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Leverage Effect in Equity Markets Around the World

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Abstract

Equity indices are known to exhibit an asymmetric leverage effect, meaning that negative returns have a greater impact on volatility than positive returns of the same magnitude. We reevaluate the presence of leverage effect in a large sample of 21 equity indices around the world. We utilize not only daily data, but also realized volatility calculated from high-frequency data. Using realized volatility as a benchmark allows us for a more precise comparison of volatility models. Moreover, we also study models based directly on realized volatility. We find that all the 21 equity indices analyzed exhibit the leverage effect. In order to investigate whether asymmetric models produce more accurate volatility forecasts than symmetric models, three pairs of volatility models are compared. Within each pair, two models are almost identical. The only difference is that one model allow for the leverage effect, whereas the other model is a restricted version, which does not allow for the leverage effect. We find that the volatility models that allow for the asymmetric leverage effect produce significantly more accurate forecasts than the symmetric volatility models.

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1 Introduction

Volatility is an important variable for participants in financial markets. It measures the dispersion of return for a given security or market index. Volatility has received great attention from investors, academics and regulators because of its role in option pricing, asset allocation, hedging and risk management in general. Furthermore, the financial world has witnessed bankruptcy and stock market crashes, which have led to huge losses. The stock market crash in 1987 and the financial crises in 2007-2008 have highlighted the importance of understanding volatility in financial markets.

It is well documented that the volatility in equity markets appears to be asymmetric. This observation, usually called the leverage effect, and first documented by Black (1976) and Christie (1982), states that a drop in the value of the stock (negative return) increases financial leverage as debt to equity ratio increases and that makes the stock riskier and consequently more volatile. However, the magnitude of the asymmetric volatility effect is too large to be caused solely by an increase in financial leverage. A study presented by Figlewski and Wang (2000) suggests that leverage changes due to changes in capital structure (such as issuance of new debt) have in fact no impact on volatility. It is therefore questionable whether the leverage effect is related to financial leverage.

Other explanations to this phenomenon are based on the existence of a time-varying premium and the volatility feedback effect (e.g. Pindyck, 1984; Engle, 1987; French et al., 1987; Campbell and Hentschel, 1992). The volatility feedback effect suggests that an increase in volatility requires a higher rate of return from the asset, which only happens by a fall in the asset price. If volatility is priced, an expected rise in volatility increases the required return on equity, which leads to a decline in the stock price. In volatility feedback effect, changes in volatility drive returns, while in the leverage effect, returns drive volatility. A study presented by Bekaert and Wu (2000) find more support for the positive feedback effect of volatility in Nikkei 225 stocks, because of the lack of causality from leverage effect. However, Bollerslev (2006) find that when using high frequency data from S&P 500 returns, five-minutes absolute returns and realized volatility over longer time, the correlation is negative between the realized volatility and the current and lagged absolute returns. This effect is lasting for several days. There are many other studies investigating and researching the presence of the leverage effect, and there is a broad agreement that the effect is present in many markets. Therefore, it is in our interest to further investigate this topic, by looking at different indices around the world and study the presence of leverage effect. Which of these theories is the most accurate explanation of this phenomenon has not been resolved yet.

The Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) model has become a popular tool for forecasting and modeling volatility. For that purpose, GARCH, E-GARCH (1,1), GJR-GARCH (1,1), TGARCH (1,1) models are often employed. For a review over volatility models and their forecasting performance see Poon & Granger (2003). Since the availability of high-frequency data, researchers and academics had rather relied on different approaches to estimate and model the volatility of returns on financial assets. Intra-day returns are often used to construct nonparametric, lower-frequency (daily) volatility measures. These so called realized volatilities are used to assess the predictive performance and adequacy of existing stochastic-volatility models (Andersen & Bollerslev, 1998) and to explore the predictability of market volatility in general. It is therefore not surprising that realized volatility estimators are also used to test the asymmetric volatility effect.

While the empirical studies on realized volatility is still ongoing, there are some facts that have been determined. It has been ascertained that the unconditional distribution of realized volatility is kurtosed and highly skewed, while the unconditional distribution of logarithmic realized volatility is nearly Gaussian (Molnár, 2012). Another fact is that the (logarithmic) realized volatility seems to be fractionally integrated. And finally, according to Ebens (1999), the realized volatility of stock indices is nonlinear in returns, which is also known as the leverage effect. Meaning that past negative return shocks have a larger impact on current volatility than previous positive shocks.

In this thesis, we will study the presence of leverage effect in 21 equity indices around the world. Further, we will investigate if the asymmetric models produce more accurate volatility forecasts than the symmetric counterpart models using daily-realized volatility as the proxy for evaluating the predictive ability for the models, instead of squared returns. The volatility models in this thesis are GARCH, GJR-GARCH, Log-GARCH and E-GARCH. We have chosen these models because they are commonly used in the literature and superior in empirical studies. We have also included the realized volatility models HAR-RV and LHAR-RV (leveraged), due to their straightforward estimation via OLS and their strong forecasting performance found in the literature (e.g. Corsi, Audrino, & Renò, 2012). From previous research, it is clear that the Autoregressive realized volatility models are superior over various GARCH models (e.g. Andersen, Bollerslev, & Huang, 2011). It will therefore be of no use to compare these models against each other, but more intuitive to compare the symmetric models with an asymmetric counterpart.

The aim of this thesis is not to find the best model for forecasting volatility, so there might be other models that are superior over the models employed in this thesis. Typically, prior research on stock market volatility focuses on one or more models and typically only one or two stocks or markets. This thesis aims to study a large number of stock markets. In our knowledge, there are no similar studies including the same number of indices and at the same time employing the more precise realized volatility estimator as a proxy in the volatility forecasting evaluation. There have been studies examining a large number of stock markets, instead of realized volatility as the proxy, they employ an imperfect volatility proxy, namely the commonly used squared returns (Evans & McMillan, 2007). Moreover, we provide in- and out-of-sample evidence about the existence of the leverage effect. We therefore argue that this paper is a great contribution to the existing volatility literature.

Empirical results indicate that the asymmetric models obtain the most accurate volatility forecasts, where the LHAR-RV model yields the lowest values for the loss functions. We therefore find, that irrespective of the employed model, adding an asymmetric component improves the fit and volatility forecasting performance. We can therefore conclude that the leverage effect should be considered as one of the reasons behind the observation of the asymmetric volatility effect.

Following this introduction part is section 2 which will present the data used in this thesis. In section 3 we present the various models applied in this thesis and the forecasting and evaluation methods that have been used. Section 4 will present the results based on our analysis. Section 5 will be a summary and a conclusion of the analysis, followed by references and appendix.

2 Data

All the data are obtained from the Oxford-Man Institute of Quantitative Finance (2017). The dataset used in this thesis consists of daily data, including realized variance, open and closing prices for 21 equity indices around the world. An overview of these indices is provided in table 1.

Table 1: Overview over the indices, including name of the index, ticker, location and number of observations.

Name	Ticker	Country	Number of observations
S&P 500	SPX	United States	4252
FTSE 100	FTSE	United Kingdom	4274
Nikkei 225	N2252	Japan	4120
German DAX	GDAXI	Germany	4308
Russel 2000	RUT	United States	4255
All Ordinaries	AORD	Australia	4253
Dow Jones Industrial Average	DJI	United States	4255
Nasdaq 100	IXIC	United States	4258
CAC 40	FCHI	France	4335
Hang Seng	HSI	Hong Kong	3925
KOSPI Composite Index	KS	South Korea	4189
AEX Index	AEX	Netherlands	4334
Swiss Market Index	SSMI	Switzerland	4260
IBEX 35	IBEX	Spain	4300
S&P CNX Nifty	NSEI	India	3677
IPC Mexico	MXX	Mexico	4257
Bovespa Index	BVSP	Brazil	4164
S&P/TSX Composite Index	GSPTSE	Canada	3669
Euro Stoxx 50	STOXX50E	Germany	4311
FTSE Straits Times Singapore	FTSTI	Singapore	3879
FTSE MIB	FTSEMIB	Italy	4292

Oxford-Man Institute for Quantitative Finance provides estimates of realized variance calculated in several different ways. In this paper, we use the most common measure of realized variance, the one calculated as a sum of squared 5-minute returns. To get the realized volatility we then take the square root of the variance. For most indices, the data covers the time period from 3rd

January 2000 to 9th January 2017. Due to different starting dates and differences in trading days in the different markets, number of observations differ accordingly. The index with highest number of observations is the French FCHI index with 4335 observations, while the index with the lowest number of observations is the Canadian GSPTSE index with 3669 observations.

In the realized volatility literature and when dealing with high frequency data, it is a common approach to use data from the opening to the closing of the market, the so called open-to-close returns. Volatility estimated from intraday or daily data do not include data from the overnight period, i.e. the period from close-to-open. This period, often called the opening jump, exhibits different dynamics than the volatility. Analysis in the main body of the paper is conducted on the open-to-close returns. It is conveniently assumed that the opening jumps are constant over time. However, the analysis is repeated with close-to-close returns for the GARCH models. The results are presented in appendix A. However, all the main results remain the same.

We define open-to-close returns as:

$$r_t = \log\left(\frac{P_t}{O_t}\right) \tag{1}$$

and close-to-close returns as:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) \tag{2}$$

where P_t are the closing price at time *t* and O_t are the open price at time *t*.

Table 2: Descriptive statistics for the daily open-to-close returns for all the indices reported in percentage. Period ranging from 3rd January 2000 to 9th January 2017. Auto.Q denotes the first order autocorrelation coefficient of returns and Auto.QR^2 denotes the first order autocorrelation coefficient of squared returns from the Automatic Portmanteau test and the corresponding p-values.

Ticker	Min	1Q	Mean	3.Q	Max	Auto.Q	P-value	Auto.QR^2	P-value	Skew	Ex. Kurt.
SPX	-9.35	-0.50	0.01	0.55	10.22	-0.08	0.0034	0.21	0.0000	-0.17	7.67
FTSE	-5.76	-0.48	-0.04	0.44	7.04	-0.03	0.2017	0.18	0.0000	-0.14	4.27
N2252	-10.56	-0.59	-0.03	0.58	11.66	-0.06	0.0452	0.25	0.0177	-0.55	10.54
GDAXI	-9.41	-0.66	-0.03	0.62	9.99	0.01	0.6754	0.21	0.0004	-0.09	4.79
RUT	-11.05	-0.73	0.01	0.79	7.78	-0.06	0.0282	0.30	0.0000	-0.26	4.24
AORD	-6.44	-0.42	0.00	0.46	3.89	-0.03	0.2852	0.23	0.0001	-0.49	3.81
DJI	-8.41	-0.47	0.02	0.55	10.75	-0.08	0.0028	0.18	0.0000	0.00	8.33
IXIC	-8.05	-0.63	-0.02	0.64	14.91	-0.06	0.0459	0.28	0.0039	0.11	7.29
FCHI	-8.12	-0.64	-0.04	0.57	7.28	-0.04	0.0634	0.24	0.0000	-0.14	4.13
HSI	-11.62	-0.55	-0.05	0.48	12.16	-0.05	0.3022	0.52	0.1350	0.08	12.70
KS	-11.78	-0.62	-0.04	0.56	8.76	-0.05	0.0316	0.21	0.0039	-0.34	5.78
AEX	-8.42	-0.56	-0.04	0.53	9.24	-0.04	0.1570	0.26	0.0000	-0.20	6.51
SSMI	-9.73	-0.48	-0.02	0.48	8.68	0.00	0.9633	0.23	0.0000	-0.29	8.26
IBEX	-7.58	-0.67	-0.05	0.61	13.04	-0.01	0.6284	0.11	0.0000	-0.03	5.28
NSEI	-13.38	-0.50	0.02	0.63	7.13	0.04	0.2525	0.35	0.0266	-0.99	10.38
MXX	-8.26	-0.59	0.04	0.71	9.95	0.09	0.0000	0.15	0.0000	0.00	5.04
BVSP	-15.92	-0.95	0.00	1.02	13.25	0.00	0.8486	0.13	0.0001	-0.17	4.90
GSPTSE	-7.72	-0.43	-0.02	0.44	6.48	0.00	0.9096	0.36	0.0003	-0.61	8.81
STOXX50E	-9.35	-0.67	-0.03	0.65	8.27	-0.03	0.2354	0.22	0.0010	-0.19	4.56
FTSTI	-7.71	-0.45	-0.03	0.39	9.47	-0.09	0.0110	0.38	0.0488	0.38	9.11
FTSEMIB	-9.19	-0.66	-0.06	0.59	8.23	-0.06	0.0066	0.16	0.0000	-0.26	3.83

Note: The Automatic Portmanteau test for serial correlation as presented by Escanciano & Lobato (2009)

The summary statistics for the intraday returns, presented in table 2, display an evidence of mild skewness and large kurtosis. Even though these are summary statistics of unconditional distribution of returns, residuals are not normally distributed even after modelling volatility as a GARCH model.[1] We therefore use the reparametrized Johnson Su distribution (JSU, see Johnson 1949a, 1949b) for all the GARCH models which are very flexible with respect to skewness and kurtosis in the residuals.

Furthermore, the p-values from the Automatic Portmanteau test for serial correlation (Escanciano & Lobato, 2009) presented in table 2 confirm that the returns series exhibit serial correlation. We therefore use as the mean equation not only a simple constant (ARMA(0,0)) but also an ARMA (1,1) model. ARMA(0,0) will be presented in the main body of the thesis. The results when we use ARMA(1,1) as a mean equation can be found in appendix C (for the in-sample results) and in appendix D (for the out-of-sample forecasting evaluation). However, all our main results remain unaffected by the choice of the mean equation. In the time period the S&P 500´s highest return was 10.22 % and the lowest was -9.35 %, with a mean return of 0.01 %.

[1] However, for the sake of brevity, we do not report these results in the paper.

Table 3: Descriptive statistics for the daily close-to-close returns for all the indices reported in percentage. Period ranging from 3rd January 2000 to 9th January 2017. Auto.Q denotes the first order autocorrelation coefficient of returns and Auto.QR^2 denotes the first order autocorrelation coefficient of squared returns from the Automatic Portmanteau test and the corresponding p-values.

Ticker	Min	1Q	Mean	3.Q	Max	Auto.Q	P-value	Auto.QR^2	P-value	Skew	Ex. Kurt.
SPX	-9.69	-0.51	0.01	0.58	10.64	-0.07	0.0067	0.21	0.0000	-0.17	7.82
FTSE	-8.93	-0.56	0.00	0.59	9.48	-0.01	0.8095	0.23	0.0001	-0.17	6.24
N2252	-12.11	-0.78	0.00	0.87	13.23	-0.03	0.2242	0.26	0.0087	-0.43	5.99
GDAXI	-11.05	-0.69	0.01	0.78	12.03	0.01	0.5995	0.17	0.0011	-0.09	5.49
RUT	-12.46	-0.76	0.02	0.87	8.76	-0.05	0.0501	0.28	0.0000	-0.31	4.50
AORD	-7.26	-0.45	0.01	0.53	4.53	0.01	0.7131	0.20	0.0000	-0.65	5.71
DJI	-8.61	-0.50	0.01	0.56	10.53	-0.07	0.0066	0.19	0.0000	-0.07	7.79
IXIC	-10.22	-0.70	0.01	0.75	13.28	-0.03	0.2295	0.21	0.0000	0.07	6.23
FCHI	-8.52	-0.71	0.00	0.75	10.44	-0.01	0.5956	0.22	0.0007	-0.08	4.49
HSI	-32.40	-0.67	0.01	0.74	13.41	0.00	0.9820	0.22	0.1092	-1.95	50.03
KS	-12.83	-0.65	0.02	0.78	11.24	0.02	0.3861	0.18	0.0002	-0.60	6.63
AEX	-9.12	-0.66	-0.01	0.68	9.57	0.01	0.7499	0.23	0.0000	-0.16	5.90
SSMI	-9.07	-0.56	0.00	0.61	10.79	0.03	0.3995	0.34	0.0012	-0.21	7.19
IBEX	-12.72	-0.75	0.00	0.77	12.87	0.02	0.4536	0.16	0.0025	-0.14	5.29
NSEI	-24.38	-0.66	0.04	0.84	18.47	0.00	0.9551	0.36	0.1696	-0.97	26.66
MXX	-8.27	-0.59	0.04	0.73	10.44	0.09	0.0000	0.16	0.0000	0.04	5.18
BVSP	-15.40	-0.98	0.03	1.10	13.36	0.01	0.5797	0.13	0.0001	-0.16	4.40
GSPTSE	-9.06	-0.43	0.02	0.56	7.52	-0.01	0.6922	0.39	0.0000	-0.66	7.98
STOXX50E	-8.77	-0.72	-0.01	0.74	10.55	-0.01	0.6046	0.22	0.0006	-0.08	4.48
FTSTI	-16.50	-0.50	0.00	0.56	8.92	0.03	0.2634	0.09	0.0023	-1.04	16.48
FTSEMIB	-13.33	-0.78	-0.02	0.79	10.76	-0.03	0.1273	0.17	0.0000	-0.24	5.09

Note: The Automatic Portmanteau test for serial correlation as presented by Escanciano & Lobato (2009)

Table 3 presents the descriptive statistics for the daily close-to-close returns. Comparing the intraday statistics with the daily close-to-close returns, we see that properties of close-to-close and open-to-close returns are rather similar.

2.1 Realized volatility

As mentioned before, modelling and forecasting volatility is one of the central issues finance. Realized volatility goes way back to 1980, when Merton showed that when data sampled at a high frequency are available, the sum of squared realizations can be used to estimate the variance of a random variable. In later years, it has been showed by Taylor and Xu (1997) and Andersen and Bollerslev (1998) and others that realized volatility can be calculated simply by summing up intraday squared returns. The daily volatility for day *t* can be written as:

$$RV_t^D = \sum_{i=1}^{N} r_{i,t}^2 \tag{3}$$

In equation (3), a day is divided in *N* equidistant periods, and $r_{i,t}$ denotes the intraday return of the *i*th interval of day *t*. RV_t^D is consistent and unbiased estimator of the daily volatility σ_t^2, when the returns have a zero mean and are uncorrelated. Superscript *D* in RV_t^D refers to the world daily, because later in the article we introduce also weekly (*W*) and monthly (*M*) realized volatility.

Theoretically, higher sampling frequency should lead to more precise estimate of realized volatility. However, choosing a very high sampling frequency, for example one second frequency, would lead to a bias in the estimated variance due to market microstructure effects. Andersen (2001) has proposed 5-min returns to compute the daily realized return, while other have found that 15-min and 25-min are optimal (Giot & Laurent, 2004) However, the most common choice in the existing literature is 5 minutes, and we therefore follow this standard choice.

Table 4: Descriptive statistics for the realized volatility for all the indices, period ranging from 3^{rd}*January 2000 to 9th January 2017. Realized volatility are annualized with the square-root-of-time rule, namely sqrt(252), reported values are in percentage. Auto.Q denotes the first order autocorrelation coefficient of realized volatility from the Automatic Portmanteau test and the p-value.*

Ticker	Min	1Q	Mean	3.Q	Max	Auto.Q	P-value	Skew	Ex. Kurt.
SPX	2.02	8.32	14.38	17.14	139.73	0.80	0.0000	3.14	17.84
FTSE	3.09	7.61	12.66	15.17	107.98	0.79	0.0000	2.90	16.06
N2252	3.64	10.04	14.95	17.96	90.20	0.73	0.0000	2.82	15.15
GDAXI	3.17	11.15	18.32	21.70	121.76	0.83	0.0000	2.48	10.74
RUT	0.00	9.13	14.34	16.64	121.40	0.77	0.0000	3.07	16.67
AORD	1.77	6.09	9.56	11.27	62.00	0.66	0.0000	2.71	12.56
DJI	2.38	8.32	14.21	16.75	147.42	0.76	0.0000	3.53	23.43
IXIC	0.00	8.85	15.34	18.77	104.11	0.84	0.0000	2.43	9.84
FCHI	3.20	10.55	16.76	20.09	113.61	0.81	0.0000	2.60	13.23
HSI	3.57	9.51	13.75	15.87	104.98	0.71	0.0000	3.46	24.67
KS	3.78	9.09	15.70	19.25	122.39	0.84	0.0000	2.46	11.71
AEX	2.38	9.31	15.27	18.35	95.56	0.83	0.0000	2.46	9.60
SSMI	4.27	8.28	12.93	14.84	102.83	0.82	0.0000	3.10	16.26
IBEX	3.21	11.31	17.52	21.34	117.84	0.78	0.0000	2.19	11.50
NSEI	3.32	10.51	16.77	19.47	217.85	0.71	0.0000	4.59	46.17
MXX	2.91	8.02	12.77	15.11	114.60	0.60	0.0000	3.41	22.05
BVSP	5.02	15.62	21.43	24.22	130.48	0.72	0.0000	3.53	21.97
GSPTSE	0.00	5.90	9.99	11.48	95.20	0.80	0.0000	3.99	26.30
STOXX50E	0.34	11.20	17.91	21.20	165.18	0.77	0.0000	3.08	19.62
FTSTI	4.26	7.73	11.15	13.12	72.88	0.76	0.0000	2.94	18.28
FTSEMIB	4.12	10.01	16.13	19.56	115.28	0.79	0.0000	2.39	11.41

Note: The Automatic Portmanteau test for serial correlation as presented by Escanciano & Lobato (2009)

Summary statistics for the realized volatility in table 4, clearly shows the presence of autocorrelation in volatility for all the indices. This was expected and it is in accordance with the empirical literature. The annualized mean for the S&P 500 index in the period from January 2000 to the start of January 2017 is 14.38 %. Meanwhile the Brazilian BVSP has the highest volatility in the time-period and the U.S.'s RUT has the lowest volatility with 21.43 % and 9.56 % respectively. From the skewness and kurtosis, we can observe that the realized volatility does not follow a normal distribution and it is skewed to the right.

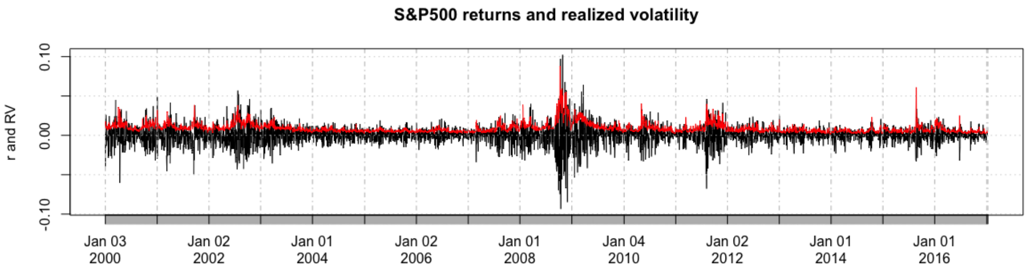

Figure 1: S&P 500 daily returns(black line) and daily realized volatility(red line) from 2000 to 2017

The time-series plot in figure 1 shows the relationship between intraday returns and volatility. We observe a highly volatile period in the end of 2008 and high fluctuations in the returns. When there are high fluctuations in returns, the volatility is also high. The highest return for the S&P 500 index was at 15th October 2008, meanwhile the lowest was at 28th October 2008. The date when the volatility was at is highest for the S&P 500 index was 10th October 2008 during the period after the Lehman Brother collapsed.

2.2 Cross-correlation

A negative cross-correlation may be interpreted as evidence for the leverage effect. Because we want to observe if the cross-correlation are negative between the returns and volatility and therefore evidence for the leverage effect we plot the cross-correlation for S&P 500 in figure 2. Upper panel in figure 2 plots the cross-correlation between the absolute returns and returns. The lower panel displays the cross-correlation between realized volatility and returns, with lags and leads ranging from -20 to 20 days,

$$corr\left(\left|r_{t,t+1}\right|, r_{t-j,t-j+1}\right) \qquad j = -20, \dots, 20. \tag{4}$$

$$corr\left(RV_{t,t+1}, r_{t-j,t-j+1}\right) \qquad j = -20, \dots, 20. \tag{5}$$

For lags 0 to 20 the cross-correlation are negative and mostly significant (all significant in the lower panel). When lags are negative the correlations are around zero and mostly not significant. For the realized volatility and returns, the effect is strongest at lag one and two, then declining as the lags increase. The impact of returns on the future realized volatility lasts for at least 20 lags, indicating long memory in volatility. This pattern is the same for all the indices (see appendix B), with the strongest effect in the Indian NSEI and the lowest effect in the Singapore's FTSTI. The cross-correlation already suggests a strong evidence for the presence of leverage effect in all the indices.

Meanwhile, the correlation between returns and future absolute returns is also highly significant, but weaker than the correlation between returns and subsequent realized volatility. We can observe that the correlation is about half compared to the lower panel in Figure 2. Altogether, both the relationship between returns and absolute returns and returns and realized volatility are an evidence for the leverage effect. Stronger relation between returns and realized volatility means that is easier to detect leverage effect when realized volatility can be utilized in the analysis.

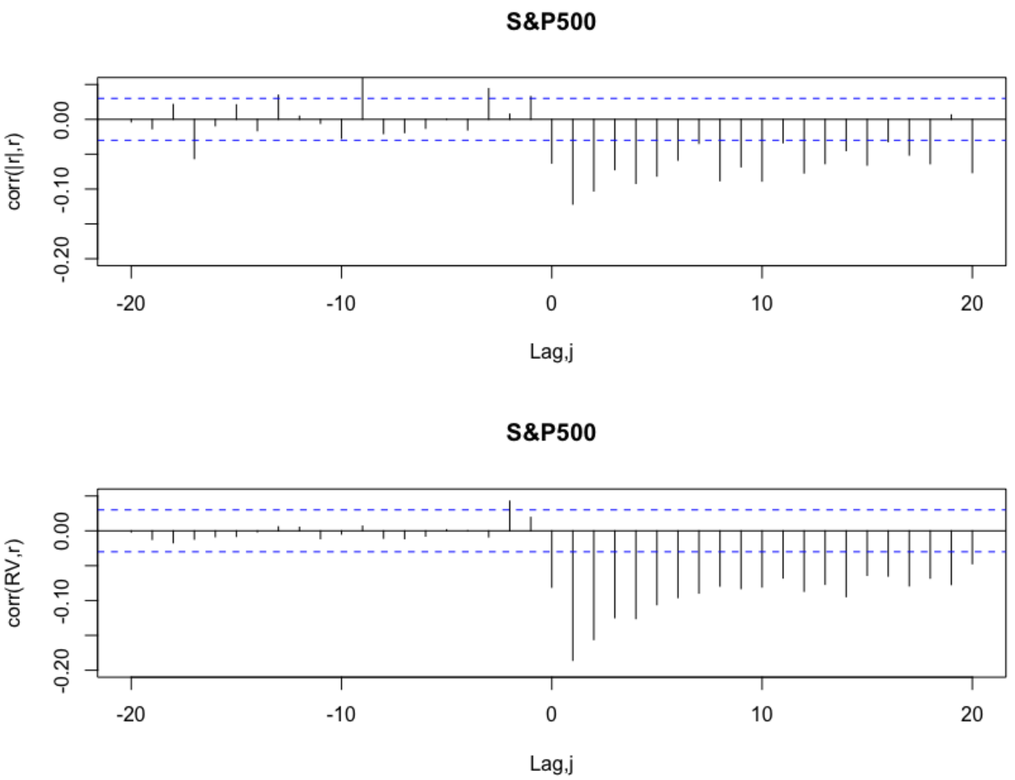

Figure 2: Cross-correlation for the S&P 500 with lags -20 to 20. The upper panel shows the cross-correlation between absolute returns and returns. The lower panel shows the cross-correlation between realized volatility and returns. The blue dotted lines indicate a 95% confidence interval under the null hypothesis of zero correlations.

3 Volatility models

Our goal is to investigate whether the leverage effect exist in the 21 stock indices we investigate. We investigate the existence of the leverage effect both in- and out-of-sample. In other words, we investigate not only whether the leverage effect exist, but also whether it is strong enough to improve out-of-sample volatility forecasts.

Many volatility models already exist in the literature. However, the goal of this study is not to compare various models, but to investigate the leverage effect. We therefore focus on the most commonly used volatility models.

We conduct our analysis by comparing three pairs of volatility models. Within each pair, two models are almost identical. The only difference is that one model allows for leverage effect, whereas the other model is a restricted version which does not allow for the leverage effect. In this section, we just present the models. Empirical comparison of these models is conducted in Section 4.

GARCH models are probably the most popular volatility models. These models are based on daily data and belong to the oldest volatility models coined in the works of Engle (1982) and Bollerslev (1986). GARCH(1,1) model and the E-GARCH(1,1) model are probably the most popular volatility models. We therefore compare the GARCH(1,1) model with the GJR-GARCH(1,1) model. These two models are identical except for GJR-GARCH model allows for the leverage effect. Similarly, we compare the E-GARCH model, which allows for the leverage effect, with its restricted version Log-GARCH, which does not allow for the leverage effect.

Emergence of high-frequency data and the concept of realized volatility allowed for a rapid development of new volatility models, usually based on the realized volatility. Probably the most popular from these models is the heterogeneous autoregressive model for realized volatility (HAR-RV) model of Corsi (2009). We therefore compare this model with its extended version which allows for the leverage effect, the LHAR-RV model.

3.1 GARCH Models

All GARCH models used are estimated via maximum likelihood. We assume that the daily returns are drawn from a reparametrized Johnson Su distribution (JSU, see Johnson 1949a, 1949b) with a constant mean and time-varying variance:

$$r_t = \mu_t + z_t$$
$$\left(1 - \sum_{i=0}^{p} \phi_i L^i\right) z_t = \left(1 + \sum_{j=0}^{q} \theta_j L^j\right) \varepsilon_t \qquad (6)$$
$$\varepsilon_t = \sigma_t \eta_t, \quad \eta_t \sim Johnson's\ Su\ (0, 1, \nu, \kappa)$$

where ϕ and θ are constants, μ denote the conditional mean of returns, σ_t denote the conditional variance of returns, ε_t denotes the innovation process, z_t is the standardized residuals, L is the backshift operator, and η_t follows the Johnson's Su distribution where ν and κ are skewness and kurtosis parameters. For the constant mean (ARMA(0,0), p and q are 0. We keep this assumption for all the GARCH models in the main body of the thesis. For the sake of robustness, we also present the estimated models and forecasting performance evaluation with ARMA(1,1) as the mean equation, p and q are then set to 1 (appendix C and D). The main results remain the same as with the constant mean equation.

3.1.1 GARCH

GARCH(1,1) is a symmetric volatility model presented by Bollerslev (1986). We can write the GARCH(1,1) model as:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \qquad (7)$$

where σ_t^2 is the conditional variance, ω is the intercept and ε_{t-1}^2 is the residual from the mean process. The parameters ω, β and α are restricted to be nonnegative with the restrictions $\alpha \geq 0, \beta \geq 0, \alpha + \beta < 1$, to ensure the positivity of conditional variance and stationarity.

3.1.2 GJR-GARCH

A model that can cope with asymmetric volatility response to negative and positive return shocks is the GJR-GARCH model proposed by Glosten et al. (1993). The GJR-GARCH can be written as follows:

$$\sigma_t^2 = \omega + [\alpha + \gamma I(\varepsilon_{t-1} > 0)]\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (8)$$

The indicator function *I* takes the value of 1 if $\varepsilon \le 0$ and 0 otherwise. This model uses the indicator function *I* to capture the asymmetric shocks on the conditional variance asymmetrically, where γ represents the leverage effect. In the GJR-GARCH model positive news has an impact of α, while negative news has an impact of $\alpha + \gamma$. Negative news has an even greater effect on the volatility than positive news if $\gamma > 0$. The parameters ω, β and α are restricted to be nonnegative with additional restriction $\alpha + \beta + 0.5\gamma < 1$, while the estimate of $\alpha + 0.5\gamma$ should be positive.

3.1.3 Log-GARCH

The Log-GARCH (1,1) has been presented in different forms by several authors, first by Geweke (1986) and Pantula (1986). In this thesis we present the Log-GARCH (1,1) model based on the model presented by Hansen and Lunde (2005). Log-GARCH (1,1) model can be written as follows:

$$\log(\sigma_t) = \omega + \gamma\,(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|] + \beta \log(\sigma_{t-1}) \quad (9)$$

Equation (9) has a logaritmic form that allows the parameters to be negative without the conditional volatility becoming negative.

3.1.4 E-GARCH

Alternative model that can cope with asymmetric volatility in response to asymmetric shocks is the Exponential GARCH (E-GARCH), which was advocated by Nelson (1991). The E-GARCH can be written as follows:

$$\log(\sigma_t^2) = \omega + [\alpha\varepsilon_{t-1} + \gamma\,(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|] + \beta \log(\sigma_{t-1}^2) \quad (10)$$

In equation (10) the coefficient α captures the sign effect, where negative shocks have greater impact than positive news of equal magnitude if $\alpha < 0$. The coefficient γ captures the size effect. Since equation (10) has a logarithmic form, no restriction for the estimated coefficients are needed.

3.2 Realized volatility models

3.2.1 HAR-RV

The HAR-RV and LHAR-RV models are estimated via OLS. The heterogeneous autoregressive model of realized volatility (HAR-RV) is an additive cascade model of different volatility components. This model is proposed by Corsi (2009) and is designed to simulate the behavior of different types of market participants. We can formulate the HAR-RV model by the following time series representation:

$$RV^{D}{}_{t+1} = c + \beta_1 RV^{D}_{t-1} + \beta_2 RV^{W}_{t-1} + \beta_3 RV^{M}_{t-1} + \varepsilon_{t+1} \tag{11}$$

Where the weekly and monthly horizons are defined by,

$$RV^{W}_{t} = \frac{1}{5}\sum_{i=0}^{4} RV^{D}_{t-i} \tag{12}$$

$$RV^{M}_{t} = \frac{1}{22}\sum_{i=0}^{21} RV^{D}_{t-i} \tag{13}$$

From the equation (11) we can see that this model predicts future volatility using a daily, a weekly and a monthly component. In practice this model has been very successful, which is impressive given its simple structure. Generally it produces more accurate forecasts than GARCH models (Andersen et al., 2011)

3.2.2 LHAR-RV

Based on the HAR-RV model, and extensions by McAleer and Medeiros (2008) and Corsi and Reno (2009) we present the leveraged HAR-RV (LHAR-RV) model. This model features asymmetry by adding leverage terms related with lagged absolute returns and lagged negative returns. The LHAR-model can be presented as follow:

$$RV^{D}_{t} = c + \beta_1 RV^{D}_{t-1} + \beta_2 RV^{W}_{t-1} + \beta_3 RV^{M}_{t-1} + \gamma_1 |r_{t-1}| + \gamma_2 r^{-}_{t-1} + \varepsilon_{t+1} \tag{14}$$

where,

$$|r_t| = absoulte\ value\ of\ returns \text{ and } r^{-}_{t} = \max(r^{-}, 0) \tag{15}$$

We are particularly interested in the coefficient γ_2, which captures the leverage effect. The reason why we include the term $|r_{t-1}|$ is the following. If we include only the term r_{t-1}^{-}, this term should be significant simply due to returns are high in absolute value, happening usually during periods of high volatility. We control for this by including the term $|r_{t-1}|$.

3.3 Forecasting procedure and evaluation

The volatility is forecasted with an estimation window of the 1000 most recent observations (trading days). Realized volatility is then forecasted one day ahead and the model parameters are re-estimated every day. The forecasts are based on a rolling window procedure, the estimation window moves one step ahead for every forecast, but the size of the estimation window is always 1000 observations. More specifically, the second forecast uses an estimation window which starts with the second observation and ends with 1001th observation.

When evaluating out-of-sample volatility forecasting performance we must choose a proxy for the true volatility. The squared returns are an often used proxy, but they provide a generally poor and very noisy proxy for the actual daily volatility. The use of realized volatility as the proxy, instead of squared returns improves the consistency of the volatility model ranking and comparison (Hansen & Lunde, 2006). Barndorff-Nielsen & Shephard (2002) shows that the realized volatility is a precise estimator of the actual volatility when microstructure noise effects are assumed to be non-existing.

Evaluating the predictive accuracy of the volatility models can be done by a various number of loss functions. When using imperfect volatility proxies, such as the squared return, using an arbitrary loss functions could lead to inconsistent volatility model ranking (Patton, 2011). The loss functions used in the thesis are the Mean Squared Error (MSE) and QLIKE which are two of the most widely used loss functions in volatility literature and the only loss functions that are robust even if imperfect volatility proxy is employed (Patton, 2011).

We are comparing the forecasts from GARCH (1.1) with the forecasts from GJR-GARCH (1.1), Log-GARCH (1.1) with the E-GARCH (1.1) and the forecasts from HAR-RV with the LHAR-RV model.

MSE is defined by,

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(\widehat{\sigma_t} - \sigma_t)^2, \tag{16}$$

QLIKE is defined by,

$$QLIKE = \frac{1}{n}\sum_{t=1}^{n}\frac{\sigma_t}{\widehat{\sigma_t}} - \ln\left(\frac{\sigma_t}{\widehat{\sigma_t}}\right) - 1, \tag{17}$$

where $\widehat{\sigma_t}$ is the forecasted realized volatility and σ_t is the observed realized volatility, and used as the proxy in the forecast evaluation.

Moreover, the model with the lowest loss function value does not necessarily imply that the model is superior over other models (Koopman, Jungbacker, & Hol, 2005). We therefore adopt the Model Confidence Set (MCS) by Hansen, Lunde & Nason (2011) to assess the relative forecasting performance between the symmetric models and the asymmetric counterpart models.

The model confidence set (MCS) determines the best set of models, M^*, instead of one superior model. Therefore, there can be several models that are equally good instead of other methods that only choose one model to be the superior one. The MCS determine the M^* from a collection of models, M^0, with the use of some criterions, typically loss functions. The procedure gives a model confidence set, $\widehat{M^*}$, which is a collection of the best models with a given level of confidence. The process of removing models from the set M^0 relies on the information from the sample about the performance of the models in the collection of models. The procedure is based on an equivalence test for equal predictive ability and an elimination rule. The test is applied to the set $M = M^0$. If the test is rejected there is evidence that the models in are not equally good. The elimination rule is than employed to remove an inferior model from M. The procedure is repeated until the equivalence test is not rejected. When the equivalence test is not rejected we have the best set of models, M^* (Hansen et al., 2011).

The MCS function is initiated on the nested models, with the symmetric models as benchmark. We employ a confidence level of 95 %, giving an alpha value of 0.05 and 10000 bootstrapped samples are used to construct the statistical test and p-values. P-values < α indicate that the model provides significantly better forecasts than the counterpart model. The null hypothesis is that the models have equal predictive ability; our alternative hypothesis is not equal predictive ability.

4 Empirical results and analysis

First, we estimate the models in-sample utilizing the full data sample in order to investigate the presence of leverage effect and to see how well the models fit the data. All the models indicate high volatility persistence as expected from volatility of equity indices. These estimated coefficients are not used in the forecasting procedure in order to avoid look-ahead bias.

4.1 GARCH models

4.1.1 GARCH (1,1)

Table 5 shows the whole sample parameter estimates for the GARCH (1,1) model together with the respective AIC values. GARCH (1,1) does not account for the leverage effect due to the symmetric form. We will use the GARCH model as a benchmark for comparison with the GJR-GARCH model. As shown in Table 5 the parameters ω, α and β in the model are all positive, ω is relatively small and not significant. The α parameter is significant for some of the stock indices at the 5 % level or higher, whereas the β parameter is significant for all the indices except for the U.S. RUT and the Swiss SSMI index.

When $\alpha + \beta$ is close to one, it implies the existence of strong volatility persistence, the average for all the indices in our sample is 0.99, which indicates a strong volatility persistence. The estimated distribution parameters κ and ν are at least significant at the 5 % level for most of the indices. This confirms negative skewness and kurtosis features, where ν determines the skewness of the distribution and κ determines the kurtosis of the distribution.

Table 5: Estimated coefficients for the GARCH (1,1) model, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

Ticker	μ	ω	α	β	ν	κ	AIC
SPX	3.80E-04[c]	1.20E-06	0.10	0.89[d]	-0.40	2.02[c]	-6.52
FTSE	-1.77E-04[a]	5.69E-07	0.10	0.90[d]	-0.52[c]	2.39[d]	-6.89
N2252	-1.71E-04	2.45E-06	0.10	0.88[d]	-0.29[d]	1.92[d]	-6.39
GDAXI	2.14E-04	1.06E-06	0.08[a]	0.92[d]	-0.36[c]	2.13[d]	-6.19
RUT	3.42E-04	2.28E-06	0.09	0.90	-1.50	4.02	-5.99
AORD	3.87E-05	4.74E-07	0.07[a]	0.92[d]	-0.84[c]	2.71[d]	-7.08
DJI	4.79E-04[c]	1.20E-06	0.11	0.89[d]	-0.33	2.04[c]	-6.58
IXIC	2.37E-04[a]	8.99E-07	0.09	0.91[d]	-0.71[d]	2.56[d]	-6.25
FCHI	7.76E-05	1.32E-06	0.08	0.91[d]	-0.21[b]	2.05[d]	-6.31
HSI	-3.14E-04[c]	7.87E-07	0.05[b]	0.94[d]	-0.024	2.00[d]	-6.64
KS	-4.02E-04[c]	5.42E-07	0.08[b]	0.92[d]	-0.48[d]	2.34[d]	-6.42
AEX	-4.97E-06	9.71E-07	0.08[a]	0.91[d]	-0.29[b]	2.03[d]	-6.52
SSMI	1.21E-05	1.61E-06	0.10	0.88	-0.30	1.98	-6.80
IBEX	2.85E-06	8.07E-07	0.07[b]	0.93[d]	-0.31[d]	2.04[d]	-6.20
NSEI	2.88E-04[a]	1.55E-06	0.08	0.91[d]	-0.26[b]	1.76[d]	-6.40
MXX	5.73E-04[c]	1.62E-06	0.08	0.91[d]	-0.31[b]	2.00[d]	-6.19
BVSP	1.65E-04	3.66E-06	0.06	0.93[d]	-0.40[b]	2.77[d]	-5.47
GSPTSE	4.34E-05	9.04E-07	0.08	0.90[d]	-0.81[c]	2.42[d]	-7.07
STOXX50	1.67E-04	1.52E-06	0.09	0.90[d]	-0.32	2.12[d]	-6.14
FTSTI	2.00E-04[b]	5.58E-07	0.10	0.90[d]	-0.08	2.20[d]	-7.04
FTSEMIB	2.25E-04	8.96E-07	0.08[b]	0.92[d]	-0.36[d]	1.97[d]	-6.23

4.1.2 GJR-GARCH (1,1)

To examine the presence of leverage effect we next estimate the GJR-GARCH (1,1) model. Table 6 shows the parameters estimated for the GJR-GARCH (1,1) and the respective AIC values. Comparing the AIC values from GARCH, we observe that the AIC values are lower for all the indices in our sample, suggesting that the GJR-GARCH model is superior to GARCH model considering the in-sample comparison.

The results also display the gamma γ parameter, which is the leverage parameter that we are interested in. Gamma is significant at the 5 % level or lower in eleven of the 21 indices and we therefore find a weak evidence of the leverage effect. The S&P index exhibits highest magnitude of the leverage effect, with an estimated value of 0.19, but it is not significant. Highest and significant at 5 % level is European STOXX50 with a gamma value of 0.17. Hong Kong's HSI index shows the lowest magnitude in our sample, with an estimated value of 0.04 and a significance level of 1 %. Estimated parameters ω and α are relatively small and mostly not significant.

When $\alpha + \beta + 0.5\gamma$ is close to one, it implies the existence of strong and growing volatility persistence, where the average for all the indices in our sample is 0.986. Moreover shape parameters κ and ν are similar as the results from GARCH, indicating fat-tails, leptokurtosis and skewness.

Table 6: Estimated coefficients for the GJR-GARCH (1,1) model, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

Ticker	μ	ω	α	β	γ	ν	κ	AIC
SPX	5.18E-05	1.59E-06	2.21E-08	0.89[d]	0.19	-0.60[b]	2.22[d]	-6.56
FTSE	-3.63E-04[b]	6.79E-07	1.40E-02	0.91[d]	0.13[b]	-0.66[c]	2.50[d]	-6.91
N2252	-3.52E-04[b]	3.09E-06	4.13E-02[b]	0.88[d]	0.11[b]	-0.35[d]	1.98[d]	-6.40
GDAXI	-4.50E-05	1.62E-06	3.64E-08	0.92[b]	0.14	-0.48	2.29	-6.22
RUT	7.21E-06	2.85E-06	3.73E-03	0.91[d]	0.13	-2.12	4.75	-6.01
AORD	3.45E-05	7.71E-07	6.22E-03	0.92[d]	0.11	-1.01	3.04	-7.10
DJI	1.90E-04	1.44E-06	6.61E-08	0.89[d]	0.19	-0.50[b]	2.21[d]	-6.62
IXIC	-2.97E-05	1.14E-06	1.01E-02	0.92[d]	0.13[c]	-0.93[c]	2.77[d]	-6.27
FCHI	-2.21E-04	1.81E-06	5.34E-08	0.91[d]	0.15	-0.37[b]	2.27[d]	-6.33
HSI	-3.84E-04[c]	8.52E-07	2.57E-02	0.94[d]	0.04[c]	-0.06	2.03[d]	-6.64
KS	-4.79E-04[d]	6.58E-07	5.10E-02	0.92[d]	0.05[b]	-0.51[d]	2.39[d]	-6.42
AEX	-2.45E-04	1.08E-06	4.00E-06	0.92[d]	0.13[d]	-0.43[c]	2.26[d]	-6.55
SSMI	-1.54E-04	1.68E-06	1.40E-02	0.89[d]	0.14[b]	-0.37[c]	2.07[d]	-6.82
IBEX	-1.71E-04	1.19E-06	5.89E-03	0.93[d]	0.10[b]	-0.40[c]	2.16[d]	-6.22
NSEI	2.13E-04	2.25E-06	5.38E-02	0.89[d]	0.08	-0.28[b]	1.79[d]	-6.41
MXX	3.92E-04[a]	1.82E-06	1.91E-02	0.92[d]	0.11	-0.36	2.14[c]	-6.20
BVSP	-1.09E-04	4.49E-06	8.49E-03	0.93[d]	0.08[d]	-0.50[c]	3.02[d]	-5.48
GSPTSE	-1.02E-04	1.17E-06	1.27E-02	0.91[d]	0.11	-0.98	2.62[d]	-7.08
STOXX50	-1.82E-04	2.20E-06	1.43E-07	0.90[d]	0.17[b]	-0.52[d]	2.39[d]	-6.17
FTSTI	-2.62E-04[c]	5.58E-07	6.76E-02	0.90[d]	0.06[c]	-0.09	2.23[d]	-7.04
FTSEMIB	-4.28E-04[b]	1.24E-06	4.87E-04	0.93[d]	0.12[b]	-0.47[d]	2.09[d]	-6.25

4.1.3 Log-GARCH (1,1)

The Log-GARCH is presented as a restricted version of the E-GARCH model, where the leverage parameter, alpha (α), is set to zero. This model will serve as our benchmark model in our comparison with the E-GARCH model. Table 7 presents the parameter estimates of the Log-GARCH and the respective AIC values.

Looking at Table 7 we can see that ω is negative and highly significant for almost all the indices. β and γ are positive and highly significant for all the indices, except for the Australia's AORD and the Swiss SSMI index, where γ is not significant at all. The β coefficient, which is less than and close to one for all the considered indices, implies a high persistence and a slow decay in the volatility shocks. The distribution parameters are consistent with the tables presented earlier in the thesis.

Table 7: Estimated coefficients for the Log-GARCH (1,1) model, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

Ticker	μ	ω	β	γ	ν	κ	AIC
SPX	3.72E-04[c]	-0.12[d]	0.99[d]	0.21[d]	-0.37[d]	1.96[d]	-6.52
FTSE	-1.61E-04[a]	-0.11[d]	0.99[d]	0.20[d]	-0.52[d]	2.39[d]	-6.89
N2252	-1.24E-04	-0.20[d]	0.98[d]	0.20[d]	-0.27[d]	1.91[d]	-6.39
GDAXI	2.22E-04	-0.10[d]	0.99[d]	0.18[d]	-0.34[d]	2.12[d]	-6.19
RUT	3.66E-04[c]	-0.12[d]	0.99[d]	0.17[d]	-1.35	3.72[d]	-5.99
AORD	3.87E-05	-0.11	0.99[d]	0.15	-0.84	2.73	-7.08
DJI	5.05E-04[d]	-0.13[d]	0.99[d]	0.22[d]	-0.31[d]	1.99[d]	-6.58
IXIC	2.31E-04[a]	-0.07[d]	0.99[d]	0.18[d]	-0.70[d]	2.48[d]	-6.25
FCHI	1.12E-04	-0.12[d]	0.99[d]	0.19[d]	-0.20[c]	2.06[d]	-6.31
HSI	-3.19E-04[d]	-0.09[d]	0.99[d]	0.12[d]	-0.03	1.98[d]	-6.64
KS	-3.92E-04[d]	-0.07[d]	0.99[d]	0.16[d]	-0.49[d]	2.36[d]	-6.42
AEX	-6.90E-07	-0.11[d]	0.99[d]	0.19[d]	-0.28[d]	2.02[d]	-6.52
SSMI	3.31E-05	-0.20	0.98[d]	0.21	-0.30[c]	1.96[d]	-6.79
IBEX	1.28E-05	-0.09[d]	0.99[d]	0.16[d]	-0.30[d]	2.03[d]	-6.20
NSEI	2.09E-04	-0.12[d]	0.99[d]	0.17[d]	-0.28[d]	1.72[d]	-6.40
MXX	5.48E-04[d]	-0.12[d]	0.99[d]	0.18[d]	-0.31[d]	1.98[d]	-6.19
BVSP	1.22E-04	-0.10[c]	0.99[d]	0.13[a]	-0.38	2.68[c]	-5.46
GSPTSE	5.85E-05	-0.13[d]	0.99[d]	0.18[d]	-0.78[c]	2.37[d]	-7.06
STOXX50	1.55E-04	-0.11[d]	0.99[d]	0.19[d]	-0.32[d]	2.12[d]	-6.13
FTSTI	-2.25E-04[d]	-0.13[d]	0.99[d]	0.21[d]	-0.08	2.17[d]	-7.04
FTSEMIB	-1.94E-04	-0.09[d]	0.99[d]	0.17[d]	-0.34[d]	1.97[d]	-6.23

4.1.4 E-GARCH (1,1)

Comparing the estimated Log-GARCH and E-GARCH we see that the E-GARCH model yields lower AIC values for all indices in our sample, suggesting E-GARCH as the superior in-sample model of the two models. E-GARCH model estimation and respective AIC values are presented in Table 8. The leverage effect effect is captured by the alpha α parameter. Moreover, ω, α, β and γ are highly significant for all the indices in our sample except for the U.K.'s FTSE index, where γ is insignificant and α is significant only at the 10 % level. The effect on volatility for a positive shock is measured by $\alpha + \gamma$, and for a negative shock the effect is measured by $\alpha - \gamma$. Therefore, the leverage effect can be measured by the α coefficient.

The results indicate a strong evidence for the leverage effect in the indices, where the U.S.'s S&P and DJI exhibit the strongest leverage effect with a value of -0.16. Again, distribution parameters are all significant and support our choice for modelling residuals via the Johnon's Su distribution.

Table 8: Estimated coefficients for the E-GARCH (1,1) model, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

Ticker	μ	ω	α	β	γ	ν	κ	AIC
SPX	3.98E-05	-0.18^{d}	-0.16^{b}	0.98^{d}	0.12^{b}	-0.64	2.23^{a}	-6.57
FTSE	-3.98E-04	-0.13	-0.10^{a}	0.99^{d}	0.14	-0.68	2.52	-6.91
N2252	$-3.69E-04^{c}$	-0.25^{d}	-0.09^{d}	0.97^{d}	0.16^{d}	-0.35^{d}	1.99^{d}	-6.41
GDAXI	-7.06E-05	-0.17^{d}	-0.11^{d}	0.98^{d}	0.13^{d}	-0.47^{d}	2.28^{d}	-6.22
RUT	-7.21E-05	-0.16^{d}	-0.10^{d}	0.98^{d}	0.12^{d}	-2.36^{b}	4.68^{d}	-6.02
AORD	3.87E-05	-0.21^{d}	-0.10^{d}	0.98^{d}	0.12^{d}	-1.10^{d}	3.20^{d}	-7.11
DJI	$1.66E-04^{a}$	-0.19^{d}	-0.16^{d}	0.98^{d}	0.13^{d}	-0.52^{d}	2.20^{d}	-6.63
IXIC	-5.50E-05	-0.11^{d}	-0.10^{d}	0.99^{d}	0.12^{d}	-0.98^{d}	2.70^{d}	-6.28
FCHI	$-2.98E-04^{a}$	-0.19^{d}	-0.13^{d}	0.98^{d}	0.11^{d}	-0.41^{d}	2.33^{d}	-6.34
HSI	$-4.11E-04^{c}$	-0.11^{d}	-0.03^{c}	0.99^{d}	0.11^{a}	-0.06	2.00^{d}	-6.64
KS	$-5.13E-04^{d}$	-0.09^{d}	-0.05^{d}	0.99^{d}	0.16^{d}	-0.53^{d}	2.43^{d}	-6.43
AEX	$-2.77E-04^{c}$	-0.14^{d}	-0.11^{d}	0.98^{d}	0.11^{d}	-0.42^{d}	2.25^{d}	-6.55
SSMI	-1.92E-04	-0.22^{d}	-0.11^{d}	0.98^{d}	0.13^{d}	-0.41^{d}	2.12^{d}	-6.82
IBEX	-2.28E-04	-0.15^{d}	-0.09^{d}	0.98^{d}	0.11^{d}	-0.40^{d}	2.17^{d}	-6.22
NSEI	9.33E-05	-0.18^{d}	-0.06^{d}	0.98^{d}	0.18^{d}	-0.30^{d}	1.77^{d}	-6.41
MXX	$3.30E-04^{a}$	-0.14^{d}	-0.09^{d}	0.98^{d}	0.15^{d}	-0.38^{c}	2.15^{d}	-6.20
BVSP	-1.58E-04	-0.13^{d}	-0.07^{d}	0.98^{d}	0.12^{d}	-0.49^{c}	2.94^{d}	-5.48
GSPTSE	$-7.87E-05^{d}$	-0.18^{d}	-0.08^{d}	0.98^{d}	0.13^{d}	-0.94^{d}	2.58^{d}	-7.08
STOXX50	$-2.52E-04^{c}$	-0.19^{d}	-0.15^{d}	0.98^{d}	0.11^{d}	-0.59^{d}	2.51^{d}	-6.18
FTSTI	$-2.97E-04^{d}$	-0.13^{d}	-0.04^{d}	0.99^{d}	0.20^{d}	-0.09	2.21^{d}	-7.04
FTSEMIB	$-4.73E-04^{d}$	-0.14^{d}	-0.10^{d}	0.98^{d}	0.12^{d}	-0.47^{d}	2.15^{d}	-6.26

4.2 HAR-RV

Recent literature on realized volatility suggest that all the terms in the HAR-RV model can be treated as observable and we can therefore estimate the parameters by OLS. Table 9 presents the regression of the Heterogeneous Autoregressive model of Realized Volatility (HAR-RV) and the respective adjusted R-squared. From Table 9 we see that all the lagged volatility values for daily, weekly and monthly are highly significant. The model puts the most weight on the daily lagged variable in fifteen of the 21 indices.

Table 9: Regression results for the HAR-RV model, reported together with the corresponding adjusted R-squared. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1% level and 0, 1% respectively.

Ticker	Intercept	β_1	β_2	β_3	R^2_{adj}
SPX	5.39E-04^b	0.37^d	0.35^d	0.20^d	0.69
FTSE	5.39E-04^d	0.36^d	0.32^d	0.23^d	0.68
N2252	8.80E-04^d	0.42^d	0.28^d	0.19^d	0.59
GDAXI	6.08E-04^c	0.44^d	0.28^d	0.21^d	0.73
RUT	5.90E-04^c	0.33^d	0.38^d	0.20^d	0.66
AORD	5.13E-04^d	0.18^d	0.43^d	0.29^d	0.56
DJI	6.33E-04^c	0.33^d	0.36^d	0.21^d	0.65
IXIC	4.65E-04^c	0.46^d	0.27^d	0.21^d	0.75
FCHI	6.90E-04^d	0.41^d	0.35^d	0.16^d	0.70
HSI	8.58E-04^d	0.23^d	0.45^d	0.20^d	0.58
KS	5.21E-04^d	0.41^d	0.33^d	0.19^d	0.74
AEX	5.31E-04^d	0.44^d	0.33^d	0.16^d	0.74
SSMI	5.56E-04^d	0.42^d	0.35^d	0.15^d	0.72
IBEX	8.85E-04^d	0.42^d	0.31^d	0.18^d	0.65
NSEI	1.27E-04^d	0.40^d	0.24^c	0.22^d	0.56
MXX	1.04E-03^d	0.23^d	0.30^d	0.31^d	0.46
BVSP	1.20E-03^c	0.32^d	0.39^d	0.18^d	0.60
GSPTSE	3.89E-04^b	0.37^d	0.34^d	0.20^d	0.70
STOXX50	8.83E-04^d	0.38^d	0.34^d	0.19^d	0.65
FTSTI	4.90E-04^d	0.35^d	0.29^d	0.28^d	0.67
FTSEMIB	7.32E-04^d	0.41^d	0.34^d	0.16^d	0.68

4.3 LHAR-RV

Including the lagged absolute value of returns and the lagged negative returns in the HAR-RV model allow us to investigate if the indices exhibit the leverage effect. The regression results and the adjusted R-squared are presented in Table 10. We see that all the lagged values for daily, weekly and monthly volatility are significant. The regression result reveals highly significant coefficients for the lagged negative returns variable (γ_2), all the indices have significant coefficients at the 0.1 % level, except the Hong Kong's HSI and Singapore's FTSTI which are significant at the 5 % and 10 % level, respectively. The results revealing a strong evidence that the indices exhibit leverage effect, with respect to the significant γ_2

coefficient for negative lagged returns. Moreover, we observe higher adjusted R-squared for all the indices but one (HSI) in the sample, compared to the HAR-RV model. Average improvement based in adjusted R-squared is 4.24 % points for all the indices, where India's NSEI and HSI exhibit the biggest and smallest improvement by 9.57 % and -0.84 % respectively. This implies that the leveraged HAR-RV is superior over the simple HAR-RV model for the in-sample analysis.

Table 10: Regression results for the LHAR-RV model, reported together with the corresponding adjusted R-squared. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

Ticker	Intercept	β_1	β_2	β_3	γ_1	γ_2	R^2_{adj}
SPX	5.68E-04^{c}	0.28^{d}	0.40^{d}	0.20^{d}	-0.03	0.16^{d}	0.73
FTSE	5.24E-04^{d}	0.27^{d}	0.34^{d}	0.24^{d}	0.03^{a}	0.11^{d}	0.71
N2252	9.39E-04^{d}	0.32^{d}	0.32^{d}	0.20^{d}	0.01	0.09^{d}	0.60
GDAXI	6.18E-04^{d}	0.34^{d}	0.32^{d}	0.22^{d}	0.00	0.12^{d}	0.75
RUT	5.95E-04^{c}	0.25^{d}	0.41^{d}	0.20^{d}	-0.02^{b}	0.14^{d}	0.70
AORD	5.88E-04^{d}	0.09^{b}	0.46^{d}	0.29^{d}	-0.03^{b}	0.13^{d}	0.58
DJI	6.28E-04^{c}	0.25^{d}	0.41^{d}	0.21^{d}	-0.03^{a}	0.15^{d}	0.69
IXIC	5.57E-04^{d}	0.35^{d}	0.30^{d}	0.19^{d}	0.02	0.14^{d}	0.79
FCHI	7.22E-04^{d}	0.30^{d}	0.39^{d}	0.16^{d}	0.01	0.12^{d}	0.73
HSI	9.94E-04^{d}	0.17^{d}	0.47^{d}	0.19^{d}	0.04^{c}	0.05^{b}	0.58
KS	5.28E-04^{d}	0.31^{d}	0.36^{d}	0.18^{d}	0.08^{d}	0.04^{b}	0.76
AEX	6.26E-04^{d}	0.32^{d}	0.35^{d}	0.17^{d}	0.03^{c}	0.11^{d}	0.77
SSMI	6.10E-04^{d}	0.31^{d}	0.36^{d}	0.18^{d}	0.02	0.11^{d}	0.75
IBEX	8.32E-04^{d}	0.32^{d}	0.34^{d}	0.19^{d}	0.01	0.10^{d}	0.68
NSEI	1.42E-03^{d}	0.22^{d}	0.26^{d}	0.21^{d}	0.09^{d}	0.17^{d}	0.61
MXX	1.00E-03^{d}	0.13^{d}	0.33^{d}	0.30^{d}	0.03^{b}	0.08^{d}	0.48
BVSP	1.20E-03^{d}	0.24^{d}	0.44^{d}	0.17^{d}	-0.00	0.10^{d}	0.63
GSPTSE	4.50E-04^{c}	0.23^{d}	0.39^{d}	0.21^{d}	0.01	0.14^{d}	0.73
STOXX50	9.03E-04^{d}	0.30^{d}	0.38^{d}	0.20^{d}	-0.01	0.13^{d}	0.68
FTSTI	6.50E-04^{d}	0.23^{d}	0.37^{d}	0.28^{d}	0.06^{d}	0.02^{a}	0.69
FTSEMIB	7.62E.04^{d}	0.27^{d}	0.39^{d}	0.17^{d}	0.02^{b}	0.12^{d}	0.72

4.4 Forecasting performance

4.4.1 GARCH and GJR-GARCH

Table 11 presents the results for the one-day ahead forecast evaluation of the GARCH(1,1) and the GJR-GARCH(1,1) models. In terms of MSE, GJR-GARCH led to lower average values of the loss function and in six cases the improvement of the GJR-GARCH model was even statistically significant. The GARCH model outperformed the GJR-GARCH model for only one index (AORD). In terms of

QLIKE loss function, GJR-GARCH is superior for nine indices and GARCH superior in three of the indices. GARCH is superior for three indices where in-sample estimated coefficients were of low magnitude (HSI and South Korea's KS) or the coefficient was insignificant (Mexico's MXX). In general GJR-GARCH models are superior over the simple GARCH for both loss functions employed in four of the indices, the U.S.'s SPX and DJI, the Europe's STOXX50 and the Singapore's FTSTI. Meanwhile, GARCH is not superior for any of the indices with respect to both loss functions.

Since there is no clear pattern in the results, we cannot conclude on which model is the superior one. These findings are consistent with the results of earlier research (Ng & McAleer, 2004), which indicated that the GJR-GARCH is not superior over the symmetric (and simpler) GARCH model. The average improvement of the MSE and QLIKE for the GJR-GARCH reported in Table 10 is just 0.58 % and 1.65 % respectively. The results imply that either the leverage effect is not very strong, or the GJR-GARCH model is not very suitable for capturing the leverage effect.

Table 11:Out-of-sample results for GARCH (1,1) and GJR-GARCH (1,1). Reported are the MSE and QLIKE lossfunctions, and the p-values from the Model Confidence Set (Hansen et al., 2011).

Ticker	GARCH	GJRGARCH	% change	P-value	GARCH	GJRGARCH	% change	P-value
	MSE	MSE	Delta	MCS	QLIKE	QLIKE	Delta	MCS
SPX	42.76	39.07	-8.64	0.0146	0.073	0.067	-8.76	0.0000
FTSE	19.70	18.57	-5.76	0.0343	0.045	0.044	-2.06	0.1589
N2252	49.63	52.45	5.68	0.0636	0.069	0.067	-2.26	0.0110
GDAXI	38.04	35.05	-7.86	0.1389	0.047	0.045	-4.74	0.0024
RUT	69.63	68.81	-1.18	0.4328	0.108	0.109	1.17	0.1943
AORD	21.65	25.27	16.74	0.0004	0.072	0.074	2.23	0.1184
DJI	44.31	39.91	-9.93	0.0088	0.073	0.067	-8.88	0.0000
IXIC	34.78	34.89	0.30	0.7719	0.067	0.068	1.21	0.2269
FCHI	33.74	33.01	-2.16	0.3493	0.045	0.043	-5.64	0.0022
HSI	28.07	28.52	1.61	0.2309	0.048	0.050	4.86	0.0000
KS	28.67	28.99	1.12	0.3451	0.052	0.055	5.73	0.0000
AEX	26.66	27.20	2.04	0.3729	0.047	0.044	-7.06	0.0000
SSMI	23.59	26.09	10.56	0.1870	0.043	0.043	-0.58	0.5554
IBEX	44.67	43.55	-2.51	0.3356	0.048	0.045	-5.60	0.0062
NSEI	44.33	43.12	-2.71	0.3132	0.052	0.052	0.49	0.3862
MXX	70.13	71.67	2.19	0.3032	0.114	0.117	3.24	0.0011
BVSP	77.51	74.37	-4.05	0.1885	0.059	0.060	2.98	0.0633
GSPTSE	27.99	28.57	2.08	0.2143	0.071	0.071	0.02	0.9382
STOXX50E	46.23	42.20	-8.73	0.0097	0.055	0.049	-10.92	0.0000
FTSTI	18.76	17.34	-7.57	0.0239	0.039	0.038	-2.00	0.0130
FTSEMIB	40.68	43.34	6.55	0.1144	0.054	0.055	1.89	0.2222

4.4.2 Log-GARCH and E-GARCH

Table 12 presents the results for the forecast evaluation of the Log-GARCH(1,1) and E-GARCH(1,1) model. The results show that E-GARCH model, with respect to the MSE loss function, performs significantly better. For 17 of the 21 indices it is the only model included in the set of superior models M^*. Log-GARCH model is not included as the only model in M^* for any of the indices in our sample. The models are found to have equal predictive ability for four of the indices, namely Hong Kong's HSI, South Korea's KS, the Swiss's SSMI and Mexico's MXX.

The results in the term of the QLIKE loss function are quite similar. The E-GARCH model is superior for 15 of the indices and the Log-GARCH is superior only for the KS and HIS index. The E-GARCH model performs significantly better then the Log-GARCH model for both loss functions for 14 of the indices.

The results clearly show that the model including the leverage effect (E-GARCH) has superior predictive ability compared to the model not exhibiting asymmetries (Log-GARCH). Other researchers have come to the same conclusion (Watt, Yadav, & Loudon, 2000), concluding that the E-GARCH is superior in predicting volatility in stock markets. Furthermore, Evans & McMillan (2007) found that the E-GARCH are marginally superior over varoius GARCH models when evaluating forecasting performance for 33 countries. The average improvement of the MSE and QLIKE for the EGARCH reported in Table 12 is 9.21 % and 5.21 % respectively.

Table 12: Out-of-sample results for Lo-gGARCH(1,1) and E-GARCH(1,1). Reported are the MSE and QLIKE lossfunctions, and the p-values from the Model Confidence Set (Hansen et al., 2011).

Ticker	logGARCH	EGARCH	% change	P-value	logGARCH	EGARCH	% change	P-value
	MSE	MSE	Delta	MCS	QLIKE	QLIKE	Delta	MCS
SPX	44.89	34.64	-22.82	0.0000	0.075	0.068	-10.03	0.0000
FTSE	19.96	18.04	-9.63	0.0004	0.046	0.044	-3.03	0.1193
N2252	44.44	40.58	-8.69	0.0211	0.068	0.064	-6.03	0.0000
GDAXI	38.28	33.10	-13.52	0.0013	0.049	0.044	-9.09	0.0000
RUT	70.66	60.09	-14.97	0.0002	0.109	0.105	-3.43	0.0332
AORD	23.47	21.17	-9.81	0.0000	0.079	0.071	-10.14	0.0000
DJI	46.50	38.73	-16.71	0.0000	0.076	0.069	-9.00	0.0000
IXIC	37.08	33.29	-10.24	0.0090	0.069	0.067	-2.72	0.0679
FCHI	34.22	30.68	-10.34	0.0008	0.046	0.042	-10.09	0.0000
HSI	25.83	26.39	2.17	0.1212	0.049	0.051	4.84	0.0000
KS	28.80	28.97	0.60	0.6000	0.051	0.055	7.34	0.0000
AEX	28.54	26.09	-8.60	0.0047	0.050	0.044	-11.77	0.0000
SSMI	24.50	24.72	0.87	0.5791	0.047	0.043	-7.94	0.0025
IBEX	43.33	38.49	-11.16	0.0005	0.047	0.043	-9.66	0.0000
NSEI	47.40	44.04	-7.09	0.0447	0.054	0.053	-2.30	0.0424
MXX	75.24	71.74	-4.65	0.1289	0.120	0.122	1.20	0.1460
BVSP	78.91	73.35	-7.05	0.0439	0.060	0.061	1.38	0.2643
GSPTSE	31.50	28.39	-9.88	0.0000	0.076	0.071	-6.40	0.0000
STOXX50E	47.64	39.99	-16.06	0.0000	0.057	0.049	-13.65	0.0000
FTSTI	19.34	18.08	-6.54	0.0029	0.044	0.042	-2.62	0.0004
FTSEMIB	44.11	39.98	-9.36	0.0088	0.058	0.055	-6.25	0.0040

4.4.3 HAR-RV and LHAR-RV

Table 13 presents the results for the forecast comparison of the HAR-RV and LHAR-RV. With respect to MSE as the loss function the LHAR-RV are included as the only model in the set of superior models (M^*) for 17 of the indices. This is a very strong evidence for the superiority of the leveraged model. The HAR-RV model never out-performs the LHAR-RV model. The models are found to have equal predictive ability for four of the indices, namely Hong Kong's HSI, South Korea's KS, Swiss's SSMI and India's NSEI. These results are consistent with results from the comparison between Log-GARCH and E-GARCH models presented earlier. The equal predictive ability for the models in both HSI and KS index are as expected, due to the fact that these indices showing the lowest magnitude of the asymmetry in the estimated models earlier in the thesis.

Meanwhile, when employing the QLIKE loss function the LHAR-RV forecasts are superior in 17 of the indices. The HAR-RV model is not selected as the superior model for any of the indices.

The results clearly show that the model including the leverage effect (LHAR-RV) has superior predictive ability compared to the model not accounting for the asymmetries (HAR-RV). Results are consistent with the results for the E-GARCH and Log-GARCH comparison, where the model including the leverage effect produces significantly more accurate forecasts then the models not accounting for the asymmetry. On average the LHAR-RV produce an improvement in the forecasts based on MSE and QLIKE at 6.23 % and 2.9 % respectively.

Table 13: Out-of-sample results for HAR-RV and leveraged HAR-RV. Reported are the MSE and QLIKE lossfunctions, percentive change and the p-values from the Model Confidence Set (Hansen et al., 2011).

Ticker	HAR-RV	LHAR-RV	% change	P-value	HAR-RV	LHAR-RV	% change	P-value
	MSE	MSE	Delta	MCS	QLIKE	QLIKE	Delta	MCS
SPX	29.91	27.59	-7.77	0.0023	0.050	0.048	-4.16	0.0002
FTSE	14.25	13.44	-5.64	0.0054	0.032	0.031	-1.95	0.0155
N2252	27.15	26.22	-3.44	0.0292	0.043	0.042	-2.79	0.0028
GDAXI	28.34	26.93	-4.97	0.0207	0.036	0.035	-2.36	0.0013
RUT	26.77	23.95	-10.52	0.0138	0.043	0.041	-4.18	0.0000
AORD	14.54	13.62	-6.33	0.0011	0.048	0.046	-5.18	0.0000
DJI	34.16	32.10	-6.00	0.0002	0.056	0.053	-4.14	0.0000
IXIC	17.92	16.21	-9.51	0.0060	0.034	0.034	-2.53	0.0066
FCHI	25.72	23.95	-6.87	0.0134	0.033	0.032	-2.44	0.0016
HSI	17.63	17.22	-2.28	0.1252	0.036	0.036	0.32	0.3109
KS	17.69	16.94	-4.23	0.0787	0.030	0.030	-1.03	0.1094
AEX	19.53	18.15	-7.05	0.0000	0.033	0.032	-3.55	0.0002
SSMI	16.43	15.47	-5.88	0.1430	0.026	0.026	0.36	0.4874
IBEX	31.40	29.54	-5.91	0.0000	0.033	0.032	-3.46	0.0000
NSEI	44.46	40.76	-8.32	0.0739	0.047	0.046	-2.88	0.0979
MXX	34.41	33.09	-3.86	0.0017	0.057	0.055	-3.52	0.0000
BVSP	46.19	43.70	-5.37	0.0000	0.034	0.033	-3.08	0.0000
GSPTSE	21.48	19.66	-8.46	0.0026	0.049	0.047	-4.10	0.0002
STOXX50E	38.35	36.16	-5.71	0.0001	0.043	0.041	-3.47	0.0000
FTSTI	6.42	6.09	-5.17	0.0037	0.019	0.018	-2.90	0.0187
FTSEMIB	26.84	24.80	-7.61	0.0000	0.035	0.034	-3.96	0.0000

5 Summary and conclusion

This thesis investigates the leverage effect in 21 equity indices around the world. We investigate not only whether leverage effect can be detected in the in-sample, but also whether allowing for the leverage effect improves out-of-sample performance of volatility models. We compare three pairs of volatility models. Within each pair, two models are almost identical. The only difference is that one model allows for the leverage effect, whereas the other model is a restricted version which does not allow for the leverage effect.

The out-of-sample analysis are based on one-step-ahead forecasts from different GARCH and HAR-RV models. Using the model confidence set by Hansen (2011) we compare the two nested model’s forecasting performance.

From the empirical in-sample results we observe that all the indices exhibit the leverage effect. As expected, the volatility models allowing for the leverage effect have a better fit to the data than the symmetric counterpart models. Unexpectedly, the GJR model does not provide a strong evidence for the leverage effect, with leverage parameter being insignificant for nine (out of 21) equity indices in the in-sample analysis.

Turning to the forecast comparison in the out-of-sample analysis, we observe the same results as for the in-sample analysis. The volatility models that allow for the leverage effect produce significantly more accurate forecasts than their symmetric counterpart models. In particular, this is the case for the E-GARCH and LHAR-RV model. The LHAR-RV model based on realized volatility calculated from high-frequency data yields the most precise forecasts. Meanwhile, for the GJR model, we cannot conclude whether the asymmetric GJR-GARCH model perform better than its symmetric counterpart, the GARCH(1,1) model. As a side result, we therefore conclude that the GJR model may not be particularly suitable for capturing the leverage effect.

Our results support findings in the earlier research, as we find evidence that all the 21 equity indices in our sample exhibit leverage effect. We add new findings to the existing literature by presenting results from a large number of markets and using the more precise realized volatility as proxy in the forecasting evaluation. Moreover, our results show that the models allowing for the leverage effect produce significantly more accurate forecasts than the symmetric counterpart models that do not allow for the leverage effect. The models including the leverage effect is therefore preferable over the symmetric models for volatility forecasting.

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Appendix A

Estimated GARCH models with close-to-close returns

A.1 GARCH (1,1)

Table A1 presents the estimated coefficients from GARCH (1,1) based on close-to-close returns.

Table A1: Estimated coefficients for the GARCH (1,1) model based on close-to-close returns, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0. 1% respectively.

Ticker	μ	ω	α	β	ν	κ	AIC
SPX	4.30E-04^{b}	1.00E-06	0.10	0.89^{d}	-0.35	1.95^{b}	-6.44
FTSE	2.92E-04^{a}	1.00E-06	0.10	0.89^{d}	-0.67	2.64^{c}	-6.46
N2252	4.05E-04^{b}	4.00E-06	0.09^{d}	0.89^{d}	-0.52^{d}	2.41^{d}	-5.75
GDAXI	6.34E-04	2.00E-06	0.094	0.91^{d}	-0.42	2.29^{a}	-5.92
RUT	5.68E-04^{d}	3.00E-06	0.08^{c}	0.90^{d}	-1.51^{b}	3.97^{d}	-5.85
AORD	3.95E-04^{c}	1.00E-06	0.08	0.91^{d}	-0.83^{c}	2.53^{d}	-6.85
DJI	4.33E.04^{b}	1.00E-06	0.10	0.89^{d}	-0.32	1.98^{b}	-6.55
IXIC	6.39E-04^{c}	1.00E-06	0.08	0.91^{d}	-0.47^{b}	2.36^{d}	-5.91
FCHI	4.19E-04	2.00E-06	0.09	0.90	-0.41	2.36	-5.95
HSI	3.18E-04^{a}	2.00E-06	0.06^{a}	0.93^{d}	-0.18^{b}	1.78^{d}	-5.95
KS	3.67E-04^{b}	1.00E-06	0.07^{b}	0.93^{d}	-0.41^{d}	1.95^{d}	-5.94
AEX	3.81E-04	2.00E-06	0.16	0.90	-0.43	2.27	-6.13
SSMI	3.97E-04^{d}	2.00E-06	0.12^{b}	0.87^{d}	-0.44^{d}	2.20^{d}	-6.43
IBEX	4.61E-04^{b}	1.00E-06	0.08	0.91^{d}	-0.40^{a}	2.21^{d}	-5.89
NSEI	7.89E-04^{d}	5.00R-06	0.11^{d}	0.86^{d}	-0.28^{c}	1.89^{d}	-5.90
MXX	6.18E-04^{d}	2.00E-06	0.08	0.91^{d}	-0.31^{b}	1.98^{d}	-6.16
BVSP	4.49E-04^{a}	4.00E-06	0.06^{d}	0.93^{d}	-0.35^{c}	2.44^{d}	-5.36
GSPTSE	4.58E-04^{c}	1.00E-06	0.09	0.90^{d}	-1.26	2.80^{b}	-6.74
STOXX50	3.77E-04	2.00E-06	0.09	0.91^{d}	-0.32	2.15^{c}	-5.94
FTSTI	3.12E-04^{b}	1.00E-06	0.09	0.91^{d}	-0.30^{a}	1.94^{d}	-6.54
FTSEMIB	3.00E-04	1.00E-06	0.09	0.91^{d}	-0.55^{b}	2.23^{d}	-5.85

A.2 GJR-GARCH (1,1)

Table A2 presents the estimated coefficients from GJRGARCH (1,1) based on close-to-close returns.

Table A2: Estimated coefficients for the GJR-GARCH (1,1) model based on close-to-close returns, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively

Ticker	μ	ω	α	β	γ	ν	κ	AIC
SPX	8.30E-05	2.00E-06	0.00E+00	0.89^{d}	0.19^{a}	-0.53^{c}	2.11^{d}	-6.48
FTSE	-2.50E-05	2.00E-06	0.00E+00	0.90^{d}	0.16	-1.01^{b}	3.07^{d}	-6.50
N2252	1.64E-04	$5.00E\text{-}06^{b}$	2.93E-02	0.88^{d}	0.12^{d}	-0.57^{d}	2.50^{d}	-5.77
GDAXI	2.78E-04	2.00E-06	0.00E+00	0.91^{d}	0.15^{b}	-0.64^{d}	2.60^{d}	-5.95
RUT	1.78E-04	$4.00E\text{-}06^{b}$	0.00E+00	0.91^{d}	0.14^{d}	-1.81^{b}	4.28^{d}	-5.88
AORD	2.57E-04	1.00E-06	9.10E-03	0.92^{d}	0.11	-1.02	2.84	-6.87
DJI	1.42E-04	1.00E-06	0.00E+00	0.90^{d}	0.18	-0.47^{b}	2.13^{d}	-6.59
IXIC	2.55E-04	2.00E-06	0.00E+00	0.92^{d}	0.15^{c}	-0.66^{b}	2.52^{d}	-5.94
FCHI	2.40E-05	3.00E-06	0.00E+00	0.90^{d}	0.17^{d}	-0.66^{d}	2.67^{d}	-5.99
HSI	1.67E-04	2.00E-06	1.36E-02	0.94^{d}	0.08	-0.19	1.81^{d}	-5.97
KS	1.95E-04	1.00E-06	2.35E-02	0.92^{d}	0.09^{b}	-0.44^{d}	1.99^{d}	-5.95
AEX	5.20E-05	2.00E-06	0.00E+00	0.91^{d}	0.16	-0.72	2.80^{b}	-6.17
SSMI	9.50E-05	3.00E-06	0.00E+00	0.88^{d}	0.20^{d}	-0.59^{d}	2.41^{d}	-6.46
IBEX	1.90E-04	2.00E-06	0.00E+00	0.92^{d}	0.13	-0.51^{b}	2.38^{d}	-5.92
NSEI	$6.09E\text{-}04^{c}$	$7.00E\text{-}06^{d}$	$3.34E\text{-}02^{c}$	0.85^{d}	0.17^{d}	-0.29^{c}	1.91^{d}	-5.91
MXX	$4.41E\text{-}04^{b}$	2.00E-06	1.82E-02	0.92^{d}	0.10	-0.37^{b}	2.13^{d}	-6.17
BVSP	1.71E-04	5.00E-06	1.02E-03	0.93^{d}	0.08^{d}	-0.40^{c}	2.58^{d}	-5.37
GSPTSE	$2.45E\text{-}04^{a}$	1.00E-06	6.10E-03	0.91^{d}	0.12	-1.41^{d}	2.93^{d}	-6.76
STOXX50	4.00E-06	1.00E-06	0.00E+00	0.91^{d}	0.17^{c}	-0.54^{c}	2.50^{d}	-5.98
FTSTI	2.32E-04	1.00E-06	5.33E-02	0.91d	0.06^{a}	-0.31^{a}	1.98^{d}	-6.55
FTSEMIB	2.90E-05	1.00E-06	8.80E-03	0.92^{d}	0.13	-0.69^{a}	2.41^{d}	-5.88

A.3 Log-GARCH (1,1)

Table A3 presents the estimated coefficients from Log-GARCH (1,1) based on close-to-close returns.

Table A3: Estimated coefficients for the Log-GARCH (1,1) model based on close-to-close returns, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1% respectively.

Ticker	μ	ω	β	γ	ν	κ	AIC
SPX	4.00E-04^{d}	-0.12^{d}	0.99^{d}	0.21^{d}	-0.34^{d}	1.92^{d}	-6.44
FTSE	3.00E-04^{b}	-0.13^{c}	0.99^{d}	0.20^{d}	-0.61^{d}	2.56^{d}	-6.46
N2252	4.00E-04	-0.19^{d}	0.98^{d}	0.20^{d}	-0.48^{c}	2.35^{d}	-5.75
GDAXI	5.91E-04^{d}	-0.10^{d}	0.99^{d}	0.19^{d}	-0.41^{d}	2.25^{d}	-5.91
RUT	5.89E-04^{b}	-0.13^{d}	0.99^{d}	0.18^{d}	-1.30	3.60^{c}	-5.84
AORD	4.11E-04^{a}	-0.11	0.99^{d}	0.17	-0.84	2.61	-6.85
DJI	4.77E-04^{d}	-0.14^{d}	0.99^{d}	0.22^{d}	-0.30^{d}	1.95^{d}	-6.55
IXIC	6.32E-04^{d}	-0.07^{d}	0.99^{d}	0.18^{d}	-0.46^{d}	2.32^{d}	-5.90
FCHI	4.10E-04^{a}	-0.11	0.99^{d}	0.19	-0.41	2.36	-5.95
HSI	3.56E-04^{b}	-0.09	0.99^{d}	0.14	-0.15	1.75^{d}	-5.95
KS	3.99E-04^{d}	-0.06^{d}	0.99^{d}	0.16^{d}	-0.41^{d}	1.96^{d}	-5.94
AEX	3.74E-04^{c}	-0.10^{d}	0.99^{d}	0.21^{d}	-0.41^{d}	2.23^{d}	-6.12
SSMI	3.93E-04^{c}	-0.21^{c}	0.98^{d}	0.24^{d}	-0.41^{d}	2.17^{d}	-6.43
IBEX	4.71E-04^{a}	-0.09^{d}	0.99^{d}	0.18^{d}	-0.39^{d}	2.20^{d}	-5.89
NSEI	7.16E-04	-0.20^{d}	0.98^{d}	0.21^{d}	-0.29^{d}	1.86^{d}	-5.90
MXX	5.90E-04^{d}	-0.11^{d}	0.99^{d}	0.18^{d}	-0.32^{d}	1.96^{d}	-6.16
BVSP	4.14E-04	-0.11^{d}	0.99^{d}	0.13^{a}	-0.34	2.38^{d}	-5.35
GSPTSE	4.80E-04^{c}	-0.12	0.99^{d}	0.18^{d}	-1.13	2.67	-6.73
STOXX50	3.68E-04^{b}	-0.10^{d}	0.99^{d}	0.18^{d}	-0.32^{d}	2.16^{d}	-5.94
FTSTI	3.25E-04^{c}	-0.11^{d}	0.99^{d}	0.19^{d}	-0.31^{d}	1.94^{d}	-6.54
FTSEMIB	3.11E-04^{a}	-0.08^{d}	0.99^{d}	0.19^{d}	-0.53^{d}	2.21^{d}	-5.85

A.4 E-GARCH (1,1)

Table A4 presents the estimated coefficients from E-GARCH (1,1) based on close-to-close returns.

Table A4: Estimated coefficients for the E-GARCH (1,1) model based on close-to-close returns, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

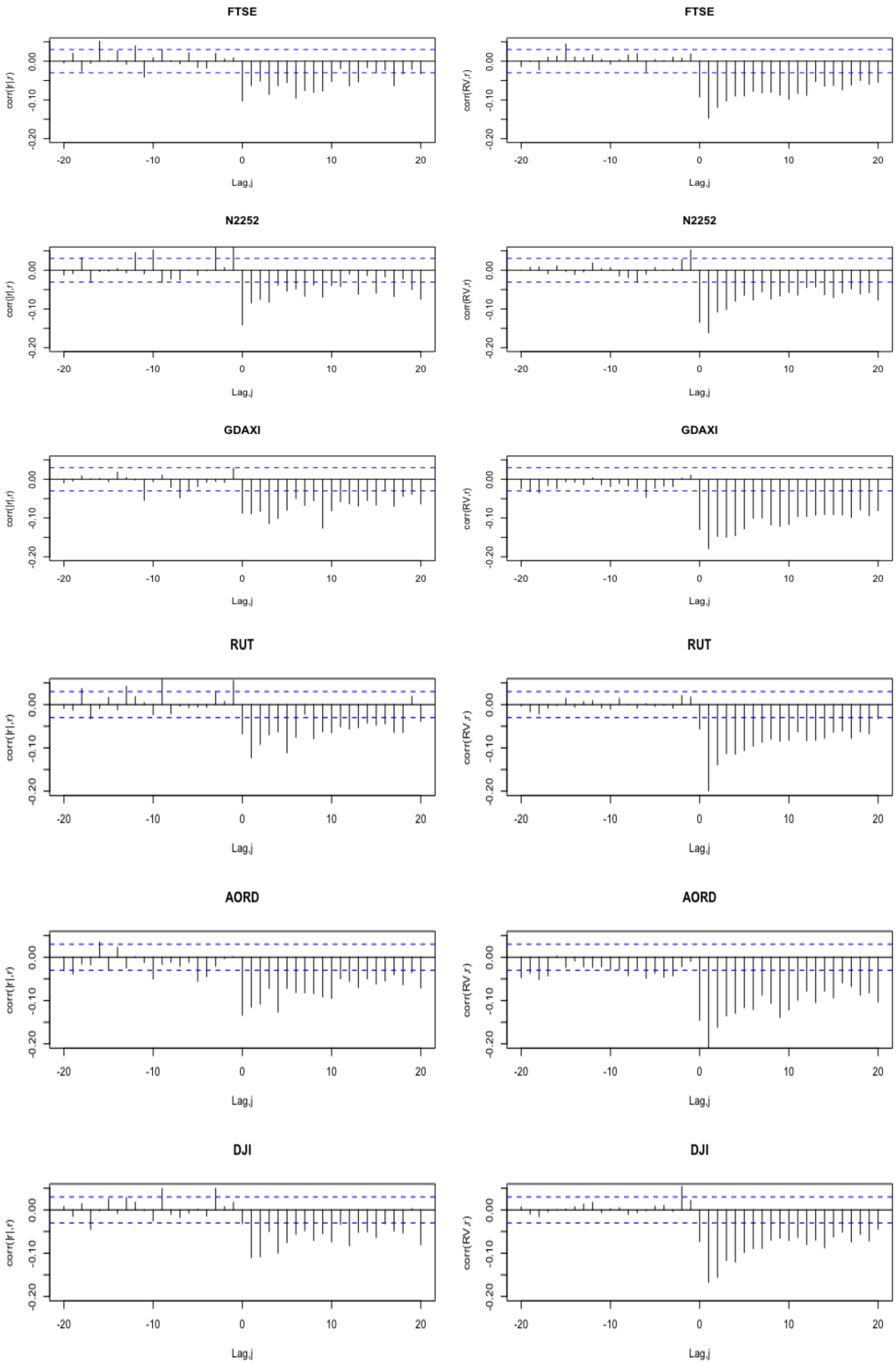
Ticker	μ	ω	α	β	γ	ν	κ	AIC
SPX	6.70E-05	-0.18^{d}	-0.16^{d}	0.98^{d}	0.11^{d}	-0.58^{d}	2.13^{d}	-6.49
FTSE	-5.80E-05	-0.16^{d}	-0.13^{d}	0.98^{d}	0.11^{d}	-1.03^{d}	3.10^{d}	-6.50
N2252	9.40E-05	-0.29^{d}	-0.10^{d}	0.97^{d}	0.17^{d}	-0.55^{c}	2.47^{d}	-5.77
GDAXI	1.98E-04	-0.17^{d}	-0.13^{d}	0.98^{d}	0.12^{d}	-0.65^{d}	2.58^{d}	-5.96
RUT	1.06E-04	-0.17^{d}	-0.11^{d}	0.98^{d}	0.12^{d}	-1.92^{c}	4.14^{d}	-5.88
AORD	2.19E-04^{b}	-0.17^{d}	-0.09^{d}	0.98^{d}	0.12^{d}	-1.08^{d}	2.94^{d}	-6.88
DJI	1.20E-04	-0.18^{d}	-0.15^{d}	0.98^{d}	0.12^{d}	-0.50^{d}	2.15^{d}	-6.59
IXIC	2.00E-04^{a}	-0.10^{d}	-0.12^{d}	0.99^{d}	0.11^{d}	-0.77^{d}	2.58^{d}	-5.94
FCHI	-1.00E-04	-0.18^{d}	-0.15^{d}	0.98^{d}	0.10^{d}	-0.78^{d}	2.82^{d}	-6.01
HSI	1.92E-04	-0.12^{d}	-0.07^{d}	0.99^{d}	0.12^{d}	-0.16^{b}	1.77^{d}	-5.97
KS	1.40E-04	-0.10^{d}	-0.08^{d}	0.99^{d}	0.15^{d}	-0.45^{d}	2.01^{d}	-5.96
AEX	3.50E-04	-0.14^{d}	-0.14^{d}	0.98^{d}	0.11^{d}	-0.78^{d}	2.91^{d}	-6.17
SSMI	4.50E-04	-0.26^{d}	-0.15^{d}	0.97^{d}	0.14^{d}	-0.61^{d}	2.45^{d}	-6.47
IBEX	1.35E-04^{a}	-0.14^{d}	-0.11^{d}	0.98^{d}	0.10^{d}	-0.55^{d}	2.47^{d}	-5.93
NSEI	4.92E-04^{d}	-0.30^{d}	-0.10^{d}	0.97^{d}	0.21^{d}	-0.31^{d}	1.91^{d}	-5.92
MXX	3.67E-04^{c}	-0.13^{d}	-0.08^{d}	0.99^{d}	0.15^{d}	-0.39^{d}	2.13^{d}	-6.17
BVSP	1.52E-04	-0.14^{d}	-0.07^{d}	0.98^{d}	0.11^{d}	-0.40^{c}	2.54^{d}	-5.37
GSPTSE	2.70E-04	-0.17^{d}	-0.10^{d}	0.98^{d}	0.12^{d}	-1.37^{d}	2.89^{d}	-6.76
STOXX50	-8.80E-05	-0.17^{d}	-0.15^{d}	0.98^{d}	0.10^{d}	-0.66^{d}	2.67^{d}	-5.99
FTSTI	1.98E-04^{b}	-0.11^{d}	-0.05^{d}	0.99^{d}	0.15^{b}	-0.32b	1.99^{d}	-6.55
FTSEMIB	2.70E-05	-0.13^{d}	-0.11^{d}	0.98^{d}	0.13^{d}	-0.72^{d}	2.48^{d}	-5.89

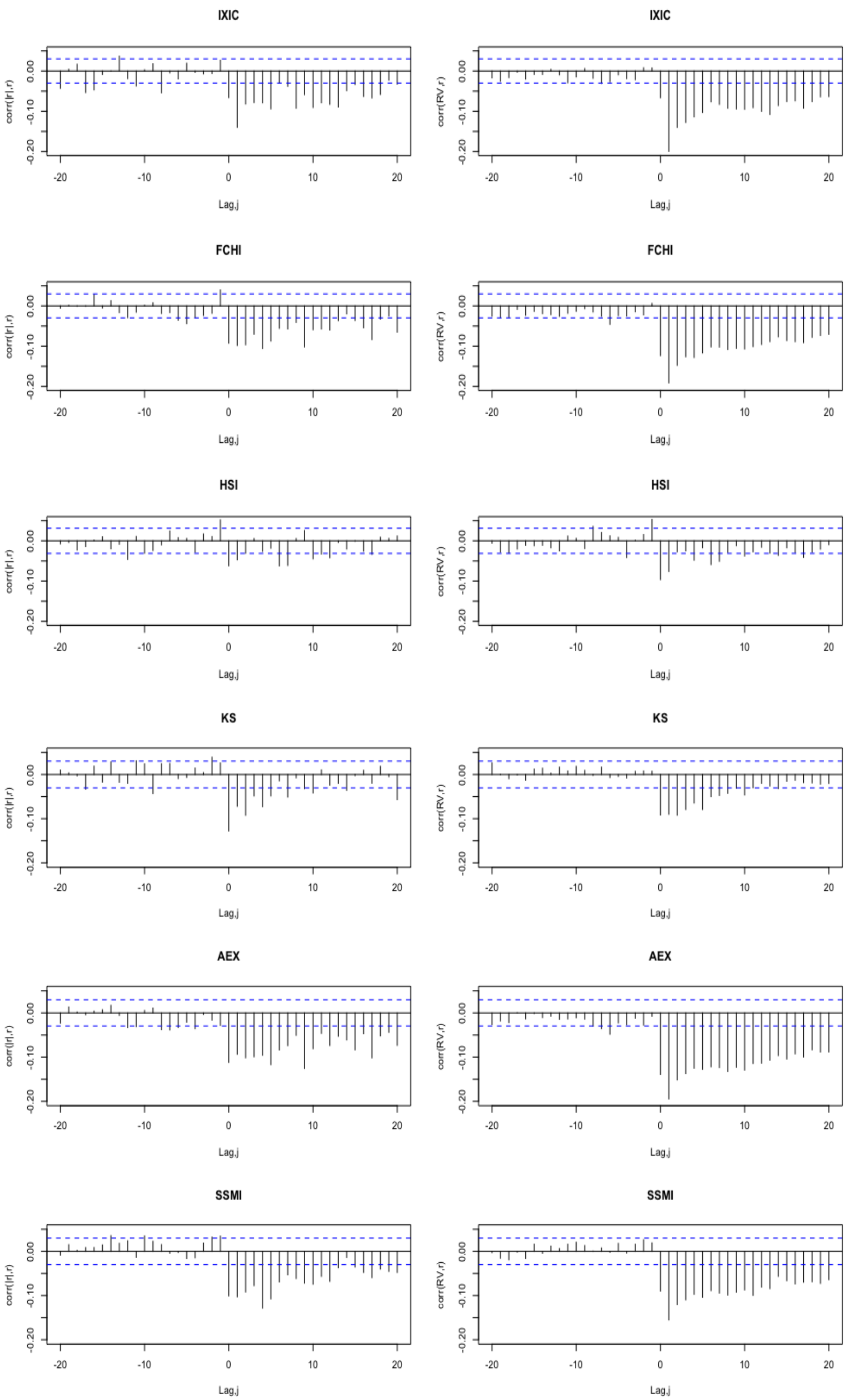
Appendix B

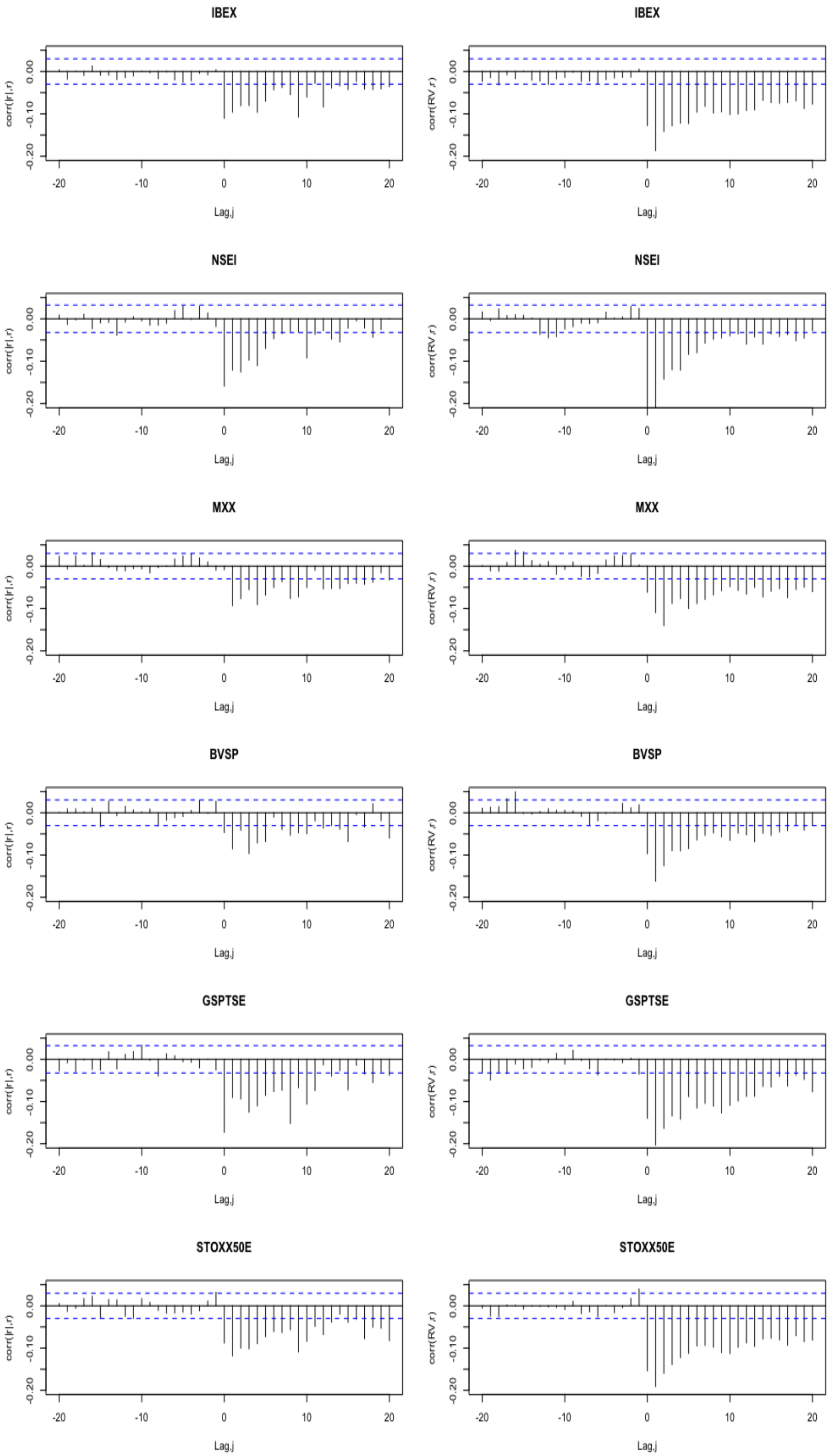
Cross-correlation for all indices

B.1 Cross-correlations for all the indices

Figure B1 plots the cross-correlations for all the indices.




IXIC
IXIC
FCHI
FCHI
HSI
HSI
KS
KS
AEX
AEX
SSMI
SSMI
corr(|r|,r)
corr(RV,r)
0.00
-0.10
-0.20
-20
-10
0
10
20
Lag,j


IBEX
IBEX
NSEI
NSEI
MXX
MXX
BVSP
BVSP
GSPTSE
GSPTSE
STOXX50E
STOXX50E
corr(|r|,r)
corr(RV,r)
Lag,j

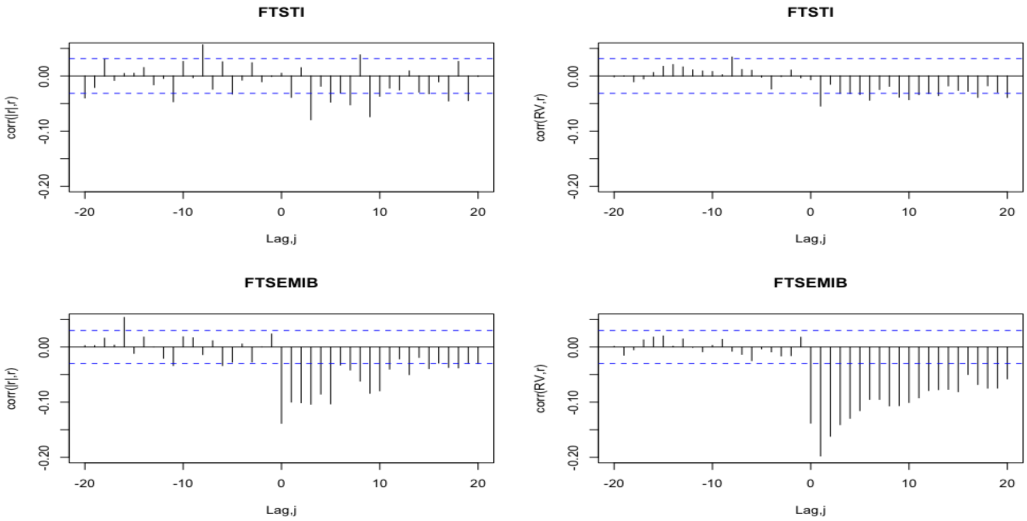


Figure B1: Cross-correlations for all the indices with lags -20 to 20. The panels to the left shows the cross-correlation between absolute returns and returns. The panels to the right shows the cross-correlation between realized volatility and returns. The blue dotted lines indicate a 95% confidence interval under the null hypothesis of zero correlations.

Appendix C

Estimated GARCH models, open-to-close returns and ARMA (1,1) as mean equation.

C.1 ARMA (1,1) GARCH (1,1)

Table C1 presents the estimated coefficients from GARCH (1,1) with ARMA (1,1) as the mean equation and based on open-to-close returns.

Table C1: Estimated coefficients for the GARCH (1,1) model and ARMA (1,1) as the mean equation, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively.

Ticker	μ	ϕ	θ	ω	α	β	ν	κ	AIC
SPX	4E-04^{c}	0.76^{d}	-0.82^{d}	1E-06	0.09	0.90^{d}	-0.52	1.98^{d}	-6.45
FTSE	3E-04^{b}	0.90^{d}	-0.94^{d}	1E-06	0.09	0.90^{d}	-0.82	2.61^{d}	-6.47
N2252	4E-04^{b}	0.80^{d}	-0.82^{d}	4E-06	0.09^{d}	0.90^{d}	-0.60^{d}	2.42^{d}	-5.75
GDAXI	7E-04^{b}	0.88^{d}	-0.90^{d}	2E-06	0.08	0.91^{d}	-0.52	2.32^{c}	-5.92
RUT	6E-04^{d}	0.81^{d}	-0.85^{d}	3E-06	0.08^{a}	0.91^{d}	-1.95	3.99^{d}	-5.86
AORD	4E-04^{c}	-0.10	0.09	1E-06	0.08	0.91^{d}	-0.85^{c}	2.59^{d}	-6.85
DJI	5E-04^{c}	0.81^{c}	-0.86^{d}	1E-06	0.10	0.89^{d}	-0.43	2.01^{c}	-6.55
IXIC	6E-04^{d}	0.78^{d}	-0.83^{d}	1E-06	0.08	0.91^{d}	-0.66^{b}	2.44^{d}	-5.91
FCHI	4E-04	0.81^{d}	-0.86^{d}	2E-06	0.08	0.91^{d}	-0.58	2.36^{c}	-5.96
HSI	3E-04^{a}	-0.69^{c}	0.71^{c}	2E-06	0.06^{a}	0.93^{d}	-0.18^{b}	1.78^{d}	-5.95
KS	4E-04^{b}	0.78^{d}	-0.81^{d}	1E-06	0.07^{c}	0.93^{d}	-0.47^{d}	1.95^{d}	-5.94
AEX	4E-04	-0.30^{a}	0.31^{a}	2E-06	0.10	0.90	-0.42	2.26	-6.12
SSMI	4E-04^{d}	-0.42^{d}	0.43^{d}	2E-06	0.12^{b}	0.87^{d}	-0.43^{d}	2.19^{d}	-6.43
IBEX	5E-04^{b}	-0.48^{c}	0.51^{c}	1E-06	0.08	0.91^{d}	-0.38^{a}	2.21^{d}	-5.89
NSEI	8E-04^{d}	-0.63^{b}	0.69^{b}	5E-06	0.11^{d}	0.86^{d}	-0.26^{c}	1.89^{d}	-5.90
MXX	6E-04^{d}	-0.36^{c}	0.43^{c}	2E-06	0.08	0.91^{d}	-0.3b	1.98^{d}	-6.16
BVSP	4E-04^{a}	-0.39^{b}	0.39^{b}	5E-06	0.06^{d}	0.93^{d}	-0.35^{c}	2.45^{d}	-5.36
GSPTSE	5E-04^{c}	-0.35^{b}	0.36^{b}	1E-06	0.09	0.90^{d}	-1.23	2.79^{b}	-6.74
STOXX50	4E-04^{a}	0.82^{d}	-0.87^{d}	2E-06	0.08	0.91^{d}	-0.46	2.19^{d}	-5.94
FTSTI	3E-04^{b}	0.70^{d}	-0.69^{d}	1E-06	0.09	0.91^{d}	-0.30^{a}	1.95^{d}	-6.54
FTSEMIB	3E-04	0.39	-0.46	1E-06	0.09	0.91^{d}	-0.66^{a}	2.26^{d}	-5.86

C.2 ARMA (1,1) GJR-GARCH (1,1)

Table C2 presents the estimated coefficients from GJR-GARCH (1,1) with ARMA (1,1) as the mean equation and based on open-to-close returns.

Table C2: Estimated coefficients for the GJR-GARCH (1,1) model and ARMA (1,1) as the mean equation, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1 % respectively

Ticker	μ	ϕ	θ	ω	α	β	γ	ν	κ	AIC
SPX	2E-04^{a}	0.59^{d}	-0.65^{d}	2E-06	1E-06	0.90^{d}	0.17	-0.59	2.10^{d}	-6.49
FTSE	4E-05	0.67^{d}	-0.70^{d}	2E-06	0E+00	0.91^{d}	0.15	-1.04^{b}	3.05^{d}	-6.50
N2252	2E-04	-0.28	0.26	5E-06^{a}	3E-02	0.89^{d}	0.12^{d}	-0.57^{d}	2.49^{d}	-5.76
GDAXI	3E-04	-0.44	0.44	2E-06	0E+00	0.91^{d}	0.15^{b}	-0.64^{d}	2.60^{d}	-5.95
RUT	3E-04	0.68^{d}	-0.71^{d}	4E-06	5E-03	0.91^{d}	0.12^{c}	-1.88^{a}	4.23^{c}	-5.88
AORD	2E-04	0.97^{d}	-0.97^{d}	1E-06	5E-03	0.92^{d}	0.11	-1.01	2.86^{c}	-6.87
DJI	2E-04	0.16	-0.22	1E-06	0E+00	0.90^{d}	0.17	-0.49^{b}	2.12^{d}	-6.59
IXIC	3E-04^{a}	0.61^{d}	-0.64^{d}	2E-06	0E+00	0.92^{d}	0.13^{c}	-0.72^{c}	2.54^{d}	-5.94
FCHI	2E-04	0.71^{d}	-0.75^{d}	2E-06	0E+00	0.91^{d}	0.15^{c}	-0.72^{d}	2.66^{d}	-6.00
HSI	2E-04	-0.68^{b}	0.70^{b}	2E-06	1E-02	0.93^{d}	0.08	-0.19	1.81^{d}	-5.97
KS	2E-04	-0.63	0.64	1E-06	2E-02	0.93^{d}	0.09^{b}	-0.44^{d}	2.00^{d}	-5.95
AEX	4E-05	-0.22	0.24	2E-06	2E-06	0.91^{d}	0.16	-0.71	2.78^{b}	-6.17
SSMI	9E-05	-0.35^{b}	0.36^{b}	3E-06	0E+00	0.88^{d}	0.20^{d}	-0.59^{d}	2.40^{d}	-6.46
IBEX	2E-04	-0.49^{a}	0.52^{b}	2E-06	1E-06	0.92^{d}	0.13	-0.50^{b}	2.39^{d}	-5.92
NSEI	6E-04^{c}	-0.52	0.59^{a}	8E-06^{d}	3E-02^{c}	0.84^{d}	0.18^{d}	-0.28^{c}	1.91^{d}	-5.92
MXX	4E-04^{b}	-0.34^{b}	0.40^{c}	2E-06	2E-02	0.92^{d}	0.11^{a}	-0.36^{b}	2.12^{d}	-6.18
BVSP	2E-04	-0.31	0.32	5E-06	1E-02	0.93^{d}	0.08^{d}	-0.40^{c}	2.58^{d}	-5.37
GSPTSE	2E-04^{a}	-0.26	0.28	5E-06	6E-03	0.91^{d}	0.12	-1.40^{d}	2.92^{d}	-6.76
STOXX50	1E-04	0.72^{d}	-0.75^{d}	5E-06	0E+00	0.91^{d}	0.15^{b}	-0.59^{c}	2.49^{d}	-5.98
FTSTI	2E-04	0.78^{a}	-0.77^{a}	1E-06	5E-02	0.91^{d}	0.07^{a}	-0.30^{a}	1.99^{d}	-6.54
FTSEMIB	6E-05	-0.07	0.01	1E-06	1E-02	0.92^{d}	0.13	-0.75^{a}	2.42^{d}	-5.88

C.3 ARMA (1,1) Log-GARCH (1,1)

Table C3 presents the estimated coefficients from Log-GARCH (1,1) with ARMA (1,1) as the mean equation and based on open-to-close returns.

Table C3: Estimated coefficients for the Log-GARCH (1,1) model and ARMA (1,1) as the mean equation, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1% level and 0.1% respectively

Ticker	μ	ϕ	θ	ω	β	γ	ν	κ	AIC
SPX	4.62E-04[b]	0.74[a]	-0.81[b]	-0.11[d]	0.99[d]	0.20[d]	-0.50[d]	1.96[d]	-6.45
FTSE	3.06E-04	0.90[d]	-0.94[d]	-0.12	0.99[c]	0.19	-0.77	2.55	-6.46
N2252	4.34E-04[c]	0.79[d]	-0.82[d]	-0.19[d]	0.98[d]	0.19[d]	-0.56[d]	2.37[d]	-5.75
GDAXI	6.32E-04[d]	0.89[d]	-0.91[d]	-0.10	0.99[d]	0.18[c]	-0.51[a]	2.30[d]	-5.92
RUT	8.82E-04[d]	0.80[d]	-0.85[d]	-0.12[c]	0.99[d]	0.17[d]	-1.76	3.67[c]	-5.85
AORD	4.28E-04[c]	0.93[d]	-0.94[d]	-0.11	0.99[d]	0.16[d]	-0.92	2.63	-6.85
DJI	4.68E-04[d]	0.80[d]	-0.85[d]	-0.13[d]	0.99[d]	0.21[d]	-0.41[d]	1.98[d]	-6.55
IXIC	6.35E-04[d]	0.79[d]	-0.84[d]	-0.07[d]	0.99[d]	0.17[d]	-0.64[d]	2.41[d]	-5.91
FCHI	4.33E-04[d]	0.81[d]	-0.86[d]	-0.11[c]	0.99[d]	0.18[d]	-0.57[b]	2.37[d]	-5.96
HSI	3.67E-04	0.96[d]	-0.96[d]	-0.09[c]	0.99[d]	0.14	-0.16[a]	1.75[d]	-5.95
KS	3.86E-04[c]	0.78[d]	-0.81[d]	-0.05[d]	0.99[d]	0.15[d]	-0.47[d]	1.96[d]	-5.94
AEX	3.81E-04[b]	-0.24[d]	0.26[d]	-0.10[d]	0.99[d]	0.21[d]	-0.40[d]	2.22[d]	-6.12
SSMI	3.91E-04[d]	-0.43[d]	0.44[d]	-0.21[c]	0.98[d]	0.24[d]	-0.41[d]	2.17[d]	-6.43
IBEX	4.75E-04[c]	-0.53[d]	0.56[d]	-0.09[d]	0.99[d]	0.18[d]	-0.37[d]	2.20[d]	-5.89
NSEI	7.25E-04[d]	-0.63[d]	0.68[d]	-0.20[d]	0.98[d]	0.22[d]	-0.28[d]	1.86[d]	-5.90
MXX	5.75E-04[d]	-0.38[d]	0.44[d]	-0.11[d]	0.99[d]	0.18[d]	-0.30[d]	1.96[d]	-6.16
BVSP	4.18E-04[a]	-0.36[d]	0.37[d]	-0.11[d]	0.99[d]	0.13[a]	-0.34	2.37[d]	5.35
GSPTSE	4.80E-04[b]	-0.38[d]	0.39[d]	-0.12	0.99[d]	0.18[d]	-1.12	2.67	-6.73
STOXX50	3.93E-04[d]	0.84[d]	-0.88[d]	-0.10[d]	0.99[d]	0.18[d]	-0.44[d]	2.17[d]	-5.94
FTSTI	3.24E-04[c]	0.68[d]	-0.67[d]	-0.11[d]	0.99[d]	0.19[d]	-0.30[d]	1.94[d]	-6.54
FTSEMIB	3.22E-04[c]	0.51[d]	-0.56[d]	-0.09[d]	0.99[d]	0.18[d]	-0.64[d]	2.24[d]	-5.86

C.4 ARMA (1,1) E-GARCH (1,1)

Table C4 presents the estimated coefficients from E-GARCH (1,1) with ARMA (1,1) as the mean equation and based on open-to-close returns.

Table C4: Estimated coefficients for the E-GARCH (1,1) model and ARMA (1,1) as the mean equation, reported together with the corresponding AIC values. Superscripts a, b, c, d indicate significance at the 10 %, 5 %, 1 % level and 0.1% respectively

Ticker	μ	ϕ	θ	ω	α	β	γ	ν	κ	AIC
SPX	1.00E-04	0.32^{d}	-0.38^{d}	-0.17^{d}	-0.15^{d}	0.98^{d}	0.11^{d}	-0.62^{d}	2.12^{d}	-6.49
FTSE	-2.00E-05	0.57^{d}	-0.59^{d}	-0.16^{d}	-0.13^{d}	0.98^{d}	0.11^{d}	-1.05^{d}	3.10^{d}	-6.5
N2252	1.16E-04	-0.31^{d}	0.29^{d}	-0.29^{d}	-0.10^{d}	0.97^{d}	0.17^{d}	-0.55^{c}	2.46^{d}	-5.77
GDAXI	1.86E-04	-0.42^{d}	0.44^{d}	-0.17^{d}	-0.13^{d}	0.98^{d}	0.12^{d}	-0.64^{d}	2.58^{d}	-5.96
RUT	2.01E-04	0.54^{d}	-0.57^{d}	-0.16^{d}	-0.10^{d}	0.98^{d}	0.12^{d}	-1.99^{c}	4.12^{d}	5.88
AORD	-1.17E-04	0.98^{d}	-0.97^{d}	-0.12^{d}	-0.11^{d}	0.99^{d}	0.10^{d}	-1.15^{d}	3.01^{d}	-6.88
DJI	1.72E-04^{b}	0.08^{d}	-0.13^{d}	-0.18^{d}	-0.14^{d}	0.98^{d}	0.12^{d}	-0.52^{d}	2.14^{d}	-6.6
IXIC	2.54E-04^{b}	0.45^{d}	-0.48^{d}	-0.10^{d}	-0.11^{d}	0.99^{d}	0.11^{d}	-0.81^{d}	2.59^{d}	-5.94
FCHI	2.50E-05	0.59^{d}	-0.62^{d}	-0.17^{d}	-0.14^{d}	0.98^{d}	0.10^{d}	-0.80^{d}	2.80^{d}	-6.01
HSI	1.97E-04	-0.67^{d}	0.69^{d}	-0.12^{d}	-0.07^{d}	0.99^{d}	0.12^{d}	-0.16^{b}	1.77^{d}	-5.97
KS	1.57E-04	0.62^{d}	-0.63^{d}	-0.09^{d}	-0.07^{d}	0.99^{d}	0.15^{d}	-0.46^{d}	2.01^{d}	-5.96
AEX	1.40E-05	-0.02	0.04	-0.14^{d}	-0.14^{d}	0.98^{d}	0.11^{d}	-0.77^{d}	2.90^{d}	-6.17
SSMI	3.50E-05	-0.33^{d}	0.34^{d}	-0.26^{d}	-0.15^{d}	0.97^{d}	0.14^{d}	-0.60^{d}	2.44^{d}	-6.47
IBEX	1.22E-04	-0.49^{d}	0.52^{d}	-0.15^{d}	-0.11^{d}	0.98^{d}	0.10^{d}	-0.54^{d}	2.48^{d}	-5.93
NSEI	4.65E-04^{c}	-0.48^{d}	0.54^{d}	-0.32^{d}	-0.11^{d}	0.96^{d}	0.21^{d}	-0.30^{d}	1.91^{d}	-5.92
MXX	3.27E-04^{b}	-0.30^{d}	0.37^{d}	-0.13^{d}	-0.09^{d}	0.99^{d}	0.15^{d}	-0.38^{c}	2.13^{d}	-6.18
BVSP	1.44E-04	-0.29^{d}	0.30^{d}	-0.14^{d}	-0.07^{d}	0.98^{d}	0.11^{d}	-0.40^{c}	2.54^{d}	-5.37
GSPTSE	2.56E-04^{b}	-0.22^{d}	0.24^{d}	-0.17^{d}	-0.10^{d}	0.98^{d}	0.12^{d}	-1.36^{d}	2.88^{d}	-6.76
STOXX50	-1.30E-05	0.57^{d}	-0.59^{d}	-0.16^{d}	-0.14^{d}	0.98^{d}	0.10^{d}	-0.67^{d}	2.66^{d}	-5.99
FTSTI	-7.57E-04^{d}	1.00^{d}	-1.00^{d}	-0.10^{d}	-0.05^{d}	0.99^{d}	0.15^{d}	-0.33^{d}	1.95^{d}	-6.55
FTSEMIB	2.00E-06	-0.21^{d}	0.16^{d}	-0.13^{d}	-0.11^{d}	0.98^{d}	0.13^{d}	-0.77^{d}	2.49^{d}	-5.89

Appendix D

Forecasting performance GARCH and GJR-GARCH with ARMA (1,1) as mean equation and open-to-close returns.

D.1 GARCH and GJR-GARCH

Table D1 presents the results for the forecast evaluation of GARCH and GJR-GARCH with ARMA (1,1) as the mean equation and based on open-to-close returns.

Table D1: Out-of-sample results for ARMA (1,1) GARCH (1,1) and GJR-GARCH (1,1). Reported are the MSE and QLIKE lossfunctions, and the p-values from the Model Confidence Set (Hansen et al., 2011).

	GARCH	GJRGARCH	% change	P-value	GARCH	GJRGARCH	% change	P-value
Ticker	MSE	MSE	Delta	MCS	QLIKE	QLIKE	Delta	MCS
SPX	41.38	40.37	-2.44	0.3183	0.072	0.068	-5.56	0.0004
FTSE	19.30	18.81	-2.54	0.2181	0.044	0.044	0.00	0.7716
N2252	48.29	51.54	6.73	0.0214	0.068	0.068	0.00	0.0953
GDAXI	37.47	35.11	-6.30	0.1791	0.047	0.045	-4.26	0.0128
RUT	68.17	69.18	1.48	0.3456	0.107	0.109	1.87	0.0507
AORD	21.57	25.40	17.76	0.0003	0.072	0.074	2.78	0.0837
DJI	43.35	40.31	-7.01	0.0659	0.073	0.068	-6.85	0.0000
IXIC	34.33	34.91	1.69	0.3272	0.067	0.068	1.49	0.1473
FCHI	32.92	33.93	3.07	0.2970	0.045	0.044	-2.22	0.1735
HSI	27.66	28.30	2.31	0.1473	0.047	0.050	6.38	0.0000
KS	28.07	28.42	1.25	0.2656	0.052	0.054	3.85	0.0000
AEX	26.27	27.10	3.16	0.2785	0.047	0.044	-6.38	0.0000
SSMI	23.34	26.06	11.65	0.1586	0.043	0.043	0.00	0.9893
IBEX	44.70	43.78	-2.06	0.3693	0.048	0.045	-6.25	0.0056
NSEI	44.65	43.59	-2.37	0.3447	0.052	0.052	0.00	0.3019
MXX	69.47	69.77	0.43	0.7272	0.114	0.117	2.63	0.0068
BVSP	76.99	74.25	-3.56	0.2145	0.058	0.060	3.45	0.0378
GSPTSE	27.97	28.66	2.47	0.1988	0.071	0.071	0.00	0.9817
STOXX50E	45.20	44.33	-1.92	0.3250	0.055	0.051	-7.27	0.0000
FTSTI	18.26	16.91	-7.39	0.0209	0.039	0.038	-2.56	0.0205
FTSEMIB	39.95	43.43	8.71	0.0260	0.053	0.055	3.77	0.1176

D.2 Log-GARCH and E-GARCH

Table D2 presents the results for the forecast evaluation of Log-GARCH and E-GARCH with ARMA (1,1) as the mean equation and based on open-to-close returns.

Table D2:Out-of-sample results for ARMA (1,1) Log-GARCH (1,1) and E-GARCH (1,1). Reported are the MSE and QLIKE lossfunctions, and the p-values from the Model Confidence Set (Hansen et al., 2011).

	logGARCH	EGARCH	% change	P-value	logGARCH	EGARCH	% change	P-value
Ticker	MSE	MSE	Delta	MCS	QLIKE	QLIKE	Delta	MCS
SPX	43.86	35.22	-19.70	0.0000	0.075	0.068	-9.33	0.0000
FTSE	19.74	18.29	-7.35	0.0028	0.045	0.044	-2.22	0.2132
N2252	42.95	40.24	-6.31	0.0385	0.067	0.064	-4.48	0.0000
GDAXI	37.81	33.06	-12.56	0.0037	0.048	0.044	-8.33	0.0000
RUT	69.61	60.29	-13.39	0.0013	0.108	0.105	-2.78	0.0221
AORD	23.51	21.18	-9.91	0.0000	0.079	0.070	-11.39	0.0000
DJI	45.83	38.93	-15.06	0.0000	0.076	0.069	-9.21	0.0000
IXIC	37.10	33.58	-9.49	0.0122	0.068	0.067	-1.47	0.1020
FCHI	33.59	30.99	-7.74	0.0086	0.046	0.042	-8.70	0.0001
HSI	25.41	26.04	2.48	0.0775	0.048	0.050	4.17	0.0000
KS	28.44	28.65	0.74	0.4229	0.051	0.054	5.88	0.0000
AEX	28.19	25.98	-7.84	0.0119	0.050	0.044	-12.00	0.0000
SSMI	24.30	24.51	0.86	0.5653	0.047	0.043	-8.51	0.0033
IBEX	43.31	38.68	-10.69	0.0001	0.047	0.043	-8.51	0.0000
NSEI	47.81	44.60	-6.71	0.0691	0.054	0.053	-1.85	0.0807
MXX	74.22	70.33	-5.24	0.0867	0.120	0.121	0.83	0.2257
BVSP	78.20	73.24	-6.34	0.0550	0.060	0.061	1.67	0.1700
GSPTSE	31.52	28.38	-9.96	0.0000	0.076	0.071	-6.58	0.0000
STOXX50E	46.74	40.52	-13.31	0.0000	0.057	0.050	-12.28	0.0000
FTSTI	18.95	17.76	-6.28	0.0028	0.043	0.042	-2.33	0.0015
FTSEMIB	43.74	39.64	-9.37	0.0077	0.058	0.054	-6.90	0.0017