THE STABILITY OF ANARCHY AND BREAKDOWN OF PRODUCTION

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In Hirshleifer's (1995) model for unitary actors, combined fighting/production abruptly breaks down when inter-group decisiveness of fighting is above a certain value (above one) or income requirements are not met. Accounting for the collective action problem, this article gives the opposite result that fighting/production is stable also for large decisiveness parameters (above one) and strict income requirements for each agent. The stable fighting/production equilibrium gets gradually easier to perturb off balance for high inter-group decisiveness, high costs of fighting, different fighting efficiencies, and equal group sizes. The equilibrium number of groups that can be sustained decreases in the inter-group decisiveness and increases in the cost of fighting.

Keywords: Production; Fighting; Anarchy; Peace; Collective action

INTRODUCTION

In an interesting contribution entitled 'Anarchy and its Breakdown', Jack Hirshleifer (1995) defined anarchy as a non-chaotic spontaneous order in which participants can seize and defend resources without regulation from above. Each contestant balances between productive exploitation of the current resource base and fighting to acquire or defend resources. He found that anarchy is sustainable only when the 'decisiveness parameter' is sufficiently low (less than one), which means that there are strongly diminishing returns to fighting effort, and when incomes exceed a viability minimum.[1] This may occur when resources are defendable, predictable, and dispersed. When decisiveness is larger than one, anarchy becomes dynamically unstable. This may occur when resources are concentrated, which may lead to 'winner-take-all' battles, and dictatorship by the strongest. As the population increases, fighting spreads and incomes fall below the viability limit. Hirshleifer (1995: 26, 48ff) exemplifies anarchy, with dispersed resources, with international struggles for control of the globe's resources, gang warfare in prohibition-era Chicago, miners versus claim jumpers in the California gold rush, the era prior to the introduction of cannons and modern fortifications in the 15th century, animal territoriality, and male elephant seals who fight to sequester 'harems' of females.

[1] In the ratio form of the Contest Success Function the decisiveness parameter is the exponent to which each effort is raised. When the exponent is zero, efforts are irrelevant and distribution is egalitarian. When the exponent is one, distribution is proportional to effort. When the exponent is infinite, winner takes all.

A basic assumption in Hirshleifer's (1995: 27) analysis is to 'treat [competing] groups as unitary actors that have somehow managed to resolve the collective action problem [among themselves].' But the collective action problem ought not be assumed away. The purpose of this article therefore is to incorporate the conflict inherent within competing groups into Hirshleifer's model. The article analyzes for groups (firms, nations, organizations, multiplicities, collectivities) the trade-off between peaceful production and fighting (such as interference struggles, strikes, lockouts, litigation, rent-seeking, or other non-productive self-serving activities). Throughout history, peaceful production has been an ideal sought by many. Firms want their employees to produce so the firm can flourish. Nations want their inhabitants to be productive to increase output, welfare and power. Alas, there is an inherent instability to universal production. Agents on all sides fight to appropriate the production of others and defend against theft by others. This article determines the balance between production and fighting for groups dependent on a variety of parameters.

For monolithic groups, earlier work has determined the equilibrium fractions of actors who specialize in production versus fighting (e.g. Grossman & Kim, 2000). Other work has determined how each of two unitary actors balances between fighting effort and productive effort.[2] This article incorporates elements of the former models in assuming that agents specialize in either production or fighting. It is related to the latter models in assuming that all production goes into a common pool that is then distributed between groups (and the agents comprising those groups) by fighting. The main conceptual innovation is to let each agent pursue one of two occupations. Each agent specializes either in production or fighting, making a decentralized choice strictly to advance his own private welfare since there is no decision-making authority at the group level.

The introduction of groups and analysis of internal incentive structures and decision processes of its members is empirically interesting and an essential analytical innovation for several reasons. Collective action among members within a group is essential to group behavior and to the outcome of competition among groups. It can seldom be abstracted away, and accounting for it generally gives different results. Groups are central to many economic phenomena (e.g. collective rent seeking[3], collective goods, joint interaction within and between countries, etc) and no less so with respect to production versus fighting. Group size *per se* is an important factor in the outcome of conflict in addition to the equilibrium fractions of producers and fighters, while group heterogeneity realistically allows for a diversity of agents in each group. Groups exist in the real world and should be modeled as such to capture the richness of within-group and inter-group phenomena. Moreover, as this article demonstrates, decisiveness in inter-group fighting leads to understanding the complex dynamic stability conditions in the equilibrium fractions of producers and fighters in each group.

This article bears particular relationship to two earlier advances by Hausken (2000a) and Hirshleifer (1995). In Hausken's (2000a) analysis, the degree of cooperation (in public goods production) within a group determines how successfully it competes with another group. If one group has many more cooperators than another, its inter-group cooperation is deemed successful. But free-rider incentives will reduce the number of cooperators. In a group with too few cooperators, inter-group cooperation is unsuccessful, and no one wants to be the first to initiate cooperation. The result is universal defection where everyone free-rides.

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[2] See Hausken (2005), Hirshleifer (1988, 1991ab, 1995), Skaperdas (1992), Skaperdas and Syropoulos (1997). When reduced to certain minimal characteristics, there is an isomorphism between a model with a 'yes/no dichotomy' on the part of each agent choosing one extreme or the other where the numbers of each type come into a balanced equilibrium, and a model in which the typical agent balances between fighting effort and productive effort assuming a scaled choice along a continuum. In the aggregate the models operate analogously. For the latter models many actors are necessary, while for the former two actors are sufficient.

[3] Hausken (2005); Katz *et al*. (1990); Nitzan (1991, 1994).

This article assumes a different incentive structure and strategy set for each agent. Here, (in contrast to Hausken, 2000a) a cooperator contributes, at a cost, to his group's fighting effort, while a defector contributes only to production. Our rationale is that in the real world, fighting is usually costlier and riskier than producing. So in this model someone who does not incur the cost of fighting is a 'free rider' – he is depending upon his neighbors to protect him against criminals and other outside predators. With no cooperators in a group, no one fights for the group and its members receive zero income or product. Similarly, with no defectors, there is no production, which gives non-positive income. This article determines the conditions under which these two extreme equilibria and combined intermediate cooperation/defection equilibria are obtainable. We further illustrate non-equilibrium behavior dependent on the cost of fighting, decisiveness of competition, the number of agents in each group, and the fighting and productive efficiencies.

My approach here takes its inspiration from, and is greatly indebted to, Hirshleifer's (1995: 26) analysis of anarchy defined not as 'chaos' but rather 'as a spontaneous order' where 'each contestant balances between productive exploitation of the current resource base and fighting to acquire or defend resources.' Assuming unitary actors, Hirshleifer (1995: 33) derives two conditions under which anarchy breaks down. The first is that 'an excessively large decisiveness parameter m (i.e. $m > 1$) leads to dynamic instability; that is, movement toward a corner solution' where one actor gets the entire product, and the other gets nothing. 'A second source of breakdown is income inadequacy. Suppose that some minimum income y is required to sustain life for an individual actor or for a group to preserve its institutional integrity. Then anarchy cannot be stable if the equilibrium of the dynamic process implies income $Y_i < y$ for either contender' (Hirshleifer, 1995: 33). Incorporating the collective action factor into the problem, this article also derives the impact of the decisiveness parameter, and of income adequacy. The two models supplement each other, one assuming unitary actors, and one accounting for collective action.

The fighting/production equilibrium analyzed in this article does not correspond to Adam Smith's division of labor, where fighting is absent. The equilibrium corresponds better with Hobbes' state of nature with 'war of each against all.' Insights from the two philosophies are useful for a theory of the division of labor between production and fighting.[4]

The next section presents the model. The section after analyzes the equilibrium fractions of producers and fighters in each group. The fourth section considers the effects of changes in the decisiveness parameter and cost of fighting, while the fifth section compares the results with Hirshleifer's. The sixth section generalizes to n groups, and describes the maximum number of groups that can be sustained given that each agent within each group has to meet a Malthusian survival level of income. The seventh section concludes.

THE MODEL

In each of two groups, with n_1 and n_2 members, an agent who chooses to 'fight/cooperate' must incur a cost c of fighting effort. If he chooses 'not-to-fight/to-defect/to-produce', he does not incur this cost. By assumption, an agent cannot be involved in both activities. A defector contributes only to production. Production of both groups is placed in a common pool and has to be fought for. Fighting is interpreted as cooperation since if no agents in a group fight for

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[4] Hirshleifer (1994) presents in a Presidential Address a related argument for the need of a dual philosophy where both Coases's and Machiavelli's theorems are natural ingredients.

the group, the group earns zero income. Thus, a cooperator contributes to the group fighting effort, and not to production. In the real world, fighting is often costlier (riskier) than producing. Not incurring the cost of fighting amounts to free-riding with respect to fighting. The cost c represents physical hardship, risk and stress associated with fighting and preparedness, the cost of injuries, expenses for acquiring the skills, tactics, and strategies of fighting, and expenses for equipment and the build-up of a fighting infrastructure. We define h_1-1 as the number of 'other' agents in the group who are fighting, q_1 is the number of 'other' agents who are producing. Letting 1 stand for the individual himself ('Ego'=Agent j), it follows that $n_1=h_1-1+q_1+1$. Fighting (cooperation) by agent j gives h_1 fighters (cooperators) in group 1. Aggregate production of the two groups is given by $(n_1-h_1)b_1+(n_2-h_2)b_2$, where h_i is the number of fighters and b_i is the productive efficiency of each worker/defector. The total product is divided and distributed between groups in accordance with their respective fighting powers $(h_i f_i)^m$, where f_i is the fighting efficiency and m is the decisiveness parameter, $i=1,2$. We use the ratio formula as our contest success function (Skaperdas, 1996; Tullock, 1980). A fighter j in group 1 thus receives an income:[5]

$$P_{1j}(S^{-j},c)=\frac{1}{n_1}\frac{\left(h_1 f_1\right)^m}{\left(h_1 f_1\right)^m+\left(h_2 f_2\right)^m}\left[(n_1-h_1)b_1+(n_2-h_2)b_2\right]-c \qquad (1)$$

where S^{-j} is the set of strategies by all the n_1-1+n_2 agents in the two groups except agent j in group 1 who chooses to fight. The subscript $1j$ refers to agent j in group 1. We divide by n_1 in equation (1) since the collective action problem is most prominent when benefits are equally divided among cooperators and defectors, while cooperators incur a larger cost of cooperation. If each individual's income were allocated within its group according to its degree of fighting-cooperation, then cooperators would earn more than defectors, the collective action problem would become less prominent, and the results would resemble Hirshleifer's (1995) analysis for unitary actors. In equation (1), aggregate group product is reduced by diversion of effort to fighting. Fighting involves both a private cost c and a social cost. The private cost is a deduction from the agent's earned share, whereas the social cost is suffered by everyone in the form of reduced group production.

Now if agent j decides to produce rather than to fight, there will be h_1-1 fighters in group 1 giving agent j an income

$$P_{1j}(S^{-j},0)=\frac{1}{n_1}\frac{\left((h_1-1)f_1\right)^m}{\left((h_1-1)f_1\right)^m+\left(h_2 f_2\right)^m}\left[(n_1-h_1+1)b_1+(n_2-h_2)b_2\right] \qquad (2)$$

The incomes to agents in group 2 are found by permuting the indices in equations (1) and (2). Fighters/cooperators confer an external benefit on producers/defectors by increasing their group's share, and an external cost on producers/defectors by reducing the total product. At the same time, fighters/cooperators impose two external costs on the adversary, namely by reducing total product and by reducing the adversary's share. The Pareto optimal allocation is zero fighters/cooperators, which gives the maximum number of producers/defectors, and to let their total product be maximized and distributed arbitrarily.

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[5] It is straightforward to endogenize the within-group sharing rule and show that egalitarian sharing is an equilibrium. See Noh (1999) for a fuller treatment of within-group sharing rules.

EQUILIBRIUM ANALYSIS

Each agent chooses the profession that gives the highest income. A necessary and sufficient condition for agent j in group 1 to fight/cooperate with his fellow cooperating group members, is $P_{1j}(S^{-j}, c) > P_{1j}(S^{-j}, 0)$. Inserting equations (1) and (2) gives

$$c < \frac{1}{n_1}\left[\frac{(h_1 f_1)^m[(n_1 - h_1)b_1 + (n_2 - h_2)b_2]}{(h_1 f_1)^m + (h_2 f_2)^m} - \frac{((h_1 - 1)f_1)^m[(n_1 - h_1 + 1)b_1 + (n_2 - h_2)b_2]}{((h_1 - 1)f_1)^m + (h_2 f_2)^m}\right]$$
$$= c_{r1}(h_1, h_2) \tag{3}$$

The analogous requirement for group 2 is found by permuting the indices. Equilibrating variables are h_1 and h_2. We first determine h_1 for group 1 assuming h_2 to be fixed, and h_2 for group 2 assuming h_1 as fixed. We secondly determine the overall equilibrium h_1 and h_2 for both groups. We thirdly carry out comparative statics of h_1 and h_2. When equation (3) is satisfied, then the marginal agent j for whom the condition is being evaluated wishes to fight; of course no current fighter wishes to produce having already decided against it. Given that agent j fights, h_1 has then increased by 1, and we may ask whether another current non-fighter wishes to switch to fighting. So long as the inequality is maintained then current non-fighters wish to become fighters. Thus, we can imagine a step-by-step process whereby the number of fighters increases until either a value of h_1 is reached at which $c=c_{r1}$ (treating h_1 as real here), or else $h_1=n_1$ is reached. If, in contrast to equation (3), we begin at $c > c_{r1}$ then the opposite occurs. Current fighters wish to switch to production, and we can again imagine a one-by-one process whereby they do so until either a value of h_1 is reached at which $c=c_{r1}$ or $h_1=0$.

Property 1

When the status (including a possible non-equilibrium situation) within group 2 is taken as given assuming $h_2 > 0$, a Nash equilibrium in fighting/production strategies for the members of group 1 is a value of h_1 such that either $h_1=0$ and $c > c_{r1}$ (an all-production stable equilibrium where group 1 loses its entire product); or $0<h_1<n_1$ and $c=c_{r1}$ (an interior stable or unstable equilibrium[6]), or $h_1=n_1$ and $c<c_{r1}$ (an all-fighting total war stable equilibrium with no production in group 1).

Proof

The proof follows from equations (1)–(3).

Property 2

For the special case that $h_2=0$, a Nash equilibrium in fighting/production strategies for the members of group 1 is a value of h_1 such that either $h_1=0$ and $c > (n_2 b_2 - b_1)/n_1$ (an all-production stable equilibrium); or $h_1=1$ and $0<c<(n_2 b_2 - b_1)/n_1$ (a stable equilibrium).

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[6] At an unstable equilibrium $h_1=h_1^u$ no agent has an incentive to switch strategy in any direction, but if h_1 is externally perturbed, h_1 either increases or decreases to a stable equilibrium $h_1=h_1^s$. In contrast, perturbation of a stable equilibrium causes movement back to $h_1=h_1^s$.

Proof

Assume $h_2=0$ and $h_1=0$. The first agent in group 1 to fight will do so if the benefit is larger than the cost. Without fighting assume that he gets a share $(1/n_1)[(n_1 - 0)b_1] = b_1$ of his group's production. This means that, in the absence of fighting, there is no redistribution across groups, and no redistribution across agents, so that all production is distributed by individual productivities. With fighting the agent incurs a cost c, but group 2 loses its entire production, and the fighting agent gets a share $(1/n_1)[(n_1 - 1)b_1 + n_2b_2] - c$ according to equation (1). Hence, the first agent will fight when $c < (n_2b_2 - b_1)/n_1$. Each non-fighting member of group 1 gets the same benefit as the one fighting member, but does not incur the cost. Hence there will only be one fighter in group 1 since the cost c cannot be negative.

Property 3

For the special case that $h_2=1$, a Nash equilibrium in fighting/production strategies for the members of group 1 is a value of h_1 such that either $h_1=0$ and $c>[f_1^m/(f_1^m+f_2^m)n_1][(n_1-1)b_1+(n_2-1)b_2]$; or $h_1=1$ and $0<c<[f_1^m/(f_1^m+f_2^m)n_1][(n_1-1)b_1+(n_2-1)b_2]$, which simplifies to $0<c<(1 - 1/n_1)b_1$ when $f_1=f_2$, $b_1=b_2$, $n_1=n_2$.

Proof

Assume $h_2=1$ and $h_1=0$. The first agent in group 1 to fight will do so if the benefit is larger than the cost. Without fighting he gets 0 since group 1's entire production is lost to group 2. With fighting the agent incurs a cost c, but the two groups will share their joint production. Inserting $h_1=h_2=1$ into equation (1) gives $[f_1^m/(f_1^m+f_2^m)n_1][(n_1-1)b_1+(n_2-1)b_2]-c$, so the agent will fight if this is positive, which is usually the case, and will otherwise not fight. Inserting $f_1=f_2$, $b_1=b_2$, $n_1=n_2$ gives $(1 - 1/n_1)b_1 - c$.

Property 4

The corner solution $h_1=n_1$ and $h_2=n_2$ is not possible. For the special case that $h_2=n_2$, a Nash equilibrium in fighting/production strategies for the members of group 1 is a value of h_1, $0 \leq h_1 < n_1$, such that $c=c_{r1}$. There will be zero fighters in group 1 if $c>[(1-1/n_1)b_1](f_1)^m/[(f_1)^m+(n_2f_2)^m]$.

Proof

Assume $h_2=n_2$ and $h_1=n_1$. This gives zero production. Using equation (1), each fighter in group 1 earns $-c$. If one agent in group 1 decides to produce rather than to fight, equation (2) gives $(b_1/n_1)((n_1-1)f_1)^m/[((n_1-1)f_1)^m+(n_2f_2)^m]$, which is positive, which means that at least one agent will switch to cooperation. Each fighter earns the same but incurs the cost c. Hence, fighters will switch to cooperation until an equilibrium is reached where each fighter is indifferent between fighting and production. With fighting he incurs a cost c, but if he switches to cooperation, his group – and thus himself – earns lower income. The equilibrium is determined by $c=c_{r1}$, using equation (3). If there were to be one fighter $h_1=1$ in group 1, his income would

be $\left[(1-1/n_1)b_1\right](f_1)^m\big/\left[(f_1)^m+(n_2f_2)^m\right]-c$. If this last fighter switches to cooperation, his income is zero.

Properties 1–4 for group 2 are found by permuting the indices.

Property 4 states that 100% fighting by all agents in both groups, $h_1=n_1$ and $h_2=n_2$, is not possible. There is no production in this hypothetical case, which gives zero income, and additionally there is a cost of fighting. However, as stated in Property 1, 100% fighting in one of the groups is possible. As a digression to the rent seeking literature, this means that over-dissipation of the rent never occurs. Production and conflict models, as discussed by Hausken (2005), are such that the agents never earn negative income, and they adjust their production versus fighting accordingly.

However, 100% production by all agents in both groups, $h_1=h_2=0$, is possible. Property 2 states that it occurs when $c > (n_2b_2 - b_1)/n_1$, which is usually a large number, and especially large if there are many agents in group 2, and few members in group 1. Such a large cost of fighting prevents its occurrence. The all-production equilibrium $h_1=h_2=0$ with income b_i to each producer occurs only if the cost c is so large that no first fighter has an incentive to incur it.

The first fighter in group 1 receives $(1/n_1)[(n_1-1)b_1+n_2b_2]-c$, does not fight if this income is less than b_1, but fights if $0<c<(n_2b_2 - b_1)/n_1$. $h_2=0$ and $h_1=1$ cause zero income to each producer in group 2 and a large income to the first fighter in group 2, and also the second fighter, in group 2, given that the cost c of fighting is not too large. This causes $h_2=2$ and $h_1=1$. When c is not too large the groups thus inch up on each other until an internal equilibrium is reached. This equilibrium falls short of the impossible all-fighting situation $h_1=n_1$ and $h_2=n_2$. Figure 1 shows the 'breakeven cost' $c_{r1}=c_r$ from equation (3) for group 1 dependent on h_1 given $h_2=400$ fighters in group 2, where $b_1=b_2=n_1=n_2=1000, f_1=f_2=1$. The value h_1^u is an unstable equilibrium value of h_1, and h_1^s is a stable equilibrium value of h_1.

Equations (1) and (2) imply that group 1 in isolation (where $h_2=n_2=0$) specializes in production alone since $c > 0$.[7] When there are no attackers against one's group, fighting within the group has no value. We exclude the possibility that an agent can get paid from an external source to fight. With egalitarian inter-group distribution ($m=0$), which means that each group
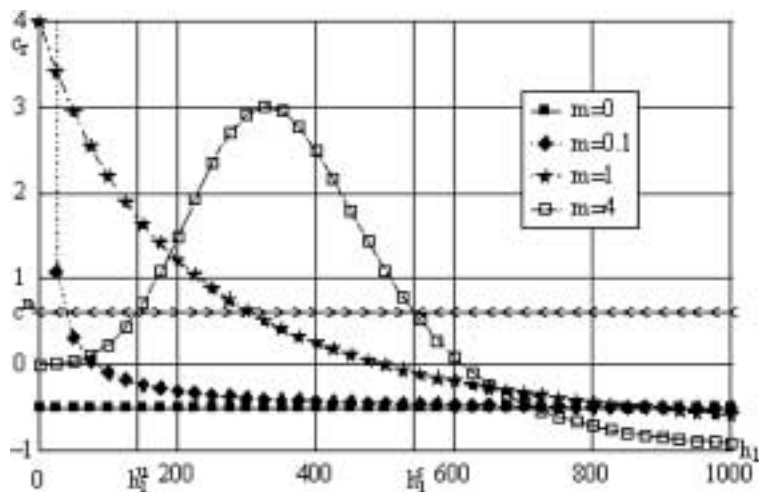

FIGURE 1   Requirement $c<c_r$ as a function of h_1 for $h_2=400$ for four values of the decisiveness m.

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[7] Contrast this with Hausken's (2000a) model where group 1 in isolation gives prisoner's dilemma characteristics when $1=b_1/n_1<c<b_1$.

gets an equal amount, equations (1) and (2) give 100% production in group 1 when $c > -b_1/2n_1$ illustrated with $c_r = -0.5$ in Figure 1. Since $c > 0$, this is always satisfied. For $0 < m < 1$ the requirement for fighting is lenient when h_1 is small, as the very first agents to fight may increase their income. c_r declines toward a small positive value as h_1 increases. For $m > 1$, h_1 considerably lower than $h_2 = 400$ causes a strict requirement for c because the benefits from fighting by agent j gets expropriated by group 2. For intermediate h_1, the requirement $c < c_r$ is lenient inducing agent j to fight even at considerable cost c. For high h_1 the incentives for agent j to free-ride increases if c is high.

Consider the curve $m = 4$ in Figure 1. When $c > 3$, h_1 falls to zero. An intermediate cost $c = c^m = 0.6$ gives three possibilities. When $0 < h_1 < h_1^u = 141$, any fighter prefers to produce and no current producer prefers to fight, so h_1 falls to 0. For $h_1^u < h_1 < h_1^s = 543$, each current producer wishes to fight, and no fighter wishes to produce, so h_1 increases to h_1^s, at which point no further incentive exists for either a fighter or a producer to switch. For $h_1 > h_1^s$ each current fighter has an incentive to produce, pushing h_1 back to h_1^s. Hence, h_1^s is a stable equilibrium, while h_1^u is an unstable equilibrium.

The overall Nash equilibrium (h_2^o, h_1^o) is determined by the four curves $h_1^s = h_1^s(h_2, \cdot)$ $h_1^u = h_1^u(h_2, \cdot)$, $h_2^s = h_2^s(h_1, \cdot)$, $h_2^u = h_2^u(h_1, \cdot)$, shown in Figure 2 for $m = 4$, $c = 0.6$. The agents inevitably move to the overall stable internal Nash equilibrium $(h_2^o, h_1^o) = (h_2^s(h_1^s, \cdot), h_1^s(h_2^s, \cdot)) = (646, 646)$. If initially located inside the 'heart' spanned partly by the thick unstable equilibrium curves $h_1^u = h_1^u(h_2, \cdot)$ and $h_2^u = h_2^u(h_1, \cdot)$, and partly by $h_1^s = h_1^s(h_2, \cdot)$ and $h_2^s = h_2^s(h_1, \cdot)$, they do so directly. If located outside the heart but sufficiently close to the diagonal in the upper right part of Figure 2, they also do so directly in a leftward and downward movement. Assume that fighting is higher in group 1 than in group 2 which gives positioning outside the heart in the upper left part of Figure 2. The unstable equilibrium curve $h_2^u(h_1, \cdot)$ along the upper boundary of the heart causes leftward movement reducing h_2. This causes less need for fighting in group 1, so h_1 decreases too. This causes leftward and downward movement outside the heart. Because of the unstable equilibrium curve $h_2^u(h_1, \cdot)$, h_2 eventually decreases to zero. We know from Property 2 than when $h_2 = 0$, h_1 can decrease to $h_1 = 0$ only with substantial
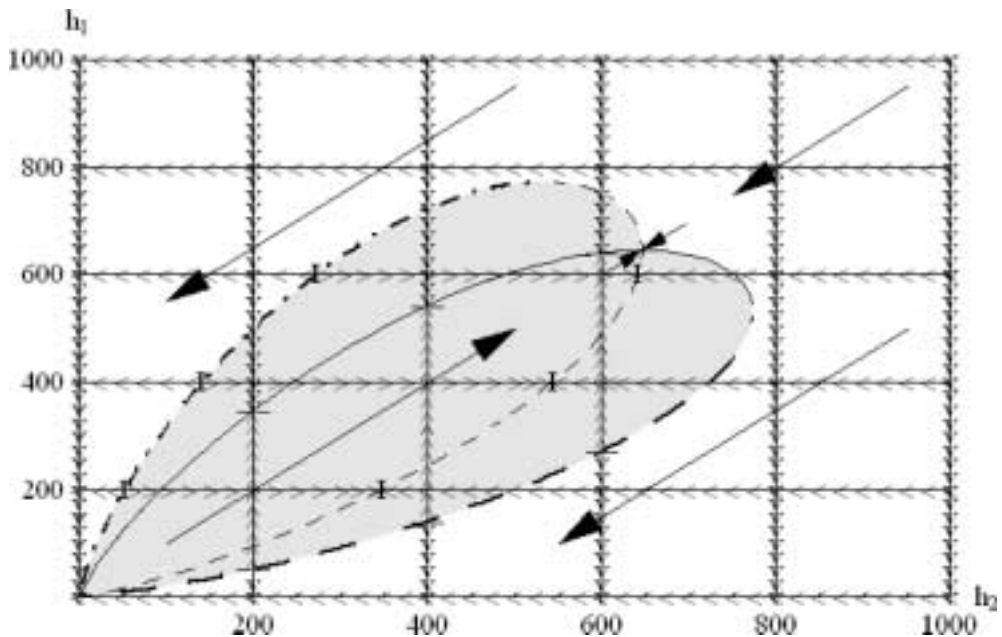

FIGURE 2 Mutual reaction curves $h_1^s = h_1^s(h_2, \cdot)$, $h_2^s = h_2^s(h_1, \cdot)$, $h_1^u = h_1^u(h_2, \cdot)$, $h_2^u = h_2^u(h_1, \cdot)$, $m = 4$, $c = 0.6$.

cost $c > (n_2 b_2 - b_1)/n_1$ of fighting, which gives $c>999$ with the given parameters. For this uncommon case the heart disintegrates to nothingness, which does not happen in Figure 2 since $c=0.6$. Usually, the cost of fighting is lower, so h_1 decreases to $h_1=1$. When $h_1=1$, Property 3 with permuted indices implies that the first agent in group 2 will switch to fighting when $0<c<(1-1/n_2)b_2$, which is clearly satisfied with the given parameters. Consequently, the leftward and downward movement outside the heart leads to either the all-production equilibrium $(h_2,h_1)=(0,0)$ for the uncommon case of substantial fighting cost, or movement to $(h_2,h_1)=(0,1)$ and thereafter $(h_2,h_1)=(1,1)$ for the more common case of moderate fighting cost. The latter common case demonstrates successful movement into the heart along the diagonal. $(h_2,h_1)=(1,1)$ is usually not an equilibrium since movement to either $(h_2,h_1)=(1,2)$ or $(h_2,h_1)=(2,1)$ causes competitive advantage to one of the groups. Hence the two groups can be expected to inch up on each other until a stable internal equilibrium is reached. This occurs when $(h_2^o,h_1^o)=(646,646)$ in Figure 2. If fighting is lower in group 1 than in group 2, which gives positioning outside the heart in the lower right part of Figure 2, the argument is analogous. h_1 decreases to zero, h_2 decreases to zero or one, causing either the stable uncommon $(h_2,h_1)=(0,0)$ or the common $(h_2,h_1)=(1,1)$ with subsequent escalation inside the heart to $(h_2^o,h_1^o)=(646,646)$.

Property 5

Assume two identical groups, which implies $c_{r1}=c_{r2}=c_r$, where $f_1=f_2$, $n_1=n_2$, $m>1$. First, one unique overall Nash all-fighting equilibrium $(h_2^o,h_1^o)=(n_2,n_1)$ is impossible. Second, when $c=c_r(h_1^s(h_2^s,\cdot),h_2^s(h_1^s,\cdot),\cdot)$ for $0<h_1^s(h_2^s,\cdot)=h_2^s(h_1^s,\cdot)<n_1=n_2$, an overall stable internal Nash equilibrium $(h_2^o,h_1^o)=(h_2^s(h_1^s,\cdot),h_1^s(h_2^s,\cdot))$ is reached given confinement to the heart. Third, when $\forall h_i$, $c>c_r(h_1,h_2,\cdot) \,\forall h_i$, $0 \le h_i \le n_i$, $i=1,2$, there exists one unique overall Nash all-production total peace equilibrium $(h_2^o,h_1^o)=(0,0)$.

Proof

The proof follows from equations (1)–(3), Properties 1–4, and applying Figures 1 and 2.

Property 5 states that the shaded area (corresponding to the heart in Figure 2) cannot exhaust the entire parameter space $\forall h_i$, since $(h_2^o,h_1^o)=(n_2,n_1)$ is impossible. As c increases, the increased cost of fighting causes the heart to be shorter and fatter.

Property 5 has implications for how the design, monitoring, and external regulation of competing groups affect strategic behavior within them. A key issue is whether location within or outside the heart occurs. Similarities in fighting levels more likely ensure confinement to the heart. Given such confinement to the heart, an internal equilibrium is eventually reached. In contrast, differences in fighting levels more likely ensure location outside the heart. Such differences may be induced by providing incentives to raise the fighting level in one group to be sufficiently larger than that of the other group, or by providing disincentives so that the fighting level in the other group gets lowered to be sufficiently lower than that of the first. This ensures the avoidance of the heart but requires sustained external inducement. Either group may deter the other unilaterally, but only temporarily. Unless external means are imposed to settle a situation outside the heart, movement through $(h_2,h_1)=(1,0)$ or $(h_2,h_1)=(0,1)$ occurs with subsequent escalation to (n_2,n_1).[8]

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[8] Figure 2 suggests that movement into, but not out of, the heart is possible. This is because $h_1^s=h_1^s(h_2,\cdot)$ and $h_2^s=h_2^s(h_1,\cdot)$ are within or on the border of the shaded area. It cannot be ruled out that parameter combinations exist incompatible with the latter event, allowing movement also out of the heart. Subsequent return to the heart through movement through $(h_2,h_1)=(1,0)$ or $(h_2,h_1)=(0,1)$ cannot be avoided, however, with subsequent escalation to (h_2^o,h_1^o).

EFFECTS OF CHANGES IN THE DECISIVENESS PARAMETER AND COST OF FIGHTING

For two identical groups, equation (3) is analytically solvable with respect to the equilibrium variable $h_i = h_1 = h_2$ when $m=2$ and $m=1$. The solutions are available on request. It can be shown that when $m=1$, c_r is always downwards sloping in h_i.[9] Assuming identical groups where $b_1 = b_2 = n_1 = n_2 = 1000$, $f_1 = f_2 = 1$, Table I shows the equilibrium fighting for decisiveness $m=1,2,4$ and costs $c=0.6$ and $c=4$ of fighting.

The upper right value $h_i^o = 646$ corresponds to Figure 2 where $m=4$ and $c=0.6$. Decreasing decisiveness m gives gradually less fighting, which makes the heart shorter and fatter. As Hirshleifer (1995: 32) shows, 'in military struggles, low m corresponds to the defense having the upper hand. On the western front in World War I, entrenchment plus the machine gun made for very low decisiveness m.' When the defense is superior to attack, an especially stable equilibrium is reached. The equilibrium (477,477) for $m=2$ is harder to tilt off balance, since the heart is shorter and fatter. Furthermore, the equilibrium (313,313) for $m=1$ is almost impossible to tilt off balance.

As the fighting cost c increases, the heart grows smaller and narrower.[10] Cost $c=4$ and $m=4$ gives $(h_2^o, h_1^o) = (308, 308)$. An example of this case is two poor groups that both have to invest heavily in production ($n_1 - h_1$ and $n_2 - h_2$ are large) in order to meet their minimum income or Malthusian survival level. The two groups cannot engage in heavy costly fighting. Each group has modest means to secure its production. Although the equilibrium is stable, not much perturbation is needed to tilt it off balance. If increased capacity for fighting is injected temporarily to one group from an outside source, that group may temporarily increase its fighting, which causes first the other group, and then both groups, to reduce their fighting. After movement through $(h_2, h_1) = (1,0)$ or $(h_2, h_1) = (0,1)$ as illustrated earlier, the groups again escalate to (308,308).

COMPARISONS OF THE RESULTS WITH HIRSHLEIFER'S

Our more stable anarchy when the decisiveness parameter, m, is low can be compared with Hirshleifer's (1995: 33) result 1 'that for an interior stable equilibrium, the decisiveness parameter must lie in the range $0 < m < 1$,' which is his 'condition for dynamic stability.' If this condition is not met (i.e. $m > 1$), Hirshleifer's (1995: 49–50, Ex. 1) analysis for unitary actors reveals dynamic instability and movement toward a corner solution where one actor gets the entire resource base and the other gets nothing. In this model, in contrast, where the collective action problem is accounted for within each group, such abrupt dynamic instability

TABLE I   Fighting $h_i^o = h_2^o = h_1^o$ for decisiveness $m=1,2,4$ and fighting costs $c=0.6$ and $c=4$.

	$m=1$	$m=2$	$m=4$
$c=0.6$	$h_i^o = 313$	$h_i^o = 477$	$h_i^o = 646$
$c=4$	$h_i^o = 101$	$h_i^o = 182$	$h_i^o = 308$

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[9] When $m=2$, c_r is downwards sloping in h_i when h_i is larger than a small number which is slightly less than one when n_i is large.

[10] Increasing b_1 and b_2 has a similar effect as decreasing c, and vice versa. We consider b_i proportional to n_i, which is often realistic and means that the benefits reaped by one agent do not reduce the benefits received by another agent. An alternative is to consider b_i as constant. Then the benefits of production are divided between the group members, giving smaller share to each as n_i increases.

for specific ranges of m does not occur. Multiple agents within each group cause a smooth trade-off between fighting and production, with no abrupt switches from dynamic stability to instability at specific parameter values, such as for $m=1$ in Hirshleifer's analysis. One agent's behavior, no matter how extreme, has limited impact on the whole group, especially when the group is large. This stands in contrast to Hirshleifer's (1995) analysis for two unitary agents where altering m has more abrupt impact causing instability and a corner solution when $m>1$. Groups that fight with each other over their joint production have a stabilizing impact on themselves and on the agents within each group, ruling out the corner solution demonstrated by Hirshleifer (1995) for large decisiveness.

For groups, as m decreases, there is a gradual increase in the degree of anarchic stability, and reduced fighting. More perturbation is needed to tilt the stable equilibrium off balance. Conversely, increasing m does not cause abrupt instability, but makes the heart in Figure 2 slimmer and longer (shaped like a carrot). This causes a gradual transition toward more fighting, and a stable equilibrium, which is more easily tilted off balance. That the stable equilibrium is more easily tilted off balance for large rather than small decisiveness correlates with the dynamic instability demonstrated by Hirshleifer (1995) for large decisiveness and unitary actors. The reason for this linkage is that reducing the number of agents to one in each group gives similar, but not equivalent, models. The similarity is that the two agents fight for their joint production in both models, applying the same contest success function with a given decisiveness parameter. The difference is that this article dichotomously assumes that an agent either fights incurring a cost c, or does not fight incurring no cost. Applying equations (1) and (2), Table II shows the group model with one agent in each group.

Inserting $n_1=n_2=1$ into Property 2 and its permuted version implies that Table II has one unique all-production equilibrium (b_1,b_2) when $c > b_2 - b_1$ and $c > b_1 - b_2$. When $b_1 = b_2$, this is always satisfied, and it is satisfied when c is sufficiently large while b_1 and b_2 are not too different. When $b_2 < b_1 - c$, agent 1 produces, and agent 2 fights and secures the entire production, which is the production of agent 1 since agent 2 does not produce. Although agent 2 secures the entire production in this case, this result does not depend on the decisiveness m as evident from Table II and equations (1) and (2) when $n_1=n_2=1$. Hence, there is no instability as that found by Hirshleifer (1995) for unitary actors. Conversely, when $b_1 < b_2 - c$, agent 2 produces, and agent 1 fights and secures the entire production of agent 2. The individual productivities b_1 and b_2 play a role in Table II, but the decisiveness m and fighting efficiencies f_1 and f_2 play no role in Table II, although these play a role when $n_1 \geq 2$ or $n_2 \geq 2$. In accordance with Property 4, three of the outcomes in Table II are possible, while the all-fighting solution $(-c,-c)$ is not possible.

In contrast to the group model, Hirshleifer (1995) assumes non-dichotomously that each agent has a resource that can be divided into fighting versus production, which generates an optimal trade-off between the two activities for each agent. In the group model in this article the trade-off between fighting and production is expressed as the number of agents in each group that chooses one rather than the other activity.

My group model gives no corner solution with all-fighting and gives no corner solution with all-production unless the cost of fighting is extremely large or the groups are small with similar

TABLE II   The Group Model with One Agent in Each Group

		Agent 2	
		Fight ($h_2=1$)	Produce ($h_2=0$)
Agent 1	Fight ($h_1=1$)	$(-c,-c)$	$(b_2-c,0)$
	Produce ($h_1=0$)	$(0,b_1-c)$	(b_1,b_2)

productivities b_1 and b_2 (Property 2 and Table II). However, high decisiveness causes considerable fighting and low income as the agents in each group strive to get greater shares of total production. In Hirshleifer (1995), one agent gets the upper hand when $m>1$, leading to a corner solution where he gets the entire product, and the other gets nothing. In my group model, no single group gets the entire joint product because the free-rider incentive for each agent prevents one group from getting sufficiently ahead of the other. A corner solution with all-fighting, therefore, is never reached. As agents within each group strive to appropriate more and more, they stop short of universal fighting, which would give no aggregate production to fight for. Thus, whereas Hirshleifer's (1995) model with unitary actors implies that anarchy breaks down for $m>1$, our group model suggests that anarchy is stable for large values of m, although it is more easily perturbed off balance. Let us formulate this as a property.

Property 6

Hirshleifer's model for unitary actors gives dynamic stability in the form of an interior stable equilibrium when the decisiveness parameter lies within the range $0<m<1$, and dynamic instability and movement toward a corner solution where one actor gets the entire resource base, and the other gets nothing, when $m>1$. In contrast, accounting for the collective action problem in a group model gives no abrupt switch from dynamic stability to dynamic instability for one specific value of the decisiveness parameter, but instead dynamic stability is preserved for large decisiveness substantially above one. Such a 'Hirshleifer effect' of dynamic instability is, however, more easily perturbed off balance by an exogenous shock for large rather than for small decisiveness.

Simulations of our 'group model' that demonstrate these properties are available on request. As two examples of asymmetry, first, reducing the size of group 1 to $n_1=500$ while keeping $n_2=1000$ causes $(h_2{}^o,h_1{}^o)=(483,500)$ and an asymmetric heart. Agents in the small group 1 fight maximally, and agents in the large group 2 fight slightly less, in accordance with Hirshleifer's (1991a) paradox of power. Second, doubling the fighting efficiency in group 1 to $f_1=2$ while keeping $f_2=1$ reduces group 1's fighting to $(h_2{}^o,h_1{}^o)=(725,469)$.

Finally, note that inter-group mobility promotes production (is peace-inducing). Imagine universal production (total peace) and that agent j in group 1 decides to fight. This causes movement of all agents to group 1, which captures all production, after which agent j switches to production. Setting the income in equation (1) for fighting equal in the two groups implies $n_1/n_2 = (h_1F_1/h_2F_2)^m$. For an alternative model of inter-group mobility see Hausken (2000b).

AN EQUILIBRIUM NUMBER OF GROUPS[11]

Hirshleifer (1995: 37–39) determines for unitary actors an equilibrium number of actors (e.g. nations) where each meets a Malthusian survival level of income (viability limit, zero-profit condition). Accounting for the collective action problem, Generalizing equations (1)–(3) to n groups gives:

$$P_{1j}(S^{-j},c)=\frac{1}{n_1}\frac{(h_1f_1)^m}{(h_1f_1)^m+\sum_{i=2}^{n}(h_if_i)^m}\left[(n_1-h_1)b_1+\sum_{i=2}^{n}(n_i-h_i)b_i\right]-c \qquad (4)$$

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[11] I am indebted to Jack Hirshleifer for the insights from his work on 'exogenous vs. endogenous variation' of the 'number of competitors' (Hirshleifer 1995: 37–39).

$$P_{1j}(S^{-j},0)=\frac{1}{n_1}\frac{\left((h_1-1)f_1\right)^m}{\left((h_1-1)f_1\right)^m+\sum_{i=2}^{n}\left(h_if_i\right)^m}\left[(n_1-h_1+1)b_1+\sum_{i=2}^{n}(n_i-h_i)b_i\right] \quad (5)$$

$$c<\frac{1}{n_1}\left[\frac{\left(h_1f_1\right)^m\left[(n_1-h_1)b_1+\sum_{i=2}^{n}(n_i-h_i)b_i\right]}{\left(h_1f_1\right)^m+\sum_{i=2}^{n}\left(h_if_i\right)^m}-\frac{\left((h_1-1)f_1\right)^m\left[(n_1-h_1+1)b_1+\sum_{i=2}^{n}(n_i-h_i)b_i\right]}{\left((h_1-1)f_1\right)^m+\sum_{i=2}^{n}\left(h_if_i\right)^m}\right]=c_r \quad (6)$$

I have performed simulations to show the stable equilibrium values $h_1{}^s$ and incomes $P_{1jc}=P_{1jd}$ dependent on the number n of groups, with $n_i=n_1=1000$, $f_i=f_1$, $b_i=b_1=n_1$. These demonstrate how the number $h_1{}^s$ of fighters increases, and the income $P_{1jc}=P_{1jd}$ decreases as the number n of competing groups increases, making the Malthusian survival level more difficult to meet. Hence, there is a maximum number of groups that can be sustained. The survival level is more easily met when the cost c of fighting for each agent increases, or the inter-group decisiveness m decreases.

CONCLUSION

In Hausken's (2000a) model for groups each agent can choose to fight/defect incurring no cost of effort, or to produce/cooperate incurring a cost of effort. Agents are propelled from universal defection (free-riding) to universal cooperation (production) given confinement within a sector. In Hausken (2000a) cooperation is required for *production* whereas in this article cooperation is required for *fighting*, and free-riders defect to production. A cooperator is allowed to contribute but is required to pay a cost of fighting. A defector contributes only to production, and pays no cost. This article shows how agents are propelled from universal production to combined fighting/production given confinement within a shape that is formed like a heart. If located outside the heart, fighting de-escalates, and the heart is entered with subsequent escalation to a combined fighting/production equilibrium. This equilibrium falls well short of the all-fighting situation that would give no production to fight for, and which never occurs. Over-dissipation is not possible, in contrast to the rent-seeking literature. With infinitely high cost of fighting, the heart disintegrates to an all-production equilibrium.

In Hirshleifer's (1995) model for unitary actors, combined fighting/production (anarchy) frequently breaks down when the decisiveness of fighting is above a certain value (above one) or available income is inadequate. The breakdown leads to a corner solution where one actor gets the entire resource base, and the other gets nothing. Accounting for the collective action problem within each group, this article gives the opposite result: fighting/production is stable even for large decisiveness parameters (above one) and strict income requirements for each agent. One agent's behavior, no matter how extreme, has limited impact on the whole group, especially when the group is large. Groups that fight with each other over their joint production have a stabilizing impact on themselves and on the agents within each group, ruling out the instability demonstrated by Hirshleifer (1995) for large decisiveness. No group captures the entire product because of the free-rider incentive for each agent. The linkage to Hirshleifer's (1995) model is that for high inter-group decisiveness in the contest success function, over a considerable range (shown by a long and narrow heart in Figure 2) fighting persists in a stable equilibrium. However, in this range equilibrium can be perturbed off balance through

a shock. As decisiveness m decreases, the heart region in Figure 2 becomes shorter and broader requiring a larger shock to be perturbed off balance. This stable fighting/production equilibrium grows gradually easier to perturb off balance as decisiveness, or costs of fighting increase, as fighting efficiencies become more different, and as group sizes approach equality.

Hirshleifer (1995: 46–48) pointed out that 'military technology' has often 'moved in the direction of higher decisiveness m, threatening dynamic stability, leading 'the most militarily effective contender to become a hegemon.' This holds for centralized unitary actors, but not for decentralized groups of voluntary agents. The collective action problem with its incentives for free-riding is often interpreted to have negative consequences. This article, however, illustrates a positive consequence of decentralized decision (in a democratic spirit) downward by each individual agent within each group when the decision concerns choice of peaceful or combative action. And the positive benefit results from free-riding, which gives a superior outcome to that of monolithic groups where all agents act identically through social pressure or dictatorial politics. 'Democratic' groups fighting for their joint production may cause some agents in the weaker group to fight harder, and some agents in the stronger group to free-ride with respect to fighting, thereby preventing one group from becoming a hegemon. History shows that agents in weak groups sometimes fight fiercely to get ahead, while agents in strong groups sometimes grow decadent and may slack-off, cutting back on fighting. One illustration of this is Hirshleifer's (1991a) paradox of power. Most empirics show that democracies seldom wage war with each other, and seldom exterminate each other. This article shows how collective action typified by free-riding within groups moderates their fighting with each other to the benefit of both. Not merely when war is inefficient but also when decisiveness of conflict is large, a considerable degree of peaceful coexistence is possible, and significant production is maintained within each group. The reader is referred to Hirshleifer (1995: 43–49) for discussions of limitations and possible extensions, many of which also hold for this article.

This article has further shown the maximum number of groups that can be sustained when each agent within each group has to meet a Malthusian survival level of income. The number of fighters increases, and the income of each decreases as the number n of competing groups increases, making the Malthusian survival level more difficult to meet. The survival level is more easily met when the cost of fighting increases, or inter-group decisiveness decreases.

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