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Cooperation and between-group competition

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Abstract

Introducing competition between groups may induce cooperation to emerge in defection games despite considerable cost of cooperation. If the groups can confine themselves to a cooperative sector, either by providing incentives to raise the cooperation level in one group, or by providing disincentives so that the cooperation level in the other group gets lowered to match that of the first, maximum degrees of cooperation can be obtained. The cooperative sector broadens as the degrees of cooperation increase, or the cost of cooperation decreases, or the group benefits of cooperation increase.

1. Introduction

The article illustrates cooperation against all odds. Imagine a group rigged such that defection is inevitable. Introducing a second group and specifying conventional competition between the groups may imply that within-group cooperation nevertheless is possible. Two-level analysis involves drawing upon ideas from collective rent seeking,[1] the analysis of the impact of product–market competition on managerial slack,[2] and the analysis of conflict between actors.[3] Each agent makes an individual decision of whether to cooperate or defect, mediated through the within/between-group structure of the model.

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[1] Katz et al. (1990), Nitzan (1991, 1994), Hausken (1995a, b, 1998), Lee (1995), Baik and Lee (1997), Rapoport and Amaldoss (1997).
[2] Winter (1971), Hart (1983), Tirole (1988: 46–47), Horn et al. (1995), Vickers (1995).
[3] Hirshleifer (1995).

2. The model

In each of two groups with n_1 and n_2 agents, an agent can choose to cooperate through incurring a cost c of effort, or to defect incurring no cost of effort. h_i cooperators in group i produce an amount h_iB_i of payoffs (products, goods, outcomes, prizes, benefits, or rewards), where B_i is the productive efficiency, i=1, 2. We assume that h_iB_i, and not the aggregate effort h_ic, is used as input in the between-group competition for group i's eventual payoff. This is so because c may be utilized differently in the two groups if the group characteristics expressed by the productive efficiencies B_1 and B_2 are different.[4] The total amount $h_1B_1+h_2B_2$ of payoffs is placed in a common pool which the two groups compete for according to the conventional ratio form (Tullock, 1967) with $(h_1B_1)^m$ and $(h_2B_2)^m$ as input, where m is a parameter.[5] Payoffs acquired by each group are distributed equally on the group members. A cooperator j in group 1 receives a payoff

$$P_{1j}(S^{-1j}, c) = \frac{1}{n_1}\frac{(h_1B_1)^m}{(h_1B_1)^m + (h_2B_2)^m}[h_1B_1 + h_2B_2] - c, \tag{1}$$

where S^{-1j} is the set of strategies by all the n_1-1+n_2 agents in the two groups except agent j in group 1 who chooses to cooperate. If agent j decides to defect rather than to cooperate, there will be h_1-1 cooperators in group 1 giving agent j a payoff

$$P_{1j}(S^{-1j}, 0) = \frac{1}{n_1}\frac{((h_1-1)B_1)^m}{((h_1-1)B_1)^m + (h_2B_2)^m}[(h_1-1)B_1 + h_2B_2]. \tag{2}$$

The payoffs to an agent in group 2 are found by permuting the indices in (1) and (2).

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[4] An example considered by Hausken (1995a: 471) is a two-island tax system where the within-group efforts are used to invest in 'social welfare, cultural training, military training and equipment, and so on,' all of which are relevant for how the group succeeds in the between-group struggle with the other group. The effort by an agent is thus not devoted directly to the between-group competition, but to the 'within-group machinery,' which may be efficient (B_i is large) or inefficient (B_i is small) in utilizing it in the between-group competition. For example, if the overall strategy, culture, equipment, or training in one group are lacking, insufficient, or inadequate, it may not matter much whether each agent cooperates because a mechanism at the group-level is not able to utilize the cooperation, which corresponds to a smaller B_i for this group.

[5] For unitary actors, Hirshleifer (1995) interprets m as a 'decisiveness parameter,' while Tullock (1980) and Nitzan (1994: 44) interpret it as 'the marginal return to lobbying outlays'. m>1 gives a disproportional advantage to group i of producing more payoffs h_iB_i than the other group, which implies that payoffs are transferred to group i, which can be interpreted as exploiting benefits from economies of scale. m<1 gives a disproportional advantage to each group of producing less payoffs than the other group. For the special case that m=1, n_1=n_2, B_1=B_2, there is no transfer of payoffs between the groups. The groups then do not appropriate each others' internally generated payoffs, and operate as if in isolation from each other. m=0 causes equal distribution of payoffs between the groups. m<0 means punishing cooperation and placing a premium on defection, which is not considered here. Consider three interpretations of m, one economic/industrial, one political, and one military. First, a low m for industrial imperiums, companies, business firms, enterprises, means that each group can defend itself easily. This can be due to stable market conditions where neither group has an incentive or opportunity to get the upper hand in the competition, where the groups have divided the market geographically or according to target consumer groups, or where heavy sunk costs in production technology, procedures, personnel training, marketing strategies etc. hamper the way in which the groups can change their interference with each other, e.g. through the entering of new markets and employment of new strategies. Second, a low m for some political groups or collective entities in a democratic constitution means wide separation of powers, bills of right, capacities, endowments, and legal entitlements among the groups, which 'reduce the decisiveness of majority supremacy, thereby tending to moderate the intensity of factional struggles. If the political system were winner take all, decisiveness m would be very high and all politics would be a fight to the death' (Hirshleifer, 1995: 32–33). Third, as Hirshleifer (1995: 32) points out, "in military struggles, low m corresponds to the defense having the upper hand. On the western front in World War I, entrenchment plus the machine gun made for very low decisiveness m. . . But in World War II, the combination of airplanes, tanks, and mechanized infantry allowed the offense to concentrate firepower more rapidly than the defense, thus intensifying the effect of force superiority."

3. Equilibrium analysis

Agent j in group 1 cooperates rather than defects when $P_{1j}(S^{-1j}, c) > P_{1j}(S^{-1j}, 0)$, which by inserting (1) and (2), gives

$$c < \frac{1}{n_1}\frac{(h_1B_1)^m[h_1B_1 + h_2B_2]}{(h_1B_1)^m + (h_2B_2)^m} - \frac{1}{n_1}\frac{((h_1-1)B_1)^m[(h_1-1)B_1 + h_2B_2]}{((h_1-1)B_1)^m + (h_2B_2)^m} = c_{\mathrm{r}}. \quad (3)$$

The analogous requirement for group 2 is found by permuting the indices. (3) can also be expressed as $c < c_{\mathrm{r}}(h_1, h_2, B_1, B_2, m, n_1)$ which is a necessary and sufficient condition for agent j in group 1 to cooperate with his fellow group members. The key equilibrating variables of interest are h_1 and h_2. We first determine h_1 for group 1 assuming h_2, B_1, B_2, m, n_1 as fixed. We secondly determine the overall equilibrium h_1 and h_2 for both groups, assuming B_1, B_2, m, n_1, n_2 as fixed. We thirdly carry out comparative statics of h_1 and h_2.

When (3) is satisfied so that $c < c_{\mathrm{r}}$ for a given number h_1 of cooperators in group 1, then the marginal agent j for whom the condition is being evaluated wishes to cooperate; of course, no current cooperator wishes to defect. Given that agent j cooperates, h_1 has now increased with 1, and we may ask whether another current non-cooperator wishes to switch to cooperation. So long as the inequality is maintained, the current non-cooperators wish to become cooperators. Thus, we can imagine a one-by-one process whereby the number of cooperators increases until either a value of h_1 is reached at which $c = c_{\mathrm{r}}$ (treating h_1 as real here), or else $h_1 = n_1$ is reached. On the other hand, if we begin at $c > c_{\mathrm{r}}$, then the opposite occurs. Current cooperators wish to switch to defection, and we can again imagine a one-by-one process whereby they do so until either a value of h_1 is reached at which $c = c_{\mathrm{r}}$ or $h_1 = 0$.

Property 1. *When the status (including a possible non-equilibrium situation) within group 2 is taken as given, a Nash equilibrium in cooperation/defection strategies for the members of group 1 is a value of h_1 such that either $h_1 = 0$ and $c = c_{\mathrm{r}}$ (an all-defection stable equilibrium); or $h_1 = n_1$ and $c < c_{\mathrm{r}}$ (an all-cooperation stable equilibrium); or $0 < h_1 < n_1$ and $c = c_{\mathrm{r}}$ (an interior stable or unstable equilibrium).*

Property 1 for group 2 is found by permuting the indices. To throw light on Property 1, assume $B_1 = B_2 = n_1 = n_2 = 1000$ agents in the two groups and that m takes on seven values in the range $0 \leq m \leq 7$. Given $h_2 = 400$ cooperators in group 2, c_{r} for group 1 is given in Fig. 1.

The familiar case of Fig. 1 is $m = 1$ which gives pure cooperation by all agents when $c < 1$ $\forall h_i \geq 0$ (necessary and sufficient requirement), and a prisoner's dilemma and pure defection when $1 = B_i/n_i < c < B_i = 1000$, $i = 1, 2$. Any value c_{r} takes on the above $c_{\mathrm{r}} = 1$ and makes the between-group model interesting. For $m < 1$, the requirement for cooperation is more lenient than $c < 1$ for $h_1 \approx 0$, as the very first agents to cooperate may increase their payoff above

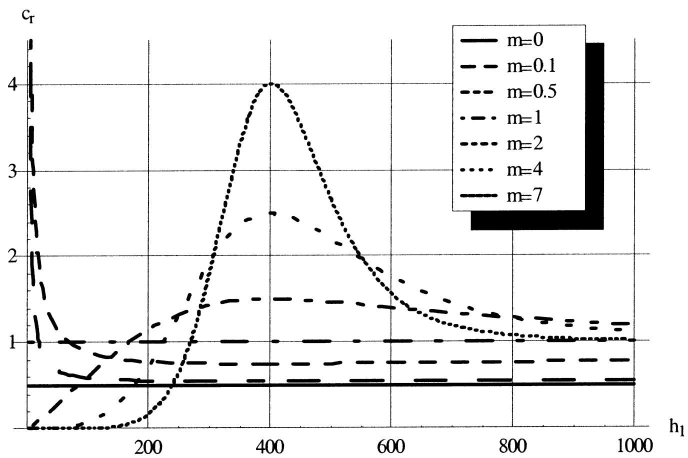

Fig. 1. Requirement $c<c_r$ as a function of h_1 for h_2=400 for seven values of *m*.

0. c_r approaches asymptotically a stricter requirement below 1 as h_1 increases. For $m>1$, h_1 considerably lower than $h_2=400$ causes a strict requirement for c because the benefits from cooperation by agent *j* get expropriated by group 2. When the payoff production in the two groups is similar, $h_1B_1\approx h_2B_2$, which gives $h_1\approx h_2$ when $B_1=B_2$, the requirement $c<c_r$ is lenient, inducing agent *j* to cooperate even at considerable cost c. This is illustrated by $c<4$ for $h_1=h_2$ and $m=7$. As $h_1>h_2$, the incentives for agent *j* to free-ride increases if c is high. To illustrate the three different cases of Property 1, Fig. 2 replicates the curve for $m=7$ from Fig. 1, considering three different values of c; $c=c^h$, $c=c^m$, and $c=c^l$.

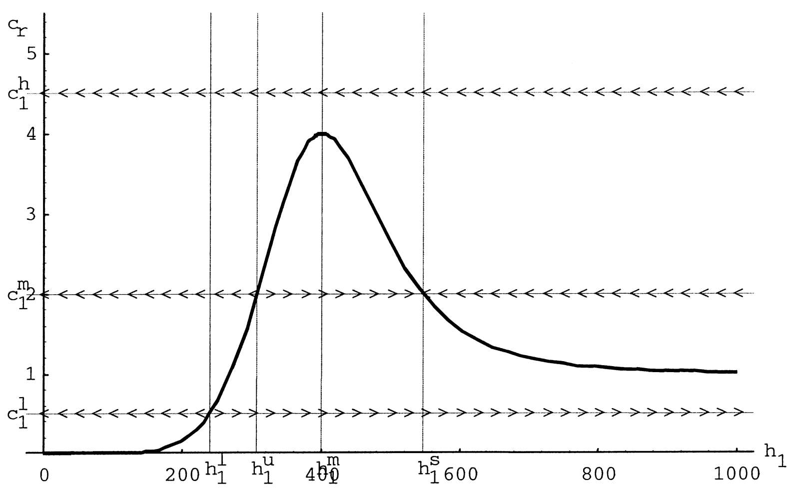

Fig. 2. Equilibrium values of h_1 for h_2=400 and m=7.

The all-defection stable equilibrium in Property 1 giving $h_1=0$ occurs when c is sufficiently high and/or there are considerably fewer cooperators h_1 in group 1 than in group 2. The necessary condition is $c>c_r$. For $c=c^h$, this happens for $\forall h_1$, for $c=c^m$, it happens for $h_1 < h_1^{\mathrm{u}}$, and for $c=c^{\mathrm{l}}$, it happens for $h_1 < h_1^{\mathrm{l}}$. The all-cooperation stable equilibrium in Property 1 giving $h_1=n_1$ occurs when c is low and there initially are many cooperators h_1 in group 1. The necessary condition is $c<c_r$. For $c=c^h$ and $c=c^m$, this never happens, and for $c=c^{\mathrm{l}}$, it happens for $h_1>h_1^{\mathrm{l}}$. The interior equilibrium in Property 1 giving $0<h_1<n_i$ occurs when $c=c_r$. For $c=c^h$, this never happens, for $c=c^m$, it happens for $h_1 = h_1^{\mathrm{u}}$ (unstable equilibrium) and $h_1 = h_1^{\mathrm{s}}$ (stable equilibrium), and for $c=c^{\mathrm{l}}$, it happens for $h_1 = h_1^{\mathrm{l}}$ (unstable equilibrium). Assume $c=c^m$. When $0 < h_1 < h_1^{\mathrm{u}}$, any cooperator wishes to defect and no current defector wishes to cooperate, so h_1 falls to 0. At $h_1 = h_1^{\mathrm{u}}$, no member has an incentive to switch in any direction. For $h_1^{\mathrm{u}} < h_1 < h_1^{\mathrm{s}}$, each current defector wishes to cooperate, and no cooperator wishes to defect, so h_1 increases to h_1^{s}, at which point no further incentive exists for either a cooperator or a defector to switch. Finally, for $h_1 > h_1^{\mathrm{s}}$, each current cooperator has an incentive to defect, pushing h_1 back to h_1^{s}. Observe that all stable interior equilibria has $\partial c_r/\partial h_i \leq 0$. The Nash equilibrium solution for the stable interior equilibrium in groups 1 and 2 can be written as

$$h_1^{\mathrm{s}} = h_1^{\mathrm{s}}(h_2, B_1, B_2, m, n_1, c) \quad \text{and} \quad h_2^{\mathrm{s}} = h_2^{\mathrm{s}}(h_1, B_1, B_2, m, n_2, c), \tag{4}$$

respectively. The overall Nash equilibrium for the agents in the two groups is given by the simultaneous solution of the two equations in (4), which gives

$$h_1^{\mathrm{o}} = h_1^{\mathrm{o}}(B_1, B_2, m, n_1, n_2, c), \qquad h_2^{\mathrm{o}} = h_2^{\mathrm{o}}(B_1, B_2, m, n_1, n_2, c). \tag{5}$$

For the example above where $h_2=400$, $m=7$, and $c^m=2$, the stable interior equilibrium is $h_1^{\mathrm{s}} \approx 547$, and the unstable interior equilibrium is $h_1^{\mathrm{u}} \approx 307$. To determine the overall equilibrium h_1^{o} and h_2^{o}, we need to determine four curves: first, the stable equilibrium value $h_1^{\mathrm{s}} = h_1^{\mathrm{s}}(h_2, \cdot)$ for all h_2, $0\leq h_2\leq 1000$; second, the unstable equilibrium value $h_1^{\mathrm{u}} = h_1^{\mathrm{u}}(h_2, \cdot)$ for all h_2, $0\leq h_2\leq 1000$; third, the stable equilibrium value $h_2^{\mathrm{s}} = h_2^{\mathrm{s}}(h_1, \cdot)$ for all h_1, $0\leq h_1\leq 1000$; fourth, the unstable equilibrium value $h_2^{\mathrm{u}} = h_2^{\mathrm{u}}(h_1, \cdot)$ for all h_1, $0\leq h_1\leq 1000$. These are shown in Fig. 3.The interesting part of Fig. 3 is the 'cooperative sector' spanned out by the thick unstable equilibrium curves $h_1^{\mathrm{u}} = h_1^{\mathrm{u}}(h_2, \cdot)$ and $h_2^{\mathrm{u}} = h_2^{\mathrm{u}}(h_1, \cdot)$. If the groups confine their initial and subsequent location (h_1, h_2) to the cooperative sector, they inevitably get propelled to the overall cooperation equilibrium $(h_1, h_2)=(1000, 1000)$.[6] Conversely, if the groups confine their initial and subsequent location outside the cooperative sector, they move to $(h_1, h_2)=(0, 0)$. This means that an overall stable internal Nash equilibrium for the two groups does not exist.

Property 2. *Assume two equivalent groups where* $B_1 = B_2$ *and* $n_1 = n_2$. *When* $c > c_r \forall h_i$, $0 \leq h_i \leq n_i$, *there exists one unique overall Nash all-defection equilibrium* $(h_1^{\mathrm{o}}, h_2^{\mathrm{o}}) = (0, 0)$. *When* $c < c_r$ *for at least one* h_i, *there exist two overall Nash equilibria. The first*

[6] Consider a random point within the cooperative sector. For any given value of h_2, a defector in group 1 will switch to cooperation, increasing h_1, and no cooperator will switch to defection. Analogously, for any given value of h_1, a defector in group 2 will also switch to cooperation, increasing h_2, and no cooperator will switch to defection. The groups will thus inch up on each other, eventually reaching $(h_1, h_2)=(1000, 1000)$.

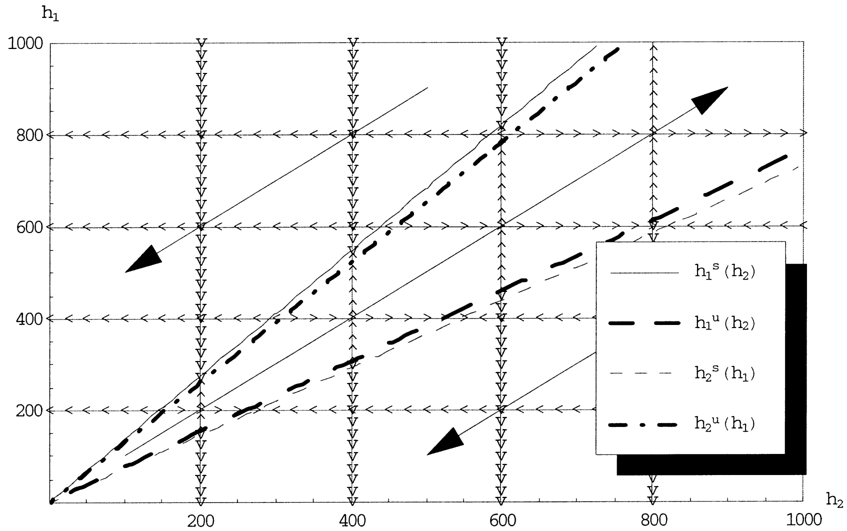

Fig. 3. Mutual reaction curves $h_1^s = h_1^s(h_2, \cdot)$, $h_2^s = h_2^s(h_1, \cdot)$, $h_1^u = h_1^u(h_2, \cdot)$, $h_2^u = h_2^u(h_1, \cdot)$.

is $(h_1^o, h_2^o) = (n_1, n_2)$ *and is reached if* h_1 and h_2 *throughout the equilibrating process lie within the cooperative sector spanned out by* $h_1^u = h_1^u(h_2, \cdot)$ and $h_2^u = h_2^u(h_1, \cdot)$. *If confinement to the cooperative sector can not be obtained, the overall Nash all-defection equilibrium* $(h_1^o, h_2^o) = (0, 0)$ *is reached.*

Property 2 has implications for how the rigging, monitoring, and external regulation of competing groups can affect strategic behavior within groups. Maximum degrees h_1 and h_2 of cooperation can be obtained by matching the cooperation levels in the groups with each other, either by providing incentives to raise the cooperation level in one group, or by providing disincentives so that the cooperation level in the other group gets lowered to match that of the first. This ensures a transition into the cooperative sector. If h_1 and h_2 initially are unequal, or the internal dynamics or speed for switching from defection to cooperation is different, movement out of the cooperative sector may occur giving $(h_1, h_2)=(0, 0)$. The exactness by which the cooperation levels in the groups are matched is more important the lower are h_1 and h_2, as indicated by the cooperative sector being narrower for low cooperation levels. Conversely, as h_1 and h_2 increase, the two groups' capacity for mutual cooperation becomes more stable to parameter fluctuations. Hence, Property 2 may still hold when $B_1 \neq B_2$ or $n_1 \neq n_2$.

W.r.t. comparative statics, the cooperative sector for given h_1 and h_2, broadens as c declines, and narrows to the line $h_1=h_2$ as c increases to the maximum value of c where $c=c_r$ has a unique solution. This happens for $h_1=h_1^m$ and corresponds to the mountain top in Fig. 2. For $c>c_r$, there is no cooperative sector and the equilibrium $(h_1^o, h_2^o) = (0, 0)$ is inevitable. Increasing B_1 and B_2 has a similar effect as decreasing c since one group in isolation has prisoner's dilemma characteristics when $1=B_i/n_i<c<B_i=1000$, $i=1, 2$. Increasing m has an effect as can be seen from Fig. 1. First, if c is low, increasing m

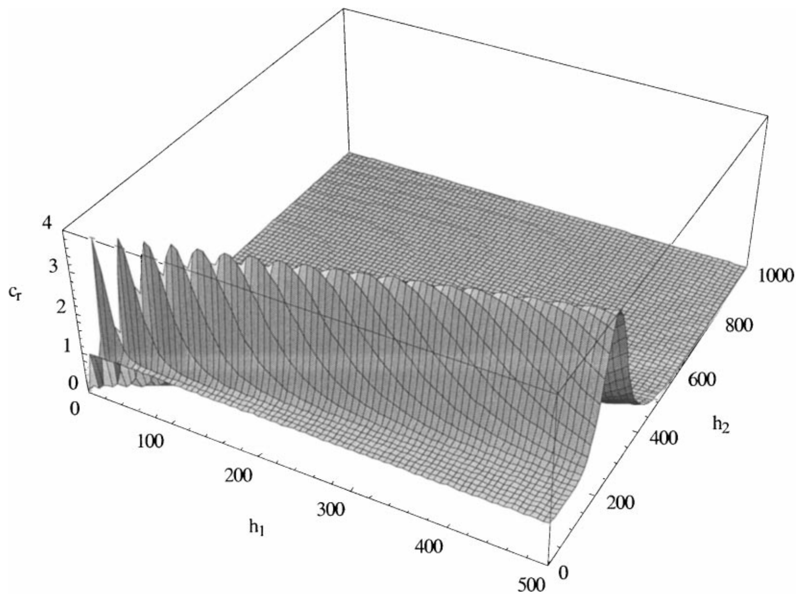

Fig. 4. c_r as a function of h_1 and h_2 for m=7, B_1=n_1=500, B_2=n_2=1000.

may imply the presence of a cooperative sector. Second, if m is too high, giving fierce between-group competition, the cooperative sector becomes narrower, making a mutual cooperation equilibrium unstable if other parameters fluctuate too much.

As an example of an asymmetry, let $B_1=n_1=500$ and $B_2=n_2=1000$.[7] Fig. 4 plots c_r as a function of h_1 and h_2 for m=7.[8]

Fig. 4 illustrates how the stable and unstable equilibria change, while being dependent on c, and how a diagonal mountain ridge in a symmetric case changes to a translate which is such that $\max(c_r)$ occurs for $h_2 \approx 250$ when $h_1=n_1=500$. Hence, the larger efficiency of production or larger group size $B_2=n_2=1000$ in group 2 must be accompanied with a smaller cooperation level h_2 in group 2 to facilitate the initiation and increase in cooperation in group 1, if c s.t. $c<c_r$ is high. If this is satisfied, the two groups move to an equilibrium with a maximum degree $h_1=n_1$ of cooperation in the smaller or less efficient group 1, and a lower degree $h_2<n_2$ of cooperation in the larger or more efficient group 2. For the smaller group 1, the payoff to each of the $h_1=n_1=500$ cooperators is $P_{1j}(S^{-1j}, c) \approx 502-c$ where

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[7] We have considered B_i proportional to n_i, which is often realistic and means that the benefits reaped by one agent do not reduce the benefits received by another agent. An alternative is to consider B_i as a constant, which means that the amount of payoffs produced by a cooperative act is divided between the group members, giving smaller share to each as n_i increases. With proportionality between B_i and n_i, varying B_i or n_i for the groups has similar effects, where we focus on varying B_i.

[8] The 'mountain ridge' in Fig. 4 is continuous and has no isolated tops, the latter being due to the resolution of the Mathematica software package used to generate the plots (PlotPoints→80). The resolution can be made arbitrarily good, but then it becomes more difficult to read the landscape. The mountain ridge is especially narrow and knife-edge sharp when h_1 and h_2 are small.

$c < c_r = 4$. For the larger group 2, the payoff to each agent v of the $h_2 \approx 250$ cooperators (adjusted in an equilibrium manner to the nearest whole number) is $P_{2v}(S^{-2v}, c) = 249 - c$ where $c < c_r = 4$, and the payoff to each defector is $P_{2v}(S^{-2v}, 0) = 245$.

Property 3. *When group 1 is smaller or less efficient than group 2, the two groups move to an equilibrium with a maximum degree $h_1 = n_1$ of cooperation in group 1, and a lower degree $h_2 < n_u$ of cooperation in group 2. The payoff to each cooperator in the smaller group 1 is larger than the payoff to each agent (cooperator or defector) in the larger group 2.* [9]

4. Conclusion

Cooperation may emerge in defection games if competition between groups is introduced and the degrees of cooperation in the groups are sufficiently matched to fall within a cooperative sector. If the groups gradually inch up on each other within a cooperative sector, no group falling behind or ahead of the other group, maximum degrees of cooperation are obtained. This may occur through providing incentives for cooperation in the least cooperative group, or providing disincentives for cooperation in the most cooperative group. A crucial point is how to get cooperation started since the cooperative sector is narrow for low degrees of cooperation. The cooperative sector broadens as the degrees of cooperation increase, or the cost of cooperation decreases, or the group benefits of cooperation increase.

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[9] Finally, note that intergroup mobility generally has a negative impact on cooperation, especially with low switching costs, because transfer of agents to the other group reduces the beneficial effect of the between-group competition. Cooperation may occur theoretically in four different but unlikely cases. The first is if the parameters and initial conditions in the groups are equivalent. The second is if the parameters and initial conditions are different but 'counteracting' each other through time such that no group is eventually more attractive than the other group. Third, intermediate degrees of cooperation can be sustained if a situation occurs and persists through time where no one has an incentive to change strategy nor to switch group. Fourth, stable exhaustive cooperation can be attained if one group absorbs the members of the other group and sustains cooperation as an equilibrium through being endowed with beneficial structural parameters.

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