to generate this column with a loop operator; for instance, a “for” loop with a counter that begins at 1 until *times*, where each row is the result of the *counter**Δ*t*, beginning at *t*=- Δ*t*.

The second is to define the boundary conditions in time (first two rows: *t*=-Δ*t* and *t*=0), which for this analysis will be all zeros for all the segments midpoints, as the string is relaxed. So at time *t*=Δ*t*, the top part of the string (first segment) begins to move as forced point according to the rig heave movement.

And third, as the first segment represents the point where the slips are holding the drill string, it is a forced point that depends on time only, and not on other displacements. Therefore, the displacements for this segment can be done independently for all the times, whether the other segments displacements are calculated or not. Similarly to the time steps, a simple “for” loop is used to calculate it for all the times (from *t*=Δ*t* until *t*=*n*Δ*t*) using the following equation:

$$XN_j = \left(1 - e^{-t_j/T_h} \right)^2 \left[AA_{h1} * \sin \left(2 \frac{2\pi}{T_h} t_j \right) + AA_{h2} * \sin \left(4 \frac{2\pi}{T_h} t_j - \pi/2 \right) \right] = m \quad (3.87)$$

Where, t_j is the time in msec defined in the first column of Section 2 in this Table, and XN_j is the displacement of the first segment of the drill string which is calculated at all times steps. The other variables are defined in Section 3.2.7. Note that the first parenthesis with the exponent, gives a soft start of the platform movement, which will increase as the time increases.

Finally after defining the three points, explained above, the displacements can begin to be calculated for the time $t=\Delta t$ and second segment, going row by row from left to right. These calculations were done using two “for” loops, the first counting the columns (segments) and the second counting the rows (times). Then inside both loops, the operator “Case” was used, which evaluates which segment number is in question and analyzes whether it corresponds to a change in cross section, the last segment of string, or a segment inside the section with constant cross section; in order to apply the corresponding numerical equation. The displacements, then, are calculated as follows for all segments in the drill string:

Segments inside sections with constant cross section (DP or BHA):

The segments that apply into this category are all the segments of constant cross section where at least half of the neighboring segments, closest to the segment in question, have the same cross section; whether in the DP or BHA section. As explained in Figure 3.3, Section 3.2.2, these segments are:

- From segment number 2 until *Last Segment of DP section* ($\#SN_{LAST-DP}$), for the DP section.
- From *Second segment in BHA* ($\#SN_{SECOND-BHA}$) until segment $N-1$, for the BHA section.

These segments will follow the Standard Numerical equation 2.9, but including the contact friction already calculated in the fifth row of Section 1 in Table 3.2 and the approximated solution if the friction shows inconsistencies (segment stopped due to friction) as follows, assuming that the segment in question is number j :

$$\begin{aligned}
 &\mathbf{If} \quad X_{j-1} + X_{j+1} - XG_j - [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] > X_j \\
 &\mathbf{Then} \quad XN_j = X_{j-1} + X_{j+1} - XG_j - [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] \\
 &\mathbf{Else} \quad \mathbf{if} \quad X_{j-1} + X_{j+1} - XG_j + [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] < X_j \\
 &\quad \mathbf{Then} \quad XN_j = X_{j-1} + X_{j+1} - XG_j + [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] \\
 &\quad \mathbf{Else} \quad XN_j = X_j
 \end{aligned}$$

As seen above, the last “else” represents the approximation made when the inconsistent self-contradictory situation of the segment being stopped due to friction appears, as explained in Figure 3.10; therefore this approximation can be easily changed by the 3 point parabolic approximation, explained in Figure 3.11, by applying the Eq. 3.85 to find more accurately where the segment stopped. Note that this change can be done for absolutely all the segments in the

string, so it is not practical to explain it all over again in the following displacements calculations.

Change of Cross section segment:

This segment midpoint is the closest to the change in cross section DP-BHA, and its segment number is represented by “*First segment of BHA*” ($\#SN_{FIRST-BHA}$), in Figure 3.3. Therefore, the displacement of this segment midpoint is found by applying the numerical equation for change in cross section (Eq. 2.19), considering the friction as usual; but instead of applying the relation of cross sections it will consider the displacement coefficients from Section 3.2.4, as follows.

$$\begin{aligned}
 &\mathbf{If} \quad DC_{DP-BHA} * X_{j-1} + DC_{BHA-DP} * X_{j+1} - XG_j - [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] > X_j \\
 &\mathbf{Then} \quad XN_j = DC_{DP-BHA} * X_{j-1} + DC_{BHA-DP} * X_{j+1} - XG_j - [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] \\
 &\mathbf{Else} \quad \mathbf{if} \\
 &\quad DC_{DP-BHA} * X_{j-1} + DC_{BHA-DP} * X_{j+1} - XG_j + [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] < X_j \\
 &\quad \mathbf{Then} \\
 &\quad XN_j = DC_{DP-BHA} * X_{j-1} + DC_{BHA-DP} * X_{j+1} - XG_j + [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] \\
 &\quad \mathbf{Else} \quad XN_j = X_j
 \end{aligned}$$

One must check that the previous equations are applied in the correct segment number, corresponding to the change in cross section; otherwise the displacements will be completely wrongly calculated.

Bottom of string Segment (Last segment of string)

This segment midpoint is at the very bottom of the drill string, represented by the segment number N in Figure 3.3. It will follow the equation for free end ending with a whole segment (Eq. 2.12), because the type of ending is chosen by the user (BHA_{end}) which will be considered in its final equation. Furthermore, this segment is considered as a change from BHA to MUD, similarly to a change in cross section but instead of using Displacements Coefficient from DP to BHA, it will use Displacements Coefficients from BHA to MUD calculated in Section 3.2.4.

This displacement is quite complex to calculate, as it has to consider the friction as usual, the type of ending (half or whole segment), and the force exerted over the bit (upward direction equal to “*Pressure to force factor*” times “*Sum of pressures*”) due to the pressure losses in the different sections. “*Sum of pressures*” is calculated in the Table “*Bit Movement and Pressure Calculations*” one time step before the displacement in question here, its symbol will be “ $G\Sigma P$ ”, for less complexity in the equations (G because is “*gammel*” or old, because it is calculated one time step behind).

Then the displacement for this midpoint is calculated as follows:

If $X_{j-1} + X_j - XG_j - [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] - G \sum P * PFF_{DS} > X_j$

Then

$$XN_j = (1 - BHA_{end}) [X_{j-1} + X_j - XG_j - [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] - G \sum P * PFF_{DS}] + BHA_{end} [DC_{BHA-MUD} * X_{j-1} - XG_j - G \sum P * PFF_{DS}]$$

Else if $X_{j-1} + X_j - XG_j + [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] - G \sum P * PFF_{DS} < X_j$

Then

$$XN_j = (1 - BHA_{end}) [X_{j-1} + X_j - XG_j + [\Delta x * \mu_{fj} * F_{buoyancy} * \sin(Dev_j)] - G \sum P * PFF_{DS}] + BHA_{end} [DC_{BHA-MUD} * X_{j-1} - XG_j - G \sum P * PFF_{DS}]$$

Else $XN_j = (1 - BHA_{end})X_j + BHA_{end} [DC_{BHA-MUD} * X_{j-1} - XG_j - G \sum P * PFF_{DS}]$

3.3.4 “Bit Movement and Pressure Calculations” Table

As seen in the Table 3.3, there are four sections in which this complementary Table is divided. Section 3, 4 and 5 objective is to calculate “Sum of pressures bar ” which itself is needed to calculate the displacement of the last segment of the drill string; therefore, these are calculated one time step delayed compared to the displacements calculations. On the other hand, section 6, basically calculates the results for the bit movement and pressure fluctuations below the bit, considering all previous calculations.

TABLE 3.3 - SECTIONS OF THE BIT MOVEMENT AND PRESSURE CALCULATIONS TABLE

BIT MOVEMENT AND PRESSURE CALCULATIONS TABLE														
Average movement of DP in one time step [m]	Average movement of BHA in one time step [m]	Volume Flow Drill Bit [lpm]	Volume Flow BHA [lpm]	Volume Flow DP [lpm]	Pressure Drill Bit [bar]	Pressure BHA [bar]	Pressure Dpt [bar]	SUM OF PRESSURES [bar]	Maximum Pressure below bit [bar]	Minimum Pressure below bit [bar]	Maximum Amplitude of Bit [m]	Minimum Amplitude of Bit [m]	Maximum Pressure below bit [numeric]	Minimum Pressure below bit [numeric]
3		4					5				6			

Section 3: Average movements in m

This section simply calculates the average movement of the whole segments of DP and BHA sections, at every time step. The average movement is defined as the sum of all the displacements within a given section divided into the amount of segments within the same

section. These are needed to calculate the volume flows in each section of DP or BHA later. They are calculated as follows:

For the DP section, from segment number 1 until *Last Segment of DP section* ($\#SN_{LAST-DP}$):

$$AM_{DP} = \frac{\sum_{j=1}^{j=\#SN_{LAST-DP}} X_j}{\#SN_{LAST-DP}} = m \quad (3.88)$$

For the BHA section, from *Second segment in BHA* ($\#SN_{SECOND-BHA}$) until segment $N-1$:

$$AM_{BHA} = \frac{\sum_{j=\#SN_{SECOND-BHA}}^{j=N-1} X_j}{N-\#SN_{FIRST-BHA}} = m \quad (3.89)$$

Note that neither the DP section nor BHA section, consider the segment where the change in cross section is present, as this represent the end of DP section and beginning of BHA section.

Section 4: Volume Flows lpm

The volume flows are calculated for the three sections of the drill string: bit, BHA and DP. As explained in Section 3.2.5, the absolute value of flow factors when multiplied by the displacement of the last segment at the bottom of the drill string, gives the volume flowing through the drill string at any time step and for any given section of the drill string. Therefore, to calculate the total volume flow, for each section, the contraflow factors must be included which when multiplied by the ‘‘Average Movement’’ (explained previously) will increase or decrease the flow, depending on the calculated average movements. So, these are calculated as follows:

For the drill bit, only the displacements for the last segment of the string are considered:

$$VF_{BIT} = FF_{bit} * |X_N - XG_N| = lpm \quad (3.90)$$

For the BHA section:

$$VF_{BHA} = |FF_{BHA} * (X_N - XG_N) + CFF_{BHA} * (AM_{BHA} - AMG_{BHA})| = lpm \quad (3.91)$$

Where, AMG_{BHA} is referred to the old value (one time step before the time in question) of average movement of the BHA section.

For the DP section:

$$VF_{DP} = |FF_{DP} * (X_N - XG_N) + CFF_{DP} * (AM_{DP} - AMG_{DP})| = lpm \quad (3.92)$$

Similarly, AMG_{DP} is referred to the old value (one time step before the time in question) of average movement of the DP section.

Section 5: Pressure losses bar

This section's aim is to determine the pressures due to the outside viscous friction on each section of the drill string. Even though, these are called factors, they will not be multiplied by the displacements to get the pressures. These factors, as explained in Section 3.2.6, were built using the equations for pressure losses in annulus from Table 2.1 (Drilling Data Handbook), so it is clearly seen that they depend on the volume flow. Therefore these pressure factors will be multiplied directly by the "Volume Flow" calculated in the previous section.

As the surge and swab pressure fluctuations below the bit will be greater if the viscous friction losses are larger; to make the calculations even more severe, the pressures for each section calculated here will consider only the maximum value of pressure obtained by either applying laminar or turbulent regime pressure factors.

Then the pressures for each section are calculated as follows, note that the calculation of these pressures are done at the same time step as the volume flow calculated before, still delayed one time step compared to the displacements:

For the bit, the pressure factor for the bit nozzles is calculated directly:

$$P_{BIT} = PF_{BIT} * VF_{BIT} = \text{bar} \quad (3.93)$$

For the BHA section, the maximum pressure between laminar and turbulent must be calculated, note that Eq. 3.59 must be applied when calculating the pressure loss for the laminar regime:

If $VF_{BHA} \neq 0$

Then $P_{BHA} = \max(PF_{BHA-TUR} * VF_{BHA}; PF_{BHA-LAM} * VF_{BHA} + ADD_{BHA-LAM}) = \text{bar}$

Else $P_{BHA} = 0 \text{ bar}$

For the DP section, similarly the maximum pressure must be calculated applying Eq. 3.62 for laminar regime:

If $VF_{DP} \neq 0$

Then $P_{DP} = \max(PF_{DP-TUR} * VF_{DP}; PF_{DP-LAM} * VF_{DP} + ADD_{DP-LAM}) = \text{bar}$

Else $P_{DP} = 0 \text{ bar}$

When applying the laminar pressure factors which have 2 terms (the first is factor of volume flow and the second term is volume flow independent); and the volume flow is zero, according to the equation, there would still be a pressure loss which is not correct. Therefore an "if" exception operator must be included here when programming,

After the calculation of the pressures for all the time steps is done, the last column of this Section can be calculated: "Sum of Pressures bar ($G\Sigma P$)". This is simply the sum of all the pressures

previously calculated for each time step. This value is very important because it considers all previous volume flows and pressures calculated, which themselves depend on the displacements already calculated as well. Its variable name states G , for “gammel” or old, this is due to its calculation being one time step delayed, because this value will be used to calculate the displacement of the last segment (N) when multiplied to the pressure to force factor as explained in Section 3.3.3, for the last segment of the drill string. This is the reason why these sections have to be calculated one time step delayed.

Section 6: Bit movement and pressure fluctuations

This section shows the two main results from the Numerical Method presented here, the pressures below the bit and the bit movement. Obviously these are calculated for all time steps and do not show the desired results directly, still some small operations should be done in order to calculate them and obtain the desired values for further analyses.

For the pressures below the bit in bar , it is more representative to define the maximum and minimum at each time step, therefore these are calculated as follows.

For the maximum pressure below bit in bar:

If $G\Sigma P > GP_{MAX\ BELOW\ BIT}$
Then $P_{MAX\ BELOW\ BIT} = G\Sigma P = bar$
Else $P_{MAX\ BELOW\ BIT} = GP_{MAX\ BELOW\ BIT} = bar$

For the minimum pressure below bit in bar:

If $G\Sigma P < GP_{MIN\ BELOW\ BIT}$
Then $P_{MIN\ BELOW\ BIT} = G\Sigma P = [bar]$
Else $P_{MIN\ BELOW\ BIT} = GP_{MIN\ BELOW\ BIT} = bar$

For the bit movement (Amplitude of bit) in m , the maximum and minimum are calculated as well considering only the displacement of the last segment midpoint of the string (N):

For the maximum amplitude of bit in m:

If $X_N > GAM_{MAX\ BIT}$
Then $AM_{MAX\ BIT} = X_N = m$
Else $AM_{MAX\ BIT} = GAM_{MAX\ BIT} = m$

For the minimum amplitude of bit m :

If $X_N < GAM_{MIN\ BIT}$
Then $AM_{MIN\ BIT} = X_N = m$
Else $AM_{MIN\ BIT} = GAM_{MIN\ BIT} = m$

Note that the prefix “ G ” refers to “gammel” or old that represents the value of the preceding variable, but one time step before. Both results, pressure fluctuations and bit movement, will be

further analyzed in Chapter 4, where results will be shown with figures and tables, with special emphasize of evaluating how different variables affect these results.

3.3.5 Matrix reordering and export to Excel.

This section only purpose is to explain the order in how the displacements must be calculated in MatLab, so afterwards the matrix calculated in MatLab can be reordered and exported to Excel. The only reason of having to reorder the matrix in MatLab is because the last segment (N) displacements need to be calculated considering the “Sum of Pressure” column, which is calculated one time step delayed in comparison to the other displacements. This time delay can be observed in the Table 3.4, where the red arrow indicates the order of calculation.

TABLE 3.4 - DISPLACEMENT CALCULATION ORDER IN MATLAB

Segment Number:	0	1	N-1																
Depth [m] (position in the well)	-																				
Deviation of the well at above position	-																				
Coeff. Of friction at above position	-																				
Numerical Friction Force in [m]	-																				
TIME	DISPLACEMENTS					Average movement of DP in one time step [m]	Average movement of BHA in one time step [m]	Volume Flow Drill Bit [lpm]	Volume Flow BHA [lpm]	Volume Flow DP [lpm]	Pressure Drill Bit [bar]	Pressure BHA [bar]	Pressure DP [bar]	SUM OF PRESSURES [bar]							
$t=-\Delta t$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
$t=0$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
$t=\Delta t$																					
.																					
.																					
$t=n\Delta t$																					

All these values are calculated one time step delayed.

After the above Table had been filled up with the correct values, another matrix is created for showing all the displacements properly, looking at the end like Table 3.1. Only then the matrix from MatLab is exported to Excel for further analysis. Note that the matrix in Excel show only numbers, but it strictly follows the Table 3.4 structure and format.

4. Results Analysis and Discussion

This chapter addresses the UiS Numerical method results from the drill string movement and pressure fluctuations below bit, when the drill string is wedged to the drill floor and no heave compensation is available. However, in order to obtain these two important results, there are many intermediate results from all the calculations that are important as well. Therefore, as these complementary results will not be any further analyzed nor discussed, they are presented in the Appendix 8.5, following the structure of the Flow Diagram in Figure 3.1.

Then, the main objective of this chapter is to show how the drill string movement and pressure fluctuations are affected by some important variables which are introduced directly by the user (explained in Section 3.1.2), and the change of the drill string length and deviation of the well. These variations will be calculated and presented with figures and tables. Note that all the results are based on the example well case presented in Section 3.1.1, which is a “Build and Hold” type of well. The well is drilled vertically from surface to the KOP, where the well is steadily and smoothly deflected until a maximum desired deviation. The established angle and direction are maintained while drilling to the target depth. The drill string has a 5” DP section followed by 200 m of 8” BHA section at the bottom, and a 12” drill bit. The drill string steel and drilling mud properties are considered as standard.

4.1 Important results and evaluated variables

These important results are extracted from the Tables generated with the Numerical Method:

- Bit displacement upwards m : As the positive direction is downwards for all the displacement calculations, these values are negative and are found in the column “Minimum Amplitude of Bit m ” located in Section 6, in Table 3.3.
- Bit displacement downwards m : are the values found in the column “Maximum Amplitude of Bit m ” located in Section 6, in Table 3.3.
- Swab effect below bit bar (pressure decrease): As these are pressure decreases, these values are negative and are found in the column “Minimum Pressure below bit bar ” located in Section 6, in Table 3.3.
- Surge effect below bit bar (pressure increase): are the values found in the column “Maximum Pressure below bit bar ” located in Section 6, in Table 3.3.

It is important to explain how the above results should be appreciated and understood; and which important values should be emphasized the most. Then, assuming any giving well case scenario with a defined wave environment, well path, drill string, etc. The rough results are presented in a

table similar to Table 3.1; so focusing on the four columns explained above and the time, one can build the following graphs, for the bit movement and the swab/surge pressures:

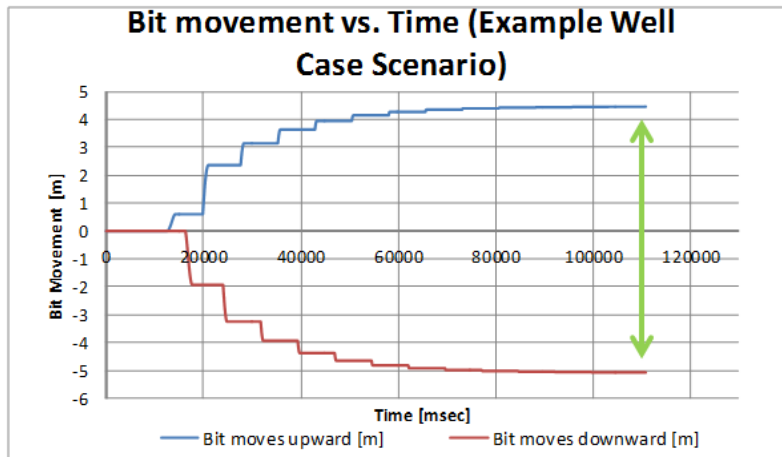


FIGURE 4.1 - BIT MOVEMENT VS TIME, FOR AN EXAMPLE WELL CASE SCENARIO

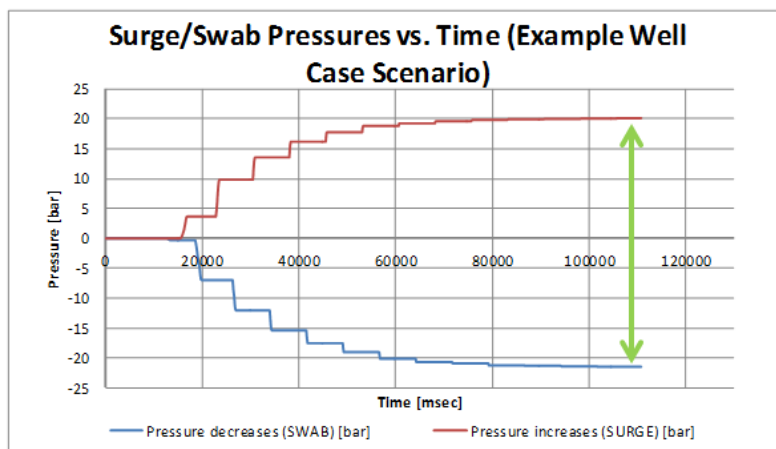


FIGURE 4.2 - PRESSURE FLUCTUATIONS VS. TIME, FOR AN EXAMPLE WELL CASE SCENARIO

In both Figures above one can clearly appreciate the trends of continuously increasing or decreasing as time goes by. This is because the forced movement of the top part of the drill string is not immediately felt at the bottom, so as the time goes by the oscillations (waves) will keep travelling downwards the string until reaching the relative maximum and minimum values. The relation between both Figures can be seen by the colors; for instance, in Figure 4.1, the upward movement of the bit (blue line) has a corresponding decreasing pressure in Figure 4.2 (blue line as well), representing the decrease in pressure (swab). The same applies for the downward movement of the bit and the corresponding increase in pressure (red line).

Even though both graphs are important to observe how the oscillations are developed in time, how the pressures are building up and the corresponding bit movement; it is of more interest to

just consider the values of pressure or bit movement when the connection of DP is finishing (adding or removing a stand). These values with respect to safety are the largest, after approx. 2 min (average time of connections, green arrows in Figures 4.1 and 4.2). So, if only these values are considered, some other graphs can be constructed, instead of “time” on the x-axis, one can introduce some other variables such as wave parameters, amount of numerical segments, etc. These important variables are explained below.

The variables that will be evaluated with respect to how they affect the bit movement and pressure fluctuations are presented in the following Table:

TABLE 4.1 - PARAMETERS ENTERED BY USER, WHICH AFFECT BIT MOVEMENT AND PRESSURE FLUCTUATION CALCULATIONS

PARAMETERS	VARIABLES	UNITS	COMMENTS
MUD – STRING PARAMETERS	Area of mud escape	$A_{MUD-ESCAPE}$ m^2	Can have two values, an absolute given area (i.e. 20 cm^2), or a relative value dependent on the wellbore (i.e. 10% of A_{well}).
	Degree of mud moving with string	$Deg_{MUD-MOVE}$ <i>fraction</i>	Is a fraction that can have a value from 0 to 1, and helps represent the viscous friction inside the drill string.
CONTRAFLOW CONDITIONS	Contraflow	<i>Contraflow</i> <i>on/off</i>	Is an on/off factor, which means that when it is turned on (value equal to 1) the “Contraflow factors” will be used to calculate the total volume flow of the section in the drill string. When it is off, the “Contraflow factors” will be zeros.
	Bit Coefficient	BIT_{coeff} <i>on/off</i>	Another on/off factor. This affects only the bit flow factor, so when it is on (value equal to 1) the area for the flow increases, otherwise when off the flow area for the bit is simply: $A_{well} - A_{MUD-ESCAPE}$
WAVE HEAVE PARAMETERS	Heave amplitude of wave	A_{hl} <i>m</i>	Both represent the most important parameters for the modeled waves and will be varied according to the North Sea environment.
	Period	T_h <i>s</i>	
NUMERICAL PARAMETERS	Amount of numerical segments	<i>N number</i>	It is freely chosen by the user and defines the amount of numerical segments in which the drill string is divided. It will be practically varied in order to appreciate its effects on the results.
	Amount of time steps	<i>Times number</i>	It is the amount of time steps that the drill string is subjected to the rig heave movement, when the connections are done. Usually one DP connection is made in 1.5 – 2 min, therefore this amount of time steps should consider the time step Δt in order to represent approximately the time spent in a connection.
	BHA type on ending	BHA_{end} <i>number</i>	It is the type of ending of the last numerical segment at the very bottom of the drill string. A value of “0” represents a half segment and “1” that is coupled to the mud.

As seen in the previous Table, there are many variables which can be changed and a lot of variations can be achieved by different combinations of them, which is not so practical. Therefore, some of these variables must be kept constant through all the calculations, but realistic values must be assigned to them in order to obtain representative results. These variables are:

- Area of mud escape will have a relative value of 10% of the bit area. Because according to the type of bit, this area might vary significantly, but in general it increases with the bit diameter. Therefore to choose an absolute value (i.e. 20 cm^2) for any kind of bit is not very realistic.
- Bit coefficient will have a value of zero (turned off), therefore the contraflow factor for the bit will consider only the area of the wellbore minus the area of mud escape, for its calculation.
- Amount of time steps is defined by the approximated time spent in making one connection. Even though it depends directly on the time step (Δt), which depends on the amount of numerical segments and speed of sound in steel, this will be calculated to approximately represent 2 minutes.
- BHA type of ending will have a value of zero (end with half segment), because when having the segment midpoint at the very bottom of the drill string, the calculations of bit displacement and pressure fluctuations at this point will be exact.

Then after having defined the previous variables, the effects of two important variables will be evaluated for a given wave condition. These are the “Degree of mud moving with string” and “Contraflow” whose effects on the bit movement and pressure fluctuations will be evaluated with non-varying wave conditions. This is explained in the Section 4.1.

Afterwards, the platform heave amplitude (assumed to follow the wave amplitude movement) and amount of numerical segments will be evaluated, given different conditions of contraflow and degree of mud moving with string. These calculations are presented in the Section 4.2.

Finally, the drill string length and well deviation will be evaluated, given the same well case scenario. This analysis is very important in order to know how the well path and drill string affects the pressure fluctuations and bit movement. The results are presented in the Section 4.3.

4.2 Effects of “Degree of mud moving with string” and “Contraflow” over the pressure below the bit and bit movement

Degree of mud moving with string:

As explained before, the mud to string friction inside the drill string is larger than on the outside due to the smaller space. In addition all the friction from the mud acts on the string inside wall. This inside mud friction can be modeled by adjusting the value of the parameter “degree of mud moving with string”. As this effect only adds mass to the string numerical segments, it is possible to calculate it exactly. Therefore, now it will be evaluated how important is this friction and how it influences the bit movement and pressure fluctuations.

The context in which this variable will be evaluated is the following:

- Wave conditions: average platform heave movement equal to 4 m and period 15 s.
- Contraflow turned off.
- String divided into 30 numerical segments.

The results are, for an approximated time of connection equal to 1.78 min :

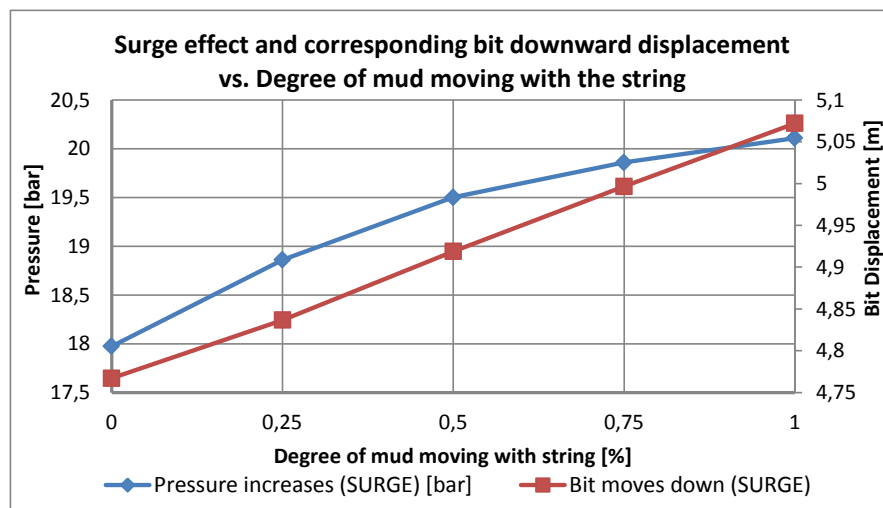


FIGURE 4.3 - SURGE EFFECT AND CORRESPONDING BIT MOVEMENT, GIVEN DIFFERENT VALUES OF THE PARAMETER “DEGREE OF MUD MOVING ALONG WITH THE STRING” FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5” DP, 200 M OF 8” BHA AND 12” BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL. (AVERAGE PLATFORM MOVEMENT EQUAL TO 4 M AND WAVE PERIOD OF 15 S)

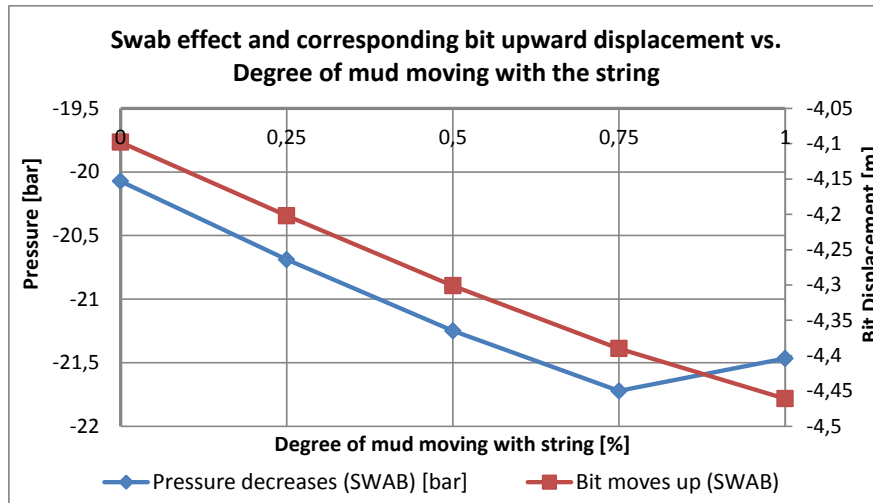


FIGURE 4.4 - SWAB EFFECT AND CORRESPONDING BIT MOVEMENT, GIVEN DIFFERENT VALUES OF THE PARAMETER “DEGREE OF MUD MOVING ALONG WITH THE STRING” FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5” DP, 200 M OF 8” BHA AND 12” BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL. (AVERAGE PLATFORM MOVEMENT EQUAL TO 4 M AND WAVE PERIOD OF 15 S)

According to the both Figures above, the surge and swab pressures are in average approx. ± 20.15 bar and the bit displacement approx. ± 4.6 m , after the drill string have been forced to move with the rig heave for 1.8 min. Even though these values are reasonable, it is not in this connection of interest to evaluate the absolute value of them, but it is rather important to analyze how the pressures and bit displacements change according to different values of the parameter “degree of mud moving along with the string”, which represent the inside viscous friction in the string.

Therefore, to see if this friction is very important an extreme form of mud friction was tried, where the all the mud inside the string is forced to follow the movement of the string ($Deg_{MUD_MOVE}=1$). As seen before, it turns out to have small, but noticeable effects. For both surge and swab effects, the increase is in average approx. 1.76 bar , which represents an increase of 8.5% from the average surge and swab pressures. Similarly for the bit displacement, the increase is in average approx. 0.33 m (7.2% increases). So, in despite the fact that this friction has a small effect over the pressure fluctuations and bit displacements; further simulations will consider the Deg_{MUD_MOVE} equal to 1, in order to obtain more conservative calculations in the sense that the calculated pressures and bit displacements will probably be larger than for the actual string.

Contraflow:

As explained in Table 4.1, this on/off variable is used to calculate the total volume flow in the different sections of the drill string. Therefore, its intend is to model an increased friction against

the drill string outer surface due to the string moving down when the mud flows upward, or due to the string moving up when the mud flows down. This gives a larger velocity difference between the mud and the drill string than when the string is stationary. However, the main part of the friction against the mud flow, giving the pressure build up below the bit, is against the stationary wall of the well because this wall has a larger surface than the string outer surface. Even though this issue of annulus flow when the drill string is moving at a significant velocity relative to the mud flow velocity needs further investigation, some preliminary results are shown here to evaluate this effect when included or not.

The context, then, in which this variable will be evaluated, is the following:

- Wave conditions: average platform heave movement equal to 4 m and period 15 s.
- Parameter “Degree of mud moving with the string” equal to 1.
- String divided into 30 numerical segments.

The results are, for an approximated time of connection equal to 1.78 min :

TABLE 4.2 - CONTRAFLOW AFFECTION UPON THE PRESSURE BELOW BIT AND BIT DISPLACEMENTS FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5” DP, 200 M OF 8” BHA AND 12” BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL.

Contraflow	Pressure increase (SURGE) [bar]	Pressure decrease (SWAB) [bar]	Bit downwards movement (SURGE) [m]	Bit upwards movement (SWAB) [m]
0 (off)	20,1097	-21,4669	5,0721	-4,461
1 (on)	30,6078	-31,7531	4,7607	-4,2238

As seen above, the pressure increases and decreases are about 50% larger than when this effect is not included. In addition, as expected, the bit movement amplitudes are reduced because this contraflow, when turned on (equal to 1), represents the additional friction against the drill string from the mud which will make the drill string oscillate less. In conclusion, as this contraflow addition to volume flow affects greatly the pressure fluctuations and is known that further research is needed to calculate realistic values of the annulus flow velocity when the drill string is significantly moving, the contraflow will be turned off (equal to zero) for all following simulations.

4.3 Effects of “Wave conditions” and “Amount of numerical segments” over the pressure below the bit and bit movement

In order to simulate the most realistic scenarios on how the drill string oscillations will be determined by different wave environments, the North Sea was chosen as it is classified as one

of the harshest and challenging offshore drilling environments. In Section 2.2.2 the North Sea environment was explained and important measured wave parameters have been presented in order to consider them here as different scenarios.

Considering Table 2.3, for 60° North Sea, the most probable wave height was 29,4 m and 14 s period. More actual information, seen in Figure 2.17, a range of 8 m to 32 m of maximum wave height is shown, with a corresponding period of 13 s to 20 s. Moreover, one must not only consider the environment characteristics; but also the drilling capabilities of the drilling rigs. For example, for the semi-submersible rig “OFFRIG PIONEER” from the company Offrig Drilling ASA, its drilling conditions are: maximum heave 10 ft and minimum period 12 s ; and for the drillship “Aban Abrahan” from the company Aban Offshore Ltd., its drilling conditions are: maximum heave 8,2 ft and minimum period 10 s.

Therefore, taking into account all the previous realistic wave parameters, and due to the assumption that the platform heave movement follows the wave amplitude; the conditions that will be simulated here will vary in the following ranges:

- Average platform heave movement: from 0,5 m to 5 m.
- Wave period: from 10 s to 16 s.

On the other hand, the amount of numerical segments (N) in which the string is divided will be included. Note that when this value is changed, the time step (Δt) also changes; for instance, if N is increased, then the space step length (Δz) and the time step (Δt) are reduced. Therefore in order to consider the approximated connection time of 2 min , the amount of time steps to be calculated (*times*) should be increased as well.

The context, then, in which the different wave conditions will be considered, is the following:

- Parameter: “Degree of mud moving with the string” equal to 1.
- Parameter: “Contraflow” turned off (equal to 0)
- String will be divided into 30, 60 and 100 numerical segments.

So, the results for different wave conditions and different numerical segments, considering an approximated connection time of 2 min , are the following:

TABLE 4.3 - SWAB/SURGE PRESSURE EFFECTS WITH THE CORRESPONDING BIT DISPLACEMENT FOR DIFFERENT WAVE CONDITIONS AND 30 NUMERICAL SEGMENTS. FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5" DP, 200 M OF 8" BHA AND 12" BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL.

Numerical Segments: N=30					
Wave Period [s]	Average Platform Heave Movement [m]	Pressure increase (SURGE) [bar]	Pressure decrease (SWAB) [bar]	Bit downwards movement (SURGE) [m]	Bit upwards movement (SWAB) [m]
10	0,5	0	0	0	0
	1	0,0413	-0,0206	0,2945	0
	2	13,615	-16,8411	3,0768	-2,7819
	2,5	20,6422	-26,3865	3,9635	-3,6707
	3	29,7099	-36,5349	4,7829	-4,4807
	3,5	39,6679	-46,9549	5,5485	-5,2311
	4	50,3722	-57,8849	6,2684	-5,9387
	5	73,2236	-80,6648	7,5901	-7,2463
12	0,5	0	0	0	0
	1	0,0096	-0,0014	0,1994	0
	2	10,0804	-10,3675	2,787	-2,3812
	2,5	13,7881	-15,5322	3,512	-3,1435
	3	18,1345	-21,5967	4,2317	-3,8789
	3,5	23,3006	-28,1404	4,9415	-4,587
	4	28,9263	-35,2119	5,6408	-5,2772
	5	42,891	-51,1806	7,0038	-6,6263
14	0,5	0	0	0	0
	1	0,0047	0	0,1434	0
	2	7,7491	-7,867	2,5806	-2,0356
	2,5	10,8331	-11,4348	3,2896	-2,7187
	3	14,1203	-15,5005	3,9489	-3,3868
	3,5	17,5123	-19,8615	4,595	-4,0486
	4	21,087	-24,4094	5,2345	-4,7029
	5	29,5814	-34,3631	6,5135	-5,9976
16	0,5	0	0	0	0
	1	0,0054	0	0,1505	0
	2	6,2176	-6,4382	2,4025	-1,7939
	2,5	8,9471	-9,6861	3,0528	-2,4365
	3	11,9244	-13,1355	3,6744	-3,0533
	3,5	15,0737	-16,4787	4,2807	-3,6538
	4	18,3314	-19,9532	4,8842	-4,2535
	5	25,0869	-27,1698	6,0939	-5,4646

TABLE 4.4 - SWAB/SURGE PRESSURE EFFECTS WITH THE CORRESPONDING BIT DISPLACEMENT FOR DIFFERENT WAVE CONDITIONS AND 60 NUMERICAL SEGMENTS. FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5" DP, 200 M OF 8" BHA AND 12" BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL.

Numerical Segments: N=60					
Wave Period [s]	Average Platform Heave Movement [m]	Pressure increase (SURGE) [bar]	Pressure decrease (SWAB) [bar]	Bit downwards movement (SURGE) [m]	Bit upwards movement (SWAB) [m]
10	0,5	0	0	0	0
	1	0,2046	-0,1024	0,3461	0
	2	13,1934	-16,9878	3,06	-2,7564
	2,5	19,8015	-26,1023	3,9105	-3,6129
	3	28,2654	-35,8875	4,7109	-4,4018
	3,5	37,8421	-46,1702	5,4658	-5,1403
	4	48,0889	-56,748	6,1787	-5,8381
	5	70,7388	-79,1042	7,4953	-7,1368
12	0,5	0	0	0	0
	1	0,0244	-0,0204	0,2293	0
	2	9,9284	-10,4284	2,7805	-2,3573
	2,5	13,4705	-15,5469	3,4853	-3,0915
	3	17,6772	-21,411	4,1858	-3,807
	3,5	22,4491	-27,8222	4,8808	-4,5013
	4	28,0147	-34,6351	5,5682	-5,1798
	5	41,0395	-49,991	6,9113	-6,5087
14	0,5	0	0	0	0
	1	0,0073	-0,0029	0,1802	0
	2	7,7328	-7,9063	2,5729	-2,012
	2,5	10,7669	-11,3387	3,2609	-2,6687
	3	14,0033	-15,2836	3,9098	-3,3211
	3,5	17,4567	-19,494	4,5473	-3,9696
	4	20,9067	-23,8585	5,1799	-4,6156
	5	29,1204	-33,6023	6,4448	-5,8984
16	0,5	0	0	0	0
	1	0,0111	-0,00062024	0,1906	0
	2	6,201	-6,4437	2,3908	-1,7691
	2,5	8,8443	-9,6015	3,022	-2,3882
	3	11,8051	-12,9894	3,6291	-2,9908
	3,5	14,8541	-16,3941	4,2258	-3,583
	4	17,9857	-19,822	4,8196	-4,1738
	5	24,6998	-26,7229	6,021	-5,376

TABLE 4.5 - SWAB/SURGE PRESSURE EFFECTS WITH THE CORRESPONDING BIT DISPLACEMENT FOR DIFFERENT WAVE CONDITIONS AND 100 NUMERICAL SEGMENTS. FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5" DP, 200 M OF 8" BHA AND 12" BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL.

Numerical Segments: N=100					
Wave Period [s]	Average Platform Heave Movement [m]	Pressure increase (SURGE) [bar]	Pressure decrease (SWAB) [bar]	Bit downwards movement (SURGE) [m]	Bit upwards movement (SWAB) [m]
10	0,5	0	0	0	0
	1	0,2288	-0,1134	0,351	0
	2	13,2971	-17,1968	3,0514	-2,7425
	2,5	19,9273	-26,2439	3,8963	-3,5962
	3	28,3112	-35,9885	4,694	-4,3826
	3,5	37,9878	-46,2604	5,4459	-5,1197
	4	48,3271	-56,9203	6,1556	-5,8148
	5	70,727	-79,1363	7,4671	-7,1102
12	0,5	0	0	0	0
	1	0,0309	-0,0249	0,2324	0
	2	10,0104	-10,5156	2,7766	-2,3481
	2,5	13,5359	-15,6382	3,479	-3,0802
	3	17,7974	-21,4697	4,1776	-3,7936
	3,5	22,6238	-27,9564	4,8701	-4,4851
	4	28,1464	-34,953	5,5551	-5,163
	5	41,3284	-50,2491	6,8942	-6,4875
14	0,5	0	0	0	0
	1	0,0069	-0,0048	0,186	0
	2	7,8103	-7,9916	2,5684	-2,0081
	2,5	10,8404	-11,4795	3,2545	-2,6625
	3	14,1306	-15,4428	3,9023	-3,3129
	3,5	17,618	-19,7157	4,5382	-3,9614
	4	21,1107	-24,0629	5,1705	-4,6064
	5	29,374	-33,8066	6,4333	-5,8881
16	0,5	0	0	0	0
	1	0,0111	-0,0014	0,1935	0
	2	6,2671	-6,519	2,3872	-1,7655
	2,5	8,9351	-9,6929	3,0163	-2,3822
	3	11,9119	-13,1113	3,6219	-2,9837
	3,5	14,9474	-16,573	4,2166	-3,5749
	4	18,1052	-19,9573	4,8108	-4,1644
	5	24,7939	-26,9347	6,0121	-5,3666

The effects of the amount of numerical segments (N) into which the string is divided is first analyzed. As there is not much difference in the pressures or bit displacements values when a certain number of N is reached, making figures instead of tables make hard to appreciate and provide accurate information regarding the differences for different N 's. As discussed earlier, this numeric method generates no error due to elastic forces within the string; the friction forces (string-mud and string-wall of well) give errors that are reduced when the space step is reduced,

meaning N is increased. This condition of no error in calculating due to elastic effects in the string is only possible when the time step (Δt) is given by $\Delta t = \Delta z/c$, where c is the speed of sound in the string material, here assumed to be steel. This determines the time step Δt when the space step Δz is chosen. Also, the total length of the drill string and of the BHA must be a whole number of $\Delta z/2$ for accurate calculations. The program therefore adjusts any given lengths to fit with these requirements, which may give actual lengths in the calculations somewhat different from the desired input lengths, as explained in Section 3.2.2.

If Table 4.3 ($N=30$) is compared to Table 4.4 ($N=60$), one can see that the pressures and bit displacements change significantly when the amount of numerical segments increases, these changes are summarized as follows:

- In general, the surge effect pressures decrease if N is increased; however the magnitude of these decreases are larger when the platform heave movement is larger and wave period shorter. For instance, the maximum pressure decrease is 2.5 bar for a platform average heave equal to 5 m and period 10 s (maximum wave amplitude and minimum period). On the other hand, the minimum decrease is 0.016 bar for a platform average heave of 2 m and period 16 s (minimum wave amplitude and maximum period). Therefore, the decreases in surge pressures obtained by increasing the amount of numerical segments will be more severe when the period is low and the platform heave movement is large.

The downward movements of the bit will, as expected, decrease in amplitudes when the amount of numerical segments increases; however, this decrease value is very small. The maximum decrease is approx. 0.1 m , when the period is the minimum (10 s) and platform heave movement is at maximum (5 m).

- In contrary, for the swab effect, when the number of numerical segments is increased the pressure will increase. Note that these pressures are negative, so when saying “increase” it means that they are less negative, but with less absolute value. As the absolute value is of more interest, this means that the swab effect is reduced when the amount of numerical segments is increased. However, the differences are not so big in magnitude; for instance, the maximum swab effect reduction is 1.56 bar when the period is 10 s and platform heave movement is 5 m (following the same relation to amplitude and period as before). Therefore, as this pressure variation is small, the upward movement variation of the bit is also expected to be without much variation.

If the same comparisons are made between Tables 4.4 ($N=60$) and 4.5 ($N=100$), one clearly see that the changes are very small, for all the wave conditions. As the maximum approx. change in pressures is 0.3 bar and 0.03 m for the bit displacement, there is no point in making further effort by dividing the string in more numerical segments. It is clear that between 60 and 100 segments the difference is negligible; however, between 30 and 60 numerical segments, the differences are

significant. Therefore is important to note that the largest variations occur when the period is low and the platform heave movement is high, corresponding to rougher weather conditions. So, as a conclusion, further analysis of the wave conditions will be done using the Table 4.4, with 60 numerical segments in order to minimize errors in calculating the friction.

To evaluate the effects of the wave period and average platform heave movement, the following figures are obtained from Table 4.4:

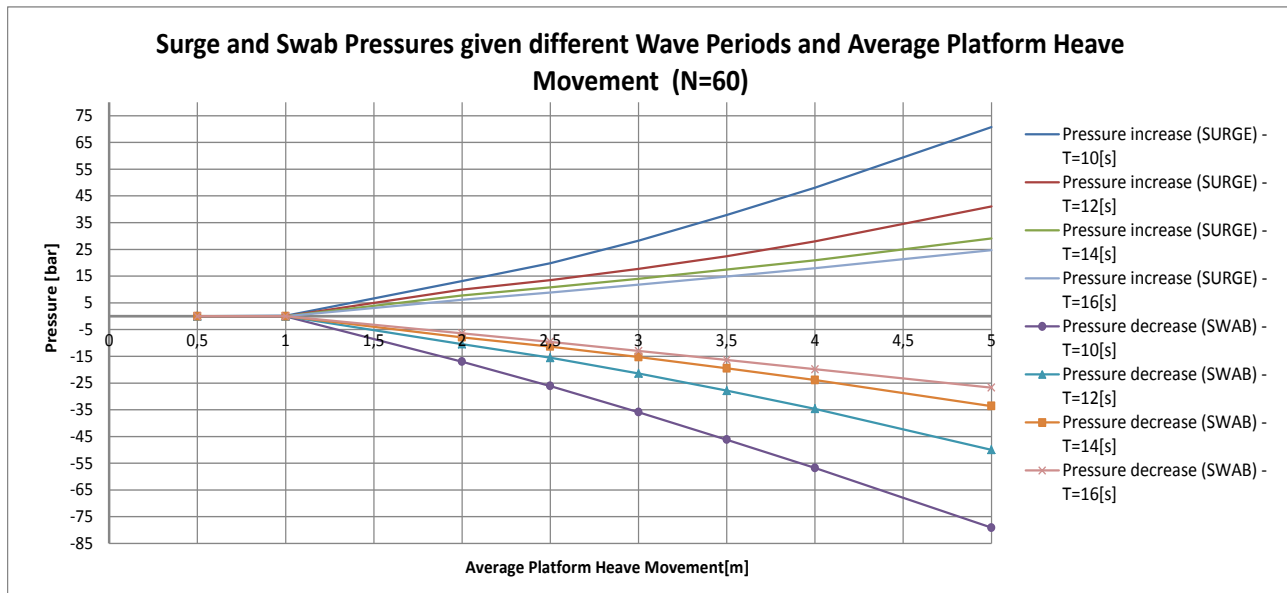


FIGURE 4.5 - PRESSURE EFFECTS FOR DIFFERENT WAVE CONDITIONS. FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5" DP, 200 M OF 8" BHA AND 12" BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL

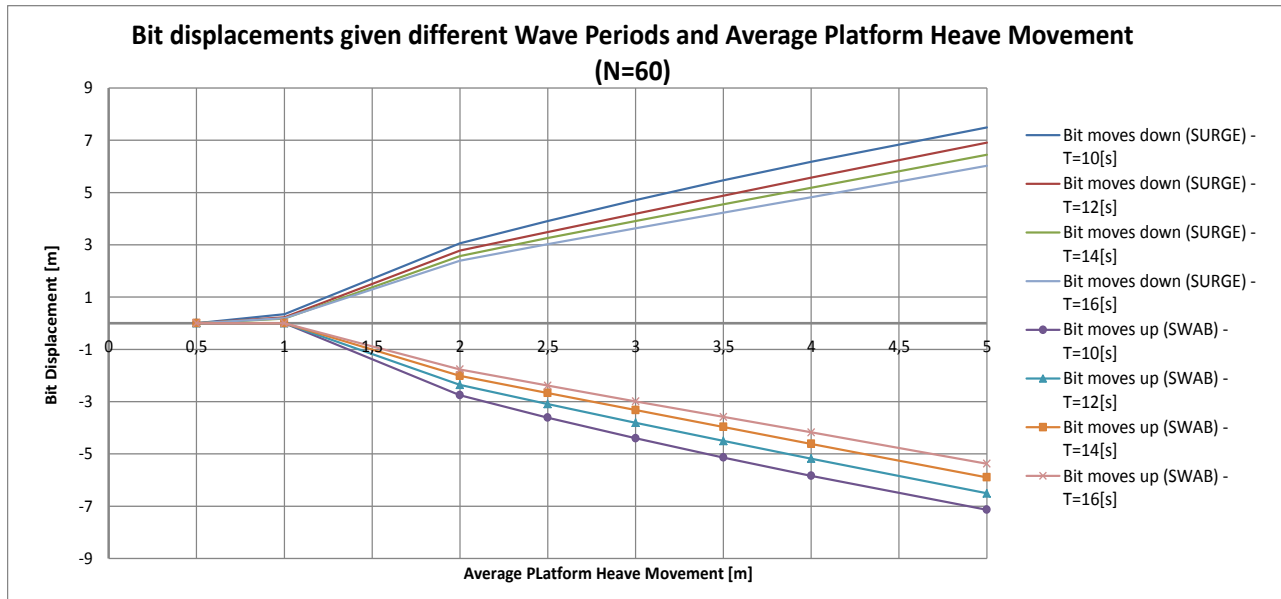


FIGURE 4.6 - BIT DISPLACEMENT FOR DIFFERENT WAVE CONDITIONS. FOR A BUILD AND HOLD DIRECTIONAL TYPE WELL GEOMETRY, WITH A 500 M VERTICAL SECTION, 500 M OF KICK OFF SECTION TO 70°. WITH A DRILL STRING MADE OF 2800 M OF 5” DP, 200 M OF 8” BHA AND 12” BIT; AND STANDARD PROPERTIES FOR DRILLING MUD AND STEEL

From the two Figures presented above, one can readily conclude and according to the theory, that when the platform heave movement is higher the pressure fluctuations are higher and therefore the bit displacements; the same occurs when the wave period is smaller. Especially, when analyzing the pressure fluctuations (Figure 4.5) it is interesting how big is the affection upon the wave period, which is in the order of ± 10 bar when the wave amplitude is 2.5 m and wave period varies from 16 s to 10 s. This pressure fluctuation increases dramatically when higher platform heaves are encountered and smaller wave periods. On the other side, the bit displacement seems to have a steady and constant increase or decrease according to the period.

4.4 Effects of “Drill String length” and “Well Deviation” upon the pressure below the bit and the bit movement

For making further analysis, the parameters “Drill String length” and “Well deviation” will be evaluated with respect to how these affect the pressure fluctuations and bit movement. For both cases, the well case scenario is the same “build and hold” type well with 500 m of vertical section until the KOP and 500 m of build-up section until the desired deviation (evaluated from 0° to 90°) and reach the target. The drill string is made of steel with a 5” DP section, 200 m of 8” BHA and a 12” drill bit. The drill string length will be changed, keeping the length of the BHA section constant, from 3000 m to 9000 m. Moreover, the parameter “Degree of mud moving with string” will be equal to 1 and “Contraflow” equal to 0. Regarding the numerical segments, the

string will be divided into 60 numerical segments. And, finally, the average platform heave movement is 4 m and wave period 12 s.

Evaluation of the “Drill String length”

In this part, as only the drill string length will be changed, the well deviation will be set equal to 70° and the parameters explained in the previous paragraph for the well case scenario will remain unchanged. The results are shown by Figure 4.7 below.

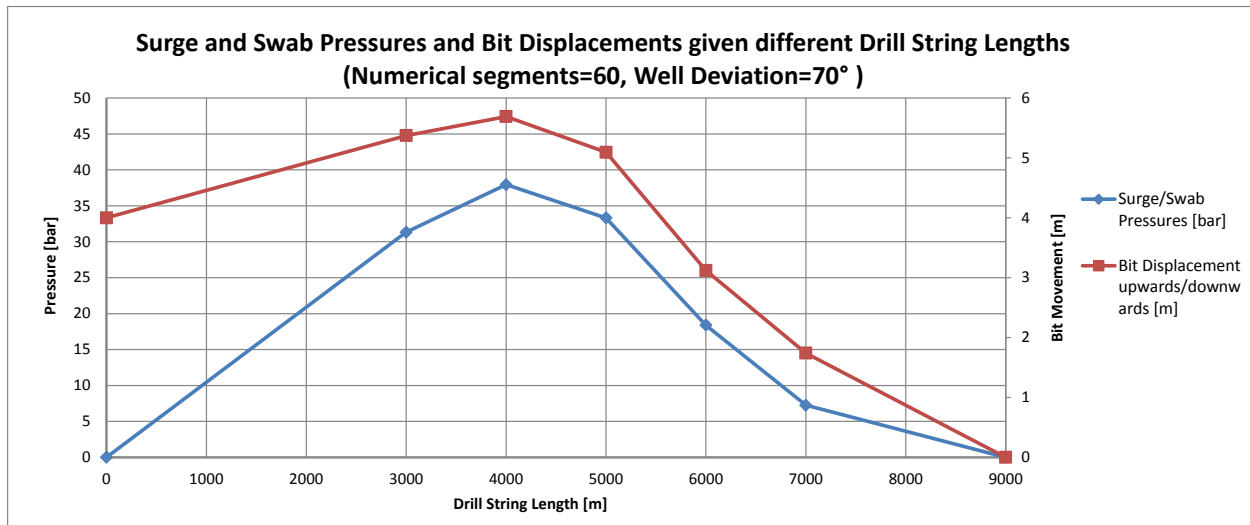


FIGURE 4.7 - SURGE/SWAB PRESURES AND BIT DISPLACEMENT GIVEN DIFFERENT DRILL STRING LENGTHS. FOR A "BUILD AND HOLD" TYPE WELL WITH A 500 M VERTICAL SECTION UNTIL THE KOP, AND A 500 M OF BUILD-UP SECTION TO REACH 70° OF WELL DEVIATION. THE DRILL STRING HAS A 5" DP SECTION, 200 M 8" BHA SECTION, AND 12" DRILL BIT. THE AVERAGE PLATFORM HEAVE MOVEMENT IS 4 M AND THE WAVE PERIOD IS EQUAL TO 12 S. THE DRILLING FLUID HAS STANDARD PROPERTIES.

In the previous figure, one can clearly observe the parabolic behavior of both the pressure fluctuations and bit movement. The pressure has a starting value of zero because no pressures can be induced when the drill string is outside the well; while, on the other hand, the bit displacement upwards and downwards starts with a value of 4 m, which is the average platform heave movement. As expected in long wells with high deviation, the pressure fluctuations and bit movements increase until reaching a maximum where the friction begins to damp down the movement of the bit, therefore also the pressure fluctuations.

Evaluation of the “Well Deviation”

In this part, the drill string length will not be changed and is set equal to 3000 m; and only the angle of deviation will change, from vertical to completely horizontal. The results are given by Figure 4.8 below.

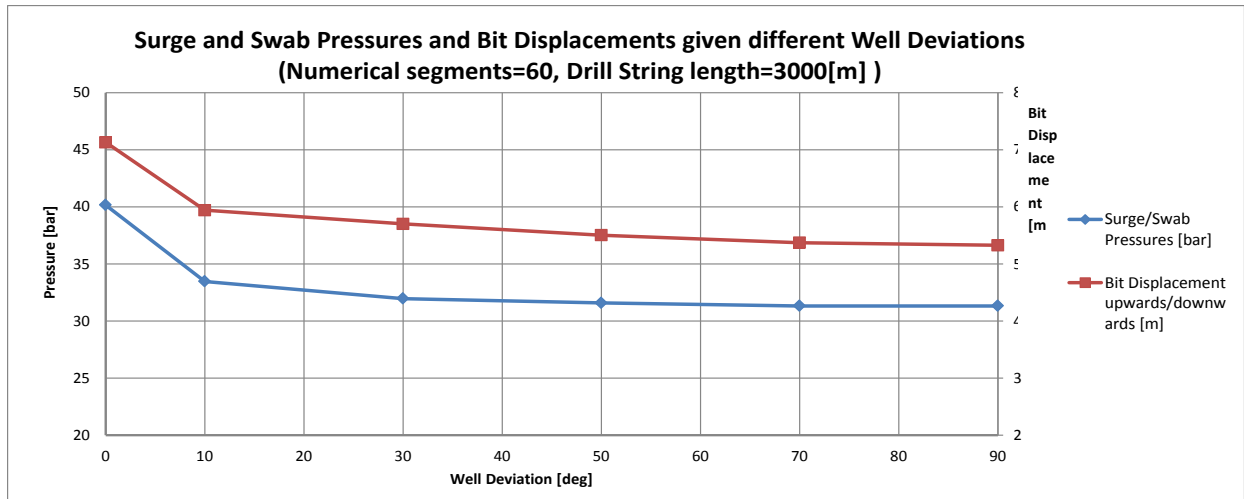


FIGURE 4.8 – SURGE/SWAB PRESSURES AND BIT DISPLACEMENT GIVEN DIFFERENT VALUES OF WELL DEVIATION. FOR A “BUILD AND HOLD” TYPE OF WELL WITH A 500 M VERTICAL SECTION UNTIL THE KOP, 500 M OF BUILD-UP SECTION TO REACH THE DESIRED WELL DEVIATION. THE DRILL STRING HAS A 2800 M 5” DP SECTION, A 200 M 8” BHA SECTION, AND A 12” DRILL BIT. THE AVERAGE PLATFORM HEAVE MOVEMENT IS 4 M AND WAVE PERIOD IS EQUAL TO 12 S. THE DRILLING FLUID HAS STANDARD PROPERTIES.

In the above figure, the biggest decrease in pressure fluctuation and bit movement occurs when the deviation changes from 0° to 10°. This happens because the program assumes a perfectly vertical well if the deviation is chosen as 0°; therefore, the contact friction would be zero as it depends upon the sine of the deviation. Then, as the well deviation increases, the contact friction increases as well; which is why we observe a decreasing tendency in both the swab/surge pressures and bit movement.

Furthermore, if the Drill String length is increased from 3000 m (in the previous Figure) to 7000 m. and the same analysis regarding the well deviation is done. Interesting results are found in order to explain better the increase of contact friction and the attenuation of the drill string oscillations as the well deviation increases towards 90°. The results are shown by Figure 4.9 below.

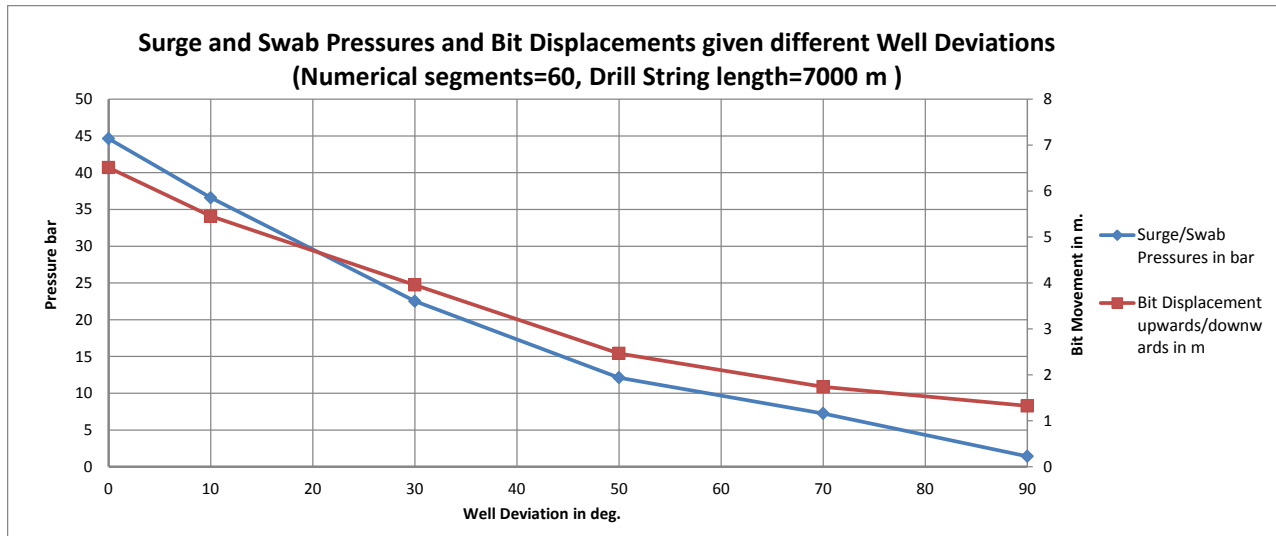


FIGURE 4.9 – SURGE/ SWAB PRESSURES AND BIT DISPLACEMENT GIVEN DIFFERENT VALUES OF WELL DEVIATION. FOR A “BUILD AND HOLD” TYPE OF WELL WITH A 500 M VERTICAL SECTION UNTIL THE KOP, 500 M OF BUILD-UP SECTION TO REACH THE DESIRED WELL DEVIATION. THE DRILL STRING HAS A 6800 M 5” DP SECTION, A 200 M 8” BHA SECTION, AND A 12” DRILL BIT. THE AVERAGE PLATFORM HEAVE MOVEMENT IS 4 M AND THE WAVE PERIOD IS EQUAL TO 12 S. THE DRILLING FLUID HAS STANDARD PROPERTIES.

By analyzing the Figure 4.9, one can readily observe that it shows better how the bit oscillation decreases towards zero as the deviation increases towards 90 degrees. As expected, due to the large drill string and high deviation the contact friction becomes so large that when the well is becoming horizontal, the pressure fluctuations and the bit oscillations are almost completely damped down.

Finally, these results show that is very important to know the magnitude of the surge and swab pressures when the drill string is hanging from the drill floor during connections (approx. 2 min), given the different well scenarios modeled in this thesis. The operator may have this information for a better planning of the operations, especially when drilling challenging reservoirs that present a narrow drilling window, as explained in Section 2.1.3. Therefore, by having these figures and weather information, one can readily find the possible surge and swab pressures below the bit when making the connections, and evaluate if these may cause kick-loss scenarios, losses or undesired kicks.

Moreover, these results can help evaluate how “safe” it is to drill in any given weather condition. In this case “safe”, would not refer to the maximum drilling conditions for a defined rig; on the other hand, it would represent a different safety limit in order to avoid wellbore instability problems, undesired well control situations, and especially undesired economic losses incurred in NPT, reservoir damage, waste of time in solving kick-loss scenarios, etc. Therefore, the question arises: Is it necessary to invest in a heave compensated drill floor in order to make challenging prospects economical and safely drilled?

5. Conclusions and Recommendations

This chapter forms the conclusion of this thesis and begins with a summary and discussion of the issues that were raised during various phases of the research work. Afterwards, the main conclusions are drawn from the results of using the UiS Numerical Method for calculating the drill string movement when it is wedged to the drill floor, its top part forced to move along with the platform heave. The chapter ends with recommendations for future research and improvements in this field.

5.1 Summary and Discussion

5.1.1 Context of the research

As the industry moves forward to improve drilling operations, apply new technologies and make challenging prospects economically drillable; this thesis main objective is to introduce a Numerical Method which might aid to a better wellbore pressure management. This pressure management is very important in the context of drilling hard reservoirs, especially in small drilling windows usually present in depleted reservoirs, HPHT wells, deep water drilling and pressurized cap rocks. Consequently, kick – loss scenarios are very frequent, which results in much NPT and might make some prospects becoming uneconomically drillable.

Therefore, as the trend is to achieve a better pressure management, for instance, with the relatively new MPD, UBO techniques; this research may represent one step to a better control and more knowledge regarding the down hole pressures. This thesis is focused on how the rig heave movement affects the pressures below the bit when the drill string is attached to the drill floor, where the induced pressure fluctuations may be sufficiently large to create kick-loss scenarios in narrow windows drilling environments.

5.1.2 Rig Movement

For a maximum workability in different weather conditions, a floating offshore drilling structure is generally designed in such a way that its vertical motion response to waves is small. However, the rig heave movement is still present and, depending on how the waves vary seasonally and regionally, may affect greatly the drilling operations. Therefore, this heave movement should be modeled as realistic as possible, in order to have a better planning of operations and to increase drilling safety.

For this thesis, a simple wave movement was modeled by using the superposition theory with two regular harmonic wave components, which had whole numbers relations between their

frequencies. Due to lack of information, it was assumed that the platform heave movement follows the wave amplitude. Therefore, as the platform heave transferred to the drill string is a function of the wave amplitude and period, several wave scenarios were analyzed according to the information regarding the wave environment in the North Sea. These scenarios will vary in wave heave amplitude (0.5 – 5 m) and wave period (10 – 16 s), in order to better represent different wave conditions.

It is important to note that more complex wave models can be used in order to better represent the North Sea waves. More accurate environment data may be found to give more accurate calculations of wave behavior at specific locations. These may lead to a construction of a map showing the most probable surge/swab pressures during connections, in different locations in the North Sea; therefore, a better drilling planning can be achieved and the possible need of a heave compensated drill floor may be better supported.

5.1.3 Inputs for the UiS Numerical Method

The inputs for the numerical calculation were divided into two groups. The first is the well case scenario data, and the second comprises the parameters that are defined by the user, which are not limited by the type of well under consideration, and where their effects on the pressure fluctuations were emphasized. All these inputs represent some limitations and simplifications made in order to calculate the pressure fluctuations and bit movement, as realistic as possible, but at the same time keeping it simple.

Well case example data:

Regarding the drill string, only a simple combination of DP, BHA and bit was considered: 2800 m of 5” DP section followed by 200 m of 8” BHA, ending with a 12” drill bit. This was done in order to have one change in diameter and only two different sections in the drill string, which makes the calculations much faster and easier; however, all diameters, weights and material can be freely chosen. What cannot be freely modified “directly” in the program is to have a more complex geometry of drill string, considering more sections, like HWDP, different materials, etc. Indeed, these changes might help making further analysis on how the stiffness of the material, sections, weights or dimensions may affect the pressure fluctuations and bit movements when the drill string is wedged to the drill floor. As explained before, changes cannot be done “directly” to the current MatLab program, because it does not allow the user to enter any desired drill string geometry directly, on the other hand, the program considers the DP and BHA sections only. Therefore, in the future it might be useful to introduce a specific section in order to let the user introduce any desired drill string geometry.

Regarding the well path, the calculations are limited to a “Build and Hold” directional type well geometry, with a 500 m vertical section followed by a 500 m of kick off section to 70 deg , then

a 2000 m straight section (with a constant deviation of 70°). All these lengths and the final deviation can also be freely chosen, but the kick off section must have a constant radius of curvature. Moreover, the distance from the bit to the well bottom, when the string is suspended on the slips, can be freely chosen.

Parameters entered by user

These are six important variables, which can be defined and changed directly by the user. They were introduced in the program in order to model specific situations which were explained in the Section 3.1.2. These variables are defined in Table 4.1 and their effects on the results were determined in Section 4.1 to 4.3, however, in the following Section 5.1.5 these are recalled. Moreover, in Section 4.4 the effects of the drill string length and well deviation upon the pressure fluctuations and bit movement were analyzed.

5.1.4 Numerical Calculations using the UiS Method

The numeric program used calculates the movements of the drill string at a number of equidistant points along the drill string, and only at equidistant points in time. The pressure fluctuations below the bit can be calculated from these calculated displacements. The method is based upon a numeric, finite element method specially designed to handle problems with long strings exposed to different external forces and/or movements of parts of the string. This method includes full and exact calculations of the string elasticity in the axial direction, and allows inclusion of linear viscous friction, and contact friction between the drill string and the walls of the well.

Calculations showed that the viscous friction on the outside of the string was very small compared to the contact friction, and a simple method using the pressure loss equations for annuli (from the Drilling Data Handbook) were used, these are presented in Table 2.1. Only contact friction was calculated accurately for the outside friction, where standard friction coefficients of 0.23 for steel against steel, and 0.3 for steel against rock were used. It is important to note that the contact friction is assumed to be zero when $\alpha=0^\circ$ (*vertical well*), which leads to an underestimation of the contact friction, because the well is never completely vertical. However, it was not regarded as an issue since the calculated amplitudes of the bit and pressure variations below bit will be larger than in reality, therefore this calculation can be regarded as conservative.

For the mud to string friction inside the drill string, a percentage of mud moving along with the string was used to evaluate the importance of this friction. Even though this friction is larger than the outside viscous friction, this friction had small effects on the pressure fluctuations, as explained in Section 4.2 (Degree of mud moving along with the string). However, the fact that the outside viscous friction effects were proven smaller than the inside mud friction, and even

smaller than the contact friction (explained in Section 3.3.1), indicates that a more accurate and complicated method to represent the outside viscous friction is not practical, as it will not change the results much.

It was also modeled an increased friction against the drill string outer surface due to the string moving down when the mud flow upward, or vice versa. This gives an increased annulus flow velocity relative to the drill string. This issue of annulus flow when the drill string is moving at a significant velocity relative to the mud flow was represented by adding the “Contraflow” parameter, which increases the pressure fluctuations about 50%. However, this result is considered as “preliminary”, because further investigation regarding this annulus flow is needed.

The increasing pressure below the bit due to friction against the mud flow upward when the bit moved down, and vice versa; strongly reduced the amplitudes of the bit movements. These were even much more reduced when the drill bit was considered as a non-leaky piston, because the elastic compression and decompression of the mud below the drill bit is then strongly opposed to the movement of the bit. But then the pressure variations below the bit became much larger due to compression of the non-escaping mud. These two effects, mud escaping by flowing up the annulus and compressed somewhat in the process, can be combined for more realistic results.

Finally, the method generates no error due to elastic forces within the string, but the friction forces at the contact surface between the string and the drilling mud, and between the string and the wall of the well, give errors that are reduced when the space step is reduced (number of segments N increased). For contact friction there are no numeric errors except for the inaccuracy in determining the moment when the drill string changes the direction it slides; this was solved by introducing the approximations explained in Section 3.3.1.

For fluid friction, even if it is due to laminar flow, there will be numeric errors that decrease if the space step length is reduced. In addition it is very complicated to calculate friction forces against the drill string when the string surface velocity keeps changing due to oscillations in the axial direction. For these preliminary calculations liquid friction was therefore included only as steady state viscous friction, giving smaller viscous friction forces than expected, and only contact friction was accurately calculated. Also, the increased maximum value of contact friction when there is no movement of the string relative to the well was neglected. This gives conservative calculations in the sense that all of the neglected friction effects will reduce the amplitudes of the string oscillations; therefore the calculated oscillations of the elastic string will probably be larger than for the actual string. The actual pressure variations at the bottom of the string will thus be smaller than the pressure variations calculated from the drill string oscillations found here.

5.1.5 Results

The most important results obtained by the application of the UiS Numerical Method are the drill string movement and pressure fluctuations below the bit. Moreover, identifying which are the most critical variables that affect these results was emphasized, in order to calculate the most realistic situation. Note that all the results are based on the well case scenario presented in Section 3.1.1. However, the length of the drill string and well deviation was also analyzed, which represent different well case scenarios.

As explained before, there are six main parameters whose criticality was evaluated:

- Area of mud escape was defined to be a constant value of 10% of the bit area, throughout all the consequent calculations. Choosing an absolute value (i.e. 20 cm²) is not too flexible, as many kinds and sizes of bits can be used.
- Bit coefficient was defined to have a value of zero (turned off) for all the consequent calculations. With this consideration the “Contraflow Factor” for the bit only considered the area of the wellbore minus the area of mud escape.
- Amount of time steps (*times*) was defined by the approximated time spent in making one connection (≈ 2 min). As it depends on the amount of numerical segments in which the string is divided (N), this amount of time steps to be calculated will vary as follows:
 - For $N = 30 \Rightarrow \Delta t = 0.02249$ s $\Rightarrow times = 120/0.02249 = 5335$ times
 - For $N = 60 \Rightarrow \Delta t = 0.01124$ s $\Rightarrow times = 120/0.01124 = 10676$ times
 - For $N = 100 \Rightarrow \Delta t = 0.006749$ s $\Rightarrow times = 120/0.006749 = 17780$ times
- BHA type of ending have a constant value of zero (end with half segment), because when having the segment midpoint at the very bottom of the drill string, the calculations of bit displacement and pressure fluctuations at this point will be exact.
- Degree of mud moving along with the string, which helped represent the mud friction inside the drill string. Given an average platform heave movement of 4 m amplitude and 15 s period, and the drill string divided into 30 numerical segments; it was shown in Figures 4.3 and 4.4, that it has a small but noticeable effect on the pressure fluctuations and bit displacement. For both surge and swab pressures, in the extreme case of making all the mud move along with the string, an increase of 8.5% from the average was seen. Similarly for the bit displacement, the increase was approx. 7.2%. Therefore, this parameter will be kept with a value equal to 1 (all mud inside drill string moves along with it), to obtain conservative results.
- Contraflow, this parameter was used to model the increased friction against the drill string outer surface due to the string moving down when the mud flows upward, or vice versa. Given an average platform heave movement of 4 m, 15 s period, and the drill string divided into 30 numerical segments; Table 4.2 shows that the surge and swab pressures are about 50% larger than when this effect is not included. Consequently, as

expected, the bit displacements are reduced because this contraflow effect increases the friction, which will make the drill string oscillate less. As this contraflow addition to volume flow greatly affects the pressure fluctuations below the bit, and this situation where the annulus volume flow velocity changes when the drill string is moving at a significantly different relative velocity needs further investigation; this contraflow effect was turned off (value equal to zero) for the further analyses.

- Amount of numerical segments (N) and wave conditions, were both evaluated together; meaning that the pressure fluctuations and bit movement were calculated considering different amounts of numerical segments (30, 60 and 90) and with different wave conditions (platform heave movement varies from 0.5 m to 5 m and period vary from 10 s to 16 s). The detailed results can be seen in Tables 4.3 to 4.5. It was explained that the numeric method generates no error due to elastic forces within the string, but the friction forces give errors which are reduced when the space step is reduced (amount of numerical segments increased). By recalculating the same situation with an increasing number of segments, it was found that a number of segments larger than 60 gave nearly the same results. For further analysis, $N=60$ was accordingly chosen to be a sufficiently accurate amount of numerical segments.

Next, the wave conditions were analyzed given that the string was divided into 60 numerical segments; the results are presented in the Table 4.4, from which Figures 4.5 and 4.6 were obtained. One can clearly observe the trend on how severe the pressure fluctuations and bit movements are when the platform heave movement increases and the period decreases. For instance, for a wave period of 14 s and platform heave of 3 m the swab/surge pressures are 14,6 bar and bit movement equal to 3,6 m; then, if the wave period decreases to 12 s for the same platform heave the swab/surge pressures are 19,5 bar and bit movement equal to 4 m. Nevertheless, at some point drilling must stop; this is decided by the drilling supervisor based on, whether the rig capabilities are being overwhelmed, or as presented here if the oscillations of the drill string in a narrow drilling window reservoir may lead to undesired kicks or losses. Thus, these pressure fluctuations are very important information for a better management of the wellbore pressures.

- Finally the drill string length and well deviation were analyzed in order to evaluate their effects on the pressure fluctuations and bit movement. The results can be seen in Figures 4.7, 4.8 and 4.9, for the same “Build and Hold” well with 5” DP, 200 m of 8” BHA and 12” Drill Bit, where the wave period was 12 s and 4 m average platform heave movement and for standard drilling mud properties. As expected for long wells with high deviation, the pressure fluctuations and bit movements increased as the drill string became longer until reaching a maximum value where the friction begins to damp down the movement of the bit, thus also the pressure fluctuations. When the well deviation was changed while keeping the drill string length constant, the pressure fluctuations and bit displacement

showed a decreasing tendency as the deviation became larger. This decrease was much larger for a long string (7000 m) than for a short one (3000 m), as expected due to the larger contact friction of the long string. As shown in Figure 4.9, for a long string, the pressure fluctuations and the bit movement are almost completely damped down as the well becomes completely horizontal.

5.2 Conclusions

Several conclusions can be drawn from the application of the UiS Numerical Method and its introduction as a tool for calculating the movement of the string:

- The UiS Numerical Method was used to calculate the pressure fluctuations and bit movements when the drill string is attached to the drill floor where no heave compensation is available. The facts that more challenging prospects are left to be drilled, technologies are evolving into a better management of wellbore pressures, and importance of precise monitor/control of pressures when the drill bit is off bottom; support the usefulness of this method. For different wave environments, well paths and drill string configurations, it may represent one step forward to a better and more precise wellbore pressure profile management in order to drill safely and efficiently (avoiding NPT, waiting on weather, losses, excessive well control situations or kick-loss scenarios).
- Considering the practical approach that has been taken to develop the UiS Numerical Method, while perhaps not definitive and given its inherent limitations and assumptions, at the very least it gives proof of the concept and provides a pretty accurate calculation of the surge/swab pressures and bit movements due to the heaving rig, whose movement is transferred to the top of the drill string; given its inherent limitations and assumptions. Also, information regarding the North Sea environment was considered and different parameters effects on the results were evaluated successfully.
- Several strengths and more flexibility are gained when programming the Numerical Method into MatLab rather than Excel. It allows the evaluation of several wave scenarios, different hydraulics considerations, drilling fluids, drill string design, well deviation, etc., and specially the ability to enter any desired amount of numerical segments in which the drill string is divided, for achieving more accuracy regarding friction considerations. Moreover, it can show readily without further complexities how all these parameters will affect the pressure fluctuations below the bit and its movement.
- Regarding the Numerical Method itself, it shows how the bottom movement of the drill string differs from the forced movement at the top, due to the drill string elasticity; as all the elastic effects and some of the effects of friction were calculated.
- The results obtained by the method can be regarded as conservative, since the real or actual pressure fluctuations will be less than the ones calculated here. Thus, it still can be used for

better planning of drilling operations, and a better understanding of the drill string oscillations, giving an enhanced wellbore pressure profile management, and a strong indication that in very sensitive reservoirs (i.e. narrow drilling window) a heave compensated drill floor may be used to drill more efficiently and safely.

5.3 Recommendations

The thesis has accomplished the initial goal of using the UiS Numerical Method to calculate the pressure fluctuations below the bit (swab/surge), when the top part of the string is forced to follow the rig movement during a stand connection. However, the possibility for further research and improvements are numerous.

The MatLab program can be more flexible regarding the well path, wave models, drill string design and the user interface. The method can be validated using field data and a full-scale experimental drilling facility, like UllRigg (Stavanger); then the method can be experimentally validated for several scenarios. The validation of the method is very important since, as results showed, a large heave motion of 2 to 3 m and periods of 10 to 16 s may result in big swab and surge pressures, which may be difficult to compensate and are very damaging for the reservoir. Therefore, by accurately quantifying the pressure fluctuations, the problem of maintaining the bottom hole pressures within acceptable limits during the make-up and break-out of the connections can be better addressed, and potential alternatives or technology may prove to be necessary when drilling challenging prospects.

Finally regarding the Numerical Method itself, the problem of annulus flow when the drill string is moving at a significant velocity relative to the mud flow velocity need further investigation.

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7. Nomenclature

ABBREVIATIONS

KOP	Kick off point
LWD	Logging While Drilling
MD	Measured Depth
MODU	Mobile Offshore Drilling Unit
MPD	Managed Pressure Drilling
MWD	Measure While Drilling
NPT	Non-Productive Time
OBM	Oil Based Mud
ROP	Rate of Penetration
SBM	Synthetic Based Mud
TTRD	Through Tubing Rotary Drilling
UBO	Under Balanced Operations
WBM	Water Based Mud
WOB	Weight on Bit
WOW	Wait on Weather

VARIABLES AND SIGNS

In MatLab	In Repport	Units	Name
A_bit_mud_escape	$A_{MUD-ESCAPE}$	m ²	Mud escape cross section area
A_rel_escape	$A_{MUD-ESCAPE}$	m ²	Mud escape cross section area
A_mud_escape_used	$A_{MUD-ESCAPE}$	m ²	Mud escape cross section area
Actual_Amp_h	AA_{h1}	m	Actual 1 st wave comp. amplitude
Actual_neg_h	ANA	m	Actual negative amplitude (down)
Actual_pos_h	APA	m	Actual positive amplitude (up)
Actual_second-Amp	AA_{h2}	m	Actual 2 nd wave comp. amplitude
Add_BHA	$ADD_{BHA-LAM}$	bar	Laminar adding factor for BHA

Heave effects on Drill String during connections

Add_DP	ADD_{DP-LAM}	bar	Laminar adding factor for DP
Adj_MDdepth_end_curved	$ADJMD_{END-CURVED}$	m	Adjusted MD at the end of curved section
Adj_depth_bit_wbottom	$ADJD_{BIT-BOTTOM}$	m	Adjusted depth from bit-well bottom
Adj_length_BHA	$ADJL_{BHA}$	m	Adjusted length of BHA
Adj_length_CSG	$ADJL_{CSG}$	m	Adjusted length of cased section
Adj_length_DP	$ADJL_{DP}$	m	Adjusted length of DP
Adj_length_curved	$ADJL_{CURVED}$	m	Adjusted length of curved section
Adj_length_dev	$ADJL_{DEV}$	m	Adjusted length of deviated section
Adj_length_vert	$ADJL_{VERT}$	m	Adjusted length of vertical section
Adj_second_wave_comp	$ADJA_{h2}$	m	Adjusted second wave comp. amp.
Amp_h	A_{h1}	m	Heave amp. of 1 st wave component
App_dens_BHA	$A\rho_{BHA}$	kg/m ³	Apparent density of BHA
App_dens_DP	$A\rho_{DP}$	kg/m ³	Apparent density of DP
BHA_Displacement_segment	$\#SN_{SECOND-BHA}$	-	# of segment where BHA begins
BHA_Displacement_coeff	DC_{BHA-DP}	-	Displacement coeff. BHA-DP
BHA_MUD_Displacement_coeff	$DC_{BHA-MUD}$	-	Displacement coeff. BHA-MUD
BHA_ending	BHA_{end}	-	Type of ending of BHA bottom
Bit_coeff	BIT_{coeff}	-	Bit Coefficient (on=1, off=0)
Bit_move_down_SURGE	-	m	Bit downwards movement (surge effect)
Bit_move_up_SWAB	-	m	Bit upwards movement (swab effect)
c_adj_BHA	$c_{adj-BHA}$	m/s	Adjusted speed of sound in BHA
c_adj_DP	c_{adj-DP}	m/s	Adjusted speed of sound in DP
c_mud	c_{MUD}	m/s	Speed of sound in drilling mud
c_steel	c_{steel}	m/s	Speed of sound in steel
CFF_BHA	CFF_{BHA}	l/min.m	Contraflow factor for BHA
CFF_DP	CFF_{DP}	l/min.m	Contraflow factor for DP
Contraflow	$Contraflow$	-	Contraflow addition to volume flow
Cross_Sec_BHA	CS_{BHA}	m ²	Material cross section of BHA
Cross_Sec_DP	CS_{DP}	m ²	Material cross section of DP

Cross_Sec_well	A_{well}	m ²	Cross section of Bit and well
DA_BHA	DA_{BHA}	m ²	Area Bit-BHA for bit contraflow
DP_Displ_segment	$\#SN_{LAST-DP}$	-	# of segment where DP ends
DP_Displ_coeff	DC_{DP-BHA}	-	Displacement coeff. DP-BHA
Damp_fac_mud	DF_{MUD}	-	Damping Factor for mud movement
Deg_mud_move	$Deg_{MUD-MOVE}$	-	Degree of mud moving with string
Dens_mud	ρ_{mud}	kg/m ³	Density of mud
Dens_steel	ρ_{steel}	kg/m ³	Density of steel
Depth_Bit_Bottom	$D_{bit-bottom}$	m	Depth from end of bit-bottom of well
Dev	Dev	deg	Deviation after curved section
Dt	Δt	msec	Time step length
Dx	Δx	m	Displacement unit
Dz	Δz	m	Space step length
Dz_BHA	Δz_{BHA}	m	Adjusted space step length for BHA
Dz_eq_mud	Δz_{MUD}	m	Equivalent space step length for mud
E	E	bar	Modulus of elasticity for steel
E_mud	E_{MUD}	bar	Equivalent modulus of elasticity for mud
FF_BHA	FF_{BHA}	l/min.m	Flow Factor for BHA
FF_DP	FF_{DP}	l/min.m	Flow Factor for DP
FF_bit	FF_{Bit}	l/min.m	Flow Factor for Bit
F_buoy	Fb	-	Buoyancy Factor for steel
Final_Adj_Length_BHA	$FADJL_{BHA}$	m	Final adjusted length of BHA
First_freq	F_{h1}	rad/msec	1 st wave comp. frequency
First_seg_BHA	$\#SN_{FIRST-BHA}$	-	First segment number in BHA
FricCoef_st_rck	$f_{steel-rock}$	-	Friction coefficient steel - rock
FricCoef_st_st	$f_{steel-steel}$	-	Friction coefficient steel – steel
In_crossec_BHA	ICS_{BHA}	m ²	Inner cross section of BHA
In_crossec_DP	ICS_{DP}	m ²	Inner cross section of DP
KOP	KOP	m	Depth to the kick off point

L_curved	L_{CURVED}	m	Length of curved section
L_dev	L_{dev}	m	Length of straight deviated section
Length_BHA	L_{BHA}	m	Length of BHA section
Length_CSG	L_{CSG}	m	Length of cased section
MDtotal_well	MD_{TOTAL}	m	Total MD of well
MUD_Displ_coeff	DC_{MUD}	-	Coefficient for mud displacement
MaxP_decrease_below_bit_SWAB		bar	Swab pressure below the bit
MaxP_increase_below_bit_SURGE		bar	Surge pressure below the bit
Min_OD_BHA	MIN_{OD-BHA}	m	Minimum outer diameter of BHA
Min_OD_DP	MIN_{OD-DP}	m	Minimum outer diameter of DP
Min_in_diam	MIN_{ID}	m	Minimum inner diameter of string
Mud_escape	-	-	Input to choose Area of mud escape
N	N	-	Amount of numerical segments
Num_segments_BHA	$N_{SEG-BHA}$	-	Number of segments in BHA section
Num_segments_CSG	$N_{SEG-CSG}$	-	Number of segments in cased section
Num_segments_DP	N_{SEG-DP}	-	Number of segments in DP section
Num_segments_below_bit	$N_{SEG-BELOW-BIT}$	-	Number of segments below the bit
Num_segments_curved	$N_{SEG-CURVED}$	-	Number of segments curved section
Num_segments_dev	$N_{SEG-DEV}$	-	Number of segments deviated sec.
Num_segments_vert	$N_{SEG-VERTICAL}$	-	Number of segments vertical section
OD_BHA	OD_{BHA}	in	Outer diameter of BHA
OD_BIT	OD_{BIT}	in	Drill bit diameter
OD_DP	OD_{DP}	in	Outer diameter of DP
PFF_DS	PFF_{DS}	m/bar	Pressure to force factor against string
PF_BHA_l	$PF_{BHA-LAM}$	bar(min/l)	Laminar P. factor for BHA
PF_BHA_t	$PF_{BHA-TUR}$	bar(min/l)1.8	Turbulent P. factor for BHA
PF_BIT_nozzle	PF_{BIT}	bar(min/l)2	Pressure factor for bit nozzles
PF_DP_l	PF_{DP-LAM}	bar(min/l)	Laminar P. factor for DP
PF_DP_t	PF_{DP-TUR}	bar(min/l)1.8	Turbulent P. factor for DP

Peak_bottom_Amp	PB_{AMP}	m	Peak to bottom wave amplitude
Second_freq	F_{h2}	rad/msec	2 nd wave comp. frequency
Second_wave_comp	A_{h2}	m	Heave amp. of 2 nd wave component
T_h	T_h	s	Wave period
Times	$Times$	-	Amount of time steps to calculate
Visc_mud	μ_{mud}	cp	Viscosity of mud
W_BHA	W_{BHA}	kg/m	Weight of BHA per unit length
W_DP	W_{DP}	kg/m	Weight of DP per unit length
W_mud_below bit	$W_{MUD-BELOW BIT}$	kg	Weight of mud below the drill bit
W_mud_in_BHA	WM_{BHA}	kg/m	Weight of mud per length unit BHA
W_mud_in_DP	WM_{DP}	kg/m	Weight of mud per length unit in DP
W_mud_in_OH	WM_{OH}	kg/m	Weight of mud in open hole
W_mud_steel_in_BHA	$W_{MUD-STEEL BHA}$	kg	Weight of mud and steel for BHA
W_mud_steel_in_DP	$W_{MUD-STEEL DP}$	kg	Weight of mud and steel for DP
YP_mud	YP_{MUD}	lb/100ft ²	Yield point of drilling mud

8. Appendixes

8.1 Derivation of the one-dimensional, second order wave equation and its general solution

The one-dimensional, second order wave equation describes the actual motion of the material in an infinitely long, straight string (tube, rod) of constant cross section, when the string material is not stressed beyond its linear elastic limit (yield limit). This material motion can be a linear motion along the string axis, or a rotation around the string axis. In both cases it is assumed that all the material molecules in any cross-section of the string performs exactly the same movement, a single parameter X is therefore sufficient to describe this movement at any given cross-section.

The parameter used to describe the material motion is usually the displacement X from a reference position, either as a displacement length along the string axis (unit: meters or any other length unit), or as a displacement angle of rotation around this axis (unit: none (radians) or degree). As all the molecules in any given cross section perform the same movement, the string behavior is completely described by specifying X as a function only of time t and position z along the axis:

$$X = X(z,t) \quad (8.1)$$

Where, the z -axis of the coordinate system is the string axis direction. The reference positions for all $X(z,t)$ are usually and most conveniently taken to be such that there is no stress in the string. That is, at time t there is no stress in the string material when $X(z,t) = 0$ for all z . This does not, however, uniquely define the reference positions, as any uniform displacement of the string, it will also give zero stress: $X(z,t) = \text{constant}$ for all z gives zero stress. Even when it is defined as: $X(z,t) = C_1t + C_2$ for all z gives zero stress in the string (all $X(z,t)$ are the same linear function of time). This gives a useful flexibility in setting up the correct equations.

Note that the equation is called one-dimensional because:

- All the material movements are in one direction only (z -direction or rotation).
- The material movement depends upon only one space coordinate, here z .

In a complete three-dimensional description of the string material movement the movements in all three independent space directions must be specified as functions of three-dimensional position and of time. Assuming a Cartesian coordinate system:

$$\begin{aligned}
 \xi &= \xi(x,y,z,t) & \xi \text{ is displacement along the } x\text{-axis} \\
 \psi &= \psi(x,y,z,t) & \psi \text{ is displacement along the } y\text{-axis} \\
 \zeta &= \zeta(x,y,z,t) & \zeta \text{ is displacement along the } z\text{-axis}
 \end{aligned} \tag{8.2}$$

Where x and y must be within the outer surface of the string, while z can have any value in an infinitely long string. To find these equations are quite complicated, one can use the much simpler Eq. (8.1) as a very good approximation in many cases. Note that X in Eq. (8.1) corresponds to ζ in Eq. (8.2), where x has been used for the space coordinate. The simplifications done in arriving at Eq. (8.1) when X is displacement along the z -axis are:

$$\begin{aligned}
 \xi &= \xi(x,y,z,t) = 0 & \text{No displacement along the } x\text{-axis} \\
 \psi &= \psi(x,y,z,t) = 0 & \text{No displacement along the } y\text{-axis} \\
 \zeta &= \zeta(x,y,z,t) = \zeta(z,t) = X(z,t) & \text{Uniform displacement along the } z\text{-axis}
 \end{aligned} \tag{8.3}$$

When X is rotation around the z -axis it is simpler to use cylinder coordinates, where any position in the string is given by distance r from string axis, angle ϕ of rotation away from a reference direction, and position z along the string axis (which is the coordinate system z -axis). The general case given by Eq. (8.2) then becomes:

$$\begin{aligned}
 \rho &= \rho(r,\phi,z,t) & \rho \text{ is displacement along the } r\text{-axis} \\
 \Phi &= \Phi(r,\phi,z,t) & \Phi \text{ is displacement along the } \phi\text{-axis} \\
 \zeta &= \zeta(r,\phi,z,t) & \zeta \text{ is displacement along the } z\text{-axis}
 \end{aligned} \tag{8.4}$$

In the case of the one-dimensional wave equation this is simplified to:

$$\begin{aligned}
 \rho &= \rho(r,\phi,z,t) = 0 & \text{No displacement along the } r\text{-axis} \\
 \Phi &= \Phi(r,\phi,z,t) = \Phi(z,t) = X(z,t) & \text{Uniform displacement along the } \phi\text{-axis} \\
 \zeta &= \zeta(r,\phi,z,t) = 0 & \text{No displacement along the } z\text{-axis}
 \end{aligned} \tag{8.5}$$

In the following discussion it is assumed that the displacement is along the z -axis, but it applies equally well to angular displacements.

The axial stress σ in the string material (stress in the z -direction) is directly proportional to the relative elongation ε (strain) in the same direction, $\sigma(z,t) = E\varepsilon(z,t)$, where E is Young's modulus of elasticity. Over a finite distance Δz the strain at time t and position z is approximately given by:

$$\varepsilon(z,t) \cong \frac{X(z + \Delta z,t) - X(z,t)}{\Delta z} \quad (8.6)$$

This is in fact an exact expression for the average strain over the distance Δz . This assumes, as mentioned before, that the reference displacement ($X = 0$) is in a string with no stress, so that $\varepsilon(z,t) = 0$ when $X(z + \Delta z,t) = X(z,t)$. When Δz goes to its limit approaching 0, the average over Δz becomes the value at z . Then at the right side of Eq. (8.6) it is given the definition of the differential of $X(z,t)$ with respect to position z :

$$\varepsilon(z,t) = \lim_{\Delta z \rightarrow 0} \frac{X(z + \Delta z,t) - X(z,t)}{\Delta z} \equiv \frac{\partial X(z,t)}{\partial z} \text{ giving } \sigma(z,t) = E \frac{\partial X(z,t)}{\partial z} \quad (8.7)$$

Figure 8.1 illustrates this, showing the displacement as a function of position along the z -axis for a given time. The wave is then “frozen” in time, hiding the dependency upon time. The figure will then show correctly the result of differentiation with respect to position z , graphically it is the tangent to the wave at the position where the differentiation is performed. This figure shows that the stress is zero at the largest and smallest displacement amplitudes, also at any local maximum and minimum. However, the displacement is not necessarily zero at the largest and/or smallest stresses. The stress is zero at positions z_{01} , z_{02} , z_{03} , z_{04} , (and far to the left and to the right of the curve). The largest stress is at position z_{MAX} , and the minimum stress (largest negative stress or compression) is at position z_{MIN} .

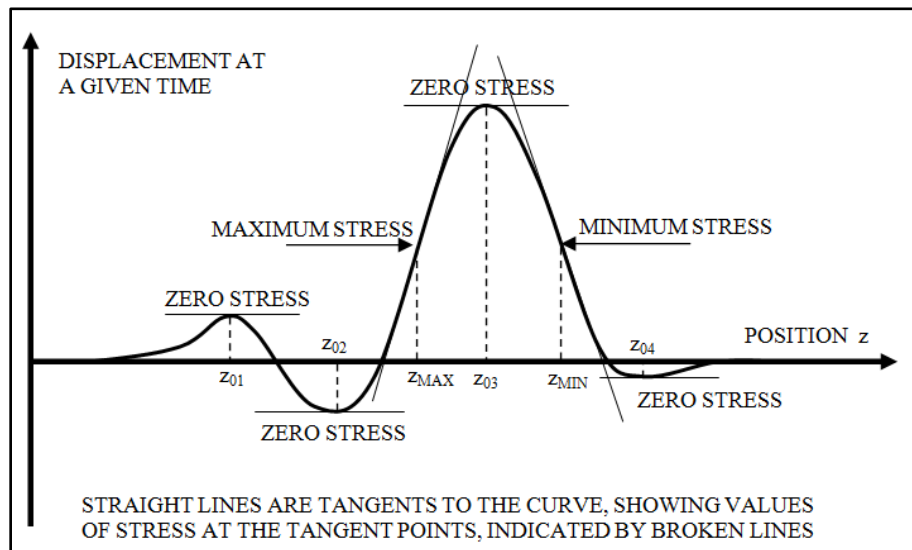


FIGURE 8.1 - DISPLACEMENT X AS A FUNCTION OF POSITION Z FOR A GIVEN (FIXED) TIME TO. RATES OF INCREASE OF TANGENTS TO THE CURVE SHOW STRESS AT THE TANGENT POINTS (WHEN MULTIPLIED BY MODULUS OF ELASTICITY).

The stress generated by the strain tries to shorten the string if the strain is positive (tension), to elongate it if it is negative (compression). Since stress is defined as force per unit area, the force

generated is given by $F_S(z,t) = A\sigma(z,t) = AE\varepsilon(z,t)$, where A is the cross section area of the string material. Consider again a short string section with length Δz . The net force $\Delta F(z,t)$ acting on this section will be the difference between the stress forces at the right and left ends of the section:

$$\Delta F(z,t) = F_S(z + \Delta z,t) - F_S(z,t) = AE\left(\frac{\partial X(z,t)}{\partial z}\right)_{z=z+\Delta z} - AE\left(\frac{\partial X(z,t)}{\partial z}\right)_{z=z} \quad (8.8)$$

This net force will accelerate the mass $\Delta m = A\Delta z\rho$ of the string section, where ρ is the density of the string material, as given by Newton's second law:

$$\Delta m a = A\Delta z\rho \frac{\partial^2 X(z,t)}{\partial t^2} \cong AE\left(\left(\frac{\partial X(z,t)}{\partial z}\right)_{z=z+\Delta z} - \left(\frac{\partial X(z,t)}{\partial z}\right)_{z=z}\right) \quad (8.9)$$

This equation is only approximately correct, as the acceleration is exactly at position z , while the net force is acting on a finite length of string, from z to $z + \Delta z$ (this equation is exact only for the average acceleration from z to $z + \Delta z$). An exact equation is obtained only in the limit when Δz goes to zero. If first divide it by the sections mass Δm , the acceleration at z is found:

$$a = \frac{\partial^2 X(z,t)}{\partial t^2} = \lim_{\Delta z \rightarrow 0} \frac{E}{\rho\Delta z} \left(\left(\frac{\partial X(z,t)}{\partial z}\right)_{z=z+\Delta z} - \left(\frac{\partial X(z,t)}{\partial z}\right)_{z=z}\right) \equiv \frac{E}{\rho} \frac{\partial^2 X(z,t)}{\partial z^2} \quad (8.10)$$

This is the one-dimensional, second order wave equation (second order because second order differentials are involved). It is often written:

$$\frac{\partial^2 X(z,t)}{\partial t^2} = c^2 \frac{\partial^2 X(z,t)}{\partial z^2}, \quad \text{where } c^2 = \frac{E}{\rho} \quad (8.11)$$

The **general solution** is:

$$X(z,t) = f(z - ct) + g(z + ct) \quad (8.12)$$

Where $f(u)$ and $g(v)$ are arbitrary functions of single variables (here: $u = z - ct$, and $v = z + ct$), the only restriction is that it must be possible to differentiate these functions. This solution is extremely general, as all possible, differentiable functions are allowed. Still, reducing the possible solution from one function $X(z,t)$ of two independent variables z and t to a sum of two functions, each of only one independent variable (u and v , respectively), is a considerable simplification.

The main property of the solution is seen as follows. The function $f(u) = f(z - ct)$, whatever it is, must have a constant value if the argument $u = z - ct = u_o = \text{constant}$. This gives that this particular value $f(u_o)$ is always found at the position along the z -axis given by $z = ct + u_o$, that is,

any possible value $f(u_0)$ of $f(u)$ is moving along the z -axis with a constant velocity $\partial z / \partial t = \partial(ct + u_0) / \partial t = c$. The same argument applies to $g(v)$, except that here the constant velocity is $-c$. This is the behavior of waves that move along the z -axis with constant shapes and constant speeds.

The general solution, given by Eq. (8.12), is therefore the sum of two waves moving with the same speed, but in opposite directions along the string axis. Of course, this wave movement cannot be detected if the displacement $X(z,t)$ is constant along the string, $X(z,t) = X(t)$. The string then is either at rest at a constant displacement ($X(z,t) = \text{constant}$), at a constant speed ($X(z,t) = at + b$), or at an accelerated movement ($X(z,t) = \text{any function of } t \text{ only, different from a constant or from } at + b$). But for any variation of $X(z,t)$ with position z , this can be observed. Moreover, no other wave velocity is possible. As sound is also a variation of displacements, it must travel at this velocity. In fact, the constant parameter c is referred to as the speed of sound in the string. Finally, an expression for the speed of sound is found, from Eq. (8.11), $c = \sqrt{E/\rho}$.

It should be noted that the solution of the wave equation for a string gives all possible movements (displacements) of the string, it is the general solution. This means that slow movements, static loading and so on are also solutions of the wave equation. The usual static calculations applied to a string being stressed by an applied load, give only an approximate solution of the problem, although in most cases of interest a very accurate solution of the final result (when a string is subjected to a static load, stress waves are set up, but due to friction these waves die out rather quickly).

The basic string parameter chosen here is the displacement X , and the waves discussed can be called displacement waves. But in practice one is really more interested in the stress in the string, the actual loading of the string material. For simplicity, it is assumed now that $g(v) = 0$, so that $X(z,t) = f(u) = f(z - ct)$. The material stress in the string is then, as discussed before, given by:

$$\sigma(z,t) = E \frac{\partial X(z,t)}{\partial z} = E \frac{\partial f(u)}{\partial z} = E \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial z} = E \frac{\partial f(u)}{\partial u} \frac{\partial(z - ct)}{\partial z} = E \frac{\partial f(u)}{\partial u} \quad (8.13)$$

Here the same argument as for the displacement can be used, for a constant value of u the stress will be constant, and this constant stress value will be found for $z = ct + u_0$. That is, it will be found at a position z that moves along the string with a velocity c . This means that a stress wave moves with the displacement wave, which is no surprise. The same argument applies to the function $g(v)$, which gives stress waves moving in the opposite direction.

It should be perfectly clear that the speed of sound in the string applies to displacement and stress waves, not to the string material. The actual velocity of the string material is defined, as usual, by:

$$v(z,t) = \frac{\partial X(z,t)}{\partial t} = \frac{\partial f(u)}{\partial t} = \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial t} = \frac{\partial f(u)}{\partial u} \frac{\partial(z-ct)}{\partial t} = -c \frac{\partial f(u)}{\partial u} \quad (8.14)$$

In all sensible circumstances the parameter $\partial f(u)/\partial u$ is much smaller than one, which gives a velocity of the material in the string much less than the speed of sound. This is seen from Eq. (8.13), a value of $\partial f(u)/\partial u$ equal to one would give a material stress equal to E . And per definition this would compress any volume of the string to zero! ($\sigma = E\Delta L/L = E$ gives $\Delta L = L$, and under compression the length of the string is reduced from L to $L - \Delta L = L - L = 0$). This is of course impossible, what this actually shows is that the theory of linear elasticity, which is an approximation to reality, breaks down under these conditions.

In practice, for steel of high quality, the stress becomes greater than the yield limit if $\partial f(u)/\partial u \geq 0.004$ (giving a stress of $\sigma = 0.004 * E = 8000$ bar for a typical value of E), and the wave equation cannot be used to describe the behavior of the string. Note that in order to avoid yield the maximum velocity of the string material will be less than $0.004 c$, which is approximately 21 m/s ($c = 5200$ m/s for steel).

It is, however, possible to describe a relaxed string traveling along with a large speed if both functions f and g are used in the solution. By including both functions in the differentiation done in Eqs. (8.13) and (8.14) gives:

$$\sigma(z,t) = E \frac{\partial X(z,t)}{\partial z} = E \frac{\partial f(u) + \partial g(v)}{\partial z} = E \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial z} + E \frac{\partial g(v)}{\partial v} \frac{\partial v}{\partial z} = E \left(\frac{\partial f(u)}{\partial u} + \frac{\partial g(v)}{\partial v} \right) \quad (8.15)$$

$$v(z,t) = \frac{\partial X(z,t)}{\partial t} = \frac{\partial f(u) + \partial g(v)}{\partial t} = \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial g(v)}{\partial v} \frac{\partial v}{\partial t} = -c \left(\frac{\partial f(u)}{\partial u} - \frac{\partial g(v)}{\partial v} \right) \quad (8.16)$$

The possibility of zero stress in the string can be achieved, while it is moving with a large velocity, by setting $\partial g(v)/\partial v = -\partial f(u)/\partial u$ which gives $\sigma(z,t) = 0$, while $v(z,t) = -2c\partial f(u)/\partial u$ which now can have any value. A simple example of this is shown below.

$$\begin{aligned} f(u) &= -520m - u = -520m - (z - ct) = ct - z - 520m & \partial f(u)/\partial u &= -1 \\ g(v) &= -520m + v = -520m + (z + ct) = ct + z - 520m & \partial g(v)/\partial v &= +1 \end{aligned} \quad (8.17)$$

$$\sigma(z,t) = E \left(\frac{\partial f(u)}{\partial u} + \frac{\partial g(v)}{\partial v} \right) = E(-1+1) = 0 \quad v(z,t) = -c \left(\frac{\partial f(u)}{\partial u} - \frac{\partial g(v)}{\partial v} \right) = -c(-1-1) = 2c$$

This can also be found by inspecting directly the displacements now obtained:

$$X(z,t) = f(u) + g(v) = ct - z - 520m + ct + z - 520m = 2ct - 1040m \quad (8.18)$$

At any given time the displacements along the string are equal, there will accordingly be no stress. Note that at $t = 0$ the displacements will be -1040 m , while at $t = 0.1\text{ s}$ they will be 0 (for steel). So they are increasing really fast, their rate of increase is, for steel, $2c = 10400\text{ m/s}$ which is nearly the escape velocity from earth (11.2 km/s). It is approximately the velocity a rocket going directly to the moon would have right outside earth's atmosphere. So it is not a completely unrealistic velocity, and it can be modeled by the wave equation. Formally, stress and velocity can now be obtained directly from X :

$$\sigma(z,t) = E \frac{\partial X(z,t)}{\partial z} = E \frac{\partial(2ct - 1040m)}{\partial z} = 0 \quad v(z,t) = \frac{\partial X(z,t)}{\partial t} = \frac{\partial(2ct - 1040m)}{\partial t} = 2c \quad (8.19)$$

The above example illustrates that the wave equation can handle a lot of possible one-dimensional problems in connection with strings, also problems that do not appear to have anything to do with waves. The physical restriction is that the stresses calculated must be within the linear elastic range of the string material.

So far it has been discussed waves on an infinitely long string, but in real life the string is always finite. This cannot be handled directly by the solution of the wave equation presented here, because the possibility of string termination is not included. A solution to this problem can be found by constructing a sum of two waves that interfere in such a way that a given part of the infinitely long string behaves in exactly the same way as an identical string, but with a finite length, would do.

For a fixed end the basic condition is that it cannot move, that is, at the end the displacement $X(z,t) = 0$ for all times t . Consider a finite string of length L , from $z = 0$ to $z = L$, and that the string ends have been fixed when the string was relaxed and with all displacement values equal to zero. Assume now that a pure wave $f(z-ct)$ is traveling along the finite string (not a sum of two waves), so that $X(z,t) = f(z-ct)$. But as this wave reach the end at $z = L$ we get into trouble, because here $X(z,t) = 0$. As the previous wave solution assumes an infinitely long string, there is nothing there to keep the end fixed, and the wave f just passes $z = L$ and travel on, now outside the region from 0 to L . The only way to obtain zero displacement at $z = L$ is to assume that another wave $h(z+ct)$ also travels along the infinitely long string, in the opposite direction, and that the sum of these waves is zero at $z = L$. This determines $h(z+ct)$, giving:

$$X(z,t) = f(z-ct) + h(z+ct) = 0 \quad , \text{ for } z = L:$$

$$X(L,t) = f(L-ct) + h(L+ct) \Rightarrow h(L+ct) = -f(L-ct) \quad (8.20)$$

As this must be true for all values of time t , it is obtained:

$$h(v) = h(L+ct) = -f(L-ct) = -f(L-(v-L)) = -f(2L-v) \quad (8.21)$$

Because $L+ct = v$ gives $ct = v-L$. The intermediary result $h(L+ct) = -f(L-ct)$ above shows clearly that the functions f and h are symmetric about $z = L$, but with opposite signs. This ensures that at $z = L$ they will always add up to zero (but not necessarily anywhere else at a given time). Figure 8.2 illustrates this.

As the wave given by $f(z-ct)$ travels up the string (here to the right - the positive direction) the wave given by $h(z+ct)$ travels down into the section from 0 to L . In this section then $f(z-ct)$ disappears at $z = L$, while an inverted copy of $f(z-ct)$ travels down the string. This is described as the original wave being reflected, but with inverted or opposite amplitudes (displacements). If a fixed end is at $z = 0$ also, the same process must repeat itself, a wave of the same shape as $h(z+ct)$, but with inverted displacements, must travel up the string, from below the string interval from 0 to L . This wave will then be an exact copy of the original wave (inverted amplitudes of inverted amplitudes gives back the original amplitudes), and it must be a distance $2L$ behind the original wave. As this process repeats itself we see that two trains of waves are required, one going up the string, the other down. Both wave trains consist of identical waves with a distance of $2L$, as shown in the figure, and one train is a mirror image of the other (actually mirrored around $z = L$, then mirrored around the z -axis to invert the amplitudes).

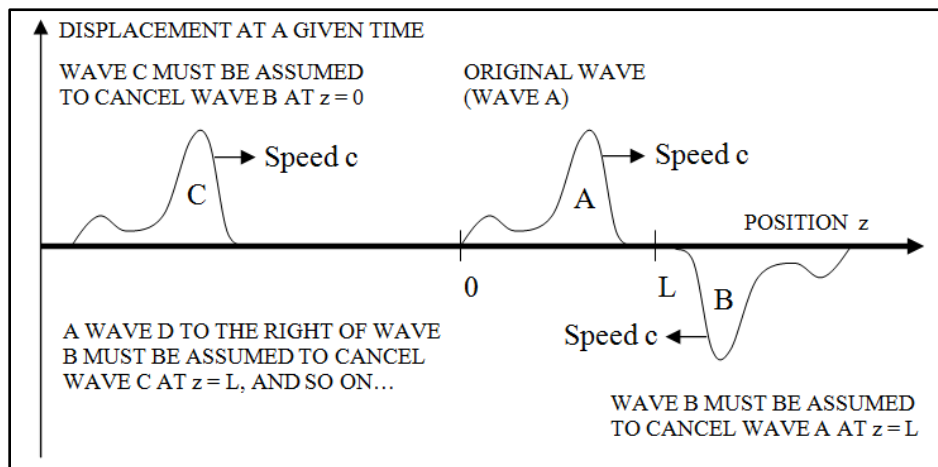


FIGURE 8.2 - STRING WITH TWO FIXED ENDS. TWO WAVE TRAINS ARE NEEDED TO GIVE ZERO DISPLACEMENT AMPLITUDES.

Assuming that the original wave $f(z-ct)$ on the finite string from 0 to L is known, then the complete equation for the displacements on an infinite string is given by:

$$X(z, t) = \sum_{j=-\infty}^{j=\infty} f(z-ct + 2jL) + \sum_{j=-\infty}^{j=\infty} h(z+ct - 2jL) = \sum_{j=-\infty}^{j=\infty} [f(2jL + z - ct) - f(2(j+1)L - (z + ct))]$$

$$\text{For } z = 0 \quad X(0, t) = \sum_{j=-\infty}^{j=\infty} [f(2jL + 0 - ct) - f(2(j+1)L - (0 + ct))] = 0 \quad (8.22)$$

$$\text{For } z = L \quad X(L, t) = \sum_{j=-\infty}^{j=\infty} [f(2jL + L - ct) - f(2(j+1)L - (L + ct))] = 0$$

Note that this is completely general, the original function $f(z-ct)$ does not need to be limited in the sense that it is non-zero over an interval which is shorter than the string length L , as shown in the figure. The equation for $z = 0$ in Eq. (8.22) may appear wrong, but since all possible values of j are considered it is correct if the value of the original wave function $f(u)$ decreases towards zero when u goes both towards very large positive and negative values.

The same arguments apply for any function $g(z+ct)$ traveling down the string, in order to model a string section with fixed ends on an infinitely long string (as required by the analytical wave solution), two trains of waves must be placed, one traveling up the string and the other down. One the finite string sections will appear as if both the original wave f and g are reflected with inverted displacement amplitudes from the fixed ends. These two waves travel independently of each other since the wave equation (without friction) is linear.

In most cases one is more interested in the stress in the string than in the displacement. The stresses due to the original wave f and the reflected wave h are, as shown before, found by differentiating with respect to z :

$$\begin{aligned} \sigma(z, t) &= E \frac{\partial X(z, t)}{\partial z} = E \frac{\partial (f(z - ct) - f(2L - (z + ct)))}{\partial z} \\ &= E \left[\left(\frac{\partial f(u)}{\partial u} \right)_{u=z-ct} \frac{\partial(z - ct)}{\partial z} - \left(\frac{\partial f(u)}{\partial u} \right)_{u=2L-z-ct} \frac{\partial(2L - (z + ct))}{\partial z} \right] \quad (8.23) \\ &= E \left[\left(\frac{\partial f(u)}{\partial u} \right)_{u=z-ct} + \left(\frac{\partial f(u)}{\partial u} \right)_{u=2L-z-ct} \right] \end{aligned}$$

It is observed that the stress amplitudes are added, not subtracted as the displacement amplitudes are. This is most clearly seen at the end ($z = L$), where $u = 2L - L - ct = L - ct$, which there is the value of u for both differentials, giving:

$$\sigma(z, t) = E \left[\left(\frac{\partial f(u)}{\partial u} \right)_{u=L-ct} + \left(\frac{\partial f(u)}{\partial u} \right)_{u=2L-L-ct} \right] = 2E \left(\frac{\partial f(u)}{\partial u} \right)_{u=L-ct} \quad (8.24)$$

At a fixed end the stress due to an incoming wave is twice what it would have been if the string had continued. This is often formulated by the rule that at a fixed end, an incoming wave is reflected unchanged. The reflected wave is added to the incoming wave, which at the fixed end gives twice the amplitude of the incoming wave. But note that this is correct only for the stress

component, the displacement wave is reflected with opposite amplitudes, giving zero displacement at the fixed end (this was the requirement at the beginning).

For a finite string with free ends a corresponding argument as presented above can be used. The requirement for a free end is that there cannot be any stress there, as there is no external mass the string end can act against. One solution is obvious from Eq's. (8.23) and (8.24). If the original wave $f(z-ct)$ is reflected at the free end with unchanged displacement amplitudes, the reflected wave is given by $f(2L-(z+ct))$, and the total stress due to both the incoming and reflected wave is given by Eq. (8.23) by changing the sign in front of f from minus to plus. At the end it is obtained from Eq. (8.24):

$$\sigma(z,t) = E \left[\left(\frac{\partial f(u)}{\partial u} \right)_{u=L-ct} - \left(\frac{\partial f(u)}{\partial u} \right)_{u=2L-L-ct} \right] = 0 \quad (8.25)$$

The full solution of a finite string with two free ends is two identical wave trains traveling along an infinitely long string, one train up the string and the other down. The only difference from the solution for fixed ends is that the two trains now have identical displacement amplitudes, not mirrored around the z -axis. Figure 8.3 shows this situation:

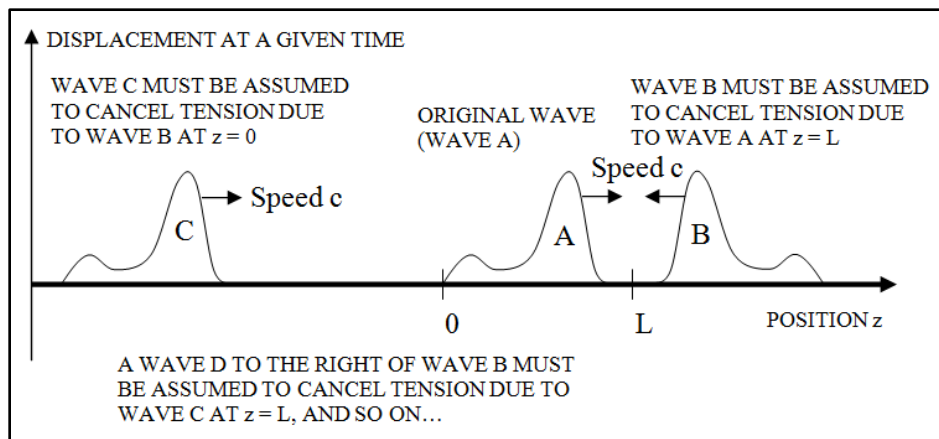


FIGURE 8.3 - STRING WITH TWO FREE ENDS, ONE AT $Z=0$ AND THE OTHER $Z=L$.

Since the distance between two components of each train is $2L$ (each component being identical to the original wave), the displacements at any given position on the string are exactly repeated when the wave train has moved this distance of $2L$, which takes a time $T = 2L/c$. This is the period of oscillation for the whole section of string (with length L). If there is one fixed and one free end the situation is somewhat different, as the wave trains now will consist of components being alternatively identical and inverted copies of the original wave, and the distance between identical copies will be $4L$. The period will therefore be twice as long, given by $T = 4L/c$.

The kinetic energy found in the original wave is given by:

$$\begin{aligned}
 E_{KIN} &= \int dE_{KIN} = \int \frac{1}{2} dm v^2 = \frac{1}{2} \int A \rho dz \left(\frac{\partial f(z-ct)}{\partial t} \right)^2 = \frac{1}{2} A \rho \int \left(\frac{\partial f(u)}{\partial u} \frac{\partial(z-ct)}{\partial t} \right)^2 dz \\
 &= \frac{1}{2} A \rho \int \left(\frac{\partial f(u)}{\partial u} (-c) \right)^2 dz = \frac{1}{2} A \rho c^2 \int \left(\frac{\partial f(u)}{\partial u} \right)^2 dz
 \end{aligned} \tag{8.26}$$

Where A is the string material cross section, ρ is the string material density and c the speed of sound in the string material. The integration should in general be over the whole infinitely long string.

The elastic energy in the original wave is given by:

$$E_{EL} = \int dE_{EL} = \int \frac{1}{2} AE \left(\frac{\partial f(z-ct)}{\partial z} \right)^2 dz = \frac{1}{2} A (\rho c^2) \int \left(\frac{\partial f(u)}{\partial u} \frac{\partial(z-ct)}{\partial z} \right)^2 dz = \frac{1}{2} A \rho c^2 \int \left(\frac{\partial f(u)}{\partial u} \right)^2 dz \tag{8.27}$$

Where $E = \rho c^2$, is Young's modulus of elasticity in the string material. But this is exactly the same result as found for the kinetic energy, the kinetic and elastic energy in a simple wave are accordingly equal. This is not the case when the complete wave is a sum of two waves traveling in opposite directions, as found on a finite string (because any initial wave will be reflected from the ends). We now find from Eq. (8.15) and (8.16):

$$E_{KIN} = \frac{1}{2} \int A \rho dz \left(\frac{\partial X(z,t)}{\partial t} \right)^2 = \frac{1}{2} A \rho \int \left(-c \left(\frac{\partial f(u)}{\partial u} - \frac{\partial g(v)}{\partial v} \right) \right)^2 dz = \frac{1}{2} A \rho c^2 \int \left(\frac{\partial f(u)}{\partial u} - \frac{\partial g(v)}{\partial v} \right)^2 dz \tag{8.28}$$

$$E_{EL} = \int \frac{1}{2} AE \left(\frac{\partial X(z,t)}{\partial z} \right)^2 dz = \frac{1}{2} A \rho c^2 \int \left(\frac{\partial f(u)}{\partial u} + \frac{\partial g(v)}{\partial v} \right)^2 dz \tag{8.29}$$

These two expressions are identical only when either $f(u)$ or $g(v)$ are equal to zero, which is the case for a simple wave. The total energy will, however, always be constant and equal to the sum of the total energy in each wave when alone:

$$E_{KIN} + E_{EL} = \frac{1}{2} A \rho c^2 \left[\int \left(\frac{\partial f(u)}{\partial u} + \frac{\partial g(v)}{\partial v} \right)^2 dz + \int \left(\frac{\partial f(u)}{\partial u} - \frac{\partial g(v)}{\partial v} \right)^2 dz \right]$$

$$= A\rho c^2 \left[\int \left(\frac{\partial f(u)}{\partial u} \right)^2 dz + \int \left(\frac{\partial g(v)}{\partial v} \right)^2 dz \right] \quad (8.30)$$

Finally friction is considered. In general it is only possible to solve differential equations if they are linear, and this is the case when the friction is linear, for instance proportional to the velocity of the string material. On the right hand side of the wave equation the sum of forces are defined, where now is added the force from friction. The friction is proportional to both the velocity and the length Δz of the piece of string considered. It is directed against the velocity, that is, if the velocity is in the positive direction the friction force is negative. This modifies Eq. (8.9) as follows:

$$A\Delta z\rho \frac{\partial^2 X(z,t)}{\partial t^2} \cong AE \left(\left(\frac{\partial X(z,t)}{\partial z} \right)_{z=z+\Delta z} - \left(\frac{\partial X(z,t)}{\partial z} \right)_{z=z} \right) - C\Delta z \frac{\partial X(z,t)}{\partial t} \quad (8.31)$$

Where, C is the constant of proportionality for friction. By dividing by $A\rho\Delta z$ and reducing Δz towards zero as done in Eq. (8.10), it is found:

$$\frac{\partial^2 X(z,t)}{\partial t^2} = \lim_{\Delta z \rightarrow 0} \frac{E}{\rho\Delta z} \left(\left(\frac{\partial X}{\partial z} \right)_{z+\Delta z} - \left(\frac{\partial X}{\partial z} \right)_z \right) - \frac{C}{A\rho} \frac{\partial X(z,t)}{\partial t} = \frac{E}{\rho} \frac{\partial^2 X(z,t)}{\partial z^2} - \frac{C}{A\rho} \frac{\partial X(z,t)}{\partial t} \quad (8.32)$$

This equation is linear and can be solved by standard methods and for different boundary conditions, for instance by Fourier analysis. Unfortunately, linear friction is not present, or at least it is not the most important form of friction in most cases of interest. Still it has been assumed to be the only type of friction present in most cases when solving problems of this type, probably mainly because it is the only form of friction possible to include in standard solution methods based upon analytical solutions.

8.2 Proof of the numerical equation for a uniform string (segment inside string and constant cross section) using the wave equation

The displacement X of the string material at any position z along a uniform string and at any time t , is given by the wave equation, defined in the previous Appendix 8.1, Eq. 8.11:

$$\frac{\partial^2 X(z,t)}{\partial t^2} = c^2 \frac{\partial^2 X(z,t)}{\partial z^2}, \quad \text{where } c = \sqrt{\frac{E}{\rho}}$$

Where, $X(z,t)$ is the displacement of the string material at position z and time t , E is Young's modulus of elasticity of the string material, and ρ is the density of the string material.

By Taylor expansion the displacement at $z_{j+1} = z_j + \Delta z$ can be calculated if the displacement at z_j is known:

$$X(z_j + \Delta z, t_i) = X(z_j, t_i) + \left(\frac{\partial X}{\partial z}\right)_{z_j, t_i} \Delta z + \frac{1}{2} \left(\frac{\partial^2 X}{\partial z^2}\right)_{z_j, t_i} \Delta z^2 + \frac{1}{6} \left(\frac{\partial^3 X}{\partial z^3}\right)_{z_j, t_i} \Delta z^3 + \frac{1}{24} \left(\frac{\partial^4 X}{\partial z^4}\right)_{z_j, t_i} \Delta z^4 + \dots \quad (8.33)$$

Where X is an abbreviation for $X(z,t)$, and the indexes z_j, t_i of the brackets indicate that the differential is evaluated at z_j and t_i . By setting $\Delta z = -\Delta z$ this gives an expression for the displacement at $z_{j-1} = z_j - \Delta z$:

$$X(z_j - \Delta z, t_i) = X(z_j, t_i) - \left(\frac{\partial X}{\partial z}\right)_{z_j, t_i} \Delta z + \frac{1}{2} \left(\frac{\partial^2 X}{\partial z^2}\right)_{z_j, t_i} \Delta z^2 - \frac{1}{6} \left(\frac{\partial^3 X}{\partial z^3}\right)_{z_j, t_i} \Delta z^3 + \frac{1}{24} \left(\frac{\partial^4 X}{\partial z^4}\right)_{z_j, t_i} \Delta z^4 - \dots \quad (8.34)$$

When all the infinite numbers of terms are included, these equations are exact. It is convenient to rewrite them by defining:

$$\begin{aligned} X_j &= X(z_j, t_i) & X_{j-1} &= X(z_j - \Delta z, t_i) & X_{j+1} &= X(z_j + \Delta z, t_i) & (8.35) \\ XG_j &= X(z_j, t_i - \Delta t) & XN_j &= X(z_j, t_i + \Delta t) \end{aligned}$$

Here an abbreviated notation for displacements in time is also shown; it will be used later on. Note that z_j and t_i are arbitrary, but fixed values of space (distance along string) and time.

It is also defined:

$${}_z X'_j = \left(\frac{\partial X}{\partial z} \right)_{z_j, t_i} \quad {}_z X''_j = \left(\frac{\partial^2 X}{\partial z^2} \right)_{z_j, t_i} \quad {}_z X_j^{(n)} = \left(\frac{\partial^n X}{\partial z^n} \right)_{z_j, t_i} \quad (8.36)$$

$${}_t X'_j = \left(\frac{\partial X}{\partial t} \right)_{z_j, t_i} \quad {}_t X''_j = \left(\frac{\partial^2 X}{\partial t^2} \right)_{z_j, t_i} \quad {}_t X_j^{(n)} = \left(\frac{\partial^n X}{\partial t^n} \right)_{z_j, t_i} \quad (8.37)$$

The two equations above ((8.33) and (8.34)) now become:

$$X_{j+1} = X_j + {}_z X'_j \Delta z + \frac{1}{2} {}_z X''_j \Delta z^2 + \frac{1}{6} {}_z X_j^{(3)} \Delta z^3 + \frac{1}{24} {}_z X_j^{(4)} \Delta z^4 + \frac{1}{120} {}_z X_j^{(5)} \Delta z^5 + \frac{1}{720} {}_z X_j^{(6)} \Delta z^6 + \dots \quad (8.38)$$

$$X_{j-1} = X_j - {}_z X'_j \Delta z + \frac{1}{2} {}_z X''_j \Delta z^2 - \frac{1}{6} {}_z X_j^{(3)} \Delta z^3 + \frac{1}{24} {}_z X_j^{(4)} \Delta z^4 - \frac{1}{120} {}_z X_j^{(5)} \Delta z^5 + \frac{1}{720} {}_z X_j^{(6)} \Delta z^6 - \dots \quad (8.39)$$

Adding these two equations gives:

$$X_{j+1} + X_{j-1} = 2X_j + {}_z X''_j \Delta z^2 + \frac{1}{12} {}_z X_j^{(4)} \Delta z^4 + \frac{1}{720} {}_z X_j^{(6)} \Delta z^6 + \dots \quad (8.40)$$

This can be solved with respect to the second differential:

$${}_z X''_j = \frac{X_{j+1} + X_{j-1} - 2X_j}{\Delta z^2} - \frac{1}{12} {}_z X_j^{(4)} \Delta z^2 - \frac{1}{720} {}_z X_j^{(6)} \Delta z^4 - \dots \quad (8.41)$$

By dropping higher order terms, including fourth order differential, a numerical equation for the second differential with respect to z is obtained:

$${}_z X''_j = \frac{X_{j-1} - 2X_j + X_{j+1}}{\Delta z^2} \quad (8.42)$$

The reason for adding the two equations is mainly that the first order differential disappears, but also that the third order differential then disappears making this numerical expression considerably more accurate.

A corresponding operation can be performed for the second order differential with respect to time, giving:

$${}_t X''_j = \frac{X_{j-1} - 2X_j + X_{j+1}}{\Delta t^2} \quad (8.43)$$

This is found by adding the two equations corresponding to Eq. (2.38) and (2.39):

$$XN_j = X_j + {}_tX'_j\Delta t + \frac{1}{2}{}_tX''_j\Delta t^2 + \frac{1}{6}{}_tX^{(3)}_j\Delta t^3 + \frac{1}{24}{}_tX^{(4)}_j\Delta t^4 + \frac{1}{120}{}_tX^{(5)}_j\Delta t^5 + \frac{1}{720}{}_tX^{(6)}_j\Delta t^6 + \dots \quad (8.44)$$

$$XG_j = X_j - {}_tX'_j\Delta t + \frac{1}{2}{}_tX''_j\Delta t^2 - \frac{1}{6}{}_tX^{(3)}_j\Delta t^3 + \frac{1}{24}{}_tX^{(4)}_j\Delta t^4 - \frac{1}{120}{}_tX^{(5)}_j\Delta t^5 + \frac{1}{720}{}_tX^{(6)}_j\Delta t^6 - \dots \quad (8.45)$$

With the simplified notation the original partial differential equation (the wave equation) becomes:

$$\frac{\partial^2 X(z,t)}{\partial t^2} = c^2 \frac{\partial^2 X(z,t)}{\partial z^2} \quad \text{Simplifies to: } {}_tX''_j = c^2 {}_zX''_j \quad , \text{ where } c = \sqrt{\frac{E}{\rho}} \quad (8.46)$$

By using the numerical approximations for ${}_tX''_j$ and ${}_zX''_j$:

$$\frac{XN_j - 2X_j + XG_j}{\Delta t^2} = c^2 \frac{X_{j-1} - 2X_j + X_{j+1}}{\Delta z^2} \quad (8.47)$$

By solving this with respect to XN_j a numerical expression for the new value of X is found:

$$XN_j = 2X_j - XG_j + \frac{\Delta t^2}{\Delta z^2} c^2 (X_{j-1} - 2X_j + X_{j+1}) \quad (8.48)$$

Setting $\Delta z = c\Delta t$ this simplifies to:

$$XN_j = 2X_j - XG_j + (X_{j-1} - 2X_j + X_{j+1}) = X_{j-1} + X_{j+1} - XG_j \quad (8.49)$$

This would usually be an inaccurate expression, with an increasing accuracy if Δz and Δt are decreased. The exact equation must include the higher order terms (all of them), and can be written as:

$$XN_j = X_{j-1} + X_{j+1} - XG_j - REST(z) + REST(t) \quad (8.50)$$

Where

$$REST(z) = \frac{1}{12}{}_zX^{(4)}_j\Delta z^4 + \frac{1}{720}{}_zX^{(6)}_j\Delta z^6 + \dots$$

$$REST(t) = \frac{1}{12}{}_tX^{(4)}_j\Delta t^4 + \frac{1}{720}{}_tX^{(6)}_j\Delta t^6 + \dots$$

It can be shown that $REST(t) = REST(z)$ if and only if $\Delta z = c\Delta t$. The exact equation then becomes:

$$XN_j = X_{j-1} + X_{j+1} - XG_j - REST(z) + REST(z) = X_{j-1} + X_{j+1} - XG_j \quad (8.51)$$

This numerical equation therefore is exact when $\Delta z = c\Delta t$. In order to prove this, the general analytical solution of the wave equation must be used (explained in Appendix 8.1), which is:

$$X(z,t) = f(z-ct) + g(z+ct)$$

where $f(u)$ and $g(v)$ are arbitrary functions of a single variable (here: $u = z - ct$, and $v = z + ct$).

$$\frac{\partial X(z,t)}{\partial t} = \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial t} + \frac{\partial g(v)}{\partial v} \frac{\partial v}{\partial t} = \frac{\partial f(u)}{\partial u} \frac{\partial(z-ct)}{\partial t} + \frac{\partial g(v)}{\partial v} \frac{\partial(z+ct)}{\partial t} = \frac{\partial f(u)}{\partial u} (-c) + \frac{\partial g(v)}{\partial v} c \quad (8.52)$$

$$\begin{aligned} \frac{\partial^2 X(z,t)}{\partial t^2} &= (-c) \frac{\partial^2 f(u)}{\partial u^2} \frac{\partial u}{\partial t} + c \frac{\partial^2 g(v)}{\partial v^2} \frac{\partial v}{\partial t} = -c \frac{\partial^2 f(u)}{\partial u^2} \frac{\partial(z-ct)}{\partial t} + c \frac{\partial^2 g(v)}{\partial v^2} \frac{\partial(z+ct)}{\partial t} \\ &= \frac{\partial^2 f(u)}{\partial u^2} (-c)^2 + \frac{\partial^2 g(v)}{\partial v^2} c^2 \end{aligned} \quad (8.53)$$

$$\text{In general:} \quad \frac{\partial^n X(z,t)}{\partial t^n} = \frac{\partial^n f(u)}{\partial u^n} (-c)^n + \frac{\partial^n g(v)}{\partial v^n} c^n \quad (8.54)$$

Differentiating with respect to z gives, by corresponding calculations:

$$\frac{\partial X(z,t)}{\partial z} = \frac{\partial f(u)}{\partial u} \frac{\partial u}{\partial z} + \frac{\partial g(v)}{\partial v} \frac{\partial v}{\partial z} = \frac{\partial f(u)}{\partial u} \frac{\partial(z-ct)}{\partial z} + \frac{\partial g(v)}{\partial v} \frac{\partial(z+ct)}{\partial z} = \frac{\partial f(u)}{\partial u} + \frac{\partial g(v)}{\partial v} \quad (8.55)$$

$$\text{In general:} \quad \frac{\partial^n X(z,t)}{\partial z^n} = \frac{\partial^n f(u)}{\partial u^n} + \frac{\partial^n g(v)}{\partial v^n} \quad (8.56)$$

By using these equations in the *REST* equations (it is here assumed that the differentials are evaluated at $z = z_j$ and $t = t_i$):

$$\begin{aligned} REST(z) &= \frac{1}{12} X_j^{(4)} \Delta z^4 + \frac{1}{720} X_j^{(6)} \Delta z^6 + \dots \\ &= \frac{1}{12} \left(\frac{\partial^4 f(u)}{\partial u^4} + \frac{\partial^4 g(v)}{\partial v^4} \right) \Delta z^4 + \frac{1}{720} \left(\frac{\partial^6 f(u)}{\partial u^6} + \frac{\partial^6 g(v)}{\partial v^6} \right) \Delta z^6 + \dots \end{aligned} \quad (8.57)$$

$$\begin{aligned} REST(t) &= \frac{1}{12} X_j^{(4)} \Delta t^4 + \frac{1}{720} X_j^{(6)} \Delta t^6 + \dots \\ &= \frac{1}{12} \left((-c)^4 \frac{\partial^4 f(u)}{\partial u^4} + c^4 \frac{\partial^4 g(v)}{\partial v^4} \right) \Delta t^4 + \frac{1}{720} \left((-c)^6 \frac{\partial^6 f(u)}{\partial u^6} + c^6 \frac{\partial^6 g(v)}{\partial v^6} \right) \Delta t^6 + \dots \\ &= \frac{1}{12} \left(\frac{\partial^4 f(u)}{\partial u^4} + \frac{\partial^4 g(v)}{\partial v^4} \right) c^4 \Delta t^4 + \frac{1}{720} \left(\frac{\partial^6 f(u)}{\partial u^6} + \frac{\partial^6 g(v)}{\partial v^6} \right) c^6 \Delta t^6 + \dots \\ &= \frac{1}{12} \left(\frac{\partial^4 f(u)}{\partial u^4} + \frac{\partial^4 g(v)}{\partial v^4} \right) \Delta z^4 + \frac{1}{720} \left(\frac{\partial^6 f(u)}{\partial u^6} + \frac{\partial^6 g(v)}{\partial v^6} \right) \Delta z^6 + \dots \end{aligned} \quad (8.58)$$

Which is identical to *REST*(z), which proves that the numerical equation is exact. Note that the term $(-c)^n$ always is positive in *REST*(t), since there are only even terms, $n = 4, 6, 8, \dots$, and that $c^n \Delta t^n = \Delta z^n$ since $c \Delta t = \Delta z$.

8.3 Physical ball spring model to find numerical equations

The physical ball-spring model is a tool for developing numerical equations for different situations in an efficient way. It can be shown that it is an exact representation of the numerical solution of the one-dimensional wave equation used here. The ball-spring model is based upon the chosen division of the string into numerical segments. In this model, it is assumed that the real string to consist of small balls positioned exactly at the segment mid points, where each ball has the mass of the whole string segment around its mid point. Thus, if the string ends with a half segment, the ball at this segment mid point (at the end of the string) will have only half the mass of the other balls. These balls are connected with mass-less springs with exactly the same elastic properties as the string (they will be extremely stiff springs). This ball-spring model is shown in the figure below, together with the string it is representing.

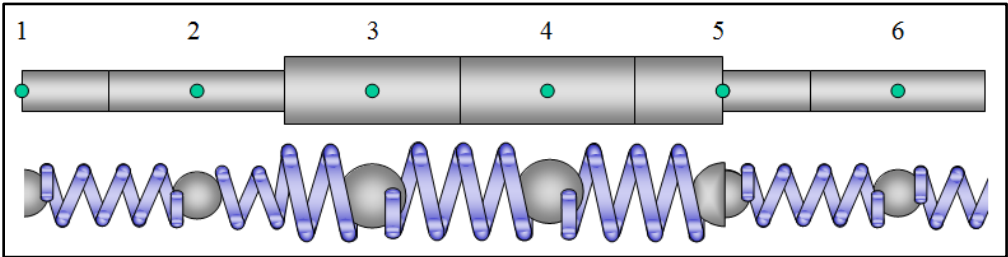


FIGURE 8.4 - PHYSICAL BALL-SPRING MODEL, REPRESENTING MIDPOINTS AND BALL MASSES.

Figure 8.4, presents a string with two free ends. The whole mass of each segment is assumed to be collected in a solid and inelastic ball at the position of the segment midpoint. At segment 1 the mass of the half segment is here shown by half a ball. At segment 5, where the segment has two different cross sections, the mass of the segment is shown as two half balls joined together. At the right end the half spring is not having any effect since it is without mass and not connected to anything at its right end.

The basic parameters of the physical ball-spring model are:

- The speed of sound in the string material: $c = \sqrt{E/\rho}$
- The density of the string material: ρ
- The modulus of elasticity of the string material: E
- The cross section of the string, may be indexed if there are several: A
- The distance between neighboring balls: Δz (8.59)
- The time step length, as given by $\Delta z/c$: Δt
- The mass of a whole segment (of length Δz): $m = A\Delta z\rho$

- The elastic force in the spring, being stretched or compressed by ΔX : $AE\Delta X/\Delta z$
- The acceleration of any ball, usually indexed by the ball number j : a_j

The expression for the elastic force is equal to that of the string, which from basic linear elastic theory is given by the stress σ times the cross section A . The stress is given by the relative elongation $\Delta X/\Delta z$ times the modulus of elasticity E . Note that by cross section, it always mean the material cross section area, which for a pipe of inner radius r and outer radius R is given by $A = \pi(R^2 - r^2)$.

The displacement of the mid point of segment j is also the displacement of ball number j (see Figure 8.4). The displacement is the distance the ball (segment mid point) moves away from its position in the initial, relaxed reference string. For ball j , X_j is its present displacement, X_{N_j} is its displacement one time step later in the future, and X_{G_j} is its displacement one time step earlier (past).

For the acceleration a numeric expression is used, based only upon the displacements. This is required, as the displacements are the only parameters being calculated directly in the numeric method used here.

$$a_j = \frac{X_{N_j} - 2X_j + X_{G_j}}{\Delta t^2} \quad (8.60)$$

The accuracy of this expression increases rapidly as Δt is decreased, in fact, the error is proportional to the second power of Δt . That is, if Δt is reduced by a factor of two, the error is reduced by a factor of four. But this does not matter so much here, because the error in this expression exactly cancel the error in representing the string as a succession of balls connected by mass-less springs.

The assumption of mass-less springs between the balls has the great advantage that there will be no oscillation and changing force from the spring when the balls are moved. The spring force is always and immediately equal to the value given by Eq. (8.59) for a relative stretching of $\Delta X/\Delta z$. The length increase $\Delta X_{j,j+1}$ between ball j and $j + 1$ is given by the displacement difference between these two balls, $\Delta X_{j,j+1} = X_{j+1} - X_j$. If X_{j+1} is larger than X_j , ball $j + 1$ has moved further in the positive direction than ball j , and the spring between them is stretched. This gives a spring force trying to pull the balls against each other; thus will pull ball j in the positive direction. If $\Delta X_{j,j+1}$ is negative, the balls are closer than in the relaxed state, and the spring is compressed, pushing the balls away from each other. This will push ball j in the negative direction. The opposite is the case for the spring to the left of ball j , when it is stretched it will pull this ball in the negative direction. The total force acting on ball j is accordingly given by:

$$F_{j,j+1} = \frac{AE}{\Delta z} \Delta X_{j,j+1} - \frac{AE}{\Delta z} \Delta X_{j-1,j} = \frac{AE}{\Delta z} [(X_{j+1} - X_j) - (X_j - X_{j-1})] = \frac{AE}{\Delta z} [X_{j+1} - 2X_j + X_{j-1}]$$

(8.61)

According to Newton's second law, assuming all forces and the acceleration are acting in the direction of the string axis: "Mass times acceleration equals the sum of forces". When there are no external forces, this gives for a segment within a string, using Eqs. (8.60) and (8.61):

$$ma_j = m \frac{XN_j - 2X_j + XG_j}{\Delta t^2} = \sum Forces = \frac{AE}{\Delta z} [X_{j+1} - 2X_j + X_{j-1}] \quad (8.62)$$

By multiplying this equation by Δt^2 and dividing by m :

$$XN_j - 2X_j + XG_j = \frac{AE\Delta t^2}{m\Delta z} [X_{j+1} - 2X_j + X_{j-1}] = X_{j+1} - 2X_j + X_{j-1} \quad (8.63)$$

where:

$$\frac{AE\Delta t^2}{m\Delta z} = \frac{AE\Delta t^2}{A\Delta z\rho\Delta z} = \frac{E}{\rho} \frac{\Delta t^2}{\Delta z^2} = c^2 \frac{1}{c^2} = 1, \text{ where } m = A\Delta z\rho \quad \frac{E}{\rho} = c^2 = \frac{\Delta z^2}{\Delta t^2} \quad (8.64)$$

By ordering Eq. (8.63) the standard equation for the new displacement is found:

$$XN_j = X_{j-1} + X_{j+1} - XG_j$$

In a similar way all the equations for free ends, fixed ends, and change of cross section, can be found from the physical ball-spring model.

8.4 MatLab program code

```
clc
clear

%PHYSICAL AND MATERIAL PARAMETERS
Dens_steel=7850;           %Density of steel kg/m3
c_steel=5172;             %Speed of sound in steel m/sec
E=Dens_steel*c_steel^2/100000 %Modulus of elasticity for steel bar
g=9.81;                   %Gravity acceleration m/sec2
Dens_mud=1200;            %Density of drilling mud kg/m3
Visc_mud=15;              %Viscosity of drilling mud cp
YP_mud=14;                %Yield Point lb/100ft2
c_mud=1000;               %Speed of sound in drilling mud m/sec
E_mud=Dens_mud*c_mud^2/100000; %Equivalent Modulus of Elasticity for mud bar
FricCoef_st_rck=0.3;      %Coefficient of friction Steel-Rock
FricCoef_st_st=0.23;     %Coefficient of friction Steel-Steel

%WELL PATH PARAMETERS AND DRILL STRING SPECIFICATION - Deviated well with buildup section and a straight deviated section
Length_BHA=200;           %Length of BHA section m
Length_CSG=1200;          %Length of Cased section m
MDtotal_well=3000;        %Total MD of well m
KOP=500;                  %Depth to kick off point m
L_curved=500;             %Length of curved section m
Dev=70;                   %Deviation after curved section deg
L_dev=2000;               %Length of straight deviated section m

Min_in_diam=0.06;         %Minimum inner diameter of string m
OD_DP=5;                  %Outer Diameter of DP in
OD_BHA=8;                 %Outer Diameter of BHA in
W_DP=30;                  %Weight of DP kg/m
W_BHA=200;                %Weight of BHA kg/m
OD_BIT=12;                %Drill bit diameter in
Depth_Bit_Bottom=20;     %Depth from bit to well bottom, when string on slips m

%CALCULATED GEOMETRY AND DRILL STRING VALUES
Cross_Sec_DP=W_DP/Dens_steel; %Material cross section of DP m2
```

Heave effects on Drill String during connections

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Cross_Sec_BHA=W_BHA/Dens_steel;           %Material cross section of BHA m2
Cross_well=pi*(OD_BIT*0.0254/2)^2;       %Cross section of bit and well m2

Min_OD_DP=sqrt(4*W_DP/(pi*Dens_steel)+Min_in_diam^2); %Minimum outer diameter of DP using its weight and minimum inner diameter of
string m
Min_OD_BHA=sqrt(4*W_BHA/(pi*Dens_steel)+Min_in_diam^2); %Minimum outer diameter of BHA using its weight and minimum inner diameter of
string m
In_crossec_DP=pi*(max(OD_DP*0.0254,Min_OD_DP))^2/4-W_DP/Dens_steel; %Inner cross section of DP m2
In_crossec_BHA=pi*(max(OD_BHA*0.0254,Min_OD_BHA))^2/4-W_BHA/Dens_steel; %Inner cross section of BHA m2

W_mud_in_DP=In_crossec_DP*Dens_mud; %Weight of mud per length unit in DP kg/m
W_mud_in_BHA=In_crossec_BHA*Dens_mud; %Weight of mud per length unit in BHA kg/m
W_mud_in_OH=pi*(OD_BIT*0.0254/2)^2*Dens_mud; %Weight of mud per length of open hole kg/m

%VALUES ENTERED BY USER!
A_bit_mud_escape=20; %Bit mud escape cross section area cm2
A_rel_escape=0.1; %Relative escape cross section area
Deg_mud_move=1; %Degree of mud moving with string (MAX=1, MIN=0)
Damp_fac_mud=0; %Damping factor for mud movement

App_dens_DP=Dens_steel+Deg_mud_move*W_mud_in_DP/Cross_Sec_DP; %Apparent density of DP material, including mud kg/m3
App_dens_BHA=Dens_steel+Deg_mud_move*W_mud_in_BHA/Cross_Sec_BHA; %Apparent density of BHA material, including mud kg/m3

%Choose the Mud Escape cross section are that will be used
Mud_escape=input('Type the number of the options: 1 Bit mud escape cross section area; 2 RELATIVE escape cross section area:');
if Mud_escape==1
    A_mud_escape_used=A_bit_mud_escape/10000; %Mud escape cross section area, using Bit mud escape cross section area m2
else
    A_mud_escape_used=A_rel_escape*pi*(OD_BIT*0.0254/2)^2; %Mud escape cross section area, using Relative Escape cross section area m2
end

%SEGMENTS AND NUMERICAL PARAMETERS
N=input('Introduce the Number of desired numerical segments:');
Dz=MDtotal_well/N; %Space step length m
c_adj_DP=c_steel*sqrt(Dens_steel/App_dens_DP); %Adjusted speed of sound in DP using apparent Density of DP(including mud) m/s
c_adj_BHA=c_steel*sqrt(Dens_steel/App_dens_BHA); %Adjusted speed of sound in BHA using apparent Density of BHA(including mud) m/s
Dt=Dz*1000/c_adj_DP; %Time step length msec (Using only c_adj_DP!!!) THE SAME TIME STEP FOR MUD!!!!
F_buoy=1-Dens_mud/Dens_steel; %Buoyancy factor
Dx=g*(Dt/1000)^2; %Displacement unit m

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Heave effects on Drill String during connections

Num_segments_BHA=round(Length_BHA/Dz); %Number of segments in BHA
 Num_segments_CSG=round(Length_CSG/Dz); %Number of segments inside cased well
 Num_segments_DP=round((MDtotal_well-Length_BHA)/Dz); %Number of segments inside cased well
 Num_segments_vert=round(KOP/Dz); %Number of segments in vertical section until KOP
 Num_segments_curved=(round((KOP+L_curved)/Dz)-Num_segments_vert); %Number of segments in curved section
 Num_segments_dev=N-Num_segments_vert-Num_segments_curved; %Number of segments in deviated straight section

Adj_length_BHA=Dz*Num_segments_BHA; %Adjusted length considering the segments and space step length m
 Adj_length_CSG=Dz*Num_segments_CSG; %Adjusted length considering the segments and space step length m
 Adj_length_DP=Dz*Num_segments_DP; %Adjusted length considering the segments and space step length m
 Adj_length_vert=Dz*Num_segments_vert; %Adjusted length considering the segments and space step length m (vertical section until KOP)
 Adj_length_curved=Dz*Num_segments_curved; %Adjusted length considering the segments and space step length m (curved section)
 Adj_length_dev=MDtotal_well-Adj_length_vert-Adj_length_curved; %Adjusted length considering the total MD and previous adjusted values m (straight deviated section)
 Adj_MDdepth_end_curved=Adj_length_vert+Adj_length_curved; %Adjusted MDdepth at the end of curved section m

Dz_eq_mud=Dz*c_mud/c_steel; %Equivalent space step length for drilling mud m
 Num_segments_below_bit=round(Depth_Bit_Bottom/Dz_eq_mud); %Number of segments below the bit (only drilling mud)
 Adj_depth_bit_wbottom=Dz_eq_mud*Num_segments_below_bit; %Adjusted depth from bit to well bottom m , considering Dz_mud and number of whole segments

Dz_BHA=Dt*c_adj_BHA/1000; %Space step length adjusted due to the adjusted speed of sound in BHA m
 Final_Adj_Length_BHA=Num_segments_BHA*Dz_BHA; %Adjusted length of BHA considering new space step length in BHA m

%TOTAL WEIGHTS OF ONE STEP LENGTH ARE CALCULATED RESPECTIVELY FOR DP, BHA AND MUD BELOW BIT.
 W_mud_steel_in_DP=(W_DP+Deg_mud_move*W_mud_in_DP)*Dz; %Weight of mud and steel for DP section CONSIDERING ONE SPACE STEP LENGTH = TOTAL/N ; kg
 W_mud_steel_in_BHA=(W_BHA+Deg_mud_move*W_mud_in_BHA)*Dz_BHA; %Weight of mud and steel for BHA section CONSIDERING ONE "BHA" SPACE STEP LENGTH=c_bha*Dt ; kg
 W_mud_below_bit=Cross_well*Dz_eq_mud*Dens_mud; %Weight of mud below drill bit and BHA, CONSIDERING ONE STEP LENGTH FOR MUD; kg

%SEGMENTS AND COEFF. OF DISPLACEMENTS
 First_seg_BHA=N-Num_segments_BHA; %First segment number in BHA

DP_Displ_coeff=2*W_mud_steel_in_DP/(W_mud_steel_in_DP+W_mud_steel_in_BHA); %Coeff for DP displacement, for DP segments, considering weight of DP+mud in one DP step length - DP/BHA change

```

DP_Displ_segment=First_seg_BHA-1; %Segment in which DP_Displ_coeff is used

BHA_Displ_coeff=2*W_mud_steel_in_BHA/(W_mud_steel_in_DP+W_mud_steel_in_BHA); %Coeff. for BHA displacement, for BHA
segments, considering weight of BHA+mud in one BHA step length - BHA/DP change
BHA_Displ_segment=First_seg_BHA+1; %Segment in which BHA_Displ_coeff is used
BHA_MUD_Displ_coeff=2*W_mud_steel_in_BHA/(W_mud_below_bit+W_mud_steel_in_BHA); %Coeff for BHA displacement, for BHA
segments, considering weight of BHA+mud in one BHA step length - BHA/MUD change
MUD_Displ_coeff=W_mud_below_bit/(W_mud_below_bit+W_mud_steel_in_BHA); %Coeff for MUD displacement, for MUD
segments, considering weight of mud in one mud step length - MUD change

%RIG HEAVE PARAMETERS
Amp_h=5; %Heave amplitude m
T_h=16; %Heave period sec
Second_wave_comp=0.2; %Heave of second wave component m

if Second_wave_comp<Amp_h/4 %Adjusted second wave component m
    Adj_second_wave_comp=Second_wave_comp;
else
    Adj_second_wave_comp=Amp_h/4;
end

Actual_Amp_h=Amp_h^2/(Amp_h+Adj_second_wave_comp); %ACTUAL WAVE COMPONENT AMPLITUDES m
Actual_second_Amp=Amp_h-Actual_Amp_h;
Actual_pos_Amp=Actual_Amp_h+Actual_second_Amp; %Positive (UPWARD) amplitude m
Actual_neg_Amp=Actual_Amp_h-Actual_second_Amp; %Negative (DOWNWARD) amplitude m
Peak_bottom_Amp=Actual_pos_Amp+Actual_neg_Amp; %Peak to bottom wave amplitude m

First_freq=4*pi/(T_h*1000); %First wave component frequency rad/msec
Second_freq=8*pi/(T_h*1000); %Second wave component frequency rad/msec

%FLOW AND CONTRAFLOW FACTORS
%Values entered by user
Contraflow=0; %Contraflow addition to volume flow (on=1, off=0)
Bit_coeff=0; %Turn off Bit coefficient (on=1, off=1)
BHA_ending=0; %Half segment=0, BHA coupled to mud=1

FF_bit=(Cross_well-A_mud_escape_used+0.5*Bit_coeff*A_mud_escape_used)*60000/(Dt/1000); %Flow Factor for Drill Bit l/min.m
FF_BHA=pi*max(OD_BHA*0.0254,Min_OD_BHA)^2*60000/(4*Dt/1000); %Flow Factor for BHA l/min.m
FF_DP=pi*max(OD_DP*0.0254,Min_OD_DP)^2*60000/(4*Dt/1000); %Flow Factor for DP l/min.m

```


$CFF_BHA = (\max(OD_BHA * 0.0254, Min_OD_BHA) / (\max(OD_BHA * 0.0254, Min_OD_BHA) + OD_BIT * 0.0254)) * Contraflow * (\text{Cross_well} - \pi * \max(OD_BHA * 0.0254, Min_OD_BHA)^2 / 4) * 60000 / (Dt / 1000);$ % Contraflow Factor for BHA l/min.m
 $CFF_DP = (\max(OD_DP * 0.0254, Min_OD_DP) / (\max(OD_DP * 0.0254, Min_OD_DP) + OD_BIT * 0.0254)) * Contraflow * (\text{Cross_well} - \pi * \max(OD_DP * 0.0254, Min_OD_DP)^2 / 4) * 60000 / (Dt / 1000);$ % Contraflow Factor for DP l/min.m

%PRESSURE FACTORS

%TURBULENT

$PF_BIT_nozzle = Dens_mud / (200000 * (60000 * A_mud_escape_used)^2);$ % Pressure factor for drill bit, nozzle eq. bar (min/l)^2
 $PF_BHA_t = (Dens_mud / 1000)^{0.8} * Visc_mud^{0.2} * Adj_length_BHA / (70696 * (OD_BIT + \max(OD_BHA * 0.0254, Min_OD_BHA) / 0.0254)^{1.8} * (OD_BIT - \max(OD_BHA * 0.0254, Min_OD_BHA) / 0.0254)^3);$ % Pressure factor for BHA, turbulent friction flow eq. bar (min/l)^1.8
 $PF_DP_t = (Dens_mud / 1000)^{0.8} * Visc_mud^{0.2} * (MDtotal_well - Adj_length_BHA) / (70696 * (OD_BIT + \max(OD_DP * 0.0254, Min_OD_DP) / 0.0254)^{1.8} * (OD_BIT - \max(OD_DP * 0.0254, Min_OD_DP) / 0.0254)^3);$ % Pressure factor for DP, turbulent friction flow eq. bar (min/l)^1.8

%LAMINAR

$\%Add_BHA = YP_mud * Adj_length_BHA / (100 * 13.26 * (OD_BIT - \max(OD_BHA * 0.0254, Min_OD_BHA) / 0.0254));$ % Adding factor (function of YP) which adds to Pressure factor for BHA, Laminar flow eq. bar
 $\%PF_BHA_l = Adj_length_BHA * Visc_mud / (100 * 408.63 * (OD_BIT + \max(OD_BHA * 0.0254, Min_OD_BHA) / 0.0254) * (OD_BIT - \max(OD_BHA * 0.0254, Min_OD_BHA) / 0.0254)^3);$ % Pressure Factor for BHA, laminar flow eq. bar (min/l)
 $\%Add_DP = YP_mud * (MDtotal_well - Adj_length_BHA) / (100 * 13.26 * (OD_BIT - \max(OD_DP * 0.0254, Min_OD_DP) / 0.0254));$ % Adding factor (function of YP) which adds to Pressure factor for DP, Laminar flow eq. bar
 $\%PF_DP_l = (MDtotal_well - Adj_length_BHA) * Visc_mud / (100 * 408.63 * (OD_BIT + \max(OD_DP * 0.0254, Min_OD_DP) / 0.0254) * (OD_BIT - \max(OD_DP * 0.0254, Min_OD_DP) / 0.0254)^3);$ % Pressure Factor for DP, laminar flow eq. bar (min/l)

$PFF_DS = 200000 * (\text{Cross_well} - A_mud_escape_used) * (Dt / 1000)^2 / (W_BHA * Dz_BHA);$ % Pressure to force factor (against drill string) m/bar

%CONTRAFLOW AREA

$\%DA_BHA = Contraflow * (\pi / 4) * (OD_BIT^2 * 0.0254^2 - OD_BHA^2 * 0.0254^2) * 60000 / (Dt / 1000);$ % Area Bit-BHA, for bit contraflow only m2
 $DA_BHA = 0;$

%NUMERIC CALCULATIONS

$times = input('Intro amount of Time Steps to be calculated:');$ % Introduce the amount of Time steps to be evaluated, INCLUDE 2ROWS FOR BOUNDARY CONDITIONS (rows)

$j = N + 18;$ % Number of columns in the matrix, including N segments
 $i = times + 5;$ % Number of rows in the matrix, including 5 initial rows

$A = zeros(i, j);$ % All matrix is filled up with zeros

%NAMES OF 5 FIRST ROWS % Preliminary calculations

```

%Name of row: "Segment number"
Seg=0;
for b=2:N+2
    A(1,b)=Seg;
    Seg=A(1,b)+1;
end

%Name of row: "Depth in meters (position in the well)"
for c=2:N+2
    A(2,c)=A(1,c)*Dz;
end

%Name of row: "Deviation in deg at depth (position in well)"
for t=2:N+2
    if A(2,t)<KOP
        A(3,t)=0;
    else
        if A(2,t)<(KOP+L_curved)
            A(3,t)=Dev*((A(2,t)-KOP)/L_curved);
        else
            A(3,t)=Dev;
        end
    end
end

%Name of row: "Coefficient of friction at depth (position of well)"
for d=2:N+2
    if A(2,d)<Length_CSG
        A(4,d)=FricCoef_st_st;
    else
        A(4,d)=FricCoef_st_rck;
    end
end

%Name of row: "Numeric Friction force in meters"
for f=2:N+2
    A(5,f)=Dx*F_buoy*A(4,f)*sin(A(3,f)*pi/180);
end

```

```

for t=1:5          %Fix column N+11 - for the last segment, so when matrix exported to excel five first rows will be in correct order
    A(t,N+11)=A(t,N+2);
    A(t,N+2)=0;
end

```

```

%Column with time steps in seconds

```

```

count_Dt=-Dt;
for e=6:i
    A(e,1)=count_Dt;
    count_Dt=A(e,1)+Dt;
end

```

```

%DISPLACEMENTS CALCULATIONS

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```

%For Segment 0 Introduce the Wave equation

```

```

for g=8:i
    A(g,2)=(1-exp(-1*A(g,1)/(T_h*1000)))^2*(Actual_Amp_h*sin(First_freq*A(g,1))+Actual_second_Amp*sin(Second_freq*A(g,1)-pi/2));
end

```

```

%SEGMENTS EQUATIONS – INCLUDE BOUNDARY CONDITIONS IN SPACE FOR DISPLACEMENT CALCULATIONS, AND TABLE FOR RESULTS IS ALSO INCLUDED HERE

```

```

for h=8:i
    for k=3:N+18
        switch k

            case First_seg_BHA+2
                if DP_Displ_coeff*A(h-1,k-1)+BHA_Displ_coeff*A(h-1,k+1)-A(h-2,k)-A(5,k)>A(h-1,k) %For DP-BHA change of CROSS SECTION
                    A(h,k)=DP_Displ_coeff*A(h-1,k-1)+BHA_Displ_coeff*A(h-1,k+1)-A(h-2,k)-A(5,k);
                else
                    if DP_Displ_coeff*A(h-1,k-1)+BHA_Displ_coeff*A(h-1,k+1)-A(h-2,k)+A(5,k)<A(h-1,k)
                        A(h,k)=DP_Displ_coeff*A(h-1,k-1)+BHA_Displ_coeff*A(h-1,k+1)-A(h-2,k)+A(5,k);
                    else
                        A(h,k)=A(h-1,k);
                    end
                end

            case N+2
                %Average movement of DP during one time step m
                add=0;
                ad=0;

```

```

for aa=2:(First_seg_BHA+1)
    add=A(h,aa)+ad;
    ad=add;
end
A(h-1,k)=add/First_seg_BHA;

case N+3                                     % Average movement of BHA during one time step m
SUM_BHA=0;
AUX_BHA=0;
for v=1:(Num_segments_BHA-1)
    SUM_BHA=A(h-1,First_seg_BHA+2+v)+AUX_BHA;
    AUX_BHA=SUM_BHA;
end
A(h-1,k)=SUM_BHA/(Num_segments_BHA-1);

case N+4                                     % Volume FLOW Drill Bit lpm
AUX=0;
SUM=0;
for u=1:(Num_segments_BHA-1)
    if sign(A(h-1,N+11)-A(h-2,N+11))==sign(A(h-1,First_seg_BHA+2+u)-A(h-2,First_seg_BHA+2+u))
        SUM=abs(A(h-1,First_seg_BHA+2+u)-A(h-2,First_seg_BHA+2+u))+AUX;
        AUX=SUM;
    else
        SUM=AUX;
    end
end
A(h-1,k)=(FF_bit*abs(A(h-1,N+11)-A(h-2,N+11)))+SUM*DA_BHA;

case N+5                                     % Volume FLOW BHA lpm
A(h-1,k)=abs(FF_BHA*(A(h-1,N+11)-A(h-2,N+11))+CFF_BHA*(A(h-1,N+3)-A(h-2,N+3)));

case N+6                                     % Volume Flow DP lpm
A(h-1,k)=abs(FF_DP*(A(h-1,N+11)-A(h-2,N+11))+CFF_DP*(A(h-1,N+2)-A(h-2,N+2)));

case N+7                                     % Pressure Drill Bit bar
A(h-1,k)=PF_BIT_nozzle*A(h-1,N+4)^2;

case N+8                                     % Pressure BHA bar
if A(h-1,N+5)~=0

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```

    A(h-1,k)=PF_BHA_t*A(h-1,N+5)^1.8;
else
    A(h-1,k)=0;
end

case N+9                                %Pressure DP bar
if A(h-1,N+6)~=0
    A(h-1,k)=PF_DP_t*A(h-1,N+6)^1.8;
else
    A(h-1,k)=0;
end

case N+10                                %SUM OF PRESSURES bar
if A(h-1,N+11)>A(h-2,N+11)
    A(h-1,k)=A(h-1,N+7)+A(h-1,N+8)+A(h-1,N+9);
else
    A(h-1,k)=-1*(A(h-1,N+7)+A(h-1,N+8)+A(h-1,N+9));
end

case N+11                                %Last segment of string (bottom point)- After SUM OF PRESSURES because SUM OF PRESSURES
is an input for calculating this value
if h==8
    if (2*A(h-1,N+1)-A(h-2,N+11)-A(5,N+11))>A(h-1,N+11)    %For t=Dt, there is no Sum of pressures, so make it like this
        A(h,N+11)=(1-BHA_ending)*(2*A(h-1,N+1)-A(h-2,N+11)-A(5,N+11));
    else
        if (2*A(h-1,N+1)-A(h-2,N+11)+A(5,N+11))<A(h-1,N+11)
            A(h,N+11)=(1-BHA_ending)*(2*A(h-1,N+1)-A(h-2,N+11)+A(5,N+11));
        else
            A(h,N+11)=A(h-1,N+11);
        end
    end
end
else
    if (A(h-1,N+1)+A(h-1,N+11)-A(h-2,N+11)-A(5,N+11)-PFF_DS*A(h-1,N+10))>A(h-1,N+11)
        A(h,N+11)=(1-BHA_ending)*(A(h-1,N+1)+A(h-1,N+11)-A(h-2,N+11)-A(5,N+11)-PFF_DS*A(h-
1,N+10))+BHA_ending*(BHA_MUD_Displ_coeff*A(h-1,N+1)+MUD_Displ_coeff*A(h-1,N+12)-A(h-2,N+11)-PFF_DS*A(h-1,N+10));
    else
        if (A(h-1,N+1)+A(h-1,N+11)-A(h-2,N+11)+A(5,N+11)-PFF_DS*A(h-1,N+10))<A(h-1,N+11)
            A(h,N+11)=(1-BHA_ending)*(A(h-1,N+1)+A(h-1,N+11)-A(h-2,N+11)+A(5,N+11)-PFF_DS*A(h-
1,N+10))+BHA_ending*(BHA_MUD_Displ_coeff*A(h-1,N+1)+MUD_Displ_coeff*A(h-1,N+12)-A(h-2,N+11)-PFF_DS*A(h-1,N+10));

```

```

else
    A(h,N+11)=(1-BHA_ending)*A(h-1,N+11)+BHA_ending*(BHA_MUD_Displ_coeff*A(h-1,N+1)+MUD_Displ_coeff*A(h-1,N+12)-A(h-2,N+11)-
    PFF_DS*A(h-1,N+10));
end
end
end

case N+12                                     %Column N+12 only zeros! Divide Table of “Numeric Calculations” and “Bit Movement and Pressure
Calculations”
    A(h,k)=0;

case N+13                                     %Max Pressure Below Bit bar
    if A(h-1,N+10)>A(h-2,N+13)
        A(h-1,N+13)=A(h-1,N+10);
    else
        A(h-1,N+13)=A(h-2,N+13);
    end

case N+14                                     %Min Pressure Below Bit bar
    if A(h-1,N+10)<A(h-2,N+14)
        A(h-1,N+14)=A(h-1,N+10);
    else
        A(h-1,N+14)=A(h-2,N+14);
    end

case N+15                                     %Max AMplitude of bit m
    if A(h,N+11)>A(h-1,N+15)
        A(h,N+15)=A(h,N+11);
    else
        A(h,N+15)=A(h-1,N+15);
    end

case N+16                                     %Min Amplitude of BI t m
    if A(h,N+11)<A(h-1,N+16)
        A(h,N+16)=A(h,N+11);
    else
        A(h,N+16)=A(h-1,N+16);
    end
end

```

```

case N+17                                     %Max Pressure below Bit numeric
if (A(h,N+12)-A(h,N+11))<A(h-1,N+17)
    A(h,N+17)=A(h,N+12)-A(h,N+11);
else
    A(h,N+17)=A(h-1,N+17);
end

case N+18                                     %Min Pressure below Bit numeric
if (A(h,N+12)-A(h,N+11))>A(h-1,N+18)
    A(h,N+18)=A(h,N+12)-A(h,N+11);
else
    A(h,N+18)=A(h-1,N+18);
end

case N+1                                       %Segment previous to last one (N-1)! (Standard Numerical eq.)
if (A(h-1,k-1)+A(h-1,N+11)-A(h-2,k)-A(5,k))>A(h-1,k)
    A(h,k)=A(h-1,k-1)+A(h-1,N+11)-A(h-2,k)-A(5,k);
else
    if (A(h-1,k-1)+A(h-1,N+11)-A(h-2,k)+A(5,k))<A(h-1,k)
        A(h,k)=A(h-1,k-1)+A(h-1,N+11)-A(h-2,k)+A(5,k);
    else
        A(h,k)=A(h-1,k);
    end
end

otherwise
if (A(h-1,k-1)+A(h-1,k+1)-A(h-2,k)-A(5,k))>A(h-1,k)           %For inside the string segments (Standard Numerical Eq.)
    A(h,k)=A(h-1,k-1)+A(h-1,k+1)-A(h-2,k)-A(5,k);
else
    if (A(h-1,k-1)+A(h-1,k+1)-A(h-2,k)+A(5,k))<A(h-1,k)
        A(h,k)=A(h-1,k-1)+A(h-1,k+1)-A(h-2,k)+A(5,k);
    else
        A(h,k)=A(h-1,k);
    end
end

end
end
end

```

```

B=zeros(i,j);
for l=1:i %Making final matrix
    for m=1:N+1
        B(l,m)=A(l,m);
    end
end
for n=1:i %Order Final Matrix to export to Excel in correct order
    for o=N+2:N+3
        B(n,o)=A(n,o+9);
    end
end
for p=1:i
    for q=N+4:N+12
        B(p,q)=A(p,q-2);
    end
end
for r=1:i
    for s=N+13:N+18
        B(r,s)=A(r,s);
    end
end
xlswrite('Thesis_1.xlsx',B,'Sheet1','A1') %Export to Excel

%PRESSURES BELOW BIT AND BIT MOVEMENTE - FOR MUD FLOW DRIVING PRESSURE
MaxP=max(B); %Create a row with the maximum values of all the columns
MinP=min(B); %Create a row with the minimum values of all the columns
%RESULTS
MaxP_increase_below_bit_SURGE=MaxP(1,N+13)
MaxP_decrease_below_bit_SWAB=MinP(1,N+14)
Bit_move_down_SURGE=MaxP(1,N+15)
Bit_move_up_SWAB=MinP(1,N+16)

```


8.5 MatLab Intermediate Results and Variable's values

As Chapter 4 shows the most important results from the UiS Numerical Method, this appendix shows the results and values from all the preliminary and intermediate calculations done in order to get the results (pressure fluctuations below bit and bit movement). Therefore, all the values and results showed here follows the same “Build and Hold” well case scenario, considering the contraflow turned off, the BHA ends with half segment, all the mud inside the string move along with the string, wave conditions: average platform heave movement is 4 m and Period 14 s , and the string is divided into 60 numerical segments. Note that the variables follow the nomenclature and units presented in Chapter 7.

Name ▲	Value	Min	Max
A	<10681x78 doubl...	<Too many elements>	<Too many elements>
AUX	0.1562	0.1562	0.1562
AUX_BHA	6.1808	6.1808	6.1808
A_bit_mud_escape	20	20	20
A_mud_escape_used	0.0073	0.0073	0.0073
A_rel_escape	0.1000	0.1000	0.1000
Actual_Amp_h	3.8095	3.8095	3.8095
Actual_neg_Amp	3.6190	3.6190	3.6190
Actual_pos_Amp	4	4	4
Actual_second_Amp	0.1905	0.1905	0.1905
Adj_MDdepth_end_curved	1000	1000	1000
Adj_depth_bit_wbottom	19.3349	19.3349	19.3349
Adj_length_BHA	200	200	200
Adj_length_CSG	1200	1200	1200
Adj_length_DP	2800	2800	2800
Adj_length_curved	500	500	500
Adj_length_dev	2000	2000	2000
Adj_length_vert	500	500	500
Adj_second_wave_comp	0.2000	0.2000	0.2000
Amp_h	4	4	4
App_dens_BHA	8.1774e+03	8.1774e+03	8.1774e+03
App_dens_DP	1.0628e+04	1.0628e+04	1.0628e+04
B	<10681x78 doubl...	<Too many elements>	<Too many elements>
BHA_Displ_segment	57	57	57
BHA_Displ_coeff	1.7079	1.7079	1.7079
BHA_MUD_Displ_coeff	1.8669	1.8669	1.8669
BHA_ending	0	0	0
Bit_coeff	0	0	0
Bit_move_down_SURGE	5.1799	5.1799	5.1799
Bit_move_up_SWAB	-4.6156	-4.6156	-4.6156
CFF_BHA	0	0	0
CFF_DP	0	0	0

Name ▲	Value	Min	Max
Contraflow	0	0	0
Cross_Sec_BHA	0.0255	0.0255	0.0255
Cross_Sec_DP	0.0038	0.0038	0.0038
Cross_well	0.0730	0.0730	0.0730
DA_BHA	0	0	0
DP_Displ_segment	55	55	55
DP_Displ_coeff	0.2921	0.2921	0.2921
Damp_fac_mud	0	0	0
Deg_mud_move	1	1	1
Dens_mud	1200	1200	1200
Dens_steel	7850	7850	7850
Depth_Bit_Bottom	20	20	20
Dev	70	70	70
Dt	11.2485	11.2485	11.2485
Dx	0.0012	0.0012	0.0012
Dz	50	50	50
Dz_BHA	57.0008	57.0008	57.0008
Dz_eq_mud	9.6674	9.6674	9.6674
E	2.0998e+06	2.0998e+06	2.0998e+06
E_mud	12000	12000	12000
FF_BHA	1.7298e+05	1.7298e+05	1.7298e+05
FF_DP	6.7570e+04	6.7570e+04	6.7570e+04
FF_bit	3.5028e+05	3.5028e+05	3.5028e+05
F_buoy	0.8471	0.8471	0.8471
Final_Adj_Length_BHA	228.0030	228.0030	228.0030
First_freq	8.9760e-04	8.9760e-04	8.9760e-04
First_seg_BHA	56	56	56
FricCoef_st_rck	0.3000	0.3000	0.3000
FricCoef_st_st	0.2300	0.2300	0.2300
In_crossec_BHA	0.0070	0.0070	0.0070
In_crossec_DP	0.0088	0.0088	0.0088
KOP	500	500	500
L_curved	500	500	500
L_dev	2000	2000	2000
Length_BHA	200	200	200
Length_CSG	1200	1200	1200
MDtotal_well	3000	3000	3000
MUD_Displ_coeff	0.0665	0.0665	0.0665
MaxP	<1x78 double>	0	1.2007e+05
MaxP_decrease_below_bit_SW...	-23.8585	-23.8585	-23.8585
MaxP_increase_below_bit_SU...	20.9067	20.9067	20.9067
MinP	<1x78 double>	-23.8585	0
Min_OD_BHA	0.1898	0.1898	0.1898
Min_OD_DP	0.0920	0.0920	0.0920
Min_in_diam	0.0600	0.0600	0.0600
Mud_escape	2	2	2
N	60	60	60
Num_segments_BHA	4	4	4

Heave effects on Drill String during connections

Name ▲	Value	Min	Max
Num_segments_CSG	24	24	24
Num_segments_DP	56	56	56
Num_segments_below_bit	2	2	2
Num_segments_curved	10	10	10
Num_segments_dev	40	40	40
Num_segments_vert	10	10	10
OD_BHA	8	8	8
OD_BIT	12	12	12
OD_DP	5	5	5
PFF_DS	1.4577e-04	1.4577e-04	1.4577e-04
PF_BHA_t	4.0010e-07	4.0010e-07	4.0010e-07
PF_BIT_nozzle	3.1305e-08	3.1305e-08	3.1305e-08
PF_DP_t	1.4003e-06	1.4003e-06	1.4003e-06
Peak_bottom_Amp	7.6190	7.6190	7.6190
SUM	0.1562	0.1562	0.1562
SUM_BHA	6.1808	6.1808	6.1808
Second_freq	0.0018	0.0018	0.0018
Second_wave_comp	0.2000	0.2000	0.2000
Seg	61	61	61
T_h	14	14	14
Visc_mud	15	15	15
W_BHA	200	200	200
W_DP	30	30	30
W_mud_below_bit	846.4719	846.4719	846.4719
W_mud_in_BHA	8.3419	8.3419	8.3419
W_mud_in_DP	10.6152	10.6152	10.6152
W_mud_in_OH	87.5591	87.5591	87.5591
W_mud_steel_in_BHA	1.1876e+04	1.1876e+04	1.1876e+04
W_mud_steel_in_DP	2.0308e+03	2.0308e+03	2.0308e+03
YP_mud	14	14	14
c_adj_BHA	5.0674e+03	5.0674e+03	5.0674e+03
c_adj_DP	4.4450e+03	4.4450e+03	4.4450e+03
c_mud	1000	1000	1000
c_steel	5172	5172	5172
count_Dt	1.2008e+05	1.2008e+05	1.2008e+05
times	10676	10676	10676