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Author: **Bjarne Haaland**	………………………………………… (signature of author)
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Stavanger, June 2017

Bjarne Haaland

Abstract

This thesis presents a method for reliability based casing design. The method is inspired by the model described by Das et al. (2015) - *A model for Well Reliability Analysis throughout the Life of a Well Using Barrier Engineering and Performance* (1). The purpose of this model is to show how reliability based casing design, based on statistical data, can be performed in practice.

Monte Carlo simulation conducted in MATLAB is the basis of the approach. Statistical data of load and strength simulates a casing burst scenario for a life cycle period of 20 years. Degradation factors accounting for casing wear and corrosion are applied.

Based on the result from the simulation, the underlying lifetime distribution was identified using the Nelson estimator method. Maximum Likelihood estimation was used to calculate parameters for the identified lifetime distribution.

The simulated data showed that the underlying lifetime distribution fitted a Weibull distribution. Reliability data such as, failure rate, failure function and survival function was found from this Weibull distribution. The presented approach, show how it is possible to quantify the reliability of a given design.

Qualitative statistical data of load and estimation of degradation factors related to the casing strength, is some of the identified challenges using reliability based design in practice.

Regulations, well integrity and load cases are important aspects of casing design. An introduction to these topics are given to get a full overview of the casing design process. The different design approaches used in well design is also explained and compared to show how reliability based design differs from other design approaches.

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List of abbreviations

ALARP	As low as reasonable possible
API	American Petroleum Institute
ASV	Annulus safety valve
CSC	Chloride stress cracking
EAC	Element Acceptance Criteria
EIC	Environmentally induced cracking
HSE	Health Safety and Environment
ISO	International Organization for Standarization
LSD	Limit States design
MASP	Maximum allowable surface pressure
MLE	Maximum Likelihood estimator
NCS	Norwegian continental shelf
NORSOK	Norsk Sokkels Konkurranseposisjon
OLF	Norwegian Oil and Gas Association
OTC	Offshore Technology Conference
PDF	Probability Density Function
PP	Pore pressure
PSA	Petroleum Safety Authority
RBD	Reliability based design
SCC	Stress corrosion cracking
SITP	Shut in tubing pressure
SLS	Serviceability limit strength
SSC	Sulphide stress cracking
TOC	Top of cement
ULS	Ultimate limit strength
WB	Well barrier
WBE	Well barrier element
WBS	Well barrier schematics
WIF	Well Integrity Forum
WIM	Well Integrity Management
WSD	Working stress design

1 Introduction

Well Integrity is defined as *"the application of technical, operational and organizational solutions to reduce risk of uncontrolled release of formation fluids and well fluids throughout the life cycle of the well"* (2). The integrity related to the *technical* part is manifested through the effectiveness of the barriers in the well. Regulations and standards define requirements for well design and well barriers. This is to ensure that an appropriate level of well integrity is present through the life cycle of the well.

Well design is the application of technical solutions that makes sure that a well can operate effective and safely during its life cycle. The main part of well design is to identify loads, and then apply a solution that is strong enough for the entire lifetime of the well.

Classic well design is based on the working stress method. In this approach the load and strength are considered deterministic values. To ensure a safety margin between the load and the strength, a safety factor is applied. The reliability for such a design is unknown and unquantifiable.

In reliability based design the uncertainties in the estimation of strength and load variables is taken into account. With appropriate chosen values for these variables, the reliability of a certain design can be measured and quantified.

1.1 Objective

The main objective of this thesis is to demonstrate how reliability based casing design can be used in practice and how it can contribute to well integrity. Special focus is on showing how one can build a probabilistic model to generate time dependent failure data for casing using Monte Carlo simulations, and how these data can be analysed by statistical methods to provide different reliability measures. This was motivated by ideas presented in *A model for Well Reliability Analysis throughout the Life of a Well Using Barrier Engineering and Performance* by Bibek Das and Robello Samuel (1).

Chapter 8 presents the model and associated theory and methodologies. Developed MATLAB codes are presented in appendix A

An introduction to regulations, well integrity, load cases and design approaches are given for the reader to understand some of the most important aspects of casing design.

1.2 Structure of thesis

- Chapter 2 presents the regulations and laws, along with the most important standards and guidelines governing the activity on the Norwegian continental shelf (NCS).
- Chapter 3 introduce the fundamentals of well integrity, well barriers and well design based on the on the recommendations given in NORSOK-D010.
- Chapter 4 present some of the most important studies regarding well integrity on the NCS. Some of the causes of these issues are also identified.

- Chapter 5 introduce the most important loads that governs the design of a well, with focus on burst, collapse and tensile loads. Special loads caused by environmental factors due to change in temperature and formation loads are mentioned
- Chapter 6 explain some of the most important casing degradation factors that must be taken into account in a design process.
- In chapter 7 working stress design and the limit states design methods are explained. The main differences between them are also pointed out.
- Chapter 8 describe reliability based design and reliability mathematics. A computer-based model for simulating reliability data and methods for identifying the underlying lifetime distribution is also presented.

2 Regulations on the Norwegian Continental Shelf (NCS)

The laws stated by the Norwegian Parliament control the petroleum activity on the NCS. The laws are manifested through the regulations and guidelines provided by the Petroleum Safety Authority (PSA), in cooperation with the Norwegian Environment Agency, Norwegian Directorate of Health and Norwegian Food Safety Authority. The regulations are divided into five main groups, and collectively covers the regulations for the petroleum activity in Norway (3).

1. Framework HSE
    - *Regulations relating to health, safety and the environment in the petroleum activities and at certain onshore facilities*

2. Management Regulations
    - *Regulations relating to management and the duty to provide information in the petroleum activities and at certain onshore facilities*

3. Facilities Regulations
    - *Regulations relating to design and outfitting of facilities in the petroleum activities*

4. Activities Regulations
    - *Regulations relating to conducting petroleum activities*

5. Working Environment Regulations
    - *Regulation regarding working hours, employment protection, etc.*

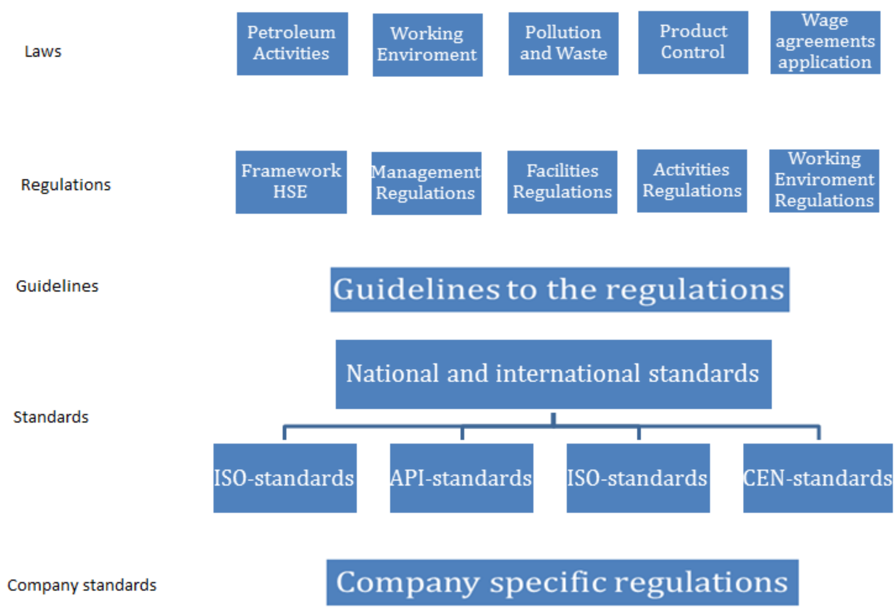


FIGURE 1: HIERARCHY OF LAWS AND REGULATIONS ON THE NCS

The Activities regulations and the facilities regulations specify the regulations concerning well integrity and well design. The regulations are normative in nature and do not specify how they shall be fulfilled.

PSA provides guidelines to help fulfil the regulations. The guidelines are not legally binding, but they demonstrate and instruct the operating companies on how they can satisfy the requirements in practice. The guidelines often refer to industry-recognized standards. The leading standards concerning well integrity, design and activities on NCS are:

- NORSOK-D010 *Well integrity in drilling and well operations* (2)
- Norwegian Oil and Gas Association (OLF) 117-*Recommended Guidelines for Well Integrity* (4)

2.1 NORSOK standard D-010

NORSOK-D010 is a Norwegian standard developed by participants in the petroleum industry. OLF-117 is owned by the petroleum industry represented by Norwegian Oil and Gas Association and The Federation of Norwegian Industries (2).

The main goal for developing the standard was to increase competitiveness by reducing cost and time in the development and operation of offshore fields. This is accomplished through specification of technical and operational requirements. The specifications are divided into different activities, and covers the whole life cycle of the well (2):

- Drilling activities
- Well testing activities
- Completion activities
- Production activities
- Abandonment activities

NORSOK-D010 refers extensively to other industry-recognized standards. The most important of these are API-standards, ISO-standards, OLF-guidelines and NORSOK standards. These must be used complementary to NORSOK-D010 in order to get a full overview of the recommended practices (2).

2.2 OLF 117 Recommended Guidelines for Well Integrity

OLF 117 was developed as a result of the *Well Integrity Survey Phase 1* conducted by the PSA in 2006. The problems discovered by this survey led to the establishment of Well Integrity Forum (WIF), represented by a broad spectre of operating companies. Together with OLF they created these guidelines to assess the findings from this survey (5). The guidelines specify procedures related to:

- Well Integrity Training of personnel
- Well Handover Documentation
- Well Barrier Schematics for the Operational Phase
- Well Integrity Well Categorization
- Well Integrity Management System

Some of these topics are also described in NORSOK-D010. In NORSOK-D010 the specifications are mainly normative. OLF-117 states detailed information on how these topics should be assessed and what information they should contain.

3 Well Integrity

NORSOK D-010 defines well integrity as *"the application of technical, operational and organizational solutions to reduce risk of uncontrolled release of formation fluids and well fluids throughout the life cycle of the well"*. There is not a global definition of well integrity. The definition from NORSOK D-010 has been adopted by many organizations and companies and have become the most widely accepted definition (6).

Well integrity is a multidisciplinary field. To assess the different parts of well integrity NORSOK-D010 states that *"a systematic approach shall be established to manage the well integrity in all stages of the life cycle of the well, from construction phase to final abandonment"* (2).

The *systematic approach* stated by NORSOK-D010 is usually assessed by a Well Integrity Management (WIM) system. The main task of a WIM system is to identify and monitor risks related to well activities. The content of a WIM system is described in NORSOK-D010 and further specified in OLF-117. The most important content of a WIM system can be illustrated as follow:

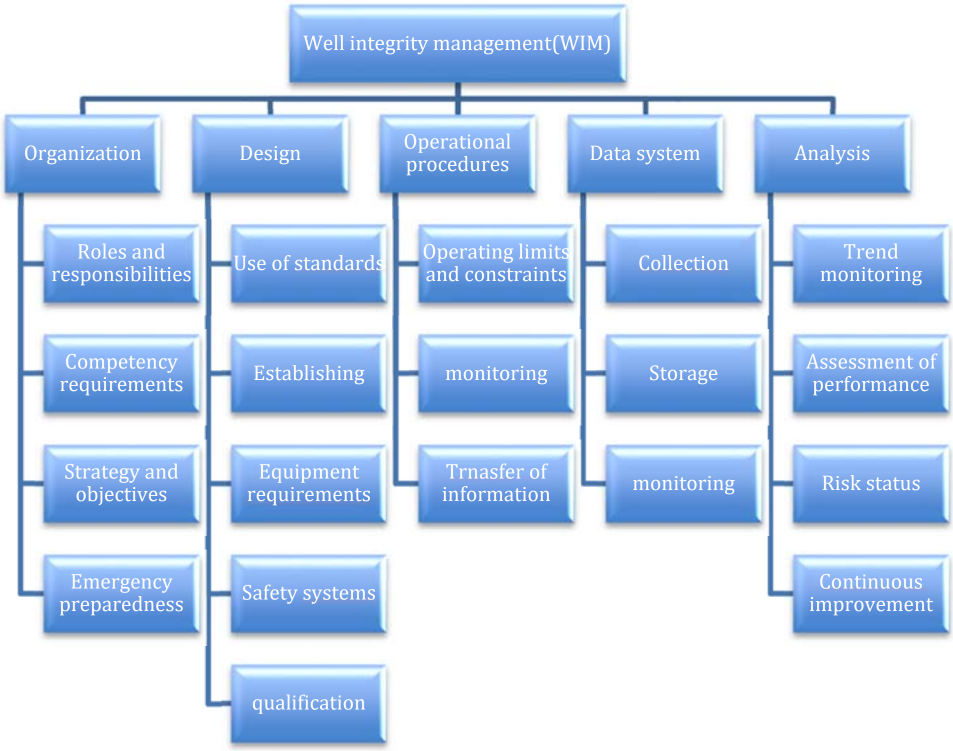


FIGURE 2: WIM SYSTEM (4)

The organizational side is addressed to the operators responsible to establish a WIM system for the activities carried out by all involved parties, and ensure that this complies with well integrity regulations. The WIM system should also define roles and objectives for all staff involved in well integrity activities. The well integrity program should assess the integrity for the complete life cycle of the well. This involve all the operational activities such as: well design, drilling, completion, production and plug and abandonment (4).

Design is related to the establishment of technical solutions for a well that is in compliance with the requirements and has an acceptable risk of failure during the life cycle of the well (4).

Operational procedures refers to the use of procedures to avoid accident and situations that compromises well integrity and HSE related accidents. The goal is to ensure that all activities are carried out in a safe and prudent manner (4).

Data system is related to information about the well. This involves data about limitations, critical parameters and well barrier schematics (WBS). It also involves monitoring of the well, along with data about risk levels and integrity status of the well (4).

Analysis should be done on the available and sampled data. The purpose of the analysis is to identify and quantify risk. The analysis data should be used to make improvement to all sides of the well activities. This include management system, planning of work, work processes, preventive maintenance and HSE work (4).

3.1 Well barriers

A functional well barrier (WB) is the fundamental part of well integrity. It is the *technical solution* that keeps pressurized fluids contained. The WB consist of several well barrier elements (WBE) that together forms an impenetrable envelope. In NORSOK-D010 a well barrier is defined as:

An envelope of one or several well barrier elements preventing fluids from flowing unintentionally from the formation into the wellbore, into another formation or to the external environment (2)*.*

NORSOK-D010 define several requirements for a WB and WBE to be functional. These are related to:

- Number of barriers
- Well barrier requirements
- Well barriers schematic (WBS)
- Elements acceptance criteria (EAC) tables
- Well design

3.1.1 Numbers of barriers

The barrier philosophy is based on the *double block and bleed principle.* The double block is represented by the primary and secondary barrier, which ensures the redundancy in case of failure in one of the WB envelopes. The bleed principle is related to the verifications and monitoring of the WB, to assure that both of them are in functioning order (7).

NORSOK-D010 specify the number of barriers for different situations:

Minimum number of well barriers	Source of inflow
One well barrier	a) Undesirable cross flow between formation zones b) Normally pressured formation with no hydrocarbon and no potential to flow to surface c) Abnormally pressured hydrocarbon formation with no potential to flow to surface (e.g. tar formation without hydrocarbon vapour)
Two well barriers	d) Hydrocarbon bearing formations e) Abnormally pressured formation with potential to flow to surface

FIGURE 3: NUMBER OF BARRIERS (2)

3.1.2 Well barrier requirements

For a well barrier to be operational according to NORSOK-D010, several requirements must be met. These requirements are specified through defined functional goals related to design, construction and verification and monitoring. This is to ensure that the WB works towards its intended purpose (2).

NORSOK-D010 states that (2):

The well barriers shall be designed, selected and with capabilities to:

- *a) Withstand maximum differential pressure and temperature it may become exposed to.*
- *b) Be pressure tested, function tested or verified by other methods.*
- *c) Ensure that no single failure of a well barrier or WBE can lead to uncontrolled flow of wellbore fluids or gases to the external environment.*
- *d) Re-establish a lost well barrier or establish another alternative well barrier.*
- *e) Operate competently and withstand the environment for which it may be expose to over time.*
- *f) Determine the physical position /location and integrity status at all times when such monitoring is possible.*
- *g) Be independent of each other and avoid having common WBE to the extent possible.*

During operation the following apply:

- *h) The double block and bleed principle shall be fulfilled for all the equipment above seabed/surface, which can be exposed to well pressure, i.e. two valves in series in all in-/outlets from the well.*
- *i) When a work string penetrates the well barrier, one of the WBEs should be able to shear the work string and seal the wellbore after having sheared the string.*
- *j) All non-shearable components in the work string shall be identified.*
- *k) When running non-shearable components through the BOP, there shall be procedures in place for handling a well control situation.*

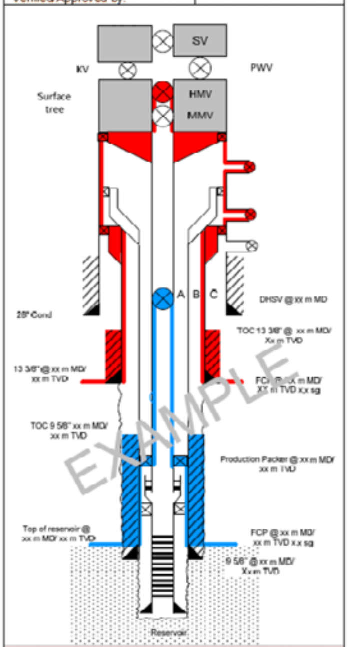
l) *When running long non shearable assemblies, there shall be an element installed that can seal the wellbore against any size assembly that penetrates the well barrier.*

3.1.3 Well barrier schematics (WBS)

A well barrier schematic is an illustration of the WBE and the WB that are present in the well at a certain time. The WBS change during the different operational phases during the life cycle of the well. NORSOK-D10 describe the information a WBS should contain (2):

a) A drawing illustrating the well barriers, with primary well barrier shown with blue colour and secondary barrier shown with red colour.
b) The formation integrity when information is part of a well barrier.
c) Reservoir/potential sources of inflow.
d) Tabulated listing of WBEs with initial verification and monitoring requirements,
e) All casing and cement. Casing and cement (including TOC) defined as WBEs should be labelled with size and depth (TVD and MD).
f) Component should be show relatively correct position in relation to each other.
g) Well information: field/installation, well name, well type, well status, well/section design pressure, revision number and date, "Prepared by", "Verified/Approved by".
h) Clear labelling of actual well barrier status – planned or as built.
i) Any failed or impaired WBE to be clearly stated.
j) A note field for important well integrity information (anomalies, exemptions, etc.).

Well information (as built)	
Field/Installation:	
Well name:	
Well type:	Producer
Well status:	Shut-in
Well design pressure:	xx bar
Revision number:	Version 1.0
Date prepared/revised:	
Prepared by: Verified/Approved by:	



Note 1: Well shut-in due to tubing leak above DHSV.

Nomenclature:
PT: Pressure test
IT: Inflow test
FCP: fracture closure pressure
XLOT: extended leak off test
AC: acceptance criteria

Well barrier elements	EAC table	Verification / Monitoring
Primary well barrier		
In-situ formation (cap rock)	51	FCP: x.x s.g. Based on field model / n/a after initial verification
Casing cement (9 5/8")	22	Length: xx mMD Cement bond logs / Daily pressure monitoring of B-annulus
Casing (9 5/8")	2	PT: xx bar with x s.g. EMW / n/a after initial verification
Production packer	7	PT: xx bar with x s.g. EMW / Continuous pressure monitoring of A-annulus
Completion string	25	PT: xx bar with x s.g. EMW / Continuous pressure monitoring of A-annulus See Note 1.
Completion string component (Chemical Injection valve)	29	PT: xx bar with x s.g. EMW / Periodic leak testing AC DHSV: xx bar/xx min
Downhole safety valve (incl. control line)	8	IT: xx bar (DHSV) PT: xx bar (control line) / Periodic leak testing AC DHSV: xx bar/xx min
Secondary well barrier		
In-situ formation (13 3/8" shoe)	51	FCP: x.x s.g. Based on XLOT / n/a after initial verification
Casing cement (13 3/8")	22	Length: xx mMD Method: Volume control / Daily pressure monitoring of C-annulus
Casing (13 3/8")	2	PT: xx bar with x s.g. EMW / Daily pressure monitoring of C-annulus
Wellhead (Casing hanger with seal assembly)	5	PT: xx bar / Daily pressure monitoring of C-annulus/ Periodic leak testing
Wellhead / annulus access valves	12	PT: xx bar / Periodic leak testing of valve AC: xx bar/xx min.
Tubing hanger (body seals and neck seal)	10	PT: xx bar / Periodic leak testing
Wellhead (WH/XT Connector)	5	PT: xx bar / Periodic leak testing
Surface tree	33	PT: xx bar / Periodic leak testing of valves AC: xx bar/xx min

FIGURE 4: EXAMPLE OF WELL BARRIER SCHEMATICS (2)

3.1.4 Element acceptance criteria (EAC) table

An EAC table is a listing that show the technical and operational information for each WBE that is present in the well. The table shows verification dates, how the WBE is monitored and the most important specifications (2).

TABLE 1 : EXAMPLE OF EAC TABLE FOR CASING (2)

Features	Acceptance criteria	See
A. Description	This element consists of casing/liner and/or tubing in case tubing is used for through tubing drilling and completion operations.	
B. Function	The purpose of casing/liner is to provide an isolation that stops uncontrolled flow of formation fluid or injected fluid between the casing bore and the casing annulus.	
C. Design construction selection	1. Casing/liner strings, including connections shall be designed to withstand all loads and stresses expected during the lifetime of the well (including all planned operations and potential well control situations). Any effects of degradations shall be included. 2. Minimum acceptable design factors shall be calculated for each load type. Estimated effects of temperature, corrosion and wear shall be included in the design factors. 3. All load cases shall be defined and documented with regards to burst, collapse and tension/compression. 4. Casing design can be based on deterministic or probabilistic models. 5. Casing exposed to hydrocarbon flow potential shall have gas-tight threads. Exception: Surface casing which is exposed or can be potentially exposed to normal gradient shallow gas.	ISO 11960 ISO 13679 ISO 10405
D. Initial test and verification	1. Casing/liner shall be leak tested to maximum differential pressure. 2. Casing/liner that has been drilled through after initial leak test shall be retested during completion activities. 3. The leak test of casing shall be performed either when cement is wet (immediately after pumping) or after cement has set up. No pressure testing should be performed while the cement is setting up.	
E. Use	Casing/liner should be stored and handled properly to prevent damage to pipe body and connections prior to installation.	
F. Monitoring	1. The A-annulus shall be continuously monitored for pressure anomalies. Other accessible annuli shall be monitored at regular intervals. 2. All casing strings shall be logged for wear after drilling if simulation indicates excessive wear which exceeds allowable wear based on casing design. Metal shavings should be collected by the use of ditch magnets.	
G. Common well barrier	1. During drilling operations with surface BOP, the annulus outside the current casing shall be monitored continuously and alarm levels be defined. 2. Actual status of the casing shall be known and confirmed capable of withstanding maximum expected pressure after expected wear. 3. Pressure test should include safety margin to cover expected wear after testing. 4. Magnet shall be in the mud return flowline to measure metal and assess changes in the nature of the metal filings. 5. If drilling through an old casing: a) Prior to drilling activity commences, casing wear log(s) should be run (calliper and/or sonic). The logs shall be verified by qualified personnel and documented. b) Logs that can identify localised (1 m interval between measurements) doglegs (gyro or similar) should be run.	

3.2 Well Design

A design process shall establish a technical solution that is verified and documented. The design shall be able to withstand all types of load that it can be exposed to during its lifetime. The risk and uncertainties shall be assessed and be as low as reasonably possible (ALARP). The design methods shall be based on recognized industry standards. The most important of them are (4):

- NORSOK standards
- ISO-standards
- API-standards
- Company specific standards
- Supplier specific standards

In NORSOK-D010 the design principle is based on the elastic deformation principle. The allowable utilization range of the casing shall be defined as the common performance envelope area defined by the intersections of (2):

a) The von Mises Ellipse
b) ISO/TR 10400:2007 or API TR 5C3, 1st edition, December 2008 formulas for burst, collapse and axial stress
c) Pipe end connection capabilities.

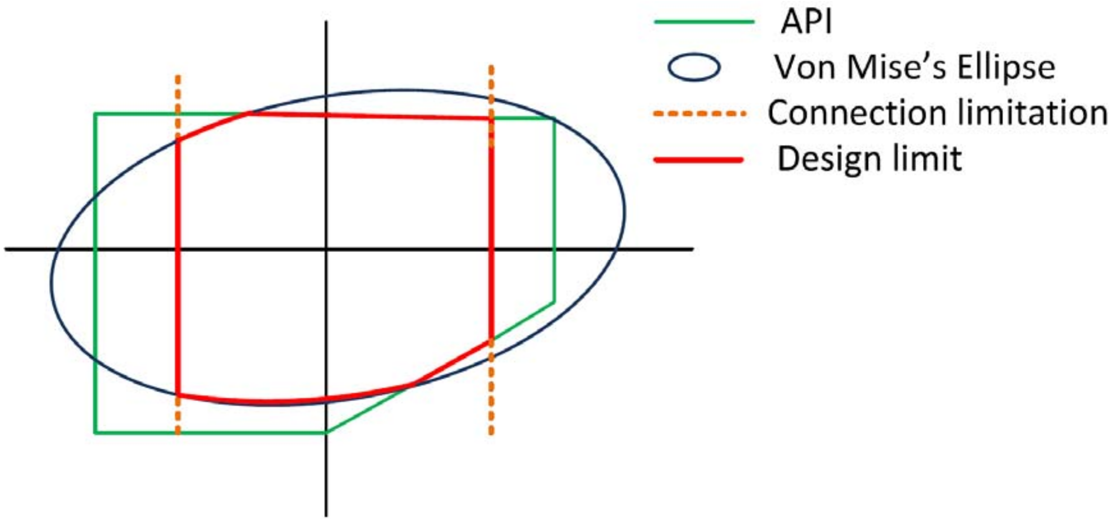


FIGURE 5: DESIGN CRITERIA (2)

4 Well Integrity status on NCS

Petroleum Safety Authority (PSA) is responsible for monitoring and controlling the petroleum activity on the NCS. An important part of this is to monitor the HSE status on the NCS. PSA have during the years conducted several surveys regarding well integrity on the NCS (3).

4.1 Well surveys on the NCS

4.1.1 PSA Well Integrity Surveys

4.1.1.1 PSA Well Integrity Survey, Phase 1

In 2006, PSA conducted a well integrity survey to identify issues related to well integrity on the Norwegian continental shelf. Seven operating companies and some onshore facilities and wells contributed to the survey (8).

The scope of this survey was to identify the causes and issues related to well integrity. A representative part of subsea wells, production wells and injection wells from the NCS was investigated. In total 406 wells were studied. This represented 21% percent of all active wells at the time (8).

The results from this study showed that 18% of the wells had integrity issues. 7% of these were fully shut in.

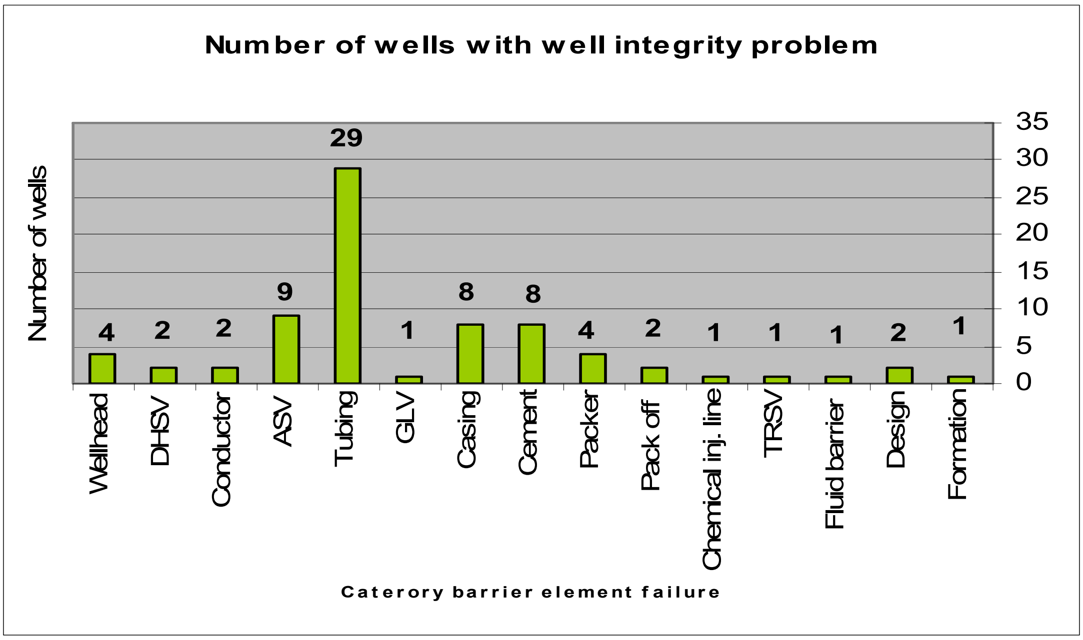


FIGURE 6: FAILURE ELEMENT CATEGORY (5)

The figure above illustrates cause of failure related to the WBE. Most of the failure were related to tubing, annulus safety valve, casing and cement. The function failure reported to these WBE where related to (5):

- Leakage above safety valve
- Tubing to annulus leakage
- ASV malfunctions.
- Casing connection leakage and collapsed casing
- No cement behind casing
- Leaks along cement bond.

Casing and tubing failures contributed with 50% percent of the total failures.

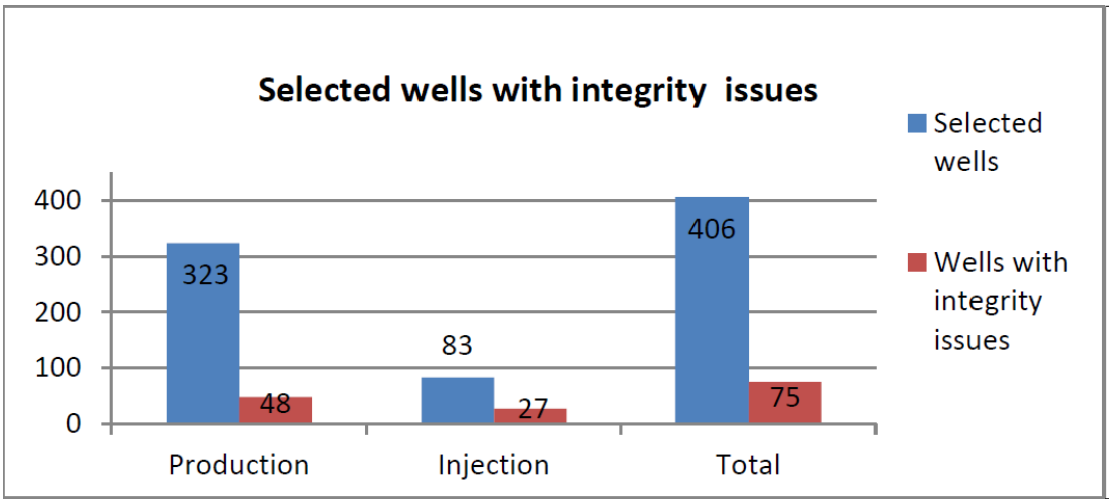


FIGURE 7: PRODUCTION VS INJECTION WELLS FAILURE (5)

It was also identified that injection wells had more integrity issues than producers. 33% injection wells had integrity issues.

The survey identified several areas of improvement based on the integrity issues of the wells. These were related to handover documentation, lack of compliance with NORSOK-D010, regular monitoring, competence, training and well documentation (5).

4.1.1.2 PSA Injection well study

PSA Well Integrity Survey Phase 1 (8), identified a high degree of integrity issues with injection wells. As a follow up to this study an injection well study was performed. The scope of the survey was to identify the possible integrity issues regarding the injection wells.

The reported barriers failure and integrity problems in the injection wells were related to:

- Quality of the injected medium
- Completion design
- Tubing hanger
- Production packer

The survey concluded that there was a need for increased focus on injection wells with potential improvement on the following (9):

- The qualification method when converting from production to injection wells
- The quality control of the injected medium with regards to well design and material composition
- Continuous monitoring of injected media.
- Design issues
- Well integrity survey with regards to logging and monitoring methods
- HSE guidelines for personnel safety in relation to CO_2 injection wells
- PBR and seal stem solutions in completion design
- Personnel competency level in relation to well design factors and degradation mechanisms

4.1.2 Assessment of sustained well integrity on the NCS (SINTEF Study)

In 2007, SINTEF performed two well integrity studies for one operator on the NCS. The study was done on eight different fields and 217 wells was studied. Leak history from 1998 until 2007 was mapped and studied (10).

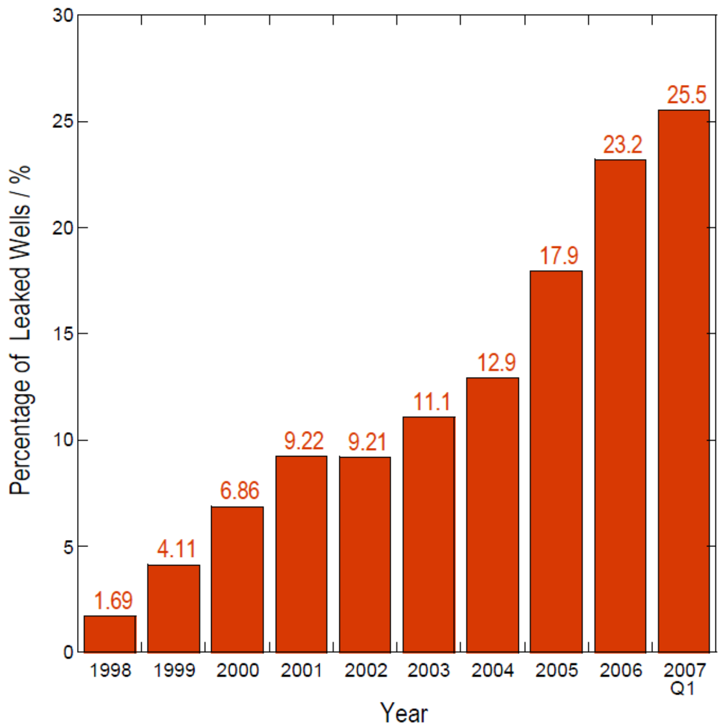


FIGURE 8: PERCENTAGE LEAKED WELLS (10)

Several reasons for the increase rate of leakage was proposed:

- Ageing of the wells
- Increased number of wells
- Improved reporting and/or awareness

- Operating outside the design envelope

The results from this study indicated many similarities to the PSA studies. Injections wells were found to be more prone to well integrity issues.

The study identified that gas lift wells experienced leakage after two years, after they were introduced to gas lift. Many of the wells where designed to operate in dry gas conditions, where the real conditions involved presence of wet gas and corrosive CO_2. They concluded that the wells were operating outside their design envelope, leading to very short-lived wells.

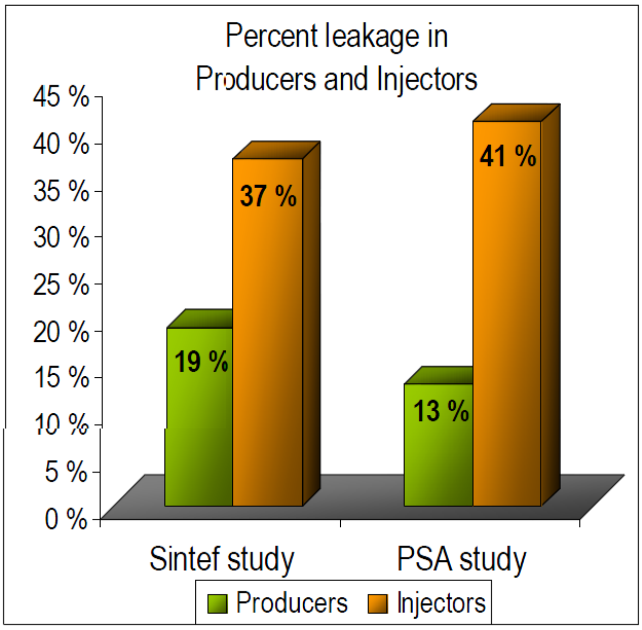


FIGURE 9: COMPARISON OF SINTEF STUDY AND PSA STUDY (10)

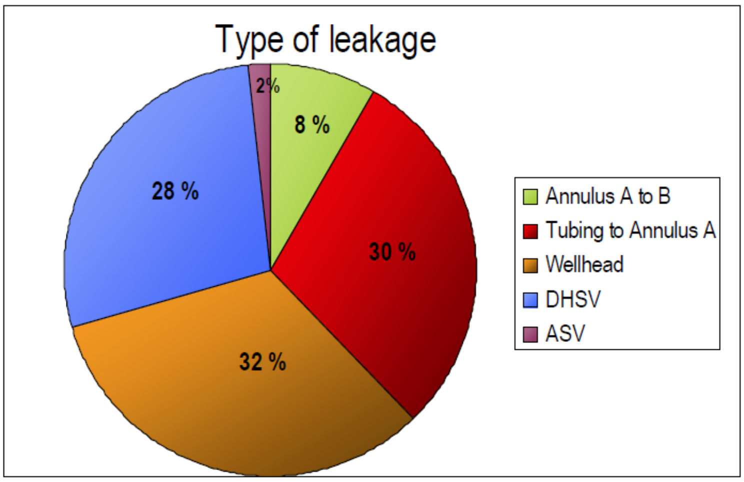


FIGURE 10: TYPE OF LEAKAGE

Identified type of leakage related to WBE elements is shown in figure 10

4.1.3 Temporary abandoned wells survey

PSA, SINTEF and Wellbarrier (company) performed a study on well integrity on temporary abandoned wells on the NCS in 2011. 193 wells from eight different operating companies was included in this survey (11).

The result from the study indicated that about one third of all the temporary abandoned wells had integrity issues of some kind. Figure 11 shows the severity of the integrity issues, based on the OLF-guidelines categorization of the integrity issues (see figure 12)

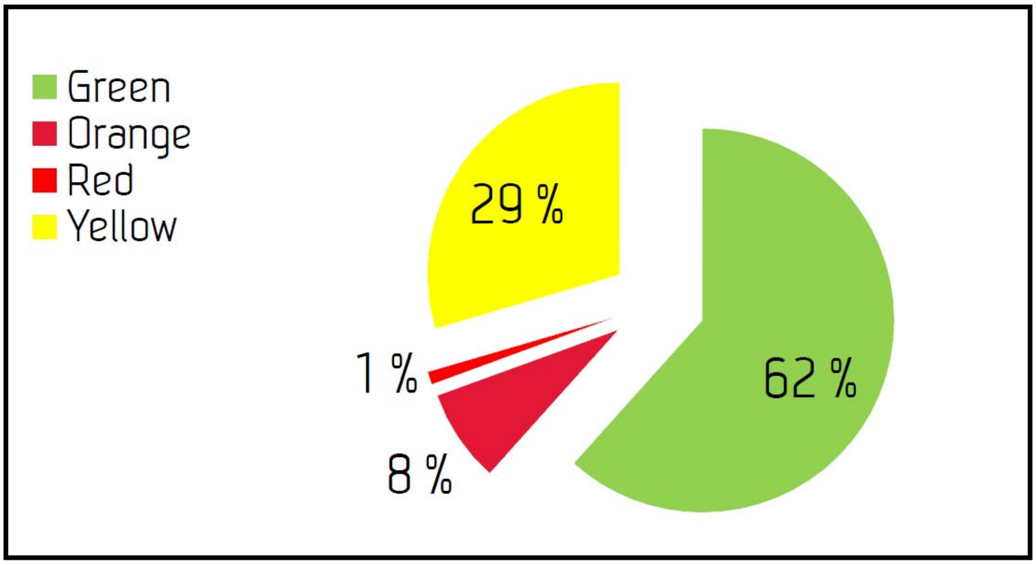


FIGURE 11: WELL INTEGRITY ISSUES (11)

4.2 Status on NCS

The initial Well Integrity Study Phase 1 led to an increased focus on well integrity on the NCS. Since 2008 one of the main priorities of PSA has been barriers and integrity, to avoid major accidents. In 2007 the Oil Industry Association (OLF) established the WIF which led to the establishment of OLF-117 *Recommended Guidelines for Well Integrity,* described under section 2.2. The OLF-117 defined a new categorization for well integrity issues, based on colour codes related to the severity. They were defined as follow:

Category	Principle
Red	One barrier failure and the other is degraded/not verified, or leak to surface
Orange	One barrier failure and the other is intact, or a single failure may lead to leak to surface
Yellow	One barrier degraded, the other is intact
Green	Healthy well - no or minor issue

FIGURE 12: CATEGORIZATION CRITERIA (4)

In 2008, this categorization was implemented to RNNP (Risk Level in Norwegian petroleum industry) reports (5). And PSA have continuously monitored the integrity based on these criteria up to this day.

The main trends based on the yearly RNNP reports is shown in the figure below.

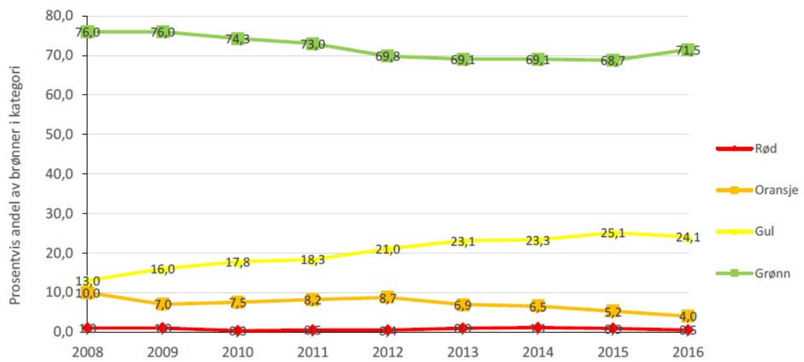


FIGURE 13: INTEGRITY ISSUES DEVELOPMENT (12)

Figure 13 shows relative constant development for the most severe category red. Orange categorized wells show a declining rate over the years, while the yellow categorized wells have an up-going trend. The green categorized wells show a downward slope until 2016. Overall there are still 28,5% of the well that have integrity issues. This accounts to 554 wells with integrity issues. Focus on well integrity will therefore also be important in the future (12).

5 Casing Loads

A load case is the description of internal pressure, external pressure, and temperature that affect the casing string at different locations. The calculation of these loads lay the foundation for choosing a casing that can provide integrity for the well during its life cycle. Identification and calculation of loads plays a major role in a design process (13).

Loads have different origin. They are usually divided into two groups:

- Intentional loads
- Accidental loads

Intentional loads are either planned or assumed present in the well. They will therefore happen with a high degree of certainty, but some uncertainty to the magnitude of them still apply. Some of the most important intentionally loads are:

- Pressure testing
- Static and running tensional load
- Static pressure loads
- Temperature loads

Accidental loads are loads that can happen. It can be difficult to identify all types of accidental loads, and which loads to assess in the design process. The uncertainty is big in both occurrence and magnitude. Some of the most important accidental load that usually are assessed are:

- Influx of gas during drilling (Kick)
- Tensional loads- top drive malfunction/ slips drop
- Leaking tubing

According to NORSOK-D010 both static and dynamic load cases for all WBE shall be established. Load cases can be very different depending on the purpose and use of a well, an injection well is exposed to different loads than a production well. There is no industry standard or consensus for defining a unified set of load cases, but some of the most normal loads for casing design will be presented in the next sections (7).

5.1 Burst loads

Burst loads arise due to higher pressure on the inside of the casing than outside of the casing. When this pressure differential exceeds the strength of the casing, it will fail. The failure normally causes a rupture along the axis of the pipe. Parting of the pipe can also occur if there is tensional loading in combination with burst load (biaxial loading) (14). The burst load is defined as:

$$p_{burst} = p_i - p_o \qquad 1$$

Where:
p_{burst}= *burst pressure*
p_i= *inside pressure*
p_o= *outside pressure*

5.1.1 Kick load

Influx of formation fluid is normally the most severe burst load during a drilling operation. Assuming that the pressure gradient outside the casing is higher than the gradient of the influx fluid inside the well, the maximum burst load will be at the top of the well. This is called the MASP (maximum allowable surface pressure). The method for calculating the MASP is based on *frack at shoe- gas to surface* criteria (13). The lower of these two criteria is the load that the design is based on.

Calculation of MASP involve estimation of several steps (13):

1. First step is to estimate the formation fluid gradient. The most conservative is to assume methane as this is the lightest gas and will result in the highest MASP. The compressible properties of gas are usually not assessed, and the gas gradient is assumed to linear with depth.
2. Determine fracture pressure at the casing shoe
3. Estimate maximum bottom hole pressure BHP in the next drilling section

Example:

TABLE 2: KICK CALCULATION

		Depth
Maximum estimated BHP gradient	b [Pa/m]	h_1 [m]
Fracture gradient at casing shoe	f [Pa/m]	h_2 [m]
Formation fluid gradient	g [Pa/m]	

For *frac-at-shoe* criteria:

$$MASP = f \cdot h_2 - g \cdot h_2 \qquad 2$$

For *gas-to-surface* criteria:

$$MASP = b \cdot h_1 - g \cdot h_1 \qquad 3$$

The pressure outside casing is created by the hydrostatic column of the fluid outside the wellhead. For a surface wellhead this is zero and for a subsea well it is the hydrostatic pressure of the water column. The burst load due to kick can be calculated as:

$$p_{burst} = p_i - p_o = MASP - p_o \qquad 4$$

5.1.2 Leaking tubing load

Leaking tubing can happen during initial well testing or in the production period. The anticipated pressure in the tubing or test string if it is shut in, is called the Shut in tubing pressure (SITP). The SITP is calculated as followed:

$$SITP = BHP - \rho_i gh \qquad 5$$

Where:
SITP=shut in tubing pressure
BHP=bottom hole pressure
ρ_i=density of fluid inside well
h=TVD to kick location
g=gravitational constant

In the case of a leaking tubing the maximum burst load will occur at the production packer. Assuming that a leak is at the top of the tubing, or that the leak is a gas that can migrate up inside the production casing. The maximum load will be the SITP plus the hydrostatic pressure exerted by the completion fluid inside the production casing. The inside pressure can be calculated as followed:

$$p_i = SITP + \rho_{cf} g h_{pp} \qquad 6$$

Where:
p_i= pressure inside production casing
SITP=shut in tubing pressure
ρ_{cf}=completion fluid density
h_{pp}=depth to production packer
g=gravitational constant

The burst pressure at the production packer depth, is dependent on the pressure outside the production casing at the production packer depth. For short time scenarios, this is usually the mud gradient above top of cement (TOC) and water gradient in the cemented section. In a long time scenario, pore pressure (PP) is usually applied (13). The outside pressure then becomes:

$$p_o = p_{pp} \qquad 7$$

Where:
p_o=pressure outside production casing
p_{pp}=pore pressure

The effective burst pressure can be calculated as followed:

$$p_{burst} = (SITP + \rho_{cf} g h_{pp}) - p_{pp} \qquad 8$$

The assumption that the maximum burst pressure is at the production packer, is only valid if the completion fluid gradient is bigger than the PP gradient. This may not always be the case.

5.1.3 Production stimulation

For wells that use injection of any fluids into the well a burst pressure will be exerted on the casing. The load can be calculated as followed:

$$p_i = p_p + \rho_i g h \qquad 9$$

Where:
ρ_i=density of fluid inside well
p_p=pump pressure
p_i=injection fluid density
h= depth
g=gravitational constant

The frictional loss due to moving fluids is not accounted for.

The pressure outside the casing can be calculated on the same method described under the leaking tubing criteria. The burst pressure becomes:

$$p_{burst} = (p_p + \rho_i g h) - p_o \qquad 10$$

5.1.4 Pressure testing

Testing of the well causes a burst pressure. The load can be calculated in the same way as for production stimulation.

5.2 Collapse loads

Collapse pressure is present when the external pressure is higher than the internal pressure. Collapse pressure is defined as:

$$p_{collapse} = p_o - p_i \qquad 11$$

5.2.1 Cement collapse

Cement collapse is a load that is relevant for both drilling casing and production casing. For big casing sizes and long cemented length, the collapse load can be critical. The load comes from the difference in hydrostatic pressure caused by density differences in the fluids. The maximum load is found at the casing shoe. It can be calculated as:

$$p_o = (\rho_{mw} \cdot h_1 + \rho_c \cdot (h_3 - h_2))g \qquad 12$$

Where:
p_o=*outside pressure*
ρ_{mw}=*mud density*
ρ_c=*cement density*
h_1= *TVD of mud*
h_2=*TVD of TOC*
h_3=*TVD of casing shoe*
g=gravitational constant

The pressure on the inside is the hydrostatic pressure exerted by the fluid inside the casing. This can be calculated as:

$$p_i = \rho_{df} \cdot h_3 \cdot g \qquad 13$$

Where:
p_i=*inside pressure*
ρ_{df}=*displacement fluid*
h_3=*TVD of casing shoe*
g=gravitational constant

Collapse pressure then become:

$$p_{collapse} = (\rho_{mw} \cdot h_1 + \rho_c \cdot (h_3 - h_2))g - \rho_{df} \cdot h_3 \cdot g \qquad 14$$

5.2.2 Drilling collapse:

Drilling collapse can happen if the fluids inside the casing are evacuated due to a thief zone. In a worst case scenario, the whole casing can be evacuated and the internal backup pressure inside the casing is set to zero. The design is normally based on a certain degree of evacuated casing. Either represented by a fraction of the casing length or equalized to the lowest estimated pore pressure in next drilling section (13). It can be calculated as followed:

$$p_o = \rho_{mw/cf} \cdot h \cdot g \tag{15}$$

Where:
p_o=outside pressure
$\rho_{mw/cf}$=mud density/completion fluid density
h=TVD depth of evaluation point
g=gravitational constant

$$p_i = \rho_{mw2} \cdot (h - l) \cdot g \tag{16}$$

Where:
p_i=outside pressure
ρ_{mw2}=mud density
h=TVD depth of evaluation point
l=drop in mud level inside casing
g=gravitational constant

The collapse pressure then become:

$$p_{collapse} = (\rho_{mw/cf} \cdot h - \rho_{mw2} \cdot (h - l))g \tag{17}$$

5.3 Tensional loads

Tensional loads are loads that affect the casing in the axial direction. When the tensional load exceeds the tensional strength, the failure will result in a parted pipe.

5.3.1 Static weight of casing string

Normal load due to the weight of the string itself can be calculated as:

$$F_a = \Delta l \cdot w_{air} + p_i \cdot A_i - p_o \cdot A_o \qquad 18$$

Where:
Δl=length of pipe
w_{air}= weight/m in air
p_i=pressure inside the pipe
A_i=area of inner radius of pipe
p_o=pressure outside the pipe
A_o=area of outer radius of pipe

Where the pressure terms are evaluated at the bottom of the pipe and represent the magnitude of the buoyed weight.

5.3.2 Dynamic forces and shock loads

Dynamic and shock loads usually occur in the operational stage of installing the casing. Stick-slip effects caused by running casing through tight spots and doglegs may cause shock loads on casing. Accidental shock loads caused by top-drive failure or slips failure can also occur. On floating rigs, additional risk of tensional load can occur when casing is fixed by slips on the rig floor, or failures in the heave compensation system. Shock load can be calculated as (15):

$$F_{shock} = \frac{w_a}{g_c} v_{run} v_{sonic} \qquad 19$$

Where:
F_{shock}=shock load axial force [lbf]
w_a=pipe weight per unit length in air [lbm/ft]
g_c=gravity constant [ft/sec^2]
v_{run}=running speed [ft/sec]
v_{sonic}=speed of sound in pipe [ft/sec]

5.3.3 Special loads

5.3.3.1 Temperature and Pressure loads

When the casing hanger is placed in the wellhead and the cement is set, temperature and pressure are the only parameters that can induce tensional load in the casing string. The change in pressure can cause either ballooning or reversed ballooning of the casing string leading to tensional or compressional loads. The difference in temperature will lead to shrinking or expansion of the steel. This can also cause additional tensional or compressional loads. (13)

5.3.3.2 Annular pressure build-up (APB)

APB is a load caused by the thermal expansion of fluids in a closed container. Under production, thermal energy is transported from the reservoir and dissipated across the well. The internal pressure in the annuli is dependent of many variables. The elasticity of production casing and production tubing, along with thermal expansion coefficient of the fluid in the closed annuli are some of the most important factors. (15)

5.3.3.3 Formation load

Production of oil from a reservoir will lead to a depressurization of the reservoir. This depressurization may cause the reservoir rock to compress. This will lead to a subduction of the overlying formation. The moving formation can exert forces that lead to both tensional and collapse load on the casing. (15)

5.4 Summary of casing loads

There are many various loads that can affect the casing during its different operational phases. In the above chapter, the loads are presented as they are uniaxial and operate indistinguishable from each other. Many loads are present at the same time, creating tri-axial/biaxial loads scenarios.

6 Casing degradation

During a well construction process of installing different casings, many of them act as a temporary barrier. Both to ensure the integrity, structural support and formation isolation. As soon as the next casing is set, many of the tasks of the former casing is completed.

For the production casing, this is not the case. The production casing acts as a barrier to ensure well integrity during the whole life cycle of the well. This makes the design task of the production casing different from the other casing strings. NORSOK D-010 states that:

The design process shall cover the complete well or section lifespan encompassing all phases from installation to permanent abandonment and include the effects of material deterioration.

In a life cycle time frame, casing degradation will have a big impact on the production casing's ability to withstand loads.

6.1 Casing wear

Along with the development of more complex wells, including long horizontal sections and multilateral wells, casing wear becomes an important factor in the design process. Casing wear is related to the drilling operation. The wear on the inside of the casing will lead to a reduction in wall thickness, leading to a decrease in strength. There are many parameters influencing the amount of wear. Some of them are (16):

- Well path (Doglegs)
- Contact forces
- Hardness of the materials
- Drilling time
- Drilling mud properties

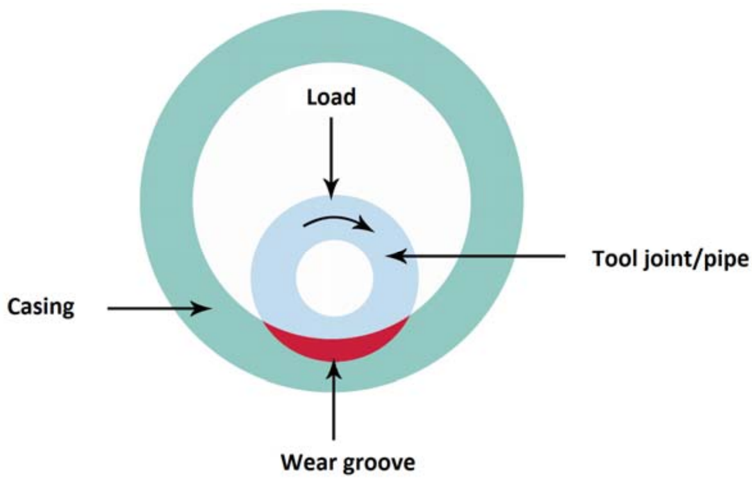


FIGURE 14: ILLUSTRATION OF CASING WEAR

6.1.1 Estimation of casing wear

Several models have been developed to predict casing wear. Bradley and Fontenot (1975)-*The prediction and Control of Casing Wear,* developed a model for estimation of casing wear caused by rotating pipe and wireline operations. Based on experimental testing they developed equations and wear coefficients for predicting the severity of the wear. They concluded that drilling rotation was the main cause of casing wear (17).

White and Dawson (1987) - *Casing Wear: Laboratory Measurements and Field Predictions* conducted experimental full-sized testing of casing wear caused by non-hard-banded tool joints on casing. The test where performed with different grades of casing, and studied in both water-based and oil based-mud with different contact forces. Based on the results from the experimental work, they developed a linear model with respect to contact force to predict casing wear (18).

Hall et al. (1994) - *Contact Pressure Threshold: An Important New Aspect of Casing Wear,* established a mathematical model that predicts casing wear in terms of hole-geometry, casing/tool-joint material, mud system, and drilling program (19). Extensive wear testing was performed and wear factors for different situations was provided. The model was also incorporated into a computer program for planning and operational use. In 2005, they further developed this study by introducing the contact pressure threshold concept. This was based on more than 475, 8 hour tests performed on casings and risers. The proposed model estimated maximum wear groove depth for a given lateral load (20).

The models mentioned above are some of the models with more extensive testing results to support their results. Several other models are also proposed, based on analytical results from logged well parameters.

Measurements of casing wear is often performed in complex wells with extensive drilling sections. This is done with calliper logging and ultrasonic imaging tools. Based on the well data and wear measurements, the casing wear models are back calculated to estimate wear factors. Even if measured field data is used, the models do not accurately estimate casing wear (19).

The inconsistency and difficulty of measuring casing wear is reflected in the choice of applying excessive safety factors leading to over-designed wells in some cases. Despite the many models and methods to estimate casing wear, an accurate method remains elusive (19).

6.1.1.1 White and Dawson Model

The White and Dawson model is based on wear efficiency, which is defined by the fraction of energy adsorbed in the wear to the total work done:

$$k = \frac{Energy\ absorbed\ in\ wear}{Total\ mechanical\ work\ done} = \frac{VH}{\mu F_n x} \qquad 20$$

Where:
k=wear efficiency
H= Brinell Hardness
μ= coefficient of friction between wearing surfaces

F_n=normal contact force
x = the sliding distance
V=volume of steel removed

The sliding distance can be calculated as follows:

$$x = 2\frac{L_h}{v_z p}\omega\pi r_{TJ} \qquad 21$$

Where:

L_h=length of drilling section
v_z=rate of penetration
p= number of tool joints
ω = rotational speed
r_{TJ}=radius of tool joint

Solving for V in equation 20, and inserting right side of equation 21 for x, gives and expression for the volume of steel removed:

$$V = \frac{k\mu F_n}{H}\cdot\frac{2L_h}{v_z p}\omega\pi r_{TJ} \qquad 22$$

Based on the volume steel removed and assumptions of wear groove geometry(see Figure 14 , the depth of groove can be calculated. The normal contact force can be estimated based on axial load and dog leg severity. The wear efficiency is usually empirical found values. Empirical values from the White and Dawson model is shown below (18). Where E is the wear efficiency.

TABLE 3: WEAR EFFICIENCY, WHITE AND DAWSON (18)

TABLE 6—AVERAGE VALUES OF WEAR EFFICIENCY AND *E/H*

Mud Type	Casing Grade	*E*	*E/H** (psi $^{-1}$ x 10 $^{-9}$)
Water	K55	0.00010	0.36
	N80	0.00025	0.81
	P110	0.00063	1.4
Oil	K55	0.0006	2.2
	N80	0.0012	3.9
	P110	0.0017	4.2

**E/H* is the wear efficiency divided by the Brinell hardness. This ratio is used for wear predictions (see Appendix).

6.1.1.2 Mitigation and Control

Mitigation of excessive wear can be done in many ways. Some of them are (21) (22):

- Mud additives and lubricants
- Selection of tool-joint materials
- Casing materials
- Non rotating drill strings/protectors

Logging and measuring metal debris from the mud system using ditch magnets can give information about the severity of the wear during drilling. This method is primarily qualitative, and it can give information in case of excessive wear (23).

6.2 Corrosion

Casing in an oil well can be exposed to corrosive environments. Over time corrosion will lead to degradation and failure of the casing string if not mitigated.

NORSOK-D010 states:

The calculation of the design factor shall take into considerations all applicable factors influencing the materials performance, with emphasis on wall thickness manufacturing tolerance, corrosion and tubular wear over the life cycle of the well.

Corrosion is defined at the deterioration of a material, usually a metal, because of a reaction with its environment. Corrosion can only occur when four elements are present (24):

- Cathode
- Anode
- Electrolyte
- Electrical current

The anodic reaction releases electrons and the cathodic reaction consumes electrons. For a corrosion process to stop, one of the four elements has to be "removed".

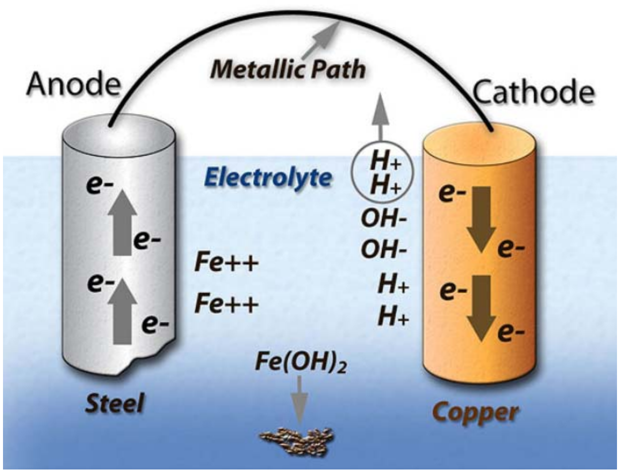


FIGURE 15: CORROSION CELL (25)

Corrosion can be categorized in different ways. For oilfield tubulars the following categorization is convenient (13):

- Uniform corrosion
- Pitting corrosion
- Environmentally induced cracking.

Uniform corrosion is a generalized and ideal of corrosion. It assumes that the corrosion is evenly distributed across the surface. This leads to a uniform material loss, resulting in a constant decrease in wall thickness. Uniformly corrosion is desirable in the sense that it is predictable and therefore easy to account for during a design process. Uniform corrosion is therefore seldom a cause to catastrophic failures (25).

Pitting corrosion is a common cause of corrosion, and is more likely to occur in reality than uniform corrosion. The detection and damage estimation of pitting corrosion is complicated due to its random nature. When pitting corrosion is initiated, it often develop rapidly and cause failure in the material in a short time. Such failure can lead to holes in the casing, thus compromising the integrity (13).

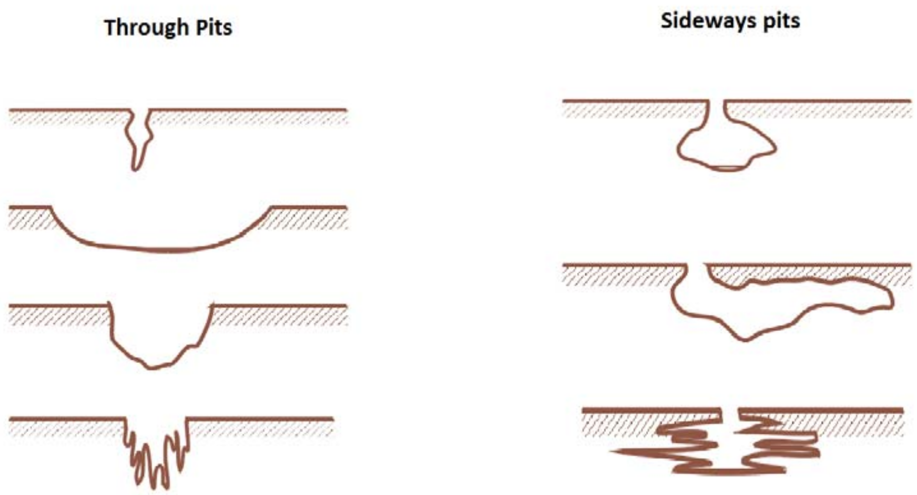


FIGURE 16: COMMON PIT SHAPES (24)

Environmentally induced cracking (EIC) occur when an alloy is under tensile stress in a corrosive environment. EIC is one of the most important corrosion mechanisms in the selection of materials. Stress corrosion cracking (SCC), sulphide stress cracking (SSC) and chloride stress cracking (CSC) are all different forms of EIC (13).

SCC is caused by stress, where the highest stressed area becomes the anode and the lower stressed area becomes the cathode. This results in cracks that can form intergranular in the metal, leading to sudden failure far below its ductile strength limit (24).

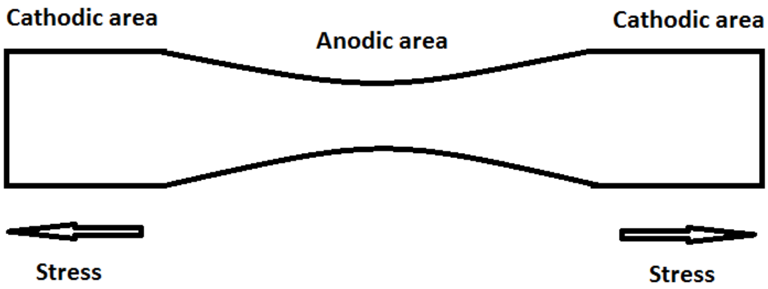


FIGURE 17: STRESS CORROSION CRACKING

SSC and CSC is stress induced cracking in the presence of sulphides or chlorides. Other chemicals can also lead to this type of corrosion. This type of corrosion is usually caused by atomic hydrogen infused into the metal matrix leading to embrittlement damage.

TABLE 4: COMBINATIONS OF SOME ALLOYS AND ENVIRONMENTS THAT PROMOTE EIC (24)

Material	Environments
Al alloys	Chlorides, moist air
Mg alloys	Chloride-chromate mixtures, moist air Nitric acid, fluorides, Sodium hydroxide
Cu alloys	Ammonia, moist air, moist sulfur dioxide
C steels	Nitrates, hydroxides, carbonates Anhydrous ammonia
High strength steels	Moist air, water, chlorides, sulfates, sulfides
Ni alloys	Hydroxides
Ti alloys	Halides, methanol

6.2.1.1 Mitigation

Corrosion can be inhibited by breaking the electrochemical reaction (13). The four main methods are:

- Coatings
- Cathode protection
- Chemical inhibitors
- Change metallurgy

For oil field tubulars chemical inhibitors and change of metallurgy are the most popular approaches. Chemical inhibitors slows the corrosion process by reducing the movement or diffusion of ions to the metallic surface, or by increasing the electrical resistance of the metallic surface (24). When chemical inhibitors are not enough to prevent corrosion, raising of the alloying content in the casing with chrome or nickel is usually done to prevent corrosion (13).

The environmental characteristics is the factor that determines the severity of the corrosion environment. There are many factors that affect this. Some of them are:

- Temperature
- Pressure
- Salinity
- PH
- Presence of H_2S and CO_2

Material selection is important to ensure that they can withstand different corrosive environment. NORSOK M-001 Material Selection covers the recommended guidelines for material selection and corrosion protection for use in well activities.

6.3 Cyclic loads

Casing loads have traditionally been assessed as a static condition. Along with more complicated wells with long drilling sections, HPHT conditions and injection wells, the casing get exposed to several cyclic loads.

Fatigue induced while running casing comes from the variations in running speed when installing the casing. Tripping speed, deceleration and axial stick slip are the main variables that control the magnitude of the load. Most cycles and minimum load is experienced by the first casing joint. Least cycles and maximum load are experienced by the casing hanger joint. This inverse relationship between the cycles and loads lowers the total impact of this kind of fatigue.

Drilling induced fatigue arises due to the vibrational forces from the drill string slamming against the inner side of the casing during drilling. The impact of this load is considered small, but in some cases it can lead to break-out of the casing connections. (26)

Temperature variations causes different kinds of cycles loads. These can be related to axial load, internal pressure load and collapse load, and usually a combination of them. The loads come from change in temperature. Some of the factors causing this is:

- Start/stop of production
- Change in production rate
- Injection of gas or water

7 Design methods

The main task of a design process is to ensure that the strength of the materials used exceed the loads that the materials are exposed to during its lifetime. Several different methodologies can be applied in a design process. Regardless of the chosen approach, there are some similarities in the design process. The design process usually consist of the following steps (13):

- Identify all load scenarios and estimate the load parameters
- Calculate the load at different positions in the string
- Calculate the strength
- Check the design and make adjustments

7.1 Working stress design (WSD)

Working stress design is the most used and widely accepted design approach for downhole tubulars in the oil industry. To account for uncertainties in both load and strength, a safety factor is applied. The equation and figure below illustrates the principle for WSD (13):

$$Design\ strenght \geq Load \times Safety\ facor \qquad 23$$

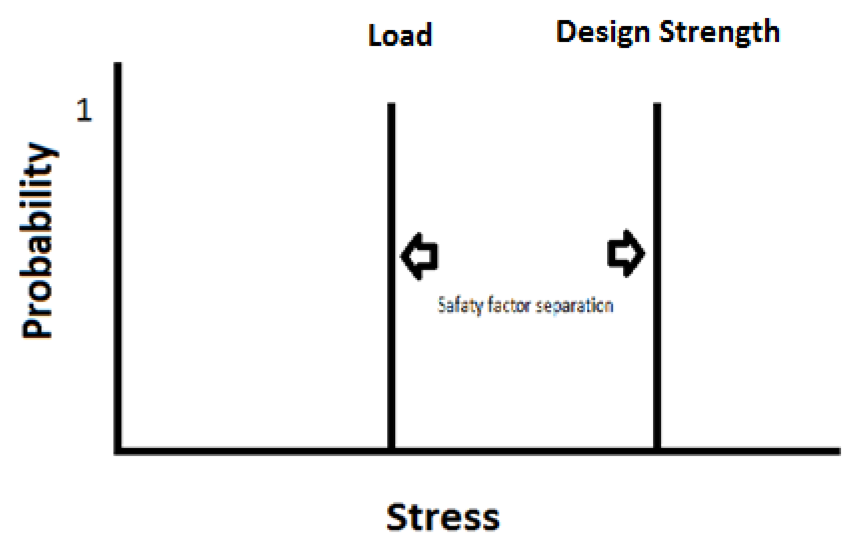


FIGURE 18: DETERMINISTIC STRENGTH AND LOAD

A typical approach for WSD is to define a set of load cases based on burst, collapse and tensional loads. The standards load cases that are assessed, are mentioned in chapter 5. The most critical load that is anticipated becomes the design load. When the design loads are determined, calculation of strength can be done (15).

The strength equations that are used in a design process are usually described as failure criteria in the literature. In a WSD the failure criteria are based on the elastic deformation principle. When the casing exceeds the elastic deformation region the casing is considered failed. A failure in WSD therefore represent a permanent deformation in the material rather than a complete failure (15). This is shown in figure 19.

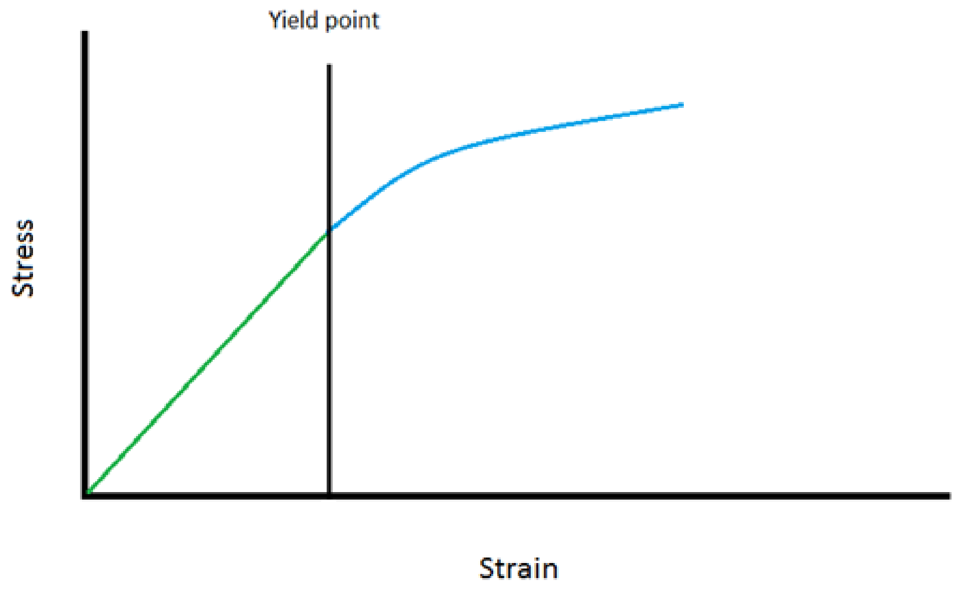


FIGURE 19: FAILURE CRITERIA WSD

7.1.1 Strength estimation in WSD

The strength estimation in a WSD is usually carried out with a one/two (biaxial) stress analysis and then outlined by a tri-axial analysis. The historical API equations and the von Mises maximum distortion energy theory shall be the basis for the strength estimation according to NORSOK-D010 (ref: figure 5). These equations are given in the ISO 10400 and API-TR5C3 standards.

7.1.1.1 Burst strength

API-burst equation is based on a one dimensional approximation of the von Mises yield criteria combined with thin wall pipe theory (27). The equation approximates the hoop stress and equates this to the yield strength (28).

$$P_y = \frac{T \cdot \sigma_Y \cdot 2t}{D_o} \qquad 24$$

Where:
P_y= Internal yield pressure
T= Manufacturing tolerance, typical set to 0,875
σ_y=Specified minimum yield strength
t= Pipe wall thickness
D_o= Outside diameter

7.1.1.2 Collapse strength

API-collapse equations are a set of four equations that are used to calculate collapse strength. The origin of the equations are based on empirical tests of large number of pipes. The API- collapse equations are based on the yield strength and the D_o/t ratio (29).

If the D_o/t is high, the strength is governed by geometrical failure, due to instability. With decreasing D_o/t ratio the strength approaches the yield limit. This is illustrated in the figure below. API also provide biaxial correction factors for decrease in collapse pressure due to axial load (13).

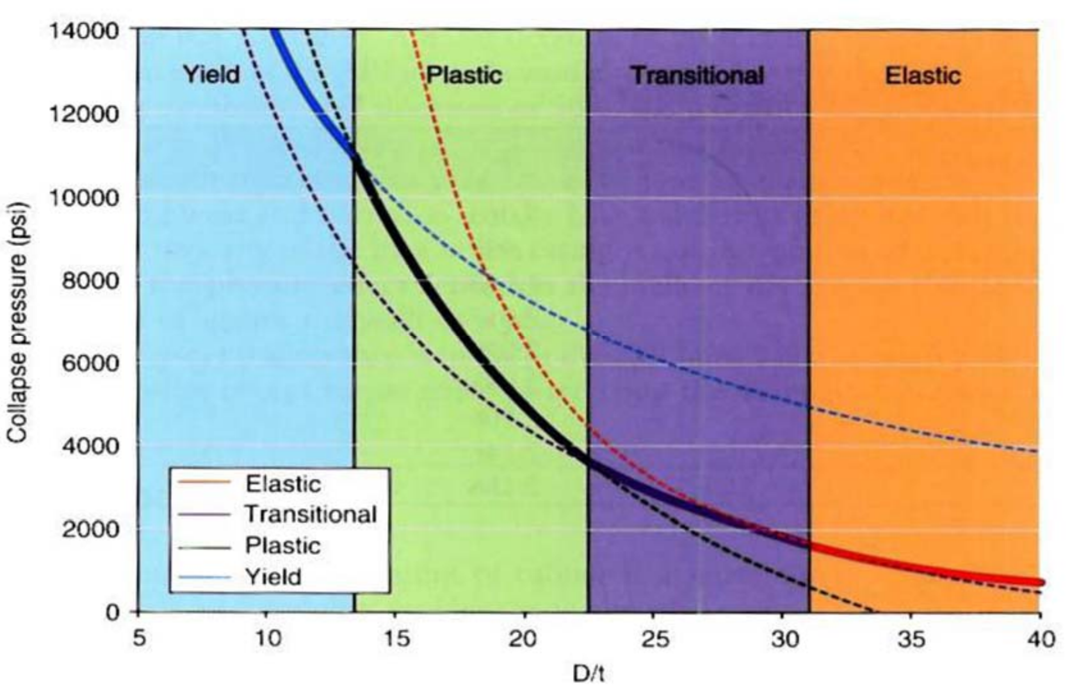


FIGURE 20: COLLAPSE PRESSURE FOR DIFFERENT GRADE(LINES) AND D_o/T RATIO (29)

7.1.1.3 Tensional Strength

API tensional strength is calculated based on assumptions that the stress is distributed evenly across the cross sectional area. It can be expressed as:

$$F_y = \sigma_Y \cdot A \qquad 25$$

Where:
F_y= Axial load
σ_y=Specified minimum yield strength
A= cross sectional area of the pipe

7.1.1.4 Von Mises criteria

In supplement to the one dimensional criteria, the tri-axial von Mises criteria is applied (14). The von Mises criteria is based on (24):

- Radial and circumferential stress as determined by the Lamè equations for thick cylinder
- Uniform axial stress due to all sources except bending
- Axial bending stress for a Timoshenko beam
- Torsional shear stress due to a moment aligned with the axis of the pipe.

The von-Mises equation:

$$\sigma_{VME} = \sqrt{(\frac{1}{2}\left((\sigma_\theta - \sigma_r) + (\sigma_r - \sigma_a) + (\sigma_a - \sigma_\theta)\right) + 3\tau^2)} \qquad 26$$

The interpretation of this equation can be seen in figure 5 (blue ellipse).

7.1.1.5 Discussion

In general, WSD have been a successful approach. The simplicity of the model makes it easy to understand and apply. Under ideal conditions when the casing, load and safety factor are within its specification, the design stands up to the ravages of time.

There are several limitations regarding WSD design. The deterministic nature is one of them. To account for the variations and uncertainties in the load and strength estimation, a safety factor is applied. On NCS these safety factors are specified by NORSOK-D010:

TABLE 5: DESIGN FACTORS NORSOK-D010

Parameter	Design factor*	Supplementary requirement/information
Burst	1,10	
Collapse	1,10	
Axial	1,25	For well testing a design factor of 1,50 should be used to cater for pulling the packer free at the end of the test.
Tri-axial	1,25	Tri-axial design factors are not relevant for connections

*The above design factors are based on wall thickness manufacturing tolerance of minus 12,5%.

However, there is not specified any risk measurement to these safety factors. Any documented reasons for choosing a specific safety factor is rarely documented (13). This result is a safety-factor consistent design, but not risk consistent design. With specified failure criteria and safety factors, the risk of failure then becomes a function of the accuracy and identifications of the load cases (13).

As an example consider the two load cases, *leaking tubing* and *pressure testing*. Pressure testing is an intentional load that will happen with 100% certainty. While leaking tubing is an accidental load with big uncertainty in appearance. If the magnitude and consequences of such load where equal, there would have been much more risk associated with pressure testing than leaking tubing due to the possibility of occurrence. Such considerations are not assessed in WSD.

The WSD method have also been criticized for being conservative. Leading to over dimensioned wells.

Consider the Barlow equation (equation 24) and assume that this equation represents the true yield limit. The equation first applies the manufacturer's tolerance T. A typical value for this is 0,875. Then it uses the nominal value of the diameter, yield strength and thickness to calculate the burst strength, which is the often underestimated compared to real values (13). On top of this a safety factor of 1.1 is applied. The numbers are shown in the table below. The measured values are provided from table 2.2 in the book *Advance Drilling and Well Technology* (13). Here it is also shown that real strength value varies statistically with a given mean and standard deviation.

TABLE 6: VALUES FROM TABLE 2.2 *ADVANCED DRILLING AND WELL TECHNOLOGY* (13)

9 ⅝ inch casing, 47lbm/ft, L80 grade			
Parameter	**Nominal value**	**Mean measured value**	**Std.dev measured values Norm.dist**
Outside diameter	9.625 in	9.635 in	0.003
Wall thickness	0.472 in	0.479 in	0.003
Yield strength	80000 psi	87000 psi	2751
Barlow strength	7846 psi	8650 psi	279
Tolerance 0,875	6865psi		
Safety factor 1.1	6241psi		
Design strength	6241psi		

The difference in burst strength is:

$$\Delta P = 8650\ psi - 6241\ psi = 2409\ psi \qquad 27$$

The probability to pick a casing with strength of less than 6241psi is:

$$P_p = 2.9515 \cdot\ 10^{-18} \qquad 28$$

When considering the probability distribution of the strength given in the table above, then the amount of joints (13meter) that must be used to expect one with burst strength less than 6241psi are:

$$N = \frac{1}{2.95 \cdot\ 10^{-18}} = 3.38 \cdot 10^{17}\ casing\ joints \qquad 29$$

This exemplifies some of the concerns regarding overdesign using WSD.

7.2 Limit States design (LSD)

Limit states design is based on a limit state rather than a working stress. The limit state term is the description of the limit where the casing does not longer perform its intended function. In LSD there are usually two addressed criteria. This is (13):

- Ultimate limit strength (ULS)
- Serviceability limit strength (SLS)

The ULS refers to the actual strength of the material. This criterion allows the material to use all of its strength capabilities. This include excessive deformations, fractures and necking of the casing. The serviceability strength is addressed to its limit where the component ceases to be operational for its intended function (30).

A collapse load exceeding the ULS would lead to rupture of the pipe leading to leakage between the annulus. A collapse load exceeding the SLS criteria would only lead to a deformation of the casing to an elliptical shape. This may lead to problems to get equipment down the well, even though the casing does not leak.

LSD therefore brings in the opportunity to make different criteria for different types of loads. In this type of design, the loads are often divided into two groups.

- Survival loads
- Operational loads

Survival loads are generally defined as infrequent loads with large uncertainties in magnitude. Survival loads can lead to catastrophically results (31). For survival load the ULS criteria is often applied. Operational loads are usually more frequent and a SLS criteria is applied for this.

Limit state design is widely accepted and used in civil engineering, mainly in steels structures and concrete design (32). It is also used in the oil industry within offshore structures and pipelines. But it has not been extensive used for well design (13). ISO 10400 has implemented equations which represent the ULS for casing under various types of loads.

LSD use safety factors to account for the uncertainties in the load estimation. The main difference between LSD and WSD are therefore the failure criteria. Where WSD only allows the tubular to reach the yield limit, LSD allow for the pipe to reach the ULS. Both the WSD and LSD are deterministic methods. But LSD bring risk into the design in a limited way, based on its categorization of different criteria for survival and operational load. The difference between WSD and LSD is illustrated in Figure 21.

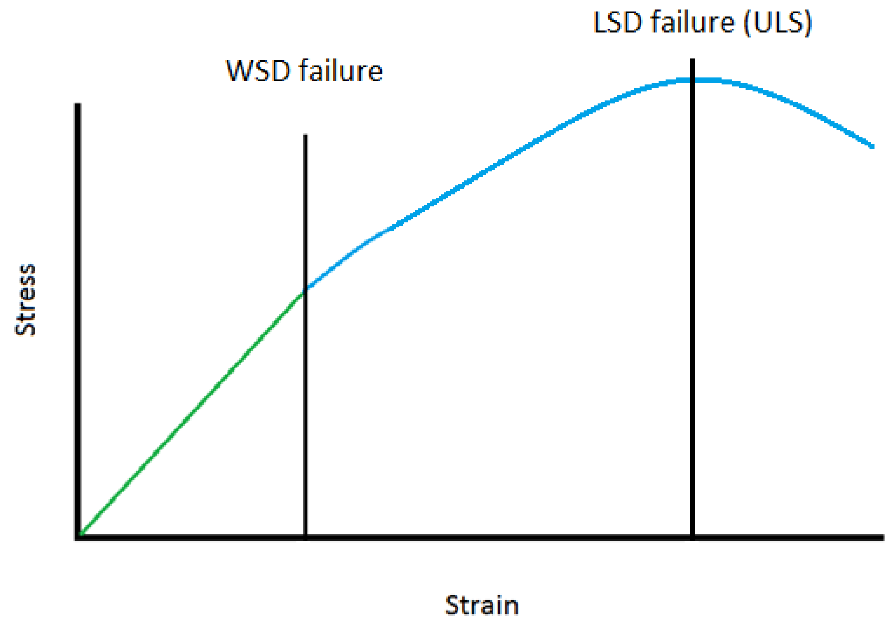


FIGURE 21: WSD FAILURE CRITERIA VS LSD FAILURE CRITERIA (ULS)

8 Reliability based design (RBD)

In Reliability based design, load and strength are explicitly recognized as random distributed variables. Every variable that affects the casing design can be represented by an underlying probability density function (PDF). A PDF shows which values that are most likely to occur, represented by the area under the graph.

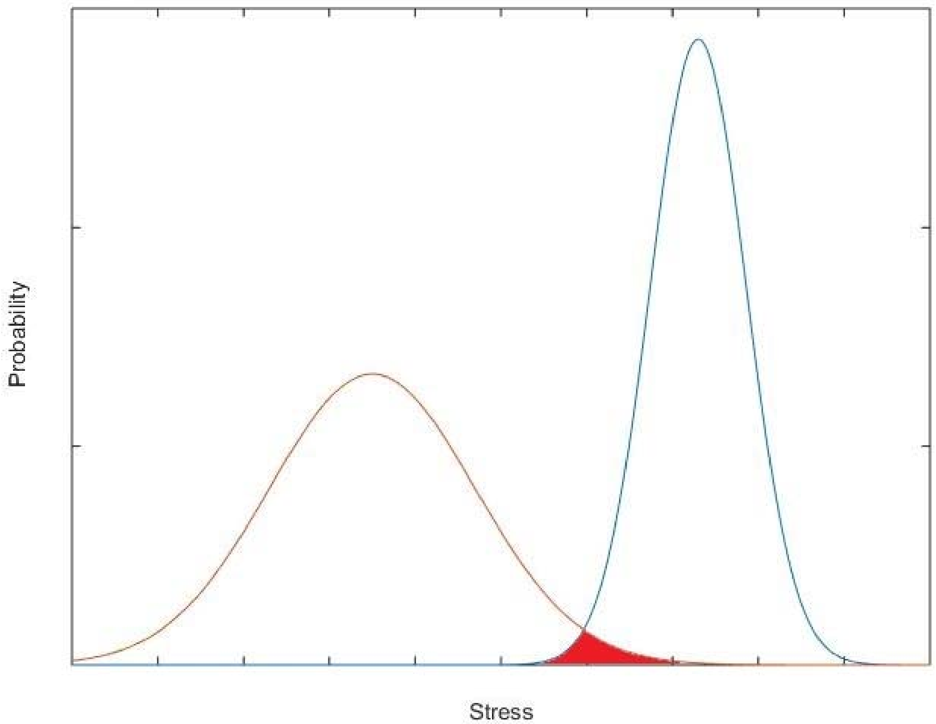


FIGURE 22: LOAD-STRENGTH INTERFERENCE

Figure 22 illustrate the PDF of the load and strength. The intersection between the load and strength shows that it exists a possibility for failure. The chance of failure is related to the red area, but do not represent the numerical value.

From chapter 5 and 6 it is clear that both load and strength estimation is based on many variables. The variabilities and uncertainties in load and strength estimation may not conform to a specific PDF, and the true distributions must be respected (13).

Both WSD and LSD can be used as failure criteria in RBD. The fundamental difference is that every variable is considered a random variable with an underlying distribution. This makes it possible to quantify chance of failure (33).

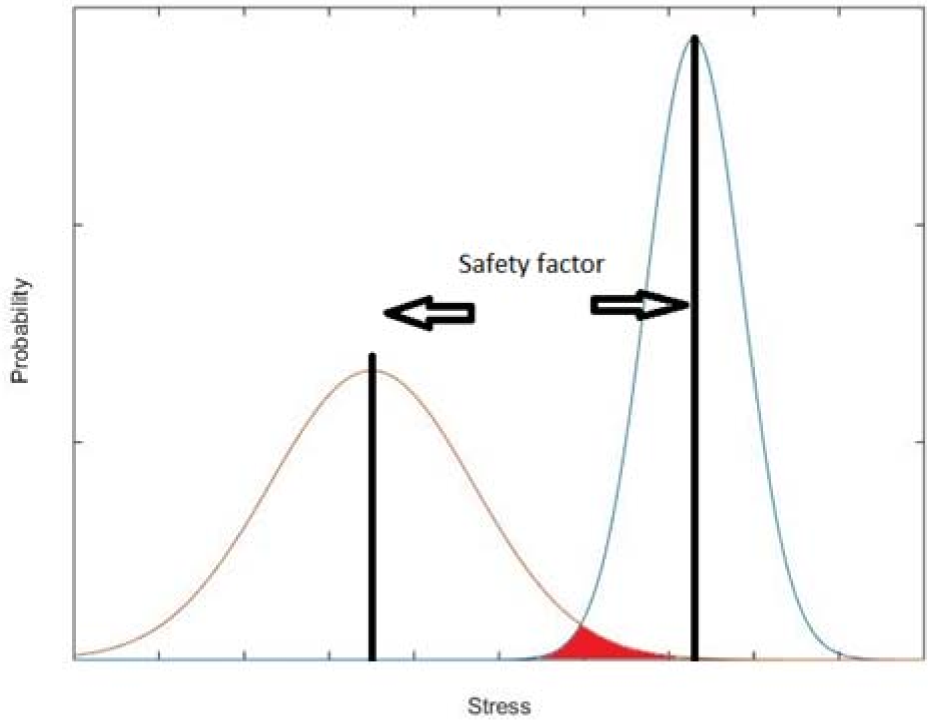


FIGURE 23: COMPARISON DETERMINISTIC VS PROBABILISTIC

The difference between a deterministic approach and RBD is illustrated in Figure 23. The red area is related to the failure probability. If the distributions were wider, the failure probability would increase. This would not be reflected in a deterministic approach.

8.1.1.1 Load estimation

Load estimations in RBD are a difficult task. Many of the parameters that affect the load are not always easily measurable. Well parameters can be correlated from different sources, such as seismic data and logged parameters for nearby wells, but the accuracy of this correlation is difficult to quantify.

The well loads are also influenced by operational procedures and human interactions. An example of this can be the possibility for a driller to discover a kick. Estimation and identification of such variables are difficult. The challenges in load estimation is reflected in the distributions, in the form of a wider shape.

8.1.1.2 Strength estimation

Strength estimation is related to the parameters that control the strength of the pipe. Some of them are (34):

- Yield strength
- Toughness
- Ductility
- Durability
- Geometrical properties

The estimation and distribution of such parameters are easily found by merely measuring and testing the pipe.

In 2007 ISO-10400 implemented equations for LSD. For the Klever-Tamano collapse equations they also added complete statistical data. The progress in this direction make it easier to apply RBD in practice. The precise estimation of strength, is reflected in a narrower distribution of the strength.

8.1.1.3 Reliability based design in oilfield tubular

Reliability based design is not a new thing. API performed one of the first statistically based design within tubing in 1963. They tested 2900 samples of casing with various, sizes, weights and grades. These data were used to develop the API collapse ratings for the plastic regime of collapse, with a target probability of 0.5 % for the equations. Although the equations did not include statistical considerations, the origin and accuracy of them were defined in a probabilistic way (35) (31).

Payne et al. (1989)-*Application of probabilistic reliability methods for tubing design,* proposed a new method to account for varieties in tubing load and tubing performance. The proposed method would be able to quantify the safety for a given design. It was also stated that classical design methods (WSD) promotes overdesign because of conservativeness in tubing performance. They also emphasized the need for improvement in data collection of load and failure data to optimize estimations (35).

Lewis et al (1994)-*Reliability based Design and Application of Drilling Tubulars,* present a RBD approach based on the LSD with equations to apply for ULS. They concluded that RBD is necessary to eliminate the conservatism of WSD (36).

Hinton (1998)-*Will Risk Based Casing Mean Safer Wells?,* discuss how RBD can lead to safer wells. The importance of a thoroughly understanding of the RBD was an important factor, along with more accurate failure equations. If these areas were assessed, he concluded that RBD design would be a safe method to use (37).

Das et al.-*A Model for Well Reliability Analysis throughout the life of a well using barrier engineering and performance,* proposed a unique well reliability model for estimating the reliability of casing design. The model takes load, strength and degradation factors into account. Based on a computer simulated approach, a stress modification factor was incorporated into the reliability equations to account for changing well conditions. They concluded that if their stress modification factor was established for different casings, it could be used to calculate the reliability for the life cycle of the well (1).

Suryanarayana et al. *A Reliability-Based Approach for survival design in Deepwater and High Pressure/High Temperature Wells*, investigated how RBD approaches could be applied to survival design of critical wells, especially in HPHT conditions. They state that WSD fails to quantify risk for survival load and can lead to outcome that is not optimal. They proposed a model with a deterministic load and RBD strength, that can be applied in design of survival loads for casing (31).

The literature on RBD seems to recognize the benefits of quantifying risk of failure. The quantification of failure along with reduced costs is some of the incentives. Some of the identified challenges are related to the complexity of RBD, along with lack of data for strength/load distributions. As for today’s status, RBD approach is still a special case and not the norm in tubular design.

8.2 Reliability mathematics

If the true Probability Density Function (PDF) for both load and strength are known, the probability of failure can be calculated. This can mathematically be expressed as (13):

$$g(\tilde{s}) = S(\tilde{x}) - L(\tilde{y}) \qquad 30$$

Where:
$g(\tilde{s})$=failure/survival distribution
$S(\tilde{x})$=strength distribution
$L(\tilde{y})$=load distribution

The probability of failure and survivor can be expressed as:

$$F(s) = P(g(s) < 0) \qquad 31$$

$$R(s) = P(g(s) > 0) \qquad 32$$

For a binary system with two possible outcomes of either failure or survivor, the relationship between failure and survival is:

$$R + F = 1 \qquad 33$$

This relationship can be illustrated as follows:

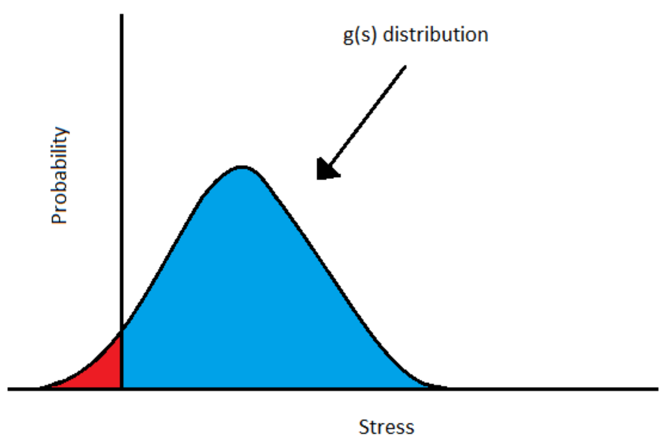


FIGURE 24: RED AREA REPRESENT THE FAILURE FUNCTION F(S), BLUE AREA REPRESENT THE SURVIVAL FUNCTION R(S)

The figure above is for a static situation and can be solved with an analytical approach. An analytical approach for a model that is time dependent with several distributions is a difficult task, and not always possible. In complicated time dependent models, it is often preferable to use a Monte Carlo simulation (38).

8.2.1.1 Well Reliability

Reliability in an oil well is a function of the reliability of all WBE that acts together in the well. For a single WBE the reliability is:

$$R_i(t) = 1 - F_i(t) \qquad 34$$

Where
$R_i(t)$ = Probability that a WBE function until time t=T (survivor function)
$F_i(t)$= Probability that a WBE has failed at time t=T (failure function)

The failure function is related to the underlying PDF. This relation is:

$$F_i(t) = \int_0^t f_i(u)\, du \qquad 35$$

Where
$F_i(t)$ = Probability that a WBE has failed at time t=T (failure function)
$f_i(u)$= Lifetime distribution

$f_i(t)$ is the PDF for the lifetime distribution. The statistical data is fully defined by the lifetime PDF. Identification of this lifetime distribution is therefore the essential task in a reliability analysis.

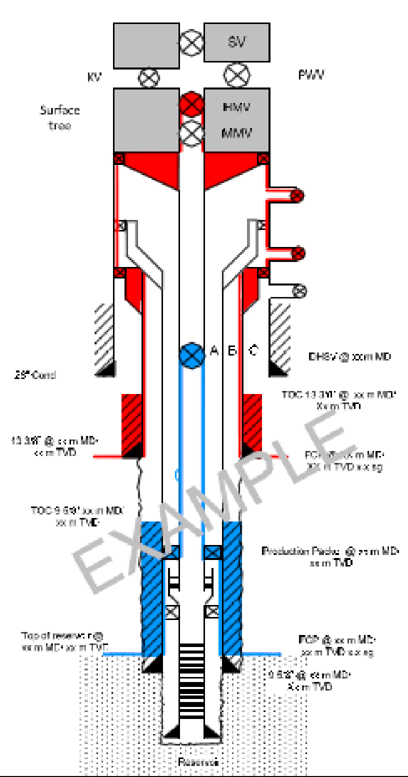
One of the most important factor in reliability modelling is the failure rate. The failure rate is defined as:

$$z_i(t) = \frac{f_i(t)}{R_i(t)} \qquad 36$$

Failure rate describe the probability for failure at time t=T, given that it has survived until time t=T.

According to NORSOK-D010 there shall always be two barriers if there is a potential of flow (2). These are called the primary barrier and the secondary barriers. Each barrier consists of several WBE. Below is an example of a well barrier schematics, followed by equations on how to calculate reliability for such a system.

Well information (as built)	
Field/Installation:	
Well name:	
Well type:	Producer
Well status:	Shut-in
Well design pressure:	xx bar
Revision number:	Version 1.0
Date prepared/revised:	
Prepared by: Verified/Approved by:	



Note 1: Well shut-in due to tubing leak above DHSV.

Nomenclature:
PT: Pressure test
IT: Inflow test
FCP: fracture closure pressure
XLOT: extended leak off test
AC: acceptance criteria

Well barrier elements	EAC table	Verification	Monitoring
Primary well barrier			
In-situ formation (cap rock)	51	FCP: x.x s.g. Based on field model	n/a after initial verification
Casing cement (9 5/8")	22	Length: xx mMD Cement bond logs	Daily pressure monitoring of B-annulus
Casing (9 5/8")	2	PT: xx bar with x s.g. EMW	n/a after initial verification
Production packer	7	PT: xx bar with x s.g. EMW	Continuous pressure monitoring of A-annulus
Completion string	25	PT: xx bar with x s.g. EMW	Continuous pressure monitoring of A-annulus See Note 1.
Completion string component (Chemical injection valve)	29	PT: xx bar with x s.g. EMW	Periodic leak testing AC DHSV: xx bar/xx min
Downhole safety valve (incl. control line)	8	IT: xx bar (DHSV) PT: xx bar (control line)	Periodic leak testing AC DHSV: xx bar/xx min
Secondary well barrier			
In-situ formation (13 3/8" shoe)	51	FCP: x.x s.g. Based on XLOT	n/a after initial verification
Casing cement (13 3/8")	22	Length: xx mMD Method: Volume control	Daily pressure monitoring of C-annulus
Casing (13 3/8")	2	PT: xx bar with x s.g. EMW	Daily pressure monitoring of C-annulus
Wellhead (Casing hanger with seal assembly)	5	PT: xx bar	Daily pressure monitoring of C-annulus/ Periodic leak testing
Wellhead / annulus access valves	12	PT: xx bar	Periodic leak testing of valve AC: xx bar/xx min.
Tubing hanger (body seals and neck seal)	10	PT: xx bar	Periodic leak testing
Wellhead (WH/XT Connector)	5	PT: xx bar	Periodic leak testing
Surface tree	33	PT: xx bar	Periodic leak testing of valves AC: xx bar/xx min

FIGURE 25: WBS EXAMPLE (2)

The primary and secondary barriers are in a parallel configuration. The single WBE's that form a well barrier envelope are in series configuration.

Reliability for the primary barrier can be expressed as:

$$R_p(t) = \prod_{i=1}^{n} R_i(t) \qquad 37$$

Where:
$R_p(t)$= Reliability of the primary barrier
$R_i(t)$= Reliability of each WBE in the primary barrier

Reliability of the secondary barrier can be expressed as:

$$R_s(t) = \prod_{k=1}^{j} R_k(t) \qquad 38$$

Where:
$R_s(t)$= Reliability of the secondary barrier
$R_k(t)$= Reliability of each WBE in the secondary barrier.

These two barriers act together in a parallel construction that increases the redundancy. The total well reliability for m barriers can be expressed as:

$$R_t(t) = 1 - \prod_{l=1}^{m=2} \big(1 - R_l(t)\big) \qquad 39$$

$$R_t(t) = \prod_{l=1}^{m=2} R_l(t) \qquad 40$$

Where
$R_t(t)$= total reliability
$R_l(t)$= reliability of each barrier envelope

The failure rate for the primary and secondary barriers can be expressed as (39):

$$z_p(t) = \sum_{i=1}^{n} z_i(t) \qquad 41$$

$$z_s(t) = \sum_{k=1}^{j} z_k(t) \quad 42$$

Where
$z_p(t)$= primary barrier failure rate
$z_s(t)$= secondary barrier failure rate

If the reliability for each WBE element is known, the reliability for the well can be calculated. The assumptions for these equations to be valid, is that each WBE is considered independent components.

This is not always the case in a real scenario. As an example, consider a situation with high APB. This can lead to a burst casing or collapsed tubing. The reliability of the casing then become dependent on tubing collapse or not. Further consideration of such problems is beyond the scope of this thesis.

What is important to emphasize is that WBS's will change during the life cycle of the well, dependent on its operational phase.

8.3 A model for Reliability based design

This section will present a computer based method for estimation of the lifetime distribution. The model is inspired by the approach described by Das Bibek and Samuel Robello in "*A Model for Well Reliability Analysis throughout the lifetime the Life of a Well Using Barrier Engineering and Performance* (1). The well data that is used is based on an example from the book *Modern Well Design* by Aadnøy, Bernt (p.166) (14). The casing parameters used is based on table 2.2 in the book *Advanced Drilling and Well Technology* (13).

The purpose of this model is to show how Monte Carlo simulation can be used to generate statistical data to create life time distributions and reliability data. This can be used if empirical data is lacking. The data does not reflect a real scenario, but is accurate enough to demonstrate the methodology. Appendix A show the full MATLAB code.

8.3.1 Method

The computer based method is based on Monte Carlo simulation conducted in MATLAB. The Monte Carlo simulation create realizations of the model. The data from these realizations is used to estimate reliability data.

The simulation takes burst failure into consideration, and asses the reliability related to the production casing.

The lifetime of the well is assumed to be 240 months (20 years). Time is simulated by increments of 1 month. For each month, the load/strength calculations taking degradation into account, are compared. During the simulation the failure can occur at any time during the 240 months. The casing that survives beyond the 240 months are right censored.

The operational stages that are simulated are:

- Drilling phase
- Production phase

The loads that are taken into account are:

- Kick load
- Leaking tubing load

The loads are assumed to take place every month.

The kick load is evaluated at the wellhead, were the anticipated load is most critical. Leaking tubing load is evaluated at production packer depth were the load is at its maximum.

TABLE 7: TIME SCHEME FOR MODEL

Operational phase	Time	Load
Drilling	2 months	Kick Load
Production	238 moths	Leaking tubing load

The structure of the model shown in Figure 26 (next page). The outer loop is the Monte Carlo simulation. The reason for using this is to generate failure data for a large number of casing strings, with random numbers chosen from different distributions related to load, strength and degradation. For each Monte Carlo iteration, a life cycle of the well is completed. During a life cycle, a failure can occur during the 240 months. If the casing survives, it will be treated as right censored in the calculations.

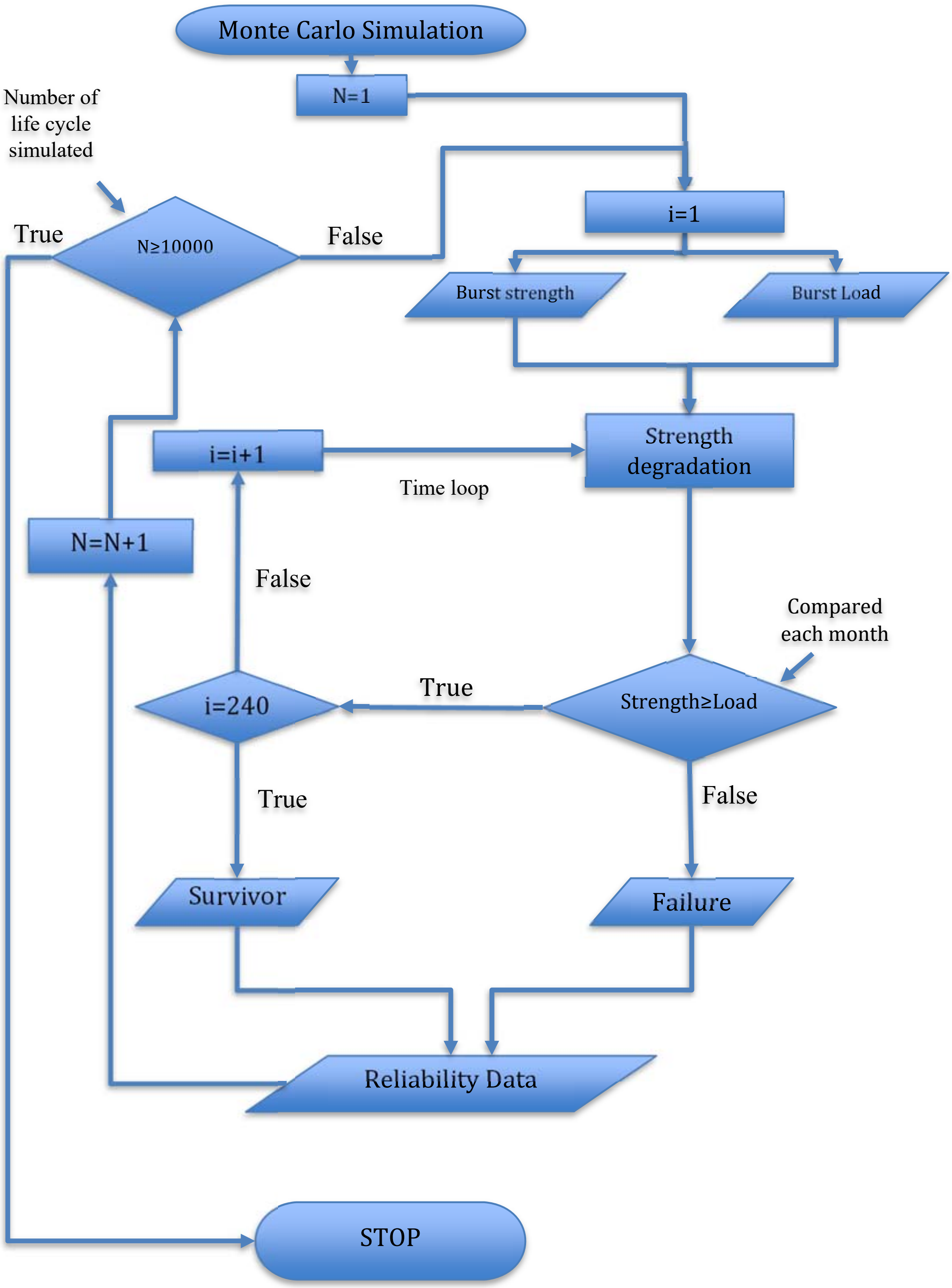
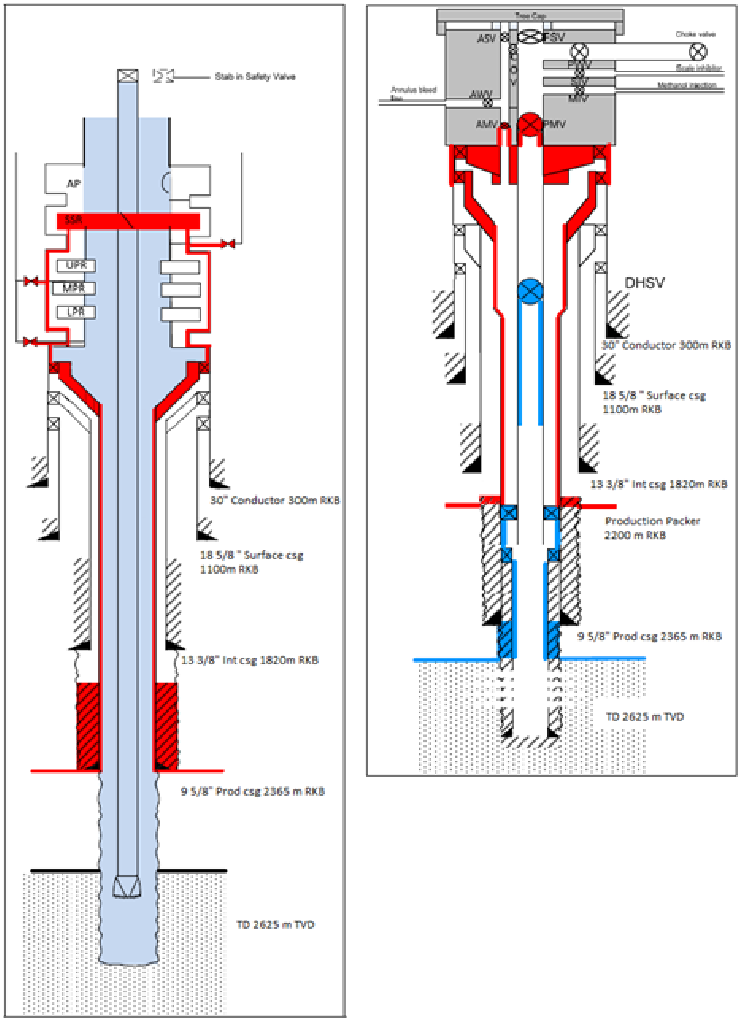


FIGURE 26: FLOW CHART OF SIMULATION

8.3.2 Well parameters and WBS

TABLE 8: WELL PARAMETERS

Parameter	**RKB depth**
Depth casing shoe	2365m
Depth to production packer	2200m
Depth to seabed	225m
Mean sea level depth	25m
Pore pressure gradient	2.00 s.g
Formation fluid gradient	0.76 s.g
Mud density	1.70 s.g
Completion Fluid density	1.10 s.g



Drilling phase

Production phase

FIGURE 27: SIMPLIFIED WBS FOR SIMULATION MODEL (2)

8.3.3 Strength estimation

The criteria used for strength failure is the Barlow equation (28). Manufacturer tolerance factor is not accounted for.

$$P_i = \frac{\sigma_Y \cdot 2t}{D_o} \qquad 43$$

Where:
σ_Y= yield strength of pipe [psi]
t = thickness of the pipe [inch]
D_o= outside diameter of the pipe [inch]
P_i= burst pressure [psi]

The casing simulated is 9 ⅝ inch L80 grade. The parameters used to calculate the strength have a normal distribution with specified values shown in the table below (13).

TABLE 9: CASING PARAMETERS

Parameter	Distribution	Mean Values	Standard Deviation
D_o	Normal	9.635 in	0.003
Wall thickness	Normal	0.479 in	0.003
Yield strength	Normal	87000 psi	2751

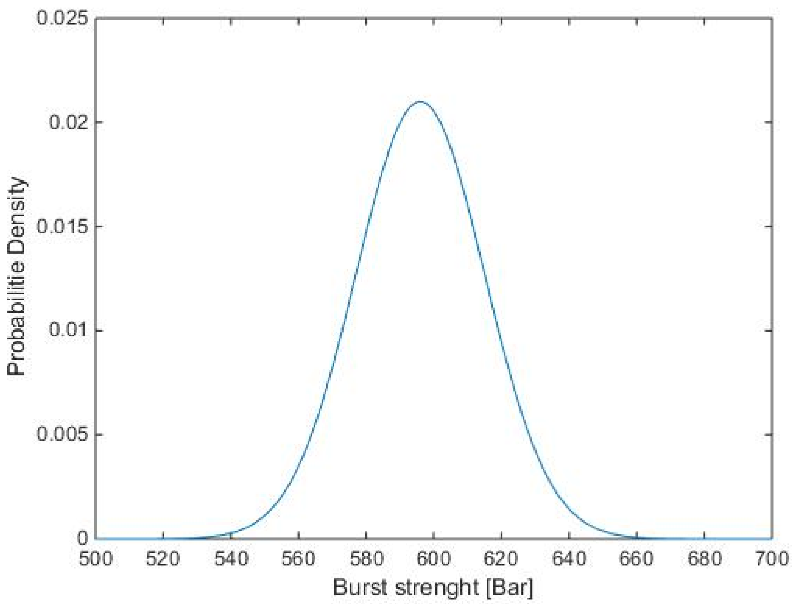


FIGURE 28: INITIAL DISTRIBUTION OF STRENGTH

8.3.4 Strength degradation factors

Degradation factors during the drilling phase is related to decrease in wall thickness due to wear. In the production phase, decrease in wall thickness is assumed related to corrosion. The corrosion is considered uniform.

The decrease in wall thickness is based on a triangular distribution, $\widetilde{T}$(a,b,c). Were *a* is the minimum value, *b* is the most likely value and *c* is the maximum value.

TABLE 10: STRENGTH DEGRADATION PARAMETERS

Wall thickness	Distribution	a	b	c
Drilling	Triangular	0.9617692	0.9746794	0.9874208
Production	Triangular	0.9980710	0.9990072	0.9997728

The decrease in burst pressure is calculated as followed:

$$P_{i+1} = P_i \cdot \widetilde{T}(a, b, c) \qquad 44$$

Where:
P_{i+1}=*burst strength in month i+1*
P_i= *burst strength in month i*
$\widetilde{T}$*(a,b,c)= random values from triangular distribution*

8.3.5 Load estimation

Load estimations are based on estimated distributions for the well parameters. The load is considered as an initial value, thus being constant over the lifetime of the well. The load parameters that are considered as distributions are:

- Pore pressure gradient
- Formation fluid gradient

TABLE 11: LOAD PARAMETERS

Parameter	Distribution	Mean	Standard deviation
Pore Pressure	Normal distribution	2.0 s.g	0.30
Formation fluid	Normal distribution	0.76 s.g	0.114

8.3.5.1 Kick load

Kick load is calculated using the formula for MASP at “gas-to-surface”, see equation 3. Pressure on the outside of the casing is based on seawater gradient to packer.

8.3.5.2 Leaking tubing load

Leaking tubing is calculated using equation 8. With completion fluid in annulus, and seawater gradient pressure on the outside of the casing.

8.4 Results

The results are based on 10000 life cycle simulations, each 240 months. Complete failure data is shown in appendix B.

8.4.1 Simulated results

A total of 2.57% of the production casings failed before the target lifetime of 240 months.

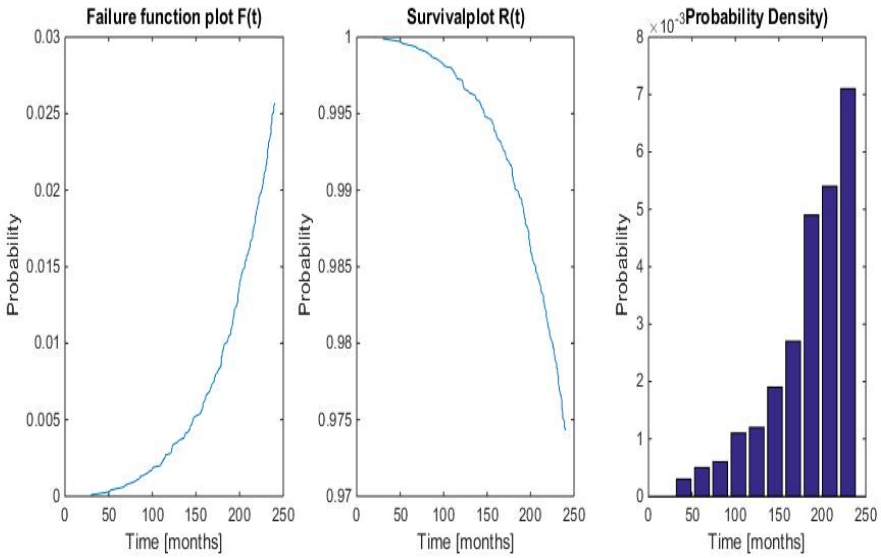


FIGURE 29: SIMULATED FAILURE DATA

The failure function F(t) shows an increasing trend with time and have a convex form. Survival function R(t) shows a decrease in survived casings with time. The lifetime PDF shows that there is a higher percentage of failures with increased age (time).

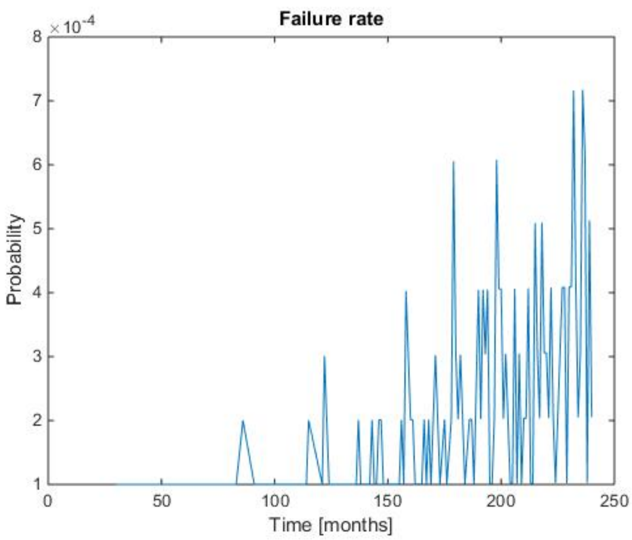


FIGURE 30: SIMULATED FAILURE RATE Z(T)

From Figure 30 it is shown that the simulated failure rate is rough. But the underlying trend clearly shows an indication of increased failure rate.

8.4.2 Identification of the life time distribution

Analysis of the failure data obtained by the simulations is done to identify the underlying lifetime distribution. Nelson plotting (hazard plotting) is used to identify the underlying distribution.

Nelson estimator:

$$N(t_i) = \sum_{j:dj=1,t_i\leq T} \frac{1}{n-j+1} \qquad 45$$

Where:
N(t)= Nelson estimator
n= number of life cycles (10000 in this case)
j= failure number, when the failures are sorted based on time at failure

The Nelson estimator is plotted against time of failure, $(N\ (t_i), t_i)$. An increasing Nelson plot indicates a lifetime distribution with increasing failure rate (38). For a large number of tests, it has been shown that the Nelson estimator approximates the cumulative failure rate Z(t) (38).

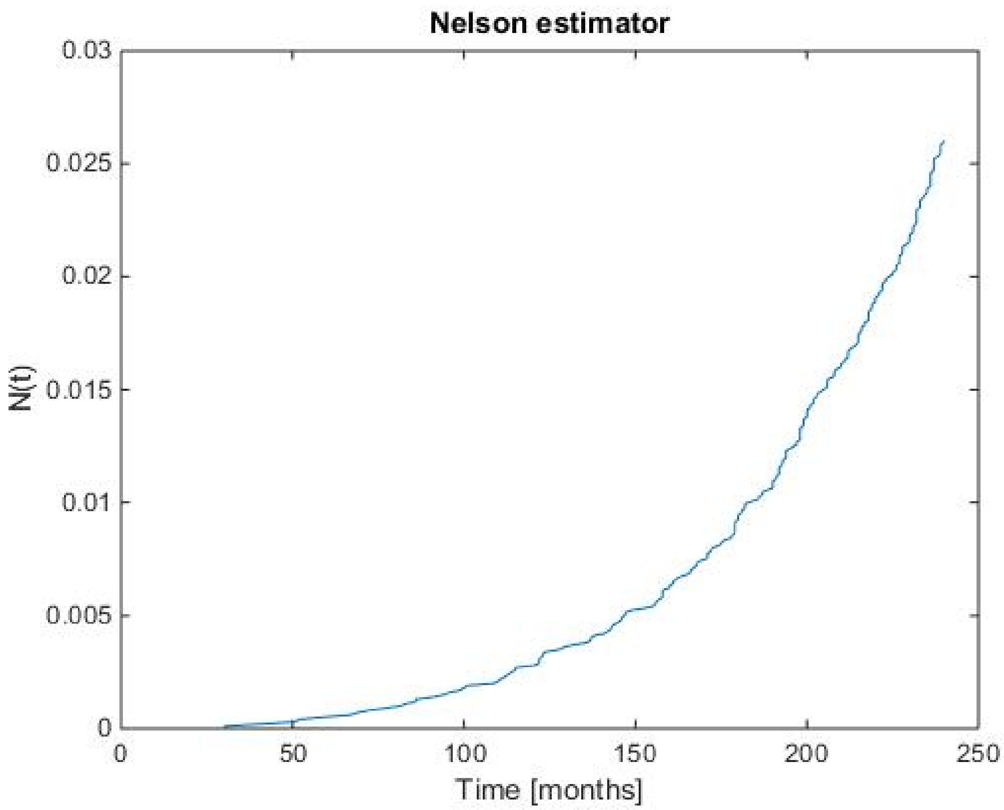


FIGURE 31: NELSON/ HAZARD PLOTTING

The Nelson estimator from the simulated data clearly shows an increasing trend. The convex form of the graph supports the assumption of an underlying increased failure rate (IFR).

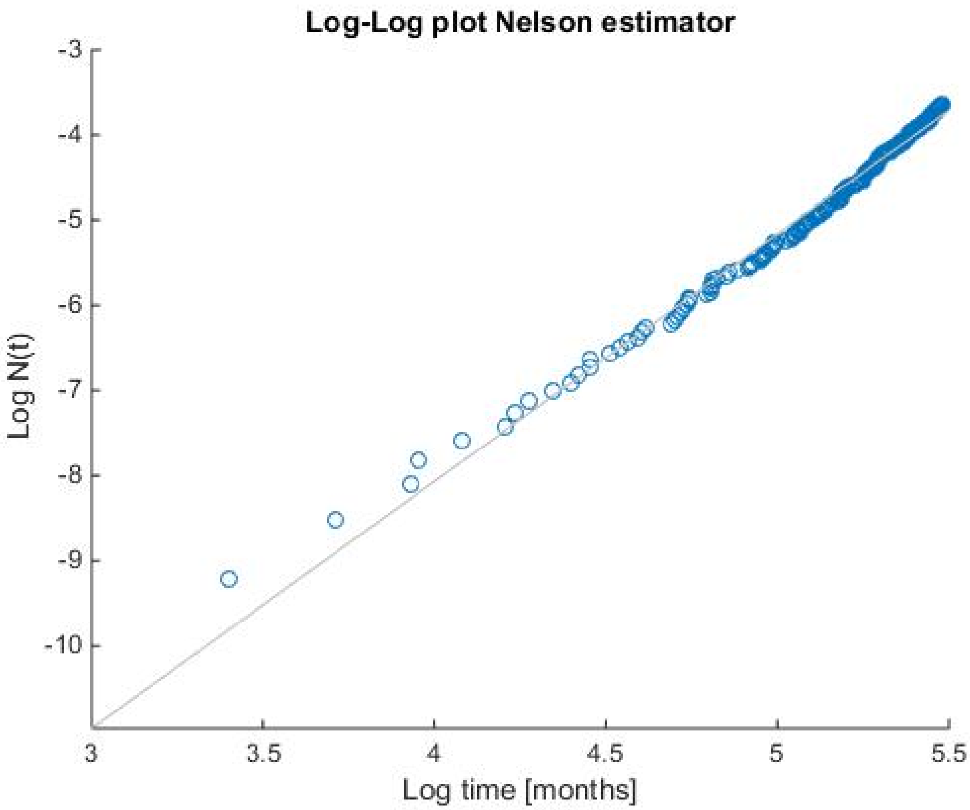


FIGURE 32: NATURAL LOGARITHMIC NELSON PLOT

The logarithmic Nelson plot falls on an approximately straight line. This indicates that the underlying distribution has an increasing failure rate (IFR) and can be described by a Weibull distribution (38).

The Weibull distribution is often used when the failure rate is increasing. A Weibull distribution is characterized by the shape parameter β and the scale parameter λ. The different reliability data can be estimated from these parameters. For the Weibull distribution, this can mathematically be expresses as (38):

$$f(t) = \lambda^{\beta} \cdot \beta \cdot t^{\beta-1} \cdot e^{-(\lambda t)^{\beta}} \tag{46}$$

$$R(t) = e^{-(\lambda t)^{\beta}} \tag{47}$$

$$F(t) = 1 - e^{-(\lambda t)^{\beta}} \tag{48}$$

$$z(t) = \lambda^{\beta} \cdot \beta \cdot t^{\beta-1} \tag{49}$$

$$Z(t) = \int_0^t z(s)ds = (\lambda t)^{\beta} \tag{50}$$

Where:
f(t)= lifetime distribution
R(t)= survivor function
F(t) =failure function
z(t)= failure rate
Z(t)= cumulative failure rate

Based on the behaviour of the simulated data shown in the logarithmic Nelson plot, a Weibull distribution was assumed. From the mathematical relation in equation 50, and the least square fitted line in figure 31, the Weibull parameter were calculated from the relation below. (In the equation below N(t)≈Z(t)).

$$\log N(t) = \beta_N \log \lambda_N + \beta_N \log t \qquad 51$$

Where:
β_N=shape parameter from Nelson plotting
λ_N=scale parameter from Nelson plotting
t= time (age)
N(t)= Nelson estimator

Shape parameter:

$$\beta_N = 2.902 \qquad 52$$

Scale parameter:

$$\gamma_N = 1.137 \cdot 10^{-3} \qquad 53$$

Nelson estimator/ hazard plotting is a good method to identify the underlying life time distribution. The estimation of the Weibull parameters from this method is quite rough (38).

Maximum likelihood estimation (MLE) is a more advanced technique that is often used to estimate distribution parameters. The MLE method maximizes the probability of getting the observed data (38).

The results from using Maximum likelihood estimator (MLE) to calculate parameters was:

Shape parameter:

$$\beta_{MLE} = 3.356 \qquad 54$$

Scale parameter:

$$\lambda_{MLE} = 1.405 \cdot 10^{-3} \tag{55}$$

Based on the parameters estimated from the MLE method. The lifetime distribution can be expressed as:

$$f(t) = 1.405 \cdot 10^{-3^{3.356}} \cdot 3.356 \cdot t^{3.356-1} \cdot e^{-(1.405 \cdot 10^{-3} t)^{3.356}} \tag{56}$$

All other representation of reliability data is defined by the lifetime distribution shown in equation 56.

8.4.3 Lifetime data from MLE parameters

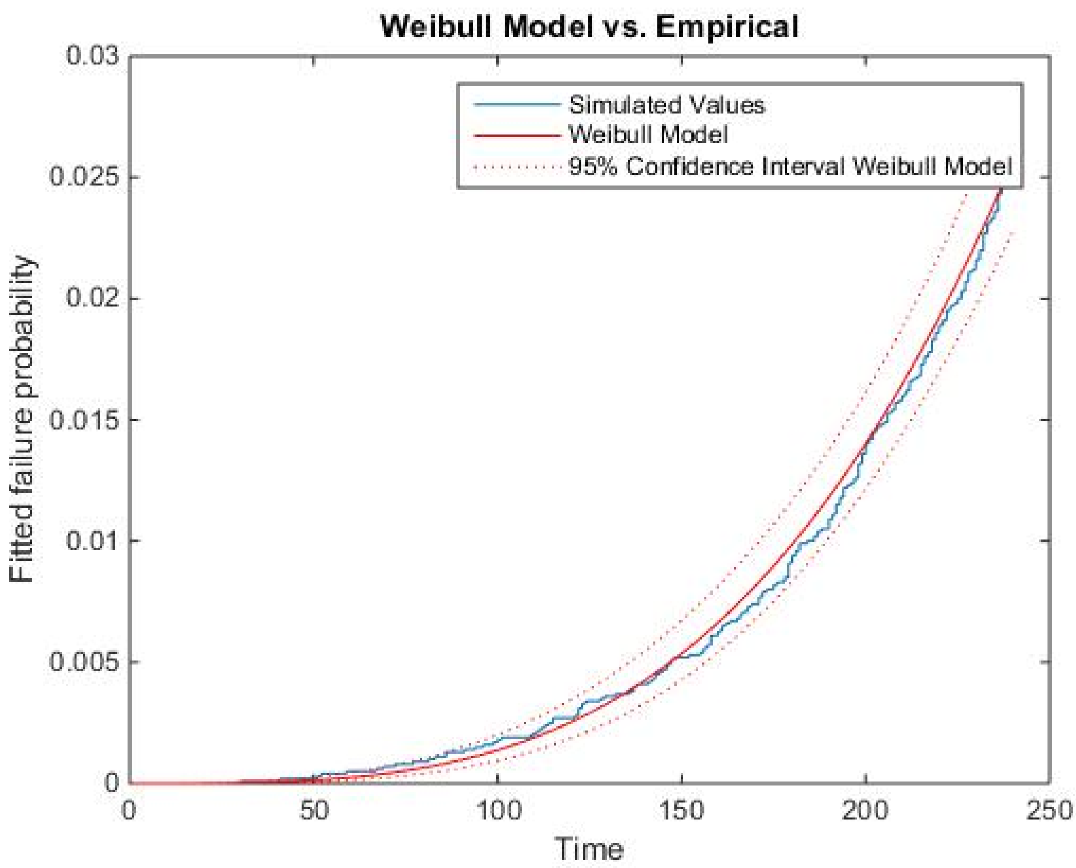


FIGURE 33: ESTIMATED MLE WEIBULL PARAMETERS VS EMPIRICAL DATA (FAILURE FUNCTION)

The blue line shows the simulated data. The red line is the fitted Weibull distribution. The red dotted lines shows the 95% confidence interval for the fitted Weibull distribution based on MLE parameters. The simulated data stay within the confidence interval during the 240 months.

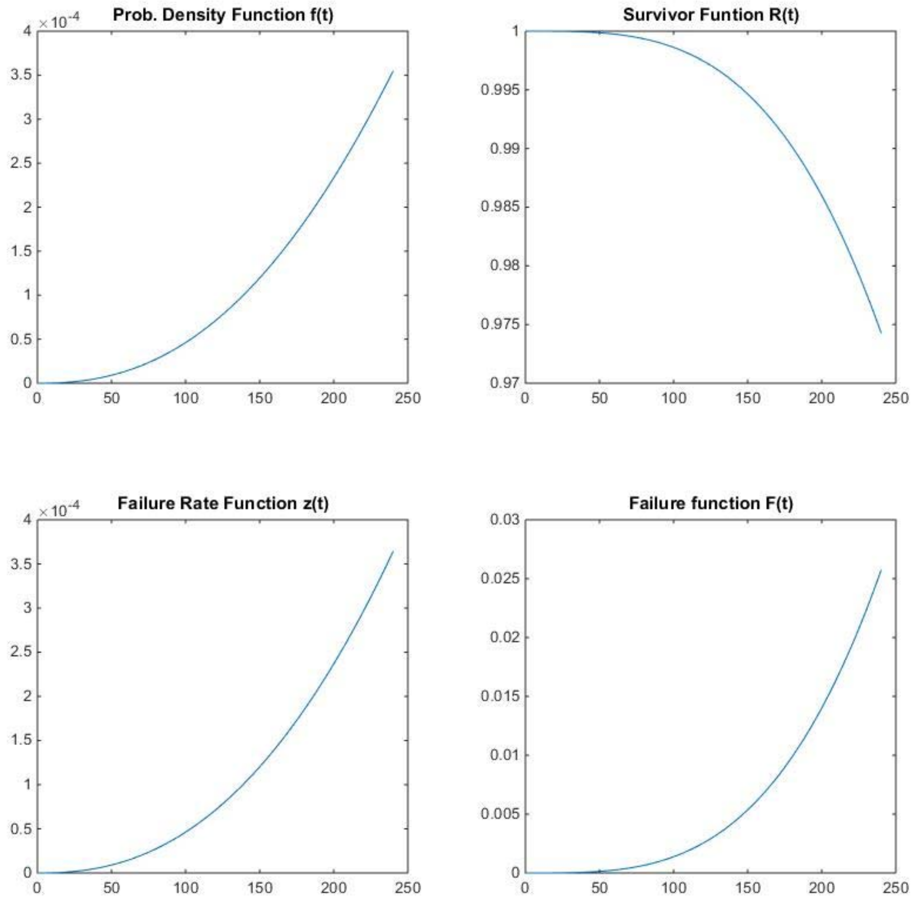


FIGURE 34: RELIABILITY DATA FROM WEIBULL DISTRIBUTION BASED ON MLE PARAMETERS

The plotted graphs above is based in the MLE parameters.

Mean time to failure for casing is estimated to:

$$MTTF = \frac{1}{\lambda_{MLE}} \Gamma\left(1 + \frac{1}{\beta_{MLE}}\right) = 639\ months \qquad 57$$

Where:
β_{MLE}*=shape parameter from MLE*
$\lambda_{;MLE}$*=scale parameter from MLE*
Γ*= gamma function*

8.4.4 Conclusion

The simulated data fit within a confidence level of 95% for the estimated Weibull distribution model. The conclusion is therefore that the lifetime simulated data can be expressed with a Weibull distribution.

8.4.5 Limitations and assumptions

The reliability model presented above have many limitations.

- Strength variables in the Barlow formula are considered independent. The dependency that may be between the thickness of the pipe and the outer radius is not assessed.

- The data used for determine the distribution of yield strength, wall thickness and outer diameter are of unknown origin. The amount of casing joints that is tested to find these values are not specified.

- Estimated initial strength of the pipe is considered to have equal value throughout the length of the well. This means that the initial strength is the same at wellhead level as it is at production packer level. In a real case scenario with different pipes this would not be the case.

- The reliability is only considered at two discrete positions. This is at the wellhead level during drilling, and at the production packer during drilling.

- The degradation factor is a multiplicative factor that is random picked from a triangular distribution. The consequence of this is that a stronger casing will on average have a higher absolute decrease in burst strength. This may not be the case in a real well.

- The values and distribution of load factors and strength decrease is not based on real observed data. And thus do not represent a real case for load and degradation uncertainties.

The model shows how reliability data can be estimated based on statistical data using Monte Carlo Simulation. There are many assumptions and limitations in the presented model, and it does not represent a realistic reliability scenario. For a complete assessment of the reliability of a well, all variables and correlations between them must be considered. The reliability must also be considered along the whole length of the well, and not just in discrete points.

9 Discussion

9.1.1.1 Regulations on the Norwegian Continental Shelf (NCS)

Operators are responsible for meeting the requirements given by the regulations. It is not stated how these requirements should be met. The individual operating companies must decide how they should meet the requirements, this gives freedom in choice of approach. A RBD can be applied as long as it is documented and well-reasoned.

However NORSOK-D010 states that design shall be based on the elastic-deformation principle. This gives indications that the design shall be based on a WSD method, even though a RBD approach with WSD criteria can be applied.

To base a design on methods that is not in compliance with NORSOK-D010 may not be easy for the companies to accept. Implementations of RBD into NORSOK-D010 in a way that allows to discriminate loads and utilize strength above yield limit, would stimulate and give confidence in the use of a RBD approach.

9.1.1.2 Well integrity

If all variables and uncertainties in a well were known, a RBD approach would add great contribution to well integrity. The risk of failure would be quantifiable and sensitivity analysis could be performed to find the critical factors in the design.

However, well integrity is a multidisciplinary field, not only a description of material properties. To quantify variables related to organizational systems, operational procedures and human behaviour may not be possible. Some questions to illustrate this is:

- What is the probability for a driller to discover a kick?
- What is the probability of a driller to exceed the operating limits of casing?
- What is the probability for a person to misunderstand operational procedures?

A RBD approach is useful in describing material integrity. To assess the integrity of a complete operational sequence other factors come into play. Such factors may be practical impossible to approach in a probabilistic manner.

9.1.1.3 Casing loads

Estimation of loads lay the foundation for the design of a well. The fundamental problem is to assign probabilistic values to underlying variables. Every well has its individual set of properties. The correlations between similar wells may not always be adequate to use. This problem is not special for RBD. All design methods use some kind of estimation for the load, and there is good reasons to think RBD will handle the uncertainties and variabilities in load better then deterministic methods.

9.1.1.4 Casing strength

Casing strength has the advantage that is it easily accessible for testing. This leads to accurate estimations for performance variability. ISO 10400 implement the limit states equations and probability parameters for some of them. This makes probabilistic strength estimations easy to apply in practice.

9.1.1.5 Casing degradation

Estimation of degradation parameters is difficult. There are many different models that describe casing wear due to drilling, but a consensus of what model to be used is absent (16). The calculation of the degraded strength if the wear is known is another consideration that must be understood. This involves study of both geometrical and steel properties. A complete understanding and estimation regarding this topic seems to be elusive.

Cyclic loads and corrosion must also be considered. To quantify a well-reasoned probability to such factors is a formidable task.

- What are the probability of initiation of pitting corrosion?
- How much degradation in strength is caused by cyclical loads?
- How does casing wear affect corrosion?

These are just some of the questions that must be taken into consideration to get a complete understanding of the casing resistance to load during its life cycle.

9.1.1.6 Working Stress Design vs Reliability based design

In chapter 4 some of the integrity issues on the NCS was shown. The PSA Well Integrity, Phase 1, identified that casing and tubing was related to 50% of the integrity issues. SINTEF study identified that casing and tubing were related to 38% of the integrity issues. In the doctor thesis *Contribution to well integrity and increased focus on well barriers from a life cycle aspect* by Birgit Vignes, some of the identified challenges related to casing and tubing leaks was (5):

- Connections performance
- Material selection
- Corrosion
- Tubing and casing installation

This may indicate that failures are not directly related to the mechanical strength of the tubular, but rather operational procedures and understanding of degradation factors in a life cycle aspect. If this is the case, the claimed accusations that WSD lead to overdesigned wells, may be true. If appropriate models and probabilistic data of connection performance, corrosion and material selection was included in a RBD. A Sensitivity analysis would have found that there is a lot higher possibility for integrity issues related to these factors. This is some of the advantage of RBD design.

9.1.1.7 Use of simulations to generate failure data

Sufficient data of barrier element failures might be difficult to obtain. As presented by Das et al. (1), one can use simulations to generate time dependent failure data. This has also been done in this thesis. However, it is important to have realistic models for load, strength and degradation factors to obtain realistic results. In a complex Monte Carlo simulation it can be difficult to identify malfunctions in the written program. A thorough understanding of this approach is therefore necessary to avoid black-box phenomena and have a reliable outcome (38).

10 Conclusion

In this thesis, a probabilistic model and simulation framework for calculating the reliability of the casing as a function of time was presented. The proposed model was inspired by the ideas presented in *A model for Well Reliability Analysis throughout the Life of a Well Using Barrier Engineering and Performance* by Bibek Das and Robello Samuel (1). Reliability based casing design methodology was combined with time degradation of casing strength in a life cycle setting. Monte Carlo simulations were then used to generate a large set of failure data for the casing. These data were then treated as "experimental data" and analysed using methods like the Nelson estimator and Maximum Likelihood estimator to find the lifetime distribution and other reliability measures. The model can be extended to include other well barrier elements, as well as considering several barriers in parallel. This was also discussed by Das et al (1).

If statistical data for all the variables affecting the casing is available, the reliability of a certain design can be calculated by the method discussed in this thesis. The dependency of the different WB and WBE must also be assessed to give accurate reliability results.

Calculation of the reliability for a given design, makes it possible to quantify probability of failure and identify critical factors in a design. Based on this, target goals for reliability can be established to give a measurable goal for well integrity. This would be of great advantage compared to working stress design, where the integrity of the well is based on the application of a safety factor, which do not quantify the risk associated with them.

The main challenge in the use of RBD is related to the availability of statistical data. The strength estimations is well studied and appropriate equations and statistical data are available. Load cases and their magnitude can also be identified and measured, but the uncertainty of them can be difficult to estimate. The most difficult task is to estimate values for time dependent variables, such as casing degradation and changes in loads. Appropriate models for estimation of this must be established to make it possible to account for such factors.

Regulations are normative, and gives the operators the responsibility for the design approach. However, the specification that a design shall be based on the elastic deformation principle in NORSOK-D010, makes it difficult for operators to pursue a RBD approach in practice.

Further work:

To be able to use RBD in practice a complete model for estimating the reliability have to be made, taking into account several barriers and barrier elements, as well as all types of failures modes and loads scenarios . Data of casing degradation and failure causes also have to be sampled and analysed to make it possible to estimate statistical values related to these variables.

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Appendix A

This appendix contain the full MATLAB codes used in the Monte Carlo simulations. The codes used can also be accessed/downloaded at:

https://www.dropbox.com/sh/fuscqrbetkaugx8/AADfksQhEneg0-Nda5nId57ga?dl=0

MATLAB code

STRENGTH ESTIMATION

```
% This function gives an output for the burst strenght for every month.

function P=strenght

d = normrnd(9.635,0.003);   %random pick from normal dsiribution of outside diamter

t = normrnd(0.479,0.003);   %random pick from normal distributon of thickness

y = normrnd(87000,2751);    %random pick from normal distribution of yield strenght
```

Drilling phase degradation

```
for i = 1:2;

a= trianglerand(0.9617692031,0.9746794345,0.9874208829,1); %Triangular distribution
t = a*t; %Thickness decrease for every month during drilling



P1(i)=y*2*t/d*(0.0689475729); %Burst strenght



end
```

Production phase degradation

```
for i=1:238;

c = trianglerand(0.9980710547,0.9990072629,0.9997728526,1); %Triangular distribution
t = c*t; %Thickness decrease for every month
y1=y*1;% Yield decreas due to temperature


P2(i)=y1*2*t/d*(0.0689475729); %Burst strenght




end
 P=[P1 P2];
```

Burst load

```
%This functin gives buest load for every month during lifetime of the well

%INPUTS
%Porepressure gradient=2.0 s.g  @ 2365m RKB
%Formation fluid density=0,76 s.g @2365m RKB
%Mud density=1,7 s.g
%Cement density=1,9 s.g
%Completion fluid density=1,10 sg
%seawater gradient=1,03 s.g


function [P1]=loads
PP_gr=2.0;
FF_gr=0.76;
CF_gr=1.1;
```

```
SW_gr=1.03;

FF_gr=normrnd(FF_gr,(FF_gr*0.15));  %Random number from normal distribution.
```

```
PP_gr=normrnd(PP_gr,(PP_gr*0.15));  %Random number from normal distribution.
```

Drilling phase

```
for i = 1:2;


  P2(i)=(PP_gr*2365*0.098-FF_gr*2140*0.098-SW_gr*0.098*225);




end
```

Production phase

```
%Tubing leak load
for i=1:238;


%Tubing leak


   P3(i)=(2365*0.098*PP_gr-209.72*FF_gr+193.55*CF_gr-215.6*SW_gr);




end
```

```
P1=[P2 P3]; %Output matrice load
```

Montecarlo Simulation

```
%Montecarlo simulation function for N numbers of lifetimes
%Compares load vs stenght

function
[load_matrix,strenght_matrix,failure_matrix_binary,failure_times]=loadvsstrenght(N)

k=N; %Number
strenght_matrix=zeros(k,240);
load_matrix=zeros(k,240);
for k = 1:k;

S=strenght;

strenght_matrix(k,:)=S;

L=loads;

load_matrix(k,:)=L;

end

failure_matrix_binary=strenght_matrix<load_matrix;

%This function outputs failure time for each casing
%Inputs to this function are number of simulation N and
%failure_matrix_binery

failure_times=[];
for j=1:k;
```

```
  for i=1:240;
   if failure_matrix_binary(j,i)==1

     failure_times=[failure_times;j,i];

   break

   end

  end

end
```

Empirical Reliability data

```
%F(t)

m=(1:length(failure_times));
m=m(:);
m=m/N;        %N number of simulation
m=[m,sort(failure_times(:,2))];



figure('Name','Distributions')
subplot(1,3,1);
plot(m(:,2),m(:,1));
title('Failure function plot F(t)')
xlabel('Time [months]')
ylabel('Probability')
```

Survivor Function plot R (t)

```
%survivor function R(t)=1-F(t)

subplot(1,3,2);
n=(1-m(:,1));
plot(m(:,2),n);
title('Survivalplot R(t)')
xlabel('Time [months]')
ylabel('Probability')
```

Probability density plot f(t)

```
[a,b]=hist(failure_times(:,end));

a=a/N;


subplot(1,3,3);
bar(b,a);
title('Probability Density)')
xlabel('Time [months]')
ylabel('Probability')
```

Probability density

```
x=sort(failure_times(:,end)');

y = zeros(size(x));

for i = 1:length(x);

y(i) = sum(x==x(i));

end
```

```
y=y';

y=y/N;

y=[y,sort(failure_times(:,end))];
figure('Name','Probability Discrete')
plot(y(:,end),y(:,1));
title('Probability Density)')
xlabel('Time [months]')
ylabel('Probability')
```

Failure rate z(t)=f(t)/R(t)

```
z=y(:,1)./n;

figure('Name','Failure rate discrete')
plot(y(:,end),z);
title('Failure rate')
xlabel('Time [months]')
ylabel('Probability')
```

```
%Nelson estemator :::::graphical mehtod for finding the underlying lifetime
%distribution:::::fairly rought method, accuercy is

function [nelson_parameters]=Nelson_estemator(failure_times,N)
```

```
p=transpose(1:length(failure_times));
z_n=[];
for i=1:length(p);

z_n=[z_n;1/(N-p(i)+1)];
```

```
end
z_n=cumsum(z_n);
z_n=[z_n,sort(failure_times(:,end))];



figure('Name','Nelson plott')
plot(z_n(:,2),z_n(:,1));
title('Nelson estimator')
xlabel('Time [months]')
ylabel('N(t)')

z_log=log(z_n);
figure('Name','Neslon plott Log')
scatter(z_log(:,2),z_log(:,1));
title('Log-Log plot Nelson estimator')
xlabel('Log time [months]')
ylabel('Log N(t)')

lsline
 [lsfit]=polyfit(z_log(:,2),z_log(:,1),1);

 nelson_parameters=[lsfit(:,1),exp(lsfit(:,end)/lsfit(:,1))];
```

Maximum Likelihood estimation

```
% Maximum likelihood estimation



function [paramEsts,Analysys]=Maximum_LH(N,failure_times)

Analysys=zeros(N,2);
Analysys(1:length(failure_times),1)=sort(failure_times(:,end));
```

```
Analysys((length(failure_times)+1):N,1)=240;
Analysys((length(failure_times)+1):N,2)=1;

paramEsts=wblfit(Analysys(:,1),'censoring',Analysys(:,end));
x=linspace(1,800,500);
y=wblcdf(x,paramEsts(1),paramEsts(2));

figure('Name','Maximumm like')
plot(x,y)
```

Survival analysis

```
%Contains plot from estimated values
[paramEsts,Analysys]=Maximum_LH(N,failure_times);
T=240;
obstime=Analysys(:,1);
```

```
failed = obstime(obstime<T); nfailed = length(failed);
survived = obstime(obstime==T); nsurvived = length(survived);
censored = (obstime >= T);
plot([zeros(size(obstime)),obstime]', repmat(1:length(obstime),2,1), ...
    'Color','b','LineStyle','-')
line([T;3e4], repmat(nfailed+(1:nsurvived), 2, 1), 'Color','b','LineStyle',':');
line([T;T], [0;nfailed+nsurvived],'Color','k','LineStyle','-')
text(T,30,'<--Unknown survival time past here')
xlabel('Survival time'); ylabel('Observation number')
```

```
x = linspace(1,240);
subplot(2,2,1);
plot(x,wblpdf(x,paramEsts(1),paramEsts(2)))
title('Prob. Density Function f(t)')
subplot(2,2,2);
plot(x,1-wblcdf(x,paramEsts(1),paramEsts(2)))
title('Survivor Funtion R(t)')
subplot(2,2,3);
```

```
wblhaz = @(x,a,b) (wblpdf(x,a,b) ./ (1-wblcdf(x,a,b)));
plot(x,wblhaz(x,paramEsts(1),paramEsts(2)))
title('Failure Rate Function z(t)')
subplot(2,2,4);
plot(x,wblcdf(x,paramEsts(1),paramEsts(2)))
title('Failure function F(t)')
```

```
subplot(1,1,1);
[empF,x,empFlo,empFup] = ecdf(obstime,'censoring',Analysys(:,2));
stairs(x,empF);
hold on;
stairs(x,empFlo,':'); stairs(x,empFup,':');
hold off
xlabel('Time'); ylabel('Proportion failed'); title('Empirical CDF')
```

```
[nlogl,paramCov] = wbllike(paramEsts,obstime,Analysys(:,2));
xx = linspace(1,T,500);
[wblF,wblFlo,wblFup] = wblcdf(xx,paramEsts(1),paramEsts(2),paramCov);
```

```
stairs(x,empF);
hold on
handles = plot(xx,wblF,'r-',xx,wblFlo,'r:',xx,wblFup,'r:');
hold off
xlabel('Time'); ylabel('Fitted failure probability'); title('Weibull Model vs. Empirical')
```

```
[M,V] = wblstat(paramEsts(1),paramEsts(2))
```

Simulation overview

```
% This script runs the montecarlo simulation for N iterations

rng('shuffle')% Reset random number genrator

N=10000;               %Input of lifetime simulated


%calling simulation matrices for strenght and load

[load_matrix,strenght_matrix,failure_matrix_binary,failure_times]=loadvsstrenght(N);
```

```
function f = trianglerand(xstart,mostlik,xstop,N)
% TRIANGLERAND Random numbers from a triangle distribution.
%    R = trianglerand(min,mostlikely,max,N) returns a vector of N draws from a
%    triangular distribution starting at min, maxpoint at mostlikely and endpoint at max.
%
%   Copyright 2003 RF - Rogaland Research
%   Author: Øystein Arild

a = mostlik-xstart;
b = xstop-xstart;

h1 = 2/a;
m1 = h1/a;


A1 = a/b;
p = A1;




f_ = (rand(N,1) < p);
ind1 = find(f_==1);
ind2 = find(f_==0);
N1 = length(ind1);

if (a == b)
  u = rand(N,1);
  f = sqrt(2*m1*u)/m1;
else
  u = rand(N1,1);
  f1 = sqrt(2*m1*u)/m1;

  h2 = 2/(b-a);
```

```
  m2 = -h2/(b-a);
  beq=h2;
  u = rand(N-N1,1);
  f2 = a+(-beq+sqrt(beq*beq+2*m2*u))/m2;
  f(ind1) = f1;
  f(ind2) = f2;
  f = f';
end

f = f + xstart;
```

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Appendix B

The table below shows the failure data-set, sorted by month of failure. Column 1 shows the number of failures, column 2 shows at what month the failure occur, and column 3 shows the associated Monte Carlo iteration. The casing that survived over 240 month were right censored in the calculations.

Lifetime data

Number of failures	Time [month]	Iteration number
1	30	6386
2	41	2907
3	51	6636
4	52	2457
5	59	6719
6	67	3645
7	69	9686
8	72	9503
9	77	6081
10	81	2091
11	83	7766
12	86	2535
13	86	3897
14	91	6785
15	94	1080
16	96	1602
17	99	9723
18	100	597
19	101	2442
20	109	343
21	110	1029
22	111	4182
23	112	4953
24	113	4023
25	114	4724
26	115	613
27	115	4877
28	121	2438
29	122	661
30	122	5219
31	122	8854
32	123	2615
33	123	8624
34	124	5860

35	128	8851
36	129	9342
37	132	9805
38	136	9588
39	137	125
40	137	581
41	138	4085
42	141	9628
43	142	5273
44	143	786
45	143	2054
46	144	7126
47	145	1298
48	146	3570
49	146	5865
50	147	1772
51	147	5985
52	148	5288
53	152	7439
54	155	6455
55	156	2312
56	156	2963
57	157	4691
58	158	56
59	158	2042
60	158	2964
61	158	6918
62	160	2813
63	160	9328
64	161	428
65	161	1294
66	162	2061
67	163	5028
68	165	3106
69	166	5446
70	166	5907
71	167	4450
72	168	59
73	168	2711
74	169	2165
75	171	6361
76	171	9204
77	171	9218
78	172	4083
79	172	8879

80	173	8441
81	175	3564
82	175	4362
83	176	6329
84	178	7444
85	178	7731
86	179	3397
87	179	4589
88	179	5464
89	179	5585
90	179	7314
91	179	8630
92	180	1295
93	180	7019
94	180	9763
95	181	6592
96	181	6936
97	182	83
98	182	4686
99	182	6147
100	184	1766
101	186	1491
102	186	4534
103	187	718
104	187	9042
105	188	188
106	190	3049
107	190	5282
108	190	5570
109	190	9459
110	191	2470
111	191	6555
112	192	2672
113	192	3222
114	192	3946
115	192	5343
116	193	6581
117	193	6777
118	193	7880
119	194	4080
120	194	5716
121	194	8200
122	194	8710
123	195	2347
124	196	1000

125	197	2496
126	197	6613
127	198	1146
128	198	3415
129	198	4371
130	198	4863
131	198	7529
132	198	9168
133	199	333
134	199	6128
135	199	6957
136	199	7306
137	200	2179
138	200	5439
139	200	6728
140	200	7905
141	201	9840
142	201	9842
143	202	1025
144	202	1446
145	202	3376
146	203	4367
147	203	9493
148	204	5505
149	205	2168
150	206	3066
151	206	5597
152	206	7414
153	206	9714
154	207	1669
155	208	3214
156	208	4454
157	208	6205
158	209	1077
159	210	5886
160	210	7433
161	211	1164
162	211	3266
163	212	1377
164	212	2007
165	212	3485
166	212	6579
167	213	8012
168	214	7513
169	215	1266

170	215	4217
171	215	4627
172	215	4984
173	215	8536
174	216	2393
175	216	3220
176	216	8732
177	217	2625
178	217	3513
179	218	2018
180	218	2657
181	218	5992
182	218	6070
183	218	6941
184	219	2233
185	219	5361
186	219	9937
187	220	2687
188	220	3849
189	220	8283
190	221	5308
191	221	5828
192	222	1031
193	222	1041
194	222	8472
195	222	9495
196	223	5326
197	223	7242
198	224	5334
199	225	3294
200	225	3306
201	226	1876
202	226	8295
203	226	8930
204	227	1007
205	227	4844
206	227	7580
207	227	8992
208	228	1072
209	228	7101
210	228	7141
211	228	9975
212	229	8156
213	230	87
214	230	2094

215	230	2498
216	230	6732
217	231	925
218	231	1365
219	231	4102
220	231	8155
221	232	324
222	232	1109
223	232	1942
224	232	4126
225	232	4257
226	232	6705
227	232	7800
228	233	541
229	233	2024
230	233	6955
231	233	7418
232	234	1437
233	234	5202
234	235	5932
235	235	8623
236	235	9900
237	236	341
238	236	710
239	236	953
240	236	5506
241	236	6760
242	236	7869
243	236	9399
244	237	1514
245	237	2930
246	237	3774
247	237	6162
248	237	6772
249	237	8637
250	238	7831
251	239	655
252	239	1258
253	239	2532
254	239	7091
255	239	8664
256	240	3448
257	240	6151