University of Stavanger

**Faculty of Science and Technology**

# MASTER'S THESIS

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## Summary

Offshore structures are exposed to environmental loads such as waves, wind, currents etc. and it is important to understand how the structures behave under different conditions. In this thesis the main focus has been on determining the long term extreme response. A case study has been performed on a semi-submersible located in the North Sea.

Because of the randomness in the ocean environment and the corresponding response, statistical methods are required to estimate extreme motions. Two different approaches have been used to estimate the long term extreme response; the all sea state approach and the environmental contour line method. The all sea state approach utilizes the long term variability of the environmental conditions and the variability of response for a given sea state. All sea states are taken into consideration. Normally a full long term analysis is performed. However, if a complicated non-linear problem is under consideration, a full long term analysis is quite time consuming and a simplified method is preferred. One such method is the environmental contour line method that utilizes the long term variability of the environmental conditions to predict the extreme sea states. To establish the extreme response, a short term analysis is performed near the sea states in proximity of the "worst" sea state.

The results obtained with the full long term analysis have been evaluated by using Monte Carlo simulations based the available hindcast data.

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## Preface and Acknowledgement

This thesis is written as a final closure of my master degree in Constructions and Material. The work has been performed at the University of Stavanger between January and June 2014. The analysis has been performed in MATLAB and Excel, and learning MATLAB has been a part of this thesis.

I would like to sincerely thank my supervisor Sverre Haver for his patience and support during these months. I am very grateful for his guidance and the information he has provided. Furthermore I would also like to thank Etienne Cheynet for the help and discussions regarding MATLAB and Ove Tobias Gudmestad for his help the last week. Finally I will also thank Sturla Henriksbø for helping me with the English grammar in this thesis.

Stavanger, 16th June 2014

Magnus Haugen Morken

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## List of Symbols and Abbreviations

| | |
|---|---|
| SLS | Serviceability limit state |
| FLS | Fatigue limit state |
| ALS | Accidental limit state |
| ULS | Ultimate limit state |
| FORM | First Order Reliability Method |
| $H_s$ | Significant wave height |
| $T_p$ | Spectral peak period |
| $\gamma_p$ , $\gamma_v$, $\gamma_e$, $\gamma_m$ | Partial safety factors |
| $x_e$ | Environmental loads |
| $x_v$ | Variable loads |
| $x_p$ | Permanent loads |
| $\sigma_c$ | Characteristic capacity of structural component |
| $\sigma_c'$ | Part of characteristic capacity that shall withstand environmental loads |
| $q$ | Annual exceedance probability |
| $x_q$ | Response/load corresponding to an annual exceedance probability |
| $T$ | Return period in years |
| $\zeta$ | Wave elevation |
| $A_i$ | Wave amplitude of component $i$ |
| $w_i$ | Circular frequency of component $i$ |
| $k_i$ | Wave number of component $i$ |
| $\epsilon_i$ | Random phase angle of component $i$ |
| $U_{10}$ | Mean wind speed |
| $\sigma_U$ | Standard deviation of wind speed |
| $I$ | Turbulence intensity for wind |
| $f$ | Frequency in Hertz |
| $S_{\Xi\Xi}$ | Wave spectrum |
| $S_{\Gamma\Gamma}$ | Response spectrum |

| | |
|---|---|
| $\Phi$ | Wave direction |
| $f_{HsTp}(h,t)$ | Joint probability density function for spectral peak period and significant wave height |
| $f_{T_p\|H_s}(t\|h)$ | Conditional distribution of spectral peak period given significant wave height |
| $f_{H_s}(h)$ | Marginal distribution of significant wave height |
| $F_{X_{3h}\|H_sT_p}(x\|h,t)$ | Distribution of 3 hour load/response maxima in a given sea state |
| $v_{0,\Gamma}^{+}(h,t)$ | Expected zero-up crossing frequency |
| $n_{3h}(h,t)$ | Expected number of global maxima during the given sea state |
| $X_{3h,\alpha}$ | Percentile response |
| $F_{X_{3h}}(x)$ | Long term distribution of 3 hour response maxima |
| $n_{3h}$ | Number of 3 hour sea states per year |
| $u_1, u_2$ | Transformed variables in the U-space |
| $\beta_c$ | Radius of the U-space |
| $\tilde{X}_s$ | Most probable largest storm maximum |
| $F_{\tilde{X}_s}(\tilde{x}_s)$ | Long term description of the most probable largest storm maximum |
| $v_i$ | Ratio between most probable largest storm response and the simulated storm response |
| $F_{X\|\tilde{X}_s}(x\|\tilde{x}_s)$ | Conditional distribution function for simulated storm response given the most probable largest storm maximum |
| $F_{x_s}(x)$ | Long term distribution of storm maximum response |

# 1 Introduction

Understanding how ocean environments interact with fixed or floating structures is vital to ensure the safety of personnel and property, as well as reducing the cost. In that context it is important to accurately predict environmental loads subjected during an offshore structure's life cycle and to ensure that it can withstand extreme environmental loads, such as waves, wind, currents etc. In addition, floating structures may have complex geometry and its behavior in different sea states can be challenging to predict.

To be able to design for these environmental loads, the Norwegian Rule and Regulations require that the characteristic loads are determined for a low probability of occurrence, i.e. that offshore structures shall withstand environmental loads that will only occur one time in a given time period.

## 1.1 Background

When observing the open sea, it behaves in a confused way, meaning that waves are propagating in different direction with different wave height and period. The sea is composed of many waves with different wave height and period that moves in different directions, also referred to as irregular sea. To describe this behavior, it is necessary to treat the sea characteristics in statistical terms. These are obtained from time-series measurements of the natural sea state (Ocean Engineering Research Group, n.d.). However, it is rare that the long term measurements are of a sufficient length and content, and therefore hindcasting is often preferred. Hindcasting uses mathematical models to generate the sea state characteristics. It can be used to extend measurements series or interpolate to places where measured data are not available. The hindcast data should be compared with measurements, and calibrated thereafter (Odland, 2013). One of the most important parameters are the significant wave height, $H_s$, which is the average of the $1/3$ highest waves within a weather window. Throughout a weather window with duration between 20 minutes and 6 hours, it is common to set the sea surface level to be constant, i.e. the significant wave height and spectral peak period is constant (DNV, 2010). To obtain estimates of extreme waves, the standard approach is to fit the data to an extreme probability distribution (Weibull, Gumbel, Generalized Pareto etc.). By extrapolation, extreme waves with a low probability of occurrence (e.g. a wave that will only occur one time in period of 100 years) can be estimated. These results are used as design conditions for offshore structures.

In this thesis we are interested in the effect of waves, i.e. the corresponding response motion of the structure. Therefore some definitions are introduced.

When exposed to waves, a floating structure might have linear and rotational response motion. The wave motion will be oscillatory, with the frequency depending on the sea state. The oscillatory linear motion is referred to as surge, sway and heave, where heave is the vertical motion and the oscillatory rotational motion is roll, pitch and yaw, where yaw is the rotation around the vertical axis (Faltinsen, 1990). In this thesis the main focus will be on heave motions.

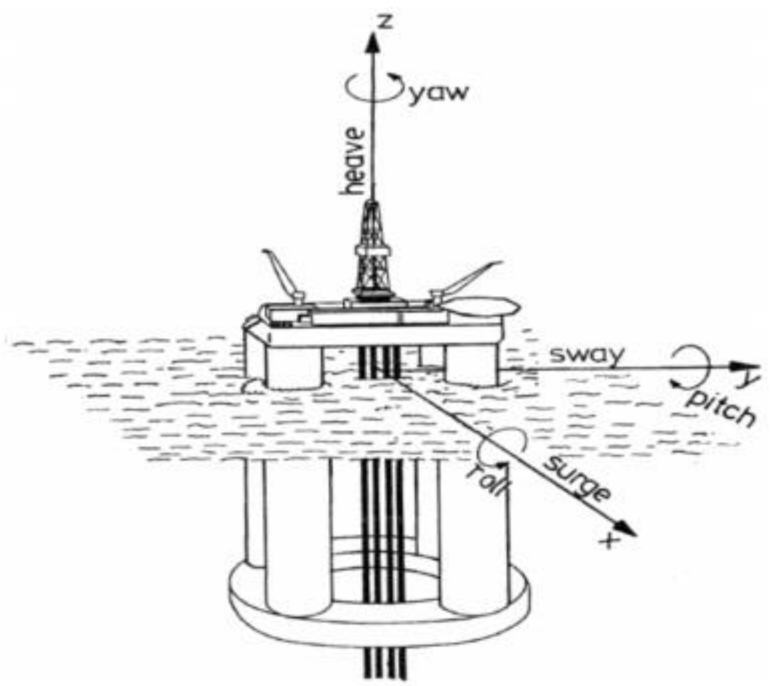


Figure 1.1: Axis system illustrating the different motions (Faltinsen, 1990).

## 1.2 Scope of the thesis

The scope for this thesis is related to predicting the long term extreme heave response for a semi-submersible located in the North Sea. Different approaches to predict extreme motion will be investigated and the underlying statistical methods introduced. A case study will be performed using the all sea state approach and the environmental contour line method.

To perform the calculations, a mathematical program is needed. MATLAB is selected due to its ability to perform numerical computations in an effective manner. Excel is also used.

## 1.3 Outline of the thesis

Chapter 2 gives a brief introduction to the different limit states that are involved in designing offshore structures.

In chapter 3 a brief introduction to environmental modeling is given. Different environmental loads are described and wave spectra are introduced. Further, a demonstration of how to describe the long term variability of ocean waves is made.

Chapter 4 reviews how the effect of waves on a structure within a stationary weather event can be described. Both the frequency domain (linear) and time domain (non-linear) are addressed.

Chapter 5 involves a discussion on how to predict the long term extreme response. The all sea state approach, the environmental contour line method and the peak over threshold approach are discussed.

In chapter 6 and 7 a case study is performed with the all sea state approach and the environmental contour line method. Extreme heave responses are estimated.

A verification of the result found in chapters 6 and 7 is performed in chapter 8. Monte Carlo simulations are used, based on the available hindcast data to estimate the heave response.

Chapter 9 contains the conclusions of the work completed and recommendations for further work.

## 2 Limit states

When designing offshore structures it is important to ensure that the structure can withstand all foreseen loads and/or responses during its lifetime. Generally, one should account for permanent, environmental and variable loads (actions). Permanent loads will not differ regarding magnitude, direction or position over specific period, e.g. weight of structure, weight of permanent ballast, equipment etc. Personnel, helicopters and stored operations are examples of variable loads due to the change in magnitude from ordinary operations. Environmental loads are induced from hydrodynamic forces, wind, ice, earthquakes etc. (Haver, 2013).

According to NORSOK (2012), the requirement for ensuring safety of offshore structures is given by:

$$\gamma_p x_p + \gamma_v x_v + \gamma_e x_e \leq \frac{\sigma_c}{\gamma_m} \qquad \text{Equation 2.1}$$

Where $x_p$, $x_v$ and $x_e$ are the permanent loads, variable loads and the environmental loads. $\sigma_c$ is the characteristic capacity of the structural component and $\gamma_p$, $\gamma_v$, $\gamma_e$ and $\gamma_m$ are partial safety factors to ensure sufficient margin between the limit state capacity and the corresponding characteristic limit state response. $x_e$ in Equation 2.1 correspond to the characteristic loads and/or response, $x_q$, with an annual exceedance probability of $q$, that correspond to a return period of $T -$ years $(\text{T} = 1/q)$. Equation 2.1 can be simplified to:

$$\gamma_e x_q \leq \frac{\sigma_c'}{\gamma_m} \qquad \text{Equation 2.2}$$

Where $\sigma_c'$ corresponds to the part of the capacity that will withstand the environmental loads.

There are four limit states that needs to be controlled in NORSOK (2012), these are:

- Serviceability limit state(SLS):

  This limit state is meant to ensure that deformation should not interrupt the functionality of normal operations of the structure. The characteristic load quantities are typically expected maximum monthly or annual value. All safety factors are typically set to 1,0 as shown in Table 2.1.

- Fatigue limit state(FLS):

  This limit state shall ensure that the structure is designed with proper margin against fatigue failure and are divided into two categories depending on the severity of the consequences. If the consequence is small, the calculated fatigue life should be equal or longer than the designed life of the structure. When the severity of the consequence is high (risk of human life, significant pollution or mayor financial consequences), the calculated fatigue life should be calculated to $n_f$-times the designed life of the structure. $n_f$ is the safety factor and varies from 2-10, where 10 implies that the structure is not accessible for inspection and repair or in splash zone.

- Ultimate limit sate(ULS):

  The ULS is used to ensure that all foreseen loads can be resisted with sufficient margin. It is usually used on a component basis and the characteristic resistance is taken as 5 % of the elastic component capacity. The characteristic load is usually taken to the value corresponding to an annual exceedance probability of $10^{-2}$. ULS is divided into different scenarios that need to be checked; A) where variable and permanent actions are governing and B) where environmental actions are governing. Their relative importance is adjusted with safety factors shown in Table 2.1.

- Accidental limit state(ALS):

  ALS is applied in connection with accidental loads, e.g. loads caused by explosions, fires and collisions. The purpose is to ensure that a given accident does not lead to full loss of the integrity of the structure. Accidental loads are loads corresponding to an annual exceedance probability of $10^{-4}$. In the Norwegian rule regime, very rare environmental loads are also checked in ALS with an annual probability of exceedance of $10^{-4}$. Additionally, the structure shall withstand environmental loads corresponding to an annual exceedance probability of $10^{-2}$ in damaged condition. Usually the plastic capacity is utilized and minor local damage is permitted in connection with ALS. The safety factors are set to 1.0 for steel structures and the recommended values for aluminum and concrete are found in EN-1999 and EN-1992.

Table 2.1: Partial action factors for limit states (NORSOK, 2012)

| Limit state | Action combinations | Permanent actions | Variable actions | Environmental Actions | Deformations actions |
|---|---|---|---|---|---|
| ULS | A | 1,3 | 1,3 | 0,7 | 1,0 |
| ULS | B | 1,0 | 1,0 | 1,3 | 1,0 |
| ALS | A | 1,0 | 1,0 | 1,0 | 1,0 |
| ALS | B | 1,0 | 1,0 | 1,0 | 1,0 |
| SLS | | 1,0 | 1,0 | 1,0 | 1,0 |
| FLS | | 1,0 | 1,0 | 1,0 | 1,0 |

In NORSOK (2012) offshore structures are controlled against overload failure in two limit states, ULS and ALS. In ULS on the Norwegian Continental Shelf, $\gamma_e$ and $\gamma_m$ is set to 1,3 and 1,15 with an annual probability of exceedance of $10^{-2}$. For ALS, $\gamma_e$ and $\gamma_m$ are usually set to 1,0 with an annual exceedance probability of $10^{-4}$. Usually ULS is governing the design as long as the relation between the load and the corresponding annual exceedance probability does not change rapidly. If this is the case, ULS might be sufficiently safe (Haver, 2013). However, the nature of the load side of the problem may vary and structures can face significantly larger characteristic loads with a low annual exceedance probability. By using a safety factor of 1,3, the design load will correspond to an annual exceedance probability of about $10^{-4}$ or lower, which equivalents the ALS requirement (Haver & Winterstein, 2008). For a new structure, all load patterns of concern regarding the $10^{-4}$ probability loads are identified. However for old structures where the load patterns have gotten worse since the structure was designed due to changed wave conditions, reservoir subsidence etc. it is possible that a bad behaving load mechanism can occur as shown in Figure 2.1. With this in mind, one should account for the probability of a bad behaving load mechanism and ALS should be applied for environmental loads to ensure robustness against overload failure.

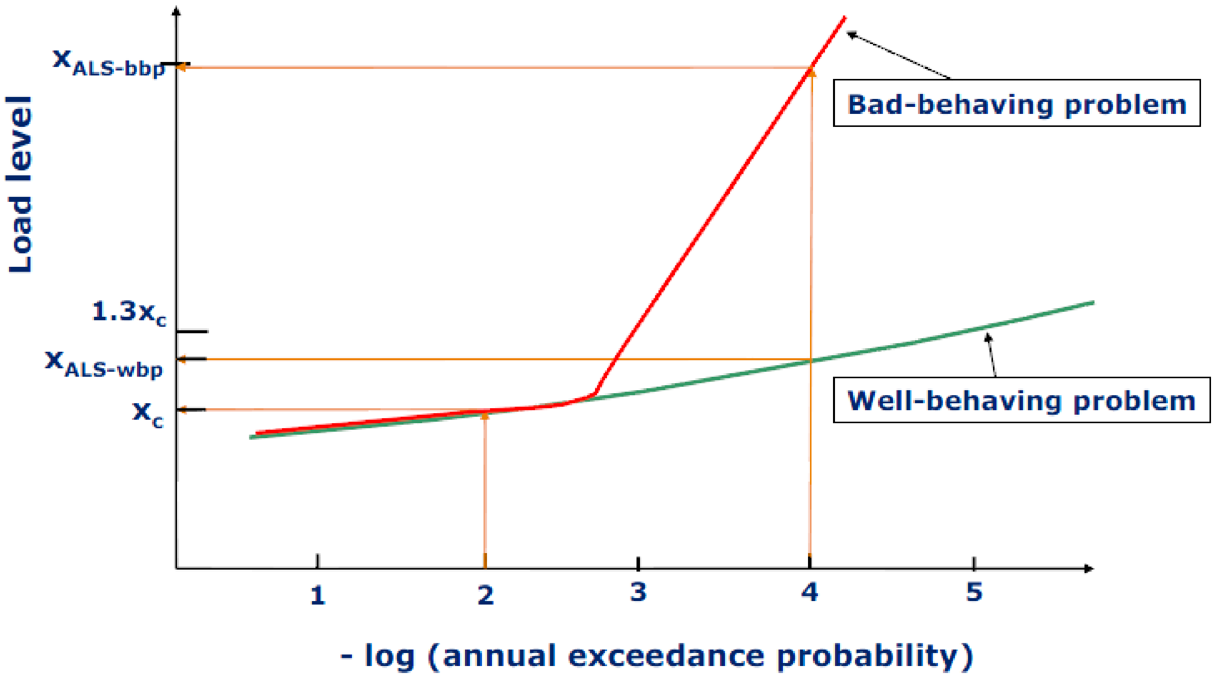


Figure 2.1: Illustration of adequacy of ULS and ALS control for a well and bad behaving load mechanism (Haver, 2013)

# 3 Environmental modeling

Offshore structures are exposed to environmental loads as waves, wind, currents etc. To understand how these environmental loads affects offshore structures it is important to accurately predict the behavior both in short period of time and in the long term. This will be discussed in the following chapter.

## 3.1 Loading of marine structures

The main focus in this thesis is on wave induced loads, but also wind and currents are important parameters when considering the loading of marine structures. Therefore a brief introduction is presented.

### 3.1.1 Waves

As discussed in chapter 1.1, the ocean waves behaves in an irregular way, see Figure 3.1.

Figure 3.1: Ocean waves behave in an irregular way (Okstad, 2012).

In order to simulate irregular sea, linear superposition is utilized where waves with different amplitude, wavelengths and propagation direction are added together. This means that the surface elevation can be modeled by a sum of sinusoidal wave components with different amplitude, frequencies and phases (Faltinsen, 1990):

$$\zeta(x,t) = \sum_{i=1}^{N} A_i \sin(w_i t - k_i x + \epsilon_i) \qquad \text{Equation 3.1}$$

Where $\zeta$, $A_i$, $w_i$, $k_i$ and $\epsilon_i$ are wave elevation, wave amplitude, circular frequency, wave number and a random phase angle between $0$ and $2\pi$ of wave component $i$. As a consequence of considering the waves as a linear superposition, the processes can be treated as Gaussian process with a mean value of zero and variance equal the sum of the components variance. For a short period of time, the wave process is stationary. This implies that the mean and variance are constant within a short period (Battjes, 1978).

### 3.1.2 Wind

Wind speeds vary with time and height above sea level. The parameters that describes the wind speeds are mean wind speed, $U_{10}$, and the standard deviation of the wind speed, $\sigma_U$. The mean wind speed is taken as the average wind speed within a given time period, 10 minutes are commonly used, at a reference height of 10m above sea level. The standard deviation of the wind speed describes the natural variability in wind speed around the mean value. For a short period of time, i.e. over a 10 minutes period, the process can be assumed stationary with constant $U_{10}$ and $\sigma_U$. Another important parameter is the turbulence intensity which is the ratio between standard deviation of the wind speed and the mean speed, $I = \frac{\sigma_U}{U_{10}}$. This describes how much the wind varies within the period of time considered (DNV, 2010).

### 3.1.3 Currents

Currents are very site specific and design values should be obtained by performing measurements on site. In most cases, measurements are conducted in different depths and the velocity is assumed linearly in between (DNV, 2010).

Currents can be divided into several components:

- Wind-generated currents –can be assumed linearly decreasing to zero at a distance of 50m below sea level.
- Tidal currents – the horizontal flow induced by tide.
- Circulational currents – the large ocean currents, e.g. the Gulf Stream in the Atlantic Ocean.

The importance of including currents in estimating loads on offshore structures can be illustrated with the Morrison equation. For a drag dominated structure the effect of current might be substantial. As an example, a wave induced horizontal particle speed with $U_w = 8m/s$ and a typical $10^{-2}$ annual probability current speed with $U_c = 1{,}4m/s$ are used.

$$\frac{\frac{1}{2}\rho C_D A(U_w+U_c)|(U_w+U_c)|}{\frac{1}{2}\rho C_D A U_w|U_w|} \approx \frac{88{,}36}{64} = 1{,}38 \qquad \text{Equation 3.2}$$

We see that the load is 38% higher when the current velocity is included. The reason for this is the cross product of the wave velocity and the current velocity (Haver, 2013).

## 3.2 Wave spectrum

In a stationary process, the wave spectrum, $S_{\Xi\Xi}(w)$, can be utilized to describe the sea surface elevation. The spectrum gives a description of the distribution of wave energy among different frequencies, $f$, alternatively in angular frequency, $w$. The relation between wave amplitude and wave spectrum can be described as following (Faltinsen, 1990):

$$\frac{1}{2}A_i = S_{\Xi\Xi}(w_i)\Delta w \qquad \text{Equation 3.3}$$

Where $A_i$ and $w_i$ are wave amplitude and frequency of wave component $i$, respectively, and $\Delta w$ is the frequency interval.

$S_{\Xi\Xi}(w)$ contains all the information about the statistical properties for $A_i$ since:

$$\sigma^2 = \int_0^\infty S_{\Xi\Xi}(w)dw \qquad \text{Equation 3.4}$$

Ocean waves are generated by wind. However, generating wave spectrum directly from wind measurements is out of reach due to the complex mechanisms on how wind generates waves. As a result, several spectral models have been developed for different wind conditions. In the following, different sea conditions will be introduced and some of the most used spectrum models.

*Wind sea* refers to waves affected and generated by local wind. When the wind blows steadily over a long period over a large area, the waves would come into equilibrium with the wind. This is called a *fully developed sea*. When the wind are reduced significantly or leaves the area, *swells* are formed. These are not affected by the local wind at the time (Stewart, 2012). Usually, both wind sea and swells are present and this interaction is called *combined sea*.

Pierson-Moskowitz Spectrum

Pierson-Moskowitz (1964) used measurements of waves from the North Atlantic to derive this spectrum and it is applicable for fully developed sea states. Haver (2013) proposed that by looking at the significant wave height and spectral peak period, the given sea state could be checked whether it fulfilled the fully developed sea conditions or not. The relation is given by:

$$t_p = 5\sqrt{h_s} \qquad \text{Equation 3.5}$$

The Pierson-Moskowitz spectrum reads:

$$S_{\Xi\Xi}(w) = \frac{\alpha g^2}{w^5} exp\left[-\beta\left(\frac{w_0}{w}\right)^4\right] \qquad \text{Equation 3.6}$$

Where $w = 2\pi f$, and $f$ is the wave frequency in Hertz, $\alpha = 0{,}0081$, $\beta = 0{,}74$, $w_o = \frac{g}{wU_{19,5}}$, and $U_{19,5}$ is the wind speed at a height of 19,5m above the sea surface. Hence, the Pierson-Moskowitz spectrum reads:

$$S_{\Xi\Xi}(w) = \frac{0{,}0081 g^2}{w^5} exp\left[-0{,}74\left(\frac{g}{wU_{19,5}}\right)^4\right] \qquad \text{Equation 3.7}$$

Later the spectrum was modified and the spectral shape was re-parameterized into two parameters, $h_s$ $and$ $t_p$, and is given by:

$$S_{\Xi\Xi}(f) = 0{,}3125 h_s^2 t_p^{-4} f^{-5} \exp\{-1{,}25 t_p^{-4} f_p^{-4}\} \qquad \text{Equation 3.8}$$

JONSWAP Spectrum

JONSWAP spectrum was proposed by Hasselman (1973). The JONSWAP spectrum extends the Pierson-Moskowitz spectrum to include fetch limited seas. It is the most common wave spectrum used in the North Sea and is as following:

$$S_{\Xi\Xi}(f) = 0{,}3125 h_s^2 t_p^{-4} f^{-5} \exp\{-1{,}25 t_p^{-4} f_p^{-4}\}(1 - \ln\gamma)\gamma^{\left\{-\frac{1}{2}\left(\frac{t_p f - 1}{\sigma}\right)^2\right\}} \qquad \text{Equation 3.9}$$

Where $\sigma$ is the width of the spectral peak and given by:

$$\sigma = \begin{cases} 0{,}07, f < f_p \\ 0{,}09, f \geq f_p \end{cases}$$ Equation 3.10

The peak enhancement factor, $\gamma$, are introduced to represent fetch limited wind sea. Torsethaugen (2004) suggest that $\gamma$ can be computed as following:

$$\gamma = 42{,}2\left(\frac{2\pi h_s}{g t_p^2}\right)^{\frac{6}{7}}$$ Equation 3.11

If no particular value for the peak enhancement factor is given, DNV (2010) recommends:

$$\gamma = 5 \; for \; \frac{t_p}{\sqrt{h_s}} \leq 3{,}6$$

$$\gamma = \exp\left(5{,}75 - 1{,}15\frac{t_p}{\sqrt{h_s}}\right) for \; 3{,}6 < \frac{t_p}{\sqrt{h_s}} < 5$$ Equation 3.12

$$\gamma = 1 \; for \; 5 \leq \frac{t_p}{\sqrt{h_s}}$$

<u>Torsethaugen Spectrum</u>

Generally a sea system will be a combined sea where the propagation direction are different with an arbitrary combination of $H_s$ and $T_p$. Torsethaugen (2004) proposed a spectrum that divided the $h_s - t_p$ plane into a wind sea dominated region and a swell sea dominated region where the boundary between these regions is given by:

$$t_{pb} = 6{,}6 h_s^{0{,}333}$$ Equation 3.13

If the spectral peak period in the sea state is close to this border, the spectrum has a single peaked form. However, if the spectral peak period differs from $t_{pb}$ the spectrum has a two peaked form where values lower than $t_{pb}$ are associated with wind growing sea and values higher than $t_{pb}$ are associated with swells (Haver, 2013).

As mentioned above, each spectrum has its own validity range. Before selecting a wave spectrum, the sea state characteristics that are most critical for the problem under consideration should be evaluated and located. Haver(2012) gives an illustration of the validity of different spectral models in Figure 3.2.

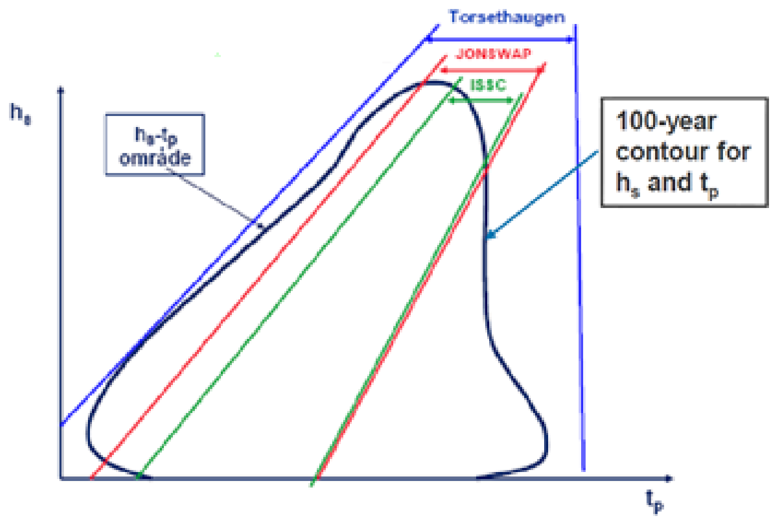


Figure 3.2: Range of validity for spectral models (Haver, 2012).

## 3.3 Long term description of the sea characteristics

For a short period of time, the wave process is stationary and can be completely characterized by the wave spectrum. On the Norwegian Continental Shelf, the most common wave spectrum is the JONSWAP which are defined by the significant wave height and the spectral peak period. Additionally, the main direction of the wave propagation, Φ, should be included.

Since the short term sea state are characterized by $h_s, t_p$ $and$ $\Phi$, the long term variability of the sea state characteristics can be described by a joint density function compressed of the same parameter and are written as following:

$$f_{H_s T_p \Phi}(h, t, \phi) = f_{\Phi}(\phi) f_{H_s T_p|\Phi}(h, t|\phi) \quad \text{Equation 3.14}$$

To describe the long term variability accurately, it is vital with a continuous sample of data. On the Norwegian Continental Shelf, wave observations with a duration of 20 minutes have been gathered every third hour for over 50 years.

### 3.3.1 Marginal distribution of Φ

The marginal distribution of Φ are difficult to model by a few parameters. A method that has proven to give good estimates is to divide the circle into a number of sectors. A common choice is to divide the circle into 12 sectors with a width of 30 degrees and associate each sector with a probability of occurrence (number of observations in one sector divided by the total number of observations). For problems that are sensitive to Φ, one should consider a finer resolution. However, by dividing the circle with a finer resolution, the amount of available data for the significant wave height and spectral peak period decreases and the establishment of the conditional distribution of significant wave height and spectral peak period are more uncertain (Wijaya & Haver, 2009).

The direction of the wave propagation is commonly assumed to be equal to the wind direction. This has been proven to be sufficient for storm seas, but for low and moderate seas the accuracy is more unknown. Information provided by hindcast data is another approach. Comparison between hindcast wave direction and measured wave direction suggest that the similarity is reasonable and it is likely that the hindcast wave direction will be used for obtaining wave direction information in the future (Haver, 2013).

### 3.3.2 Conditional distribution of $T_p$ given $H_s$

Experience indicates that the conditional distribution of $T_p$ given $H_s$ and can be well described by a log normal distribution (Haver & Nyhus, 1986).

$$f_{T_p|H_s}(t|h) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\left[\frac{lnt-\mu}{\sigma}\right]^2\right\} \qquad \text{Equation 3.15}$$

Where $\mu = E[\ln T_p]$ and $\sigma^2 = VAR[lnT_p]$. By dividing the hindcast data into intervals for $H_s$, say 0,5m, the expected value and variance for $lnT_p$ are found for each interval and continuous functions are fitted to the data.

If the wave propagation is included, this is done for each sector.

### 3.3.3 Marginal distribution of $H_s$

The hybrid model was proposed by Haver & Nyhus (1986). The hybrid model consists of a log-normal model for $h \leq \eta$ and a 2-parameter Weibull model for $h > \eta$ and usually gives a very good fit to the observations. By utilizing the hybrid model, an accurate description of the waves can be made for both the lower tail and the upper tail of the probability distribution. This implies that this model is very good for both estimating extreme waves (upper tail) and waves regarding marine operations (lower tail). The hybrid model is as following:

$$f_{H_s}(h) = \begin{cases} \frac{1}{\sqrt{2\pi}\delta h} \exp\left[-0{,}5\frac{(lnh-\lambda)^2}{\delta^2}\right] & ; h \leq \eta \\ \frac{\beta h^{\beta-1}}{\alpha^\beta} exp\left[-\left(\frac{h}{\alpha}\right)^\beta\right] & ; h > \eta \end{cases}$$ Equation 3.16

Where $\delta$ and $\lambda$ are mean and variance of the variable $lnHs$ and the Weibull parameters $\alpha$ and $\beta$ are estimated such that the model is continuous at $h = \eta$ for both cumulative and probability distribution. To obtain a good fit, various values for $\eta$ are calculated and the corresponding values for $\alpha$ and $\beta$ are found. For each $\eta$, a goodness of fit are performed, e.g. Chi square-test, to ensure the least error. The value of $\eta$ that gives least error are selected.

If we are only interested in the upper tail of the probability distribution, DNV (2010) suggests a 3-parameter Weibull distribution, see e.g. Nordenstrøm (1973) or Bury (1975). The 3-parameter Weibull distribution is as following:

$$F_{H_s}(h) = 1 - exp\left(-\left[\frac{(h-\gamma)}{\alpha}\right]^\beta\right)$$ Equation 3.17

Where $\alpha$ is the scale parameter, $\beta$ is the shape parameter and $\gamma$ is the location parameter. Since observations below the location parameter are neglected, not all data are taken into consideration.

If the wave propagation is included, this is done for each sector.

# 4 The response problem

So far the environmental modeling has been addressed. However, it is the consequences of the environmental characteristics that are of most interest, i.e. the corresponding response of the structure. To predict the long term extreme response, the long term response distribution have to be found by combining the short term response distribution for a given sea state with the long term variation of the sea state characteristics.

Often the short term response distribution is more challenging to obtain. Therefore, several approaches will be introduced to determine this. For linear problems, the best way is to use the frequency domain. If one has a non-linear problem, it would be impossible to solve using the frequency domain and the time domain is more suited. Both approaches will be addressed in the following.

## 4.1 Frequency domain

For each frequency there is a linear relation between response amplitude and wave amplitude. The transfer function, $h_{\Xi\Gamma}(f_i)$, characterizes the relation between the wave process , $\Xi(t)$, and the response process , $\Gamma(t)$. Transfer function, also called response amplitude operator (RAO), indicates which effect a given sea state has on a vessel or rig. The following is based on Haver (2013). If the wave spectrum for a given sea state is known and the transfer function, the response spectrum is given by:

$$S_{\Gamma\Gamma}(f;h,t) = h_{\Xi\Gamma}(f_i)^2 S_{\Xi\Xi}(f;h,t) \quad \text{Equation 4.1}$$

Due to the relation between the wave process and the response process it can be modeled by a Gaussian process.

The variance, $\sigma_\Gamma^2$, and expected zero-up-crossing frequency, $v_{0,\Gamma}^+$, can be found from the following:

$$\sigma_\Gamma^2(h,t) = m_{\Gamma\Gamma}^0(h,t) \quad \text{Equation 4.2}$$

$$v_{0,\Gamma}^+(h,t) = \sqrt{\frac{m_{\Gamma\Gamma}^2(h,t)}{m_{\Gamma\Gamma}^0(h,t)}} \quad \text{Equation 4.3}$$

Where the spectral moments are defined by:

$$m_{\Gamma\Gamma}^{j}(h,t) = \int_{0}^{\infty} f^{j}\, s_{\Gamma\Gamma}(f;h,t)df \qquad \text{Equation 4.4}$$

Under the linear assumption the response process can be described by a Gaussian process due to that the wave process is Gaussian. Therefore the global response maxima, i.e. the largest response maxima between two zero-up-crossings, can be modeled by a Rayleigh distribution as a conditional distribution given the sea characteristics:

$$F_{X|H_sT_p}(x|h,t) = 1 - \exp\left\{-\frac{1}{2}\left[\frac{x}{\sigma_\Gamma(h,t)}\right]^2\right\} \qquad \text{Equation 4.5}$$

Assuming all global responses during the sea state are independent, the distribution of the largest response during 3 hour can be written as:

$$F_{X_{3h}|H_sT_p}(x|h,t) = \left\{1 - \exp\left\{-\frac{1}{2}\left[\frac{x}{\sigma_\Gamma(h,t)}\right]^2\right\}\right\}^{n_{3h}(h,t)} \qquad \text{Equation 4.6}$$

Where the duration of the stationary sea state is 3 hour and $n_{3h}(h,t)$ the expected number of global response maxima during the given sea state, i.e.:

$$n_{3h}(h,t) = 3600\frac{s}{h}\cdot 3h \cdot v_{\Gamma,0}^{+} \qquad \text{Equation 4.7}$$

When $n_{3h}(h,t)$ increases, Equation 4.6 can be approximated by a Gumbel distribution:

$$F_{X_{3h}|H_sT_p}(x|h,t) = exp\left\{-exp\left[-\left(\frac{x-\gamma}{\beta}\right)\right]\right\} \qquad \text{Equation 4.8}$$

Where $\gamma$ is the most probable largest response amplitude during 3 hours and the parameters are:

$$\gamma = \sigma_\Gamma(h,t)\sqrt{2\ln n_d(h,t)} \qquad \text{Equation 4.9}$$

$$\beta = \frac{\sigma_\Gamma(h,t)}{\sqrt{2\ln n_d(h,t)}} \qquad \text{Equation 4.10}$$

Alternatively, a higher percentile value can be adopted to estimate the most probable largest response:

$$X_{3h,\alpha} = \gamma - \beta\ln[-\ln(\alpha)] \qquad \text{Equation 4.11}$$

## 4.2 Time domain

By adopting a step-by-step procedure in the time domain, non-linear response can be achieved. This is needed when solutions from the frequency domain no longer are available. The different steps in this approach are shortly introduced with the simplest system, one-degree-of-freedom formulation of the equation of motion. The following is based on Haver(2014).

Equation of motion is as following:

$$m x(t) + c\dot{x}(t) + k\ddot{x}(t) = Q(t)$$ Equation 4.12

The relation between $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are as following:

$$\dot{x}(t) = \frac{dx}{dt}$$ Equation 4.13

$$\ddot{x}(t) = \frac{d\dot{x}}{dt}$$ Equation 4.14

This implies that if we know the acceleration from $t$ to $t + h$, the displacement and velocity can be determined given the initial conditions are known:

$$\dot{x}(t+h) = \dot{x}(t) + \int_0^h \ddot{x}(t+\varepsilon)d\varepsilon$$ Equation 4.15

$$x(t+h) = x(t) + \int_0^h \dot{x}\,(t+\varepsilon)d\varepsilon$$ Equation 4.16

A well-known method for solving the equation of motion in the time domain is the Newmark $\beta - \mathrm{methods}$, see Newmark (1959):

$$\dot{x}(t+h) = \dot{x}(t) + (1-\gamma)h\ddot{x}(t) + \gamma h\ddot{x}(t+h)$$ Equation 4.17

$$x(t+h) = x(t) + h\dot{x}(t) + \left(\frac{1}{2} - \beta\right)h^2\ddot{x}(t) + \beta h^2\ddot{x}(t+h)$$ Equation 4.18

Most common is to assume that between $t$ and $t + h$, the acceleration is constant and is equal to the average of acceleration at interval ends, i.e. $\gamma = 0{,}5$ and $\beta = 0{,}25$:

$$\ddot{x}(\varepsilon) = \frac{1}{2}\big(\ddot{x}(t) + \ddot{x}(t+h)\big);\;\; t \le \varepsilon \le t+h$$ Equation 4.19

To find the expression for displacement we introduce Equation 4.19 into Equation 4.15 and insert this to Equation 4.16:

$$x(t+h) = x(t) + \int_0^h \left[\dot{x}(t) + \frac{1}{2}\varepsilon\big(\ddot{x}(t) + \ddot{x}(t+h)\big)\right] d\varepsilon$$

$$= x(t) + \dot{x}(t)h + \frac{1}{4}h^2(\ddot{x}(t) + \ddot{x}(t+h))$$ Equation 4.20

By setting $\varepsilon$ equal to the step length $h$, we can calculate the next step ahead of the known state. Further we will denote $x(t), \dot{x}(t), \ddot{x}(t), x(t+h), \dot{x}(t+h)$ and $\ddot{x}(t+h)$ by $x_i$, $\dot{x}_i$, $\ddot{x}_i$, $x_{i+1}, \dot{x}_{i+1}$ and $\ddot{x}_{i+1}$:

$$\dot{x}_{i+1} = \dot{x}_i + \frac{1}{2}(\ddot{x}_i + \ddot{x}_{i+1})h$$ Equation 4.21

$$x_{i+1} = x_i + \dot{x}_i h + \frac{1}{4}h^2(\ddot{x}_i + \ddot{x}_{i+1})$$ Equation 4.22

Rewriting Equation 4.22 to get the expression for acceleration at time $i+1$:

$$\ddot{x}_{i+1} = \frac{4x_{i+1}}{h^2} - \frac{4x_i}{h^2} - \frac{4\dot{x}_i}{h} - \ddot{x}_i$$ Equation 4.23

Introducing Equation 4.23 into Equation 4.21 to get velocity at $i+1$:

$$\dot{x}_{i+1} = \dot{x}_i + \frac{1}{2}\left(\ddot{x}_i + \frac{4x_{i+1}}{h^2} - \frac{4x_i}{h^2} - \frac{4\dot{x}_i}{h} - \ddot{x}_i\right)h = \frac{2x_{i+1}}{h} - \frac{2x_i}{h} - \dot{x}_i$$ Equation 4.24

Equation of motion at $i+1$:

$$m\ddot{x}_{i+1} + c\dot{x}_{i+1} + kc = Q_{i+1}$$ Equation 4.25

Introducing Equation 4.23 and Equation 4.24 into Equation 4.25:

$$m\left(\frac{4x_{i+1}}{h^2} - \frac{4x_i}{h^2} - \frac{4\dot{x}_i}{h} - \ddot{x}_i\right) + c\left(\frac{2x_{i+1}}{h} - \frac{2x_i}{h} - \dot{x}_i\right) + kx_{i+1} = Q_{i+1}$$ Equation 4.26

From this we can find the displacement at $i+1$:

$$x_{i+1} = \frac{\left[Q_{i+1} + m\ddot{x}_i + \dot{x}_i\left(\frac{4m}{h} + c\right) + x_i\left(\frac{4m}{h} + \frac{2c}{h}\right)\right]}{\left(\frac{4m}{h^2} + \frac{2c}{h} + k\right)}$$ Equation 4.27

Going through this step-by-step method, the displacement at $i+1$ can be calculated using Equation 4.27. Once the displacement is known, the velocity and acceleration at $i+1$ can be found from Equation 4.23 and Equation 4.24. By knowing the initial conditions for the structure, the response history can be obtained using time domain simulations.

By evaluating the sea surface elevation for a given 3 hour duration and generating a wave spectrum, the time domain simulations can be used to establish a short term response distribution. Hence, water particles velocity and acceleration surrounding the submerged parts of the structure are calculated. For each time step, the load vector is calculated for the corresponding parts of the structure. This is used to solve the equation of motion shown above and a time series of nodal displacements are obtained. If the structural motions are small, this is done rather quickly. When this information is gathered, we can estimate the global maxima distribution, or alternatively the 3 hour maximum response from each simulation.

By conducting N numbers of simulations, say 30 times, we obtain 30 simulated 3 hour maximum responses. To create a short term distribution, a distribution function must be selected. A Gumbel distribution corresponds well to a time-series containing maxima's (Oosterbaan, 1994).

$$F_{X_{3h}|H_sT_p}(x|h,t) = exp\left\{-exp\left[\frac{x-\gamma}{\beta}\right]\right\} \qquad \text{Equation 4.28}$$

Unlike in the linear problem, both $\gamma$ and $\beta$ are unknown. However, the maximum response of each simulation, $X_{3h}$, is known and from this the expected value, $\bar{X}$, and standard deviation, $\sigma_x$, can be calculated:

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_{3\mathrm{h}_i} \qquad \text{Equation 4.29}$$

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(X_{3\mathrm{h}_i} - \bar{X}\right)^2} \qquad \text{Equation 4.30}$$

$\gamma$ and $\beta$ can be written as a function of the variance and standard deviation (Haver, 2013):

$$E[X_{3h}] = \gamma + 0{,}57722\beta = \bar{X} \qquad \text{Equation 4.31}$$

$$STD[X_{3h}] = 1{,}28255\beta = \sigma_x \qquad \text{Equation 4.32}$$

By solving the equations, the parameters are found and short term distribution for a 3 hour response maxima are established with time domain simulations.

# 5 Different approaches to estimate characteristic responses

Characteristic loads/responses are estimated to correspond to a given exceedance probability. There are several ways of doing this, e.g. the all sea state approach, the peak over threshold approach, the environmental contour line method etc. The selection of methods to be used depends on the nature of the problem under consideration and the environmental characteristics around the location of the structure. Some of the most frequently used method will be introduced in the following.

## 5.1 The all sea state approach

In the 1950s, the long term wave and response analyses where introduced to the field of naval architecture, see e.g. Jasper (1956). This approach combined the short term response variability with the long term variability of the sea characteristics. With the all sea state approach, the distribution is established with all sea states. In the past it has been challenging to describe the long term variability of the sea characteristics accurately due to lack of information regarding significant wave height and spectral peak period. However, when the petroleum activity increased the amount of available site-specific data increased. This resulted in major improvements regarding the joint modeling of the environmental characteristics like significant wave height and spectral peak period, see e.g. Haver & Nyhus(1986).

Previously we have shown that the wave situation for a given sea state can be described by wave spectrum and maximum response can be found for the duration of the storm. However, it is desirable to calculate the long term distribution for a 3 hour maximum response. To be able to do this, a long term contribution is needed. This can be obtained from the wave history of a selected area, which contains information about how many waves with a certain significant wave height and spectral peak period that has appeared, see chapter 3.3. The long term distribution of responses in a random 3 hour sea state is then given by:

$$F_{X_{3h}}(x) = \iint\limits_{h\,t} F_{X_{3h}|H_sT_p}(x_q|h,t) f_{H_sT_p}(h,t)\,dt\,dh \qquad \text{Equation 5.1}$$

The characteristic response, $x_q$, that corresponds to the $q$-probability is given by:

$$1 - F_{X_{3h}}(x_q) = \frac{q}{n_{3h}}$$ Equation 5.2

Where $n_{3h}$ is the number of 3 hour sea states per year (2920) and the exceedance probability corresponds to the requirements in ULS and ALS, see chapter 2.

## 5.2 The environmental contour line method

When a non-linear problem is under consideration, many short term distributions have to be solved either by time domain simulations or model testing. This can prove to be very challenging and time consuming since many wave conditions must be analyzed. Therefore, a simplified approach has been established, called the environmental contour lines method. This method uses the environmental description of the sea state characteristics to establish a contour line with respect to significant wave height and spectral peak period that correspond to a given annual exceedance probability. This implies that any sea state along the contour line has the same probability of occurrence (Baarholm et al., 2010).

The contour lines can be established by using the First Order Reliability Method (FORM). By utilizing the long term wave climate description, the significant wave height and spectral peak period can be transformed from the physical space into a U-space. The transformation can be done with Rosenblatt transformation scheme, see e.g. Madsen et al (1986).

$$F_{H_s}(h) = \Phi(u_1)$$ Equation 5.3

$$F_{T_p|H_s}(t|h) = \Phi(u_2)$$ Equation 5.4

Where $\Phi$ is the standard normal distribution and the transformed variables $u_1$ and $u_2$ are independent. The contour lines in the U-space can therefore be written as:

$$u_1^2 + u_2^2 = \beta_c^2$$ Equation 5.5

Where the radius of the U-space, $\beta_c$, corresponds to the inverse of the standard normal distribution for a given exceedance probability, $q$:

$$\beta_c = -\Phi^{-1}(q)$$ Equation 5.6

The corresponding values for $u_1$ and $u_2$ along the circle can be found from simple geometry (Baarholm et al., 2010):

$$u_1 = \beta_c \cos(\theta)$$ Equation 5.7

$$u_2 = \beta_c \sin(\theta)$$ Equation 5.8

When all the values for $u_1$ and $u_2$ are found around the circle, the corresponding values for $H_s$ and $T_p$ can be found from Equation 5.3 and Equation 5.4. In Figure 5.1 the relation between the U-space and physical space are illustrated.

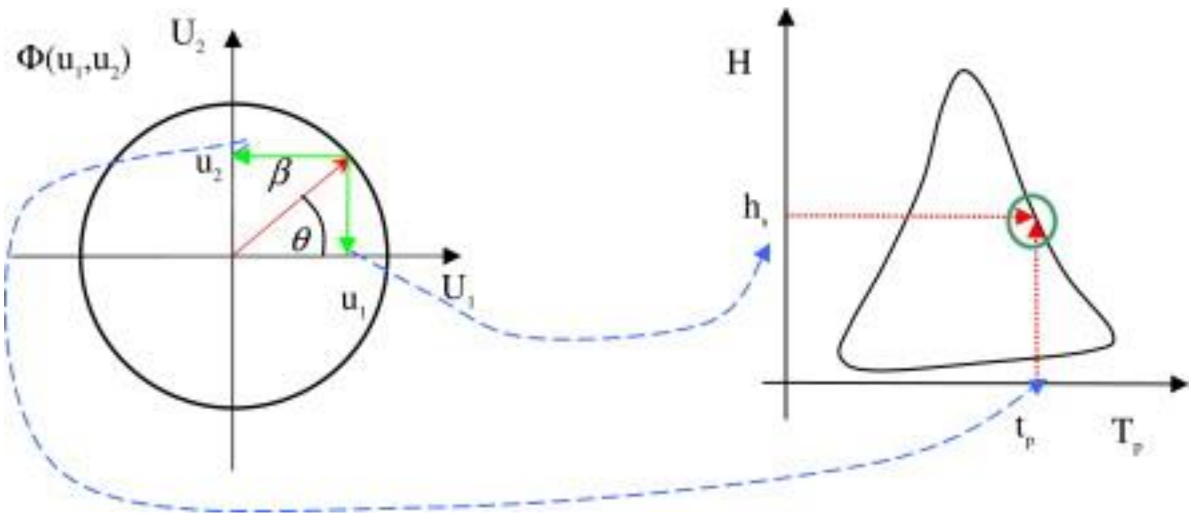


**Figure 5.1: Transformation from U-space to $H_s - T_p$ plane (Baarholm et al., 2010).**

When the contour lines for a given exceedance probability are created, we select some of the worst sea states to be further evaluated. Time domain simulations or model tests are performed to the selected sea states to establish which sea states that creates the largest response on the structure. When the worst sea state is located, further simulations or model tests shall be performed for the given sea state to establish the response distribution.

If the response distribution is very narrow, the estimated extreme response can be taken as the mean value, $\mu_x$, see Figure 5.2 (a), since the short term variability is very low. However, in reality the response distribution is not narrow and if the short term variability is neglected, the extreme response is typically underestimated by 10-15 % (Haver, 2013). To get a proper estimate of the extreme response this variability has to be accounted for.

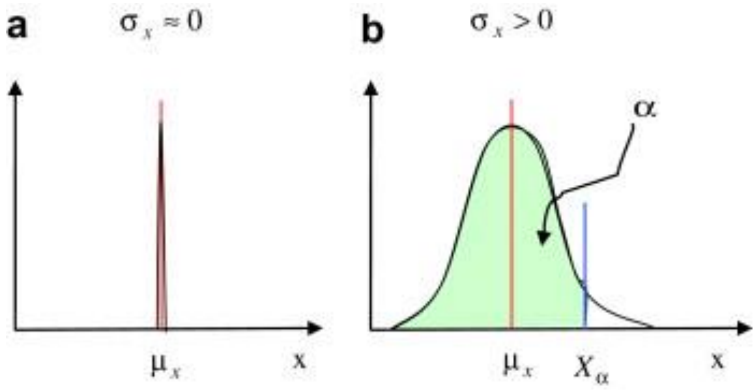


**Figure 5.2: a) Narrow response distribution with the extreme response equal the mean value, $\mu_x$.b) response distribution with higher short term variability and an example with the use of percentile response (Baarholm et al., 2010).**

Several methods are used, e.g. multiplying the most probable largest response with a predetermined factor or calculating the expected largest response as a high percentile value of the 3 hour extreme response distribution. When estimating the contour lines corresponding to an exceedance probability of $10^{-2}$, a factor between 1,1 and 1,3 are recommended or a percentile of 85-95%. For lower exceedance probabilities, e.g. $10^{-4}$, a percentile of 90-95% is recommended. Since environmental contour lines are a simplified approach, it should if possible be verified by a long term analysis (NORSOK, 2007).

## 5.3 The peak over threshold approach

Contrary to the all sea state approach, in the peak over threshold approach only sea states with a significant wave height above a given threshold are used. By applying a higher threshold to the significant wave height we can obtain the storm history, see Figure 5.3. By introducing a step function to each storm, the distribution function for the largest response in each step can be found. Each step represents a stationary weather window, with constant significant wave height and spectral peak period. The steps are increasing to the maximum in each storm and decreasing on the other side down to the threshold, see Figure 5.4.

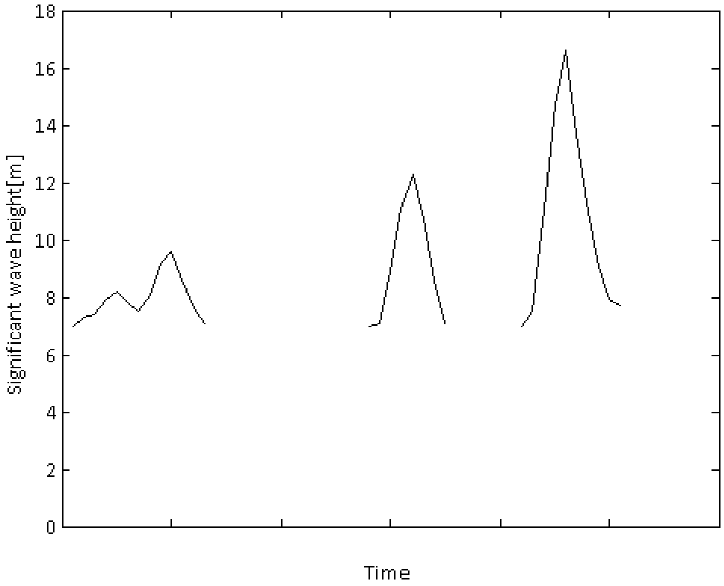


Figure 5.3 :Example of a storm history with a threshold of 7m.

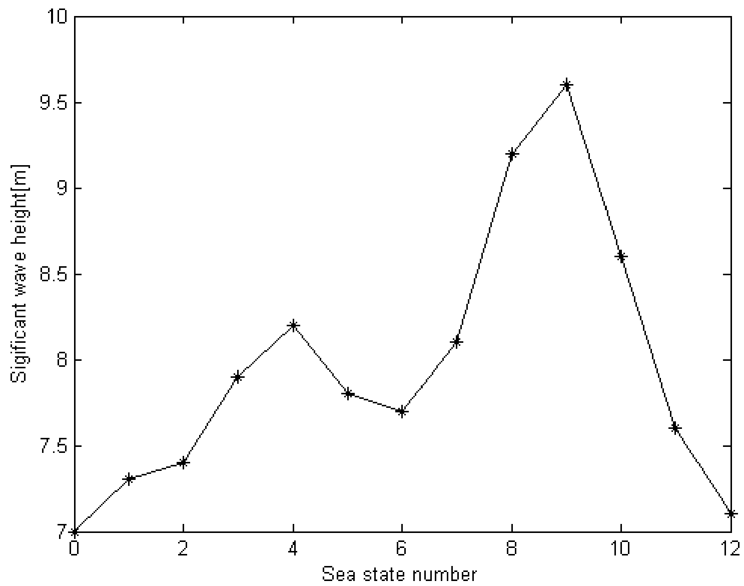


Figure 5.4: Example of the steps in one storm with a threshold of 7m.

The extreme value for each step in a given storm, $k$, are assumed to follow a Gumbel distribution and can be obtained by:

$$F_{X_m|k}(x|k) = \exp\left\{-exp\left\{-\left(\frac{x - \propto_{m,k}}{\beta_{m,k}}\right)\right\}\right\} \qquad \text{Equation 5.9}$$

By assuming that each step can be treated as statistically independent, the distribution function for the response maxima in a given storm, $X_a$, can be written as:

$$F_{X_a|k}(x) = P(X_a \le x) = P[(X_1 \le x) \cap (X_2 \le x) \cap \dots (X_m \le x)]$$

$$= \prod_{m=1}^{m_{max}} F_{X_m|k}(x|k) = \exp\left\{-\sum_m exp\left\{-\left(\frac{x - \propto_{m,k}}{\beta_{m,k}}\right)\right\}\right\}$$ Equation 5.10

From Equation 5.10, the most probable largest storm maximum, $\tilde{X}_s$, can be estimated by $\frac{\partial^2 F_{X_a|k}(\tilde{x})}{\partial x^2} = 0$. By applying this to all the storms, we get a sample of the most probable response maximum for each storm.

To describe the long term description of the most probable largest response, both a 3-parameter Weibull model and a generalized Pareto model has proven to describe the data well (Tromans & Vanderschuren, 1995). In the following, a 3-parameter Weibull distribution is selected:

$$F_{\tilde{X}_s}(\tilde{x}_s) = 1 - exp\left\{-\left(\frac{\tilde{x}_s - \tilde{x}_{s,0}}{\lambda}\right)^{\rho}\right\}$$ Equation 5.11

Where the shape parameter, $\rho$, and scale parameter, $\lambda$, are estimated from the data sample and $\tilde{x}_{s,0}$ are chosen as a reasonable value from the data sample.

If we observe the storm, we would notice that the actual response maximum varies around the most probable response maximum. To account for this variability, we can generate a possible observation for each step in every storm. By using Monte Carlo simulation on each step in storm $k$, we get a new simulated response history. By replacing $F_{X_m|k}(x|k)$ with $u_{m,k}$ and let this be randomly uniformed between 0 and 1, a new realization of the response maximum are achieved.

$$x_{m,k} = \propto_{m,k} - \beta_{m,k}\ln(-\ln(u_{m,k})$$ Equation 5.12

The simulated response maximum in each storm is found by:

$$x_s = max[x_{m,k}], for\ m = 1\ to\ number\ of\ steps\ in\ storm\ k$$ Equation 5.13

By looking at the relation between the most probable storm response and the simulated storm response, $v_i$, the variability regarding the most probable storm response and the effect of non-observed storms can be accounted for (Tromans & Vanderschuren, 1995).

$$v_i = \frac{x_{s,i}}{\tilde{x}_{s,i}}, for\ i = 1\ to\ number\ of\ storms$$ Equation 5.14

The ratio of $v$ is assumed to follow a Gumbel distribution:

$$F_V(v) = \exp\left\{-\exp\left\{-\frac{(v-\alpha_V)}{\beta_V}\right\}\right\}$$ Equation 5.15

For the estimation of the Gumbel parameters, see Bury (1975). By simple transformation:

$$F_{X_s|\tilde{X}_s}(x_s|\tilde{x}_s) = P[X_s \leq x_s|\tilde{X}_s \leq \tilde{x}_s] = P[V\tilde{x}_s \leq x_s] = P\left[V \leq \frac{x_s}{\tilde{x}_s}\right] = F_V\left(\frac{\tilde{x}_s}{\tilde{x}}\right)$$ Equation 5.16

The conditional distribution function for $X_s$ given $\tilde{X}_s$ can be written as:

$$F_{X|\tilde{X}_s}(x|\tilde{x}_s) = \exp\left\{-\exp\left\{-\frac{(x_s-\alpha_V\tilde{x}_s)}{\beta_V\tilde{x}_s}\right\}\right\}$$ Equation 5.17

The long term distribution of storm maximum response is found by:

$$F_{x_s}(x) = \int_{\tilde{x}_s} F_{X_s|\tilde{X}_s}(x|\tilde{x}_s) f_{\tilde{X}_s}(\tilde{x}_s)d\tilde{x}_s$$ Equation 5.18

Where the response corresponding to an exceedance probability $q$ is found from:

$$1 - F_{X_s}(x_q) = \frac{q}{n_s}$$ Equation 5.19

Where $n_s$ is number of storms per year. The $q$-probability corresponds to the requirement in NORSOK (2012), see chapter 2.

## 5.4 The difference between the all sea state and the peak over threshold approach

Peak over threshold is frequently used in hurricane governed areas like the Gulf of Mexico. The reason why this approach is favorable in these areas is because the design conditions are based on hurricanes. To establish a reliable joint description of the weather characteristics, a very long measurement series has to be established. Since the weather between hurricanes is usually of a good nature, it is more accurate to estimate extreme responses from the storm history. In the Gulf of Mexico the duration of each step are usually set to be 0,5 hours. If the peak over threshold method were to be used in the North Sea, a step size of 3 hours is more likely since hindcast data are available with that time frame. Because of the nature of the environmental conditions in the North Sea, the most common approach is the all sea state (Haver, 2013).

All sea state approach is more conservative than peak over threshold approach. The reason is that all stationary weather windows are assumed statistically independent and the correlation of nearby sea states are neglected. As a result, estimates are on the safe side.

The estimated heave response is given for different amount of observations for the two approaches. In the all sea state approach, the estimated heave response will only be exceeded in one random 3 hour sea state for a given return period. This is contrary to the peak over threshold approach that predicts the response that will be exceeded in only one storm for a given return period.

# 6 The all sea state approach

## 6.1 Example of application of the all sea state approach

To illustrate the concept of the all sea state approach, the extreme heave motion response for a semi-submersible located in the North Sea will be analyzed. We will assume that the motion of the semi-submersible can be described linearly, i.e. the heave motion is linearly related to the wave process. The relation between wave process and response process is given in Figure 6.1.

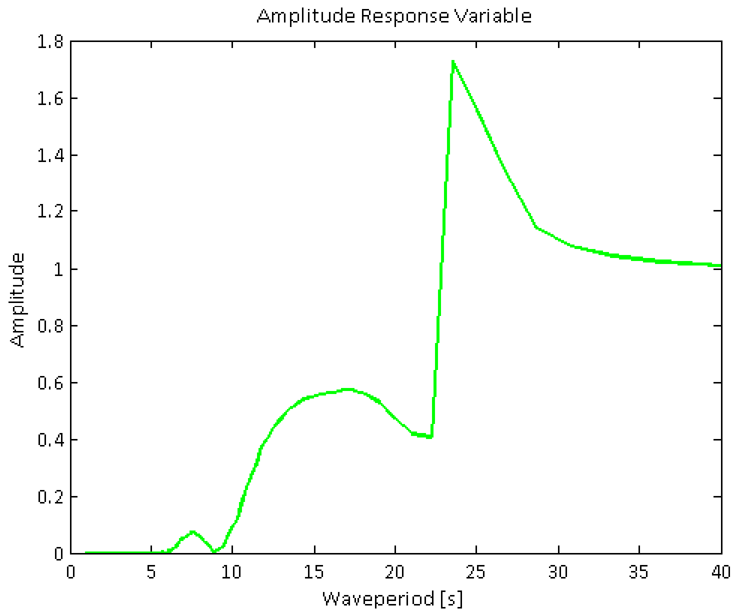


Figure 6.1: Response amplitude operator for selected structure in heave.

From the selected area in the North Sea, a hindcast data series for the significant wave height and spectral peak period are available. The data includes values for every 3 hour stationary weather situation from September 1957 to June 2013. It is assumed that all waves are propagating in the same direction.

## 6.2 Environmental description

The long term variation of wave climate can be described by a joint probability density of $H_s$ and $T_p$, $f_{HsTp}(h,t)$, which can be estimated from the hindcast data. A scatter diagram for this particular area is obtained from the hindcast data, shown in Table 6.1.

**Table 6.1: 55,75 year scatter diagram for the selected area in the North Sea .**

| Hs \ Tp | 0-1 | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 | 9-10 | 10-11 | 11-12 | 12-13 | 13-14 | 14-15 | 15-16 | 16-17 | 17-18 | 18-19 | 19-20 | 20-21 | 21-22 | 22-23 | Sum |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-0,5 | 0 | 0 | 0 | 5 | 52 | 84 | 61 | 104 | 80 | 62 | 37 | 8 | 8 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 504 |
| 0,5-1 | 0 | 0 | 2 | 127 | 888 | 1824 | 2929 | 2853 | 2249 | 1565 | 839 | 368 | 240 | 87 | 44 | 25 | 27 | 11 | 9 | 4 | 2 | 0 | 0 | 14093 |
| 1-1,5 | 0 | 0 | 0 | 13 | 1027 | 2669 | 3792 | 5340 | 5926 | 4064 | 2619 | 1588 | 802 | 424 | 200 | 134 | 79 | 37 | 28 | 6 | 8 | 4 | 1 | 28761 |
| 1,5-2 | 0 | 0 | 0 | 0 | 192 | 1814 | 3350 | 3765 | 4896 | 4736 | 3445 | 2467 | 1519 | 855 | 368 | 219 | 158 | 61 | 43 | 15 | 17 | 4 | 9 | 27933 |
| 2-2,5 | 0 | 0 | 0 | 0 | 16 | 420 | 2132 | 2909 | 3326 | 3762 | 3359 | 2402 | 1641 | 1077 | 514 | 284 | 147 | 50 | 53 | 13 | 17 | 6 | 2 | 22130 |
| 2,5-3 | 0 | 0 | 0 | 0 | 0 | 65 | 807 | 2151 | 2582 | 2690 | 2848 | 2444 | 1736 | 1041 | 584 | 346 | 203 | 84 | 58 | 13 | 5 | 3 | 4 | 17664 |
| 3-3,5 | 0 | 0 | 0 | 0 | 0 | 5 | 217 | 1083 | 2072 | 2246 | 2346 | 2009 | 1591 | 965 | 609 | 345 | 180 | 61 | 59 | 12 | 13 | 2 | 3 | 13818 |
| 3,5-4 | 0 | 0 | 0 | 0 | 0 | 0 | 46 | 347 | 1425 | 1827 | 1835 | 1643 | 1297 | 894 | 553 | 340 | 167 | 92 | 60 | 7 | 10 | 0 | 1 | 10544 |
| 4-4,5 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 81 | 677 | 1289 | 1502 | 1346 | 1052 | 757 | 473 | 301 | 163 | 81 | 38 | 13 | 4 | 1 | 0 | 7781 |
| 4,5-5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 | 247 | 808 | 1193 | 1152 | 814 | 537 | 369 | 253 | 120 | 75 | 43 | 4 | 3 | 0 | 0 | 5633 |
| 5-5,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 66 | 385 | 853 | 1007 | 691 | 425 | 258 | 155 | 89 | 48 | 30 | 2 | 1 | 0 | 1 | 4012 |
| 5,5-6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 25 | 130 | 474 | 797 | 667 | 339 | 232 | 153 | 78 | 31 | 28 | 3 | 0 | 1 | 0 | 2958 |
| 6-6,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 58 | 255 | 539 | 547 | 300 | 174 | 109 | 81 | 41 | 19 | 1 | 1 | 0 | 0 | 2132 |
| 6,5-7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13 | 116 | 347 | 487 | 301 | 117 | 90 | 42 | 31 | 19 | 1 | 1 | 1 | 0 | 1566 |
| 7-7,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 33 | 190 | 378 | 308 | 108 | 64 | 41 | 27 | 20 | 0 | 0 | 0 | 0 | 1170 |
| 7,5-8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 87 | 178 | 272 | 105 | 59 | 21 | 16 | 20 | 0 | 0 | 0 | 0 | 767 |
| 8-8,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 44 | 123 | 213 | 107 | 45 | 17 | 15 | 14 | 0 | 1 | 0 | 0 | 580 |
| 8,5-9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 54 | 146 | 94 | 38 | 5 | 8 | 13 | 1 | 0 | 0 | 0 | 370 |
| 9-9,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 30 | 85 | 72 | 35 | 17 | 7 | 11 | 1 | 0 | 0 | 0 | 263 |
| 9,5-10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 11 | 46 | 55 | 27 | 16 | 4 | 5 | 0 | 2 | 0 | 0 | 167 |
| 10-10,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 21 | 51 | 25 | 10 | 6 | 5 | 0 | 0 | 0 | 0 | 120 |
| 10,5-11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 18 | 19 | 10 | 6 | 7 | 0 | 0 | 0 | 0 | 65 |
| 11-11,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 3 | 11 | 13 | 4 | 2 | 2 | 0 | 0 | 0 | 0 | 38 |
| 11,5-12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 8 | 4 | 6 | 2 | 1 | 0 | 0 | 0 | 0 | 22 |
| 12-12,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 2 | 8 | 1 | 2 | 0 | 0 | 0 | 0 | 19 |
| 12,5-13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 5 |
| 13-13,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 5 | 0 | 1 | 0 | 0 | 0 | 0 | 7 |
| 13,5-14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 4 |
| 14-14,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 14,5-15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 0 | 0 | 0 | 0 | 3 |
| 15-15,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15,5-16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16-16,5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 2 |
| 16,5-17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Sum | 0 | 0 | 2 | 145 | 2175 | 6881 | 13337 | 18649 | 23578 | 23636 | 21764 | 18455 | 13871 | 9103 | 5133 | 3089 | 1697 | 801 | 593 | 96 | 85 | 22 | 21 | 163133 |

The significant wave height and spectral peak period can be described by a joint distribution function, $f_{HsTp}(h,t)$, where $f_{HsTp}(h,t) = f_{Hs}(h) f_{Tp|Hs}(t|h)$. $f_{Hs}(h)$ and $f_{Tp|Hs}(t|h)$ are fitted to the observations separately.

For estimating extreme responses where the upper tail of the probability distribution are of most interest DNV (2010) suggests a 3-parameter Weibull distribution for modeling $F_{Hs}(h)$:

$$F_{Hs}(h) = 1 - exp\left(-\left[\frac{(h-\gamma)}{\alpha}\right]^{\beta}\right)$$ Equation 6.1

In the present case, a good fit is obtained with $\gamma = 0{,}75$. This indicates that all observations with $h_s \leq 0{,}75\ m$ are neglected in further calculations. From the hindcast data, there are 159 917 observations above the threshold. By finding the expected value and variance of the observations, the scale parameter, $\alpha$, and shape parameter, $\beta$, can be estimated from the hindcast data. In the present case, the expected value $E[x] = 1{,}976m$ and variance $VAR[x] = 2{,}5524m$ where $x = h - \gamma$.

$$E[x] = \alpha\Gamma\left(1 + \frac{1}{\beta}\right)$$ Equation 6.2

$$VAR[x] = \alpha^2\left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2\right]$$ Equation 6.3

By dividing Equation 6.2 with Equation 6.3 we find $\beta = 1{,}2623$. Substituting $\beta$ into Equation 6.2 and $\alpha = 2{,}1265$ are found for the hindcast data. The distribution of significant wave height is shown in Figure 6.2:

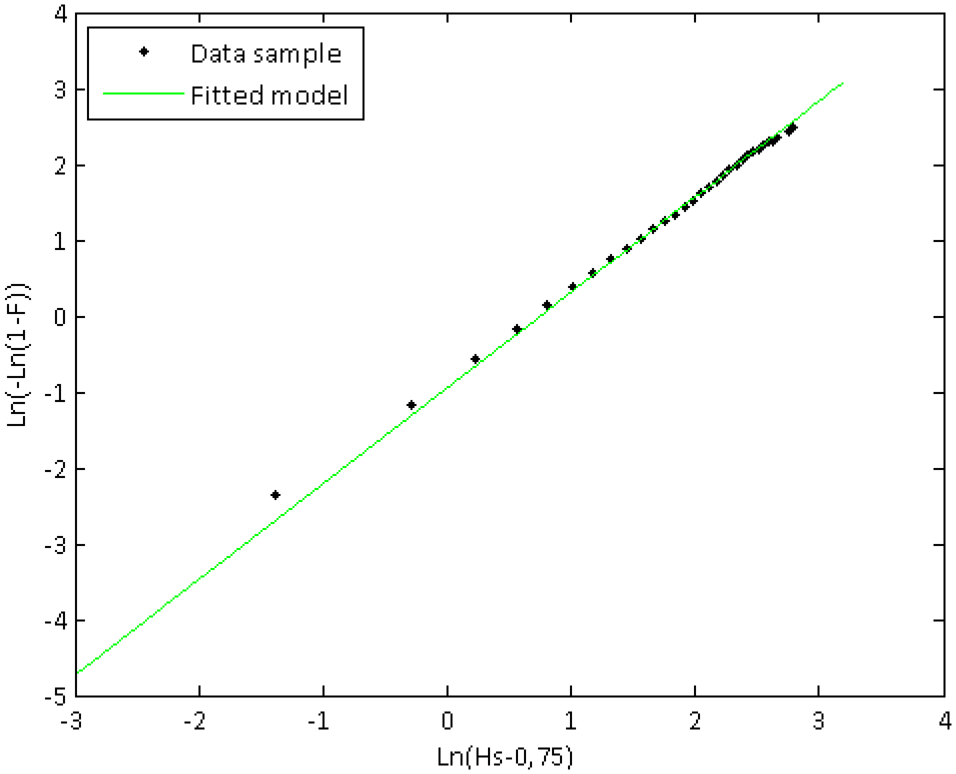


Figure 6.2: Distribution of significant wave height.

Figure 6.2 illustrates that the lower tail of the probability distribution doesn't correspond well to the data. Since we are interested in extreme sea states, i.e. the upper tail, the adaption of this area is of importance and the fit to the lower tail are not of interest. The significant wave height with a corresponding return period of $T$ years can be found by:

$$F_{Hs}(h) = 1 - \frac{1}{T \cdot n_{3h}} \qquad \text{Equation 6.4}$$

Where $n_{3h}$=2920, is the number of 3 hours observations per year and. The significant wave height for 100 year and 10 000 year are as following:

$$H_s^{100} = F_{Hs}^{-1}\left(1 - \frac{1}{100 \cdot 2920}\right) = 16.56m \qquad \text{Equation 6.5}$$

$$H_s^{10\,000} = F_{Hs}^{-1}\left(1 - \frac{1}{10000 \cdot 2920}\right) = 20{,}99m \qquad \text{Equation 6.6}$$

Another approach is to only utilize the data above the location parameter. In this case, the number of observations are 159 917. This gives an annual number of observations above the location parameter of $\frac{159917}{55,75} = 2868,47$. This implies that over a period of 100 and 10 000 years the amount of observations will be less than if observations below the location parameter also are taken into consideration. The significant wave height corresponding to a return period of 100 year and 10 000 year with observation below 0,75 m neglected are:

$$H_s^{100} = F_{Hs}^{-1}\left(1 - \frac{1}{100 \cdot 2868,47}\right) = 16,55m \qquad \text{Equation 6.7}$$

$$H_s^{10\,000} = F_{Hs}^{-1}\left(1 - \frac{1}{10000 \cdot 2868,47}\right) = 20,97m \qquad \text{Equation 6.8}$$

From the calculations we can see that the difference in significant wave height are marginal when the annual number of observations are 2920 and 2868,47. However, if the location parameter is high the effect will be larger. Therefore, when estimating the significant wave height using a 3-parameter Weibull distribution, it is important to check how many annual observations above the location parameter that are expected and check the effect on the estimates.

So far we have only looked at the distribution of significant wave height. To accurate estimate the extreme sea state, the conditional distribution of $T_p$ given $H_s$ must also be taken into account. The conditional distribution of $T_p$ given $H_s$ can be described by a log normal distribution:

$$f_{Tp|Hs}(t|h) = \frac{1}{\sqrt{2\pi}\sigma t} \exp\left\{-\frac{(lnt - \mu)^2}{\sigma^2}\right\} \qquad \text{Equation 6.9}$$

Where $\mu = E[lnTp]$ and $\sigma^2 = VAR[lnTp]$.

From the data set, the values of $\mu$ and $\sigma^2$ are calculated for each class of $H_s$ with an interval of 0,5 m. Continues functions are fitted to these values to obtain estimates for extreme sea states:

$$\mu = a_1 + a_2 h_s^{a_3} \qquad \text{Equation 6.10}$$

$$\sigma^2 = b_1 + b_2 \exp(-b_3 h_s^{b_4}) \qquad \text{Equation 6.11}$$

From the calculated values, a good fit are obtained using the curve fitting tool in MATLAB and are as following:

$\mu = 1{,}195 + 0{,}8585 h_s^{0{,}2425}$ Equation 6.12

$\sigma^2 = 0{,}005 + 0{,}0707 \exp(-0{,}07826 h_s^{1{,}674})$ Equation 6.13

How well the data fits the model is found from the regression line. In regression, the $R^2$ value is a statistical measure of how well the regression line approximates the data points. If $R^2$ equals 1, the regression line fits the data perfectly. In this case, the $R^2$ values for $\mu$ and $\sigma^2$ are 0,9959 and 0,9918. To ensure that $\sigma^2$ always is positive, $b_1$ is set to be 0,005. This is not a requirement, but a recommended practice in developing the function. However, the effect of not locking $b_1$ and let it be selected from the data sample are very small. An example of the consequence is illustrated in Appendix B. The fit of the continuous function to the hindcast data are shown in Figure 6.3 and Figure 6.4.

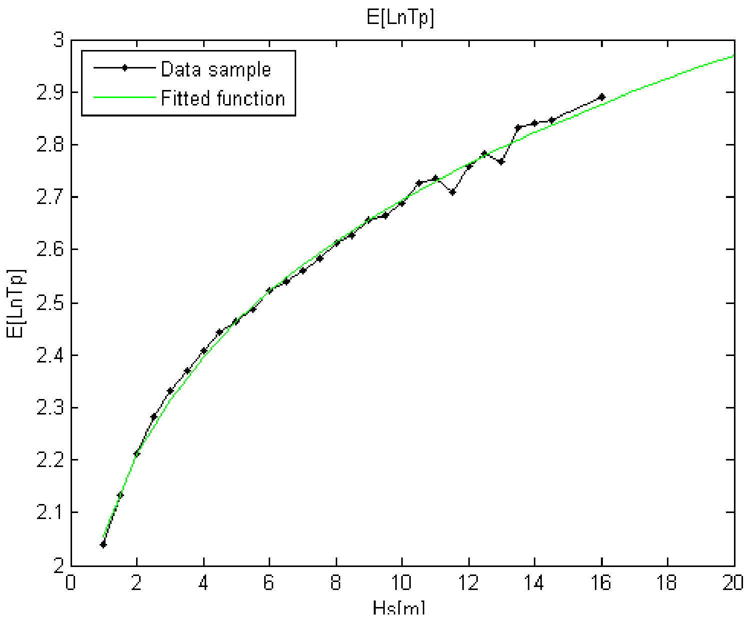


**Figure 6.3: Distribution of E[LnTp] and fitted model.**

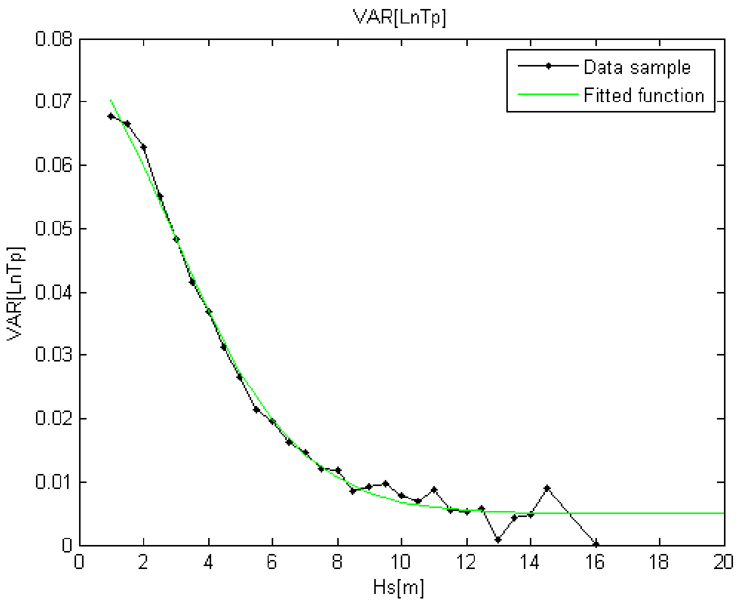


Figure 6.4: Distribution of VAR[LnTp] and fitted model.

When the extreme significant wave height is estimated, the corresponding mean spectral peak period can be found as following (Wikipedia, n.d.):

$$E\left[T_p|H_s\right] = exp\left[\mu + \frac{1}{2}\sigma^2\right] \qquad \text{Equation 6.14}$$

The fit of Equation 6.14 to the data sample are illustrated in Figure 6.5.

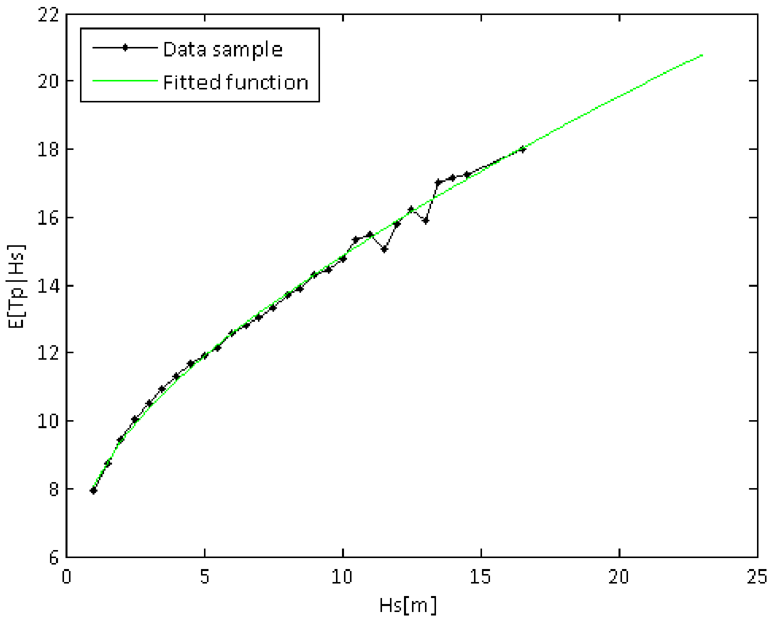


Figure 6.5: Distribution of E[Tp|Hs] and fitted model.

The sea state characteristics corresponding to a return period of 100 years and 10 000 years are listed in Table 6.2:

Table 6.2: Sea state characteristics corresponding to T-years return period.

| Return period | Extreme sea states | |
|---|---|---|
| | $H_s$[m] | $T_p$[s] |
| 100 years | 16,56 | 18,09 |
| 10 000 years | 20,99 | 20,01 |

**Sensitivity of Weibull distribution**

To predict an accurate estimate for the extreme waves, the fit of the Weibull distribution must be correct. By selecting $\gamma = 1{,}5$, there are 119 775 observations above the threshold. This means that over 25 % of the measurements are neglected. The parameters are found as shown above. The cumulative distribution is then as following:

$$F_{Hs}(h) = 1 - exp\left(-\left[\frac{(h-1{,}5)}{1{,}8212}\right]^{1{,}379}\right)$$ Equation 6.15

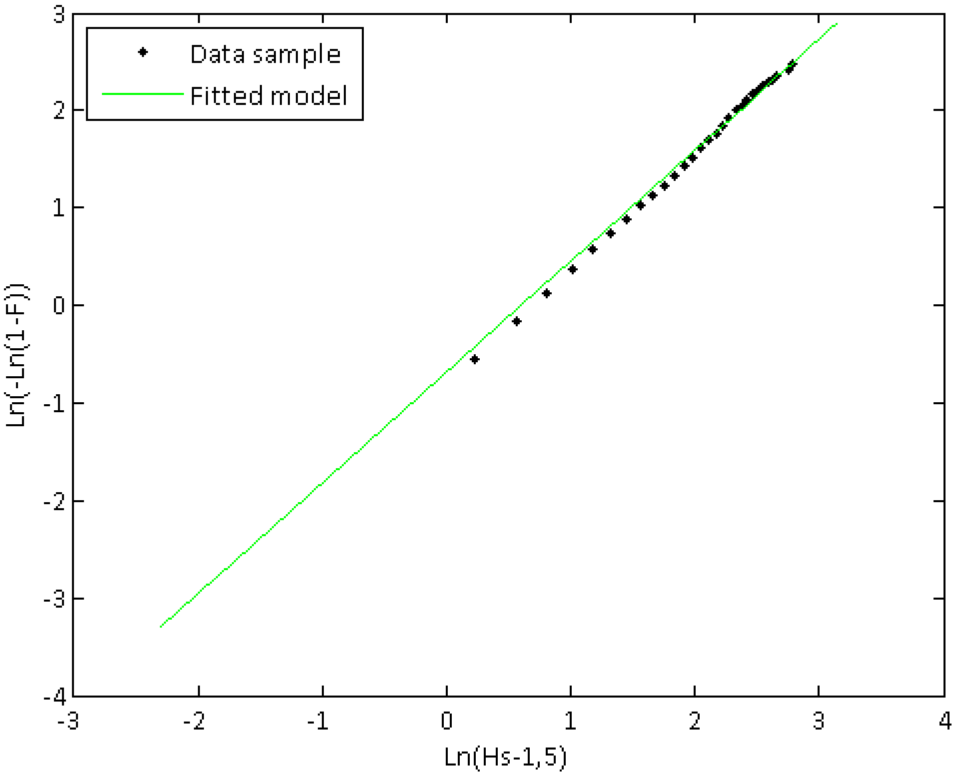


Figure 6.6: Distribution of significant wave height with $\gamma = 1{,}5m$

Figure 6.6 illustrates the Weibull fit to the data. Compared with the original estimates with $\gamma = 0{,}75m$ that gave a 100 year wave of 16,56m, the 100 year wave with $\gamma = 1{,}75m$ is

18,36m. The difference is nearly 11 % which indicates that the 3-parameter Weibull distribution is sensitive and without experience in adapting data to the distribution the result can be misleading.

Since 25 % of the observations are neglected, the annual amount of observations decreases to $\frac{119775}{55,75} = 2148,43$. This indicates that the exceedance probability increases when observations are neglected. From Table 6.3 we can see that the difference in significant wave height for a return period of 100 years is 0,36m. The difference is not large. However, when compared with the result when $\gamma = 0,75$m the difference where only 0,01m when the amount of observations decreased. This indicates that when the location parameter increases it is important to take into consideration that number of observations decreases, directly affecting the exceedance probability.

**Table 6.3: Significant wave height with location parameter 1,5m and different amount of storms pr. year.**

| Return Period | $H_s$[m] | Number of 3 hour storms pr. year | Exceedance probability |
|---|---|---|---|
| 100 years | 18,36 | 2920 | $1/100 \cdot 2920$ |
| | 18,0 | 2148,43 | $1/100 \cdot 2148,43$ |
| 10 000 years | 23,68 | 2920 | $1/10000 \cdot 2920$ |
| | 23,33 | 2148,43 | $1/10000 \cdot 2148,43$ |

## 6.3 Long term distribution of 3 hour extreme response maxima

In designing offshore structures, one of the main objectives is to understand the effects of environmental loads and how structures behave in different sea states. One approach is to estimate the response maxima during a random 3 hour stationary sea state. In the North Sea, all the hindcast data is given for a weather window of 3 hours and within the significant wave height and spectral peak period are constant. Equation 4.8 gives the distribution of the largest response in a 3 hour stationary sea state. The long term distribution of 3 hour response maxima can then be established by including the joint probability function:

$$F_{X_{3h}}(x_q) = \iint_{h\,t} F_{X_{3h}|H_sT_p}(x_q|h,t) f_{HsTp}(h,t)dtdh \qquad \text{Equation 6.16}$$

We now divide significant wave height and spectral peak period into classes, where the intervals are $\Delta h = 0{,}5m$ and $\Delta t = 1s$. The mid-value of each class are denoted with $i$ for significant wave height, $h_{si}$ , and $j$ for spectral peak period, $t_{pj}$, and the probability density for a sea state $f_{HsTp}(h,t)$ with $p_{ij}(h_{si}, t_{pj})$. Equation 6.16 can be rewritten to:

$$F_{X_{3h}}(x_q) = \sum_i \sum_j F_{X_{3\text{h}}|H_sT_p}(x_q|h_{si}, t_{pj}) p_{ij}(h_{si}, t_{pj}) \qquad \text{Equation 6.17}$$

The joint long term description of the sea state characteristics can be established as the product of the marginal distribution for $H_s$, and the conditional distribution of $T_p$ given $H_s$. By utilizing the fitted distributions, non-observed sea states are taken into consideration:

$$p_{ij}(h_{si}, t_{pj}) = p_i(h_{si}) p_{j|i}(t_{pj}|h_{si}) \qquad \text{Equation 6.18}$$

Where $p_i(h_{si})$ and $p_{j|i}(t_{pj}|h_{si})$ can be obtained from Equation 6.1 and Equation 6.9:

$$p_i(h_{si}) = F_{H_s}\left(h_{si} + \frac{\Delta h}{2}\right) - F_{H_s}\left(h_{si} - \frac{\Delta h}{2}\right) \qquad \text{Equation 6.19}$$

$$p_{j|i}(t_{pj}|h_{si}) = F_{T_p|H_s}\left(t_{pj} + \frac{\Delta \text{t}}{2}\middle|h_{si}\right) - F_{T_p|H_s}\left(t_{pj} - \frac{\Delta \text{t}}{2}\middle|h_{si}\right) \qquad \text{Equation 6.20}$$

Since it's unnecessary and time consuming to include unlimited number of $H_s$ and $T_p$, the upper limits are set at $F_{H_s}(25) = 1$ and $F_{T_p|H_s}(35|h_{si}) = 1$, for all $i$. This implies that we neglect occurrences with $h_s > 25m$ and also for $t_p > 35s$. The complete values of the joint probability density function can be seen in Appendix A.

For the short term response there are different approaches available. In the case of a simple response problem where the transfer function is known, the short term response can be solved in the frequency domain. For each sea state, Equation 4.8 is solved:

$$F_{X_{3h}|H_sT_p}(x|h_{si}, t_{pj}) = exp\left\{-exp\left[\frac{x - \gamma(i,j)}{\beta(i,j)}\right]\right\} \qquad \text{Equation 6.21}$$

Where $\gamma(i,j) and\ \beta(i,j)$ are obtained with Equation 4.9 and Equation 4.10.

The long term distribution of 3 hour response extremes can then be expressed as:

$$F_{X_{3h}}(x) = \sum_i \sum_j \left[exp\left\{-exp\left[\frac{x - \gamma(i,j)}{\beta(i,j)}\right]\right\}\right] p_{ij}(h_{si}, t_{pj}) \qquad \text{Equation 6.22}$$

The response maxima's that correspond to the requirements in ULS and ALS, with an annual exceedance probability of $10^{-2}$ and $10^{-4}$ are found by:

$$F_{X_{3h}}(x_q) = 1 - \frac{q}{n_{3h}}$$ Equation 6.23

The corresponding values are shown in Table 6.4:

**Table 6.4: Long term extreme heave response in a random 3 hour sea state**

| Return period | Heave response [m] |
|---|---|
| 100 years | 9,16 |
| 10 000 years | 14,22 |

From Table 6.2 we see that the estimated 10 000 year significant wave height is approximately 27% higher than the 100 year wave. However, for the estimated heave response the ratio is approximately 55%. Since the estimated heave period of the 10 000 year significant wave height is close to the natural period of the semisubmersible, it is natural that the effect on the structure is more severe than for the 100 year wave.

### 6.3.1 Short term response

The extreme sea states for 100 years and 10 000 years are found in section 6.2. However, it is interesting to evaluate which response this given sea state would give if only the short term approach are utilized. For this example, the 100 year wave are used with $H_s = 16,56m$ and $T_p = 18,05s$. The Gumbel distribution is shown in Figure 6.7:

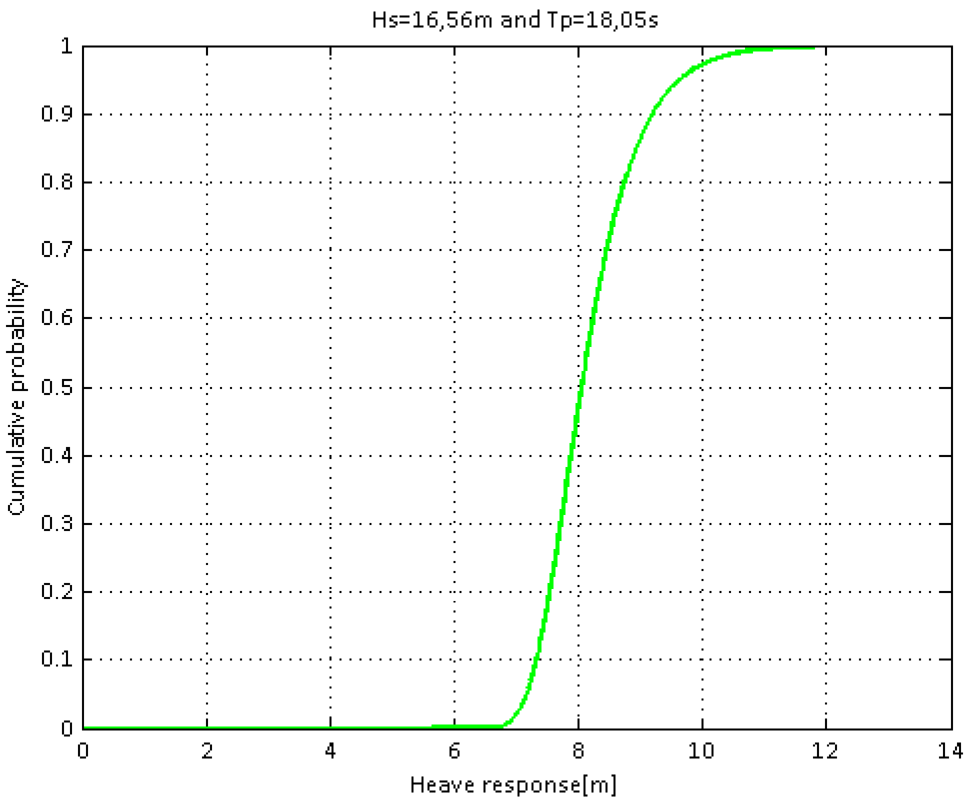


Figure 6.7: Short term distribution for given 100 years sea state.

The most likely largest response are found to be 7,83m in this given sea state, see Table 6.5. The calculated long term response maximum was found to be 9,16m. This corresponds to a probability of not being exceeded of 89,45 % in this given sea state. This agrees reasonably well with recommended percentiles when using environmental contour method (NORSOK, 2007). An analysis using environmental contour lines will be performed in the next chapter.

Table 6.5: Heave response when only the short term is utilized.

| Probability of not being exceeded [%] | Heave response [m] |
|---|---|
| 50 | 7,83 |
| 89,45 | 9,16 |

# 7 The environmental contour lines approach

So far we have conducted a long term analysis with the all sea state approach. In our case it entailed a simple response problem that made it possible to solve the short term response problem in the frequency domain. In other cases where the response problem is complicated and the frequency domain no longer available it is more difficult to obtain the short term response distribution. Thus, time domain simulations or model test have to be used. This is where the environmental contour lines approach is attractive. By using this method, an estimate for the long term extreme response can be obtained without performing a long term analysis. The process will be shown in this chapter.

## 7.1 Establishing contour lines

As discussed in chapter 5.2, environmental contour lines are established from the long term description of the wave conditions. In the all sea state approach, the coupled distribution of the long term description of the wave conditions and short term response distribution are utilized to estimate the most probable largest response. On contrary, in the environmental contour line approach the extreme sea states are located first and used to estimate the most probable largest response. We will now establish the contour lines for 100 and 10 000 years return period.

The contour lines corresponds to certain exceedance probability. When transformed into the U-space, the radius of the circle, $\beta_c$, corresponds to a given exceedance probability. This implies that every points on the circle has the same probability density (Haver & Winterstein, 2008). $\beta_c$ for a return period of 100 and 10 000 years are found to be 4,4983 and 5,3951 respectively. In the U-space values for $u_1$ and $u_2$ are found along the enitre circle with Equation 5.7 and Equation 5.8. The corresponding values of $H_s$ in the physical plane are found from the marginal distribution of $H_s$:

$$\Phi(u_1) = F_{H_s}(h) = 1 - exp\left\{-\left[\frac{h-\gamma}{\alpha}\right]^{\beta}\right\} \quad \text{Equation 7.1}$$

$$h = \alpha\{-ln[1-\Phi(u_1)]\}^{\frac{1}{\beta}} + \gamma \quad \text{Equation 7.2}$$

$$u_1 = \Phi^{-1}\left\{1 - exp\left[-\left(\frac{h-\gamma}{\alpha}\right)^{\beta}\right]\right\} \quad \text{Equation 7.3}$$

From the conditional distribution of $T_p$ given $H_s$ we find the corresponding values of $T_p$:

$$\Phi(u_2) = \Phi\left[\frac{lnt + \mu}{\sigma}\right] \qquad \text{Equation 7.4}$$

$$t = \exp(\mu + \sigma u_2) \qquad \text{Equation 7.5}$$

$$u_2 = \frac{lnt - \mu}{\sigma} \qquad \text{Equation 7.6}$$

When all the points around the circle in the U-space are transformed into the physical plane, the contour lines for return period of 100 and 10 000 years are created, see Figure 7.1.

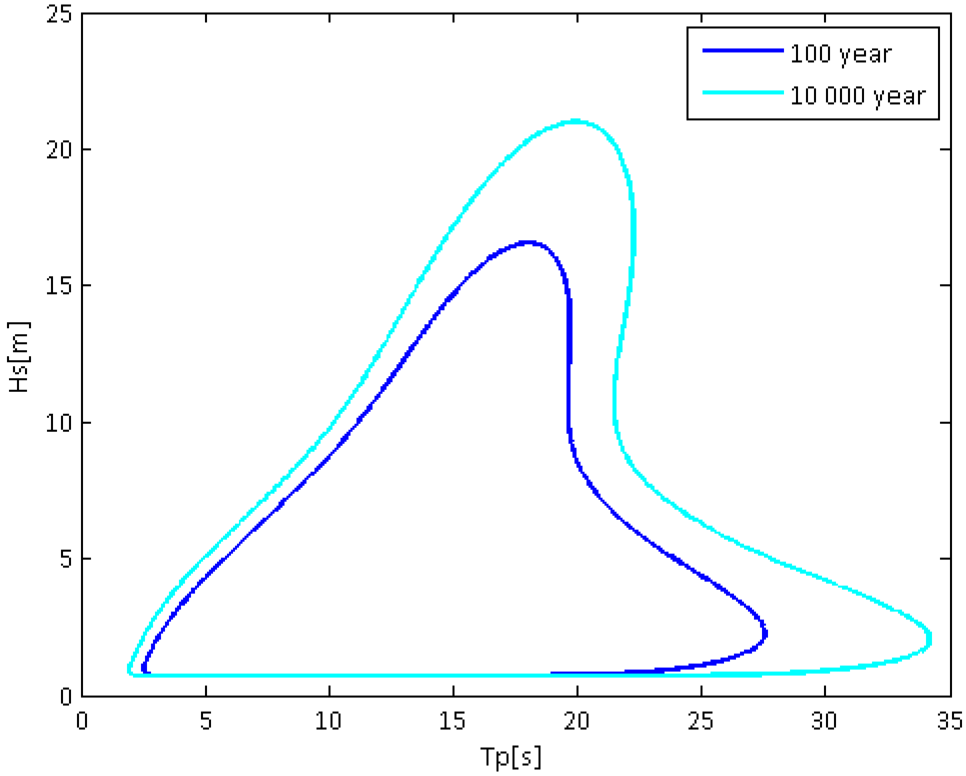


**Figure 7.1: Contour lines for selected area.**

## 7.2 Most probable response when short term variability are neglected

When the contour lines are established, we select several sea states close to the assumed worst sea state. Usually the worst sea states are close to the highest significant wave height along the contour line where the most energetic waves are expected. For complex non-linear response problems the short term response distribution is the most challenging and the frequency domain are no longer valid. By conducting several 3 hour time domain simulations for the worst sea state, e.g. 30 times, we get a sample of 30 simulated 3 hour response extremes. These are then used to establish the short term response distribution.

Model test are also one possibility. By utilizing the worst sea state along the contour line, several model tests can be performed and a series of 3 hour responses are obtained. For these results the short term response distribution can be established. Since the upper tail of the probability distribution is of interest, the model tests should be performed multiple times, say 20-30 times to get a valid description of the upper tail.

In our example, a linear response problem is considered. The transfer function is known and the short term response distribution of the 3 hour maximum can be solved in the frequency domain as discussed in chapter 4.1, see Equation 4.5 to Equation 4.8. The short term response distribution will follow a Gumbel distribution and the most probable largest response, see Equation 4.9, in each sea state are calculated and shown in Table 7.1 and

Table 7.2.

Table 7.1: Worst sea state characteristics with a return period of 100 years

| Significant wave height [m] | Spectral peak period [s] | Most probable response [m] |
|---|---|---|
| 15,62 | 15,62 | 6,9 |
| 16 | 16,5 | 7,1 |
| 16,3 | 17,02 | 7,39 |
| 16,56 | 18,05 | 7,83 |
| 16,47 | 18,51 | 8,01 |
| 16,11 | 19,01 | 8,17 |
| 15,8 | 19,24 | 8,21 |
| 15,2 | 19,49 | 8,15 |

Table 7.2: Worst sea state characteristics with a return period of 10 000 years

| Significant wave height [m] | Spectral peak period [s] | Most probable response [m] |
|---|---|---|
| 20,81 | 19 | 10,3 |
| 20,96 | 19,52 | 10,88 |
| 20,99 | 20,01 | 11,36 |
| 20,9 | 20,5 | 11,87 |
| 20,62 | 21,02 | 12,41 |
| 20,12 | 21,49 | 12,79 |
| 19,69 | 21,74 | 12,89 |
| 19 | 22 | 12,79 |

We see that the sea state that provides the largest most probable response are slightly different than the extreme sea states that was estimated in chapter 6.2, see Table 6.2. Since the natural period of the structure under consideration is close to 23s, it is natural that the worst period regarding heave response is to the right of the peak sea state, closer to the natural period.

Table 7.3: Most probable largest heave response when neglecting short term variability.

| | 100 years | 10 000 years |
|---|---|---|
| Most probable response [m] | 8,21 | 12,89 |

Compared with the result found from the full long term analysis, we see that the heave response in Table 7.3 is underestimated with about 10 %. This implies that the short term variability cannot be neglected.

## 7.3 Taking into account the short term variability

As shown above, the most likely heave response obtained from contour lines are underestimated compared to a full long term analysis. This indicates that the short term variability is of importance when estimating response.

One approach to account for the short term variability is to select a higher percentile as the short term characteristic response as shown in Equation 4.11. By using the worst sea states located in Table 7.1 and

Table 7.2, a higher percentile, $\alpha$, are used to estimate the corresponding response. In Figure 7.2 different values for $\alpha$ are used and the corresponding response are illustrated and compared with the response obtained with the all sea state approach. A return period of 100 years is used. It shows that the worst response are obtained by using a $\alpha - \text{percentile}$ between 75% and 80%.

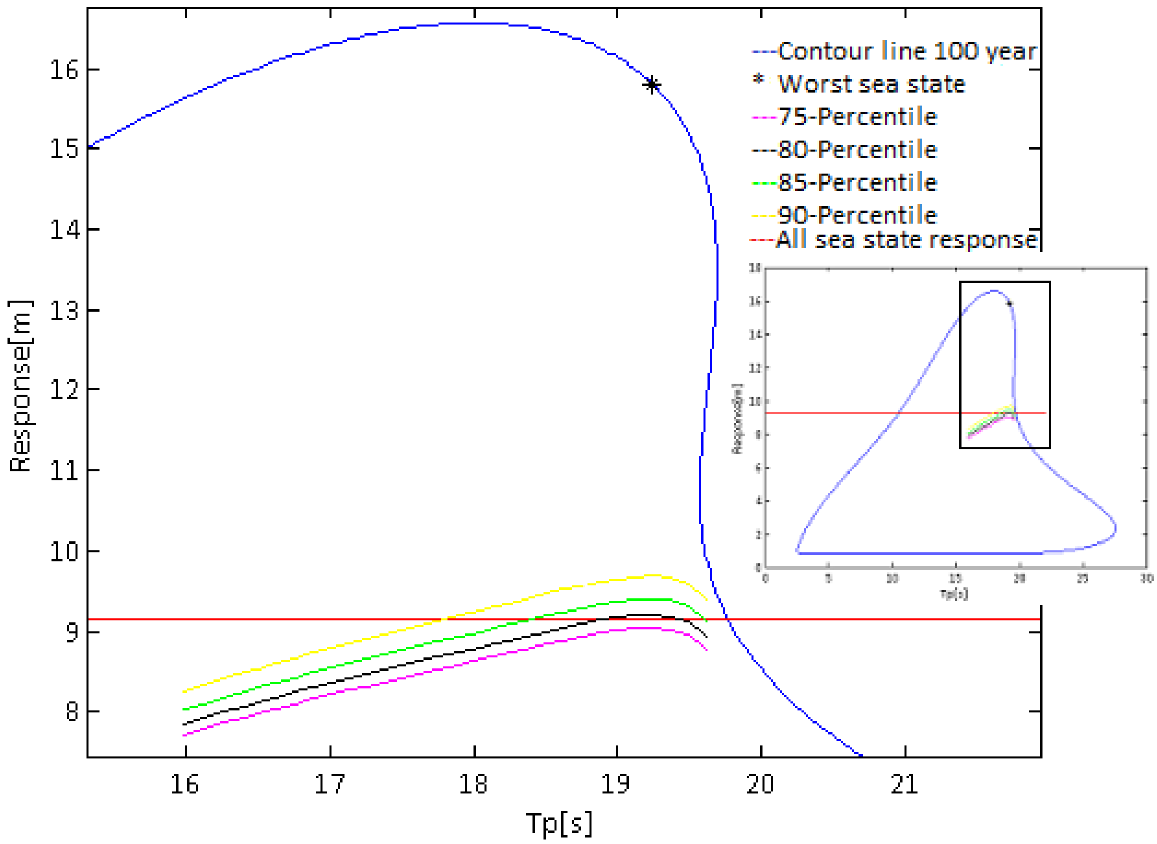


Figure 7.2: Different values for the $\alpha-\text{percentile}$ and the corresponding response compared with response obtained with the all sea state approach for a return period of 100 years.

Since a full long term analysis already has been completed, the $\alpha-\text{percentile}$ response shall be equal to the heave response found using full long term analysis. In Equation 7.8 and Equation 7.9 the worst sea states for 100 and 10 000 year return period along the contour line are used to estimate the correct $\alpha-\text{precentile}$.

$$F_{X_{3h}|H_sT_p}(x|h,t) = exp\left\{-exp\left[-\left(\frac{x-\gamma}{\beta}\right)\right]\right\} \qquad \text{Equation 7.7}$$

$$F_{X_{3h}|H_sT_p}^{100} = exp\left\{-exp\left[-\left(\frac{9{,}16-8{,}2137}{0{,}6444}\right)\right]\right\} = 0{,}7943 \qquad \text{Equation 7.8}$$

$$F_{X_{3h}|H_sT_p}^{10\,000} = exp\left\{-exp\left[-\left(\frac{14{,}22-12{,}8852}{1{,}0348}\right)\right]\right\} = 0{,}7593 \qquad \text{Equation 7.9}$$

The establishment of the $\alpha-\text{precentile}$ value is case specific and generally the $\alpha-\text{precentile}$ is higher for a non-linear problem (DNV, 2010). In this case the percentile corresponding to 100 year return period is found to be 79,4 %.This is lower than the recommended values from NORSOK (2007), which are in the range of 85-95 %. The heave response, $X_{3h,\alpha}$, for the worst sea state along the contour line gives only a 5% difference between selected percentile and recommended percentile($\frac{X_{3h,90}}{X_{3h,79}}$). This suggests that in this case, the percentile recommended in NORSK (2007) is slightly conservative.

# 8 Verification of the case studies

We assume that the model used in the long term analysis is true, i.e. the estimated heave response is true. By utilizing all of the observations from the hindcast data, 163 133, the most probable heave response for each observations can be calculated with Equation 4.9. The calculated most probable heave response are divided into intervals of 0,5m and plotted on a Weibull paper, see Figure 8.1, where $x$ is the most probable heave response.

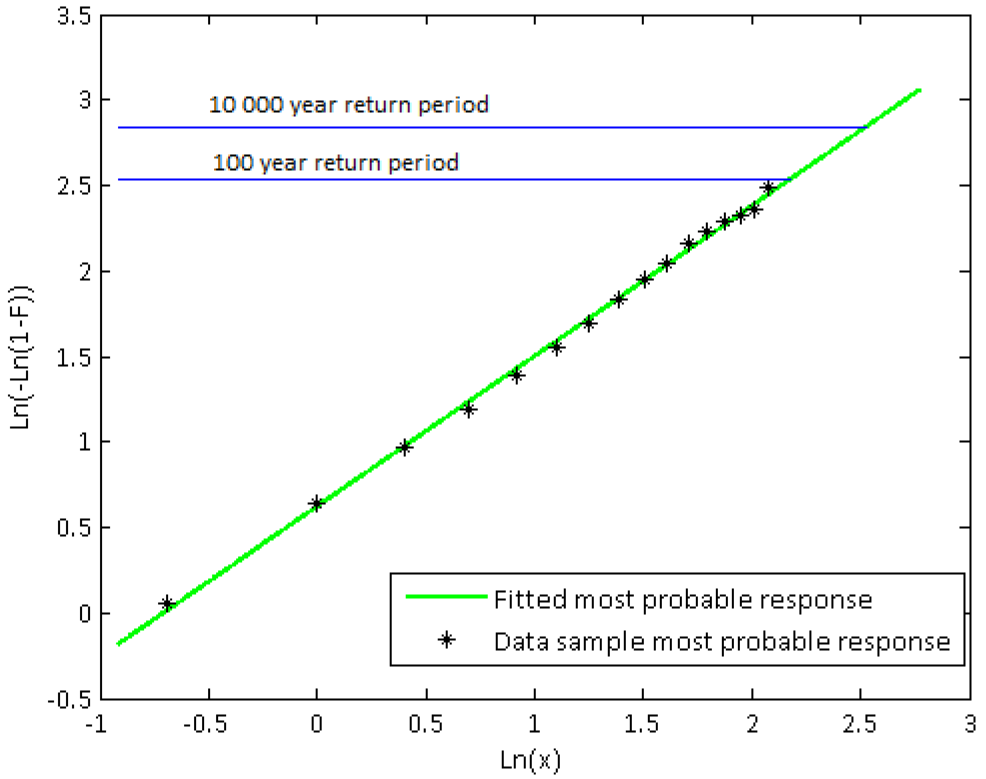


Figure 8.1: Most probable response from hindcast data.

We can generate a possible heave response for each 3-hour sea state by using the Monte Carlo simulation. A random number, $u_i$, between 0 and 1 is generated for the number $i$ sea state. By replacing $F_{X_{3h}|H_sT_p}(x|h,t)$ with $u_i$ in Equation 4.8 and solving with respect to $x$, the expression for the 3 hour extreme response can be found as following:

$$x_i = \gamma_i - \ln(-\ln(u_i)$$ Equation 8.1

We can generate 5 series with a size corresponding to the size of the hindcast data. In theory, all of these samples could have been the most probable heave responses from the observations. The 5 series are plotted on a Weibull paper, same as the most probable response, see Figure 8.2.

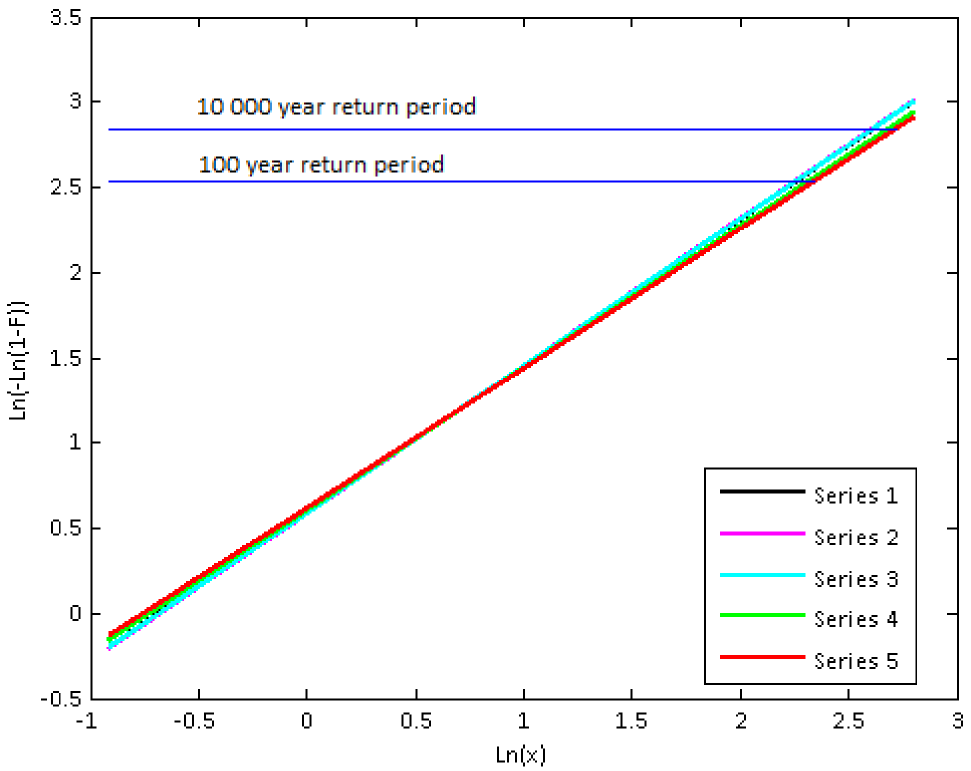


Figure 8.2: 5 series of simulated heave response.

From the 5 series, 5 different estimates of the simulated heave response are found. The simulated heave response corresponding to an exceedance probability of $10^{-2}$ is scattered from 9,49m to 10,37m with an average of 9,8m. For exceedance probability of $10^{-4}$ the results varies from 13,61m to 15,9m with an average of 14,7m. Improved accuracy on the simulated heave response can be achieved by running more simulations.

Table 8.1: Most probable response and simulated response corresponding to a return period of 100-and 10 000 year.

| Return period | Most probable response[m] | Simulated response[m] |
|---|---|---|
| 100 year | 8,79 | 9,8 |
| 10 000 year | 12,53 | 14,17 |

Table 8.1 shows that the simulated response is close to the heave response estimated from the full long term analysis, 7% higher for a return period of 100 years. It is interesting to see that the simulated heave response for a return period of 10 000 years coincides with the response from the long term analysis. The result gathered here indicates that the estimated heave response from the full long term analysis is accurate.

# 9 Conclusion and further work

In this thesis the use of statistical methods has been applied to estimate extreme sea states and extreme responses on a semi-submersible located in the North Sea. The theoretical background for describing waves and the corresponding response in a stationary weather event and the long term variability of ocean waves have been addressed. In addition, several approaches to estimate the long term extreme response have been discussed. Some conclusions regarding the case studies can be made:

- In the long term description of the wave characteristics, the marginal distribution of significant wave height has been determined with a 3-parameter Weibull distribution. Two different location parameters have been selected to see the effect on estimating extreme wave height. It indicates that the selection of location parameter is important and should be selected with care.
- Use of environmental contour lines is an approximated method to estimate extremes and it is especially attractive with complex response problems that require time domain simulations or model testing. In the case study performed, the transfer function were known and the short term response where solved using the frequency domain. The most probable heave response was underestimated with approximately 10% compared with the result from the long term analysis. This indicated that the short term variability cannot be neglected. To include the short term variability, a $\alpha - \text{precentile}$ response was found. The $\alpha - \text{precentile}$ was found to be around 75-80%. These values are lower than the suggested values given in NORSOK (2007), indicating that the $\alpha - \text{precentile}$ is case sensitive and for the case study, slightly conservative.
- Monte Carlo simulations are performed on the available hindcast data. Five simulations are conducted to verify the results from the long term response analysis. The simulated heave response for a return period of 100 years differs with only 7% from the long term analysis. For a return period of 10 000 years, the heave response differs with only 0,5%. This point towards that the estimates obtained from the all sea state approach are correct.

The result from the case study is shown in Table 9.1.

Table 9.1: Summary of the results obtained in the case study.

| Method | Exceedance probability | Extreme sea state characteristics | | | Type of response |
|---|---|---|---|---|---|
| | | $H_s$[m] | $T_p$[s] | Heave response[m] | |
| All Sea state | $10^{-2}$ | 16,56 | 18,09 | 9,16 | Characteristic |
| | $10^{-4}$ | 20,99 | 20,01 | 14,22 | Characteristic |
| Environmental Contour lines | $10^{-2}$ | 15,8 | 15,8 | 8,21 | Most probable |
| | $10^{-2}$ | 15,8 | 15,8 | 9,16 | 79-percentile |
| | $10^{-4}$ | 19,69 | 21,74 | 12,89 | Most probable |
| | $10^{-4}$ | 19,69 | 21,74 | 14,22 | 76-percentile |
| Verification | $10^{-2}$ | - | - | 9,8 | Simulated |
| | $10^{-4}$ | - | - | 14,17 | Simulated |

Due to time limitations, the case study was not performed with the peak over threshold approach. As a next step, it is recommended to conduct an analysis with this method and compare with the results found with the all sea state approach.

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# Appendix A – Joint probability density function for selected area in the North Sea

| Hs \ Tp | 0,5 | 1,5 | 2,5 | 3,5 | 4,5 | 5,5 | 6,5 | 7,5 | 8,5 | 9,5 | 10,5 | 11,5 | 12,5 | 13,5 | 14,5 | 15,5 | 16,5 | 17,5 | 18,5 | 19,5 | 20,5 | 21,5 | 22,5 | 23,5 | 24,5 | 25,5 | 26,5 | 27,5 | 28,5 | 29,5 | 30,5 | 31,5 | 32,5 | 33,5 | 34,5 | SUM |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1,0 | 3,19E-11 | 2,18E-08 | 2,37E-05 | 8,61E-04 | 6,11E-03 | 1,71E-02 | 2,68E-02 | 2,92E-02 | 2,48E-02 | 1,78E-02 | 1,14E-02 | 6,71E-03 | 3,72E-03 | 1,97E-03 | 1,02E-03 | 5,12E-04 | 2,54E-04 | 1,25E-04 | 6,12E-05 | 2,98E-05 | 1,45E-05 | 7,08E-06 | 3,46E-06 | 1,70E-06 | 8,37E-07 | 4,15E-07 | 2,07E-07 | 1,03E-07 | 5,22E-08 | 2,65E-08 | 1,35E-08 | 6,98E-09 | 3,63E-09 | 1,90E-09 | 1,01E-09 | 1,49E-01 |
| 1,5 | 8,42E-11 | 1,40E-09 | 3,95E-06 | 2,67E-04 | 2,95E-03 | 1,15E-02 | 2,33E-02 | 3,11E-02 | 3,12E-02 | 2,56E-02 | 1,83E-02 | 1,18E-02 | 7,04E-03 | 3,98E-03 | 2,16E-03 | 1,14E-03 | 5,89E-04 | 2,99E-04 | 1,50E-04 | 7,49E-05 | 3,72E-05 | 1,84E-05 | 9,13E-06 | 4,52E-06 | 2,25E-06 | 1,12E-06 | 5,60E-07 | 2,81E-07 | 1,42E-07 | 7,20E-08 | 3,68E-08 | 1,89E-08 | 9,79E-09 | 5,12E-09 | 2,71E-09 | 1,72E-01 |
| 2,0 | 9,06E-11 | 1,37E-10 | 4,41E-07 | 5,91E-05 | 1,04E-03 | 5,66E-03 | 1,49E-02 | 2,41E-02 | 2,81E-02 | 2,60E-02 | 2,03E-02 | 1,40E-02 | 8,87E-03 | 5,24E-03 | 2,94E-03 | 1,59E-03 | 8,33E-04 | 4,28E-04 | 2,16E-04 | 1,08E-04 | 5,34E-05 | 2,63E-05 | 1,29E-05 | 6,33E-06 | 3,11E-06 | 1,52E-06 | 7,50E-07 | 3,70E-07 | 1,83E-07 | 9,13E-08 | 4,56E-08 | 2,29E-08 | 1,16E-08 | 5,94E-09 | 3,07E-09 | 1,55E-01 |
| 2,5 | 5,77E-11 | 5,85E-11 | 3,34E-08 | 9,93E-06 | 2,95E-04 | 2,35E-03 | 8,16E-03 | 1,63E-02 | 2,22E-02 | 2,30E-02 | 1,97E-02 | 1,45E-02 | 9,60E-03 | 5,85E-03 | 3,35E-03 | 1,83E-03 | 9,60E-04 | 4,90E-04 | 2,45E-04 | 1,21E-04 | 5,86E-05 | 2,83E-05 | 1,35E-05 | 6,46E-06 | 3,07E-06 | 1,46E-06 | 6,97E-07 | 3,32E-07 | 1,59E-07 | 7,63E-08 | 3,68E-08 | 1,78E-08 | 8,67E-09 | 4,26E-09 | 2,12E-09 | 1,29E-01 |
| 3,0 | 2,41E-11 | 2,41E-11 | 1,56E-09 | 1,19E-06 | 6,58E-05 | 8,12E-04 | 3,88E-03 | 9,79E-03 | 1,59E-02 | 1,87E-02 | 1,75E-02 | 1,38E-02 | 9,51E-03 | 5,95E-03 | 3,44E-03 | 1,88E-03 | 9,79E-04 | 4,92E-04 | 2,40E-04 | 1,15E-04 | 5,41E-05 | 2,51E-05 | 1,16E-05 | 5,29E-06 | 2,41E-06 | 1,09E-06 | 4,97E-07 | 2,25E-07 | 1,02E-07 | 4,66E-08 | 2,13E-08 | 9,77E-09 | 4,51E-09 | 2,09E-09 | 9,84E-10 | 1,03E-01 |
| 3,5 | 6,92E-12 | 6,92E-12 | 4,45E-11 | 9,34E-08 | 1,09E-05 | 2,25E-04 | 1,57E-03 | 5,20E-03 | 1,03E-02 | 1,41E-02 | 1,46E-02 | 1,23E-02 | 8,90E-03 | 5,70E-03 | 3,32E-03 | 1,80E-03 | 9,22E-04 | 4,51E-04 | 2,13E-04 | 9,77E-05 | 4,39E-05 | 1,94E-05 | 8,45E-06 | 3,65E-06 | 1,56E-06 | 6,67E-07 | 2,84E-07 | 1,21E-07 | 5,12E-08 | 2,18E-08 | 9,27E-09 | 3,96E-09 | 1,70E-09 | 7,35E-10 | 3,21E-10 | 7,98E-02 |
| 4,0 | 1,37E-12 | 1,37E-12 | 1,78E-12 | 4,31E-09 | 1,24E-06 | 4,80E-05 | 5,21E-04 | 2,40E-03 | 6,04E-03 | 9,80E-03 | 1,15E-02 | 1,05E-02 | 7,98E-03 | 5,24E-03 | 3,07E-03 | 1,64E-03 | 8,20E-04 | 3,87E-04 | 1,74E-04 | 7,59E-05 | 3,21E-05 | 1,33E-05 | 5,40E-06 | 2,16E-06 | 8,59E-07 | 3,39E-07 | 1,33E-07 | 5,18E-08 | 2,02E-08 | 7,87E-09 | 3,07E-09 | 1,20E-09 | 4,71E-10 | 1,86E-10 | 7,40E-11 | 6,03E-02 |
| 4,5 | 1,85E-13 | 1,85E-13 | 1,86E-13 | 1,02E-10 | 8,97E-08 | 7,36E-06 | 1,37E-04 | 9,36E-04 | 3,14E-03 | 6,27E-03 | 8,50E-03 | 8,59E-03 | 6,92E-03 | 4,68E-03 | 2,75E-03 | 1,45E-03 | 7,01E-04 | 3,16E-04 | 1,35E-04 | 5,48E-05 | 2,15E-05 | 8,21E-06 | 3,06E-06 | 1,12E-06 | 4,04E-07 | 1,44E-07 | 5,11E-08 | 1,80E-08 | 6,32E-09 | 2,21E-09 | 7,74E-10 | 2,71E-10 | 9,51E-11 | 3,35E-11 | 1,19E-11 | 4,46E-02 |
| 5,0 | 1,67E-14 | 1,67E-14 | 1,67E-14 | 1,09E-12 | 3,68E-09 | 7,57E-07 | 2,70E-05 | 2,97E-04 | 1,41E-03 | 3,63E-03 | 5,88E-03 | 6,70E-03 | 5,82E-03 | 4,08E-03 | 2,42E-03 | 1,25E-03 | 5,84E-04 | 2,49E-04 | 9,91E-05 | 3,73E-05 | 1,34E-05 | 4,63E-06 | 1,56E-06 | 5,10E-07 | 1,64E-07 | 5,20E-08 | 1,63E-08 | 5,06E-09 | 1,56E-09 | 4,80E-10 | 1,47E-10 | 4,52E-11 | 1,39E-11 | 4,27E-12 | 1,32E-12 | 3,25E-02 |
| 5,5 | 1,00E-15 | 1,00E-15 | 1,00E-15 | 5,24E-15 | 7,59E-11 | 4,79E-08 | 3,76E-06 | 7,32E-05 | 5,30E-04 | 1,85E-03 | 3,75E-03 | 4,96E-03 | 4,74E-03 | 3,49E-03 | 2,10E-03 | 1,07E-03 | 4,77E-04 | 1,91E-04 | 7,04E-05 | 2,41E-05 | 7,80E-06 | 2,41E-06 | 7,15E-07 | 2,06E-07 | 5,78E-08 | 1,59E-08 | 4,31E-09 | 1,15E-09 | 3,06E-10 | 8,05E-11 | 2,11E-11 | 5,52E-12 | 1,44E-12 | 3,78E-13 | 9,95E-14 | 2,33E-02 |
| 6,0 | 3,88E-17 | 3,88E-17 | 3,88E-17 | 4,40E-17 | 6,94E-13 | 1,70E-09 | 3,46E-07 | 1,34E-05 | 1,61E-04 | 8,18E-04 | 2,16E-03 | 3,45E-03 | 3,71E-03 | 2,92E-03 | 1,79E-03 | 9,02E-04 | 3,86E-04 | 1,45E-04 | 4,88E-05 | 1,50E-05 | 4,32E-06 | 1,17E-06 | 3,01E-07 | 7,44E-08 | 1,78E-08 | 4,16E-09 | 9,52E-10 | 2,14E-10 | 4,75E-11 | 1,04E-11 | 2,27E-12 | 4,94E-13 | 1,07E-13 | 2,31E-14 | 5,04E-15 | 1,65E-02 |
| 6,5 | 7,34E-19 | 7,34E-19 | 7,34E-19 | 7,36E-19 | 2,49E-15 | 3,09E-11 | 1,97E-08 | 1,74E-06 | 3,82E-05 | 3,02E-04 | 1,10E-03 | 2,22E-03 | 2,77E-03 | 2,39E-03 | 1,52E-03 | 7,61E-04 | 3,12E-04 | 1,09E-04 | 3,34E-05 | 9,18E-06 | 2,31E-06 | 5,38E-07 | 1,18E-07 | 2,46E-08 | 4,93E-09 | 9,53E-10 | 1,79E-10 | 3,30E-11 | 5,97E-12 | 1,07E-12 | 1,88E-13 | 3,29E-14 | 5,74E-15 | 9,98E-16 | 1,74E-16 | 1,16E-02 |
| 7,0 | 2,84E-105 | 3,74E-58 | 2,37E-37 | 1,30E-25 | 3,17E-18 | 2,66E-13 | 6,52E-10 | 1,52E-07 | 6,80E-06 | 9,11E-05 | 4,90E-04 | 1,30E-03 | 1,96E-03 | 1,89E-03 | 1,27E-03 | 6,41E-04 | 2,54E-04 | 8,28E-05 | 2,30E-05 | 5,58E-06 | 1,21E-06 | 2,41E-07 | 4,43E-08 | 7,66E-09 | 1,25E-09 | 1,97E-10 | 2,99E-11 | 4,40E-12 | 6,32E-13 | 8,93E-14 | 1,24E-14 | 1,71E-15 | 2,34E-16 | 3,11E-17 | 4,45E-18 | 8,01E-03 |
| 7,5 | 2,02E-124 | 6,90E-69 | 2,69E-44 | 2,24E-30 | 1,37E-21 | 1,04E-15 | 1,21E-11 | 8,69E-09 | 8,86E-07 | 2,18E-05 | 1,84E-04 | 6,77E-04 | 1,29E-03 | 1,44E-03 | 1,05E-03 | 5,41E-04 | 2,09E-04 | 6,40E-05 | 1,61E-05 | 3,45E-06 | 6,47E-07 | 1,08E-07 | 1,65E-08 | 2,33E-09 | 3,09E-10 | 3,87E-11 | 4,64E-12 | 5,36E-13 | 6,01E-14 | 6,58E-15 | 7,06E-16 | 7,50E-17 | 7,93E-18 | 6,10E-19 | 0,00E+00 | 5,49E-03 |
| 8,0 | 9,89E-146 | 6,82E-81 | 4,24E-52 | 9,40E-36 | 2,11E-25 | 1,89E-18 | 1,27E-13 | 3,23E-10 | 8,38E-08 | 4,12E-06 | 5,81E-05 | 3,12E-04 | 7,76E-04 | 1,04E-03 | 8,46E-04 | 4,56E-04 | 1,75E-04 | 5,08E-05 | 1,17E-05 | 2,22E-06 | 3,59E-07 | 5,08E-08 | 6,41E-09 | 7,35E-10 | 7,80E-11 | 7,74E-12 | 7,26E-13 | 6,51E-14 | 5,62E-15 | 4,71E-16 | 3,85E-17 | 2,90E-18 | 4,14E-19 | 0,00E+00 | 0,00E+00 | 3,73E-03 |
| 8,5 | 1,09E-168 | 6,69E-94 | 1,34E-60 | 1,21E-41 | 1,33E-29 | 1,72E-21 | 7,80E-16 | 8,01E-12 | 5,80E-09 | 6,13E-07 | 1,53E-05 | 1,25E-04 | 4,25E-04 | 7,07E-04 | 6,59E-04 | 3,81E-04 | 1,49E-04 | 4,18E-05 | 8,94E-06 | 1,52E-06 | 2,15E-07 | 2,58E-08 | 2,71E-09 | 2,54E-10 | 2,16E-11 | 1,70E-12 | 1,25E-13 | 8,71E-15 | 5,78E-16 | 3,68E-17 | 2,23E-18 | 2,79E-19 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 2,51E-03 |
| 9,0 | 1,55E-192 | 1,66E-107 | 1,51E-69 | 6,98E-48 | 4,39E-34 | 9,37E-25 | 3,16E-18 | 1,42E-13 | 3,06E-10 | 7,33E-08 | 3,38E-06 | 4,38E-05 | 2,09E-04 | 4,46E-04 | 4,91E-04 | 3,14E-04 | 1,27E-04 | 3,55E-05 | 7,23E-06 | 1,13E-06 | 1,42E-07 | 1,47E-08 | 1,31E-09 | 1,02E-10 | 7,07E-12 | 4,47E-13 | 2,61E-14 | 1,43E-15 | 7,36E-17 | 3,54E-18 | 1,86E-19 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 1,68E-03 |
| 9,5 | 2,31E-216 | 3,32E-121 | 1,27E-78 | 2,94E-54 | 1,07E-38 | 3,81E-28 | 9,80E-21 | 1,97E-15 | 1,31E-11 | 7,31E-09 | 6,39E-07 | 1,35E-05 | 9,29E-05 | 2,60E-04 | 3,47E-04 | 2,51E-04 | 1,09E-04 | 3,11E-05 | 6,20E-06 | 9,15E-07 | 1,05E-07 | 9,70E-09 | 7,49E-10 | 4,96E-11 | 2,89E-12 | 1,51E-13 | 7,16E-15 | 3,15E-16 | 1,30E-17 | 4,94E-19 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 1,11E-03 |
| 10,0 | 3,02E-239 | 1,75E-134 | 1,72E-87 | 1,53E-60 | 2,75E-43 | 1,50E-31 | 2,79E-23 | 2,44E-17 | 4,94E-13 | 6,42E-10 | 1,07E-07 | 3,70E-06 | 3,73E-05 | 1,39E-04 | 2,30E-04 | 1,94E-04 | 9,32E-05 | 2,79E-05 | 5,60E-06 | 8,06E-07 | 8,73E-08 | 7,44E-09 | 5,17E-10 | 3,02E-11 | 1,53E-12 | 6,80E-14 | 2,72E-15 | 9,98E-17 | 3,41E-18 | 8,13E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 7,32E-04 |
| 10,5 | 2,08E-260 | 6,81E-147 | 7,30E-96 | 1,56E-66 | 1,05E-47 | 7,24E-35 | 8,62E-26 | 3,04E-19 | 1,78E-14 | 5,24E-11 | 1,64E-08 | 9,28E-07 | 1,37E-05 | 6,87E-05 | 1,43E-04 | 1,43E-04 | 7,76E-05 | 2,51E-05 | 5,26E-06 | 7,63E-07 | 8,10E-08 | 6,60E-09 | 4,29E-10 | 2,30E-11 | 1,05E-12 | 4,17E-14 | 1,47E-15 | 4,68E-17 | 1,33E-18 | 5,31E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 4,78E-04 |
| 11,0 | 2,55E-279 | 4,00E-158 | 1,59E-103 | 4,46E-72 | 7,60E-52 | 5,19E-38 | 3,35E-28 | 4,24E-21 | 6,64E-16 | 4,21E-12 | 2,40E-09 | 2,18E-07 | 4,66E-06 | 3,15E-05 | 8,31E-05 | 1,00E-04 | 6,26E-05 | 2,25E-05 | 5,04E-06 | 7,62E-07 | 8,20E-08 | 6,63E-09 | 4,20E-10 | 2,15E-11 | 9,25E-13 | 3,41E-14 | 1,10E-15 | 3,19E-17 | 8,27E-19 | 3,45E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 3,11E-04 |
| 11,5 | 9,55E-296 | 5,02E-168 | 2,27E-110 | 4,29E-77 | 1,22E-55 | 6,23E-41 | 1,79E-30 | 7,10E-23 | 2,71E-17 | 3,46E-13 | 3,45E-10 | 4,88E-08 | 1,50E-06 | 1,35E-05 | 4,53E-05 | 6,62E-05 | 4,84E-05 | 1,96E-05 | 4,85E-06 | 7,84E-07 | 8,84E-08 | 7,33E-09 | 4,68E-10 | 2,38E-11 | 1,00E-12 | 3,58E-14 | 1,11E-15 | 3,04E-17 | 7,56E-19 | 2,22E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 2,00E-04 |
| 12,0 | 0,00E+00 | 1,37E-176 | 2,22E-116 | 1,46E-81 | 4,56E-59 | 1,31E-43 | 1,37E-32 | 1,49E-24 | 1,25E-18 | 3,00E-14 | 4,98E-11 | 1,07E-08 | 4,58E-07 | 5,49E-06 | 2,32E-05 | 4,14E-05 | 3,57E-05 | 1,66E-05 | 4,59E-06 | 8,13E-07 | 9,85E-08 | 8,62E-09 | 5,72E-10 | 2,99E-11 | 1,27E-12 | 4,55E-14 | 1,40E-15 | 3,78E-17 | 9,27E-19 | 1,43E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 1,28E-04 |
| 12,5 | 0,00E+00 | 6,44E-184 | 1,31E-121 | 1,64E-85 | 3,83E-62 | 4,74E-46 | 1,52E-34 | 3,91E-26 | 6,59E-20 | 2,78E-15 | 7,34E-12 | 2,30E-09 | 1,36E-07 | 2,13E-06 | 1,13E-05 | 2,45E-05 | 2,50E-05 | 1,35E-05 | 4,24E-06 | 8,35E-07 | 1,11E-07 | 1,05E-08 | 7,40E-10 | 4,07E-11 | 1,81E-12 | 6,66E-14 | 2,10E-15 | 5,75E-17 | 1,41E-18 | 2,73E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 8,18E-05 |
| 13,0 | 0,00E+00 | 3,62E-190 | 3,74E-126 | 5,32E-89 | 6,65E-65 | 2,83E-48 | 2,34E-36 | 1,28E-27 | 3,98E-21 | 2,77E-16 | 1,11E-12 | 4,93E-10 | 3,92E-08 | 7,91E-07 | 5,24E-06 | 1,38E-05 | 1,67E-05 | 1,05E-05 | 3,77E-06 | 8,37E-07 | 1,23E-07 | 1,28E-08 | 9,80E-10 | 5,79E-11 | 2,73E-12 | 1,06E-13 | 3,51E-15 | 1,00E-16 | 2,52E-18 | 5,76E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 5,18E-05 |
| 13,5 | 0,00E+00 | 1,59E-195 | 4,01E-130 | 4,22E-92 | 2,14E-67 | 2,60E-50 | 4,85E-38 | 5,11E-29 | 2,71E-22 | 2,96E-17 | 1,73E-13 | 1,06E-10 | 1,11E-08 | 2,85E-07 | 2,33E-06 | 7,41E-06 | 1,07E-05 | 7,81E-06 | 3,22E-06 | 8,12E-07 | 1,34E-07 | 1,54E-08 | 1,30E-09 | 8,36E-11 | 4,26E-12 | 1,78E-13 | 6,25E-15 | 1,89E-16 | 5,01E-18 | 1,16E-19 | 3,63E-21 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 3,27E-05 |
| 14,0 | 0,00E+00 | 3,64E-200 | 1,25E-133 | 6,93E-95 | 1,14E-69 | 3,39E-52 | 1,28E-39 | 2,40E-30 | 2,05E-23 | 3,34E-18 | 2,76E-14 | 2,28E-11 | 3,09E-09 | 9,96E-08 | 9,98E-07 | 3,81E-06 | 6,48E-06 | 5,54E-06 | 2,63E-06 | 7,56E-07 | 1,41E-07 | 1,81E-08 | 1,69E-09 | 1,19E-10 | 6,65E-12 | 3,01E-13 | 1,14E-14 | 3,70E-16 | 1,05E-17 | 2,64E-19 | 6,82E-21 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 2,05E-05 |
| 14,5 | 0,00E+00 | 3,02E-204 | 9,01E-137 | 2,03E-97 | 9,17E-72 | 5,89E-54 | 4,13E-41 | 1,29E-31 | 1,69E-24 | 3,97E-19 | 4,50E-15 | 4,90E-12 | 8,49E-10 | 3,40E-08 | 4,14E-07 | 1,89E-06 | 3,78E-06 | 3,76E-06 | 2,06E-06 | 6,74E-07 | 1,42E-07 | 2,05E-08 | 2,13E-09 | 1,67E-10 | 1,02E-11 | 5,05E-13 | 2,08E-14 | 7,28E-16 | 2,22E-17 | 6,00E-19 | 1,56E-20 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 1,28E-05 |
| 15,0 | 0,00E+00 | 6,68E-208 | 1,24E-139 | 9,25E-100 | 1,01E-73 | 1,28E-55 | 1,56E-42 | 7,69E-33 | 1,49E-25 | 4,91E-20 | 7,46E-16 | 1,05E-12 | 2,30E-10 | 1,13E-08 | 1,66E-07 | 9,02E-07 | 2,12E-06 | 2,45E-06 | 1,54E-06 | 5,76E-07 | 1,38E-07 | 2,23E-08 | 2,58E-09 | 2,24E-10 | 1,52E-11 | 8,23E-13 | 3,70E-14 | 1,41E-15 | 4,66E-17 | 1,36E-18 | 3,52E-20 | 8,80E-22 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 7,93E-06 |
| 15,5 | 0,00E+00 | 3,11E-211 | 2,78E-142 | 5,95E-102 | 1,41E-75 | 3,31E-57 | 6,65E-44 | 4,99E-34 | 1,39E-26 | 6,26E-21 | 1,25E-16 | 2,27E-13 | 6,16E-11 | 3,70E-09 | 6,52E-08 | 4,17E-07 | 1,15E-06 | 1,53E-06 | 1,11E-06 | 4,72E-07 | 1,27E-07 | 2,32E-08 | 3,01E-09 | 2,90E-10 | 2,17E-11 | 1,29E-12 | 6,38E-14 | 2,65E-15 | 9,52E-17 | 3,01E-18 | 8,48E-20 | 2,17E-21 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 4,90E-06 |
| 16,0 | 0,00E+00 | 2,52E-214 | 9,06E-145 | 4,97E-104 | 2,39E-77 | 9,79E-59 | 3,12E-45 | 3,46E-35 | 1,35E-27 | 8,16E-22 | 2,13E-17 | 4,86E-14 | 1,63E-11 | 1,19E-09 | 2,49E-08 | 1,87E-07 | 5,99E-07 | 9,24E-07 | 7,64E-07 | 3,70E-07 | 1,13E-07 | 2,31E-08 | 3,35E-09 | 3,59E-10 | 2,96E-11 | 1,95E-12 | 1,05E-13 | 4,79E-15 | 1,87E-16 | 6,41E-18 | 1,95E-19 | 5,68E-21 | 0,00E+00 | 0,00E+00 | 0,00E+00 | 3,01E-06 |
| 16,5 | 0,00E+00 | 3,11E-217 | 3,91E-147 | 5,07E-106 | 4,66E-79 | 3,21E-60 | 1,57E-46 | 2,52E-36 | 1,35E-28 | 1,08E-22 | 3,63E-18 | 1,04E-14 | 4,27E-12 | 3,73E-10 | 9,28E-09 | 8,19E-08 | 3,03E-07 | 5,38E-07 | 5,08E-07 | 2,79E-07 | 9,61E-08 | 2,21E-08 | 3,56E-09 | 4,24E-10 | 3,88E-11 | 2,81E-12 | 1,66E-13 | 8,25E-15 | 3,51E-16 | 1,31E-17 | 4,32E-19 | 1,29E-20 | 4,09E-22 | 0,00E+00 | 0,00E+00 | 1,84E-06 |
| 17,0 | 0,00E+00 | 5,26E-220 | 2,09E-149 | 6,02E-108 | 1,02E-80 | 1,14E-61 | 8,41E-48 | 1,91E-37 | 1,39E-29 | 1,46E-23 | 6,20E-19 | 2,20E-15 | 1,10E-12 | 1,16E-10 | 3,39E-09 | 3,49E-08 | 1,49E-07 | 3,03E-07 | 3,26E-07 | 2,03E-07 | 7,85E-08 | 2,02E-08 | 3,63E-09 | 4,80E-10 | 4,84E-11 | 3,86E-12 | 2,50E-13 | 1,36E-14 | 6,29E-16 | 2,54E-17 | 9,07E-19 | 2,91E-20 | 8,72E-22 | 0,00E+00 | 0,00E+00 | 1,12E-06 |
| 17,5 | 0,00E+00 | 1,13E-222 | 1,32E-151 | 8,06E-110 | 2,42E-82 | 4,33E-63 | 4,71E-49 | 1,49E-38 | 1,45E-30 | 1,98E-24 | 1,06E-19 | 4,65E-16 | 2,82E-13 | 3,52E-11 | 1,21E-09 | 1,45E-08 | 7,13E-08 | 1,66E-07 | 2,02E-07 | 1,42E-07 | 6,18E-08 | 1,77E-08 | 3,55E-09 | 5,19E-10 | 5,78E-11 | 5,06E-12 | 3,59E-13 | 2,13E-14 | 1,07E-15 | 4,68E-17 | 1,81E-18 | 6,29E-20 | 1,96E-21 | 7,56E-23 | 0,00E+00 | 6,81E-07 |
| 18,0 | 0,00E+00 | 2,93E-225 | 9,50E-154 | 1,19E-111 | 6,17E-84 | 1,72E-64 | 2,73E-50 | 1,19E-39 | 1,54E-31 | 2,70E-25 | 1,82E-20 | 9,76E-17 | 7,14E-14 | 1,06E-11 | 4,26E-10 | 5,89E-09 | 3,33E-08 | 8,80E-08 | 1,22E-07 | 9,61E-08 | 4,68E-08 | 1,50E-08 | 3,33E-09 | 5,39E-10 | 6,60E-11 | 6,34E-12 | 4,92E-13 | 3,17E-14 | 1,74E-15 | 8,23E-17 | 3,44E-18 | 1,29E-19 | 4,38E-21 | 1,37E-22 | 0,00E+00 | 4,11E-07 |
| 18,5 | 0,00E+00 | 8,79E-228 | 7,59E-156 | 1,89E-113 | 1,67E-85 | 7,17E-66 | 1,63E-51 | 9,76E-41 | 1,65E-32 | 3,70E-26 | 3,10E-21 | 2,04E-17 | 1,79E-14 | 3,12E-12 | 1,47E-10 | 2,34E-09 | 1,51E-08 | 4,55E-08 | 7,10E-08 | 6,30E-08 | 3,43E-08 | 1,22E-08 | 3,01E-09 | 5,37E-10 | 7,22E-11 | 7,60E-12 | 6,44E-13 | 4,52E-14 | 2,68E-15 | 1,38E-16 | 6,20E-18 | 2,50E-19 | 9,11E-21 | 3,02E-22 | 0,00E+00 | 2,47E-07 |
| 19,0 | 0,00E+00 | 2,98E-230 | 6,61E-158 | 3,20E-115 | 4,72E-87 | 3,09E-67 | 9,99E-53 | 8,10E-42 | 1,80E-33 | 5,10E-27 | 5,30E-22 | 4,22E-18 | 4,42E-15 | 9,09E-13 | 4,97E-11 | 9,13E-10 | 6,73E-09 | 2,29E-08 | 4,03E-08 | 4,01E-08 | 2,43E-08 | 9,62E-09 | 2,62E-09 | 5,14E-10 | 7,59E-11 | 8,73E-12 | 8,07E-13 | 6,15E-14 | 3,96E-15 | 2,19E-16 | 1,07E-17 | 4,61E-19 | 1,80E-20 | 6,41E-22 | 1,64E-23 | 1,48E-07 |
| 19,5 | 0,00E+00 | 1,12E-232 | 6,20E-160 | 5,73E-117 | 1,40E-88 | 1,37E-68 | 6,25E-54 | 6,83E-43 | 1,96E-34 | 7,04E-28 | 9,02E-23 | 8,69E-19 | 1,08E-15 | 2,61E-13 | 1,66E-11 | 3,49E-10 | 2,93E-09 | 1,13E-08 | 2,23E-08 | 2,48E-08 | 1,67E-08 | 7,32E-09 | 2,20E-09 | 4,75E-10 | 7,68E-11 | 9,64E-12 | 9,69E-13 | 8,01E-14 | 5,58E-15 | 3,33E-16 | 1,74E-17 | 8,11E-19 | 3,40E-20 | 1,30E-21 | 4,91E-23 | 8,84E-08 |
| 20,0 | 0,00E+00 | 4,60E-235 | 6,21E-162 | 1,08E-118 | 4,28E-90 | 6,25E-70 | 3,99E-55 | 5,83E-44 | 2,16E-35 | 9,74E-29 | 1,53E-23 | 1,78E-19 | 2,63E-16 | 7,42E-14 | 5,44E-12 | 1,31E-10 | 1,25E-09 | 5,42E-09 | 1,20E-08 | 1,49E-08 | 1,12E-08 | 5,40E-09 | 1,79E-09 | 4,23E-10 | 7,48E-11 | 1,02E-11 | 1,12E-12 | 1,00E-13 | 7,53E-15 | 4,85E-16 | 2,73E-17 | 1,36E-18 | 6,09E-20 | 2,49E-21 | 9,33E-23 | 5,26E-08 |
| 20,5 | 0,00E+00 | 2,04E-237 | 6,58E-164 | 2,12E-120 | 1,35E-91 | 2,93E-71 | 2,59E-56 | 5,03E-45 | 2,40E-36 | 1,35E-29 | 2,60E-24 | 3,62E-20 | 6,33E-17 | 2,08E-14 | 1,76E-12 | 4,85E-11 | 5,23E-10 | 2,55E-09 | 6,32E-09 | 8,72E-09 | 7,24E-09 | 3,87E-09 | 1,41E-09 | 3,65E-10 | 7,04E-11 | 1,05E-11 | 1,24E-12 | 1,20E-13 | 9,75E-15 | 6,76E-16 | 4,08E-17 | 2,18E-18 | 1,04E-19 | 4,54E-21 | 1,83E-22 | 3,11E-08 |
| 21,0 | 0,00E+00 | 9,75E-240 | 7,37E-166 | 4,33E-122 | 4,43E-93 | 1,40E-72 | 1,71E-57 | 4,39E-46 | 2,68E-37 | 1,88E-30 | 4,40E-25 | 7,34E-21 | 1,51E-17 | 5,78E-15 | 5,62E-13 | 1,76E-11 | 2,15E-10 | 1,18E-09 | 3,25E-09 | 4,98E-09 | 4,58E-09 | 2,69E-09 | 1,07E-09 | 3,05E-10 | 6,41E-11 | 1,04E-11 | 1,33E-12 | 1,39E-13 | 1,21E-14 | 9,05E-16 | 5,86E-17 | 3,35E-18 | 1,71E-19 | 7,94E-21 | 3,36E-22 | 1,84E-08 |
| 21,5 | 0,00E+00 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,20E-24 | 1,27E-24 | 1,48E-21 | 3,58E-18 | 1,59E-15 | 1,77E-13 | 6,31E-12 | 8,68E-11 | 5,32E-10 | 1,64E-09 | 2,78E-09 | 2,82E-09 | 1,82E-09 | 7,97E-10 | 2,47E-10 | 5,64E-11 | 9,90E-12 | 1,37E-12 | 1,55E-13 | 1,46E-14 | 1,17E-15 | 8,07E-17 | 4,93E-18 | 2,69E-19 | 1,33E-20 | 6,01E-22 | 1,08E-08 |
| 22,0 | 0,00E+00 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,47E-24 | 1,48E-24 | 2,98E-22 | 8,41E-19 | 4,32E-16 | 5,51E-14 | 2,23E-12 | 3,45E-11 | 2,36E-10 | 8,09E-10 | 1,52E-09 | 1,70E-09 | 1,20E-09 | 5,75E-10 | 1,94E-10 | 4,82E-11 | 9,17E-12 | 1,37E-12 | 1,67E-13 | 1,69E-14 | 1,45E-15 | 1,07E-16 | 6,98E-18 | 4,05E-19 | 2,13E-20 | 1,02E-21 | 6,33E-09 |
| 22,5 | 0,00E+00 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 2,30E-24 | 6,15E-23 | 1,96E-19 | 1,16E-16 | 1,70E-14 | 7,77E-13 | 1,35E-11 | 1,03E-10 | 3,92E-10 | 8,13E-10 | 9,99E-10 | 7,77E-10 | 4,05E-10 | 1,49E-10 | 4,01E-11 | 8,23E-12 | 1,33E-12 | 1,74E-13 | 1,89E-14 | 1,73E-15 | 1,37E-16 | 9,51E-18 | 5,88E-19 | 3,28E-20 | 1,67E-21 | 3,70E-09 |
| 23,0 | 0,00E+00 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 4,03E-24 | 1,58E-23 | 4,55E-20 | 3,11E-17 | 5,17E-15 | 2,67E-13 | 5,20E-12 | 4,43E-11 | 1,86E-10 | 4,26E-10 | 5,75E-10 | 4,89E-10 | 2,78E-10 | 1,11E-10 | 3,24E-11 | 7,19E-12 | 1,25E-12 | 1,76E-13 | 2,04E-14 | 2,00E-15 | 1,69E-16 | 1,25E-17 | 8,23E-19 | 4,87E-20 | 2,62E-21 | 2,16E-09 |
| 23,5 | 0,00E+00 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 6,42E-24 | 8,76E-24 | 1,05E-20 | 8,26E-18 | 1,56E-15 | 9,10E-14 | 1,98E-12 | 1,88E-11 | 8,73E-11 | 2,20E-10 | 3,25E-10 | 3,02E-10 | 1,87E-10 | 8,08E-11 | 2,55E-11 | 6,11E-12 | 1,14E-12 | 1,73E-13 | 2,15E-14 | 2,25E-15 | 2,03E-16 | 1,59E-17 | 1,11E-18 | 6,97E-20 | 3,98E-21 | 1,26E-09 |
| 24,0 | 0,00E+00 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 9,95E-24 | 1,04E-23 | 2,43E-21 | 2,18E-18 | 4,68E-16 | 3,07E-14 | 7,46E-13 | 7,85E-12 | 4,03E-11 | 1,12E-10 | 1,81E-10 | 1,83E-10 | 1,23E-10 | 5,76E-11 | 1,97E-11 | 5,07E-12 | 1,02E-12 | 1,65E-13 | 2,19E-14 | 2,45E-15 | 2,35E-16 | 1,97E-17 | 1,46E-18 | 9,66E-20 | 5,83E-21 | 7,30E-10 |
| 24,5 | 0,00E+00 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,50E-23 | 1,51E-23 | 5,71E-22 | 5,73E-19 | 1,39E-16 | 1,03E-14 | 2,79E-13 | 3,25E-12 | 1,84E-11 | 5,59E-11 | 9,88E-11 | 1,09E-10 | 7,96E-11 | 4,04E-11 | 1,49E-11 | 4,12E-12 | 8,90E-13 | 1,54E-13 | 2,19E-14 | 2,61E-15 | 2,66E-16 | 2,36E-17 | 1,85E-18 | 1,30E-19 | 8,29E-21 | 4,26E-10 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | SUM | 1,00E+00 |

**Figure A.0.1: joint probability distribution for selected area in the North Sea.**

# Appendix B –New conditional distribution of $T_p$ given $H_s$

When estimating the conditional distribution of $T_p$ given $H_s$, the parameter $b_1$ are usually set to be 0,005 when continuous function are fitted to the data sample of $VAR[lnTp]$ for each class of $H_s$. We will investigate what the consequences are if this parameter are decided on background of the available data. A full long term 3 hour response analysis is conducted and response maxima corresponding to a return period of 100- and 10 000 years are found.

The marginal distribution of $H_s$ is the same, i.e. the significant wave height corresponding to a return period of 100 and 10 000 year is 16,56m and 20,99m.

| | |
|---|---|
| $\sigma^2 = b_1 + b_2 \exp(-b_3 h_s^{b_4})$ | Equation B.1 |
| $\sigma^2 = 0{,}00492 + 0{,}0709\exp(-0{,}07931 h_s^{1{,}665})$ | Equation B.2 |

In Figure B.0.1 the fit between the variance from the data sample and Equation B.2 are shown.

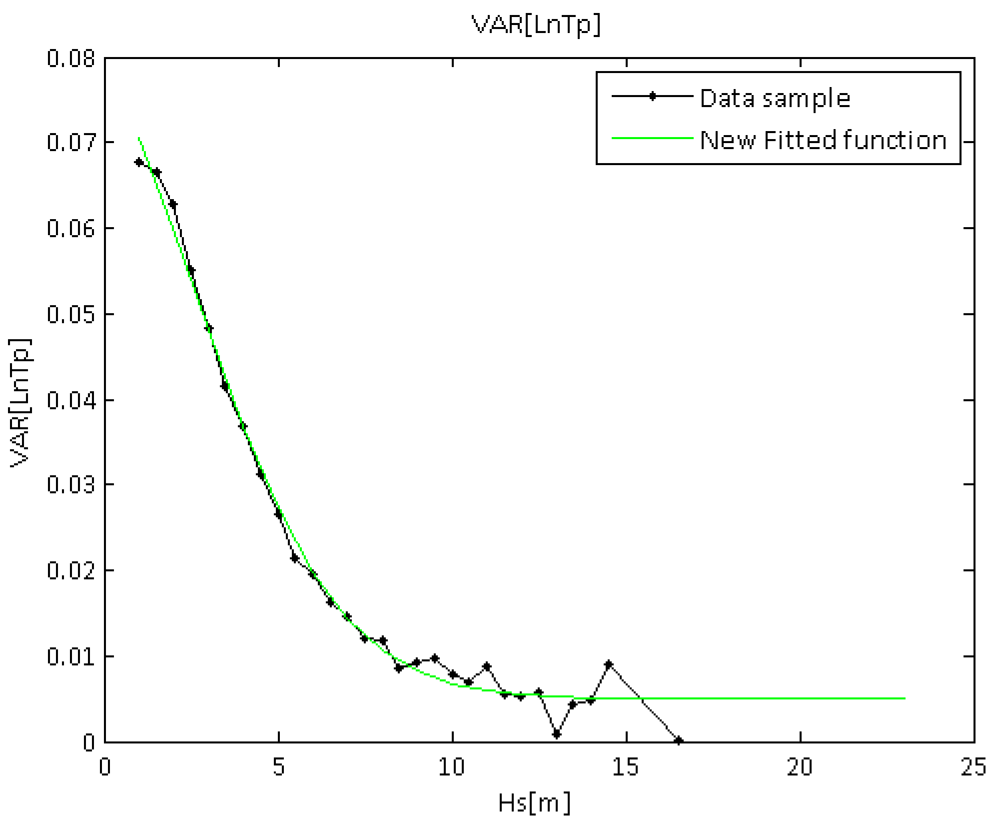


Figure B.0.1: Distribution of VAR[LnTp] from data sample and with new fitted function.

The corresponding spectral peak period are found with Equation 6.14 and the extreme sea states for a return period of 100 and 10 000 years are shown in Table B.1.

**Table B.1: Extreme sea states with the new conditional distribution of $T_p$ given $H_s$**

| Return Period [years] | Extreme Sea States | |
|---|---|---|
| | Hs | Tp |
| 100 | 16,56m | 18,05s |
| 10 000 | 20,99m | 19,96s |

A full long term 3 hour response analysis is conducted. The procedure is the same as in chapter 6.2 with the parameters in the fitted model of $\sigma^2$ changed according to Equation B.2.

**Table B.2: Comparison between heave response found in chapter 6.3 and heave response with the new conditional distribution of $T_p$ given $H_s$.**

| Return Period [years] | Heave [m] | Previous result for heave [m] |
|---|---|---|
| 100 | 9,15 | 9,16 |
| 10 000 | 14,2 | 14,22 |

The spectral peak period and the estimated heave response is almost the same as found in chapter 6.2. This implies that the importance of parameter $b_1$ is not of a critical matter. However, it is important to make sure that the fitted function for $VAR[LnTp]$ stays positive within all of the classes of $H_s$.

# Appendix C –Matlab and Excel files

All files are available on CD-ROM.

**Matlab:**

- Shortterm_response.m
- Jonswap.m
- Longterm_variability.m
- Contour_line_method.m
- Verification_of_case_studies.m
- Jonswap_ver.m
- Plotting.m (only available on CD-ROM)

**Excel** (only available on CD-ROM)**:**

- All sea state approach and verification
- Contour line method
- Transfer function
- NORA10, hindcast data

**Shortterm_response.m**

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%                                                                     %%%
%%%            SHORT TERM RESPONSE IN THE FREQUENCY DOMAIN              %%%
%%%                                                                     %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;close all;clc;
f = linspace(0.025,1,196);
hs = 1:24.5;
tp = 0.5:34.5;
wavespectrum = Jonswap(f,hs,tp);
%response spectrum
%Response amplitude from 1s to 40s.
RAO = xlsread('Transfer function','RAO vs freq','I2:I197');
RAO2 = RAO.^2';
%Response spectrum:
Response = zeros(size(wavespectrum));
for ii = 1:length(f);
    for jj =1:length(hs);
        for kk = 1:length(tp);
            Response(ii,jj,kk) = RAO2(ii).*wavespectrum(ii,jj,kk);
        end
    end
end
%% Spectral moments and zero-up-crossing frequency
%Spectral moments
%0 moment, variance:
m0=zeros(numel(hs),numel(tp));
for jj = 1:length(hs);
    for kk = 1:length(tp);
        m0(jj,kk) = trapz(f,Response(:,jj,kk));
    end
end
%2 moment:
m2=zeros(numel(hs),numel(tp));
for jj = 1:length(hs),
    for kk = 1:length(tp),
        Response2 = f'.^2.*Response(:,jj,kk);
        m2(jj,kk) = trapz(f,Response2);
    end
end
%Expected zero-up-crossing frequency:
zerofreq=sqrt(m2./m0);
%Standard deviation:
stddev=sqrt((m0));
%% gumbel distribution
%Expected number of global maxima
expectedmaxima=3600*3*zerofreq;
%Largest amplitude
Lamplitude=stddev.*sqrt(2*log(expectedmaxima));
beta=stddev./(sqrt(2*log(expectedmaxima)));
%% Contour lines, heave response with alfa percentile near "worst" state
precentile1=0.75; precentile2=0.80; precentile3=0.85; precentile4=0.90;
heaveresponse1=Lamplitude-beta*log(-log(precentile1));
heaveresponse2=Lamplitude-beta*log(-log(precentile2));
heaveresponse3=Lamplitude-beta*log(-log(precentile3));
heaveresponse4=Lamplitude-beta*log(-log(precentile4));
```

### Jonswap.m

```
function [Result] = Jonswap(f,hs,tp)
fp = 1./tp;

Result = zeros(length(f),length(hs),length(tp));
% sigma = zeros(1,length(f));
for ii = 1:length(f),
  for jj = 1:length(hs),
      for kk = 1:length(tp),

        if f(ii)*tp(kk)<= 1,
            sigma = 0.07;
        else
            sigma = 0.09;
        end
%         %From DNV RP-c205
        if (tp(kk)/sqrt(hs(jj)))<=3.6,
            gama = 5;
        elseif 3.6 < (tp(kk)/sqrt(hs(jj))) < 5,
            gama = exp(5.75-1.15*(tp(kk)/sqrt(hs(jj))));
        else (tp(kk)/sqrt(hs(jj)))>= 5,
            gama = 1;
        end
        Result(ii,jj,kk) = 0.3125*hs(jj)^2*tp(kk)*(f(ii)./fp(kk))^(-5)*...
            exp(-1.25*(f(ii)./fp(kk))^(-4))*(1-0.287*log(gama))*gama^...
            (exp(-0.5*((f(ii)-fp(kk))/(fp(kk)*sigma))^2));
      end
  end
end
```

## Longterm_variability.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%             Conditional distribution of Tp given Hs and             %%%
%%%                     marginal distribution of Hs                     %%%
%%%      Calculation of 100 and 10000 year sigificant wave height       %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;close all;clc;
    hs=1;
    tp=0:1:35;
  % Conditional distribution of Tp given Hs
 %  Expected value
    for ii=1:length(hs);
        u(ii)=1.195+0.8585*hs(ii)^0.2425;
    end
 % variance
    for ii=1:length(hs);
        v(ii)=0.00492+0.0709*exp(-0.0791*hs(ii)^1.665);
    end
 %standard devitation
    for ii=1:length(hs);
        v2(ii)=sqrt(v(ii));
    end
 %Performed manually for each interval of Hs and pasted into Excel
        C=logncdf(tp,u,v2);
        probac=[C(1) zeros(1,length(tp)-1)];

        for jj=1:length(tp)-1;
            probac(jj)=C(jj+1)-C(jj);
        end
%% Marginal distribution of Hs, pasted into Excel
for ii=1:length(hs);
           F(ii)=1-exp(-((hs(ii)-0.75)/2.1262)^1.2623);
end
proba=[F(1) zeros(1,length(hs)-1)];
for ii=1:length(hs)-1;
    proba(ii)=F(ii+1)-F(ii);
end
%% Calculation of 100 year and 10 000 year significant wave
syms x
solve('(1-exp(-((x-0.75)/2.1262)^1.2623))=0.999996575342466')
syms z
solve('(1-exp(-((z-0.75)/2.1262)^1.2623))=0.999999965753425')
```

## Contour_line_method.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%                                                                     %%%
%%%                  Environmental Contour line method                  %%%
%%%                                                                     %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc, clear all;
%Inputs:
alfa=2.1265;
beta=1.2623;
lamda=0.75;
%Probability corresponding to return period of 100 and 10 000 years
q_100=1/(2920*100);
q_10000=1/(2920*10000);
r_1=-norminv(q_100,0,1);
r_2=-norminv(q_10000,0,1);
% Imaginary plane
tetha=linspace(0,2*pi,1000);
u1_1=r_1*cos(tetha);
u1_2=r_2*cos(tetha);
normu1_1=normcdf(u1_1,0,1);
normu1_2=normcdf(u1_2,0,1);
u2_1=r_1*sin(tetha);
u2_2=r_2*sin(tetha);
%Physical plane:
h_100=lamda+((-log(1-normu1_1)).^(1/beta))*alfa;
h_10000=lamda+((-log(1-normu1_2)).^(1/beta))*alfa;
%expected value
expected_1=1.195+0.8585*(h_100).^0.2425;
expected_2=1.195+0.8585*(h_10000).^0.2425;
%Variance and standard deviation
variance_1=0.005+0.0707*exp(-0.07826*h_100.^1.674);
variance_2=0.005+0.0707*exp(-0.07826*h_10000.^1.674);
standarddv_1=sqrt(variance_1);
standarddv_2=sqrt(variance_2);
%Physical plane
t_100=exp(expected_1+standarddv_1.*u2_1);
t_10000=exp(expected_2+standarddv_2.*u2_2);
% Illustration
figure(1)
plot(t_100,h_100,'Linewidth',2)
hold on
plot(t_10000,h_10000,'c','Linewidth',2)
ylabel('Hs[m]');
xlabel('Tp[s]');
hleg3=legend('100 year','10 000 year');
set(hleg3,'Location','NorthEast')
%% Heave response from all sea state and higher precentile response
x_allseastate=9.16;
tp1=0:0.001:22;
t_selected=xlsread('Contour line method','sheet1','C4:C33');
response_075=xlsread('Contour line method','sheet1','e4:e33')';
response_080=xlsread('Contour line method','sheet1','f4:f33')';
response_085=xlsread('Contour line method','sheet1','g4:g33')';
response_090=xlsread('Contour line method','sheet1','h4:h33')';
figure(2)
h1=plot(t_100,h_100,'b');
```

```
hold on
h2=plot(tp1,x_allseastate,'r');
h3=plot(t_selected,response_075,'m');
h4=plot(t_selected,response_080,'k');
h5=plot(t_selected,response_085,'g');
h6=plot(t_selected,response_090,'y');
h7=plot(19.24,15.8,'k*');
xlabel('Tp[s]');
ylabel('Response[m]');
hleg1=legend([h1 h2(1) h3 h4 h5 h6 h7],'Contour line 100 years',...
'all sea state response','75 percentile','80 percentile','85 percentile'...
,'90 percentile','Worst sea state');
set(gcf,'Color','w');
set(hleg1,'Location','Best');
figure(3)
hold on
h1=plot(t_100,h_100,'b');
h2=plot(tp1,x_allseastate,'r');
h3=plot(t_selected,response_075,'m');
h4=plot(t_selected,response_080,'k');
h5=plot(t_selected,response_085,'g');
h6=plot(t_selected,response_090,'y');
h7=plot(19.24,15.8,'*k');
xlabel('Tp[s]');
ylabel('Response[m]');
hleg2=legend([h1 h2(1) h3 h4 h5 h6 h7],'Contour line 100 years',...
'all sea state response','75 percentile','80 percentile','85 percentile'...
,'90 percentile','Worse sea state');
xlim([15 21])
ylim([7 17])
set(gcf,'Color','w');
set(hleg2,'Location','Best')
```

University of Stavanger

## Verification_of_case_studies.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%                                                                     %%%
%%%             Verification of case studies by using Monte Carlo       %%%
%%%      simulations on available hindcast data from selcted area in    %%%
%%%                             the North Sea                           %%%
%%%                                                                     %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear all
clc
% Inputs
f = linspace(0.025,1,196);
combo_hstp=xlsread('All sea state approach and verification'...
,'Verification','A3:B163135');
n_seastates=length(combo_hstp(:,1));
RAO = xlsread('Transfer function','RAO vs freq','I2:I197');
RAO2 = RAO.^2;
RAO3=RAO2';
Lamplitude=zeros(length(n_seastates));
sim_3h_resp=zeros(length(n_seastates));
%Wavespectrum
for j=1:n_seastates
    hs=combo_hstp(j,1);
    tp=combo_hstp(j,2);
wave_spectrum=Jonswap_ver(f,hs,tp);
%Response spectrum
Response=RAO3.*wave_spectrum;
%spectral moments
m0=trapz(f,Response);
m2=trapz(f,Response.*f.^2);
%Expected zero-up-crossing frequency:
zerofreq=sqrt(m2./m0);
%Standard deviation:
stddev=sqrt(abs(m0));
%% gumbel distribution
%Expected number of global maxima
expectedmaxima=3600*3*zerofreq;
%Largest amplitude
Lamplitude(j)=stddev.*sqrt(2*log(expectedmaxima));
beta=stddev./(sqrt(2*log(expectedmaxima)));
sim_3h_resp(j)=Lamplitude(j)-beta*(log(-log(rand())));
end
mpm=Lamplitude';
sim_response=sim_3h_resp';
```

## Jonswap_ver.m

```
function [Result] = Jonswap_ver(f,hs,tp)

        if (f*tp)<= 1,
            sigma = 0.07;
        else
            sigma = 0.09;
        end
%         %From DNV RP-c205
        if (tp/sqrt(hs))<=3.6,
            gama = 5;
        elseif 3.6 < (tp/sqrt(hs)) < 5,
            gama = exp(5.75-1.15*(tp/sqrt(hs)));
        else (tp/sqrt(hs))>=5,
            gama = 1;
        end
        Result= 0.3125.*(hs.^2).*tp.*(f*tp).^(-5).*...
            exp(-1.25.*(f.*tp).^(-4)).*(1-0.287.*log(gama)).*gama.^...
            (exp(-0.5.*((f.*tp-1)/sigma).^2));
end
```

## Appendix D – Description of thesis

Title: **A comparison of various approaches for predicting extreme wave induced response for design of offshore structures**

Student: Magnus Haugen Morken

**Background**

Environmental response (action effect) for design of offshore structures is defined in terms of a maximum permitted annual exceedance probability. The basic control against overload failure is the ULS control, but at the Norwegian Continental Shelf we are additionally required to check the structures against accidental loads, ALS control. For ULS the maximum permissible exceedance probability is $10^{-2}$ per year, while the corresponding requirement for ULS is $10^{-4}$ per year.

In order to predict actions or action effects corresponding to given annual exceedance probabilities per year, the exceedance probabilities for all important weather events in an arbitrary year need to be accounted for. Two essential different approaches are available: i) All sea state approach, and ii) Peak-over –threshold approach. Both methods have their advantages and disadvantages.

**Scope of work**

The purpose of this thesis is to investigate adequacy of various approaches for predicting long term extremes by comparing the estimated characteristic action effects.

The necessary weather information will be given by the Norwegian hindcast data base, NORA10, giving weather characteristics every 3 hours from 1957 – 2011.

Below a possible division into sub-tasks is given.

1. Introduce briefly the various limit states involved in design of marine structures with emphasis given to the limit states ensuring robustness against overload failure. The

discussion shall also include a consideration of the relative importance of the two overload limit states.

2. Discuss briefly the loading of marine structures and discuss how one can describe the action effect in a stationary weather event (of 3-hour duration) for a linear system (frequency domain assessment) and a non-linear system (time domain assessment). Establish the expression for the distribution function of the largest response during 3 hours.
3. Discuss how you can find the long term distribution of 3-hour maximum response. Show how you can estimate the action effect corresponding to an annual exceedance probability of q.
4. Establish the joint long term description of significant wave height and spectral peak period from the NORA10 data base.
5. The response amplitude operator (modulus of transfer function) of platform heave of a semi-submersible is available. Estimate the heave amplitudes corresponding to $10^{-2}$ and $10^{-4}$ annual exceedance probabilities, respectively.
6. Validate the estimate results by doing various implementations of the all sea state approach. Available methods are long term analysis in time domain using NORA10 data directly and environmental contour method with contours from the joint distribution of $H_s$ and $T_p$.
7. Discuss how you can perform a long term action effect analysis using the peak over threshold approach. Discuss advantages and disadvantages relative to the all sea state approach.
8. Estimate heave extremes corresponding $10^{-2}$ and $10^{-4}$ exceedance probabilities using the peak over threshold approach and compare results.
9. Summarize the investigation in conclusions pointing out major learning's of this investigation.

The candidate may of course select another scheme as the preferred approach for solving the requested problem.

The work may show to be more extensive than anticipated. Some topics may therefore be left out after discussion with the supervisor without any negative influence on the grading. This will most likely be to skip 8.

The candidate should in his report give a personal contribution to the solution of the problem formulated in this text. All assumptions and conclusions must be supported by mathematical models and/or references to physical effects in a logical manner. The candidate should apply all available sources to find relevant literature and information on the actual problem.

The report should be well organised and give a clear presentation of the work and all conclusions. It is important that the text is well written and that tables and figures are used to support the verbal presentation. The report should be complete, but still as short as possible.

The final report must contain this text, an acknowledgement, summary, main body, conclusions, suggestions for further work, symbol list, references and appendices. All figures, tables and equations must be identified by numbers. References should be given by author and year in the text, and presented alphabetically in the reference list. The report must be submitted in two copies unless otherwise has been agreed with the supervisor.

The supervisor may require that the candidate should give a written plan that describes the progress of the work after having received this text. The plan may contain a table of content for the report and also assumed use of computer resources. As an indication such a plan should be available by early March.

From the report it should be possible to identify the work carried out by the candidate and what has been found in the available literature. It is important to give references to the original source for theories and experimental results.

The report must be signed by the candidate, include this text, appear as a paperback, and - if needed - have a separate enclosure (binder, diskette or CD-ROM) with additional material.

Supervisor: Sverre Haver, Statoil ASA.