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TITLE:

Oil Price and Equity Markets: Modeling Co-Movement and Conditional Value at Risk.

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Abstract

This paper studies the co-movement between oil prices and stock markets during the period 2006 – 2017 utilizing quantile regression. The studied stock indices are AEX, BOVESPA, CAC40, DAX30, EUROSTOXX50, FTSE100, SMI, S&P500 and TSX60, and the United States Oil Fund ETF represents the oil price. We investigate the co-movement and find a positive and significant co-movement between oil returns and stock market returns across quantiles for the stock market return distribution in all indices examined. The estimated coefficients from the quantile regression exhibit a U-shape, meaning that the dependence between oil returns and stock returns is strongest for high and low quantiles of the stock market distribution. However, we show that this U-shaped pattern disappears after we include implied volatility as an additional explanatory variable. Next, we find that the co-movement between oil and equity is asymmetric for most indices, with higher dependence in the lowermost quantiles. Finally, we find that the contribution of oil prices to value at risk of stock indices vary over time and is asymmetric, meaning that oil price risk contribute differently to the long position in the stock market than to the short position.

Preface

This master thesis completes our Master of Science in Business Administration with specialization in Applied Finance at UiS Business School.

The recent year's turmoil in oil price has been a constant headline in the Norwegian financial media. As finance students in Norway the oil price and its effect on the economy and stock prices has been a regular talking point. This has sparked an interest and led us toward a study of oil price and equity markets.

The subject of this paper is to model the co-movement between oil prices and stock market indices across the return distribution using quantile regression, additionally we explore the value at risk in stock indices, conditional on oil price returns. The process has been a challenging, yet rewarding one. We have immersed ourselves in previous research on the subject of the relationship between oil prices, the economy and stock markets, and found it to be most interesting.

We would like to thank our supervisor associate professor Peter Molnar at UiS Business School for his useful guidance, constructive feedback and engaging discussions as well as taking a keen interest in our research. We would also like to thank him for being available at short notice and always answering e-mails swiftly.

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1. Introduction

Since the oil shortages in the 1970s, a large amount of research has focused on the relationship between the price of oil and the economy. Many researchers have documented a negative relationship between oil and real economic activity (Hamilton, 1983, 2011; Jimenez-Rodriguez & Sanchez, 2005). The price of oil was viewed as an exogenous variable affecting the economy. However Hamilton (1983) showed the oil price Granger-caused many U.S. macroeconomic variables before 1973 but not after, thereby arguing the oil price was endogenous after the structural break in 1973. Many researchers treat oil price as an important factor in understanding fluctuation in stock prices even if there is no consensus about the relationship (Kilian & Park, 2009).

A significant amount of research has studied the relationship between oil price and stock markets. The effect of oil price changes on the real economy should be reflected in the stock markets as it is an important factor as input or output price in many sectors, as well as an indicator of economic activity (Cunado & Perez de Gracia, 2005). Considerable amounts of research have found evidence of a negative impact from rising oil prices on various industry sectors as well as the whole stock market. Positive shocks to oil prices depress real stock returns on the S&P 500 (Sadorsky, 1999). Similarly, a negative impact from increased oil prices on real stock returns in the US, Canada, Japan and the UK was found by Jones and Kaul (1996). Oil price changes have a negative impact on equity returns from all industry sectors except mining and oil and gas (Nandha & Faff, 2008). Sharp increases in precautionary demand leading to sharp increases in oil price are immediately reflected in lower stock prices (Kilian & Park, 2009). Many others support these findings, see Driesprong, Jacobsen, and Maat (2008), Filis, Degiannakis, and Floros (2011), Gjerde and Sættem (1999), Miller and Ratti (2009) and Nandha and Faff (2008) among others. Park and Ratti (2008) and Bjornland (2009) both found a positive relationship between oil prices and stock markets in oil-exporting Norway. Wang, Wu, and Yang (2013) provide more evidence that there is a positive relationship between stock returns and oil price in oil exporting countries. Others have not found any significant relationship between oil prices and real stock returns (Apergis & Miller, 2009; Henriques & Sadorsky, 2008; Huang, Masulis, & Stroll, 1996). Some recent studies have reported that the relationship between oil and equity is non-linear and changes over time in response to global events such as wars and financial crises (Filis et al., 2011; Miller & Ratti, 2009). Although there is an abundance of research into this topic there is no clear agreement as to whether the relationship between oil and equity is positive, negative or non-existent (Reboredo & Ugolini, 2015).

The use of quantile regression to model stock market and oil price have been limited. Agbeyegbe (2015a) investigates the relationship between oil price and its volatility by examining the relationship between the United States oil Fund ETF (USO) returns and its implied volatility index the CBOE Crude Oil ETF Volatility Index (OVX). He finds a negative relationship between the OVX and USO returns, the relationship depends on the quantile of the return distribution. In particular, there is an inverted U-shaped dependency between returns and implied volatility across quantiles. Linear quantile regression and copula quantile regression has been utilized by Agbeyegbe (2015b) to study the return-volatility relationship between major US stock indices and their corresponding implied volatility indices (IV). Again the relationship between stock returns and implied volatility depends on considered quantile, and the relationship across quantiles is of an inverted U-shape (Agbeyegbe, 2015b). Similarly, Badshah (2013) finds that the relationship between stock indices and their implied volatility is asymmetric, it is higher in the upper quantiles and thus strongly underestimated by ordinary regression for these quantiles. Although not oil price related, Jareño, Ferrer, and Miroslavova (2016) use quantile regression methodology to examine individual companies in the U.S. and their sensitivity to interest rate changes and Tsai (2012) finds a negative relationship between exchange rates and Asian stock markets. These studies provide support for the importance and the practical potential quantile regression has in studying the asymmetric relationship between returns and various variables. Lee and Zeng (2011) examine the oil-stock relationship and find that the linkage between the two variables differs greatly across the quantiles. Using weekly data from 2000 to 2014 Reboredo and Ugolini (2015) use an unconditional quantile dependency model to analyze the oil-stock relationship. They find that oil price and stock markets co-move before the financial crisis of 2008 and that this co-movement strengthened considerably during the crisis. (Ding, Kim, & Park, 2016) examine the causal relationship between oil market (represented by WTI returns) and stocks market (represented by S&P 500 returns) in the tails of the distributions and find a strong causal relationship.

Little research using value at risk outside financial sector have been done. Reboredo (2015) uses daily data to examine co-movement and systemic risk between oil and clean energy stocks using a GARCH(1,1) model to calculate the conditional value at risk. The findings from his research show that oil and renewable energy stocks have an interdependent relationship, with positive time-varying dependence and symmetric tail dependence, indicating that oil price and renewable energy stocks move together during both market booms and busts. This is similar to our research that shows a positive time-varying relationship between oil price and stock indices.

Our contribution to the existing literature is the analysis of the dependence structure of equity indices for various countries with regards to the oil price. We also include implied volatility of the respective stock market as an explanatory variable. We find that oil price has a positive and significant impact on all indices included in the analysis. First, we show that the impact of oil returns on various quantiles of stock returns exhibit a very clear U-shaped relationship, with dependence being strongest for high and low quantiles. However, next we show that this U-shaped relationship is to a large degree caused by not controlling to implied volatility as a state variable. After we include implied volatility of a respective stock index as an additional explanatory variable, the U-shaped relationship becomes much less pronounced.

We also find that most stock indices exhibit an asymmetric structure of co-movement with highest co-movement with oil price when the stock market is bearish, and that implied volatility accounts for most of the variation in the estimated coefficients across quantiles. Lastly, we find that contribution of the oil price to the value at risk is not symmetric, meaning if the price of oil increases 5% one day the contribution to value at risk in a short position is not the same as the contribution to value at risk in a long position in the market if the oil price decreases 5%.

The rest of the paper is organized as follows: Section 2 describes the data used, its properties and descriptive statistics, Section 3 describes the methodology and models used, Section 4 presents the empirical results of the research and lastly, Section 5 concludes.

2. Data

The data analyzed in this paper is comprised of indices for equity markets in nine different countries, the implied volatility index for each corresponding market index and the United States Oil Fund (USO) exchange traded fund, representing the US oil market. All time series are obtained from the Thomson Reuters DataStream service and are denominated in U.S. dollars. For both stock market indices and oil market we calculate logarithmic returns r_{it} from the closing prices spot prices p_t as:

$$r_{it} = 100 * \ln(\frac{p_t}{p_{t-1}}) \qquad (1)$$

Table 1: Overview of stock indices and their respective implied volatility (IV) indices used in this paper.

Index	Ticker	IV Index	Ticker	Period	Country
AEX	.AEX	AEX IV	.VAEX	10.04.06 - 27.03.17	Netherland
BOVESPA	.BVSP	BOVESPA IV	.VXEWZ	16.03.11 - 27.03.17	Brazil
CAC 40	.FCHI	CAC 40 IV	.VCAC	10.04.06 - 27.03.17	France
DAX 30	.GDAXI	DAX 30 IV	.V1XI	10.04.06 - 27.03.17	Germany
EUROSTOXX 50	.STOXXE50E	STOXX50 IV	.V2TX	10.04.06 - 27.03.17	Europe
FTSE 100	.FTSE	FTSE 100 IV	.VFTSE	10.04.06 - 27.03.17	UK
SMI	.SSMI	SMI IV	.V3X	10.04.06 - 27.03.17	Switzerland
S&P 500	.SPX	New VIX	.VIX	10.04.06 - 27.03.17	USA
TSX 60	.SPTSE	TSX 60 IV	.GSPVIXC	18.10.10 - 27.03.17	Canada
USO	.USO			10.04.06 - 27.03.17	USA

The sample period starts at 10.04.06 because this is the date the United States Oil Fund ETF was created and therefore its earliest available data point. The sample period for the BOVESPA and TSX60 differs from the rest because there is limited implied volatility data available. All calculations, tables and graphs are calculated and created using Stata. Dates are synchronized since non-trading days varies depending on national holidays in the various markets. The number of observations will vary from variable to variable in the sample. Days when one variable is missing an observation in the time series, are excluded from the analysis.

In this analysis, we use logarithmic returns from the equity indices and the oil price, while we use daily closing price for the implied volatility. This is because equity indices are non-stationary as shown in this papers' "Stationarity" section. We use daily data because less research has been conducted with such frequent data. Moreover, implied volatility indices spike in times of high market uncertainty and subsequently fall back quite rapidly to its mean-reverting previous state, meaning the IV can change significantly even at a daily scale. As Figure 3 shows these spikes can be rather short and a weekly or monthly interval would not capture these spikes. Also, using monthly or weekly returns will decrease the number of observations significantly. As we study not only the average relationship between the oil and stock market returns but also their relationship in extreme quantiles, sufficient number of observations is paramount for our analysis.

In addition to implied volatility as a control variable for systematic risk, we considered credit default swaps for the 5-year government bonds issued by the respective countries. However, the explanatory power of this variable was negligible in comparison to the implied volatility. We also considered the slope[1] of the bond yield curve, but we didn't find statistically significant results. Therefore, we exclude these variables and only report the results for implied volatility.

[1] The slope of the bond yield curve was calculated as the difference between one-month and one-year government bonds.

2.1 Market indices

The time series we use are comprised of daily data. The goal of our analysis is to study the contemporary relation between stock market returns and oil returns. Different trading hours for the considered stock market and the U.S. oil fund could be problematic if this difference is very large. Therefore, we only include stock indices from the American and European continents and exclude indices with a time difference of more than five hours from the closing price of the U.S. oil fund.

Table 2: Trading hours for the market indices the paper is based on.

Index	Trading hours	Time zone
AEX	09:00 – 17:40	UTC + 01:00
BOVESPA	09:00 – 17:00	UTC – 03:00
CAC 40	09:00 – 17:30	UTC + 01:00
DAX 30	09:00 – 17:30	UTC + 01:00
EUROSTOXX 50	09:00 – 17:30	UTC + 01:00
FTSE 100	09:00 – 16:30	UTC
S&P 500	09:00 – 16:00	UTC – 05:00
TSX 60	09:00 – 16:00	UTC – 05:00
USO	09:00 – 16:00	UTC – 05:00

*Note: Closing hours change for the indices in countries observing daylight savings time and Opening hours reported here are at the end of the period of our study.

2.2 United States oil fund ETF

In this paper, the United States Oil fund (USO) is used as a proxy for the oil price. The exchange traded fund (ETF) is designed to track the daily price movements of West Texas Intermediate (WTI) sweet crude delivered to Crushing, Oklahoma ("United States Oil Fund," 2017). The fund gives access to smaller investors who normally wouldn't have access to the oil market. Investing indirectly via oil companies would not be ideal since individual companies may not necessarily have returns that follow crude oil due to idiosyncratic risk (Murdock & Richie, 2008). Therefore, the fund invests in a mixture of oil futures contracts as well as other oil interests to replicate crude oil. Future contracts also represent a good indicator of market expectations for the future spot price of oil. These contracts are financial instruments that lock in a price at a predetermined future date where the holders must sell or buy at the predetermined price and volume. Exchange traded funds are unable to completely replicate the spot price of oil since such funds cannot invest in physical oil. Replacing physical oil with future contract makes the fund subjected to "roll yield", which decreases the returns in contango markets and increases during backwardation (Haugorm, Langeland, Molnar, & Westgaard, 2014). The fund rolls contracts during a 4-day period starting approximately 14-days before expiration. The Fund is also subjected to quantity risk, where the number of contracts available deviates from the purchasing requirement and tracking error, which occurs when the fund invests in Treasury securities and cash equivalents. (Murdock & Richie, 2008).

2.3 Implied Volatility

The first to propose using options to measure implied volatility was Brenner and Galai (1989) who created the Sigma index. Stating the need for a robust measurement of expected volatility in the market where stock futures and options are used by investors to hedge against market and interest rate volatility. The original VIX methodology was introduced by Whaley (1993), which was based on options prices from the S&P 100 (Whaley, 2008). Later the "VIX methodology" issued by the Chicago Board Options Exchange (CBOE, 2014) has become the foundation of most implied volatility indices around the world. All the implied volatility indices in our sample are created using the "VIX methodology" or very similar methods.

The CBOE Volatility Index (VIX) is an index that measures the implied or forward-looking 30-day volatility. The idea of the index is to calculate the squared root of the price of variance by constructing a portfolio of options on the underlying stocks in the S&P 500 index. The portfolio of options is continually calculated and rebalanced to always provide implied 30-day volatility of the S&P 500. If the market is "bearish" and expects higher volatility due to market uncertainty, the price of options will increase as demand for options as insurance for the market uncertainty increases.

The index is calculated from two portfolios of out of money call and put options, with its exposure to price variations eliminated by delta hedging. The two portfolios reflect the price of variance for one near-term and one next-term portfolio, regularly "rolling over" to the next contracts. The price of variance σ can be defined as the forward price of a strip of options, calculated in the equation:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} G(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad (2)$$

Where T is the time to expiration, F is the forward price level of variance defined as the strike price at which the absolute price difference between the call and put is the smallest. K_0 is the first strike price below the forward index level for the near- and next-term options. K_i is the strike price of out of money option i, a call and a put. ΔK_i is the half difference between the strike prices of either side of the puts and calls K_i. R is the risk-free rate and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike price K_i.

The selected options are out of money S&P500 calls and puts centered around an at the money strike price K_0. Only S&P 500 quoted with non-zero bid prices are used and as volatility rises and falls the strike price range of options with non-zero bids expand and contract. Therefore, the number of options in the VIX calculations tend to change over time.

Equation 2 is applied to the near-term and next-term options with time to expiration T_1 and T_2 which gives a price for variance for the near-term and the next-term σ_1 and σ_2.

Lastly, the 30-day weighted average of σ_1 and σ_2 is calculated. Then the square root of that value is multiplied by 100.

$$VIX = 100 * \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right\} * \frac{N_{365}}{N_{30}}} \quad (3)$$

When the near-term option has less than a week to expiration the index rolls over to the next position, maintaining a constant 30-days to expiration.

2.4 Stationary and Non-Stationary Time Series

To avoid the problem of spurious regression result we test every time series for stationarity with the help of unit root tests. This is not important for the volatility index, because from economic insights we know that it should be mean-reverting and stationary.

A violation of stationarity in econometric models could lead to invalid results (Enders, 2010). We use an augmented Dickey-Fuller (ADF) unit root test, which is an expanded form of dickey-fuller test (Dickey & Fuller, 1981) and the Phillips-Perron (PP) test (Phillips & Perron, 1988) to determine the presences of a unit roots. Including both strengthens the robustness of our conclusion.

Reasons for choosing the augmented versions of the dickey-fuller test is that not all time-series can be represented as an autoregressive process. If the process is of a higher order autoregressive process, the residuals will be serial correlated and produce incorrect test results. The ADF-test can be formally defined as:

$$\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{I=2}^{n} \beta_i \Delta y_{t-i} + e_t \qquad (4)$$

The PP-test is a more comprehensive test that builds on the Dickey-Fuller test. Like the ADF-test it tries to include for a higher order autoregressive process, but with a nonparametric correction that accommodates for weakly dependent and heteroscedastic data (Phillips & Perron, 1988). The method estimates the non-augmented Dickey-Fuller test for unit root equation:

$$\Delta y_t = \alpha y_{t-1} + x_t' \delta + \varepsilon_t \qquad (5)$$

The null hypothesis in the ADF and PP-test is $H_0 : y_t \sim I(1)$ that the process contains a unit root and alternative hypothesis $H_t : y_t \sim I(0)$ when the process does not contain a unit root. If we reject the null, the time series is assumed to be stationary. Non-stationarity can be corrected for by differencing between the consecutive observations to make the non-stationary time series stationary, by simply calculating the daily log returns.

The test results confirm the reasonable assumptions that stock indices closing prices are non-stationary and we accept the null hypothesis I(0). After obtaining the log returns, the null is rejected I(1) and variables are confirmed to be stationary. Implied volatility indices based on the stock indices in our dataset are also checked for stationarity, given the nature of our regression analysis this is particularly important. All IV-indices levels and the log returns of IV-indices are confirmed to be I(1) at 5% significant level and therefore stationary. We can go ahead and use the levels in the regression analysis as its ideal.

Table 3: Results with p-values from stationarity test. Hypothesis testing both Augmented Dickey-Fuller and Phillips-Perron test for unit root is: H:0 Non-stationary, H:1 Stationary. We reject the null if the p-value is below 0.05.

	ADF		PP	
Stock indices	Level	% Δ	Level	% Δ
AEX	0.43	0	0.12	0
BOVESPA	0.54	0	0.55	0
CAC 40	0.16	0	0.22	0
DAX 30	0.77	0	0.80	0
EUROSTOXX 50	0.14	0	0.19	0
FTSE 100	0.12	0	0.19	0
S&P 500	0.95	0	0.97	0
SMI	0.72	0	0.77	0
TSX 60	0.48	0	0.46	0
USO	0.66	0	0.67	0
IV Indices				
AEX	0	0	0	0
BOVESPA	0.001	0	0.001	0
CAC 40	0	0	0	0
DAX 30	0	0	0	0
STOXX 50	0	0	0	0
FTSE 100	0	0	0	0
S&P 500	0	0	0	0
SMI	0	0	0	0
TSX 60	0	0	0.001	0

Figure 1 shows clearly that oil price and the S&P 500 are not stationary. We therefore calculate logarithmic returns to correct for this and confirm that the new variables are stationary as seen in Figure 2. Figure 3 shows implied volatility of S&P 500 and DAX 30 indices. Both these plots, as well as formal tests, confirm that implied volatility indices are stationary. Therefore, in our analysis, we utilize directly the closing prices of the implied volatility indices. Moreover, implied volatility has obviously a very strong relation with the distribution of returns.

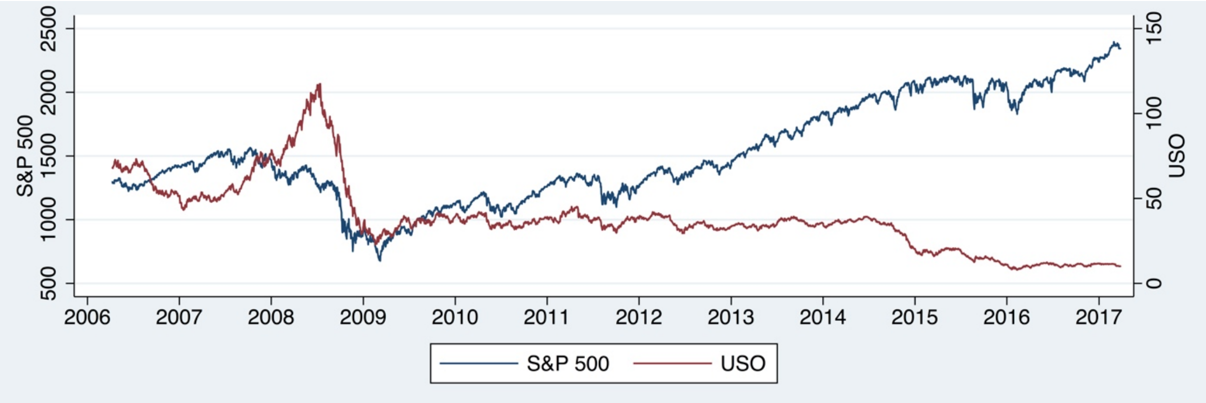


Figure 1: Line plot graph of S&P 500 and USO daily closing prices in the period of the study.

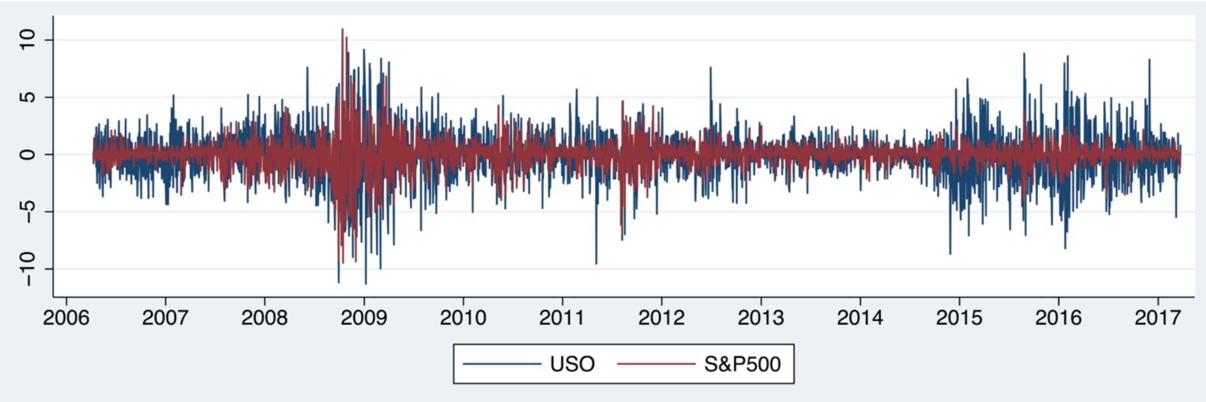


Figure 2: Line plot graph of S&P 500 and USO daily log returns in the period of the study.

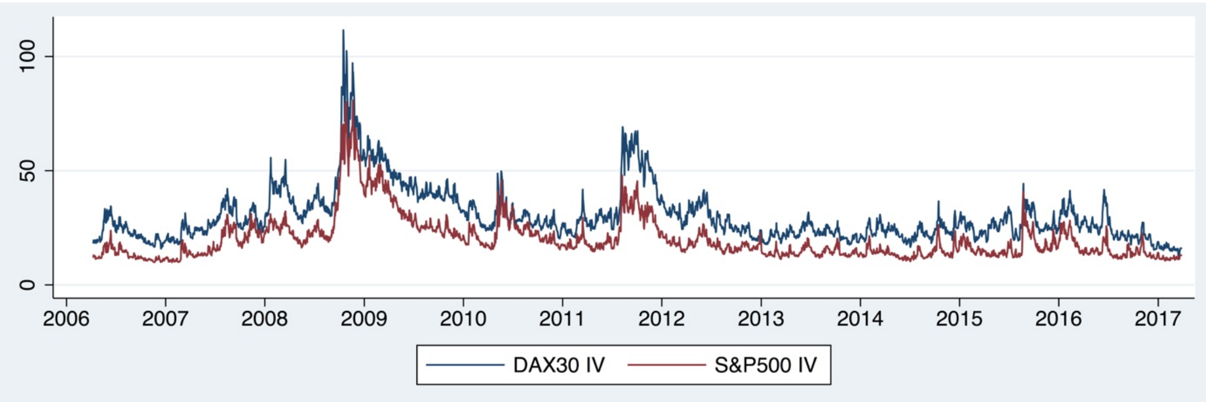


Figure 3: Line plot graph of S&P 500 and DAX 30 implied volatility index with daily closing prices in the period of the study.

2.5 Descriptive statistics

Table 4 and Table 5 show the descriptive statistics of stock market log returns and the implied volatility indices of the underlying stock market which is used in our study. The mean percentage daily log returns are very close to zero for all variables, but the standard deviation is slightly higher for the BOVESPA and USO indicating higher variation in the returns. The higher standard deviation for the BOVESPA index might be explained by the fact that Brazil is one of the BRICS emerging markets and not an OECD country like the rest represented in this study. The country has also suffered from political and economic turmoil in the period of our study (Cruz & Leite, 2015). All the stock indices are negatively skewed indicating a fat left tail on the distributions of returns. This complies with the hypothesis that when the market is bearish, extreme negative returns are more pronounced than the extreme positive returns during the bullish markets. All stock market indices exhibit leptokurtic properties.

Table 4: Descriptive statistics of stock indices log returns.

Index	Mean	Std dev	Max	Min	Skew	Kurtosis	N
AEX	-0.001	1.63	12.32	-11.86	-13.89	11.18	2830
BOVESPA	0.004	2.36	16.86	-17.96	-18.57	9.71	2834
CAC 40	-0.005	1.73	12.14	-11.74	-3.11	9.60	2832
DAX 30	0.021	1.67	12.37	-9.60	-8.32	8.88	2830
STOXX 50	-0.008	1.74	11.97	-11.10	-8.49	8.99	2832
FTSE 100	-0.005	1.52	12.22	-11.51	-23.81	12.37	2828
S&P 500	0.021	1.27	10.96	-9.47	-33.53	13.62	2757
SMI	0.012	1.25	10.02	-7.51	-1.38	8.88	2828
TSX 60	0.003	1.50	9.93	-13.79	-70.60	12.62	2830
USO	-0.070	2.20	9.17	-11.30	-12.56	5.25	2746

Table 5 reports descriptive statistics for the implied volatility indices in levels. From the table we see that the indices have a very high maximum values when compared to the mean and minimum. All indices are positively skewed, most above 1.79, with the exception of the BOVESPA and TSX60 for which the financial crisis of 2008-2009 is not included. Stochastic properties of volatility gives the index jump characteristics when volatility increases. This property is more sudden in tranquil periods when values are low, the jumps to large values are more prominent (Kaeck & Alexander, 2010). The added information of IV indices in a quantile regression will show how changes in expected volatility will impact oil price returns explanatory effect on stock prices.

Table 5: Descriptive statistics of implied volatility indices levels.

IV indices	Mean	Std dev	Max	Min	Skew	Kurtosis	N
AEX	28.93	13.23	109.25	7.42	1.97	7.99	2861
BOVESPA	33.20	9.37	72.83	16.67	0.89	3.59	1574
CAC 40	30.05	11.43	104.63	0.45	1.79	8.01	2861
DAX 30	30.31	11.93	111.57	11.85	2.08	9.05	2861
STOXX 50	30.00	12.35	117.31	11.99	1.71	7.32	2861
FTSE 100	26.05	11.97	100.68	6.61	1.88	8.22	2793
S&P 500	20.00	9.54	80.86	9.89	2.38	10.49	2757
SMI	18.72	7.28	74.89	8.64	2.35	10.90	2861
TSX 60	14.87	4.71	35.37	3.03	1.33	5.34	1681

Table 6 presents the correlations between the stock market returns, oil price returns and implied volatility levels. The results are somewhat similar, most stock indices and the USO are correlated which is expected based on the importance of oil to the macroeconomy and stock market in previous research. The implied volatility indices have a much lower correlation to both stock indices and the USO. With a weak correlation structure between the independent variables and VIF-values around one (see Appendix 6.1), multicollinearity does not present a problem in the analysis. The absence of multicollinearity is vital for an understanding of how the two variable effects the stock market returns. High multicollinearity distorts the standard errors and decreases the significance of the findings in the regression analysis.

Table 6: Correlations matrix of indices, USO and implied volatility as denoted in equation 7.

	USO	IV
AEX	0.4	0
USO	1	-0.02
BOVESPA	0.4	-0.3
USO	1	-0.02
CAC	0.38	0.02
USO	1	-0.02
DAX	0.37	0.02
USO	1	-0.02
STOXX	0.37	0.03
USO	1	-0.01
FTSE	0.4	0.02
USO	1	-0.02
S&P	0.43	0.03
USO	1	-0.02
SMI	0.32	0
USO	1	-0.01
TSX	0.57	0.03
USO	1	0.05

3. Methodology

In this paper, we model the co-movement between stock markets, oil price and implied volatility. To analyze this relationship we employ quantile regression (Koenker & Bassett, 1978). This approach will allow us to look at the co-movement for different stock market situations (bearish/bullish). First, in our analysis, we estimate ordinary least squares (OLS) regression for use as a benchmark comparison with the quantile regression models (QRM). The OLS estimates the conditional mean of the independent variable and its effect on the dependent variable, whereas the QRM estimates the conditional quantiles of the dependent variable (Koenker & Hallock, 2001). The advantage of this approach is its capacity to permit estimation of various quantile functions in a conditional distribution. Investigation of various quantile regressions is particularly useful when the conditional distribution is heterogeneous (Tsai, 2012).

Additionally, we utilize the framework for conditional value at risk (CoVaR) developed by Adrian and Brunnermeier (2016). This approach allows us to calculate the value at risk in stock indices conditional on bearish and bullish returns in oil price (0.05 and 0.95 quantile returns) thus furthering our understanding of the relationship between stock markets and oil price.

3.1 Ordinary Least Squares

First, we utilize ordinary least squares regression to investigate the relationship between oil price and equity indices and to obtain a benchmark for a comparison with the quantile regressions. The simple linear regression models we utilize can be represented by the equation:

$$r_{i,t} = \alpha_i + \beta_1 r_{USO,t} + \varepsilon_{i,t} \quad (6)$$

Where $r_{i,t}$ is the log returns of stock market index i at time t, α_i is the intercept and β_1 is the coefficient for the oil price log returns r_{USO} at time t and $\varepsilon_{i,t}$ is the error term.

We extend the simple linear regression by adding a second explanatory variable, the implied volatility of the stock market which was used as the dependent variable. This is to control for the impact of implied volatility of the stock market on the returns of this stock market.

$$r_{i,t} = \alpha_i + \beta_1 r_{USO,t} + \beta_2 IV_{i,t-1} + \varepsilon_{i,t} \quad (7)$$

Equation 7 is similar to the simple linear regression in equation 6 with the addition of the β_2 as the coefficient of implied volatility at time t-1. The reasoning why stock market and oil markets are included with the same time index t, whereas the implied volatility is included with the index t-1, is very similar to the reasoning in Adrian and Brunnermeier (2016). We are interested in the contemporary relationship between the stock market return and oil return. Therefore, both of these indices are included with the same time index. In addition, we want to investigate this relationship conditional on any information available at time t-1, in our case implied volatility at time t-1. In other words, we are trying to answer the question what is the relationship between the stock market return and oil return tomorrow given what we know today (in particular the implied volatility today).

This is particularly important when examining VaR later in the analysis, where the IV indices serve as a state variable capturing the risk. The VaR denotes the loss level for the stock index which we expect to exceed with the probability $\tau \in (0,1)$. Measuring loss at time t given what we know such that $\Pr(r_t < VaR_t | \Omega_{t-1}) = \tau\%$ where Ω_{t-1} denotes the information set available at time $t-1$ (Haugom, Ray, Ullrich, Veka, & Westgaard, 2016).

3.2 Quantile Regression

We utilize quantile regression introduced by Koenker and Bassett (1978). This will let us look at the structure of dependency across quantiles between equity indices and oil price. This approach will extend our picture to a more accurate and complete understanding of the variables joint distribution. The approach will also be far more robust to leptokurtosis, non-normality and skewness than OLS (Davino, Furno, & Vistocco, 2014).

Just as the sample mean is defined as the solution to the problem of minimizing a sum of squared residuals, the sample median (0.5 quantile) is defined as the solution to minimizing a sum of absolute residuals. The symmetry of the piecewise linear absolute value function implies that the minimization of the sum of absolute residuals must equate to the number of positive and negative residuals, assuring that there are the same number of observations above and below the median. Since the symmetry of the absolute value yields the median, minimizing a sum of asymmetrically weighted absolute residuals, giving different weights to positive and negative residuals will yield the quantiles (Koenker & Hallock, 2001). We use this to look at the co-movement of oil price and equity markets by estimating regression coefficients for the different quantiles of the return distribution of the equity indices. Of special interest here, is the relationship in the tails of the distribution, the lower and higher quantiles.

The simple quantile regression in our analysis:

$$r_{i,t}^{(\tau)} = \alpha_i^{(\tau)} + \beta_1^{(\tau)} r_{USO,t} + \varepsilon_{i,t}^{(\tau)} \tag{8}$$

The multiple quantile regression in our analysis:

$$r_{i,t}^{(\tau)} = \alpha_i^{(\tau)} + \beta_1^{(\tau)} r_{USO,t} + \beta_2^{(\tau)} VIX_{i,t-1} + \varepsilon_{i,t}^{(\tau)} \tag{9}$$

Where $r_{i,t}^{(\tau)}$ is the log return of stock index *i* at time *t* for the τ-th quantile, $\alpha_i^{(\tau)}$ is the intercept for the τ-th quantile, $\beta_1^{(\tau)} r_{USO,t}$ is the coefficient of co-movement with the oil price at time *t* for the τ-th quantile and $\beta_2^{(\tau)} VIX_{i,t-1}$ is the coefficient for the implied volatility at time *t-1* for the τ-th quantile and $\varepsilon_{i,t}^{(\tau)}$ is the error term for the τ-th quantile.

These equations can be estimated for the quantile range $\tau \in (0,1)$ to provide an image of the structure of dependence in different market situations (bull/bear).

3.3 Conditional Value at Risk

Value at risk (VaR) is the particular conditional quantile on the distribution of stock market returns i and can be used as a risk measure of the loss level that is expected to be exceeded with a probability $\tau \in (0,1)$, if the portfolio is held at a given time period (Alexander, 2008).

$$\Pr\left(r_{i,t} \leq VaR_{i,t}^{(\tau)}\right) = \tau\% \tag{10}$$

In this paper, we focus on the co-movement of stock markets and oil price, thus we want to analyze the VaR of stock market returns taking into account the oil price and its effect on stock markets. Adrian and Brunnermeier (2016) proposed a measure of systematic risk, ΔCoVaR defined as the change in the VaR of the financial system conditional on an institution being under distress relative to its median state. We use their methodology in a model where we measure the systematic risk of a stock index. Instead of defining ΔCoVaR as the change in the VaR of a financial institution we measure the stock market risk conditional on the oil market being under distress relative to its median state. Extending the VaR, we can rewrite it to CoVaR where the "co" means the VaR of the stock market i loss level is conditional on some potential bad event $C(\cdot)$ occurring in the oil price $i|C(r_{USO})$ with the probability $\tau\%$. Formally defined as:

$$\Pr\left(r_{i,t} \leq CoVaR_{i|C(r_{USO,t})}^{(\tau)} \middle| C(r_{USO})\right) = \tau\% \tag{11}$$

We use quantile regression to calculate the CoVaR because of its efficient use of data, we can calculate both a time-varying variant and CoVaR constant over time. The constant over time-variant is simply the predicted values from the quantile regression of the stock market $r_i^{(\tau)}$ on the losses of oil price $r_{USO}^{(\tau)}$ for the $\tau\%$ quantile.

$$r_{i|USO}^{(\tau)} = \hat{\alpha}^{(\tau)} + \hat{\beta}_{USO}^{(\tau)} r_{USO} \tag{12}$$

Where $r_{i|USO}^{(\tau)}$ is the predicted value for a $\tau\%$-quantile of the stock market conditional on the oil price returns r_{USO}. CoVaR is therefore the predicted values from the quantile regression of stock market return losses on the losses of oil price returns, which gives the value at risk for the stock market conditional on the oil price.

$$CoVaR_{i|USO}^{(\tau)} = VaR_{i|USO}^{(\tau)} = \alpha^{(\tau)} + \hat{\beta}_{USO}^{(\tau)} VaR_{USO}^{(\tau)} \tag{13}$$

VaR_{USO} is simply the τ%-quantile of oil price. Thus, we can find the oil price contribution to the constant $CoVaR$ by calculating:

$$\Delta CoVaR_{USO}^{(\tau)} = \hat{\beta}_{USO}^{(\tau)}(VaR_{i|USO}^{(\tau)} - VaR_{i|USO}^{(median)}) \qquad (14)$$

In our analysis, we focus on the stock markets time-varying CoVaR and the oil price contribution to the CoVaR. To capture this time-varying joint distribution of stock market and oil price we calculate VaR and $\Delta CoVaR$ as a function of a state variable. We use the implied volatility as the state variable to captures the daily changing risk expectation in the market. We add the subscript t to indicate the time varying $CoVaR_{i|USO,t}^{(\tau)}$ and $VaR_{USO,t}^{(\tau)}$ and calculate the time variations conditional on the state variable IV_{t-1}.

We start with calculating the quantile regression for:

$$r_{USO,t}^{(\tau)} = \alpha_{USO}^{(\tau)} + \gamma_{USO}^{(\tau)} IV_{t-1} + \varepsilon_{USO,t}^{(\tau)} \qquad (15)$$

$$r_{i,t}^{(\tau)} = \alpha_{i|USO}^{(\tau)} + \beta_{i|USO}^{(\tau)} r_{USO,t} + \gamma_{i|USO}^{(\tau)} IV_{t-1} + \varepsilon_{i|USO,t}^{(\tau)} \qquad (16)$$

Then the predicted values from Equation 15 and 16 are used to obtain:

$$VaR_{USO,t}^{(\tau)} = \hat{\alpha}_{USO}^{(\tau)} + \hat{\gamma}_{USO}^{(\tau)} IV_{t-1} \qquad (17)$$

$$CoVaR_{i|USO,t}^{(\tau)} = \hat{\alpha}_{i|USO} + \hat{\beta}_{i|USO}^{(\tau)} VaR_{USO,t} + \hat{\gamma}_{i|USO}^{(\tau)} IV_{t-1} \qquad (18)$$

Last, we calculate the oil price contribution

$$\Delta CoVaR_{USO,t}^{(\tau)} = CoVaR_{i|r_{USO}=VaR_{USO}^{(\tau)}}^{(\tau)} - CoVaR_{i|r_{USO}=VaR_{USO}^{(Median)}}^{(\tau)} \qquad (19)$$

$$= \hat{\beta}_{i|USO}(VaR_{USO,t}^{(\tau)} - VaR_{USO,t}^{(Median)}) \qquad (20)$$

ΔCoVaR calculated here is the change in the VaR of the stock market conditional on oil price being under distress (returns in the 0.05 or 0.95 quantile) relative to its median state.

4. Results

4.1 Ordinary Least Squares Regression

The results from the simple OLS are summarized in Table 7 and multiple OLS in Table 8. The coefficients represent the relationship between the variables in the nine markets, in both tables, the coefficient (β_1) for oil price returns is always positive and significant. In addition, the multiple regression coefficient (β_2) is not significant and very low.

Table 7: Simple ordinary least squares regression results of Equation 6:
$r_{i,t} = \alpha_i + \beta_1 r_{USO,t} + \varepsilon_{i,t}$.

	α	β_1	R^2
AEX	0.02	0.29***	0.16
BOVESPA	0.04	0.47***	0.19
CAC 40	0.01	0.30***	0.14
DAX 30	0.04	0.28***	0.13
EUROSTOXX 50	0.01	0.30***	0.14
FTSE 100	0.01	0.28***	0.16
S&P 500	0.04	0.25***	0.18
SMI	0.02	0.18***	0.10
TSX 60	0.03	0.39***	0.32

*** 1% significant level, ** 5% significant level and * 10% significant level.*

Table 8: Multiple ordinary least squares regressions results of Equation 7:
$$r_{i,t} = \alpha_i + \beta_1 r_{USO,t} + \beta_2 IV_{i,t-1} + \varepsilon_{i,t}.$$

	α	β1	β2	R^2
AEX	-0.02	0.29***	0.001	0.16
BOVESPA	-0.18	0.40***	0.005	0.16
CAC 40	-0.12	0.30***	0.005	0.14
DAX 30	-0.07	0.28***	0.004	0.14
EUROSTOXX 50	-0.15	0.30***	0.005	0.14
FTSE 100	-0.07	0.28***	0.003	0.16
S&P 500	-0.06	0.25***	0.005	0.18
SMI	0.01	0.18***	0	0.10
TSX 60	0.01	0.31***	0	0.32

**** 1% significant level, ** 5% significant level and * 10% significant level.*

This means that for all markets investigated there is positive and significant co-movement with oil prices. Supporting the hypothesis that increased oil prices indicate increased economic activity. We see that the coefficients of the BOVESPA and TSX60, decrease somewhat from the single to the multiple regression.

In the multiple regression, none of the coefficients for the implied volatility are significant. This indicates that the implied volatility of the underlying stock market has none to a minor effect on average returns of equity indices. R^2 increases just slightly when implied volatility is included in the regression, this indicates the explanatory power of implied volatility on expected average return is very small. These results are similar to Agbeyegbe (2015a) who implemented quantile regression to analyze the return-volatility relationship between oil price returns and its implied volatility and found it to be asymmetric. These OLS regressions are used as a benchmark comparison and next, we estimate the quantile regression models to further investigate the relationship between oil and equity in different market conditions.

4.2 Simple Quantile Regression

Figure 4 shows the results from the simple quantile regression. The graphs plot the intercept and coefficient for equation 8 across the 0.05 – 0.95 quantile range. The green full line is the quantile regression result and the gray area its corresponding 95% confidence interval, the thick dotted line is the OLS regression with its 95% confidence interval marked by the thin dotted lines above and below.

From Figure 4, we see that the impact of oil price returns on the AEX is positive and significant for all quantiles of the distribution. Furthermore, the relationship strengthens for the lower and upper quantiles resulting in a U-shaped dependency structure across quantiles. The clear increase in co-movement seen for the 0.05 and the 0.95 quantile, means oil price has the largest impact on this index when this stock market is "bearish" or "bullish". For the BOVESPA we see that the impact of oil price on the index is positive and significant across all quantiles. Unlike the AEX, the BOVESPA has a very even dependency structure across quantiles situated around the OLS line, with a slight increase in the lower- and uppermost quantiles. Brazil produced more than 3.2 million barrels of oil per day in 2016 ("U.S. Energy Information Administration," 2017) and has the highest coefficients for co-movement across all quantiles in our sample. Continuing with the CAC40, we again find that the impact of the oil price is positive and significant for all quantiles of the distribution. Looking at the dependency structure we find a U-shape matching that of the AEX as well as a further similarity in the fact that they both have the highest co-movement (with the oil price) for the 0.05 quantile. For the DAX30 we also observe a positive and significant impact of the oil price for all quantiles. The structure of dependency is U-shaped and similar to that of the AEX and CAC40. Moreover, the DAX30 also has the highest co-movement for the lower quantiles. The trend of positive and significant impact across all quantiles continues with the EUROSTOXX50. The U-shaped dependency structure and peak in co-movement for the lowest quantiles is also similar to that of other European indices. For the FTSE100, we identify a positive and significant influence from the oil price. Additionally, we find a U-shaped dependency structure across quantiles. Like most other indices in the sample the highest co-movement is found in the lowermost quantiles. The SMI continues the trend of positive and significant impact from oil, however in contrast to other European indices it does not display a clear rise in co-movement toward the upper quantiles. Like the other European indices however, co-movement is highest for "bearish" market behavior. For the S&P 500 the impact of the oil price is also positive and significant for all quantiles. The dependency structure is U-shaped across the distribution and similarly to the European markets the highest co-movement is found in the lowermost quantiles. Finally, we have the TSX60, in accordance with the other indices in our sample we find the oil price has a positive and significant impact across all quantiles. In contrast to most other indices in the sample however the structure of the dependency across quantiles does not form a clear U-shaped pattern, coefficients are quite even across all quantiles except the lower- and uppermost where they display a slight increase.

We see a clear tendency in our results of co-movement between oil and equity indices rising toward the lower- and uppermost quantiles of the return distribution. Additionally, there is a clear U-shaped pattern in the dependency structure across quantiles for the indices AEX, CAC40, DAX30, EUROSTOXX50, FTSE100 and S&P 500. These indices display a very similar relationship in both the shape and magnitude of dependency toward oil prices with a rise in co-movement when stock markets are decidedly bearish or bullish. Furthermore, all these indices display the highest coefficients in the lowermost quantiles, this is also true for the SMI. This index does not display the same rise toward upper quantiles as the others do. For the final two indices, the BOVESPA and TSX60 no such U-shaped relationship is discernable, though they both display increasing coefficients in the lower and upper quantiles, the coefficients for the intermediate quantiles lie evenly distributed along the OLS for these two indices. Both of these indices use a shorter window of data because of the limited availability of implied volatility data for use in the next regression equation. However, increasing the timespan for these two does not give them a clear U-shape, and decreasing the timespan for other indices in the sample does not remove their U-shaped pattern. Both Canada and Brazil are large exporters of oil, something that might explain their high and evenly distributed co-movement with oil ("U.S. Energy Information Administration," 2017)

In short, all indices except the BOVESPA, TSX60, and SMI display a rise in co-movement with oil price toward the lower and upper quantiles of the return distribution with the very highest coefficients being found in the lowermost quantiles. This means the co-movement with oil is decidedly higher for these indices when the stock markets are bearish or bullish. For these indices, a 1% change in the price of oil is associated with a change in the stock index of between 0.32% and 0.35% in the 0.05 quantile. For the 0.95 quantile, the same change would be associated with a change of between 0.26% and 0.31%. For the BOVESPA a 1% change in the oil price would indicate a change of between 0.40% and 0.45% for the entire quantile range, for the TSX60 the same numbers are 0.30% - 0.35%. The impact of oil price returns is positive and significant across all quantiles for all indices

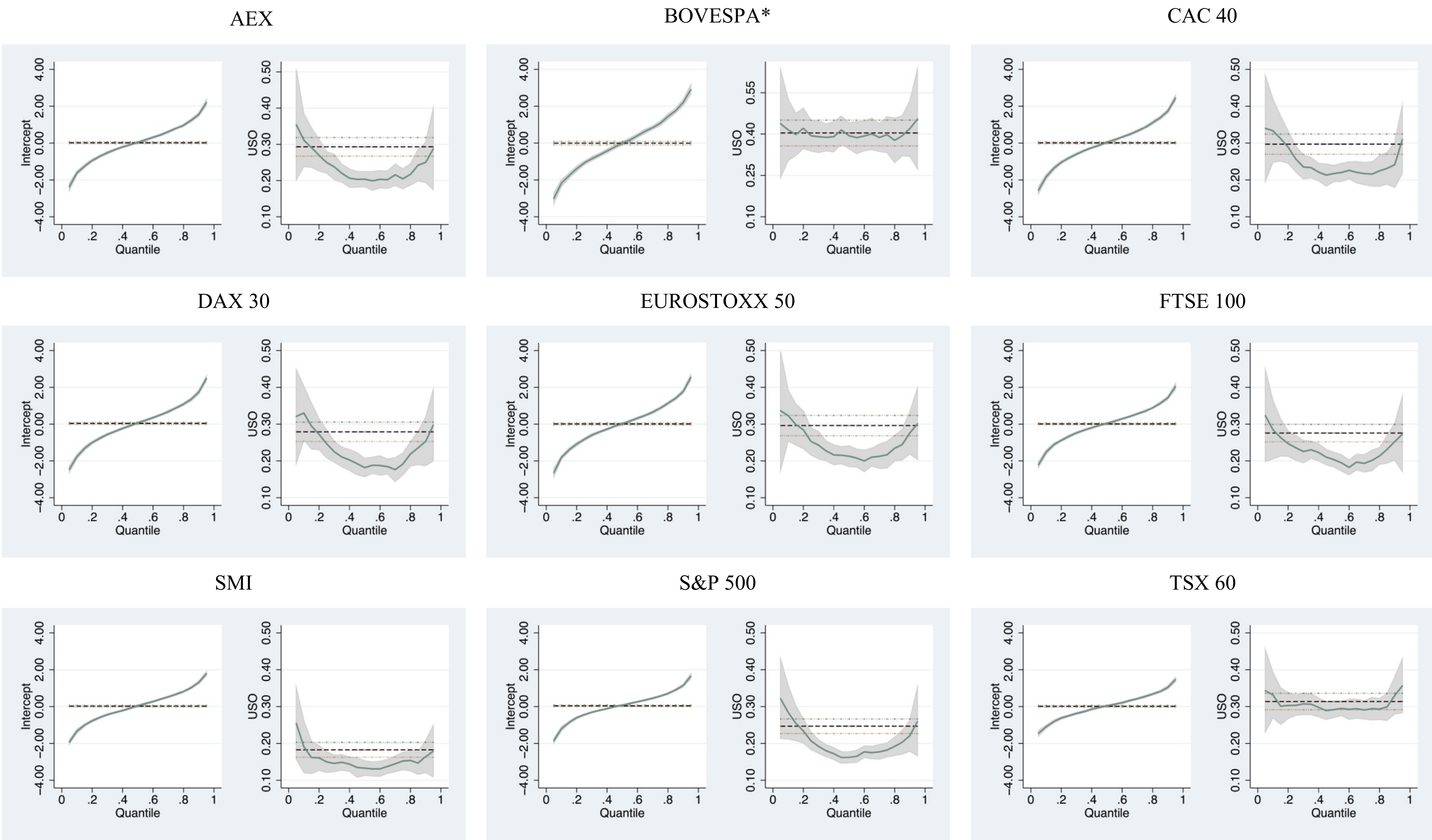


Figure 4: Graphical illustration of Equation 8: $r_{i,t}^{(\tau)} = \alpha_i^{(\tau)} + \beta_1^{(\tau)} r_{USO,t} + \varepsilon_{i,t}^{(\tau)}$*. USO log returns as the independent and stock index log returns as a dependent variable in quantile regression. Coefficient (*β_1*) on Y-axis and quantiles (*τ*) on X-axis. *Note: Different coefficient scale.*

4.3 Multiple Quantile Regression

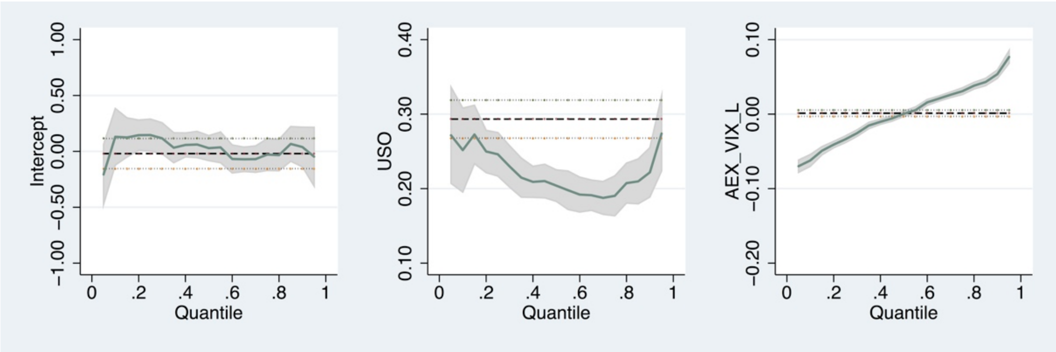
Figure 5 shows results from the multiple quantile regression. The graphs plot the coefficients for Equation 9 across the quantiles 0.05 - 0.95. Again, the green full line plots the quantile regression coefficients and the gray area is its corresponding 95% confidence interval. The dotted line represents the OLS estimate and the small dotted lines its 95% confidence interval.

The impact of the oil price on AEX returns is positive and significant across all quantiles, with the highest coefficients found in the tails of the distribution. The U-shape found in the simple quantile regression is still present after the introduction of implied volatility as a second independent variable. The BOVESPA displays quite even dependence with the oil price across quantiles before having a slight increase in the 0.95 quantile. All coefficients are positive and significant across all quantiles, hovering around 0.4, meaning there is little change from the simple quantile regression. For the CAC40 the dependence structure with the oil price is positive across all quantiles, the relationship is strongest in the lower quantile (0.05) meaning this index has the highest co-movement with oil price for bearish market situations. Continuing with the DAX30 we see a relationship that is very similar to that of the AEX. The coefficients are positive and significant across all quantiles with the highest co-movement found in the tails of the distribution. The DAX30 has kept its U-shaped relationship found in the simple quantile regression, meaning it displays increased co-movement in both bearish and bullish markets. Next, the EUROSTOXX50 also displays a positive dependence with the oil price across all quantiles, with the highest co-movement being found in the lower tail of the distribution. The FTSE100 displays a structure of dependence similar to the EUROSTOXX50. Coefficients are positive and significant across the entire distribution with the highest co-movement found in the lower tail. Similarly, the SMI has a positive and significant dependence structure across all quantiles. Co-movement with oil price is very flat across the quantiles with a slight increase in the lower tail of the return distribution. Continuing with the S&P500 coefficients are positive and significant across all quantiles with the highest co-movement found in the lowermost quantile (0.05) and then decreasing across the distribution toward the 0.95 quantile. The TSX60 has a positive and significant dependence structure across the distribution with the highest co-movement found in the lower quantiles and decreasing slightly toward the uppermost quantiles.

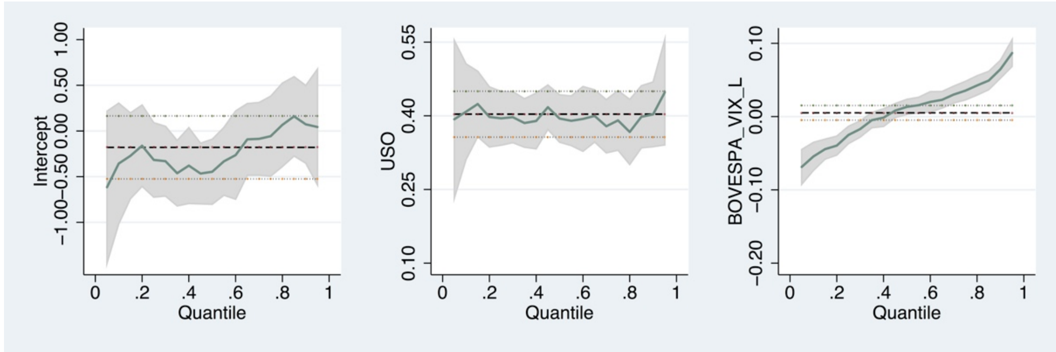
Compared to the results of the simple quantile regression, the only indices that keep a clear U-shaped dependency structure are the AEX and the DAX30, while it is to a limited extent discernable in the EUROSTOXX50. The CAC40, FTSE100, SMI, S&P500 and TSX60 display similar patterns of dependency with the highest co-movement found in the lowermost quantiles and decreasing steadily toward the uppermost quantiles of the distribution. Hence, these stock markets co-move with the oil price most strongly when the economy is “bearish”. In common for the indices AEX, CAC40, DAX30, EUROSTOXX50, FTSE100, S&P500 and the SMI is the fact that for all or most of the quantile range 0.05-0.95 the OLS regression will overestimate the relationship with the oil price. For the TSX60 and BOVESPA, however, the OLS regression would be a good approximation across most quantiles. The BOVESPA and the TSX60 also have the highest coefficients for co-movement with the oil price, which is logical when taking into consideration the fact that these two economies are large net exporters of oil ("U.S. Energy Information Administration," 2017).

In summary, we find a positive and significant dependence structure across all quantiles for all indices. We find that there is significant co-movement between equity indices and oil price, especially in bear markets. The clear U-shaped pattern of co-movement, however, disappears for most indices except the AEX and DAX30 when the implied volatility is added to the regression as a control variable. This indicates that for most indices the U-shape is not real but a consequence of not controlling for implied volatility. In addition, all indices except the BOVESPA and TSX60 have notably lower coefficients of co-movement with oil price in the tails of the distribution, when implied volatility is taken into consideration. The impact of the implied volatility is appreciably uniform throughout all indices and most of the variation in quantiles can be explained by the implied volatility.

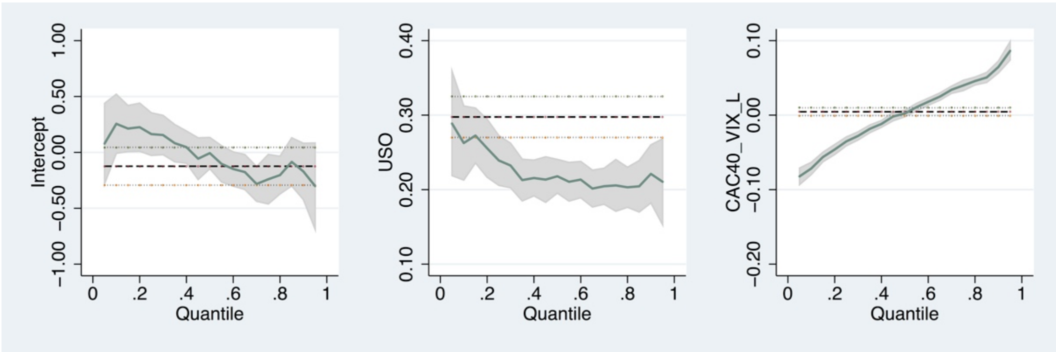
AEX



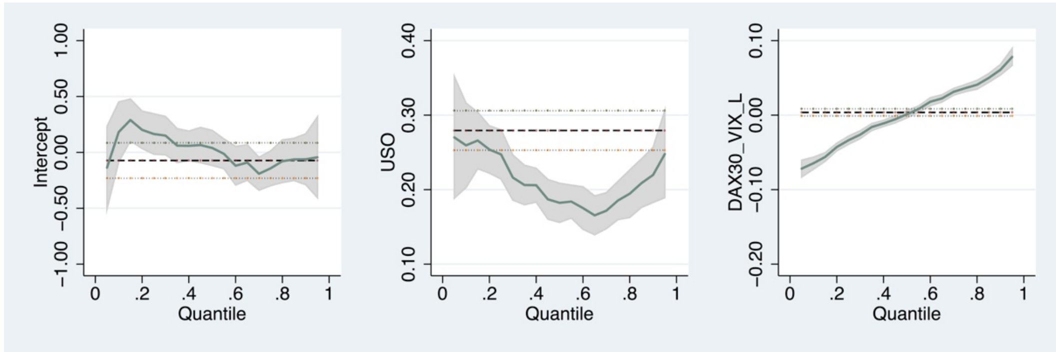
BOVESPA*



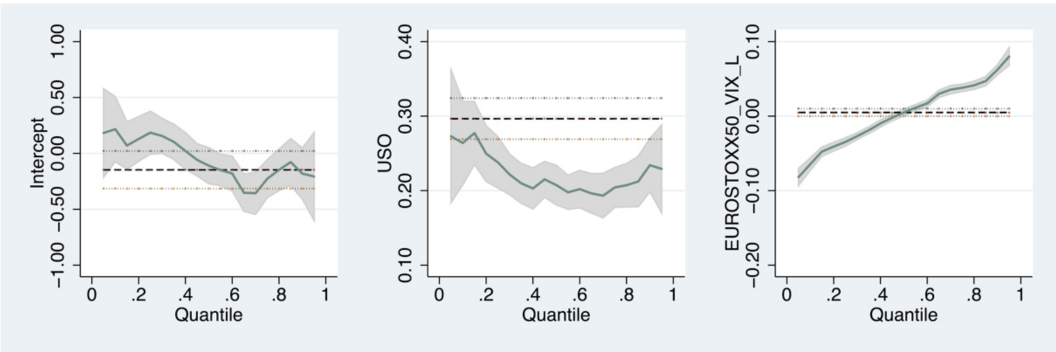
CAC 40



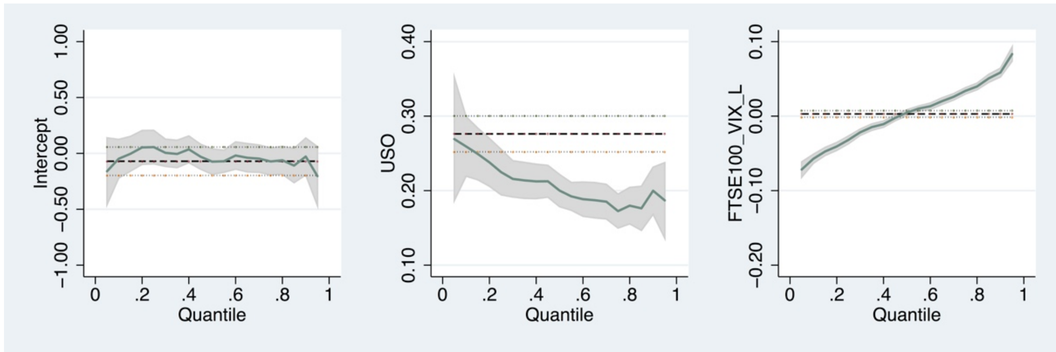
DAX 30



EUROSTOXX 50



FTSE 100



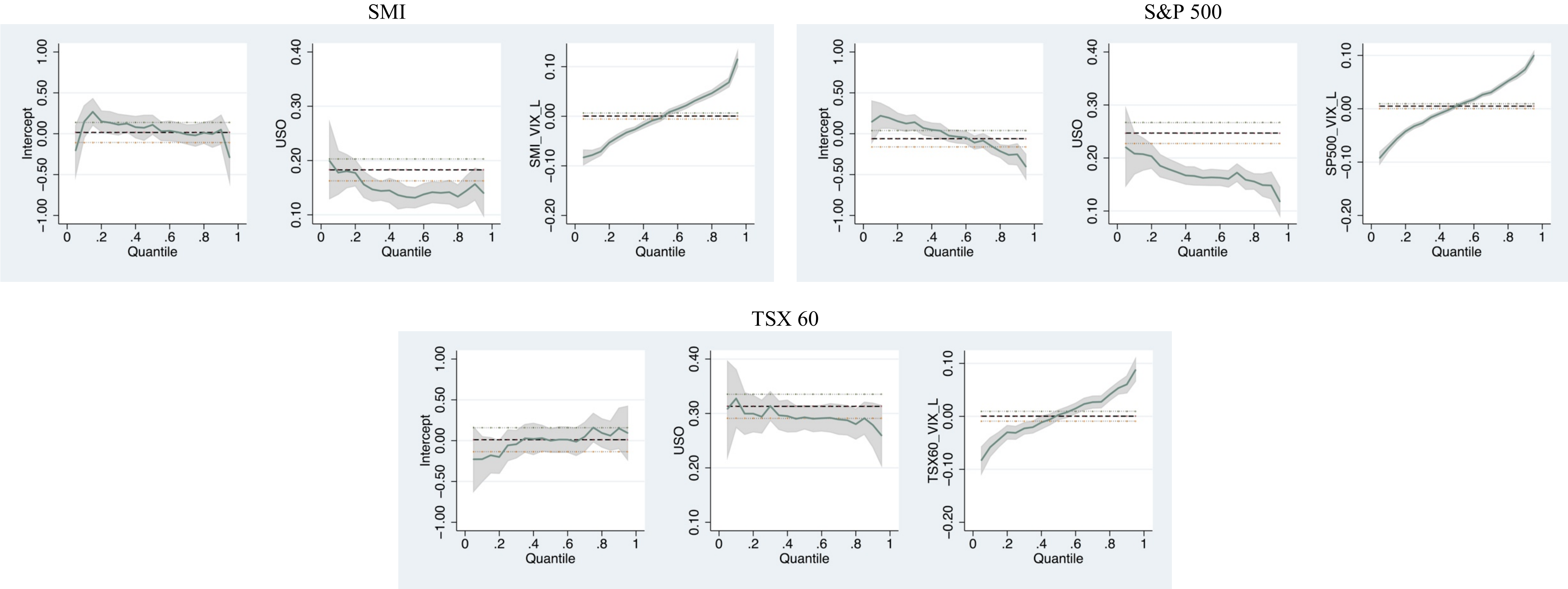


Figure 5: Graphical illustration of Equation 9: $r_{i,t}^{(\tau)} = \alpha_i^{(\tau)} + \beta_1^{(\tau)} r_{USO,t} + \beta_2^{(\tau)} VIX_{i,t-1} + \varepsilon_{i,t}^{(\tau)}$*. Oil price returns, implied volatility levels as independent variable and stock index returns as dependent variable in quantile regression. Coefficient (β_l) on Y-axis and quantiles (τ) on X-axis. *Note: Different scale on USO coefficient for BOVESPA.*

4.4 Conditional Value at Risk

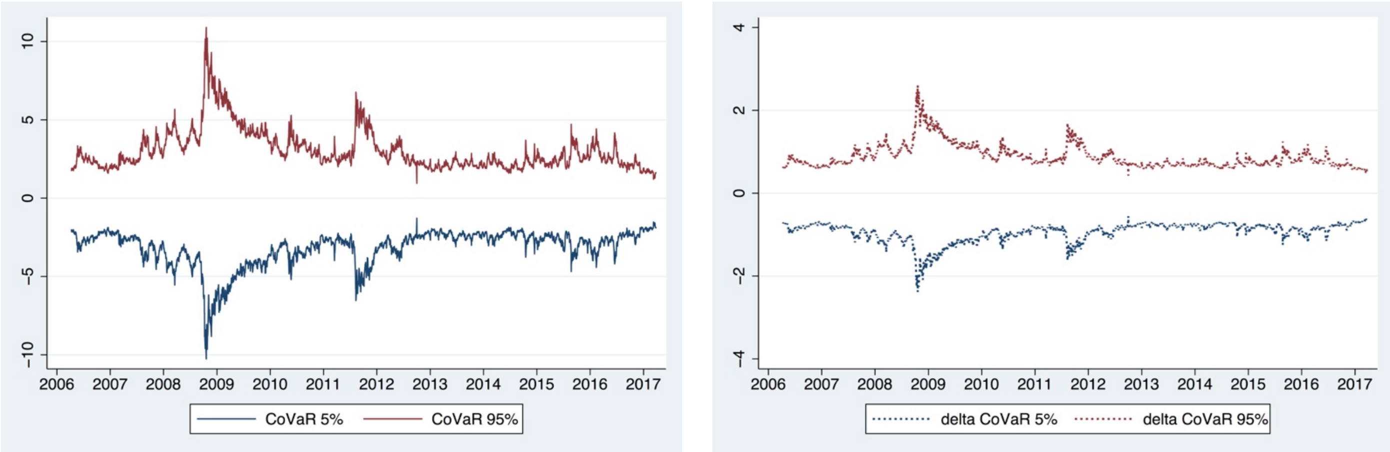
In Figure 6 the CoVaR is on the left and the ΔCoVaR is on the right, the top line shows the rate of loss at 95% quantiles for a short position in the market and the bottom line shows the rate of loss for 5% quantiles for a long position in the market. Both CoVaR and ΔCoVaR follow the state variable very closely, this is because the extent to which the time-series of CoVaR of stock market differs from the oil price VaR time-series depends on the magnitude of the state variable coefficient γ (Girandi, 2013). We observe that in times of financial distress the value at risk increases. Both the financial crisis of 2008/2009 and the debt problems in Europe in 2011 are the two largest spikes in systematic risk. We can also observe that the CoVaR is very similar across the stock indices in our sample. Since CoVaR follows the implied volatility so closely, it also holds the same mean-reverting properties. For all stock indices, the mean loss rate for 5% CoVaR is slightly higher than the mean loss rate for 95% CoVaR. Except for the BOVESPA index, where the mean loss rate is higher for the 95% CoVaR, which can be interpreted as higher value at risk for a short position than for a long position. Mean percentage CoVaR is around 2.5% to 3.5% depending on the different indices for the 5% value at risk. In the 95% CoVaR S&P 500 is the only index that's lower than 2%, with the rest ranging from 2.16% for the TSX 60 up to 3.1% for the EUROSTOXX 50. BOVESPA deviates somewhat from the rest with a higher CoVaR at 4.21% for 5% and 4.22% at 95%.

The oil price contribution to CoVaR, ΔCoVaR is asymmetric in magnitude across indices. This is no surprise as most indices display an asymmetric dependency structure across quantiles. Since the estimated coefficients in the lower quantile are higher, the contribution to CoVaR from oil price VaR is much stronger in the 5%, than the 95% CoVaR. The AEX and DAX30 (U-shaped dependency structure) have symmetrically distributed co-movement with oil and this is reflected in the ΔCoVaR for these two indices being very similar for both the 5% and 95% level. The BOVESPA has steady high co-movement across quantiles with an increase for the uppermost quantiles. Remember that oil is an important factor in the Brazilian economy as they are the world's 9th largest oil exporter (U.S. Energy Information Administration, 2017).The BOVESPA has the highest maximum ΔCoVaR in the 95% level of all indices and the highest mean ΔCoVaR in our study for both the 5% and 95% level over time. The rest of the indices in the sample have asymmetrically distributed coefficients of co-movement with oil, their co-movement being highest for bearish markets, this results in the 5% ΔCoVaR being higher than the 95%.

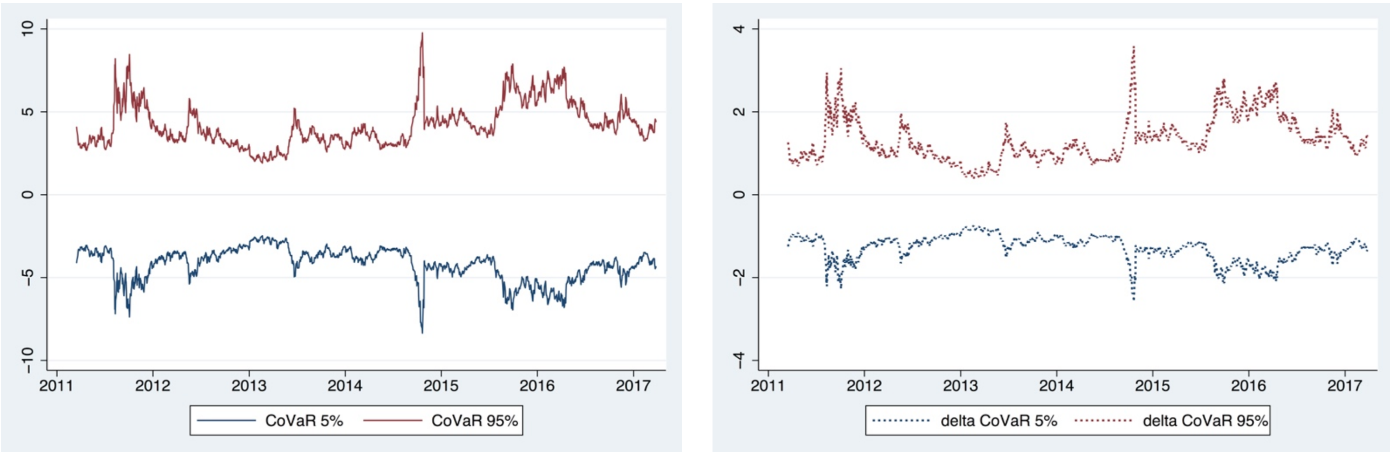
We see that the BOVESPA stands out when it comes to the magnitude of ΔCoVaR. This is in accordance with results reported in Table 9. The limited period covered by this index means it has not been calculated for the 2008-2009 financial crisis when contribution from oil price bear/bull events peak for all other indices, however, it still displays the highest mean ΔCoVaR for both 5% and 95% in mean as well as the highest peak in the 95% and among the highest in the 5%. This suggests the value at risk in the BOVESPA is highly exposed to oil price return tail events. The indices AEX, CAC40, DAX30, EUROSTOXX50, FTSE100 and TSX60 all have a 5%ΔCoVaR mean around 1%. The SMI and S&P500 stand out as the ones least influenced by oil price lower tail events with a mean 5%ΔCoVaR of just 0.7% and 0.76% respectively. In the 95% ΔCoVaR we find that the mean contribution to CoVaR for the indices AEX, CAC40, DAX30, EUROSTOXX50, FTSE100 and TSX60 are between 0.6% and 0.88%. The SMI and S&P500 again stand out as the two indices whose CoVaR is least influenced by oil price events with a 95% ΔCoVaR of 0.48% and 0.38% respectively.

In summary, we find that there is a difference between the indices, when it comes to the 5% and 95% ΔCoVaR some are more affected than others, and some indices are affected at different magnitudes by bearish and bullish oil price returns. This is an important insight that can help investors manage their risk exposure.

AEX

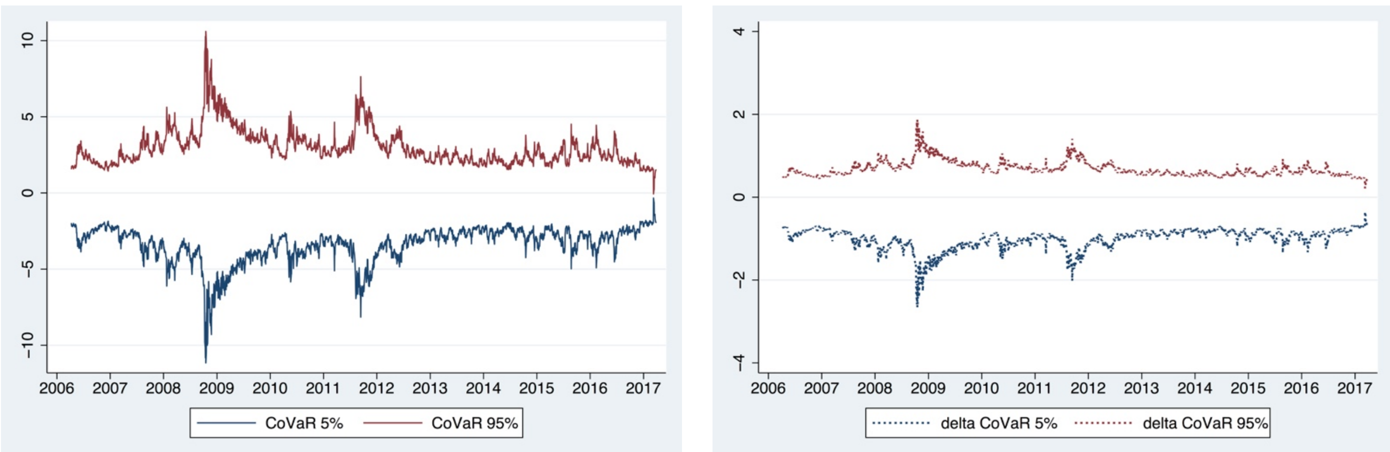


BOVESPA

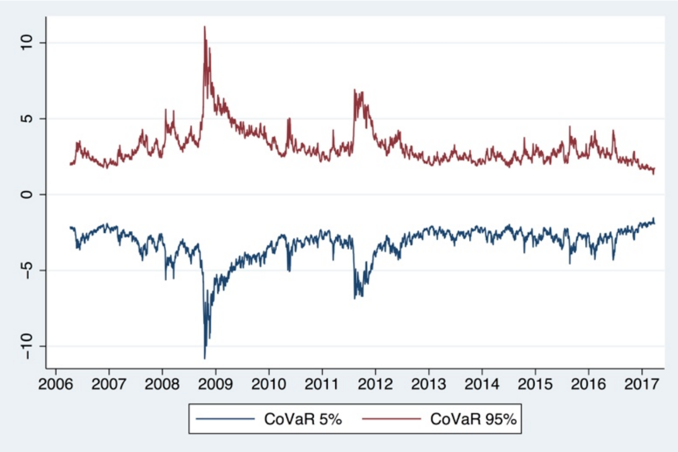


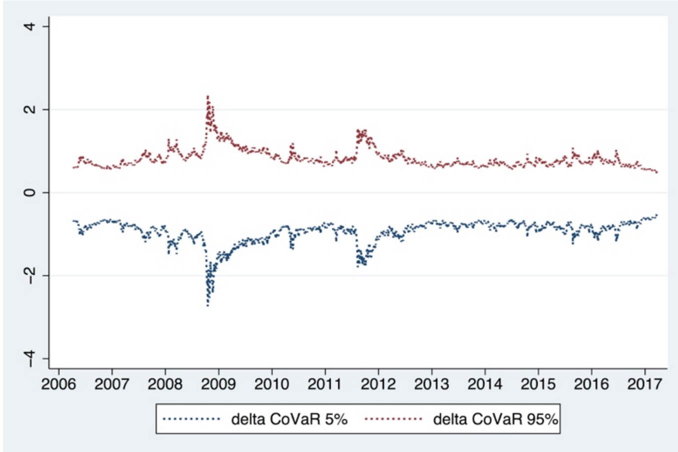
*Note: Shorter time period

CAC 40

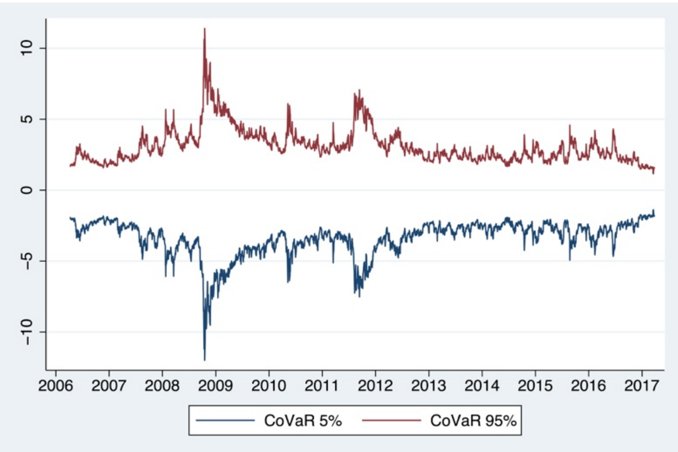


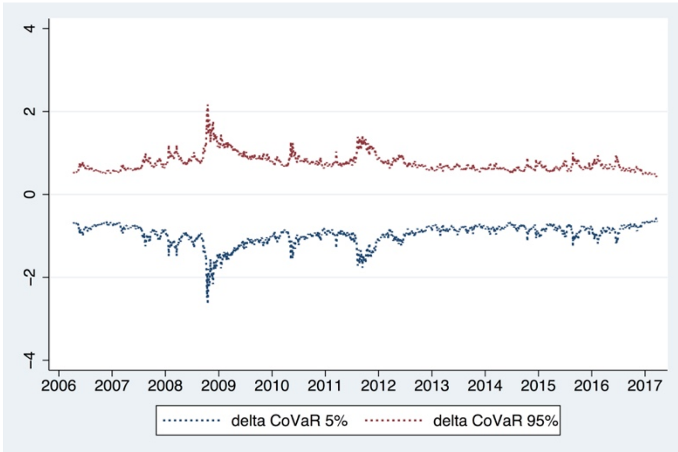
DAX 30



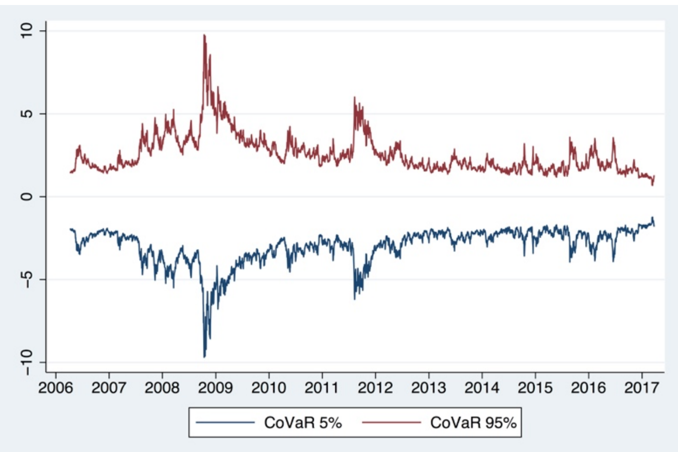


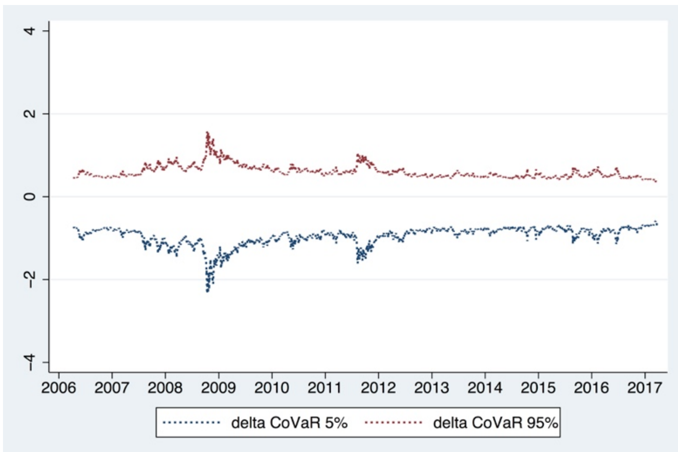
EUROSTOXX 50



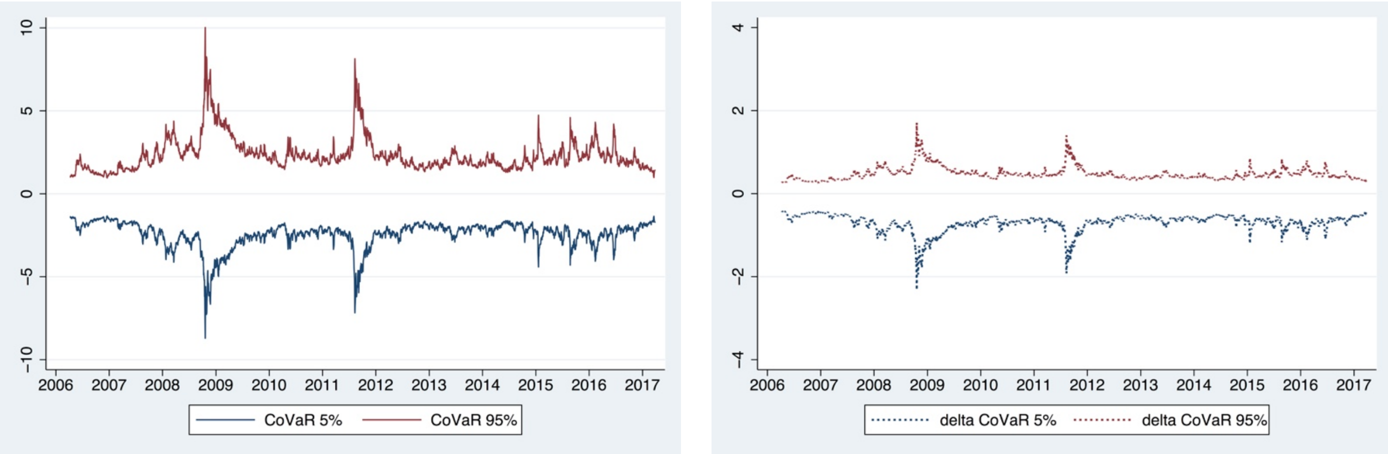


FTSE 100

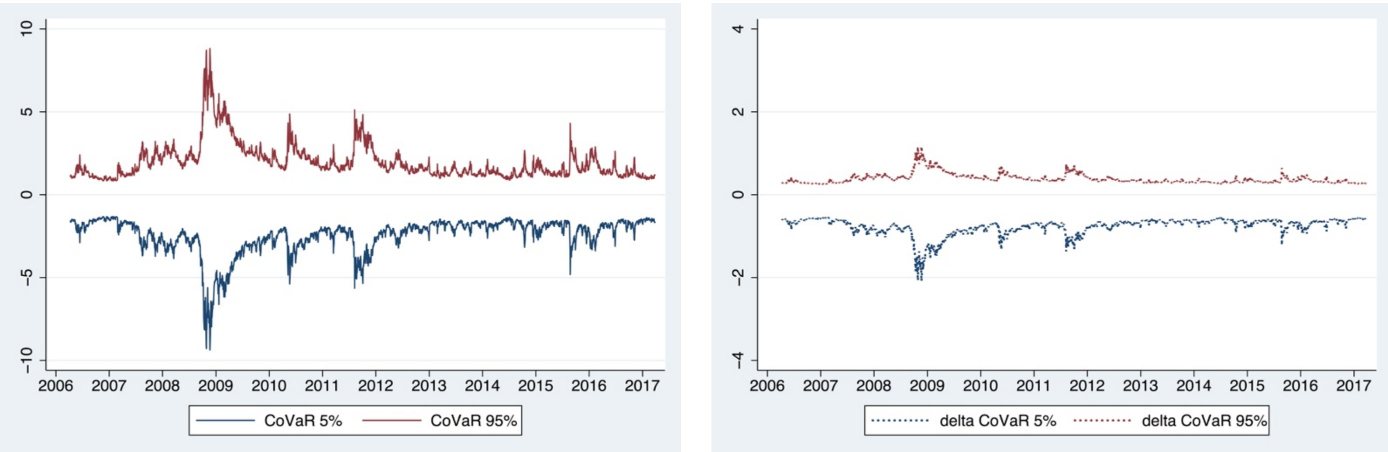




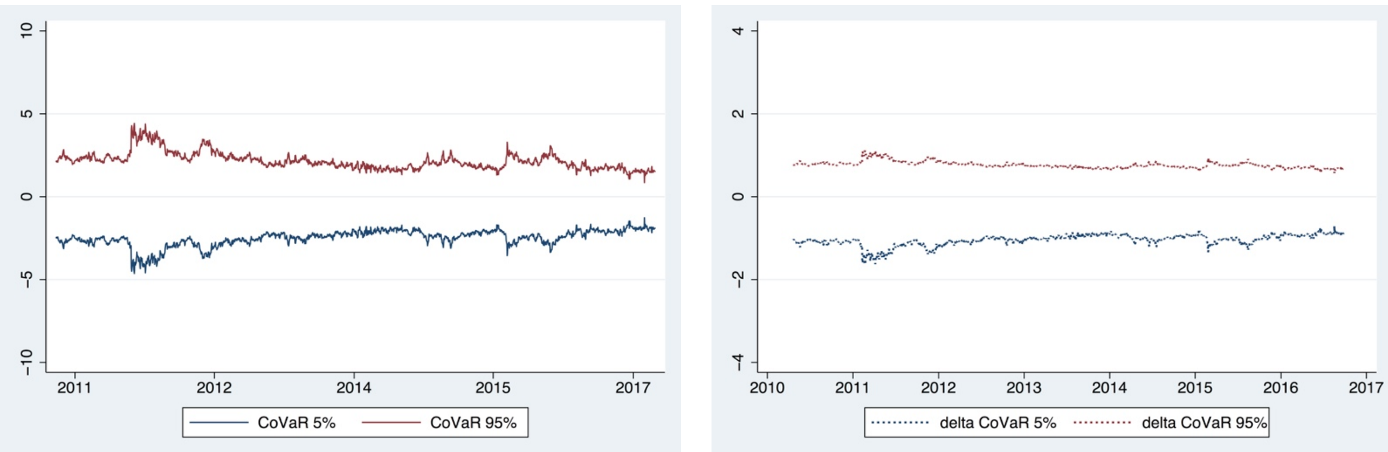
SMI



S&P 500



TSX 60



*Note: shorter time period

Figure 6: Graphical illustration of 5% and 95% CoVaR on the left and ΔCoVaR on the right

Table 9: Result statistics for time-varying CoVaR

	5% CoVaR				95% CoVaR			
	Mean	Std dev	Min	Max	Mean	Std dev	Min	Max
AEX	-3.18	1.17	-10.26	-1.28	3.05	1.29	0.94	10.90
BOVESPA	-4.21	0.98	-8.35	-2.48	4.22	1.31	1.91	9.77
CAC 40	-3.41	1.18	-11.16	-0.33	2.97	1.17	-0.06	10.60
DAX 30	-3.25	1.11	-10.82	-1.53	3.13	1.17	1.33	11.08
EUROSTOXX 50	-3.40	1.20	-11.99	-1.39	3.10	1.20	1.16	11.39
FTSE 100	-2.98	1.08	-9.69	-1.24	2.60	1.16	0.68	9.77
SMI	-2.46	0.81	-8.71	-1.34	2.33	1.00	0.95	10.02
S&P 500	-2.46	1.08	-9.37	-1.31	1.97	1.07	0.83	8.81
TSX 60	-2.50	0.49	-4.63	-1.27	2.16	0.52	0.86	4.43
	5% ΔCoVaR				95% ΔCoVaR			
AEX	-0.94	0.24	-2.38	-0.55	0.88	0.28	0.42	2.60
BOVESPA	-1.28	0.31	-2.58	-0.75	1.32	0.53	0.38	3.59
CAC 40	-1.01	0.25	-2.66	-0.35	0.68	0.18	0.21	1.87
DAX 30	-0.94	0.26	-2.74	-0.53	0.82	0.23	0.47	2.36
EUROSTOXX 50	-0.96	0.24	-2.65	-0.57	0.75	0.20	0.42	2.17
FTSE 100	-0.95	0.22	-2.33	-0.59	0.60	0.16	0.34	1.57
SMI	-0.70	0.21	-2.32	-0.41	0.48	0.16	0.26	1.71
S&P 500	-0.76	0.21	-2.08	-0.54	0.38	0.12	0.25	1.16
TSX 60	-1.05	0.14	-1.64	-0.7	0.77	0.08	0.57	1.11

5. Conclusion

This paper investigates co-movement between stock market indices and oil price. The data consists of daily stock, implied volatility and oil prices for the indices AEX, BOVESPA, CAC40, DAX30, EUROSTOXX50, FTSE100, SMI, S&P500, TSX60 and the USO. The co-movement is estimated using not only ordinary regression, but also quantile regression (Koenker & Bassett, 1978) which allow us to examine how the co-movement differs in different market situations. Additionally, we estimate CoVaR, value at risk conditional on oil price returns and ΔCoVaR introduced by Adrian and Brunnermeier (2016) for all indices.

We find a positive co-movement between stock market indices and oil price for the period studied. Univariate quantile regressions between oil returns and stock returns suggest a U-shaped structure of dependency across quantiles for 6 of the 9 indices examined. However, we find that this relationship disappears or is weakened when controlling for implied volatility of the stock market. This means that most of the variation in estimated coefficients across quantiles can be explained by implied volatility. The AEX and DAX30 kept their U-shaped dependency structure in the quantile regression. In other words, for these indices there is a rise in co-movement in both bearish and bullish markets. The CAC40, FTSE100, SMI, S&P500 and TSX60 all display similar slightly asymmetric dependency structures across quantiles. For these indices, the co-movement is stronger in bearish markets. For the BOVESPA the results are quite similar for all models estimated suggesting a high co-movement with oil in all market conditions. These results support the notion of oil price as an indicator of economic activity.

Finally, we find that there are differences in the 5% and 95% ΔCoVaR for indices, meaning the contribution to value at risk from bearish oil price returns to a long position in the market is not necessarily the same as the contribution to value at risk from an equally sized bullish oil price return to a short position in the market. Consequently, we conclude ΔCoVaR is a valuable tool for investors in determining the time-varying value at risk.

6. Appendix

6.1 Descriptive Test

Table 10: Test of assumptions when using OLS in Equation 7: $r_{i,t} = \alpha + \beta_1 r_{USO,t} + \beta_2 IV_{i,t-1} + \varepsilon_{i,t}$

	White test	BP-test	BG-test	JB-test	VIF
AEX	694.38	4.61	2.61	8858	1
BOVESPA	128.07	14.11	1.38	4691	1
CAC 40	656.21	2.71	2.63	6558	1
DAX 30	589.96	1.24	2.21	3951	1
EUROSTOXX 50	593.31	2.96	2.34	5233	1
FTSE 100	666.86	3.95	3.17	1.4e+04	1
S&P 500	579.73	6.38	3.26	1.2e+04	1
SMI	494.74	14.98	2.47	5159	1
TSX 60	187.96	0.96	1.42	1.7e+04	1.001
USO				588	

*Note: White test for heteroscedasticity in the residuals. The test reports chi value and we reject the null at a 5% significant level, the regression is clearly heteroscedastic. Breusch-Godfrey LM test for Autocorrelation with 50 lags. Test reports the F-value which indicates that there is autocorrelation. Jarque-Bera test for normality, with null-hypothesis that the residuals are normally distributed.

6.2 Simple Quantile Regression Table

Table 11: Results from Equation 8: $r_{i,t}^{(\tau)} = \alpha_i^{(\tau)} + \beta_1^{(\tau)} r_{USO,t} + \varepsilon_{i,t}^{(\tau)}$.

	τ	α	β1	R^2
AEX	0.05	-2.37***	0.35***	0.12
	0.10	-1.61***	0.31***	0.09
	0.25	-0.72***	0.25***	0.06
	0.50	0.04	0.20***	0.05
	0.75	0.80***	0.20***	0.05
	0.90	1.55***	0.25***	0.06
	0.95	2.19***	0.29***	0.08
BOVESPA	0.05	-3.00***	0.44***	0.07
	0.10	-2.17***	0.41***	0.87
	0.25	-1.08***	0.39***	0.09
	0.50	-0.01	0.39***	0.08
	0.75	1.08***	0.40***	0.08
	0.90	2.20***	0.42***	0.10
	0.95	2.90***	0.45***	0.11
CAC 40	0.05	-2.56***	0.34***	0.12
	0.10	-1.83***	0.33***	0.09
	0.25	-0.80***	0.26***	0.05
	0.50	0.04	0.22***	0.05
	0.75	0.85***	0.22***	0.05
	0.90	1.73***	0.24***	0.06
	0.95	2.44***	0.31***	0.08
DAX 30	0.05	-2.45***	0.32***	0.11
	0.10	-1.76***	0.33***	0.09
	0.25	-7.66***	0.25***	0.05
	0.50	0.07	0.18***	0.04
	0.75	0.88***	0.19***	0.04
	0.90	1.74***	0.25***	0.06
	0.95	2.47***	0.30***	0.08
EUROSTOXX 50	0.05	-2.63***	0.34***	0.12
	0.10	-1.81***	0.32***	0.09
	0.25	-0.82***	0.25***	0.05
	0.50	0.02	0.21***	0.04
	0.75	0.87***	0.22***	0.05
	0.90	1.77***	0.28***	0.06
	0.95	2.53***	0.30***	0.08
FTSE 100	0.05	-2.20***	0.32***	0.12
	0.10	-1.52***	0.28***	0.09
	0.25	-0.66***	0.24***	0.07
	0.50	0.04*	0.20***	0.06
	0.75	0.70***	0.20***	0.06
	0.90	1.43***	0.25***	0.07
	0.95	2.04***	0.27***	0.08
S&P 500	0.05	-1.84***	0.32***	0.15
	0.10	-1.20***	0.28***	0.11
	0.25	-0.44***	0.21***	0.08
	0.50	0.07***	0.16***	0.06
	0.75	0.57***	0.18***	0.07
	0.90	1.14***	0.22***	0.09
	0.95	1.64***	0.26***	0.09
SMI	0.05	-1.92***	0.25***	0.11
	0.10	-1.33***	0.19***	0.06
	0.25	-0.59***	0.15***	0.04
	0.50	0.05**	0.13***	0.03
	0.75	0.66***	0.15***	0.04
	0.90	1.31***	0.16***	0.04
	0.95	1.78***	0.18***	0.04
TSX 60	0.05	-1.45***	0.34***	0.19
	0.10	-1.10***	0.33***	0.18
	0.25	-0.50***	0.30***	0.17
	0.50	0.04*	0.29***	0.16
	0.75	0.54***	0.29***	0.16
	0.90	1.04***	0.33***	0.20
	0.95	1.45***	0.35***	0.19

**** 1% significant level, ** 5% significant level and * 10% significant level.*

6.3 Multiple Quantile Regression Table

Table 12: Results from Equation 9: $r_{i,t}^{(\tau)} = \alpha_i^{(\tau)} + \beta_1^{(\tau)} r_{USO,t} + \beta_2^{(\tau)} VIX_{i,t-1} + \varepsilon_{i,t}^{(\tau)}$.

	τ	α	β1	β2	R^2
AEX	0.05	-0,21	0.27***	-0.07***	0.25
	0.10	0.13	0.25***	-0.06***	0.18
	0.25	0.15*	0.25***	-0.03***	0.09
	0.50	0.03	0.20***	0.001	0.05
	0.75	-0.03	0.19***	0.03***	0.09
	0.90	0.04	0.22***	0.05***	0.16
	0.95	-0.05	0.27***	0.08***	0.23
BOVESPA	0.05	-0.62	0.39***	-0.07***	0.12
	0.10	-0.36	0.41***	-0.05***	0.12
	0.25	-0.32	0.39***	-0.03***	0.10
	0.50	-0.45**	0.39***	0.01**	0.08
	0.75	-0.06	0.39***	0.04***	0.09
	0.90	0.07	0.40***	0.06***	0.14
	0.95	0.04	0.45***	0.09***	0.18
CAC 40	0.05	0.08	0.29***	-0.08***	0.24
	0.10	0.26*	0.26***	-0.07***	0.17
	0.25	0.16	0.24***	-0.04***	0.08
	0.50	-0.01	0.22***	0.002	0.05
	0.75	-0,24**	0.21***	0.04***	0.08
	0.90	-0.17	0.22***	0.06***	0.15
	0.95	-0.30	0.21***	0.09***	0.21
DAX 30	0.05	-0.14	0.27***	-0.07***	0.22
	0.10	0.18	0.26***	-0.06***	0.16
	0.25	0.16	0.25***	-0.03***	0.08
	0.50	0.04	0.18***	0.001	0.04
	0.75	-0.14	0.19***	0.04***	0.08
	0.90	-0.06	0.22***	0.06***	0.15
	0.95	-0.04	0.25***	0.08***	0.21
EUROSTOXX 50	0.05	0.18	0.27***	-0.08***	0.23
	0.10	0.22	0.26***	-0.06***	0.16
	0.25	0.19*	0.24***	-0.03***	0.08
	0.50	-0.11	0.21***	0.005**	0.04
	0.75	-0.23***	0.20***	0.04***	0.09
	0.90	-0.18	0.23***	0.06***	0.16
	0.95	-0.21	0.23***	0.08***	0.22
FTSE 100	0.05	-0.16	0.27***	-0.07***	0.24
	0.10	-0.05	0.26***	-0.06***	0.19
	0.25	0.06	0.22***	-0.03***	0.09
	0.50	-0.07	0.20***	0.01**	0.06
	0.75	-0.07	0.17***	0.03***	0.09
	0.90	-0.03	0.20***	0.06***	0.18
	0.95	-0.20	0.19***	0.08***	0.24
S&P 500	0.05	0.15	0.22***	-0.09***	0.27
	0.10	0.22**	0.21***	-0.07***	0.20
	0.25	0.12**	0.19***	-0.03***	0.11
	0.50	-0.02	0.16***	0.01**	0.06
	0.75	-0.15**	0.16***	0.04***	0.12
	0.90	-0.25***	0.15***	0.07***	0.23
	0.95	-0.40***	0.12***	0.10***	0.32
SMI	0.05	-0.20	0.20***	-0.08***	0.20
	0.10	0.15	0.18***	-0.08***	0.15
	0.25	0.14*	0.16***	-0.04***	0.06
	0.50	0.07	0.13***	-0.004	0.03
	0.75	-0.02	0.14***	0.04***	0.06
	0.90	0.05	0.16***	0.07***	0.12
	0.95	-0.2*	0.14***	0.11***	0.16
TSX 60	0.05	-0.23	0.31***	-0.08***	0.24
	0.10	-0.23	0.33***	-0.06***	0.21
	0.25	-0.06	0.29***	-0.03***	0.18
	0.50	0	0.29***	0.003	0.16
	0.75	0.16	0.29***	0.03***	0.17
	0.90	0.15	0.28***	0.06***	0.24
	0.95	0.10	0.26***	0.09***	0.25

***** 1% significant level, ** 5% significant level and * 10% significant level.**

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