# The battle of the sexes when the future is important

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**Abstract**

Contrary to the widespread belief that game repetition induces conciliatory behavior, in a repeated battle of the sexes where player 1 values the future and player 2 is myopic, player 1 is more inclined through conflicting behavior to risk a conflict in the present when the future is important, and/or there are many periods left in the game.

*Keywords:* Battle of the sexes; Repeated game; Discounting; Conflict; Toughness

Should players choose conflict today to reap benefits tomorrow? The results in today's literature are mixed. Folk theorem arguments (Fudenberg and Maskin, 1986) exemplified by repeating the prisoner's dilemma (Axelrod, 1984) are often taken to imply cooperation in long-term relationships. This result is often applied uncritically out of context. Skaperdas and Syropoulos (1996) equip each agent with a resource, which can be allocated into production versus arms. They show that increased importance of the future may harm cooperation.

This article considers neither the prisoner's dilemma nor resource division into production versus arms, but considers the repeated battle of the sexes in Table 1 where $a_1 \geq b_1 \geq t_1$, $b_2 \geq a_2 \geq t_2$ and where $a_1 \geq d_1$, $b_2 > d_2$, o r $a_1 > d_1$, $b_2 \geq d_2$. Does increased importance of the future induce conciliatory

Table 1
Two-person two-strategy game

| | I | II |
|---|---|---|
| I | $a_1$=4, $a_2$=3 | $t_1$=2, $t_2$=2 |
| II | $d_1$=1, $d_2$=1 | $b_1$=3, $b_2$=4 |

or conflicting behavior when players have incomplete information[1] about how tough they are? Are the players indifferent to the discount factor? Which role does the number of game repetitions play? This article answers these questions.

Players 1 and 2 have discount factors $0 \le \delta_1 \le 1$ and $\delta_2 = 0$, which means that player 2 is 100% myopic.[2] Consider $(a_1,a_2)=(4,3)$ preferred by row player 1, the starting point of our analysis, by assuming that both players have a common conjecture that they will both play their first strategy. Column player 2 has an incentive to switch strategy from I to II. Player 1 resists switching from I to II in each period with a probability of $0 \le \alpha_1 \le 1$, which expresses how tough player 1 is. If he is tough, the threat point $(t_1,t_2)$ is reached where both are worse off. Player 1 is weak with a probability of $1-\alpha_1$, and resists only when his expected payoff from resisting is larger than when acquiescing. If player 1 is weak, player 2's preferred $(b_1,b_2)=(3,4)$ is reached. Player 1 gets to know player 2's choice before he chooses.[3] In a one-period game, player 1 acquiesces to a challenge if he is weak. A weak player 2 thus challenges when:

$$(1-\alpha_1)b_2 + \alpha_1 t_2 > a_2 \Rightarrow \alpha_1 < \frac{b_2 - a_2}{b_2 - t_2}. \tag{1}$$

Player 2 is tough with a probability of $0 \le \alpha_2 \le 1$ (is 100% certain to challenge) and weak with a probability of $1-\alpha_2$ (challenges only when his expected payoff from challenging is larger than when not challenging). In a two-period game, a weak player 1 acquiesces in period 1 when:

$$t_1 + \delta_1\big((1-\alpha_2)a_1 + \alpha_2 b_1\big) < b_1 + \delta_1 b_1 \Rightarrow \alpha_2 > 1 - \frac{b_1 - t_1}{\delta_1(a_1 - b_1)}. \tag{2}$$

On the left side of the inequality in Eq. (2), the term $\delta_1\alpha_2 b_1$ follows since a weak player 1 acquiesces in period 2 to a tough player 2, which emerges with a probability of $\alpha_2$. A weak player 2 challenges in period 1 when Eq. (1) is satisfied, and acquiesces otherwise. He challenges in period 2 if player 1 acquiesces in period 1, and does not challenge in period 2 if player 1 resists in period 1. When Eq. (1) is not satisfied, the weak player 1 resists with probability 1 in period 1, and the weak player 2 does not

[1] This allows a realistic role for reputation, which presumes several players in the game, that at least one player has private information that persists over time, that this player is likely to take several actions in sequence, and that the player is unable to commit in advance to the sequence of actions he will take (Wilson, 1985, p. 29).

[2] This equivalently "corresponds to a sequence of short run players." Player 2 plays "a short-run best response in each period. The best possible commitment for 'player 1' is to the Stackelberg strategy for the corresponding static game" (Celentani et al., 1996, p. 691).

[3] The subsequent analysis is similar to Fudenberg and Tirole's (1991, pp. 369–374) analysis of the chain store game.

Table 2
Conciliatory versus conflicting behavior

| | $\alpha_2<\alpha_2(N,\delta_1)$ | $\alpha_2>\alpha_2(N,\delta_1)$ |
|---|---|---|
| $\alpha_1<\alpha_1(N)$ | Player 1 resists, player 2 challenges: $(t_1,t_2)$ | Player 1 acquiesces, player 2 challenges: $(b_1,b_2)$ |
| $\alpha_1>\alpha_1(N)$ | Player 1 resists, player 2 acquiesces: $(a_1,a_2)$ | Player 1 acquiesces, player 2 acquiesces: $(d_1,d_2)$ |

challenge in period 1. When Eq. (1) is satisfied, the weak player 1 randomizes, which requires that when player 1 resists in period 1, the weak player 2 randomizes in a way that makes the weak player 1 indifferent in his randomizing in period 1. This requires that the posterior probability that player 1 is tough, conditional on fighting, equals Eq. (1) as an equality i.e., $\alpha_1=(b_2-a_2)/(b_2-t_2)$. Defining $\beta$ as the conditional probability that a weak player 1 resists in period 1, applying Bayes' rule gives:

$$Pr(tough/resist)=\frac{Pr(tough)\cap Pr(resist)}{Pr(resist)}=\frac{\alpha_1}{\alpha_1+(1+\alpha_1)\beta}=\frac{b_2-a_2}{b_2-t_2}$$
$$\Rightarrow\beta=\frac{\alpha_1(a_2-t_2)}{(1-\alpha_1)(b_2-a_2)}. \tag{3}$$

The total probability that player 1 resists in period 1 is:

$$\alpha_1\cdot 1+(1-\alpha_1)\frac{\alpha_1(a_2-t_2)}{(1-\alpha_1)(b_2-a_2)}=\frac{\alpha_1(b_2-t_2)}{(b_2-a_2)}. \tag{4}$$

Applying analogous reasoning to that leading to Eq. (1), a weak player 2 challenges in period 1 when:

$$\left(1-\frac{\alpha_1(b_2-t_2)}{(b_2-a_2)}\right)b_2+\frac{\alpha_1(b_2-t_2)}{(b_2-a_2)}t_2>a_2\Rightarrow\alpha_1<\left(\frac{b_2-a_2}{b_2-t_2}\right)^2=\alpha_1(2), \tag{5}$$

and acquiesces otherwise. In a three-period game where $\alpha_1>((b_2-a_2)/(b_2-t_2))^2$, a weak player 1 resists a challenge and a weak player 2 does not challenge. If $((b_2-a_2)/(b_2-t_2))^3<\alpha_1<((b_2-a_2)/(b_2-t_2))^2$, a weak player 1 randomizes and a weak player 2 does not challenge. If $\alpha_1<((b_2-a_2)/(b_2-t_2))^3$, a weak player 1 randomizes and a weak player 2 challenges. In a finitely repeated game, a weak player 2 does not challenge until the first period where $N$ periods remain and:

$$\alpha_1<\left(\frac{b_2-a_2}{b_2-t_2}\right)^N=\alpha_1(N). \tag{6}$$

Hence, the size of $\alpha_1$ required to deter a challenge[4] from player 2, when $\alpha_2$ is sufficiently small, shrinks geometrically at the rate $(b_2-a_2)/(b_2-t_2)$ as $N$ increases. When $N$ is large even a very small $\alpha_1$ may deter the challenge. A weak player 1 deters the challenge when[5]

[4] If $(a_1,a_2)=(4,3)$ represents a monopoly situation for player 1, the term "entry deterrence" expresses that player 2 does not challenge.

[5] Analogously to (9.2) player 1 receives $b_1$ if player 2 is tough in period $N$, and $t_1$ if player 2 is tough in periods 2 to $N-1$. See Fudenberg and Tirole (1991, pp. 373-381) for mixed-strategy reputations.

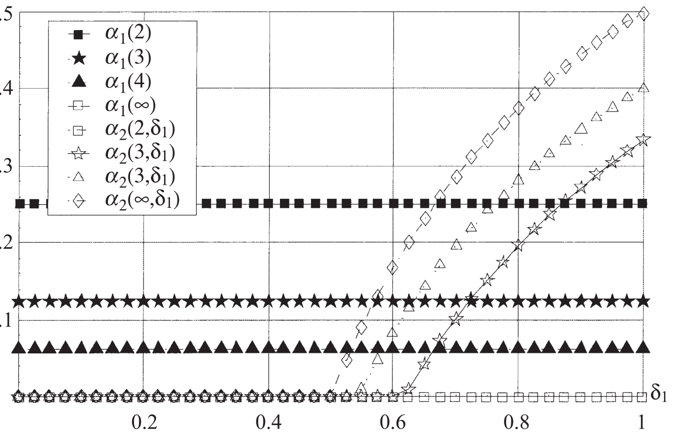


Fig. 1. $\alpha_1(N)$ and $\alpha_2(N,\delta_1)$ as functions of $\delta_1$ for various $N$.

$$t_1 + \delta_1((1-\alpha_2)a_1 + \alpha_2 t_1)\left(\frac{1-\delta_1^{N-2}}{1-\delta_1}\right) + \delta_1^{N-1}((1-\alpha_2)a_1 + \alpha_2 b_1) > b_1\left(\frac{1-\delta_1^{N}}{1-\delta_1}\right)$$

$$\Rightarrow \alpha_2 < \frac{a_1\delta_1\left(1-\delta_1^{N-1}\right) - b_1\left(1-\delta_1^{N}\right) + t_1(1-\delta_1)}{\delta_1\left[a_1\left(1-\delta_1^{N-1}\right) - b_1\delta_1^{N-2}(1-\delta_1) - t_1\left(1-\delta_1^{N-2}\right)\right]} = \alpha_2(N,\delta_1), \quad N \geq 2. \tag{7}$$

Eqs. (6) and (7) establish, uniquely dependent on $\alpha_1$ and $\alpha_2$, how the game is played and which payoffs ($a_1$,$a_2$), ($t_1$,$t_2$), and ($b_1$,$b_2$) accrue to the players in each period. Player 2 challenges when player 1's toughness $\alpha_1$ is low. Player 1 resists when player 2's toughness $\alpha_2$ is low (see Table 2).

Figs. 1 and 2 illustrate with numbers in Table 1. $\alpha_1(N)$ is independent of $\delta_1$, and decreases in $N$. Player 2's challenge does not depend on $\delta_1$ (due to myopia). When many periods $N$ remain, player 2 does not challenge even when $\alpha_1$ is small, although decreasing $\alpha_1$ further causes player 2 to challenge. $\alpha_2(N,\delta_1)$

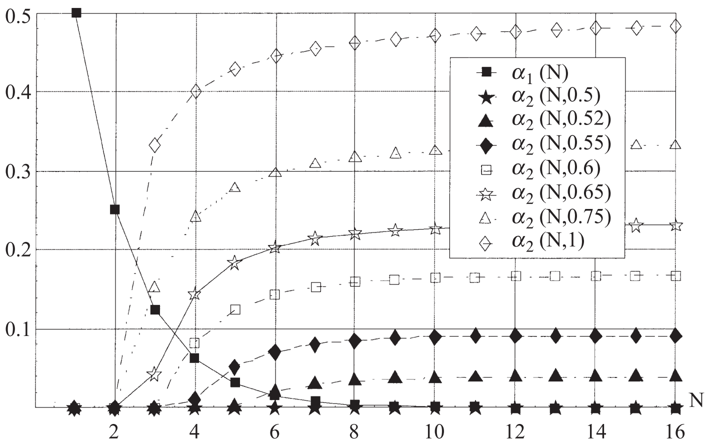


Fig. 2. $\alpha_1(N)$ and $\alpha_2(N,\delta_1)$ as functions of $N$ for various $\delta_1$.

increases in both $N$ and $\delta_1$. Player 1 resists the challenge when $\alpha_2$ is low. As $N$ or $\delta_1$ increases, player 1 resists the challenge even when $\alpha_2$ is high, but not when too high.

The equilibrium strategies are: player 2 either challenges always ($\alpha_1$ is low), or starts the challenge in period 2 or thereafter ($\alpha_1$ is intermediate), or challenges never ($\alpha_1$ is high). Player 1 either resists always ($\alpha_2$ is low), or resists halfway through the game and thereafter acquiesces ($\alpha_2$ is intermediate), or acquiesces in all periods ($\alpha_2$ is high).

Eq. (7) expresses that for a given $\alpha_2$, player 1 is more inclined to resist a challenge when the future is important ($\delta_1$ is large) and the game has many periods ($N$ is large). The myopic player 2 is, for a given $\alpha_1$, more inclined to challenge the fewer periods $N$ left of the game. Inserting $N$=2 and $N$=$\infty$ into Eq. (7) gives Eqs. (2) and (8), respectively. When $N$=$\infty$, player 1 deters the challenge in an infinitely repeated game when:

$$t_1 + \delta_1 \frac{(1-\alpha_2)a_1 + \alpha_2 t_1}{1-\delta_1} > \frac{b_1}{1-\delta_1}$$
$$\Rightarrow \alpha_2 < 1 - \frac{b_1 - t_1}{\delta_1(a_1 - t_1)} = \alpha_2(\infty, \delta_1) \Leftrightarrow \delta_1 > \frac{b_1 - t_1}{(1-\alpha_2)(a_1 - t_1)}. \tag{8}$$

Summing up, player 1 is more inclined through conflicting behavior and deterrence to risk a conflict in the present when the future is more important ($\delta_1$ is large), and/or there are many periods $N$ left in the game, given that player 2 is sufficiently inclined not to challenge ($\alpha_2$ is small).

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