# Production versus safety in a risky competitive industry

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Abstract: Each of two firms has a resource that can be converted into safety versus productive investment in the first stage, with Bertrand competition on price in the second stage of a two-stage game. The firms produce differentiated products in a risky environment. If risks are negligible, investing more in safety decreases the price, and producing more increases the price. The results depend on whether risks get reduced concavely or convexly. With concave (convex) risk reduction, higher safety investment by the competitor causes higher (lower) own safety investment. With concave (convex) risk reduction, lower firm loyalty by consumers implies lower (higher) safety investment, higher product substitutability implies higher (lower) safety investment, and more adverse implications of the competitor's productive investment on the demand intercept of the firm implies lower (higher) safety investment. When each firm independently maximises profit in a Nash equilibrium, safety investment is lower than when a social planner maximises social welfare and when maximising joint industry profits. The impact of the income, substitution, and interdependence effects on safety investment and price is finally analysed.

**Keywords:** profit; safety; production; risk; accidents; competition; two-stage game; prices.

## 1 Introduction

Firms face risks due to internal factors related to production, equipment failure, human failure, due to interaction with other firms within the industry, or external factors. The latter can be societal changes in general, or targeted action such as crime, theft, espionage, hacking, blackmail, donation, terrorism. Firms maximise profits and often consider safety concerns as constraints imposed by law and regulations. This article takes the perspective that firms come equipped with resources, beyond sunk costs and fixed costs that can be converted into productive investment or safety investment. There is a tradeoff between the two. Too much productive investment increases the risk which may prove costly. Too much safety investment decreases the risk, but also decreases the profit, which may also prove costly. This article intends to understand the factors that affect the tradeoff, realising the multiple pathways from causes through intermediate effects and to ultimate effects.

Asche and Aven (2004) discuss the hypothesis that safety and accident risk is in general not adequately incorporated into the economic planning and decision processes. They demonstrate 'that safety measures have a value in an economic sense', and consider 'the business incentives for investing into safety', for one firm in isolation. This article analyses these incentives considering competition between two firms. To understand safety investment, firms need to be analysed in isolation, with externalities, and as strategically interacting with other firms. Accounting for strategic interaction between several firms usually generates insights that cannot be gained from one firm. The reason is that competition between firms may be strong or weak, products may be differentiated, externalities need to be modelled in a mutually dependent manner, production in one firm may affect one firm or cause differential risk on multiple firms, safety investment may benefit one firm or all firms in varying degrees, and incentives vary across firms.

We analyse two firms producing differentiated products in a two-stage game. In the first stage the firms choose safety investment and thus productive investment, independently and simultaneously. In the second stage they choose prices independently, assuming Bertrand competition without collusion in the product market. Two stage models like this are common in the literatures on trade associations and joint ventures. Typically, information is shared in the first period, with Cournot or Betrand competition in the second stage. See e.g., Kirby (1988), Novshek and Sonnenschein (1982), Gal-Or and Ghose (2005), Shapiro (1986) and Vives (1990).

To locate this research within a broader context, see Calow (1998), Fischhoff et al. (1981) and Jones-Lee (1989) for economic approaches to safety. Much literature focuses on public safety in a variety of senses. See Feber et al. (2003) for the economic effects of road safety improvements, Rienstra et al. (2000) for an economic evaluation of traffic safety measures for transport companies, and Swinbank (1993) for the economics of food safety. Some literature focuses on specific industries where safety concerns are prominent, in varying degrees affecting the public at large. See Thomas (1999) for economic and safety pressures on nuclear power, Rose (1990) for economic determinants of airline safety performance related to profitability and product quality, Ravi et al. (2002) for well safety and economics related to cement design in oil and gas production, and Kjellen et al. (1997) for economic effects of implementing internal control of health, safety and environment in an aluminium plant.

Some literature balances safety and costs. Oi (1974, 1995) considers the economics of product safety, and which price is acceptable for safety, Kroger and Fischer's (2000) balances safety and economics, and Gibson (1978) questions whether major hazards should be prevented at all costs. Further, see Hale (2000) for regulations of safety, Kotz and Schafer (1993) for economic incentives to accident prevention, Pape (1997) for the tolerability of risk in the application of ALARP, and Viscusi (1986, 1989, 1993) for safety through markets, market incentives for safety, and the value of risk to life and health. For a managerial review considering economic analysis as one of several inputs, see Aven (2003) and Hertz and Thomas (1983). Aven and Kørte (2003) consider the use of cost/benefit analyses and expected utility theory to support decision-making, Hausken (2002) merges game theory and probabilistic risk analysis, and Marcus et al. (1993) consider economic and behavioural perspectives on safety.

Differing from most of the approaches above, this article takes one firm's perspective in competition with another firm. A firm seeks to maximise profit, but has incentives to focus on safety due to the presence of risk. The risk is affected by both firms' independent investments in safety versus production. The approach is intended to be useful for firms which implicitly, through complex decision making, need to make tradeoffs between safety and productive investment. There is a need to understand the factors that are influential. The article is also intended to be useful for regulators and policy makers who need to understand how firms think, or implicitly think. Finally the article is intended to be useful for the academic community which continuously strives to push the cutting edge research frontiers to enhance our insight into safety versus productive investments from a firm's perspective.

The article mainly considers risks that are present and are affected by strategic choices by the two firms, e.g., due to modes of production. However, we also briefly consider the risk impact of three external effects causing risky threats; the income, substitution, and interdependence effects. Such effects may be other firms, societal changes in general, or targeted action against one or both firms, e.g., crime or terrorism.

Section 2 presents the model. Section 3 analyses the model. Section 4 considers social welfare and joint industry profits. Section 5 assumes sequential entry. Section 6 considers the income, substitution, and interdependence effects. Section 7 concludes.

#### 2 The model

Consider a market of two firms with resources  $R_i$  and  $R_j$ ,  $i, j = 1, 2, i \neq j$ , which may be capital goods or labour. Resource  $R_i$  can be converted into productive investment  $t_i$ , at unit conversion cost  $c_i$ , and safety investment  $s_i$ , at unit conversion cost  $d_i$ ,

$$R_i = c_i t_i + d_i s_i, \quad i = 1, 2$$
 (1)

Each firm produces a differentiated product in a two-stage non-cooperative game. As a practical aid it may be convenient to think of a product as a consumption good such as oil, and the resource  $R_i$  as a capital good such as oil drilling equipment. Alternatively, the product may be a consumption good such as fish, and the resource  $R_i$  a capital good such as fish, and the resource  $R_i$  a capital good such as fishing nets. In the first stage, the firms simultaneously and independently choose optimal levels of safety investments  $s_i$  and  $s_j$ , which are inserted into (1) to yield optimal productive investments  $t_i$  and  $t_j$ . In the second stage, they choose prices  $p_i$  and  $p_j$ 

simultaneously and independently. We apply backward induction to determine the subgame perfect equilibrium, which is a Bertrand-Nash solution since the firms compete choosing prices.

The demand  $q_i$  facing each product is

$$q_i = a_i - b_1 p_i + b_2 p_j + B_i, \quad 0 \le b_2 < b_1$$
(2)

as suggested by, e.g., McGuire and Staelin (1983). Assuming linearity in self and cross-price effects,  $b_1$  measures how price sensitive consumers are to firm *i*'s product, which reduces demand  $q_i$ , and  $b_2$  measures product substitutability, which increases demand  $q_i$ . That is,  $b_1 = b_2$  means homogenous products, and  $b_2 = 0$  means two monopolists. The initial intercept of demand is  $a_i$  for firm *i*, and  $B_i$  is the potential shift in the demand. The quantity  $q_i$  demanded from firm *i* increases with firm *i*'s productive investment  $t_i$ , and decreases with firm *j*'s productive investment  $\alpha t_j$ , where  $0 \le \alpha < 1$  expresses that own effects exceed cross effects. The quantity  $q_i$  decreases with the risk  $r_i = r_i(s_i, s_j)$  of accidents, which is the probability  $v_i$  of accidents times the magnitude *m*, that is  $r_i = v_i m$ . We thus define

$$B_i = t_i - \alpha t_j - r_i \tag{3}$$

#### **3** Analysing the model

Firm *i*'s profit function is

$$\Pi_{i} = p_{i}q_{i} = p_{i}\left(a_{i} - b_{1}p_{i} + b_{2}p_{j} + t_{i} - \alpha t_{j} - r_{i}\right)$$
(4)

where we have inserted (2) and (3). The Appendix determines the second stage equilibrium price

$$p_{i} = \frac{2b_{1}\left(a_{i} + t_{i} - \alpha t_{j} - r_{i}\right) + b_{2}\left(a_{j} + t_{j} - \alpha t_{i} - r_{j}\right)}{4b_{1}^{2} - b_{2}^{2}}$$
(5)

Substituting the second stage price into the profit function and differentiating with respect to the first stage decision variables gives a profit function and first order condition,

$$\Pi_i = b_1 p_i^2, \quad \frac{\partial \Pi_i}{\partial s_i} = 2b_1 p_i \frac{\partial p_i}{\partial s_i} = 0 \tag{6}$$

Without risk  $r_i = r_j = 0$ , firm *i* invests  $t_i = R_i / c_i$  in production, zero in safety, enjoying maximum price and profit. With risk, setting the first order condition in (A2) equal to zero gives

Proposition 1 For two firms with independent risks defined s.t.  $\partial r_i / \partial s_j = 0$ , the optimal level  $s_i$  of safety investment is given by

$$\partial r_i / \partial s_i = \left[ -\left(2b_1 - b_2 \alpha\right) d_i / c_i \right] / (2b_1) < 0 \tag{7}$$

The inequality follows since we always have  $2b_1 - b_2\alpha > 0$  since  $0 \le b_2 < b_1$  and  $0 \le \alpha < 1$ . We assume that the inequality holds generally. It means that increased productive investment increases the risk, and safety investment decreases the risk.

Proposition 2 For two firms with interdependent risks defined s.t.  $\partial r_i / \partial s_j < 0$ , the optimal level  $s_i$  of safety investment is given by

$$\partial r_i / \partial s_i = \left[ -\left(2b_1 - b_2 \alpha\right) d_i / c_i - b_2 \partial r_j / \partial s_i \right] / (2b_1)$$
(8)

Proposition 2 causes  $\partial r_i / \partial s_i$  to be larger (less negative) than in Proposition 1. This means that if the other firm's safety investment reduces one's own risk, then one's own safety investment has less impact on one's risk. Inserting the optimal  $s_i$  in Propositions 1 and 2 into (4) and (5) applying (1) for  $t_i$ , gives the price and profit.

Applying (A2) gives the following comparative statics.

**Proposition 3** 

- 1 A firm's price decreases in own safety investment if the impact on risks is small. If the impact on risks is sufficiently negative and  $d_i / c_i$  is small, then the price increases.
- 2 A firm's price decreases in the competitor's safety investment if  $\alpha < b_2 / (2b_1)$  and there is no impact on risks. If the impact on risks is sufficiently negative and  $d_j / c_j$  is small, then the price increases.

This means that if risks are negligible, investing more in safety decreases the price, and producing more increases the price. Conversely, if risks are affected sufficiently by both safety and productive investment, investing more in safety increases the price, and producing more decreases the price.

Let us first consider the favourable case where own productive investment has no impact on the risks of the two firms. Then the price always increases in own productive investment, and it increases more if consumers are price sensitive or disloyal to firm *i*'s product ( $b_1$  is large) if product substitutability is low ( $b_2$  is low) and if own effects of investment on demand exceed cross effects considerably ( $\alpha$  is low). If own productive investment has sufficiently large impact on the risks of one or both firms, then the price decreases. Especially the impact on one's own risk contributes to reducing the price.

The ratio of the unit conversion costs  $d_i / c_i$  of safety investments boosts  $2b_1 - b_2 \alpha$  in the numerator of (A2) which has a negative impact on price. When the cost  $d_i$  of safety investment is large, the impact on price is negative, and positive if  $d_i$  is small. Ensuring a price increase through the other firm's productive investment is more difficult, though it is possible if  $2b_1\alpha - b_2 < 0$  and there is no impact on risks. If the requirement is not satisfied, or if the impact on risks is sufficiently large, then the price decreases.

To determine how the firms react to each other's productive and safety investments, we must consider the second derivatives. We assume

$$A1: \quad \frac{\partial^2 r_i}{\partial s_i \partial s_j} > 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_i} = 0 \tag{9}$$

The first inequality assumes that since  $\partial r_i / \partial s_i < 0$  and  $\partial r_j / \partial s_j < 0$ , intensified safety investment by both firms reduces the marginal impact, causing a positive second cross

derivative. The second expression assumes that the joint operation of increased safety and productive investment within one firm cancel each other out, causing zero impact on risk.

Our second assumption A2I assumes that safety investment reduces risk in an increasingly beneficial and convex manner,

$$A2I: \quad \frac{\partial^2 r_i}{\partial s_i^2} < 0, \quad \frac{\partial^2 r_i}{\partial t_i^2} > 0, \quad \frac{\partial^2 r_i}{\partial s_j^2} < 0, \quad \frac{\partial^2 r_i}{\partial t_j^2} > 0 \tag{10}$$

which means that low investments have low impact on risks. High safety investment is needed to constrain the risk, and then the risk is constrained dramatically. This is likely with simple technology or when limited competence is needed for production. Safety measures may then quickly exceed essential thresholds and cause beneficial results. Similarly, high production causes a dramatic increase in risk, e.g., due to deleterious ripple effects with larger volumes, higher complexity, and more activity.

Our alternative second assumption A2D assumes that safety investment reduces risk in a decreasingly beneficial and concave manner,

$$A2D: \quad \frac{\partial^2 r_i}{\partial s_i^2} > 0, \quad \frac{\partial^2 r_i}{\partial t_i^2} < 0, \quad \frac{\partial^2 r_i}{\partial s_j^2} > 0, \quad \frac{\partial^2 r_i}{\partial t_j^2} < 0 \tag{11}$$

which means that low investments have high impact on risks. As safety investment increases, there is diminishing return, e.g., because opportunities for safety are low. Some risk phenomena may be such that these are hard to regulate or control, regardless how much resources and effort are invested. Similarly, increasing production impacts risk diminishingly. A2D applies in simple production plants with low complexity where modest safety measures are all that is needed, and where large production volumes can be implemented with modest risk exposure.

Proposition 4

- 1 With increasing risk return on safety investment (A1 and A2I) higher safety investment by the competitor causes higher own safety investment. The reaction functions are upward sloping,  $\partial s_i / \partial s_i > 0$ .
- 2 With decreasing risk return on safety investment (A1 and A2D) higher safety investment by the competitor causes lower own safety investment. The reaction functions are downward sloping,  $\partial s_i / \partial s_j < 0$ .

Proposition 4 is illustrated in Table 1, where  $s_j \uparrow$  means that  $s_j$  increases, and  $t_i \downarrow$  means that  $t_i$  decreases. Note that the diagonal elements are equivalent.

**Table 1**Illustration of Proposition 4

		Own firm		
		A1 and A2I	A1 and A2D	
Competitor	$s_j$	$s_i \uparrow, t_i \downarrow$	$s_i \downarrow, t_i \uparrow$	
	$t_j$	$s_i \downarrow, t_i \uparrow$	$s_i \uparrow, t_i \downarrow$	

Summing up, with increasing risk return on investment, the two firms' safety investments reinforce each other, and the two firms' productive investments reinforce each other. Conversely, with decreasing risk return on investment, the two firms' safety investments

work in the opposite direction, and the two firms' productive investments work in the opposite direction.

Proposition 5

- 1 lower firm loyalty (higher b<sub>1</sub>) implies lower safety investment  $\partial s_i / \partial b_1 < 0$  when A2I and  $\partial r_i / \partial s_i << 0$
- 2 conversely, lower firm loyalty implies higher safety investment  $\partial s_i / \partial b_1 > 0$  when A2D and  $\partial r_i / \partial s_i \ll 0$ .

A steeper demand schedule  $b_1$  means a lower level of demand at a given price, which means lower firm loyalty. Assume substantial impact of safety investment on risk reduction,  $\partial r_i / \partial s_i \ll 0$ . When safety investment reduces risk in an increasingly beneficial and convex manner,  $\partial^2 r_i / \partial s_i^2 \ll 0$ , firm *i* invests less in safety. This seems due to a cashing in effect. Investment in safety has such substantial impact that resources can be converted into production instead. Conversely, when  $\partial^2 r_i / \partial s_i^2 \ll 0$ , firm *i* invests more in safety. The decreasingly beneficial impact of safety investment implies that firm *i* must take on the burden of investing in safety to satisfy the safety concerns.

#### Proposition 6

- 1 higher product substitutability (higher  $b_2$ ) implies higher safety investment  $\partial s_i / \partial b_2 > 0$  when A2I and  $\partial r_i / \partial s_i << 0$
- 2 conversely, lower product substitutability implies lower safety investment  $\partial s_i / \partial b_2 < 0$  when A2D and  $\partial r_i / \partial s_i << 0$ .

Higher product substitutability causes more competition between the two firms. Assume again substantial impact of safety investment on risk reduction,  $\partial r_i / \partial s_i << 0$ , which causes  $\partial p_i / \partial s_i > 0$ . When safety investment reduces risk in an increasingly beneficial and convex manner,  $\partial^2 r_i / \partial s_i^2 < 0$ , firm *i* invests more in safety, and less in production. Conversely, when safety investment reduces risk in a decreasingly beneficial and concave manner,  $\partial^2 r_i / \partial s_i^2 < 0$ , firm *i* invests less in safety.

#### Proposition 7

- 1 higher  $\alpha$  implies lower safety investment  $\partial s_i / \partial \alpha < 0$  when A2I and  $\partial r_i / \partial s_i << 0$
- 2 conversely, higher  $\alpha$  implies higher safety investment  $\partial s_i / \partial \alpha > 0$  when A2D and  $\partial r_i / \partial s_i \ll 0$ .

The parameter  $\alpha$  measures the adverse implications of the competitor's productive investment on the demand intercept of firm *i*. Higher  $\alpha$  implies lower volumes of sales. Assume substantial impact of safety investment on risk reduction,  $\partial r_i / \partial s_i << 0$ , which causes  $\partial p_i / \partial s_i > 0$ . When safety investment reduces risk in an increasingly beneficial and convex manner,  $\partial^2 r_i / \partial s_i^2 < 0$ , firm *i* invests less in safety, which seems due to a cashing in effect. Resources are converted into production instead. Conversely, when safety investment reduces risk in a decreasingly beneficial and concave manner,  $\partial^2 r_i / \partial s_i^2 < 0$ , firm *i* invests more in safety.

Proposition 8 Safety and productive investments have no impact on the initial intercept  $a_i$  of demand for firm *i*, interpreted as firm size,  $\partial s_i / \partial a_i = 0$ .

## 4 Social welfare and joint industry profits

Introducing a social planner, we derive the inverse demand function s.t.  $p_i = F_i (q_i, q_j)$ , and assume symmetry s.t. subscripts can be removed. We define social welfare as

$$SW = 2\int_0^{q^*} F(q,q)dq = 2\int_0^{q^*} \frac{a+B-q}{b_1-b_2}dq, \quad B = t(1-\alpha) - r(s,s)$$
(12)

Applying (3) and (5) at the symmetric equilibrium gives  $q^* = b_1 p^* = b_1 (a + B) / (2b_1 - b_2)$  and  $\partial q^* / \partial s = (\partial B + \partial s) b_1 / (2b_1 - b_2)$ . Applying Leibniz's Theorem gives

$$\frac{\partial SW}{\partial s} = \frac{\partial}{\partial s} \int_0^{q^*} 2\frac{a+B-q}{b_1-b_2} dq = \frac{2}{b_1-b_2} \left[ \int_0^{q^*} \frac{\partial B}{\partial s} dq + (a+B-q^*) \frac{\partial q^*}{\partial s} \right]$$

$$= 2\frac{b_1(a+B)(3b_1-2b_2)}{(2b_1-b_2)^2(b_1-b_2)} \frac{\partial B}{\partial s} = 0$$
(13)

Inserting t = (R - ds) / c,  $\partial B / \partial s = 0$ , and  $\partial r / \partial s_i = \partial r / \partial s_j = \partial r / \partial s$  when symmetry gives

$$B = \frac{(1-\alpha)(R-ds)}{c} - r(s,s),$$

$$\frac{\partial B}{\partial s} = -\frac{(1-\alpha)d}{c} - \left[\frac{\partial r}{\partial s_i} + \frac{\partial r}{\partial s_j}\right] = 0 \quad \Rightarrow \left(\frac{\partial r}{\partial s}\right)^{SW} = -\frac{(1-\alpha)d}{2c}$$
(14)

For productive investment we analogously get

$$\frac{\partial B}{\partial t} = (1 - \alpha) - \left[\frac{\partial r}{\partial t_i} + \frac{\partial r}{\partial t_j}\right] = 0 \quad \Rightarrow \left(\frac{\partial r}{\partial t}\right)^{SW} = \frac{(1 - \alpha)}{2} \tag{15}$$

When maximising joint industry profits without price coordination, prices are  $p^* = (a + B) / (2b_1 - b_2)$  and joint industry profits according to (6) are

$$\Pi^{JP} = \Pi_{i} + \Pi_{j} = \frac{2b_{1}(a+B)^{2}}{(2b_{1}-b_{2})^{2}}, \quad \frac{\partial\Pi^{JP}}{\partial s} = \frac{4b_{1}(a+B)}{(2b_{1}-b_{2})^{2}}\frac{\partial B}{\partial s} = 0,$$

$$\frac{\partial\Pi^{JP}}{\partial t} = \frac{4b_{1}(a+B)}{(2b_{1}-b_{2})^{2}}\frac{\partial B}{\partial t} = 0$$
(16)

Hence maximising joint industry profits gives the same result as maximising social welfare.

When each firm maximises profit, (6), (A2),  $\partial p_i / \partial s_i = 0$ , and symmetry imply

$$-(2b_{1}-b_{2}\alpha)d_{i}/c_{i}-2b_{1}\partial r_{i}/\partial s_{i}-b_{2}\partial r_{j}/\partial s_{i}=0$$

$$\Rightarrow \left(\frac{\partial r}{\partial s}\right)^{NE} = -\frac{(2b_{1}/b_{2}-\alpha)d}{\left[(2b_{1}/b_{2})+1\right]c} < \left(\frac{\partial r}{\partial s}\right)^{SW}$$

$$\tag{17}$$

Equations (6), (A2),  $\partial p_i / \partial s_i = 0$ , and symmetry analogously imply

$$2b_{1} - b_{2}\alpha - 2b_{1}\partial r_{i}/\partial t_{i} - b_{2}\partial r_{j}/\partial t_{i} = 0$$

$$\Rightarrow \left(\frac{\partial r}{\partial t}\right)^{NE} = \frac{2b_{1}/b_{2} - \alpha}{\left[\left(2b_{1}/b_{2}\right) + 1\right]} > \left(\frac{\partial r}{\partial t}\right)^{SW}$$
(18)

Proposition 9 When each firm independently maximises profit in a Nash equilibrium, safety investment is lower than when a social planner maximises social welfare, which gives the same result as when maximising joint industry profits,  $(\partial r / \partial s)^{NE} < (\partial r / \partial s)^{SW} \Rightarrow s^{NE} < s^{SW}$ .

#### 5 Sequential entry

Consider a three stage game. In stage 1 the leader incumbent firm *i* chooses  $s_i$ , which determines  $t_i$  from (1). In stage 2 the follower entrant firm *j* chooses  $s_j$ . In stage 3 both firms choose prices  $p_i$  and  $p_j$ . Stage 3 has the same solution as stage 2 in Section 3, and stage 2 for firm *j* has the same first order condition as (6). The first order condition for firm *i* is

$$\frac{\partial \Pi_i}{\partial s_i} = 2b_1 p_i \frac{\partial p_i}{\partial s_i} + 2b_1 p_i \frac{\partial p_i}{\partial s_j} \frac{\partial s_j}{\partial s_i} = 0$$
(19)

Applying (A2) and Proposition 3 give

$$\frac{\partial p_{i}}{\partial s_{j}} \frac{\partial s_{j}}{\partial s_{i}} \begin{cases} > 0 \text{ when } A2I \text{ and } (2b_{1}\alpha - b_{2})d_{j}/c_{j} - 2b_{1}\partial r_{i}/\partial s_{j} - b_{2}\partial r_{j}/\partial s_{j} > 0 \\ < 0 \text{ when } A2I \text{ and } (2b_{1}\alpha - b_{2})d_{j}/c_{j} - 2b_{1}\partial r_{i}/\partial s_{j} - b_{2}\partial r_{j}/\partial s_{j} < 0 \\ < 0 \text{ when } A2D \text{ and } (2b_{1}\alpha - b_{2})d_{j}/c_{j} - 2b_{1}\partial r_{i}/\partial s_{j} - b_{2}\partial r_{j}/\partial s_{j} > 0 \\ > 0 \text{ when } A2D \text{ and } (2b_{1}\alpha - b_{2})d_{j}/c_{j} - 2b_{1}\partial r_{i}/\partial s_{j} - b_{2}\partial r_{j}/\partial s_{j} < 0 \end{cases}$$

$$(20)$$

Proposition 10

- 1 for those cases in (20) where  $(\partial p_i / \partial s_j) (\partial s_j / \partial s_j) > 0$ , safety investments  $s_i$  and  $s_j$  are larger for sequential entry than in the simultaneous game
- 2 conversely, when  $(\partial p_i / \partial s_j) (\partial s_j / \partial s_j) < 0$ , safety investments  $s_i$  and  $s_j$  are smaller for sequential entry than in the simultaneous game.

Consider the first line in (20) which applies when safety investment reduces risk in an increasingly beneficial and convex manner, and when  $\partial r_j / \partial s_j \ll 0$ . In this case safety investments  $s_i$  and  $s_j$  are larger for sequential entry than in the simultaneous game.

#### 6 Income effect, substitution effect, and interdependence effect

We have so far assumed that risks are present and affected by strategic choices by the two firms, e.g., due to modes of production. In this last section, we briefly consider the risk impact of three external effects, please see Hausken (2006). Such effects causing risky threats may be other firms, societal changes in general, or targeted action against one or both firms. Examples are crime, theft, espionage, hacking, blackmail, donation, terrorism.

We first consider the income effect. Enders and Sandler (2003) show how 'freezing terrorist's assets reduces their income. A firm's safety investment may be passive defensive mechanisms, or active offensive mechanisms designed for eliminating or deterring some or all external risks, which may benefit one or both firms. This causes  $\partial r_i / \partial s_i$  and  $\partial r_i / \partial s_j$  to be more negative than before since safety eliminates or deters part of the external threat. This causes  $\partial p_i / \partial s_i$  to be larger according to (A2). We also assume

$$A3: \quad \frac{\partial^2 r_i}{\partial s_i \partial s_j} > 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial t_j} < 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_i} < 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_j} < 0 \tag{21}$$

We second consider the substitution effect. Enders and Sandler (2003) show how 'the installation of screening devices in US airports in January 1973 made skyjackings more difficult, thus encouraging terrorists to substitute into other kinds of hostage missions or to stage a skyjacking from an airport outside of the USA. This causes  $\partial r_i / \partial s_j$  to be less negative than before, and may be positive, since safety investment by the other firm makes one's own firm more liable to the external threat. This causes  $\partial p_i / \partial s_i$  to be smaller according to (A2).  $\partial r_i / \partial t_i$  is more positive than before since productive investment makes one's own firm more liable to the external threat. We also assume

$$A4: \quad \frac{\partial^2 r_i}{\partial s_i \partial s_j} > 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial t_j} < 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_i} = 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_j} >> 0 \tag{22}$$

We third consider the interdependence effect, considered by Kunreuther and Heal (2003) exemplified within the airline industry, computer networks, fire protection, theft protection, bankruptcy protection, vaccinations. In varying degrees, one target's safety investment benefits all targets. This causes  $\partial r_i / \partial s_j$  to be more negative than before since safety investment by the other firm benefits one's own firm increasingly. This causes  $\partial p_i / \partial s_j$  to be larger according to (A2).  $\partial r_i / \partial t_j$  is more positive than before since productive investment by the other firm increases one's own risk increasingly. We also assume

$$A5: \quad \frac{\partial^2 r_i}{\partial s_i \partial s_j} > 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial t_j} = 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_i} = 0, \quad \frac{\partial^2 r_i}{\partial t_i \partial s_j} = 0$$
(23)

 Table 2
 Income, substitution, and interdependence effects

Effect	$\partial r_i / \partial s_i$	$\partial r_i / \partial s_j$	$\partial p_i / \partial s_i$
Income	More negative	More negative	Larger
Substitution		Less negative, or positive	Smaller
Interdependence		More negative	Larger

#### 7 Conclusions

The article analyses two firms which produce differentiated products in a risky environment and compete on price to maximise profit. Each firm has a resource that be converted into safety versus productive investment. The tradeoff depends on a variety of factors which this article intends to understand and illuminate. Choosing the firm's perspective and incentives for safety investment is essential since many of today's societal mechanisms are generated by the interests of firms. Firms' incentives have received modest attention in earlier research compared, e.g., by the perspective that safety concerns are constraints imposed by law and regulations which are often designed from a societal perspective. Both perspectives are needed.

If risks are negligible, the article shows that investing more in safety decreases the price, and producing more increases the price. Conversely, if risks are affected sufficiently by both safety and productive investment, safety investment is cheap, and production is expensive, then investing more in safety increases the price, and producing more decreases the price.

Although risks get reduced by safety investment, the results of the article depend strongly on whether risks get reduced concavely (increasingly beneficial) or convexly (decreasingly beneficial). The former is likely with simple technology or when limited competence is needed for production. As safety measures exceed certain limits, beneficial results may quickly follow. The latter is likely for risk phenomena that are hard to regulate or control, and when opportunities for safety are low, regardless how much resources and effort are invested.

With concave (convex) risk reduction, higher safety investment by the competitor causes higher (lower) own safety investment. Three results follow for the case that risks are substantially reduced by safety investment. With concave (convex) risk reduction, lower firm loyalty by consumers implies lower (higher) safety investment, higher product substitutability implies higher (lower) safety investment, and more adverse implications of the competitor's productive investment on the demand intercept of the firm implies lower (higher) safety investment.

Safety and productive investments have no impact on the initial intercept of demand for a firm, interpreted as firm size. We show that when each firm independently maximises profit in a Nash equilibrium, safety investment is lower than when a social planner maximises social welfare, which gives the same result as when maximising joint industry profits. The article also considers the case of sequential entry.

The main results of the article apply for risks that are present and affected by strategic choices by the two firms, e.g., due to modes of production. The last section briefly considers the risk impact of three external effects, e.g., crime or terrorism targeting one or both firms. The income effect, where safety investment by one or both firms reduce the externally generated risk, causes safety investment to have more impact, and the price increases. The substitution effect causes safety investment by the other firm to make one's own firm more liable to the external threat, which reduces the price of one's own product. The interdependence effect causes safety investment by the other firm to benefit both firms, which increases the price.

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## Appendix

We first solve the second stage for price. Setting the derivative of (4) with respect to  $p_i$  equal to zero gives

$$p_{i} = \left(a_{i} + b_{2}p_{j} + t_{i} - \alpha t_{j} - r_{i}\right) / (2b_{1}) = q_{i} / b_{1}$$
(A1)

Interchanging *i* and *j* and solving the two price reaction functions gives (5). Inserting from (1) that  $t_i = (R_i - d_i s_i) / c_i$  and  $t_j = (R_j - d_j s_j) / c_j$ , and differentiating gives

$$\frac{\partial p_i}{\partial s_i} = \frac{-(2b_1 - b_2\alpha)d_i / c_i - 2b_1\partial r_i / \partial s_i - b_2\partial r_j / \partial s_i}{\left(4b_1^2 - b_2^2\right)},$$

$$\frac{\partial p_i}{\partial s_j} = \frac{\left(2b_1\alpha - b_2\right)d_j / c_j - 2b_1\partial r_i / \partial s_j - b_2\partial r_j / \partial s_j}{\left(4b_1^2 - b_2^2\right)}$$
(A2)

## Proof of Proposition 4

The second derivatives are as follows, inserting the assumptions A1, A2I, A2D:

$$\frac{\partial^2 p_i}{\partial s_i^2} = \frac{-2b_1 \partial^2 r_i / \partial s_i^2 - b_2 \partial^2 r_j / \partial s_i^2}{\left(4b_1^2 - b_2^2\right)} \begin{cases} > 0 \text{ when } A2I \\ < 0 \text{ when } A2D \end{cases}$$
(A3)

$$\frac{\partial^2 p_i}{\partial s_i \partial t_i} = \frac{-2b_1 \partial^2 r_i / \partial s_i \partial t_i - b_2 \partial^2 r_j / \partial s_i \partial t_i}{4b_1^2 - b_2^2} = 0$$
(A4)

$$\frac{\partial^2 p_i}{\partial s_i \partial s_j} = \frac{-2b_1 \partial^2 r_i / \partial s_i \partial s_j - b_2 \partial^2 r_j / \partial s_i \partial s_j}{\left(4b_1^2 - b_2^2\right)} < 0$$
(A5)

Applying the implicit function approach, total differentiation of (6) gives

$$\frac{\partial^2 p_i}{\partial s_i^2} ds_i + \frac{\partial^2 p_i}{\partial s_i \partial t_i} dt_i + \frac{\partial^2 p_i}{\partial s_i \partial s_j} ds_j = 0,$$

$$\frac{\partial^2 p_i}{\partial t_i \partial s_i} ds_i + \frac{\partial^2 p_i}{\partial t_i^2} dt_i + \frac{\partial^2 p_i}{\partial t_i \partial s_j} ds_j = 0$$
(A6)

which are solved to yield

$$\frac{\partial s_i}{\partial s_j} = -\frac{\frac{\partial^2 p_i}{\partial t_i^2} \frac{\partial^2 p_i}{\partial s_i \partial s_j} - \frac{\partial^2 p_i}{\partial s_i \partial t_i} \frac{\partial^2 p_i}{\partial t_i \partial s_j}}{\frac{\partial^2 p_i}{\partial s_i^2} - \left(\frac{\partial^2 p_i}{\partial s_i \partial t_i}\right)^2} = -\frac{\frac{\partial^2 p_i}{\partial s_i \partial s_j}}{\frac{\partial^2 p_i}{\partial s_i^2}} \begin{cases} > 0 \text{ when } A2I \\ < 0 \text{ when } A2D \end{cases}$$
(A7)

where the assumptions are inserted.

*Proof of Proposition 5 for*  $b_1$ 

We first determine

$$\frac{\partial^2 p_i}{\partial s_i \partial b_l} = -2 \frac{\left(4b_l^2 + b_2^2\right) \left(d_i / c_i + \partial r_i / \partial s_i\right) + 4b_l b_2 \left(-\alpha d_i / c_i + \partial r_j / \partial s_i\right)}{\left(4b_l^2 - b_2^2\right)}$$

$$> 0 \text{ when } \partial r_i / \partial s_i << 0$$
(A8)

Applying the implicit function approach, total differentiation of (6) gives

$$\frac{\partial^2 p_i}{\partial s_i^2} ds_i + \frac{\partial^2 p_i}{\partial s_i \partial t_i} dt_i + \frac{\partial^2 p_i}{\partial s_i \partial b_1} db_1 = 0,$$

$$\frac{\partial^2 p_i}{\partial t_i \partial s_i} ds_i + \frac{\partial^2 p_i}{\partial t_i^2} dt_i + \frac{\partial^2 p_i}{\partial t_i \partial b_1} db_1 = 0$$
(A9)

which are solved to yield

$$\frac{\partial s_{i}}{\partial b_{1}} = -\frac{\frac{\partial^{2} p_{i}}{\partial t_{i}^{2}} \frac{\partial^{2} p_{i}}{\partial s_{i} \partial b_{1}} - \frac{\partial^{2} p_{i}}{\partial s_{i} \partial t_{i}} \frac{\partial^{2} p_{i}}{\partial t_{i} \partial b_{1}}}{\frac{\partial^{2} p_{i}}{\partial s_{i}^{2}} \frac{\partial^{2} p_{i}}{\partial t_{i}^{2}} - \left(\frac{\partial^{2} p_{i}}{\partial s_{i} \partial t_{i}}\right)^{2}}{\left|\frac{\partial s_{i}}{\partial s_{i}}\right|^{2}} = -\frac{\frac{\partial^{2} p_{i}}{\partial s_{i} \partial t_{i}}}{\frac{\partial s_{i}}{\partial s_{i}}} \begin{cases} < 0 \text{ when } A2I \text{ and } \partial r_{i} / \partial s_{i} << 0 \\ > 0 \text{ when } A2D \text{ and } \partial r_{i} / \partial s_{i} << 0 \end{cases}$$
(A10)

where the assumptions are inserted.

*Proof of Proposition 6 for*  $b_2$ 

We first determine

$$\frac{\partial^2 p_i}{\partial s_i \partial b_2} = \frac{4b_1 b_2 \left(\alpha d_i / c_i + \partial r_i / \partial s_i\right) + \left(4b_1^2 + b_2^2\right) \left(-\alpha d_i / c_i + \partial r_j / \partial s_i\right)}{\left(4b_1^2 - b_2^2\right)}$$
(A11)
< 0 when  $\partial r_i / \partial s_i \ll 0$ 

Applying the implicit function approach, total differentiation of (6) gives

$$\frac{\partial^2 p_i}{\partial s_i^2} ds_i + \frac{\partial^2 p_i}{\partial s_i \partial t_i} dt_i + \frac{\partial^2 p_i}{\partial s_i \partial b_2} db_2 = 0,$$

$$\frac{\partial^2 p_i}{\partial t_i \partial s_i} ds_i + \frac{\partial^2 p_i}{\partial t_i^2} dt_i + \frac{\partial^2 p_i}{\partial t_i \partial b_2} db_2 = 0$$
(A12)

which are solved to yield

$$\frac{\partial s_{i}}{\partial b_{2}} = -\frac{\frac{\partial^{2} p_{i}}{\partial t_{i}^{2}} \frac{\partial^{2} p_{i}}{\partial s_{i} \partial b_{2}} - \frac{\partial^{2} p_{i}}{\partial s_{i} \partial t_{i}} \frac{\partial^{2} p_{i}}{\partial t_{i} \partial b_{2}}}{\frac{\partial^{2} p_{i}}{\partial s_{i}^{2}} \frac{\partial^{2} p_{i}}{\partial t_{i}^{2}} - \left(\frac{\partial^{2} p_{i}}{\partial s_{i} \partial t_{i}}\right)^{2}}{\left(\frac{\partial^{2} p_{i}}{\partial s_{i} \partial b_{2}}\right)^{2}}$$

$$= -\frac{\frac{\partial^{2} p_{i}}{\partial s_{i} \partial b_{2}}}{\frac{\partial^{2} p_{i}}{\partial s_{i}^{2}}} \begin{cases} > 0 \text{ when } A2I \text{ and } \partial r_{i} / \partial s_{i} <<0 \\ < 0 \text{ when } A2D \text{ and } \partial r_{i} / \partial s_{i} <<0 \end{cases}$$
(A13)

where the assumptions are inserted.

Proof of Proposition 7 for  $\alpha$ We first determine

$$\frac{\partial^2 p_i}{\partial s_i \partial \alpha} = \frac{b_2 d_i / c_i}{\left(4b_1^2 - b_2^2\right)} > 0 \tag{A14}$$

Applying the implicit function approach, total differentiation of (6) gives

$$\frac{\partial^2 p_i}{\partial s_i^2} ds_i + \frac{\partial^2 p_i}{\partial s_i \partial t_i} dt_i + \frac{\partial^2 p_i}{\partial s_i \partial \alpha} d\alpha = 0,$$

$$\frac{\partial^2 p_i}{\partial t_i \partial s_i} ds_i + \frac{\partial^2 p_i}{\partial t_i^2} dt_i + \frac{\partial^2 p_i}{\partial t_i \partial \alpha} d\alpha = 0$$
(A15)

which are solved to yield

$$\frac{\partial s_i}{\partial \alpha} = -\frac{\frac{\partial^2 p_i}{\partial t_i^2} \frac{\partial^2 p_i}{\partial s_i \partial \alpha} - \frac{\partial^2 p_i}{\partial s_i \partial t_i} \frac{\partial^2 p_i}{\partial t_i \partial \alpha}}{\frac{\partial^2 p_i}{\partial s_i^2} - \left(\frac{\partial^2 p_i}{\partial s_i \partial t_i}\right)^2} = -\frac{\frac{\partial^2 p_i}{\partial s_i \partial \alpha}}{\frac{\partial^2 p_i}{\partial s_i^2}} \begin{cases} < 0 \text{ when } A2I \\ > 0 \text{ when } A2D \end{cases}$$
(A16)

where the assumptions are inserted.

Proof of Proposition 8 for  $a_i$ 

Applying the implicit function approach, total differentiation of (6) gives

$$\frac{\partial^2 p_i}{\partial s_i^2} ds_i + \frac{\partial^2 p_i}{\partial s_i \partial t_i} dt_i + \frac{\partial^2 p_i}{\partial s_i \partial a_i} da_i = 0,$$

$$\frac{\partial^2 p_i}{\partial t_i \partial s_i} ds_i + \frac{\partial^2 p_i}{\partial t_i^2} dt_i + \frac{\partial^2 p_i}{\partial t_i \partial a_i} da_i = 0$$
(A17)

which are solved to yield

$$\frac{\partial s_i}{\partial a_i} = -\frac{\frac{\partial^2 p_i}{\partial t_i^2} \frac{\partial^2 p_i}{\partial s_i \partial a_i} - \frac{\partial^2 p_i}{\partial s_i \partial t_i} \frac{\partial^2 p_i}{\partial t_i \partial a_i}}{\frac{\partial^2 p_i}{\partial s_i^2} \frac{\partial^2 p_i}{\partial t_i^2} - \left(\frac{\partial^2 p_i}{\partial s_i \partial t_i}\right)^2} = -\frac{\frac{\partial^2 p_i}{\partial s_i \partial a_i}}{\frac{\partial^2 p_i}{\partial s_i^2}} = 0$$
(A18)

where the assumptions are inserted. Differentiation of (6) and inserting

$$\frac{\partial p_i}{\partial a_i} = \frac{2b_1}{4b_1^2 - b_2^2} > 0, \quad \frac{\partial p_i}{\partial a_j} = \frac{b_2}{4b_1^2 - b_2^2} > 0, \quad \frac{\partial^2 p_i}{\partial t_i \partial a_i} = \frac{\partial^2 p_i}{\partial t_i \partial a_j} = 0$$
(A19)

gives

$$\frac{\partial^2 \Pi_i}{\partial t_i \partial a_i} = 2b_1 p_i \frac{\partial^2 p_i}{\partial t_i \partial a_i} + 2b_1 \frac{\partial p_i}{\partial a_i} \frac{\partial p_i}{\partial t_i} < 0 \text{ when } \partial r_i / \partial t_i >> 0$$
(A20)

$$\frac{\partial^2 \Pi_i}{\partial t_i \partial a_j} = 2b_1 p_i \frac{\partial^2 p_i}{\partial t_i \partial a_j} + 2b_1 \frac{\partial p_i}{\partial a_j} \frac{\partial p_i}{\partial t_i} < 0 \text{ when } \partial r_i / \partial t_i >> 0$$
(A21)