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The mediating role of a teacher's use of semiotic resources in pupils' early algebraic reasoning

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Abstract

This paper focuses on the semiotic resources used by an experienced sixth-grade teacher when her pupils are working on a mathematical task involving written text and the two inscriptions of figure and diagram. Socio-cultural analytical constructs such as semiotic bundle, space of joint action and togethering are applied in order to enable and frame the collective activity of the teacher and pupils. Four extracts from different situations in the classroom illustrate the important role of both teacher gestures and pupil gestures, interacting with other modalities such as speech and inscription, in the process of making sense of pupils' appropriation of coordinating two dimensions in a diagram. It is argued that the nature of the mathematical task is an important entry point into early algebraic reasoning. The study emphasises the mediating role of the dynamics of semiotic bundles produced in teacher–pupil dialogues as a promising way to address the fundamental relationships between mathematics, pupil and teacher in a classroom context in order to provoke pupil involvement and engagement when experiencing mathematics.

Keywords Pointing gestures · Semiotic bundle · Early algebraic reasoning · Space of joint action · Didactic triangle · Mediation

1 Introduction

Research has, over the past couple of decades, paid considerable attention to mathematics teaching development (Bjuland and Jaworski 2009; Jaworski 1994; Stigler and Hiebert 1999). Jaworski (1994) introduced a teaching model derived from case studies of teachers which seemed to capture important elements of the teaching. Her model, proposed as the teaching triad (management of learning, sensitivity of students, mathematical challenge), illustrates the complex relationship between mathematics, student and teacher in a classroom context. In 1999 Stigler and Hiebert together with researchers and educators reviewed and discussed video recordings from classrooms in Germany, Japan and the United States made for the TIMSS video study (Stigler and Hiebert 1999). This review identified two patterns of teaching not conducive to student learning. In the German classes, teachers exhibit ownership over the mathematics and introduce it for their students with little student involvement; while in the US lessons, teachers' attention is mainly on the discussion or interaction between students and teachers with little focus on the mathematics. A general research question emerging from these findings would be to explore how teachers might be empowered to become aware of and work on relationships between themselves (the teacher), their students and the mathematics.

In order to approach such a question, this paper focuses on how a class of sixth-grade pupils experience early algebraic reasoning through a teacher's use of semiotic resources (e.g. speech, gestures, body position and inscription) when involved with a mathematical task. The particular task has been introduced as "the diagram task" in previous publications by the author (e.g. Bjuland et al. 2008a). The term early algebraization (Cai and Knuth 2011) will be defined below. It is argued that the diagram task can be used as an important entry point into pupils' early algebraic reasoning since it challenges the pupils to make transitions between written text and the two inscriptions of figure and diagram.

The starting point is to present a case study of a particular teacher called Agnes and illustrate how her pupils were encouraged to experience mathematics through the particular diagram task. Agnes organised weekly work-shops in her classroom in which her pupils were challenged to work on problem-solving tasks in small groups. The diagram task is just one example of a problem that was introduced in one workshop. Four extracts of the classroom dialogue from this particular workshop are analysed to illustrate the mediating role of Agnes's semiotic resources produced in the teacher–pupil dialogues from different situations in this lesson.

The four extracts of classroom dialogue have previously been used to address a teacher's communicative strategies through discourse and gestures (Bjuland et al. 2008b, 2010). In this paper, using a socio-cultural analytical framework the modalities of gestures, speech and inscription form part of the resources activated in the collective teacher–pupil communication in a Grade 6 classroom. The focus is particularly on the mediating role of Agnes's gestures, interacting with other modalities, in forming the pupils' approach to early algebraic reasoning.

The following research question is addressed: To what extent can a teacher's use of semiotic resources play a mediating role in pupils' early algebraic reasoning while solving a written mathematical task involving the two inscriptions of figure and diagram?

Researchers in the field of mathematics education have recently focused on various semiotic resources utilised within classrooms when students have been working on mathematical problems related to functions (Arzarello et al. 2009; Radford 2009) and when the students and the teacher explore the first terms of the odd number sequence (Radford and Roth 2011) in a Grade 2 classroom.

Inspired by Arzarello et al. (2009), the model of semiotic bundle and the notion of semiotic games are used as tools in the analysis. Radford's (2009) notion of semiotic means of objectification has also been applied in order to capture the active process of sense-making that takes place in the teacher–pupil dialogues when the sixth grade pupils try to solve the diagram task. One crucial aspect of the analyses of the four classroom dialogues is to focus on the didactic triangle with its components mathematics, student and teacher. The didactic triangle (see Fig. 1) illustrates that mathematical teaching–learning processes are not separate (Steinbring 2005). In order to capture the collective thinking from joint activity in the interactions from class- room dialogues with Agnes and her pupils, the analyses of the four extracts apply the two socio-cultural concepts space of joint action and togethering presented by Radford and Roth (2011). All the analytical constructs introduced in this paragraph will be explained below.

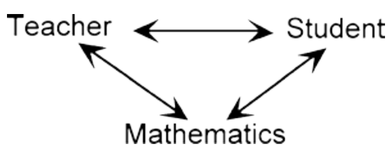


Fig. 1 The didactic triangle

2 Theoretical background

Research involving processes of mathematical interactions has become more and more prominent in the last 25 years (Bjuland and Jaworski 2009; Jaworski 1994; Stigler and Hiebert 1999). The theory of didactical situations developed by Guy Brousseau has played a crucial role in mathematics teaching developmental research since the theory emphasises the close relationship between three essential elements of mathematical teaching–learning processes (Steinbring 2005). These three elements (mathematics, student, teacher) have been represented as vertices in the didactic triangle (Fig. 1).

The didactical situation has been used as a model for focusing on the negotiations and interactions between teacher and students in the learning context. The didactical contract refers to the rules that regulate and constitute the roles between actors in the learning environment (Hansson 2010). Steinbring (2005) emphasises that the mathematics– student–teacher triplet cannot be reduced to the individual components: the three different elements depend on the mutual relations and interactions between the components.

Inspired by Radford and Roth (2011), this paper adopts a socio-cultural approach to classroom interaction, focusing on the forms of knowing that emerge when teacher and pupils engage in joint activity. This activity perspective is an "alternative to contemporary constructivist conceptions of classroom interaction" (Radford and Roth 2011, p. 227). These authors claim that a distinct trait of activity based on Leont'ev's perspective runs against the dualism between the mind and the social. Activity is stimulated by the collective effort made by the participants, articulating around the object of activity. In this paper, the object of activity for Agnes and her pupils is to participate in the sense-making process in order to solve the diagram task. For the teacher Agnes, it seems important to allow her pupils to experience the mathematics themselves by focusing on the meaning of coordinating two dimensions in a diagram.

Mediation is a crucial term within a socio-cultural perspective. According to Carlsen (2008, p. 31), this term is used "to describe how humans interact with cultural tools in action". He suggests that cultural tools consist of both psychological and physical aspects (for more details, see Carlsen op. cit.). The term mediation is here applied to identify how the semiotic resources used by Agnes might play a mediating role for her pupils in order to deal with early algebraic reasoning.

2.1 The diagram task situated as an entry point to early algebra

In recent years, researchers have started to explore issues related to students’ algebraic thinking in earlier grades in preparation for the introduction of more formal aspects of algebra in later grades (Cai and Knuth 2011). According to Cai and Knuth, the development of algebraic thinking in the earlier grades should attend to deeper underlying structures of mathematics. Such thinking, also termed early algebraization, “requires the development of particular ways of thinking, including analyzing relationships between quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, proving, and predicting” (2011, p. ix).

Carraher and Schliemann (2007) recommend introducing elementary school children to algebraic ideas through the use of multiple representations. This view is confirmed by Blanton and Kaput (2011) who illustrate that functional thinking in the elementary grades is a promising route into algebra. In an earlier study, Blanton and Kaput (2004) revealed that elementary school teachers were able to supervise children’s use of pictures, symbols, tables, graphs and words across the grades in gradually more sophisticated ways. The children were able to make sense of data and interpret functional relationships. These findings from early algebra research suggest that “students’ flexibility with multiple representations both reflects and promotes deeper mathematical insights” (Blanton and Kaput 2011, p. 9).

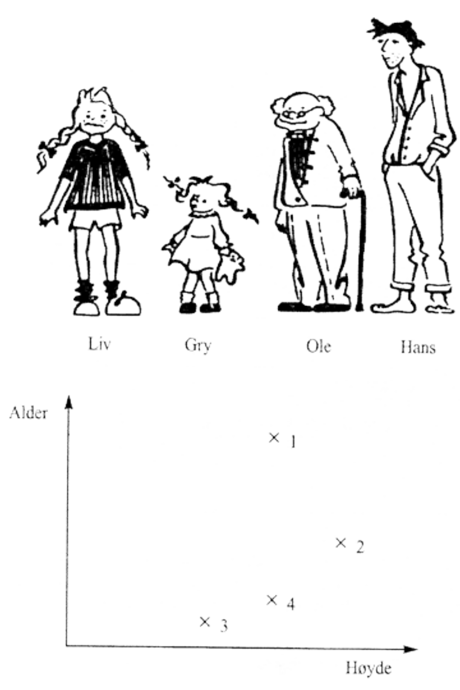
In the research discussed in this paper, the term early algebraic reasoning is used to encompass a clearly focused entry point into algebra when sixth-grade pupils are challenged when solving the diagram task to make transitions between written text and the inscriptions of figure and diagram. Carlsen (2009, p. 55) defines inscription as “the notion used to label drawings, signs, diagrams, and graphs that are made explicit and inscribed on paper or on the chalkboard”. He argues that inscriptions can be understood as accessible signs produced by a teacher or his/her pupils in situ to solve mathematical problems.

The diagram task was originally introduced by the Shell Centre for Mathematical Education (Bell et al. 1986). In the original task seven persons stand in a queue at a bus stop. The task used here is an adapted version previously used in the Norwegian KIM project study [KIM: Kvalitet i matematikkundervisningen (quality in mathematics teaching)] (Gjone 1997).

The written task has four parts. First, the question or instruction is given as written text. The two inscriptions of a figure and a diagram, respectively, constitute parts 2 and 3. Finally, the answer part is a list of names (from the figure) with open spaces for the pupils to write numbers 1, 2, 3, 4 (from the diagram) that correspond to persons in the figure. The pupils in Agnes’s class were working in groups of two and three on the diagram task (see below):

Part 1: Write down which person corresponds to each of the points in the diagram (the Norwegian words alder and høyde are translated as age and height, respectively).

Parts 2 and 3:



Part 4:

Liv corresponds to point… Gry corresponds to point… Ole corresponds to point… Hans corresponds to point…

The figure is a drawing of four people with differing gender, age and height, illustrated by clothing, use of glasses, stick, long and short hair, etc. The diagram shows a Cartesian coordinate system with two axes: the vertical one indicating age, and the horizontal axis indicating height. There is no indication of units, but on both axes there is an arrow, indicating increasing age and height, respectively. In this diagram four points are marked with a cross and labelled 1, 2, 3 and 4. The written text in the beginning of the task prepares the pupils to make transitions between the two inscriptions of figure and diagram, while the written text in the answer part asks the pupils to link the name of each person from the figure with a number label in the Cartesian diagram, expecting them to write their answers in a ready written list.

In a previous work we have commented on the design of this particular task, which may provoke some confusion for the pupils since the dimension of height is located along the horizontal axis (Bjuland et al. 2008a). In Norwegian text-books this dimension is customarily located along the vertical axis.

2.2 Multimodality and the role of gestures in the classrooms

The focus on mathematical teaching–learning processes from the perspective of multimodality has received current attention among researchers in the field of mathematical education (Radford et al. 2009). This perspective involves "the range of cognitive, physical, and perceptual resources that people utilize when working with mathematical ideas" (Radford et al. 2009, p. 91). From this wide range of resources that might be involved when using a multimodal approach, Radford (2003) proposed a more focused notion of semiotic means of objectification which include, among other things, gestures, speech, graphs, formulas, tables and drawings. According to Radford, these means help students to undergo a process of objectification. He explains this notion by stating that "objectifying something is an active process of sense-making that takes place in the interplay of sensuous activity and its continuous revised interpretations" (Radford 2009, p. 119).

Following Radford (2003, 2009), gestures are here considered as one of the semiotic resources applied by both teacher and pupils in mathematics teaching and learning. To analyse gestures, Arzarello et al. (2009) introduced a model proposed as the semiotic bundle. They claim that this construct encompasses classical semiotic analyses which have focused on sign and semiotic resources, semiotic systems (Ernest 2006) and registers of semiotic representations (Duval 2006). Inspired by Peirce's comprehensive notion of sign, Arzarello et al. (2009, p. 100) define a semiotic bundle as a "system of signs…that is produced by one or more interacting subjects and that evolves in time". Typically, this construct refers to signs produced by the teacher and her pupils in a discussion of a mathematical question or while pupils are solving a problem. This enlarged notion defines a semiotic system as comprising speech, gestures and inscriptions and their relationships through the activities of pupils and teacher in the classroom. In this way, the role of gestures is framed in mathematical activities with other semiotic resources within a multimodal approach.

Another important notion proposed by Arzarello and his colleagues (2009) is the semiotic game of the teacher. This construct constitutes a crucial "strategy in the process of appropriation of the culturally shared meaning of signs" (p. 107). In the bundle of signs, the teacher coordinates with the semiotic resources applied by pupils when she, for instance, makes use of the same gestures as the pupils and when she helps the pupils to use precise mathematical language by rephrasing the pupils' utterances.

McNeill (1992, p. 11), an influential figure in gesture research, has proposed that gestures can be defined as "movements of the arms and hands…closely synchronized with the flow of speech". Radford (2009) emphasises that gestures considered in isolation have little interest as far as human cognition is concerned. He claims that "the cognitive possibilities of gestures can only be understood in the broader context of the interplay of the various sensuous aspects of cognition" (p. 123), implying that gestures are genuine constituents of thinking.

In order to capture the mutual relations and interactions between the components of the didactic triangle, the analyses of classroom dialogues below will focus on two central socio-cultural concepts presented by Radford and Roth (2011). The space of joint action draws on the idea of capturing the collective dimensions of thought and feeling in teacher–student interaction. More specifically here, Agnes and her pupils, who are participating in the activity of working on the diagram task, are different—both cognitively and emotionally. As the interaction unfolds, the structure of the space of joint action is mainly made up of a complex and sensuous coordination and tuning of the semiotic resources speech, gesture and inscription. The concept of togethering is used to capture ‘‘the ethical commitment participants make to engage in and produce activity’’ (Radford and Roth 2011, p. 227). The idea of togethering does not mean that Agnes and her pupils are just getting together to do something. Rather, the concept is used to capture joint practical activity with ‘‘the purpose of realizing a collective motivated object’’ (p. 236). Agnes and her pupils have to engage in a process of objectification in which there is a collective responsibility to realise the object of activity.

3 Method

Agnes, the teacher selected for this case study, was a participant in a 3-year developmental research project, learning communities in mathematics (LCM), conducted at a university in Norway. LCM strived to improve the learning and teaching of mathematics in classrooms from the first grade to the 13th grade by developing communities of inquiry among researchers (didacticians) and teachers. A substantial amount of data was collected from workshops, classrooms, planning meetings, etc. Details about the LCM project can be found in earlier publications (Bjuland and Jaworski 2009; Jaworski et al. 2007).

Agnes was selected from among the participants in the LCM project partly due to her long experience as a primary teacher, about 35 years in service, but also because of her engagement and interest in mathematics. Agnes gives her pupils the opportunity to experience mathematics through mathematical tasks in a problem-solving context. The diagram task was introduced at a project workshop for the participants in the LCM project. One week later, Agnes used this task in a classroom workshop context in a lesson with her sixth-grade pupils. The class consisted of 27 pupils (13 girls and 14 boys) aged 11 or 12 years. In this study, a 19-min video clip from the lesson has been chosen to illustrate the mediating role of the semiotic resources used by Agnes when her pupils are working on this task. The lesson has the following organisational structure: (1) introduction and presentation of the diagram task by the teacher (00:00–04:28), (2) pupils’ work in collaborating in small groups of two and three (04:28–13:47), and (3) conclusion of the task as a full class (13:47–18:47).

One group of three boys used approximately 2 min finding a solution to the diagram task (04:28–06:40). It was therefore possible to move the video camera to another group, consisting of two girls who had spent some minutes on preparatory work and therefore started to work on the task (07:14–13:47) only after the boys had come up with a solution.

In earlier work (e.g. Bjuland et al. 2008a), a dialogical approach to analysing communication and cognition has been applied in order to identify pupils’ reasoning strategies and their pointing gestures when working on the diagram task. There is not space here to present this approach with its principles in detail (e.g. the sequentiality, joint construction and act–activity interdependency, Linell 1998). However the sequential organisation of discourse is crucial here in order to understand how the two different semiotic resources, gestures and speech, are used simultaneously in the teacher–pupil dialogue. This dialogical principle takes “into account how a particular utterance is related to the previous utterance as well as to the sub-sequent one” (Bjuland et al. 2010, p. 886). The transcribed video clip has been divided into numbered utterances in which an “utterance lasts as long as a speaker holds the floor” (Bjuland et al. 2008a, p. 281).

In Bjuland et al. (2008a) we identified the following pointing gestures from the dialogues of two groups, consisting of three boys and two girls, respectively: repeated pointing (subject repeatedly points to the same object), consecutive pointing (pointing to different objects, often from one representation to another one), held point (duration-long pointing, subject points and holds her/his finger on an object for some seconds), and point-slide (subject points and moves her/his finger/hand continuously within or between two representations). This last pointing gesture has been more specifically identified as linear point-slide or circular point-slide, respectively. We have described linear point-slide as the pointing when subjects “point and move their fingers along a line, for example, along the figurative representation of the task or along one of the axes of the Cartesian coordinate system” (Bjuland et al. 2008a, p. 280). The circular point-slide “comprises pointing with a circular movement of the hand, for example, between two semiotic representations” (p. 280).

In this study the focus is not only on speech and gesture. The focus here is broader, illustrating mathematical teaching–learning processes from the components of the didactic triangle. The core of mathematics in the particular task for this study is the coordination of the two dimensions of age and height in the diagram. An important question for the analyses of the dialogues will be to focus on how early algebraic reasoning is developed or mediated to the pupils so that they have an opportunity to experience and appropriate the reasoning themselves. In order to avoid the coordination of the two dimensions in a diagram remaining with the teacher, it is therefore crucial to focus on the mediating role of Agnes’s use of semiotic resources in her pupils’ early algebraic reasoning. More specifically, the semiotic resources consist here of the following elements: speech, gestures, body position and the inscriptions and written texts of the mathematical task. The semiotic bundle is the system of signs or the cluster of semiotic resources that are produced in the teacher–pupil dialogues in order to establish *a space of joint action* and *togethering* (Radford and Roth 2011), capturing the collective thinking from the joint activity.

4 Analysing the classroom dialogues

Four extracts of dialogues from Agnes's classroom are analysed from a multimodal perspective. The analyses focus on Agnes and her pupils' activities in semiotic bundles (Arzarello et al. 2009) made of speech, pointing gestures, body position inscriptions and written text. The notion of semiotic game is also addressed in the analyses in order to illustrate whether Agnes and her pupils appropriate the culturally shared meanings of gestures and speech in order to make sense of the mathematical task. The analytical constructs of space of joint action and togethering are used to capture the collective sense-making process established by Agnes and her sixth grade pupils in their attempt at agreeing a common object for the activity.

4.1 Presenting the diagram task

The first extract illustrates Agnes in action with her pupils at the start of the lesson. Agnes uses an overhead projector to project the diagram task onto the screen. The pupils are sitting in a semicircle, looking at the screen. Agnes is also sitting in the semicircle, close to the projector:

15 Agnes: Look, this is about Liv and Gry and Ole and Hans [*Agnes points to the screen from her location in the semicircle*]. Do you notice anything about Liv and Gry and Ole and Hans? Can you see any differences between them? [*Kari has raised her hand*.] Kari what do you see?

16 Kari: They have different heights.

17 Agnes: That's right. Mm. Can you see some more differences? [Several pupils have raised their hands.] Sofie?

18 Sofie: Different age.

19 Agnes: Yes, that's clear that they are different ages.
Yes, then you know that these four people have been out for a walk and, and then we're going to try to find out where the different people are [*Agnes goes from her chair towards the screen*.] Who is number one? [*Pointing at point 1 followed by a circular point-slide up to the figure*.] Who is number two? [*The circular point-slide from the figure ends in pointing at point 2*.] Who is number three? [*From point 2 with a decreasing circular point-slide without reaching the figure, pointing at point 3*.] And who is number four? [*From point 3, a decreasing circular point-slide before pointing at point 4*.] Hm! How can we find out this? [*Agnes goes from the screen and stands closer to the pupils and the projector.]*

20 Pupil: Number one.

21 Agnes: Don't say it aloud yet, don't say it aloud [*Agnes raises her right hand while speaking*]. Now I have thought that you should go in groups [*Agnes turns off the projector*]. And you should try to find out who the different people are. Be sure to read the task carefully.

While within the semicircle, both Agnes and her pupils are concerned with the screen (15)–(19) in order to acquire a more overall view of the task. Agnes's pointing gesture (15) and her communicative strategy of posing open questions provokes the pupils to introduce the dimensions of height and age into the dialogue. Agnes is contextualising the task by locating it in a concrete life situation (19). In these utterances, the modality of speech is the crucial semiotic resource, showing the initiative–response–feedback (IRF) structure in teacher–pupil dialogues (Mehan 1979; Sinclair and Coulthard 1975).

The teacher's body movement, going from the semi- circle to the screen, shows a clear shift in the presentation of the task: from an overview of the task to a closer focus on the two inscriptions of figure and diagram. Agnes's speech and gestures in conjunction with her body position are coordinated in order to help her pupils to stay focused on these inscriptions. More specifically, the dialogue illustrates the close relationship between Agnes's questions and her simultaneous pointing to the diagram followed by gradually decreasing circular point-slides between the diagram and the figure. The semiotic bundle (Arzarello et al. 2009) is here produced by the synchronised process of the teacher–pupil communication (linguistic activity), the teacher's pointing gestures, and the inscriptions of figure and diagram made visible by the artefact of an overhead projector.

Agnes's open questions coordinated with her body movement away from the screen (19) indicate that, without her going into the mathematics, the pupils are encouraged to explore the diagram task in collaborative working groups. This is also emphasised in her utterance and with gesture (21) as a response to one of the pupils (20).

The TIMSS video study (Stigler and Hiebert 1999) suggested that there is a danger that a teacher may introduce the mathematics without giving his/her pupils the opportunities to experience the mathematics. If we look more closely at Agnes's presentation as far as the mathematics is concerned, we observe that she focuses on (a) the inscription of the figure, (b) the transition between the figure and the diagram, and (c) the dimensions of age and height. However, as already pointed out in Bjuland et al. (2008b), Agnes does not put much emphasis on the diagram. For example, she does not examine the points in the diagram by posing questions such as: What does a point represent in a diagram? The pupils are invited to read the task carefully and experience the mathematics themselves by exploring the diagram task in small groups. Agnes is giving her pupils the opportunity to make transitions between the inscriptions of figure and diagram attained through actions (e.g. pointing gestures) so that they are able to locate persons from the figure to points in the diagram by the coordination of the two dimensions age and height in the diagram.

Two extracts of the dialogues from the group work are presented below in order to illustrate how three boys and two girls work on the diagram task and experience the mathematics.

4.2 Gestures and speech from the dialogue of the three boys

The boys quickly arrived to a solution of the diagram task without any supervision by Agnes. First, they found the extreme location and placed Gry as point 3. Second, they compared Gry and Liv and argued that Liv, being the oldest girl, corresponded to point 4. Third, they moved to Ole, as is made clear in the brief extract of the dialogue below. The pupil–pupil discussion illustrates how the boys' gestures and their speech are coordinated when making transitions between figure and diagram:

44 Jon: But then he, he is oldest [*Points at Ole*], then he is second [*Points at Hans*], then she is third [*Points at Liv*], and then she is fourth [*Points at Gry*]. Did you see this?

45 Per: Yes but look, age that's up isn't it? [*Slides along the vertical axis, linear point-slide.*] And he is oldest [Points at Ole], then he must be one [*Points at point 1*].

46 Jon: Mmm.

47 Per: Yes. [*Brief sliding along horizontal axis, linear point-slide*.] Number two [*Points at point 2, Points at Hans*] is then [and then him]

48 Tor: [Yes but (…)]

49 Per: the tallest [*Repeated pointing at Hans*], that's him. [Points at point 2.] Then he is two [*Points at Hans*].

50 Tor: Yes, four and then [*Circular point-slides between diagram and figure, points at Liv, points at Gry*].

51 Per: And then she is three [*Points at Gry*] and four [*Points at Liv*].

The boys have already made suggestions for solutions for Gry and Liv. However, Jon with his four consecutive pointing gestures directed towards the four people shown on the figure seems to be occupied only with the one-dimensional perspective (age). In the following I will argue for the fact that Per's utterance coordinated with his pointing gesture identified as linear point-slide (45) suggests a turning point in the dialogue. In one respect it is possible to interpret the meaning of pointing gestures as simply supporting the easy use of pronouns rather than proper names. However, I will argue that the pointing gestures indicate elements of the complexity of Per's thinking from one dimension to the coordination of two dimensions. Per is attuned to the one-dimensional perspective introduced by Jon, but Per's utterances (45), (47), (49) and (51) show that he is coordinating the two dimensions of age and height in the diagram. Even though Per does not explicitly mention that the arrows on the axes in the diagram indicate increasing height and age, it seems from his linear point-slides along the axes as if he makes sense of the arrows in this way. These pointing gestures illustrate that Per is very focused on the two dimensions, indicating that his point-slides are important semiotic resources that play a mediating role for his thinking.

Per's utterances suggest solutions for Ole and Hans, and the recapitulation of solutions for Liv and Gry are coordinated with his linear sliding along the vertical and horizontal axis, respectively. Jon with his one-dimensional perspective and Tor with his moving of the hand to and fro between figure and diagram (circular point- slides) indicate uncertainty about the point locations corresponding to the two female drawings. In one respect, it is possible to argue that the semiotic bundle (Arzarello et al. 2009) is mainly produced by Per from his oral explanation coordinated with his pointing gestures when focusing on the inscriptions of figure and diagram. However, I argue for the fact that the three boys' utterances are not made up of three different and juxtaposed individual perspectives. The dialogue illustrates a collective activity and the participation of the boys creates a space of joint action (Radford and Roth 2011). Indicators for this joint action can be seen from the dialogue since the three boys are attuned to each other's perspective; they are acting together for the benefit of solving the task. They all contribute to produce this bundle of signs in order to apply the reasoning strategy of coordination between the two dimensions of age and height.

4.3 Agnes in dialogue with two girls

The two girls' work (Eli and Mia) on the diagram task can be summarised in four episodes (identified in Bjuland et al. 2008a, p. 284): (1) approaching the task (2.45 min); (2) first dialogue with their teacher (2.00 min); (3) new approach to the task (1.05 min); and (4) second dialogue with their teacher (0.43 min). Earlier analyses (Bjuland et al. 2008a) have shown that Eli sticks to a one-dimensional perspective when approaching the task (1), and that the two pupils seem to be little attuned to each other's perspectives. Mia, after having made a wrong suggestion, used her pencil to slide along the horizontal and vertical axis, respectively. This linear point-slide to both axes illustrates signs of coordination and Mia then suggests placing Gry as point 3. The dialogue below has been selected from their first dialogue with Agnes:

96 Mia: We didn't understand it [*Agnes stands behind the two girls, Mia and Eli*].
97 Agnes: Didn't you understand it? [*The task.*]
98 Mia: No. [*Erasing her written solution.*]
99 Agnes: No. Mm. But what have you looked at?
100 Eli: We have looked at the height [*Moving her pencil around without any specific pointing or sliding*] [because Hans is highest there.]
101 Mia: [It tells that height there and age there.]
102 Agnes: Have you looked at the age?
103 Eli: That Gry, she is the youngest.

:

113 Eli: But I didn't understand what these labels meant.
114 Agnes: No. These tell which people they are [*Linear point-slide along the figure*]. One of those is number three [*Linear point-slide along the figure followed by pointing at point 3 in the diagram*]. One of those is number four [*Linear point-slide the figure followed by pointing at point 4 in the diagram*]. One of those is number two [*Linear point-slide along the figure followed by pointing at point 2 in the diagram*] and so on, aren't they?
115 Eli: Okay, but those then?
116 Agnes: Yes the points one, two, three, four. Those are four different points.
117 Eli: Should we write the name to those points? [*Moving her pencil between the diagram and the written text*.]
118 Agnes: Yes, you should only write one, two, three or four on these, according as where you find that those are the different [persons].
119 Eli: [Okay.]

The dialogue illustrates the mutual relationship of the three components of the didactic triangle (Steinbring 2005), showing how the two girls and Agnes are focusing on the mathematics. The two girls (particularly Eli) have problems capturing the connection and making the coordination between the two dimensions of height and age. The utterances (96)–(103) illustrate Agnes's open questioning style, inviting the pupils to express what they have discussed so far in the solution process, and her body position allows her to make an overview of the task. Eli confirms her focus on the dimension height (one-dimensional perspective), and her gestures are not focused on any particular inscription (100). The dialogue illustrates how Agnes tries to take part in the two girls' learning process, attempting to pose open questions without employing any gestural resources.

In the continuation of the dialogue, Agnes shifts from the linguistic activity of questioning to the employment of gestural resources when making an explanation (114). The linear point-slides along the figure followed by a pointing at a point in the diagram (repeated three times) illustrate the semiotic resource of the gestures, making the important connection between the two inscriptions of figure and diagram. Agnes is here making the transition from figure to diagram, as opposed to her initial presentation to the full class at the start of the lesson when she used circular point- slides in order to move from a point in the diagram to the figure.

This shift from circular point-slide to linear point-slide, making the transitions from figure to diagram and vice versa, indicates Agnes's important role in the pupils' process of objectification. The semiotic bundle (explanations, linear point-slides and inscriptions) seems to challenge the two girls to experience and appropriate the coordination of the two dimensions of age and height in the diagram.

The participation of Agnes and the two girls has created a space of joint action (Radford and Roth 2011), illustrating the collective process of appropriating the culturally shared meaning of the diagram task. The shift from circular point-slide to linear point-slide also illustrates the semiotic game of Agnes (Arzarello et al. 2009) since with her pointing gestures she helps the two girls to focus more on the transitions between figure and diagram. The dialogue also shows togethering (Radford and Roth 2011), illustrating the committed engagement of Agnes and the two girls in the joint activity of solving the task. Agnes is tuned into the idea of Eli's one-dimensional perspective, and through her explanation and pointing gestures tries to help Eli to make sense of coordinating the two dimensions of age and height.

Agnes adheres to her teaching strategy of giving her pupils the opportunity to engage with the mathematics of the diagram task. More specifically, the two girls are not directly told about the coordination of the dimensions of height and age. After having been guided by Agnes with the open questions and her linear point-slides, Mia suggests a correct solution to the task (episode 3). However, the two girls have a second dialogue with their teacher in order for Agnes to confirm their suggested solutions. This illustrates that the two girls were not confident in their solutions. The lack of confidence suggests that they still need more time to reach an understanding of the coordination between two axes in a diagram.

4.4 Concluding the diagram task

The concluding teacher–pupil discussion, where pupils present mathematical solutions based on the small-group work, consists of one ongoing episode with five thematic sequences: (1) the location of Ole (1.13 min), (2) the location of Gry (0.43 min), (3) the location of Hans (1.21 min), (4) the location of Liv (0.40 min), and (5) Agnes concluding the teacher–pupil discussion (0.54 min). The dialogue below is chosen from the third thematic sequence in order to illustrate Agnes's role as a coordinator, giving pupils the opportunity to present their solutions and explanations.

In the continuation of the dialogue (sequence 3), Agnes is still sitting in the semicircle. One of the boys has just responded to Agnes's open question and showed his answer for Hans on the transparency:

194 Agnes: But you [*singular you*], what did you [*plural you*] think when you found out that Hans should be number two?
195 Odd: We thought that he was tall, and he [*Hans*] was much younger than Ole.
196 Agnes: Mm. Yes, so therefore he should be there.
Is there anyone else that thought about it? [*Silence, 6 sec*.] Leo, what did you think?
197 Leo: Eeh, no I (…)
198 Agnes: Eeh, yes, Is there anyone else that thought about it? Let's see, Hans is number two. He had to be there. Why couldn't Hans be there [*Points at point 1, diagram*]. Why couldn't Hans be there, Eli? [*The teacher chose Eli among several pupils who raised their hands*.]
199 Eli: Since he, or if Ole, he is the oldest and then couldn't he [*Hans*], since he [*Hans*] is the youngest [of these *two*].
200a Agnes: Mm. Yes.

The utterances (194)–(198) illustrate that the pupils are first challenged to explain their solutions, then allowed a waiting time (of 6 s) in which to make individual considerations before one of the pupils, Leo, is invited to make a comment. The two why-questions (198) are crucial moments in the dialogue. When Agnes is posing the first why question, she moves from the semicircle to the transparency and points at the diagram. The coordination of this bundle of signs (why-question, pointing gesture, body position) illustrates that Agnes, by focusing on Hans as a candidate at point 1, challenges a visual misconception. The teacher uses this particular pointing gesture at point 1 in the diagram to provoke the pupils to consider that Hans, who is the tallest person, corresponds to point 1 which is located highest in the diagram. Agnes's gesture is here used in coordination with her why-question to provoke the pupils to reject this suggestion and hopefully come up with an explanation of coordinating the two dimensions age and height.

The preceding analyses of dialogues have suggested that semiotic bundles have contributed so that the pupils have been challenged to experience the coordination of two dimensions in a diagram. The semiotic bundle (Arzarello et al. 2009) produced here (194)–(198) illustrates more clearly how the bundle of signs play a mediating role for the pupils to deal with the mathematics. Having found a solution for Hans, the pupils are challenged by Agnes's why question and pointing gestures to argue for their solution.

When Agnes repeats the why-question, she chooses Eli from among many pupils to respond to this challenging why-question. Eli is one of the pupils from the girl group. Analyses from this group (third extract of dialogue) revealed that Eli had difficulties in coordinating the two dimensions of age and height. From Eli's comparison of Hans and Ole due to their ages (199), it seems that Eli still has a one-dimensional perspective without giving a proper explanation.

5 Discussion and conclusion

The analyses of the classroom dialogues have revealed how the mathematics (the coordination of the two dimensions age and height in a diagram) creates a focused collective activity between Agnes and her pupils, illustrating the mutual relationship between the three components of the didactic triangle (Steinbring 2005). The sixth-grade pupils have been engaged in early algebraic reasoning through the nature of the diagram task, which involves the transitions between written text and the two inscriptions of figure and diagram. These transitions may provide a promising route into algebraic ideas (Carraher and Schliemann 2007). This view has also been expressed by Blanton and Kaput (2004, 2011), whose findings, when studying young children's functional thinking, have revealed that children's flexibility with multiple representations has stimulated them to reflect and promote deeper mathematical insights.

The model of the semiotic bundle and the notion of semiotic game, adopted from Arzarello and colleagues (2009), show promise as analytic tools in order to enable and frame the collective activity of Agnes and her pupils to make sense of the diagram task. The analyses of the classroom dialogues have revealed that the pupils in both groups underwent a process of objectification (Radford 2009) by making sense of the diagram task from their linguistic activity and pointing gestures. The collective activities consist of a close combination of gestures and inscriptions co-timed with the pupils' and the teacher's utterances. The component of the semiotic bundle (speech, gestures, body position and inscription) appears to be active at the same time in Agnes and her pupils' ongoing activity.

The analyses have revealed the seemingly crucial role of both teacher gestures and pupil gestures, interacting with other modalities of the semiotic bundle. This has been particularly illustrated in the dialogue of the three boys when pointing gestures have been used in coordination with speech to make transitions between figure and diagram. It seems that the boys have used the important strategy of semiotic games (Arzarello et al. 2009) since they elaborate on the pointing gestures produced by Agnes (circular point-slide). They use linear point-slides which are more focused on both axes in the diagram. The nature of this particular task also seems to play a crucial role in the boys' production of these pointing gestures, helping them in the process of coming up with a solution on the diagram task.

The first extract of the dialogue illustrates that Agnes presents the diagram task without first delving into the mathematics. Her pupils are challenged to experience the mathematics themselves in order to make the transitions within and between the two inscriptions of figure and diagram. However, Agnes is more directly involved in the solution process of the two girls. The semiotic game produced by Agnes, by shifting from circular point-slide in her presentation of the task to linear point-slide in her dialogue with the two girls, constitutes a more focused attempt to help the girls to gain an awareness of the reasoning strategy of coordination of two dimensions. From the concluding discussion of the task with the whole class, Agnes also invites Eli from the girls' group to respond to the challenge of considering Hans as a candidate at point 1. This mathematical challenge of provoking a visual misconception, emphasising the focus on the coordination of two dimensions, might arise from Agnes's knowledge of the conversation with Eli in the small-group dialogue. This indicates Agnes's sensitivity to Eli's experience with the diagram task, illustrating the close and interrelated relationship between elements of the didactic triangle (Steinbring 2005).

The analyses of the four extracts of dialogues have focused mainly on the collective cognitive process of understanding the task. However, it is important to emphasise that the pupils' processes of objectification also involve an emotional-affective component, underpinning the willingness of both Agnes and her pupils to reach a solution on the diagram task. The two examples summarised above, which show Agnes's sensitivity in helping the two girls to reach a solution on the diagram task, illustrate that individual participants in a collective activity behave differently—both cognitively and emotionally (Radford and Roth 2011). According to these authors, the object of activity is refracted differently among the individuals who participate in the activity. The analyses of dialogues in this paper have confirmed this view. For the pupils the object of activity has mainly been to reach a solution on the diagram task, while Agnes reveals that the collective activity for her is directed toward the reaching of a common object (solving the diagram task, involving both pupils' explanations and justifications and stimulating early algebraic reasoning through the coordination of two dimensions in a diagram). Two socio-cultural constructs, space of joint action and togethering (Radford and Roth 2011) have been used in the analyses to illustrate that Agnes and her pupils have a collective responsibility and commitment to reach the common object of activity.

Inspired by the didactic triangle (Steinbring 2005), an overarching aim with this paper has been to focus on mathematics teaching development through the question about how teachers might be encouraged to become aware of their relationships with students' involvement in experiencing mathematics. The paper partly addresses this complex question by presenting a case study of the teacher, Agnes. The analyses of classroom extracts have illustrated how her use of speech (verbal explanations and questions), pointing gestures and inscriptions can mediate her sixth- grade pupils to experience early algebraic reasoning through the entry point of the diagram task. The pupils have been engaged with pointing gestures used within or between the inscriptions of figure and diagram with a particular focus on coordination of the two dimensions of age and height. Through these detailed analyses, it can be observed how the semiotic resources used by Agnes become interwoven during the pupils' activity of solving a particular task situated in a problem-solving context.

The research question addressed in this paper was: To what extent can a teacher's use of semiotic resources play a mediating role in pupils' early algebraic reasoning while solving a written mathematical task involving the two inscriptions of figure and diagram? The analyses suggest that teachers together with their pupils can create a space of joint action, being aware of the collective activity from the dynamics of semiotic bundles (speech, gestures, body position and inscription) produced in teacher–pupil dialogues. There is a need for teachers and pupils to establish togethering, in order to capture the cognitive, emotional and ethical commitment that is necessary for being engaged and produce the activity.

This case study illustrates that the production of semiotic bundles, and also the use of semiotic games, seem to constitute processes of objectification for pupils to appropriate early algebraic reasoning. In this paper, the algebraic reasoning is particularly related to transitions between the two inscriptions of figure and diagram in order to really appropriate the meaning of the coordination between two axes in a diagram.

The notion of a didactic triangle, emphasising the dependent relationship between mathematics, student and teacher lies at the heart of the concerns addressed in mathematics teaching developmental research. The complex relationship between these three components in a classroom context is also emphasised in the reviews and discussions of classroom recordings made for the TIMSS video study (Stigler and Hiebert 1999) and from the teaching triad proposed by Jaworski (1994). This paper suggests that a promising way to address the fundamental relationships within the didactic triangle is to allow pupils to engage with early algebraic reasoning through teacher encouragement. The dynamics of semiotic bundles (speech, gestures, body position and inscription) play a crucial role for the collective thinking from joint activity in the interactions from teacher–pupil dialogues. Future research might put more emphasis on how such bundles might provoke more pupil involvement and engagement when experiencing mathematics.

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