

Production, safety, fighting, and risk

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Abstract: Two agents make a trade-off between production and safety investment, fighting for joint production. Ceteris paribus, if agent 1 has a higher unit cost of production, lower emphasis on safety causes more fighting and higher utility for agent 1, and less production and safety effort by agent 1.

Keywords: production; safety; fighting; risk.

Risk and safety issues gain importance in today's world, but have not received much attention in economics.¹ This article intends to change that with a simple model. Two agents convert resources into production, safety investment, and fighting.² The trade-off between production and safety takes the Cobb-Douglas form. Firms face risks due to internal factors related to production, equipment failure, human failure, interaction with other firms, crime, theft, espionage, hacking, blackmail, terrorism.

Safety concerns are often considered as constraints imposed by law and regulations. In contrast, Asche and Aven (2004) argue "that safety measures have a value in an economic sense", and Viscusi (1986) considers market incentives for safety. Recent changes in US accounting laws have made CEOs liable to legal malpractice if accounting information is fraudulent. Firms with finite resource constraints are thus led to determine optimal investments in information assurance technologies versus production technologies. The former reduces the risk of legal malpractice. This article intends to understand the factors that influence the trade-off between safety and productive investment.

Each agent i has a resource R_i (e.g., a capital good, or labour) transformable into three kinds of efforts. The first is productive effort E_i designed to generate production from resources currently controlled. The second is fighting effort F_i designed to acquire the production of others, or repel others as they attempt to do the same. The third is safety effort G_i designed to ensure safe production. With unit conversion costs a_i, b_i, c_i of transforming R_i into E_i, F_i, G_i , this gives

$$R_i = a_i E_i + b_i F_i + c_i G_i \Rightarrow E_i = (R_i - b_i F_i - c_i G_i) / a_i \quad (1)$$

Without risk, assume a simple production function where agent i produces $Y_i = E_i$, with no need for safety effort.³ With risk, the production function is

$$Y_i = E_i^\beta G_i^{1-\beta} = \left((R_i - b_i F_i - c_i G_i) / a_i \right)^\beta G_i^{1-\beta}, \quad \beta \in [0, 1] \quad (2)$$

where β is a parameter that scales the importance of productive effort relative to safety effort. $\beta = 1$ means no safety effort. As β decreases from 1, safety effort gains increased importance, and the agent faces a trade-off between E_i and G_i . As a practical aid it may be convenient to think of production as generating a consumption good such as oil, and the resource R_i as a capital good such as oil drilling equipment. Alternatively, the product may be a consumption good such as fish, and the resource R_i a capital good such as fishing nets.

The total production $Y_1 + Y_2$ is placed in a common pool for capture.⁴ The two agents fight with each other for the total production. Agent i gets a ratio $F_i^m / (F_1^m + F_2^m)$, known as the contest success function (e.g., Skaperdas, 1996), where m is the decisiveness parameter.⁵ Multiplying this ratio with the total production gives the utility

$$U_i = \frac{F_i^m}{F_1^m + F_2^m} \left[\left(\frac{R_1 - b_1 F_1 - c_1 G_1}{a_1} \right)^\beta G_1^{1-\beta} + \left(\frac{R_2 - b_2 F_2 - c_2 G_2}{a_2} \right)^\beta G_2^{1-\beta} \right] \quad (3)$$

To determine the first order conditions, agent 1 chooses F_1 and G_1 , and agent 2 chooses F_2 and G_2 , simultaneously and independently, to maximise utility. The FOCs are

$$\begin{aligned} \frac{\partial U_1}{\partial F_1} = 0 &\Rightarrow F_2^m G_1^\beta G_2^m (R_1 - b_1 F_1 - c_1 G_1) \left(\frac{R_2 - b_2 F_2 - c_2 G_2}{a_2} \right)^\beta \\ &= G_1 G_2^\beta \left(\frac{R_1 - b_1 F_1 - c_1 G_1}{a_1} \right)^\beta \times \\ &\quad \left[F_2^m m (c_1 G_1 - R_1) + b_1 F_1 (F_1^m \beta + F_2^m (m + \beta)) \right], \\ \frac{\partial U_2}{\partial F_2} = 0 &\Rightarrow F_1^m G_2^\beta G_1^m (R_2 - b_2 F_2 - c_2 G_2) \left(\frac{R_1 - b_1 F_1 - c_1 G_1}{a_1} \right)^\beta \\ &= G_2 G_1^\beta \left(\frac{R_2 - b_2 F_2 - c_2 G_2}{a_2} \right)^\beta \times \\ &\quad \left[F_1^m m (c_2 G_2 - R_2) + b_2 F_2 (F_2^m \beta + F_1^m (m + \beta)) \right], \end{aligned} \quad (4)$$

$$\frac{\partial U_1}{\partial G_1} = 0 \Rightarrow G_1 = \frac{(R_1 - b_1 F_1)(1 - \beta)}{c_1}, \quad \frac{\partial U_2}{\partial G_2} = 0 \Rightarrow G_2 = \frac{(R_2 - b_2 F_2)(1 - \beta)}{c_2} \quad (5)$$

Inserting (5) into (4), the two equations in (4) imply

$$b_1 F_1^{m+1} (m R_2 - b_2 F_2 (m + 1)) + b_2 F_2^{m+1} (m R_1 - b_1 F_1 (m + 1)) = 0 \quad (6)$$

With symmetry, this gives

$$F = \frac{mR}{b(m+1)}, \quad G = \frac{R(1-\beta)}{c(m+1)}, \quad E = \frac{R\beta}{a(m+1)}, \quad U = \frac{R\beta^\beta (1-\beta)^{1-\beta}}{a^\beta c^{1-\beta} (m+1)} \quad (7)$$

Proposition 1: With symmetry, increased fighting decisiveness m increases fighting concavely, and decreases production, safety effort, and utility convexly. Production is proportional to β , safety effort is proportional to $1 - \beta$, and all efforts are inverse proportional to their unit costs and proportional to the resource. The utility is proportional to $\beta^\beta(1 - \beta)^{1-\beta}$, which is U formed with minimum at $b = 1/2$, inverse proportional to the weighted product $a^\beta c^{1-\beta}$ of unit costs, independent of the unit cost of fighting, and proportional to the resource.

Increased unit cost b of fighting reduces fighting by both agents, with no impact on the utility with decisiveness $m = 1$, (4) solves to

$$\begin{aligned}
 F_1 &= \frac{a_2^\beta c_2^{1-\beta} R_1 + a_1^\beta c_1^{1-\beta} R_2}{2a_2^\beta c_2^{1-\beta} b_1 + 2a_1^{\beta/2} a_2^{\beta/2} \sqrt{b_1 b_2} c_1^{(1-\beta)/2} c_2^{(1-\beta)/2}}, \\
 F_2 &= \frac{a_2^\beta c_2^{1-\beta} R_1 + a_1^\beta c_1^{1-\beta} R_2}{2a_1^\beta c_1^{1-\beta} b_2 + 2a_1^{\beta/2} a_2^{\beta/2} \sqrt{b_1 b_2} c_1^{(1-\beta)/2} c_2^{(1-\beta)/2}}, \\
 \frac{F_1}{F_2} &= \frac{U_1}{U_2} = \left(\frac{a_1}{a_2}\right)^{\frac{\beta}{2}} \left(\frac{c_1}{c_2}\right)^{\frac{1-\beta}{2}} \sqrt{\frac{b_2}{b_1}}
 \end{aligned} \tag{8}$$

where E_1, E_2, G_1, G_2 follow from (1) and (5).

Proposition 2: Ceteris paribus, if agent 1 has a higher unit cost of production ($a_1 > a_2$), lower emphasis on safety (higher β) causes more fighting and higher utility for agent 1, $\partial(F_1 / F_2) / \partial\beta > 0$, where $F_1 / F_2 = U_1 / U_2 > 1$, and less production and safety effort by agent 1, $\partial(E_1 / E_2) / \partial\beta < 0$, $\partial(G_1 / G_2) / \partial\beta < 0$, $E_1 / E_2 < 1$, $G_1 / G_2 < 1$.

Ceteris paribus means that all the other parameters for the two agents are equal. Proposition 2 is partly related to Hirshleifer's (1991, p.177) paradox of power which states that "poorer or smaller combatants often end up improving their position relative to richer or larger ones.... The explanation is that initially poorer contenders are rationally motivated to fight harder, to invest relatively more in conflictual activity". In Proposition 2, the poorer contender is the one with the higher unit cost of production. The novelty of Proposition 2 is to determine the impact of accounting for safety. With lower emphasis on safety, the impetus to focus on production becomes stronger. However, agent 1 is already disadvantaged by the higher unit cost of production, and becomes more disadvantaged by the lower emphasis on safety. Consequently, agent 1 fights harder relative to agent 2. Conversely, with higher emphasis on safety (lower β), agent 1's production disadvantage is partly offset. Agent 1 still fights more than agent 2, but not so much more.

Proposition 3: Ceteris paribus, if agent 1 has a higher unit cost of safety effort ($c_1 > c_2$), lower emphasis on safety (higher β) causes less fighting and lower utility for agent 1, $\partial(F_1 / F_2) / \partial\beta < 0$, where $F_1 / F_2 = U_1 / U_2 > 1$, and more production and safety effort by agent 1, $\partial(E_1 / E_2) / \partial\beta > 0$, $\partial(G_1 / G_2) / \partial\beta > 0$, $E_1 / E_2 < 1$, $G_1 / G_2 < 1$.

Proposition 3 is the opposite of Proposition 2. Agent 1 is now disadvantaged by a higher unit cost of safety effort. This induces him to fight harder than agent 2. Lower emphasis on safety makes agent 1 less disadvantaged. He still fights more than agent 2, but not so much more.

The upshot of Propositions 2 and 3 is that if agent 1 is disadvantaged by higher unit costs of production or safety effort, he will fight harder than agent 2. His fighting gets reduced by higher safety emphasis if disadvantaged with respect to unit cost of production, and gets reduced by lower safety emphasis if disadvantaged with respect to unit cost of safety effort.

Inserting equal parameters $a_1 = a_2$ and $c_1 = c_2$ into (1), (5), (8) gives

$$\frac{E_1}{E_2} = \frac{G_1}{G_2} = \frac{(2 + \sqrt{b_1/b_2})R_1 - \sqrt{b_1/b_2}R_2}{(1 + 2\sqrt{b_1/b_2})R_2 - R_1}, \quad \frac{F_1}{F_2} = \frac{U_1}{U_2} = \sqrt{\frac{b_2}{b_1}} \quad (9)$$

Proposition 4: Ceteris paribus, if agent 1 has a higher unit cost of fighting ($b_1 > b_2$), he fights less, produces less, and invests less in safety than agent 2, $E_1/E_2 = G_1/G_2 = F_1/F_2 = U_1/U_2 = \sqrt{b_2/b_1}$, independently of the emphasis on safety.

In this case, agent 1 is disadvantaged where it hurts most, namely in his ability to fight. He responds by lowering not only his fighting effort, but also by lowering his production effort and safety effort equally much, although he is not disadvantaged in those respects. He consequently suffers lower utility. The trade-off between investment in production versus safety effort plays no role due to the equal parameters.

Proposition 5: Ceteris paribus, if agent 1 enjoys a higher resource than agent 2 ($R_1 > R_2$), he produces more and invests more in safety effort, $E_1/E_2 = G_1/G_2 = (3R_1/R_2 - 1)/(3 - R_1/R_2)$, but fights equally much and receives the same utility as agent 2, $F_1/F_2 = U_1/U_2 = 1$.

This result follows since the joint production is placed in a common pool for capture. As agent 1 gets advantaged by a higher resource, agent 2 cuts down on production and safety effort in order to match agent 1's fighting. The two fighting levels are matched causing equal utilities until agent 2 at the extreme produces nothing and invests nothing in safety effort. For the uncommon case that all parameters are equal except that $R_1 > 3R_2$, a corner solution follows where agent 2 allocates all his resources to fighting. The two agents thus fight exclusively over agent 1's production, where agent 1 fights more and enjoys higher utility. Table 1 exemplifies the propositions.

Table 1 Exemplification of Propositions 2–5

	E_1/E_2	G_1/G_2	$F_1/F_2 = U_1/U_2$
$a_1 = 2a_2, \beta = 1/2$	0.42	0.84	1.19
$a_1 = 2a_2, \beta = 3/4$	0.38	0.76	1.30
$c_1 = 2c_2, \beta = 1/2$	0.84	0.42	1.19
$c_1 = 2c_2, \beta = 3/4$	0.92	0.46	1.09
$b_1 = 2b_2$	0.71	0.71	0.71
$R_1 = 2R_2$	5	5	1

References

- Asche, F. and Aven, T. (2004) 'On the economic value of safety', *Risk Decision and Policy*, Vol. 9, No. 3, pp.253–267.
- Calow, P. (Ed.) (1998) *Handbook of Environmental risk assessment and management*, Blackwell Sciences, Oxford.
- Feber, D.J., Feldmeier, J.M. and Crocker, K.J. (2003) 'The economic effects of road safety improvements: an insurance claims analysis', *Journal of Risk and Insurance*, Vol. 70, No. 4, pp.651–664.
- Fischhoff, B., Lichtenstein, S., Slovic, P., Derby, S.L. and Keeney, R.L. (1981) *Acceptable Risk*, Cambridge University Press, Cambridge.
- Grossman, H.I. (1991) 'A general equilibrium model of insurrections', *American Economic Review*, Vol. 81, No. 4, pp.912–921.
- Hausken, K. (2005) 'Production and conflict models versus rent seeking models', *Public Choice*, Vol. 123, No. 1, pp.59–93.
- Hirshleifer, J. (1991) 'The paradox of power', *Economics and Politics*, Vol. 3, No. 3, pp.177–200.
- Hirshleifer, J. (2001) *The Dark Side of the Force: Economic Foundations of Conflict Theory*, Cambridge University Press, Cambridge.
- Jones-Lee, M.W. (1989) *The Economics of Safety and Physical Risk*, First Blackwell, Oxford.
- Kotz, H. and Schafer, H-B. (1993) 'Economic incentives to accident prevention: an empirical study of the German sugar industry', *International Review of Law and Economics*, Vol. 13, No. 1, pp.19–33.
- Oi, W.Y. (1974) 'Economics of product safety', *Bell Journal of Economics*, Vol. 5, No. 2, pp.689–695.
- Oi, W.Y. (1995) 'Safety at what price', *American Economic Review*, Vol. 85, No. 2, pp.67–71.
- Rose, N.L. (1990) 'Profitability and product quality – economic determinants of airline safety performance', *Journal of Political Economy*, Vol. 98, No. 5, pp.944–964.
- Skaperdas, S. (1996) 'Contest success functions', *Economic Theory*, Vol. 7, No. 2, pp.283–290.
- Skaperdas, S. and Syropoulos, C. (1997) 'The distribution of income in the presence of appropriative activities', *Economica*, Vol. 64, No. 253, pp.101–117.
- Swinbank, A. (1993) 'The economics of food safety', *Food Policy*, Vol. 18, No. 2, pp.83–94.
- Thomas, S. (1999) 'Economic and safety pressures on nuclear power: a comparison of Russia and Ukraine since the break-up of the Soviet Union', *Energy Policy*, Vol. 27, No. 13, pp.745–767.
- Viscusi, W.K. (1986) 'Market incentives for safety', *Harvard Business Review*, Vol. 63, No. 4, pp.133–138.

Notes

- 1 See Calow (1998), Fischhoff et al. (1981), and Jones-Lee (1989) for economic approaches to safety. Much literature focuses on public safety. See Feber et al. (2003) for the economic effects of road safety improvements, Swinbank (1993) for the economics of food safety, Thomas (1999) for economic and safety pressures on nuclear power, Rose (1990) for economic determinants of airline safety, Oi (1974, 1995) for the economics of product safety, Kotz and Schafer (1993) for economic incentives to accident prevention.
- 2 Simpler models without safety investment have been analysed in the economics and conflict literature. See e.g., Hausken (2005), Hirshleifer (2001), Grossman (1991) and Skaperdas and Syropoulos (1997).
- 3 This amounts to inserting $h = 1$ in Hirshleifer's (1995a, p.31) production function.
- 4 This assumption, caused by a need for simplicity and analytical tractability, is often realistic, but not always.
- 5 $m = \infty$ means winner takes all, $m > 1$ gives a disproportional advantage of appropriating or defending more than the other agent, $m = 1$ means proportional distribution, $0 < m < 1$ gives a disproportional advantage of appropriating or defending less than the other agent, $m = 0$ causes equal distribution between the agents, and $m < 0$ means punishing appropriating or defending.