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Abstract

Pipelines are some of the main components of the production and transportation of hydrocarbons in offshore Oil & Gas Industry. The design, installation and operation of pipelines involve many challenging tasks. The recent decline in oil price is putting pressure on future subsea projects and leads to the stop or postponement of operators investment. While keeping the project costs as low as possible, an installation of pipelines should be done economically. As the water depth increases the loads on the pipeline changes and new challenges are experienced. A pipeline may for instance rotate during installation when there is some plastic curvature introduced or an inline structure is attached to it that causes a rotational movement. Plastic curvature can be formed due to bending of pipe beyond elastic point on the stinger (S-lay) or residual strains remain during unspooling the pipe (Reel Lay). The rotation of pipe is not desirable during the installation for inline structures i.e. Wyes or Tees at seabed which may become difficult to access and connect to. The rotation of pipelines can in other cases be desirable in case of installation of residual curvature sections to control thermal buckling and expansion effects. The main objective of this thesis is to explain the rotation phenomena for pipelines and its dependency on different laying parameters. The next objective is to compare experimental results with the analytical energy equations.

Up until now, little information, literature and public data is available on the subject of rotation of pipelines, and most of the recent information on pipeline rotation has been performed by Geir Endal et al. (Statoil).

Three different approaches have been used to assess the pipeline rotation touch down point (TDP). The first Approach is the experimental approach. An experimental approach is applied in the thesis to study the parameters of pipeline rotation. A small model test rig was built replicating the real life pipeline installation by reel-lay and S-Lay. The experiments include all the basic pipeline installation parameters i.e. stinger/ramp, rollers, top vessel tension, bottom horizontal tension, catenary shape of the pipeline from tip of the stinger to TDP and nominal curvature of the pipeline from tip of stinger to TDP. Building the test rig took a good portion of the thesis duration. Different pre-bend sections were created with different residual curvatures lengths and strains. The pipeline installation was simulated by pulling the pipeline from the bottom side and measurement of rotation angles at TDP for various parameters were performed.

The Second Approach assessed was the analytical approach. The Analytical approach is based upon energy minimization presented by (Endal, et al., 2014). The rotation angle at TDP was calculated using the same experimental test parameters except for the pipeline bottom end boundary condition, which is free-end in the analytical approach while partially free in the experimental test.

The Third approach combined the experimental and analytical methods. The nominal curvature of pipeline was calculated by using the catenary shape of the pipeline for each top tension and water depth. The nominal curvature was then used in the analytical calculations to predict the rotation angle at TDP.

Finally, the three approaches were compared and conclusions made. Uncertainties related to the experiments were discussed and recommendations for further work outlined. Applications of the residual curvature method were also briefly discussed.

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Nomenclature

K_r	Residual curvature
ε_r	Residual strain
r	Outer radius of the pipe
α	Coefficient of nominal curvature equation ins simplified energy method
β	Coefficient of nominal curvature equation ins simplified energy method
$\varepsilon_{nom(max)}$	Maximum nominal strain
L	Length of suspended pipeline length
$\phi(x)$	Roll or Rotation angle
W_{tot}	Total work done
W_B	Work due to bending
W_R `	Work due to rotation
M_B	Bending moment due to bending
$k(s)$	Nominal curvature of the pipe
M_R	Bending moment due to rotation
G	Shear modulus of elasticity
I_T	Polar moment of inertia
K_{res}	Residual curvature
ε_{res}	Residual strain
L_{curve}	Residual curvature length
Ht	Horizontal tension
w	Submerged weight per unit length
E	Young modulus
I	Second moment of inertia
T	Top vessel tension (barge tension)
A	Coefficient of nominal curvature equation ins modified energy method
γ	Coefficient of nominal curvature equation ins modified energy method
σ	Bending stress
R	Radius of curvature
DF	Design factor
σ_{eq}	Equivalent stress (Von Mises)
f_y	Yield stress of the material
T_h	Horizontal component of top tension
z	Height above seabed
T_v	Vertical component of top vessel tension
t	Wall thickness of pipe
ds	Arc length
d_i	Residual of fitting

Abbreviation

DPs	Dynamic Positioning System
TDP	Touch down point
RCS	Residual Curvature Strain
RCL	Residual Curvature Length
RCM	Residual Curvature Method
RS	Residual Strain
Dn	Deflection number
BCs	Boundary Conditions
WD	Water depth
ROV	Remotely operated vehicle
ID	internal diameter
OD	Outer diameter

1 Introduction

After the discovery of Offshore Oil& Gas, Subsea pipelines got more attention. Offshore drilling reaching ultra-deep water up to 3500 meter sets new challenges for subsea pipelines. Pipelines have challenges both in design, installation and operations phase. Due to economic benefits, Pipelines are preferred to transport oil and gas, tieback to platforms, to shore, or to other pipelines are cost-effective.

From the structural point of view, pipelines fall into the category of slender marine structures i.e. small ratio of cross-sectional area to length. They range from small diameter inter-field connections (flowlines) to large diameters pipelines from the offshore fields to shore or across the oceans (trunk lines). Pipelines represents a very reliable, safe, cost-effective and environmental friendly mode of transportation.

Pipelines represents a major component of the offshore industry. A successful installation is possible if the statics and the dynamics of the Pipelines is well understood. Extensive structural integrity analysis is done before installation of pipelines in deep and harsh water for production and transportation of hydrocarbons. The motivation in Pipeline installation research is the economic and technical challenges increase with increase of water depth, seabed topography, high pressure and temperature. In deep and ultra-deep water, the environmental conditions are harsh. The unreachability to structures and pipelines in ultra-deep water makes the repairs very costly. Thus, during installation of pipeline and inline structures, it should be installed according to planned operation.

The Rotation and Residual plastic curvature of pipelines chosen for the study during installation of pipelines. The large catenary section of the pipeline will be expose, to sea loads, Residual strains in overbend, Strains in sagbend, barge tension and pipeline weight are important parameters that need special attention during the installation process. Limitation in Barge tension and capacity are key challenges to deal with. Traditionally in the Oil & Gas industry, there is very less open literature to control and benefit from these phenomena. Thus, this thesis will do comprehensive study with knowing the physics and experimentation of Rotation and Residual plastics curvature of pipelines. It will also focus on the benefits in terms of cost, design and life of pipelines.

1.1 Pipeline Rotation during installations:

Rotation in the pipeline follow the principal minimization of the potential energy. There are of two types of energy minimizations: strain energy minimization and gravity energy minimization (Vaughan & Nystrom, 2016). The Strain energy minimization phenomena appears when plastic strains are present in the pipe, Gravity energy minimization dominates due to top-heavy structures, which are unstable and eccentric loadings (like inline tees/wyes, or inline pigging structures, etc.) acting downward, causes torsion and leads to increased rotation.

Other Potential triggers of pipeline rotation may be the torsion created from laying in curves, slack in tensioner machines etc.

1.2 Objective

The motivation of this thesis is to carry out a parametric study of pipeline rotation during installation in general and problems faced during installation. This study of rotation of pipeline will help the pipeline industry implement the cost effective solutions. This will reduce the project cost and benefits the subsea industry. Applications of rotation in residual curvature method i.e. uneven seabed, thermal buckling control etc. will improve the efficiency of pipelines during its life cycle. This thesis deals with the detail analysis of pipeline rotation. The thesis Objectives are:

1. Building an S-lay installation test rig for future projects and study purpose.
2. Study and understand the physics of rotation of pipelines during installation.
3. Experimental Study effect of residual curvature strain and residual curvature length on the rotation of pipelines.
4. Experimental study of Rotation of pipelines during installation with changing parameters i.e. varying water depth, laying vessel tension, varying lower boundary conditions (Fixed, and partially free boundary conditions with torque measurement) and inline structure. The pipeline to be stimulated by moving the pipeline as to be represented real life vessel movement.
5. Analytically calculate the rotation of pipeline at TDP by employing strain energy minimization principle of pipeline using the present literature.
6. Use of pipe catenary shape for each load and water depth to calculated the nominal curvature of the pipe. The experimental nominal curvature of the pipeline to be used in analytical calculation to predict the pipeline rotation at TDP. In analytical energy method, the analytical nominal curvature is predicted through the assuming pipe suspended section as stiffened catenary system.
7. Comparison of three approaches and brief explanation to each result.

The scope of this experimental thesis were defined during the meeting with supervisors. This thesis scope constraint by three parameters, shown in figure 1-1.

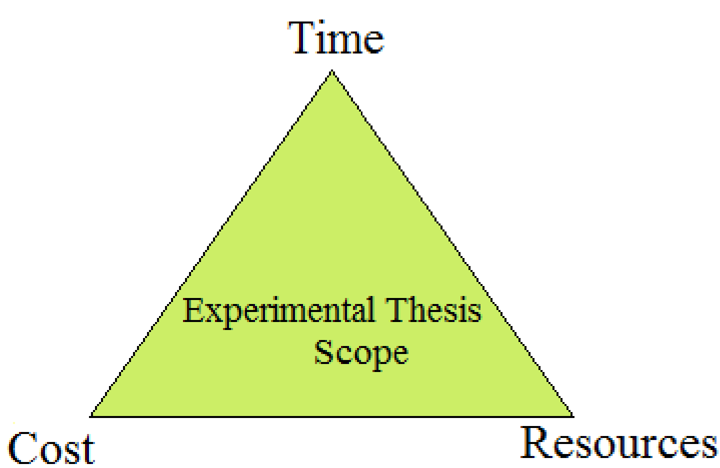

Figure 1-1Experimental Thesis Scope

Time: This thesis has the duration of five and half months. All the work related to experiment and report writing were completed within this time period. The time to get the equipment and to build test rig were time consuming. Each experiment test also took considerable amount of time to complete.

Cost: The equipment and experiment test rig cost were covered by the University of Stavanger. Due to limited funds for master thesis, a best possible set up was built to illustrate Pipeline installation and explain rotation of pipelines.

Resources: This master thesis is done by only the author. The manpower was one of the big issues in the test rig building and experiments. It took enormous amount of effort and energy to build the test rig, and do the experiment. The experiment is performed in university material laboratory pool. The availability of the pool was limited due to other thesis.

1.3 Structure of Thesis Report

This thesis follows the Harvard-Anglia Ruskin University Citation style for referencing and citation. The organization structure of this thesis is as follows:
<u>Section 2</u>: It deals with the short introduction of the pipeline installation methods used in subsea pipeline industry.
<u>Section 3</u>: This section illustrates about the Analytical approaches used to initial predict the rotation angle of pipeline at TDP.
<u>Section 4</u>: It relates to brief theoretical aspects that were study and used for the building of the experimental test rig. A S-lay installation method is used for the experimental test rig.
<u>Section 5</u>: This section focuses on the manufacturing of test rig. It gives the brief discussion for all the work carried out to build the test rig. New devices and technique were developed to do model scale testing.
<u>Section 6</u>: It presents the scaling laws and number used for the experiment. The non-dimensional number help to compare the experiment with real life scenario.
<u>Section 7</u>: It describes the experimental test performed on the test rig. Each experimental test was assigned with a number and laying parameters were reported in this section.
<u>Section 8</u>: This section has three parts. First discussion on the experimental tests: Effect of laying parameters is exclusively discussed and comparison to the different projects has been written as a source of validation. Second from the Analytical: rotation angle is calculated using section 3 analytical approach. Third part includes calculation for nominal curvature of the pipeline and used in analytical equation to find rotation angles at TDP. Finally, results comparisons, from three approaches has been presented.
<u>Section 9:</u> It points out the uncertainties during the experiment that were beyond the control or the limitations of equipment used.
<u>Section 10:</u> It conclude the thesis with the Conclusions and Recommendations for future work.

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2 Offshore Pipe laying Installation

Pipeline installation is one of the challenging task in offshore industry. It requires high level of engineering and control for the best efficiency between the cost and scheduling of operation. During the pipeline installation, Pipeline control is necessary. Pipeline Control defines as pipe deflections from the sea surface to seabed, vessel or barge six-degree motion control using anchor or Dynamic Positioning System (DPs), Pipe laying configuration according to plan route and pipe behavior at touch down Point (TDP) on the sea bottom. Any Pipeline project is divided in three categories:

- Design
- Installation
- Operation

In design phase, the pipeline properties and path are discussed in detail. During the design, the pipe properties are determined i.e. pipe diameter, pipe material, wall thickness, thermal insulations, and coatings depend upon fluid flowing properties i.e. temperature, pressure, density, phase mixture information, corrosion properties, chemical properties. Then pipe is analyzed for available vessel and equipment limitations. For complex installations of pipelines, complex stress-strain analyses are performed to know the effects during installation. The method of installations of pipelines can change pipe designing parameters.

2.1 Definition of Pipe laying installation

Pipeline installation can be defined as
"Operation of installing a pipeline on seabed from sea surface barge, the pipeline is laid –off from the vessel with a speed and a departure laying angle $\alpha = (0, \pi/2)$, relative to sea surface."
In case of S-lay method, the departure angle depends on the water depth with the stinger. It varies from low departure angle for shallow water to high departure angle for deep waters. In J-lay method the departure angle is $\pi/2$ radians. In Reeling, the angle varies depending upon vessel and laying parameters.

2.2 History

The offshore pipeline industry started during the World War II with the project name *Operation Pluto (Pipe line Underwater the Ocean)* by the British engineers and defense forces for provide gasoline to the allied forces. Pipeline of 3inch (75mm) were used to reel on Floating drums named *Conundrums*. These pipelines delivered 781 000m^3 of gasoline during war. (Searle, 2004).In 1950 and 1960 the offshore pipe industry starting to develop in Gulf of Mexico for onshore supply of oil. In 1986 due to low oil prices, the offshore pipe laying industry were severely hit. Due to boom in oil prices in 1990s and new projects up to 500 meters' water depth, make the new kick start to Pipelaying industry. Now the pipe laying industry is present in many parts of world like North Sea, West Africa, Gulf of Mexico, South East Asia and Brazil.

2.3 Pipelaying construction methods

Different Pipelaying methods are applied but up till now these four methods dominates the Pipelaying industry. The choice is depending upon the water depth, pipeline properties and cost. The laying rate for S-Lay is 5 km per day, for J-lay 1-1.15 km per day and for reeling is 14km per day. The Pipeline installation methods are:

1. S-Lay
2. Reeling
3. J-Lay
4. Towing

These methods based upon principles of:

- Allowable bending stresses
- Allowable axial stresses
- Avoidance of kinking

The main companies in installation of pipelines are Subsea 7, Technip, Heerma, Saipem, Allseas Group and J.Ray McDermott. Following is brief description of each method:

2.4 S-Lay

This is most common way of installation of pipelines. The pipeline installation starts in a plane parallel to laying vessel and get the shape of character "S" on its way to seabed, shown in Figure 2-1. Pipes of 12m-24m are constructed and coated onshore, these pipes then brought to Pipelaying vessel where they undergo series of welding stations called as *firing line* to Non Destructive Testing and coatings to make long sections of pipelines. These pipelines then leave the vessel from its stern. The Pipelines are held in a tension by *tensioners*, Tensioners are like rolling convers belt with rubber pads that are operated hydraulic to press the pipe and keep the pipe sag bend in allowable range. When one weld is finish the vessel move forward with a speed called as *pay-out speed* for another pipe to be welded. The pipeline leaves the vessel with a sloping ramp called as *Stinger*. Stinger is open structures, which could be rigid or articulated. The stinger has rollers to provide frictionless and control curvature of the pipe laying. The world largest S-laying vessel Solitaire has 130-meter length of stinger shown in figure 2-2. Shorter length of stinger can buckle the pipe. The upper curvature of pipe on the stinger is called as overbend. The pipe leaves the stinger with a *departure angle, which* varies according to water depth and tension to the pipe. *Inflection point* starts right after the pipe leaves the stinger.

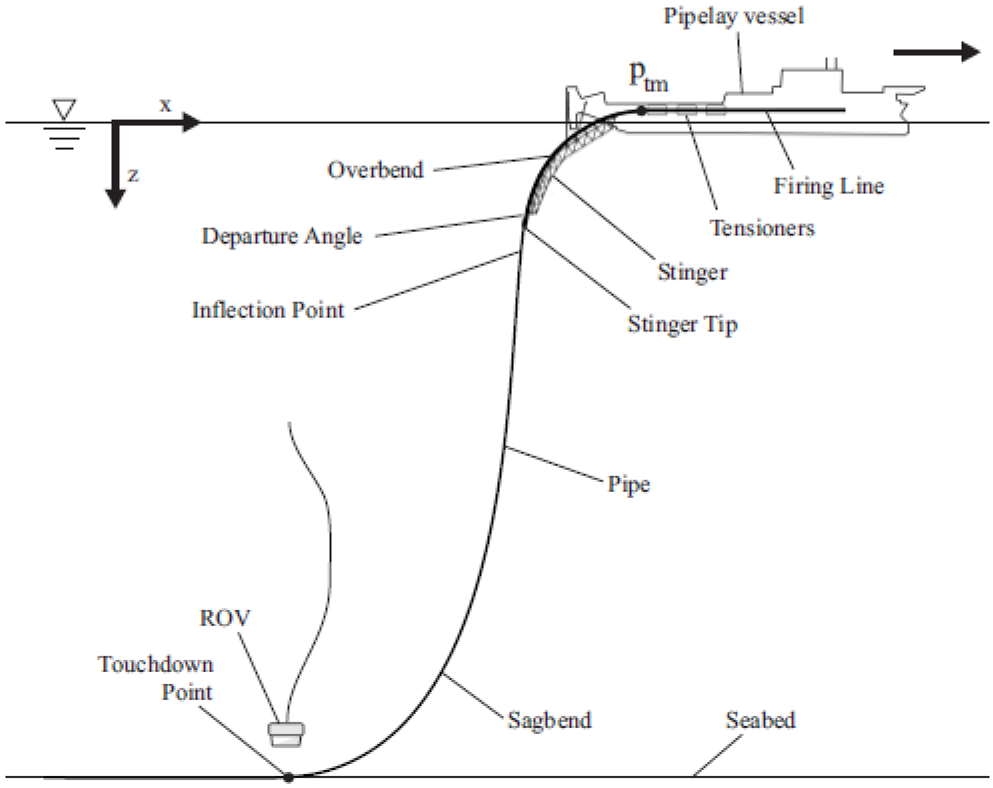

Figure 2-1 S-lay installation Schematic, from (Kyriakides & Corona, 2007)

The pipe due to its weight undergoes a curvature near to seabed, called as sagbend. It is one of the critical region of pipeline. It need be assured that pipe has allowable bending radius in this region. This curvature is controlled by top tension of the pipe. Tension needs to be optimized as excessive tension can cause the plasticity of pipe in overbend region. Modern S-lay vessel have Dynamic positioning system for control their position in the sea.

Figure 2-2 S-lay installation vessel Solitaire. 300m long without stinger and has capacity to a holding force of 1050 t. Courtesy of The Allseas Group.

2.5 Reeling

Reeling is efficient method for installation of pipelines. Due to small diameter requirement, reeling is suitable for cables, umbilicals, flexible and rigid pipe up to 18 inches' diameter.

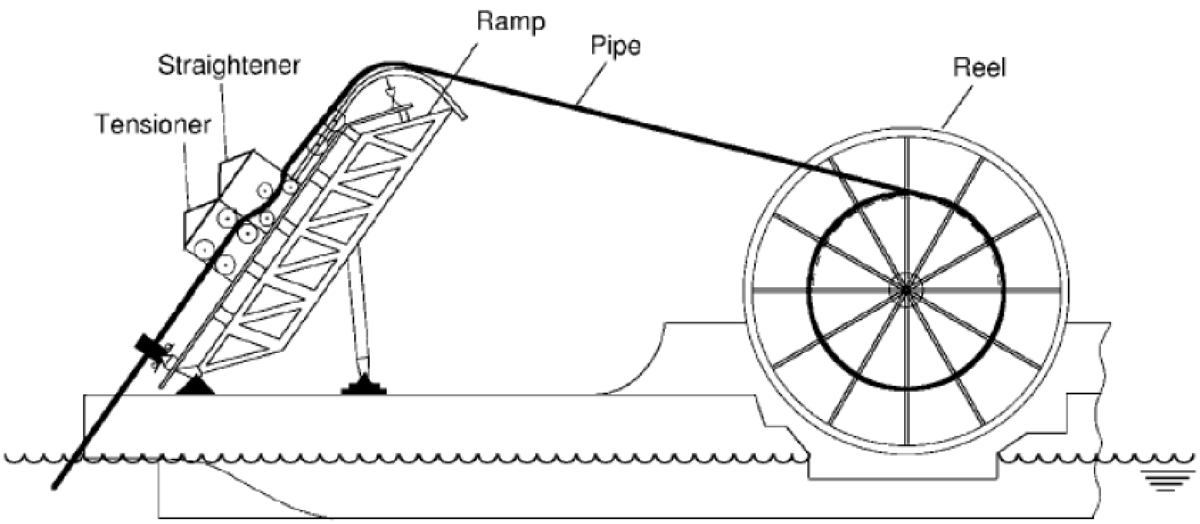

Figure 2-3 Reeling installation Schematic from (Kyriakides & Corona, 2007)

All the pipe construction i.e. assembly, welding, inspection and coating is carried out onshore and pipes are spooled on big diameter reel drum fixed on the reel vessel, shown in Figure 2-3. The vessel then moves to installation site and unspooling of pipes started. The vessel moves forward with attached straighter and tensioners. The reeling and unreeling process of pipe induce large strains in the pipe in range of 2%-3%, which needs to mechanically straighten out during unreeling. As mention in (Kyriakides & Corona, 2007) with 12 inch of pipe on a reel drum of 8.23 m, a strain of 1.93% and 16-inch pipe on a reel drum of 8.23m, a strain of 2.41% is attained. Thus mechanical properties i.e. wall thickness must be chosen carefully to avoid local buckling. The reeling of pipe lines has advantage of installation time and overall cost benefit. Reel vessels may have horizontal reel and pipe is laid into the sea overs stinger like S-Lay, while Vertical drum reel vessels has tower similar to J-lay type installation.

The concrete coated pipes cannot be used in reeling. Relatively heavy wall thickness is needed to avoid the pipe flattening (Mousselli, 1981).

In Reeling, the reel drum is permanently located on the vessel, and it has to be "recharge" with pipes after laying off all the pipe or another vessel taker her place which is not possible due to high cost. This cause stops in installation, making it inefficient both in time and money. The Seven Oceans, owned by Subsea 7 is the stat of the art reeling vessel, Shown in Figure 2-4.

Figure 2-4 Seven Ocean Reeling Vessel, 4-16-inch pipe laying capacity up to 3500 ton of pipe weight, Courtesy of Subsea 7

2.6 J-Lay

The method is named due to having pipe installation resembles letter "J". Pipe are welded while having support on the vertical towers and leaves the ship nearly vertically. The angle of Pipe laying from vessel is between 0-15 degrees. This removes the Overbend region in J-lay compare to S-Lay. J-lay method schematic shown in Figure 2-6.

J-lay method reduces the stinger requirements, which are used for deep water in S-lay. The stinger used in J-lay has only to change the angle of pipeline with respect to vertical orientation. Due to having one welding and one inspection section in J-Lay normally, long sections of onshore welded pipes are used in laying process to increase the efficiency and time. The method has slow day rate of pipe laying compared to S-lay. Also the long heavy vertical tower causes the instability problems to the vessel.

The J-lay method is attractive as bending stresses are low and forces require for station keeping of the vessel are within capable range. However, the tension in the pipe is high and the span length of pipe is reduced. The method is not good for shallow water. One of the largest J-Lay vessel is Saipem 700(S-700) shown in Figure2-5.

Figure 2-5 a J-Lay semisubmersible vessel for 4''-32" pipe lay, Saipem 7000, Courtesy of Saipem SpA

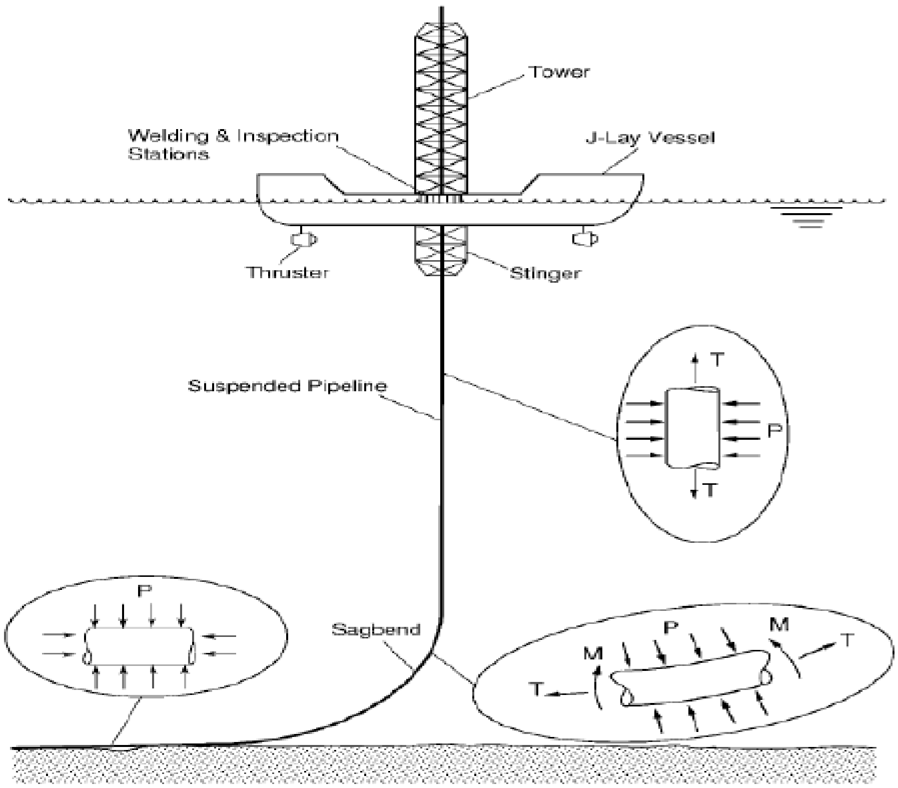

Figure 2-6 J lay Installation schematic from (Kyriakides & Corona, 2007)

2.7 Towing

Towing installation method is use for installation of small length of pipelines usually less than 4km (7km has installed too). The pipe construction and inspections had done onshore, while pipe towed to the installation site via tow or pull vessels. This method normally used for small several flowlines constructed together. The towing is usually by two vessels, one at the leading end of the pipe while second at the rear end of the pipe. The towing method generally divided into following categories:

- Bottom tow
- Off-bottom tow
- Controlled depth tow
- Surface tow

The selection of the method dependent up the length, seabed conditions and pipeline material properties. The Pipeline constructed onshore, which has advantages i.e. low equipment cost, easy accessibility and no weather restrictions. The pipe buoyancy controlled to tow it at specified depth of water so that the strain on the pipeline is within the design criteria. While due to limitations of sizes, the method does not have wide applications. The length of the pipe should be straight and intervention is required for the bends in the pipe.

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3 Analytical Approaches

This section is describing the analytical work and approach adopted in this study.
Pipeline roll dynamics during installation operations is of a vital interest and requires careful consideration of the most important parameters. The primary source of pipeline rotation is the plastic strain in the pipeline. which is introduced during both the S-lay and Reel-Lay installations.

In S-lay the plastic strain is introduced in so-called overbend region on the stinger. As the pipeline moves down passing the inflection point and reaching the sagbend region, it tends to rotates rather than to undergo reverse bending. This is due to strain energy minimization.

Let`s consider plastic strains in the overbend region during laying of pipelines. This plastic strain induces curvature in the pipeline, which is dependent on the radius of stinger and plastic strain. As mentioned in (Bynum Jr & Havik, 1981), if the pipeline would not roll then it additional work for reversing the positive plastics bending (caused in overbend) into the negative elastic bending (in sagbend due to its weight). The work required for reversing curvature would be less if the pipeline rotates. The lateral forces due to, for example, sideways current, waves, barge yaw or surge have influence on the pipeline rotation too. So, it is necessary to compare:

I. The elastic strain energy of torsion
II. The plastic strain energy of Bending

In (Bynum Jr & Havik, 1981), the pipeline roll is calculated by equating the internal and external work done for the two positions. The resulting equations are solved for unknown "Roll angle".

I. Internal Work: is equal to the sum of torsional strain energy of pipe, tensile strain energy due to bottom force and, total strain due to bending or flexural energy (plastic and elastic).
II. External Work: is due to bottom force applied (which is multiplied by horizontal distance between two positions of pipeline) and the work due to gravity force at the COG of the sag-bend.

However, in (Bynum Jr & Havik, 1981) the calculations procedures were not reported.

3.1 A Simplified Analytical Approach

In (Endal, et al., 1995) & (Endal, et al., 2014) , a simplified analytical approach is used, based on energy minimization by considering the torsional and bending effects in pipeline only. This approach did not consider the effect of rotation due to minimization of gravity potential energy. The assumptions in simplified analytical approach are:

I. The pipeline rolls between the inflection and TDP. L represents the length of pipe from the inflection point to TDP.
II. The pipeline residual curvature is assumed to be formed on the stinger over bend region due to exceeding nominal strain. the residual curvature K_r can be found:

$$K_r = \frac{\varepsilon_r}{r} \qquad (eq3.1)$$

Where ε_r is residual strain and r is the outer radius of the pipe.

III. The total under bend curvature of the pipeline is the sum of nominal curvature and residual curvature. It can be represented by equation:

$$K_{tot}(x) = k_0(x) + K_r cos\theta(x) \qquad (eq3.2)$$

here $K_{tot}(x)$ is the total curvature of the pipe, $K_{nom}(x)$ is the nominal pipeline curvature and $\theta(x)$ is the roll angle.

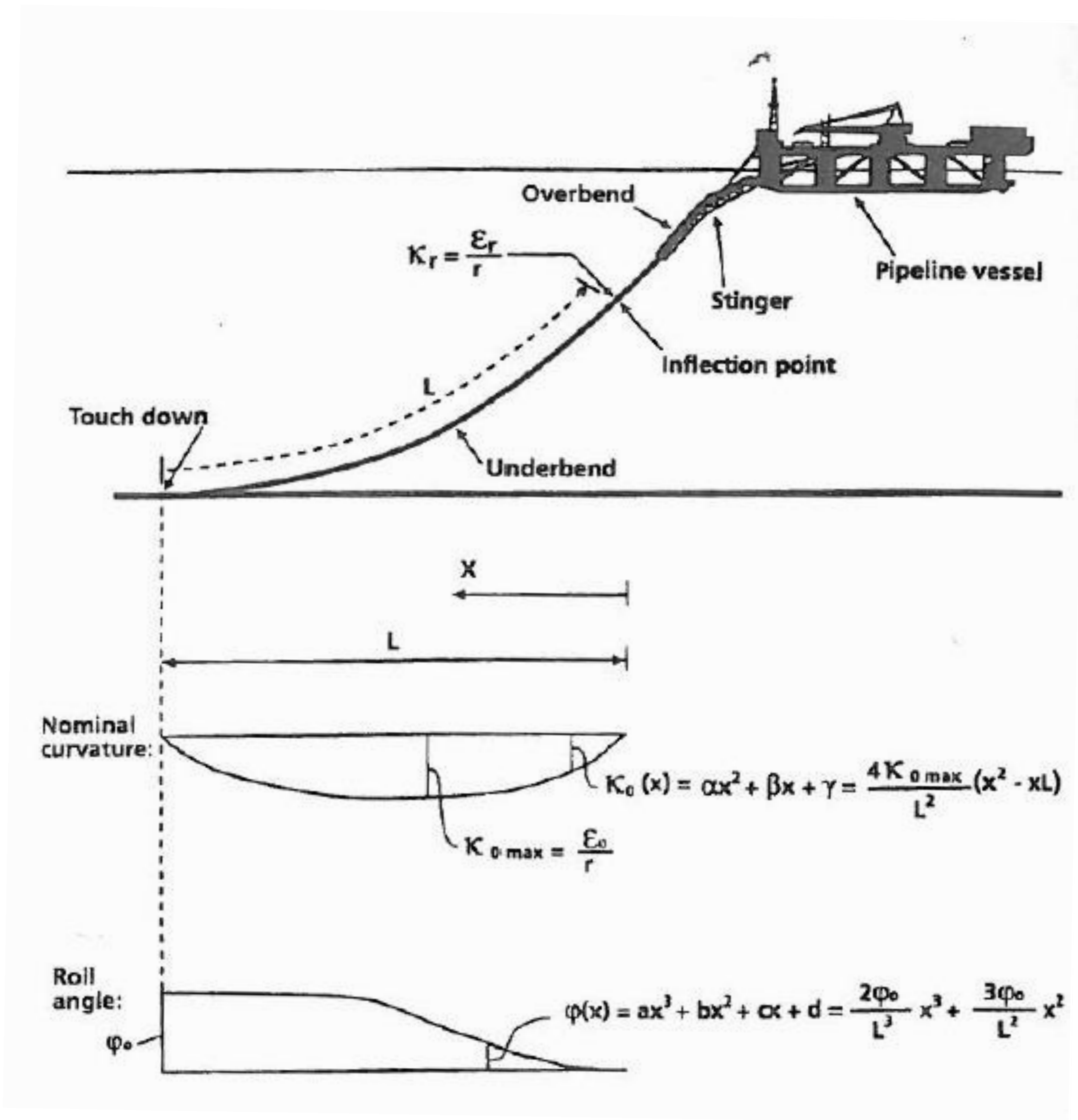


Figure 3-1 Simplified Energy Approach for S-lay installation, from (Endal, et al., 1995)

$k_o(x)$ is described as second order polynomial nominal curvature with zero value at at the inflection point and TDP as can be seen from the Figure 3-1.

$$k_o(x) = \alpha x^2 + \beta x + \gamma \qquad (eq3.3)$$

The coefficients can be solved for boundary conditions. For S-lay installation method we have boundary conditions i.e. $k_o(0) = k_o(L) = 0$, The maximum nominal curvature is $k_{o(max)} = \frac{\varepsilon_{nom(max)}}{r}$. So the above equations become:

$$k_o(x) = \frac{4k_{nom(max)}}{L^2}(x^2 - xL) \qquad (eq3.4)$$

For the Roll angle, a 3rd order polynomial is used to represent it along the pipeline in the underbend region (Endal, et al., 1995).

$$\phi(x) = ax^3 + bx^2 + cx + d \qquad (eq3.5)$$

The above equation coefficients can be found with the boundary condition $(\phi(0) = \phi'(0) = \phi''(x) = 0, \phi(L) = \phi_0,)$, so it becomes

$$\phi(x) = \frac{2\phi_0}{L^3}x^3 + \frac{3\phi_0}{L^2}x^2 \qquad (eq3.6)$$

Then the total work done in rotation of pipeline is combination of bending work and rolling work.

$$W_{tot}(\phi_0) = W_B(\phi_0) + W_R(\phi_0) \qquad (eq3.7)$$

Work done due to the bending can be calculated as:

$$W_B(\phi_0) = \int_0^L M_B(x) \cdot K_{tot}(x) \qquad (eq3.8)$$

Where $M_B(x)$is the bending moment, it can be written as:

$$M_B(x) = EI \cdot K_{tot}(x) \qquad (eq3.9)$$

Work done due to rolling can be calculated as:

$$W_R(\phi_0) = \int_0^L M_R(x) \cdot \phi(x) \qquad (eq3.10)$$

Where $M_R(x)$is the roll moment, it can be written as:

$$M_R(x) = GI_T \cdot \frac{d\phi}{dx} \qquad (eq3.11)$$

Substituting this into equation 3.7 the total work done:

$$W_{tot}(\phi_0) = GI_T \cdot \frac{6}{5} \cdot \phi_0^2 + EI \int_0^L [4k_{0\max}\left(\frac{x}{L} - \frac{x^2}{L^2}\right) + k_r cos\left(3\phi_0 \frac{x}{L} - 2\phi_0 \frac{x^3}{L^3}\right)]^2 \qquad (eq3.11)$$

By assuming ϕ_0 from 0 to 180^0 degree and solving for the total work done, the minimum value of work is giving the roll angle.
Note that we are only concerned with the rotation along x-axis i.e. these rotations do not cause any on-bottom instabilities and straight configurations along path are attained.

3.2 Modified Analytical Approach

The nominal curvature can also be found using catenary theory for pipelines (Endal, et al., 2014).This approach was used to predict for 12-inch ID pipe to calculate the rotation angle at TDP. This approach assumed that the pipe is free to rotate at TDP while fixed at laying vessel. However, in reality, pipeline is not free to rotate at the bottom and has some seabed friction. So this approach gives higher angle for the pipeline rotation at seabed. Figure 3.2 shows the pipeline spooling off from the reeling and the corresponding Nominal curvature and roll angle estimation is shown Figure 3-2.
The main points of the approach are following:

- The pipeline rotation is between the top vessels to the touch down point. Only the region of pipe from the sea surface to touch down point is to be considered and is represented by length L.
- The residual curvature of the pipeline due to under-straightening region is defined as K_{res},

$$K_{res} = \frac{\varepsilon_{res}}{r} \qquad (eq3.12)$$

Where ε_{res}residual strain and r is the outer pipeline radius.

- The pipeline total curvature can be found from the equation:

$$K_{tot}(s, \emptyset_0) = k(s) + K_{res}\cos\big(\emptyset(s, \emptyset_0)\big)\, if\ s \leq L_{curve} \qquad (eq3.13)$$
$$K_{tot}(s, \emptyset_0) = k(s)\ \ otherwise$$

Here $K_{tot}(s, \emptyset_0)$ is the total curvature of the pipe, $\emptyset(s, \emptyset_0)$ is the rotation angle, $L_{curve}$is the length of residual curvature section and k(s) is the nominal curvature of the pipe along the suspended section. At TDP, s=0 and s=L at the vessel top.

- If the pipeline nominal curvature is expressed completely according to catenary theory than the curvature will be maximum at seabed which is not true for pipelines. The pipelines have bending stiffness which makes them differ from catenary theory. So, curvature is taken zero at the sea surface (laying vessel) and seabed. It is assumed that pipe is simply supported at both ends. This deduce following expression for zero curvature at seabed (s=0) and sea surface(s=L) (Endal, et al., 2014):

$$K_{nom}(s) = \frac{A}{(A^2 + s^2)} - \frac{A}{(A+d)^2} \cdot e^{\left(\frac{s}{\gamma} - \frac{L}{\gamma}\right)} - \frac{A}{(A+s)^2} \cdot \frac{e^{\left(\frac{L}{\gamma} - \frac{s}{\gamma}\right)}}{e^{\left(\frac{L}{\gamma}\right)}} \qquad (eq3.14)$$

Here:
d is water depth

A=$\frac{Ht}{w}$; is horizontal Lay tension (Ht) divided by pipe submerged weight (w)

$\gamma = \frac{EI}{T} \cdot 0.5$, E= young Modulus, I= second moment of inertia, T=top tension (barge tension)

The calculations performed in this approach form the stiffened catenary calculation for pipeline laying are described by the (D.A & D.R., 1968).

- Roll angle along suspended section of the pipeline approximated by second degree polynomial as shown in figure 3.2:

$$\phi(s) = bs^2 + cs + d \qquad (eq3.15)$$

The boundary condition are ($\phi(L) = \phi'(0) = 0, \phi(0) = \phi_0,$), so it becomes

$$\phi(x) = \frac{-\phi_0}{L^2} s^2 + \phi_0 \qquad (eq3.16)$$

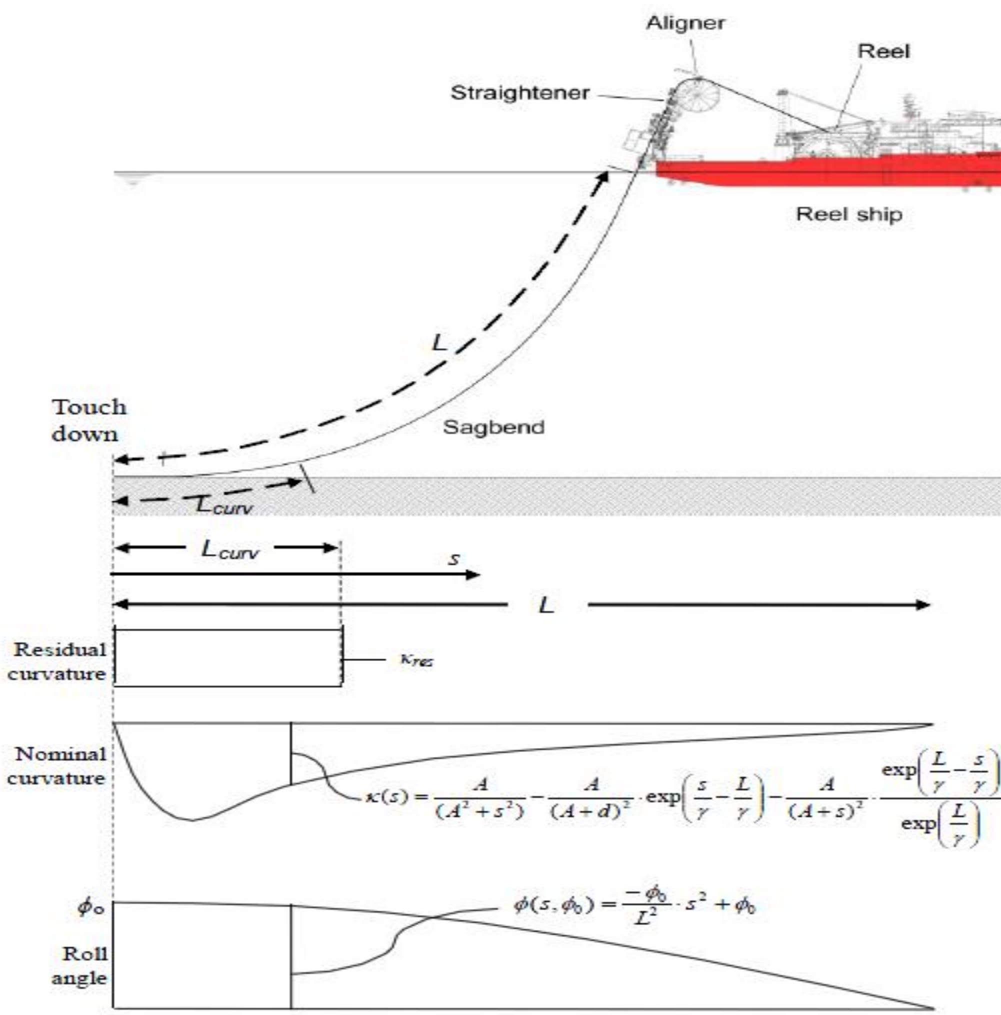


Figure 3-2 Modified analytical approach for pipeline roll prediction, from (Endal, et al., 2014)

The total work done from top vessel (sea surface) to TDP is composed of two components: due to roll and due to bending:

$$W_{tot}(\phi_0) = W_B(\phi_0) + W_R(\phi_0) \qquad (eq3.17)$$

The work done by the bending:

$$W_B(\phi_0) = \int_0^{Lcurve} EI \cdot [k(s) + k_{res} \cdot \cos\big(\phi(s,\phi_0)\big)]^2 \, ds \qquad (eq3.18)$$

$$+ \int_0^{Lcurve} EI \cdot k(s) \, ds$$

$$if \; L_{curv} < L$$

$$W_B(\phi_0) = \int_0^{Lcurve} EI \cdot [k(s) + k_{res} \cdot \cos(\phi(s,\phi_0))]^2 \, ds \qquad otherwise$$

Note that the k(s) and $\cos(\phi(s,\phi_0)$ are calculated from the equation 3.14 and equation 3.16 respectively.

The work due to roll of the pipeline is:

$$W_R(\phi_0) = \int_0^{L} M_{\emptyset}(s,\phi_0) \cdot \frac{d}{ds}\big(\phi(s,\phi_0)\big) \, ds = \int_0^{L} G \cdot I_T \cdot [\frac{d}{ds}(\phi(s,\phi_0))]^2 \, ds \qquad (eq3.19)$$

Where G is the shear modulus of pipeline, I_T is the polar moment of inertia for the pipeline. The total work done due to roll contribution and bending contribution can be get by putting equation

$$W_{tot}(\phi_0) = \int_0^{Lcurve} EI \cdot [k(s) + k_{res} \cdot \cos\big(\phi(s,\phi_0)\big)]^2 \, ds \qquad (eq3.20)$$

$$+ \int_0^{Lcurve} EI \cdot k(s) \, ds + \int_0^{L} G \cdot I_T \cdot [\frac{d}{ds}(\phi(s,\phi_0))]^2 \, ds$$

The angle ϕ_0 was assumed to vary from 0 degree to 180 and work done for each ϕ_0 is calculated. The graph would be plotted shows how work done changes with ϕ_0. The value of ϕ_0 at which work done is minimum, would be value of rotation angle at TDP.

The above expression for the work done was evaluated using, a Mathcad program varying the angle from 0 to 180 degree.

3.3 Initial Calculations for the experiment

The initial angle for the experiment has been made using the simplified energy method and modified energy method. The experiments were performed using the initial guess for the rotation angle form these two methods. The limitations for pipelines used in experimentations were:

- Pipe could have maximum go 0.35% residual strain as indicated from the uniaxial tensile test of the pipe using for the experiment, see Appendix C.
- Each pipe has length of 3meters, so the residual curvature length of the maximum 2m could be used to allow connection of pipes.

Taking into account these limitations, parametric study pipes were pre-bend according to residual curvature strain and residual curvature lengths. The Mathcad file for the calculations along with all the parameters is attached in the Appendix A and Appendix B. Predicted angle was calculated as initial guess for given pipe laying parameters for different residual strains and residual lengths. This enables the study of the influence of different parameters on rotation of the pipeline.
The effect of residual strain and residual lengths on the rotation has been analyzed independently on each other. Rotation Angles for different residual strain had been calculated for different residual lengths by using simplified energy method and modified energy method.

3.3.1 Simplified Energy Method

Following calculation of the angle at TDP is carried out using simplified energy method. The Mathcad file with all pipe laying parameters is attached in appendix A.
The total work done is calculated from equation 3.11. It is to be noted that simplified energy approach does not account for the length of residual curvature. Thus a rotation of angle for the residual strain is being calculated and presented in following graphs for residual strain of 0.15%, 0.20%, 0.26% and 0.30%. The rotation at TDP is the angle at which total work done, is minimum.
The results of the analysis are shown in Figures 3.3-3.6.

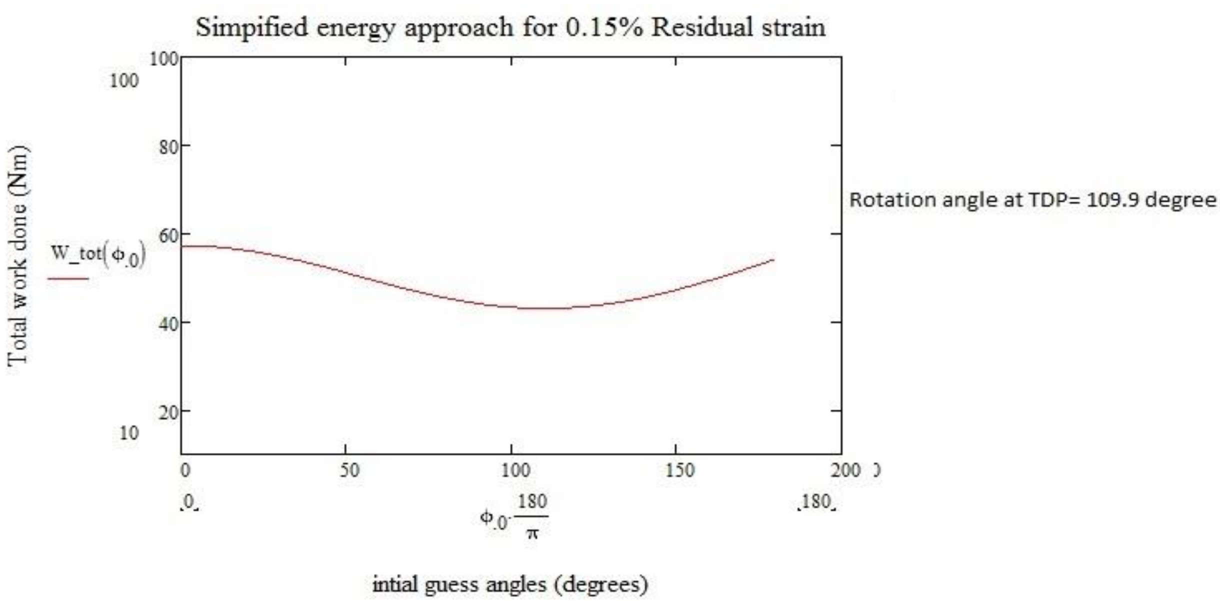


Figure 3-3 Angle prediction for 0.15% Residual curvature strain using simplified energy method

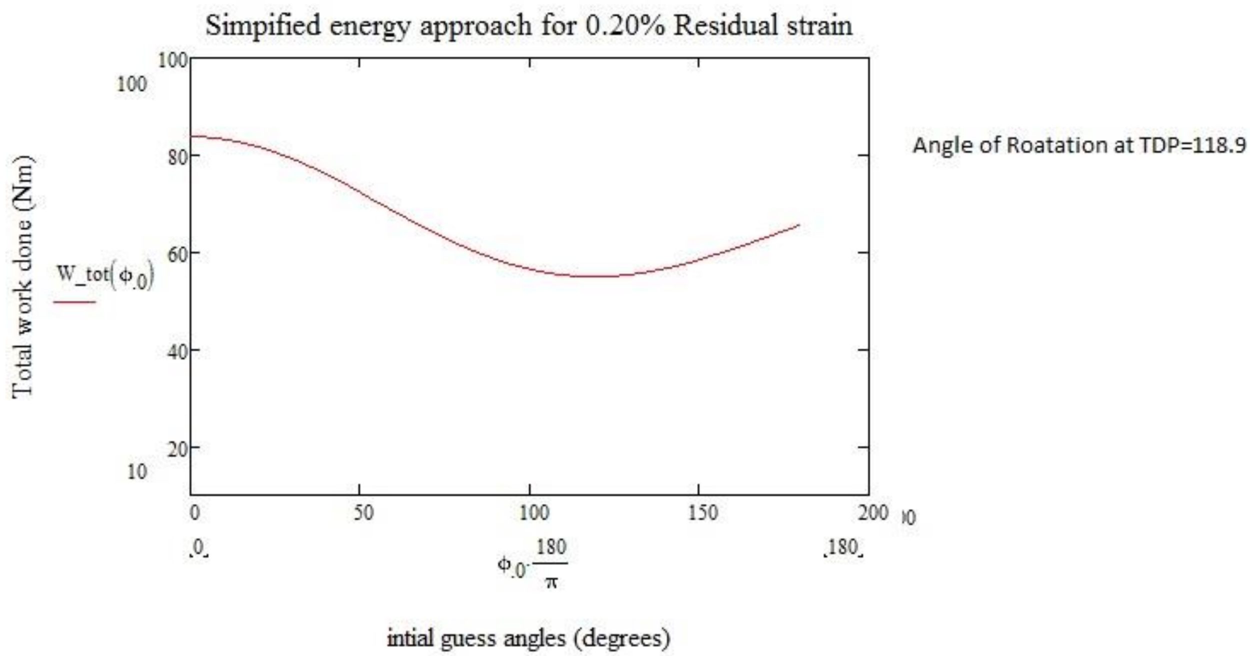


Figure 3-4 Angle prediction for 0.20% Residual curvature strain using simplified energy method

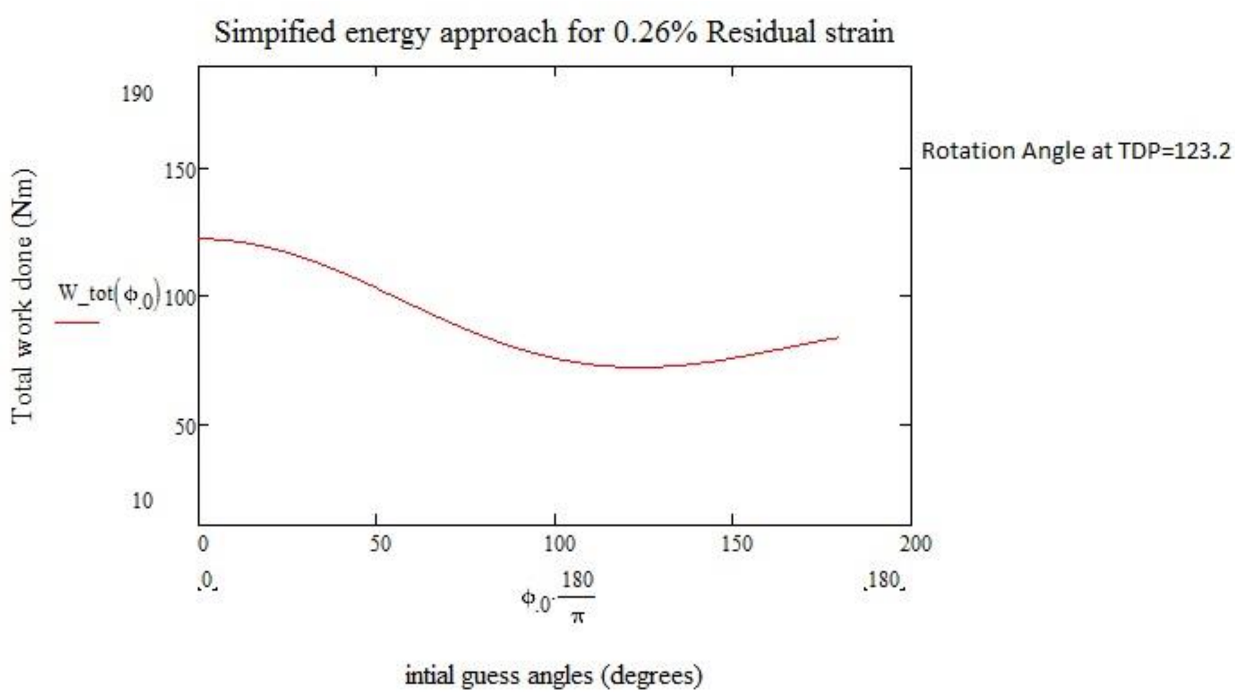


Figure 3-5 Angle prediction for 0.26% Residual curvature strain using simplified energy method

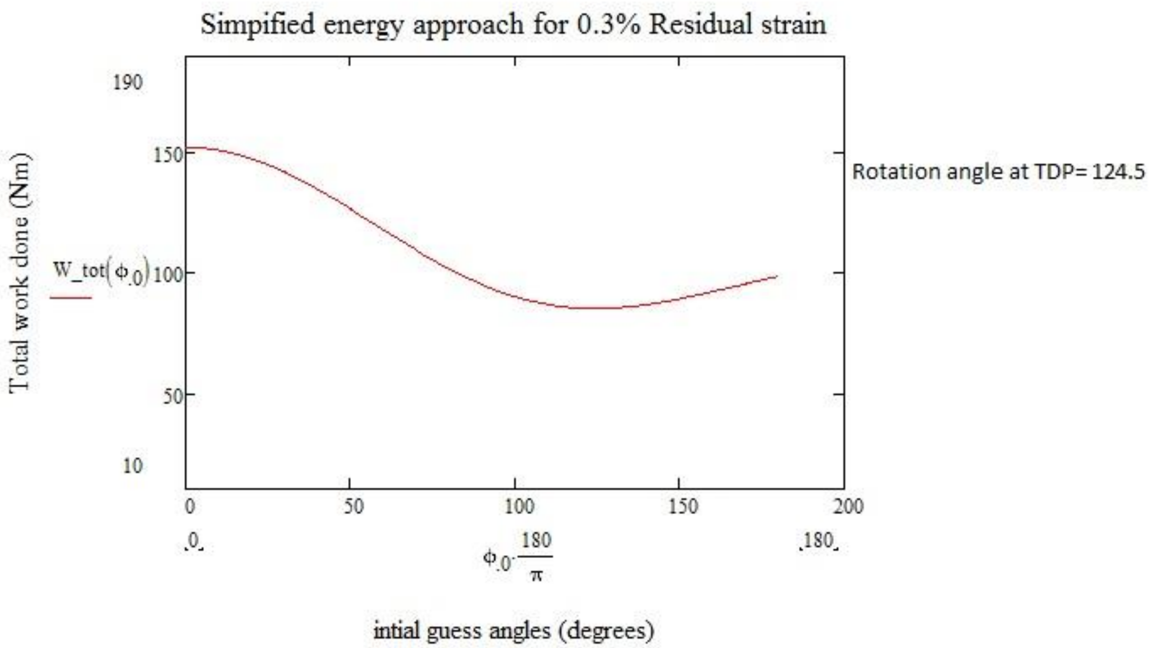
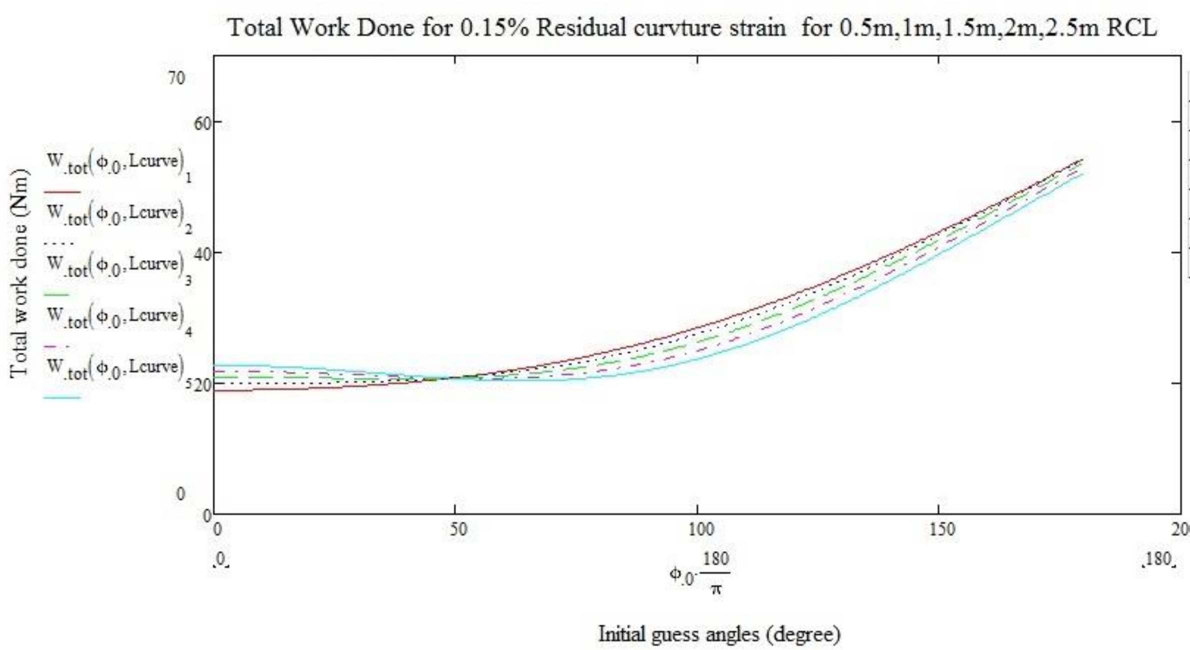


Figure 3-6 Angle prediction for 0.30% Residual curvature strain using simplified energy method

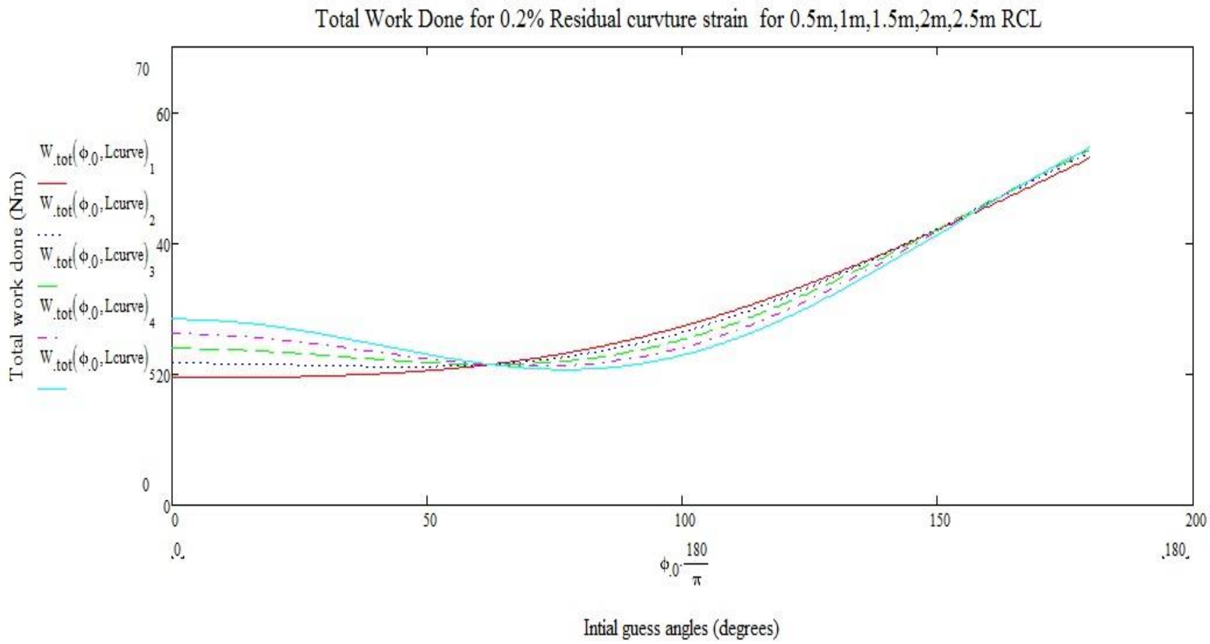
3.3.2 Modified Energy Approach

The rotation angle at TDP is calculated using equation 3.20 from the modified energy approach. This approach accounts for the residual curvature strain and residual curvature length. The residual curvature length (RCL) of 0.5m, 1m, 1.5m, 2m and 2.5m were used to calculate the total work done for the 0.15%, 0.2%, 0.25% and 0.3%.
The results of the analysis are shown in Figures 3.7-3.10.



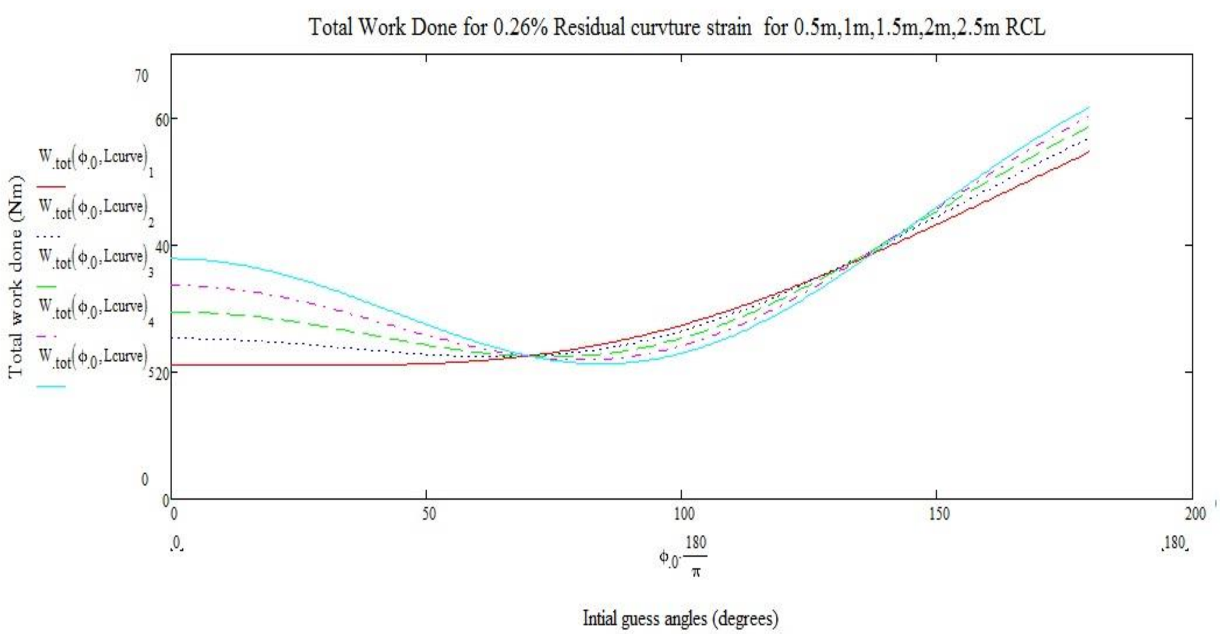
RCL	Rotation angle
m	degree
0.5	0
1	0
1.5	37.1
2	54.1
2.5	63.8

Figure 3-7 Angle prediction for 0.15% Residual curvature strain using modified energy method



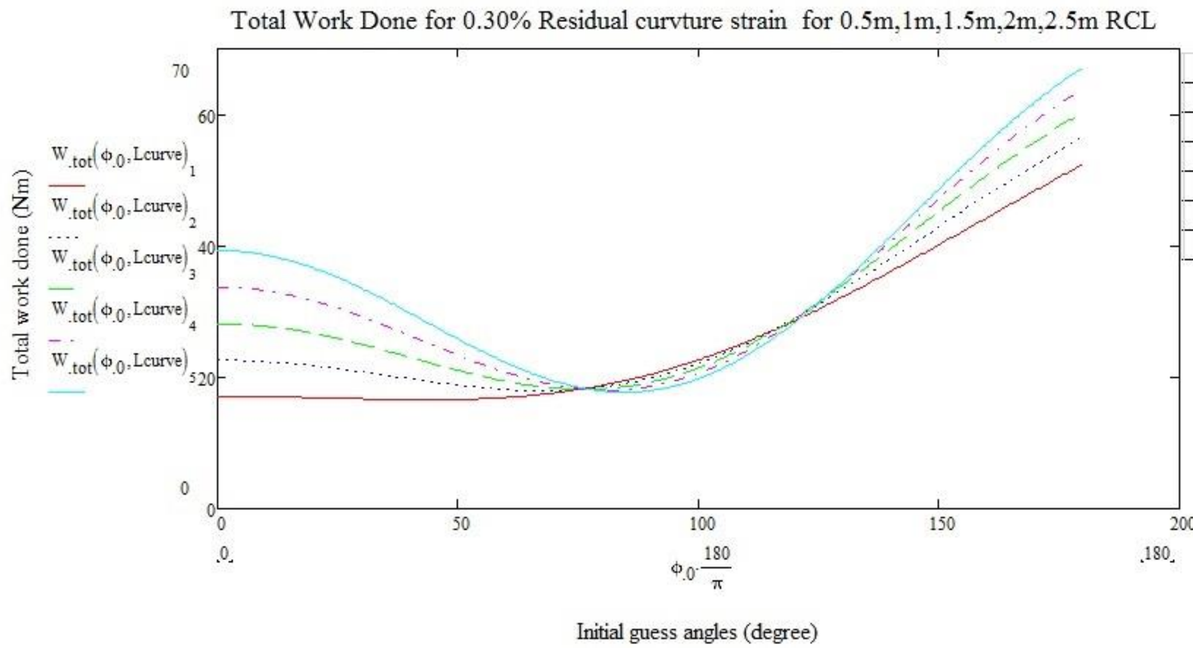
RCL	Rotation angle
m	degree
0.5	0.0
1	45.1
1.5	62.4
2	71.4
2.5	77.3

Figure 3-8 Angle prediction for 0.20% Residual curvature strain using modified energy method



RCL	Rotation angle
m	degree
0.5	31.0
1	63.3
1.5	74.0
2	80.1
2.5	84.3

Figure 3-9 Angle prediction for 0.26% Residual curvature strain using modified energy method



RCL	Rotation angle
m	degree
0.5	43.7
1	67.3
1.5	75.9
2	80.9
2.5	84.6

Figure 3-10 Angle prediction for 0.30% Residual curvature strain using modified energy method

The selected residual curvatures were:

- 0.15% of residual curvature strain with 2m of residual curvature length
- 0.26% of residual curvature strain with 2m of residual curvature length
- 0.30% of residual curvature strain with 0.84 m length.

3.4 Residual Curvature

When a stress is applied to any material, it deforms. If the stress is within elastic range i.e. below the material yield point, the material fully recovers its shape. These displacements or deformations are not due to breaking of chemical bonds between the atoms but just stretching of atoms. In elastic region, the material behavior can be described as according to Hooke's law:

The elastic modulus can be found from the slope of stress-strain as shown in Figure 3-11. It shows the nature of bonding between the material atoms.

When the stresses increases beyond the yield point in the material, the material undergoes Plastic deformations that is permanent and stays even the stress applied becomes zero. The plastic deformations are due to slip between the atoms planes. The residual strains correspond to residual stresses. The residual stresses are induced in many structures and components manufacturing process.

During installation pipeline undergoes bending beyond the yield point, so permanent curvatures are formed. These permanent curvatures are referred to as residual curvatures. The residual curvatures in pipeline can be formed due to uneven seabed, higher strains in sagbend and higher strains in overbend region on the stinger. The consequence is that the pipelines are bended beyond elastic limit to form residual curvature and today, residual curvatures are widely used (Statoil Patent) (Statoil, 2002).

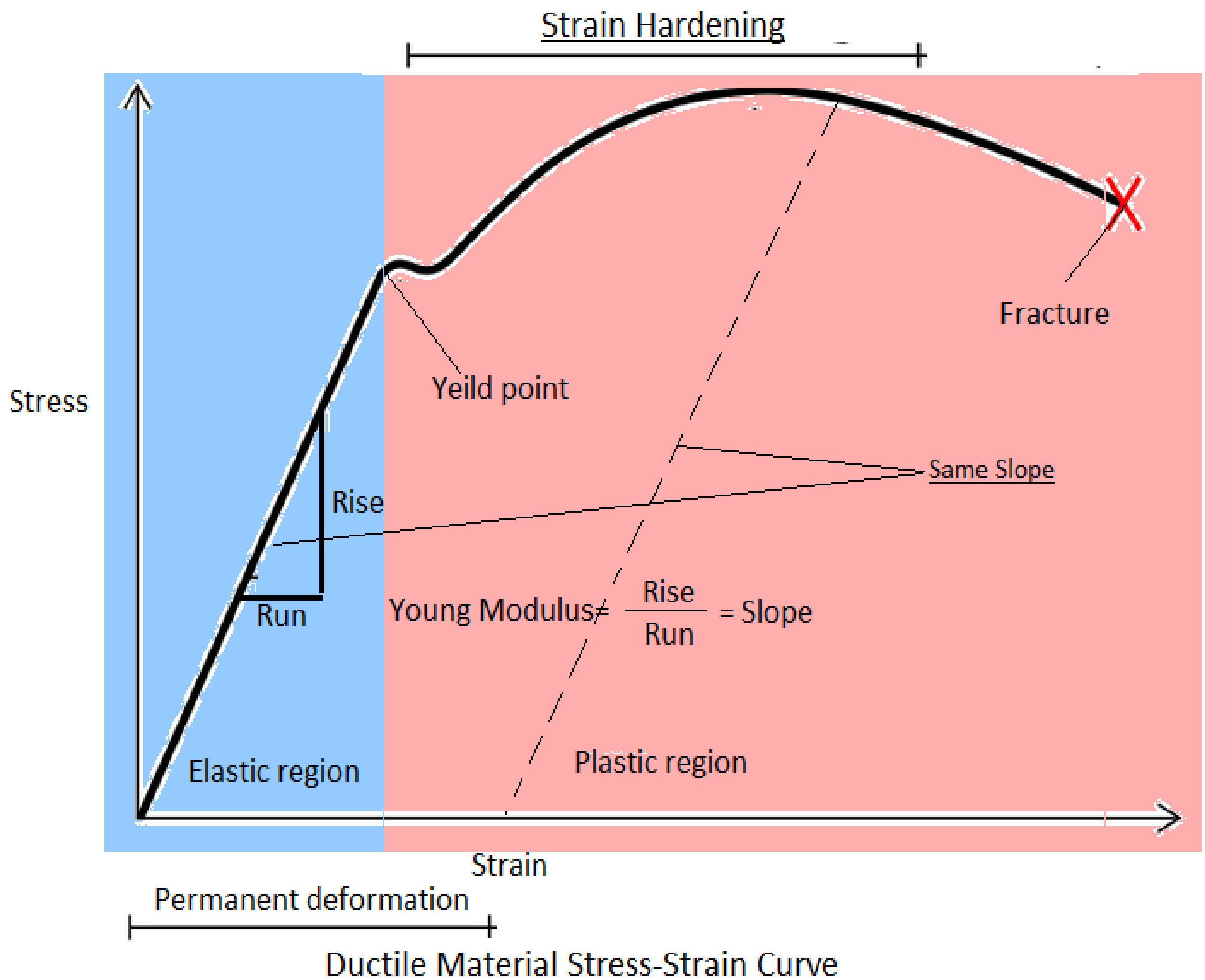


Figure 3-11 Ductile Material Stress-Strain Curve.

In the experimental set up of this work, residual curvatures are produced by inducing permanent deformation. The nominal static bending strain obtained from uniaxial tensile tests on the pipes was applied and corresponding residual strain. Note that the residual strain is a result of the elastic unloading (same slope as the stress-strain curve in elastic region).

4 Theory of Experimental Set Up

The experimental design set up is built upon the S-Lay installation method. The basic theory involves in building the S-lay installation is discussed in this section so that fundamental so experimental set up must be known.

4.1 Design of stinger:

The rotation of pipelines phenomena occurred in reel lay and s-lay installation of pipelines. Therefore, to stimulate the experiment a stinger need to design. A stinger is structure which use to lay pipes through an angle from the vessel. It is used in S-lay types of simulations.

4.1.1 Overbend Region

The region from the vessel deck to the tensioners, over the vessel ramp to the stinger to the lift-off point where the pipe is no longer supported by the any structure .The stinger radius should be large so that pipe does not go into plastic region. According to (Mousselli, 1981) the bending stress cannot be 85% of SMYS (specified minimum yield stress). The bending strain corresponding to bending stress can be calculated from:

From the Elastic Flexure Stress equation

$$\sigma = \frac{Mr}{I} \qquad (eq4.1)$$

Here σ is the maximum stress value, M is bending moment, r is the outer pipe radius and I is the second movement of interia.

The radius of curvature, R for beam for the linear elastic material.

$$R = \frac{M}{EI} \qquad (eq4.2)$$

Here E is the young modulus.

From stress strain relationship

$$\sigma = E\epsilon \qquad (eq4.3)$$

Combining equation 4.1, 4.2 and 4.3 and rearranging them, we get:

$$\varepsilon = \frac{r}{R} \qquad (eq4.4)$$

Where ε is bending strain, r is the radius of pipe and R is the radius of curvature.

The bend radius is also given by equation (Mousselli, 1981):

$$R = \frac{E \cdot D}{2\sigma \cdot DF} \qquad (eq4.4)$$

Where,

E=Elastics modulus of pipe

D is the diameter of pipe

σ is the specified yield stress,

DF=Design Factor (usually 0.85)

the above equations are required for the designing of the stinger. It can be seen that large radius is require for large pipelines to operate pipe within elastic limits.
According to DNV-OS-F101 Section H 300, in static loading the safety Criterion I for strain is according to Table 4-1. It includes the loading of Bending, axial and rollers. Also in case of combine loading i.e. static and dynamic, the safety criteria II from table 1 needs to be satisfy.

Table 4-1 Strain Safety criteria in overbend for different materials. (DNV, August 2012)

Simplified criteria, overbend				
Criterion	X70	X65	X60	X52
I	0.270%	0.250%	0.230%	0.205%
II	0.325%	0.305%	0.290%	0.260%

4.2 Stinger

The main problem in Pipelaying is the supporting the pipe during laying from the vessel. Thus this required a structure called as stinger. The stinger is large truss structure. The stinger extents deep into the water and have high environmental loads from the waves and currents. The stinger serves as a support for pipeline in the overbend region. It avoids uncontrolled bending and buckling of a pipe. Commonly used stingers are (Universitetsforlaget, 1974):

- Rigid stingers
- Articulated stingers

4.3 Rigid Stingers

Rigid stingers are of three types (Universitetsforlaget, 1974): rigid stinger firmed connection at the vessel stern, rigid stinger firmed connection at the vessel side, rigid stinger hinged connection and articulated stinger.

4.3.1 Rigid stinger firmed connection at the vessel stern

These types of stingers are constructed for avoiding the wave forces directly and hoist from the water easily when not in operation. Shown in Figure 4-1a.

4.3.2 Rigid stinger firmed connection at the vessel side

Forces on the stinger are small but during vessel sideways movement, there will be large forces on the stinger due to absent of horizontal support at the end of stinger. Thus this will cause large load on the front end connection point of the stinger. Also the pitch and heave motion of the vessel have load effect on the stinger. Shown in Figure 4-1b.

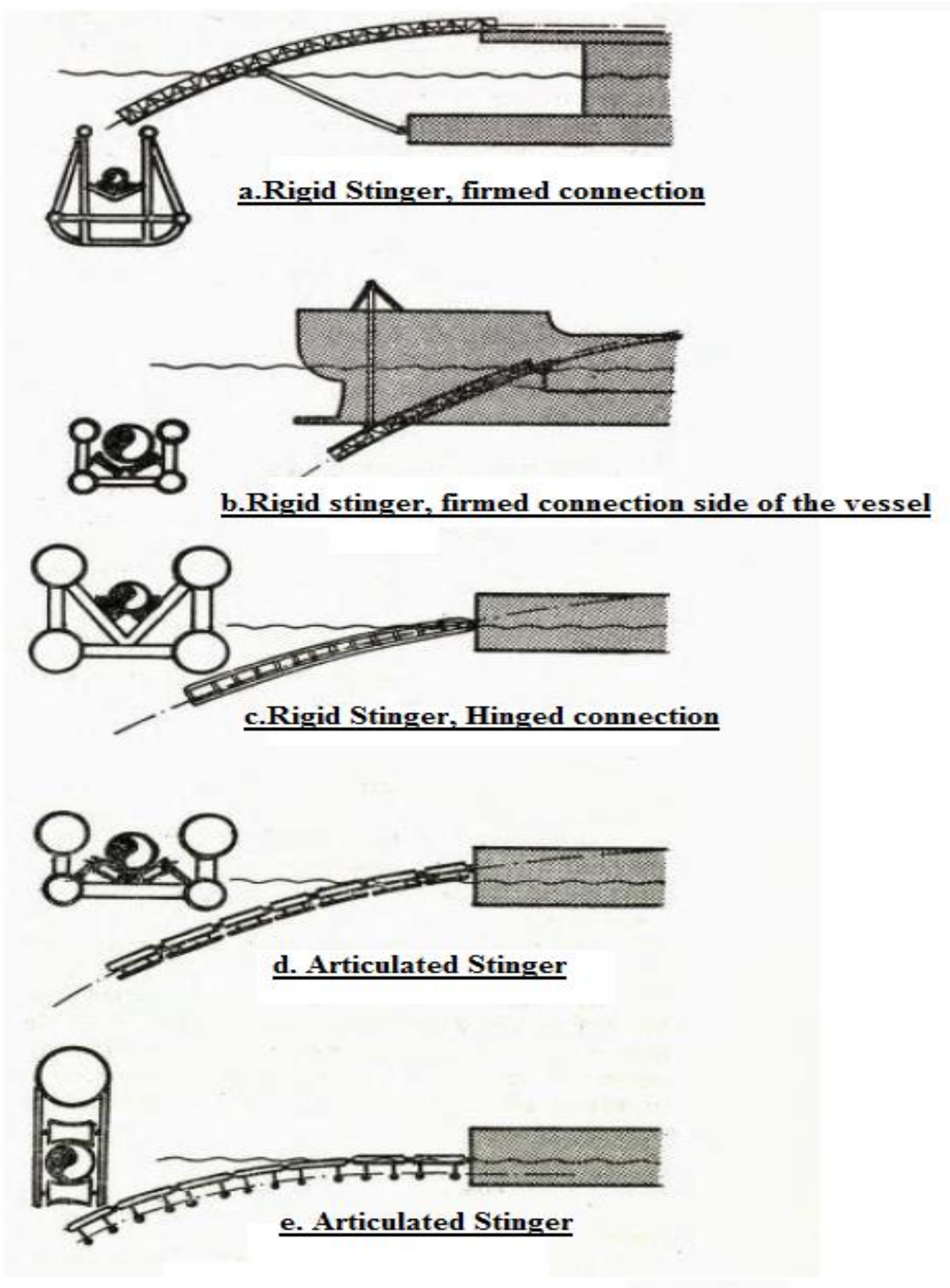


Fig. 4-1(a-e) Stinger different types. Translated picture from (Universitetsforlaget, 1974).

4.3.3 Rigid stinger hinged connection

The rigid stinger consists of several elements connected in a long and arced shape. The connection point between the vessel and stinger is through hinged. This makes the relatively high stinger movement thus making more bending of pipes during laying. A safety check is required for pipe bending within allowable limits. Shown in Figure 4-1c.

4.3.4 Articulated Stinger

The articulated stinger elements which are connected in series give restricted degree of freedom in vertical, horizontal and torsional flexibility. They are designed to increase the water depth and weather capabilities due to vertical and lateral degree of freedom respectively. Shown in Figure 4-1d and 4-1e.

4.4 Selection of stinger

Important factors in choosing and designing the stinger are:

I. Stinger length
II. Wave conditions
III. Loadings on pipeline (static and dynamics)
IV. Control capabilities

For the experimental test rig, rigid stinger is used to build. It is because the rigid stinger is easy to manufacture in work shop.

4.5 Sagbend

It is from the inflection point to the touch down point (TDP). According to (DNV, August 2012) in section 13, G 300, for combine static and dynamic loadings, the equivalent stress in sagbend and at stinger tip should fallow the equation:

$$\sigma_{eq} < 0.87 * f_y$$

Here σ_{eq} =Equivalent stress (Von Mises), f_y =yield stress of the material.
While all load effect, factors are equal to one.
The yield stress of material f_y can be found from (DNV, August 2012) in section 5, C 300 given as:

$$f_y = \left(\text{SMYS} - f_{y,temp}\right) * \alpha_U$$

Where,
$f_{y,temp}$ are the de-rating values for temperature compensation. It is different for different materials. One example is shown in (DNV, August 2012) in section 5, C 304, Figure 2 for C-MN and 22Cr-25Cr.
α_U is the strength factor for materials. It can be found from below table 4-2.

Table 4-2 Material strength Factor, from Table 5-4, section 5, C306, (DNV, August 2012)

Material Strength Factor		
Factor	Normally	Supplementary requirement U
α_U	0.96	1

4.6 Forces During installation

The tension forces are applied from the vessel to control the bending of pipe in sagbend. The pipe has its own weight, w which also act as a load. The applied tension force determines the length of the free span pipe from vessel to touch down point (TDP). The tension force does not have much influence on the overbend region of the pipe. The simplest way for the calculation of the tension and sagbend is by using the natural catenary model, shown in

Figure 4-2.. The horizontal component T_h of the applied axial tension remains the same throughout the pipe length while vertical component T_v decrease from the top to the bottom due to submerged weight of pipe (Bai & Bai, 2005).

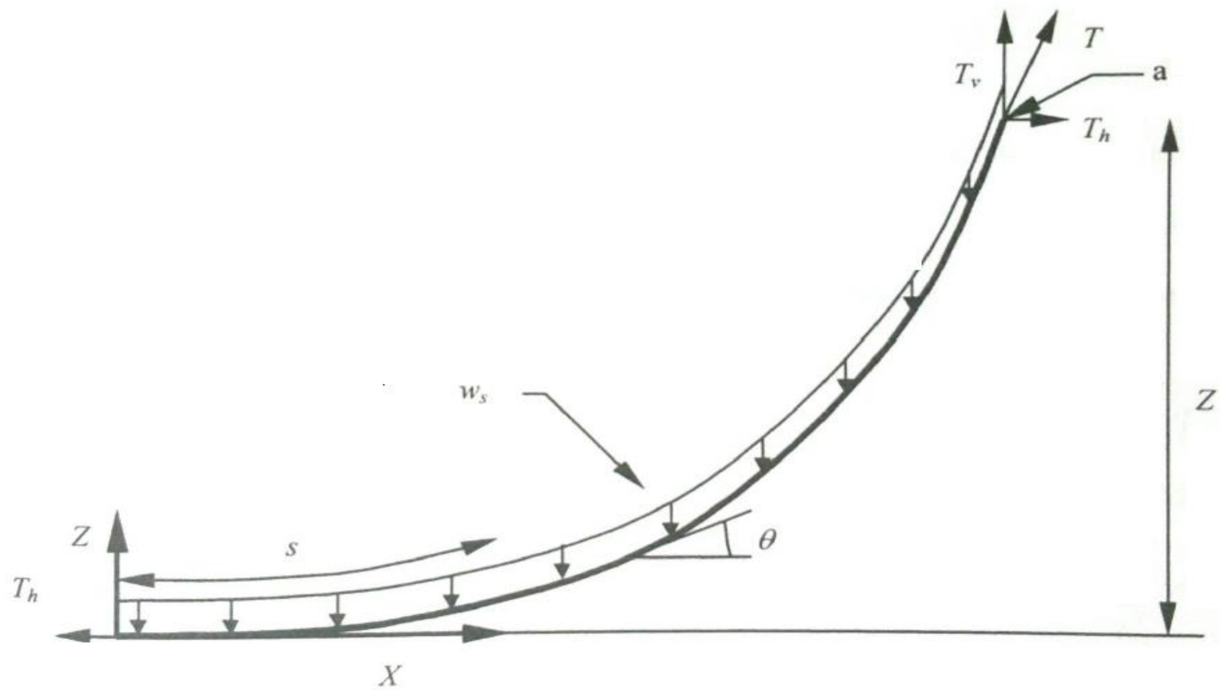


Figure 4-2 Forces in installation on pipe by assuming simple catenary model, (Bai & Bai, 2005)

The shape of catenary can be expressed as

$$z = \frac{T_h}{w_s}\left(\cosh\left(\frac{x \cdot w_s}{T_h}\right) - 1\right)$$

Where z is the height above seabed, w_s is submerge weight of the pipe, T_h is the horizontal tension.

The highest curvature, R is at the TDP and can be calculated:

$$\frac{1}{R} = \frac{w_s}{T_h}$$

The strain ε at the bottom can be found:

$$\varepsilon = \frac{r}{R}$$

Where r is the radius of pipe.

The vertical component of tension T_v can be expressed as:

$$T_v = w_s \cdot s$$

Where s is the length of suspended part of pipe.

$$s = z\sqrt{1 + 2\frac{T_h}{zw_s}}$$

The angle θ is the angle between the pipeline and x-y plane

$$tan\ \theta = \frac{T_v}{T_h}$$

Then horizontal tension force can be expressed as

$$T_h = \frac{z \cdot w_s}{tan^2\,\theta}(1 + \sqrt{1 + tan^2\theta}\,)$$

The axial tension at any point s along the pipe (catenary) can be found by:

$$T = \sqrt{{T_h}^2 + {T_v}^2}$$

The natural catenary method has some advantages and disadvantages and disadvantages.
Its main advantages are:

- Simple formulas
- Easy starting point
- Results are acceptable far away from the ends

While disadvantages are:

- Wrong results at the pipe ends
- Dynamic loads are ignored
- Ignores pipe bending stiffness

4.7 Models for pipe behavior in free span

There are four most prominent methods for defining the behavior of pipe in free span region between the top vessel and TDP.
These Methods are:

I. Beam method
II. Non-linear Beam method
III. Natural catenary method
IV. Stiffened catenary method
V. Finite element method

The applications, boundary conditions and validity of each method is given in table 4-3. The section 3.2 described in the thesis is based upon the Stiffened catenary method for the pipelines.

Table 4-3 Pipe stress analysis methods Comparison. From (Mousselli, 1981)

Methods	Applications	Boundary Conditions	Validity
Beam method	Shallow waters	Satisfied	Small deflections
Non-linear Beam method	All depths	Satisfied	General
Natural catenary method	Deep water	Not	Away from ends, small stiffness
Stiffened catenary method	Deep water	Satisfied	Small stiffness
Finite element method	All depths	Satisfied	General

5 Experimentation Designing

The experiment were carried out at scale down model of real life installation vessel. It is challenging and difficult task. The installation of pipelines is itself complicated phenomena therefore it requires many parameters to control. In the designing of experiment equipment and defining parameters, an assessment was done to make sure that an experiment covers all the important and controlling factors in laying of real life installation vessel.
A large amount of time was spent upon designing and manufacturing of experiment equipment. The important parameters in designing of experiment was:

1. Pipe selection and testing
2. Preparation of the Pool
3. Design of stinger
4. Rollers placements on stinger
5. Strain gauges
6. Load cell
7. Applying tension force
8. Pre-bending or Residual curvatures of pipes
9. Inline structure
10. Control of water depth
11. Torque measurement
12. Digitalization of pictures

5.1 Pipe selection and testing

5.1.1 Selection of Pipes

For execution of small scale model, a proper pipe selection was required. After a lot of market survey, it was found that 10 millimeters copper pipes are smallest radius pipes available in Norway. The steel pipes are very much stiff and not suitable for dry model test of pipes. Steel pipes requires large height to model the practical pipeline installation while copper are lesser stiffer at lower height. Thus the laboratory space is limited so copper pipes were considered. Annealed straight copper pipes with wall thickness of 0.8 mm and length of 3 meters were used in experiments. The pipes are manufactured according to European standard of EN1057. The material and geometrical properties of the pipe is given in Table 5-1.

5.1.2 Testing of Pipes

University of Stavanger has the facility for the uniaxial tensile testing machine. For the uniaxial testing, the guidelines of IS0 2009 Standard should be followed for accurate results, so testing is performed according to (ISO, 2009) Mechanical testing of metals, Annex E “Types of test pieces to be used for tubes”, shown in Figure 5-1 and Figure 5-2. The (ISO, 2009) specify the shape and dimensions of the test piece. Since the pipes have wall thickness less than 4mm so test piece of length 300 mm were taken (Annex C, ISO 2009). The stress-strain curve obtained from the test is shown in Figure 5-3. The laboratory test report for pipe is attached in Appendix C.

Figure 5-1 three Test samples from the pipes, placed in a tensile testing machine

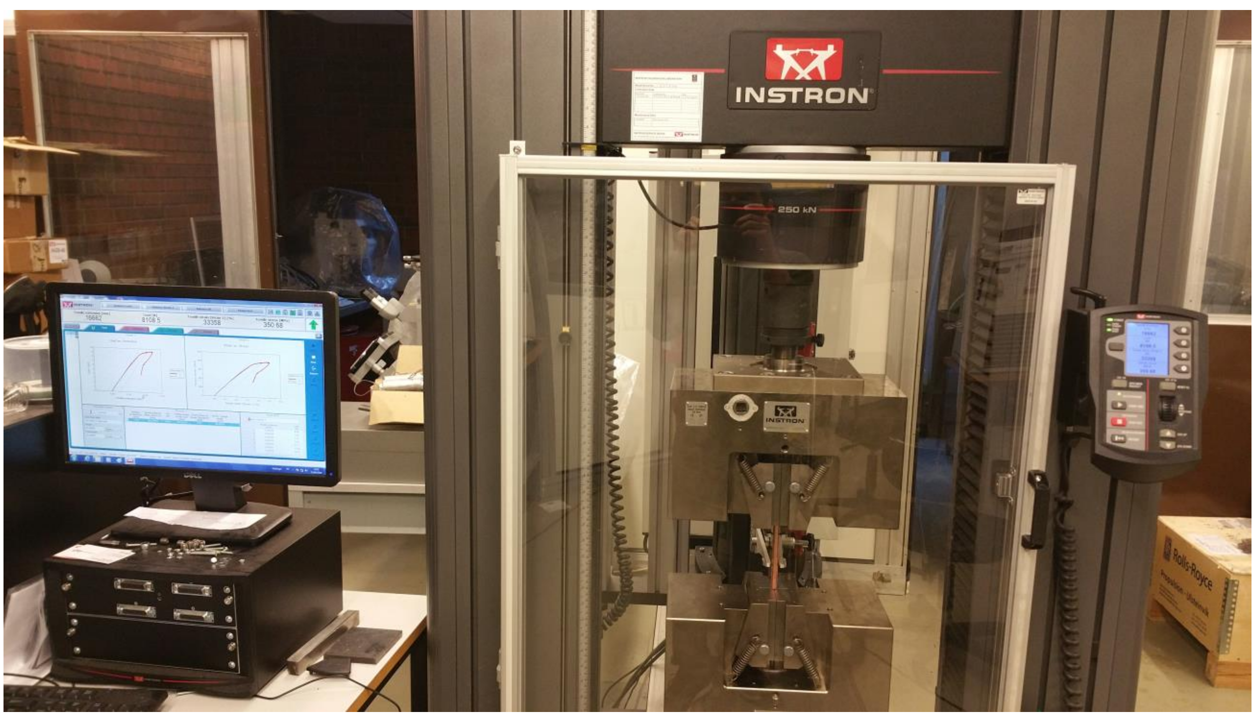


Figure 5-2 Uniaxial Tensile testing in progress

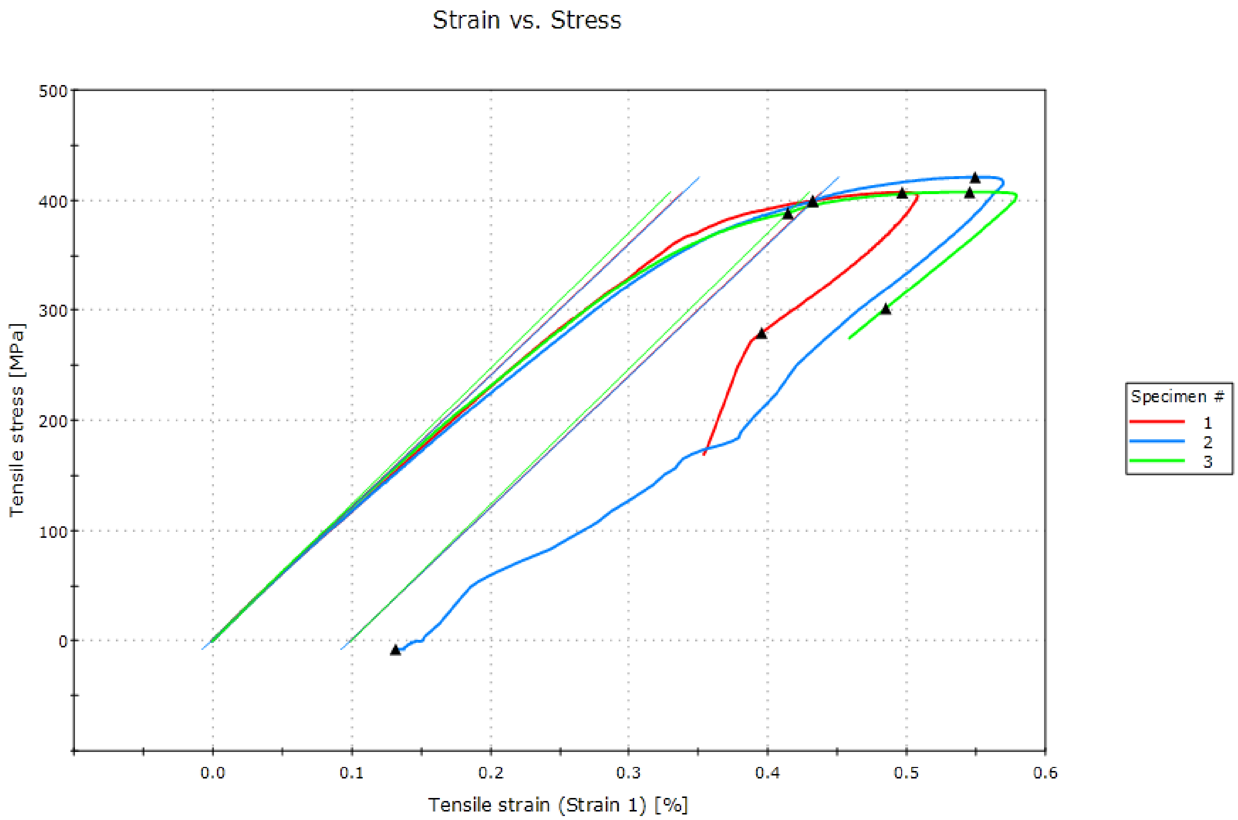


Figure 5-3 Stress-Strain curve for the test sample of pipe

Table 5-1 Experiment Pipe properties.

Pipe used in experiment Properties			
Outer diameter*	10 mm	Length of each pipe*	3m
Wall thickness*	0.8 mm	Poisson ratio***	0.33
Young Modulus**	120.97 x10^9Pa	Weight per unit length*	0.1901Kg/m or 1.863N/m
Yield Strength**	395.56 x10^6Pa	Density***	8.94kg.dm^3
*Measured values		**Uniaxial tensile testing values	
***EN1057 (R290) Standard values			

5.1.3 Connection of Pipes

The pipes are made of copper, which has low melting point, so to make connections of pipes soldering is required. For rigid connections of pipes, small pieces of 8mm copper pipes are used at joining point as shown in Figure 5-4. It has been observed that without using inner smaller diameter pipe near connection of pipes, the pipes connections were not stiff and durable. The soldering was done by using propane gas burner and filler metal (solder). The two supports structure were made for alignment of the pipes during soldering. During the experiment tests, the pipes were disconnected to add new pipes, as shown in Figure 5-5. The pipes were also painted for more clear visualization.

Figure 5-4 Soldering of pipes, Small diameter pipe is used inside at connections

Figure 5-5 Heating of pipe joint for disconnecting the pipe

5.2 Preparation of the Pool

The experiment needs to be carried at the height of minimum of 4 meters. For this purpose, the pool at the material department is used for the experimentation. The limitations of pool space were considered. The experiment was carried out in dry test so empty pool availability was big task due to other experimentations in the pool. Due to limitations of space, the working area for the experiment was 3m depth, 2 m width and 12 m length. The catenary shape of pipe was required to determine for the nominal curvature of the pipeline for different applied load and water depth. Following work were performed during pool preparations

- Sketching the experiment layout according to pool dimensions
- Emptying and Cleaning the pool
- Painting the pool wall with white color
- Building of the grid with size of 15cm x 15cm

The white color gives good visibility. The size of the grid helps to know the position of the pipeline from the choose origin. The wall before and after dimensioning is shown in Figure 5-6.

Figure 5-6 Pool wall preparation. Before and After preparation for experiment tests.

5.3 Design of stinger

The experiment was done by using S-lay installation method. The stinger structure should keep the pipe within elastic limits. The data results obtained from the tensile testing of the three samples pipe indicates that the pipe start deviation from hook`s law after 0.13% of strain.

From the uniaxial test, it was observed from data that the pipe starts to deviate from the proportionality line when the nominal strain reaches the value of 0.13%. The stinger radius calculated as:

From equation 4.4

$$\varepsilon = \frac{r}{R}$$

Here r=0.005m,$\varepsilon = 0.13\%$, R is the radius of stinger.

$$R = \frac{r}{\varepsilon} = \frac{0.005}{0.0013} = 3.85m$$

To be on safe side a stinger of radius 4m is made, which gives the 0.125% of strain. The length of stinger was chosen 3.84 m which gives the maximum departure angle of 55 degrees.

The manufacturing of stinger is difficult task. A rigid stinger type is used for this experiment. First the radius of 4m were marked on card board and the card board were cut through the radius, shown in Figure 5-7.

Figure 5-7 Stinger marking of radius of card board

Three hard wooden plates are used to make stinger by cuttings them in round shape of radius 4m, shown in Figure5-8 and Figure 5-9.

Figure 5-8 Wooden board marking according to radius of stinger

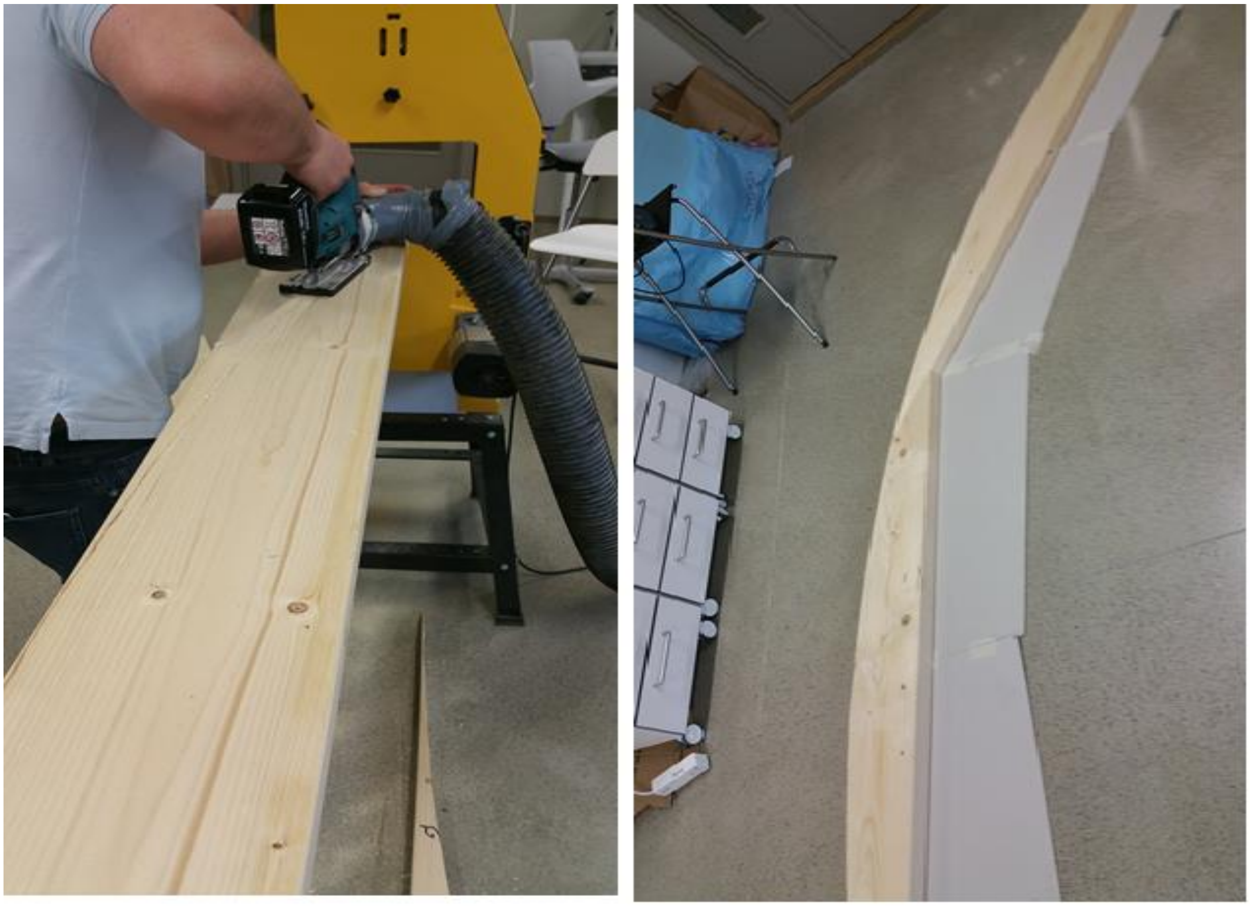


Figure 5-9 Cutting of wooden board.

To execute the experiment of pipe laying, there were two options:

1. Continuous welding/soldering of pipes as pipe moves on stinger
2. The stinger structure connected to long straight section to accommodate the long pipes

In real life scenario, the first options is used as hundreds of kilometers of pipes are laid and one complete welding of pipe section takes 8-15mints. While for experimental setup, the option 2 was choose. A section of 18m pipeline was made by soldering the six pipes. To support the 18m pipeline section, the stinger was connected to long sections of two straight wooden board structure as shown in Figure 5-10.

Figure 5-10 stinger connected to long straight wooden boards.

5.3.1 Rollers placements on stinger

The stinger is made of wood material, which has high friction. The stinger in real life has adjustable rollers for smooth laying of pipelines. The rollers of 20 mm length are place on the stinger for the smooth laying of pipeline. This also helps in knowing the correct horizontal tension and vessel tension effect on the pipeline. The placement of rollers on the stinger is shown in Figure 5-11.

Figure 5-11 Rollers placement on the stinger

5.4 Measurements

The strain gauges and load cell were used for measurements. The measurements were recorded using Spider 8 Panel board and computer.

5.4.1 Strain gauges

The strain gauges used to check the residual strains remained after the bending of pipe. The strain gauges that used in the thesis were HBM K-LY41-3/120. These types of stain gauges measure the strains in one direction either on x or y direction. The strain gauge placed at the center of pipe during bending to measure the maximum strain.

The strain gauges used two strain gauges one active and dummy. The dummy strain gauge used to compensate the temperature changes. Each strain gauge has three wires. The wires from active and dummy strain gauges were connected to 15-pin port through soldering as shown in Figure 5-12. Spider 8 panel was used to measure the strain gauges' output. Catman Easy 4.1.2 software used to calibrate, read and display the results. The results are displayed in µm/m. The strain gauges scaled according to gauge factor. The gauge factor of 2.00 used as specified by the manufacturing lot of strain gauges.

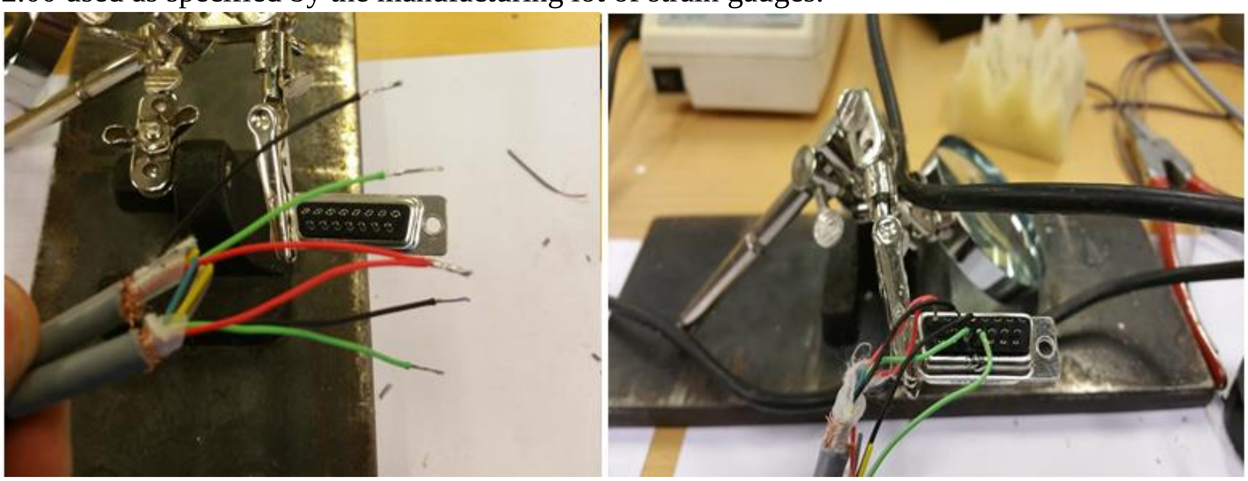


Figure 5-12 Soldering of active and dummy strain gauges ton 15-pin port

Figure 5-13 Placement of strain gauge. 1 shows the chemical used for strain gauges. 2 Roughing of pipe surface with sand paper. 3 cleaning with acetone and fine cotton paper. 4 placing of strain gauge

The placement of the strain on the pipe requires great attention and care. According to (HBM, 2016), if the strain gauge mounted five-degree angle away from the maximum strain point, then the measure strain will be 1% less than actual strain. The glue used to stick the strain gauge is highly adhesive and can be dangerous if sticks to hand. First, the pipe was cleaned using the acetone liquid and fine cotton paper. Then the surface of pipe was roughening by the fine sand paper. The pipe was cleaned again to remove any metallic dust. Then the strain gauge was place on the pipe through sticking tape. Finally, one drop of sticking glue was placed on the pipe to stick the strain gauge on the pipe and held for two minutes. The procedure is shown in Figure 5-13.

5.4.2 Load cell

The load cell was used to measure the horizontal tension and spring stiffness. The load cell used from the HBM instrument with the name of RSCA 50, shown in Figure 5-14. The load cell has the capacity to measure the load up to 50 kg (490N). It can measure the tension and compression. The load cell receives the signals in the form of V and convert it into the kilograms. The CatmanEasy V4.1.2 software converts the signals, shown in Figure 5-16. The load cell is connected to pc via 8 channel Sider 8 panel board.

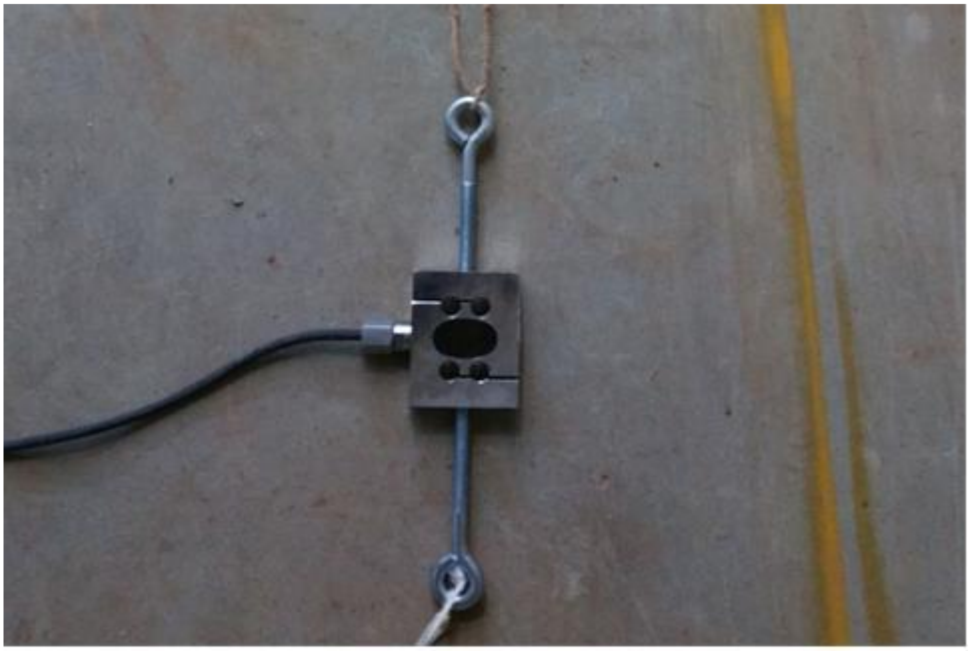


Figure 5-14 Load cell RSCA 50

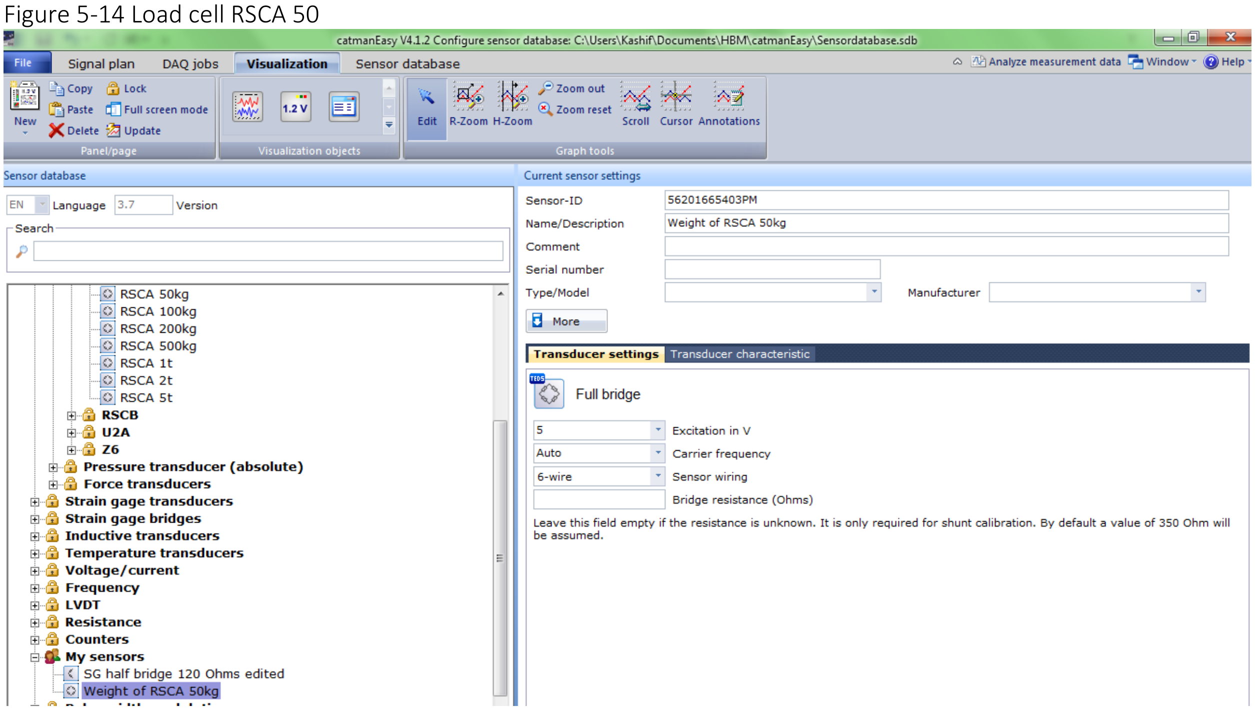


Figure 5-15 Load cell configuration in Catman Easy 4.1.2 software

5.5 Applying tension force

The tension force was applied at the end of pipe by using a sting. The sting runs over the three pulleys. Due to having limited space, a special type of arrangement was made to ensure the accurate measurement of tension force applied to the pipe. Three pulleys were used to apply tension naming P1, P2 and P3. The schematic of tension mechanism is shown in the Figure 5-16. The pulleys and load used in experiments are shown in Figure 5-17 and Figure 5-18.

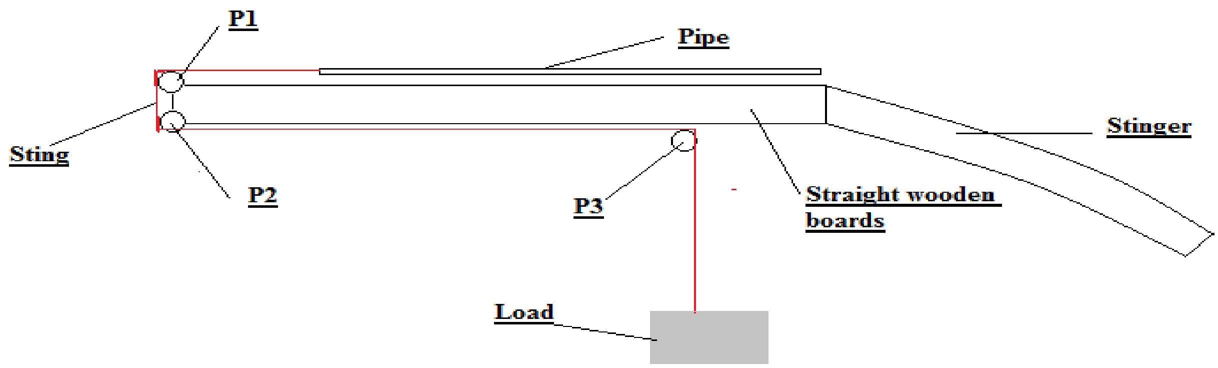


Figure 5-16 Tension mechanism schematic

This arrangement works well for all the experimental tests.

Figure 5-17 Pulley P1 and P2 applying tension through sting at the end of pipe

After the soldering of pipe on the pool floor to make long section of 18m, the pipeline need to be lifted upwards on the stinger. This was very sensitive operation as the pipe can go into the plastic deformations. It was done in a way that tension force applied the end of the pipe and, pipe was pulled upward from the stinger. Due to rollers on the stingers, the pipe experience little friction.

During the simulation of pipe laying with residual curvature the tension force was constant. Before starting the simulation, the load (shown in Figure 5-18) was touching the pool floor, while during simulation of experiment the load keep moving upwards. At the end of experiment as the residual curvature section touch the pool floor, the load was near the pulley P3. Calculations were performed so that the sting should be long enough to ensure that the constant tension must be applied during the pipe laying simulation.

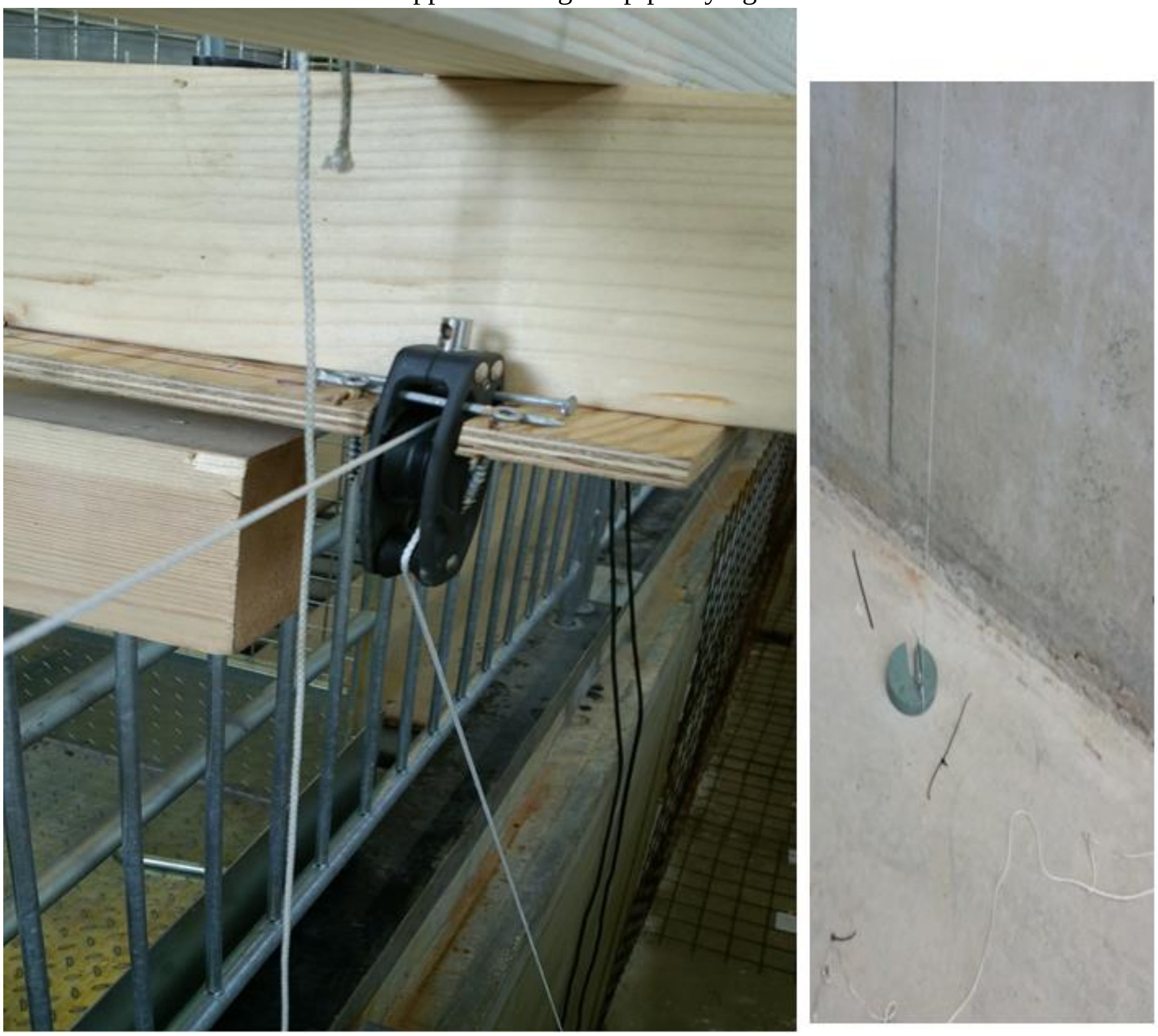


Figure 5-18 tension pulley P3 connected to the load

5.6 Pre-bending or Residual curvatures of pipes

One of the main theme of this thesis study is to determine the rotation of pipelines with different residual curvatures of pipelines. It has been observed that residual curvature and inline structures are one of the main reasons of the pipeline rotation. During simulation of pipelines in the pool it has been observed that the straight pipes does not rotates.

It has been observed that in real life scenario the pipes have residual curvature in S-lay due to stinger and loads configurations while in Reeling it is bend using 3-point bending (Statoil, 2002). To run the experimental simulation with residual curvature, a pre-bending of the pipe has to be done for different residual strains. The tensile testing stress strain curves indicates about how much of nominal strains can induce how much of plastic strains. For example, nominal strains up to 0.2% will not induce any plastic strain in the pipe. As explained in section 3.4, a plastic or residual strain of 0.2% can be achieved by drawing a line with slope equal to young modulus from 0.2 % strain, where this line intersects the stress-strain curve, that is the amount of nominal strains need to be applied. For example, for 0.26% residual strain the 0.55% of nominal strain is required.

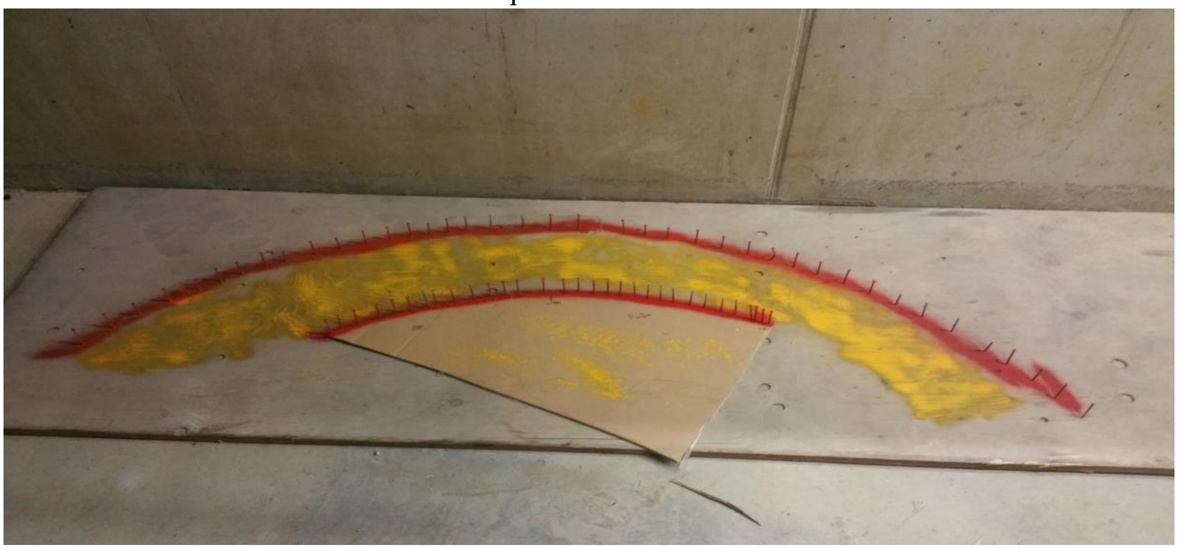


Figure 5-19 Pipe bending set up used to create residual curvature sections

The bending of pipe was difficult job as the pipe is made of copper, which is ductile material and has small diameter of 10mm. In market the bending machine for 10mm copper pipes is not available. Thus to bend the pipe at different radii and length, a setup is built to bend the pipes, shown in figure 5-19. The procedure of bending a pipe was consisting of following:

1. It was decided that the residual curvature sections of 0.15%, 0.26% and 0.30% residual strain would be used in the experimental tests.
2. By knowing the residual strain levels, corresponding nominal strain were calculated using the stress-strain graphs obtained from uniaxial test of the pipe (See section 5.1.2).
3. Once the nominal strain level ε, was found, the radius of curvature was calculated using the equation.

$$\varepsilon = \frac{r}{R}$$

   Here r is outer radius of pipe, R is the radius of curvature of the pipe.
4. The radius of curvature was drawn on hard board with desired residual curvature length and steel nails were used to mark the boundary.
5. A straight pipe was bend in such a way that it follows the curvature marked by the nails (shown in red color in Figure 5-20).
6. In Figure 5-20, two different level of residual curvature stains were achieved for two different residual lengths.

The Figure 5-20 shows the residual curvature sections of pipes with with 0.0%, 0.15%, 0.26% and 0.30% of Residual strains. These bending of pipes were performed according to above mention procedure.

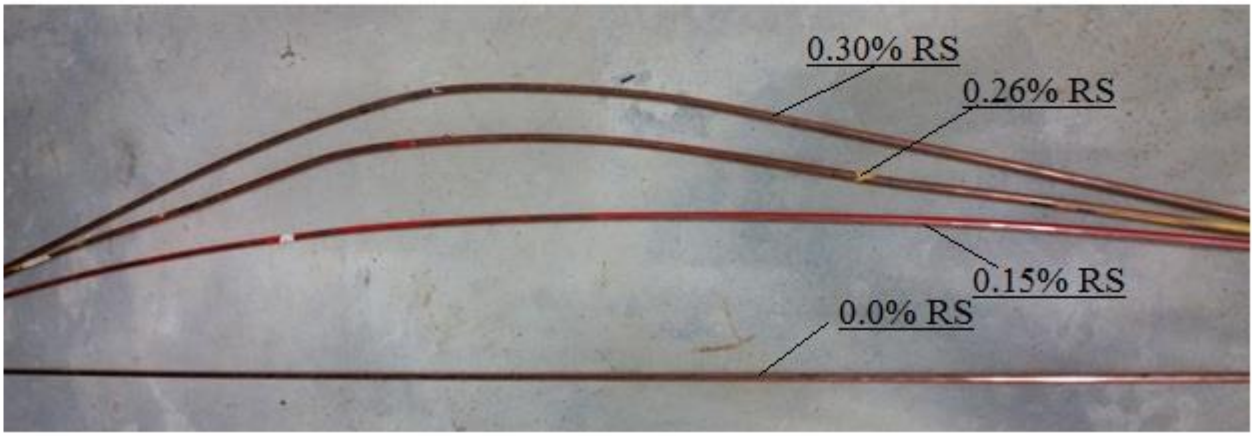


Figure 5-20 Different residual curvatures strains used in experiment

5.6.1 Inline structure

During the installation of pipes, the pipes tends to rotate when there is attached inline structure. An inline structure could be inline tees, pig launcher or wyes. Inline structure is installed with pipe laying. To stimulate the experiment with the inline structure attached to the pipe, a small model structure is made and attached to the pipeline to see the effect of pipe rotation, as shown in Figure 5-21.

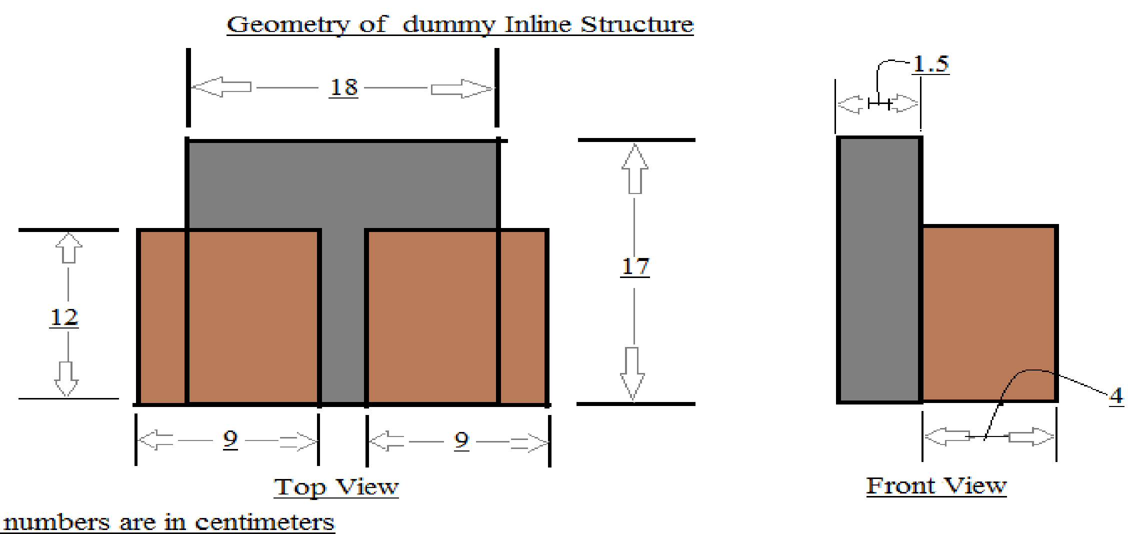


Figure 5-21 Inline structure geometry used in experiment.

5.7 Boundary conditions

There were two boundaries conditions for the pipeline in the experiment. First for the top end of pipeline at stinger and second for the bottom end of pipeline at the seabed. At the top end of pipeline, the tension was applied at the end of pipeline through three pulleys system as described in section 5.5, shown in Figure 5-22. While at the bottom end of the pipeline, there were two options:

- the end of pipeline has fixed to rotate (no degree of rotation)
- the end of pipeline has partially free end to rotate with torque measuring device

The fixed end of pipelines was achieved by fixing the pipe with the long wooden plate through the nail, shown in left side of Figure 5-23. While the partially free to rotate by attaching the torque measuring device, shown in right side of Figure 5-23. The torque measuring device is explained in section 5.7.1.

Figure 5-22 Top boundary condition for pipe. Sting is applying the tension force.

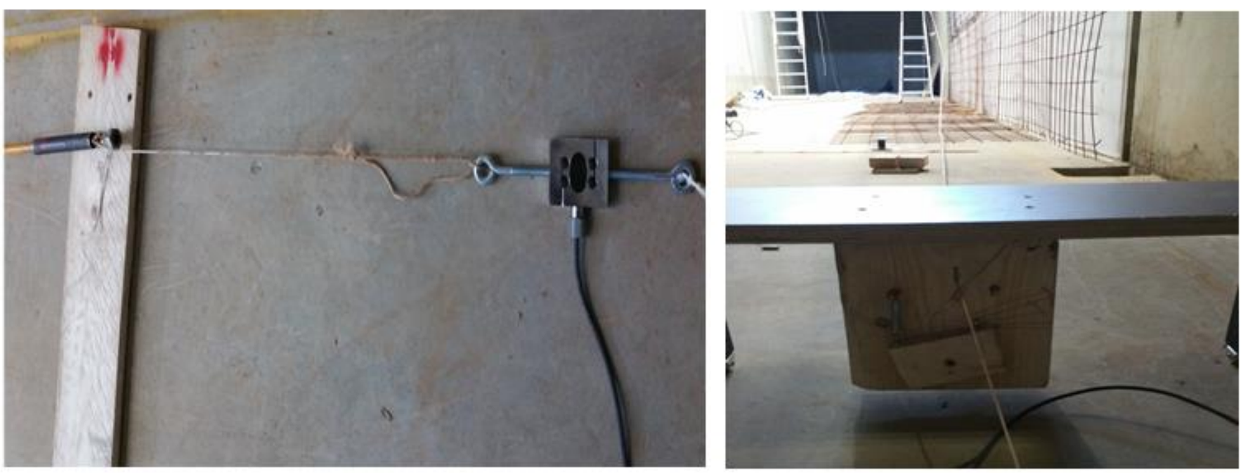


Figure 5-23 boundaries Condition at the bottom. Left side picture: Fixed to rotation. Right side picture: Free to rotate

5.7.1 Torque measurement device

During the experiment it was difficult to know how much a torque is applied by the different pipe configurations with the free end bottom boundary condition. Thus a torque measuring device was needed to measure torque. A torque measuring device was design and built, It was based on mechanical potential energy of the spring. In the device the pipe rotation was constrained by the spring elongation. There were mainly two types of springs i.e. compression springs and tension springs, as shown in left side of Figure 5-24. Tension spring were chosen for the device due to limitations of space. In tension spring two spring were under study with spring 1 and spring 2, shown in right side of Figure 5-24, to find the maximum stiffness k by using the mass-spring formula

$$F = kx$$

Where F is applied force, k is stiffness constant and x is the displacement. After applying force from the load cell for each spring with fixing from one end while elongating from other end, along known displacement, the stiffness of the spring was calculated as shown in Figure 5-25. The experiment was repeated three times for both springs and three elongations were measured. Results are plotted in Figure 5-26 and Figure 5-27 for spring 1 and spring 2 respectively. Since spring 2 has high stiffness constant, k than spring 1. So spring 2 was chosen for the experiment.

Figure 5-24 springs types, Right side picture shows the spring 1 and spring 2.

Figure 5-25 Finding spring stiffness by applying force from load cell along known displacements

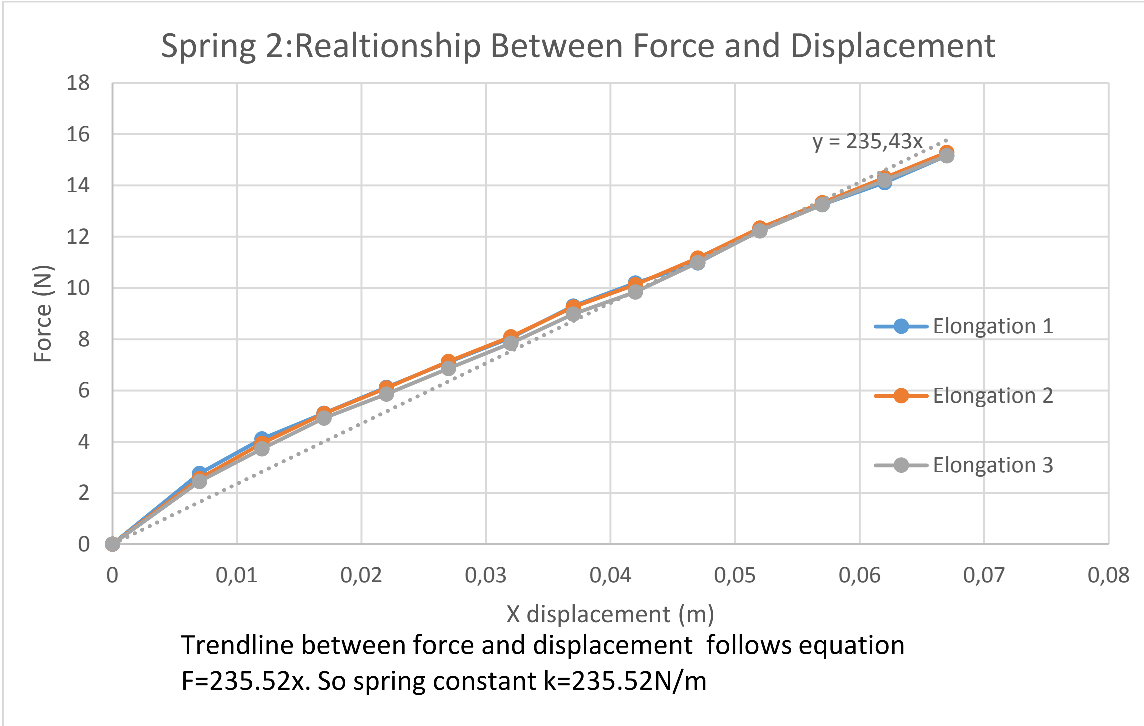


Figure 5-26 spring type 1, Force and displacement relationship

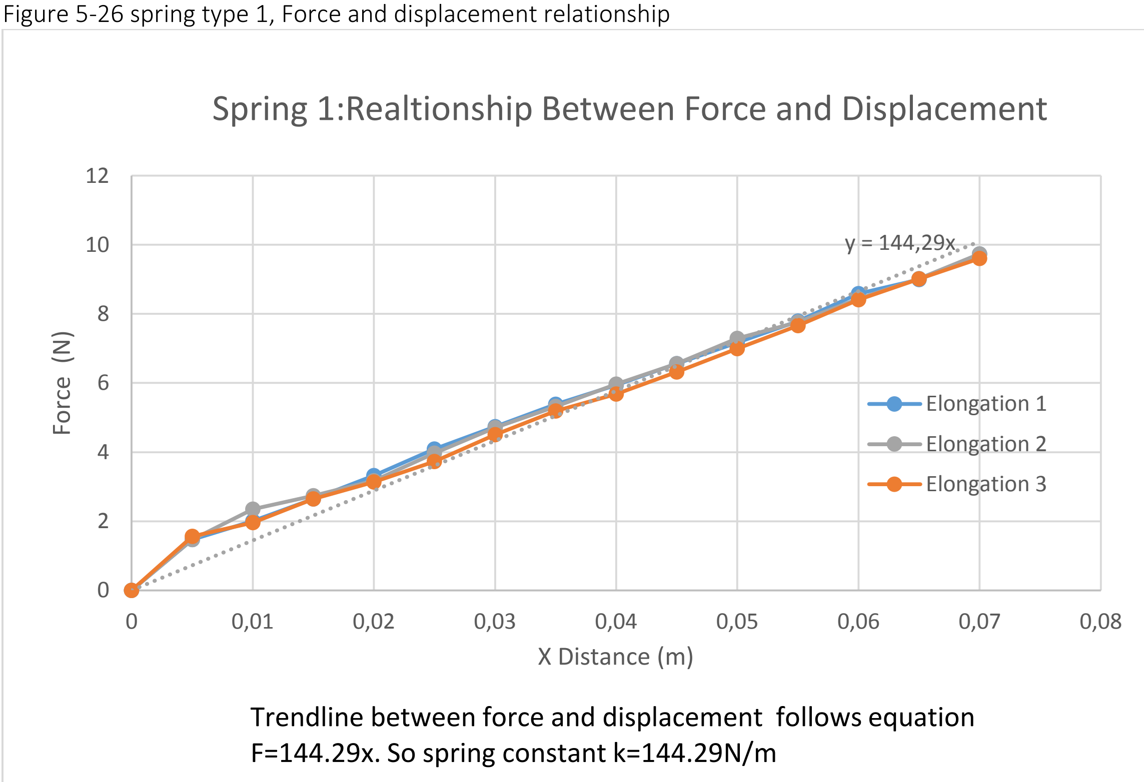


Figure 5-27 spring type 2: Relationship between the Force and displacement

The supporting structure was made to measure the applied torque, shown in Figure 5-28. The structure has small wheels at four ends for the movement along horizontal axis. The pipe was fixed with small flat wooden slab through a screw (shown in picture). The small wooden slab was free to rotate clockwise and anti-clockwise. At the end of small wood slab, a screw was tightened which is connected to spring elongation end. As the pipe rotates the small wood slab rotates, making the spring to elongate. By measuring the spring elongation displacement, the force can be measured. By knowing the force and perpendicular distance, the pipe torque was found from the formula:

$$T = d * F = dFsin\theta$$

Here T is the torque applied by the pipe, d is the moment arm and had a value of 0.04m, F is the applied force. The force applied perpendicular to moment arm in the device, so $sin\theta$ becomes unity. The value of torque applied by the pipe is the same as the torque resist by the springs. In this report, resisting torque or applied torque words are used, which have same meanings.

In Figure 5-28, there is only one spring attach to measure torque has shown, while the number of springs could two or three. It depends upon the how much torque pipe is applying to rotate. Note that the maximum allowed angle for all the experimental tests was 90 degree, if pipe is rotating more than 90 degree, then number of springs were increased to measure the torque.

Figure 5-28 Torque measuring device with attached spring

5.8 Digitalization of pictures

The photos obtained from the experiment were digitalized to get rotation and nominal curvature of the pipeline. An open source program known as *WebPlotDigitizer* is used for this purpose (Rohatgi, 2016). This program is developed on HTML5 and can work for variety of plots. The *WebPlotDigitizer* gave the efficient way of measuring data points from the catenary shape of the pipeline. Due to dimensioning of the wall, the data points obtained from the WebPlotDigitizer were accurate. The data points obtained from the *WebPlotDigitizer* shows excellent results. The process of digitalization of pictures was time consuming and tedious.

6 Scaling

Dimensionless ratios have advantages to correlate the parameters defined on experimental setup to equivalent parameters made in real life. This makes the study inexpensive when doing small scale experiment and getting the same results as for real life situations.
In offshore industry these experiment has been quite useful as the cost of operations in offshore is highly expensive as compared to laboratory experimentation.
In any physical model there three types of similarity required between simulation or experiment and real life situations. Such as:

- Geometric Similarity: All the physical parameters must be same scale ratio i.e. same shape. It applies to elastic deformations and environment surroundings.
- Kinematics Similarity: Velocities of model experiment and real life scenario must be same. e.g. velocities in x-y direction of certain flow have to be same ratio in experimental model in case of circular motion of certain flows.
- Dynamic Similarity: it is similarity of forces i.e. Inertia forces, Surface forces, Elastic Forces, Pressure force, Gravitational forces. The ratio of different forces between experimental model and full scale model must be same.

Scaling can be done in many ways depending upon full scale model type and efficiency. For example, as shown in Figure 3.1. In fluids flow model study Reynolds number and Froude numbers are considered most appropriate to scale the fluid properties and its flow characteristics. Where as in solid mechanics where displacement/deformation is very less and structural analysis is performed, displacement, strains, elasticity are used for non-dimensionlization.
Exact scaling of models is complex in subsea pipes installation. Stresses that are due to pipeline motions, and hydro dynamics loads are most important to consider. The experimental setup is based on principle of partial geometric similarity. For the transformation of model test into full scale, all the basic equations and their boundary conditions are converted into non-dimensional form. As any mechanical dimensions have three fundamentals qualities i.e. length, force and time. For pipeline and laying vessel, it is required to have some identical scaling factors for the three fundamentals qualities (Clauss & Saroukh, 1996). Following characteristics of pipelines are to be considered during dimensioning:

- Pipe diameter
- Pipe bending stiffness
- Weight per unit length
- Water depth

Due to inertial forces, viscous forces, Impact loads, oscillating vessel movement makes difficult to achieve full similarity between model and real life pipe line installation.

6.1 Deflection Number

Study of hydrostatic stresses on different tube were carried out in water and air, with fundamental equations and good accuracy. This results in defining the deflection of pipes. (Universitetsforlaget, 1974) (Clauss & Kuhnlein, 1974) (Clasuu, et al., 1991)

$$Dn = \frac{w \ \cdot L^3}{E \cdot t \cdot d^3} = \frac{w^* \ \cdot L^{*3}}{E^* \cdot t^* \cdot OD^{*3}}$$

Here w is submerged weight of pipe per unit length, L is the suspended length of pipe, E is the young modulus, t is wall thickness and OD is water depth.
Calculation of deflection number is performed in Mathcad and shown in Appendix D. The scaling ratio is calculated with installation of 36-inch steel pipe and 20-inch steel pipe.

7 Experiment Test results

The tests were carried out with different residual curvatures strain, residual curve length, top tension and other variables. The pipe is pulled from the bottom. The purpose of these experiments is to study the behavior of the residual curvature sections and inline structures on the pipe rotation.

Different residual curvature section was created with different length. First, test with the laying configuration of the pipe according to S-lay were performed with the straight pipes. After that the calculations were performed taking into account the pool space limitations, so residual curvature pipe behavior could be observed at touch down point (TDP). The experiments were carried out with pipes with different residual curvature just after the inflection point. For each residual curvature pipe, the pipe is disconnected and new pipe is added. Disconnection of pipes was done by heating the soldered joint and then adding a new pipe with different residual curvature section by soldering. The experiments were work intensive. They were given a great importance for control and accurate measurements of the pipeline installation parameters.

7.1 Catenary shape of the pipe

Different models for pipe installations exist, the simplest model is the natural catenary model. The experimental set up is in the agreement with the natural catenary model and the pipes were selected in such a way that they would have catenary shape in experiments. It was found that copper pipes have sufficient flexibility to such water depth 4-5m. Steel pipes were not suitable for laboratory experiments due to high stiffness. The equation for natural catenary model is (Bai & Bai, 2005):

$$z = \frac{T_h}{w_s}\left(\cosh\left(\frac{x \cdot w_s}{T_h}\right) - 1\right) \qquad (eq7.1)$$

Where z is the shape of catenary, T_h is the horizontal force, w_s is the submerged weight, and x is horizontal distance along the length of pipe.

The dimensioning of the experiments was done to study the shape of the pipes and rotation. High resolution pictures were taken and combined to form picture sequence following the pipe from leaving the stinger to TDP. These pictures were put into open source software *WebPlotDigitizer* and data points were generated as shown in Figures 7-1, 7-2 and 7-3. The generated data points were used to draw the pipe profile from pipeline leaving the stinger to the TDP. Similarly, by using the natural catenary equation as describe above, shape of the catenary profile is plotted. Comparison between the experimental and natural catenary curves for pipes of different wall thickness is provided in Figures 7.4 and 7.5 below. Note that the experimental curves are mirrored to enable direct comparison with the natural catenary curves.

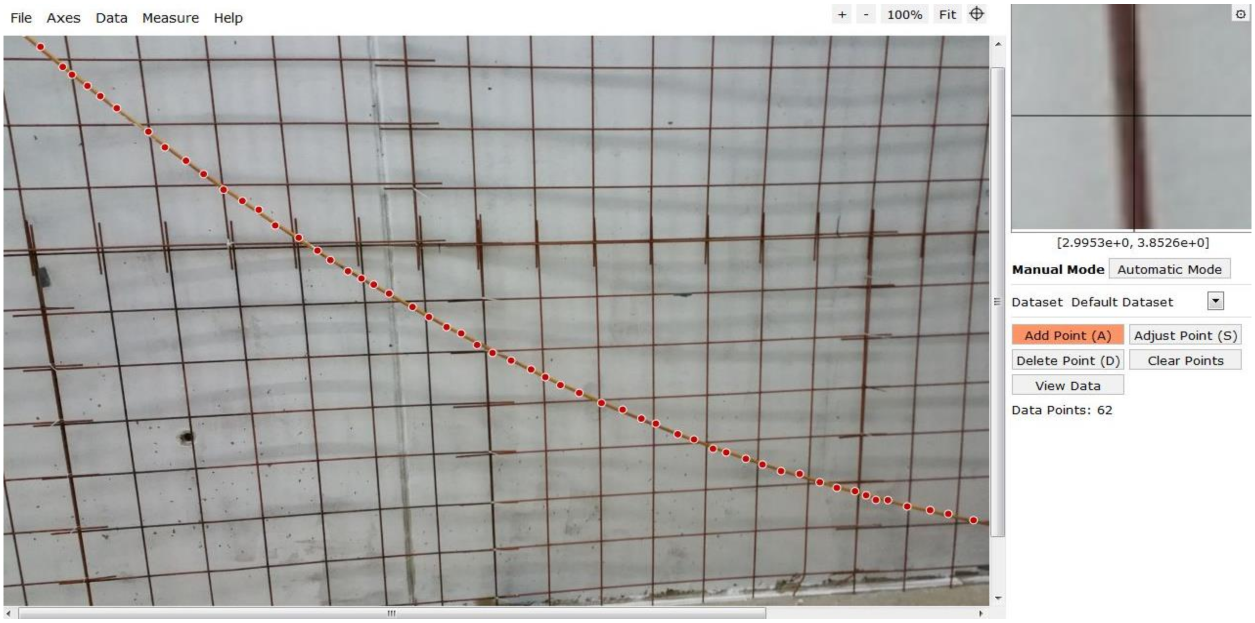


Figure 7-1 Digitization of picture to obtained shape of pipe, pipe at near TDP

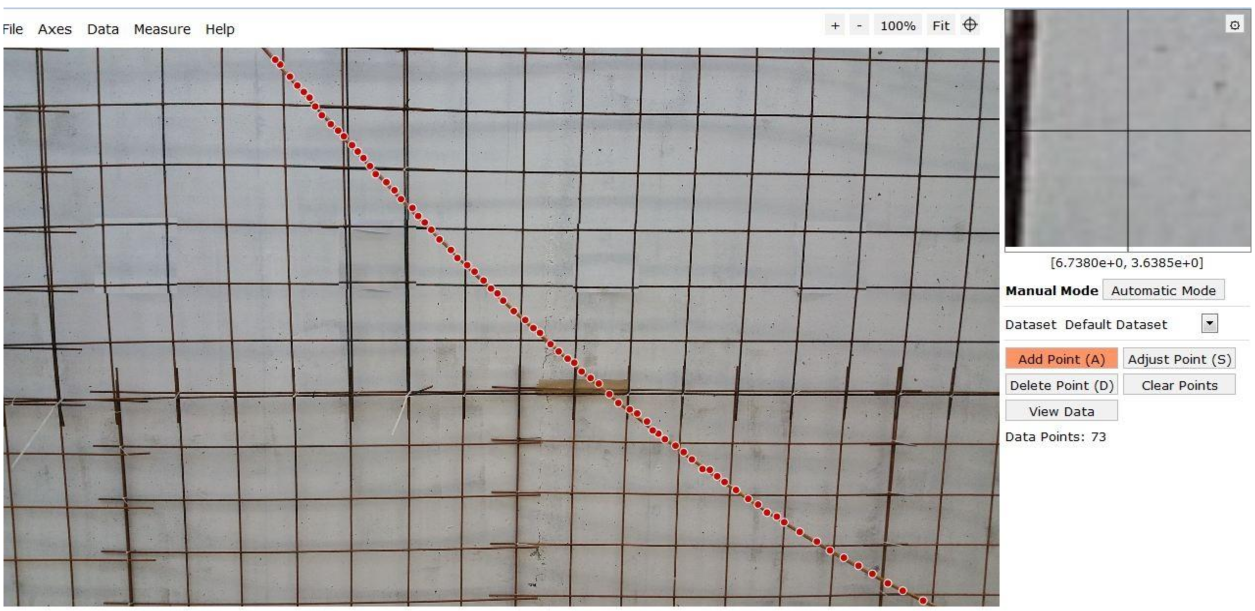


Figure 7-2 Digitization of picture to obtained shape of pipe, pipe at near center of suspended section

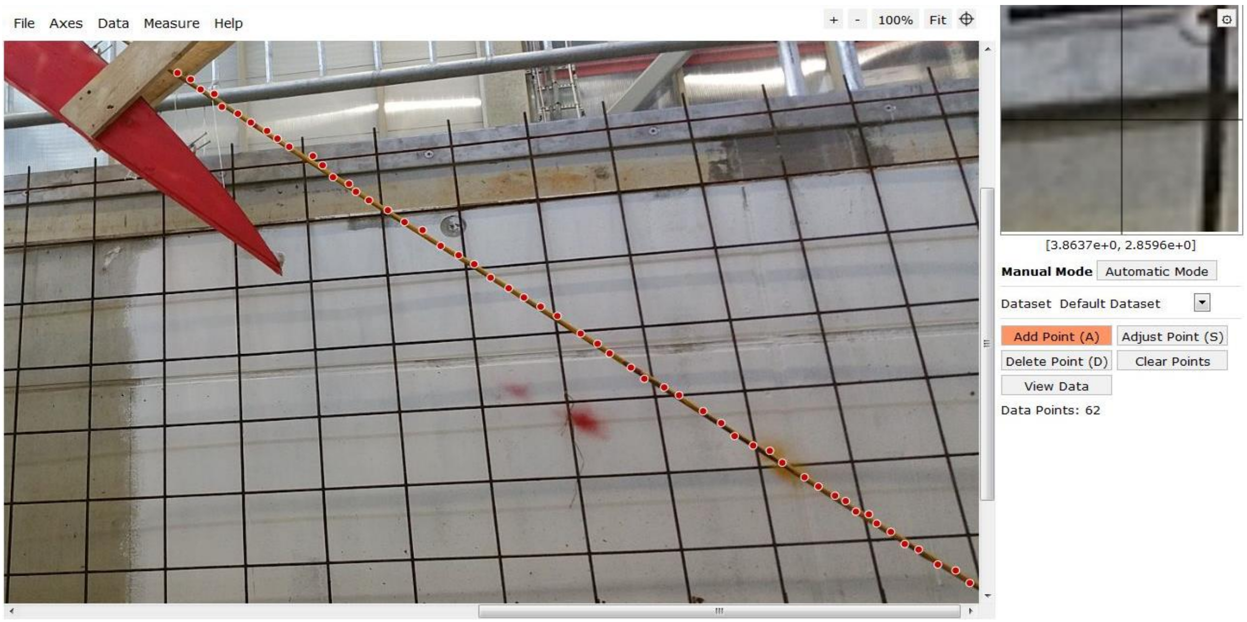


Figure 7-3 Digitization of picture to obtained shape of pipe, pipe at the stinger tip

The Figure 7-4 and Figure 7-5 shows that the pipeline somehow follows the shape of catenary model. This validates that selection of copper pipe for model testing is correct and pipe results can be compared to industrial pipeline installation.

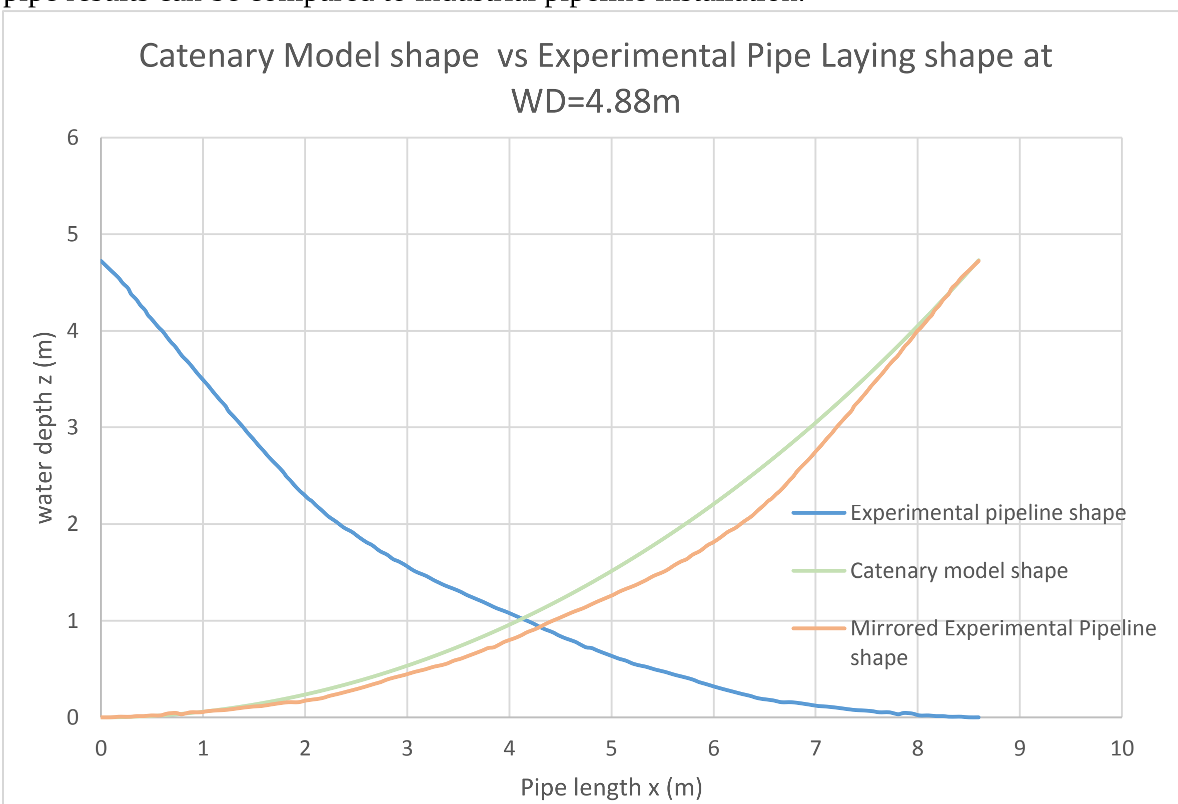


Figure 7-4 Catenary Model shape vs Experimental Pipe laying shape for WD: 4.88m

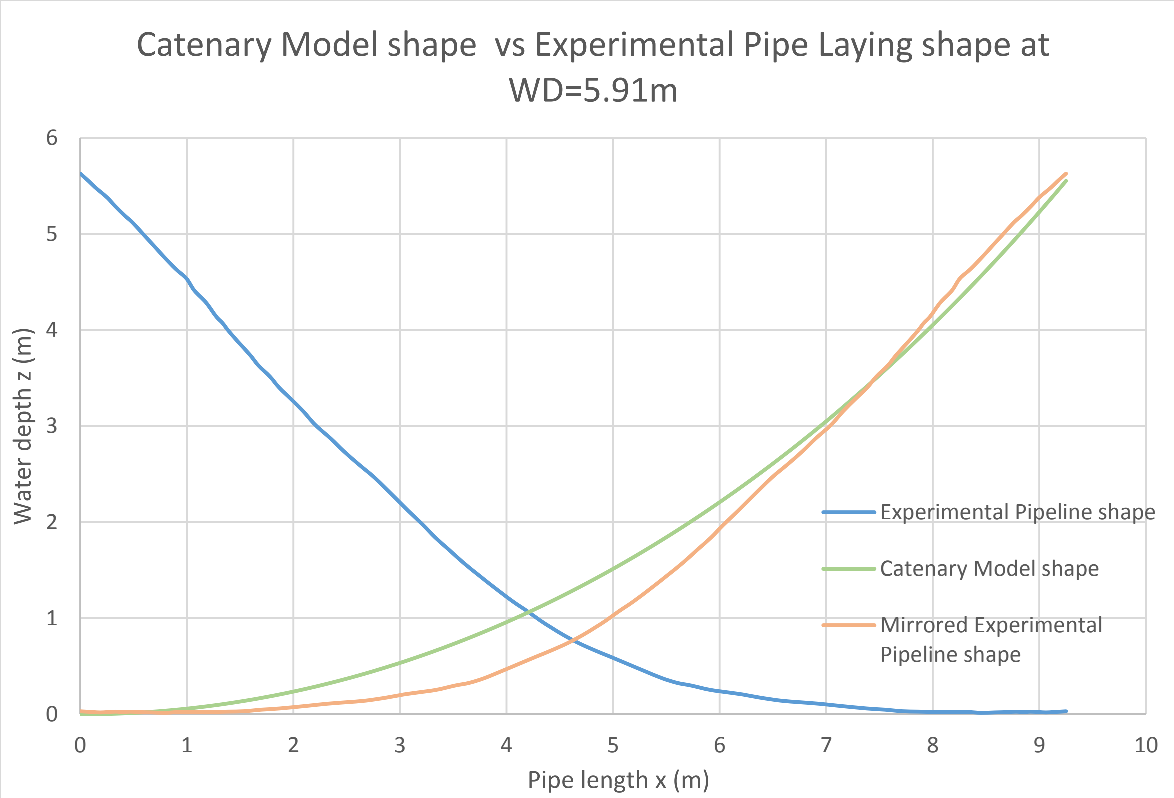


Figure 7-5Catenary Model shape vs Experimental Pipe laying shape for WD: 5.91m

7.2 Tension Calculation for the Experiment

The tensioners at the ship vessel apply tension to the pipeline after the welding connections of pipes, but before the stinger. Tensioners at the installation vessel for S-lay are of two types:

1. Horizontal tensioners
2. Vertical tensioners

The tension in the pipeline compromises of horizontal component and vertical component. Horizontal component remains more or less constant after leaving the stinger, while the vertical component varies from maximum at stinger tip, to the zero at bottom. The total tension in the pipe is the vector sum of these components.

Pipeline installation software “OFFPIPE” was used to analyze the tension along the pipeline. Typical distribution is shown in Figure 7.6. In this graph the tensioners are placed at 60m length and the stinger is between 110m and 180m. The horizontal force was constant from the stinger tip to the TDP while the vertical component changes its value from 800kN at stinger tip to zero at TDP. This corresponds to the own weight of the pipe in sagbend (excluding of the pipe on the stinger), and is more than twice the difference between top tension and the bottom tension. The pipe is long and thin but has some bending stiffness, and when this interacts with the stinger it means that not all of the pipe’s self-weight contributes to the tension.

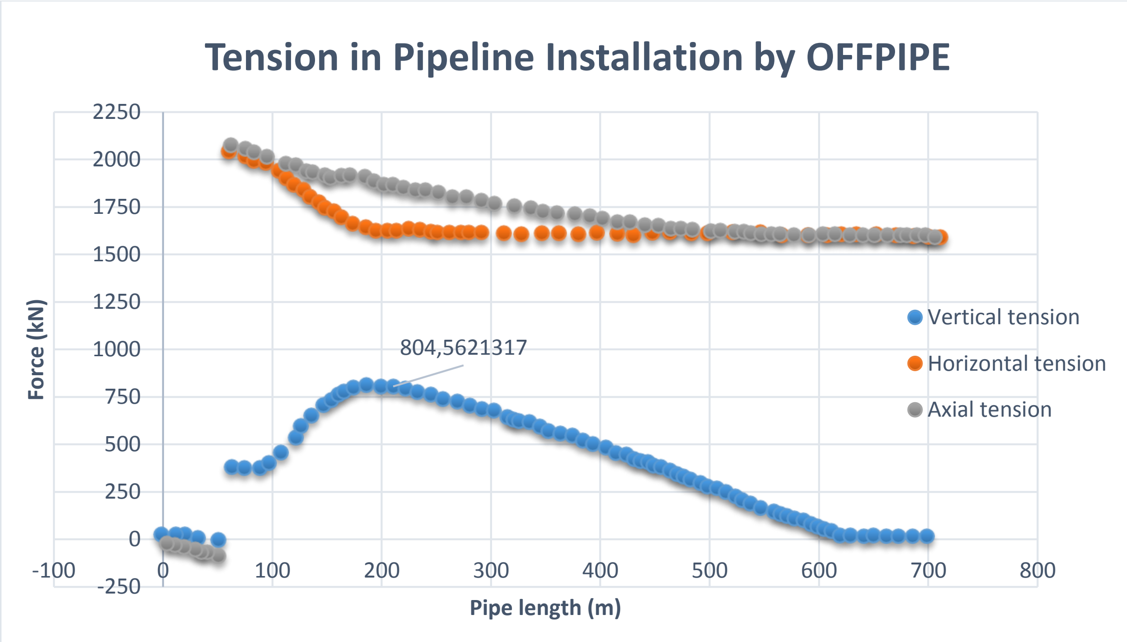


Figure 7-6 OFFPIPE Analysis of pipeline installation tension

The tension in the experimental set up for pipe laying is shown in Figure7.7. The top tension and horizontal tension values are used in the analytical calculations. The load cell connected at the end of the pipeline measures the horizontal force electronically and converted into kilogram force. The load was applied through 3 pullies and was varied in the experiments. The length of the pipe was 18m from the point of constant load to load cell. The horizontal tension is larger than the constant load due to friction and pipe elasticity in the residual curvature. It is assumed that the pipe follows the catenary model. The vertical component of the tension is calculated from the catenary equation:

$$T_v = w_s \cdot L \qquad (eq7.2)$$

Here T_v is the vertical component of the tension, w_s is the submerged weight of the pipe per unit length and L is the suspeneded length of the pipe. Since the experiment is carried out in dry condition, the dry weight of the pipe per unit length is considered in the above equation. The maximum value of Vertical component can be found by putting the total suspended length of the pipe.

The horizontal tension in the experiment is calculated by:

$$T_h = load\ cell\ value(Kg)\ \cdot 9.8\ m/s^2 \qquad (eq7.3)$$

The axial tension or top tension in the pipe along its suspended length is the vector sum of the horizontal and the vertical components of tension force. The top tension can be calculated as:

$$Top\ tension = Tt = \sqrt{{T_v}^2 + {T_h}^2} \qquad (eq7.4)$$

For the experimental set up T_v was calculated from the equation 7.2 while T_h calculated from the equation 7.3.

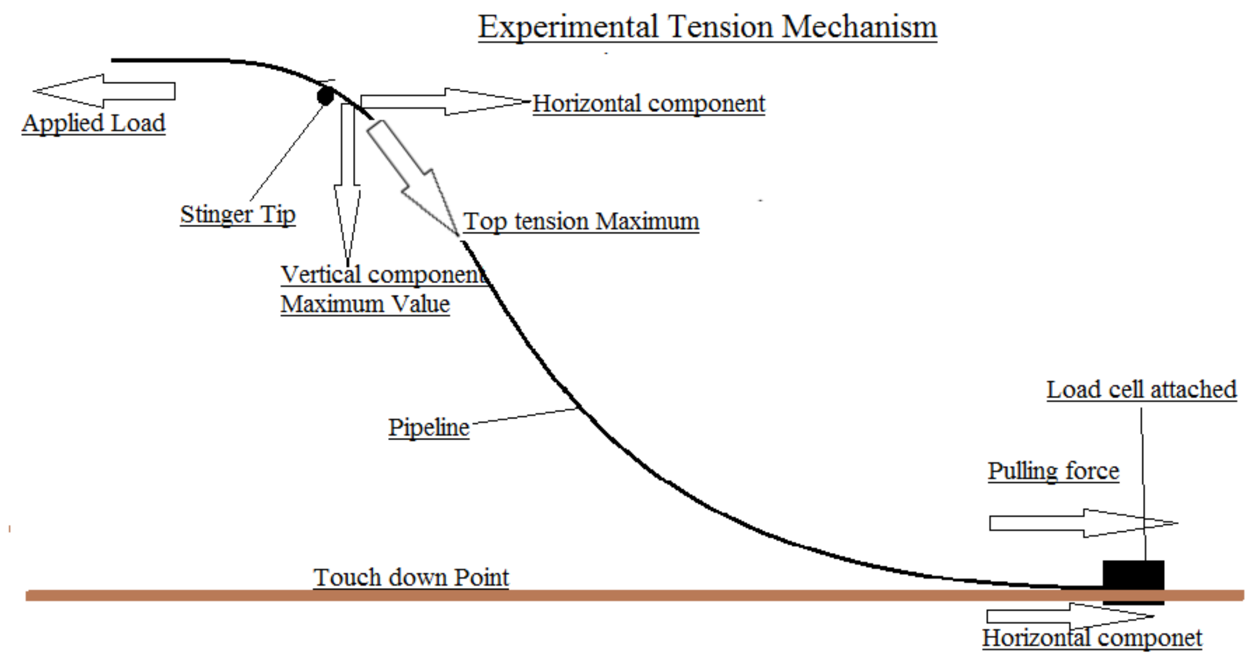


Figure 7-7 Measurements of the tension in the experiments

7.3 Test with free partially Rotation BC at Water Depth 4.88m

Experimental tests were performed with partially free bottom boundary condition.

7.3.1 Test1

This test was carried out with the parameters as in Table 7-1, the tension calculated and measured is provided in Table 7-2 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-3

Table 7-1 Test 1 pipeline installation parameters

Boundary condition bottom	Free to move with resisting torque measurement
Water depth (m)	4.88
Residual curvature strain (%)	0.26
Nominal strain at stinger (%)	0.125
Length from inflection point to TDP (m)	8.92
Residual curvature length (m)	2
Number of springs used to measure resisting torque	3

Table 7-2 Test 1, tension calculated and measured.

Applied Load (N)	Horizontal Tension (N)	Vertical tension(N)	Total tension(N)
20N	35.28	16.13	38.79

Table 7-3 Test 1, Torque exerted by pipe

Pipe Movement (m)	Spring displacement (m)	K (N/m)	F (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0055	235.43	3.884595	0.04	0.155384	15.53838
2	0.0055	235.43	3.884595	0.04	0.155384	15.53838
3	0.0095	235.43	6.709755	0.04	0.26839	26.83902
3.79	0.0152	235.43	10.73561	0.04	0.429424	42.94243
4	0.025	235.43	17.65725	0.04	0.70629	70.629

It can be seen from the Table 7-3, that the maximum torque is at the touch down point (TDP). Spring attached accounts for the pipe torsional rigidity and the pipe rotates. The maximum angle is at the TDP point and equals 79.78 degrees. This is illustrated in Figure 7-8.

Figure 7-8 Test 1, Pipe at TDP. Right side picture shows the pipe forming the angle with the x-axis obtained by picture digitalization.

7.3.2 Test 2

This test was performed with the parameters as in Table 7-4, the tension calculated and measured is provided in Table 7-5 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-6

Table 7-4 Test 2 pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Residual curvature strain (%)	0.26
Length from inflection point to TDP (m)	8.94
Residual curvature length (m)	2
Number of springs used to measure resisting torque	1

Table 7-5 Test 2, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
20N	35.57	16.13	39.06

Table 7-6 test 2, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m))	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.006	235.43	1.41258	0.04	0.056503	5.65032
2	0.0095	235.43	2.236585	0.04	0.089463	8.94634
3	0.0145	235.43	3.413735	0.04	0.136549	13.65494
3.79	0.035	235.43	8.24005	0.04	0.329602	32.9602
4	0.0475	235.43	11.18293	0.04	0.447317	44.7317

In this test only one spring was attached to the pipe, simulating lower torsional rigidity, the rotation angle is larger compared to test 1, when 3 springs were used.
The maximum torque occurs at TDP resulting in rotation angle of 89.99 degrees. This is shown in Figure 7-9.

Figure 7-9 Test 2, Pipe at TDP. Right side picture shows the pipe forming the angle with the x-axis obtained by picture digitalization.

7.3.3 Test 3

This test was carried with the parameters as in Table 7-7, the tension calculated and measured is provided in Table 7-8 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-9. In this test, the top tension was increased.

Table 7-7 Test 3 pipeline installation parameters

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	9.18
Residual curvature strain (%)	0.26
Residual curvature length (m)	2
Number of springs used to measure resisting torque	1

Table 7-8 Test 3, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
40N	60.27	17.16	62.66

Table 7-9 Test 3, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0096	235.43	2.260128	0.04	0.090405	9.040512
2	0.0115	235.43	2.707445	0.04	0.108298	10.82978
3	0.0185	235.43	4.355455	0.04	0.174218	17.42182
3.79	0.0225	235.43	5.297175	0.04	0.211887	21.1887
4	0.0325	235.43	7.651475	0.04	0.306059	30.6059

In this test it was observed that Increase in top tension, causes decrease in rotation. The angle of rotation at TDP in this test was 60.85 degrees and shown in Appendix F, Figure F1.

7.3.4 Test 4

This test was carried out with the parameters as in Table 7-10, the tension calculated and measured is provided in Table 7-11 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-12.

Table 7-10 Test 4 pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.82
Residual curvature strain (%)	0.15
Residual curvature length (m)	2
Number of springs used to measure resisting torque	3

Table 7-11 Test 4, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
10N	16.07	15.31	22.20

Table 7-12 Test 4, Torque exerted by pipe

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0035	235.43	2.472015	0.04	0.098881	9.88806
2	0.0039	235.43	2.754531	0.04	0.110181	11.01812
3	0.0046	235.43	3.248934	0.04	0.129957	12.99574
3.79	0.0049	235.43	3.460821	0.04	0.138433	13.84328
4	0.0055	235.43	3.884595	0.04	0.155384	15.53838

The experiment shows that there was no significant pipe rotation, only 15.12 degrees at the TDP. This experiment was carried out with three springs attached in torque measuring device, opposing the rotation of the pipe. The pipeline curvature remains mainly in the vertical plane, which is illustrated in Appendix F, Figure F2.

7.3.5 Test 5

This test was carried out with the parameters as in Table 7-13, the tension calculated and measured is provided in Table 7-14 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-15

Table 7-13 Test 5, pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.82
Residual curvature strain (%)	0.15
Residual curvature length (m)	2
Number of springs used to measure resisting torque	1

Table 7-14 Test 5, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
10N	16.27	15.31	22.34

Table 7-15 Test 5, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0055	235.43	1.294865	0.04	0.051795	5.17946
2	0.006	235.43	1.41258	0.04	0.056503	5.65032
3	0.0065	235.43	1.530295	0.04	0.061212	6.12118
3.79	0.0085	235.43	2.001155	0.04	0.080046	8.00462
4	0.0095	235.43	2.236585	0.04	0.089463	8.94634

In this experiment the pipe has only one resisting spring, hence a lower torsional stiffness and the rotation angle is larger compared to test 4. Rotation angle at TDP pipeline is 31.92 degrees as can be seen in Appendix F, Figure F3

7.3.6 Test 6

This test was carried with the parameters as in Table 7-16, the tension calculated and measured is provided in Table 7-17 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-18.

Table 7-16 Test 6 pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.92
Residual curvature strain (%)	0.15
Residual curvature length (m)	2
Number of springs used to measure resisting torque	1

Table 7-17 Test 6, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
20N	32.44	15.95	36.15

Table 7-18 Test 6, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0035	235.43	0.824005	0.04	0.03296	3.29602
2	0.0041	235.43	0.965263	0.04	0.038611	3.861052
3	0.0046	235.43	1.082978	0.04	0.043319	4.331912
3.79	0.0055	235.43	1.294865	0.04	0.051795	5.17946
4	0.0075	235.43	1.765725	0.04	0.070629	7.0629

In this test the tension was increased from 10 N to 20 N and the tendency of pipe to rotate decreases. The angle of rotation of the pipe at TDP is 21.43 degrees, shown in Appendix F, Figure F4.

7.3.7 Test 7

This test was carried out with the parameters as in Table 7-19, the tension calculated and measured is provided in Table 7-20 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-21.
This experiment was carried out with connected block that simulates the laying of pipeline with inline structure.

Table 7-19 Test 7, pipeline installation parameters

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.83
Residual curvature strain (%)	0.15
Residual curvature length (m)	2
Number of springs used to measure resisting torque	2
Inline structure weight (kg)	0.759

Table 7-20 Test 7, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
10N	15.68	15.31	21.92

Table 7-21 Test 7, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0035	235.43	1.64801	0.04	0.06592	6.59204
2	0.0095	235.43	4.47317	0.04	0.178927	17.89268
3	0.0145	235.43	6.82747	0.04	0.273099	27.30988
3.79	0.0195	235.43	9.18177	0.04	0.367271	36.72708
4	0.021	235.43	9.88806	0.04	0.395522	39.55224

At the TDP the pipe and the inline structure rotate 90 degrees and this is shown in Figure 7.10.

Figure 7-10 Test 7: Left side: start of the experiment. Right Side picture: Pipe is at TDP, pipe and inline structure rotated 90 degrees.

7.3.8 Test 8

This test was conducted with the parameters as in Table 7-22, the tension calculated and measured is provided in Table 7-23 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-24. This test was performed with larger residual strain of 0.3% with the curved section of 0.84 m only.

Table 7-22 Test 8 pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.94
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84
Number of springs used to measure resisting torque	1

Table 7-23 Test 8, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
20N	29.40	16.04	33.49

Table 7-24 Test 8, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0025	235.43	0.588575	0.04	0.023543	2.3543
2	0.0035	235.43	0.824005	0.04	0.03296	3.29602
3	0.0055	235.43	1.294865	0.04	0.051795	5.17946
3.79	0.0095	235.43	2.236585	0.04	0.089463	8.94634
4	0.0135	235.43	3.178305	0.04	0.127132	12.71322

The rotation angle at TDP is 53.28 degrees, shown in Appendix F, Figure F5

7.3.9 Test 9

This test was conducted with the parameters as in Table 7-25, the tension calculated and measured is provided in Table 7-26 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-27. This test was performed with larger residual strain of 0.3% with the curved section of 0.84 m. Compared to the previous test, in test 9, the top tension force was lower.

Table 7-25 Test 8 pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88

Length from inflection point to TDP (m)	8.84
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84
Number of springs used to measure resisting torque	1

Table 7-26 Test 9, tension calculated and measured

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
10N	14.11	15.35	20.85

Table 7-27 Test 9, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0037	235.43	0.871091	0.04	0.034844	3.484364
2	0.0056	235.43	1.318408	0.04	0.052736	5.273632
3	0.0095	235.43	2.236585	0.04	0.089463	8.94634
3.79	0.0125	235.43	2.942875	0.04	0.117715	11.7715
4	0.0174	235.43	4.096482	0.04	0.163859	16.38593

The pipe rotation angle at TDP is 60.38 degrees, shown in in Appendix F, Figure F6.

7.3.10 Test 10

This test was conducted with the parameters as in Table 7-28, the tension calculated and measured is provided in Table 7-29 and the torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-30. This test was performed with larger residual strain of 0.3% with the curved section of 0.84 m. Compared to the test 8, in this test the top tension force was larger.

Table 7-28 Test 10, pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Water depth (m)	4.88
Length from inflection point to TDP (m)	9.17
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84
Number of springs used to measure resisting torque	1

Table 7-29 Test 10, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
40N	57.92	17.08	60.38

Table 7-30 Test 10, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0005	235.43	0.117715	0.04	0.004709	0.47086
2	0.0009	235.43	0.211887	0.04	0.008475	0.847548
3	0.0015	235.43	0.353145	0.04	0.014126	1.41258
3.79	0.0034	235.43	0.800462	0.04	0.032018	3.201848
4	0.0095	235.43	2.236585	0.04	0.089463	8.94634

The rotation angle at TDP is 24.86 degrees and the test is shown in Figure 7-11.

Figure 7-11 Test 10, rotation angle at TDP.

7.4 Test with Fixed Rotation BC at Water Depth 4.88m

7.4.1 Test 11

This test was carried out with the parameters as in Table 7-31 and the tension calculated and measured is provided in Table 7-32

Table 7-31 Test 11, pipeline installation parameters.

Boundary condition bottom	Fixed End
Water depth (m)	4.88
Length from inflection point to TDP (m)	9.17
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

Table 7-32 Test 11, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
40N	58.90	17.08	61.33

The pipeline end at the bottom was fixed against the rotation. The rotation angle at TDP was 12.51 degrees and the test is shown in Figure 7.17.

Figure 7-12 Test 10, rotation angle at TDP.

7.4.2 Test 12

This test was carried out with the parameters as in Table 7-33 and the tension calculated and measured is provided in Table 7-34

Table 7-33 Test 12 pipeline installation parameters.

Boundary condition bottom	Fixed End
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.94
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

The tension measured and calculated for this experiment is:

Table 7-34 Test 12, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
20N	31.56	16.04	35.40

The angle of rotation of the pipe was 17.77 degrees, shown in Appendix F, Figure F7

7.4.3 Test 13

This test was carried out with the parameters as in Table 7-35 and the tension calculated and measured is provided in Table 7-36

Table 7-35 Test 13 pipeline installation parameters.

Boundary condition bottom	Fixed End
Water depth (m)	4.88
Length from inflection point to TDP (m)	8.82
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

Table 7-36 Test 13, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
10N	14.31	15.35	20.99

Compared to the previous test, in test 12 with fixed end, the tension in the pipe is reduced. The angle of rotation at TDP increased to 20.35 degrees, shown in Appendix F, Figure F8

7.5 Test with Partially Free Rotation BC at Water Depth 5.91m

7.5.1 Test 14

This test was carried out with the parameters as in Table 7-37 and the tension calculated and measured is provided in Table 7-38. The torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-39

Table 7-37 Test 14 pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Top tension (N)	20
Water depth (m)	5.91
Length from inflection point to TDP (m)	10.13
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

Table 7-38 Test 14, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
20N	34.40	17.55	38.62

Table 7-39 Test 14, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0015	235.43	0.70629	0.04	0.028252	2.82516
2	0.0025	235.43	1.17715	0.04	0.047086	4.7086
3	0.0055	235.43	2.58973	0.04	0.103589	10.35892
3.79	0.0095	235.43	4.47317	0.04	0.178927	17.89268
4	0.0115	235.43	5.41489	0.04	0.216596	21.65956

The angle of rotation at TDP was 49.32 degrees and the test is shown in Figure 7.20.

Figure 7-13 Test 14, WD=5.91m, rotation angle at TDP.

7.5.2 Test 15

This test was carried out with the parameters as in Table 7-40 and the tension calculated and measured is provided in Table 7-41. The torque the pipe exerted to the spring system measured at different pipe movement is given in Table 7-42.

Table 7-40 Test 15, pipeline installation parameters.

Boundary condition bottom	Free to move with torque measurement
Top tension (N)	40
Water depth (m)	5.91
Length from inflection point to TDP (m)	10.54
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

Table 7-41 Test15, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
40N	61.84	19.64	64.88

Table 7-42 Test 15, Torque exerted by pipe.

Pipe Movement (m)	Spring displacement (m)	Spring stiffness K (N/m)	Force (N)	Moment arm (m)	Torque (Nm)	Torque (Ncm)
1	0.0015	235.43	0.353145	0.04	0.014126	1.41258
2	0.002	235.43	0.47086	0.04	0.018834	1.88344
3	0.0035	235.43	0.824005	0.04	0.03296	3.29602
3.79	0.0065	235.43	1.530295	0.04	0.061212	6.12118
4	0.0115	235.43	2.707445	0.04	0.108298	10.82978

The angle of rotation at TDP was 28.84 degrees, shown in Appendix F, Figure F9

7.6 Test with Fixed Rotation BC at Water Depth 5.91m

7.6.1 Test 16

This test was carried out with the parameters as in Table 7-43 and the tension calculated and measured is provided in Table 7-44. Fixed boundary condition at the bottom was applied.

Table 7-43 Test 16 pipeline installation parameters.

Boundary condition bottom	Fixed
Water depth (m)	5.91
Length from inflection point to TDP (m)	10.13
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

Table 7-44 Test 16, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
20N	33.52	17.55	37.83

Figure 7-14 Test 16, Fixed Boundary condition, Rotation angle at TDP

The angle of rotation at TDP was 22.60 degrees and the test is shown in Figure 7-15.

7.6.2 Test 17

This test was carried out with the parameters as in Table 7-45 and the tension calculated and measured is provided in Table 7-46. Fixed boundary condition at the bottom was applied.

Table 7-45 Test 17 pipeline installation parameters.

Boundary condition bottom	Fixed
Water depth (m)	5.91
Length from inflection point to TDP (m)	10.54
Residual curvature strain (%)	0.30
Residual curvature length (m)	0.84

Table 7-46 Test 17, tension calculated and measured.

Applied Load (N)	Horizontal Tension(N)	Vertical tension(N)	Total tension(N)
40N	61.84	19.64	64.88

The rotation angle at TDP was 20.74 degrees and the test is Appendix F, Figure F10.

7.7 Summary of Experimental Test Results

During experiments different laying parameters were varied and effects were observed in angle rotation of pipelines. There are total seventeen experimental tests are reported in this study.

The test results are summarized in Tables 7-47 to 7-50.

Table 7-47 Summary of test 1-10, WD=4.8m, partially free to rotate BC

Summary of Experimental Test Results							
Water depth :4.88m							
Bottom boundary condition: Partially Free to rotate with torque measurement							
Test Number	Applied Load(N)	Horizontal Tension (N)	Top Tension (N)	Residual Curvature strain (%)	Residual Curvature length (m	Resisting Torque (Ncm)	Angle at TDP (Degree)
Test 1	20	35.28	38.79	0.26	2	70.63	79.78
Test 2	20	35.57	39.06	0.26	2	44.73	89.99
Test 3	40	60.27	60.66	0.26	2	30.609	60.85
Test 4	10	16.07	22.20	0.15	2	15.54	15.12
Test 5	10	16.27	22.34	0.15	2	8.95	31.92
Test 6	20	32.44	36.15	0.15	2	7.06	21.43
Test 7(0.783Kg Inline structure)	10	15.68	21.92	0.15	2	39.55	90.00
Test 8	20	29.40	33.49	0.30	0.84	12.71	53.28
Test 9	10	14.11	20.85	0.30	0.84	16.39	60.38
Test 10	40	57.92	60.38	0.30	0.84	5.48	24.86

Table 7-48 Summary of test 11-13, WD=4.8m, fixed to rotate BC

Water depth :4.88m					
Bottom boundary condition: Fixed to rotation					
Test Number	Horizontal Tension (N)	Top Tension (N)	Residual strain (%)	Residual length (m)	Angle at TDP (Degree)
Test 11	58.90	61.33	0.30	0.84	12.51
Test 12	31.56	35.40	0.30	0.84	17.77
Test 13	14.31	20.99	0.30	0.84	20.35

Table 7-49 Summary of test 14-15, WD=5.91m, partially free to rotate BC

Water depth :5.91m						
Bottom boundary condition: Free to rotate with torque measurement						
Test Number	Horizontal Tension (N)	Top Tension (N)	Residual strain (%)	Residual length (m)	Resisting Torque (Ncm)	Angle at TDP (Degree)
Test 14	34.40	38.62	0.30	0.84	21.66	49.32
Test 15	61.84	64.88	0.30	0.84	10.83	28.84

Table 7-50 Summary of test 16-17, WD=5.91m, fixed to rotate BC

Water depth :5.91m					
Bottom boundary condition: Fixed to rotation					
Test Number	Horizontal Tension (N)	Top Tension (N)	Residual strain (%)	Residual length (m)	Angle at TDP (Degree)
Test 16	33.52	37.83	0.30	0.84	22.60
Test 17	61.84	64.88	0.30	0.84	20.74

8 Discussion and Comparison of results between approaches

This section discussed the experimental, analytical and combination experimental-analytical approaches for the rotation of pipelines. The parametric study of experimental tests results is discussed for understanding the key factor involve in rotation of pipelines. The analytical approach described in chapter 3, has been used to calculate the rotation angle at TDP while keeping the laying parameters same as for experimental test. The nominal curvature of the pipeline has been calculated and used in analytical approach to predict angles of rotation while keeping the laying parameters same. The comparison between three above mention approaches is discussed and logical reasons to difference of rotation angles at TDP in between the approaches are reported.

8.1 Discussion Parameters study of Experimental Tests Results

Understanding the rotation of the pipeline behavior through varying one parameter at a time is challenging. The pipeline rotation is dependent upon many interlinked parameters. A comparison related to different projects has been presented with the test results. It is to be noted that no data related to rotation of pipelines was found. There have been few research papers that give general overview about the rotation of pipelines. This section gives interpret the experimental test results and shows the dependency of different parameters.

8.1.1 Boundary Conditions

Boundary conditions for the pipeline during installation had large influences on the rotation of pipe. The pipe with free end for rotation has the maximum degree of rotation depending upon other laying parameters.

During the experiments, it was observed that the when the pipe was allowed to rotate freely without any torsional resistance which was problematic. Almost all of the pipe Residual curvature sections were rotated with the free bottom end to 90 degrees at Touch down point (TDP). Higher residual curvature sections rotated more than 90 degrees before the touch down point. In reality a pipeline has torsional stiffness and to compensate for this, a special, torsional device was designed and built. The torsional device restricted the pipeline rotation by resisting the torque applied by the pipe, such that the rotation was under 90 degrees.

During the experiments with fixed bottom end, it was observed that the pipeline end, at the position where the load is applied through three pullies system, the pipeline had not rotated at that end. So this means that only one boundary condition at the top end of pipeline i.e. pipe is rotationally fixed.

Two options for bottom boundary conditions at the TDP were considered and discussed:

I. Fixed end at the TDP with the pipeline being prevented to rotate

II. Partially free end at the TPD with torque applied.

The Figure 8.1 and Figure 8.2 below shows comparison between fixed and restricted boundary condition. It can be observed that the partially free bottom boundary condition with resisting torque applied by the pipe clearly results in larger angles of rotation at TDP. For water depth of 4.88 meters, at around 21N top tension, the angle of rotation is 60 degrees for partially free bottom end with torque applied, while the rotation is only 20 degrees for fixed leading end. Similar trend is seen for water depth 5.91 meters; at 38N top tension, the partially free leading end pipe rotated about 50 degrees while the pipe with fixed bottom end rotated only 22 degrees.

This shows that it is important to have accurate values for the sea bottom soil - pipe friction, for the correct analysis of the rotation of pipeline. Pipe soil friction significantly affects the rotation at TDP. The pipe-soil lateral resistance for sand, rocky or soft clay is different and the torsional stiffness of pipe behave differently with different seabed lateral resistance.

Installation of an inline structure during Pipelaying influences rotation as expected. In the close proximity of the structure, the bottom end will behave as being fixed, however this effects diminishes with increasing distance from inline structure

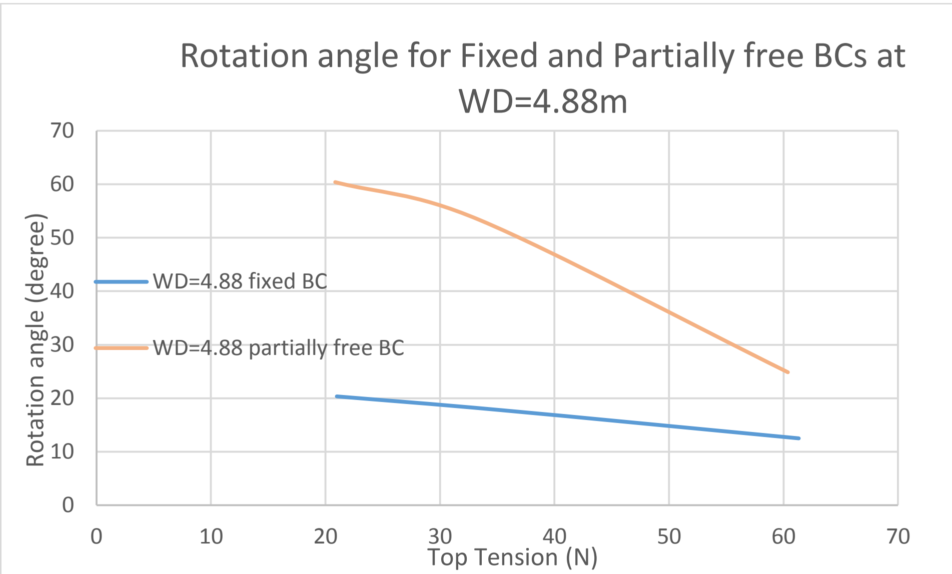


Figure 8-1 Rotation angle at TDP for different Boundary conditions at WD=4.88m, 0.3% RCS

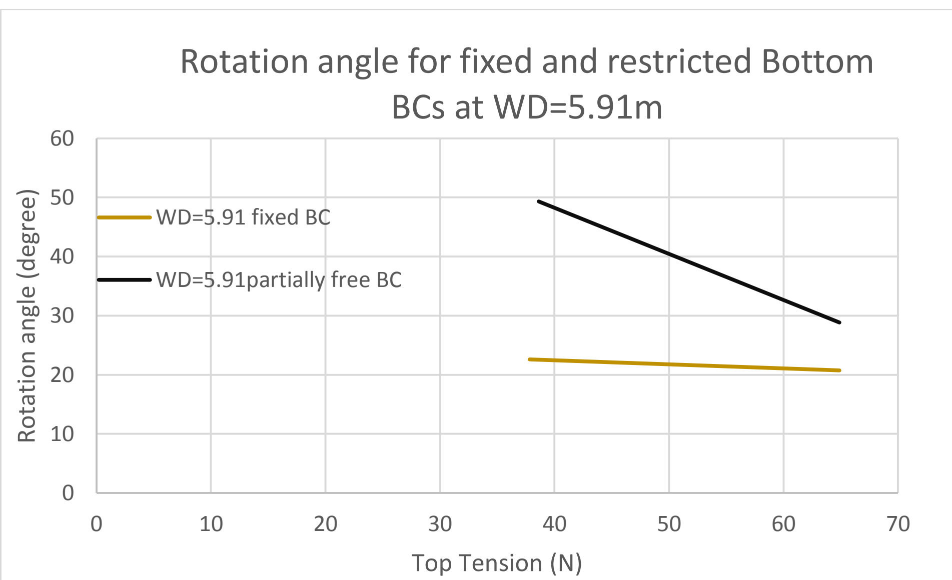


Figure 8-2 Rotation angle at TDP for different Boundary conditions at WD=5.91m, 0.3%RCS

The boundary conditions also vary during the start and the end of pipeline installation. As cable is connected to the pipeline during start of pipeline installation for laying according to the planned route, the cable can rotate the pipeline due to waves and current loads. One end of the cable is connected to an anchor on the seabed while the other end is connected to pipeline on the vessel. The cable is free to rotate, thus can make the pipeline leading end to rotate. Similarly, after installation of last pipe, the cable is used to control of stresses in sagbend, which can also cause the pipe rotation. This is case for S-lay and reeling installation method.

In (Endal, et al., 2014) the 3D FEA analysis of pipeline rotation during reeling had been performed. The analysis was done 12’’ ID and 14’’ID pipelines with clad coatings. Finite Element Analysis is performed in ABAQUS. The analysis considers two different options for bottom boundary conditions (BC):

I. Free rotation at the end of pipeline with no rotational resistance
II. Fixed bottom end at end of the pipe with high rotational resistance

The results are shown in Figure 8.3 which is taken from (Endal, et al., 2014). It can be seen that keeping all parameters same except the boundary condition, the angle of rotation for the pipe at TDP was different. The pipeline with the free end rotates 1.41 radians (80 degree) while the rotation angle for the fixed end is 0.23 radians (23 degrees) at TDP. In this analysis the torque values were not reported, so it was not possible to compare the values with those obtained in the experimental work reported in this thesis.

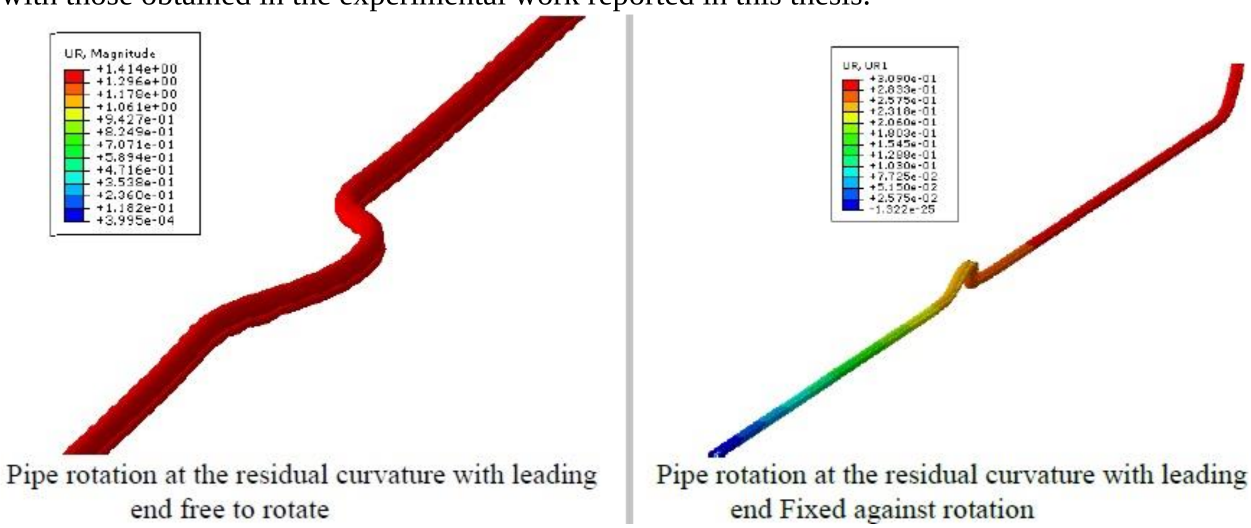


Figure 8-3 Abaqus Analysis of free end BC and fixed end BC, from (Endal, et al., 2014)

8.1.2 Top Tension Effect

With the increase in tension, there is a decreased of curvature in the sagbend of the pipe. By increasing the applied load, which increases the horizontal bottom tension and eventually pipe top tension. The results presented in table 7-47 to 7-50, shows the relationship between the top vessel tension and pipeline rotation. It can be seen that increasing the vessel tension will cause the pipe to rotate less from Figure 8-4. This is because the increase of the top tension increases the span length, as the pipe leaves the stinger early.

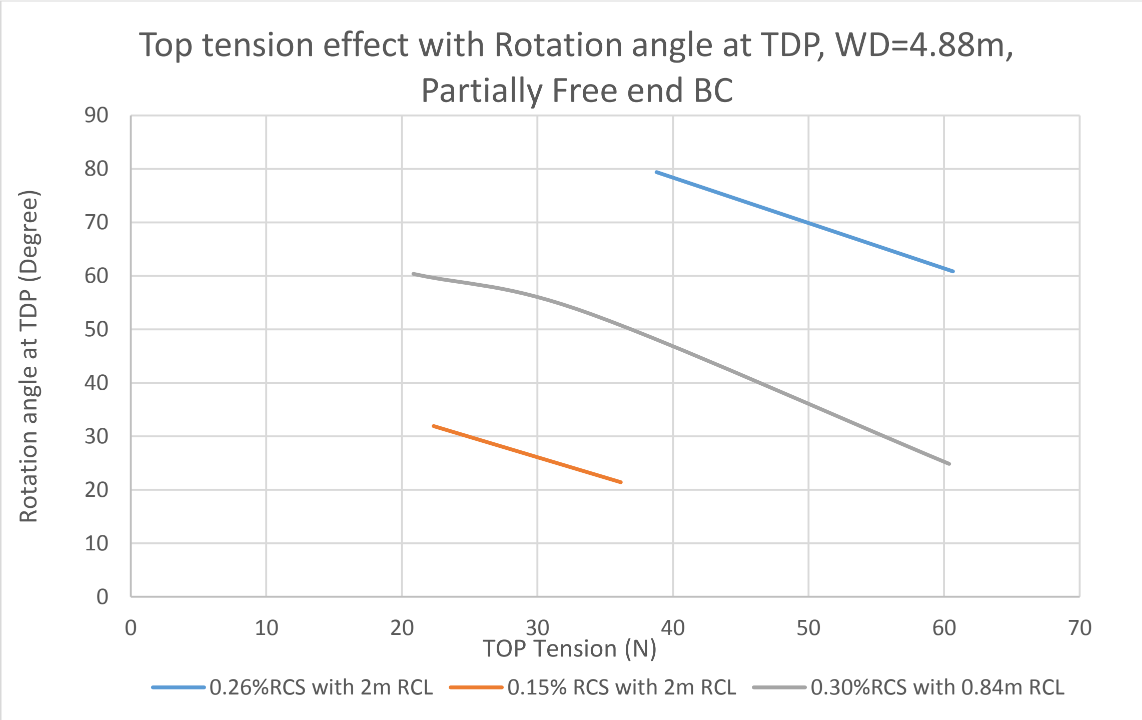


Figure 8-4 Relationship between Top tension & Rotation angle at TDP

It must be noted that the torque by the pipe applied to the device also decreased as the top tension of the pipe is increased, this is explained in next topic. The top tension in the pipe make the pipe to lay straight on the seabed. As Pipeline rotation is a consequence of the minimum potential energy principle, so high top tension restrains the pipe rotation. The high top tension changes the equilibrium forces, increasing the by restraining rotation at TDP i.e. pipe is more stable in less degree of rotation at TDP.

8.1.3 Applied Torque Effect

The applied torque applied by the pipe at the bottom end to the device is measured by the elongation of springs of the device. There are two types of trends present in applied torque with respect to angle of rotation at TDP.

When the load applied by the three pullies system is constant and the number of springs is low, the pipe rotates more. This is shown in Table 8.1. For a given residual curvature strain (RCS), the number of springs attached, which is proportional to the torque applied at the bottom end, plays a very important role and for the same top tension, pipeline rotation changes significantly. It can be seen that for 0.26% of RCS, the applied torque of 70.63 Ncm and 44.73 Ncm correspond to 79.40 and 89.99 degrees of rotation respectively. Similarly, for 0.15% of RCS the applied torque of 15.54 Ncm and 8.95 Ncm correspond to 15.12 and 31.92 degrees of rotation respectively.

Table 8-1 Effect of resisting torque for same applied load for different RCS

Water depth :4.88m							
Bottom boundary condition: Free to rotate with torque measurement							
Test Number	Horizontal Tension (N)	Top Tension (N)	Number of Springs attached	Residual Curvature strain (%)	Residual Curvature length (m)	Resisting Torque (Ncm)	Angle at TDP (Degree)
Test 1	35.28	38.79	3	0.26	2	70.63	79.40
Test 2	35.57	39.06	1	0.26	2	44.73	89.99
Test 4	16.07	22.20	3	0.15	2	15.54	15.12
Test 5	16.27	22.34	1	0.15	2	8.95	31.92

Applied torque is plotted against the top tension for different residual curvature strain values and the graph is shown in Figure 8.5. It can be seen that larger values of top tension correspond to lower values of applied torque and the trend is the same for different residual curvature strain. This seems to be valid as the vessel top tension increases the pipe resistance to rotate. large diameters will eventually have larger torsional stiffness and will rotate less. The torque applied by the pipe was depended upon the residual curvature strain and residual curvature length. From the figure 8-5, it can be seen that the higher residual curvature strain and larger residual curvature length gives more torque in the pipe to rotate. Although the pipe with 0.30% residual strain is higher than 0.26% residual strain but the residual curvature length is small for 0.3%residual strain, thus make it to apply less torque than the 0.26% of residual strain. In case of 0.26% Residual strain and 0.15% residual strain, both have same residual curvature length of 2m, the 0.26% residual strain tends to rotate more which is validation to theory.

As the pipe has torsional resistance at the bottom end due to seabed soil friction, uneven sea bed, buoyancy, and submerged weight per unit mass that cause the pipe to rotate less. In (Damsleth, et al., 2000) the rotation challenges during the Statoil Åsgard Field in the haltenbanken area discussed especially installation of Midgard Tees (inline structure). It is mention there that at TDP, the pipeline penetrates into the seabed soil due to seabed soil nature. This makes the pipeline to have passive approach against the rotation. The pipeline-seabed torsional resistance increase over few kilometers until there is a steady point where the torsional moment due to sagbend, tensioners tension slip and seabed-pipeline soil friction are in a state of equilibrium.

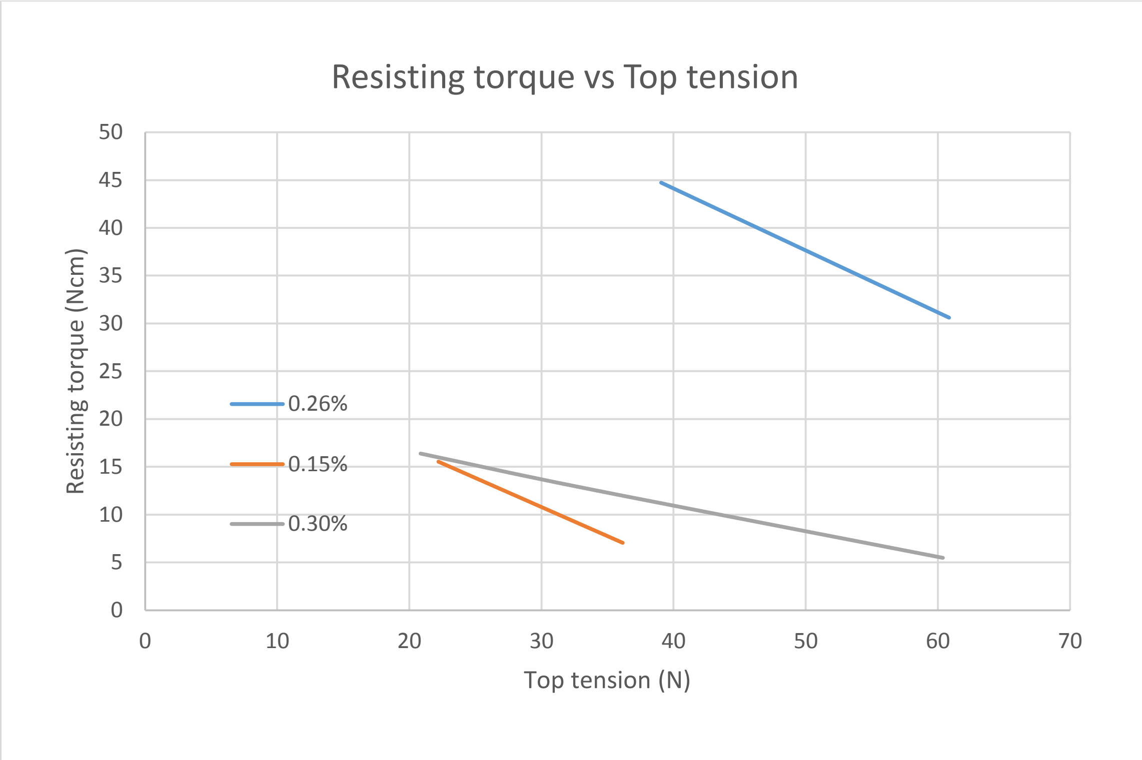


Figure 8-5 resisting torque vs Top tension for different Residual curvature sections

In (Endal, et al., 2014) during analytical and Finite element method calculations the soil-seabed friction is ignored. The seabed is considered flat and pipe end is free to rotate. But it is mention there that results obtained from the calculation would be exaggerated due to these simple assumptions. Thus to replicate the real life scenario, it was worthy to design and build the torque measuring device. These results give an important aspect that the during pipe rotation analyses the seabed soil friction and contact forces between the pipe and seabed soil surface needs to be consider carefully. The pipe torsional stiffness due to its geometry and material needs to be accounted in the analyses.

It is mention in (Damsleth, et al., 2000) that during FEA analysis of pipeline rotation, the seabed soil-pipeline friction was ignored as it required high computational power and time. The numerical solution with these condition takes longer time to reach the steady state.

The experiments are dry tests so torque generated by the pipe is only due to the residual curvature. From experiment it is seen that the torque if high for higher water depth, as illustrated in the Figure 8-6. In real, the torque may be generated due to following:

- Laying curves
- Offset of the vessel
- Waves and current forces
- Vessel dynamics
- Misalignment of the pipes

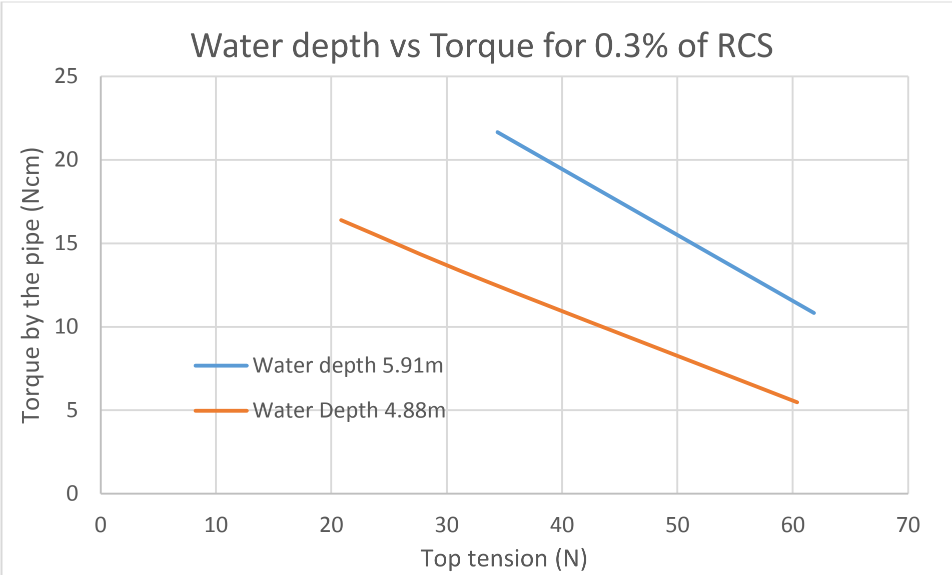


Figure 8-6 Water depth and torque relationship

8.1.4 Residual Curvature Strain

The residual curvature is one of the main causes of the pipeline rotation. If there are two pipes, laying from S-lay. One has residual curvature strain while the other one is straight. During laying the pipe with residual curvature has high potential energy than the pipe with no residual curvature. At sagbend, the residual curvature pipe opposes the reverse bending and will be in a state of high potential energy while the second pipe, bends to reach the lowest potential energy and will be in equilibrium. Thus for residual pipe it is more easy to rotate then reverse bending to reach its lowest potential energy. The degree of rotation depends upon the length of plastic region and amount of plastic strain.

The experiment results were satisfied as they completely follow the physics of rotation. From the experimental measurements it was found that two pipes with the same residual curvature length but different residual curvature strain showed different behavior. This can have observed in the Figure 8-7. For 0.26% RCS and 0.15% RCS, the top tension of 38 N the angle of rotation at TDP is 79 Degree and 7 degrees respectively.

The residual curvature in the experiment were pre-bend before laying of the pipeline. While in real life, residual curvature is produced during installation. In S-lay installation the exceeding nominal strain in the overbend region on the stinger gives the residual curvature. The residual curvature strain level depends upon the level of nominal strain in the overbend region. For example, in the installation of 42'' pipeline during the Åsgard Transport offshore with S-lay, the stinger was configured with 0.25% of nominal tensile strain which gives 0.05%-0.065% of plastic or residual strain (Endal & Verley, 2000). Due to large diameter of pipeline the 0.05% residual strain had rotate the pipe up to 80 degrees. While in Reel-lay Installation method, an adjustment in the straighteners during unspooling can produce the residual curvature of desired radius and length. The straighteners system consists of three-point bending and it is controlled hydraulically. This method first implemented on 26 km long 14''-16'' pipes in the Norwegian sea in 2012. (Endal & Egeli, 2014)

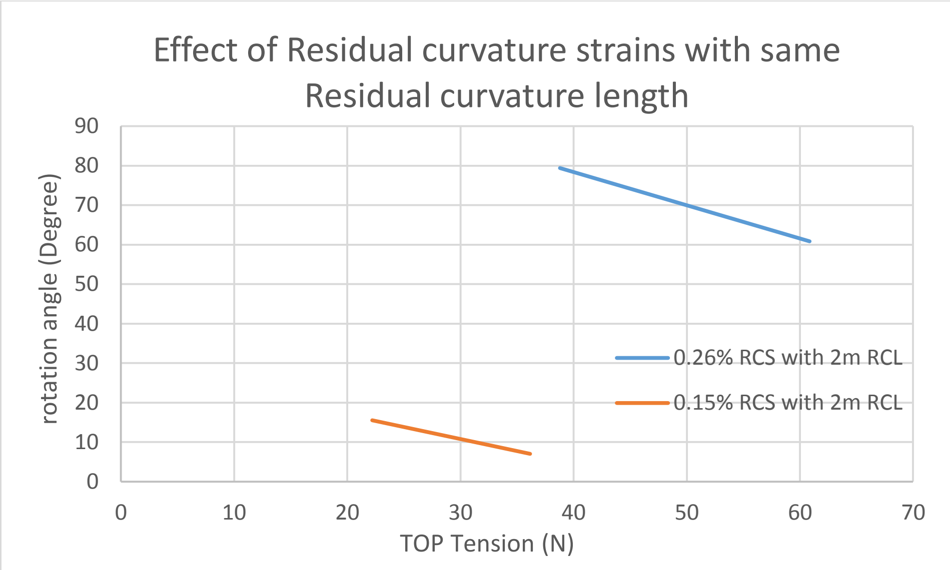


Figure 8-7 Effect of residual curvature strain with same residual curvature length

8.1.5 Residual curvature length effect

The residual curvature length has a significant influence on the pipeline rotation. It had been observed during experiments that when the residual curvature strain is high but the residual curvature length is small, the pipeline will have less rotation at the seabed as shown in figure 8-8. The two pipes with residual curvature strain (RCS) of 0.3% and 0.26% have the corresponding residual curvature length (RCL) of 0.84m and 2m. It could be seen that the pipe with 0.26% RCS have high angle of rotation then the pipe with the 0.30% of RCS even pipe with residual curvature strain of 0.30% has is more. These results validate with the theory of rotation i.e. with increase in residual curvature length there is large value of minimization of energy in rotation.

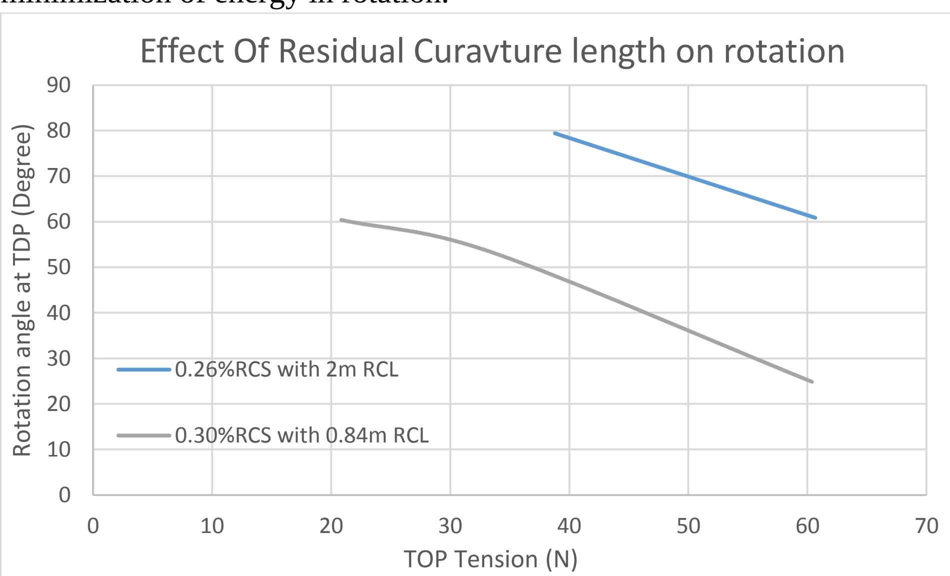


Figure 8-8 Effect of Residual Curvature length

In (Endal & Egeli, 2014), describes Statoil Skuld project in the Norwegian Sea using Subsea 7-reel ship "Seven Ocean. It has shown that changing the length of residual curvature while keeping the residual strain constant, the angle of rotation changes at TDP. It was found that the residual curvature length of 50m-100m rotates around at angle of 80^0-100^0 degree at TDP. The Figure 8-9 shows the relationship between the different residual curvatures length for 0.2% of local residual strain for 16'' inch pipeline. The residual curvature lengths were 50m, 70 and 100m. The residual curvature length of 70m as it rotates around 90 degrees at TDP. The 0.2% of residual strain was obtained for 16''inch pipe by bending it around 100m bending radius, which gives 0.41% nominal strain. The pipeline was rotated by 90^0 during installation which make the residual curvature section to be laid in horizontal plane. The as-laid survey from the Remotely Operated Vehicle (ROV) data confirms that residual curvature section rotated 90 degrees.

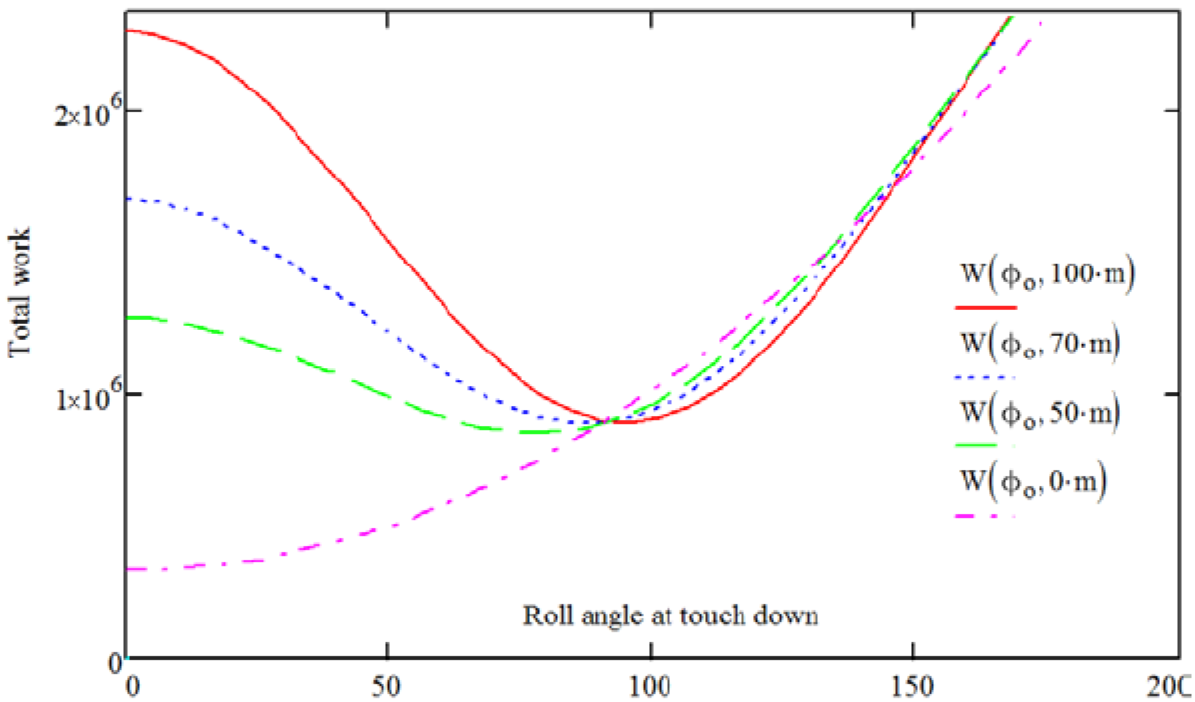


Figure 8-9 Prediction of rotation angle at TDP, Statoil Skuld Project, from (Endal & Egeli, 2014)

8.1.6 Water depth

Increasing water depth amplifies the pipeline rotation. This is due to larger suspended length of the pipeline exposed to external and internal forces. During the experimentation it has been observed that the rotation angle increases for increase in water depth while keeping all other parameters i.e. RCS, RCL, Applied Load and boundary conditions constant.

For fixed bottom Boundary condition, with top tension around 60N, the rotation angle is 12 degrees and 21 degrees for water depths of 4.88m and 5.91m respectively, shown in Figure 8- 10. Similarly, for partially free bottom boundary condition the top tension around 60N, the rotation angle is 24 degrees and 28 degrees for water depths of 4.88m and 5.91m respectively, Shown in Figure 8-11.

It is mention in (Endal & Verley, 2000) for 42'' inch Åsgard Project (1998-1999) where the water depth varies from 65m to 370m, the pipeline rotation was mainly observed when the water depth exceeds about 300m, shown in Figure 8-12. Åsgard Project employs S-lay installation method with fixed Stinger configuration. This rotation causes a problem in installing 8 inline tees for future tie-in to other fields. The rotational tolerance angles for inline tees (connection points) were ± 15 degrees but it was preferable within ± 3 degrees. Measured rotation angles were 40 degrees and 80 degree of rotation when measured through as-laid and touch down monitoring respectively.

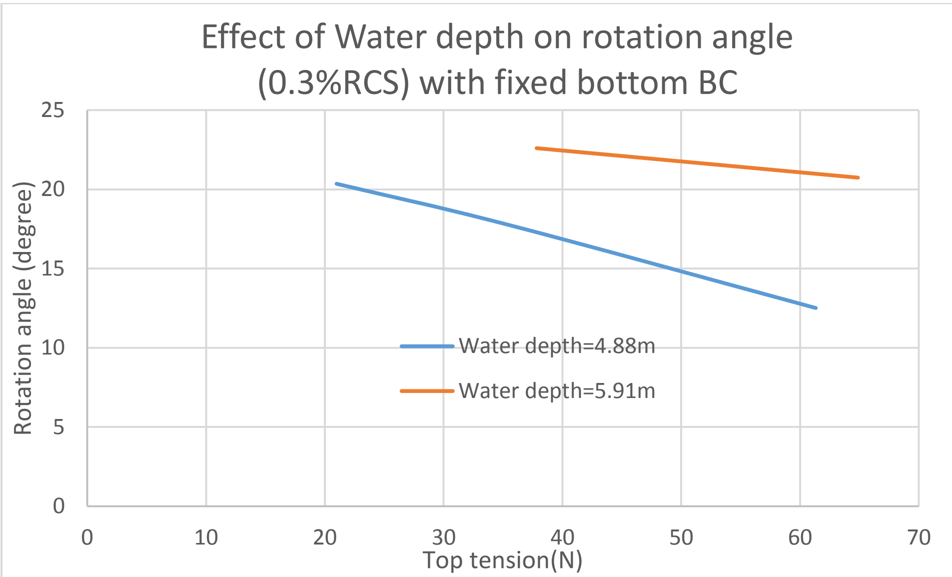


Figure 8-10 effect of water depth on rotation angle with fixed bottom BC

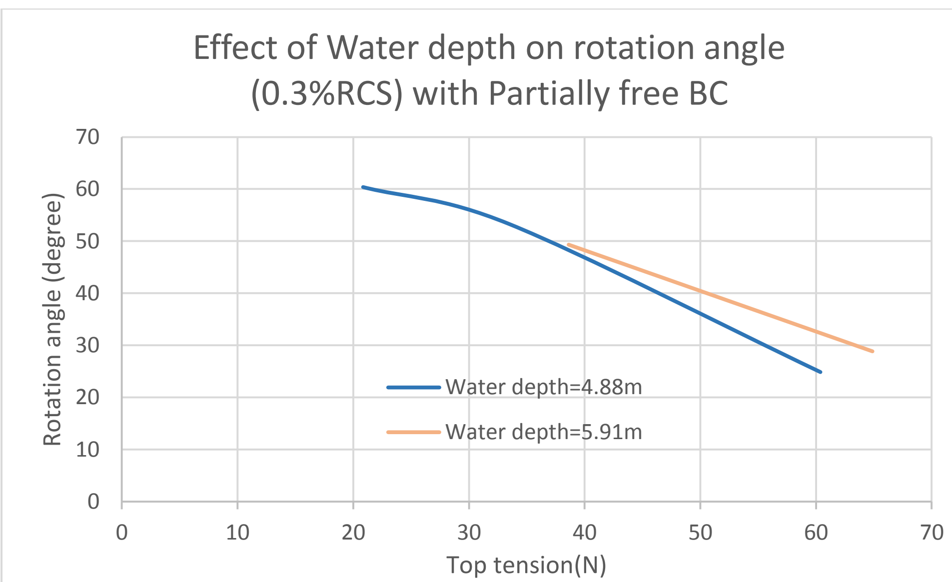


Figure 8-11 effect of Water depth with partially free BC.

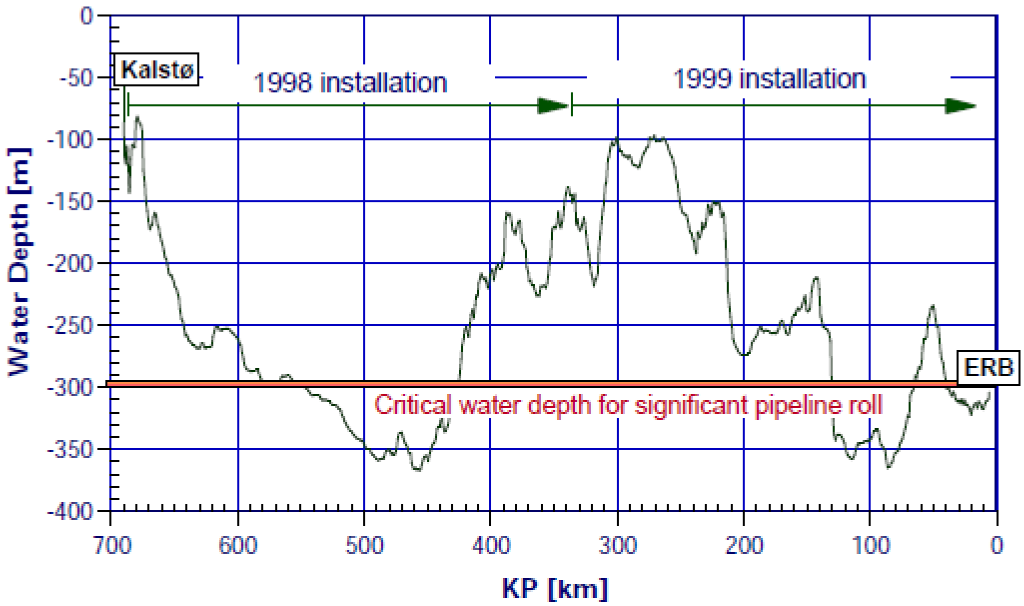


Figure 8-12 Critical water depth for high rotation during installation for Åsgard Transport Pipeline, from (Endal & Verley, 2000)

The Figure 8-13 ((Endal & Verley, 2000) illustrate the effect of increasing the water depth on residual curvature strain for initiation of 10-degree rotation at TDP. The figure 8-12 is for 42-inch pipeline. For example, a 0.18% of residual strain is required for the 200m of water to rotate as compared to 0.10% of residual strain is required for 300m water depth.

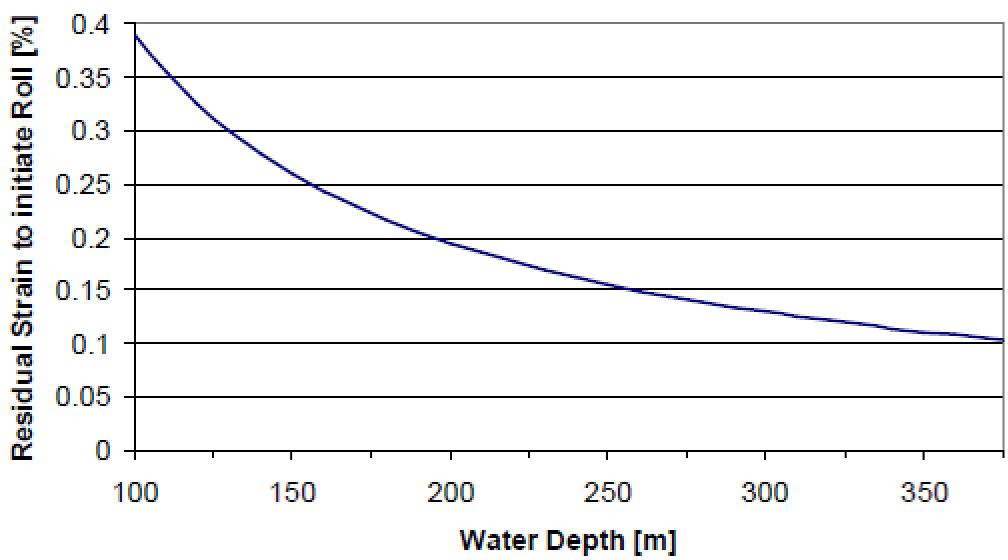


Figure 8-13 Residual strain requirement for having 10 degree of roll for Åsgard transport pipeline. From (Endal & Verley, 2000)

8.1.7 Inline structure

The inline structures provide more economical ways by directing the output of oil fields into existing operations oil fields for processing and production. These inline structures are installed during pipeline installation and involves many challenges tasks. There are other inline structures that required to be installed at the start, in the middle or at the completion of the pipeline system. They are mainly divided into two groups:

I. Inline structures: either ends of pipe e.g. PLET, PLEM
II. End Structures: middle of pipelines e.g. Inline tees, Inline SLEDS, WYES

The inline structure shows minimization of gravitational potential energy for the pipeline. This pipeline tends to rotate to achieve the minimum gravitational potential energy. The 0.783kg weight of dummy structure attached in test 7. From the table 8-2, it can be seen that pipe with inline structure rotates 90 degrees while pipe without inline structure rotates less.

Table 8-2 2 Effect of inline structure on the rotation of pipeline

Water depth :4.88m								
Bottom boundary condition: Free to rotate with torque measurement								
Test Number	Applied Load (N)	Horizontal Tension (N)	Top Tension (N)	Residual Curvature strain (%)	Residual Curvature length (m)	Resisting Torque (Ncm)	Angle at TDP (Degree)	Inline structure mass (Kg)
Test 4	10	16.07	22.20	0.15	2	15.54	15.12	--
Test 5	10	16.27	22.34	0.15	2	8.95	31.92	--
Test 7	10	15.68	21.92	0.15	2	39.55	90.00	0.783

Ideally the inline structure should be installed as shown in Figure 8-14. In research papers by the (Damsleth, et al., 2000) and (Endal & Verley, 2000), the rotation of inline tees challenges were discussed for Statoil Åsgard project. During the project, the pig launcher was rotated about 30 degrees before it was 10 m above the seabed but after 1.5km of pipe is laid, the pig launcher was rotated about 170 degrees (Damsleth, et al., 2000).

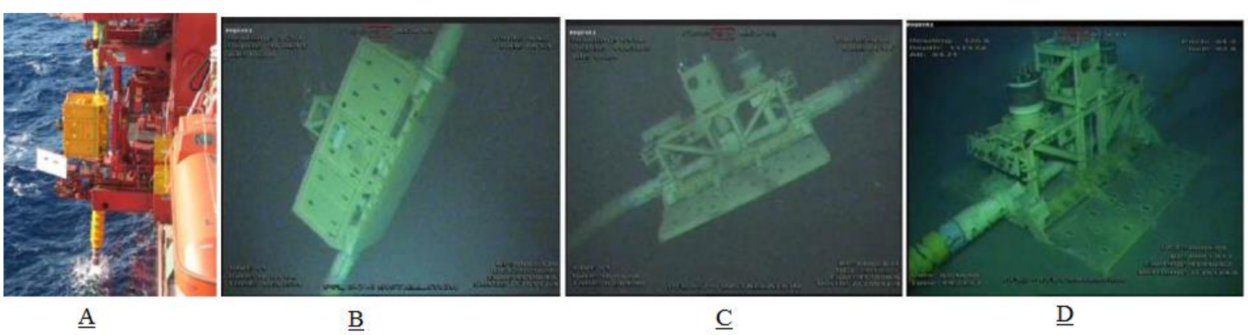


Figure 8-14 Installation of inline SLED from sea surface to seabed from (Huang, et al., May, 2009)

8.2 Analytical Calculation: Modified Energy Method

Analytical calculations based on (Endal, et al., 2014) as discussed in section 3 with topic name "modified energy method". Note that in (Endal, et al., 2014) the pipeline is free to rotate at the bottom leading end while in experiments it was partially fixed with torque measurement.

The total work done is calculated for each test using equation 3.20 from section 3 and angle of rotation is obtained. The parameters that varied for each test were residual curvature strain, residual curvature length, top vessel tension, bottom horizontal tension and length of the pipeline from the stinger tip to the TDP.

8.2.1 For Water depth=4.88m

The total work done and rotation angle calculated by using the modified energy method for water depth of 4.88m is shown in Figure 8-15 and Figure 8-16. Summary of rotation angle is given in Table 8-3.

Table 8-3 Summary of Analytical results for with same parameter test 1-10, except BC is changed

Summary of Analytical Calculations					
Water depth :4.88m					
Bottom boundary condition: Free End to rotate					
Test Number	Horizontal Tension (N)	Top Tension (N)	Residual Curvature strain (%)	Residual Curvature length (m	Analytical rotation angle (Degree)
Test 1	35.28	38.79	0.26	2	77.4
Test 2	35.57	39.06	0.26	2	77.4
Test 3	60.27	60.66	0.26	2	77.1
Test 4	16.07	22.20	0.15	2	54.3
Test 5	16.27	22.34	0.15	2	54.1
Test 6	32.44	36.15	0.15	2	50.9
Test 8	29.40	33.49	0.30	0.84	63.9
Test 9	14.11	20.85	0.30	0.84	64.7
Test 10	57.92	60.38	0.30	0.84	60.3

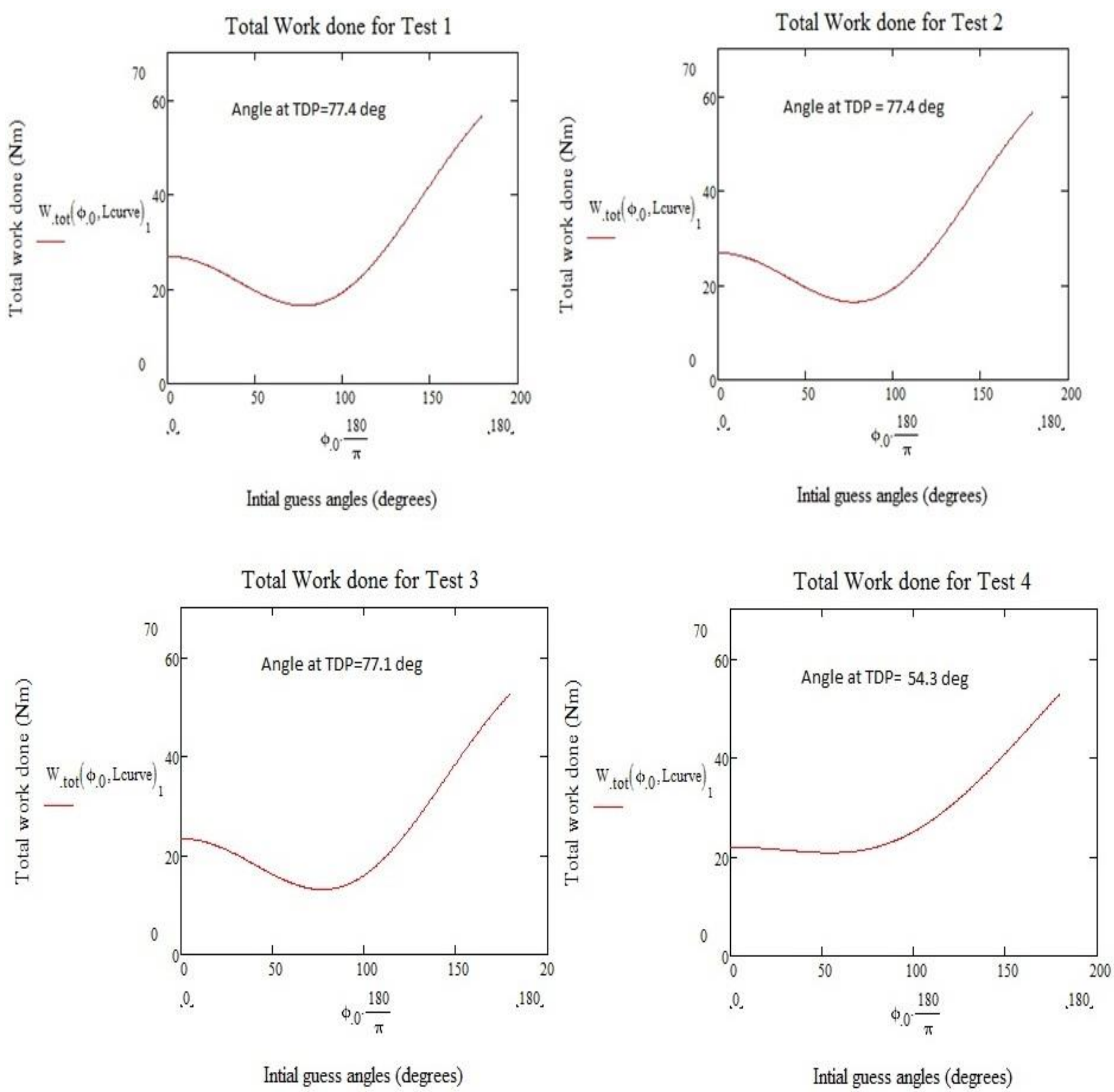


Figure 8-15 Rotation angle at TDP by modified energy method for the test 1, test 2, test 3 and test 4.

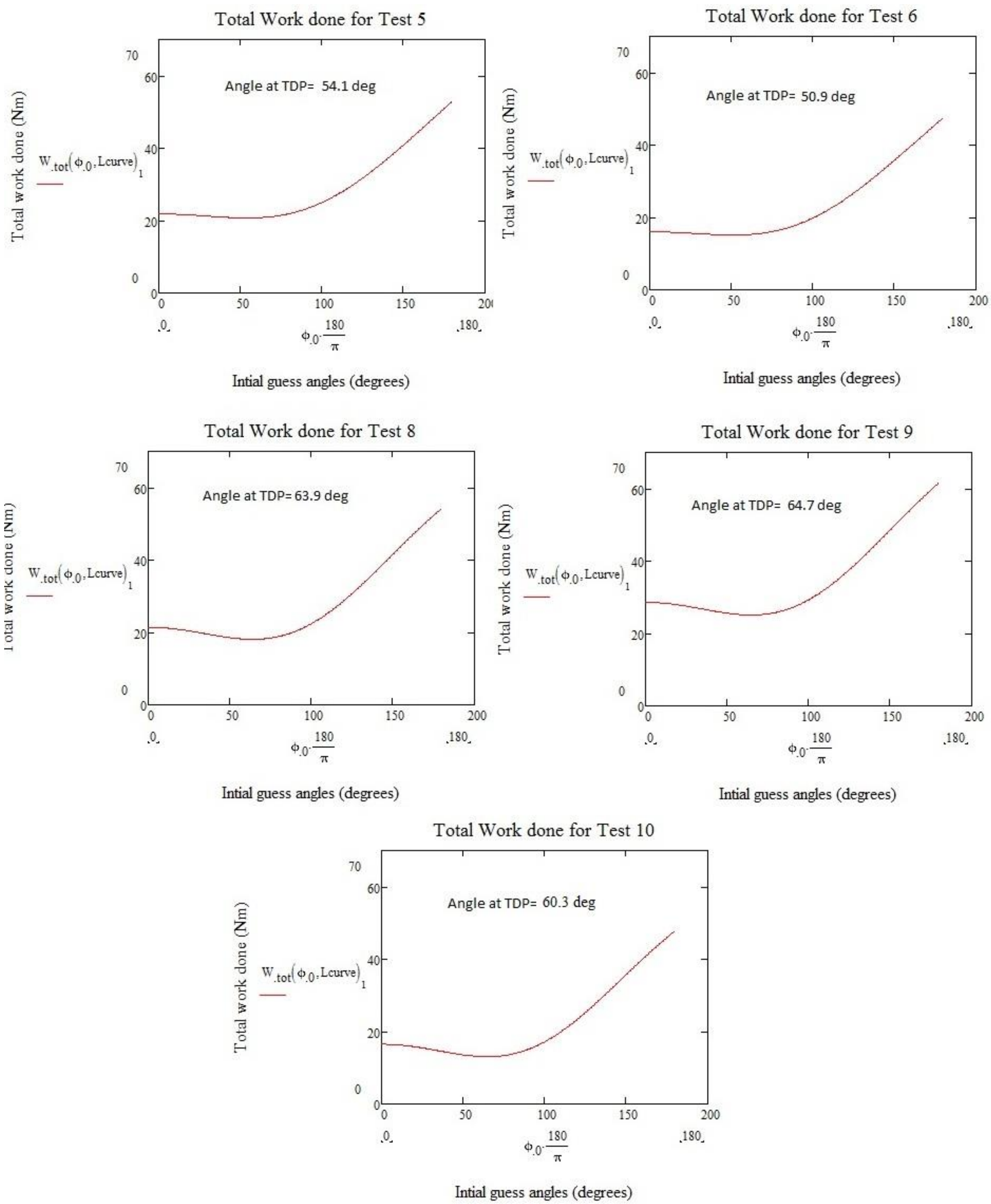


Figure 8-16 Rotation angle at TDP by modified energy method for test 5, test 6, test 8, test 9 and test 10.

8.2.2 For Water depth=5.91m

Similarly, the work done and rotation angle for the water depth of 5.91 m had been calculated and presented in Figure 8-17. The summary along with laying parameters is shown in Table 8-4.

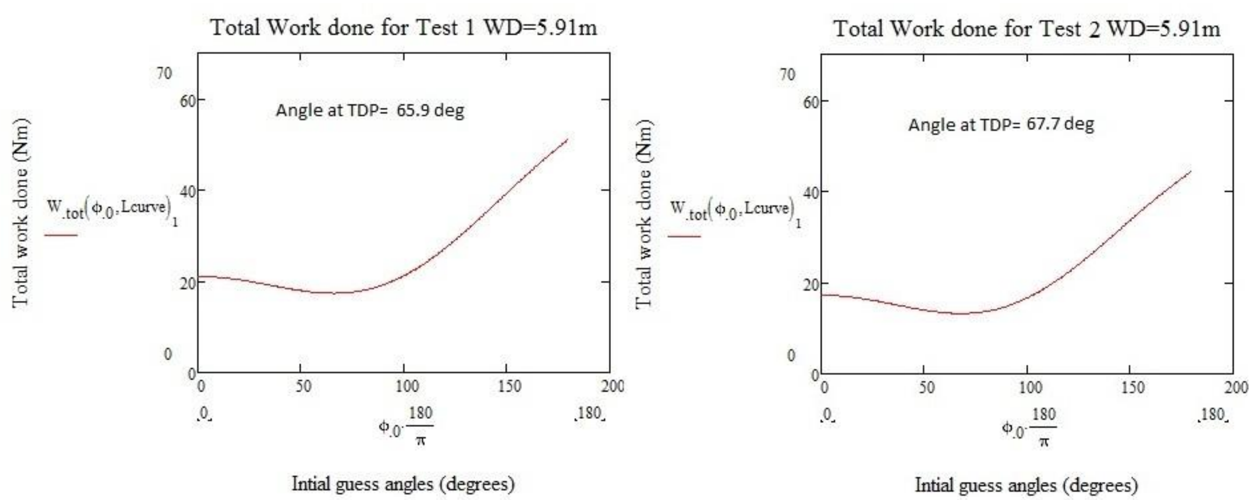


Figure 8-17 Rotation angle at TDP by modified energy method for test 1 and test 2

Table 8-4 Summary of Analytical results for with same parameter for test 14-15, except BC is changed

Summary of Analytical Calculations					
Water depth :5.91m					
Bottom boundary condition: Free End to rotate					
Test Number	Horizontal Tension (N)	Top Tension (N)	Residual strain (%)	Residual length (m)	Analytical rotation angle (Degree)
Test 14	34.40	38.62	0.30	0.84	65.9
Test 15	61.84	64.88	0.30	0.84	67.7

The modified energy method equations along with parameters is listed in Appendix B. The parameters for each test were different and listed in section 7. The experimental tests 11,12,16 and 17 had fixed bottom end boundary conditions, so rotation angle for them is not calculated. As modified energy method does not account for fixed bottom end of pipeline, thus these tests can be ignored.

8.3 Nominal Curvature from the experiment

During the experiments the nominal curvature of the pipe laying was determine. The nominal curvature of the pipeline mainly depends upon following factors:

- Water depth

- Top vessel tension
- Submerged weight per unit of the pipe
- Pipeline material and geometric properties

Since the copper pipe of 10 mm used for the whole experiment (explained section 5.1), so water depth and Top vessel tension were the influencing factor of the nominal curvature along the pipeline suspended section in these experiments.

The nominal curvature of the pipeline give comparison to the nominal curvature given by the (Endal, et al., 2014), which is used in the equation for finding analytical rotation angle at TDP.

There are many ways to find the curvature from the experimental data but following method found to be most appropriate (Vaughan, 2016). The method compromises of following:

a) Take the pictures of the pipe laying from stinger tip to the Touch down point with the grid background used in experiment.
b) Picture were digitalized using software WebPlotDigitizer (Rohatgi, 2016). The horizontal and vertical data points were generated.
c) The horizontal and vertical points plotted on graph to get catenary shape of the pipe. The catenary shape fitted by third order polynomial equation to satisfy the shape of the pipe catenary.
d) The arc length ds of the pipe were calculated by taking difference of data points (x and z data points) and Pythagorean theorem, shown in Figure 8-18 i.e.

$$ds = \sqrt{dx^2 + dz^2}$$

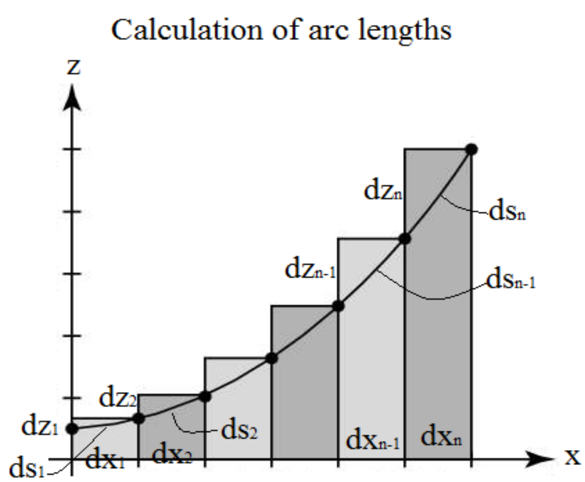


Figure 8-18 arc lengths calculation methodology schematic

e) The angle theta was calculated by solving:

$$cos\theta = \frac{dx}{ds} \ \ or \ \ sin\theta = \frac{dz}{ds}$$

Note that both equation give same angle theta.

f) The nominal curvature kappa k(s) calculated by numerically differentiate the angle theta with respect to arc length i.e.:

$$k(s) = \frac{d\theta}{ds}$$

g) Total length of suspended pipe section obtained by adding all small arc length i.e.

$$S = ds_1 + ds_2 \dots \dots ds_n$$

h) A graph has plotted between the nominal curvature kappa k(s) and Total arc length S along the pipeline.

Since the experiments were performed for three different applied loads and two different water depths. So five nominal curvature of the pipeline were calculated.

8.3.1 Water depth 4.88m with 10n Applied load

The catenary shape of the pipeline for applied load of 10N is obtained as shown in figure 8-19 from the digitalization of the pictures.

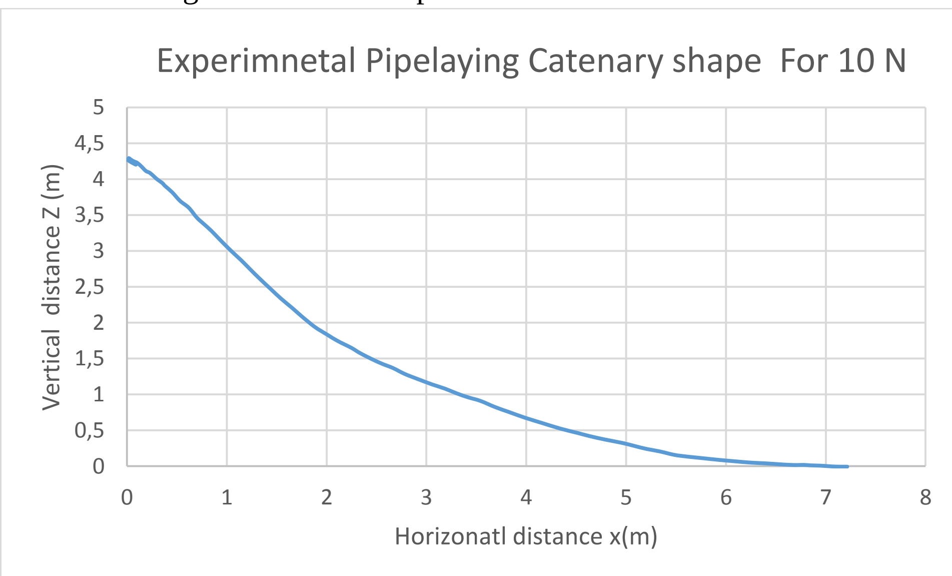


Figure 8-19 Pipe catenary for the applied load of 10N, WD=4.88m

The nominal curvature of the pipe is generated through the steps discuss above. The data points, arc lengths, angle theta and nominal curvature for applied of 10N at water depth of 4.88m is attached in Appendix E-1.

The nominal curvature data points for the pipeline is calculated and drawn on Matlab as shown in Figure 8-20. The curve fitting of the nominal curvature curve is done with quadratic, cubic and 4th degree of polynomial shown in figure 8-21.

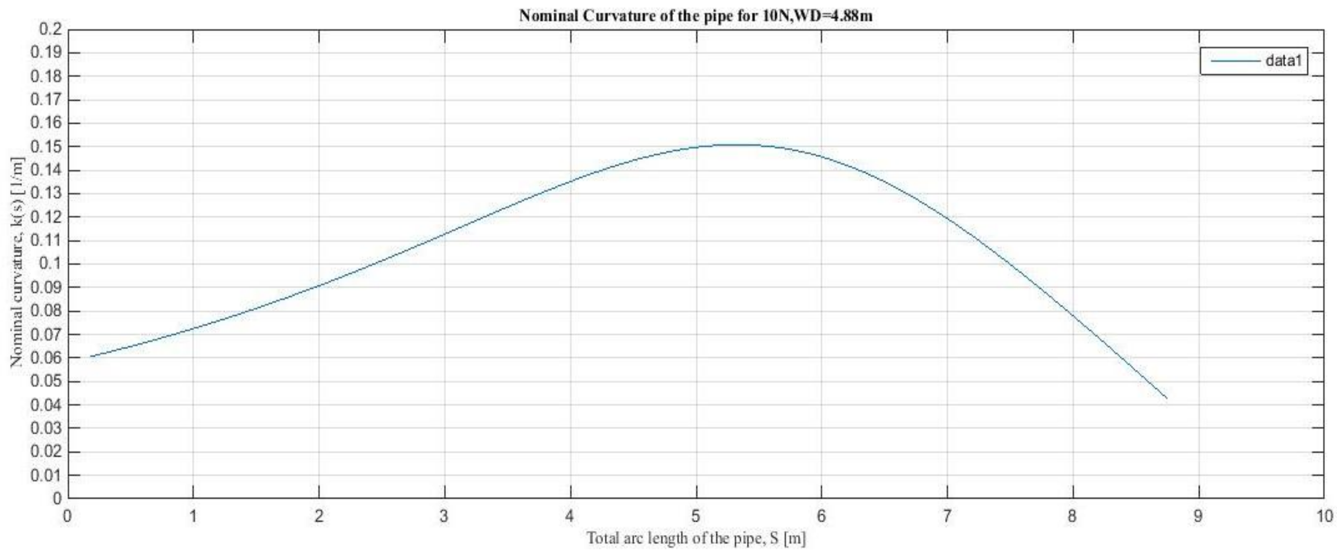


Figure 8-20 Experimental Nominal curvature curve of the pipe for Applied load of 20N, WD=4.88m

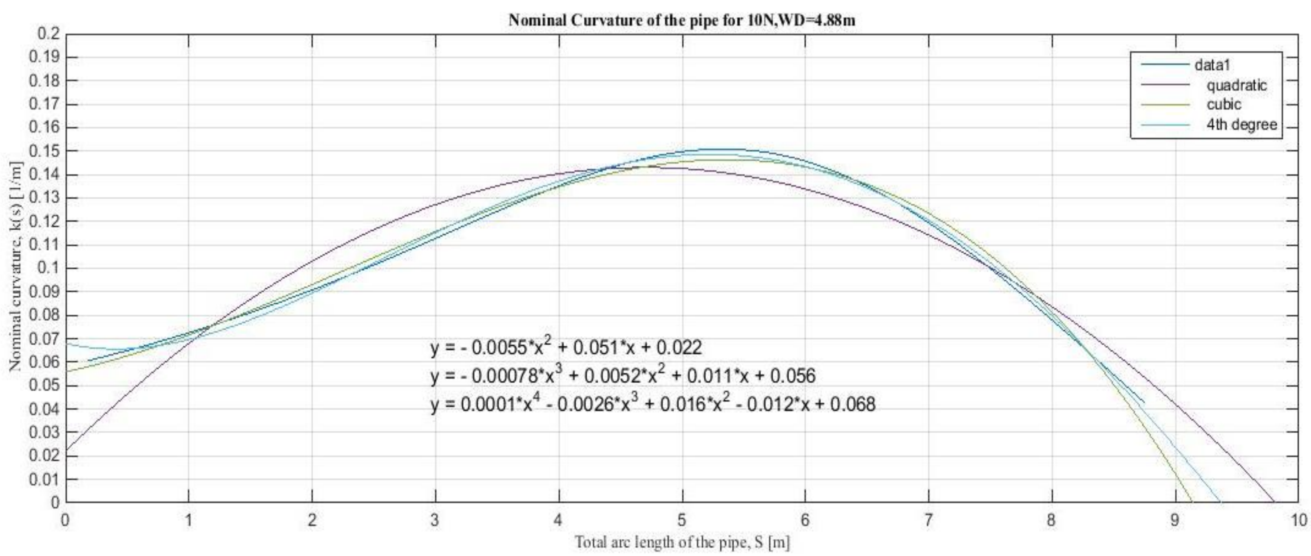


Figure 8-21 Experimental Nominal curvature curve fitting for applied load of 20N, WD=4.88m

It can be seen from the Figure 8-21 that none of polynomial is completely fit the curve, thus to know how much each polynomial was fitted good, norm of the residual was calculated. Matlab fitted the pipeline original nominal curve by predicting the values at each point for different polynomials (Mathworks, 2016). The difference between the original value and predicted value is called as residual. It is defined as

$$d_i = y_{ioriginal} - y_{i\,predicted}$$

The norm of the residual is defined as

$$norm(d) = \sqrt{\sum_{i=1}^{n} {d_i}^2}$$

Norm of residual is the sum of all the residuals at each point of the curve. It is used to know "goodness of the fit". Thus, the lower the norm of the residual, better the curve is fitted.

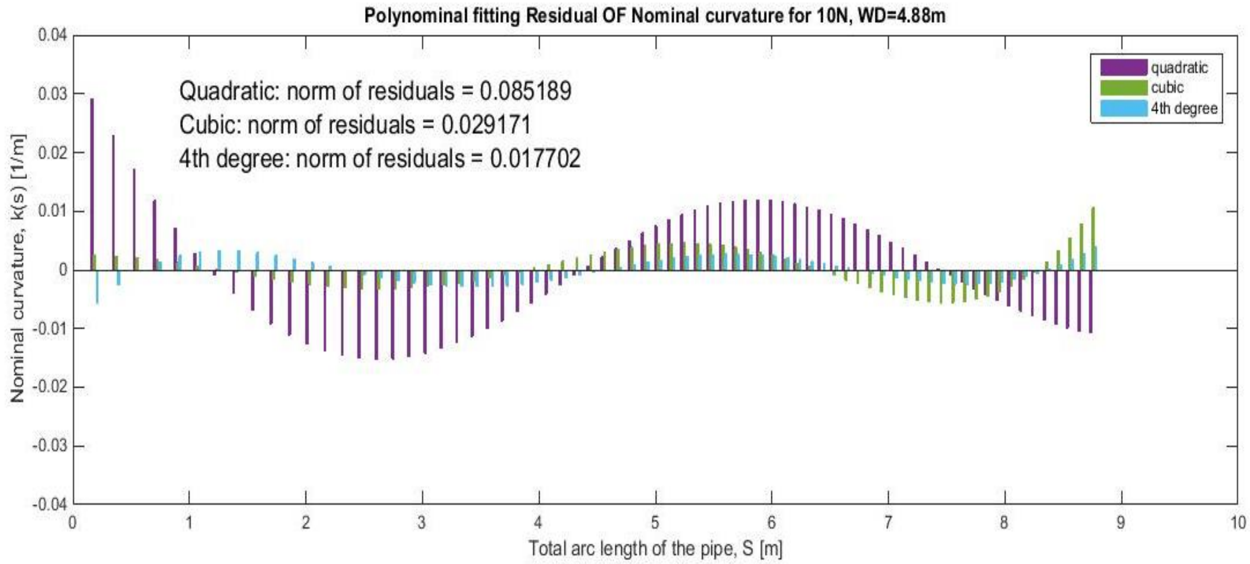


Figure 8-22 Residuals of each curve fitted for applied load of 10N, WD=4.88m

From Figure 8-22, it can be seen that 4th degree of polynomial has norm of residual of 0.0177. Thus 4th degree of polynomial represents the nominal curve. During fitting it has been observed that there is no significant difference if the order of the polynomial is increased to further.

The 4th degree of polynomial for nominal curvature is used in modified energy equation to calculate the total work done. It replaces the equation 3.14 from the section 3. The applied load of 10N at water depth of 4.88m was applied in test 4, test 5 and test 9. Since the test 4 and test 5 are almost same except the bottom boundary resisting torque condition. This difference does not account in modified energy method calculation due to assumption of free end of pipe at bottom boundary condition in energy method.

The total work done from modified energy method is calculated for 0.15% of Residual curvature strain and 0.26% of residual curvature strain with corresponding 2m and 0.84m residual curvature length respectively ash shown in figure 8-23 and figure 8-24. The angle of rotation is 65.1 degree for 0.15% RCS and 86.4 degree for 0.30%. A Mathcad sheet is attached shows the equation used for nominal curvature in modified energy method. See Appendix B and Appendix E-6

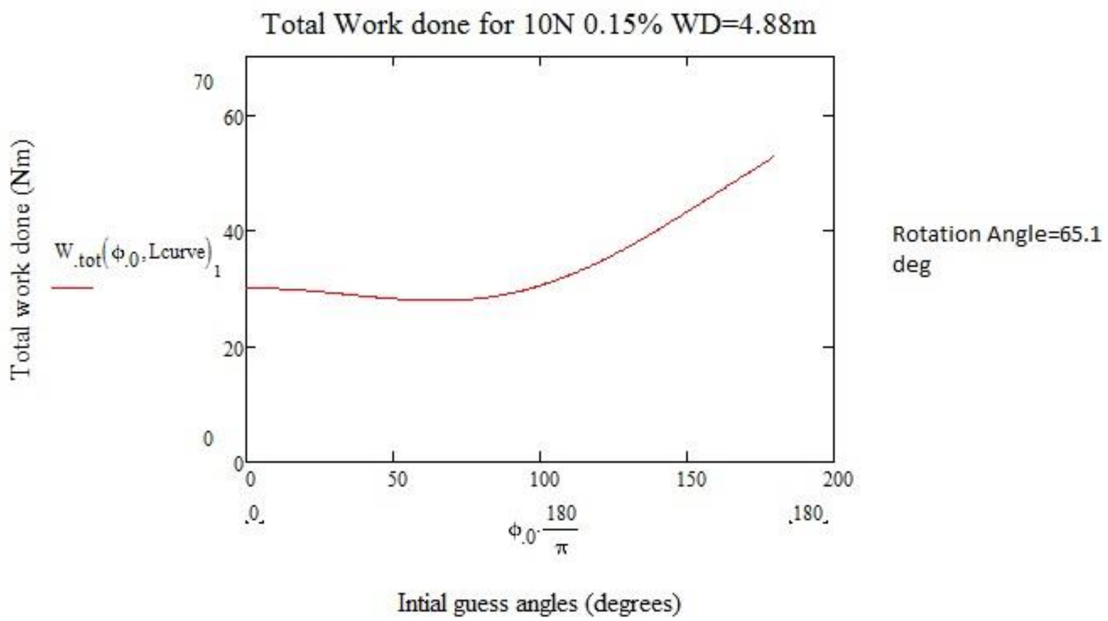


Figure 8-23 Total work done from experimental nominal curvature for Applied of 10N, 0.15% RCS, WD=4.88m

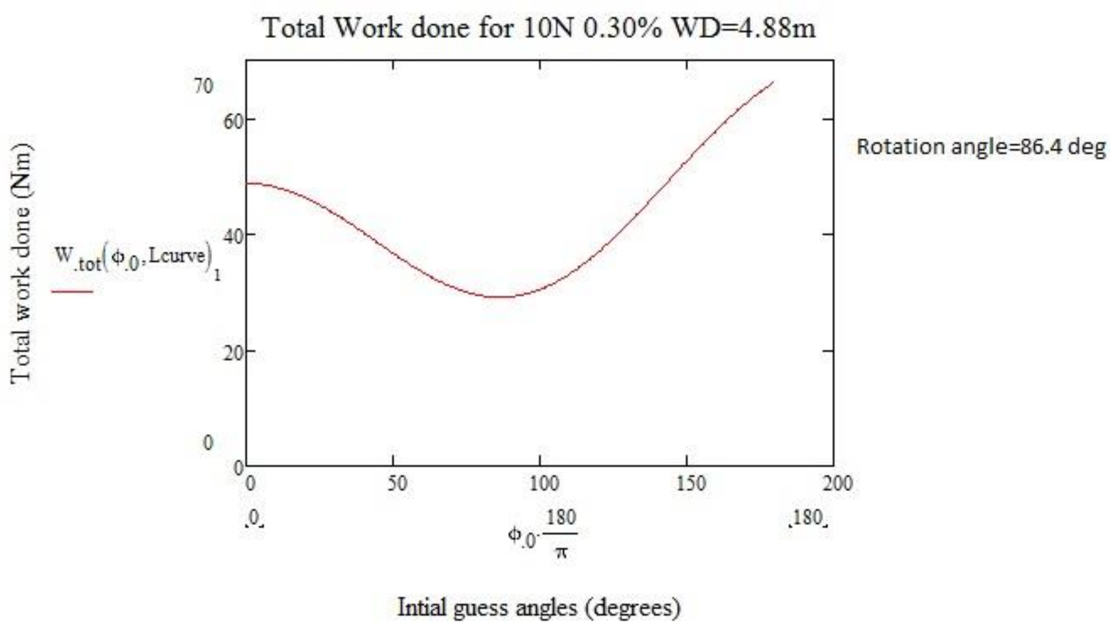


Figure 8-24 Total work done from experimental nominal curvature for Applied of 10N, 0.30% RCS, WD=4.88m

8.3.2 Water depth 4.88m with 20N Applied load

The catenary shape for applied load of 20N at water depth of 4.88m was obtained as shown in figure 8-25 from the digitalization of the pictures.

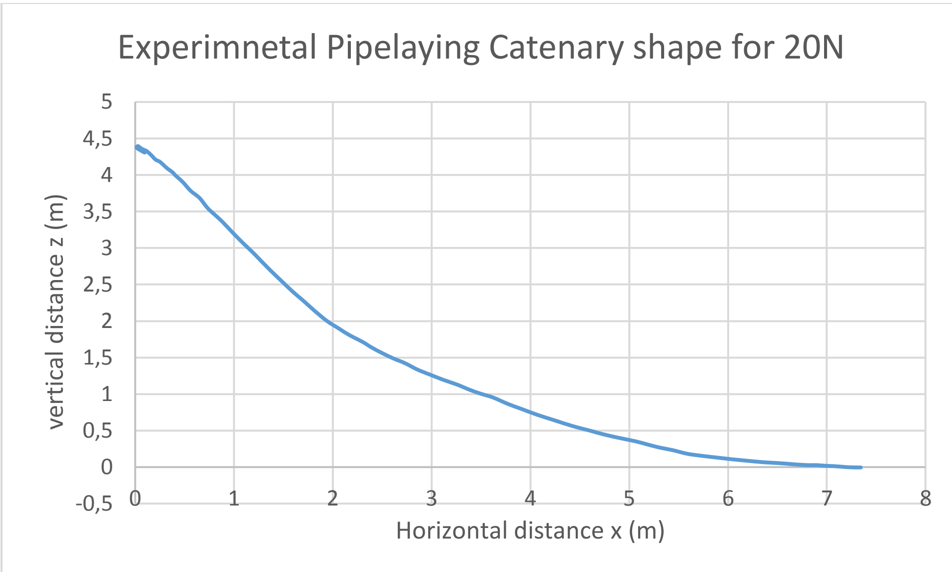


Figure 8-25 Experimental Pipe catenary for the applied load of 20N, WD =88m

The pipeline experimental nominal curvature calculated for applied load of 20N and water depth of 4.88m as shown in figure 8-26. All data points and corresponding calculated values for the calculation of nominal curvature for the applied load 20N attached in Appendix E-2.

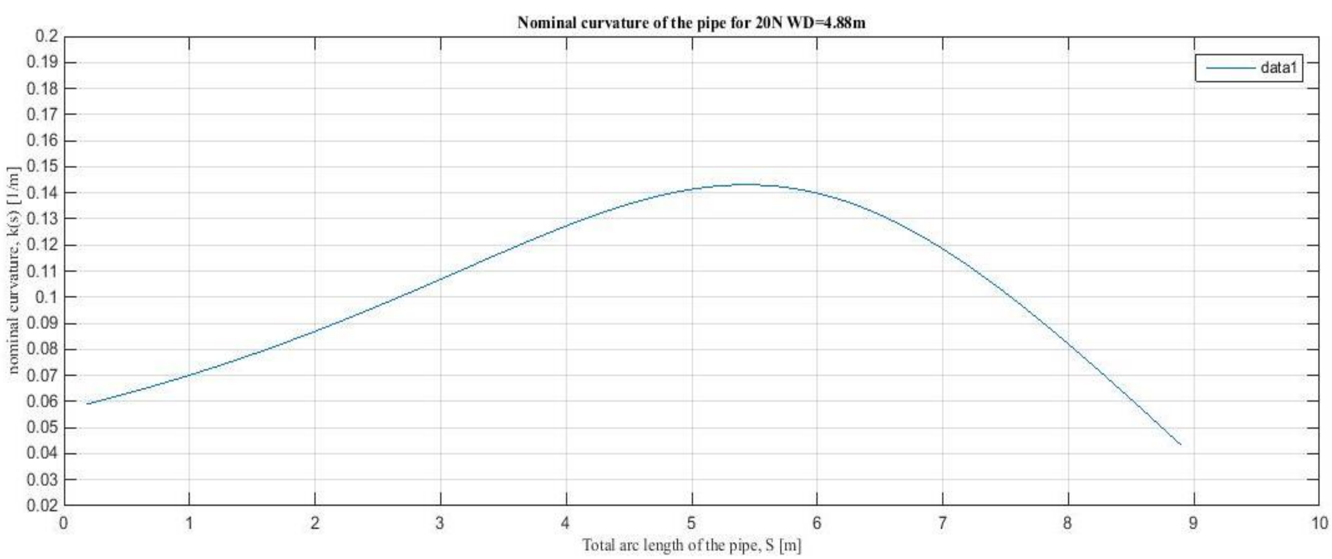


Figure 8-26 Experimental Nominal curvature curve of the pipe for Applied load of 20N, WD=4.88m

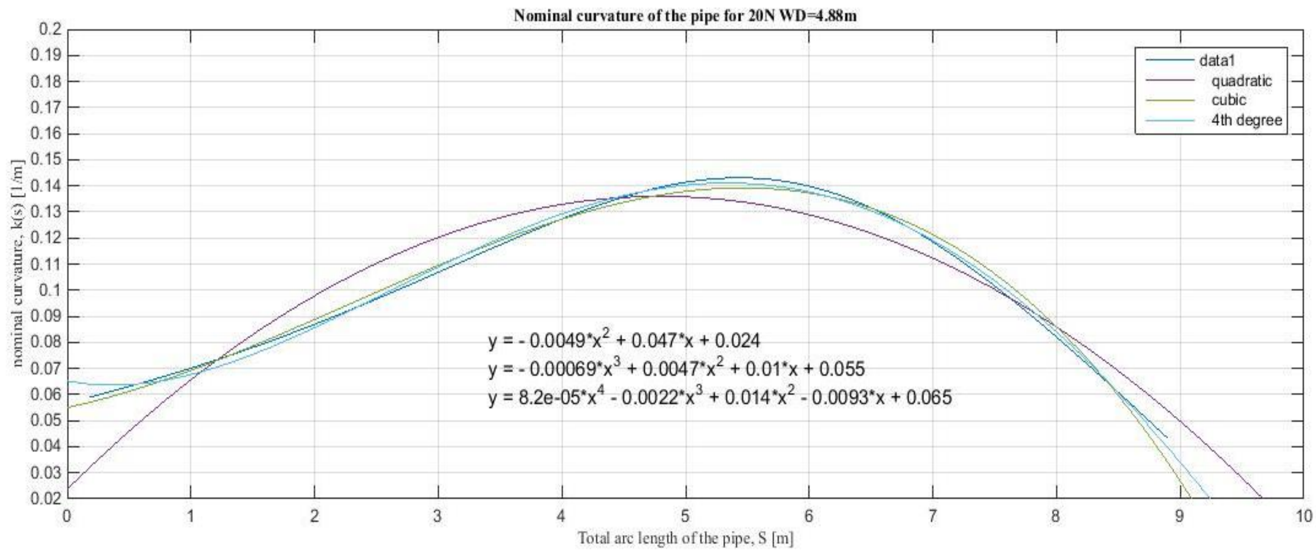


Figure 8-27 Experimental Nominal curvature curve fitting for applied load of 20N, WD=4.88m

Experimental Nominal curvature is fitted for quadratic, cubic and 4th degree of polynomial as shown in figure 8-27. The norm of residual is shown in Figure 8-28.

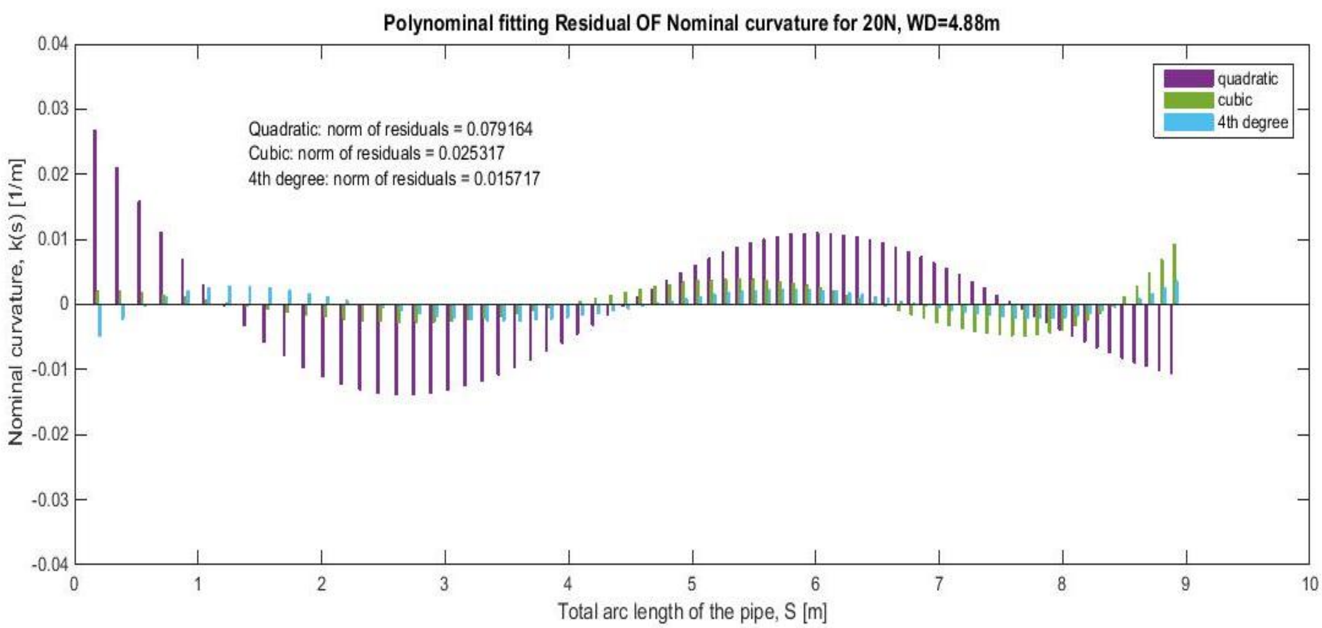


Figure 8-28 Residuals of each curve fitted for applied load of 20N, WD=4.88m

Since 4th degree of polynomial had norm of residual 0.0157, which shows it is good fit to experimental pipeline nominal curvature curve for 20N with water depth of 4.88m.

The 4th degree of polynomial equation was used as a nominal curvature for the rotation angle calculation for modified energy method. The test1, test2, test 6 and test 8 were performed with applied load of 20N with water depth of 4.88m. The test 1 and test2 are same except the bottom boundary resisting torque conditions that has no significant in modified energy method due to assumption of free end of pipe at bottom boundary condition. The corresponding angle at TDP for test1 or test2, test 6 and test8 calculated from modified energy method with 0.26% RCS, 0.15% RCS and 0.30% RCS are shown in Figure 8-29, Figure 8-30 and Figure 8-31 respectively.

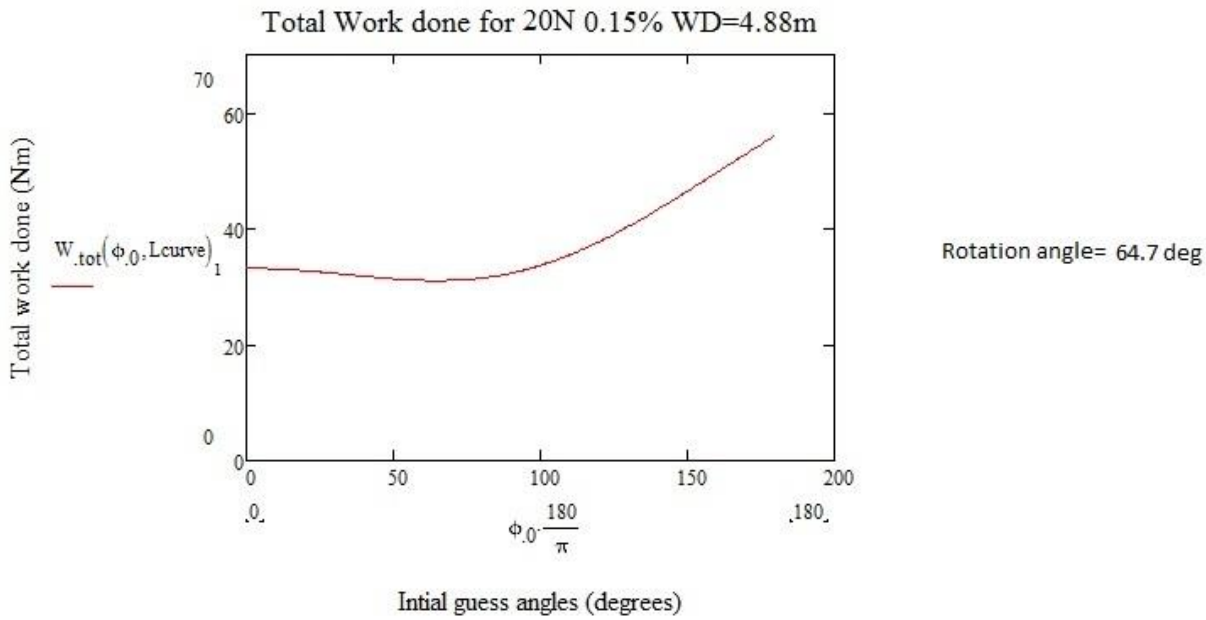


Figure 8-29 Total work done from experimental nominal curvature for Applied of 20N, 0.15% RCS, WD=4.88m

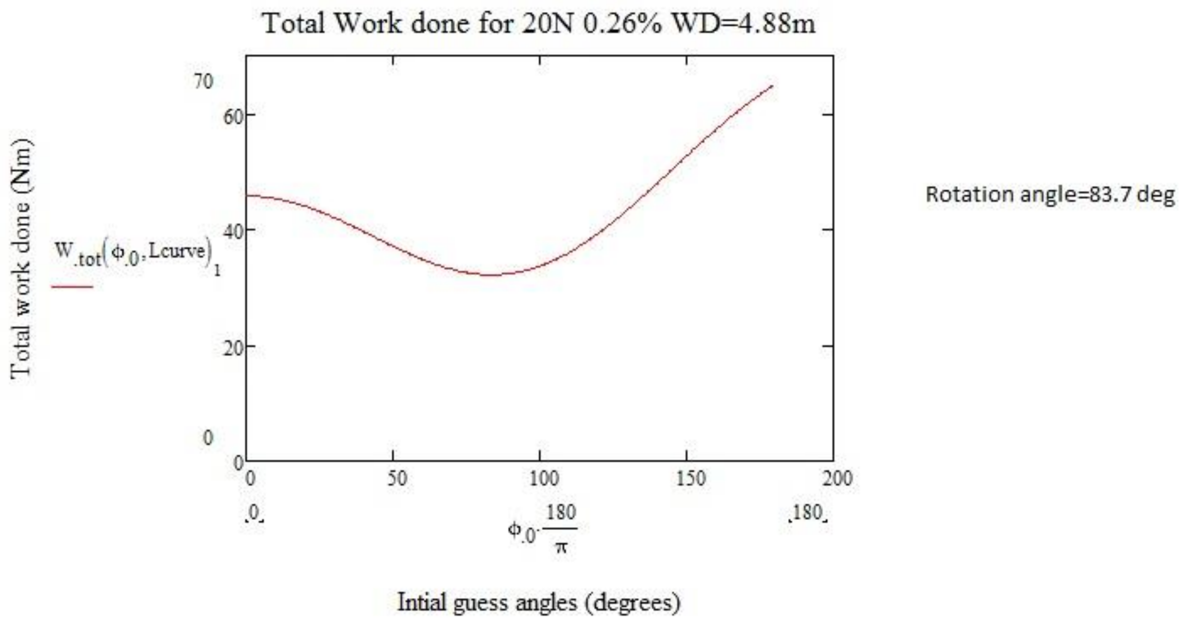


Figure 8-30 Total work done from experimental nominal curvature for Applied of 10N, 0.26% RCS, WD=4.88m

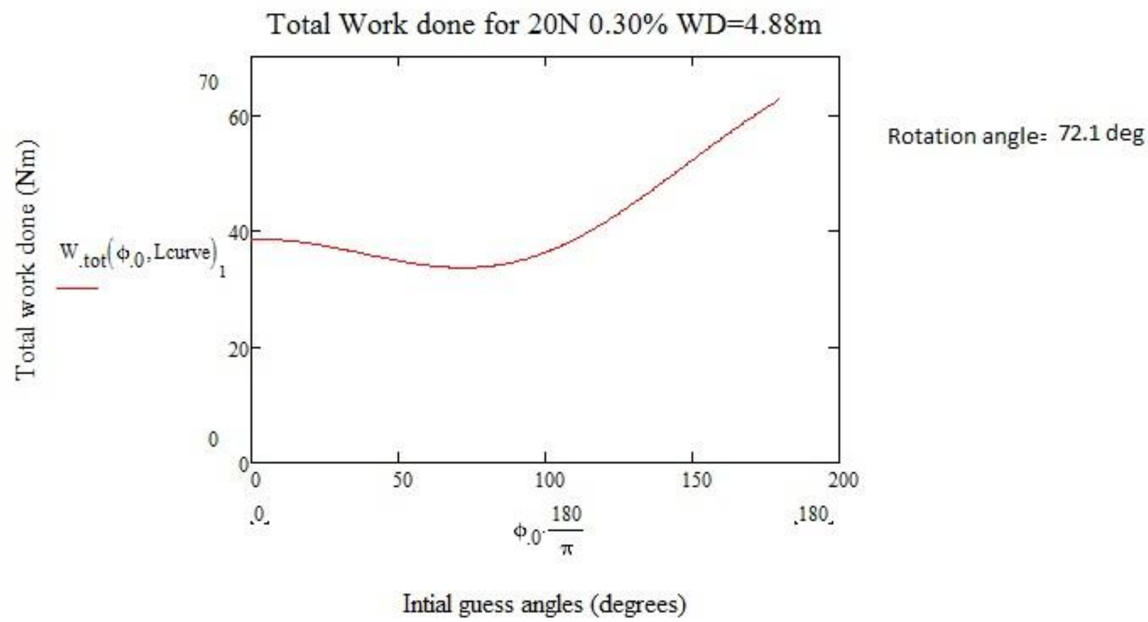


Figure 8-31 Total work done from experimental nominal curvature for Applied of 20N, 0.30% RCS, WD=4.88m

8.3.3 Water depth 4.88m with 40N Applied load

The catenary shape for applied load of 40N at water depth of 4.88m was obtained as shown in figure 8-32 from the digitalization of the pictures.

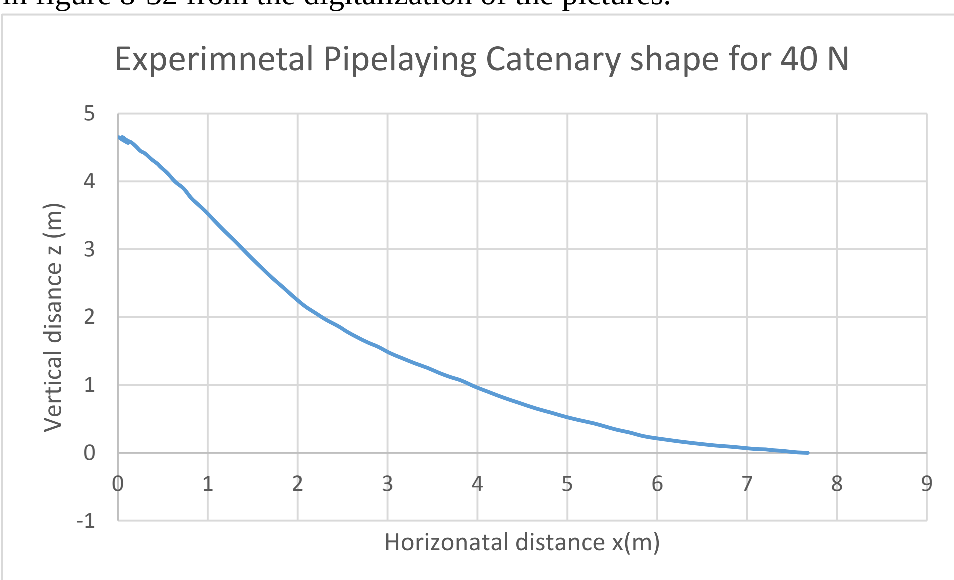


Figure 8-32 Experimental Pipe catenary for the applied load of 20N, WD =4.88m

The corresponding experimental nominal curvature of the pipeline is shown in figure 8-33. All data points and corresponding calculated values for the calculation of nominal curvature for the applied load of 40N are attached in Appendix E-3.

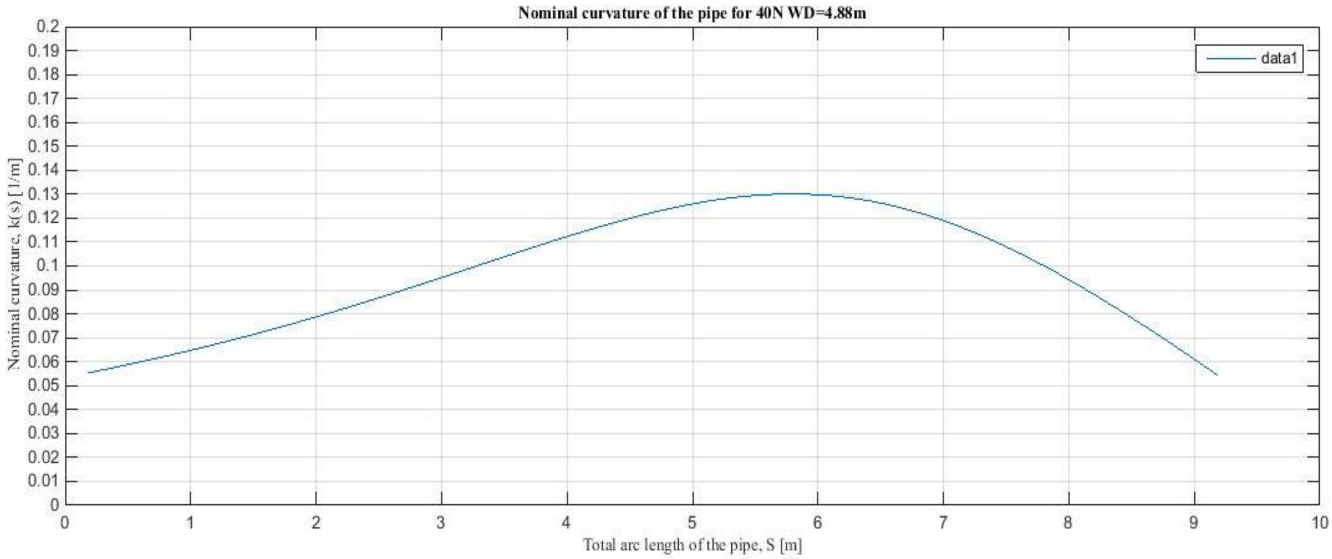


Figure 8-33 Experimental Nominal curvature curve of the pipe for Applied load of 40N, WD=4.88m

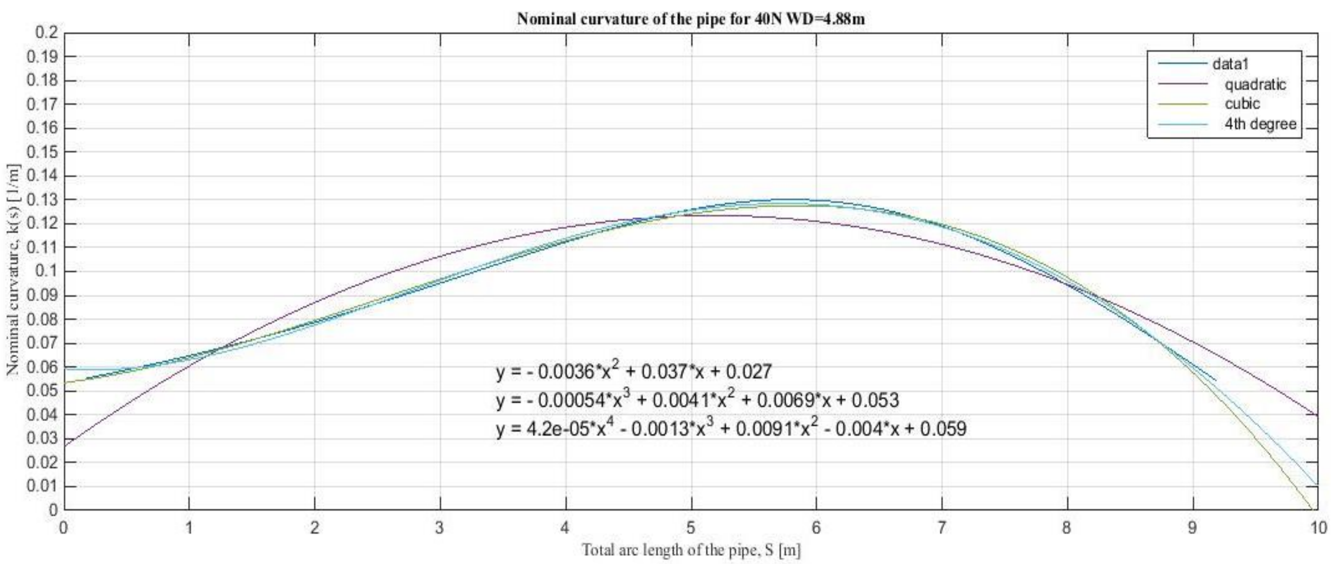


Figure 8-34 Experimental Nominal curvature curve fitting for applied load of 40N, WD=4.88m

The experimental nominal curvature is fitted for quartic, cubic and 4th degree of polynomial as shown in figure 8-34.

The norm of residual was calculated for each polynomial curve fitting, shown in figure 8-35. The norm of residual for the 4th degree polynomial is 0.01197 so 4th degree Polynomial was choosing for the nominal curvature equation in modified energy method for work done calculation.

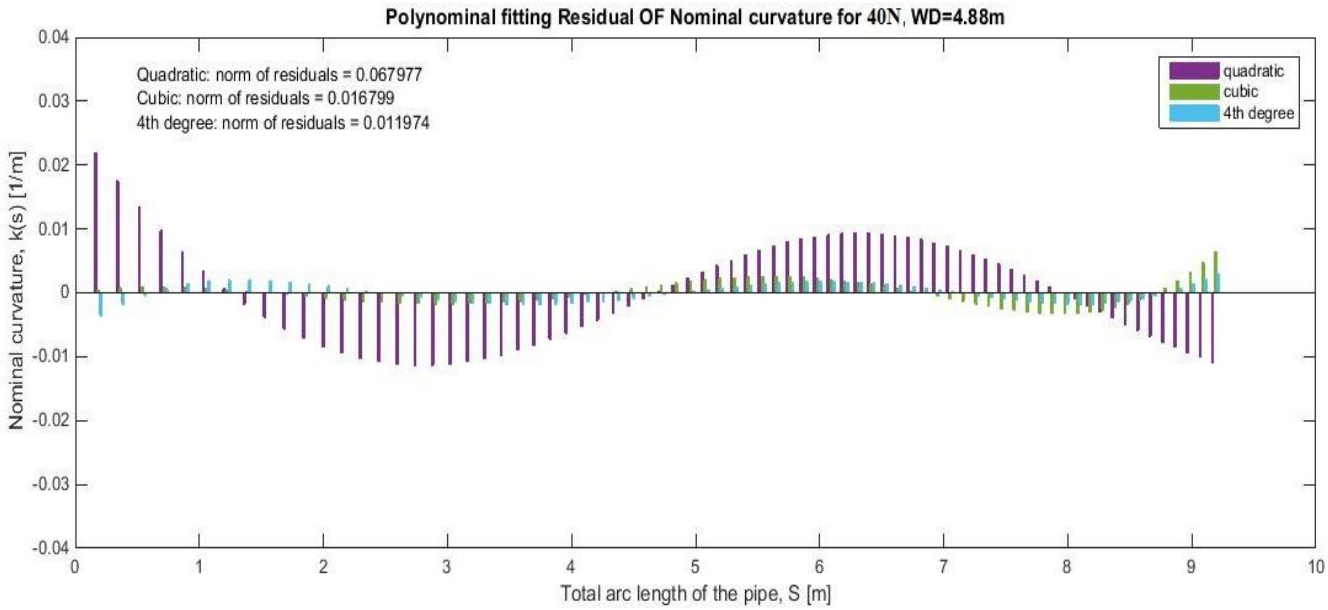


Figure 8-35 Residuals of each curve fitted for applied load of 40N , WD=4.88m

The test 3 and test 10 were performed with applied load of 40N at water depth of 4.88m. The total work done was calculated for those test by putting 4th degree polynomial nominal curvature. The test 3 has 0.26% RCS with 2m RCL and test 10 has 0.30% RCS with 0.84m RCL. The test 3 total work done curve is shown in figure 8-36 and test 10 total work done curve is shown in figure 8-37.

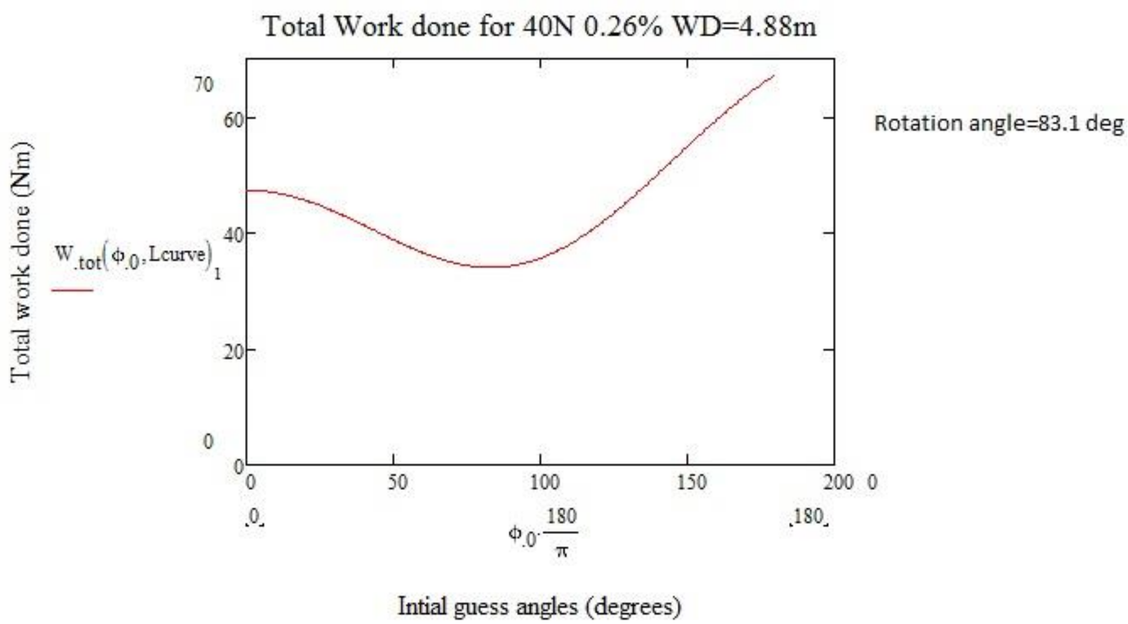


Figure 8-36 Total work done from experimental nominal curvature for Applied of 40N, 0.26% RCS, WD=4.88m

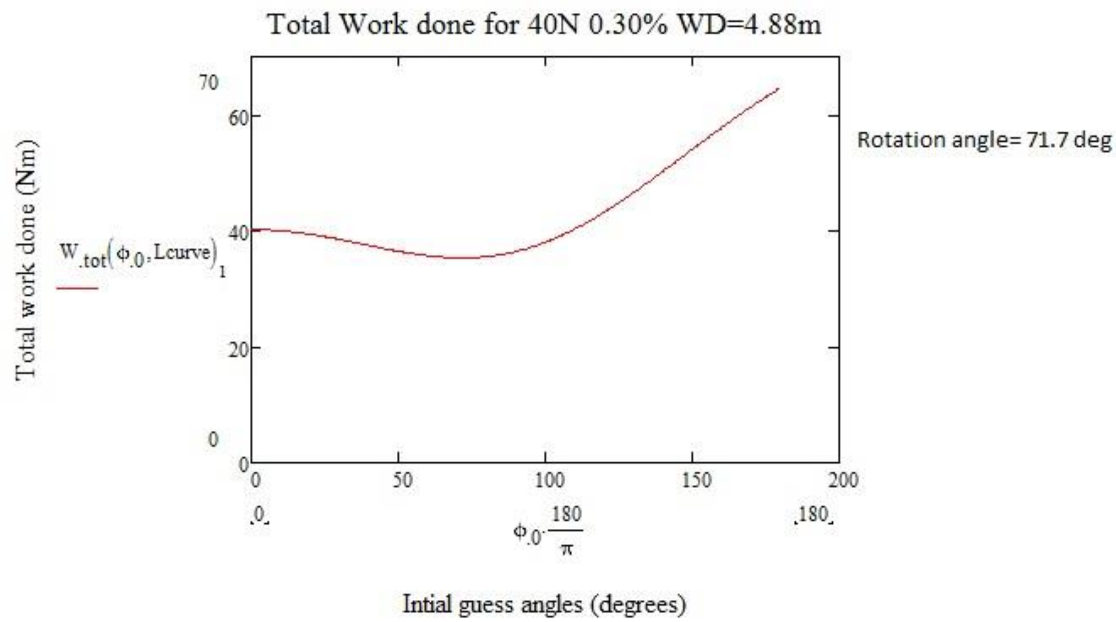


Figure 8-37 Total work done from experimental nominal curvature for Applied of 40N, 0.30% RCS, WD=4.88m

8.3.4 Water depth 5.91m with 20N Applied load

The catenary shape for applied load of 20N at water depth of 5.91m was obtained as shown in figure 8-38 from the data points obtained by digitalization of the pictures.

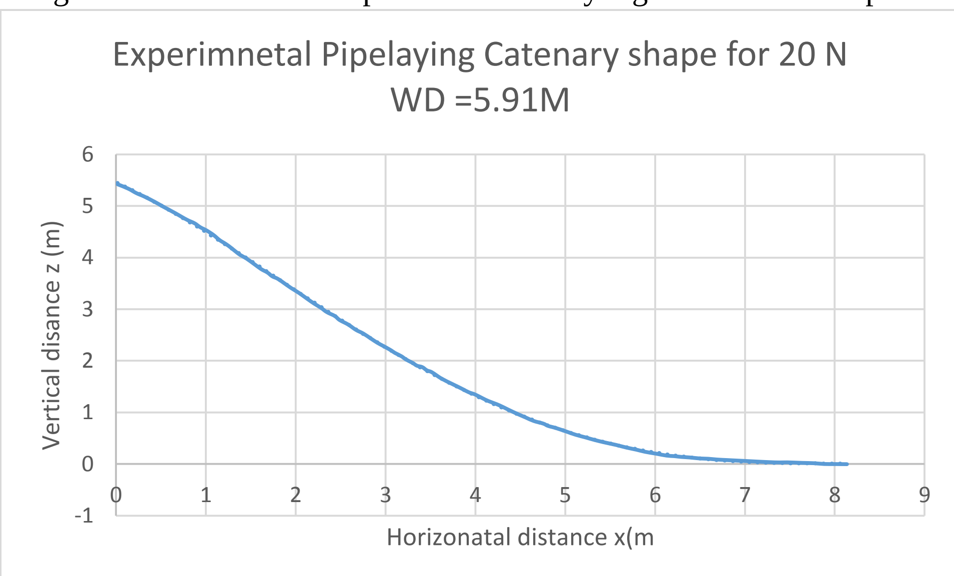


Figure 8-38 Experimental Pipe catenary for the applied load of 20N,WD =5.91m

The experimental nominal curvature of the pipeline calculated as shown in figure 11-39. The corresponding data points and calculations are attached in Appendix E-4.

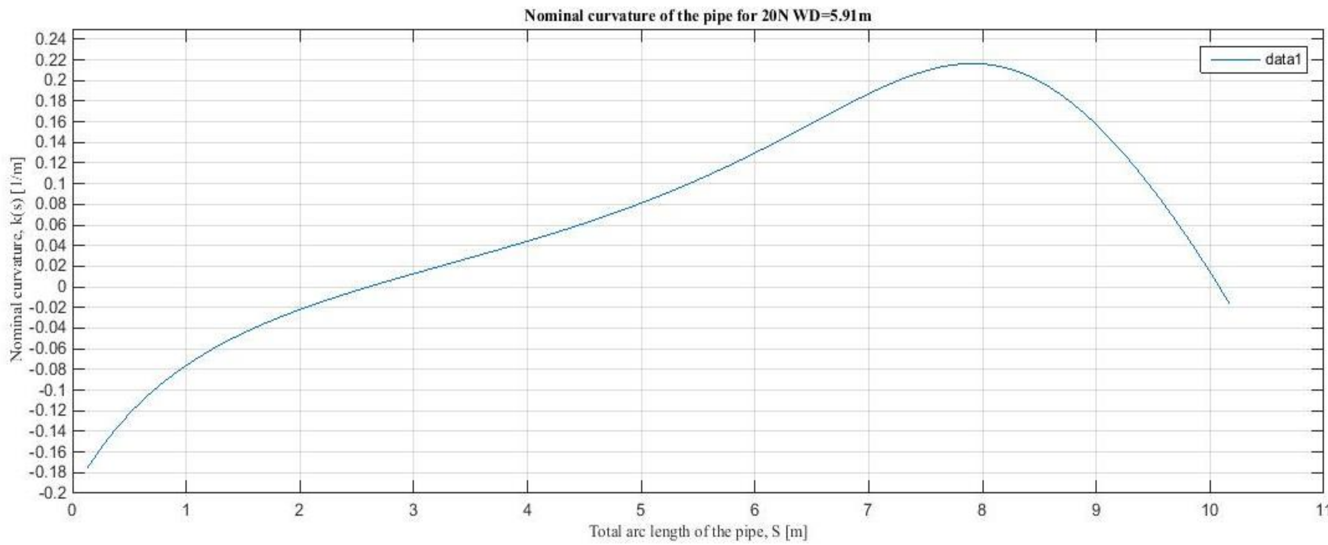


Figure 8-39 Experimental Nominal curvature curve of the pipeline for Applied load of 20N, WD=5.91m

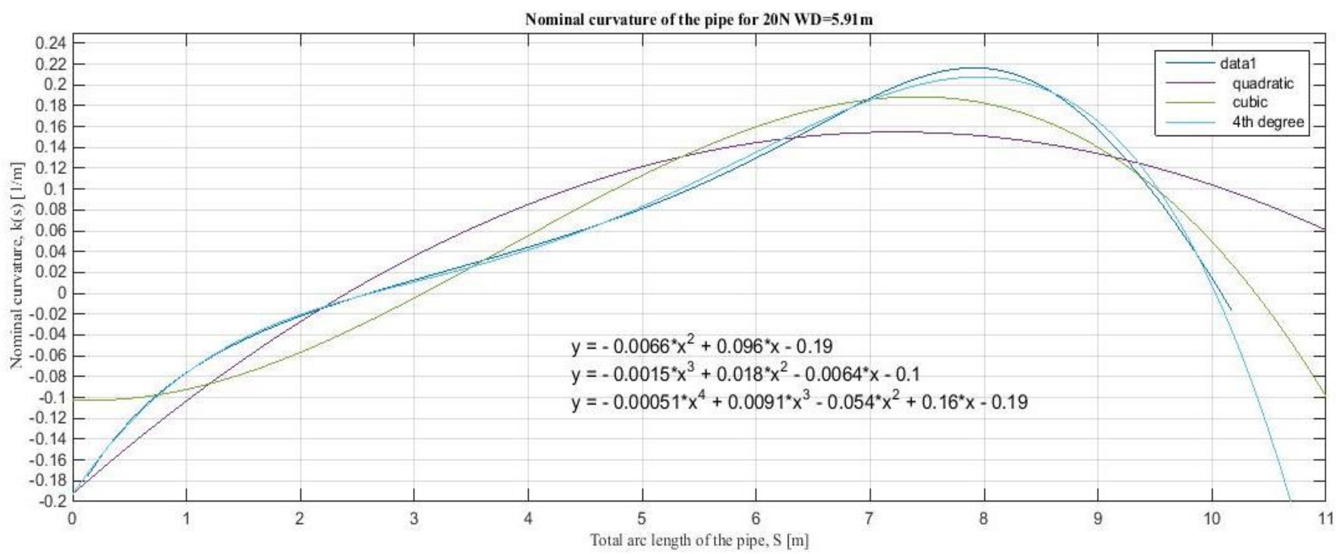


Figure 8-40 33 Experimental Nominal curvature curve fitting for applied load of 20N , WD=5.91m

The experimental nominal curvature was fitted to quadratic, cubic and 4th degree polynomial, shown in figure 8-40.

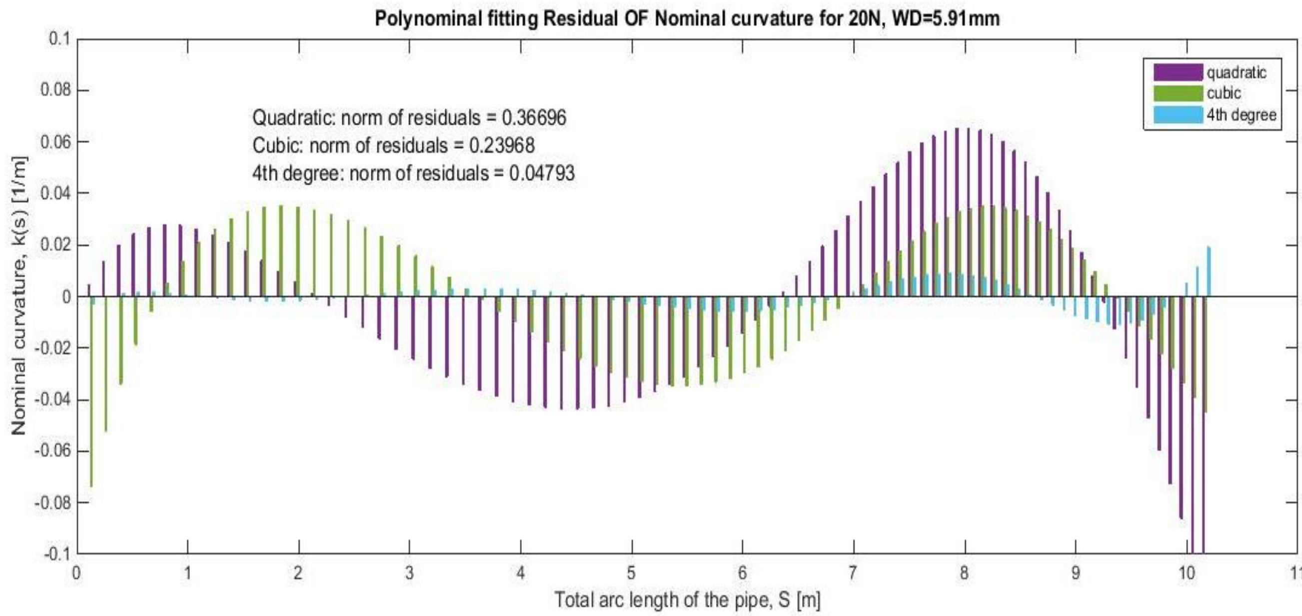


Figure 8-41 Residuals of each curve fitted for applied load of 20N , WD=5.91m

From figure 8-41, it is clear that 4th degree of polynomial has the minimum "norm of residual" with a value of 0.04793. thus this 4th degree of polynomial best represents the nominal curvature of the pipeline and it is used in total work calculation.

The test 14 was performed for the 20N at water depth of 5.91m with 0.30% residual curvature strain and 0.84m of residual curvature length. The wok done from modified energy method along with initial guess angles is shown in figure 8-42.

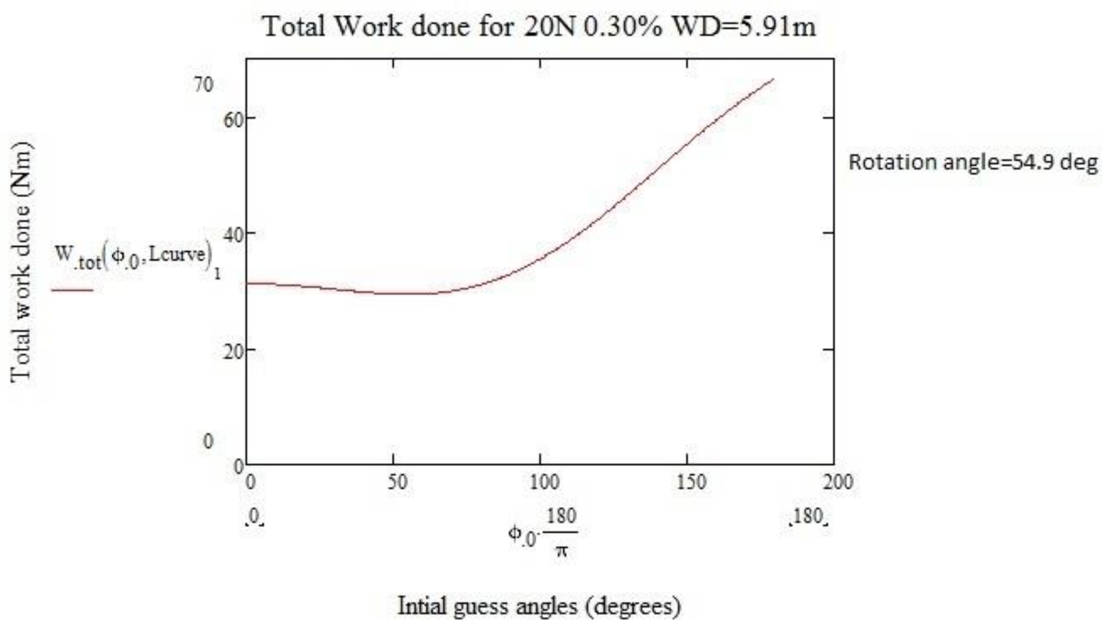


Figure 8-42 Total work done from experimental nominal curvature for Applied of 20N, 0.30% RCS, WD=5.91m

8.3.5 Water depth 5.91m with 40N Applied load

The catenary shape for applied load of 40N at water depth of 5.91m was obtained as shown in figure 8-43 from the data points obtained by digitalization of the pictures.

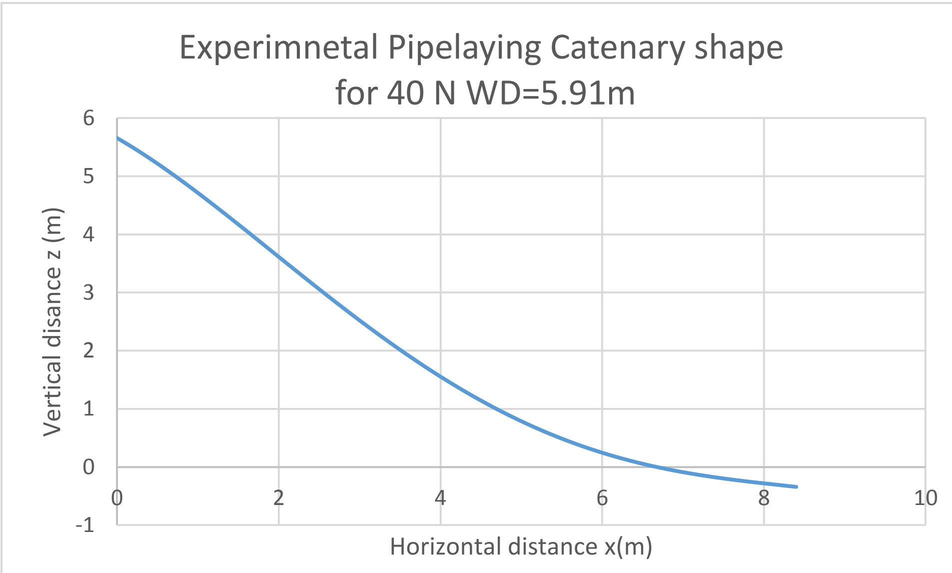


Figure 8-43 Experimental Pipe catenary for the applied load of 40N,WD =5.91m

The experimental nominal curvature was calculated for applied load of 40N with water depth of 5.91m, shown in Figure 8-44. The corresponding data points and calculations are attached in data is listed in Appendix E-5.

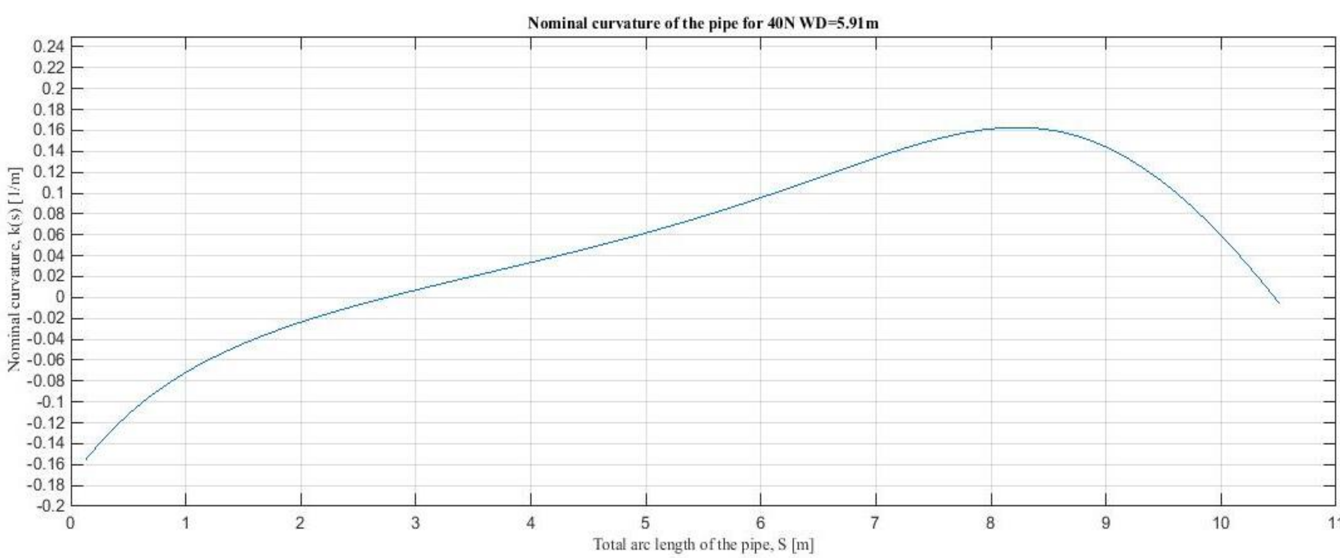


Figure 8-44 Experimental Nominal curvature curve of the pipeline for Applied load of 40N, WD=5.91m

The experimental nominal curvature is fitted, shown in figure 8-45.

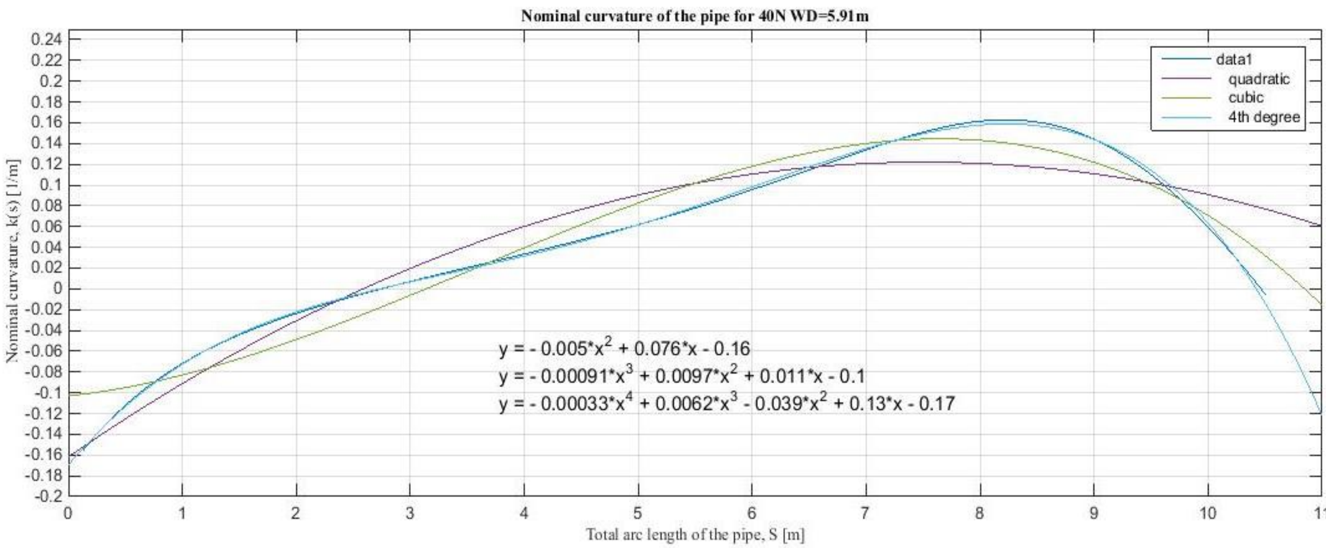


Figure 8-45 33 Experimental Nominal curvature curve fitting for applied load of 40N , WD=5.91m

The norm of residual is minimum for the 4th degree of polynomial with a value of 0.023088, shown in figure 8-46.

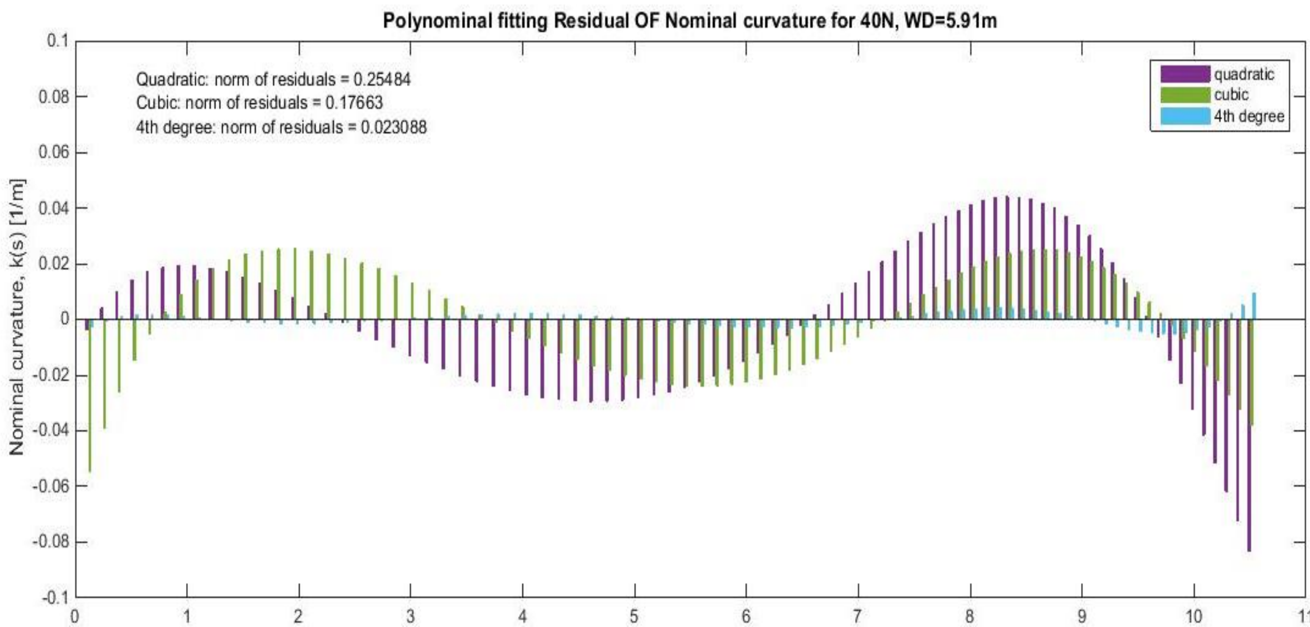


Figure 8-46 Residuals of each curve fitted for applied load of 40N , WD=5.91m

The 4th degree of polynomial is used instead of equation 3.14 in modified energy method. The test 15 was performed for the 40N at water depth of 5.91m with 0.30% residual curvature strain and 0.84m of residual curvature length. The total work done for test 14 with experimental nominal curvature is shown in Figure 8-47.

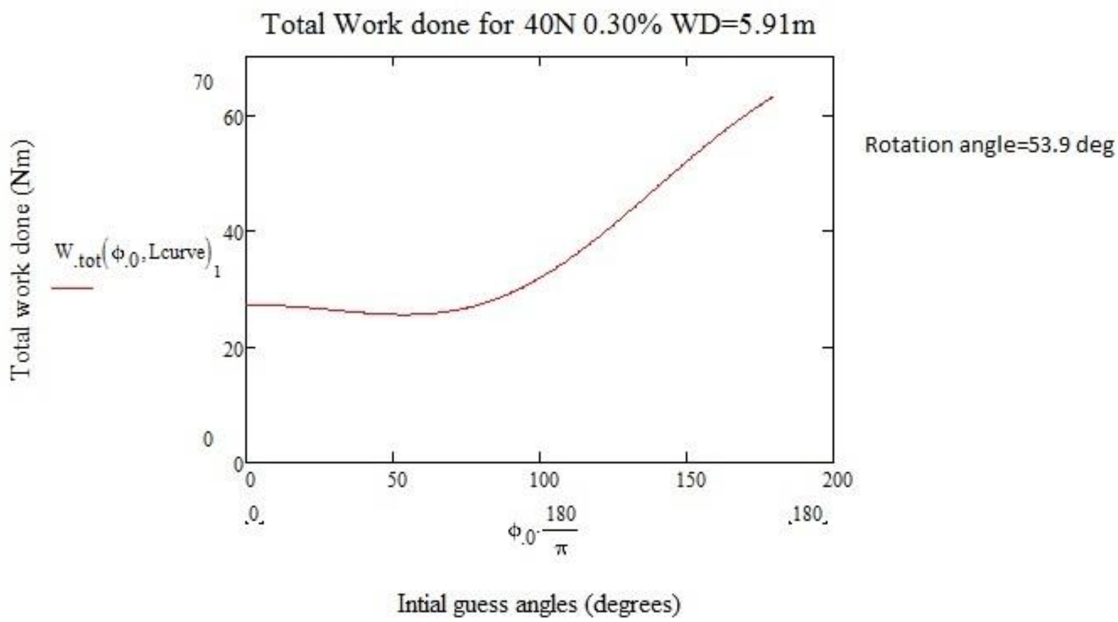


Figure 8-47 Total work done from experimental nominal curvature for Applied of 40N, 0.30% RCS,WD=5.91m

8.4 Discussion on the experimental nominal curvatures

The modified equations use the stiffened catenary method to find the nominal curvature of the pipeline. While in the experiments the nominal curvature was obtained for each case by measurement and calculation. There is different trend of nominal curvature for each case at different water depth. The explanation of those trends is explained in two following paragraphs.

The nominal curvature for water depth of 4.88m water, obtained from measuring the shape of pipeline from the stinger tip to TDP. There is overbend region on the stinger and it ends at the stinger tip. After the pipeline leaves the stinger tip it starts to bend in reverse direction until bends maximum at sagbend region. That is why the nominal curvature starts from minimum value to maximum value at sagbend and again minimum at TDP, shown in Figure 8-20, Figure 8-26 and Figure 8-33. As the applied load is increased the pipeline tends to become more straight and the nominal curvature value decreases. Remember that nominal curvature is defined as:

$$k(x) = \frac{1}{R}$$

Where R is the bending radius. The more the nominal curvature, the lesser the bending radius, and higher strains at the sagbend or vice versa. As seen from the figures that the nominal curvature decrease for higher applied load thus making less strains in the pipeline at the sagbend region. The physics of pipeline installation also states that for higher top tension the strain in sagbend would be less.

The nominal curvature for water depth of 5.91m, the data points were measured while pipeline was on the stinger. Thus the region on the stinger is overbend which is opposite of sagbend. This is reason that the experimental nominal curvature for the pipeline for water depths starts from negative value with zero value after the tip of stinger, maximum value at the sagbend region and again minimum positive at TDP, shown in Figure 8-39 and Figure 8-

44. Also the nominal value increases with increase in water depth while other parameters were kept same, thus validating with the theory that the strain increases as water depth increases. When the applied load was increased, the nominal curvature decrease, thus increasing the bending radius and decreasing the nominal strain in the sagbend, which is also the validation of pipeline installation principles.

It was important to note that the usual curvature definitions familiar form the beam theory does not apply in suspended pipeline section since the pipeline problem is highly non-linear i.e. the angle theta is not necessarily small.

8.5 Comparison of Results.

Now there are three different results comparisons.

1. Experimental measurements of rotation angle at TDP.
2. Analytical Calculation from Modified Energy Method.
3. Experimental calculation of Nominal Curvature to be used in Modified Energy Method.

The Table 8-5 and Table 8-6 shows the comparison of the angles results from the three approaches at two different water depth.

Table 8-5 Comparison of all three approaches results for test 1-10.

Results Comparison at Water depth :4.88m								
Bottom boundary condition: Free to rotate with torque measurement							Free end	Free end
Test numbering	Pipeline Laying properties		Residual curvature section		Experimental result		Analytical results	Exp. Nominal with Analytical method
Test Number	Applied Load (N)	Top Tension (N)	RCS (%)	RCL (m)	Resisting Torque (Ncm)	Rotation angle at TDP (Degree)	Rotation angle at TDP (Degree)	Rotation angle at TDP (Degree)
Test 1	20	38.79	0.26	2	70.63	79.78	77.4	83.7
Test 2	20	39.06	0.26	2	44.73	89.99	77.4	83.7
Test 3	40	60.66	0.26	2	30.609	60.85	77.1	83.1
Test 4	10	22.20	0.15	2	15.54	15.12	54.3	65.1
Test 5	10	22.34	0.15	2	8.95	31.92	54.1	65.1
Test 6	20	36.15	0.15	2	7.06	21.43	50.9	64.7
Test 8	20	33.49	0.30	0.84	12.71	53.28	63.9	72.1
Test 9	10	20.85	0.30	0.84	16.39	60.38	64.7	86.4
Test 10	40	60.38	0.30	0.84	5.48	24.86	60.3	71.7

The result comparison between the analytical results and experimental nominal curvature with analytical method shows that the rotation angle is higher for the experimental determine nominal curvature. This shows that the assumptions and equations in the Analytical methods for nominal curvature using stiffened catenary method are less conservative than the real life scenario. Since pipeline bottom end is free to rotate in both of analytical calculations, so there should be less rotation angle with resisting torque as measured from the experiment.

During the start of experiment, it was observed that with free boundary condition (assumption of analytical methods) the pipeline rotates to 180 degrees. Even changing the tension, the pipeline rotation behavior was same. This is due to distance from the pipe bottom end to the residual curvature section was 6m which is small, no soil-pipeline friction and the test is carried out dry. Thus resisting torque device provide the mechanism to do parametric study of pipeline laying. The comparison between the analytical equations and experimental values shows the analytical is based upon assumptions which may not reflect the practical pipe laying.

The experimentally determined nominal curvature is more conservative and reflects more real life scenarios. Since it is seen from comparison that with free end of the pipeline the pipe rotates more than predication from the modified energy method. It can be observed that analytical method does not show much dependency of the rotation angle on the top vessel tension and bottom horizontal tension, while they effect the Pipe laying rotation angle. From the test 8 and test 10 the rotation angle changes from 53.28 degree to 24.86 degree while analytical results shows the rotation angle change from the 72.1 degree to 71.7 degree.

Table 8-6 Comparison of all three approaches results for test 14-15

Results Comparison at Water depth :5.91m								
Bottom boundary condition: Free to rotate with torque measurement							Free end	Free end
Test numbering	Pipeline Laying properties		Residual curvature section		Experimental result		Analytical results	Exp. Nominal with Analytical eqs
Test Number	Applied Load (N)	Top Tension (N)	RCS (%)	RCL (m)	Resisting Torque (Ncm)	Angle at TDP (Degree)	Analytical rotation angle (Degree)	Rotation angle at TDP (Degree)
Test 14	34.40	38.62	0.30	0.84	21.66	49.32	65.9	54.9
Test 15	61.84	64.88	0.30	0.84	10.83	28.84	67.7	53.9

As the water depth increases the rotation angle increases both in pure analytical and experimental approach. Note that for test 8 and test 14 the rotation angle decreases from 53.28 degree to 49 .32 degrees because the resisting torque increased from 12.71 Ncm to 21.66 Ncm.

The experimentally nominal curvature for the water depth of 5.91m was calculated from the data points with the pipeline on the stinger i.e. small portion of overbend region was include, while for water depth of 4.88m the nominal curvature was calculated from the data points while the pipeline is on the stinger tip. The experimentally measured nominal curvature for water depth 5.91m is less conservative than the analytical calculated nominal curvature which calculates the suspended section of pipeline with stiffened catenary method. Also the nominal curvature of the pipeline is considered to be zero at stinger tip an d at TDP in analytical modified energy method nominal curvature.

The analytical modified energy method also does not account for the unbalanced weight distribution of the pipe. the modified energy method calculates bending and twisting elastic energy, it is implicitly assumed that gravitational potential energy has no effect. But in the experiment the residual curvature sections stick out from the undisturbed pipeline axis, which may be enough to challenge this assumption. The rotation angle observed experimentally may be combination of strain energy minimization and gravitational energy minimization.

In (Vaughan & Nystrom, 2016) , it is discussed that for the pipeline rotation may not simply assumed to be strain energy minimization as done in analytical calculation. Any significant eccentric weight also influences the pipeline rotation.

8.6 Applications of Rotation and Residual curvature

The residual curvature section of the pipeline as many advantages during installation. The residual curvature can be generated in S-lay from the rollers adjustment or stinger adjustments (Endal & Nystrom, 2015). The implementation of residual curvature had been performed by the Statoil Skuld project and the method to produce residual curvature had been patented by (Statoil, 2002). Here a glimpse of residual curvature advantages is given as the topic is out of scope for this thesis. The residual curvature section can be helpful in following:

1. Uneven seabed
2. Control global buckling

In (Endal & Nystrom, 2015) many Benefits related to residual curvature sections are discussed.

8.6.1 Uneven seabed

When the pipe is installed on the uneven seabed, it exposed to bending and free spans. These bending moments can cause the uncontrolled bending and potential threat to pipe integrity. These uncontrolled bending in the pipe also cause the pipeline flow assurance issues during operations. Mitigating measures are often required to avoid this. These measures like seabed preparation with rocks and pipeline route adjustments are consider being costly (Endal, et al., 2015). Thus Residual curvature method is considered to be cost effective.

Figure 8-48 Residual curvature section on Uneven seabed

The suspended pipe free span can be adapted according to the seabed topography. The figure 8-48 give shows the natural pipeline behavior in comparison with the residual curvature section pipe behavior. The residual curvature section pipe here is over-straightened. This section needs to rotated 180 degrees during installation of pipeline at TDP, if the residual section produced on reel-lay and S-lay stinger. The figure 8-49 shows an example of uneven seabed as the iceberg scar on the pipe route. The installation of residual curvature section needs to be analyze both economically and technically.

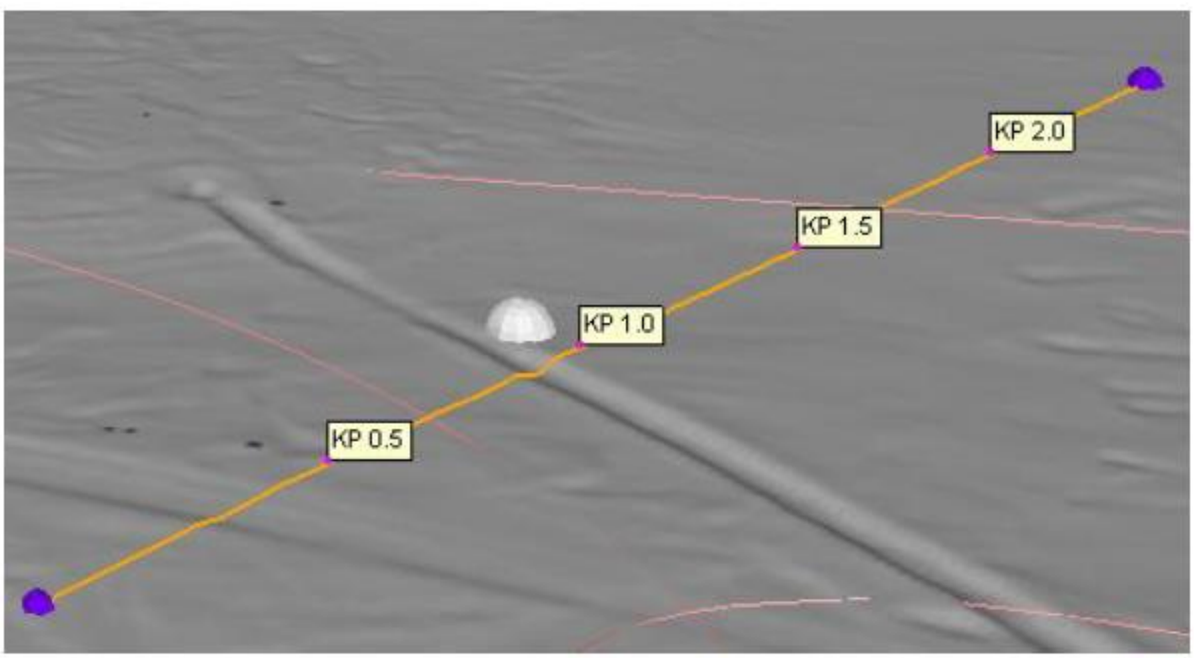


Figure 8-49 Iceberg scar on the pipeline route. From (Endal, et al., 2015)

8.6.2 Global Buckling

The length of offshore pipelines is in hundreds of Kilometers. Pipelines take the hydrocarbons fluid from the wellhead to sea surface for processing. The hydrocarbons fluid

contains high pressure and high temperatures. E.g. temperature greater 90^{o} C or even up to 150^{0} C. It is due to thermal gradient of earth crust in reservoir. Pipeline experiences axial expansion forces due to high pressure and high temperature changes of hydrocarbons. These changes are normally caused at the start up and end of hydrocarbons flow operations. Since the pipes are fixed at the ends, so pipeline thermal expansion can cause the pipe to buckle. Mitigating measures are needed for the pipelines to keep the stresses and strains within acceptable range. Commonly used measures include snake-lay method, trigger beam method to trigger that global pipe buckling at low axial forces. In (Endal & Egeli, 2014) it is shown that the residual curvature is better other mitigating measures. The figure 8-50 illustrate the advantage of using residual curvature pipe. The axial force corresponding to the displacement of the residual curvature section is measured. Since low axial force is required to trigger global buckling. The global buckle shares the thermal expansion and pipe has stresses within acceptable range.
Since Global buckling, itself is huge topic. It requires detail analysis and lot of experimentations. Since it's not included in the scope of this study, so measurements related to global buckling are not shown here. It can be part of future studies with the same experimental set up.

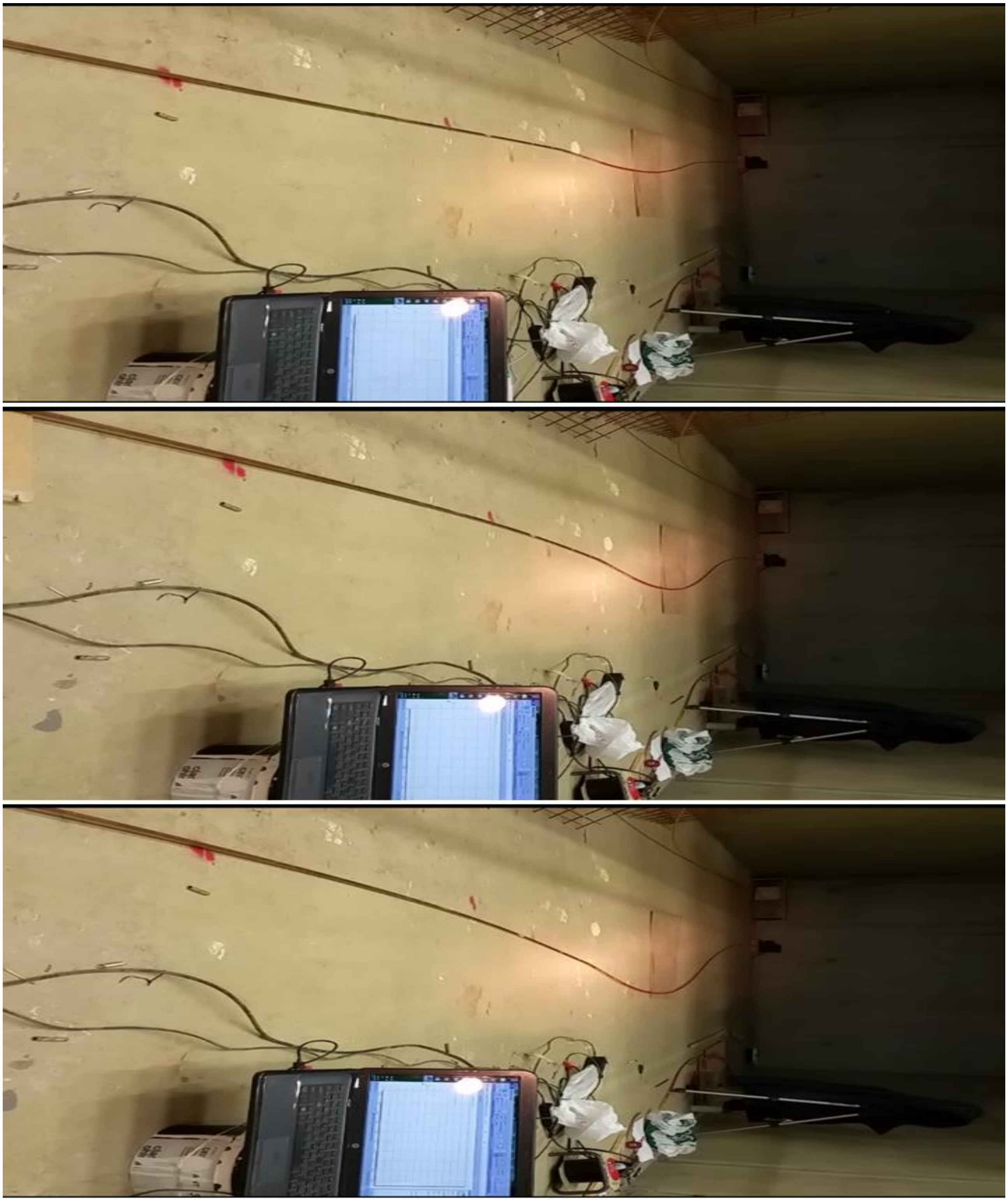


Figure 8-50 Measuring the displacement of the Residual curvature section to applied axial force

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9 Uncertainties in the experiment

Every measurement involves some level of uncertainties, errors and accuracy. Although great care is taken during experiments and every experiment was repeated minimum two times to several times to get the measurements. Still there exist uncertainties or errors, which are beyond the control or equipment limitations. It is necessary to discuss the errors and their sources for better understanding of the experiment and its set up. The uncertainties discussed here are particularly related to experiments performed. Uncertainty is defined as:

$$measurement = best\ estimate \pm uncertainty$$

The uncertainties or errors can be mainly divided into following groups:

1. Equipment Uncertainties
2. Calculation Uncertainties
3. Personnel uncertainties

9.1 Equipment Uncertainties

Any measurement from the equipment can measure the quantity to certain degree of fineness. Measurements can be greater than the equipment limitation. Similarly, any errors in the equipment cause error in measurements.

During the experiment, the load cell was tested against the known loads and it was found that the load cell shows the 0.150 kg-0.300kg more value than the actual one. For example, the 10N, 20N and 40N loads were hung to load cell to measure the load, it was found that the load cell shows the value of 1.259kg, 2.241kg and 4.187kg as load cells measures values in kg. These loads when converted to Newton, they come out to be 12.3382N, 21.9618N and 41.0326N. Since for this purpose an average value of 0.229kg was subtracted from every value of load cell. The load cell has the capacity to measure tension as well as compression loads. Similarly, during the calculation of resisting torque by the pipe, a Vernier caliper is used to measure the spring extension from its rest position. A Vernier caliper of almost zero error was used but it may contain small error. The height from touch down point to top stinger known as water depth is measured from the locally used meter rod. Locally used Meter rod may contain errors too. The pipeline mechanical properties were found different in tensile testing as according to the given by manufacturer standard. This might be due to tensile machine testing or manufacturer defect.

the uncertainty due to residual curvature section can also be caused. As during initial bends, the pipeline bends were monitor with strain gauges. The strain gauge measures the pipe bending only in one direction, so it may be possible that maximum bending point was not there where the strain gauge is placed. Similarly, later the pipes were bend from the curve shape which may not produce accurate residual curvature bends. Although great care is taken during the bends and results shows that pipe residual curvature sections behave in a satisfied manner.

The friction force between the pipe and stingers rollers also caused uncertainty on the effect of the applied load and top vessel tension. The pipe is made of copper material which is little flexible, so when residual curvature section was attached to the pipeline and pipe is pulled at the bottom end, some of the force is absorbed by the pipe residual curvature section and it tends to become straight. But this error can be neglected as it is due to material property and its value is small.

9.2 Calculation Uncertainties

The calculations errors could occur during the experiment. Although every calculation performed are checked many times and discuss with supervisor. In these experiments, these types of errors would be very less to occur. Most of calculations are performed on powerful mathematics software "Mathcad". Each simulation of analytical expressions is performed many times and each parameter is input carefully in analytical energy equations. The calculation of nominal curvature is checked thoroughly and curves/trends were discussed with supervisors. Most of the calculations were performed at the end of thesis to verify the results that were included in the thesis. The analytical modified energy equations and nominal curvatures tables are also attached in Appendix section for the verifications of the results. The significant figures error may be considered as different calculations having different least counts.

It has been observed during digitalization of pictures for angle measurement at TDP, there exist is uncertainty of $\pm$ 3 degrees. It was due to the limitations of the software used. It was solved in a way that the angle measurement is done minimum four times and most appeared value is counted for the experiment.

The experiments values are more precise since experiments are repeated many times. The experimental results values after repetition can be considered as true so results values can be taken as accurate too.

9.3 Personnel uncertainties

During the experiments, a personnel uncertainty could be occurring. Since Author perform most of experimental work, so this error could occur. This error or uncertainty has less chance to occur, as during the start and some times in the experiment, my external supervisor was present to see the measurements. Also the result obtained from the experiments were discussed. Each experiment is performed two or more times, then values are measured and reported.

10 Conclusions and Recommendations

10.1 Conclusions

During the installation of pipeline from the S-Lay, it had been observed that pipeline rotates while touching the sea bottom. This rotation is not acceptable when the inline structures have the maximum allowance of ±15 degrees (Damsleth, et al., 2000). This thesis run a real time simulation for Pipeline installation on a small scale. All the fundamentals parameters required for pipeline installation had been implemented in a control manner. The thesis involved many challenging tasks during design and experiment test.

The main conclusions of the thesis are:

Building a test rig model for S-lay installation, replicating real life scenarios for future thesis work and education purposes at the university lab.

The analytical equations lack the gravitational energy minimization terms, which should be included in case unbalanced weight distribution of the pipe due to inline structure or residual curvature section.

The important parameters that effect the pipeline rotation were: Boundary conditions, Residual curvature strain, Residual curvature length, Top vessel tension, Torque applied by the pipe, Water depth and Inline structure. These parameters were found to have interlinked effects on the rotation of pipelines and make it difficult to do parametric study. A detail study of each parameter effecting the rotation of pipes with the help test results graphs and tables.

The boundary conditions at the seabed has an influence on the pipe rotation. The analytical approaches assumptions of free bottom pipeline end are found overestimated and predict more rotation angle then actual one. For pipeline rotation the most important parameters are residual curvature strain and residual curvature length. it has been observed through literature survey that research papers only include residual curvature strain and do not includes the residual curvature length in FEM Analysis. While the experiments had shown that 0.26% RCS pipe with 2m RCL rotates more than the 0.30% RCS pipe with 0.84m RCL, even though the 0.26% RCS pipe has 0.04% less residual curvature strain than 0.30% RCS pipe. Similarly, It has been measured during the experiments that the with the increase in Top vessel tension, there is decrease in rotation angle at TDP and vice versa.

A simple torque measuring device was design and build to stimulate the soil seabed-pipeline friction. Couples of tests were performed by varying the bottom resisting torque and angle or rotation is measured. The experiments show that the resisting torque on the seabed has inverse relationship with rotation of pipelines.

The experimental results conclude that with increase in water depth, this is increase in rotation of pipeline angle at TDP. Also from the experiments it is observed that Inline structures cause the pipeline to rotate. This rotation is due to unstable and eccentric loading on the pipeline.

Catenary shape of the pipeline is measured for applied load and water depth, by digitalization of high quality pictures. The digitalization of pictures generates the data points which were used to generate nominal curvature for the pipeline. The nominal curvature curve is fitted using different degrees of polynomial and minimum norm of residual curve is used as an equation for nominal curvature. The nominal curvature equations are used analytical equations to obtained the rotation angle at TDP.

All the three results i.e. experimental, pure analytical and experimental nominal curvature with analytical equations, are compared. It has been concluded that analytical equations are more conservative than the actual pipeline rotation at TDP. The analytical equations need modifications.

These experiments also include some uncertainties and errors, such as load cell limitations, a glimpse of application for rotation and residual section is presented through experiment schematics.

The building of test rig from the scratch and designing of experimental components took lot of time. Finding the solution of experimental hurdles, took considerable amount of time, which put the limits on the number of experimental tests. Due to nature of the experiment, the manpower was key issue in the experiment.

10.2 Recommendations

The S-lay rig has been built with fixed stinger type. The test rig is a small porotype model while including all the fundamentals components of S-lay installation. This thesis can be foundation for future works to study cost and time effective solutions of pipeline installation challenges. The build test rig can save time for further study as the base has been established in this thesis.

By doing the experiments and reading the project based research papers from the industries, I have found that following areas has opportunities for research:

1. Build a 3-point bending machine that can creates the residual curvature section for small pipes without effecting the ovalization of the pipe. as for 10mm copper pipes the market does not have any bending machine. The machine could resemble Statoil 3-point bending method Patent for reeling (Statoil, 2002).
2. A series of experimentation can be done to study the effect of different residual curvature sections on mitigating the global thermal buckling. See figure 8-50.
3. Similarly, experimentation and analysis can be done to study the effect of different residual curvature sections in tie with subsea manifolds instead tie in spool. The spool may be of 3-leg, 4-leg or 5-leg structure. The cost related to spool is much higher than residual curvature sections. This is new topic as not much work done in that area. More guidance can be taken from (Nystrom, et al., 2015).

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APPENIDX A: MATHCAD File for modified energy method calculations

Rotation angle Calculation using simplified analytical method (from endal et 1995)

This Mathcad sheet is used to solve the Total work done by using simplified Analytical Approach for pipeline rotation

Pipeline data

Outer Diameter $OD := 0.01$

Wall thicknes $t := 0.0008$

Radius of pipe $r := \frac{OD}{2}$

Moment of Ineria $I := \frac{\pi}{4} \cdot \left[r^4 - (r - t)^4\right]$

Young Modulus $E := 120.97 \cdot 10^9$

Bending Stifness $EI := E \cdot I$ $EI = 29.817$

Possion Ration $\upsilon := 0.33$

Shear Modulus $G := \frac{E}{2(1 + \upsilon)} = 4.548 \times 10^{10}$

Polar moment of area $J := \frac{\pi}{2}\left[r^4 - (r - t)^4\right]$

Torsional stifness $GJ := G \cdot J$ $GJ = 22.419$

Installation parameters

Residual Strain $\varepsilon res := 0.26\%$

Residual curvature $k_{res} := \frac{\varepsilon res}{r}$

Maximum Nominal strain $\varepsilon nom := 0.125\%$

Nominal curvature maximum $k_{0max} := \frac{\varepsilon nom}{r}$

Length of caternary pipe $L := 8.6$

Rotation Angle

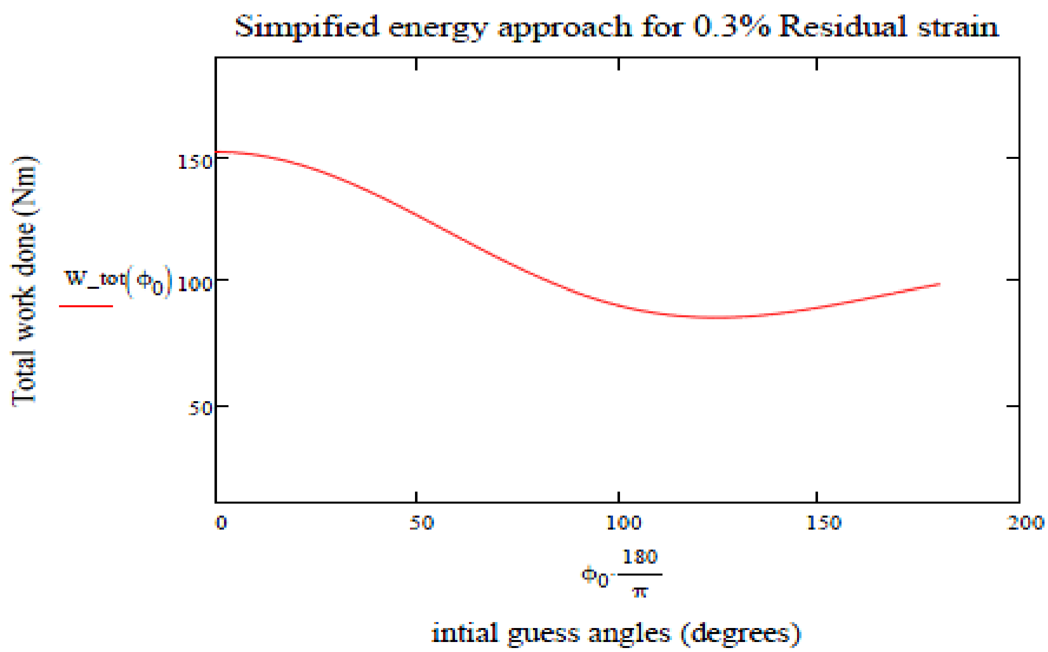
Angle of rotation from 0 to 180 degree with 0.5 degree of increment

$$\phi_0 := 0\text{deg}, 0.5\text{deg} .. 180\text{deg}$$

Total Work done

Total Work done due to bending and roll contribution of pipe is

$$W\_tot(\phi_0) := \frac{6}{5} \cdot \frac{GJ}{L} \cdot \phi_0^{\,2} + EI \cdot \int_0^L \left[4 \cdot k_{0max} \left(\frac{x}{L} - \frac{x^2}{L^2} \right) + k_{res} \cdot \cos\left(3 \cdot \phi_0 \cdot \frac{x^2}{L^2} - 2 \cdot \phi_0 \cdot \frac{x^3}{L^3} \right) \right]^2 dx$$



Rotattion Angle at TDP

Initial guess for roll angle $\phi_0 := 2$

Roll Angle minimization $\phi_{angle} := \text{Minimize}(W\_tot, \phi_0)$

Roll angle in radians at TDP $\phi_{angle} = 2.172$

Roll angle in degree at TDP $Angle := \phi_{angle} \cdot \frac{180}{\pi}$

$Angle = 124.457$

Corresponding minimum Total work $W\_tot(\phi_{angle}) = 85.079$

APPENIDX B: MATHCAD File for Modified energy method calculations

APPENDIX B

Calculation of Rotation angle: Modified Energy Method (from endal et 2014)

This Mathcad sheet is used to solve the Total work done by using modified Analytical Approach for pipeline rotation at TDP.

All the calculations for the tests were performed in this sheet while changing the parameters for each test.

Pipeline data

Outer Diameter	$OD := 0.01$
Wall thicknes	$t := 0.0008$
Radius of pipe	$r := \frac{OD}{2}$
Moment of Ineria	$I := \frac{\pi}{4}\cdot\left[r^4 - (r-t)^4\right]$
Young Modulus	$E := 120.97\cdot 10^9$
Pipe submerged weight	$w := 0.1901\cdot 9.8 = 1.863$
Bending Stiffness	$EI := E\cdot I \qquad EI = 29.817$
Possion Ration	$\upsilon := 0.33$
Shear Modulus	$G := \frac{E}{2(1+\upsilon)}$
Polar moment of area	$J := \frac{\pi}{2}\left[r^4 - (r-t)^4\right]$
Torsional stifness	$GJ := G\cdot J \qquad GJ = 22.419$

Installation parameters

Residual Strain	$\varepsilon res := 0.30\%$

The residual curvature strain for the exepriments is 0.15%, 0.26% and 0.30%. By changing the ε.res above, total work done is calculated for different residual curvature strains.

Residual curvature $k_{res} := \frac{\varepsilon res}{r}$

Top vessel tension $Tt := 33.49$

Bottom horizontal tension $Ht := 29.40$

Length of caternary pipe $L := 8.6$

Water depth $WD := 4.88$

Calculation parameters

Gamma expression $\gamma := \sqrt{\left(\frac{EI}{Tt}\right)}$

Horizontal tension divided by pipe submerged weight $A := \frac{Ht}{w}$

Rotation Angle

Roll angle equation $\phi(s, \phi_0) := \frac{-\phi_0}{L^2} \cdot s^2 + \phi_0$

Initial guess for Angle of rotation from 0 to 180 degree with 0.5 degree of increment $\phi_0 := 0deg, 0.5deg .. 180deg$

Anaytical expression for the pipeline nominal curvature

Nominal Curvature based on Stiffened catenary theory with additional pipeline assumptions

$$k(s) := \frac{A}{A^2 + s^2} - \left[\frac{A}{(A + WD)^2} \cdot \exp\left(\frac{s}{\gamma} - \frac{L}{\gamma}\right)\right] - \frac{A}{(A + s)^2} \cdot \frac{\exp\left(\frac{L}{\gamma} - \frac{s}{\gamma}\right)}{\exp\left(\frac{L}{\gamma}\right)}$$

The experimnetal nominal curvature obtained from digitization of the pictures were instered here while deletling the analytical nominal curvature of the pipe.

Work done Calculation

ORIGIN := 1

Lenght of Residual curavture with 0.5 meter increment

$$Lcurve := \begin{pmatrix} 0.5 \\ 1 \\ 1.5 \\ 2 \\ 2.5 \\ 3 \end{pmatrix}$$

Bending work done

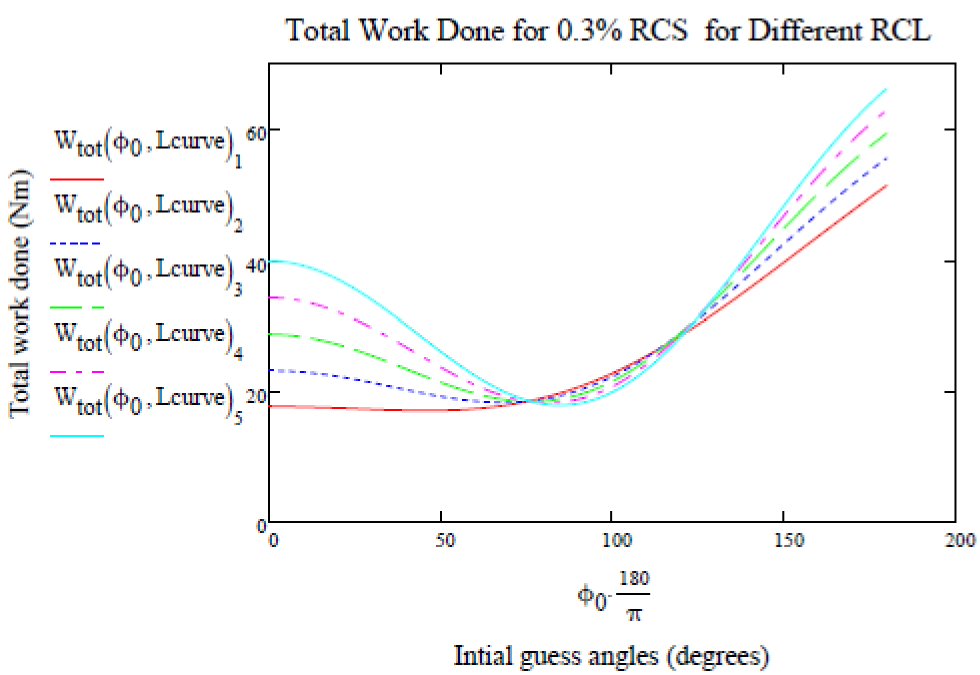
$$W_B(\phi_0, Lcurve) := \overrightarrow{\left[\int_0^{Lcurve} EI\cdot\left(k(s) + k_{res}\cdot\cos(\phi(s,\phi_0))\right)^2 ds + \int_{Lcurve}^{L} EI\cdot k(s)\, ds\right]}$$

Roll work done

$$W_R(\phi_0) := \int_0^L GJ\cdot\left(\frac{d}{ds}\phi(s,\phi_0)\right)^2 ds$$

Total Work Done

$$W_{tot}(\phi_0, Lcurve) := W_R(\phi_0) + W_B(\phi_0, Lcurve)$$



Rotattion Angle at TDP

The rotation angle at Touch point point can be found by finding the minimum work done. Below for each case the minimum work is calculated and corresponding roll angle.

initial guess rotation angle $\phi_0 := 1$

Given

The roatation angle is where the work done is minimum i.e. first derivative gives zero value

$$\frac{d}{d\phi_0} W_{tot}(\phi_0, G) = 0$$

$$f(G) := \text{Find}(\phi_0)$$

Rotation of angøe at TDP corresponding to RCL

$$\text{AngleAtMinEnergy} := \overrightarrow{f(\text{Lcurve})} = \begin{pmatrix} 45.808 \\ 68.296 \\ 76.58 \\ 81.424 \\ 84.9 \\ 87.734 \end{pmatrix} \cdot \text{deg}$$

$$\text{MinEnergy} := \overrightarrow{W_{tot}(\text{AngleAtMinEnergy}, \text{Lcurve})} \quad \text{MinEnergy} = \begin{pmatrix} 17.038 \\ 18.281 \\ 18.427 \\ 18.218 \\ 17.884 \\ 17.538 \end{pmatrix}$$

APPENIDX C: Pipe laboratory axial testing report

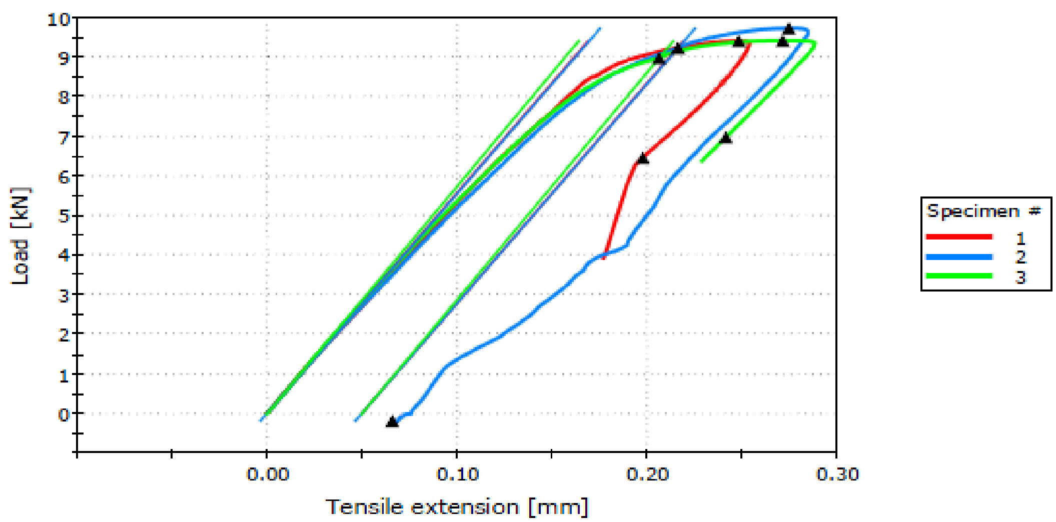
11 April 2016

INSTRON

Instron Applications Laboratory

This template is suitable for creating test procedures that comply with ISO 6892-1: 2009. Test rates and control are set according to "Method A" recommended ranges. Template is intended for specimens that produce a clearly-defined linear elastic region and homogeneous deformation. Default calculated results include Rp 0.2, Fm, Rm and A.

Load vs. Extension



	Modulus (E-Modulus) [GPa]	Tensile stress at Offset yield 0.2% [MPa]	Fm [N]	Tensile stress at Max load [MPa]	Tensile stress at Break (Standard) [MPa]	Strain 1 gauge length [mm]
1	119.9	399.32959	9409.7	406.95453	279.7	49.96555
2	119.6	399.02348	9728.5	420.74579	281.3	49.96463
3	123.4	388.33850	9417.0	407.27209	301.9	49.75263

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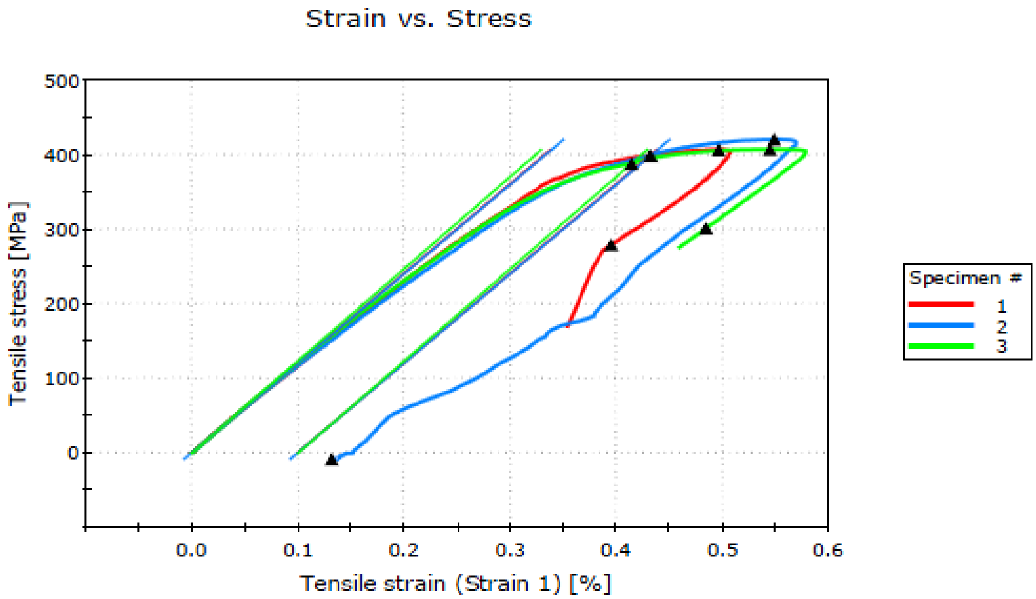
11 April 2016

Graph 2



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APPENIDX D: Scaling calculations

Scaling calculation for deflection number

This Mathcad sheet is used to solve calculate the Deflection number for experimentally used copper pipe of 10mm, 36 inch steel pipe and 20 inch steel pipe.

Copper pipe 10mm used in experiment

Pipeline data

Outer Diameter $OD := 0.01m$

Wall thicknes $t := 0.0008m$

Radius of pipe $r := \frac{OD}{2}$

Moment of Ineria $I := \frac{\pi}{4} \cdot \left[r^4 - (r - t)^4\right]$

Young Modulus $E := 120.97 \cdot 10^9 \cdot \frac{N}{m^2}$

Bending Stifness $EI := E \cdot I$

$EI = 29.817 \frac{m^3 \cdot kg}{s^2}$

Length of caternary pipe $L := 9m$

Dry weight
per unit length of pipe $w := 1.863 \cdot \frac{N}{m}$

Deflection number

$$Dn := \frac{w \cdot L^3}{E \cdot t \cdot OD^3}$$

$$Dn = 14.034$$

For 36 inch steel pipe

Outer Diameter $OD1 := 0.91 \cdot m$

Wall thicknes $t1 := 0.0312268m$

Radius of pipe $r1 := \frac{OD1}{2}$

Moment of Ineria $I1 := \frac{\pi}{4} \cdot \left[r1^4 - (r1 - t1)^4 \right]$

Young Modulus $E1 := 210 \cdot 10^9 \cdot \frac{N}{m^2}$

Bending Stifness

$$EI := E1 \cdot I1$$

$$EI = 1.75 \times 10^9 \frac{m^3 \cdot kg}{s^2}$$

Length of catemary pipe $L1 := 888.33m$

Density of steel pipe $\rho_s := 7850 \cdot \frac{kg}{m^3}$

Density of water $\rho_w := 1025 \cdot \frac{kg}{m^3}$

Area of steel pipe $A1 := \pi \cdot \left[r1^2 - (r1 - t1)^2 \right]$

$$A1 = 0.086\, m^2$$

Gravitional acceleration $g := 9.8 \cdot \frac{m}{s^2}$

$$Dn1 = 14.034$$

For 20 inch steel pipe

Outer Diameter $OD2 := 0.508m$

Wall thicknes $t2 := 0.017655m$

Radius of pipe $r2 := \frac{OD2}{2}$

Moment of Ineria $I2 := \frac{\pi}{4} \cdot \left[r2^4 - (r2 - t2)^4 \right]$

Young Modulus $E1 := 210 \cdot 10^9 \cdot \frac{N}{m^2}$

Bending Stifness $EI := E1 \cdot I2$

$$EI = 1.719 \times 10^8 \, \frac{m^3 \cdot kg}{s^2}$$

Length of catemary pipe (m) $L2 := 494.8 \cdot m$

Density of steel pipe $\rho_s := 7850 \cdot \frac{kg}{m^3}$

Density of water $\rho_w := 1025 \cdot \frac{kg}{m^3}$

Area of steel pipe $A2 := \pi \cdot \left[r2^2 - (r2 - t2)^2 \right]$

$$A2 = 0.027 \, m^2$$

Gravitional acceleration $g := 9.8 \cdot \frac{m}{s^2}$

Dry weight
per unit length of pipe (N/m) $w2 := A2 \cdot \rho_s \cdot g$

$$w2 = 2.092 \times 10^3 \, \frac{kg}{s^2}$$

Submerged weight
per unit length of pipe (N/m) $sw2 := w2 - \rho_w \cdot \pi \cdot r2^2 \cdot g$

$$sw2 = 56.307 \, \frac{kg}{s^2}$$

Deflection number

$$Dn2 := \frac{sw2 \cdot L2^3}{E1 \cdot t2 \cdot OD2^3}$$

$$Dn2 = 14.034$$

We can see that the Deflection number for the 10mm copper pipe, 36 inch steel pipe and 20 inch steel pipe is same.
The deflection number is 14.034

Note that the suspended length of the pipe is changed for different applied load and water depth, hence the deflection number can be determined for different suspended length by using above formula sheet and varying the suspended length.

Copper pipe10mm and 36 inch steel pipe

To compare the deflection result of the experimnetal pipe with the 36 inch steel pipe, the suspended length for the 36 inch pipe should be 888.33m.
the scaling ratio is 1:98.7

Copper pipe10mm and 20 inch steel pipe

To compare the deflection result of the experimnetal pipe with the 36 inch steel pipe, the suspended length for the 20 inch pipe should be 498.9m
the scaling ratio is 1:55.38

APPENDIX E-Nominal Curvature Calculations

- **APPENDIX E-1**

Here x, z are data points. dx and dz are Differentiation of x and z. ds is the arc length. θx and θz are theta angle, calculate by solving θx=invcos(dx/ds) and θz=invsin(dz/dx). They must be same. dθx or dθz is the theta differentiation. Kappa is the nominal curvature obtained by solving kappa=dθ/ds. S is the total arc length, obtained by adding all small arc lengths.

Data points and nominal curvature calculations for Applied load 10N, water depth =4.88m									
x	z	dx	dz	ds	θx	θz	dθx or dθz	kappa	S
0.0000	4.3986	0.1000	0.1567	0.1859	1.0027	1.0027	0.0113	0.0607	0.1859
0.1000	4.2419	0.1000	0.1528	0.1826	0.9914	0.9914	0.0115	0.0631	0.3685
0.2000	4.0891	0.1000	0.1491	0.1795	0.9799	0.9799	0.0118	0.0656	0.5480
0.3000	3.9400	0.1000	0.1453	0.1764	0.9681	0.9681	0.0120	0.0682	0.7244
0.4000	3.7947	0.1000	0.1417	0.1734	0.9561	0.9561	0.0123	0.0709	0.8978
0.5000	3.6530	0.1000	0.1380	0.1704	0.9438	0.9438	0.0126	0.0736	1.0683
0.6000	3.5150	0.1000	0.1344	0.1675	0.9312	0.9312	0.0128	0.0765	1.2358
0.7000	3.3806	0.1000	0.1309	0.1647	0.9184	0.9184	0.0131	0.0793	1.4005
0.8000	3.2497	0.1000	0.1274	0.1620	0.9054	0.9054	0.0133	0.0823	1.5625
0.9000	3.1223	0.1000	0.1240	0.1593	0.8920	0.8920	0.0136	0.0853	1.7218
1.0000	2.9983	0.1000	0.1206	0.1567	0.8784	0.8784	0.0138	0.0884	1.8784
1.1000	2.8777	0.1000	0.1172	0.1541	0.8646	0.8646	0.0141	0.0915	2.0325
1.2000	2.7605	0.1000	0.1139	0.1516	0.8505	0.8505	0.0143	0.0946	2.1841
1.3000	2.6465	0.1000	0.1107	0.1492	0.8362	0.8362	0.0146	0.0978	2.3333
1.4000	2.5358	0.1000	0.1075	0.1468	0.8216	0.8216	0.0148	0.1010	2.4801
1.5000	2.4283	0.1000	0.1044	0.1445	0.8067	0.8067	0.0151	0.1042	2.6247
1.6000	2.3240	0.1000	0.1013	0.1423	0.7917	0.7917	0.0153	0.1074	2.7670
1.7000	2.2227	0.1000	0.0982	0.1402	0.7764	0.7764	0.0155	0.1106	2.9072
1.8000	2.1245	0.1000	0.0952	0.1381	0.7609	0.7609	0.0157	0.1138	3.0452
1.9000	2.0293	0.1000	0.0923	0.1361	0.7452	0.7452	0.0159	0.1170	3.1813
2.0000	1.9370	0.1000	0.0894	0.1341	0.7292	0.7292	0.0161	0.1201	3.3154
2.1000	1.8476	0.1000	0.0865	0.1322	0.7131	0.7131	0.0163	0.1231	3.4476
2.2000	1.7611	0.1000	0.0837	0.1304	0.6969	0.6969	0.0164	0.1261	3.5780
2.3000	1.6774	0.1000	0.0809	0.1286	0.6804	0.6804	0.0166	0.1290	3.7067
2.4000	1.5965	0.1000	0.0782	0.1270	0.6638	0.6638	0.0167	0.1317	3.8336
2.5000	1.5183	0.1000	0.0756	0.1253	0.6471	0.6471	0.0168	0.1343	3.9590
2.6000	1.4427	0.1000	0.0730	0.1238	0.6303	0.6303	0.0169	0.1368	4.0828
2.7000	1.3698	0.1000	0.0704	0.1223	0.6133	0.6133	0.0170	0.1392	4.2051
2.8000	1.2994	0.1000	0.0679	0.1209	0.5963	0.5963	0.0171	0.1413	4.3259
2.9000	1.2315	0.1000	0.0654	0.1195	0.5792	0.5792	0.0171	0.1433	4.4454
3.0000	1.1661	0.1000	0.0630	0.1182	0.5621	0.5621	0.0171	0.1451	4.5636
3.1000	1.1031	0.1000	0.0606	0.1169	0.5450	0.5450	0.0171	0.1466	4.6805

3.2000	1.0425	0.1000	0.0583	0.1158	0.5278	0.5278	0.0171	0.1480	4.7963
3.3000	0.9842	0.1000	0.0560	0.1146	0.5107	0.5107	0.0171	0.1490	4.9109
3.4000	0.9282	0.1000	0.0538	0.1136	0.4936	0.4936	0.0170	0.1499	5.0245
3.5000	0.8744	0.1000	0.0516	0.1125	0.4766	0.4766	0.0169	0.1505	5.1370
3.6000	0.8227	0.1000	0.0495	0.1116	0.4597	0.4597	0.0168	0.1508	5.2486
3.7000	0.7732	0.1000	0.0474	0.1107	0.4428	0.4428	0.0167	0.1509	5.3593
3.8000	0.7258	0.1000	0.0454	0.1098	0.4261	0.4261	0.0165	0.1506	5.4691
3.9000	0.6804	0.1000	0.0434	0.1090	0.4096	0.4096	0.0164	0.1502	5.5781
4.0000	0.6370	0.1000	0.0415	0.1083	0.3932	0.3932	0.0162	0.1494	5.6864
4.1000	0.5955	0.1000	0.0396	0.1076	0.3770	0.3770	0.0160	0.1484	5.7939
4.2000	0.5559	0.1000	0.0378	0.1069	0.3611	0.3611	0.0157	0.1471	5.9008
4.3000	0.5182	0.1000	0.0360	0.1063	0.3454	0.3454	0.0155	0.1456	6.0071
4.4000	0.4822	0.1000	0.0342	0.1057	0.3299	0.3299	0.0152	0.1438	6.1128
4.5000	0.4479	0.1000	0.0326	0.1052	0.3147	0.3147	0.0149	0.1417	6.2179
4.6000	0.4154	0.1000	0.0309	0.1047	0.2998	0.2998	0.0146	0.1394	6.3226
4.7000	0.3845	0.1000	0.0293	0.1042	0.2852	0.2852	0.0143	0.1369	6.4268
4.8000	0.3552	0.1000	0.0278	0.1038	0.2709	0.2709	0.0139	0.1342	6.5306
4.9000	0.3274	0.1000	0.0263	0.1034	0.2570	0.2570	0.0136	0.1313	6.6340
5.0000	0.3011	0.1000	0.0248	0.1030	0.2434	0.2434	0.0132	0.1282	6.7370
5.1000	0.2763	0.1000	0.0234	0.1027	0.2302	0.2302	0.0128	0.1249	6.8398
5.2000	0.2528	0.1000	0.0221	0.1024	0.2174	0.2174	0.0124	0.1214	6.9422
5.3000	0.2307	0.1000	0.0208	0.1021	0.2050	0.2050	0.0120	0.1178	7.0443
5.4000	0.2099	0.1000	0.0195	0.1019	0.1929	0.1929	0.0116	0.1140	7.1462
5.5000	0.1904	0.1000	0.0183	0.1017	0.1813	0.1813	0.0112	0.1101	7.2479
5.6000	0.1721	0.1000	0.0172	0.1015	0.1701	0.1701	0.0108	0.1061	7.3493
5.7000	0.1549	0.1000	0.0161	0.1013	0.1594	0.1594	0.0103	0.1019	7.4506
5.8000	0.1388	0.1000	0.0150	0.1011	0.1491	0.1491	0.0099	0.0977	7.5517
5.9000	0.1238	0.1000	0.0140	0.1010	0.1392	0.1392	0.0094	0.0934	7.6527
6.0000	0.1098	0.1000	0.0130	0.1008	0.1297	0.1297	0.0090	0.0890	7.7536
6.1000	0.0968	0.1000	0.0121	0.1007	0.1208	0.1208	0.0085	0.0845	7.8543
6.2000	0.0846	0.1000	0.0113	0.1006	0.1123	0.1123	0.0080	0.0800	7.9549
6.3000	0.0733	0.1000	0.0105	0.1005	0.1042	0.1042	0.0076	0.0754	8.0555
6.4000	0.0629	0.1000	0.0097	0.1005	0.0966	0.0966	0.0071	0.0707	8.1559
6.5000	0.0532	0.1000	0.0090	0.1004	0.0895	0.0895	0.0066	0.0661	8.2563
6.6000	0.0442	0.1000	0.0083	0.1003	0.0829	0.0829	0.0062	0.0614	8.3567
6.7000	0.0359	0.1000	0.0077	0.1003	0.0767	0.0767	0.0057	0.0566	8.4570
6.8000	0.0282	0.1000	0.0071	0.1003	0.0711	0.0711	0.0052	0.0519	8.5572
6.9000	0.0211	0.1000	0.0066	0.1002	0.0659	0.0659	0.0047	0.0471	8.6574
7.0000	0.0145	0.1000	0.0061	0.1002	0.0611	0.0611	0.0042	0.0423	8.7576
7.1000	0.0084	0.1000	0.0057	0.1002	0.0569	0.0569			8.8578
7.2000	0.0027								

- ## APPENDIX E-2

Here x, z are data points. dx and dz are Differentiation of x and z. ds is the arc length. θx and θz are theta angle, calculate by solving θx=invcos(dx/ds) and θz=invsin(dz/dx). They must be same. dθx or dθz is the theta differentiation. Kappa is the nominal curvature obtained by solving kappa=dθ/ds. S is the total arc length, obtained by adding all small arc lengths

Data points and nominal curvature calculations for Applied load 20N, water depth =4.88m									
x	z	dx	dz	ds	θx	θz	dθx or dθz	kappa	S
0	4.5218	0.1000	0.1556	0.1849	0.9995	0.9995	0.0109	0.0590	0.1849
0.1	4.3663	0.1000	0.1519	0.1819	0.9886	0.9886	0.0112	0.0613	0.3668
0.2	4.2144	0.1000	0.1483	0.1788	0.9774	0.9774	0.0114	0.0637	0.5456
0.3	4.0661	0.1000	0.1447	0.1759	0.9660	0.9660	0.0116	0.0661	0.7215
0.4	3.9214	0.1000	0.1412	0.1730	0.9544	0.9544	0.0119	0.0685	0.8945
0.5	3.7802	0.1000	0.1377	0.1701	0.9426	0.9426	0.0121	0.0711	1.0647
0.6	3.6426	0.1000	0.1342	0.1674	0.9305	0.9305	0.0123	0.0737	1.2320
0.7	3.5084	0.1000	0.1308	0.1647	0.9181	0.9181	0.0126	0.0763	1.3967
0.8	3.3775	0.1000	0.1275	0.1620	0.9056	0.9056	0.0128	0.0790	1.5587
0.9	3.2501	0.1000	0.1242	0.1594	0.8928	0.8928	0.0130	0.0818	1.7181
1	3.1259	0.1000	0.1209	0.1569	0.8797	0.8797	0.0133	0.0846	1.8750
1.1	3.0050	0.1000	0.1177	0.1544	0.8664	0.8664	0.0135	0.0875	2.0294
1.2	2.8873	0.1000	0.1145	0.1520	0.8529	0.8529	0.0137	0.0903	2.1815
1.3	2.7728	0.1000	0.1114	0.1497	0.8392	0.8392	0.0140	0.0932	2.3312
1.4	2.6614	0.1000	0.1083	0.1474	0.8252	0.8252	0.0142	0.0962	2.4786
1.5	2.5531	0.1000	0.1053	0.1452	0.8111	0.8111	0.0144	0.0991	2.6238
1.6	2.4479	0.1000	0.1023	0.1430	0.7967	0.7967	0.0146	0.1020	2.7668
1.7	2.3456	0.1000	0.0993	0.1410	0.7821	0.7821	0.0148	0.1050	2.9078
1.8	2.2463	0.1000	0.0964	0.1389	0.7673	0.7673	0.0150	0.1079	3.0467
1.9	2.1498	0.1000	0.0936	0.1370	0.7523	0.7523	0.0152	0.1108	3.1837
2	2.0562	0.1000	0.0908	0.1351	0.7371	0.7371	0.0153	0.1136	3.3187
2.1	1.9654	0.1000	0.0880	0.1332	0.7218	0.7218	0.0155	0.1164	3.4519
2.2	1.8774	0.1000	0.0853	0.1314	0.7063	0.7063	0.0157	0.1191	3.5834
2.3	1.7921	0.1000	0.0826	0.1297	0.6906	0.6906	0.0158	0.1217	3.7131
2.4	1.7095	0.1000	0.0800	0.1281	0.6748	0.6748	0.0159	0.1243	3.8412
2.5	1.6294	0.1000	0.0774	0.1265	0.6589	0.6589	0.0160	0.1267	3.9677
2.6	1.5520	0.1000	0.0749	0.1249	0.6429	0.6429	0.0161	0.1290	4.0926
2.7	1.4771	0.1000	0.0724	0.1235	0.6268	0.6268	0.0162	0.1312	4.2161
2.8	1.4047	0.1000	0.0700	0.1221	0.6106	0.6106	0.0163	0.1332	4.3381
2.9	1.3347	0.1000	0.0676	0.1207	0.5943	0.5943	0.0163	0.1351	4.4588
3	1.2671	0.1000	0.0652	0.1194	0.5780	0.5780	0.0163	0.1368	4.5782
3.1	1.2019	0.1000	0.0629	0.1182	0.5617	0.5617	0.0163	0.1383	4.6964
3.2	1.1390	0.1000	0.0607	0.1170	0.5453	0.5453	0.0163	0.1397	4.8133

3.3	1.0783	0.1000	0.0585	0.1158	0.5290	0.5290	0.0163	0.1408	4.9292
3.4	1.0198	0.1000	0.0563	0.1148	0.5127	0.5127	0.0163	0.1417	5.0439
3.5	0.9635	0.1000	0.0542	0.1137	0.4964	0.4964	0.0162	0.1424	5.1577
3.6	0.9094	0.1000	0.0521	0.1128	0.4802	0.4802	0.0161	0.1428	5.2704
3.7	0.8573	0.1000	0.0501	0.1118	0.4641	0.4641	0.0160	0.1431	5.3822
3.8	0.8072	0.1000	0.0481	0.1110	0.4481	0.4481	0.0159	0.1431	5.4932
3.9	0.7592	0.1000	0.0461	0.1101	0.4322	0.4322	0.0157	0.1428	5.6033
4	0.7130	0.1000	0.0442	0.1093	0.4165	0.4165	0.0156	0.1423	5.7127
4.1	0.6688	0.1000	0.0424	0.1086	0.4010	0.4010	0.0154	0.1416	5.8213
4.2	0.6264	0.1000	0.0406	0.1079	0.3856	0.3856	0.0152	0.1406	5.9292
4.3	0.5858	0.1000	0.0388	0.1073	0.3704	0.3704	0.0150	0.1394	6.0365
4.4	0.5470	0.1000	0.0371	0.1067	0.3554	0.3554	0.0147	0.1380	6.1432
4.5	0.5098	0.1000	0.0355	0.1061	0.3407	0.3407	0.0145	0.1364	6.2493
4.6	0.4744	0.1000	0.0338	0.1056	0.3263	0.3263	0.0142	0.1345	6.3548
4.7	0.4406	0.1000	0.0323	0.1051	0.3121	0.3121	0.0139	0.1324	6.4599
4.8	0.4083	0.1000	0.0307	0.1046	0.2981	0.2981	0.0136	0.1301	6.5645
4.9	0.3776	0.1000	0.0292	0.1042	0.2845	0.2845	0.0133	0.1276	6.6687
5	0.3483	0.1000	0.0278	0.1038	0.2712	0.2712	0.0130	0.1250	6.7725
5.1	0.3205	0.1000	0.0264	0.1034	0.2583	0.2583	0.0126	0.1221	6.8759
5.2	0.2941	0.1000	0.0251	0.1031	0.2456	0.2456	0.0123	0.1191	6.9790
5.3	0.2690	0.1000	0.0238	0.1028	0.2334	0.2334	0.0119	0.1159	7.0818
5.4	0.2453	0.1000	0.0225	0.1025	0.2214	0.2214	0.0115	0.1126	7.1843
5.5	0.2227	0.1000	0.0213	0.1022	0.2099	0.2099	0.0112	0.1092	7.2866
5.6	0.2014	0.1000	0.0201	0.1020	0.1987	0.1987	0.0108	0.1056	7.3886
5.7	0.1813	0.1000	0.0190	0.1018	0.1879	0.1879	0.0104	0.1019	7.4904
5.8	0.1623	0.1000	0.0179	0.1016	0.1776	0.1776	0.0100	0.0982	7.5920
5.9	0.1443	0.1000	0.0169	0.1014	0.1676	0.1676	0.0096	0.0943	7.6934
6	0.1274	0.1000	0.0159	0.1013	0.1580	0.1580	0.0091	0.0903	7.7946
6.1	0.1115	0.1000	0.0150	0.1011	0.1489	0.1489	0.0087	0.0863	7.8958
6.2	0.0965	0.1000	0.0141	0.1010	0.1402	0.1402	0.0083	0.0822	7.9967
6.3	0.0824	0.1000	0.0133	0.1009	0.1319	0.1319	0.0079	0.0780	8.0976
6.4	0.0691	0.1000	0.0125	0.1008	0.1240	0.1240	0.0074	0.0737	8.1984
6.5	0.0566	0.1000	0.0117	0.1007	0.1166	0.1166	0.0070	0.0695	8.2991
6.6	0.0449	0.1000	0.0110	0.1006	0.1096	0.1096	0.0066	0.0651	8.3997
6.7	0.0339	0.1000	0.0103	0.1005	0.1030	0.1030	0.0061	0.0608	8.5002
6.8	0.0236	0.1000	0.0097	0.1005	0.0969	0.0969	0.0057	0.0564	8.6007
6.9	0.0139	0.1000	0.0092	0.1004	0.0913	0.0913	0.0052	0.0520	8.7011
7	0.0047	0.1000	0.0086	0.1004	0.0860	0.0860	0.0048	0.0475	8.8015
7.1	-0.0039	0.1000	0.0081	0.1003	0.0813	0.0813	0.0043	0.0431	8.9018
7.2	-0.0120	0.1000	0.0077	0.1003	0.0769	0.0769	0.0769		9.0021
7.3	-0.0198								

APPENDIX E-3

Here x, z are data points. dx and dz are Differentiation of x and z. ds is the arc length. θx and θz are theta angle, calculate by solving θx=invcos(dx/ds) and θz=invsin(dz/dx). They must be same. dθx or dθz is the theta differentiation. Kappa is the nominal curvature obtained by solving kappa=dθ/ds. S is the total arc length, obtained by adding all small arc lengths

Data points and nominal curvature calculations for Applied load 40N, water depth =4.88m									
x	z	dx	dz	ds	θx	θz	dθx or dθz	kappa	S
0	4.6000	0.1000	0.1526	0.1824	0.9907	0.9907	0.0101	0.0553	0.1824
0.1	4.4474	0.1000	0.1493	0.1797	0.9806	0.9806	0.0103	0.0573	0.3621
0.2	4.2981	0.1000	0.1460	0.1770	0.9703	0.9703	0.0105	0.0592	0.5391
0.3	4.1521	0.1000	0.1428	0.1743	0.9598	0.9598	0.0107	0.0613	0.7134
0.4	4.0093	0.1000	0.1396	0.1717	0.9492	0.9492	0.0109	0.0633	0.8851
0.5	3.8697	0.1000	0.1364	0.1692	0.9383	0.9383	0.0111	0.0655	1.0543
0.6	3.7333	0.1000	0.1333	0.1666	0.9272	0.9272	0.0113	0.0676	1.2209
0.7	3.6000	0.1000	0.1302	0.1642	0.9159	0.9159	0.0115	0.0698	1.3851
0.8	3.4698	0.1000	0.1272	0.1618	0.9045	0.9045	0.0117	0.0721	1.5469
0.9	3.3426	0.1000	0.1242	0.1594	0.8928	0.8928	0.0119	0.0744	1.7064
1	3.2184	0.1000	0.1212	0.1571	0.8810	0.8810	0.0120	0.0767	1.8635
1.1	3.0972	0.1000	0.1183	0.1549	0.8689	0.8689	0.0122	0.0790	2.0184
1.2	2.9789	0.1000	0.1154	0.1527	0.8567	0.8567	0.0124	0.0814	2.1710
1.3	2.8636	0.1000	0.1125	0.1505	0.8442	0.8442	0.0126	0.0838	2.3216
1.4	2.7510	0.1000	0.1097	0.1484	0.8316	0.8316	0.0128	0.0862	2.4700
1.5	2.6413	0.1000	0.1069	0.1464	0.8188	0.8188	0.0130	0.0886	2.6164
1.6	2.5344	0.1000	0.1042	0.1444	0.8058	0.8058	0.0132	0.0911	2.7608
1.7	2.4302	0.1000	0.1015	0.1425	0.7927	0.7927	0.0133	0.0935	2.9033
1.8	2.3288	0.1000	0.0988	0.1406	0.7794	0.7794	0.0135	0.0959	3.0439
1.9	2.2300	0.1000	0.0962	0.1387	0.7659	0.7659	0.0136	0.0983	3.1826
2	2.1338	0.1000	0.0936	0.1370	0.7522	0.7522	0.0138	0.1007	3.3196
2.1	2.0402	0.1000	0.0910	0.1352	0.7385	0.7385	0.0139	0.1031	3.4548
2.2	1.9492	0.1000	0.0885	0.1335	0.7245	0.7245	0.0141	0.1054	3.5883
2.3	1.8607	0.1000	0.0860	0.1319	0.7104	0.7104	0.0142	0.1076	3.7202
2.4	1.7747	0.1000	0.0836	0.1303	0.6963	0.6963	0.0143	0.1098	3.8506
2.5	1.6911	0.1000	0.0812	0.1288	0.6819	0.6819	0.0144	0.1120	3.9794
2.6	1.6099	0.1000	0.0788	0.1273	0.6675	0.6675	0.0145	0.1140	4.1067
2.7	1.5311	0.1000	0.0765	0.1259	0.6530	0.6530	0.0146	0.1160	4.2326
2.8	1.4546	0.1000	0.0742	0.1245	0.6384	0.6384	0.0147	0.1179	4.3571
2.9	1.3804	0.1000	0.0720	0.1232	0.6237	0.6237	0.0147	0.1197	4.4803
3	1.3084	0.1000	0.0697	0.1219	0.6090	0.6090	0.0148	0.1213	4.6023
3.1	1.2387	0.1000	0.0676	0.1207	0.5942	0.5942	0.0148	0.1229	4.7229
3.2	1.1711	0.1000	0.0654	0.1195	0.5794	0.5794	0.0149	0.1243	4.8424

3.3	1.1057	0.1000	0.0633	0.1184	0.5645	0.5645	0.0149	0.1256	4.9608
3.4	1.0423	0.1000	0.0613	0.1173	0.5496	0.5496	0.0149	0.1267	5.0781
3.5	0.9811	0.1000	0.0592	0.1162	0.5348	0.5348	0.0148	0.1277	5.1943
3.6	0.9219	0.1000	0.0572	0.1152	0.5199	0.5199	0.0148	0.1285	5.3095
3.7	0.8646	0.1000	0.0553	0.1143	0.5051	0.5051	0.0148	0.1291	5.4238
3.8	0.8093	0.1000	0.0534	0.1134	0.4904	0.4904	0.0147	0.1296	5.5372
3.9	0.7559	0.1000	0.0515	0.1125	0.4757	0.4757	0.0146	0.1299	5.6496
4	0.7044	0.1000	0.0497	0.1117	0.4611	0.4611	0.0145	0.1301	5.7613
4.1	0.6547	0.1000	0.0479	0.1109	0.4465	0.4465	0.0144	0.1300	5.8722
4.2	0.6068	0.1000	0.0461	0.1101	0.4321	0.4321	0.0143	0.1298	5.9823
4.3	0.5607	0.1000	0.0444	0.1094	0.4178	0.4178	0.0142	0.1294	6.0917
4.4	0.5163	0.1000	0.0427	0.1087	0.4037	0.4037	0.0140	0.1289	6.2005
4.5	0.4736	0.1000	0.0411	0.1081	0.3897	0.3897	0.0138	0.1281	6.3086
4.6	0.4325	0.1000	0.0395	0.1075	0.3758	0.3758	0.0137	0.1272	6.4161
4.7	0.3931	0.1000	0.0379	0.1069	0.3621	0.3621	0.0135	0.1260	6.5230
4.8	0.3552	0.1000	0.0364	0.1064	0.3487	0.3487	0.0133	0.1248	6.6294
4.9	0.3189	0.1000	0.0349	0.1059	0.3354	0.3354	0.0131	0.1233	6.7353
5	0.2840	0.1000	0.0334	0.1054	0.3223	0.3223	0.0128	0.1217	6.8407
5.1	0.2506	0.1000	0.0320	0.1050	0.3095	0.3095	0.0126	0.1199	6.9457
5.2	0.2186	0.1000	0.0306	0.1046	0.2969	0.2969	0.0123	0.1180	7.0503
5.3	0.1880	0.1000	0.0293	0.1042	0.2846	0.2846	0.0121	0.1159	7.1545
5.4	0.1588	0.1000	0.0279	0.1038	0.2725	0.2725	0.0118	0.1137	7.2583
5.5	0.1308	0.1000	0.0267	0.1035	0.2607	0.2607	0.0115	0.1114	7.3618
5.6	0.1042	0.1000	0.0254	0.1032	0.2491	0.2491	0.0112	0.1089	7.4650
5.7	0.0787	0.1000	0.0242	0.1029	0.2379	0.2379	0.0109	0.1063	7.5679
5.8	0.0545	0.1000	0.0231	0.1026	0.2270	0.2270	0.0106	0.1036	7.6705
5.9	0.0314	0.1000	0.0220	0.1024	0.2163	0.2163	0.0103	0.1008	7.7729
6	0.0094	0.1000	0.0209	0.1022	0.2060	0.2060	0.0100	0.0979	7.8751
6.1	-0.0115	0.1000	0.0199	0.1020	0.1960	0.1960	0.0097	0.0950	7.9770
6.2	-0.0314	0.1000	0.0188	0.1018	0.1863	0.1863	0.0094	0.0919	8.0788
6.3	-0.0502	0.1000	0.0179	0.1016	0.1770	0.1770	0.0090	0.0887	8.1804
6.4	-0.0681	0.1000	0.0170	0.1014	0.1679	0.1679	0.0087	0.0855	8.2818
6.5	-0.0850	0.1000	0.0161	0.1013	0.1593	0.1593	0.0083	0.0822	8.3831
6.6	-0.1011	0.1000	0.0152	0.1012	0.1509	0.1509	0.0080	0.0789	8.4842
6.7	-0.1163	0.1000	0.0144	0.1010	0.1430	0.1430	0.0076	0.0755	8.5853
6.8	-0.1307	0.1000	0.0136	0.1009	0.1353	0.1353	0.0073	0.0720	8.6862
6.9	-0.1443	0.1000	0.0129	0.1008	0.1281	0.1281	0.0069	0.0685	8.7870
7	-0.1572	0.1000	0.0122	0.1007	0.1212	0.1212	0.0066	0.0650	8.8877
7.1	-0.1694	0.1000	0.0115	0.1007	0.1146	0.1146	0.0062	0.0615	8.9884
7.2	-0.1809	0.1000	0.0109	0.1006	0.1084	0.1084	0.0058	0.0579	9.0890
7.3	-0.1918	0.1000	0.0103	0.1005	0.1026	0.1026	0.0055	0.0542	9.1895

7.4	-0.2021	0.1000	0.0097	0.1005	0.0971	0.0971	0.0051	0.0506	9.2900
7.5	-0.2118	0.1000	0.0092	0.1004	0.0921	0.0921			9.3904
7.6	-0.2210								

- ## APPENDIX E-4

Here x, z are data points. dx and dz are Differentiation of x and z. ds is the arc length. θx and θz are theta angle, calculate by solving θx=invcos(dx/ds) and θz=invsin(dz/dx). They must be same. dθx or dθz is the theta differentiation. Kappa is the nominal curvature obtained by solving kappa=dθ/ds. S is the total arc length, obtained by adding all small arc lengths

Data points and nominal curvature calculations for Applied load 20N, water depth =5.91m									
x	z	dx	dz	ds	θx	θz	dθx or dθz	kappa	S
0	5.4100	0.1000	0.0821	0.1294	0.6874	0.6874	-0.0227	-0.0227	0.1294
0.1	5.3279	0.1000	0.0860	0.1319	0.7101	0.7101	-0.0204	-0.0204	0.2613
0.2	5.2419	0.1000	0.0896	0.1343	0.7305	0.7305	-0.0182	-0.0182	0.3955
0.3	5.1524	0.1000	0.0929	0.1365	0.7487	0.7487	-0.0163	-0.0163	0.5320
0.4	5.0594	0.1000	0.0960	0.1386	0.7650	0.7650	-0.0145	-0.0145	0.6706
0.5	4.9634	0.1000	0.0988	0.1406	0.7794	0.7794	-0.0128	-0.0128	0.8112
0.6	4.8646	0.1000	0.1014	0.1424	0.7923	0.7923	-0.0113	-0.0113	0.9536
0.7	4.7632	0.1000	0.1037	0.1441	0.8036	0.8036	-0.0099	-0.0099	1.0977
0.8	4.6595	0.1000	0.1058	0.1456	0.8135	0.8135	-0.0087	-0.0087	1.2433
0.9	4.5537	0.1000	0.1076	0.1469	0.8222	0.8222	-0.0075	-0.0075	1.3902
1	4.4461	0.1000	0.1093	0.1481	0.8296	0.8296	-0.0063	-0.0063	1.5383
1.1	4.3368	0.1000	0.1107	0.1492	0.8360	0.8360	-0.0053	-0.0053	1.6875
1.2	4.2262	0.1000	0.1118	0.1500	0.8413	0.8413	-0.0043	-0.0043	1.8375
1.3	4.1143	0.1000	0.1128	0.1508	0.8456	0.8456	-0.0034	-0.0034	1.9883
1.4	4.0015	0.1000	0.1136	0.1513	0.8489	0.8489	-0.0025	-0.0025	2.1396
1.5	3.8879	0.1000	0.1142	0.1518	0.8514	0.8514	-0.0016	-0.0016	2.2914
1.6	3.7738	0.1000	0.1145	0.1520	0.8530	0.8530	-0.0008	-0.0008	2.4434
1.7	3.6592	0.1000	0.1147	0.1522	0.8539	0.8539	0.0000	0.0000	2.5956
1.8	3.5445	0.1000	0.1147	0.1522	0.8539	0.8539	0.0007	0.0007	2.7478
1.9	3.4298	0.1000	0.1146	0.1521	0.8532	0.8532	0.0015	0.0015	2.8999
2	3.3152	0.1000	0.1142	0.1518	0.8517	0.8517	0.0022	0.0022	3.0517
2.1	3.2010	0.1000	0.1137	0.1514	0.8495	0.8495	0.0029	0.0029	3.2031
2.2	3.0873	0.1000	0.1131	0.1509	0.8466	0.8466	0.0036	0.0036	3.3540
2.3	2.9742	0.1000	0.1122	0.1503	0.8430	0.8430	0.0043	0.0043	3.5044
2.4	2.8620	0.1000	0.1113	0.1496	0.8387	0.8387	0.0050	0.0050	3.6540
2.5	2.7507	0.1000	0.1102	0.1488	0.8338	0.8338	0.0056	0.0056	3.8028
2.6	2.6405	0.1000	0.1089	0.1479	0.8281	0.8281	0.0063	0.0063	3.9506
2.7	2.5316	0.1000	0.1076	0.1469	0.8218	0.8218	0.0070	0.0070	4.0975
2.8	2.4240	0.1000	0.1061	0.1458	0.8148	0.8148	0.0077	0.0077	4.2433

2.9	2.3180	0.1000	0.1045	0.1446	0.8072	0.8072	0.0083	0.0083	4.3879
3	2.2135	0.1000	0.1027	0.1434	0.7989	0.7989	0.0090	0.0090	4.5312
3.1	2.1108	0.1000	0.1009	0.1421	0.7899	0.7899	0.0097	0.0097	4.6733
3.2	2.0099	0.1000	0.0990	0.1407	0.7802	0.7802	0.0104	0.0104	4.8140
3.3	1.9109	0.1000	0.0969	0.1393	0.7698	0.7698	0.0110	0.0110	4.9533
3.4	1.8140	0.1000	0.0948	0.1378	0.7588	0.7588	0.0117	0.0117	5.0911
3.5	1.7192	0.1000	0.0926	0.1363	0.7471	0.7471	0.0124	0.0124	5.2274
3.6	1.6265	0.1000	0.0903	0.1348	0.7346	0.7346	0.0131	0.0131	5.3621
3.7	1.5362	0.1000	0.0880	0.1332	0.7215	0.7215	0.0138	0.0138	5.4953
3.8	1.4482	0.1000	0.0856	0.1316	0.7077	0.7077	0.0145	0.0145	5.6269
3.9	1.3627	0.1000	0.0831	0.1300	0.6932	0.6932	0.0152	0.0152	5.7569
4	1.2796	0.1000	0.0805	0.1284	0.6780	0.6780	0.0159	0.0159	5.8853
4.1	1.1991	0.1000	0.0780	0.1268	0.6622	0.6622	0.0166	0.0166	6.0121
4.2	1.1211	0.1000	0.0753	0.1252	0.6456	0.6456	0.0172	0.0172	6.1373
4.3	1.0458	0.1000	0.0727	0.1236	0.6284	0.6284	0.0179	0.0179	6.2609
4.4	0.9731	0.1000	0.0700	0.1220	0.6105	0.6105	0.0185	0.0185	6.3830
4.5	0.9031	0.1000	0.0672	0.1205	0.5920	0.5920	0.0191	0.0191	6.5035
4.6	0.8359	0.1000	0.0645	0.1190	0.5728	0.5728	0.0197	0.0197	6.6225
4.7	0.7714	0.1000	0.0617	0.1175	0.5531	0.5531	0.0203	0.0203	6.7400
4.8	0.7097	0.1000	0.0590	0.1161	0.5328	0.5328	0.0208	0.0208	6.8561
4.9	0.6507	0.1000	0.0562	0.1147	0.5120	0.5120	0.0213	0.0213	6.9708
5	0.5945	0.1000	0.0534	0.1134	0.4907	0.4907	0.0217	0.0217	7.0842
5.1	0.5411	0.1000	0.0507	0.1121	0.4690	0.4690	0.0221	0.0221	7.1963
5.2	0.4904	0.1000	0.0479	0.1109	0.4469	0.4469	0.0224	0.0224	7.3072
5.3	0.4425	0.1000	0.0452	0.1097	0.4244	0.4244	0.0227	0.0227	7.4169
5.4	0.3973	0.1000	0.0425	0.1087	0.4018	0.4018	0.0228	0.0228	7.5256
5.5	0.3548	0.1000	0.0398	0.1076	0.3789	0.3789	0.0229	0.0229	7.6332
5.6	0.3150	0.1000	0.0372	0.1067	0.3560	0.3560	0.0229	0.0229	7.7399
5.7	0.2778	0.1000	0.0346	0.1058	0.3331	0.3331	0.0229	0.0229	7.8457
5.8	0.2432	0.1000	0.0321	0.1050	0.3102	0.3102	0.0227	0.0227	7.9507
5.9	0.2112	0.1000	0.0296	0.1043	0.2875	0.2875	0.0225	0.0225	8.0550
6	0.1816	0.1000	0.0271	0.1036	0.2650	0.2650	0.0221	0.0221	8.1586
6.1	0.1545	0.1000	0.0248	0.1030	0.2429	0.2429	0.0217	0.0217	8.2616
6.2	0.1297	0.1000	0.0225	0.1025	0.2212	0.2212	0.0212	0.0212	8.3641
6.3	0.1072	0.1000	0.0203	0.1020	0.2000	0.2000	0.0205	0.0205	8.4662
6.4	0.0869	0.1000	0.0181	0.1016	0.1795	0.1795	0.0198	0.0198	8.5678
6.5	0.0688	0.1000	0.0161	0.1013	0.1597	0.1597	0.0190	0.0190	8.6691
6.6	0.0527	0.1000	0.0142	0.1010	0.1406	0.1406	0.0181	0.0181	8.7701
6.7	0.0385	0.1000	0.0123	0.1008	0.1225	0.1225	0.0172	0.0172	8.8708
6.8	0.0262	0.1000	0.0106	0.1006	0.1053	0.1053	0.0161	0.0161	8.9714
6.9	0.0156	0.1000	0.0089	0.1004	0.0892	0.0892	0.0150	0.0150	9.0718

7	0.0067	0.1000	0.0074	0.1003	0.0742	0.0742	0.0138	0.0138	9.1721
7.1	-0.0007	0.1000	0.0060	0.1002	0.0604	0.0604	0.0125	0.0125	9.2722
7.2	-0.0068	0.1000	0.0048	0.1001	0.0479	0.0479	0.0112	0.0112	9.3724
7.3	-0.0116	0.1000	0.0037	0.1001	0.0367	0.0367	0.0098	0.0098	9.4724
7.4	-0.0152	0.1000	0.0027	0.1000	0.0269	0.0269	0.0083	0.0083	9.5725
7.5	-0.0179	0.1000	0.0019	0.1000	0.0185	0.0185	0.0068	0.0068	9.6725
7.6	-0.0198	0.1000	0.0012	0.1000	0.0117	0.0117	0.0052	0.0052	9.7725
7.7	-0.0210	0.1000	0.0006	0.1000	0.0065	0.0065	0.0036	0.0036	9.8725
7.8	-0.0216	0.1000	0.0003	0.1000	0.0029	0.0029	0.0019	0.0019	9.9725
7.9	-0.0219	0.1000	0.0001	0.1000	0.0010	0.0010	0.0002	0.0002	10.0725
8	-0.0220	0.1000	0.0001	0.1000	0.0008	0.0008	-0.0017	-0.0017	10.1725
8.1	-0.0221	0.1000	0.0002	0.1000	0.0025	0.0025			10.2725
8.2	-0.0223								

- **APPENDIX E-5**

Here x, z are data points. dx and dz are Differentiation of x and z. ds is the arc length. θx and θz are theta angle, calculate by solving θx=invcos(dx/ds) and θz=invsin(dz/dx). They must be same. dθx or dθz is the theta differentiation. Kappa is the nominal curvature obtained by solving kappa=dθ/ds. S is the total arc length, obtained by adding all small arc lengths.

Data points and nominal curvature calculations for Applied load 40N, water depth =5.91m									
x	z	dx	dz	ds	θx	θz	dθx or dθz	kappa	S
0	5.6570	0.1000	0.0816	0.1291	0.6847	0.6847	-0.0201	-0.1557	0.1291
0.1	5.5754	0.1000	0.0850	0.1313	0.7048	0.7048	-0.0182	-0.1383	0.2604
0.2	5.4903	0.1000	0.0882	0.1334	0.7229	0.7229	-0.0164	-0.1229	0.3937
0.3	5.4021	0.1000	0.0912	0.1353	0.7393	0.7393	-0.0148	-0.1090	0.5290
0.4	5.3109	0.1000	0.0939	0.1372	0.7541	0.7541	-0.0133	-0.0966	0.6662
0.5	5.2170	0.1000	0.0964	0.1389	0.7673	0.7673	-0.0119	-0.0855	0.8052
0.6	5.1205	0.1000	0.0988	0.1406	0.7792	0.7792	-0.0106	-0.0754	0.9457
0.7	5.0218	0.1000	0.1009	0.1420	0.7898	0.7898	-0.0094	-0.0662	1.0878
0.8	4.9209	0.1000	0.1028	0.1434	0.7992	0.7992	-0.0083	-0.0579	1.2312
0.9	4.8181	0.1000	0.1045	0.1447	0.8075	0.8075	-0.0073	-0.0502	1.3758
1	4.7136	0.1000	0.1060	0.1458	0.8147	0.8147	-0.0063	-0.0432	1.5216
1.1	4.6075	0.1000	0.1074	0.1467	0.8210	0.8210	-0.0054	-0.0367	1.6683
1.2	4.5001	0.1000	0.1086	0.1476	0.8264	0.8264	-0.0045	-0.0306	1.8159
1.3	4.3916	0.1000	0.1096	0.1483	0.8309	0.8309	-0.0037	-0.0250	1.9643
1.4	4.2820	0.1000	0.1104	0.1489	0.8347	0.8347	-0.0029	-0.0197	2.1132
1.5	4.1717	0.1000	0.1110	0.1494	0.8376	0.8376	-0.0022	-0.0146	2.2626
1.6	4.0606	0.1000	0.1115	0.1498	0.8398	0.8398	-0.0015	-0.0099	2.4124
1.7	3.9491	0.1000	0.1118	0.1500	0.8412	0.8412	-0.0008	-0.0053	2.5624
1.8	3.8373	0.1000	0.1120	0.1502	0.8420	0.8420	-0.0001	-0.0009	2.7126

1.9	3.7253	0.1000	0.1121	0.1502	0.8422	0.8422	0.0005	0.0034	2.8628
2	3.6132	0.1000	0.1119	0.1501	0.8417	0.8417	0.0011	0.0075	3.0129
2.1	3.5013	0.1000	0.1117	0.1499	0.8405	0.8405	0.0017	0.0115	3.1628
2.2	3.3896	0.1000	0.1113	0.1496	0.8388	0.8388	0.0023	0.0155	3.3124
2.3	3.2783	0.1000	0.1108	0.1492	0.8365	0.8365	0.0029	0.0194	3.4616
2.4	3.1675	0.1000	0.1101	0.1488	0.8336	0.8336	0.0035	0.0233	3.6104
2.5	3.0574	0.1000	0.1094	0.1482	0.8301	0.8301	0.0040	0.0272	3.7586
2.6	2.9480	0.1000	0.1085	0.1475	0.8261	0.8261	0.0046	0.0311	3.9062
2.7	2.8395	0.1000	0.1075	0.1468	0.8215	0.8215	0.0051	0.0349	4.0530
2.8	2.7320	0.1000	0.1064	0.1460	0.8164	0.8164	0.0057	0.0389	4.1990
2.9	2.6256	0.1000	0.1052	0.1451	0.8107	0.8107	0.0062	0.0428	4.3441
3	2.5204	0.1000	0.1039	0.1442	0.8045	0.8045	0.0068	0.0468	4.4883
3.1	2.4165	0.1000	0.1025	0.1432	0.7977	0.7977	0.0073	0.0509	4.6315
3.2	2.3140	0.1000	0.1010	0.1421	0.7905	0.7905	0.0078	0.0550	4.7737
3.3	2.2130	0.1000	0.0994	0.1410	0.7826	0.7826	0.0083	0.0592	4.9147
3.4	2.1136	0.1000	0.0978	0.1399	0.7743	0.7743	0.0089	0.0635	5.0546
3.5	2.0158	0.1000	0.0961	0.1387	0.7654	0.7654	0.0094	0.0678	5.1933
3.6	1.9197	0.1000	0.0943	0.1374	0.7560	0.7560	0.0099	0.0722	5.3307
3.7	1.8254	0.1000	0.0924	0.1362	0.7461	0.7461	0.0105	0.0768	5.4669
3.8	1.7330	0.1000	0.0905	0.1349	0.7356	0.7356	0.0110	0.0813	5.6017
3.9	1.6425	0.1000	0.0885	0.1336	0.7247	0.7247	0.0115	0.0860	5.7353
4	1.5539	0.1000	0.0865	0.1322	0.7132	0.7132	0.0120	0.0907	5.8675
4.1	1.4674	0.1000	0.0844	0.1309	0.7012	0.7012	0.0125	0.0955	5.9984
4.2	1.3830	0.1000	0.0823	0.1295	0.6887	0.6887	0.0130	0.1004	6.1279
4.3	1.3007	0.1000	0.0802	0.1282	0.6757	0.6757	0.0135	0.1052	6.2561
4.4	1.2205	0.1000	0.0780	0.1268	0.6622	0.6622	0.0140	0.1101	6.3829
4.5	1.1426	0.1000	0.0757	0.1254	0.6482	0.6482	0.0144	0.1150	6.5083
4.6	1.0668	0.1000	0.0735	0.1241	0.6338	0.6338	0.0149	0.1198	6.6324
4.7	0.9933	0.1000	0.0712	0.1228	0.6189	0.6189	0.0153	0.1246	6.7552
4.8	0.9221	0.1000	0.0689	0.1215	0.6036	0.6036	0.0157	0.1292	6.8767
4.9	0.8532	0.1000	0.0667	0.1202	0.5879	0.5879	0.0161	0.1338	6.9968
5	0.7865	0.1000	0.0644	0.1189	0.5719	0.5719	0.0164	0.1381	7.1158
5.1	0.7221	0.1000	0.0621	0.1177	0.5554	0.5554	0.0167	0.1423	7.2335
5.2	0.6601	0.1000	0.0598	0.1165	0.5387	0.5387	0.0170	0.1462	7.3500
5.3	0.6003	0.1000	0.0575	0.1153	0.5217	0.5217	0.0173	0.1498	7.4653
5.4	0.5428	0.1000	0.0552	0.1142	0.5044	0.5044	0.0175	0.1531	7.5795
5.5	0.4876	0.1000	0.0529	0.1131	0.4869	0.4869	0.0176	0.1560	7.6927
5.6	0.4347	0.1000	0.0507	0.1121	0.4692	0.4692	0.0178	0.1584	7.8048
5.7	0.3840	0.1000	0.0485	0.1111	0.4515	0.4515	0.0178	0.1604	7.9159
5.8	0.3355	0.1000	0.0463	0.1102	0.4337	0.4337	0.0178	0.1618	8.0261
5.9	0.2892	0.1000	0.0442	0.1093	0.4158	0.4158	0.0178	0.1626	8.1354

6	0.2450	0.1000	0.0420	0.1085	0.3981	0.3981	0.0177	0.1628	8.2439
6.1	0.2030	0.1000	0.0400	0.1077	0.3804	0.3804	0.0175	0.1624	8.3516
6.2	0.1630	0.1000	0.0380	0.1070	0.3629	0.3629	0.0173	0.1613	8.4586
6.3	0.1250	0.1000	0.0360	0.1063	0.3456	0.3456	0.0169	0.1595	8.5649
6.4	0.0890	0.1000	0.0341	0.1057	0.3287	0.3287	0.0166	0.1569	8.6705
6.5	0.0549	0.1000	0.0323	0.1051	0.3121	0.3121	0.0161	0.1537	8.7756
6.6	0.0227	0.1000	0.0305	0.1045	0.2960	0.2960	0.0156	0.1496	8.8802
6.7	-0.0078	0.1000	0.0288	0.1041	0.2803	0.2803	0.0151	0.1449	8.9842
6.8	-0.0366	0.1000	0.0272	0.1036	0.2653	0.2653	0.0144	0.1393	9.0878
6.9	-0.0638	0.1000	0.0256	0.1032	0.2508	0.2508	0.0137	0.1331	9.1911
7	-0.0894	0.1000	0.0242	0.1029	0.2371	0.2371	0.0130	0.1261	9.2939
7.1	-0.1136	0.1000	0.0228	0.1026	0.2241	0.2241	0.0121	0.1184	9.3965
7.2	-0.1364	0.1000	0.0215	0.1023	0.2120	0.2120	0.0113	0.1100	9.4988
7.3	-0.1579	0.1000	0.0203	0.1020	0.2007	0.2007	0.0103	0.1010	9.6008
7.4	-0.1782	0.1000	0.0193	0.1018	0.1904	0.1904	0.0093	0.0913	9.7027
7.5	-0.1975	0.1000	0.0183	0.1017	0.1811	0.1811	0.0082	0.0810	9.8043
7.6	-0.2158	0.1000	0.0175	0.1015	0.1729	0.1729	0.0071	0.0701	9.9059
7.7	-0.2333	0.1000	0.0167	0.1014	0.1657	0.1657	0.0059	0.0586	10.0073
7.8	-0.2500	0.1000	0.0161	0.1013	0.1598	0.1598	0.0047	0.0465	10.1085
7.9	-0.2661	0.1000	0.0156	0.1012	0.1551	0.1551	0.0034	0.0339	10.2098
8	-0.2818	0.1000	0.0153	0.1012	0.1517	0.1517	0.0021	0.0208	10.3109
8.1	-0.2970	0.1000	0.0151	0.1011	0.1496	0.1496	0.0007	0.0072	10.4120
8.2	-0.3121	0.1000	0.0150	0.1011	0.1488	0.1488	-0.0007	-0.0068	10.5132
8.3	-0.3271	0.1000	0.0151	0.1011	0.1495	0.1495			10.6143
8.4	-0.3422								

- **APPENDIX E-6**

The 4th degree polynomial used to fit the nominal curvature for different applied load and water depth used in modified energy are shown here. There used in math cad sheet as shown in Appendix B as nominal curvature instead of analytical nominal curvature equation.

Experimental Nominal curvature

The experimentally nominal curvatures were calcluated from the digitilaaation of the pictures. Following nominal curvatures were replaced from the nominal curvature calcluated analytical in modified energy equation and total work done curve is generated.

Water depth=4.88m with 10N Applied load

The data points and related calculations for this nominal curvature are shown in Appendix E1

Experimental nominally curvture $$k(s) := 0.0001 \cdot s^4 - 0.0026 \cdot s^3 + 0.016 \cdot s^2 - 0.012 \cdot s^1 + 0.068$$

Water depth=4.88m with 20N Applied load

The data points and related calculations for this nominal curvature are shown in Appendix E2

Experimental nominally curvture $$k(s) := 0.000082 \cdot s^4 - 0.0022 \cdot s^3 + 0.014 \cdot s^2 - 0.0093 \cdot s^1 + 0.065$$

Water depth=4.88m with 40N Applied load

The data points and related calculations for this nominal curvature are shown in Appendix E3

Experimental nominally curvture $$k(s) := 0.000042 \cdot s^4 - 0.0013 \cdot s^3 + 0.0091 \cdot s^2 - 0.004 \cdot s^1 + 0.059$$

Water depth=5.91m with 20N Applied load

The data points and related calculations for this nominal curvature are shown in Appendix E4

Experimental nominally curvture $$k(s) := -0.00051 \cdot s^4 + 0.0091 \cdot s^3 - 0.054 \cdot s^2 + 0.16 \cdot s - 0.19$$

Water depth=5.91m with 40N Applied load

The data points and related calculations for this nominal curvature are shown in Appendix E5

Experimental nominally curvture $$k(s) := -0.00033 \cdot s^4 + 0.0062 \cdot s^3 - 0.039 \cdot s^2 + 0.13 \cdot s - 0.17$$

APPENDIX F-Experimental Test Figures

These are the figures used to measure the rotation angle at TDP by digitization of pictures from *WebPlotDigitizer*. The corresponding installation parameters are given section 7.

Figure F-1 for test 3, See Section 7.3.3 for installation parameters.

Figure F2- for test 4, See Section 7.3.4 for installation parameters.

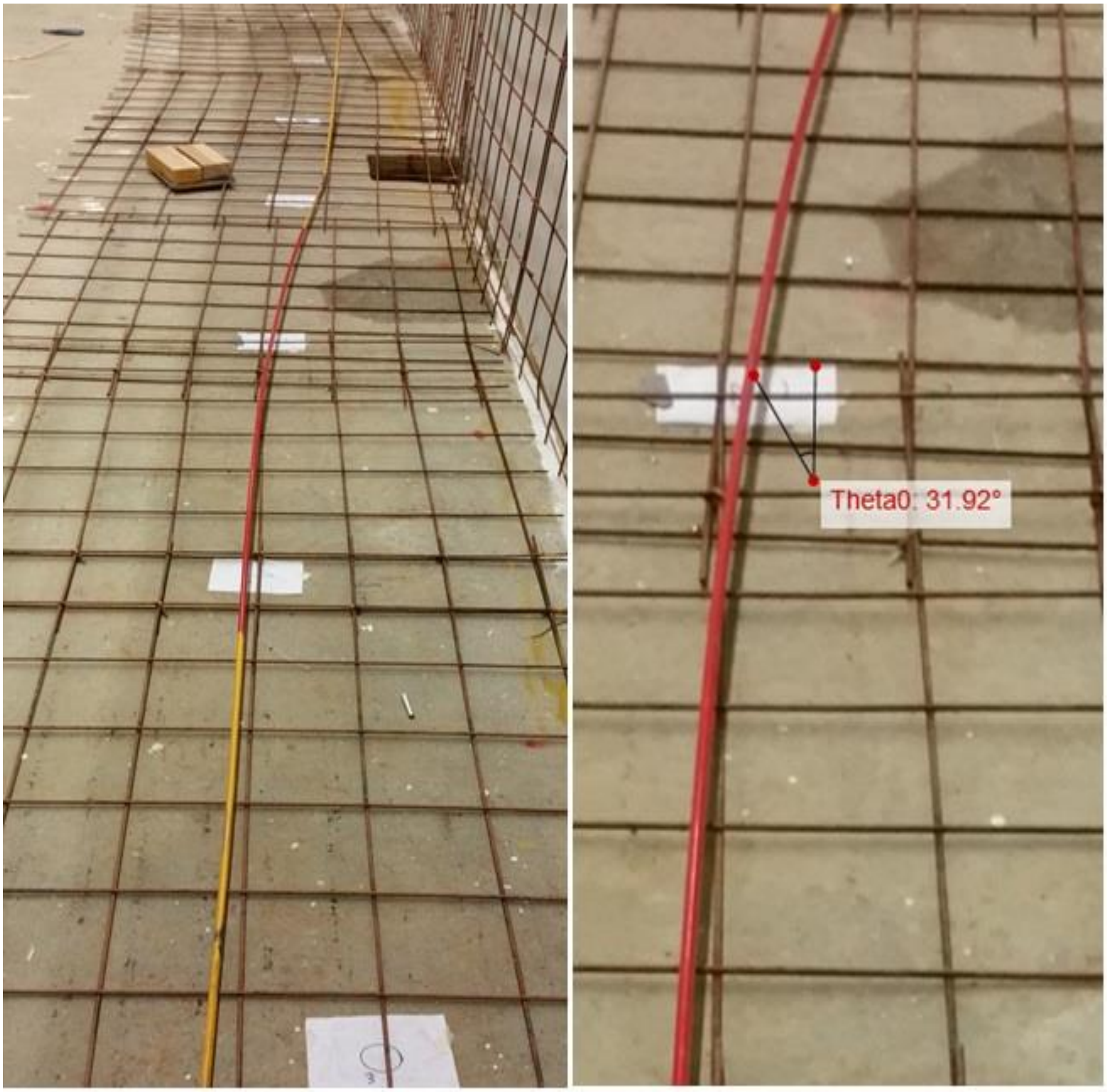

Figure F3- for test 5, See Section 7.3.5 for installation parameters.

Figure F4- for test 6, See Section 7.3.6 for installation parameters.

Figure F5- for test 8, See Section 7.3.8 for installation parameters.

Figure F6- for test 9, See Section 7.3.9 for installation parameters.

Figure F7- for test 12, See Section 7.4.2 for installation parameters.

Figure F8- for test 13, See Section 7.4.3 for installation parameters.

Figure F9- for test 15, See Section 7.5.2 for installation parameters.

Figure F10- for test 17, See Section 7.6.2 for installation parameters.