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Production and conflict models versus rent-seeking models

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A production and conflict (P&C) model and a rent-seeking (RS) model are compared for one group, two groups and K groups. Adding a new agent enlarges the pie in the P&C model, but causes the fixed size pie to be allocated on one more rent seeker in the RS model. The total production or rent is distributed within and between groups according to the within-group and between-group decisiveness. Productive and fighting efficiencies and group sizes play a role. The collective action problem is more severe for the RS model. As group size increases, the ratio of within-group to between-group fighting increases marginally toward a constant for the P&C model, while it increases convexly for the RS model. Adding an additional agent to each of two groups is more detrimental to the utilities in RS groups than in P&C groups, while adding a second group of agents when there is already one group of agents gives the reverse result. The severe between-group fighting in the P&C model for many groups causes the P&C model to be preferable for few groups, while the RS model is preferable for many groups. Applications are considered to intergroup migration, inside versus outside ownership, divestitures, mergers and acquisitions, multidivisional versus single-tier firms and U form versus M form of economic organization.

1. Introduction

The production and conflict literature[1] and the rent-seeking literature[2] continue to grow and blossom. In the former, each agent allocates his resource between production and fighting. In the latter each agent fights for an external fixed rent. This article compares the two models systematically for agents in one group, two groups, and K equally large groups. Within-group and between-group fighting, and utilities, differ for the two models dependent on within-group and between-group decisiveness, group sizes, production and fighting efficiencies, and the sizes of the resource and rent. Comparison is necessary since the differing assumptions of the models have differing im-plications which may induce framing effects. Applications are considered to intergroup migration, inside versus outside ownership, divestitures, mergers and acquisitions, multidivisional versus single-tier firms and U form versus M form of economic organization.

The production and conflict literature emerges to extend the focus on production and consumption in economic theory. Grossman (1991), Hausken (2004), Hirshleifer (2001), Skaperdas (1992) and others argue that in addition to producing commodities, agents may appropriate goods produced by others. Typically two or many unitary agents in one group are considered. Hausken (2000a,b) extends to agents in two groups. This article develops a richer model where each agent allocates his resource into fighting[3] within his group, fighting between groups and production. This accounts separately for two levels of conflict, and for production.

Early contributions to the rent-seeking literature are by Krueger (1974), Posner (1975), Tullock (1967), reviewed by Nitzan (1994). There is no production, but the agents fight for an external fixed rent. Katz, Nitzan and Rosenberg (1990) and Nitzan (1991a,b) extend to two and n groups, respectively, known as collective rent seeking. As Garfinkel (2004) notes, the approach typically "treats the two levels of conflict as one," where "each member's contribution to his respective group's effort in the inter-group conflict jointly determines the outcome of both that conflict and the intra-group conflict." Exceptions are Bös (2002), Garfinkel (2004), Inderst, Müller and Wärneryd (2002), Katz and Tokatlidu (1996), Müller and Wärneryd (2001) and Wärneryd (1998). This article is related to the exceptions but is more general. It accounts for both within-group and between-group decisiveness which determines within-group and between-group distribution.

The similarities between production and conflict (P&C) models and rent-seeking (RS) models are considerable. Both models focus on conflict as such, between agents and between groups. The objective of each agent is a maximum share of either the total production or the rent. The within-group and between-group decisiveness, which determines distribution of production or rent within and between groups, play a role in both models. The group sizes and the number of groups of course also play a role in both models. To reach his objective each agent must fight in both models, and this article makes a distinction between fighting within a group, and fighting between groups. The within-group and between-group fighting efficiencies play a role in both models. In the P&C model each agent also has an interest in production, since if no one produces, each gets zero utility. In the RS model there is no production, but each agent is concerned about how large a cost of rent seeking to incur. Collectively it would be beneficial to incur low cost of rent seeking (fighting). If no one fights, the first to fight negligibly gets the en-tire rent. Hence some degree, and often a considerable degree, of fighting occurs.

The differences between the models should also be outlined. The P&C model applies for agents and groups (firms, enterprises, divisions, institutions, collectivities, etc.) involved in production. The rent or prize from predatory activity is not given but is an endogenous result of the agents' productive activities. Similarly, the cost function for predatory activity is not given, but represents the productive activity foregone. There is thus a productive tradeoff between predatory activity and consumption goods. Endogenizing the rent implies that "overdissipation" always occurs, i.e., since fighting is

Pareto-inefficient, the agents would always do better in aggregate never engaging in it. Adding a new agent means adding a new producer, but also a new fighter who fights within and between groups for the total production. Similarly, adding a new group means adding a new group of producers, who fight within and between groups. The size of the pie to be shared thus increases with the number of productive agents. The size of the resource and the productive efficiency are thus essential. The focus in the P&C model is on how an agent allocates his resource between production and fighting (appropriation). A typical result is that an agent with low productive efficiency may allocate a larger fraction of his resource to fighting.

The rent-seeking (RS) model applies for agents and groups involved in rent seeking. The rent or "aggregate revenue" is exogenously given, and the cost function is suitably specified. Summing over all agents, this implies that the aggregate costs incurred may well exceed the value of the rent. Hence the "paradox of overdissipation" as widely discussed in the RS literature. Adding a new agent means adding a new rent seeker or fighter, who is not involved in production, but fights within and between groups for the rent which gets shared with yet another agent. Similarly, adding a new group means adding a new group of rent seekers, who fight within and between groups for the rent. The size of the pie to be shared is thus fixed and essential. The focus is on the rent-seeking efforts of each agent, and whether the rent gets dissipated by rent seeking. Examples of rents are competition for budgets by interest groups (parties, localities, industries, etc.), struggles for government support between different industries, an R&D budget, promotion and election opportunities, government allocation of public goods such as sanitation, and employment and welfare opportunities.

The similarities and differences between P&C models and RS models raise concern about how the models are applied to various phenomena. Understanding the underlying logic of the two models is imperative as a basis for interpreting results in various application areas. The article demonstrates what it is in the inherent logic of P&C models and RS models that generate different results. It is shown how and why the Nash equilibria differ in the two models. Care should be exercised to avoid that too far-reaching conclusions are made in various application areas without explicating the different premises of the two models.

Six applications are considered. For intergroup migration Hausken's analysis (Hausken, 2000b) of a P&C model is contrasted with the RS model. For inside versus outside ownership, Müller and Wärneryd's analysis of a RS model (Müller & Wärneryd, 2001) is generalized and contrasted with the P&C model. The same is done for mergers and acquisitions. For divestitures, multi-divisional versus single-tier firms, and U form versus M form of economic organization, Inderst et al.'s analysis of a RS model (Inderst et al., 2002) is generalized and contrasted with the P&C model.

The next section presents the P&C model for one group, two groups, and K equally large groups. The section that follows does the same for the RS model. The suceeding three sections compare the P&C model and RS model for one group, two groups, and K equally large groups. Last few sections consider the six applications. The final section concludes. Appendix C extends the P&C model dynamically.

2. A Production and Conflict Model

First consider one group with size n_1. As formulated by Hirshleifer (1995a:30) and Skaperdas and Syropoulos (1997:102), agent i in group 1 has a resource R_1 transformable into two kinds of efforts. The first is productive effort E_i designed to generate production from resources currently controlled. The second is fighting effort F_i designed to acquire the production of others, or repel others as they attempt to do the same. With unit conversion costs a_1 and b_1 of transforming R_1 into E_i and F_i, we get $R_1 = a_1 E_i + b_1 F_i$. Assume a simple production function where agent i produces $(R_1 - b_1 F_i)/a_1$, and the group produces $\sum_{i=1}^{n_1} (R_1 - b_1 F_i)/a_1$.[4] The agents fight with each other with within-group decisiveness m_1. Agent i gets a ratio $F_i^{m_1} / \sum_{i=1}^{n_1} F_i^{m_1}$, known as the contest success function (Hirshleifer, 2001; Skaperdas, 1996; Tullock, 1967), and utility

$$U_i = \frac{F_i^{m_1}}{\sum_{i=1}^{n_1} F_i^{m_1}} \sum_{i=1}^{n_1} \frac{R_1 - b_1 F_i}{a_1}. \tag{2.1}$$

Second consider two groups with sizes n_1 and n_2. As in the one-group model, agent i in group k has a resource R_k transformable into productive effort E_{ki} and fighting effort. One innovation in this article is to distinguish between two kinds of fighting effort. The first is fighting effort F_{ki} within group k, where agent i fights with decisiveness m_k with all the other agents within group k for a largest possible ratio of group k's ratio of the production. The second is fighting effort G_{ki}, where all the agents in group k compile their efforts into a group fighting effort $(\sum_{i=1}^{n_k} G_{ki})^m$ directed against the other group to obtain a largest possible ratio of the total production, where m is the decisiveness of between-group fighting.[5]Formally, where a_k, b_k, c_k are unit conversion costs, agent i in group k divides his resource R_k into three kinds of effort:

$$R_k = a_k E_{ki} + b_k F_{ki} + c_k G_{ki}, \ i = 1, \ldots, n_k, \ k = 1, 2. \tag{2.2}$$

c_k operates as b_k, but transforming into between-group effort instead of within-group effort. $1/a_k$, $1/b_k$, $1/c_k$ are the efficiencies. The production function for agent i in group k is

$$Y_{ki} = E_{ki} = (R_k - b_k F_{ki} - c_k G_{ki})/a_k. \tag{2.3}$$

The total production $[\sum_{i=1}^{n_1} Y_{k_1} + \sum_{i=1}^{n_2} Y_{k_2}]$ is placed in a common pool for capture.[6] Each agent is involved in two independent fights to get a largest possible ratio of the total productions.[7] In the one fight agents in both groups choose G_{1i} and G_{2i} independently to maximize utility, taking the other agents' genitive; choices of G_{1i} and G_{2i}, and all choices of F_{1i} and F_{2i}, as given. The fighting effort for group k is $(\sum_{i=1}^{n_k} G_{ki})^m$. Applying the ratio formula gives

$$\left(\sum_{i=1}^{n_k} G_{ki}\right)^m \Bigg/ \left[\left(\sum_{i=1}^{n_1} G_{1i}\right)^m + \left(\sum_{i=1}^{n_2} G_{2i}\right)^m\right]$$

to group k. Multiplying this ratio with the total production gives group k's utility. In the other fight agents in both groups choose F_{1i} and F_{2i} independently to maximize utility, taking the other agents' choices of F_{1i} and F_{2i}, and all agents' choices of G_{1i} and G_{2i}, as given. As in the one-group game, agent i's objective is to obtain a largest possible ratio of group k's utility. He thus gets a ratio$F_{ki}^{m_k} / \sum_{i=1}^{n_k} F_{ki}^{m_k}$, which is multiplied by the previous group ratio and the total production to give his utility, i.e.

$$U_{ki} = \frac{F_{ki}^{m_k}}{\sum_{i=1}^{n_k} F_{ki}^{m_k}} \frac{\left(\sum_{i=1}^{n_k} G_{ki}\right)^m}{\left[\left(\sum_{i=1}^{n_1} G_{1i}\right)^m + \left(\sum_{i=1}^{n_2} G_{2i}\right)^m\right]} \times \left[\sum_{i=1}^{n_1} \frac{R_1 - b_1 F_{1i} - c_1 G_{1i}}{a_1} + \sum_{i=1}^{n_2} \frac{R_2 - b_2 F_{2i} - c_2 G_{2i}}{a_2}\right]. \quad (2.4)$$

Third consider K groups with equal group sizes n. Agent i in group k has a resource $R = aE_{ki} + bF_{ki} + cG_{ki}$ transformable into productive effort E_{ki}, within-group fighting effort F_{ki}, and between-group fighting effort G_{ki}, with efficiencies $1/a$, $1/b$, $1/c$. The production function for agent i in group k is $Y_{ki} = E_{ki} = (R - bF_{ki} - cG_{ki})/a$. The total production $\sum_{k=1}^{K} \sum_{i=1}^{n} Y_{ki}$ is placed in a common pool for capture. The n agents in group k fight as a collective against all the agents in the $K-1$ other groups. W.l.o.g. we analyze agent i in group 1, with fighting efforts F_{1i} and G_{1i}. All agents in groups $2, \ldots, K$ choose equal fighting efforts F_2 and G_2. The ratio formula gives

$$\left(\sum_{i=1}^{n} G_{1i}\right)^m \Bigg/ \left[\left(\sum_{i=1}^{n} G_{1i}\right)^m + (K-1)(nG_2)^m\right]$$

to group 1. Analogously to Equation (2.4), agent i's utility in group 1 is

$$U_{1i} = \frac{F_{1i}^{m_1}}{\sum_{i=1}^{n} F_{1i}^{m_1}} \frac{\left(\sum_{i=1}^{n} G_{1i}\right)^m \left[\sum_{i=1}^{n} \frac{R - bF_{1i} - cG_{1i}}{a} + (K-1)n \frac{R - bF_2 - cG_2}{a}\right]}{(\sum_{i=1}^{n} G_{1i})^m + (K-1)(nG_2)^m} \quad (2.5)$$

3. A Rent-Seeking Model

First consider a rent-seeking model for one group with size n_1.[8] Agent i has a resource r_i which is exclusively transformed into rent-seeking (fighting) f_i with unit conversion cost b_1. Hence $r_i = b_1 f_i$, which is a cost incurred. As in the P&C model the agents fight with each other with within- group decisiveness m_1, where agent i gets a ratio $f_i^{m_1} / \sum_{i=1}^{n_1} f_i^{m_1}$ of the rent S. Agent i's utility is[9]

$$u_i = \frac{f_i^{m_1} S}{\sum_{i=1}^{n_1} f_i^{m_1}} - b_1 f_i. \tag{3.1}$$

Second consider two groups with sizes n_1 and n_2. Agent i in group k has a resource $r_k = b_k f_{ki} + c_k g_{ki}$, $i = 1, \ldots, n_k$, $k = 1, 2$, where b_k and c_k are unit conversion costs, transformable into two kinds of rent-seeking (fighting). The first is fighting effort f_{ki} within group k, where agent i fights with decisiveness m_k with all the other agents within group k for a largest possible ratio of group k's ratio of the rent S, determined by $f_{ki}^{m_k} / \sum_{i=1}^{n_k} f_{ki}^{m_k}$. The second is fighting effort g_{ki}, where all the agents in group k compile their efforts into a group fighting effort $(\sum_{i=1}^{n_k} g_{ki})^m$ directed against the other group to obtain a largest possible ratio of the total rent S, determined by

$$\beta_k = \left(\sum_{i=1}^{n_k} g_{ki}\right)^m \Bigg/ \left[\left(\sum_{i=1}^{n_1} g_{1i}\right)^m + \left(\sum_{i=1}^{n_2} g_{2i}\right)^m\right]$$

wnere m is the decisiveness of between-group fighting. The agent incurs a cost $b_k f_{ki}$ of within-group fighting, and a cost $c_k g_{ki}$ of between-group fighting. Analogously to the P&C model, agent i's utility in group k is

$$u_{ki} = \frac{f_{ki}^{m_k}}{\sum_{i=1}^{n_k} f_{ki}^{m_k}} \frac{\left(\sum_{i=1}^{n_k} g_{ki}\right)^m S}{\left[\left(\sum_{i=1}^{n_1} g_{1i}\right)^m + \left(\sum_{i=1}^{n_2} g_{2i}\right)^m\right]} - b_k f_{ki} - c_k g_{ki}. \tag{3.2}$$

Third consider K groups with equal group sizes n. Agent i in group k has a resource $r = bf_{ki} + cg_{ki}$ and incurs a cost bf_{ki} of within-group fighting and a cost cg_{ki} of between-group fighting, with efficiencies $1/b$, $1/c$. W.l.o.g. we analyze agent i in group 1, with fighting efforts f_{1i} and g_{1i}. All agents in groups $2, \ldots, K$ choose equal fighting efforts f_2 and g_2. The ratio formula gives

$$\left(\sum_{i=1}^{n} g_{1i}\right)^m \Bigg/ \left[\left(\sum_{i=1}^{n} g_{1i}\right)^m + (K-1)(ng_2)^m\right]$$

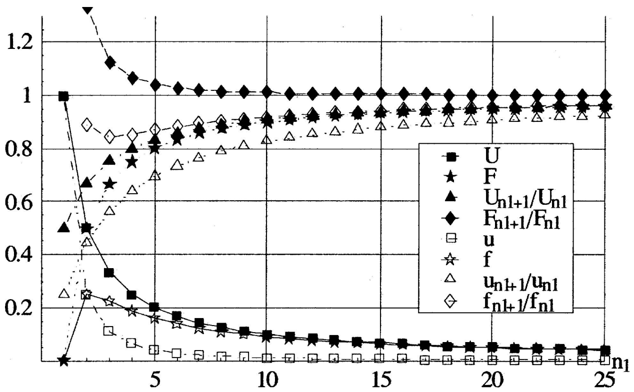

Figure 1. Variables for one group as functions of group size n_1, adding one group member.

to group 1. Analogously to Equation (3.2), agent i's utility in group 1 is

$$u_{1i} = \frac{f_{1i}^{m_1}}{\sum_{i=1}^{n} f_{1i}^{m_1}} \frac{\left(\sum_{i=1}^{n} g_{1i}\right)^m S}{\left[\left(\sum_{i=1}^{n} g_{1i}\right)^m + (K-1)(ng_2)^m\right]} - bf_{1i} - cg_{1i}. \quad (3.3)$$

4. Comparing the One-Group Production and Conflict Model and Rent-Seeking Model

For the one-group production and conflict model (P&C model) the FOC $\partial U_i/\partial F_i = 0$ gives the $U_i = U$ and $F_i = F$ listed in rows 2 and 3 in the left column of Table 1, where identical agents behave equivalently in equilibrium. Increasing group size n_1 or decisiveness m_1 causes more fighting F and lower utility U. F increases toward a horizontal asymptote in n_1 and m_1,enabled by production. There is diminishing return to investment into fighting as n_1 or m_1 increases. U decreases toward zero in n_1 and m_1, which appear multiplicatively in the denominator. See Figure 1 (for $m_1 = 1$) and Figure 2 (for $n_1 = 5$), where $a_1 = b_1 = R_1 = S = 1$. Rows 5, 6 and 7 show the impact of adding an extra agent to the group, expressed as $n_1 + 1$. This causes increased fighting F_{n_1+1} at a smaller rate as n_1 or m_1 increases, where F_{n_1+1}/F_{n_1} decreases toward 1 in n_1 and m_1. I t also causes decreased utility U_{n_1+1}, where U_{n_1+1}/U_{n_1} increases toward 1 in n_1 and decreases toward $(n_1 - 1)/n_1$ in m_1. Adding an agent when the decisiveness m_1 is large is more detrimental to the utility U_{n_1+1} than adding an agent when the group is already large. In a group with low decisiveness m_1, each agent finds low fighting and high production optimal since production is distributed in an egalitarian manner. As m_1 increases, production distribution becomes less egalitarian, and fighting increases, diminishingly since some production is needed to generate utility.

Table 1. Equilibrium variable solutions for the one/two-group P&C model and RS model

Production and conflict model	Rent seeking model
$U=\frac{R_1}{a_1\left[1+(n_1-1)m_1\right]}$	$u=\frac{n_1-m_1(n_1-1)}{n_1^2}S$
$F=\frac{(n_1-1)m_1R_1}{b_1\left[1+(n_1-1)m_1\right]}$	$f=\frac{m_1(n_1-1)S}{b_1n_1^2}$
$\frac{F}{U}=\frac{a_1(n_1-1)m_1}{b_1}$	$\frac{f}{u}=\frac{m_1(n_1-1)}{b_1\left[n_1-m_1(n_1-1)\right]}$
$\frac{U_{n_1+1}}{U_{n_1}}=\frac{1+(n_1-1)m_1}{1+n_1m_1}$	$\frac{u_{n_1+1}}{u_{n_1}}=\frac{\left[(n_1+1)-m_1n_1\right]n_1^2}{\left[n_1-m_1(n_1-1)\right](n_1+1)^2}$
$\frac{F_{n_1+1}}{F_{n_1}}=\frac{n_1\left[1+(n_1-1)m_1\right]}{(n_1-1)\left[1+n_1m_1\right]}$	$\frac{f_{n_1+1}}{f_{n_1}}=\frac{n_1^3}{(n_1-1)(n_1+1)^2}$
$\frac{(F/U)_{n_1+1}}{(F/U)_{n_1}}=\frac{n_1}{n_1-1}$	$\frac{(f/u)_{n_1+1}}{(f/u)_{n_1}}=\frac{n_1\left[n_1-m_1(n_1-1)\right]}{(n_1-1)\left[(n_1+1)-m_1n_1\right]}$
$U_1=\frac{(c_2a_1)^{\frac{m}{m+1}}\left[(c_1a_2)^{\frac{m}{m+1}}+(c_2a_1)^{\frac{m}{m+1}}\right]^{-1}\sum_{k=1}^{2}\frac{n_kR_k}{a_k}}{(m+1)n_1\left[1+(n_1-1)m_1+(n_2-1)m_2\right]}$	$u_1=\frac{\left[(n_1-m)(c_1A_2)^m+n_1(c_2A_1)^m\right]A_1\beta_1S}{n_1\left[(c_1A_2)^m+(c_2A_1)^m\right]}$
$U_1=\frac{\left[n_1+n_2\right]R_1}{2(m+1)n_1\left[n_1+n_2-1\right]}$	$u_1=\frac{\left[(n_1-m)n_1^{2m-1}+n_2^{2m}\right]n_2^{2m}S}{n_1^2\left[n_1^{2m}+n_2^{2m}\right]^2}$
$F_1=\frac{a_1(n_1-1)m_1\left[n_1R_1/a_1+n_2R_2/a_2\right]}{b_1n_1(m+1)\left[1+(n_1-1)m_1+(n_2-1)m_2\right]}$	$f_1=\frac{m_1(n_1-1)\beta_1S}{b_1n_1^2}$
$F_1=\frac{(n_1-1)\left[n_1+n_2\right]R_1}{(m+1)n_1\left[n_1+n_2-1\right]}$	$f_1=\frac{(n_1-1)n_2^{2m}S}{n_1^2\left[n_1^{2m}+n_2^{2m}\right]}$
$G_1=\frac{a_1m(c_1a_2)^{m/(m+1)}\left[n_1R_1/a_1+n_2R_2/a_2\right]}{c_1(m+1)n_1\left[(c_1a_2)^{m/(m+1)}+(c_2a_1)^{m/(m+1)}\right]}$	$g_1=\frac{m(c_1A_2)^mA_1\beta_1S}{c_1n_1\left[(c_1A_2)^m+(c_2A_1)^m\right]}$
$G_1=\frac{m\left[n_1+n_2\right]R_1}{2(m+1)n_1}$	$g_1=\frac{mn_2^{2m}S}{n_1^{3-2m}\left[n_1^{2m}+n_2^{2m}\right]^2}$
$\frac{F_1}{G_1}=\frac{c_1(n_1-1)m_1\left[(c_1a_2)^{m/(m+1)}+(c_2a_1)^{m/(m+1)}\right]}{b_1m(c_1a_2)^{m/(m+1)}\left[1+(n_1-1)m_1+(n_2-1)m_2\right]}$	$\frac{f_1}{g_1}=\frac{c_1(n_1-1)m_1\left[(c_1A_2)^m+(c_2A_1)^m\right]}{b_1mn_1A_1(c_1A_2)^m}$
$\frac{F_1}{G_1}=\frac{2(n_1-1)}{m\left[n_1+n_2-1\right]}$	$\frac{f_1}{g_1}=\frac{(n_1-1)\left[n_1^{2m}+n_2^{2m}\right]}{mn_1^{2m-1}}$
$\frac{F_1}{U_1}=\frac{a_1(n_1-1)m_1\left[(c_1a_2)^{m/(m+1)}+(c_2a_1)^{m/(m+1)}\right]}{b_1(c_2a_1)^{m/(m+1)}}$	$\frac{f_1}{u_1}=\frac{(n_1-1)m_1\left[(c_1A_2)^m+(c_2A_1)^m\right]}{b_1n_1\left[(n_1-m)(c_1A_2)^m+n_1(c_2A_1)^m\right]A_1}$
$\frac{F_1}{U_1}=2(n_1-1)$	$\frac{f_1}{u_1}=\frac{(n_1-1)\left[n_1^{2m}+n_2^{2m}\right]}{\left[(n_1-m)n_1^{2m-1}+n_2^{2m}\right]}$
$\frac{G_1}{U_1}=\frac{a_1m(c_1a_2)^{m/(m+1)}\left[1+(n_1-1)m_1+(n_2-1)m_2\right]}{c_1(c_2a_1)^{m/(m+1)}}$	$\frac{g_1}{u_1}=\frac{m(c_1A_2)^m}{c_1\left[(n_1-m)(c_1A_2)^m+n_1(c_2A_1)^m\right]}$

(*Continued on next page*)

Table 1. (*Continued*)

Production and conflict model	Rent seeking model
$\dfrac{G_1}{U_1} = m\left[n_1 + n_2 - 1\right]$	$\dfrac{g_1}{u_1} = \dfrac{mn_1^{2m-1}}{(n_1 - m)n_1^{2m-1} + n_2^{2m}}$
$\dfrac{F_1}{F_2} = \dfrac{(n_1-1)n_2 a_1 b_2 m_1}{(n_2-1)n_1 a_2 b_1 m_2} = \dfrac{(n_1-1)n_2}{(n_2-1)n_1}$	$\dfrac{f_1}{f_2} = \dfrac{b_2 n_2^2 m_1 (n_1-1)(c_2 A_1)^m}{b_1 n_1^2 m_2 (n_2-1)(c_1 A_2)^m} = \dfrac{(n_1-1)n_2^{2+2m}}{(n_2-1)n_1^{2+2m}}$
$\dfrac{G_1}{G_2} = \dfrac{n_2}{n_1}\left(\dfrac{c_2 a_1}{c_1 a_2}\right)^{1/(m+1)} = \dfrac{n_2}{n_1}$	$\dfrac{g_1}{g_2} = \dfrac{n_2 c_2 A_1}{n_1 c_1 A_2} = \dfrac{n_2^3}{n_1^3}$
$\dfrac{U_{1,n1+1,n2+1}}{U_1} = \dfrac{n_1\left[1 + \sum_{k=1}^{2}(n_k - 1)m_k\right]\left[\sum_{k=1}^{2}\dfrac{(n_k+1)R_k}{a_k}\right]}{(n_1+1)\left[1 + n_1 m_1 + n_2 m_2\right]\left[\dfrac{n_1 R_1}{a_1} + \dfrac{n_2 R_2}{a_2}\right]}$	$\dfrac{u_1^*}{u_1} = \dfrac{\dfrac{\left[(n_1^* - m)(c_1 A_2^*)^m + n_1^*(c_2 A_1^*)^m\right]A_1^*\beta_1^*}{n_1^*\left[(c_1 A_2^*)^m + (c_2 A_1^*)^m\right]}}{\dfrac{\left[(n_1 - m)(c_1 A_2)^m + n_1(c_2 A_1)^m\right]A_1\beta_1}{n_1\left[(c_1 A_2)^m + (c_2 A_1)^m\right]}}$
$\dfrac{U_{1,n1+1,n2+1}}{U_1} = \dfrac{n_1\left[n_1 + n_2 - 1\right]\left[n_1 + n_2 + 2\right]}{(n_1+1)\left[n_1 + n_2\right]\left[n_1 + n_2 + 1\right]}$	$\dfrac{u_1^*}{u_1} = \dfrac{\dfrac{\left[(n_1 + 1 - m)(n_1+1)^{2m-1} + (n_2+1)^{2m}\right](n_2+1)^{2m}}{(n_1+1)^2\left[(n_1+1)^{2m} + (n_2+1)^{2m}\right]^2}}{\left[(n_1 - m)n_1^{2m-1} + n_2^{2m}\right]n_2^{2m}\Big/\left(n_1^2\left[n_1^{2m} + n_2^{2m}\right]^2\right)}$
$\dfrac{F_{1,n1+1,n2+1}}{F_1} = \dfrac{n_1^2\left[1 + \sum_{k=1}^{2}(n_k - 1)m_k\right]\left[\sum_{k=1}^{2}\dfrac{(n_k+1)R_k}{a_k}\right]}{(n_1^2-1)\left[1 + n_1 m_1 + n_2 m_2\right]\left[\dfrac{n_1 R_1}{a_1} + \dfrac{n_2 R_2}{a_2}\right]}$	$\dfrac{f_{1,n1+1,n2+1}}{f_1} = \dfrac{n_1^3\beta_1^*}{(n_1-1)(n_1+1)^2\beta_1}$
$\dfrac{F_{1,n1+1,n2+1}}{F_1} = \dfrac{n_1^2\left[n_1 + n_2 - 1\right]\left[n_1 + n_2 + 2\right]}{(n_1^2-1)\left[n_1 + n_2\right]\left[n_1 + n_2 + 1\right]}$	$\dfrac{f_{1,n1+1,n2+1}}{f_1} = \dfrac{n_1^3(n_2+1)^{2m}\left[n_1^{2m} + n_2^{2m}\right]}{(n_1-1)(n_1+1)^2 n_2^{2m}\sum_{k=1}^{2}(n_k+1)^{2m}}$
$\dfrac{G_{1,n1+1,n2+1}}{G_1} = \dfrac{n_1\left[\dfrac{(n_1+1)R_1}{a_1} + \dfrac{(n_2+1)R_2}{a_2}\right]}{(n_1+1)\left[\dfrac{n_1 R_1}{a_1} + \dfrac{n_2 R_2}{a_2}\right]}$	$\dfrac{g_{1,n1+1,n2+1}}{g_1} = \dfrac{\dfrac{(c_1 A_2^*)^m A_1^*\beta_1^*}{(n_1+1)\left[(c_1 A_2^*)^m + (c_2 A_1^*)^m\right]}}{\dfrac{(c_1 A_2)^m A_1\beta_1}{n_1\left[(c_1 A_2)^m + (c_2 A_1)^m\right]}}$
$\dfrac{G_{1,n1+1,n2+1}}{G_1} = \dfrac{n_1\left[n_1 + n_2 + 2\right]}{(n_1+1)\left[n_1 + n_2\right]}$	$\dfrac{g_{1,n1+1,n2+1}}{g_1} = \dfrac{(n_1+1)^{2m-3}(n_2+1)^{2m}\left[n_1^{2m} + n_2^{2m}\right]^2}{n_1^{2m-3}n_2^{2m}\left[(n_1+1)^{2m} + (n_2+1)^{2m}\right]^2}$
$\dfrac{(F_1/G_1)_{n1+1,n2+1}}{F_1/G_1} = \dfrac{n_1\left[1 + (n_1-1)m_1 + (n_2-1)m_2\right]}{(n_1-1)\left[1 + n_1 m_1 + n_2 m_2\right]}$	$\dfrac{(f_1/g_1)^*}{f_1/g_1} = \dfrac{\dfrac{n_1^2\left[(c_1 A_2^*)^m + (c_2 A_1^*)^m\right]}{A_1^*(c_1 A_2^*)^m}}{\dfrac{(n_1^2-1)\left[(c_1 A_2)^m + (c_2 A_1)^m\right]}{A_1(c_1 A_2)^m}}$
$\dfrac{(F_1/G_1)_{n1+1,n2+1}}{F_1/G_1} = \dfrac{n_1\left[n_1 + n_2 - 1\right]}{(n_1-1)\left[n_1 + n_2 + 1\right]}$	$\dfrac{(f_1/g_1)^*}{f_1/g_1} = \dfrac{n_1^{2m}\left[(n_1+1)^{2m} + (n_2+1)^{2m}\right]}{(n_1^2-1)(n_1+1)^{2m-2}\left[n_1^{2m} + n_2^{2m}\right]}$

(*Continued on next page*)

Table 1. (*Continued*)

Production and conflict model	Rent seeking model
$\dfrac{U_1}{U}=\dfrac{\dfrac{(c_2a_1)^{\frac{m}{m+1}}\left[(c_1a_2)^{\frac{m}{m+1}}+(c_2a_1)^{\frac{m}{m+1}}\right]^{-1}\sum_{k=1}^{2}\frac{n_kR_k}{a_k}}{(m+1)n_1[1+(n_1-1)m_1+(n_2-1)m_2]}}{R_1/(a_1[1+(n_1-1)m_1])}$	$\dfrac{u_1}{u}=\dfrac{\dfrac{\left[(n_1-m)(c_1A_2)^m+n_1(c_2A_1)^m\right]A_1\beta_1}{\left[(c_1A_2)^m+(c_2A_1)^m\right]}}{[n_1-m_1(n_1-1)]/n_1}$
$\dfrac{U_1}{U}=\dfrac{[n_1+n_2]}{2(m+1)[n_1+n_2-1]}$	$\dfrac{u_1}{u}=\dfrac{\left[(n_1-m)n_1^{2m-1}+n_2^{2m}\right]n_2^{2m}}{\left[n_1^{2m}+n_2^{2m}\right]^2}$
$\dfrac{F_1}{F}=\dfrac{a_1[1+(n_1-1)m_1][n_1R_1/a_1+n_2R_2/a_2]}{(m+1)n_1[1+(n_1-1)m_1+(n_2-1)m_2]R_1}$	$\dfrac{f_1}{f}=\beta_1$
$\dfrac{F_1}{F}=\dfrac{[n_1+n_2]}{(m+1)[n_1+n_2-1]}$	$\dfrac{f_1}{f}=\dfrac{n_2^{2m}}{n_1^{2m}+n_2^{2m}}$
$\dfrac{F_1/U_1}{F/U}=\dfrac{(c_1a_2)^{\frac{m}{m+1}}+(c_2a_1)^{\frac{m}{m+1}}}{(c_2a_1)^{\frac{m}{m+1}}}=1+\left(\dfrac{c_1a_2}{c_2a_1}\right)^{\frac{m}{m+1}}$	$\dfrac{f_1/u_1}{f/u}=\dfrac{[n_1-m_1(n_1-1)]\left[(c_1A_2)^m+(c_2A_1)^m\right]}{n_1\left[(n_1-m)(c_1A_2)^m+n_1(c_2A_1)^m\right]A_1}$
$\dfrac{F_1/U_1}{F/U}=2$	$\dfrac{f_1/u_1}{f/u}=\dfrac{\left[n_1^{2m}+n_2^{2m}\right]}{\left[(n_1-m)n_1^{2m-1}+n_2^{2m}\right]}$

$$A_k=\frac{n_k-m_k(n_k-1)}{n_k^2},\quad \beta_1=\begin{cases}\dfrac{(c_2A_1)^m}{(c_1A_2)^m+(c_2A_1)^m} & \text{when } m_1\le\dfrac{n_1}{n_1-1} \text{ and } m_2\le\dfrac{n_2}{n_2-1}\\ 1 \text{ when } m_1\le n_1/(n_1-1) \text{ and } m_2>n_2/(n_2-1)\\ 0 \text{ when } m_1>n_1/(n_1-1) \text{ and } m_2\le n_2/(n_2-1),\end{cases} \tag{E}$$

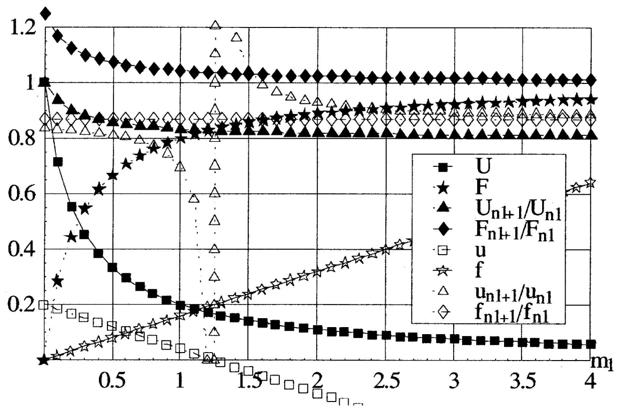

Figure 2. Variables for one group as functions of decisiveness m_1, adding one group member.

For the one-group rent seeking model (RS model) the FOC $\partial u_i/\partial f_i = 0$ gives the $u_i = u$ and $f_i = f$ listed in rows 2 and 3 in the right column of Table 1. The collective action problem is more detrimental in the RS model. Whereas adding a new agent enlarges the pie in the P&C model, it causes the fixed size pie S to be allocated on one more rent seeker in the RS model.

Hence utility u_1 decreases more severely toward zero in n_1 in the RS model. When $m_1 = 1$, u_1 decreases with $1/n_1^2$ while U_1 decreases only with $1/n_1$ in the P&C model. Correspondingly, fighting f decreases with $(n_1 - 1)/n_1^2$ approaching zero, caused by the constraint of the fixed sized rent, while it increases diminishingly in the P&C model, approaching a positive constant. The detrimental collective action problem induces each agent to reduce his cost of fighting f to avoid low utility. Hence in Figure 1, f_{n_1+1}/f_{n_1} increases toward 1 in n_1, while F_{n_1+1}/F_{n_1} decreases toward 1 in n_1. Fighting f increases linearly and utility u decreases linearly in the decisiveness m_1, in contrast to the steeper but diminishing increases and decreases for F and U in the P&C model, see Figure 2.[10] The reason is each agent's interest to keep bounds on his cost of fighting f to ensure high utility in the RS model.

5. Comparing the Two-Group Production and Conflict Model and Rent-Seeking Model

For the two-group P&C model the FOCs (Appendix A) imply the solution in Table 1. For the two-group RS model calculating $\partial u_{1i}/\partial f_{1i} = 0$ and $\partial u_{2i}/\partial f_{2i} = 0$, calculating $\partial u_{1i}/\partial g_{1i} = 0$ and $\partial u_{2i}/\partial g_{2i} = 0$ and equating the two equivalent square brackets, and letting all agents in group k incur equal rent-seeking cost $g_{ki} = g_k$ in equilibrium causing $u_{ki} = u_k$ gives the solution in Table 1. The expressions for group 2 are found by permuting the indices.

Table 1 allows for a detailed comparison of the two models dependent on variation in the 14 parameters n_1, n_2, m, R_1, R_2, S, m_1, m_2, a_1, a_2, b_1, b_2, c_1 and c_2. An exhaustive analysis would take us beyond the space constraints of this article. We choose to selectively consider variation in group size $n_1 = n_2$ and between-group decisiveness m, with some subsequent discussion.

For the second of the two expressions in Table 1, where $m_1 = m_2 = a_1 = a_2 = b_1 = b_2 = c_1 = c_2 = 1$, $R_1 = R_2$, within-group fighting F_1 approaches a constant as n_1 increases for the P&C model. In contrast, f_1 decreases by $1/n_1^{1+2m}$, i.e. even more severely than in the one-group RS model. The difference is more pronounced for the utilities. U_1 decreases by $1/n_1$, while u_1 decreases by $1/n_1^{2+2m}$. The collective action problem grows more severe in the two-group RS model due to the fixed-sized S allocated on an additional group, Figures 3–6 set $R_1 = R_2 = S = 1$. Figure 3 illustrates for $m = 1$, where f_1 decreases, F_1 increases marginally toward a constant and u_1 decreases more severely than U_1. The logarithm to base 10 of the variables is chosen along the vertical axis to account for the considerable differences.

For between-group fighting the difference is even more dramatic. G_1 approaches a constant as n_1 increases for the P&C model. In contrast, g_1 decreases by $1/n_1^{3+2m}$. Taking the ratio of within-group to between-group

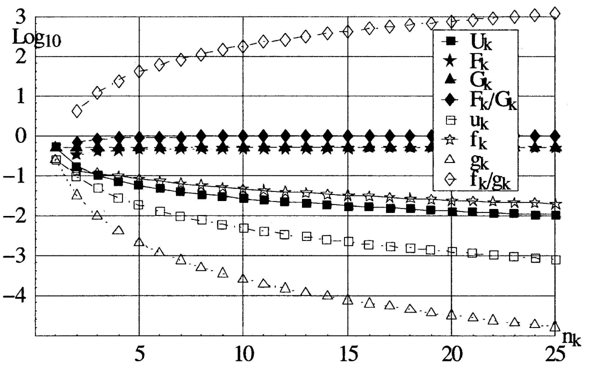

Figure 3. Variables for two groups as functions of group size $n_k = n_1 = n_2$.

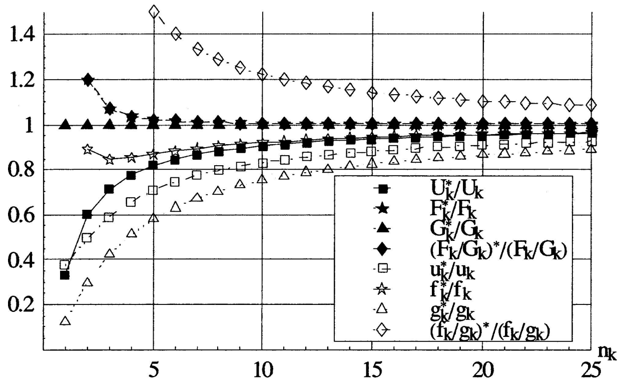

Figure 4. Variables for two groups as functions of $n_k = n_1 = n_2$, adding one group member.

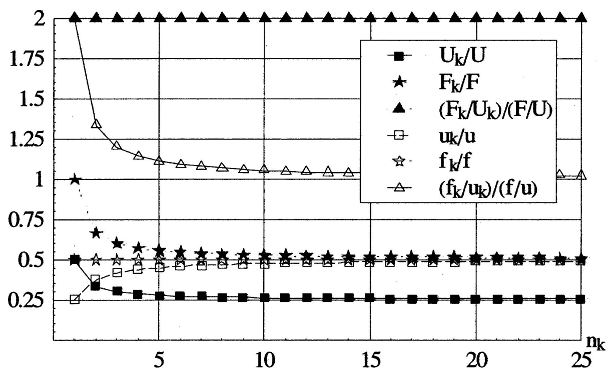

Figure 5. Comparing the variables for two groups and one group as functions of $n_k = n_1 = n_2$.

fighting, F_1/G_1 increases marginally toward a constant as n_1 increases, while f_1/g_1 increases quadratically in n_1, see Figures 3, 7 and 8. Whereas within-group and between-group fighting are comparably sized for the P&C model, between-group fighting is negligible compared with within-group fighting in

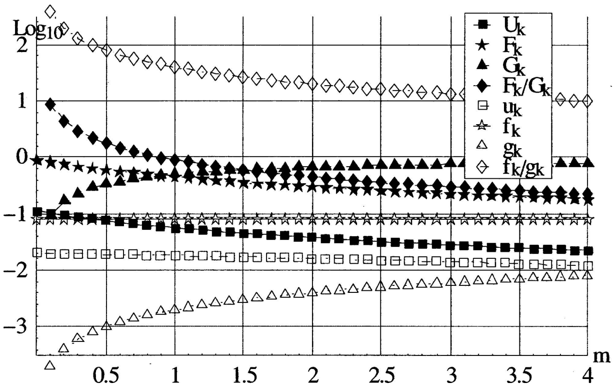

Figure 6. Variables for two groups as functions of between-group decisiveness m.

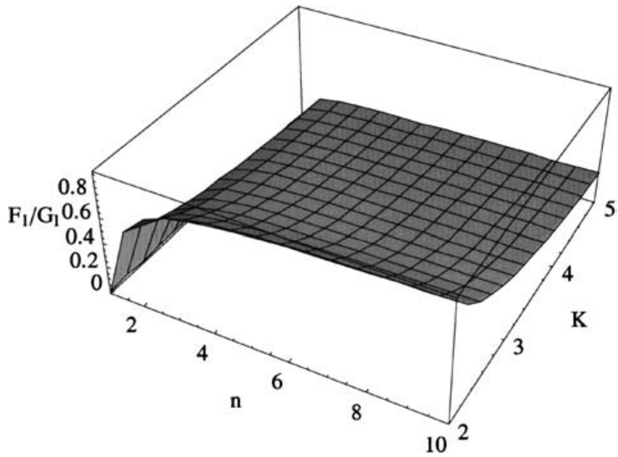

Figure 7. The ratio F_1/G_1 as function of K and n for $m = m_1 = a = b = c = 1$.

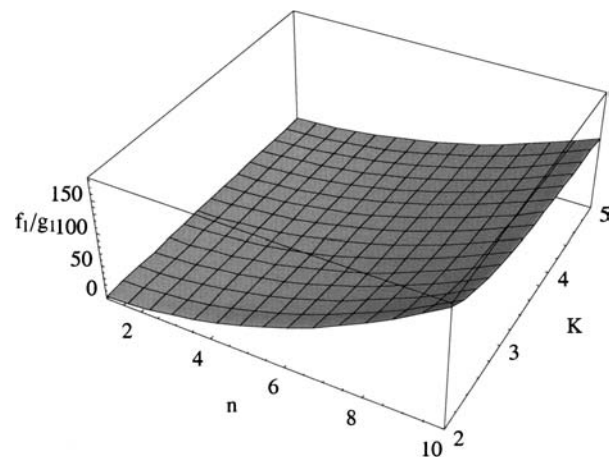

Figure 8. The ratio f_1/g_1 as function of K and n for $m = m_1 = b = c = 1$.

the RS model, e.g., $n_1 = n_2 = 10$ gives $F_1/G_1 = 18/19$ and $f_1/g_1 = 180$. That agents in the RS model cut back so severely on between-group fighting g_1 illustrates a free-rider problem for each agent in the collective conflict with the other group. No agent takes on a noticeable burden g_1 of bringing home a ratio of the rent S to his group, but takes on some burden f_1 to ensure a personal ratio of his group's ratio of the rent. The distinction between the two levels of conflict gives a more explicit account of Olson's observation of free-riding (Olson, 1965) in a large group, and of the similar observation in the collective rent-seeking literature where the two levels of conflict are effectually considered as one, i.e, each agent free-rides in his collective fight g_1 with the other group, while his fighting f_1 with his fellow group members is more a tradeoff of getting a large ratio and avoiding costly within-group fighting.

Although free-riding is also there in the P&C model, it is not that detrimental. Whereas the RS model assumes a fixed external rent independent of the number of agents fighting for it, in the P&C model every new agent is a poten-tial, and in fact an actual, producer. A crucial question in the very fast grow-ing rent-seeking literature is whether the rent gets dissipated by rent-seeking. Table 1 shows that rent dissipation may easily happen giving negative u and u_k for quite standard parameter values, when m_k is a bit large, or m is a bit large. Adding another agent in a RS model is thus a considerable liability, perhaps reminiscent of adding another person to a sinking ship, where the fixed rent S is to be shared with yet another person. In the production and conflict literature the notion of "dissipation of production" or "dissipation of produced goods" has not been introduced, though authors have compared the lower utility with fighting with the higher utility without fighting. The utilities U and U_1 in Table 1 are never negative, and they only approach zero asymptotically when the parameters take on extreme values. Adding another agent in a P&C model is also a liability, but a far smaller one since the new agent is also a producer.
This is illustrated in Figure 4 where U_k^*/U_k for the P&C model lies above u_k^*/u_k for the RS model. The * signifies that one agent is added. All the eight ratios in Figure 4 approach one asymptotically. The
decrease of F_k^*/F_k toward one and increase of f_k^*/f_k toward one have the same explanation as for the one- group models.

We have observed that adding an additional agent to each of two groups is more detrimental to the utilities in RS groups than in P&C groups. Adding a second group of agents when there is already one group of agents, however, gives the reverse result of being more detrimental to the utilities in P&C groups than in RS groups. The short explanation is that G_1 is much larger and costly in the P&C model than is g_1 in the RS model. More explicitly, in the RS model the only focus of each agent is whether to fight internally (f_1) or externally (g_1). The rent is provided for free, like manna from heaven. There is no concern where the rent comes from. It arrives mysteriously on the scene, and the agents fight for it. They fight fiercely for it within each group. Adding another group reduces the within-group fighting f_1 and only marginally increases the between-group fighting (g_1 is low), which is beneficial. Agents are not vulnerable for exploitation of something they have produced, since they do not produce, and there is nothing in the other group that the agents want to appropriate. The only focus of the agents is the fixed rent external to both groups. In contrast, in the P&C model each agent has a third concern of production (E_1) in addition to fighting internally (F_1) and externally (G_1). Without production the agents have nothing to fight for. The agents have to produce (E_1) to avoid zero utility. Production makes an agent vulnerable for exploitation by other agents who may not produce. Hence each agent has to fight internally (F_1) with other agents in his group. However, if there is a second group, agents also have to fight externally (G_1) as a collective with this group. We have shown that this external fighting G_1 with the other group is much fiercer than the external fighting g_1 in the RS model, especially as group size n_1 increases, i.e., F_1/G_1 is close to unity while f_1/g_1 typically increases convexly/dramatically in n_1. In the P&C model, each agent prefers to acquire something explicit in the other group, i.e. production or produced goods, and prefers to avoid that agents in the other group acquire one's own production. Although the total production is placed in a common pool for capture, this common pool is generated by each group, and is not external to the groups in the sense that a rent is external. This generates a large G_1.[11] Figure 5 illustrates the difference. For groups above a certain size the parameter values reduce internal fighting to ca. 50% for both models when adding a second group. The sizable and negligible between-group fighting G_1 and g_1 reduce the utilities to 25% for the P&C model and 50% for the RS model.

Between-group fighting G_1 and g_1 increase, and utilities U_1 and u_1 decrease in the between-group decisiveness m. Within-group fighting F_1 decreases while f_1 decreases, is constant or increases in m, see Figure 6 where $n_1 = n_2 = 5$. Inspecting Table 1 reveals that the within-group decisiveness m_k operates similarly in the two-group models and in the one-group models, as discussed under the section *comparing the one-group production and conflict model and rent-seeking model*. Although adding an agent or a group is never beneficial from an agent's genitive point of view in the two models, reducing both the between-group and within-group decisiveness m and m_k to zero, $m = m_k = 0$, causes an agent in the P&C model to be indifferent w.r.t. adding an agent or a group. The reason is that all fighting ceases, $G_1 = F_1 = 0$, and each agent can enjoy his own production $E_k = R_k/a_k$ undisturbed. In the RS model this is not the case. Although $m = m_k = 0$ causes all fighting to cease, $g_1 = f_1 = 0$, adding an agent or a group inevitably gives a smaller ratio of the fixed sized rent to each agent. Increasing b_k causes lower F_k and f_k, increasing c_k causes lower G_k and g_k and increasing a_k causes lower production E_k. Appendix A considers three corner solutions when m_k, m and a_k take extreme values.

Table 2. Equilibrium variable solutions for the K-group P&C model and RS model

Production and conflict model	Rent seeking model
$U_1 = \dfrac{R}{a[1+(K-1)m][1+K(n-1)m_1]}$	$u_1 = \dfrac{[Kn-(K-1)m][n-m_1(n-1)]S}{K^2n^3}$
$F_1 = \dfrac{K(n-1)m_1R}{b[1+(K-1)m][1+K(n-1)m_1]}$	$f_1 = \dfrac{(n-1)m_1S}{bKn^2}$
$G_1 = \dfrac{(K-1)mR}{c[1+(K-1)m]}$	$g_1 = \dfrac{(K-1)m[n-m_1(n-1)]S}{cK^2n^3}$
$\dfrac{F_1}{G_1} = \dfrac{cK(n-1)m_1}{b(K-1)m[1+K(n-1)m_1]}$	$\dfrac{f_1}{g_1} = \dfrac{cKn(n-1)m_1}{b(K-1)m[n-m_1(n-1)]}$
$\dfrac{U_{1,n+1}}{U_1} = \dfrac{[1+K(n-1)m_1]}{[1+Knm_1]}$	$\dfrac{u_{1,n+1}}{u_1} = \dfrac{n^3[K(n+1)-(K-1)m][n+1-m_1n]}{(n+1)^3[Kn-(K-1)m][n-m_1(n-1)]}$
$\dfrac{F_{1,n+1}}{F_1} = \dfrac{n[1+K(n-1)m_1]}{(n-1)[1+Knm_1]}$	$\dfrac{f_{1,n+1}}{f_1} = \dfrac{n^3}{(n-1)(n+1)^2}$
$\dfrac{G_{1,n+1}}{G_1} = 1$	$\dfrac{g_{1,n+1}}{g_1} = \dfrac{n^3[n+1-m_1n]}{(n+1)^3[n-m_1(n-1)]}$
$\dfrac{U_{1,K+1}}{U_1} = \dfrac{[1+(K-1)m][1+K(n-1)m_1]}{[1+Km][1+(K+1)(n-1)m_1]}$	$\dfrac{u_{1,K+1}}{u_1} = \dfrac{K^2[(K+1)n-Km]}{(K+1)^2[Kn-(K-1)m]}$
$\dfrac{F_{1,K+1}}{F_1} = \dfrac{(K+1)[1+(K-1)m][1+K(n-1)m_1]}{K[1+Km][1+(K+1)(n-1)m_1]}$	$\dfrac{f_{1,K+1}}{f_1} = \dfrac{K}{K+1}$
$\dfrac{G_{1,K+1}}{G_1} = \dfrac{K[1+(K-1)m]}{(K-1)[1+Km]}$	$\dfrac{g_{1,K+1}}{g_1} = \dfrac{K^3}{(K-1)(K+1)^2}$

6. Comparing the K-Group Production and Conflict Model and Rent-Seeking Model

For the K-group P&C model the FOCs (Appendix B) imply the solution in Table 2. For the K-group RS model calculating $\partial u_{1i}/\partial f_{1i} = 0$, $\partial u_{1i}/\partial g_{1i} = 0$, and setting $f_{1i} = f_1$, $g_{1i} = g_1$ in equilibrium causing $u_{ki} = u_k$, gives the solution in Table 2. Permuting the indices gives the group 2 expressions. Table 2 reduces to the one-group model in Table 1 when $K = 1$, $n = n_1$, $a = a_1$, $b = b_1$, $c = c_1$. One group naturally gives $G_1 = g_1 = 0$. Within-group fighting F_1 and f_1 decrease with $1/K$ for both models. Production across all groups in the P&C model causes between-group fighting G_1 to increase marginally in K toward a constant, while the fixed rent in the RS model reduces g_1 by $1/K$. The ratio F_1/G_1 decreases by $1/K$, while f_1/g_1 decreases toward a constant as K increases, as illustrated in Figures 7 and 8 as functions of the number K of groups and the number n of agents in each group, where $m = m_1 = a = b = c = 1$. The severe between-group fighting G_1 in the P&C model for many groups has a detrimental impact on the utility U_1

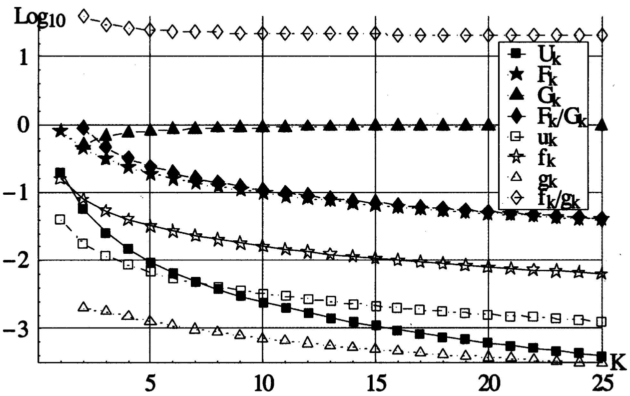

Figure 9. Variables for K groups as functions of the number K of groups, $n = 5$.

which decreases by $1/K^2$ while u_1 decreases by $1/K$. This surprising effect makes it more beneficial to raise the number K of groups in the RS model than in the P&C model. Figure 9 illustrates where $R = S = 1$. With $n = 5$ agents in each group, u_1 is low in the RS model for $K = 1$ and $K = 2$, Figures 1 and 3. As K increases, the utilities U_1 and u_1 become comparable for $K \approx 7$ where the curves cross, i.e., the P&C model is preferable for few groups, while the RS model is preferable for many groups.

7. Intergroup Migration

Setting $U_1 = U_2$ and $n_1 + n_2 = N$ for the P&C model, Table 1 implies

$$\frac{n_1}{n_2} = \left(\frac{c_2 a_1}{c_1 a_2}\right)^{\frac{m}{m+1}}, \quad n_1 = \frac{(c_2 a_1)^{\frac{m}{m+1}}}{(c_1 a_2)^{\frac{m}{m+1}} + (c_2 a_1)^{\frac{m}{m+1}}} N,$$
$$n_2 = \frac{(c_1 a_2)^{\frac{m}{m+1}}}{(c_1 a_2)^{\frac{m}{m+1}} + (c_2 a_1)^{\frac{m}{m+1}}} N. \tag{7.1}$$

Consistent with the result of Hausken (2000b), n_1/n_2 decreases in the productive efficiency $1/a_1$, and increases in the between-group fighting efficiency $1/c_1$. Increasing m increases the group size disparity. Agents leave the group with high productive efficiency, and migrate to the group with high between-group fighting efficiency.

Setting $u_1 = u_2$ for the RS model, Table 1 implies

$$\frac{n_1 A_2 (c_1 A_2)^m}{n_2 A_1 (c_2 A_1)^m} = \frac{(n_1 - m)(c_1 A_2)^m + n_1 (c_2 A_1)^m}{(n_2 - m)(c_2 A_1)^m + n_2 (c_1 A_2)^m}. \tag{7.2}$$

To facilitate analytical solution, inserting $n_k \gg m$ gives[12]

$$\frac{A_1}{A_2} \approx \left(\frac{c_1}{c_2}\right)^{\frac{m}{m+1}} \Rightarrow \left\{ \frac{n_1}{n_2} \approx \left(\frac{c_2}{c_1}\right)^{\frac{m}{2m+2}} \text{ when } m_k = 1, \right.$$

$$\left. \frac{n_1}{n_2} \approx \left(\frac{c_2}{c_1}\right)^{\frac{m}{m+1}} \text{ when } m_k = 0 \right\}. \tag{7.3}$$

With indecisive within-group fighting $m_k = 0$ and equal productive efficiencies ($a_1 = a_2$), the two models operate equivalently. Within-group fighting is then dispensed with, and although between-group fighting differ, the group size ratio n_1/n_2 is the same. With decisive within-group fighting $m_k = 1$, the ratio n_1/n_2 in Equation (7.3) is the square root of the ratio in Equation (7.1). Hence a superior between-group fighting efficiency $1/c_k$ operates more efficiently in the P&C model, reducing the group size of the other group more severely than in the RS model. Production is generated within each group, while the rent is external to the two groups causing no group to be inferior or superior to the other with respect to production. Agents in an unproductive group focus strongly on between-group fighting, which has a detrimental effect on the productive group, causing more flight from the productive group, and hence smaller group size for the productive group than in a RS model with no differential production. Hence group sizes n_1 and n_2 tend to differ more in a P&C model than in a RS model.

8. Inside Versus Outside Ownership

If there is no outside owner, there is one group of n_1 agents who only fight one another. If there is an outside owner, there are two groups, and the n_1 members of the firm fight the outside owner, which is a group with n_2 agents, as well as one another. To start simplistically, consider one outside owner, i.e. $n_2 = 1$, not involved in production, i.e. $a_2 = \infty$ which implies $E_2 = 0$ and $b_2F_2 + c_2G_2 = R_2$. The total deadweight loss of fighting for the P&C model is $D_O = n_1(b_1F_1 + c_1G_1) + n_2(b_2F_2 + c_2G_2)$.[13] Inserting from Table 1 gives

$$\begin{aligned}
\lim_{\substack{a_2\to\infty \\ n_2\to 1}} D_O &= \lim_{\substack{a_2\to\infty \\ n_2\to 1}} \{n_1(b_1F_1 + c_1G_1) + n_2R_2\} \\
&= \lim_{\substack{a_2\to\infty \\ n_2\to 1}} \left\{ \frac{1}{(m+1)} \left[\frac{a_1(n_1-1)m_1\left[\frac{n_1R_1}{a_1} + \frac{n_2R_2}{a_2}\right]}{[1+(n_1-1)m_1+(n_2-1)m_2]} \right.\right. \\
&\quad \left.\left. + \frac{a_1m(c_1a_2)^{\frac{m}{m+1}}\left[\frac{n_1R_1}{a_1} + \frac{n_2R_2}{a_2}\right]}{(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}} \right] + n_2R_2 \right\} \\
&= \frac{n_1(n_1-1)m_1R_1}{[1+(n_1-1)m_1]} + \frac{mn_1R_1}{(m+1)[1+(n_1-1)m_1]} + R_2. \qquad (8.1)
\end{aligned}$$

For inside ownership the total deadweight loss of fighting is $D_I = n_1 bF$,which when inserting F from Table 1 gives the first of the three terms in Equation (8.1). Inside ownership is unconditionally preferable for the P&C model, evidenced historically by the need for antitrust legislation.

This contrasts Müller and Wärneryd's claim that one outside owner (not involved in production) is always preferable to inside ownership for rent-seeking firms (Müller and Wärneryd, 2001). Their observation is that outside ownership, through adding a second level of conflict, mitigates distributive conflict within firms. We have seen that this occurs through an unreasonably high ratio f_1/g_1 of within-firm to between-firm fighting. Inserting from Table 1 the total deadweight loss d_O of fighting under outside ownership for the RS model is

$$\begin{aligned} d_O &= n_1(b_1 f_1 + c_1 g_1) + n_2(b_2 f_2 + c_2 g_2) \\ &= \frac{\left[\frac{m_1(n_1-1)(c_2A_1)^m}{n_1} + \frac{m_2(n_2-1)(c_1A_2)^m}{n_2} + \frac{m(c_1A_2)^m(c_2A_1)^m[A_1+A_2]}{[(c_1A_2)^m+(c_2A_1)^m]}\right]S}{(c_1A_2)^m + (c_2A_1)^m}. \end{aligned} \tag{8.2}$$

Inserting f from Table 1, the loss under inside ownership is $d_I = n_1 b_1 f|_{\beta_1=1} = m_1(n_1-1)S/n_1$, where $\beta_1 = 1$, since group 1 enjoys the entire rent S. This gives advantage to outside over inside ownership when

$$d_O - d_I = \frac{(c_1A_2)^m\left[-\frac{m_1(n_1-1)}{n_1} + \frac{m_2(n_2-1)}{n_2} + \frac{m(c_2A_1)^m[A_1+A_2]}{[(c_1A_2)^m+(c_2A_1)^m]}\right]S}{(c_1A_2)^m + (c_2A_1)^m} < 0, \tag{8.3}$$

which when $c_1 = c_2 = 1$ simplifies to

$$\frac{m_1(n_1-1)}{n_1} - \frac{m_2(n_2-1)}{n_2} > \frac{mn_2^{2m-2}\left[n_1^2+n_2^2\right]}{n_1^2\left[n_1^{2m}+n_2^{2m}\right]}. \tag{8.4}$$

Hence outside ownership is preferable to inside ownership when the within-firm decisiveness m_1 in group 1 is large, and m is small or large which reduces the RHS of Equation (8.4), and not intermediate. Hausken (2002a) analyzes for general n_1, n_2, m_2 and m. Equation (8.4) simplifies to Müller and Wärneryd's inequality[14] (Müller & Wärneryd, 2001:533) when $m_2 = m = 1$, and to $n_1^2 > n_1 + 1$ when $n_2 = 1$, which is always satisfied when $n_1 > 1$, favoring outside ownership for these parameter values.

9. Divestitures

For the P&C model, the total deadweight loss D_D of fighting in two separate firms is

$$D_D = n_1 b_1 F_{1D} + n_2 b_2 F_{2D} = \frac{n_1(n_1-1)m_1R_1}{[1+(n_1-1)m_1]} + \frac{n_2(n_2-1)m_2R_2}{[1+(n_2-1)m_2]}, \tag{9.1}$$

where F_{kD} is given by F in Table 1 for the two firms. Divestiture is preferable when

$$\begin{aligned}D_O - D_D = {} & \frac{[a_1(n_1-1)m_1 + a_2(n_2-1)m_2]\left[\frac{n_1R_1}{a_1}+\frac{n_2R_2}{a_2}\right]}{(m+1)[1+(n_1-1)m_1+(n_2-1)m_2]} \\ & + \frac{m\left[a_1(c_1a_2)^{\frac{m}{m+1}} + a_2(c_2a_1)^{\frac{m}{m+1}}\right]\left[\frac{n_1R_1}{a_1}+\frac{n_2R_2}{a_2}\right]}{(m+1)\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right]} \\ & - \frac{n_1(n_1-1)m_1R_1}{[1+(n_1-1)m_1]} - \frac{n_2(n_2-1)m_2R_2}{[1+(n_2-1)m_2]} \end{aligned} \tag{9.2}$$

is positive, where inserting $a_1 = a_2$ gives

$$\begin{aligned}D_O - D_D = {} & \frac{m[n_1R_1+n_2R_2]}{(m+1)[1+(n_1-1)m_1+(n_2-1)m_2]} \\ & + \frac{\{(n_2-1)m_2[1+(n_2-1)m_2]n_1R_1 + (n_1-1)m_1[1+(n_1-1)m_1]n_2R_2\}}{[1+(n_1-1)m_1+(n_2-1)m_2][1+(n_1-1)m_1][1+(n_2-1)m_2]}, \end{aligned} \tag{9.3}$$

which is always positive favoring divestiture, as between-division fighting is quite costly. This presupposes that the two newly created firms exist in isolation from each other, e.g. by serving different markets, so that no costly fighting occurs between the firms.

For the RS model, using Table 1, the total loss d_D of fighting in two separate firms is

$$\begin{aligned}d_D = n_1b_1f_1 + n_2b_2f_2 & = \frac{m_1(n_1-1)S}{\beta(n_1+n_2)} + \frac{m_2(n_2-1)S}{\beta(n_1+n_2)} \\ & = \frac{[m_1(n_1-1)+m_2(n_2-1)]S}{n_1+n_2}, \end{aligned} \tag{9.4}$$

where each firm k enjoys a rent $\beta_k S = n_k S/(n_1+n_2)$. Divestiture is preferable when

$$d_O - d_D = \frac{\left[\left[\frac{m_1(n_1-1)}{n_1} - \frac{m_2(n_2-1)}{n_2}\right]\frac{[n_2(c_2A_1)^m - n_1(c_1A_2)^m]}{(n_1+n_2)} + \frac{m(c_1A_2)^m(c_2A_1)^m[A_1+A_2]}{[(c_1A_2)^m+(c_2A_1)^m]}\right]S}{[(c_1A_2)^m+(c_2A_1)^m]} > 0, \tag{9.5}$$

which is satisfied when $m_1 = m_2$ and $n_1 = n_2$ since the first bracket in the numerator equals zero. Comparable within-group decisiveness and sizes of the two divisions justifies divestiture. Letting division 2's size n_2 approach infinity, we have $\mathrm{Lim}_{n_2\to\infty} A_2 = 0$, and Equation (9.5) becomes

$$\operatorname*{Lim}_{n_2\to\infty}(d_O - d_D) = \left[\frac{(m_1-m_2)n_1 - m_1}{n_1}\right]S\beta > 0 \quad \text{when } m_1 > \frac{m_2n_1}{(n_1-1)}, \tag{9.6}$$

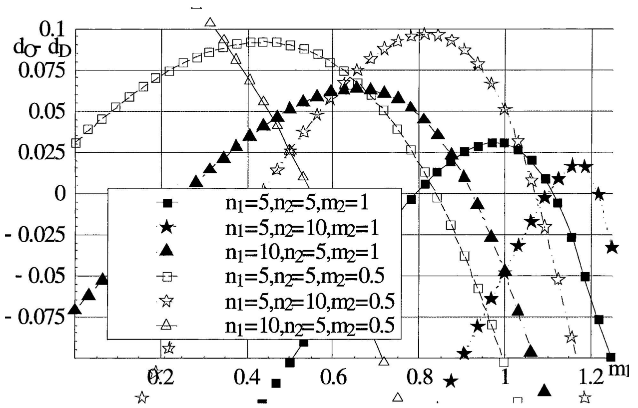

Figure 10. Divestitures: $d_O - d_D$ as a function of m_1.

which is not satisfied when $m_1 = m_2$, compatibly with Inderst et al.'s analysis (Inderst et al., 2002). However, Equation (9.6) is satisfied when m_1 is somewhat larger than m_2. More generally, the mathematical logic of the first two brackets in Equation (9.5) is such that $d_O - d_D$ typically exhibits an in-verse U form, e.g. as a function of m_1, see Figure 10. The inverse U shifts leftwards toward lower m_1 as division 1's size n_1 increases, or m_2 decreases. A large division size n_1 is counterbalanced with a low within-group decisiveness m_1 to justify divestiture. Increasing group size n_1 exacerbates the public goods problem, reducing the expenditure on the interdivision conflict, unless the within-group decisiveness m_1 is reduced accordingly to reduce the expenditure on the intradivision conflict.[15]

10. Mergers and Acquisitions

Consider two firms of sizes n_1 and n_2. Using for the P&C model the utilities $U = U_{1m}$ (substituting n_1 with $n_1 + n_2$) and $U_1 = U_{1nm}$ from Table 1, an agent in firm 1 prefers the merger if

$$U_{1m} - U_{1nm} = \frac{R_1}{a_1[1 + (n_1 + n_2 - 1)m_1]}$$

$$- \frac{(c_2a_1)^{\frac{m}{m+1}}\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right]^{-1}\sum_{k=1}^{2}\frac{n_kR_k}{a_k}}{(m+1)n_1[1 + (n_1 - 1)m_1 + (n_2 - 1)m_2]} > 0 \Rightarrow$$

$$m_1 < \frac{R_1[1 + (n_2 - 1)m_2]n_1(m+1)\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right] - a_1\left[\frac{n_1R_1}{a_1} + \frac{n_2R_2}{a_2}\right](c_2a_1)^{\frac{m}{m+1}}}{a_1\left[\frac{n_1R_1}{a_1} + \frac{n_2R_2}{a_2}\right](c_2a_1)^{\frac{m}{m+1}}(n_1 + n_2 - 1) - R_1(n_1 - 1)n_1(m+1)\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right]}, \tag{10.1}$$

i.e. if the within-firm fighting after the merger is indecisive (m_1 is low) or the between-firm fighting before the merger is decisive (m is high).[16]

Using for the RS model the utilities $u = u_{1m}$ (substituting n_1 with $n_1 + n_2$ and S with S_M for monopoly rent) and $u_1 = u_{1nm}$, from Table 1 an agent in firm 1 prefers the merger if

$$\begin{aligned} u_{1m} - u_{1nm} &= \frac{n_1 + n_2 - m_1(n_1 + n_2 - 1)}{(n_1 + n_2)^2} S_{\mathrm{M}} \\ &- \left[\frac{(n_1 - m)(c_1 A_2)^m + n_1 (c_2 A_1)^m}{n_1[(c_1 A_2)^m + (c_2 A_1)^m]^2} \right] A_1 (c_2 A_1)^m S > 0, \end{aligned} \quad (10.2)$$

where inserting $m_1 = m_2, n_1 = n_2, c_1 = c_2$ gives

$$\begin{aligned} m_1 &< \frac{2n_1\left[1 - \left(1 - \frac{m}{2n_1}\right)\frac{S}{S_{\mathrm{M}}}\right]}{\left[(2n_1 - 1) - 2(n_1 - 1)\left(1 - \frac{m}{2n_1}\right)\frac{S}{S_{\mathrm{M}}}\right]} \\ \Leftrightarrow m &> 2n_1\left(1 - \frac{[2n_1 - m_1(2n_1 - 1)]S_{\mathrm{M}}}{2[n_1 - m_1(n_1 - 1)]S}\right). \end{aligned} \quad (10.3)$$

If the RHS of the second expression in Equation (10.3) is negative, which happens when the within-firm decisiveness m_1 is large, an agent in firm 1 does not prefer the merger. Figure 11 illustrates, assuming very small firm sizes $n_1 = n_2 = 2$, $S_{\mathrm{M}} = S$. The curve for the P&C model approaches $1/(2(n_1 - 1))$ as m_1 approaches infinity, which is a small number, especially as the firm size n_1 increases, suggesting frequent merger preference. In contrast, the curve for the RS model increases convexly,and more so as the firm size n_1 increases, eventually approaching a vertical asymptote when the denominator on the RHS of the right equation in Equation (10.3) equals zero, suggesting frequent nonmerger preference. The differences between the models become more pronounced when the firm sizes $n_1 = n_2$ increase. Figure 12 illustrates as a function of m_1.[17,18]

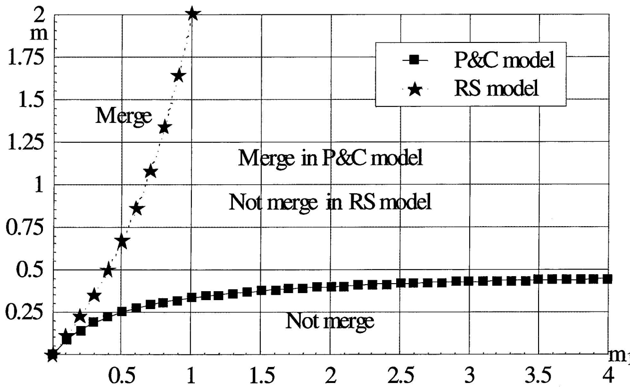

Figure 11. Mergers and acquisitions: P&C model versus RS model for $n = 2$, $S = S_{\mathrm{M}}$.

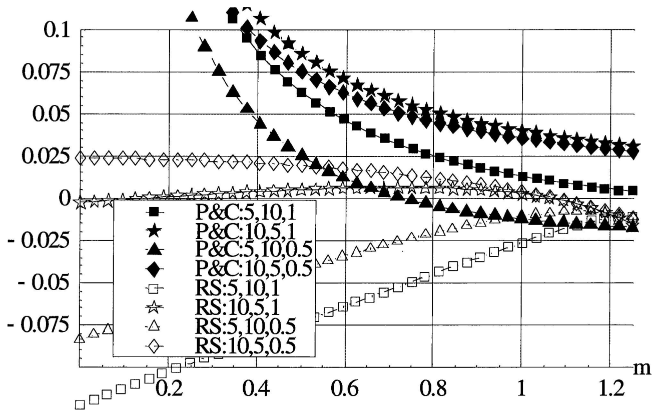

Figure 12. $U_{1m} - U_{1nm}$ (P&C) and $u_{1m} - u_{1nm}$ (RS) as functions of m; 5, 10, 1 means $n_1 = 5, n_2 = 10, m_2 = 1$.

11. Multidivisional Firms Versus Single-Tier Firms

The total cost of fighting in a multidivisional (MD) firm with K divisions and n agents in each division is $C_{\text{P\&C}} = Kn[bF_1 + cG_1]$ for the P&C model. In a single-tier (ST) firm with Kn agents the cost is $C_{\text{P\&Cs}} = KnbF$. Inserting from Tables 1 and 2 gives the difference

$$C_{\text{P\&C}} - C_{\text{P\&Cs}} = Kn\left[\frac{K(n-1)m_1}{[1+(K-1)m][1+K(n-1)m_1]} + \frac{(K-1)m}{[1+(K-1)m]} - \frac{(Kn-1)m_1}{[1+(Kn-1)m_1]}\right]R. \tag{11.1}$$

For the RS model the costs are $C_{\text{RS}} = Kn[bf_1 + cg_1]$ and $C_{\text{RSs}} = Knbf$, with difference

$$C_{\text{RS}} - C_{\text{RSs}} = \frac{(K-1)[m[n-m_1(n-1)] - m_1 n]S}{Kn^2}. \tag{11.2}$$

Figure 13 illustrates as a function of K, where $n = 5$. The P&C model typically favors the ST firm over the MD firm. The reverse is often the case for the RS model, e.g., $m = m_1$ causes $C_{\text{RS}} - C_{\text{RSs}}$ to be negative, as Inderst et al. (2002) also find, favoring the ST firm. The cost difference between MD firms and ST firms is typically larger for the P&C model than for the RS model, mainly due to the larger cost of between-division fighting in the P&C model. The cost difference is especially large when the between-division decisiveness m is large and the within-division decisiveness m_1 is small, which makes the ST firm favorable even for the RS model.

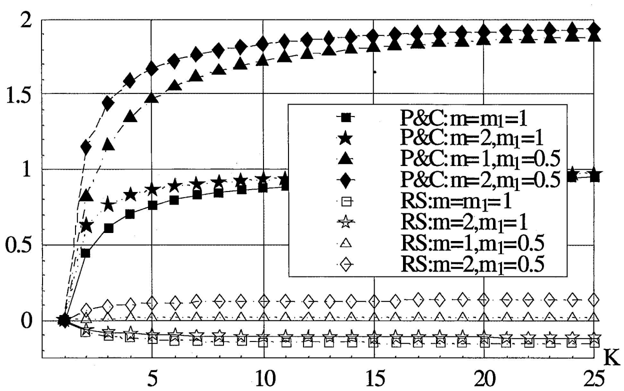

Figure 13. MD firms versus ST firms: $C_{\text{P\&C}} - C_{\text{P\&Cs}}$ and $C_{\text{RS}} - C_{\text{RSs}}$ for various m and m_1, $n_1 = 5$.

12. U Form Versus M Form of Economic Organization

Chandler (1966), Williamson (1975) and Inderst et al. (2002) analyze the merits of the U form versus M form of economic organization. The U form assumes K divisions or functions or groups such as manufacturing, marketing and sales. Each division is subdivided into n products or brands or regions. In contrast, the M form organizes products into divisions and functions into subdivisions. The cost of fighting under the two organizational forms can be determined. For the U form the cost is $C_{\text{P\&C}} = Kn[bF_1(K,n)+cG_1(K,n)]$ for the P&C model. For the corresponding M form, obtained by interchanging K and n in the expressions for F_1 and G_1 in Table 2, the cost is $Kn[bF_1(n,K)+cG_1(n,K)]$. The difference $C_{\text{P\&CUM}}$ in cost between the U form and M form is

$$\begin{aligned} C_{\text{P\&CUM}} &= Kn[bF_1(K,n) + cG_1(K,n) - bF_1(n,K) - cG_1(n,K)] \\ &= \frac{(K-n)Kn[m(1+(K-1)(n-1)m_1) - m_1]R}{[1+(K-1)m][1+K(n-1)m_1][1+(n-1)m][1+(K-1)nm_1]}, \end{aligned} \tag{12.1}$$

which depends crucially on the sign of $(K-n)$. For the RS model the corresponding expression is

$$\begin{aligned} C_{\text{RSUM}} &= Kn[bf_1(K,n) + cg_1(K,n) - bf_1(n,K) - cg_1(n,K)] \\ &= \frac{(n-K)[Knm_1 + ((K-1)(n-1)m_1 - Kn)m]S}{K^2n^2}, \end{aligned} \tag{12.2}$$

which depends crucially on the sign of $(n-K)$. Figure 14 illustrates as a function of K, for $n = 5$. The P&C model almost always gives preference for

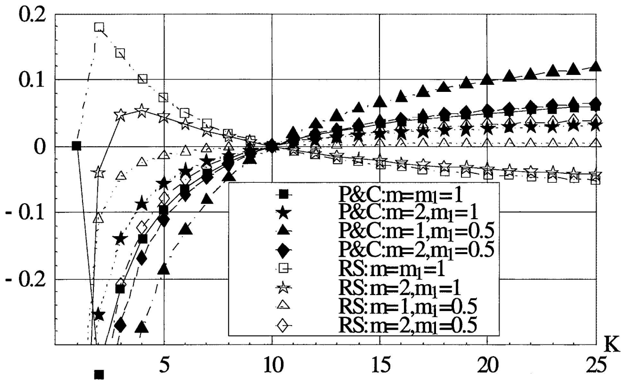

Figure 14. U form versus M form: $C_{\text{P\&CUM}}$ and C_{RSUM} for various m and m_1, $n_1 = 10$.

the M form when $K > n$ so that the firm has many functions K (divisions) and few brands n (units within each division).[19] The main reason is that within-division fighting F_1 decreases with $1/K$, while between-division fighting G_1 increases marginally in K, toward a constant. Hence many divisions K reduce the overall cost of fighting, which may explain the proliferation of the M form with many divisions.The reverse is often the case for the RS model, e.g.,$m = m_1$ causes C_{RSUM} to be negative when $K > n$, as Inderst et al. (2002) also find,[20] favoring the U form. Although both within-division and between-division fighting f_1 and g_1 decrease with $1/K$, both f_1 and g_1 decrease with $1/n^2$. Hence, few divisions K with many products n within each division is often most cost-efficient. The exception, which is quite likely, is when m_1 is low and m is above a certain level, see Equation (12.2). Egalitarian within-group distribution allows the number n of products to increase without detrimental effects. In this case within-division fighting f_1 is low anyway, and increasing K is most cost efficient for $K > n$.

13. Conclusion

This article compares the production and conflict (P&C) model and the rent-seeking (RS) model for agents in one group, two groups and K equally large groups. In the P&C model each agent allocates his resource between production and fighting, and in the RS model each agent incurs a cost of rent-seeking (fighting). The total production or rent is distributed within and between groups according to the within-group and between-group decisiveness. Both models distinguish between how much each agent fights within and between groups, and both productive and fighting efficiencies and group sizes play a role.

The second and third sections present the models. In the fourth section the FOCs are determined and the models are compared for one group. Adding a new agent enlarges the pie in the P&C model, but causes the fixed size pie to be allocated on one more rent seeker in the RS model. The collective action problem is more detrimental in the RS model. The utility decreases more severely in group size in the RS model than in the P&C model, but both approach zero. Increasing group size causes fighting to increase toward a constant in the P&C model, enabled by concomitant production, and causes fighting to decrease toward zero in the RS model, caused by the constraint of the rent. Increasing the within-group decisiveness increases fighting asymptotically toward a constant and decreases the utility asymptotically toward zero in the P&C model, while these increases and decreases are linear in the RS model, the latter eventually going negative.

The fifth section compares the models for two groups. The collective action problem is even more severe for the RS model, within-group fighting and utility decreasing more quickly toward zero as group size increases. As the group size increases, the ratio of within-group to between-group fighting realistically increases marginally toward a constant for the P&C model, while it increases convexly for the RS model, which causes unrealistically little between-group fighting. That no one takes on a noticeable burden of bringing home a ratio of the rent to his group in the RS model constitutes the biggest difference between the two models. One implication is that adding an additional agent to each of two groups is more detrimental to the utilities in RS groups than in P&C groups, while adding a second group of agents when there is already one group of agents gives the reverse result.

The sixth section compares the models for K equally large groups, which is analytically tractable. Within-group fighting decreases with $1/K$ for both models. Between-group fighting increases marginally in K toward a constant, and in the RS model decreases by $1/K$. The severe between-group fighting in the P&C model for many groups causes the P&C model to be preferable for few groups, while the RS model is preferable for many groups.

Six applications are considered. For intergroup migration group sizes tend to differ more in a P&C model than in a RS model, since agents leave a productive group, and since a superior fighting efficiency is more influential in a P&C model. Whereas inside ownership is unconditionally preferable over outside ownership for the P&C model, contrasting Müller and Wärneryd's (2001) result, for the RS model it is preferable when the within-group decisiveness is small and the between-group decisiveness is intermediate. Divestiture is always preferable in the P&C model when the productive efficiencies in the two groups are equal. For the RS model divestiture is justified with comparable within-group decisiveness and sizes of the two divisions, but not otherwise, but a large division size may compensate for a low within-group decisiveness to justify divestiture. For mergers and acquisitions an agent in firm 1 in the P&C model prefers the merger if the within-firm fighting after the merger is indecisive or the between-firm fighting before the merger is decisive. For the RS model an agent in firm 1 does not prefer the merger if the between-group decisiveness is above a minimum level, recalling that the cost of between-group fighting is low. The P&C model typically favors single-tier firms. In contrast, the RS model often favors multidivisional firms, except when the between-division decisiveness is large and the within-division decisiveness is low. The P&C model almost always gives preference for the M form when the firm has many functions K (divisions) and few brands n (units within each division). The reverse is often the case for the RS model, except, which is quite likely, when the within-division decisiveness is low and the between-division decisiveness is above a certain level.[21]

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Appendix A: Analyzing the Two-Group P&C Model

Setting the derivative of U_{1i} in Equation (2.4) w.r.t. F_{1i} equal to zero, assuming identical agents in both groups so that $F_{1i} = F_1$ and $F_{2i} = F_2$ in equilibrium, gives

$$\frac{\partial U_{1i}}{\partial F_{1i}} = 0 \Rightarrow \frac{(n_1-1)m_1}{n_1^2 F_1}\left[n_1\frac{R_1 - b_1F_1}{a_1} - \sum_{i=1}^{n_1}\frac{c_1G_{1i}}{a_1} + n_2\frac{R_2 - b_2F_2}{a_2} - \sum_{i=1}^{n_2}\frac{c_2G_{2i}}{a_2}\right] - \frac{b_1}{n_1a_1} = 0. \tag{A1.1}$$

Similarly calculating $\partial U_{2i}/\partial F_{2i} = 0$, and equating the two equivalent square brackets, gives

$$\frac{F_1}{F_2} = \frac{(n_1-1)n_2a_1b_2m_1}{(n_2-1)n_1a_2b_1m_2}$$

$$F_1 = \frac{a_1(n_1-1)m_1}{n_1b_1}\frac{\left[\frac{n_1R_1}{a_1} + \frac{n_2R_2}{a_2} - \sum_{i=1}^{n_1}\frac{c_1G_{1i}}{a_1} - \sum_{i=1}^{n_2}\frac{c_2G_{2i}}{a_2}\right]}{[1+(n_1-1)m_1+(n_2-1)m_2]}, \tag{A1.2}$$

where F_2 is found by permuting the indices. Inserting F_1 and F_2 into Equation (2.4) and rearranging gives

$$U_{1i} = \frac{\left[\frac{n_1R_1}{a_1} + \frac{n_2R_2}{a_2} - \sum_{i=1}^{n_1}\frac{c_1G_{1i}}{a_1} - \sum_{i=1}^{n_2}\frac{c_2G_{2i}}{a_2}\right]\left(\sum_{i=1}^{n_1}G_{1i}\right)^m}{n_1[1+(n_1-1)m_1+(n_2-1)m_2]\left[\left(\sum_{i=1}^{n_1}G_{1i}\right)^m + \left(\sum_{i=1}^{n_2}G_{2i}\right)^m\right]}. \tag{A1.3}$$

Calculating the FOCs $\partial U_{1i}/\partial G_{1i} = 0$ and $\partial U_{2i}/\partial G_{2i} = 0$, equating the two equivalent square brackets as above, and setting $G_{ki} = G_k$ and $U_{ki} = U_k$ gives Table 1.

There are three corner solutions when $m_1 = m_2 = m_k,\ m,\ a_1 = a_2 = a_k$ take extreme values. First, with extremely decisive within-group fighting (very large m_k), the large F_k drives E_k and G_k in Equation (2.2) to zero causing $F_k = R_k/b_k$ and zero utility $U_k = 0$. Second, with extremely decisive between-group fighting (very large m), the large G_k gets constrained by Equation (2.2) to $G_k = R_k/c_k$, causing $U_k = 0$. Third, with extremely high conversion cost a_k of transforming resources R_k into productive effort E_k, agents in group k cease to produce ($E_k = 0$), and within-group and between-group fighting get constrained by Equation (2.2) to $R_k = b_kF_k + c_kG_k$, where applying F_1/G_1 in Table 1 gives

$$F_1 = \frac{((c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}})(n_1-1)m_1R_1}{b_1[((c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}})(n_1-1)m_1 + m(c_1a_2)^{\frac{m}{m+1}}[1+(n_1-1)m_1+(n_2-1)m_2]]}.$$

$$G_1 = \frac{m(c_1a_2)^{\frac{m}{m+1}}[1+(n_1-1)m_1+(n_2-1)m_2]R_1}{c_1[((c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}})(n_1-1)m_1 + m(c_1a_2)^{\frac{m}{m+1}}[1+(n_1-1)m_1+(n_2-1)m_2]]}. \tag{A1.4}$$

Appendix B: Analyzing the K-Group P&C Model

Setting the derivative of U_{1i} in Equation (2.5) w.r.t. F_{1i} equal to zero, and assuming identical agents in all groups so that $F_{1i} = F_1$ in equilibrium, gives the FOC

$$\frac{\partial U_{1i}}{\partial F_{1i}} = 0 \Rightarrow \frac{(n-1)m_1}{n^2F_1}\left[n\frac{R-bF_1}{a} - \sum_{i=1}^{n}\frac{cG_{1i}}{a} + (K-1)n\frac{R-bF_2}{a} - (K-1)\sum_{i=1}^{n}\frac{cG_{2i}}{a}\right] - \frac{b}{na} = 0. \tag{A2.1}$$

Setting $F_1 = F_2$ in equilibrium and solving w.r.t. F_1 gives

$$F_1 = F_2 = \frac{(n-1)m_1}{nb}\frac{\left[KnR - \sum_{i=1}^{n} cG_{1i} - (K-1)ncG_2\right]}{[1+K(n-1)m_1]}. \tag{A2.2}$$

Inserting $F_1 = F_2$ into Equation (A2.1) and rearranging gives

$$U_{1i} = \frac{1}{n}\frac{(\sum_{i=1}^{n} G_{1i})^m}{(\sum_{i=1}^{n} G_{1i})^m + (K-1)(nG_2)^m} \times \frac{[KnR - \sum_{i=1}^{n} cG_{1i} - (K-1)ncG_2]}{a[1+K(n-1)m_1]}. \tag{A2.3}$$

Calculating the FOC $\partial U_{1i}/\partial G_{1i} = 0$, and setting $G_{1i} = G_1 = G_2$ and $U_{1i} = U_1$ gives the results shown in Table 2.

Appendix C: The Dynamic Extension of the Production and Conflict Model

This section extends the conflict model dynamically, and more generally than Reuveny and Maxwell (2001).[22] Assume that the population in group k grows according to

$$\frac{\partial n_k}{\partial t} = \phi_k n_k U_k - \gamma_k n_k, \quad k = 1, 2, \tag{A3.1}$$

where $n_k U_k$ is the utility of group k given by Table 1, ϕ_k is a growth rate that adjusts the growth of group k due to birth associated with the utility $n_k U_k$, and γ_k is the death rate.[23] The size of group k increases if the RHS in Equation (A3.1) is positive, decreases if it is negative and is constant otherwise giving a steady state solution. There are four possible corner steady state solutions, i.e. $(n_1, n_2) = (0, 0), (0, \infty), (\infty, 0), (\infty, \infty)$, and there may or may not exist an internal steady state solution. For the latter, using Equation (A3.1) to solve $\{\partial n_1/\partial t = 0, \partial n_2/\partial t = 0\}$ gives

$$\frac{n_1}{n_2} = \frac{\phi_1 \gamma_2}{\phi_2 \gamma_1} \left(\frac{c_2 a_1}{c_1 a_2} \right)^{\frac{m}{m+1}}. \tag{A3.2}$$

The size of group 1 increases if $\partial n_1/\partial t > 0$, i.e.

$$\phi_1 \frac{\left[\frac{n_1 R_1}{a_1} + \frac{n_2 R_2}{a_2}\right](c_2 a_1)^{\frac{m}{m+1}}}{[1 + (n_1 - 1)m_1 + (n_2 - 1)m_2](m+1)\left[(c_1 a_2)^{\frac{m}{m+1}} + (c_2 a_1)^{\frac{m}{m+1}}\right]} - \gamma_1 n_1 > 0. \tag{A3.3}$$

When $m_1 = m_2 = 0$, inserting Equation (A3.2) into Equation (A3.3) eliminates n_1 and n_2 and Equation (A3.3) becomes

$$\left[\frac{R_1}{a_1}\phi_1 \gamma_2 (c_2 a_1)^{\frac{m}{m+1}} + \frac{R_2}{a_2}\phi_2 \gamma_1 (c_1 a_2)^{\frac{m}{m+1}}\right] > \gamma_1 \gamma_2 (m+1)\left[(c_1 a_2)^{\frac{m}{m+1}} + (c_2 a_1)^{\frac{m}{m+1}}\right]. \tag{A3.4}$$

Hence there is no internal solution, and the size n_1 of group 1 increases to infinity if the LHS is larger than the RHS in Equation (A3.4). This happens when the resources R_1 and R_2 increase, the conversion costs a_1 and a_2 of transforming resources into productive effort decrease, the growth rates ϕ_1 and ϕ_2 increase, the death rates γ_1 and γ_2 decrease,and the between-group decisiveness m of fighting decreases. Without within-group fighting ($m_1 = m_2 = 0$)

there is nothing to constrain unlimited growth if the parameters are otherwise beneficial. This is not the case with within-group fighting ($m_1 > 0$ and/or $m_2 > 0$). Using Equations (A3.2) and (A3.3) gives the steady state solution

$$n_1 = \frac{\left[\dfrac{\left[\frac{R_1}{a_1}\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + \frac{R_2}{a_2}\phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right]}{\gamma_1\gamma_2(m+1)\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right]} + m_1 + m_2 - 1\right]\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}}}{m_1\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + m_2\phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}}, \tag{A3.5}$$

where $\mathrm{Lim}_{m_1\to\infty} n_1 = 1$, which means that infinitely decisive fighting within group 1 reduces the group size n_1 to one agent. Permuting the indices or using Equation (A3.2) gives the size n_2 of group 2.

Contrary to Equation (A3.4) it is usually the case, also when $m_1 = m_2 = 0$, that groups do not grow without bounds. One common mechanism of resource adjustment is the logistic form

$$\frac{\partial R}{\partial t} = \xi(n_1 + n_2)R\left[1 - \frac{(n_1 + n_2)R}{K}\right], \tag{A3.6}$$

where ξ is the growth rate of the resource R, and K is the resource-carrying capacity[24]. The steady state solution of Equation (A3.6) is $R = K/(n_1 + n_2)$, where the total size $n_1 + n_2$ of the two groups grows until the resource-carrying capacity K cannot sustain further growth. Hence for a given resource R for each agent, there is a maximum number $n_1 + n_2$ of agents that can be supported by K. With $R = R_1 = R_2$, inserting $R = K/(n_1 + n_2)$, Equation (A3.2) and $m_1 = m_2 = 0$ in Equation (A3.5) gives

$$n_1 = \frac{K\phi_1\left[\frac{1}{a_1}\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + \frac{1}{a_2}\phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right](c_2a_1)^{\frac{m}{m+1}}}{\gamma_1(m+1)\left[\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + \phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right]\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right]}. \tag{A3.7}$$

Assuming within-group fighting ($m_1 > 0$ and/or $m_2 > 0$), inserting $R = K/(n_1 + n_2)$ and Equation (A3.2) into Equation (A3.5) gives a second order equation with solution

$$\begin{aligned} n_1 &= \frac{-1 + m_1 + m_2 + \sqrt{(1 - m_1 - m_2)^2 + 4\left[m_1\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + m_2\phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right]H}}{2\left[m_1\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + m_2\phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right]} \\ &\quad \times \phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}}, \\ H &= \frac{K\left[\frac{1}{a_1}\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + \frac{1}{a_2}\phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right]}{\gamma_1\gamma_2(m+1)\left[\phi_1\gamma_2(c_2a_1)^{\frac{m}{m+1}} + \phi_2\gamma_1(c_1a_2)^{\frac{m}{m+1}}\right]\left[(c_1a_2)^{\frac{m}{m+1}} + (c_2a_1)^{\frac{m}{m+1}}\right]}. \end{aligned} \tag{A3.8}$$

where the size n_2 of group 2 is found by permuting the indices.

A characteristic of Equation (A3.1) is that no agents are killed.[25] For killing to be present in a dynamic model, the size of the other group has to be present on the RHS with a negative sign, causing direct reduction in group size. The classical example is Lanchester (1916) square warfare, $dn_1/dt = -\mu_1 n_2$, $dn_2/dt = -\mu_2 n_1$, where μ_k is the combat effectiveness coefficient, and the loss rate of one group is proportional to the size of the other group.[26] Although the size n_2 of group 2 is not present with a negative sign in the expression for U_1 in Equation (2.4), and thus not present on the RHS of Equation (A3.1), group 2 can cause a significant reduction in U_1 over time. If U_1 decreases sufficiently so that the RHS in Equation (A3.1) becomes and stays negative, group 1 eventually goes extinct, due to the death rate. Summing up, without within-group fighting and without resource constraints, a group either grows to infinite size or goes extinct. Within-group fighting prevents infinite growth, and infinitely decisive within-group fighting reduces the group size to one. Logistic resource growth places additional limitations on group growth. The dynamic model is such that agents may lose their production and property but not their life, in contrast to Lanchester war models and evolutionary biology models.

Notes

1. Anderton, Anderton and Carter (1999), Cothren (2000), Grossman (1991), Grossman and Kim (1995, 2000), Hausken (2000a,b, 2004), Hirshleifer (1991, 1995a, 2000, 2001), Neary (1997), Noh (1998), Reuveny and Maxwell (2001), Rider (1993, 1999), Skaperdas (1992), Skaperdas and Syropoulos (1997, 2001), Usher (1987, 1992), Usher and Engineer (1987).
2. Baik and Lee (1997), Baik and Shogren (1995), Hausken (1995a,b, 1998), Katz et al. (1990), Lee (1995), Müller and Wärneryd (2001), Nitzan (1991a,b, 1994) and Rapoport and Amaldoss (1999).
3. The term "fighting" is to be understood as a metaphor. As Hirshleifer (1995a:28) puts it, "falling also into the category of interference struggles are political campaigns, rent-seeking maneuvers for licenses and monopoly privileges (Tullock, 1967), commercial efforts to raise rivals costs (Salop and Scheffman,1983), strikes and lockouts, and litigation all being conflictual activities that need not involve actual violence." Fighting is a subcategory of competition. We prefer to use the narrower and therefore more precise word fighting, which can be substituted with synonyms such as struggle, conflict, battle, etc.
4. This amounts to inserting $h = 1$ in Hirshleifer's production function (Hirshleifer, 1995a:31).
5. This paper assumes no interaction between resources devoted to within-group and between-group fighting, which we believe is superior to treating the two levels of conflict as one. Partial interaction is difficult to handle analytically. Modifications are necessary to analyze cases where an agent's investment in personal fighting power may be used to help the group in fighting with the other group, or hurt the group by internal predation.
6. This assumption, caused by a need for simplicity and analytical tractability, is often realistic, but not always.
7. The sequencing of the two fights is not essential. The same result follows if $\partial U_{1i}/\partial F_{1i} = 0$ and $\partial U_{2i}/\partial F_{2i} = 0$ are calculated first, and thereafter $\partial U_{1i}/\partial G_{1i} = 0$ and $\partial U_{2i}/\partial G_{2i} = 0$, or vice versa.

8. Jack Hirshleifer (private communication) has suggested that a P&C model can be used to analyze RS problems. Imagine that the consumption good X can be produced as usual. But if agents attempt to acquire X in a predatory way, a special set of circumstances apply. To wit, there is an extra exogenous prize in a fixed amount x*, also in units of X. Agents then trade off between producing X themselves, or engaging in rent-seeking to increase their share of x*, or to increase their probability of winning if it is an all-or- nothing game.
9. R_1 is a constant in the P&C model and is not common to subtract to determine the utility U_i. The tradeoff in the optimization is between choosing a large F_i to get a large share of the total production, and a small F_i to ensure high production. In contrast, r_i is a variable cost in the RS model, and since there is no production, r_i needs to be subtracted to determine the utility u_i. The tradeoff in the optimization is between choosing a large f_i to get a large share of the rent S, and a small f_i to ensure a low cost r_i of rent-seeking. The rent S is fully dissipated by rent-seeking when $m_1 \geq n_1/(n_1 - 1)$.
10. The asymptote for u_{n_1+1}/u_{n_1} is caused by the utilities turning negative as m_1 increases.
11. The large G_1 is also consistent with findings in the production and conflict literature, where Hausken (2000b) shows the benefit of moving to a more appropriative and less productive group and appropriate goods produced by the most productive group.
12. The absolute values of n_1 and n_2 follow trivially using $n_1 + n_2 = N$.
13. An alternative is $\sum_{k=1}^{2} n_k(F_k + G_k)$, measuring costs in "effort exerted" rather than "resources expended."
14. We interpret $v(n_2) = A_2$ for $n = n_2$ in their notation.
15. With comparable groups sizes two divisions may coexist beneficially if the division with (in)decisive within-group fighting m_k has low (high) between-group fighting eficiency $1/c_k$, which is mutually beneficial.
16. Indecisive between-group fighting ($m = 0$) causes a stricter requirement to m_1 to ensure the merger than decisive between-group fighting ($m = 1$). If the denominator in Equation (8.1) is negative, which happens when m is large, an agent in group 1 always prefers the merger.
17. Most mergers are beneficial for the P&C model, except e.g. merging a large group $n_2 = 10$ with beneficial $m_2 = 0.5$ into a small group $n_1 = 5$ with high m_1. Group 1 would then suffer more from being three times as large than from the between-group fighting when nonmerged. Most mergers are not beneficial for the RS model, except e.g. merging a small group $n_2 = 5$ with beneficial $m_2 = 0.5$ into a large group $n_1 = 10$ with low m_1. Group 2 gets too large a ratio of the rent when nonmerged, to the dissatisfaction of group 1.
18. Hausken (2002b) analyzes mergers of K firms with a RS model.
19. A rare exception is when K and n and m are unreasonably high and m_1 is unreasonably low.
20. ln their notation this is verified by inserting $m = m_1 = \gamma$, $K = A$, $B = n$ into their last unnumbered equation before Proposition 4.
21. I thank Jack Hirshleifer for most of the insights that follow in this footnote. Especially in connection with the applications, a distinction between "fighting" and "arming" in future work may turn out to be important. Classical contributions are by Lanchester (1916) for war and Richardson (1939) for armament. Most studies that make the distinction tend to concentrate on arming, except Hirshleifer (1995b: Section 6) and Grossman (2003). In dealing with the two different "regimes", under "armed peace" (low-intensity conflict) there is no actual fighting (no battle destruction, no conquest, etc.), while under "fighting" or "war" (high-intensity conflict) these effects do occur combined with a choice of how much resources to devote to conflict. Within each regime, decisions are made "in the shadow of" a reversion to the other regime. E.g., peacetime arms expenditures may reduce the cost of mobilizing forces in wartime, and wartime efforts may be aimed at securing

better terms of peace. A difficult analytic challenge is what determines these reversions or transitions, i.e. when do fighting and or arming begin and end.

22. They abstract from the within-group collective action problem, which in our model means setting $m_1 = m_2 = 0$, and they let each group allocate between harvesting effort from a common resource stock and conflict effort. Although the models are built up differently, with different interpretations of the parameters, it turns out to be possible through five assumptions to make the expression for U_1 in Table 1, multiplied by the number n_1 of agents in group 1, equivalent to Reuveny and Maxwell's Equation (14) (Reuveny and Maxwell, 2001). First we assume no within-group fighting, i.e. $m_1 = m_2 = 0$. Second, we interpret the unit conversion cost c_k of transforming resources into fighting effort to be the inverse of Reuveny and Maxwell's "efficiency of conflict effort" α_k, that is $c_k = 1/\alpha_k$ (Reuveny and Maxwell, 2001:724). Third, we interpret the unit conversion cost a_k of transforming resources into productive effort to be the inverse of Reuveny and Maxwell's "efficiency of harvesting" β, and equivalent for both groups, that is $a_1 = a_2 = 1/\beta$. Fourth, we interpret each agent's resource R_k as equivalent to Reuveny and Maxwell's "resource stock" R, and equivalent for both groups, that is $R_1 = R_2 = R$. Fifth, we set the between-group decisiveness $m = 1$. Reuveny and Maxwell (2001:732) eventually introduce m for the case that $\alpha_1 = \alpha_2$, obtaining a factor $m + 1$ in the denominator which corresponds to $m + 1$ in the denominator in U_1 in Table 1.
23. Mathematically, ϕ_k corresponds to Reuveny and Maxwell's ϕ and γ_k corresponds to $-\varepsilon$, where Reuveny and Maxwell (2001:726) define ϵ as "the difference between natural birth rate and death rate." (The population biology literature is variable regarding positive and negative signs.) Care should be exercised when setting up differential equations, or time-dependent equations of motion, to avoid that the static optimization loses validity. We are thus reluctant to include more than these two terms on the RHS of Equation (A3.1).
24. Reuveny and Maxwell (2001:726) also assume logistic growth, but deduct harvest.
25. Reuveny and Maxwell (2001:735) apply their model to Easter Island and argue with references that "the loser in the conflict often lost his property but not his life".
26. See Hausken (1995b) and Hausken and Moxnes (2002) for recent work on Lanchester warfare. A second example is Lanchester (1916) linear warfare expressed as $dn_1/dt = -\mu_1 n_2 n_1, dn_2/dt = -\mu_2 n_1 n_2$. A third example is competitive exclusion in population biology, expressed as $dn_1/dt = \alpha_1 n_1 - \beta_1 n_1^2 - \mathcal{X}_1 n_1 n_2, dn_2/dt = \alpha_2 n_2 - \beta_2 n_2^2 - \mathcal{X}_2 n_1 n_2$, where $\alpha_k, \beta_k, \mathcal{X}_k$ are coefficients. A fourth example is the predator – prey system $dn_1/dt = \alpha_1 n_1 - \beta_1 n_1 n_2, dn_2/dt = -\alpha_2 n_2 + \beta_2 n_1 n_2$, where group 1 is the prey and group 2 the predator. In general, arbitrarily complex expressions can be built up on the RHS for the case under scrutiny. See Braun (1983) for further examples.

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