

# **Real-life Connections in Japan and the Netherlands: National teaching patterns and cultural beliefs**

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## ***Abstract***

The TIMSS 1999 Video Study revealed that Japan had the lowest (of the seven participating countries) amount of real-life connections in the eight grade mathematics classrooms, whereas the Netherlands had the highest amount of connections with real life. This article examines more closely how these ideas were actually implemented by teachers in these two countries. A comparison of the national teaching patterns and the cultural beliefs concerning real-life connections indicate that Japanese teachers might actually follow some of the ideas and principles of the Dutch tradition of Realistic Mathematics Education to a stronger degree than Dutch teachers.

## ***Introduction***

National patterns of teaching do exist, and the results from a large-scale study like TIMSS can be used to explore global and national patterns of teaching (Giwin et al., 2005). The general report from the TIMSS 1999 Video Study (Hiebert et al., 2003) concluded that the Netherlands had the highest amount of connections to real life, whereas Japan had the lowest amount of such connections. This is an interesting finding, and when compared with the achievements in the student assessment, one might suspect a certain connection here. In the student assessment in TIMSS 1999, Japan was ranked 5th and the Netherlands 7th, with respect to the 8th grade students scores in mathematics. Both countries were therefore among the highest achieving, although Japan had a significantly higher achievement than the Netherlands. Teaching is a complex activity, and there are apparently other issues that have stronger influence on the achievements than the amount of real-life connections. There might also be differences in the way teachers connect mathematics with real life, and the real-life connections as such could also be of different kinds. The more in-depth analysis of a qualitative study could reveal some of these issues that are easily lost in the numbers in a large comparison study like the TIMSS 1999 Video Study.

In this article, I am going to examine a selection of videos from the TIMSS 1999 Video Study,

with a particular focus on the videos from the Netherlands and Japan. The report from the TIMSS Video Study focus on the amount of real-life connections only. I am going to focus more on the nature of these connections, in order to reveal some presumably interesting patterns of interaction. These patterns of interaction will be compared with the cultural scripts that were reported in the TIMSS 1999 Video Study, and with cultural significances that has been found in other studies about teaching in Japan and the Netherlands. Cultural scripts might be described as a set of culturally common beliefs that the teachers have internalized by participating in the communities of practice that they have encountered in the educational systems of their country (Givvin et al., 2005). This would therefore be an analysis of the connections between the national teaching patterns and the cultural scripts concerning the connection of mathematics with real life. In this article, however, I will refer to cultural beliefs rather than cultural scripts. My use of the concept “cultural beliefs” mainly refers to the ideas and theories that have been reported to influence teaching in these two countries. The reason for using the term cultural beliefs rather than cultural scripts is that I cannot be absolutely certain that these culturally common beliefs that I am referring to have been internalized by the teachers whose lessons I analyze. The main questions I will address in this article are:

- How do Japanese and Dutch teachers connect mathematics with real life?
- How do these national patterns comply with the cultural beliefs?

This study was conducted in May 2003 while the author was in residence at UCLA and at LessonLab as a member of the TIMSS 1999 Video Study of Mathematics. Thanks are due to Jim Stigler and Ron Gallimore for opening the doors at LessonLab to make this article possible, and to Angel Chui and Rossella Santagata for assisting with all practical issues.

## ***Theoretical background***

### **Real-life connections**

The idea of connecting school mathematics with everyday life, daily life or real life is widespread, and researchers have addressed the issue in different ways (cf. Brenner & Moschkovich, 2002). Theories in general pedagogy, as well as in mathematics education in particular, seem to support the idea of connecting mathematics with something the pupils know and are familiar with in order to enhance learning. These ideas are implemented to various degrees in curriculum papers and frameworks around the world.

Before I approach the issue of real-life connections as such, I start by looking at the question from a more general angle. Mathematics might be described as a kind of universal knowledge. The theories of mathematics can be applied to various problems, theoretical as well as practical. Research has shown, however, that many people have difficulties applying their mathematical knowledge to real and practical problems. There might also be a strict distinction between school mathematics and street mathematics (Nunes et al., 1993). Researchers have therefore addressed the issue of transfer of learning from one context to another (Evans, 1999). From a situated perspective (cf. Lave & Wenger, 1991), knowledge is strongly connected with a social situation or context. When children learn mathematics in school, this might be experienced as some kind of school practice where the knowledge is not viewed as applicable outside of the

mathematics classroom.

When examining scholarly publications within the area of mathematics education, we come across a variety of concepts like everyday life, daily life, real life, real world, realistic as well as contextual, situated and other concepts that are directly or indirectly related (cf. Boaler, 1997; Brenner and Moschkovich, 2002; Lave and Wenger, 1991; Wistedt, 1992). In an attempt to approach this field, a proper question to address might be: “What do we mean with real life/world?”

First, we need to acknowledge that we are dealing with philosophical issues concerning the conception of reality here. The ways in which these concepts are used indicate a distinction between the real world and the mathematical world. The real or physical world is the world of objects and events that are familiar to us, and that we can describe in our natural language (Smith, 2002). The connections between these two worlds can be described as something mysterious (Penrose, 1994), and this might provide one of the possible explanations as to why so many students find mathematics meaningless or hard to understand.

Real-life connections can be defined as connections between the mathematics that is taught in school and the outside world. The conception of the outside world is not trivial. The everyday life of the pupils is often limited, and if one would focus only on issues contained in the everyday life of pupils, mathematics would become limited. There is also the aspect of different pupils having different experiences of the outside world. I therefore do not wish to limit real or everyday life to the pupils’ conception of the outside world. My suggestion is to adopt a view of real life as everything that is connected with, or might be encountered in, the outside world. This would imply that the real-life connections in school mathematics in many cases are not part of the pupils’ everyday life as such. As a result of this, real-life connections will not automatically provide more meaning to pupils. When examining curriculum papers in different countries, it appears that a goal for school mathematics should not be only to reflect the pupils’ everyday life, but also to prepare them for their future vocational life and life in society. Having introduced this goal, real-life connections could provide meaning from a cultural or societal view. They might not be directly meaningful to the pupils though, because they are not necessarily connected with the everyday life of the pupils.

Having defined how the concept is used in this article, I have to point out that this does not imply a suggestion that making real-life connections is the proper or “best” way of teaching mathematics. In some instances, direct connections with real life can make it harder for the pupils to understand. This might be due to culturally related issues or other (cf. Bransford et al., 2000). Situations and problems from real life are also, in many instances, more complex than problems from a mathematics textbook.

### **Definitions from the TIMSS 1999 Video Study**

This article is based on the TIMSS 1999 Video Study, so I will therefore have a closer look at the definitions of concepts made in this study. All the lessons of the Video Study were coded, and the coding team made a distinction between real life connections/applications, and whether they were set up as a problem or not. The coding team chose not to make a distinction between real-life connections and real-life applications, although these two differ. These two categories

were chosen: real-life connections or applications in problems, and real-life connections in non-problem situations. The definition of the real life connection/application – non-problem (RLNP) was presented like this:

The teacher and/or the students explicitly connect or apply mathematical content to real life/the real world/experiences beyond the classroom. For example, connecting the content to books, games, science fiction, etc. This code can occur only during Non-Problem (NP) segments.

As we can see here, real life is compared to real world or experiences beyond the classroom. This is a quite vague description, but it is somewhat clarified by the examples on how these connections could be made.

The by far most frequently occurring of the two is simply called real life connections, and they appeared in actual problems in class. There was made a distinction between situations where the real life connection appeared in the problem statement or set-up, or if the real life connection was brought up during the discussion or work with the problems. The definition of these kinds of real life connections, called RLC, was:

Code whether the problem is connected to a situation in real life. Real life situations are those that students might encounter outside of the mathematics classroom. These might be actual situations that students could experience or imagine experiencing in their daily life, or game situations in which students might have participated.

Real life is then whatever situation a student might encounter outside of the mathematics classroom, actual situations or imagined situations that the students might experience. A situation was coded RLC whenever a reference was made to the outside world (directly or indirectly) in a problem that the pupils worked with or discussed.

The distinction between RLC and RLNP helps answering two initial questions:

- Are there any connections to real life?
- Are these connections related to a problem or not?

Teaching mathematics by letting the pupils work with problems in a meaningful context has been presented as an alternative to the more traditional formal training of mathematical methods and formulas. This might be justified by the idea that cognition in general, and learning in particular, is situated (cf. Lave & Wenger, 1991). Some important elements of situated learning would therefore be an authentic context, cooperation and social interaction. This might provide some of the answers to the question of why one should use real-life connections, or why one should connect with everyday life.

By closely observing student activities, experiences, interests, and daily endeavors, one may be able to capture situations whose everydayness makes them potentially powerful departure points for establishing bridges to academic mathematics. Such bridging between the everyday and the academic may then consist of integrating the genuine, meaningful, and engaging origin of the problem (children's experiences) with guidance for developing and using mathematical tools (possibly ad hoc at the beginning) to help students make deeper sense of the problems [...]. The bridges also provide ways to return to the everyday situations with more powerful knowledge about handling and approaching them (Arcavi, 2002, p. 16).

The issue of motivation often comes up in this discussion, and although others have emphasized

different aspects, Arcavi's words stand as a reasonable answer to the question of why. Another question that is reasonable to ask is how this connection could be or should be carried out.

## Cultural beliefs

There are some national patterns in the teaching of mathematics in Japan and the Netherlands that should be brought to attention (for more comprehensive reviews of teaching in these two countries, see the sources referred to below).

In the Netherlands, schools have a large degree of freedom when it comes to developing their own curriculum. The textbook has a strong influence on the daily teaching for most teachers, and the final exam (in secondary school) strongly influences the content of the textbooks. There are three main textbook series in Dutch secondary school, and all three cover mainly the same content. The textbooks contain lots of extra material for the teacher, and it is therefore not necessary for the teacher to make use of other sources than the textbook in the daily teaching. Most of the problems in the textbooks are context problems, and all three textbook series more or less reflect the ideas and principles of RME (van den Heuvel-Panhuizen & Wijers, 2005).

Mathematics teaching in the Netherlands is strongly influenced by Realistic Mathematics Education (RME), and this constitutes some of the cultural background and cultural beliefs that one might expect to be observable in the classroom practice. This started around 1970 and has its roots in the so called Wiskobas project. The theoretical framework of RME is strongly influenced by the ideas of Hans Freudenthal (van den Heuvel-Panhuizen & Wijers, 2005). One of Freudenthal's main ideas was that mathematics should be taught as an activity (Freudenthal, 1968), and it should be connected to reality and the children's experiences (van den Heuvel-Panhuizen & Wijers, 2005). Another central idea is that the pupils should be actively involved in the reconstruction or re-invention of the mathematical ideas. A core activity in the process of reinvention is to mathematize the reality of the learner (Freudenthal, 1991). When activity is mentioned within RME, it is mainly pointing at mathematization, which is viewed as the most important mathematical activity. Treffers (1987) later made a distinction between "horizontal" and "vertical" mathematization. In the activity of horizontal mathematization, the pupils invent mathematical tools that can assist them in the process of solving problems from real-life situations. In vertical mathematization, however, the activity is concerning the process of reorganization within the mathematical system itself (van den Heuvel-Panhuizen & Wijers, 2005).

Context problems, as they are often called, would normally serve as a qualitative introduction to certain mathematical concepts. These context problems could be both realistic problems and problems from real world. RME has a clear distinction between horizontal and vertical mathematization, but still there is a common misconception that RME only has a focus on real-world problems. This misconception might be due to the somewhat confusing term *realistic*, which has a distinct meaning in the Dutch tradition. In RME, the word realistic refers to something we can refer to or imagine, more than it refers to the authenticity (van den Heuvel-Panhuizen, 2003; van den Heuvel-Panhuizen & Wijers, 2005). When working with these context problems, the pupils are guided by the teacher through a process of reinvention, and in this process organization and mathematization are important activities (Freudenthal, 1991; Gravemeijer, 1994; Gravemeijer and Doorman, 1999; Jaworski, 1995; van Amerom, 2002).

These ideas are strongly connected with ideas of constructivism and activity theory.

Six main principles of teaching mathematics can be identified as communal in RME (vanden Heuvel-Panhuizen & Wijers, 2005):

- the principle of activity
- the principle of reality
- the principle concerning levels of understanding
- the intertwinement principle
- the interaction principle
- the guidance principle

When comparing the situation in the Netherlands with Japan, there are several obvious differences. There are still some strong national patterns that are visible in most Dutch mathematics classrooms, and these national patterns are formed, among other things, by the influence of the textbooks, the final exam and the tradition of RME. Dutch schools are free to develop their own curriculum, and overall, they have a large degree of freedom. Japan, on the other hand, has a national curriculum which serve as a national standard, and Japan's Ministry of Education, Science and Culture has a strong influence on the educational system in the entire country. Japanese teachers are involved in the development of the national curriculum, and they are also strongly involved in researching how to teach (Stevenson et al., 1998). This particular approach has often been referred to as lesson study (Fernandez & Yoshida, 2004). In this approach, teachers continually develop, revise, demonstrate and share their teaching ideas with their colleagues (Stevenson et al., 1998).

Because of the high performance of Japanese students in all international comparison tests, there has been a strong interest in the Japanese educational system. Results from studies such as the TIMSS 1999 Video Study might be interpreted in a way that this high performance is mainly due to the quality of the mathematics lessons that we can observe in Japanese classrooms. This quality is not only due to the competence of the teachers, but more by the way Japanese teaching aims at conceptual rather than procedural understanding (Stigler & Hiebert, 1999). Research has indicated, however, that the explanation is far more complex (cf. Knipprath, 2004; Schümer, 1999). Schümer (1999) suggests that it would be valuable to observe and analyze the teaching and learning that takes place outside of the classroom. The parents appear to have an important role in the Japanese educational system, and the role of the mother has received particular emphasis in some studies. The image of the Japanese mother, and the way in which she is highly absorbed in her children's education, appear to be somewhat exaggerated. Still, Japanese children seem to get more help with their homework than children in the United States (Stevenson & Stigler, 1992). Knipprath (2004) discusses the support of the parents and the community. She claims that this support has mainly been solicited by the schools, and it has generally not been a matter of active involvement in the children's learning (Knipprath, 2004, p. 105).

## **National patterns in the TIMSS video studies**

In *The Learning Gap* (Stevenson & Stigler, 1992), the results of the SIMS study are discussed. A major idea in studies like this is to study teachers and teaching practices in different countries in order to improve teaching. In 1995 another large international study was conducted: The

TIMSS student assessment. It was comparing the students' knowledge and skills in mathematics and science, by country. This study was followed by a video study, which was the first study to use video technology to investigate and compare classroom teaching on a country wide basis (Hiebert et al., 2003).

The next TIMSS student assessment, the TIMSS 1999, was followed by another video study, this time in a much larger scale than before. The study included recording of more than 600 lessons from 8<sup>th</sup> grade classrooms in 7 countries: Australia, Czech Republic, Hong Kong SAR, Japan, Netherlands, Switzerland and United States. The Japanese videos were collected in the 1995 study and re-analyzed. All videos were transcribed, and the transcriptions were translated, coded and analyzed. In this article, I have used the transcripts from these lessons as they appeared in the database from Lesson Lab. I have decided not to make any adjustments or corrections of grammatical or other kind. In 1995 as well as in 1999, Japan was among the highest achieving countries in the student assessment part of TIMSS. When I call the Japanese pupils high achieving in the following, this is what I mean. In this article I will focus almost exclusively on the TIMSS 1999 Video Study and not the student assessment.

When it came to how the mathematical problems were presented and worked on, the coding team explored several aspects, including (Hiebert et al., 2003, pp. 83-84):

- ***The context in which problems were presented and solved:*** Whether the problems were connected with real-life situations, whether representations were used to present the information, whether physical materials were used, and whether the problems were applications (i.e., embedded in verbal or graphic situations).
- ***Specific features of how problems were worked on during the lesson:*** Whether a solution to the problem was stated publicly, whether alternative solution methods were presented, whether students had a choice in the solution method they used, and whether teachers summarized the important points after problems were solved.
- ***The kind of mathematical processes that were used to solve problems:*** What kinds of process were made visible for students during the lesson and what kinds were used by students when working on their own.

The issue of real-life situations has been addressed in the following way (Hiebert et al., 2003, p. 84):

The appropriate relationship of mathematics to real life has been discussed for a long time (Davis and Hersh, 1981; Stanic and Kilpatrick, 1988). Some psychologists and mathematics educators have argued that emphasizing the connections between mathematics and real-life situations can distract students from the important ideas and relationships within mathematics (Brownell, 1935; Prawat, 1991). Others have claimed some significant benefits of presenting mathematical problems in the context of real-life situations, including that such problems connect better with students' intuitions about mathematics, they are useful for showing the relevance of mathematics, and they are more interesting for students (Burkhardt, 1981; Lesh and Lamon, 1992; Streefland, 1991).

When comparing average percentage of problems per eight-grade mathematics lesson that were set up with the use of real-life connections, there were some interesting differences. In Netherlands, 42 percent of the lessons were set up using real-life connections, whereas only 40 percent using mathematical language and symbols only. This was the most special result in the study, where the other six countries differed between 9 and 27 percent real-life connections. It is also interesting to see that only 9 percent of the Japanese lessons had real-life connections.

In all the countries, if teachers made real-life connections, they did so at the initial presentation of the problem rather than only while solving the problem. A small percentage of eighth-grade mathematics lessons were taught by teachers who introduced a real-life connection to help solve the problem if such a connection had not been made while presenting the problem (Hiebert et al., 2003, p. 85).

A larger percentage of applications was discovered in the Japanese classrooms (74%), than in the Netherlands (51%). These applications might or might not be presented in real-life settings (Hiebert et al., 2003, p. 91).

Another interesting point is connected with the mathematical processes. In Japanese classrooms 54% of the problems were classified as having to do with making connections. In the Netherlands this was only the case in 24% of the problems (Hiebert et al., 2003, p. 99, figure 5.8). When it comes to “using procedures”, i.e. involving problem that was typically solved by applying a procedure or a set of procedures, this was the case in only 41% of the problems in Japan, and 57% in the Netherlands (Hiebert et al.).

Since the Japanese students were higher achieving than the Dutch students, and the Japanese lessons had a lower percentage of real-life connections, one might assume that the use of real-life connections did not have any positive effect learning. The image is far more complex though (as Knipprath, 2004; Schümer, 1999 and others indicate), and this is an example of how a qualitative analysis can provide information that contributes to the information from a quantitative study in a positive way.

## ***Choice of material***

This study is based on a new analysis of a selection of videos from the TIMSS 1999 Video Study. For the methodological issues concerning the collection and initial analysis of these videos I only refer to the official report (Hiebert et al., 2003). In this section I am going to discuss the considerations and choices that were made in my own selection of videos.

Japan and the Netherlands were selected for analysis here because they were extreme cases when it comes to real-life connections, and extreme cases can provide interesting information. It is a common misunderstanding that one cannot generalize from a single case, or only a few selected cases. A careful selection of cases can, however, be used to falsify propositions and thereby have general significance towards stimulating further investigations and building theory (Flyvbjerg, 2006).

The focus of this study was to investigate how Japanese and Dutch teachers connect mathematics with real-life, and the relationship between these teaching patterns and the cultural beliefs concerning real-life connections. From the main report from the TIMSS 1999 Video Study we learn that such connections were frequently used in the Netherlands, but more infrequently in Japan. It was therefore expected that the Dutch lessons would involve much focus on guided reinvention and realistic problems, being some of the main ideas in the tradition of Realistic Mathematics Education, whereas the Japanese lessons would not. In response to this focus, I selected videos with at least one real-life connection, as coded by the coding team at the Lesson Lab. After a brief analysis of this first selection of videos, a smaller subset of videos was selected for further analysis. In this last round of selection, I picked lessons where different methods of teaching and classroom organization were used. When

lessons with equal or similar content and/or structure were found, only one was selected for further analysis. I ended up with three Dutch lessons and three Japanese lessons.

## ***A closer look at the national patterns***

### **The Dutch lessons**

The Dutch lessons had a high percentage of real life connections in the TIMSS 1999 Video Study, much more than any of the other participating countries. The lessons often included a large number of problems connected with real life. From the analysis of videos, a pattern seemed to emerge. In most of the lessons I analyzed, the teacher reviewed problems from the textbook together with the class. It seemed as if the pupils had already worked on the problems before, and the pupils were asked questions related to the answers of the problems. When working on problems, they mainly worked individually, but they might also be seated in groups. What struck me was that the teachers were very focused on the textbook. These textbook problems almost exclusively had some kind of real life context. Most of the real life connections were textbook tasks presented by the teacher addressing the whole class. This was the case in most of the lessons. An example of this can be found in the lesson M-NL-021<sup>1</sup>, where the teacher went through problems like this in the entire lesson:

Teacher: Now another possibility with percentages. I have an item in the store. At present it costs three hundred ninety-eight guilders. Next week, that same item will cost only three hundred twenty guilders. With what percentage has that item been reduced in price, Grietje?

Student: Um, seventy-eight guilders was subtracted.

T: Seventy-eight guilders was subtracted, yes.

S: Eight, uhm divide it by the old amount times one hundred.

T: So – yes. By which – by which number?

S: Three hundred and ninety-eight and then times one hundred.

T: By three hundred and ninety-eight and then times one hundred. And that gives you the solution.

As we can see, the teacher read the problem from the book, and asked a pupil to give the solution. The session appears to be a review of the pupils' homework. Some of the problems were more complex, and they might include figures and tables. In this particular lesson, many problems were collected from statistical material, like in a problem on the wine imports to Netherlands in 1985, introducing picture diagram, bar diagram and line diagram. Other problems focused on temperatures, amounts of umbrellas sold on a rainy celebration day, coffee consumption in the Netherlands, etc. The contexts of these textbook tasks had an authentic appearance, and the numbers and figures presented appeared to be realistic.

One of the other lessons, M-NL-031, was different when methods of work were concerned. In this lesson the class worked with probability. The teacher had divided the pupils into smaller groups, and the groups were given different assignments. One of the groups flipped coins and wrote down the results, another group should roll dice and yet another group were told to look outside the window and write down how many men and women that passed. The groups worked five minutes with each task, and then moved to the next station. The pupils were then supposed to use these data and calculate the chance (the fraction and the percentage). The real-life

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1 This refers to how the videos were identified in the database at LessonLab.

connections in this lesson were different from the previous in that they did not work with textbook tasks only. Here other sources were involved, and these sources provided a set of data that the students had gathered themselves. During the group-work, they encountered several real-life applications and connections in non-problem settings.

The third and final Dutch lesson that was selected for analysis (M-NL-050) focused on exponential growth. The main problem in the lesson was concerning the growth of duckweed:

T: Uhm... A piece of five centimeters by five centimeters of duckweed in the pond, it's really annoying duckweed. It doubles. But the owner of the pond doesn't have the time to clean it. He takes...

S: Sick?

T: No, he takes three months of vacation. Now, the question is... the pond, with an area of four and a half square meters. Will it be completely covered in three months or not?

S: Yes.

S: ()

T: Shh. This is the spot that has duckweed at this moment. It doubles each week, no, and the pond is in total four and a half square meters, and the time that he's gone on vacation is three months. So the question now is whether the pond has grown over or not.

The pupils were then asked to use their calculators. After the pupils had worked with it for a while, the teacher asked them what they have come up with:

T: Who says it's full after three months?

S: No idea why, but it's full.

T: Uhm, who doesn't?

S: ()

T: And, uhm, who says "I don't know"?

S: Ha ha.

T: Uhm, so there are six. I have six unknown, no one for not full, and, uhm, so there are twenty-five for full. Uhm, Paul, how did you come up with full? What did you try, what did you do?

S: I don't know.

The teacher then tried to figure out how the pupils had been thinking and what calculations had been made. They eventually came up with a formula for calculating the growth during the twelve weeks. At the end of the twelfth week, they found out it was two to the twelfth. Then they had to convert square meters into square centimeters. After a discussion on this, the teacher summed it all up:

T: Uhm, so you must make sure that, in the end, you are comparing. So, or the answer that you came up with... that'll be twenty-five thousand times four, so that is somewhere close to hundred thousand, and so it's full. This is something that will be explained in Biology. In economics, well, then you will get the following: that the doubling of bacteria, then you get something like this ( ... ).

The context presented in this lesson was also authentic, and duckweed could be encountered as problematic in real life. Nowadays many children grow up in cities, and they might never experience duckweed as a problem in ponds. In real life, the issue would probably have been to clean the pond rather than do calculations concerning the growth. When we look at the problem context in this way, it would seem as a wrapping of mathematical theories and considerations rather than a genuine real-life connection.

From the statistical analysis of the TIMSS 1999 Video Study we get the impression that real-life connections are important in Dutch schools. This also fits with the theories of Realistic Mathematics Education, which represent an important element of the cultural beliefs in Dutch mathematics education, and our analysis of videos also supports this view. It is, however, worth noticing that a large proportion of the real-life connections in the Dutch classrooms were real-life connections in textbook problems.

An important principle in the tradition of RME is that of guided reinvention. An integral amount of student activity should therefore be included in the work on real-life connected problems or realistic problems as they are often called in this tradition. This was not so evident in the sample we have seen. Here it seemed to be more teacher talk in connection with a review of textbook problems than a process of guided reinvention of mathematical concepts. In several of the Dutch lessons I have analyzed, the teaching was rather traditional – with a strong focus on solving problems from the textbook. This strong position of the textbook was also reported by van den Heuvel-Panhuizen and Wijers (2005).

## **The Japanese lessons**

The most striking element of the Japanese lessons was their structure. They were extremely well structured, and mathematics lessons in Japan would often follow exactly the same pattern in corresponding lessons all over the country (Stigler & Hiebert, 1999). In the TIMSS 1999 Video Study, it was easy to find examples of this. On several occasions, lessons from different schools and different teachers were almost exactly the same. A Japanese lesson would often focus on one problem only, and this would often be a rich problem and a “making connections” - problem (Hiebert et al., 2003).

An example of such a lesson is M-JP-022. In this lesson, the teacher started off with a short introduction to the concept “center of gravity”. Here he gave a comment about the importance of center of gravity in sports, like baseball or soccer. This comment was marked as RLNP-situation in the Video Study. Then, he showed how to find the center of gravity in a book, balancing a textbook on a pencil. All along he discussed with the pupils, and he let them try and figure out where the center of gravity was. Through this process of discovery, he was leading them into ever more precise mathematical formulations.

Next, he challenged them to find the center of gravity in a triangle, and this was the main focus for the entire lesson. First the object was simply to find the center of gravity by balancing a paper triangle on a pencil. Then, as the teacher stated, it was time to look at this more mathematically:

T: Okay this time open your notebooks. Uh let's try drawing one triangle.

(pupils are drawing in their notebooks)

T: Okay. If it were a cardboard you can actually tell saying it's generally around here where it is using a pencils and such. Okay it's written in your notebooks. It's written on the blackboard. You can't exactly cut them out right? You can't exactly cut them out. And without cutting them out ... I want you to look for like just now where the balancing point is, ... that's today's lecture. Using this cardboard from just now ... in many ways. I will give you just one hint. It'll be difficult to say at once here, so on what kind of a line does it lie? ... On what kind of a line does the point lie? Please think about that.

So, first they found the center of gravity by testing on a cardboard, then the next challenge was to find this center (mathematically) without cutting out the triangles. The pupils were given time to think and discuss, and they played around with pencil and triangle. Then the teacher formed groups of six, and the pupils discussed further in groups. The teacher walked around and commented on the work. He asked them to draw lines or points on the cardboard and try it out to see if it balanced. Some pupils discovered that their solutions were wrong. The teacher interrupted the work by presenting to the class one false solution that one pupil tried:

T: Okay. It's okay. Just for a second, sorry Shinohara. Shinohara just tried with the bisectors of angles right? The bisectors of angles. And ... when you try it like this

S: ( )

T: unfortunately it doesn't balance. Um ... at the bisector of the angle please look up front for a second those of you facing the back. Group one girls, look ... look for a second. Let's see ... if you go like this at the bisector of an angle, Shinohara.

S: Yes?

T: Look over here. If you are asked whether it balances?

S: Um

T: Uh huh. This side ended up little ... heavy right? It ended up heavy. That's why even if you go like this it doesn't balance. So the areas are the same ... unless the areas are the same ... it's no good, is it?

The pupils continued trying out their theories on the cardboards. From time to time, the teacher interrupted by showing some of the pupils' solutions on the blackboard. The pupils got plenty of time to think and try things out, and the teacher mainly used the pupils' ideas and answers in a reconstruction of the theory. Eventually they reach a proof, and the teacher sums it all up in a sentence. In the end he reviews the essence of the lesson again.

Such an approach could be observed in many Japanese lessons. The pupils got lots of time to work with one problem at a time, and very often, the pupils were actively involved in the reinvention of theory. Sometimes the pupils were also given the opportunity to present their solutions and methods on the blackboards, and the class would discuss which method they would prefer. Quite often the mathematical content of a lesson would be purely mathematical, as this lesson was, except for the tiny comment on center of gravity in sports. It is not known if this lesson was the introduction to the topic, so it is impossible to claim that the pupils were really discovering or reinventing the methods and theories connected with center of gravity. The pupils seemed to be enthusiastic about the activity though, and they got the opportunity to see the link between theory and practice. They were also given the opportunity to discuss their choices of methods and solutions. Even though much of the teaching was arranged as the teacher discussing with the whole class, the pupils were active.

In M-JP-035, the approach was a bit different. They were working on congruence and similarity, and the teacher had given the pupils a homework assignment:

T: Okay. Ah...then up to now ... up to the previous lesson we were learning about congruent geometric figures, ... but today we'll study something different. As I was saying in the last class ... I said we'll think about geometric figures with the same shape but different sizes, and I was asking you to bring such objects to the class if you find any at home.

Not all the pupils brought things, but some had brought angle rulers, some protractors and erasers, and one brought origami paper. The teacher had also brought some things, and she used this to introduce the topic:

T: Okay. Then, next I'm going to talk ... all right? What similarity means is that the figure whose size is expanded or reduced is similar to the original figure. Then, well a few minutes ago I introduced the objects you have brought to the class. I, too, have brought something. What I have brought is ... some of you may have this bottle at home. Do you know what this is? Yasumoto, do you know?

S: ()

T: What? You don't know what kind of bottle this is? Taka-kun do you know?

S: A liquor bottle.

T: A liquor bottle. A ha ha ... that's right. It's a whisky bottle. Whisky ... a whisky is a liquor which ... we all like. Cause we even call it Ui-suki (we like).

S: A ha ha.

T: A ha ha. Did you get it? Then, ... about these whisky bottles ... look at these. They have the same shape don't they. They do, but have different sizes. Well, I have borrowed more bottles from a bottle collector. This.

S: A ha ha.

T: This.

S: A ha ha.

T: See ... then I wondered if there were more different sizes so I went to a liquor store yesterday. And, they did have one which contains one point five liter of ... one point five liter of whisky, but it was too expensive so I didn't buy it. As you can see that these whisky bottles ... have the same shape ... but they come in various sizes. All of these bottles are called similar figures.

The teacher started with connecting to real life through the examples of things the pupils had brought, and then went on to present some things she had brought herself. This could be described as examples from the teacher's own real or everyday life. She had also brought a couple of squid airplanes, with different sizes. And she had brought a toy dog. The pupils were shown how to draw this dog in a larger scale, using rubber bands. After this demonstration, the teacher went into the mathematics underlying this activity. The pupils were asked to draw geometrical figures like quadrilaterals and triangles in larger scales. At the end of the lesson, the pupils were guided towards discovering that the angles are equal in these expanded figures, and that they are therefore similar. The teacher also introduced a symbol for similarity.

In the last lesson M-JP-034 from Japan that I looked into, they also worked with similarity. This

teacher gave many examples from real life, and he asked the pupils to give examples also. Some of the examples he came with were the desks in the classroom, negatives of a film, fluorescent light and different sizes of batteries. All along, there was a dialog with the class. In these lessons, real-life connections appeared to be merely used in the introduction of a new topic or problem.

As could be observed from some of the Japanese lessons, the teacher would often start off with one or a few real life examples and gradually move towards the mathematical concepts. The real-life situations were often used as motivational examples, and the intention was not (it appeared) to solve real life problems.

### ***Teaching patterns and beliefs revisited***

My initial questions were how Japanese and Dutch teachers make real-life connections when teaching mathematics, and how these national teaching patterns comply with the cultural beliefs concerning this issue. A large international comparison study like the TIMSS 1999 Video Study might provide some insight into these issues. Japanese teachers appear to emphasize real-life connections quite seldom, whereas this is something that is emphasized a lot in Dutch classrooms. This appears to be consistent with the ideas and principles of Realistic Mathematics Education (RME) in the Netherlands, whereas the Japanese tradition focus on other issues. In this article I have presented nine lessons from the TIMSS 1999 Video Study, and I have focused on some episodes and points from these lessons. These examples reveal some interesting additional information to this picture.

There was a clear pattern in the Dutch classrooms that the teacher would spend much time reviewing textbook problems, and this was also confirmed by the official report (Hiebert et al., 2003). The first Dutch lesson, M-NL-021, is a typical example of this. The real-life connections were almost exclusively visible in problem situations where the problems were textbook tasks and the teacher was addressing the whole class. The one exception was when the teacher made a remark concerning one of the problems. This strong focus on textbook tasks fits with the teaching patterns revealed in the TIMSS 1999 Video Study, but such a strong focus on the textbook appears to be in contrast with some of the main principles of RME. The idea of guided reinvention for instance, which is emphasized in RME, was not so visible in the lessons I analyzed. Another central idea in RME is that of mathematization, and Freudenthal claimed that there is no mathematics without mathematizing (Freudenthal, 1973). Generally spoken, mathematization describes the activity of organizing mathematical matter or matter from reality. I suppose this is possible to do when you solve textbook problems, but a focus on this process was not so visible in the Dutch lessons. In the Japanese teaching, however, both these principles were visible.

One of the Dutch lessons, M-NL-031, contained a more extensive activity where the pupils worked in groups, but although being based on a more open task, it didn't seem to represent the ideas mentioned above. In the last lesson I focused on from the Dutch classrooms, M-NL-050, the main focus was on a real-life connected problem. The problem was concerning growth of duckweed, and it seemed to be a textbook task presented by the teacher addressing the whole class. This problem was discussed and worked on for the main part of the lesson, and here we could observe what I would call reinvention.

In the collection of Japanese videos there were few real-life connections altogether. Still, the lessons I have presented here are examples of how the Japanese teachers would often use an approach that is in compliance with the ideas and principles of RME. In the first lesson presented here, the teacher made the problem realistic to the pupils through his introduction, and the pupils were then guided through a process of reinvention of the theory. In the next, we saw examples where quite a lot of connections were made to real life, some of them being by things the pupils had brought, or other pupil initiatives, and some where real-life connections made by the teacher presenting her everyday life examples. The teacher would normally address the whole class. In conclusion, some Japanese classes involved a method of work that is strongly related to the ideas of RME, and although this seemed to be exceptions, the teachers would sometimes make explicit real-life connections in their lessons.

Based on previous knowledge about the role of RME, I expected the Dutch classrooms to contain activities where the pupils were mathematizing and reinventing mathematical theories through realistic or real-life connected problems. In the lessons I have seen, they were working with real-life connected problems (mostly in textbooks), but often in a traditional way. Some of the lessons from Japan had adopted the ideas of reinvention and mathematizing in a more visible way that what could be observed in the Dutch videos, although they did not contain so many coded real-life connections. In the Japanese lessons the pupils' ideas and solution methods were taken into account, and the pupils would often take an active part in the discussion of which methods to use. The pupils in these classrooms seemed much more involved and active than what could be observed in the Dutch videos.

The textbooks have a strong position in Dutch schools (van den Heuvel-Panhuizen & Wijers, 2005). All the main textbook series in the Netherlands present a lot of context problems (most with a real-life connection), so one might assume that Dutch classrooms – through the textbooks – would involve many real-life connections in problems. The Dutch textbooks follow the ideas of Realistic Mathematics Education, but this influence from RME was not so apparent in the classrooms. The Japanese teachers were often using other sources than the textbook in their lessons, and they were also concerned with organizing activities where the pupils could discover the procedures and theories for themselves (Hiebert et al., 2003).

In this article we have seen how teachers in these two countries carry out the connections with real life in their teaching of mathematics. Through a more in-depth analysis of a selection of videos from the TIMSS 1999 Video Study, I have hopefully shown that real-life connections are more than numbers. It is interesting to know that Japanese classrooms have 9% real-life connections and Dutch classrooms 44%, but it is even more interesting to go deeper into these teaching patterns. When studying the teaching patterns in these two countries more closely, and comparing this with the cultural beliefs, it seems like the Japanese teaching patterns corresponds better with the cultural beliefs from the Dutch tradition than the Dutch patterns. The teaching patterns that I refer to are also reported in the main report of the TIMSS 1999 Video Study (Hiebert et al., 2003), but my interpretations of the connections between these teaching patterns and the cultural beliefs can of course be discussed.

Such a discussion of the relationship between the national teaching patterns and the cultural beliefs in these two countries could have been interesting to follow up with the perspective of other researchers as well. Most important, however, is that these observations and analyses imply that real-life connections are not trivial, and much more emphasis should be given to how

they are carried out and presented in mathematics classrooms, and the connections between these practices and the beliefs (personal and cultural) concerning them.

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## **Biography**

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