GAME THEORETIC ANALYSIS OF TWO-PERIOD-DEPENDENT DEGRADED MULTISTATE RELIABILITY SYSTEMS

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A system of two components is analyzed as a two-period game. After period 1 the system can be fully operational, in two states of intermediate degradation, or fail. Analogously to changing failure rates in dependent systems analyzed with Markov analysis, unit costs of defense and attack, and contest intensities, change in period 2. As the values of the two intermediate states increase from zero which gives the series system, towards their maxima which gives the parallel system, the defender becomes more advantaged, and the attacker more disadvantaged. Simulations illustrate the players' efforts in the two time periods and utilities dependent on parametric changes. The defender withdraws from defending the system when the values of both degraded states are very low. The attacker withdraws from attacking the system when the values of both degraded states are very high. In the benchmark case the defender prefers the one-period game and the attacker prefers the two-period game, but if the attacker's unit cost of attack is large for one component, and the value of the degraded system with this component operational is above a low value, the defender prefers the two-period game to obtain high utility in period 2 against a weak attacker. When the values of the degraded states are above certain low values, the players exert higher efforts in period 1 of a two-period game than in a one-period game, as investments into the future to ensure high versus low reliability in period 2.

Keywords: Two-period game; dependent system; defense; attack; contest success function; reliability theory; multi-state system; degraded system; intermediate functioning; series system; parallel system.

1. Introduction

As the world becomes more complex, its reliable functioning becomes more challenging to ensure. After the September 11, 2001 attack it became more evident that whereas one set of players works to ensure system reliability, another set of players opposes system reliability. Unfortunately, systems analysis has a long tradition of analyzing only one player, i.e., the defender maximizing system reliability facing exogenously fixed factors related to technology, nature, weather, culture, etc.

From a game-theoretic point of view, the current state of affairs where the systems that surround us are analyzed mainly with nongame-theoretic tools, is highly unsatisfying. This paper thus provides a game-theoretic analysis. A system consists of components. Examples are roads, bridges, tunnels, power supply, telecommunications systems, water supply, political and economic institutions, businesses, schools, hospitals, recreational facilities, and other assets.

Components can be arranged in series, parallel, or various mixed types. For example, a single telecommunication line may run in series through two routers, both of which have to function for the system to function. Further, two parallel bridges may cross a river, one of which is needed to ensure transport across the river. A plethora of strategic considerations abound for such systems. The defender may defend one component, the other component, or both components with some relative emphasis, and analogously for the attacker. Further, the players can choose their efforts in the present, or in the future dependent on the outcome of the strategic interactions in the present. For example, the defender of a telecommunication line will be intent on protecting both routers in the present since if one fails, the line will not function in the future. Further, if one bridge is destroyed in the present, the other bridge may get better protection in the future, but the other bridge may also get attacked more fiercely in the future if the attacker is intent on blocking transport across the river.

The main contribution of the paper vis-a-vis the existing literature is to provide an understanding of how the strategic interactions in the present are linked to the strategic interactions in the future for systems such as those described above. This is done by analyzing a defender and an attacker in a two-period game. The first period expresses the present and the second period expresses the future. One or several components can fail dependent on the strategic interactions in the first period. This in turn impacts the strategic interactions in the second period. Furthermore, looking ahead to the second period before the game starts influences the players' strategies in the first period.

Systems where the future states of affairs depend on the present states of affairs are referred to as dependent systems (Ebeling, 1997). Dependent systems have a long tradition of being analyzed using Markov analysis, which is unrelated to game theory.[a] A simple definition of a stochastic process with the Markov property is

[a]First the number of system states is specified (for example, in a system with n components where each component can operate or fail, there are 2^n states). Second the reliability is determined based on the system configuration. Third a rate diagram is designed where each node represents a state and each branch with an arrow specifies a transition rate (failure rate) expressed with a parameter. Fourth an equation is formulated for the probability of being in each state at time $t + \Delta t$ which equals the probability of being in the state at time t, adding or subtracting the probabilities of moving into or out of the state from neighboring states when accounting for the transition rates. Fifth each equation is reformulated as a differential equation. The number of equations equals the number of states minus 1. The probability of being in the last state equals 1 minus the sum of the probabilities of being in the other states. Examples of systems analyzed with Markov analysis are load sharing systems, standby systems, degraded systems, and multistate systems (Ebeling, 1997, 108ff). For example, if one component fails in a load sharing system, the failure rates increase for the remaining components. A standby component may experience a low or zero failure rate in its standby state, and a higher failure rate when operational (which may or may not equal the failure rate of the originally operating component).

that the conditional probability distribution of future states of the process depends only upon the present state.[b] Markov analysis has proven highly successful applied to reliability analysis. This paper is concerned with two limitations of Markov analysis. First, enabling each of two players to choose strategies in each of two-periods violates the Markov property since players are free to choose future strategies that are not conditioned on their present strategies. Generally, any theory involving intentional action (e.g., game theory) violates the Markov property. Second, this paper relaxes the constraint in Markov modeling where the transition rates between different states are kept constant through time.

This paper enables players to exert efforts to impact the system reliability as time progresses. That is, we analyze how players choose strategies through time to impact the reliability of dependent systems. We consider a dependent system of two independent components which can be in four states assigned four different values, i.e., fully operational, two states of intermediate degradation, and failure. The formulation is general enough to comprise the series system, the parallel system, allowing arbitrary permutations of the values of the two intermediate states. The system is analyzed for general parameter values with backward induction as a two-period game. We determine how two players make strategic decisions through time to impact the system reliability. Players allocate resources in the sense of substituting efforts across components and across time. Determining the nature of such substitutions is of substantial interest, see e.g., Enders and Sandler (2003), Hausken (2006), and Keohane and Zeckhauser (2003).

The paper answers questions such as whether players exert high efforts in the first period to position themselves for the second period, whether they are so weakened that they withdraw from the game, or whether they prefer the game to last one-period or two-periods. One player, the defender, maximizes the system reliability. The other player, the attacker, minimizes the system reliability. Each component's reliability depends on the relative levels of defense and attack and on the contest intensity. Each player's utility depends additively on the system reliability in the two-periods, with a discount parameter varying between 0 and 1 for the second period. The unit costs of defense and attack, and the contest intensities, vary across components, and change dependent on the system state, analogously to failure rates changing in Markov analysis dependent on the system state.

The need to consider complex systems which cannot be represented with combinations of series and parallel configuration is substantial. Simon (1969) suggests handling complexity applying "near decomposability." Bier (1995) apply aggregation to reduce the number of parameters in complex systems. Ebeling (1997, 90ff) proposes decomposition or enumeration. Hausken (2010) analyzes complex systems applying game theory. Let us consider a few examples.

[b] Another definition is that of memorylessness where, conditional on the present state of the system, its future and past are independent. See e.g., Taylor and Karlin (1998) for further definitions.

For example, assume that a military system consists of two components which are an army and an air force. If the army's equipment (tanks, etc.) is destroyed through an attack, the system becomes less operational, but not nonoperational. The capacity for ground maneuvers is reduced, which can to some extent be compensated for by the air force's air strikes and use of helicopters. In contrast, if the air force's equipment (airplanes, etc.) is destroyed, the system also becomes less operational. Air strikes cannot be conducted, but pilots can be assigned for ground maneuvers. If both the army and air force are eliminated, the system fails. This system is neither a series nor parallel system since none of these allow for intermediate degrees of operation. However, if the intermediate degrees of operation are removed, the failure criterion "the system fails if and only if 'army' AND 'air force' fail" expresses an AND gate relative to a parallel system. Analogously, if the intermediate degrees of operation are removed, the failure criterion "the system fails if 'army' OR 'air force' fail" expresses an OR gate relative to a series system.

Alternatively, envision a human with one lung or one kidney. It has proven possible to ascend the world's Seven Summits (including Mount Everest) and complete an Ironman with one lung (Sean Swarner, Ohio). Hence a human can be highly functional in many regards with one lung. However, reports also exist of substantial degradation when possessing only one lung, especially during activities requiring lung capacity. These two examples cannot be represented with combined series/parallel configurations. The series system is fully operational if both components are operational, and otherwise fails. The parallel system is fully operational if at least one component is operational, and otherwise fails.

Other examples are a computer with two processors which may function partially if one processor fails, a two-engine aircraft which may operate in an intermediate state if one engine fails, a racecar driver with two mechanics with different competencies who may race partially successfully if one mechanic falls sick, and a stadium with two entrances which may operate to some extent if one entrance gets blocked (spreading entries over a longer time period). See e.g., Ebeling (1997, 117ff) for further examples in the literature on degraded systems. These examples cannot be represented with combined series/parallel configurations. The series system is fully operational if both components are operational, and otherwise fails. The parallel system is fully operational if at least one component is operational, and otherwise fails.

In this paper a model is presented which describes both a military force with an army and an air force, a person with two lungs, and the other examples. A system with two components is considered. If both components operate, the system is in a successful state which is assigned a high value. If both components fail, the system is in a failure state which is assigned a low value. The innovation of this paper is to assign one intermediate value to the system if one component operates and the other fails, and to assign one possibly different intermediate value to the system if one component fails and the other operates. The system can thus be in four states. Conventional series/parallel reliability theory describes this system as a series system if the two intermediate values equal the value if both components fail, describes this system as a parallel system if the two intermediate values equal the value if both components operate, and otherwise offers no description.

For the military force assume that if the army operates and the air force fails, then the system value of this state is estimated (by expert opinion) to be 75% of the system value if both the army and the air force operate. Further assume that if the army fails and the air force operates, then the system value of this state is estimated to be 40% of the system value if both the army and the air force operate. These two estimates are realistic in situations where the army is more important than the air force. Assume for the person with two lungs that if one lung operates and the other lung fails, then the system values of these two states are estimated to be 60% of the system value if both lungs operate. This paper shows for such systems how a defender and an attacker should exert efforts to defend and attack two components when accounting for the four system values.

For multi-state system reliability, see Lisnianski and Levitin (2003). For degraded systems see Ebeling's (1997, 117ff) Markov analysis of a system which can be fully operational, degraded, ot failed. See Zio and Podofillini (2003) for Monte Carlo simulation of the effects of different system performance levels on the importance of multi-state components. Ramirez-Marquez and Coit (2005) use Monte-Carlo simulation to approximate multi-state two-terminal reliability. Next step is to incorporate strategic defenders and attackers into the analysis of multi-state and degraded systems.

In earlier research Levitin (2007) considers the optimal element separation and protection in a complex multi-state series–parallel system, and suggests an algorithm for determining the expected damage caused by a strategic attacker. Hausken and Levitin (2009) present a minmax optimization algorithm. The defender minimizes the maximum damage the attacker can inflict thereafter. The defender has multiple defense strategies which involve separation and protection of system elements. The attacker also has multiple attack strategies against different groups of system elements. A universal generating function technique is applied for evaluating the losses caused by system performance reduction. Levitin and Hausken (2009) introduce three defensive measures, i.e., providing redundancy, protecting genuine elements, and deploying false elements and analyze the optimal resource distribution among these measures in parallel and k-out of-N systems. Levitin (2009) considers optimizing defense strategies for complex multi-state systems.

Azaiez and Bier (2007) consider the optimal resource allocation for security in reliability systems. Bier *et al.* (2005) analyze the protection of series and parallel systems with components of different values. They specify optimal defenses against intentional threats to system reliability, focusing on the tradeoff between investment cost and security. Bier *et al.* (2006) assume that a defender allocates defense to a collection of locations while an attacker chooses a location to attack. Hausken (2008) considers defense and attack for series and parallel reliability systems. Dighe *et al.*

(2009) consider secrecy in defensive allocations as a strategy for achieving more cost-effective attacker deterrence.

Section 2 presents the model. Section 3 solves the model. Section 4 analyzes three special cases. Section 5 simulates the solution. Section 6 concludes.

2. The Model

Consider a system of two independent components $i = \mathrm{A}, \mathrm{B}$. A defender and an attacker play a two-period game. In both periods both players make their strategic choices simultaneously and independently. Before the second period both players know the strategies chosen and the outcome of the first period. In period j, $j = 1, 2$, the defender exerts effort t_{ij} at unit cost c_i to defend component i, where t_{ij} is the defender's free choice variable. Analogously, the attacker exerts effort T_{ij} at unit cost C_i to attack component i, where T_{ij} is the attacker's free choice variable. Defense and attack are interpreted broadly. Defense means protecting against the attack, and maintaining and adjusting the system to prevent that it breaks down. Attacking means attacking the system, which may get aided by natural factors (technology, weather, temperature, humidity, etc.) to ensure that the system breaks down. We formulate the reliability p_{ij} of component i in period j as a contest between the defender and attacker. The most common functional form is the ratio form (Tullock, 1980)

$$p_{ij} = \frac{t_{ij}^{m_i}}{t_{ij}^{m_i} + T_{ij}^{m_i}}, \tag{1}$$

where $\partial p_{ij}/\partial t_{ij} > 0$, $\partial p_{ij}/\partial T_{ij} < 0$, and $m_i \geq 0$ is a parameter for the intensity of the contest. Equation (1) is commonly used in the rent seeking literature where the rent is an asset which corresponds to reliability in this paper. There is conflict over reliability between the defender and the attacker, just as there is conflict over a rent between contending players. See Tullock (1980) for the use of m_i, Skaperdas (1996) for an axiomatization, Nitzan (1994) for a review, Hirshleifer (1995) for illustration, usefulness, and application, and Hausken (2005) for recent literature. At the limit, with infinitely much defensive effort, and finite offensive effort, component i is 100% reliable. The same result follows with finite defensive effort and zero offensive effort. At the other limit, with infinitely much offensive effort, and finite defensive effort, component i is 0% reliable. The same result follows with finite offensive effort and zero defensive effort. The sensitivity of p_{ij} to t_{ij} increases as m_i increases. When $m_i = 0$, the efforts t_{ij} and T_{ij} have equal impact on the reliability regardless of their size which gives 50% reliability, $p_{ij} = 1/2$. $0 < m_i < 1$ gives a disproportional advantage of exerting less effort than one's opponent. When $m_i = 1$, the efforts have proportional impact on the reliability. $m_i > 1$ gives a disproportional advantage of exerting more effort than one's opponent. This is often realistic in praxis, as evidenced by benefits from economies of scale. Finally, $m_i = \infty$ gives a step function where "winner-takes-all."

Table 1. Two component system in four states.

State	Component A	Component B	Reliability	Value
1	Operates	Operates	$p_{A1}p_{B1}$	s
2	Fails	Operates	$(1-p_{A1})p_{B1}$	d_B
3	Operates	Fails	$p_{A1}(1-p_{B1})$	d_A
4	Fails	Fails	$(1-p_{A1})(1-p_{B1})$	f

The system with two components can be in the four states shown in Table 1 in period 1.

Each state has a probability shown in the reliability column. The four states have values s, d_B, d_A, f, respectively, where $s \geq d_i \geq 0 \geq f$. If component A fails while component B operates, the system is degraded at an intermediate state with value d_B. Conversely, if component B fails while component A operates, the system is degraded at an alternative intermediate state with value d_A. If both components fail, the value is f which is zero or negative.

The series system is expressed with $s > 0$ and $d_i = f = 0$, which operates only in state 1, and fails in states 2, 3, 4. The defender's utility is $u = sp_{A1}p_{B1} - c_At_{A1} - c_Bt_{B1}$, and the attacker's utility is $U = s(1-p_{A1}p_{B1}) - C_AT_{A1} - C_BT_{B1}$. The parallel system is expressed with $s = d_i > 0$ and $f = 0$ which operates in states 1, 2, 3, and fails in state 4. The defender's utility is $u = s[1-(1-p_{A1})(1-p_{B1})] - c_At_{A1} - c_Bt_{B1}$, and the attacker's utility is $U = s(1-p_{A1})(1-p_{B1}) - C_AT_{A1} - C_BT_{B1}$. We thus present the general reliability

$$p(p_{Aj}, p_{Bj}) = p_{Aj}p_{Bj} + \frac{d_B}{s}(1-p_{Aj})p_{Bj} + \frac{d_A}{s}p_{Aj}(1-p_{Bj}) + \frac{f}{s}(1-p_{Aj})(1-p_{Bj}). \quad (2)$$

The players' first period utilities are

$$\begin{aligned} u_1 &= sp(p_{A1}, p_{B1}) - c_At_{A1} - c_Bt_{B1}, \\ U_1 &= s(1-p(p_{A1}, p_{B1})) - C_AT_{A1} - C_BT_{B1}. \end{aligned} \quad (3)$$

The players' first period strategic choices determine both their first period utilities and the system state at the start of period 2. Each time period can be short or long, e.g., one minute, one month, one shift, one season. Components are not repairable.[c] Hence the strategies the players choose for period 1 have to account for the combinations of possibilities in which the components may operate or fail in the two-periods. If one or both components fail in period 1, since the failed component(s) cannot be repaired or replaced before the commencement of period 2, the players need to assess their defense and attack in period 2 to account for which of

[c]The justification for this assumption is that repairing or replacing failed components can be complicated for economical and logistical reasons, and may require competence and time, which we assume is impossible both during the periods and in the transition from period 1 to period 2. Future research may model how components can be repaired.

the four states follows after period 1. In period 2 the players also make their strategic choices simultaneously and independently, knowing the outcome and choices in period 1.

If both components operate after period 1 (state 1), then the unit costs of defense and attack remain unchanged and the players make strategic choices t_{A2}, t_{B2}, T_{A2}, and T_{B2}. If component A fails (state 2), we assume that the unit costs of defense and attack for component B change to c_{BF} and C_{BF}, and the contest intensity changes to m_{BF}. Analogously, if component B fails (state 3), the unit costs of defense and attack for component A change to c_{AF} and C_{AF}, and the contest intensity changes to m_{AF}. The contest intensity is

$$p_{i2F} = \frac{t_{i2F}^{m_{iF}}}{t_{i2F}^{m_{iF}} + T_{i2F}^{m_{iF}}}, \tag{4}$$

where $\partial p_{ij}/\partial t_{ij} > 0$, $\partial p_{ij}/\partial T_{ij} < 0$, $m_{iF} \geq 0$, and t_{i2F} and T_{i2F} are the efforts. If period 2 starts in state 2, the system has been degraded to d_B in period 1. The defender exerts effort t_{B2F} to obtain high contest success for component B. In contrast, in state 2 the attacker enjoys a guaranteed benefit $s-d_B$ from the degradation and exerts effort T_{B2F} to obtain low contest success for component B. The reasoning if component B fails (state 3) is analogous. Finally, if both components fail in the first period (state 4), the defender gets the failure utility f, the attacker gets the utility $s-f$, and no efforts are exerted. The defender and attacker values after period 2 are shown in Table 2.

The players' utilities over the two-periods are

$$\begin{aligned} u &= sp(p_{A1}, p_{B1}) - c_A t_{A1} - c_B t_{B1} + \delta p_{A1} p_{B1} (sp(p_{A2}, p_{B2}) - c_A t_{A2} - c_B t_{B2}) \\ &\quad + \delta(1 - p_{A1}) p_{B1} (d_B p_{B2F} - c_{BF} t_{B2F}) + \delta p_{A1} (1 - p_{B1})(d_A p_{A2F} - c_{AF} t_{A2F}) \\ &\quad + \delta(1 - p_{A1})(1 - p_{B1})(f - 0), \\ U &= s(1 - p(p_{A1}, p_{B1})) - C_A T_{A1} - C_B T_{B1} + \Delta p_{A1} p_{B1} (s(1 - p(p_{A2}, p_{B2})) \\ &\quad - C_A T_{A2} - C_B T_{B2}) + \Delta(1 - p_{A1}) p_{B1} (s - d_B + d_B(1 - p_{B2F}) - C_{BF} T_{B2F}) \\ &\quad + \Delta p_{A1} (1 - p_{B1})(s - d_A + d_A(1 - p_{A2F}) - C_{AF} T_{A2F}) \\ &\quad + \Delta(1 - p_{A1})(1 - p_{B1})(s - f - 0), \end{aligned} \tag{5}$$

where δ and Δ are time discount parameters.

Table 2. Defender and attacker values after period 2.

State	Component A	Component B	Reliability after period 1	Defender value after period 2	Attacker value after period 2
1	Operates	Operates	$p(p_{A1}, p_{B1})$	$sp(p_{A2}, p_{B2})$	$s(1 - p(p_{A2}, p_{B2}))$
2	Fails	Operates	$(1 - p_{A1})p_{B1}$	$d_B p_{B2F}$	$s - d_B + d_B(1 - p_{B2F})$
3	Operates	Fails	$p_{A1}(1 - p_{B1})$	$d_A p_{A2F}$	$s - d_A + d_A(1 - p_{A2F})$
4	Fails	Fails	$(1 - p_{A1})(1 - p_{B1})$	f	$s - f$

3. Solving the Model

The two players have four strategic choice variables in period 1, and eight strategic choice variables in period 2. We analyze pure strategy Nash equilibria. We solve the game with backward induction starting with period 2. Differentiating gives $\partial u/\partial t_{A2} = \partial u/\partial t_{B2} = \partial u/\partial t_{A2F} = \partial u/\partial t_{B2F} = 0$ and $\partial U/\partial T_{A2} = \partial U/\partial T_{B2} = \partial U/\partial T_{A2F} = \partial U/\partial T_{B2F} = 0$. Solving the eight equations gives

$$
\begin{gathered}
T_{A2} = \frac{m_A\left(\frac{C_A}{c_A}\right)^{m_A}\left(d_A - f + (s-d_B)\left(\frac{C_B}{c_B}\right)^{m_B}\right)}{C_A\left(1+\left(\frac{C_A}{c_A}\right)^{m_A}\right)^2\left(1+\left(\frac{C_B}{c_B}\right)^{m_B}\right)}, \\
T_{B2} = \frac{m_B\left(\frac{C_B}{c_B}\right)^{m_B}\left(d_B - f + (s-d_A)\left(\frac{C_A}{c_A}\right)^{m_A}\right)}{C_B\left(1+\left(\frac{C_B}{c_B}\right)^{m_B}\right)^2\left(1+\left(\frac{C_A}{c_A}\right)^{m_A}\right)}, \\
T_{A2F} = \frac{m_{AF} d_A\left(\frac{C_{AF}}{c_{AF}}\right)^{m_{AF}}}{C_{AF}\left(1+\left(\frac{C_{AF}}{c_{AF}}\right)^{m_{AF}}\right)^2}, \quad T_{B2F} = \frac{m_{BF} d_B\left(\frac{C_{BF}}{c_{BF}}\right)^{m_{BF}}}{C_{BF}\left(1+\left(\frac{C_{BF}}{c_{BF}}\right)^{m_{BF}}\right)^2}, \\
t_{A2} = \frac{C_A}{c_A}T_{A2}, \quad t_{B2} = \frac{C_B}{c_B}T_{B2}, \quad t_{A2F} = \frac{C_{AF}}{c_{AF}}T_{A2F}, \quad t_{B2F} = \frac{C_{BF}}{c_{BF}}T_{B2F}, \\
p_{A2} = \frac{\left(\frac{C_A}{c_A}\right)^{m_A}}{1+\left(\frac{C_A}{c_A}\right)^{m_A}}, \quad p_{B2} = \frac{\left(\frac{C_B}{c_B}\right)^{m_B}}{1+\left(\frac{C_B}{c_B}\right)^{m_B}}, \quad p_{A2F} = \frac{\left(\frac{C_{AF}}{c_{AF}}\right)^{m_{AF}}}{1+\left(\frac{C_{AF}}{c_{AF}}\right)^{m_{AF}}}, \\
p_{B2F} = \frac{\left(\frac{C_{BF}}{c_{BF}}\right)^{m_{BF}}}{1+\left(\frac{C_{BF}}{c_{BF}}\right)^{m_{BF}}}, \\
p(p_{A2}, p_{B2}) = \frac{s\left(\frac{C_A}{c_A}\right)^{m_A}\left(\frac{C_B}{c_B}\right)^{m_B} + d_B\left(\frac{C_B}{c_B}\right)^{m_B} + d_A\left(\frac{C_A}{c_A}\right)^{m_A} + f}{s\left(1+\left(\frac{C_A}{c_A}\right)^{m_A}\right)\left(1+\left(\frac{C_B}{c_B}\right)^{m_B}\right)}.
\end{gathered} \tag{6}
$$

The assumption $s \geq d_i \geq 0 \geq f$ implies $T_{A2} \geq 0$ and $T_{B2} \geq 0$, and thus all the expressions in (6) are positive. The second-order conditions are satisfied when

$$\left(\frac{m_j - 1}{m_j + 1}\right)^{1/m_j} < \frac{C_j}{c_j} < \left(\frac{m_j + 1}{m_j - 1}\right)^{1/m_j}, \quad j = A, B, AF, BF \tag{7}$$

which is a range stretching from below to above $C_j/c_j = 1$. As an example, (7) is always satisfied with an infinitely large range for the commonly used contest intensity $m_j = 1$.

Interestingly, the second period strategic choice variables do not depend on the first period strategic choice variables, only on the parameters. This means that the two-period game gives the same result as a corresponding one-period game where the players choose their 12 strategies simultaneously and independently. The intuition for this result is that the players' strategic choices in period 2 are independent for the four states that are possible after period 1. For state 1 the strategic choice variables are $t_{A2}, t_{B2}, T_{A2}, T_{B2}$. For state 2 the strategic choice variables are t_{B2F} and T_{B2F}. For state 3 the strategic choice variables are t_{A2F} and T_{A2F}. For state 4 there are no strategic choice variables since the system fails. Hence the players do not need to know the outcome of period 1 in order to play period 2. However, the probabilities of the four states depend on how period 1 is played, so the players account for the outcome of period 2 for each of the four states when determining their strategies in period 1. Thus the expressions for $t_{A2}, t_{B2}, T_{A2}, T_{B2}$ are valid for a one-period system of two components as described in Table 1. Analogously, the expressions for $t_{A2F}, t_{B2F}, T_{A2F}, T_{B2F}$ are valid for a one-period system of one target.

We rewrite (5) as

$$\begin{aligned} u &= f(1+\delta) + q_1 p_{A1} p_{B1} + q_2 p_{B1} + q_3 p_{A1} - c_A t_{A1} - c_B t_{B1}, \\ U &= (s-f)(1+\Delta) - Q_1 p_{A1} p_{B1} - Q_2 p_{B1} - Q_3 p_{A1} - C_A T_{A1} - C_B T_{B1}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} q_1 &= s - d_B - d_A + f + \delta\{sp(p_{A2}, p_{B2}) - c_A t_{A2} - c_B t_{B2} - d_B p_{B2F} + c_{BF} t_{B2F} \\ &\quad - d_A p_{A2F} + c_{AF} t_{A2F} + f\}, \\ q_2 &= d_B - f + \delta\{d_B p_{B2F} - c_{BF} t_{B2F} - f\}, \\ q_3 &= d_A - f + \delta\{d_A p_{A2F} - c_{AF} t_{A2F} - f\}, \\ Q_1 &= s - d_B - d_A + f + \Delta\{sp(p_{A2}, p_{B2}) + C_A T_{A2} + C_B T_{B2} - d_B p_{B2F} \\ &\quad - C_{BF} T_{B2F} - d_A p_{A2F} - C_{AF} T_{A2F} + f\}, \\ Q_2 &= d_B - f + \Delta\{d_B p_{B2F} + C_{BF} T_{B2F} - f\}, \\ Q_3 &= d_A - f + \Delta\{d_A p_{A2F} + C_{AF} T_{A2F} - f\}. \end{aligned} \tag{9}$$

Solving the first-order conditions presented in (A.1) in Appendix A when $m_A = m_B = 1$ gives

$$\begin{aligned} T_{A1} &= \frac{q_3 + (q_1+q_3)Q_B}{c_A(1+Q_A)^2(1+Q_B)}, \quad T_{B1} = \frac{q_2 + (q_1+q_2)Q_A}{c_B(1+Q_B)^2(1+Q_A)}, \\ t_{A1} &= Q_A T_{A1}, \quad t_{B1} = Q_B T_{B1}, \\ p_{A1} &= \frac{Q_A}{1+Q_A}, \quad p_{B1} = \frac{Q_B}{1+Q_B}, \end{aligned} \tag{10}$$

$$u = f(1+\delta) + \frac{q_1 Q_A Q_B}{(1+Q_A)(1+Q_B)} + \frac{q_2 Q_B}{1+Q_B} + \frac{q_3 Q_A}{1+Q_A} - c_A Q_A T_{A1} - c_B Q_B T_{B1},$$

$$U = (s-f)(1+\Delta) - \frac{Q_1 Q_A Q_B}{(1+Q_A)(1+Q_B)} - \frac{Q_2 Q_B}{1+Q_B} - \frac{Q_3 Q_A}{1+Q_A} - C_A T_{A1} - C_B T_{B1},$$

where

$$
\begin{aligned}
Q_B &= \frac{Q_L + \sqrt{Q_{R1} + Q_{R2}}}{2c_B\{C_A(Q_1+Q_2)(q_1+q_3) + c_A Q_2(Q_1+Q_3)\}},\\
Q_L &= C_A C_B q_1(q_1+q_2) + C_A\{C_B(q_1+q_2) - c_B(Q_1+Q_2)\}q_3\\
&\quad + c_A\{C_B q_2(Q_1+Q_3) - c_B Q_2 Q_3\},\\
Q_{R1} &= 4c_B C_B\{C_A(q_1+q_2)q_3 + c_A q_2 Q_3\}\{C_A(Q_1+Q_2)(q_1+q_3)\\
&\quad + c_A Q_2(Q_1+Q_3)\},\\
Q_{R2} &= [C_A\{C_B(q_1+q_2)(q_1+q_3) - c_B(Q_1+Q_2)q_3\} + c_A\{C_B q_2(Q_1+Q_3)\\
&\quad - c_B Q_2 Q_3\}]^2,\\
Q_A &= \frac{C_A\{q_3 + (q_1+q_3)Q_B\}}{c_A\{Q_3 + (Q_1+Q_3)Q_B\}}. \qquad (11)
\end{aligned}
$$

Appendix A shows that the second-order conditions are always satisfied when $m_A = m_B = 1$.

Let us finally consider boundary solutions. The interior solution above is valid when $u \geq 0$ and $U \geq 0$ in (10). When $u < 0$ or $U < 0$, no pure-strategy Nash equilibrium exists. Analyzing mixed strategy equilibria is beyond the scope of this paper. The next sections show that $u < 0$ is possible, e.g., for the series system when the defender is disadvantaged with a large unit effort cost, and that $U < 0$ is possible, e.g., for the parallel system when the attacker is disadvantaged with a large unit effort cost. The case $u < 0$ is disastrous for the defender since it cannot earn positive utility. We assume that the defender withdraws in this case, exerting zero effort and earning zero utility, while the attacker exerts negligible effort earning utility $s(1+\Delta)$. Of course, if the defender knows that the attacker exerts negligible effort, the defender can exert positive effort and earn positive utility. But, if the attacker knows that, it can exert positive effort and earn positive utility. In the absence of a pure-strategy Nash equilibrium exists, the assumption of withdrawal is plausible. Analogously, for the case $U < 0$, we assume that the attacker withdraws exerting zero effort and earning zero utility, while the defender exerts negligible effort earning utility $s(1+\delta)$.

4. Analyzing Three Special Cases

Let us consider three special benchmark cases with straightforward interpretations. The first is the egalitarian case $m_i = m_{iF} = 0$ causing zero efforts and thus 50% probability of failure for each component in each period. This case illustrates how the players' utilities depend on the system configuration (series, parallel, etc) and the weights assigned to period 2 expressed with δ and Δ. Cases 2 and 3 assume $\delta = \Delta = 0$, which makes period 2 irrelevant. Case 2 assumes that the defender and attacker have the same unit effort cost for each component. Hence the players are equally strong (in the sense that no one is advantaged in costs of exerting effort) and we determine how the efforts and utilities depend on the system configuration. Case 3 assumes that the defender has the same unit defense cost for both components, and that the attacker has the same unit attack cost for both components. This case shows how differently advantaged players in terms of unit effort cost exert different efforts and earn different utilities dependent on the system configuration. $\delta > 0$ and $\Delta > 0$ cause analytical expressions which are too voluminous to be interpreted, but are illustrated with simulations in the next section.

First, inserting $m_i = m_{iF} = 0$ into (1)–(9) gives

$$t_{ij} = T_{ij} = t_{i2F} = T_{i2F} = 0, \quad p_{ij} = p_{i2F} = \frac{1}{2}, \quad p(p_{Aj}, p_{Bj}) = \frac{s + d_B + d_A + f}{4s},$$
$$u = \frac{s + d_B + d_A + f}{4} + \delta\left(\frac{s + 3d_B + 3d_A + 5f}{16}\right), \tag{12}$$
$$U = \frac{3s - d_B - d_A - f}{4} + \Delta\left(\frac{15s - 3d_B - 3d_A - 5f}{16}\right), \quad u + U = (1 + \delta + \Delta)s.$$

The utilities are not affected by efforts in egalitarian contests, so the players choose zero efforts which cause 50% reliability and zero deadweight loss. The utilities are positive in both periods.

The series system, $d_i = f = 0$, causes $u = s/4 + \delta s/16$ and $U = 3s/4 + 15\Delta s/16$ since the defender is disadvantaged and the attacker is advantaged. As d_A and d_B increase giving value to the intermediate states, the defender gets increasing advantage and the attacker decreasing advantage. Increasing d_A and d_B from 0 to s gives a transition from the series system to the parallel system where $u = 3s/4 + 7\delta s/16$ and $U = s/4 + 9\Delta s/16$. In the one-period game where $\delta = \Delta = 0$ the players' utilities $s/4$ and $3s/4$ are interchanged when replacing the series system with the parallel system. Such interchange does not occur in the two-period game where $\delta = \Delta = 1$. The defender gets the low utilities $s/16$ and $7s/16$ in period 2 of the series and parallel systems, respectively. In contrast, the attacker gets the high utilities $15s/16$ and $9s/16$ in period 2 of the series and parallel systems, respectively. The reason is the probability that one or both components fail in the transition from period 1 to period 2, which advantages the attacker. As m_i increases above zero, the sum of u and U is less than $(1 + \delta + \Delta)s$ since the players choose positive efforts.

Second, inserting $C_i = c_i$ and $\delta = \Delta = 0$ into (6)–(11) gives

$$t_{A1} = T_{A1} = \frac{s + d_A - d_B - f}{8c_A}, \quad t_{B1} = T_{B1} = \frac{s + d_B - d_A - f}{8c_B}, \quad p_{i1} = \frac{1}{2},$$
$$p(p_{A1}, p_{B1}) = \frac{s + d_B + d_A + f}{4s}, \tag{13}$$
$$u = \frac{d_B + d_A + 2f}{4}, \quad U = \frac{2s - d_B - d_A}{4}, \quad u + U = \frac{s + f}{2}.$$

The equal unit costs cause the same player efforts. With intermediate degradation levels, the players earn equal utilities $u = U = (d_B + d_A + 2f)/4$ when

$s = d_B + d_A + f$. Equal unit effort costs for the defender and attacker implies that an interior solution always exists in (13).

Third, inserting $c_i = c$, $C_i = C$, $d_i = d$, $f = 0$, $\delta = \Delta = 0$ into (6)–(11) gives

$$T_{i1} = \frac{\left(1 - \frac{C}{c}\right) d + \frac{C}{c} s}{c\left(1 + \frac{C}{c}\right)^3}, \quad t_{i1} = \frac{C}{c} T_{i1}, \quad p_{i1} = p(p_{A1}, p_{B1}) = \frac{\frac{C}{c}}{1 + \frac{C}{c}},$$
$$u = \frac{\left(\frac{C}{c}\right)^2 \left(\left(\frac{C}{c} - 1\right) s + 4d\right)}{\left(1 + \frac{C}{c}\right)^3}, \quad U = \frac{\left(3\frac{C}{c} + 1\right) s - 4\frac{C}{c} d}{\left(1 + \frac{C}{c}\right)^3}. \tag{14}$$

The ratio C/c is preserved through the strategies, the contest successes, and the utilities. The player with the lowest unit effort cost exerts the highest effort. The effort increases in s. When the attacker is advantaged with $C < c$, the attacker exerts higher effort to attack the challenging parallel system ($d = s$) compared with attacking the series system ($d = 0$). The defender prefers the parallel system where $d = s$, and hence u increases in d and U decreases in d. The attacker prefers the series system where $d = 0$. Regarding boundary solutions, the defender withdraws e.g., when $d = 0$ (series system) and the defender is disadvantaged with $c > C$. The attacker withdraws e.g., when $d = s$ (parallel system) and the attacker is disadvantaged with $C > c$.

5. Simulating the Solution

Figure 1 plots the 12 efforts t_{A1}, t_{B1}, t_{A2}, t_{B2}, t_{A2F}, t_{B2F}, T_{A1}, T_{B1}, T_{A2}, T_{B2}, T_{A2F}, T_{B2F} and two utilities u and U as functions of the value of the intermediate degradation level d_A for various d_B when $c_i = C_i = c_{iF} = C_{iF} = m_i = m_{iF} = \delta = \Delta = s = 1$, $f = 0$. The titles on the vertical axis are as specified in the legend box. The symmetry in the top two panels causes equal period 2 efforts $t_{i2} = T_{i2} = 0.125$ since $d_A = d_B$, whereas $t_{i2F} = T_{i2F} = d_i/4$ increases linearly in d_i. The defender is disadvantaged in a series system with $d_A = d_B = 0$ since both components have to be defended for the system to operate. The defender withdraws when $d_A = d_B < 0.07$ earning zero utility when the degraded system has low value. The defender's period 1 effort increases from 0.10 to 0.20 as $d_A = d_B$ increases from 0.07 to 1. It becomes more important for the defender to protect both elements in period 1 as $d_A = d_B$ increases, in the hope that both survive until period 2. The advantaged attacker's period 1 effort is higher. If the attacker succeeds destroying both components in period 1, it earns a large utility. The large period 1 efforts by the two players reflect a finding also found in the conflict literature (Hausken, 2007), where both players exert large efforts in the early periods of a repeated game to reap benefits in the later periods of the game. The attacker suffers lower utility as the value $d_A = d_B$ of the degraded system increases. The division of the attacker's utility U with 2 is for scaling purposes.

The middle panels assume $d_B = 0$ which disadvantages the defender more than in the top panels. The defender withdraws earning zero utility when

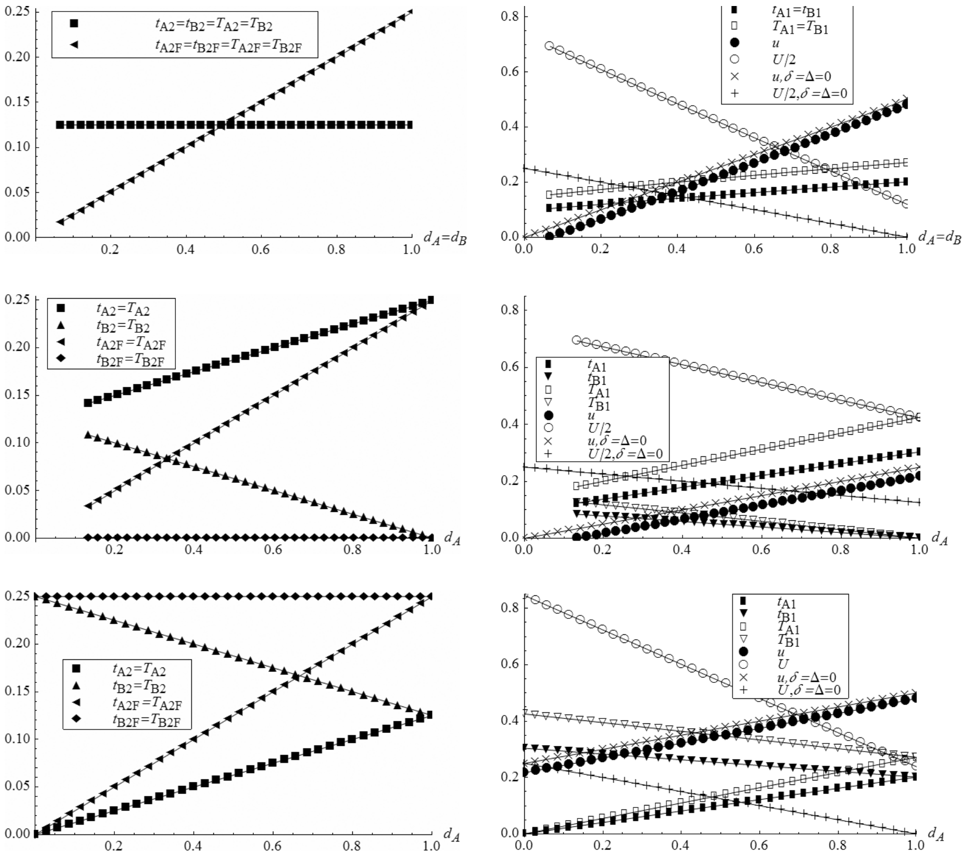

Fig. 1. Efforts and utilities as functions of d_A for various d_B when $c_i = C_i = c_{iF} = C_{iF} = m_i = m_{iF} = \delta = \Delta = s = 1$, $f = 0$. Top panels: $d_A = d_B$. Middle panels: $d_B = 0$. Bottom panels: $d_B = 1$.

$d_A = d_B < 0.13$. The low d_B causes higher efforts for component A in both periods than when $d_A = d_B$, and the efforts increase in the value of d_A. The efforts $t_{AF} = T_{AF}$ increase most, and the efforts for component B decrease.

The bottom panels assume $d_B = 1$ which disadvantages the attacker maximally in the parallel system when $d_A = d_B = 1$. The defender always earns positive utility. The high d_B causes higher efforts for component B, and lower efforts for component A, than in the middle panels.

The last two curves marked with $\times$ and $+$ in the rightmost panels are the defender and attacker utilities when period 2 is discounted to have no value, $\delta = \Delta = 0$. This effectively means that period 1 is the last period and the period 1 efforts in the rightmost panels can be read off from the period 2 efforts in the leftmost panels, $t_{i1} = t_{i2}$ and $T_{i1} = T_{i2}$. Equation (6) does not depend on δ and Δ and hence the period 2 efforts remain unchanged as δ and Δ decrease from 1 to 0. In all the panels the defender prefers to discount period 2 to $\delta = \Delta = 0$, while the attacker does not. The reason, as expressed in Sec. 4, is the positive probability that one or both components fail in the transition from period 1 to period 2, which advantages the attacker. In the top panels the defender's period 1 efforts 0.125 when $\delta = \Delta = 0$ are lower than when $\delta = \Delta = 1$ when $d_A = d_B > 0.25$. When $d_A = d_B > 0.25$, the degraded system is valuable for the defender which exerts high efforts in period 1 when period 2 is important. Such efforts are costly causing low defender utility in the two-period game. When $d_A = d_B < 0.25$, the defender exerts higher efforts in the one-period game to win the game. In the middle panels the logic is the same for component A, but for component B where $d_B = 0$ the defender exerts higher efforts in the one-period game to win the game. In the bottom panels where $d_B = 1$ the defender exerts higher efforts in period 1 in the two-period game with $\delta = \Delta = 1$ since period 2 is important.

Figure 2 assumes the same parameter values as in Fig. 1 but makes the attacker disadvantaged with triple unit cost $C_A = C_{AF} = 3$ of attacking component A.

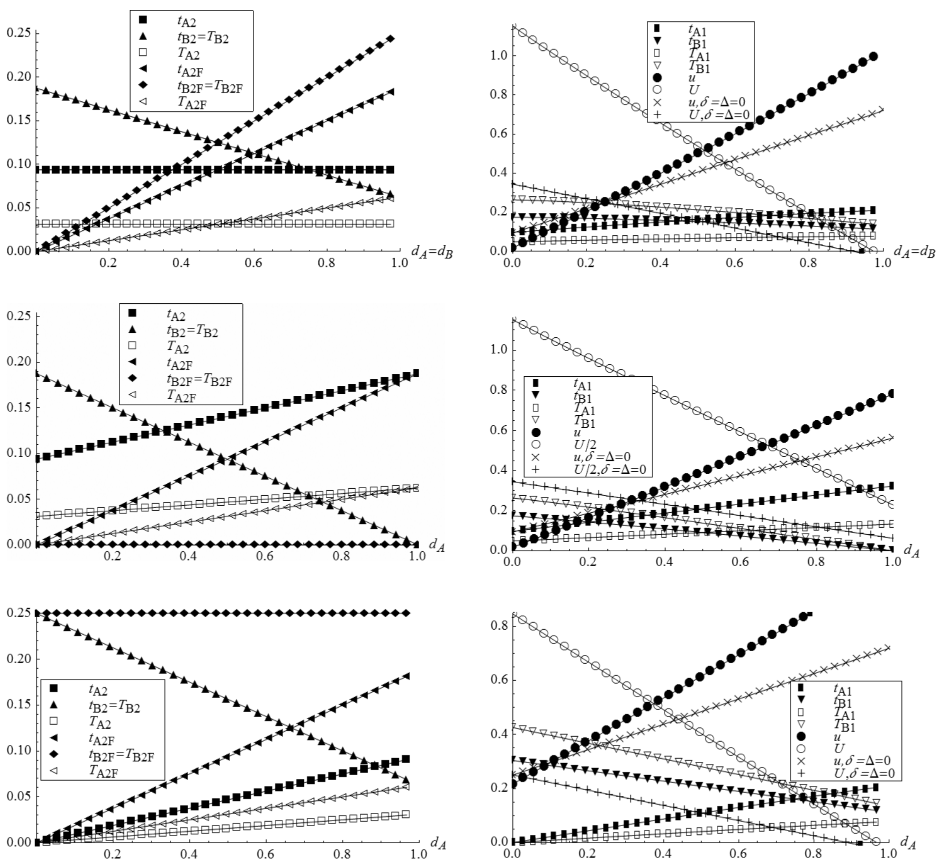

Fig. 2. Efforts and utilities as functions of d_A for various d_B when $c_i = C_B = c_{iF} = C_{BF} = m_i = m_{iF} = \delta = \Delta = s = 1, C_A = C_{AF} = 3$, $f = 0$. Top panels: $d_A = d_B$. Middle panels: $d_B = 0$. Bottom panels: $d_B = 1$.

The attacker thus exerts lower efforts for component A, earns lower utility. The defender never withdraws. The efforts for component A are constant or decrease, and the efforts for component B decrease. The attacker substitutes into attacking component B more than in Fig. 1 when d_A is small, but as d_A increases, even that becomes costly and exerting efforts into the important component A becomes essential. In the top panels the attacker withdraws when $d_A = d_B > 0.97$. In the middle panels the attacker is more advantaged and does not withdraw. In the bottom panels with $d_B = 1$ it becomes especially important to exert high efforts t_{B1} and T_{B1} for the valuable component B in period 1, and particularly when d_A is low. The disadvantaged attacker withdraws when $d_A > 0.97$.

The attacker prefers the two-period game over the one-period game in all panels. The defender prefers the two-period game when d_A is above a certain low value.

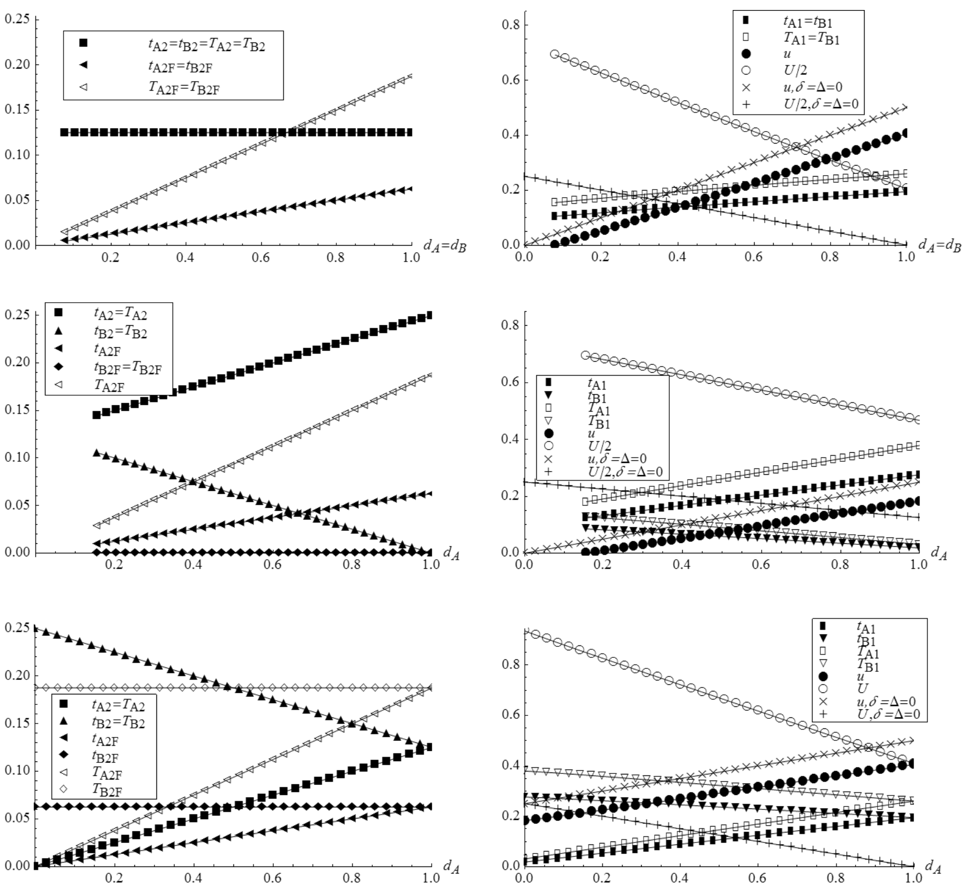

Fig. 3. Efforts and utilities as functions of d_A for various d_B when $c_i = C_i = C_{iF} = m_i = m_{iF} = \delta = \Delta = s = 1, c_{AF} = c_{BF} = 3,\ f = 0$. Top panels: $d_A = d_B$. Middle panels: $d_B = 0$. Bottom panels: $d_B = 1$.

The reason, in contrast to Fig. 1, is the high unit attack cost $C_A = C_{AF} = 3$ which disadvantages the attacker in period 2 in the contest over component A. The defender then prefers to engage in the contest also in period 2 to reap the benefits against a weak attacker.

Figure 3 assumes the same parameter values as in Fig. 1 except assumes triple unit costs $c_{AF} = c_{BF} = 3$ of defending a sole component in period 2 given that the other component failed in period 1. This assumption is analogous to Ebeling's (1997, 111ff) assumption of increased failure rate for one remaining component in a load sharing system provided that the other component has failed. In the top panels the defender withdraws when $d_A = d_B < 0.08$. In the middle panels the especially disadvantaged defender withdraws when $d_A = d_B < 0.16$ and earns low utility. In the bottom panels the defender does not withdraw.

The attacker prefers the two-period game over the one-period game in all panels. In contrast, the defender prefers the one-period game over the two-period game in all panels, and with a clearer margin than in Figs. 1 and 2. The reason is the high unit defense cost $c_{AF} = c_{BF} = 3$ of defending one sole remaining component in period 2 provided that the other component failed in period 1.

6. Conclusion

The paper analyzes how players choose strategies through time to impact the reliability of dependent systems. The system has two independent components, each of which can operate or fail. The system can be in four states, i.e., fully operational, two states of intermediate degradation, and failure. Special cases are the series system where the two degraded states have zero value, the parallel system where the two degraded states have the same value as the fully operational state, and various intermediately degraded systems. Each component is protected by a defender which maximizes its reliability subtracting the defense costs, and attacked by an attacker which maximizes its unreliability subtracting the attack costs. Each component's reliability depends on the relative levels of defense and attack and on the contest intensity. Each player's utility depends additively on the system reliability in two time periods, with a time discount parameter for the second period. The unit costs of effort and the contest intensities vary across players and components, and change dependent on the system state. Such parameter changes are analogous to changes in failure rates when dependent systems are analyzed with Markov analysis (Ebeling, 1997, 108ff). The two-period game is analyzed with backward induction.

Each player makes one effort decision for each component in period 1, and hence four decisions are made in period 1. In period 2 four effort decisions are made if the system is fully operational after period 1. If the system fails after period 1, it remains in the failed state in period 2. If one component fails in period 1, unit costs of defense and attack, and the contest intensity, change in period 2 and both players make one effort decision for the other component in period 2. Hence eight effort decisions are made in period 2.

The attacker prefers the series system which fails if one component fails, and the defender prefers the parallel system since the successful operation of one component is sufficient. We show how the defender withdraws from defending the system when the values of both degraded states are very low, and how the attacker withdraws from attacking the system when the values of both degraded states are very high.

The paper analyzes the impact of letting the values of the two intermediate states vary between zero and the value of the fully operational state. As the values of the two intermediate states increase from zero, the defender becomes more advantaged, and the attacker more disadvantaged. We present simulations to illustrate the players' efforts in the two-periods and utilities dependent on parametric changes, and in particular changes in the values of the intermediate states, assuming that these are equal, or that one value is zero and the other varies, or that one value equals the value of the fully operational state while the other value varies.

The time duration of each time period can be short or long as dictated by the nature of the system and the manner in which failures occur dependent on the players' efforts. Since components may fail in period 1, with positive probability period 2 starts with one or both components failed. Hence in the benchmark case the defender prefers the one-period game while the attacker prefers the two-period game. However, if the attacker's unit cost of attack is large for one component, and the value of the degraded system with this component operational is above a low value, the defender prefers the two-period game. The defender then benefits in period 2 against a weak attacker.

When the values of the degraded states are above certain low values, the players exert higher efforts in period 1 of a two-period game than in a one-period game (or a game where period 2 is discounted to have no value). The reason is that the high efforts exerted in period 1 are investments into the future, to ensure either high (for the defender) or low (for the attacker) reliability in period 2. Similar results are found in the conflict literature (Hausken 2007) where one player's ability to dictate its preferred solution early in a repeated game, may benefit later in the game. If the defender through high efforts ensures that one or both components survive as operational into period 2, positive reliability may also be enjoyed in period 2.

This paper has been concerned with two limitations of Markov analysis. First, we have enabled players to choose efforts strategically, which violates the Markov property. Second, we have relaxed the constraint in Markov modeling where the transition rates between different states are kept constant through time. Future research may relax another constraint by allowing changing the number of states and the definition of “Failure” and “Success” of the system as time progresses.

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Appendix A: First- and Second-Order Conditions

Differentiating the utilities in (8) with respect to the first period strategic choice variables gives

$$\begin{aligned}
\frac{\partial u}{\partial t_{A1}} &= \frac{m_A t_{A1}^{m_A-1} T_{A1}^{m_A}\left((q_1+q_3)t_{B1}^{m_B}+q_3T_{B1}^{m_B}\right)}{(t_{A1}^{m_A}+T_{A1}^{m_A})^2(t_{B1}^{m_B}+T_{B1}^{m_B})} - c_A = 0,\\
\frac{\partial u}{\partial t_{B1}} &= \frac{m_B t_{B1}^{m_B-1} T_{B1}^{m_B}\left((q_1+q_2)t_{A1}^{m_A}+q_2T_{A1}^{m_A}\right)}{(t_{B1}^{m_B}+T_{B1}^{m_B})^2(t_{A1}^{m_A}+T_{A1}^{m_A})} - c_B = 0,\\
\frac{\partial U}{\partial T_{A1}} &= \frac{m_A T_{A1}^{m_A-1} t_{A1}^{m_A}\left((Q_1+Q_3)t_{B1}^{m_B}+Q_3T_{B1}^{m_B}\right)}{(t_{A1}^{m_A}+T_{A1}^{m_A})^2(t_{B1}^{m_B}+T_{B1}^{m_B})} - C_A = 0,\\
\frac{\partial U}{\partial T_{B1}} &= \frac{m_B T_{B1}^{m_B-1} t_{B1}^{m_B}\left((Q_1+Q_2)t_{A1}^{m_A}+Q_2T_{A1}^{m_A}\right)}{(t_{B1}^{m_B}+T_{B1}^{m_B})^2(t_{A1}^{m_A}+T_{A1}^{m_A})} - C_B = 0.
\end{aligned} \tag{A.1}$$

The second-order conditions inserting $m_A = m_B = 1$ are

$$\begin{aligned}
\frac{\partial^2 u}{\partial t_{A1}^2} &= -\frac{2c_A^2(1+Q_A)(1+Q_B)}{q_3+(q_1+q_3)Q_B},\\
\frac{\partial^2 u}{\partial t_{A2}^2} &= -\frac{2c_B^2(1+Q_A)(1+Q_B)}{q_2+(q_1+q_2)Q_A},
\end{aligned} \tag{A.2}$$

which are satisfied as negative for two reasons. First, it follows from $t_{A1} = Q_A T_{A1}$ and $t_{B1} = Q_B T_{B1}$ in (10) that $Q_A \geq 0$ and $Q_B \geq 0$. Second, requiring $T_{A1} \geq 0$ and $T_{B1} \geq 0$ in (10) implies $q_3+(q_1+q_3)Q_B \geq 0$ and $q_2+(q_1+q_2)Q_A \geq 0$, which in turn is consistent with $Q_A \geq 0$ in the last expression in (11) when $Q_3+(Q_1+Q_3)Q_B \geq 0$.

Notation

t_{ij}	Defender's effort to protect component i in period j, $i = A, B, j = 1, 2$
t_{A2F}	Defender's period 2 effort to protect component A when component B fails in period 1
t_{B2F}	Defender's period 2 effort to protect component B when component A fails in period 1
T_{ij}	Attacker's effort to attack component i in period j
T_{A2F}	Attacker's period 2 effort to attack component A when component B fails in period 1
T_{B2F}	Attacker's period 2 effort to attack component B when component A fails in period 1

c_i	Defender's unit cost of effort for component i
c_{AF}	Defender's period 2 unit cost of effort for component A when component B fails in period 1
c_{BF}	Defender's period 2 unit cost of effort for component B when component A fails in period 1
C_i	Attacker's unit cost of effort for component i
C_{AF}	Attacker's period 2 unit cost of effort for component A when component B fails in period 1
C_{BF}	Attacker's period 2 unit cost of effort for component B when component A fails in period 1
p_{ij}	Reliability of component i in period j
p_{A2F}	Reliability of component A in period 2 when component B fails in period 1
p_{B2F}	Reliability of component B in period 2 when component A fails in period 1
$p(p_{Aj}, p_{Bj})$	Reliability of system with two components with reliabilities p_{Aj} and p_{Bj} in period j
m_i	Attacker–defender contest intensity for component i
m_{AF}	Attacker–defender contest intensity for component A in period 2 when component B fails in period 1
m_{BF}	Attacker–defender contest intensity for component B in period 2 when component A fails in period 1
s	Value of system where both components operate
d_A	Value of degraded system where component A operates and component B fails
d_B	Value of degraded system where component B operates and component A fails
f	Value of system where both components fail
δ	Defender's time discount parameter for period 2
Δ	Attacker's time discount parameter for period 2
u	Defender's utility
U	Attacker's utility.