University of Stavanger

Faculty of Science and Technology

MASTER'S THESIS

Study program: PETROLEUM TECHNOLOGY Specialization: DRILLING	Spring semester, 2012. Open
Writer: MOHAMED MUSE ALI	………………………………………… (Writer's signature)
Faculty supervisor: STEINAR EVJE	
Titel of thesis: INVESTIGATION OF TRANSIENT SCENARIOS IN UNDERBALANCED DRILLING	
Credits (ECTS): 30	
Key words: INVESTIGATION, TRANSIENT UNDERBALANCED DRILLING	Pages: 60 + enclosure: 19 Stavanger, 15.06.2012

ABSTACT

When drilling underbalanced wells, the standard way consists to drill with a very low mud weight in order to obtain a well pressure lower than pore pressure. The flow of the fluids from the reservoir up to the tubing is the most cases in two phases, liquid and gas. The pressure at any point in the tubing string is influenced by many factors. By modeling some simple transient scenarios in UBD for the understanding purpose of the physical nature of the flow from the reservoir from which the crude oil or gas is produced. The pressure analysis is the most valuable data from wells which to likely elucidate essentially concerns in the transient the dynamic relation between the producing rate, the bottomhole pressure and the pore pressure. There is a correlation between how easy would be an interpretation of results from simulations and how good and the quality of reservoir description characteristic in the model. One the challenging task remains on obtaining a reliable and accurate description of multiphase flow from a large scale data and heterogeneous reservoir where different types of flow regime coexist in the same wellbore at the same time. Due to the complexity nature of the multiphase flow regime, we studied the drift flux model as well as the fundamental physics behind t9he flow such as mass conservation and momentum conservation in order to simulate realistic models. There are several scenarios simulated in this project by which it is possible to gain information both about the reservoir characteristics and flow behavior in underbalanced wells; the most important are:

- Basic case model
- Drilling into the reservoir with reservoir inflow
- New connection with or without reservoir flow

After comprehensive analysis of the different simulated models, we were able to visualize whether one specific region is operating in the frictional dominated region – due to the mud circulation or gas influx - or hydrostatic dominated side. We play around with rates and productivity index model to visualize the differences existing between these regions in order to minimize bottomhole pressure fluctuations witnessed during pipe connection.

ABLE OF CONTENTS

ACKNOWLEDGEMENT

This thesis was written as a final part of my two long years to acquire a master's degree in Petroleum Engineering at the University of Stavanger in Norway. I have chosen to write about *Transient Scenarios in Underbalanced Drilling*. The project is done under the supervision of Professor Steinar Evje, professor of petroleum department at the University of Stavanger. I am deeply grateful to Dr. Steinar Evje, and thank him for all the help and guidance I received during the whole semester. Furthermore, I would like to thank my family as well as my friends for all their support during my whole study period at the University of Stavanger.

LIST OF FIGURES

1. INTRODUCTION

1.1 Background

What is underbalanced drilling? When the effective circulating downhole pressure of the drilling fluid - which is equal to the hydrostatic pressure of the fluid column, plus pump pressure, plus associated friction pressures - is less than the effective near bore formation pore pressure.
Conventionally, wells are drilled overbalanced, which provides the primary well control mechanism. Imposed wellbore pressure arises from three different mechanisms:

1. Hydrostatic pressure of materials in the wellbore due to the density of the fluid used (mud) and the density contribution of any drilled cuttings.
2. Dynamic pressure from the fluid movement due to circulating friction of the fluid used and the relative fluid motion caused by surge/swab of the drill pipe .
3. Imposed pressure, with occurs due to the pipe being sealed at surface resulting in an area with pressure differential (e.g., a rotating head or stripper element).

Underbalanced drilling is defined as drilling with the hydrostatic head of the drilling fluid intentionally designed to be lower than the pressure of the formations being drilled. The hydrostatic head of the fluid may naturally be less than the formation pressure or it can be induced. The induced state may be created by adding natural gas, nitrogen or air to the liquid phase of the drilling fluid. Whether the underbalanced status is induced or natural, the result may be an influx of formation fluids which must be circulated from the well and controlled at surface. Underbalanced drilling in practical terms will result in flow from one or more zones into the wellbore (this is more likely, however, to be solely from one zone as cross-flow is likely to result) or where the potential for flow exists.
The lower hydrostatic head avoids the build-up of filter cake on the formation as well as the invasion of mud and drilling solids into the formation. This helps to improve productivity of the reservoir and reduce related drilling problems.

When comparing underbalanced drilling with conventional drilling it soon becomes apparent that an influx of formation fluids must be controlled to avoid well control problems. In underbalanced drilling, the fluids from the well are returned to a closed system at surface to control the well. With the well flowing, the BOP system is kept closed while drilling, whereas in comparison to conventional drilling fluids return to an open system with the well open to atmosphere.

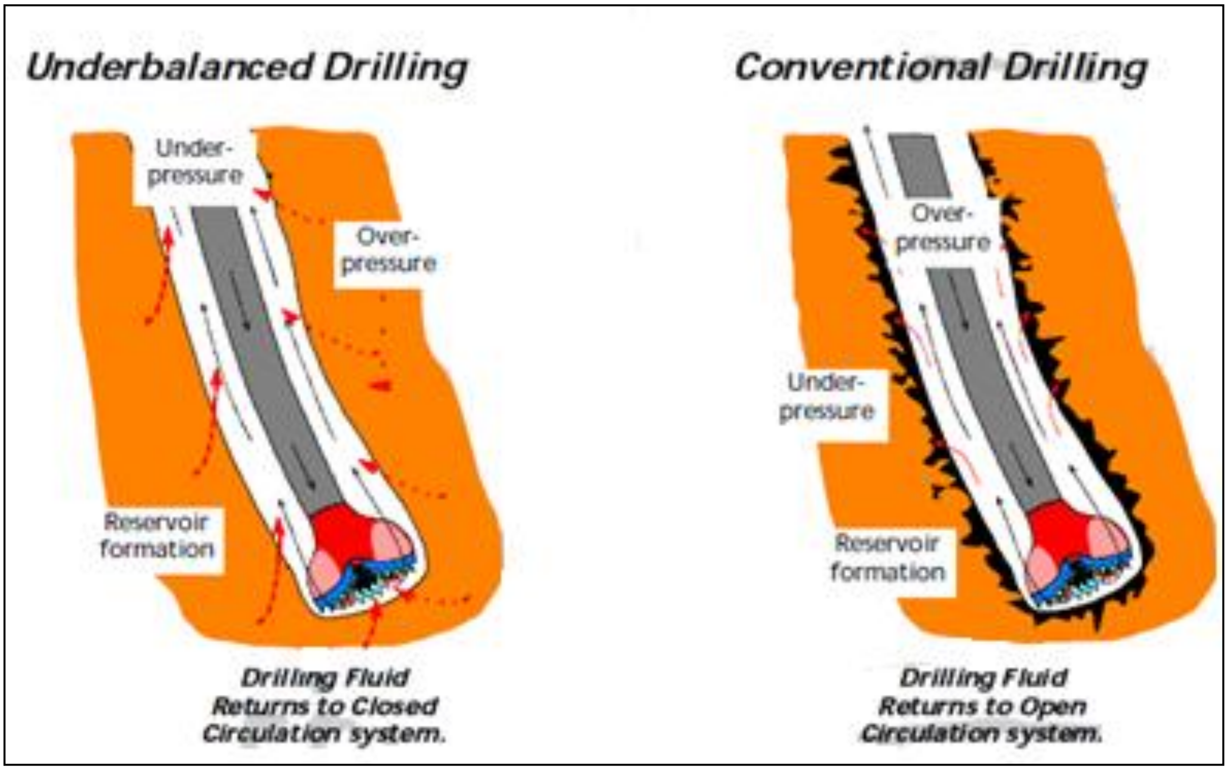

Figure 1: Near wellbore condition during Underbalanced and conventional (overbalanced) drilling

Drilling Technical Advantages

No mud losses are encountered during underbalanced drilling. Simple water based fluid systems have been used. If torque and drag are an issue, then an oil based mud system can also be used. Gas solubility in oil needs to be considered when using oil based mud systems Penetration rates increase by 2 to 5 fold when drilling underbalanced. This is still a function of formation and bit selection. The amount of drawdown has a direct impact on the rate of penetration. Bit life is increased. Because the well is drilled faster and the removal of cutting from the bit face is more efficient. No differential stuck pipe occurs when drilling underbalanced. There is no wall cake and no over pressure to push the pipe against the wall of the wellbore. It does not mean that no stuck pipe occurs when drilling underbalanced. Key seating and junk as well as hole collapse can still cause stuck pipe even in underbalanced drilled wells. Inefficient hole cleaning as a result of the multiphase flow can also cause stuck pipe in underbalanced drilled wells.

1.2 Objectives

The objective of this thesis is two fold

a) To review the UBD technologies
   - Advantages of UBD
   - History and principle of UBD
   - Illustration of UBD equipment

b) To simulate few simple transient scenarios phenomenon in UBD
   - Visualize the behavior of basic cases in the model
   - Drill a gas well with reservoir inflow
   - Make a connection with reservoir inflow in a gas well

1.3 Scope and Methodology

The scope of the study is based on literature study and simulation. The activities are:

- Thesis starts out with a literature review on status on underbalanced drilling and especially focus on transient scenarios that can occur in UBO operations
- Description of the drift flux model and AUMSV scheme based on the current status of the given Matlab code.
- Simulation :
  - Basic case in the model (Define well geometry, Reservoir and well pressure properties)
  - Different scenarios (assume the first annulus nitrogen injection at some point in the well Use base case model but a PI model must be included. Also a rate variable should be introduced to distinguish when the reservoir starts producing.
  - Drilling ahead. At some depth, the reservoir model is allowed to start producing. A new steady situation must be achieved.
  - New connection with reservoir flow
    - Open connection
      - Here we must explore what the difference would be if we were operating on the friction dominated region or hydrostatic dominated side. (Play around with rates and productivity index model) to visualize the differences.
- Summary & Recommendations.

2. LITERATURE REVIEW

2.1 Formation damage

Mechanisms of Formation Damage [2].

Mechanisms of damage common to both horizontal and vertical wells could include:

- Fluid-fluid incompatibilities-reaction of invaded mud filtrate with in situ fluids (oil or formation brine) to form scales, insoluble precipitates, asphaltic sludges or stable emulsions.
- Rock-fluid incompatibilities contact with potentially swelling (I, e., smectitic clay) or deflocculatable (ie., kaolinite clay) minerals by non-equilibrium aqueous phase solutions may have the potential to severely reduce near wellbore permeability.
- Solids invasion-the invasion of artificial solids contained in die drilling fluid (i.e., weighting agents or artificial bridging agents) or the invasion of formation solids (rnicrofines) generated by the milling action of the drill bit on the formation. The permanent entrainment of these solids in the formation can have a severely reducing effect on permeability in some situations.
- Phase trapping/blocking-the invasion and permanent entrapment of high oil or water phases at the near wellbore region can have a substantially reducing effect on oil or gas productivity, particularly for certain types of formation.
- Chemical adsorption/wettability alteration-most drilling fluids contain a variety of chemical additives to improve mud performance and character. In some cases these additives may be incompatible with the formation fluids or rock, or exhibit a high propensity for physical adsorption. This can result in a number of undesirable phenomena such as permeability reductions due to physical polymer adsorption, or wettability alterations due to surfactant adsorption.
- Fines migration-the actual internal movement of formation fines

2.2 Why Underbalanced Drilling?

The objectives of underbalanced drilling can be broken down into two main categories [3]:

- Maximising hydrocarbon recovery
- Minimising drilling problems

These categories illustrate underbalanced drilling operations are performed. There are also specific advantages and disadvantages of performing a drilling operation underbalanced. These can be summarized as follows:

Advantages:	Disadvantages:
• Increased ROP • Decreased formation damage • Eliminate risk of differential sticking • Reduce risk of loss circulation • Less weight on bit required • Improved bit life • Tight hole problems may be reduced • Reduced cutting size increasing hole cleaning capability	• Wellbore stability • Wellbore consolidation • Increased drilling costs (depending on system used) • Compatibility with conventional MWD systems • Spontaneous counter current imbibition effects • Gravity drainage in horizontal wells • Possible near wellbore mechanical damage • Discontinuous underbalanced conditions • Generally higher risk with more inherent problems • String weight is increased due to reduced buoyancy • Possible excessive borehole erosion • Possible increased torque and drag

2.2.1 Maximizing Hydrocarbon Recovery

The two main objectives of underbalanced drilling can be subdivided as follows:

Reduced formation damage

No invasion of solids or mud filtrates into the reservoir formation.

Early production

Well is producing as soon as the reservoir is penetrated with a bit. This could be a disadvantage if hydrocarbon production cannot be handled or stored on site or if the required export lines are not available.

Reduced Stimulation

As there is no filtrate or solids invasion in an underbalanced drilled reservoir, the need for reservoir stimulation is eliminated. It has been noted in wells drilled underbalanced that stimulation with fluids significantly reduces the productivity of the reservoir. An acid wash carried out on an underbalanced drilled well, reduced productivity from [4] 20MMscft/day to 2 MMscft/day. The full benefits of underbalanced drilling were never regained.

Enhanced recovery

Due to the increased productivity of an underbalanced drilled well combined with the ability to drill infill wells in depleted fields; the recovery of bypassed hydrocarbons is possible. This can significantly extend the life of a field. The improved productivity of the wells also leads to a lower drawdown, which can, in turn, reduce water coning.

2.2.2 Drilling Problems

Differential sticking

The absence of an overburden on the formation combined with the lack of any filter cake serves to prevent the drillstring from becoming differentially stuck. This is especially useful when drilling with coiled tubing because of the lack of tool joint connections. This increases the stand-off in the borehole.

No Losses

In general, a reduction of the hydrostatic pressure in the annulus reduces the fluid losses into a reservoir formation. In underbalanced drilling, the hydrostatic pressure is reduced to a level where losses do not occur.

Improved Penetration Rate

The lowering of the overpressure over the formation pressure has a significant effect on penetration rate. The reduction in the so-called chip hold down effect also has a positive impact on bit life. The increased penetration rate combined with the effective cuttings removal from the face of the bit leads to a significant increase in bit life. In underbalanced drilled wells, sections have been drilled with one bit where as in overbalanced drilled wells [1] (3, 4 or even 5) bits were used

Reduction of ECD (equivalent circulation density) in extended reach wells

The drilling of long horizontal or near horizontal sections creates more and more friction pressure in the annulus. This friction pressure acts on the bottom of the well and slowly increases the overpressure over the formation interval. This results in a reduction of ROP (rate of penetration) and increases the potential for losses. Underbalanced drilling provides an opportunity for a reduction in annular friction losses by allowing the reservoir energy to push fluids out of the hole.

2.3 Underbalanced drilling history and principle

The first underbalanced drilling operations were carried out early in1800'S in Pennsylvania. Moreover at the beginning of the 1970'S, UBD was not so popular due to of development of the rotary drilling. But later in 1980'S, it became more and more popular and was performed in different type of reservoirs namely the depleted, fractured reservoirs (South Texas Austin Chalk) and formations of highly quality. Horizontal wells, as well as wells with a narrow window between pore and fracture pressures were good candidates for UBD. Moreover since 1990'S the tendency is oriented for more and more use of underbalanced operations. **.** [4]

As stated earlier, the main purpose of UBD consists in drilling with a pressure below pore pressure, allowing production of hydrocarbon from reservoirs while drilling. Underbalanced conditions are likely achieved either by lowering the mud weight, if the pore pressure is enough high, or by using different types of drilling fluid like pure water, oil, water mixed nitrogen or foam for very low pore pressures. It's very important to prevent formation from collapse while drilling following this equation:

$$P_{collapse} < P_{well} < P_{pore}. \qquad (1.1)$$

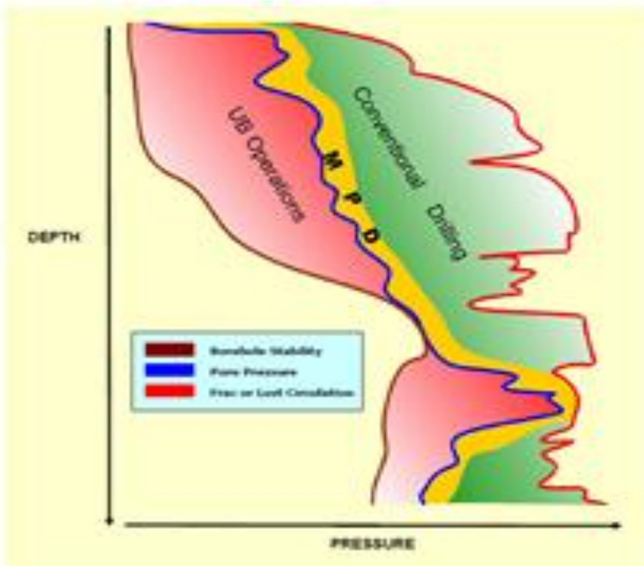

Figure 2: Conventional and underbalanced pressure profiles [4].

2.4 UBD Equipment

UBD operations include three main important surface equipment: [5] [4]

- Rotating Control Devices (RCDs) are set to isolate the annulus contain annular fluids under pressure and divert returns to pressure control and fluid management systems. Here is presented (Fig) one of the most Schlumberger advanced RCDs products. It connects to the top of the blowout preventer (BOP) in the wellhead, and allow rotary drilling and stripping out of a well with positive annular pressure. The RCD is a critical component in every managed pressure drilling (MPD) and underbalanced drilling operation because neither application is possible without one.
- Drilling Choke Manifold opening is set to keep enough pressure in well. The desired back pressure from the choke is either pre-calculated, available in the table based on mud weight, well depth and flow rate, or dynamically provided based on downhole pressure measurements or well flow model.
- Hydrocarbons Separation Equipment is needed to differentiate multiphase flow from wellbore.

All the extra- equipment is placed on the top of conventional one. Rig'S BOP and Rig chokes are still in place for more serious well control challenges.

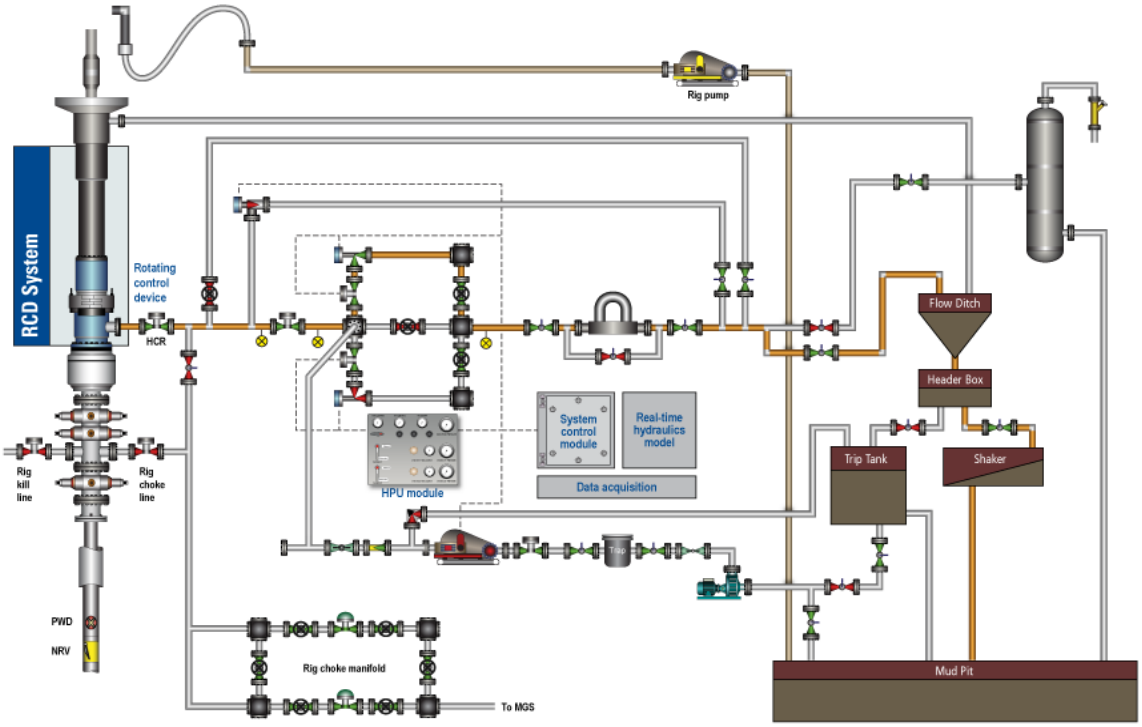

Figure 3: High-pressure RCDs from Schlumberger [5].

High-pressure rotating control device (RCD) is likely to improve safety and performance features for high-pressure, onshore, and offshore MPD and UBD operations. Most important elements in RCD could be summarized as follow:

- remotely operated seal clamps and low-pressure risers
- dual sealing element pressure sensors and integrated sealed-bearing assemblies
- low-profile models to accommodate rigs with limited space below the drill floor

High-pressure RCDs provide multiple sealing options, including sealing elements composed of different compounds, a dual-seal element adaptor for an uninterrupted wellbore seal, and a remote locking system to eliminate the need to send personnel below the rig floor to latch and unlatch manual clamps.

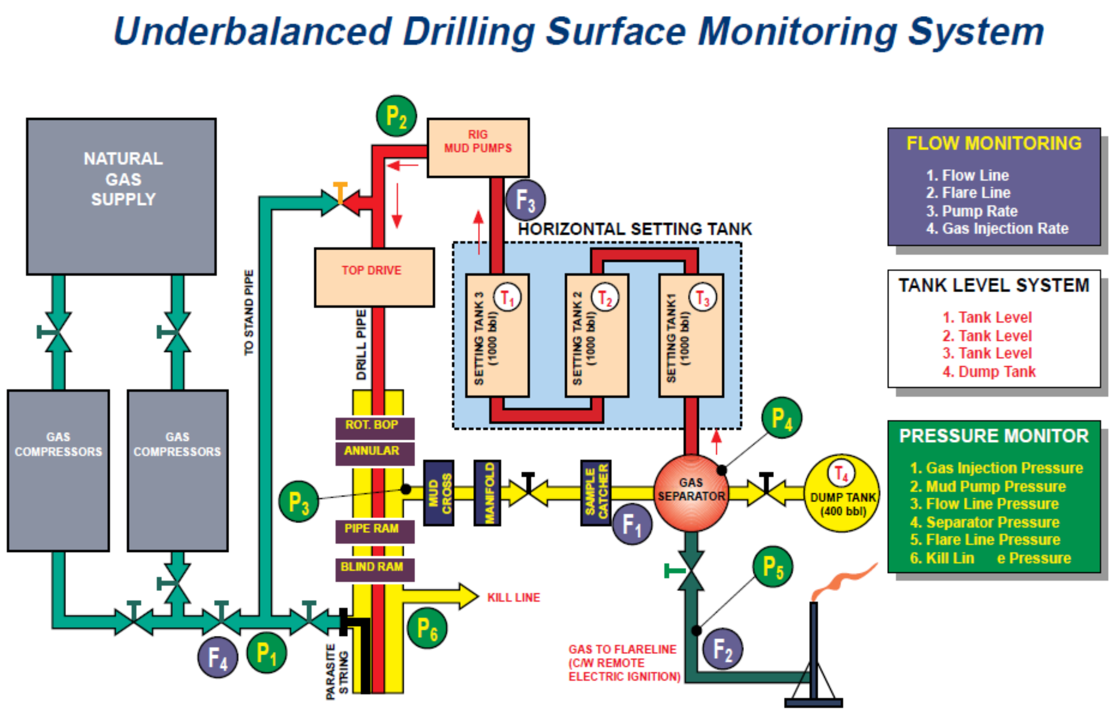

Figure 4: Underbalanced drilling surface monitoring system

Underbalanced drilling surface monitoring system with appropriated software could provide tasks such as:

- monitors all critical parameters at one central location
- real-time monitoring
- allows onsite configuration
- supports multiple displays

2.5 UBD Techniques

The most popular method leading to obtain underbalanced situation and at the same time allowing to maintain all benefits of drilling fluid , higher viscosity and yield strength, is to use a multiphase flow drilling fluid. It exists essentially two principal ways for gasifying the drilling fluid and to obtain underbalanced conditions. [9].

2.5.1 Gas Injection through Standpipe

The most popular method of aeration is to inject gas or multi-phase fluids through the drillstring. Figure 5 illustrates the process of gas injection through standpipe. These multi-phase fluids are intentionally injected in order to decrease the density of reservoir fluid. The gas in this case is introduced into the fluid at the surface before it enters the drillpipe, or downhole into the liquid at the annulus. The most common gases used include air and nitrogen. Moreover it's admitted that the second most used method consists of gas injection through a parasitic string or liner into the annulus. It is also possible to mix both above mentioned techniques as a possible combination. Injection through the drillstring has the advantages of using smaller upper hole and casing sizes and permits further gas expansion to facilitate better cuttings transport particularly in high deviated and horizontal wells. Furthermore, this method does not demand a huge gas volume for a given underbalanced situation than if the work consisted to perform a gas lift operation introduced trough the annulus. Despite all those advantages, there is a quite big concern in the industry if oxygen is introduced inside the

Wellbore leading thereby corrosion on the internal bore of drill pipe and other drilling tools such as downhole tools, motors or MWD's equipment.

Here we are only discussing UBD operations with Jointed Pipe, while Coiled Tubing applications are not taken into account in this project.

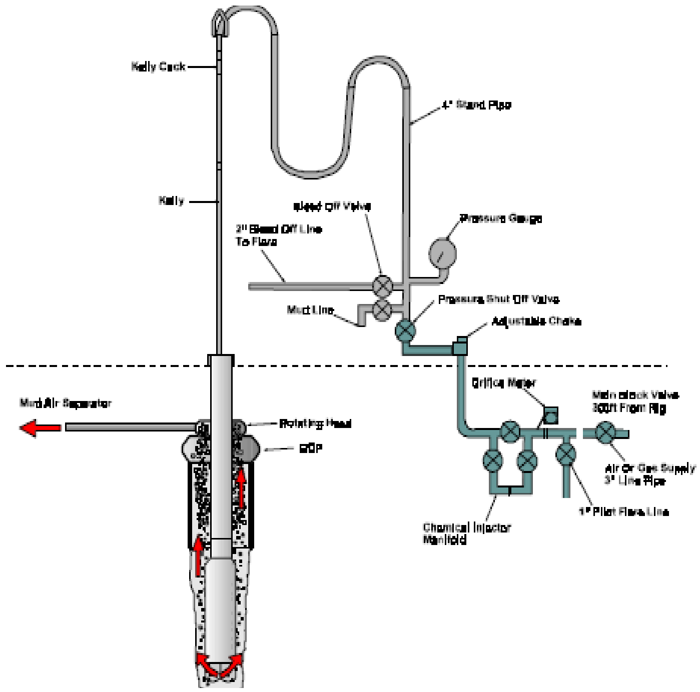

Figure 5: Injection through Standpipe.

2.5.2 Gas Injection through Parasitic String

Figure 6 illustrates gas injection through parasitic string. One of the most challenges about aerated fluid systems remain their stability during injection. It's reported that such kind of instability induces pressure surges while drilling, making connections and tripping. It results that we lost underbalanced conditions and we witness the destabilization of the whole wellbore leading to periodically overbalanced situations. Transient connections and while tripping, aerated fluids will lose its gas and go flat. There are some common techniques, such as adding more gas before connections, used to minimize the effects of pressure surge. When circulating mud, cuttings transport ability as well as fluid properties and hole size contribute to reduce the effect of pressure surges during drilling. Successful continuous gas injection without turn off pumps during drilling and connections will facilitate to lighten the mud column and thereby reducing pressure surges. Most commonly used method to achieve this goal remains gas injection through parasitic strings.

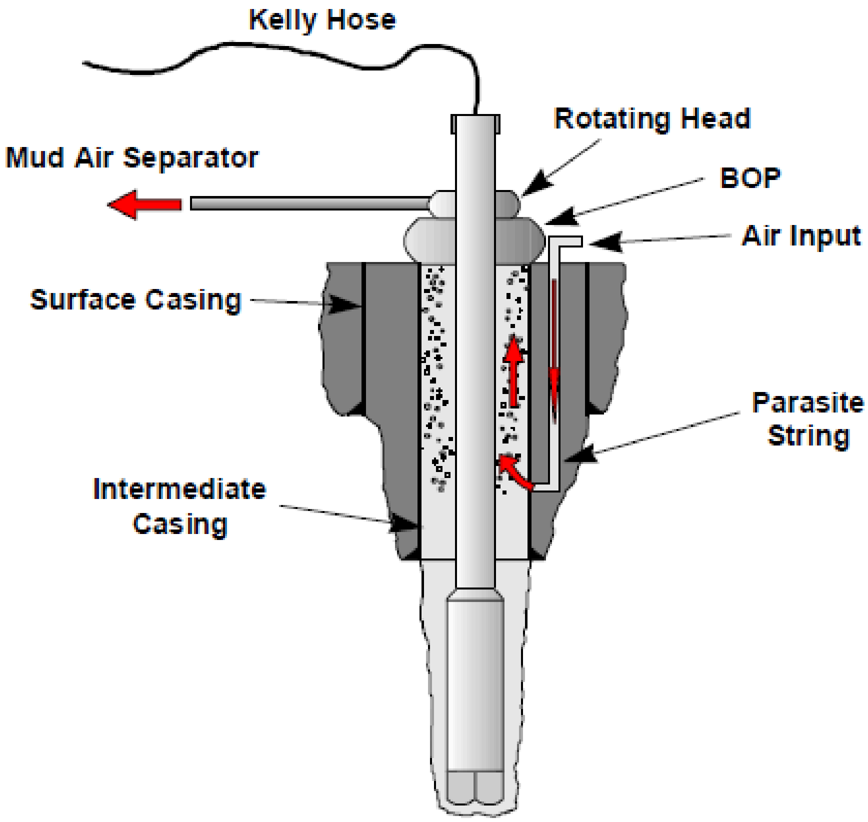

Figure 6: Injection trough parasite string

Difference between gas injection through the standpipe and gas injection through a parasitic string remains cost and the rate of injected gas. The second method demands higher gas rates and higher gas costs to realize a given underbalanced pressure condition. The opportunity for applying gas injection through a parasitic string in a re-entry drilling operation is quite restricted, except if there is causing or quite large bore production tubing. *There has to be sufficient clearance between the casing to which a parasite string is attached and both the casing and open hole section in which it is run, to accommodate the tubing and its injection sub*. Mechanical damage is big a risk during parasite string running. There is a specific concern related to deviate wells. Due to its large annular volume, it is possible to inject a huge volume of gas through parasite string. Another advantage, of gas injection through a parasite string, consists that drilling fluid is allowed to keep its incompressible properties inside the drillstring and thereby allowing efficient operation of mud pulse telemetry MWD's and downhole motors.

2.5.3. Transient System

Jointed Pipe underbalanced drilling is dealing with a non-steady-state system and demands particulars drilling procedures, circulating system considerations, specialized equipment, and on-site personnel coordination. The effects of non-steady-state, jointed pipe drilling must be properly handled and controlled in order to maximize benefits, while drilling expansive is kept as low as possible. The most common operating procedures for conventional drilling are modified to adapt UBD with jointed pipe. For instance, particular measures are needed as the drilling operations are going forward, based both for the need for safety of personnel and in order to avoid serious formation damage to the reservoir. It is required to handle the effect of the annular hydrostatic and frictional effects of the transient system in order to keep an acceptable underbalanced condition, a proper hole cleaning and BHA power requirements. Drillpipe connections and tripping with jointed pipe affects the bottomhole pressure (BHP) and proper measures are needed to deal and minimize the effects. BHP is influenced by many interdependent factors. Injection liquid and gas types and rates, reservoir fluid inflow performance, well configuration, drillpipe movement as well as downhole temperature have direct effect on the bottomhole pressure.

2.5.4. Bottomhole Pressure

At the conception stage, gas and/or multiphase flow modeling is necessary to evaluate circulation system data. Injection fluids should be studied in conjunction with reservoir fluids, at a variety of conditions, to determine BHP operating limits. Operating limits are very critical and demand particular attention so that every single operation should be executed one after another, allowing the circulating system to be in place. As we would like to optimize circulation system, factors such as actual well conditions, reservoir pressure variations, and reservoir inflow performance are most helpful parameters one has to deal with. Bottomhole pressure is defined as result of the hydrostatic pressure of the annular fluids plus pressure drops due to the friction, plus the inertial pressure of fluid acceleration.

$$P_{bh} = P_h + P_f + P_a \qquad (2.1)$$

With:

- P_{bh}: Bottomhole pressure.
- P_h: hydrostatic pressure.
- P_f: pressure drops due to friction.
- P_a: fluid acceleration pressure.

2.6 Operation process during UBD

2.6.1 Unloading a Well

According to Salim et al. [11], unloading a well is very a useful operation in the oil industry. The operation is carried out after workover jobs, kill procedures or stimulation operations in order to unload liquid from the drillstring and thereby setting the well again in production. Furthermore Watson et al. [10] describes the unloading procedure as injection of small quantities of nitrogen into the liquid to reduce the hydrostatic liquid column. As the lighter fluid flow from bottom of the well up to the annulus, bottom hole pressure decrease until the drillpipe contain only injected fluid (nitrogen) leading therefore the well to be ready to start production fluids.

2.6.2 Drilling into the reservoir

The intention of this part of project is to get an overview over reservoir behavior while drilling on it. Starting drilling into reservoir implies effective control of annular BHP and flow control from wellbore as the reservoir inflow can destabilize the whole circulation system. Saponja [7] describes the initial energy presents in the reservoir as flowing in suddenly and abundantly because high permeability and formation being putting in production for the first time. However lack of reservoir deliverability model in UBD is compensate by experience and observation of various data collected in different fields. Huge fluid inflow from pay zone combined with maximum use of initial energy present could destabilize the whole circulation system and do not allow stable and continued injection rate of nitrogen. Moreover naturally fractured and high permeable formations usually exhibit huge losses, which can exacerbate well control problems or lead to differential or mechanical sticking, hence undermining the whole underbalanced operations particularly when proceeding connections. Connection time should be minimized to avoid large volume inflow from reservoir.

2.6.3 Shutting down nitrogen

In many UBD cases nitrogen is used as drilling fluid because of its major advantage not to be flammable [1] in the presence of other hydrocarbons gases. This significantly reduces the probability of downhole fire. Moreover nitrogen has the ability to prevent corrosion which is major concern in the oil industry. The primary function of the nitrogen is to reduce the fluid density and should not be seen as a hole cleaning means. As mention early in chapter 2, there various injection possibilities:

- Through a Standpipe.
- Injection through Parasitic String.

Kelly Falk et al. [12] carried out a comparison between those two injection methods and concluded that injection through a standpipe uses less nitrogen than a parasite injection operation. They assert that the nitrogen injection point is often located at some point in the build section, in a horizontal or deviated well, with injection through parasitic String case. However an obvious question will be: how the flow from wellbore will behave when we shut down injection of nitrogen thereby we stop the circulation? Could the initial energy presents in the reservoir will enough to sustain flow from wellbore? And at the end of the day, we will be able to analyze the similarities and differences with and without nitrogen injection.

2.6.4 New connection with reservoir inflow

Pipe connections are characteristic of transitions UBD [13] and remain a major source of bottomhole pressure fluctuations when using jointed pipe technology for underbalanced drilling. This process requires injection stop aimed new pipe connection. Observed disturbances of the circulation system are accompanied with significant fluctuations of bottomhole pressure. As direct consequence of connection, both annular fluid velocity and the frictional back pressure component related with the motion of the fluid from downhole to the surface are reduced. However producing reservoir could lead a huge inflow from the formation into the wellbore in addition to hydrocarbon naturally inflowing due to underbalanced situation. The annulus can witness a significant accumulation of different phases of liquid such as slug and we may need to apply large hydrostatic back pressure in order to lift these fluids up to the surface when connection is completed and circulation resumed. In this case the transient system may require a large backpressure applied to the reservoir leading thereby overbalanced situation. Therefore it is important a real time bottomhole monitoring during underbalanced drilling.

Minimizing transient effects include the ability of well-trained rig crew to reduce connection time," the appropriate placement of multiple drill string floats to avoid extended periods of time to bleed internal string pressure down to facilitate these rapid connections maintaining annular flow during the connection to avoid fluid fall back and to minimize bottomhole low pressure reductions due to an elimination of frictional back pressure effects, and the use of large rigs capable of drilling with double or triple pipe stands to minimize the physical number of connections required “.

2.6.5 Open or Closed Annulus

The choice to use open or [9] closed annulus during connections is reliant on on the type of underbalanced well being drilled. For a well that is capable to flowing freely under its own reservoir energy, the annulus should remain open to avoid high shut-in surface pressures and unnecessary increases in BHP. The annulus should be shut in for wells with insufficient energy to maintain flow during connections and underpressured wells that produce insignificant volumes of liquid. This reduces annular fluid separation and stores the annular gas phase energy, which assists removal of liquid slugs formed during shut in. For overpressured or high deliverability gas wells the annulus should be shut in to avoid loss of flow controlling liquid and minimize surface surges post connection.

3 MULTI-PHASE FLOW AND WELL PRODUCTIVITY MODELS

In this chapter, multi-phase flow model and well productivity models will be reviewed. These models are the one implemented in Matlab and thus they are used for simulation study in Chapter 4

The drift-flux is an important part when it is come modeling multiphase flow in wells. Combined with AUSMV Scheme, it's very useful tool to do following tasks [4]:

- A discretization
  - The well is divided into a certain number of boxes to have a discrete presentation of pressure and temperatures
- A set of conservation laws
  - Conservation of mass (for each box)
  - Conservation of momentum (Newton second law)
- Closure laws
  - Density models (as function of temperature and pressure)
  - Friction models
- A numerical method
  - A method for solving the conservation laws for all the boxes in the whole well to obtain the result

3.1 Drift flux models

The drift-flux includes mainly tree equations, two equations for the mass conservation of each phase and the third one equation for momentum conservation.

3.1.1 Mass balance

Following equations are about to express the mass conservation of each phase and mixture momentum.

$$\frac{\partial}{\partial t}[\rho_l\alpha_l]+\frac{\partial}{\partial x}[\rho_l\alpha_l\upsilon_l]=0 \tag{3.1}$$

$$\frac{\partial}{\partial t}[\rho_g\alpha_g]+\frac{\partial}{\partial x}[\rho_g\alpha_g\upsilon_g]=0 \tag{3.2}$$

Where,

In the equations, we use the index $\kappa \in \{g, l\}$ to indicate either the gas (g) or liquid phase (l) phase. For each phase, are defined as flows:

- ρ_k - Density,
- υ_k - Velocity,
- α_k - Volume fraction,
- q_{fric} - Wall friction momentum source,
- p - Pressure common to both phases,
- q_{gmix} - Wall friction momentum source for mixture,
- ∂_t and ∂_x - The symbols and corresponds to the derivatives in time and position respectively

3.1.2 Momentum balance

Here we have the conservation of the mixture momentum:

$$\frac{\partial}{\partial t}[\rho_g\alpha_g\upsilon_g+\rho_l\alpha_l\upsilon_l]+\frac{\partial}{\partial x}[\rho_g\alpha_g\upsilon^2{}_g+\rho_l\alpha_l\upsilon_l^2+p]=-q \tag{3.3}$$

We can now describe this model in more detail. First, we assume there is no mass transfer between the phases. Furthermore, for computational purposes we assume an analytical slip law of the form

$$\upsilon_g = K\upsilon_{mix}+S \tag{3.4}$$

Where $\upsilon_{mix}=\alpha_l\upsilon_l+\alpha_g\upsilon_g$ the mixture average velocity and K, S is are flow-dependent parameters.

We assume that the liquid density has the form.

$$\rho_l = \rho_{l,o} + \frac{p - p_{l,o}}{a^2{}_l} \qquad (3.5)$$

Where $a_l = 1000$ is the velocity of sound in the liquid phase and $\rho_{l,o}$ and $p_{l,o}$ are given constants. Here we will assume that $\rho_{l,o} = \frac{1000 \ kg}{m^3}$ and $p_{l,o} = 1\text{bar}$ for the gas density we assume the form

$$\rho_g = \frac{p}{a^2{}_g} \qquad (3.6)$$

Where $a_g = 316$ m/s, is the velocity of the sound in gas phase.

The volume fractions are related by

$$\alpha_l + \alpha_g = 1 \qquad (3.7)$$

Mass transfer between the phases is not mentioned. In addition, dynamic energy transfers are neglected; here we considered isentropic or isothermal flows. Consequently the pressure could be expressed as flows:

$$p = p(\rho_g) = p(\rho_l) \qquad (3.8)$$

Finally for source q, we have

$$q = F\omega + F_g \qquad (3.9)$$

Where $F_g = g(\alpha_l\rho_l + \alpha_g\rho_g)\sin\theta$ represents the gravity, where g is the gravitational constant and θ is the inclination. The viscosity forces liquid and forces between the wall and the fluids are taken into account through the frictional force term F_ω given by the following simple model.

3.1.3 Friction model

Friction model is expressed as followed,

$$F_{\omega} = \frac{32\upsilon_{mix}\mu_{mix}}{d^2}, \tag{3.10}$$

Where d is the inner diameter and the mixed viscosity μ_{mix} is given by

$$\mu_{mix} = \alpha_l\mu_l + \alpha_g\mu_g . \tag{3.11}$$

And the viscosity for liquid and gas are assumed to be $\mu_l = 5.10^{-2}$ Pa s and $\mu g = 5.10^{-6}$ Pa s respectively.

Actual model could be written as flowing equations:

$$\partial_t\ \omega + \partial_x\ F(\omega) = G(\omega) \tag{3.12}$$

Where U is the vector of conserved variables, f the vector of fluxes and q is the vector of sources. The vectors are expressed as flowed:

$$\omega = \begin{pmatrix} \rho_l\alpha_l \\ \rho_g\alpha_g \\ \rho_g\alpha_g\upsilon_g + \rho_l\alpha_l\upsilon_l \end{pmatrix}, \qquad F(\omega) = \begin{pmatrix} \rho_l\alpha_l\upsilon_l \\ \rho_g\alpha_g\upsilon_g \\ \rho_g\alpha_g\upsilon^2{}_g + \rho_l\alpha_l\upsilon_l^2 + p \end{pmatrix}, \qquad G(\omega) = \begin{pmatrix} 0 \\ 0 \\ -q \end{pmatrix} \tag{3.13}$$

It could be express also as flow,

$$\partial_t \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} + \partial_x \begin{pmatrix} \upsilon_l\omega_1 \\ \upsilon_g\omega_2 \\ \upsilon^2{}_l\omega_1 + \upsilon^2{}_g\omega_2 + p(\omega_1, \omega_2) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -q \end{pmatrix} \tag{3.14}$$

Where $\omega_1 = \rho_l \alpha_l$, $\omega_g = \rho_g \alpha_g$ and $w_1 = \rho_l \alpha_l \upsilon_l + \rho_g \alpha_g \upsilon_g$ note that the pressure $p = (\omega_a, \omega_2)$ is passive variable obtained from conservative variables ω_1 and ω_2. Additional equations are mentioned here.

$$\alpha_g \ \rho_g \ \ll \ \alpha_l \ \rho_l, \tag{3.15}$$

For two phases where $\alpha_g \epsilon\ (0,1)$ the following approximated sound velocity has been devised:

$$\omega^2 = \frac{p}{\alpha_g \, \rho_l \ (1-k\alpha_g)} \tag{3.16}$$

The corresponding given eigenvalues is given by:

$$\lambda_1 = v_l - \omega, \qquad \lambda_2 = v_g \qquad \lambda_3 \ = \ v_l + \omega \tag{3.17}$$

The first and third eigenvalue correspond to pressure pulses propagating downstream and upstream while the second eigenvalue corresponds the wave speed of the gas volume traveling downstream. For pure gas liquid regions ($\alpha_g \ = 0$), we have

$$\lambda_1 = v_l \ - a_l \qquad\qquad \lambda_3 = v_l \ + a_l \tag{3.18}$$

where a_l ,is the sound velocity of the liquid phase. These eigenvalues correspond to pressure pulses propagating upstream and downstream. Similarity, for pure gas regions ($\alpha_g \ = 1$) we have

$$\lambda_1 = v_g \ - a_g \qquad\qquad \lambda_3 = v_g \ + a_g \tag{3.19}$$

Where α_g is the sound velocity of gas phase.

3.2 Flux – Splitting Schemes

Instead of discretization the flux F (see equation drift flux) directly, we would like to work the convection and pressure terms separately in the discretization procedure. The natural splitting of the total flux into convective and pressure parts is given by,

$$F = F_c + F_p = \begin{pmatrix} \rho_l \alpha_l \upsilon_l \\ \rho_g \alpha_g \upsilon_g \\ \rho_g \alpha_g \upsilon^2{}_g + \rho_l \alpha_l \upsilon_l^2 + p \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix} \tag{3.20}$$

A finer treatment of the convective flux can be written as flow:

$$F = F_{c,l} + F_{c,g} = \rho_l \alpha_l \upsilon_l \begin{pmatrix} 1 \\ 0 \\ \upsilon_l \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ \upsilon_g \end{pmatrix} \tag{3.21}$$

We can now, describe the AUMSV Scheme.

3.2.1 Description of AUMSV Scheme.

AUSM is defined [15] as Advection Upstream Splitting Method schemes for hyperbolic systems of conservation laws and do not allow any analytical calculation of the Jacobian. The AUSM system is very simple and involves two mass conservation equations, one for each phase, and a common momentum equation. The AUSM scheme is dealing with a given two- phase flow model and gives exact resolution of contact discontinuities (mass fronts), but do not avoid oscillatory approximations of acoustic waves. Moreover E. Steinar and K.K. Fjelde proposed a hybrid FVS/FDS (flux-vector splitting / flux-difference splitting) scheme, indicated as AUSMV, which is a combination of AUSM and FVS. The purpose of this scheme could be stated as flow:

- The scheme should be able to handle transition between two-phase (gas–liquid) flow and pure liquid or pure gas flow without introducing negative mass.
- The scheme should be able to handle general slip relations (unequal fluid velocities).
- For many applications it is desirable that the time step associated with the scheme is not restricted to the speed of the fastest wave, which can be very high in pure liquid regions.

3.2.2 Discretization in time and space.

The term fluid is a common word used for both gases and liquids. It is necessary to have a model[14] as a part of the numerical solution of a drift- flux formulation of the two phase flow conservation equations.

Now we can discretize the drift- flux model by solving the equation numerically. In the figure the general node denoted by j has boundary A with its neighbor node $(\mathrm{j}-1)$ in the negative x- direction and boundary B with its neighbor node $(\mathrm{j}+1)$ in the positive direction.

The time is distributed into small time steps; Δt. Spatial discretization is conducted in axial direction, where the length of each segment is denoted Δx. The value of these steps in time and space must satisfied CFL-condition,

$$\Delta t = CFL \frac{\Delta x}{max(||\lambda_1|,|\lambda_2|,|\lambda_3|}, \tag{3.22}$$

Where $\lambda_1, \lambda_2, and\ \lambda_3$ eigenvalues are correspond to pressure pulses propagating upstream and downstream.

In order to compute a numerical solution a set of computational point along x is established. Also, a number of time steps are needed. The numerical solution will be computed numbers for each computational point and each time step. In most practical applications the computational domain is subdivided into grid cells and the computational points will be cell centers. Furthermore the boundary conditions are defined. First, the initial state must be given, i.e. numbers for all computational point at time t=0. Secondly, specifications at the boundaries of the computational domain are required. Frequently no flow exterior boundaries are specified.

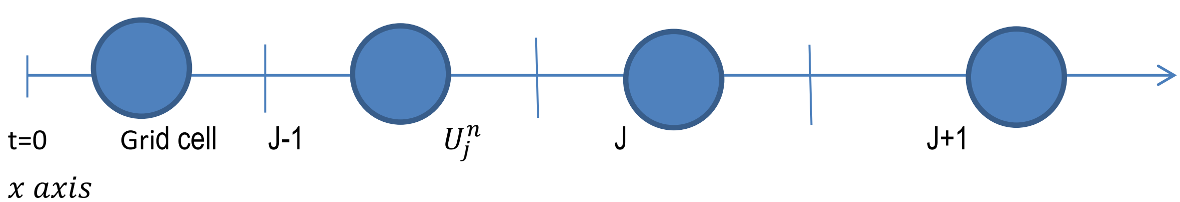

Figure 7: Grid cell

3.2.3 Numerical solution and Matlab code.

The numerical solution of continuous model is defined as flows:

$$\partial_t \, Q + \partial_x \, F(Q) = G(Q) \qquad (3.23)$$

Another version of the conservation equations could be expressed as follow,

$$Q_j^{n+1} = Q_j^n - \frac{\Delta t}{\Delta x}\left(F_{j+\frac{1}{2}}^n\left(Q_j^n, Q_{j+1}^n\right) - F_{j-\frac{1}{2}}^n\left(Q_{j-1}^n, Q_j^n\right)\right) + \Delta t.\, G\left(Q_j^n\right) q_j^n \qquad (3.24)$$

Where $F_{j\pm\frac{1}{2}}^n$ are fluxes at upper and lower of lower part of box j. and $\Delta t = t^{n+1} - t^n$ and

$\Delta x = x_{j+\frac{1}{2}} - x_{j-\frac{1}{2}}$.

Δt: Time step

$n+1$: New time

n: Old time

In the figure Δx denote the length of box.

The previous equations as well as the flux-splitting equations constitute the core of the Matlab code [18]. Before computations at time step $n+1$ the value of the solution at all computational point at the previous time step n must be calculated. In other word the new flux is equal to the old flux plus inflow from formation (source term, q).

Applied in different parts of multiphase flow, steady state and transient, Matlab is very useful tool that can compute various parameters such as primitive variables in each segment of the well. The numerical simulation could handle variation of flux at the inlet and outlet:

- Mud circulation rate
- Gas injection rate
- Choke pressure (outlet)

However we may include some assumptions which could fit for the model which we are dealing with rather than the reality of multiphase flow system. Hence we may consider the flowing simplifications:

- Isotherm system.
- No mass transfer between the phases.

3.2.4 Procedure to compute primitive variables

For a given flux U, the main task of Matlab code consists to compute the following 7unknown variables:

- Volume of fractions: α_l , α_g
- Densities of gas and liquid: ρ_l, ρ_g
- Velocities of different phases: , υ_l, υ_g
- Pressure: P

For computing the seven equations, it is oblivious that we need seven equations. One gets the three first equations from drift-flux definition. Moreover, for a given flux U, Matlab code starts computing first of all the pressure because we can obtain pressure from the conserved variables from mass conservation. The value of pressure is obtained using a second order equation by computing the previous mentioned equations. After computing P, one could easily calculate the density of the respective phases as well as volume fractions and velocities by using their respective equations.

For a given flux U, Matlab code starts computing first of all the pressure using proceed step by step in order to compute various variables α_l , α_g, P, υ_l,and υ_g.

Hence equation (3.7), $\alpha_l + \alpha_g$ =1, is used as main tool.

We multiply up and down in the equation by ρ_l and the one hand $\rho_g$and the other hand, and we obtain an equation as function of pressure P,

$$\frac{\alpha_l\rho_l}{\rho_l(P)}+\frac{\alpha_g\rho_g}{\rho_g\,(P)}=1 \qquad (3.25)$$

We can express the previous equation,

$\frac{U_1}{\rho_l(P)}+\frac{U_2}{\rho_g\,(P)}=1$ with ρ_l, ρ_g known and U defined as $U=\alpha\rho$.

We can insert equations (3.5) and (3.6) in equation (3.23) we get,

$$\frac{U_1}{\frac{P}{a_g^2}}+\frac{U_2}{\frac{(P-P_O)}{a_l^2}\rho_{l,o}}=1$$

$\rho_l(P)$ and $\rho_g(P)$ can easily solved with help of equations (3.5) and (3.6).

Moreover equations $\alpha_g = \frac{U_1}{\rho_g(P)}$ and $\alpha_l = 1 - \alpha_g$ help to solve volume fractions.

And finally in order to solve U_l and U_g, we introduce two equations with two unknowns.

$$U_3 = U_1 U_g + U_2 U_l$$

$$U_g = K(\alpha_l U_l + \alpha_g U_g) + k_1$$

3.3 Well productivity model

A realistic productivity index in multiphase flow is provided [4] by Vogel equation for underbalanced and wells producing for several years. Therefore, in most cases, the reservoir in which a well is drilled underbalanced is at saturated conditions and very well characterized. However, the Vogel equation is very complex to implement and we may resort to a simpler model to predict the gas influxes from formation while drilling.

3.3.1 Gas Inflow and productivity index

The compressible nature of gas given in the inflow performance relations (IPR) (figure 8) [18] do not allow being a straight line. However, for simplicity reason the considered model in this thesis remain linear and assumed to be an extension of the steady state relationship derived from Darcy's Law, using an average value for the properties of the gas between the pore pressure and wellbore pressure, leads to:

$$Q_g = K_{wellres}(P_{pore} - P_{well}) \tag{3.26}$$

where

$K_{wellres}$ is a constant and

P_{well} as follow:

$$P_{coll} < P_{well} < P_{pore}$$

With P_{coll} : the collapse pressure.

Remark

This relationship is valid at low flow rates, but becomes invalid at higher flow rates since non-Darcy (or turbulent) flow effects begin to be observed.

3.3.2 Well collapse management during connection operation

Assume that during connection, a large amount of gas influx into the well. This as a result reduces the density of the well bore and hence the well pressure.

The gas / mud mixture density will behave according to the following relation:

$$\rho_{mix} = \alpha_l \rho_{mud} + \alpha_g \rho_g \tag{3.27}$$

The well pressure is given as:

$$P_{well} = \rho_{mix} * g.h \tag{3.28}$$

When the gas influx volumes increase, the well pressure decreases. This will be lower than the collapse pressure. If this is the case, then the formation around the wellbore will be collapsed into the well. In order to manage /control well instability problem, it is important to apply back pressure so that the pressure will be higher than the collapse pressure.

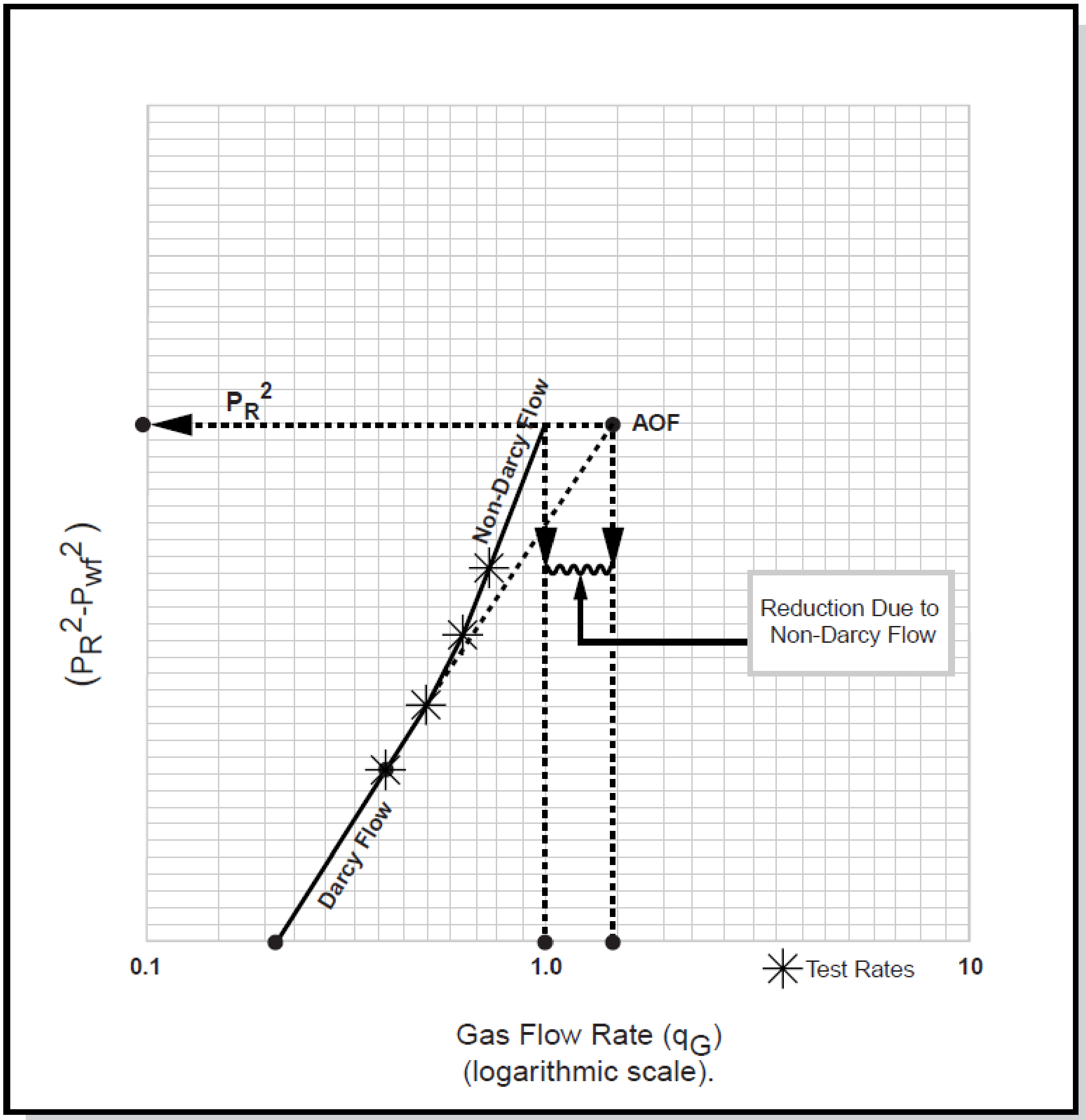

Figure 8: Gas well deliverability reduced by non- Darcy flow pressure losses [19].

A more realistic e relation between well pressure and influx rate is given as follow:

$$Q_g = K_{wellres}\left(p_{Rav}{}^2 - p_{bh}{}^2\right)^n \qquad (4.2)$$

where 0.5 <n <1.0 and

- p_{bh} is the bottomhole pressure.
- p_{Rav} is the average reservoir pressure.
- Q_g is the gas flow rate for a given bottomhole pressure.

4 BASIC CASES MODEL

In this chapter we will simulate basic cases in which the Matlab code is built originally.

We will study the behavior of:

- The gas volume fraction
- flow across the choke
- the velocity and gas and liquid mass rates

4.1 Simulation arrangement

4.1.1 Well geometry

A vertical well of 2000 is considered measured from RKB. It is filled with a two phase fluid which has the following densities:

- $\rho_l = 1000\ sg.$
- $\rho_g = 1\ sg$

The well is divided in 25 boxes simulated in 2500 seconds.

4.1.2 Reservoir /formation properties

For the previous some initiations prior the simulations are done. Hence the following data are the input parameters for this gas reservoir:

- Pore Pressure: P_{pore}= 150 bar
- Fracture Pressure: P_{frac} =200 bar
- Collapse Pressure: P_{coll}=110 bar
- Gas density is
- Water injection rate estimated 22 kg/S
- Nitrogen injection rate 2 kg/S
- With choke constant pressure 1 bar

4.2 Simulation results and interpretations

Here we are circulating mud at the same time we are injecting gas and the different parameters behavior is reported. The different parameters are simulated either as function of depth or as function of time.

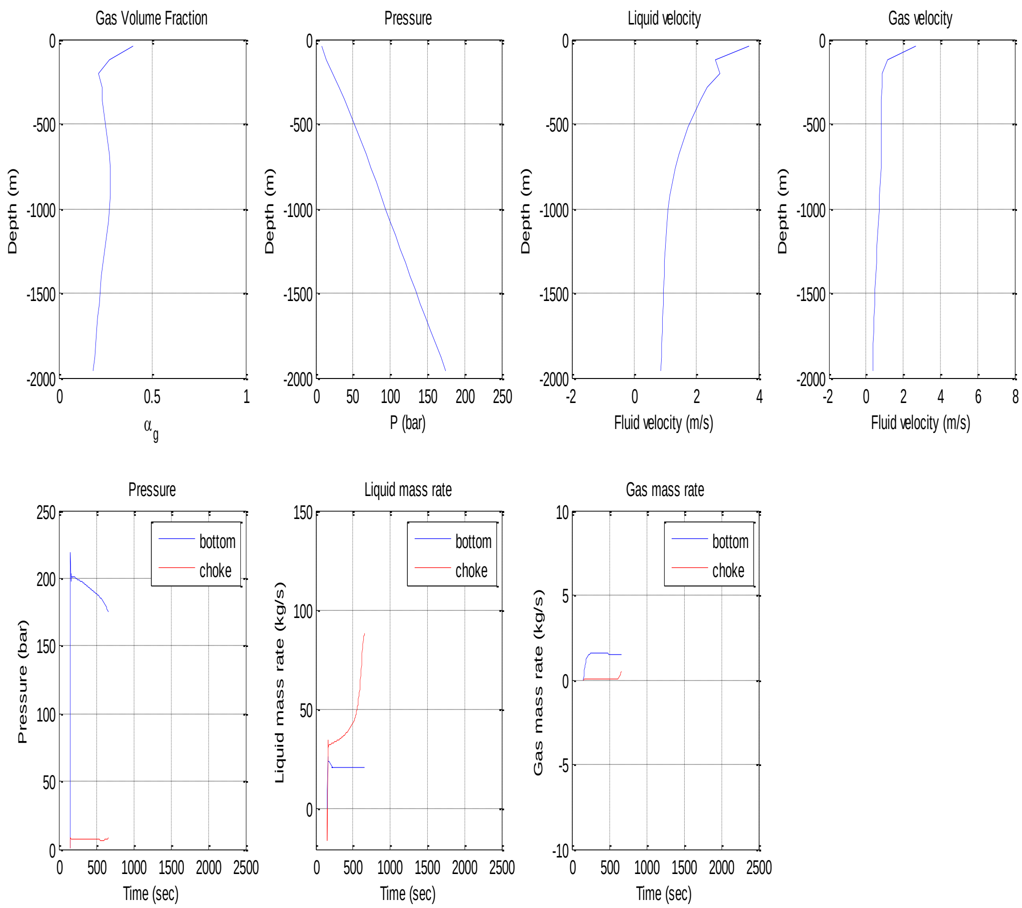

Figure 9: Well behavior at 700 seconds and upon starting nitrogen injection and mud circulation in the well step1.

The gas volume fraction verifies the inversely proportionality relationship between the pressure and volume of a gas as stated in Boyle's law [15]. Hence when pressure is high, gas volume fraction is compressed and velocities of gas and liquid are not very high. However, the less pressure in the system the more expanded is the gas volume fraction. When the pressure drops, the gas releases and expands. The reduced pressure also means greater flow rate and greater velocity, which will increase the wall friction contribution.

Decreasing pressure will provide greater volume fraction of gas as well as lighter fluid mixture and greater liquid and gas velocity. Reduced pressure will reduce the static pressure loss contribution.

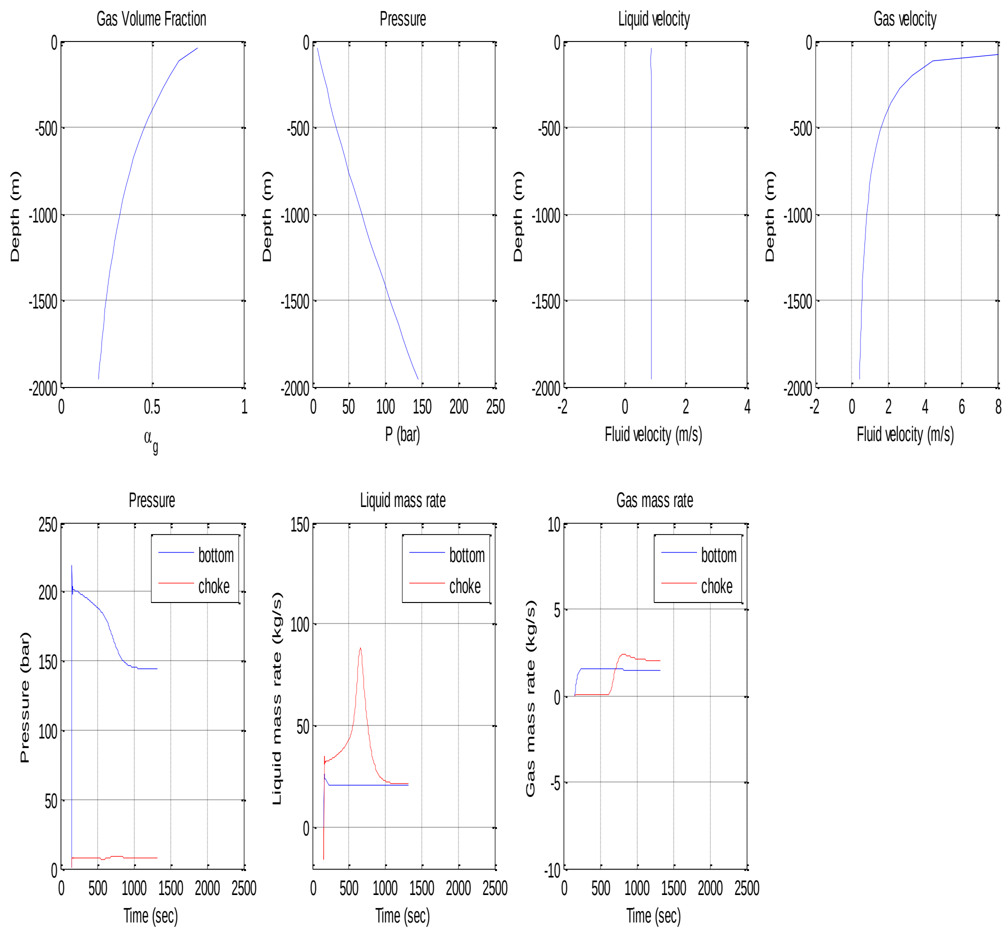

Figure 10 : The wellbore reaches a steady stead condition after 1300 sec, resulting from constant injection and constant flow rate.

At the beginning, lower density mud is circulated, and the mud is mixed with injected gas in the wellbore allowing to reduce its equivalent density and hence its hydrostatic pressure along the wellbore. When the gas injection volumes increase, the well pressure decreases. This will lead a lower hydrostatic pressure observed at the beginning the bottom hole pressure in figure 10.

As we are injecting constant rate of liquid and gas, the system stabilizes and verifies the conservation law: produced rates are equal the injected rates. Moreover under stationary conditions, the mass flow rates as well as velocities and pressures are constant with a constant cross section in the well. Knowing production rates at standard conditions

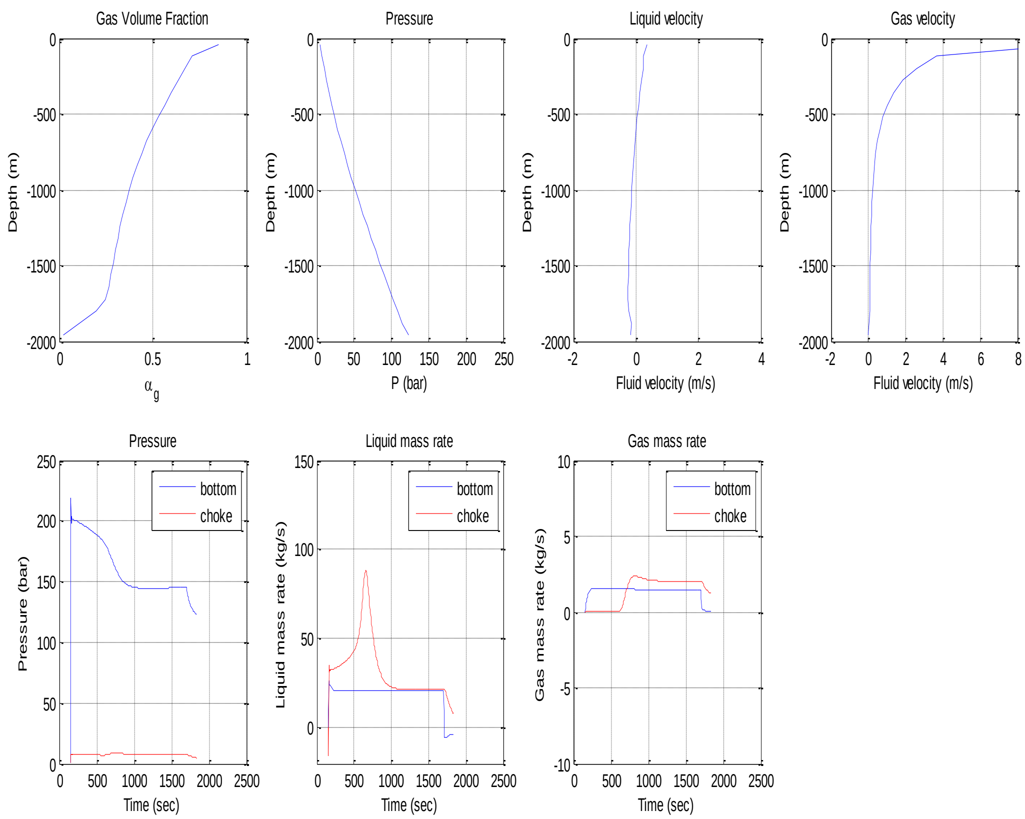

Figure 11: At the beginning of connection at 1700 sec.

In figure 11; there is not much change in choke liquid mass rate. The change starts at around 250 sec. where the curve starts to increase steadily before it jumps to its maximum at around 600 sec. After this, the curve drops until time 100 sec where it is constant until time 1600. Consequently, the curve drops to its minimum at time 1800 sec. After the steady state situation, we stop mud circulation and gas injection and start making pipe connection. Consequently we are witnessing pressure drop due to the circulation stop of the fluid column:

- Pressure is only hydrostatic dominated during connection; frictional contributions as well as fluid acceleration pressure are not represented.
- Lack of friction of the flowing fluid is affected by kinetic energy in the system and the velocity of Liquid of the heavy materials accumulate in the bottom of the well leading to negative velocity fluid.

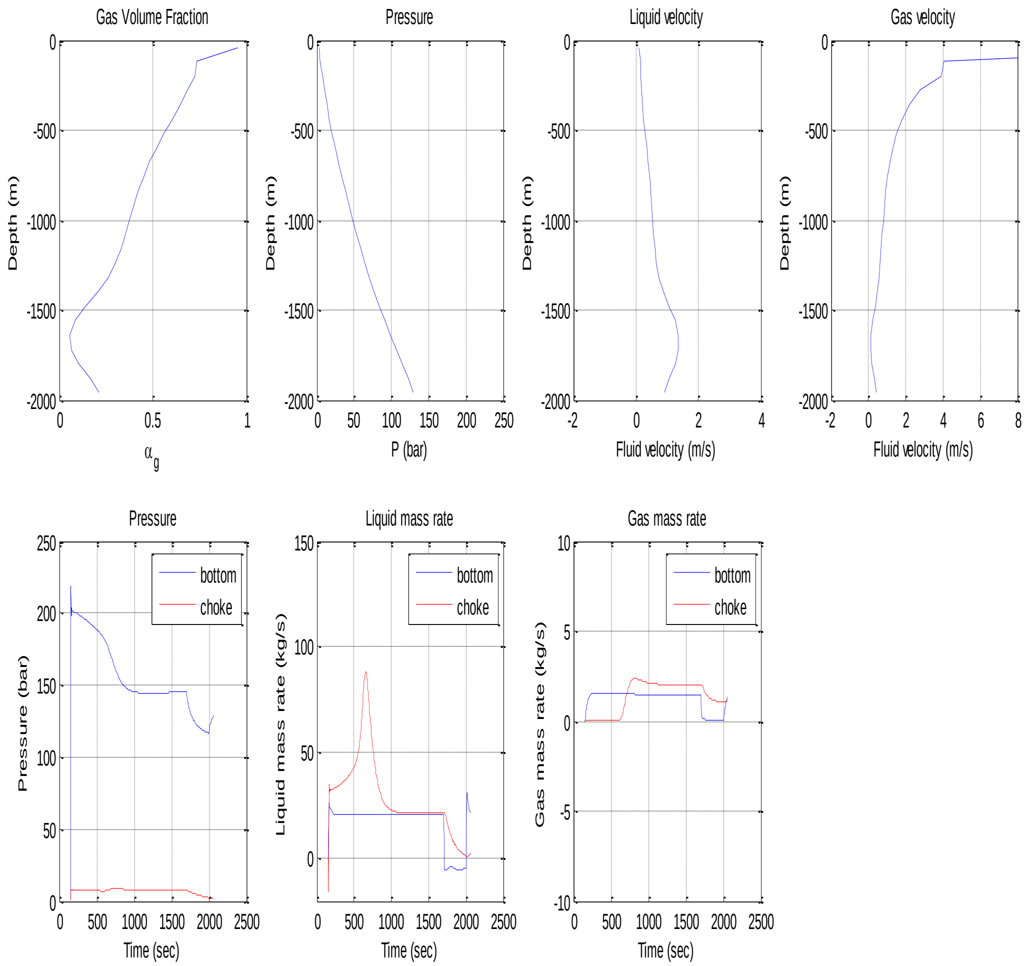

Figure 12: The end of the connection.

In figure 12 the mud circulation is stopped, we are witnessing a bottom hole pressure drop. This is due to the hydrostatic and frictional pressure drop i.e. a very lower ECD presence in the well. In order to manage /control well instability problem, it is important to make connection as quickly as possible. As we are making connection and the liquid and gas injections are stopped, produced liquid and gas drop before they increase again after resuming injections and circulation. Upon restarting injection and circulation, the fluid rate out of the choke continuously increases until we reached the maximum allowable producing rate limited by opening size of the choke and decrease gradually until it became constant.

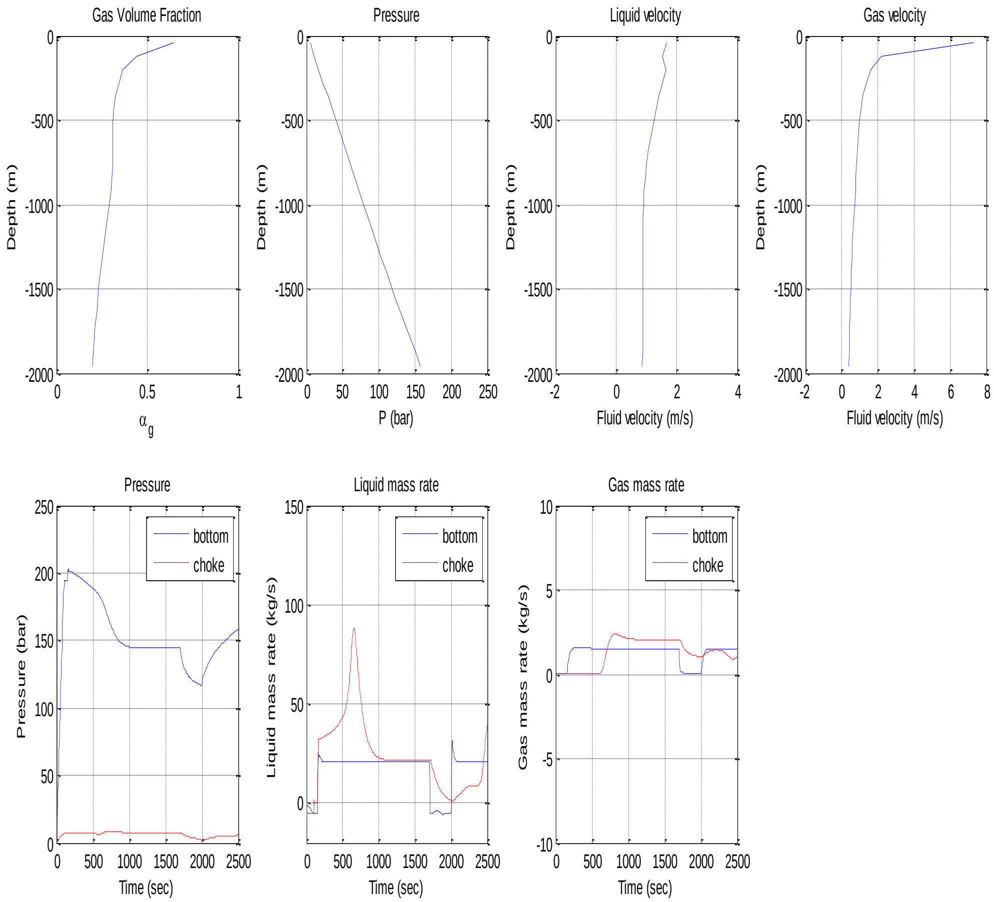

Figure 13: Final result of Basic cases

We may summarize expected bottomhole pressure as function time in the different drilling steps in the figure 13 as the choke is fully closed (choke pressure is nearly zero).

5. SIMULATION SCENARIOS

5.1 Well geometry

A vertical well is considered with the following data:

- Outside diameter of 8-1/2 in.
- Drillstring of 5 in.
- Well depth of 2000 m is divided into 25 boxes of 80 m.

5.2 Drilling into Reservoir

A producing of a single phase gas well, from a shallow reservoir, flows up in a tubing string. The total well depth is 2000 m TD (True depth).The drilling goes ahead and the reservoir model is allowed to start producing. We keep the circulation rate at 22kg/s and the nitrogen rate remains at 2kg/s. As the reservoir starts producing gas, we kept the choke constant around 1 bar in order to maintain BHP above collapse pressure. The result of the simulation is reported here after:

5.3 Simulation result with reservoir inflow

As the gas starts flowing into the wellbore at 3000 sec and the reservoir is assumed to produce with a productivity constant, Kwellres = 0.00000001$\frac{kg}{S\ bar}$..

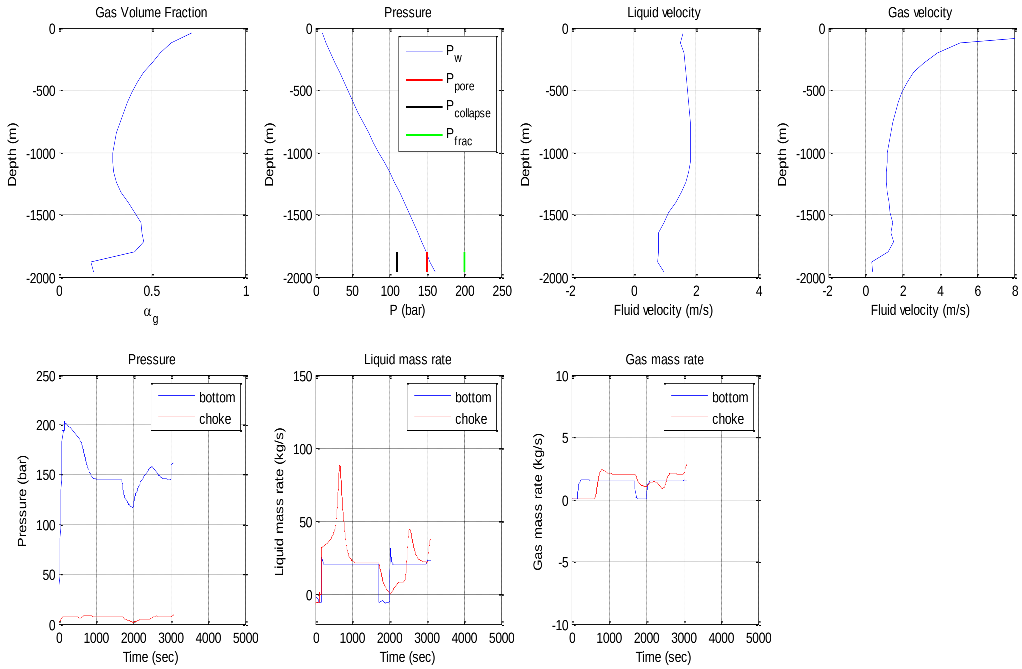

Figure 14: Simulation of producing gas well, step1.

Pressure suddenly jumped-up due to the gas influx which filing the annulus over 50 % and the well became overbalanced. In this case the pressure is only frictional dominated. Furthermore the gas influx as well as the non- stop mud circulation in the well allows lightening the mixed density increasing hence the respective velocities of different fluids in annulus.

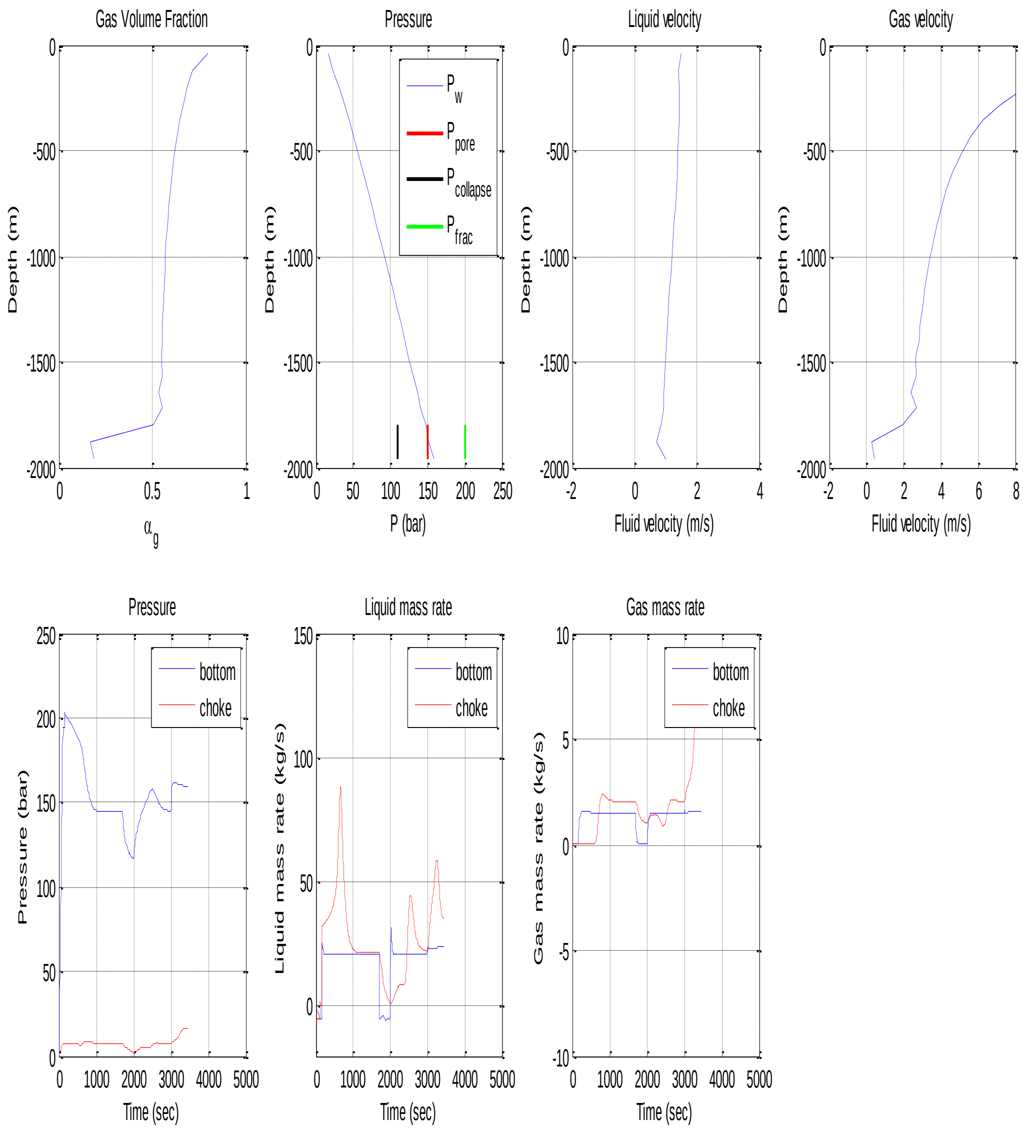

Figure 15 Simulation of producing gas well step2, at 3500 sec.

After 3000 sec figure 16 shows exponential increased in gas mass rate at the outlet of the choke. This is due to continuously gas influx from reservoir. Influx occurs when the reservoir pressure is less than the wellbore pressure. Gas has a much lower density than the drilling mud (water in our case) in the well and it cannot expand beyond the well boundaries. As the gas migrates upwards it will maintain its initial pressure leading hence pressure increase in the whole well.

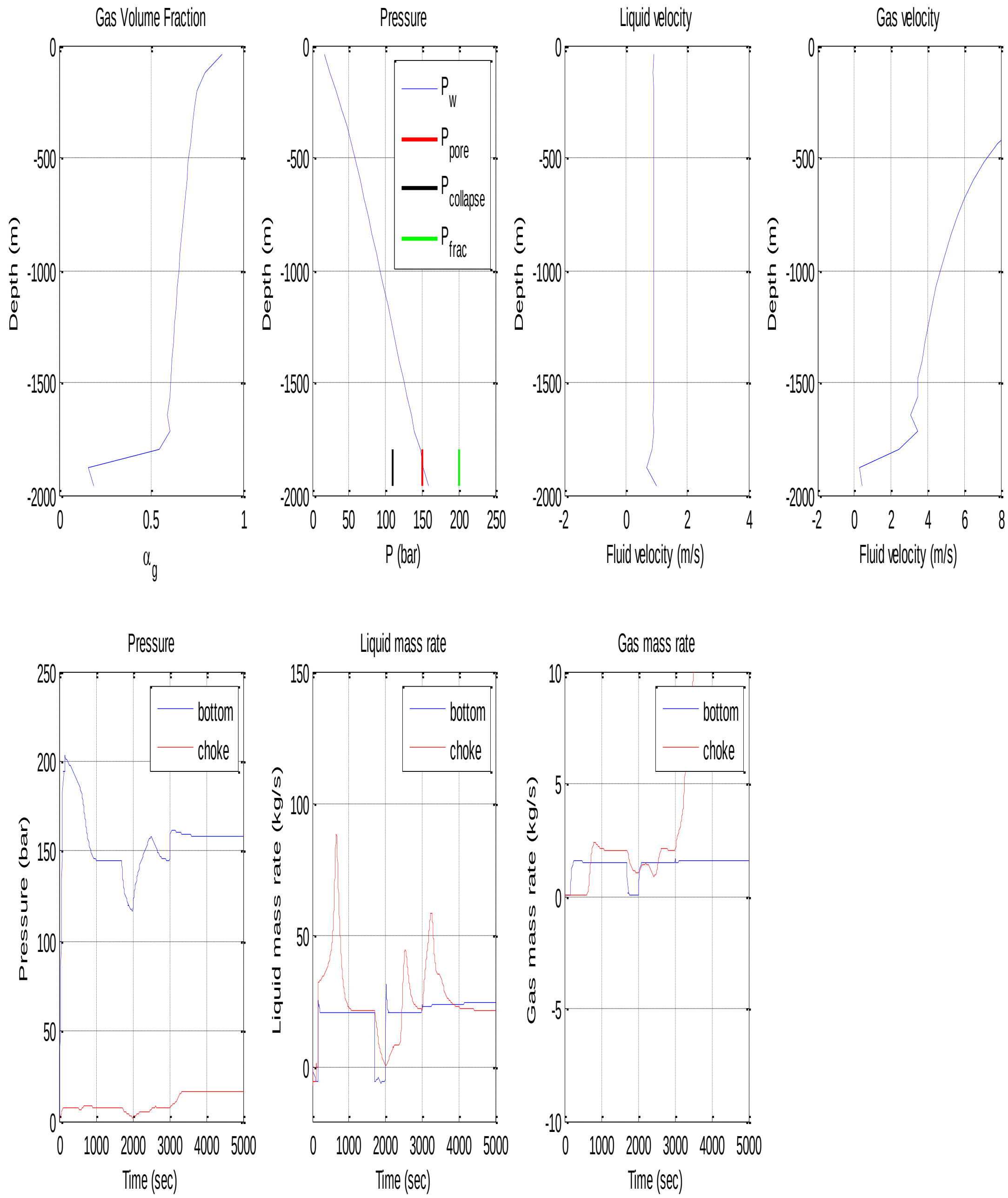

Figure 16: Simulation of producing gas well, step3.

After the wellbore being disturbed by the gas influx, the pressure builds up and the tendency goes to a more homogenous situation, a new steady state situation is achieved. This steady state is imposed by the gas density which is dominating the wellbore. In figure 11 the steady state is allowed by the mud density which is filling the wellbore and the annulus during the drilling whiteout reservoir influx.

5.4 New connection with or without reservoir flow

The following scenarios are carried with taking into account the bit penetration into the gas formation and circulation of the drilling fluid until the gas inflow into the annulus is significant, and detected at the surface. This kind of drilling condition is a necessary; otherwise, it would be very difficult to analyze any gas inflow. The well properties under which simulations are run remain the same as previous simulations but the changing parameters consist only Kwellres, constant including in the productivity index equation at the same time we are making connection at t=3500 sec in different cases:

- Without influx from reservoir, Kwellres=0 $\frac{kg}{S\ bar}$
- With small influx , Kwellres = 0.00000001$\frac{kg}{S\ bar}$
- Strong influx Kwellres = 0.00000001*10 $\frac{kg}{S\ bar}$
- Pressure response as well as velocities behavior are reported in the figures below.

5.4.1 No influx

Here we assume that there is no influx of gas from reservoir with, Kwellres= 0 $\frac{kg}{S\ bar}$

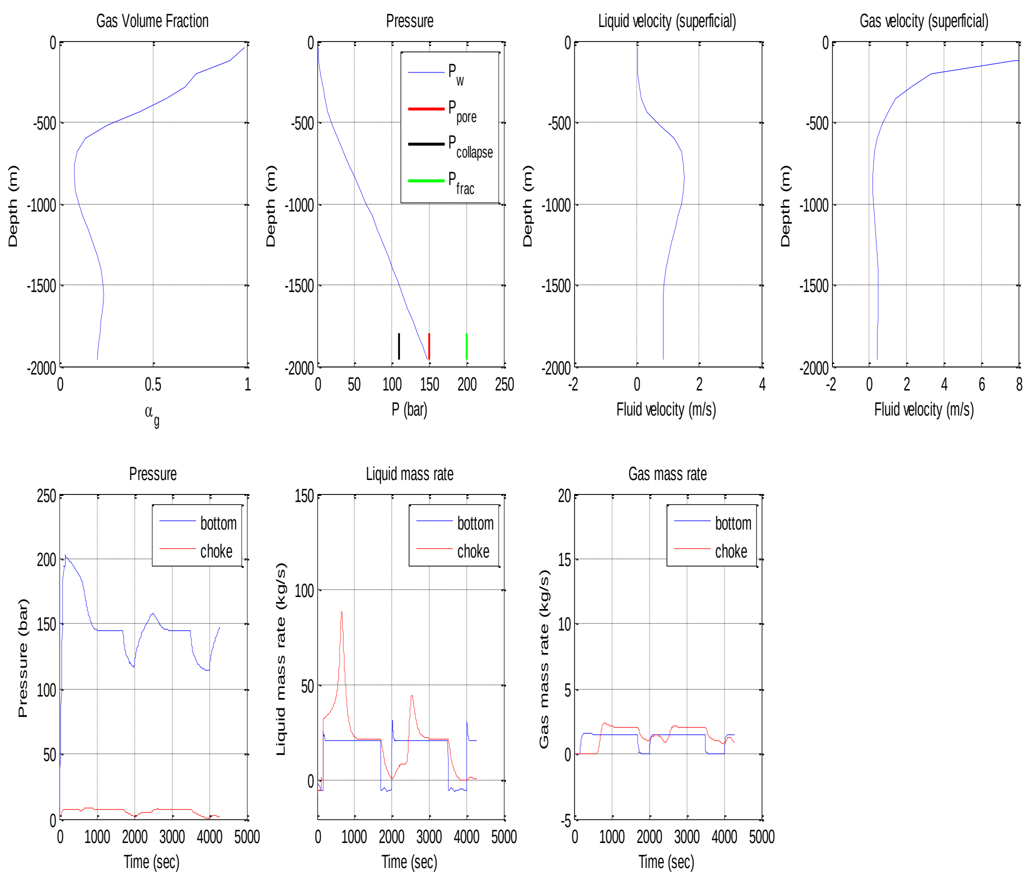

Figure 17: Making connection without influx step1

This case is similar to the one simulated in Base case see figure11 and figure 12. Here we are making connection as we stopped the circulation and injection. Furthermore the shape of pressure curve between 3000 sec and approximately 4500 sec is comparable the one obtained between 1500 sec and 2500 sec. we can differentiate the two important regions:

- The hydrostatic dominated region is characterized by pressure drop due to the lack of circulation and injection.
- The friction dominated region is described by a pressure build up upon resuming circulation and injection.

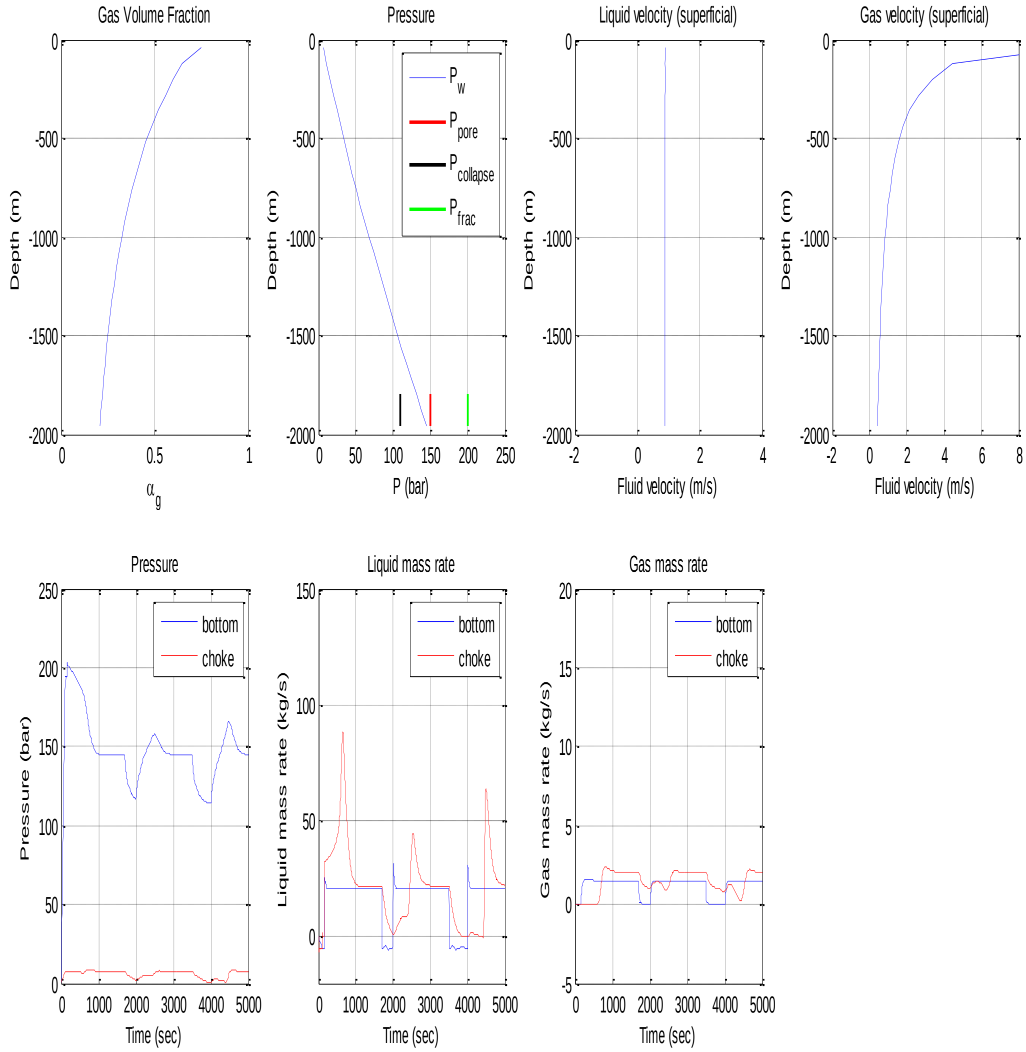

Figure 18: Making connection without influx step2

At the final stage of the simulation the wellbore pressure exceeds temporarily over the pore pressure becoming hence overbalanced. We may explain this overbalanced condition as following:

One must bear in mind that the formation pressure in open hole sections is influenced the pressure behavior. If there is no formation fluid inflow, borehole pressures are directly dependent of mud weight and could easily keep below the formation pressure.

5.4.2 Small influx

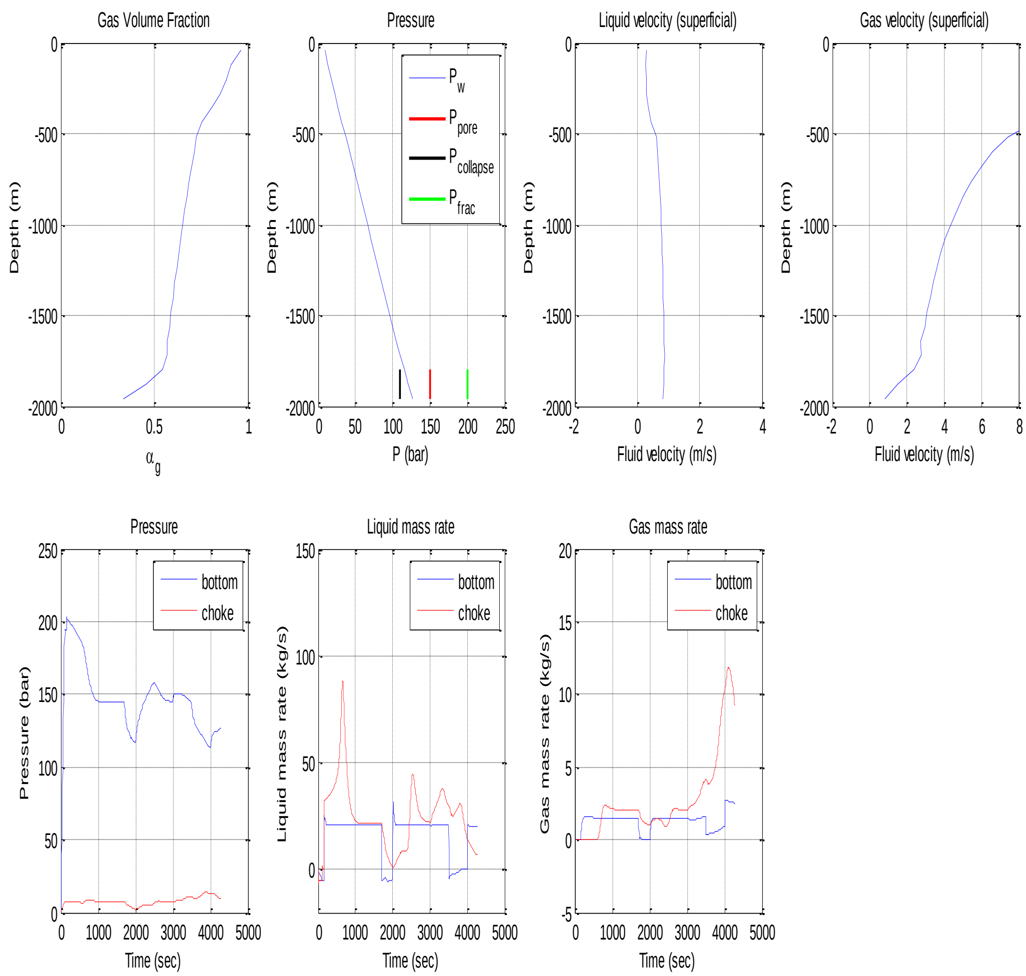

Figure 19: Making connection with small influx step1

With small gas influx, we can observe that the borehole pressure drops during connection due to the lowered mixed density. Formation gas influx interacts with drilling fluids which effect borehole pressure as well as influx rate. When mud circulation stops, the gas influx helps to lift mud staying in the annulus up to the surface. This process leads decreasing pore pressure when not circulating. The backpressure was held constant 1 bar all the time in order to safeguard the surface equipment from excess production rates or pressures. This also increases the BHP. The available backpressure is restricted by the pressure rating of the equipment and formation upstream of the choke. With a mixed density in the borehole, bottomhole pressure could easily be increased by decreasing the gas injection rate.

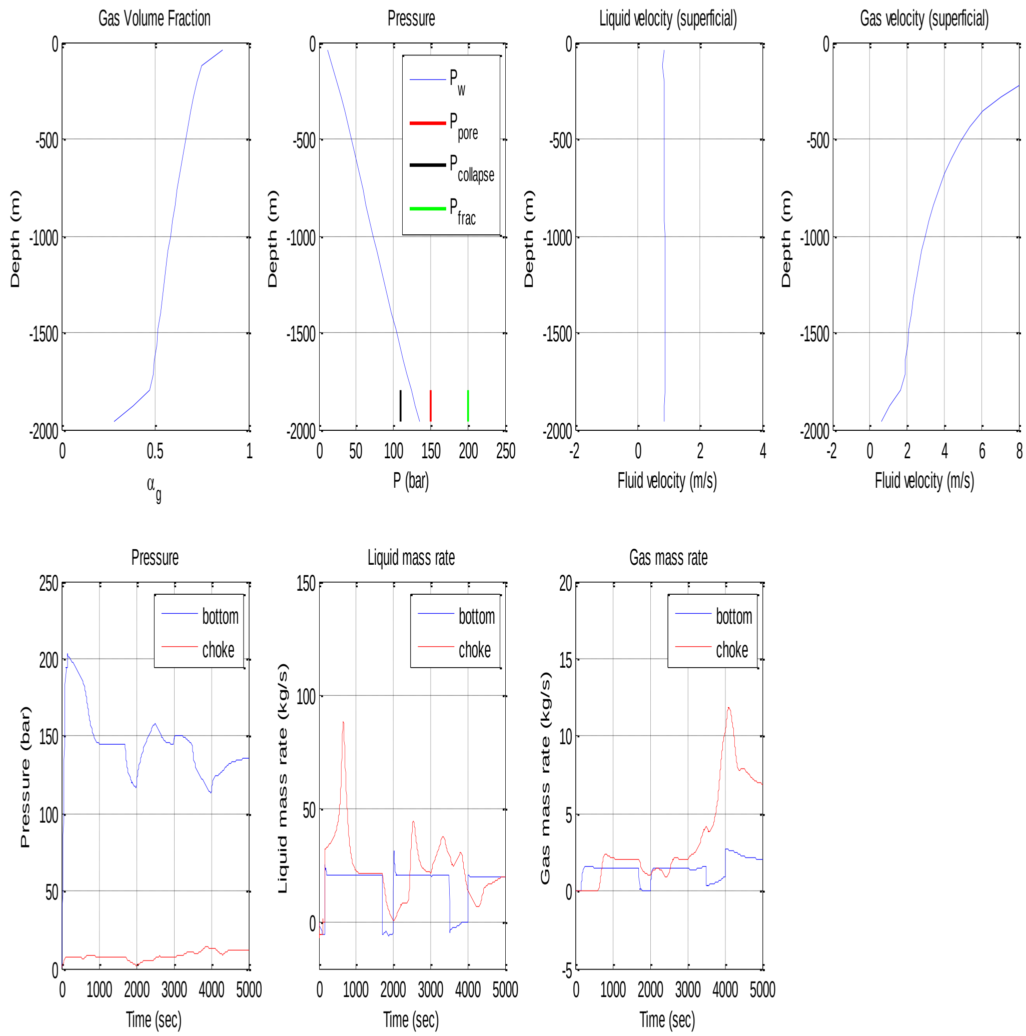

Figure 20: Making connection with small influx step2

Again upon restarting circulation at 4000 sec, the pressure builds up and increases smoothly but still stay below the pore pressure. This changes the borehole pressure and the gas production rate affects the ECD (equivalent circulation density). One can easily see that the ECD of the wellbore increases with increasing depth. In this vertical well, we can see with small influx that a homogeny permeable zone close to the bit not to be a factor to destabilizing the whole well in underbalanced drilling. However we must be ready in order to handle an unpredictable situation when pressure changes occur such as tripping, or connections.

5.4.3 Strong influx

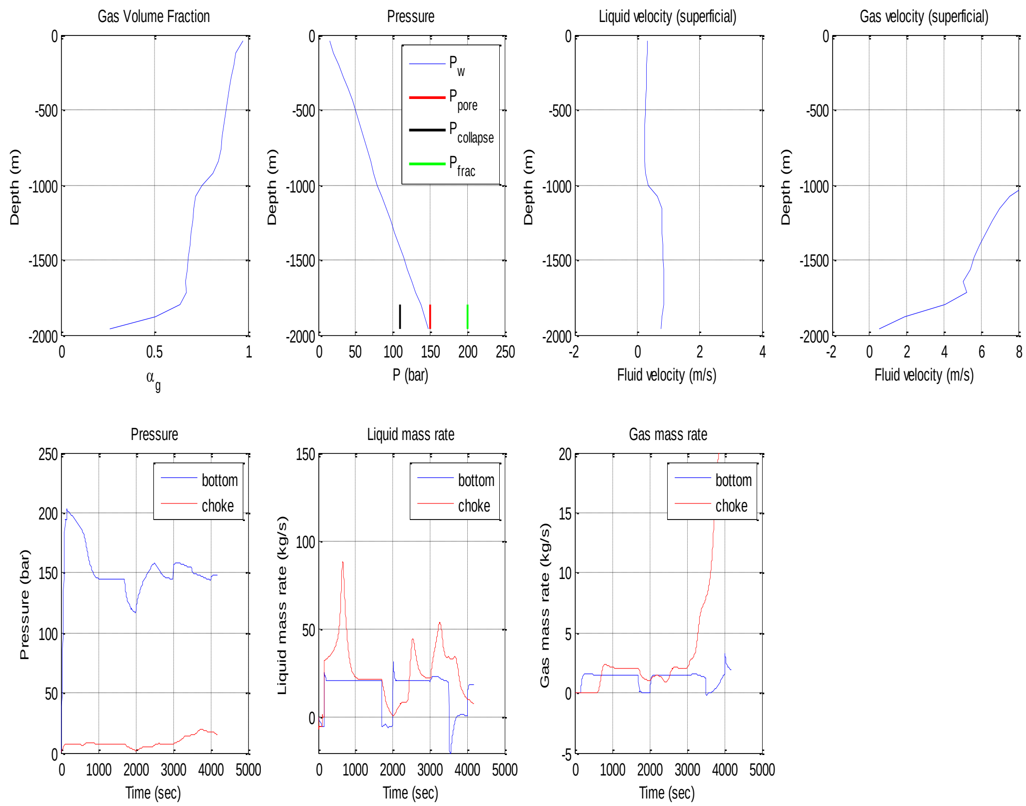

Figure 21: Making connection with" Strong"influx step1.

With a strong influx in the borehole, the tendency looks like a semi-steady state both in respective velocities and pressure which satisfying the underbalanced conditions. The gas mass rate seems being controlled by a surface choke pressure held constant 1 bar, which has the objective of keeping the rate essentially constant and within the capacity of the surface equipment. Lack of proper surface equipment may be the major source of problem associated with such huge inflow from wellbore and could exacerbate handling the flow rate measurement in accuracy compared to the pressure measurement. The determination of reservoir properties depends on how the flow rate both quantities and quality being accurately measured.

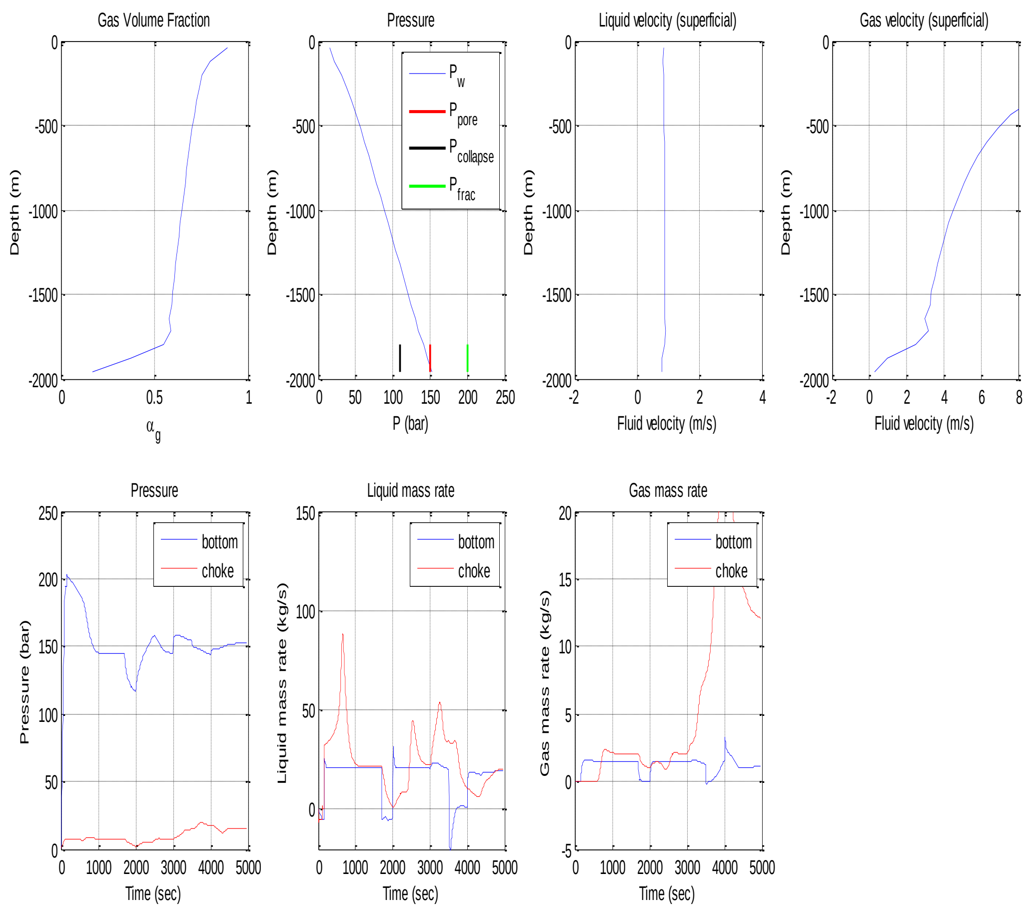

Figure 22: Making connection with" strong "influx step2.

The final step is characterized with a wellbore pressure quite under pore pressure as well as a drop of gas mass rate due to the huge amount of gas inflow rate and perhaps the homogenized tendency predominating. It seems that this case is quite different from above mentioned others connections. It is not too difficult to understand that the well pressure gradient is nearly the same as the exposed formation to drilling. Here we do not have a pressure drop during the connection. The pressure drop is compensated by the sudden and strong gas influx which imposes its density allowing hence the wellbore to keep a semi-steady state condition.

6. DISCUSSION OF RESULTS

Underbalanced Drilling can be defined as drilling activity employing appropriate equipment and controls where the pressure exerted in the wellbore is intentionally less than the pore pressure in any part of the exposed formation with the intention of bringing formation fluids to the surface.

During this work we simulate different phases of the wellbore status with or without reservoir gas influx. The purpose of basic cases simulated in 4.3 was to illustrate the basic physics behind different parameters such as Boyles law Moreover the most challenging part of the work remained when the mud circulation and injection are stopped and connection is made.

At this stage the circulation system is working on the hydrostatic dominated, and it may be not stable because the bottomhole pressure quickly drops due to reduction in the hydrostatic pressure caused by gas influx from wellbore.

During the connection a huge amount of gas could enter in the wellbore, at this moment the circulation system is a frictional dominated and the bottomhole pressure witness fluctuations and it would be difficult to maintain the wellbore pressure under pore pressure; not just because of the loss of ECD when circulation is stopped, but because formation fluids, gas in particular, accumulate in the annulus, perturbing the pressure balance and have to be circulated out through the choke before resuming circulation system . Hence re-establishing in the formation fluids an underbalanced condition downhole can take up several hours of Non Productive Time per connection. We may need a continuous circulation system which allows the ECD to be kept constant and the underbalance balance pressure to be more easily controlled. By circulating continuously during connections, there are no accumulated fluids to circulate out and the underbalanced downhole pressure regime is maintained throughout, allowing drilling to restart without delay. Furthermore, with reservoir fluids flowing into the wellbore, it is intrinsically safer to have uninterrupted circulation, particularly, while making connections. Some critical situations such as potential poor cuttings transport, as well as, and pack-off tendencies may lead to stuck pipe situations. It is important to detect such symptoms in order to be able to reduce non-productive time and increase both drilling safety and efficiency. When the formation pressure is quite high, adaptive lower density mud is need in the wellbore pressure below the pore pressure of the formation. The main task of nitrogen gas injected into the drilling mud was to reduce its equivalent density and hence its hydrostatic pressure along the well depth. The gas inflow rate from the formation is a criterion to determine how good the reservoir actually produces without significant influences by the various wellbore conditions. The model well behavior studied in this thesis has the goal to provide a more comprehensive picture on a reservoir gas as it flows to the surface would result in gas expansion. We try to keep the reservoir as simple as possible. We did not overcomplicate and we avoided compromising the whole simulation based on forecasts which could change significantly.

7. SUMMARY AND RECOMMENDATIONS

7.1 Summary

Some simple underbalanced transient scenario simulations are carried out in this project. The transient effects of underbalanced drilling are qualitatively described while the drift-flux and governing equations as well as AUSMV scheme are taking into account. There is not only transient phenomena observed that have been described but some steady state resulting from transient scenarios include this work. Wellbore flow-rate is controlled by a surface choke which has the objective of keeping the rate essentially constant and within the capacity of the surface separator. One of the major challenges associated with a well with an underbalanced condition is the fact that the flow rate measurement must have very high accuracy compared to the pressure measurement; otherwise there will be a real safety risk for the whole rig. Moreover the determination of reservoir parameters requires on both quantities and the quality of the data being measured on the surface. The pressure variation in the annulus is crucial for the gas density, volumetric gas flow rate and viscosity. The wellbore pressure variation is basically dependent on these quantities and two phase fluid properties, thus for a producing wellbore some parameters are likely interdependent such as calculations of fluid fraction, flow regime, pressure and phase velocities. When we after all determine these parameters one after one separately in the calculation, we may get a good picture not only locally but the whole well. In addition if gas and liquid flow rates are given, together with pressure and gas density and flow regimes, it is possible to visualize fluid fractions and pressure gradient behavior for each of the drillpipe section using a simulator.

7.2 Recommendations

For the further work we would like to suggest some others parameters to take into account:

- One can include the effect of temperature changes in the well over time.
- Real time downhole data can be used to calibrate the flow models (since they are not perfect representations of reality).
- A more realistic productivity index model with oil influx should be implemented.
- Extend the simulations to deviated wells with underbalanced condition.

REFERENCES

[1]. Gas Research Institute (GRI), *Underbalanced Drilling Manual*, published by Gas Research Institute, Chicago Illinois (1997).

[2]. O.B. BENNION, et al. "*underbalanced drilling and formation damage – Is a Total Solution*?", published by Hycal Energy Research Laboratories Ltd

[3]. Leading Edge Advantage, "*Introduction to Underbalanced Drilling",* published by Leading Edge Advantage International Ltd (2002).

[4]. Dr. Fjelde K.K.: Advanced Well Technology, lectures of Stavanger University College (2011).

[5]. Schlumberger. www.slb.com/services/drilling/mpd_ubd/rotating_control_devices.aspx#.

[6]. Dr. Carlos Perez-Tellez,: IMPROVED BOTTOMHOLE PRESSURE CONTROL, FOR UNDERBALANCED DRILLING OPERATIONS, Ph.D Dissertation, Louisiana State University (2003).

[7]. [5]. Saponja, J.: "*Challenges with Jointed Pipe Underbalanced Operations*", paper 37066 Presented at the SPE International Conference on Horizontal Well Technology held in Calgary, Canada 18-20 Nov. 1996.

[8]. [6]. Lage, A.C.V.M.: *Two-Phase Flow Models and Experiments for Low-Head and Underbalanced Drilling*, Ph.D Dissertation, Stavanger University College (2000).

[9]. Thomas Dahl & Bart Vos.: *Underbalanced Drilling Manual version 1.* 0 published by Baker Hughes Inteq November 1999

[10]. Dr R. Watson et al.: "*Calculating a "Rule of Thumb" – Unloading a Well*", OTC paper 15059 Presented at the Offshore Technology Center held in Houston, Texas, U.S.A., 5–8 May 2003.

[11]. [P. Salim & J. Li.: BJ Services Company " *Simulation of Liquid Unloading From a Gas Well With Coiled Tubing Using a Transient Software*" SPE paper 124195 Presented at Annual Technical Conference an Exhibition held in New Orleans, Louisiana, USA, 4-7.October 2009.

[12]. Kelly Falk et al.: "*An Overview of Underbalanced Drilling Applications in Canada"* SPE paper 30129 presented at the European Formation Damage Conference held In The Hague, The Netherlands, 15-16 May 1995.

[13]. Dr. Brant Bennion, F. Brent Thomas Hycal Energy Research Laboratories, "*UNDERBALANCED DRILLING:A RESERVOIR DESIGN PERSPECTIVE* " Presented at the 7th Annual Petroleum Society/SPE Conference on Horizontal Well Technology, November 3, 1999, Calgary, Alberta, Canada.

[14]. A.Lage et al. "Underbalanced Drilling Dynamics: Two- Phase Flow Modeling and Experiments'", published with Stavanger U. and RF-Rogaland Research.

[15]. E. Steinar and K.K.Fjelde: "On a rough AUSM scheme for a one-dimensional
two-phase model", RF-Rogaland Research, Thormøhlensgt. 55, N–5008 Bergen, Norway
Received 5 September 2001; accepted 11 January 2002.

[16]. Evje, S., and Fjelde, K. K." Hybrid _ux-splitting schemes for a two-phase _Flow model. Journal
of Computational Physics "175, 2 (2002), p.678-680.

[17]. Wikipedia: http://en.wikipedia.org/wiki/Boyle's_law

[18]. H.F Litlehamar:" Well Wontrol Procedures and Simulations", master dissertation, Stavanger, University of Stavanger, (2011).

[19]. Heriot – Watt University, "Production Technology 2", published by department of petroleum engineering page 10.

APPENDICES: MATLAB CODES

main.m

```
% Transient two-phase code based on AUSMV scheme: Gas and Water
% The code can handle area changes. The area changes are defined inside
% the cells such that the where the fluxes are calculated, the geometry is
% uniform.

clear;

% Geometry data/ Must be specified
welldepth = 2000;
nobox = 25; %Number of boxes in the well
nofluxes = nobox+1;
dx = welldepth/nobox; % Boxlength
%dt = 0.005;

% Welldepth array
x(1)= -1.0*welldepth+0.5*dx;
for i=1:nobox-1
 x(i+1)=x(i)+ dx;
end


% open zone   % sev 2/5-2012
zone_u = -2000;
zone_l = -1800;
Lzone = (x<=zone_l) & (x>=zone_u);
Pcoll = 110*10^5;
Ppore = 150*10^5;
Pfrac = 200*10^5;
%Kwellres = 0.00000001;  % liten influx
Kwellres = 0.0;  % stor influx

dt= 0.01;  % Timestep
dtdx = dt/dx;
time = 0.0;
endtime = 5000; % Rime for end of simulation
nosteps = endtime/dt;  %Number of total timesteps
timebetweensavingtimedata = 5;  % How often in s we save data vs time for
plotting.
nostepsbeforesavingtimedata = timebetweensavingtimedata/dt;

% Slip parameters used in the gas slip relation. vg =Kvmix+S
k = 1.2;
s = 0.5;

% Viscosities (Pa*s)/Used in the frictional pressure loss model.
viscl = 0.05; % Liquid phase
viscg = 0.000005; % Gas phase

% Density parameters. These parameters are used when finding the
```

```
% primitive variables pressure, densities in an analytical manner.
% Changing parameters here, you must also change parameters inside the
% density routines roliq and rogas.

% liquid density at stc and speed of sound in liquid
  dstc = 1000.0;   %Base density of liquid, See also roliq.
  pstc = 100000.0;
  al = 1000; % Speed of sound/compressibility of liquid phase.
  t1 = dstc-pstc/(al*al);
% Ideal gas law constant
  rt = 100000;

% Gravity constant

  grav = 9.81;

% Well opening. opening = 1, fully open well, opening = 0 (<0.01), the well
% is fully closed. This variable will control what boundary conditions that
% will apply at the outlet (both physical and numerical): We must change
% this further below in the code if we want to change status on this.

  wellopening = 1.0


% Specify if the primitive variables shall be found either by
% a numerical or analytical approach. If analytical = 1, analytical
% solution is used. If analytical = 0. The numerical approach is used.
% using the itsolver subroutine where the bisection numerical method
% is used.

  analytical = 1;


% Specify how many updates of plots during simulation
Nplot = 100;
teller = 1;


% Specify which box to calculate pressure and mass flow rates for during
% time
Xpos1 = 1;
Xpos2 = nobox;

% Define and intilalize flow variables




% Check area: MERK HVORDAN VI KAN FORANDRE AREALET.
% Gemoetry below is 8.5" X 5" /typical 8 1/2" section well.

   for i = 1:nobox
    do(i) = 0.2159;
    di(i) = 0.127;
    areal(i) = 3.14/4*(do(i)*do(i)- di(i)*di(i));
```

```
    arear(i) = 3.14/4*(do(i)*do(i)- di(i)*di(i));
    area(i) = 3.14/4*(do(i)*do(i)- di(i)*di(i));
    ang(i)=3.14/2;
   end

% The code below can be activated if one wants to introduce area changes inside
% the well.
%     for i = 12:nobox-1
%      do(i)=0.1;
%      di(i)=0.0;
%      areal(i+1) = 3.14/4*(do(i)*do(i)- di(i)*di(i));
%      arear(i) = 3.14/4*(do(i)*do(i)- di(i)*di(i));
%      end
%     arear(nobox) = arear(nobox-1);



% Now comes the intialization of the physical variables in the well.
% First primitive variables, then the conservative ones.
    for i = 1:nobox
% Here the well is intialized. We start out with a horizontal well and
% lift this gradually up later during a 100 sec period to get the initial
% condtions in the well prior to starting the injection of fluids at the
% bottom. The extension letter o refers to the table representing the
% values at the previous timestep (old values).

        % Density of liquid and gas:
        dl(i) = 1000.0;
        dg(i)= 1.0;
        %"Old" density is set equal to new density to calculate new values
        %based on the old ones:
        dlo(i)= dl(i);
        dgo(i)=dg(i);
        % Velocity of liquid and gas at new and previous timesteps:
        vl(i) = 0.0;
        vlo(i)= 0.0;
        vg(i)= 0.0;
        vgo(i)= 0.0;
        %The pressure in the horisontal pipe is the same
        %all over:
        p(i) = 100000.0;
        po(i) = p(i);
        %Phase volume fractions of gas and liquid:
        eg(i)= 0.0;    %Gas
        ego(i)=eg(i);
        ev(i)=1-eg(i); % Liquid
        evo(i)=ev(i);

        vg(i)=0.0;
        vgo(i)=0.0;
        vl(i)=0.0;
        vlo(i)=0.0;

        % Variables related to the velocity of the flux boundaries at old
        %and new times, and on the left and right side of the boxes
        % reflecting that area changes can take part inside cells (i.e :
```

```
        % (A x v)left = (A x v)right, continuity equation.
        vgr(i)=0.0;
        vgor(i)= 0.0;
        vgl(i)= 0.0;
        vgol(i)= 0.0;

        vlr(i)=0.0;
        vlor(i)=0.0;
        vll(i)=0.0;
        vlol(i)=0.0;

  % Conservative variables:

       qv(i,1)=dl(i)*ev(i)*(areal(i)+arear(i))*0.5;
       qvo(i,1)=qv(i,1);

       qv(i,2)=dg(i)*eg(i)*(areal(i)+arear(i))*0.5;
       qvo(i,2)=qv(i,2);

       qv(i,3)=(qv(i,1)*vl(i)+qv(i,2)*vg(i))*(areal(i)+arear(i))*0.5;
       qvo(i,3)=qv(i,3);

    end

% Intialize fluxes between the cells/boxes

for i = 1:nofluxes
  for j =1:3
   flc(i,j)=0.0; % Flux of liquid over box boundary
   fgc(i,j)=0.0; % Flux of gas over box boundary
   fp(i,j)= 0.0; % Pressure flux over box boundary
  end
end



%  Main program. Here we will progress in time. First som intializations
% and definitions to take out results. The for loop below runs until the
% simulation is finished.

countsteps = 0;
counter=0;
printcounter = 1;
pbot(printcounter) = p(1);
pchoke(printcounter)= p(nobox);
pcasingshoe(printcounter)=p(25);
liquidmassrateout_bot(printcounter) = 0;
gasmassrateout_bot(printcounter)=0;
liquidmassrateout_choke(printcounter) = 0;
gasmassrateout_choke(printcounter)=0;
timeplot(printcounter)=time;

for i = 1:nosteps
   countsteps=countsteps+1;
   counter=counter+1;
```

```
   time = time+dt;

% First of all a dirty trick is used in order to make the well vertical.
% The pipe was initialised for a horisontal case. However, for a vertical
% case we would need a steady state solver. Since the programmer in this
% case is quite lazy, he rather chose to adjust the gravity constant g from
% zero to 9.81 m/s2 during 100 seconds (corresponds to hoisting the well
% from a horisontal postion to vertical case.

% If we want to simulate a horizontal well. Just comment/deactivate the
% code below.

       if (time <= 100)
         g = grav*time/100;
       else
         g = grav;
       end



% Then a section where specify the boundary conditions.
% Here we specify the inlet rates of the different phases at the
% bottom of the pipe in kg/s. We interpolate to make things smooth.
% It is also possible to change the outlet boundary status of the well
% here. First we specify rates at the bottom and the pressure at the outlet
% in case we have an open well.

% The code below is an example of an open connection

if (time < 150)

  inletligmassrate=0.0;
  inletgasmassrate=0.0;

elseif ((time>=150) & (time < 160))
  inletligmassrate = 22*(time-150)/10;
  inletgasmassrate = 2.0*(time-150)/10;

elseif ((time >=160) & (time<1700))
  inletligmassrate = 22;
  inletgasmassrate = 2.0;

elseif ((time>=1700)& (time<1710))
  inletligmassrate = 22-22*(time-1700)/10;
  inletgasmassrate = 2.0-2.0*(time-1700)/10;
elseif ((time>=1710)&(time<2000))
  inletligmassrate =0;
  inletgasmassrate =0;

elseif ((time>=2000)& (time<2010))
  inletligmassrate= 22*(time-2000)/10;
  inletgasmassrate= 2.0*(time-2000)/10;
elseif ((time>2010) & (time<3500))
  inletligmassrate= 22;
  inletgasmassrate= 2.0;
```

```
elseif ((time>=3500) & (time<3510))
  inletligmassrate= 22-22*(time-3500)/10;
  inletgasmassrate= 2-2*(time-3500)/10;
elseif ((time>3510) & (time<4000))
  inletligmassrate= 0;
  inletgasmassrate= 0;

elseif ((time>=4000)& (time<4010))
  inletligmassrate= 22*(time-4000)/10;
  inletgasmassrate= 2.0*(time-4000)/10;
elseif (time>4010)
  inletligmassrate= 22;
  inletgasmassrate= 2.0;

end

% Below, test code for onephase connection. Commented out. Friction
% model seems reasonable.
% if (time < 150)
%
%   inletligmassrate=0.0;
%   inletgasmassrate=0.0;
%
% elseif ((time>=150) & (time < 160))
%   inletligmassrate = 44*(time-150)/10;
%   inletgasmassrate = 0;
%
% elseif ((time >=160) & (time<300))
%   inletligmassrate = 44;
%   inletgasmassrate = 0;
%
% elseif ((time>=300)& (time<310))
%   inletligmassrate = 44-44*(time-300)/10;
%   inletgasmassrate = 0;
% else
%   inletligmassrate=0;
%   inletgasmassrate = 0;
% end




% specify the outlet pressure /Physical. Here we have given the pressure as
% constant. It would be possible to adjust it during openwell conditions
% either by giving the wanted pressure directly (in the command lines
% above) or by finding it indirectly through a chokemodel where the wellopening
% would be an input parameter. The wellopening variable would equally had
% to be adjusted inside the command line structure given right above.

 pressureoutlet = 100000.0; %choke pressure

% Based on these boundary values combined with use of extrapolations techniques
% for the remaining unknowns at the boundaries, we will define the mass and
% momentum fluxes at the boundaries (inlet and outlet of pipe).
```

```
% inlet fluxes first.

     flc(1,1)= inletligmassrate/areal(1);
     flc(1,2)= 0.0;
     flc(1,3)= flc(1,1)*vlo(1);

     fgc(1,1)= 0.0;
     fgc(1,2)= inletgasmassrate/areal(1);
     fgc(1,3)= fgc(1,2)*vgo(1);

     fp(1,1)= 0.0;
     fp(1,2)= 0.0;
     fp(1,3)= po(1)+0.5*(po(1)-po(2)); %Interpolation used to find the
% pressure at the inlet/bottom of the well.



% Outlet fluxes (open & closed conditions)

    if (wellopening>0.01)
        flc(nofluxes,1)= dlo(nobox)*evo(nobox)*vlo(nobox);
        flc(nofluxes,2)= 0.0;
        flc(nofluxes,3)= flc(nofluxes,1)*vlo(nobox);

        fgc(nofluxes,1)= 0.0;
        fgc(nofluxes,2)= dgo(nobox)*ego(nobox)*vgo(nobox);
        fgc(nofluxes,3)= fgc(nofluxes,2)*vgo(nobox);

        fp(nofluxes,1)= 0.0;
        fp(nofluxes,2)= 0.0;
        fp(nofluxes,3)= pressureoutlet;
    else


         flc(nofluxes,1)= 0.0;
         flc(nofluxes,2)= 0.0;
         flc(nofluxes,3)= 0.0;

         fgc(nofluxes,1)= 0.0;
         fgc(nofluxes,2)= 0.0;
         fgc(nofluxes,3)= 0.0;

         fp(nofluxes,1)=0.0;
         fp(nofluxes,2)=0.0;
         fp(nofluxes,3)= po(nobox)-0.5*(po(nobox-1)-po(nobox));

    end

% Now we will find the fluxes between the different cells.
% NB - IMPORTANE -  Note that if we change the compressibilities/sound velocities
of
% the fluids involved, we need to do changes inside the csound function.
```

```
     for j = 2:nofluxes-1
      cl = csound(ego(j-1),po(j-1),dlo(j-1),k);
      cr = csound(ego(j),po(j),dlo(j),k);
      c = max(cl,cr);
      pll = psip(vlor(j-1),c,evo(j));
      plr = psim(vlol(j),c,evo(j-1));
      pgl = psip(vgor(j-1),c,ego(j));
      pgr = psim(vgol(j),c,ego(j-1));
      vmixr = vlol(j)*evo(j)+vgol(j)*ego(j);
      vmixl = vlor(j-1)*evo(j-1)+vgor(j-1)*ego(j-1);

      pl = pp(vmixl,c);
      pr = pm(vmixr,c);
      mll= evo(j-1)*dlo(j-1);
      mlr= evo(j)*dlo(j);
      mgl= ego(j-1)*dgo(j-1);
      mgr= ego(j)*dgo(j);

      flc(j,1)= mll*pll+mlr*plr;
      flc(j,2)= 0.0;
      flc(j,3)= mll*pll*vlor(j-1)+mlr*plr*vlol(j);

      fgc(j,1)=0.0;
      fgc(j,2)= mgl*pgl+mgr*pgr;
      fgc(j,3)= mgl*pgl*vgor(j-1)+mgr*pgr*vgol(j);

      fp(j,1)= 0.0;
      fp(j,2)= 0.0;
      fp(j,3)= pl*po(j-1)+pr*po(j);
     end

% Fluxes have now been calculated. We will now update the conservative
% variables in each of the numerical cells.



     for j=1:nobox

%      vmixfric = vlo(j)*evo(j)+vgo(j)*ego(j); % Mixture viscosity
%      viscmix =  viscl*evo(j)+viscg*ego(j);
      densmix = dlo(j)*evo(j)+dgo(j)*ego(j);

      a2 = arear(j);
      a1 = areal(j);
      avg = (a2+a1)*0.5;

      pressure=p(j);

%     We calculate the frictional gradient by calling upon the dpfric function.
      friclossgrad =
dpfric(vlo(j),vgo(j),evo(j),ego(j),dlo(j),dgo(j),pressure,do(j),di(j),viscl,viscg
);


%     calculate potential influx of gas calling up influx_gas (new!!!)
```

```
      reswell_gas =
influx_gas(Kwellres,ego(j),Ppore,pressure,zone_u,zone_l,time,x(j));


%     Here is the updating of the conservative variables/where we solve the
%     two mass conservation equations and the third momentum equation.
      qv(j,1)=qvo(j,1)-dtdx*((a2*flc(j+1,1)-a1*flc(j,1))...
                           +(a2*fgc(j+1,1)-a1*fgc(j,1))...
                           +(avg*fp(j+1,1)-avg*fp(j,1)));

      qv(j,2)=qvo(j,2)-dtdx*((a2*flc(j+1,2)-a1*flc(j,2))...
                           +(a2*fgc(j+1,2)-a1*fgc(j,2))...
                           +(avg*fp(j+1,2)-avg*fp(j,2)))...
                           +dt*reswell_gas;

      qv(j,3)=qvo(j,3)-dtdx*((a2*flc(j+1,3)-a1*flc(j,3))...
                           +(a2*fgc(j+1,3)-a1*fgc(j,3))...
                           +(avg*fp(j+1,3)-avg*fp(j,3)))...
                  -dt*avg*((friclossgrad)+g*densmix);
     end

     %Simple friction model for only pipe/laminar flow:
     %(32*vmixfric*viscmix/(do(j)*do(j))+g*densmix);



% Section where we find the physical variables (pressures, densities etc)
% from the conservative variables. Some trickes to ensure stability


     for j=1:nobox


% Remove the area from the conservative variables to find the
% the primitive variables from the conservative ones.

      qv(j,1)= qv(j,1)/(areal(j)+arear(j))*2.0;
      qv(j,2)= qv(j,2)/(areal(j)+arear(j))*2.0;

      if (qv(j,1)<0.00000001)
        qv(j,1)=0.0;
      end

      if (qv(j,2)< 0.00000001)
        qv(j,2)=0.0000001;
      end


% Below, we find the primitive variables pressure and densities based on
% the conservative variables q1,q2. One can choose between getting them by
% analytical or numerical solution approach specified in the beginning of
% the program.

    if (analytical == 1)
```

```
      % Coefficients:
      a = 1/(al*al);
      b = t1-qv(j,1)-rt*qv(j,2)/(al*al);
      c = -1.0*t1*rt*qv(j,2);

      % Analytical solution:
       p(j)=(-b+sqrt(b*b-4*a*c))/(2*a);  % Pressure
       dl(j)= dstc + (p(j)-pstc)/(al*al); % Density of liquid
       dg(j) = p(j)/rt;                   % Density of gas
    else
      %Numerical Solution:
      [p(j),error]=itsolver(po(j),qv(j,1),qv(j,2)); % Pressure
      dl(j)=rholiq(p(j)); % Density of liquid
      dg(j)=rogas(p(j)); % Density of gas

      % Incase a numerical solution is not found, the program will write out
"error":
      if error > 0
         error
      end
    end


 % Find the phase volume fractions based on new conservative variables and
 % updated densities.

      eg(j)= qv(j,2)/dg(j);
      ev(j)=1-eg(j);

%     Reset average conservative varibles in cells with area changes inside.

      qv(j,1)=qv(j,1)*(areal(j)+arear(j))/2.0;
      qv(j,2)=qv(j,2)*(areal(j)+arear(j))/2.0;



%     The section below is used to find the primitive variables vg,vl
%    (phase velocities) based on the updated conservative variable q3 and
%     the slip relation.

%      Deactivated code below, old code for no slip cond & no area change.
%      vg(j)=qv(j,3)/(dl(j)*ev(j)+dg(j)*eg(j));
%      vl(j)=vg(j);

% Part where we interpolate in the slip parameters to avoid a
% singularities when approaching one phase gas flow.
% In the transition to one-phase gas flow, we need to
% have a smooth transition to no-slip conditions.

      xint = (eg(j)-0.75)/0.25;
      k0 = k;
      s0 = s;
      if ((eg(j)>=0.75) & (eg(j)<=1.0))
        k0 =1.0*xint+k*(1-xint);
        s0 = 0.0*xint+s*(1-xint);
      end
```

```
      if (eg(j)>=0.999999)
        k1 = 1.0;
        s1 = 0.0;
      else
        k1 = (1-k0*eg(j))/(1-eg(j));
        s1 = -1.0*s0*eg(j)/(1-eg(j));
      end
 %     help1 = dl(j)*ev(j)*k1+dg(j)*eg(j)*k0;
 %     help2 = dl(j)*ev(j)*s1+dg(j)*eg(j)*s0;

 %     vmixhelp = (qv(j,3)-help2)/help1;
 %     vg(j)=k0*vmixhelp+s0;
 %     vl(j)=k1*vmixhelp+s1;
%       help1 = qv(j,3)/(dl(j)*ev(j)+dg(j)*eg(j));
%
%       vll(j)= help1/areal(j);
%       vlr(j)= help1/arear(j);
%       vgl(j)= vll(j);
%       vgr(j)= vlr(j);


 %    Below we operate with gas vg and liquid vl velcoities specified
 %    both in the right part and left part inside a box. (since we have
 %    area changes inside a box these can be different. vgl is gas velocity
 %    to the left of the disconinuity. vgr is gas velocity to the right of
 %    the discontinuity.
 %

      help1 = dl(j)*ev(j)*k1+dg(j)*eg(j)*k0;
      help2 = dl(j)*ev(j)*s1+dg(j)*eg(j)*s0;

      vmixhelpl = (qv(j,3)/areal(j)-help2)/help1;
      vgl(j)=k0*vmixhelpl+s0;
      vll(j)=k1*vmixhelpl+s1;

      vmixhelpr = (qv(j,3)/arear(j)-help2)/help1;
      vgr(j)=k0*vmixhelpr+s0;
      vlr(j)=k1*vmixhelpr+s1;

 %  Averaging velocities.

      vl(j)= 0.5*(vll(j)+vlr(j));
      vg(j)= 0.5*(vgl(j)+vgr(j));

    end



% Old values are now set equal to new values in order to prepare
% computation of next time level.
    for j = 1:nobox
     po(j)=p(j);
     dlo(j)=dl(j);  %Liquid density
     dgo(j)=dg(j);  %Gas density
```

```
      vlo(j)=vl(j);  %Liquid velocity
      vgo(j)=vg(j);  %Gas velocity
      ego(j)=eg(j);  %Gas fration
      evo(j)=ev(j);  %Liquid fraction.

      vlor(j)=vlr(j);
      vlol(j)=vll(j);
      vgor(j)=vgr(j);
      vgol(j)=vgl(j);

       for m =1:3
        qvo(j,m)=qv(j,m);
       end
     end


% Section where we save some timedependent variables in arrays.
% e.g. the bottomhole pressure. They will be saved for certain
% timeintervalls defined in the start of the program in order to ensure
% that the arrays do not get to long!

  if (counter>=nostepsbeforesavingtimedata)
    printcounter=printcounter+1;
    time
    pbot(printcounter)= p(Xpos1);
    pchoke(printcounter)=p(Xpos2);
    pcasingshoe(printcounter)=p(25); %NB THIS MUST BE DEFINED IN CORRECT BOX
%     liquidmassrateout(printcounter)=dl(nobox)*ev(nobox)*vl(nobox)*area(nobox);
%     gasmassrateout(printcounter)=dg(nobox)*eg(nobox)*vg(nobox)*area(nobox);

liquidmassrateout_bot(printcounter)=dl(Xpos1)*ev(Xpos1)*vl(Xpos1)*arear(Xpos1);
    gasmassrateout_bot(printcounter)=dg(Xpos1)*eg(Xpos1)*vg(Xpos1)*arear(Xpos1);

liquidmassrateout_choke(printcounter)=dl(Xpos2)*ev(Xpos2)*vl(Xpos2)*arear(Xpos2);

gasmassrateout_choke(printcounter)=dg(Xpos2)*eg(Xpos2)*vg(Xpos2)*arear(Xpos2);
    timeplot(printcounter)=time;
    counter = 0;


  end


  % include plotting of gas-volume fraction, pressure, fluid velocities

  if ( (i/1000) == teller )

  % gas volume fraction
  subplot(2,4,1)
  plot(eg,x);
  title('Gas Volume Fraction');
  xlabel('\alpha_g')
  ylabel('Depth (m)')
  axis([0 1  -welldepth 0]);
  grid on
```

```
drawnow;
hold off

% pressure
subplot(2,4,2)
plot(p/10^5,x); hold on;
plot((Ppore/10^5)*ones(1,length(x(Lzone))),x(Lzone),'-r','LineWidth',2);
plot((Pcoll/10^5)*ones(1,length(x(Lzone))),x(Lzone),'-k','LineWidth',2);
 plot((Pfrac/10^5)*ones(1,length(x(Lzone))),x(Lzone),'-g','LineWidth',2);
title('Pressure');
xlabel('P (bar)')
ylabel('Depth (m)')
axis([0 250  -welldepth 0]);
grid on
legend('P_{w}','P_{pore}','P_{collapse}','P_{frac}');
drawnow;
hold off

% vl
subplot(2,4,3)
plot(ev.*vl,x);
title('Liquid velocity (superficial)');
xlabel('Fluid velocity (m/s)')
ylabel('Depth (m)')
axis([-2 +4  -welldepth 0]);
grid on
drawnow;
hold off

 % vg
subplot(2,4,4)
plot(eg.*vg,x);
title('Gas velocity (superficial)');
xlabel('Fluid velocity (m/s)')
ylabel('Depth (m)')
axis([-2 +8  -welldepth 0]);
grid on
drawnow;
hold off

% Bottomholde pressure as function of time

subplot(2,4,5)
plot(timeplot,pbot/100000);
hold on
plot(timeplot,pchoke/100000,'-r')
title('Pressure');
xlabel('Time (sec)')
ylabel('Pressure (bar)')
axis([0 endtime  0 250]);
grid on
legend('bottom','choke')
drawnow;
hold off

subplot(2,4,6)
```

```
  plot(timeplot,liquidmassrateout_bot);
  hold on
  plot(timeplot,liquidmassrateout_choke,'-r');
  title('Liquid mass rate');
  xlabel('Time (sec)')
  ylabel('Liquid mass rate (kg/s)')
  axis([0 endtime  -20 +150]);
  grid on
  legend('bottom','choke')
  drawnow;
  hold off

  subplot(2,4,7)
  plot(timeplot,gasmassrateout_bot)
  hold on
  plot(timeplot,gasmassrateout_choke,'-r')
  title('Gas mass rate');
  xlabel('Time (sec)')
  ylabel('Gas mass rate (kg/s)')
  axis([0 endtime  -5 +20]);
  grid on
  legend('bottom','choke')
  drawnow;
  hold off

  teller = teller+1;
  end




end

% end of stepping forward in time.




% Printing of resultssection


countsteps % Marks number of simulation steps.


% Plot commands for variables vs time.
%plot(timeplot,pbot/100000)
%plot(timeplot,pchoke/100000)
%plot(timeplot,pcasingshoe/100000)
%plot(timeplot,liquidmassrateout)
%plot(timeplot,gasmassrateout)
%plot(vg)

%Plot commands for variables vs depth/Only the last simulated
```

```
%values/endtime is visualised

%plot(vl,x);
%plot(vg,x);
%plot(eg,x);
%plot(p,x);
%plot(dl,x);
%plot(dg,x);
```

Csound.s

```
function mixsoundvelocity = csound(gvo,po,dlo,k)
% Note that at this time k is set to 1.0 (should maybe be
% included below

temp= gvo*dlo*(1.0-gvo);
a=1;
if (temp < 0.01)
  temp = 0.01;
end

cexpr = sqrt(po/temp);

if (gvo <= 0.5)
 mixsoundvelocity = min(cexpr,1000);
else
 mixsoundvelocity = min(cexpr,316);
end
```

dpfric.m

```
function friclossgrad =
dpfric(vlo,vgo,evo,ego,dlo,dgo,pressure,do,di,viscl,viscg)

%friclossgrad =
%dpfric(vlo,vgo,evo,ego,dlo,dgo,pressure,do,di,viscl,viscg)
% Works for two phase flow. The one phase flow model is used but mixture
 % values are introduced.

 rhol = rholiq(pressure);
 rhog = rogas(pressure);
 vmixfric = vlo*evo+vgo*ego;
 viscmix =  viscl*evo+viscg*ego;
 densmix = dlo*evo+dgo*ego;

 % Calculate mix reynolds number
 Re = ((densmix*abs(vmixfric)*(do-di))/viscmix);

 % Calculate friction factor. For Re > 3000, the flow is turbulent.
 % For Re < 2000, the flow is laminar. Interpolate in between.

 if (Re<0.001)
```

```
   f=0.0;
 else
  if (Re >= 3000)
   f = 0.052*Re^(-0.19);
  elseif ( (Re<3000) & (Re > 2000))
   f1 = 24/Re;
   f2 = 0.052*Re^(-0.19);
   xint = (Re-2000)/1000.0;
   f = (1.0-xint)*f1+xint*f2;
  else
   f = 24/Re;
  end
 end

  friclossgrad = ((2*f*densmix*vmixfric*abs(vmixfric))/(do-di));


end
```

itsover.m

```
function [press,error] = itsolver(p,qv1,qv2)

% The numerical solver implementeted here for solving the equation f(x)= 0
% "wellpressure(p)= 0" is called the
% Method of Halving the Interval (Bisection Method)

% You will not find exact match for f(x)= 0. Maybe f(x) = 0.0001. By using
% ftol we say that if f(x)<ftol, we are satisfied. Since our function
% gives results in Pascal, we say that ftol = 1000 Pa gives us a quite good
% answer.

 ftol = 0.00001;
 ftol = 0.001;

 % Specify the search interval". xguess is the pressure you guess for the
 % pressure. (Remember x is in Pa). 1 Bar = 100 000 Pa.

 % Set number of iterations to zero

  noit = 0;
  error = 1.0; % Error is set to zero because we havent any input information
yet.

  i = 0;
  while (error > 0)
      i = i+1;
      xguess = p;
      xint = 150000*i;
      x1 = xguess-xint/2.0;
```

```
      x2 = xguess+xint/2.0;

 f1 = qv1-rholiq(x1)*(1.0-qv2/rogas(x1));
 f2 = qv1-rholiq(x2)*(1.0-qv2/rogas(x2));

 % First include a check on whether f1xf2<0. If not you must adjust your
 % initial search intervall. If error is 1 and zero pbot, then you must
 % adjust the intervall here.


 if (f1*f2)>=0
     error = 0;
     if (f1<ftol)
         press=x1;
     elseif (f2<ftol)
         press=x2;
     else
         error=1
     end

 else
 % start iterating, we are now on the track.
     x3 = (x1+x2)/2.0;
     f3 = qv1-rholiq(x3)*(1.0-qv2/rogas(x3));

     while (f3>ftol | f3 < -ftol)
        noit = noit +1 ;

        if (f3*f1) < 0
           x2 = x3;
        else
           x1 = x3;
        end

        x3 = (x1+x2)/2.0;
        f3 = qv1-rholiq(x3)*(1.0-qv2/rogas(x3));
        f1 = qv1-rholiq(x1)*(1.0-qv2/rogas(x1));

     end
     error = 0;
     press = x3;
     noit;
 end
  end
```

pm.m

```
function pmvalue = pm(v,c)


  if (abs(v)<=c)
    pmvalue = -1.0*(v-c)*(v-c)/(4*c)*(-2.0-v/c)/c;
  else
    pmvalue = 0.5*(v-abs(v))/v;
```

```
  end
end
```

pp.m

```
function pmvalue = pp(v,c)

  if (abs(v)<=c)
    pmvalue = (v+c)*(v+c)/(4*c)*(2.0-v/c)/c;
  else
    pmvalue = 0.5*(v+abs(v))/v;
  end
end
```

psim.m

```
function pmvalue = psim(v,c,alpha)

  if (abs(v)<=c)
    pmvalue = -1.0*alpha*(v-c)*(v-c)/(4*c)+(1-alpha)*(v-abs(v))/2;
  else
    pmvalue = 0.5*(v-abs(v));
  end
end
```

psip.m

```
function pmvalue = psim(v,c,alpha)

  if (abs(v)<=c)
    pmvalue = -1.0*alpha*(v-c)*(v-c)/(4*c)+(1-alpha)*(v-abs(v))/2;
  else
    pmvalue = 0.5*(v-abs(v));
  end
end
```

rholiq.m

```
function [rhol] = rholiq(pressure)
%Simple model for liquid density
p0 = 100000.0; % Assumed

rhol = 1000.0 + (pressure-p0)/(1000.0*1000.0);
```

```
end
```

rogas.m

```
function rhog = rogas(pressure)

%Simple gas density model. Temperature is neglected.
% rhogas = pressure / (velocity of sound in the gas phase)^2 = pressure /
% rT --> gas sound velcoity = SQRT(rT)

  rhog = pressure/100000.0;
```