



FACULTY OF SCIENCE AND TECHNOLOGY

## MASTER'S THESIS

Study Program/Specialization:  Master of Science in Petroleum Engineering, Drilling, and Well Engineering	Spring Semester, 2017   Open
Student name: <b>ELEONU OSITADINMA TOCHI</b>  Contact: <a href="mailto:ositaeleonu@yahoo.com">mailto:ositaeleonu@yahoo.com</a>	----- Student's signature
Faculty Supervisor: Prof. <b>KJELL KÅRE FJELDE</b> ,  Contact: University of Stavanger  External supervisors:	
Thesis Title:  <b>RELIABILITY BASED CASING DESIGN</b>	
Key Words:  Casing design, Reliability, Working Stress design  Risk Analysis, Monte Carlo Simulation, Burst Pressure, Probability of failure	Pages: 103  + enclosure: 33  Total: 136 pages  15 <sup>th</sup> June 2017
Credits(ECTS): 30	

## **Acknowledgement**

The success of this thesis required the guidance and support of people, hence its accomplishment could have been impossible without them.

My deep appreciation first goes to God almighty the author and finisher of my faith.

Secondly, my special thanks go to Professor Kjell Kåre Fjelde for his supervision and assistance without which this thesis could not have been possible. I am extremely grateful for having the opportunity to work with one of the best lecturers in the Petroleum Engineering department of University of Stavanger. I wish to also extend my gratitude to Professor Mesfin Belaney for his academic and moral support and to Professor Bernt Aadnoy for a good foundation in Well design.

I sincerely thank my loving husband, Augustine Uche Nkole for his patience. My appreciations also go to the rest of my loving family, my parents, Elder and Mrs. Gabriel U. Eleonu, my parents-in-law, Mr. and Mrs. Clement Ugbaga Nkole, my sisters Dr. Mrs. Ekeoma Imaga Okorie, Chinwe Eleonu, and Ezinne Ugbaga, and my brothers, Joshua Eleonu, Oke Ugbaga, Chinaza Ugbaga etc.

Finally, my gratitude goes to all the professors and staff of the department of petroleum engineering at the University of Stavanger, my friends in Nigeria, Norway and UK for their help, support, and encouragement in all ramifications.

## **Abstract**

It is a well-known fact that wells are increasingly becoming extremely expensive and difficult to drill as well as develop due to increasing water depth, temperatures, and pressures. All these result in an increasing load that subsurface structures must bear. This has resulted in the advent of better equipment and technology to give a cost effective and safety assured project. The birth of better techniques and ideas to cheaply and safely drill wells is still ongoing. This ongoing quest can affect any area of the petroleum industry. This thesis is one of such and is aimed at improving casing design for Well integrity assurance.

This work presents the application of reliability based method in casing design/selection, relative to the traditional approach known as working stress design (WSD), which is mainly used and have its setbacks. Firstly, the conventional casing design method is discussed. The traditional working stress design depends on the load and strength of a tubular to generate a safety margin used for design. In essence, the expected loads are kept way below the strength in order to avoid failure. This failure could have a low probability of occurrence but a severe consequence if it does occur or vice versa. The issues with the traditional approach are that it promotes overdesign and is too rigid. Also, the associated risk to the design is rarely quantified giving a non-risk consistent design. In fact, in a situation where the scenario causing failure does not occur, changes cannot be made. In other words, it does not properly account for uncertainties present during a casing design. For instance, it is challenging to obtain acceptable design factor using WSD for deepwater, HPHT wells with high magnitude load. On the other hand, the reliability based approach (RBD) measures the uncertainties and risk of failure of the design. The objective of this study is to demonstrate how to perform the RBD method in casing design. The RBD method was implemented by developing probability distributions for load and strength using limit state equations such as the burst load equations, the hydrostatic pressure equations, and the Barlow's yield strength equations. This study evaluated and compared the two casing design approaches to determine the best in terms of quantifying the uncertainties and risk in the design. A burst scenario due to kick was developed to demonstrate this method. Furthermore, a model for the load and different models for the strength were considered. Monte Carlo simulations were performed using Matlab and the codes developed can be found in the Appendix. The methodology can be used to predict the probability of failure, analyze risks and perform a sensitivity analysis. One challenge with this approach is that it takes some time to get familiar with the documentation of the methodology. Another issue is that it is difficult to know what input parameters to use for the uncertain input distributions.

# Table of Contents

ACKNOWLEDGEMENT.....	1
ABSTRACT .....	2
<b><u>TABLE OF CONTENTS .....</u></b>	<b><u>3</u></b>
LIST OF TABLES.....	7
LIST OF FIGURES.....	8
SYMBOLS.....	11
ABBREVIATIONS/ ACRONYMS.....	12
<b><u>CHAPTER 1 .....</u></b>	<b><u>14</u></b>
1.1 INTRODUCTION.....	14
1.2 Scope of Study, Background of problem .....	16
1.3 Objective of the application of reliability in casing design.....	17
1.4 Structure of the Thesis.....	17
1.5 Definition of important terms .....	18
<b><u>CHAPTER 2 .....</u></b>	<b><u>20</u></b>
2.1 Well Design.....	20
2.1.1 CASING DESIGN AND CRITERIA <sup>[13]</sup> .....	21
2.1.2 CASING TYPE AND SIZES.....	23
2.2 Casing Properties, <sup>[12]</sup> .....	25
2.3 Casing Setting depth selection <sup>[13]</sup> .....	25
2.2.1 KICK TOLERANCE/KICK MARGIN <sup>[13]</sup> .....	26
2.4 Well Integrity .....	27
2.5 Major loads.....	29
2.5.1 BURST <sup>[13]</sup> .....	29
2.5.2 COLLAPSE <sup>[13]</sup> .....	30
2.5.3 TENSILE LOAD.....	30
2.5.4 BIAXIAL LOADING .....	31
2.5.5 BENDING LOAD .....	31
2.5.5 TORSION AND SHEAR .....	32
2.6 Mechanisms generating loads.....	32
2.7 Design example of burst load calculation, 1D load model .....	35
2.8 Specifications and Strength of Casing with respect to burst load model <sup>[18]</sup> .....	37

2.9	Limit State Equations for Burst Strength <sup>[5, 18]</sup> .....	37
2.10	Management guidelines, Government regulation, and NORSOK .....	39
2.11	Factors that degrade casing strength .....	39

**CHAPTER 3 ..... 41**

3.1	Methods of Casing design.....	41
3.2	Working Stress Design, WSD .....	41
3.3	Limit State Design, LSD .....	41
3.4	Working Stress Design steps <sup>[11]</sup> .....	44
3.5	Designing Safety factors, SF.....	45
3.6	Working Stress Design example .....	45
3.7	Merits of Deterministic Approach <sup>[8, 20]</sup> .....	47
3.8	Limitations of Deterministic Approach <sup>[8]</sup> .....	47

**CHAPTER 4 ..... 48**

4.1	Reliability .....	48
4.2	Application of Reliability-Based Design in casing Design.....	50
<b>4.2.1</b>	<b>THE VARIABLE PARAMETERS IN THE STRENGTH MODEL .....</b>	<b>50</b>
<b>4.2.2</b>	<b>THE VARIABLE PARAMETERS IN THE LOAD MODEL.....</b>	<b>51</b>
4.3	Calculation methods for RBD.....	52
4.4	Reliability Based Design versus Working Stress Design.....	52
4.5	Reliability Based Design versus Limit State Method.....	53
4.6	Merits of the Reliability-Based Design approach are <sup>[9]</sup> : .....	53
4.7	Limitations of Reliability Based Design with respect to casing design .....	54
4.8	Historical Development of Reliability-Based Design in casing design .....	54
4.9	Risk or Unexpected Events .....	56

**CHAPTER 5 ..... 57**

5.1	Monte Carlo Simulation.....	57
5.2	Why use a Monte Carlo Simulation for RBD casing burst strength design?.....	57
5.3	Monte Carlo Simulation Work Steps .....	57
<b>5.3.1</b>	<b>A. DEFINING AN APPROPRIATE MODEL; .....</b>	<b>58</b>
5.4	Using the result.....	65
<b>5.4.1</b>	<b>SENSITIVITY ANALYSIS: .....</b>	<b>65</b>
5.5	Some of the assumptions in Monte Carlo Simulation of the casing design: .....	66
5.6	Dependent variable and Correlations:.....	66
5.7	How to calculate Probability of Failure.....	67

<b>CHAPTER 6 .....</b>	<b>68</b>
<b>6.1 THE CASE STUDY: AN APPLICATION OF MONTE CARLO SIMULATION FOR CASING RELIABILITY DESIGN .....</b>	<b>68</b>
6.1.1 DEFINING A MODEL: .....	68
6.1.2 DATA GATHERING:.....	71
6.1.3 THE INPUT DISTRIBUTIONS .....	73
6.1.4 SAMPLING THE INPUT DISTRIBUTIONS .....	73
6.1.5 PRESENTATION OF THE RESULT .....	73
<b>CHAPTER 7 .....</b>	<b>74</b>
7.1 Presentation and Discussion of Results.....	74
CASE 1: BASE CASE .....	74
CASE 2 SHOWS DIFFERENT CASES OF CASING FAILURE WITH VARYING DEGREES OF PF .....	77
CASE 2A. ....	77
CASE 2B. ....	80
CASE 3; SIMULATION OF THE STRENGTH MODELS.....	83
BARLOW’S MODEL PDF, CDF, AND STATISTICAL QUANTITIES; .....	85
VON MISES' MODEL; PDF, CDF AND STATISTICAL QUANTITIES .....	86
KLEVER STEWART’S MODEL: PDF, CDF, AND STATISTICAL QUANTITIES .....	87
PASLAY’S MODEL; PDF, CDF AND STATISTICAL QUANTITIES .....	89
MOORE’S MODEL; PDF, CDF AND STATISTICAL QUANTITIES.....	90
NADAI’S MODEL PDF, CDF, AND STATISTICAL QUANTITIES WITH YIELD AND TENSILE STRENGTH; .....	91
7.2 Application of Sensitivity Analysis .....	92
APPLICATION OF THE SENSITIVITY ANALYSIS ON THE LOAD MODEL .....	92
APPLICATION OF THE SENSITIVITY ANALYSIS ON THE STRENGTH MODEL.....	94
7.3 Compare Working Stress Design to Reliability based Approach, RBD.....	96
7.4 Limitations / Drawbacks of Monte Carlo Simulation in application of Reliability Based Design	98
<b>CHAPTER 8 .....</b>	<b>99</b>
<b>8.1 CONCLUSION AND RECOMMENDATIONS .....</b>	<b>99</b>
8.1.1 CONCLUSION.....	99
8.1.2 RECOMMENDATIONS.....	101
<b>REFERENCES.....</b>	<b>102</b>
APPENDIX A: THE MATLAB CODES.....	104
THE FUNCTION FILE .....	104

<b>CASE 1 CODES</b> .....	<b>105</b>
<b>CASE 2A CODES</b> .....	<b>108</b>
<b>CASE 2B CODES</b> .....	<b>111</b>
<b>CASE 3 CODES</b> .....	<b>114</b>
<b>SIMULATING WITH BARLOW'S MODEL</b> .....	<b>114</b>
<b>SIMULATING WITH VON MISES' MODEL</b> .....	<b>117</b>
<b>SIMULATING WITH KLEVER STEWART'S MODEL</b> .....	<b>120</b>
<b>SIMULATING WITH PASLAY'S MODEL</b> .....	<b>123</b>
<b>SIMULATING WITH MOORE'S MODEL</b> .....	<b>126</b>
<b>SIMULATING WITH NADAI'S MODEL</b> .....	<b>129</b>
<b>CODES FOR THE LOAD SENSITIVITY ANALYSIS</b> .....	<b>132</b>
<b>CODES FOR STRENGTH SENSITIVITY ANALYSIS</b> .....	<b>134</b>

## **LIST OF TABLES**

TABLE 1: DIFFERENCES BETWEEN THE WORKING STRESS DESIGN, WSD AND THE LOAD AND RESISTANCE FACTOR DESIGN, LRFD. ....	42
TABLE 2: COMMON SAFETY FACTORS, MODIFIED AFTER <sup>[11]</sup> .....	45
TABLE 3: THE DIFFERENCE BETWEEN THE WSD AND RBD <sup>[22]</sup> . ....	52
TABLE 4: THE DIFFERENCES BETWEEN THE LOAD AND RESISTANCE FACTOR DESIGN, LRFD AND THE RELIABILITY BASED DESIGN, RBD .....	53
TABLE 5: WELL DATA, SOURCE: <sup>[13]</sup> . ....	71
TABLE 6: THE CASING PROPERTIES AND PARAMETERS FOR SIMULATION (INPUT DATA). ....	72
TABLE 7: INPUT DISTRIBUTIONS FOR CASE 1 STOCHASTIC SIMULATIONS.....	74
TABLE 8: THE STATISTICAL QUANTITIES FROM CASE 1 SIMULATION.....	75
TABLE 9: SUMMARIZES THE INPUT DATA AND DISTRIBUTIONS FOR CASE 2A STOCHASTIC SIMULATIONS.....	77
TABLE 10: THE STATISTICAL QUANTITIES FROM CASE 2A. SIMULATION .....	78
TABLE 11: INPUT DATA AND DISTRIBUTIONS FOR CASE 2B STOCHASTIC SIMULATION .....	80
TABLE 12: THE STATISTICAL QUANTITIES FROM CASE2B. SIMULATION .....	81
TABLE 13: COMPARISON OF THE PROBABILITIES OF FAILURE OF THE DIFFERENT CASES. ....	82
TABLE 14: INPUT DATA AND DISTRIBUTIONS FOR CASE 3, STRENGTH MODEL'S COMPARISON.....	84
TABLE 15: THE STATISTICAL QUANTITIES FOR BARLOW'S FORMULA .....	85
TABLE 16: THE STATISTICAL QUANTITIES FOR VON MISES' FORMULA. ....	86
TABLE 17: THE STATISTICAL QUANTITIES FOR KLEVER STEWART'S FORMULA. ....	87
TABLE 18: THE STATISTICAL QUANTITIES FOR PASLAY'S FORMULA.....	89
TABLE 19: THE STATISTICAL QUANTITIES FOR MOORE'S FORMULA.....	90
TABLE 20: THE STATISTICAL QUANTITIES FOR NADAI'S FORMULA. ....	91
TABLE 21: SUMMARIZES THE RESULT OF THE SIMULATION WITH DIFFERENT STRENGTH MODELS. ....	92
TABLE 22: DATA FOR THE SENSITIVITY ANALYSIS ON LOAD MODEL.....	93
TABLE 23: STANDARD DEVIATION FOR EACH CURVE IN FIGURE 44 (LOAD SENSITIVITY ANALYSIS) 94	94
TABLE 24: SUMMARIZES THE STEPS FOR THE STRENGTH SENSITIVITY ANALYSIS.....	95
TABLE 25: STANDARD DEVIATION FOR EACH CURVE IN FIGURE 45 (STRENGTH SENSITIVITY ANALYSIS).....	96



## LIST OF FIGURES

FIGURE 1: ILLUSTRATING THE COMPLETED VERTICAL WELL SCHEMATICS, WITH CASINGS OF DIFFERENT SIZES, PRODUCTION TUBING, ETC.....	21
FIGURE 2: SHOWS A TYPICAL CASING PROGRAM, MUD WEIGHT HAS TO BE SELECTED TO BE BETWEEN THE PORE AND FRACTURE PRESSURE. EACH TIME WE HAVE TO WEIGH UP MUD, A NEW CASING HAS TO BE RUN <sup>[13]</sup> .....	23
FIGURE 3: DEMONSTRATES THE SELECTION OF CASING SHOE DEPTH, USING BOTTOM-UP PRINCIPLE BASED ON MUD WEIGHT, SOURCE: <sup>[13]</sup> .....	26
FIGURE 4: SHOWS THE RESULTING BURST LOAD AND THE REDUCED WELL INTEGRITY, SOURCE [13].....	28
FIGURE 5: ILLUSTRATING THE BURST LOAD AND THE REDUCED WELL INTEGRITY WITH THE CASING TOP AS WEAK POINT <sup>[13]</sup> .....	28
FIGURE 6: BURST PRESSURES WITH GAS KICK TAKEN WITH FULL WELL INTEGRITY, SOURCE <sup>[13]</sup> ....	29
FIGURE 7: THE ELLIPSE OF PLASTICITY, SAME AS ILLUSTRATED IN <sup>[13]</sup> .....	31
FIGURE 8: SHOWS THE CURVES OF PORE PRESSURE, FRACTURE PRESSURE, AND MUD WEIGHT IN DIFFERENT HOLE SECTIONS. THE LITHOLOGY COLUMN SHOWS WHAT FORMATIONS THAT WILL BE DRILLED THROUGH <sup>[19]</sup> .....	32
FIGURE 9: A PRODUCTION WELL WITH A SUBSEA WELLHEAD SHOWING A, B AND C ANNULUS AND THE DIRECTION OF HEAT LOSS IN THE WELLBORE.....	34
FIGURE 10: REPRESENTS THE WELL SCHEMATICS.....	36
FIGURE 11: ILLUSTRATES THE WSD WITH DETERMINISTIC LOAD EFFECT AND DETERMINISTIC RESISTANCE <sup>[8]</sup> .....	43
FIGURE 12: WORKING STRESS DESIGN PROCESS MODIFIED AFTER <sup>[8]</sup> .....	44
FIGURE 13: REPRESENTS THE WELL SCHEMATICS.....	47
FIGURE 14: SHOWS THE RBD WITH DETERMINISTIC LOAD EFFECT, RESISTANCE DISTRIBUTION AND THE PROBABILITY OF FAILURE AT THE OVERLAP <sup>[8]</sup> .....	49
FIGURE 15: ILLUSTRATING RBD WITH DISTRIBUTION LOAD- DISTRIBUTION STRENGTH RELATIONSHIP, THERE IS NO OVERLAP <sup>[3]</sup> .....	50
FIGURE 16: ILLUSTRATING RBD WITH DISTRIBUTION LOAD- DISTRIBUTION STRENGTH RELATIONSHIP, THERE IS NO OVERLAP <sup>[11]</sup> . THIS FIGURE IS ALSO A COMPARISON OF WSD TO RBD, DEMONSTRATING THE STRICTNESS OF THE WSD WITH RESPECT TO QUANTIFYING THE UNCERTAINTIES.....	52
FIGURE 17: SHOWS THE STEPS INVOLVED IN DEFINING A MODEL.....	58
FIGURE 18: SHOWS THE DISTRIBUTION TYPES WHERE (A) AND (B) REPRESENT THE MINIMUM AND MAXIMUM VALUES RESPECTIVELY FOR THE UNIFORM DISTRIBUTION, (A) DENOTES THE MINIMUM VALUE, (B) DENOTES MOST LIKELY VALUE AND (C) DENOTES THE MAXIMUM VALUE FOR THE TRIANGULAR DISTRIBUTION. M AND $\Sigma$ REPRESENT THE MEAN AND STANDARD DEVIATION OF THE NORMAL DISTRIBUTION RESPECTIVELY <sup>[29]</sup> .....	59
FIGURE 19: ILLUSTRATING THE ADDITION OF THE INPUT DISTRIBUTIONS <sup>[30]</sup> .....	60
FIGURE 20: ILLUSTRATING THE MULTIPLICATION OF THE INPUT DISTRIBUTIONS <sup>[30]</sup> .....	61

FIGURE 21: THE SCHEMATIC PROCEDURE OF THE MONTE CARLO SIMULATION. MODIFIED AFTER <sup>[28]</sup> .....	62
FIGURE 22: SHOWS THE MONTE CARLO TECHNIQUE OF SAMPLING, HOW THE INPUT VARIABLES ARE GENERATED <sup>[28]</sup> .....	63
FIGURE 23: MONTE CARLO SAMPLING <sup>[31]</sup> .....	63
FIGURE 24: ILLUSTRATING THE STATISTICAL TERMS USED FOR RESULT PRESENTATION <sup>[30]</sup> .....	65
FIGURE 25: TRIANGLE AND NORMAL DISTRIBUTIONS RESPECTIVELY <sup>[29]</sup> .....	73
FIGURE 26: SHOWS THE PDF PLOT OF CASE 1, WITH OIL DENSITY. THERE IS NO OVERLAP BETWEEN THE CURVES AND THEREFORE NO CHANCE OF FAILURE.....	75
FIGURE 27: SHOWING CDF PLOT OF CASE 1, WITH THE OIL DENSITY.....	76
FIGURE 28: PDF PLOT OF CASE 2A, WITH GAS-FILLED WELL. HERE, THE OVERLAP BETWEEN THE TAIL END OF THE LOAD CURVE AND THE START LINE OF THE STRENGTH CURVE INDICATES A FAILURE, WITH A VALUE FOR THE PROBABILITY OF FAILURE.....	78
FIGURE 29: THE CDF PLOT OF CASE 2, WITH LOW GAS DENSITY (0.3-0.5SG). .....	79
FIGURE 30: SHOWS A HIGHER PROBABILITY OF FAILURE COMPARED TO THE PREVIOUS PLOT OF CASE 2A.....	81
FIGURE 31: SHOWS THE CDF CURVE FOR CASE 2B. TAKING A CLOSE LOOK AT THE TWO CURVES, ONE WILL NOTICE AN OVERLAP BETWEEN THEM AT THEIR TAIL ENDS. THE GAS DENSITY RANGE IS 0.1-0.3SG. ....	82
FIGURE 32: PDF PLOT FROM BARLOW' STRENGTH MODEL SIMULATION. ....	85
FIGURE 33: CDF PLOT FROM BARLOW' STRENGTH MODEL. ....	85
FIGURE 34: PDF PLOT FROM THE VON MISES' STRENGTH MODEL SIMULATION. ....	86
FIGURE 35: CDF PLOT FROM THE VON MISES' STRENGTH MODEL. ....	86
FIGURE 36: PDF PLOT FROM THE KLEVER STEWART'S STRENGTH MODEL SIMULATION. THE YIELD STRENGTH IS USED IN THE TOP PLOT, WHILE THE ULTIMATE TENSILE STRENGTH IS USED IN THE BOTTOM PLOT. ....	87
FIGURE 37: CDF PLOT FROM THE KLEVER STEWART'S STRENGTH MODEL SIMULATION. THE YIELD STRENGTH IS USED IN THE TOP PLOT, WHILE THE MINIMUM TENSILE STRENGTH IS USED IN THE BOTTOM PLOT. ....	88
FIGURE 38: PDF PLOT FROM THE PASLAY'S STRENGTH MODEL SIMULATION.....	89
FIGURE 39: CDF PLOT FROM THE PASLAY'S STRENGTH MODEL SIMULATION.....	89
FIGURE 40: PDF PLOT FROM MOORE'S STRENGTH MODEL SIMULATION. ....	90
FIGURE 41: CDF PLOT FROM MOORE'S STRENGTH MODEL SIMULATION. ....	90
FIGURE 42: PDF PLOT FROM THE NADAI'S STRENGTH MODEL SIMULATION. THE YIELD STRENGTH IS USED IN THE LEFT PLOT, WHILE THE MINIMUM TENSILE STRENGTH IS USED IN THE RIGHT PLOT.....	91
FIGURE 43: CDF PLOT FROM THE NADAI'S STRENGTH MODEL SIMULATION. THE YIELD STRENGTH IS USED IN THE LEFT PLOT, WHILE THE MINIMUM TENSILE STRENGTH IS USED IN THE RIGHT PLOT .....	91
FIGURE 44: SHOWS THE EFFECT OF THE PARAMETERS ON THE SURFACE PRESSURE. ....	93
FIGURE 45: SHOWS THE EFFECT OF THE PARAMETERS IN STRENGTH.....	95

FIGURE 46: COMPARISON BETWEEN THE PROBABILISTIC PRESSURES AND DETERMINISTIC PRESSURES OF CASE1 LOAD AND STRENGTH. OIL DENSITY =0.7-0.9SG, PORE PRESSURE = 1.6SG ..... 97

FIGURE 47: COMPARISON BETWEEN THE PROBABILISTIC PRESSURES AND DETERMINISTIC PRESSURES OF CASE2B LOAD AND STRENGTH. THE OVERLAP INDICATES CASING FAILURE. GAS DENSITY= 0.1-0.3SG, PORE PRESSURE=1.6SG ..... 97

## **SYMBOLS**

$\sigma_y$	minimum yield strength, psi
$P_b$	Burst pressure, psi
$D$	Outside Diameter of a Casing,
$t$	thickness,
$\mu$	Mean
$\sigma$	Standard deviation
$f(\vec{x})$	joint probability density function of the variables in $\vec{x}$ ,
$g(\vec{x})$	limit state function
$\vec{x}$	vector of random variables
$\beta$	Reliability Index
$\Phi$	standard normal cumulative distribution function,
$P$	Probability of failure
$K_{dr}$	Correction factor for pipe deformation
$t_{nom}$	Nominal tubular wall thickness
$t_{min}$	Minimum tubular wall thickness without crack deformations
$D$	inside diameter of a Casing
$L(\vec{x})$	Load
$S(\vec{x})$	Strength
P10	10 <sup>th</sup> Percentile
P50	50 <sup>th</sup> Percentile
P90	90 <sup>th</sup> Percentile
$L_f$	Load safety factor
$S_f$	Strength safety factor

## **ABBREVIATIONS/ ACRONYMS**

AFE	Annular Fluid Expansion
AFOSM	Advanced First Order Second Moment
APB	Annular Pressure Buildup
BOP	Blowout Preventer
CDF	Cumulative Distribution Function
COV	Coefficient of Variance
CSG	Casing
DF	Design Factor
ECD	Equivalent Circulation Density
FORM	First Order Reliability Method
FOSM	First Order Second Moment
Ft.	Feet
ID	Inner Diameter
In.	Inches
HPHT	High Pressure High Temperature
HSE	Health, Safety and Environment
LRFD	Load and Resistance Factor Design
LSA	Limit State Analysis or Load and Strength Analysis
LSE	Limit State Equation
LSM	Limit State Method
MCS	Monte Carlo Simulation
MD	Measured Depth
MIYP	Minimum internal yield pressure
NORSOK:	Norsk Sokkels Konkuransesposisjon
OAT	One At A Time
OD	Outer diameter
OOIP	Original Oil in Place
PDF	Probability density function
PDO	Plan for Development and Operations
PERT	Project Evaluation and Review Technique
PPore	Pore Pressure
P&A	Plug and Abandonment
P10	10 <sup>th</sup> Percentile
P50	50 <sup>th</sup> Percentile
P90	90 <sup>th</sup> Percentile
QA/QC	Quality Assurance and Quality Control
RBD	Reliability-based Design

SA	Sensitivity Analysis
SF	Safety factor
STD	Standard deviation
SORM	Second Order Reliability Method
TD	Target Depth
TRF	Technical Recovery Factor
VME	Von Mises stress
WSD	Working Stress Design

# Chapter 1

---

## 1.1 INTRODUCTION

During the drilling and development of a well, casings are immediately run into the drilled section of the hole for multiple reasons. Casings are the cylindrical steel structures installed in a drilled well. They are used to secure the drilled well, provide access to the reservoir and to produce its contents. It is also used for formation fluid isolation which can cause kick. It is used for maintaining the integrity of the well. It prevents hole cave-ins. It prevents contamination of the drilling and treatment fluid. It provides connection and support for well head equipment. It is also used to contain pressure. It provides means to install completion, production and subsurface equipment. Casing sizes determine the final wellbore size for production. Although today, the traditional method is that the casing sizes are established from the required tubing size, being the bottom-up design. However, prior to running a casing, a design is carried out to determine which type of casings that are required to withstand loads. The casing parameters are casing weight, outer diameter, thickness, and steel type. Different casings will have different strengths. Today, the daily rig cost in Norway is approximated to be \$170000<sup>[1]</sup> and casing cost approximately \$2000 per metric ton [2].

‘Tubulars represent about 16% of an average drilling cost and it accounts for the second largest percentage of total cost of a drilling project.’<sup>[3]</sup>

This magnanimous cost of tubular and its significant contribution to the overall drilling project and well Integrity makes it necessary to refine design approaches. The design method must ensure a safety, at the same time a designer may attempt not to be too conservative to reduce costs. The basics of all casing design methods is to balance the casing cost with design reliability, in other words, the right casing design for a given situation should be economical as well as reliable.

In casing design, a traditional approach known as Working Stress Design, WSD is mainly used. This approach is based on specific margins known as safety factors that are between maximum anticipated field loads and the rating of the casing. The purpose of the traditional engineering design is to provide minimum degrees of serviceability and safety during the lifetime of a structure<sup>[4]</sup>. There is also a very good understanding of the Working Stress Design as it has made its name in the industry in promoting safety and reducing failure. However, one of the problems with this approach is that it lacks robustness as it is associated with unanalyzed uncertainties that can lead to over design or under design solutions. There are uncertainties present in the loading of a structure, material properties of a structure, environmental impact on that structure/system and the models<sup>[4]</sup>. Another problem with the traditional approach is that it is very strict in the casing rating because it assumes the worst case load scenario<sup>[3]</sup>. The burst pressure for an installed tubular is

uncertain as it is influenced by random factors. In a deterministic calculation of a casing burst pressure, all parameters and factors are assumed to be known and exact models are used to calculate the strength and load. These exact models assume a tubular burst/failure if the burst pressure equals or surpasses the strength<sup>[5]</sup>. In other words, this deterministic approach uses single point estimate of the input parameters. Therefore, in order to ensure a safe design, a worst case allowable input variables for the casing in agreement with the product specification is used<sup>[5]</sup>. The actual burst pressure can be higher or lower than the deterministic estimated value. As a result, the deterministic predicted burst pressure is not reliable because the estimation approach does not explore and understand the uncertainties associated with the design and therefore does not give a risk consistent design.

One objective of this study is to introduce how reliability based casing design can be performed. It will be discussed in relation to the classical working stress design. A burst scenario shall be considered for the demonstration of this method. The reliability based method shall account for variations in field load and casing performance. We shall also develop the probability distributions for loads and strengths. The probability of failure shall be quantified from the load and strength distributions. More so, in this study, we shall evaluate the two Casing design approaches with regards to their merits and limitations; to determine the best in terms of cost effectiveness, reduced uncertainty and safety in application.

The reliability-based approach provides a theoretical basis for taking the uncertainties into account in a complete decision scheme and to assess a system/ structure/component's ability to stay safe and functional during their lifecycle<sup>[4]</sup>. Therefore, a probabilistic estimate of load and strength will account for the uncertainties in the design, that is, the input variables and the result. It will make it possible to create a relationship between the load and a probability of burst. In this way, the approach provides a likely burst or failure pressure instead of a worst possible burst pressure given by WSD<sup>[5]</sup>. The purpose of using a probabilistic approach to load and strength estimation have been discussed fully in next chapters.

The origin and use of the word *reliability* was in 1816 where it was used synonymously with *repeatability*<sup>[6]</sup>. At that time, a test was regarded reliable if exact results were obtained after repeated running times. In the 1920s, Dr. Walter A. Shewhart promoted product improvement by suggesting the use of statistics in process control. Coincidentally, Waloddi Weibull was working on fatigue and its statistical models. Later on in the 1940s, the U.S military proposed a definition for the word reliability<sup>[6]</sup>. A lot of understanding and application of reliability in engineering has happened since then. Reliability is a sub-discipline in systems engineering. Today, the reliability based approach in engineering currently has its application in engineering and structural design to improve cost, safety, and insight into uncertainties. In the Petroleum industry, the reliability-based approach to problem-solving and decision-making is an emerging technology. The Blade Energy



Partners have applied it in the understanding of Annular Pressure Buildup, APB mitigation strategies<sup>[7]</sup>. Other applications include; a more recent work by the Blade Energy Partners, *A reliability-based Approach for Survival design in Deepwater and high pressure/temperature wells*<sup>[8]</sup>.

The aim of a probabilistic burst pressure model is to calculate the probability of failure,  $P_f$ <sup>[5]</sup>. According to ISO TR 10400, the stochastic burst load can be calculated using either a variable load model or with a deterministic load model, so long as a probabilistic approach is used to calculate the strength. The probability of failure,  $P_f$  can be calculated using a mathematical software like Oracle crystal ball. It can also be calculated using methods like the First Order Reliability Method, FORM, the Second Order Reliability Method, SORM, the First Order Second Moment method, FOSM and the Advanced First Order Second Moment method, AFOSM. One can also use the Monte Carlo simulations, MCS to obtain  $P_f$ . The mentioned methods are the various reliability analysis methods that can be utilized for engineering or research analysis. The FORM, SORM, FOSM, and AFOSM can be implemented using the optimization tools in Excel. However, the results obtained from MCS are superior to results from other methods like the FORM and SORM.

## **1.2 Scope of Study, Background of problem**

In the conventional casing design method, WSD, a safety margin is assigned and maintained between the two values of load and strength to ensure the design reliability. As earlier stated, this approach to casing design is seen to be too strict and inflexible. [Figure 11](#) illustrates this strictness. Also, the WSD has some notable limitations of which some of them are already mentioned like its inability to analyze the uncertainties in a design. The load uncertainty arises from inaccurate load estimation while the strength uncertainty arises from manufacturer's fault from workmanship, inherent variable material properties and tubular installation challenges/ handling<sup>[9]</sup>. More so, WSD lacks the ability to quantify any associated risks<sup>[8]</sup>. In other words, it lacks a risk assessment<sup>[8]</sup>. Another setback with the conventional method is that one cannot really quantify the benefits of a high safety margin if used. Also, in this method, the safety factor works well when it is supported by many experiences. However, in the absence of historical experiences, or in the introduction of new materials or new fields, then the basis of judgement to define safety factor will be lacking<sup>[9]</sup>. Consequently, the safety of the materials cannot be substantially assured. In addition, the basis for the design factor is often not well-founded because design factors have been passed down through the years and rarely subjected to any field or analytical verification. In addition, the use of design factors may not fit in well when dealing with high magnitude loads with extreme severity upon occurrence.

Therefore, to ensure an optimal casing selection, it is important that a casing design method balances the casing cost by offering a design reliability. In other words, a correct casing design for a given condition should be reliable as well as economical.

The scope of this work is to model the accurate design problem. Here, the design variables, the expected load, and the casing strength are defined in terms of two random variables, S and L. S represents the casing strength or the tubular rating while L represents the expected load which the casing shall be subjected to during the well's life. The objective of the design is to ensure that  $S > L$  at all times during the Well life. This defines the probability of success,  $P_s$ .

### **1.3 Objective of the application of reliability in casing design**

The objective is to demonstrate how to perform a reliability based casing design for a burst scenario. In this study, we will define the load effect and strength using probability distributions, run simulations using Monte Carlo and obtain the statistical results. These statistical results shall be presented in probability distribution function curves, PDF, cumulative density function curves, CDF, probability of failure,  $P_f$ , mean and spread (P90-P10). The second objective is to demonstrate how to perform the sensitivity analysis in casing design, perform one and obtain results. We shall evaluate the uncertainties in the load and strength. From our results, we shall determine the dominating parameters on our models with respect to uncertainty. Based on this information, we can determine the reliability of our casing design and at the same time achieve the overall safety of the well development project. The knowledge obtained from this study can be used to;

- To eliminate or minimize the occurrence of failure or its frequency<sup>[10]</sup>.
- Find out the causes of failure, correct the cause or find a way to cope with uncorrected failures.
- Analyze the reliability of the design and data<sup>[10]</sup>.
- Reduce the level of uncertainty in the design.

### **1.4 Structure of the Thesis**

This thesis is made up of eight chapters. The first chapter covers the introduction, the objective, scope, and background of this study. The second chapter addresses the literature review on well design, narrowing it down to casing design. Here, the major loads considered in the design and the mechanisms generating these loads are discussed. Also, this chapter explains the limit state equations, guidelines used for the casing design and the factors that degrade the casing strength. In chapter three, the focus is on the traditional method of casing design, known as the working stress design. It explains its limitations to giving a robust design. Chapter four discusses the literature review on the reliability approach to casing design. It also addresses the merits and limitations of this approach with respect to other methods of casing design. The fifth chapter discusses the literature review on Monte Carlo simulation. It explains the Monte Carlo

simulation method in RBD application. Chapter six and seven demonstrate the reliability calculation method using Monte Carlo simulation. In chapter seven, the results from the simulation are presented and interpreted. The final chapter concludes the study and recommends areas for future study.

### 1.5 Definition of important terms

**Risk;** According to the Oxford dictionary, Risk is defined as the possibility of occurrence of loss, injury or any unwanted event. It is also defined as the product of the probability and consequences of an incident of failure taking place. There are lots of similar definitions for risk and it is mathematically stated as;

$$Risk = Probability \times Consequence$$

**Load** is the pressure exerted on a casing which can lead to burst or collapse.

**The probability of Failure,  $P_f$ ;** is a term defining a value which quantifies risk. It is used to evaluate the consequences of failure<sup>[4]</sup>. The Probability of failure,  $P_f$  is calculated using the formula<sup>[11]</sup>:

$$P_f = P[R(\tilde{x}) < 0] \quad \text{Equation 1}$$

$$P_f = P[(S(\tilde{x}) - L(\tilde{x})) < 0] \quad \text{Equation 2}$$

Where  $S(\tilde{x})$  is the distribution of strength from limit state,  $L(\tilde{x})$  is the distribution of load effect,  $R(\tilde{x})$  is the limit state function and  $\tilde{x}$  is a vector of random variables that determine strength and load effect. Some recommendations for  $P_f$  for high magnitude design loads of tubulars by<sup>[8]</sup> are:  $10^{-6}$  to  $10^{-5}$  for high consequence failures and  $10^{-3}$  to  $10^{-2}$  for low consequence failures.

**A deterministic** approach is one which assumes all variables which dictates a tubular performance property are known with certainty<sup>[12]</sup>.

**The probabilistic** approach uses the distributions of geometric and material property values to compute a distribution of performance property values<sup>[12]</sup>.

**Sour service**<sup>[13]</sup>; refers to the special treatment given to a casing steel to withstand corrosion and hydrogen embrittlement. This treatment options can be to increase the casing thickness, making the casing steel a stainless quality, making the casing steel soft rather than hard, and proper material selection.

**High pressure high temperature Well;** Well with expected shut-in pressure exceeding 690bar and a static bottom hole temperature higher than 150°C.

**Riser margin** is the additional fluid density needed to compensate for the difference in pressure between the drilling fluid in the riser and seawater when the riser is disconnected.

# Chapter 2

---

## 2.1 Well Design

Well design is an activity carried out in an early stage of the well planning. Here the needs, objectives, and criteria for the well are defined. One of the purposes of this is to simplify the Well planning and its development in the long run. The benefits of Well design are cost reduction and to ensure well integrity. Also, a good Well engineering design can ensure that the well can withstand unforeseen, abnormal events <sup>[13]</sup>.

The overall Well design comprises of:

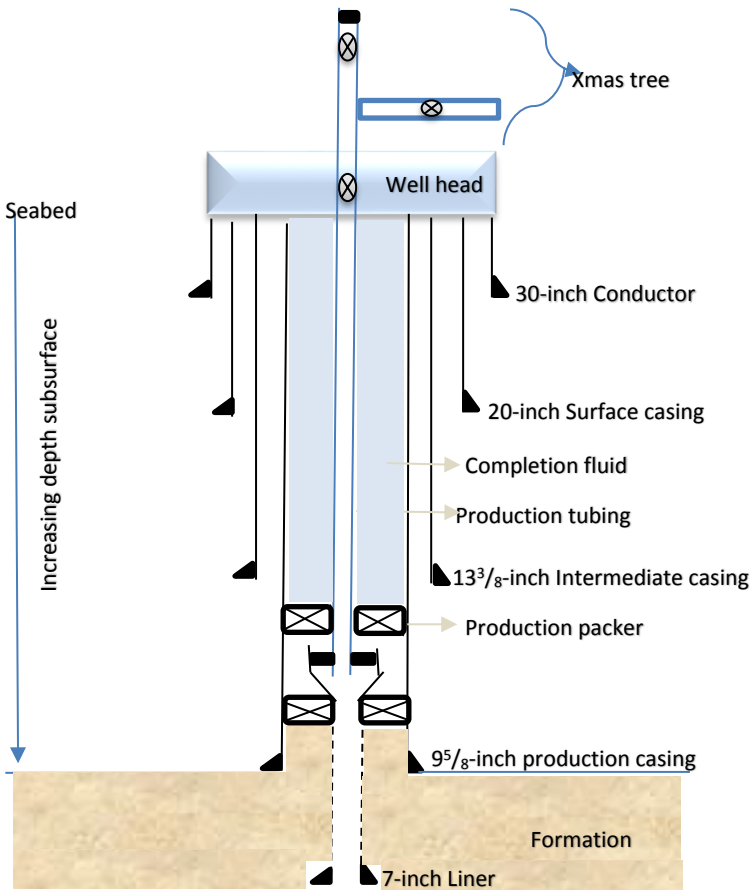
- Casing design: types and setting depths
- Completion design
- Drilling design: Well Profile and direction, Cementing, Drilling Fluids and Hydraulics<sup>[11]</sup>

The Well design process can be further divided into these stages<sup>[14]</sup>;

- I. Establish well objectives
- II. Obtain PDO or consent to drill.
- III. A collection of data, such as seismic surveys, offset well information.
- IV. Establish subsurface pressure regimes.
- V. Establish pore pressure.
- VI. Establish fracture gradients (relevant in casing seat selection).
- VII. Prepare a drilling program.
- VIII. Rig specification, jack-up, semi-submersible, etc.
- IX. Prepare Authorization for Expenditure, AFE

Figure 8 shows an example of a drilling program. The comprehensive well design is used for the Well Construction. The well Construction is a systematic process where the well is drilled in sections. A drilled hole section is cased off before continued drilling of a new hole section with a reduced bit size. A typical offshore or onshore well has the conductor, which may be preinstalled in a template for a subsea well, the surface casing, the intermediate casing, the production casing and the liner. This describes the mainly used casing arrangement. A well may have a different configuration than the one listed. For instance, a well may have a tie-back casing or do without a liner. Casings have different sizes to suit different purposes. A production tubing is installed inside the production tubing for production when all necessary casings have been installed as can be seen in Figure 1. This tubing is anchored to the production casing with the aid of a packer. The tubing-casing annulus is filled with completion fluid to prevent corrosion and minimize differential pressure. Once the well

is constructed and completed, it is then handed over for production. This thesis shall focus on the casing design aspect and the important factors to consider in casing design.



**Figure 1:** Illustrating the completed vertical well schematics, with casings of different sizes, production tubing, etc.

### 2.1.1 Casing Design and Criteria<sup>[13]</sup>

Casing design is one of the key activities of well design. In the basics of casing design, the casing strength is assessed relative to burst, collapse, tensional load, biaxial and triaxial forces imposed on it<sup>[13]</sup> <sup>[15]</sup>. In other words, a casing design involves designing each casing string with care to ensure that it can withstand these loads to achieve the overall well integrity throughout the well's lifetime <sup>[15]</sup>. The well's lifetime can be as long as 5 decades on average<sup>[15]</sup>. This assessment is carried out on these casing types namely, conductor string, surface, intermediate, production casing and liner to be used in the Well. At the end, one must ensure that the design fulfills the conflicting requirement of burst and collapse while not going beyond the tensile strength.

The formation's pore and fracture pressure are the basis for the casing design. In an exploration well, the pore and fracture pressure are approximated, but for development well, they are obtained from the record of already drilled wells. It is important that a casing program is flexible to allow future addition of casing strings if needed.

In the traditional casing design, the Well Engineer optimizes the process to select cheapest casing strings strong enough to withstand the predicted and occurring loads over time. There are several loads or conditions that the casing string can be exposed to. These loads are addressed in the next section. The design itself is a function of these factors <sup>[14]</sup>:

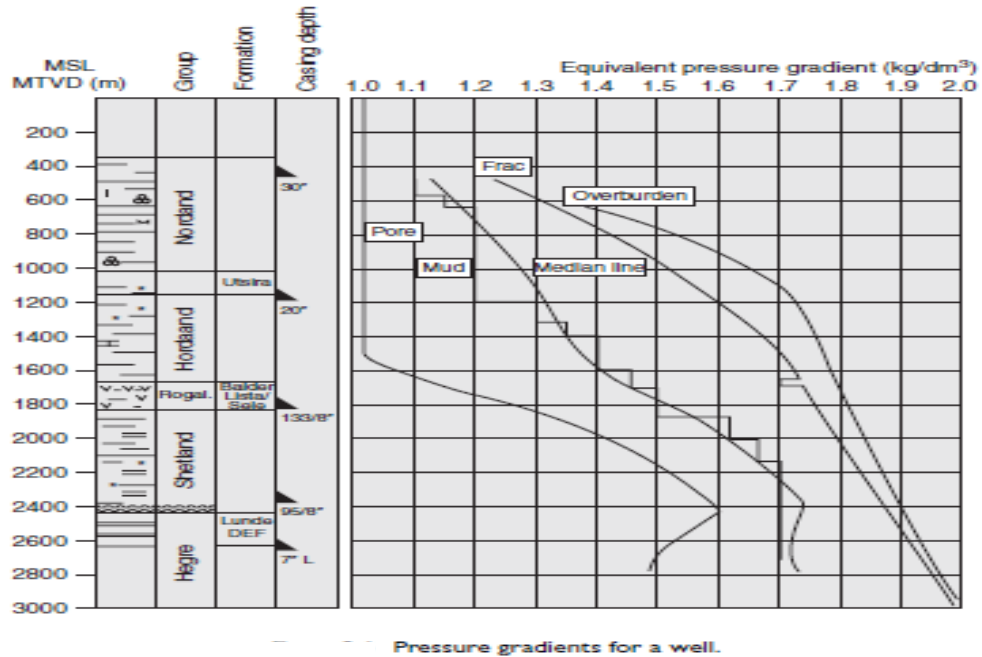
- Loads to be encountered during the lifetime of the well from drilling, completions, production, injection, testing, and interventions.
- Assumed fracture pressure during planning which is verified by formation integrity tests (strength of formation).
- Formation pore pressure
- Availability and costs of the casing.
- Casing lifetime when exposed to production (the perforated region).

The loads to be encountered throughout the lifetime of the well are defined by company policies and/or government regulations and experience<sup>[14]</sup>. As a result, it varies greatly within the industry. The assumed maximum loads are then multiplied with safety factors to obtain values that can be used to pick out a casing type with the required strength. The Safety factor is also usually stated by government regulation or company policy. In this work, the loads and safety factors used are based on the NORSOK standards<sup>[16]</sup>. During the casing design, the aim is geared towards satisfying the safety factors/safety margins.

According to NORSOK D-010 design criterion, a selected casing shall have a higher-pressure rating than the pressure it can withstand especially in the presence of corrosive media. The following are the recommended safety factors by NORSOK, applicable to undamaged casing<sup>[16]</sup>:

- Burst= 1.1
- Collapse=1.1
- Tension load=minimum design factor of 1.3
- Triaxial load=1.25

The API design factors are discussed in the next chapter.



**Figure 2:** Shows a typical casing program, mud weight has to be selected to be between the pore and fracture pressure. Each time we have to weigh up mud, a new casing has to be run<sup>[13]</sup>.

### 2.1.2 Casing type and sizes

The list includes standard casing sizes<sup>[13]</sup>.

Casings	Common Casing sizes in diameter / Alternative size, inch		Corresponding hole Sizes, inch	Example of setting depths from seabed, meter
Conductor casing	30		<u>36</u>	<u>50-100</u>
Surface casing	20	18 <sup>-5/8</sup>	<u>26</u>	<u>300-1200</u>
Intermediate casing	13 <sup>-3/8</sup>	16	17 <sup>-1/4</sup>	<u>1300-2000</u>
Production casing	9 <sup>-5/8</sup>	10 <sup>-3/4</sup>	12 <sup>-1/4</sup>	<u>2500-4500</u>
Production liner	<u>7</u>	5 <sup>-1/2</sup>	8 <sup>-1/2</sup>	
Tieback casing	<u>Any size</u>			
Intermediate liner	<u>Any size</u>			

#### Conductor pipe<sup>[14]</sup>;

This is the first casing installed during a well construction. It has the largest diameter compared to the other casings. For subsea well, it may be pre-installed in a template. its functions include:

- Seal off weak formation below the seabed.



- It serves as a conduit for mud returns.
- It serves as foundation for the wellhead
- It bears the weight of all other casings

### **Surface Casing<sup>[13]</sup>;**

This is the next Casing installed after the conductor. It is designed to carry the weight of the Blow Out Preventer, BOP, and the wellhead. It also isolates any shallow unconsolidated sand zone and fresh water zones.

### **Intermediate Casing;**

This next casing is used to isolate problematic formation like mobile formation known to cause drilling problems, such as stuck pipe, differential sticking, lost circulation, etc. it is also used to isolate high-pressure intermediate zones. This casing is designed to have sufficient strength to ensure that the next open hole section is drilled safely. A number of intermediate casing strings may be required depending on the problems that are encountered during well drilling. However, it must give sufficient integrity for the drilling of the problematic zones.

### **Production Casing<sup>[13]</sup>;**

This casing can be the last casing string installed. In this case, it is run through the pay zone. In the case where a production liner will be installed, it is set just above the pay zone in the caprock. In fact, the manner of setting depends on the type of completion selected for the well, open or cased completion. Also, when cementing this casing, a thoroughness is required to ensure that the annulus across the productive interval is sealed off, to prevent the passage of reservoir fluid along the outer side of the casing. The production casing is designed to have full well integrity throughout the well's lifetime. It is also designed to withstand corrosion (sour serviced) in the case of producing corrosive reservoir fluids. In a case where the production liner will be set, the casing will be designed to permit the further drilling of the hole. Its functions include;

- It isolates the production zone from other zones, for example, a water-bearing zone
- It protects the tubing from high-pressure fluids.
- It provides the basis for Well Completion.

### **Production liner<sup>[13]</sup>;**

This is usually the last casing string set. Consequently, it has the smallest diameter compared to all other casing strings. This casing string is short, unlike the other casings, it does not extend to the surface of the well but is hung off into the previous casing to provide an overlap. It is set to seal across the production zone, with its shoe set deep in the productive zone. The liner is designed for

full integrity to withstand high fluid pressure throughout the lifetime of the well. The benefit with the liner is its low cost and short installation time.

### **Tieback Casing<sup>[13]</sup>;**

A tieback casing is used to extend the liner string to the surface of the well. It may also be installed to protect other casings from highly corrosive gases like H<sub>2</sub>S and Co<sub>2</sub>, or to increase the Well Integrity for flow testing.

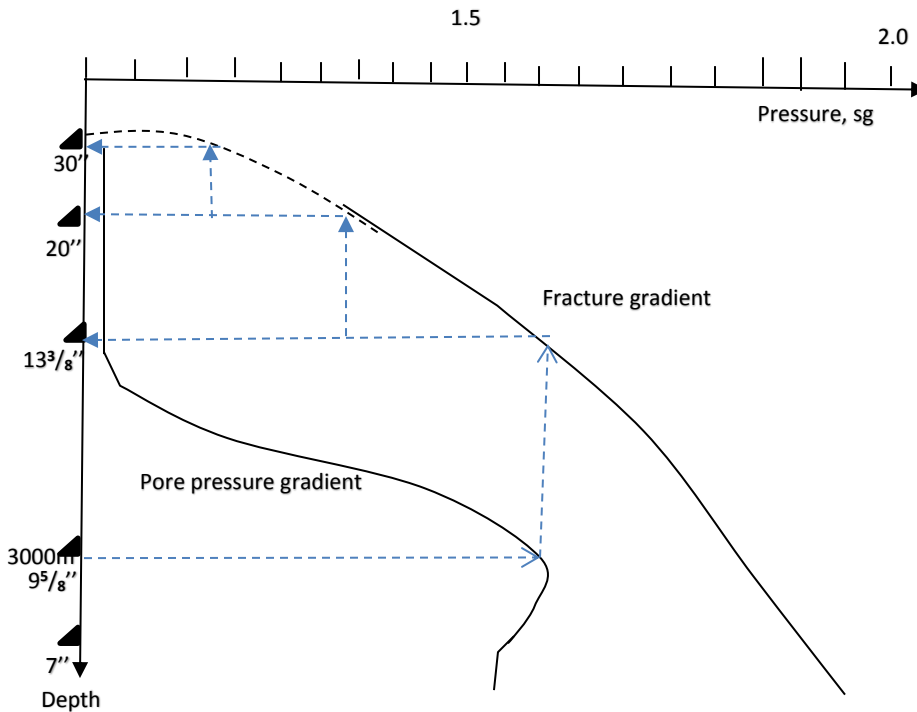
### **2.2 Casing Properties,<sup>[12]</sup>**

Casings are manufactured in all kinds of sizes, lengths, weights, and grades. Casings can also be custom made to suit a type of environment, for example, a sour-serviced casing manufactured for a corrosive environment. Casings couplings are also manufactured in variety. API, the drilling data handbook and ISO booklet (API Standard 5A, API Bulletin 5C2)<sup>[5, 17, 18]</sup> contains casing specifications of different sizes, lengths, weights, connections, grades and steel quality. Casings also have different steel quality, for example, L80, P110, and Q125. The casing performance properties refer to its axial strength, internal pressure resistance, and collapse resistance. An example could be a 9<sup>5</sup>/<sub>8</sub> L80 casing with 47lbs/ft. weight, a burst strength of 473bar and a collapse strength of 328bar. We are also given information about inner diameter, ID, outer diameter, OD and wall thickness, t.

### **2.3 Casing Setting depth selection<sup>[13]</sup>**

A well-known criterion for choosing a casing shoe setting depth should be that the next open hole section will be drilled safely and successfully. The choice of the casing seat depth depends on the pore and fracture pressure, therefore, these factors are used as a guide during the selection<sup>[13]</sup>. Also, the concept of kick margin plays a critical role when selecting casing shoe depth and is further discussed in the next section. Furthermore, the casing shoe must be set in a competent formation that is not prone to collapse and can withstand high pressure or mechanical impact from the kick, formation subsidence, etc. For example, the shale formation is competent enough for setting casing shoes, unlike the loose sand formation. For the conductor pipe, it is good to set it deep with considerations that the shoe is set at a depth such that it does not fracture when drilling the next open hole section (26-inch hole section). The shallower formation, especially the installation depths of the conductor and surface casing is not hydrocarbon bearing but may contain shallow gas pockets. For the Surface casing, in the absence of hydrocarbon, the shoe is also set as deep as possible within the boundaries of avoiding shoe fracture during the drilling of next open hole section. In the presence of hydrocarbon, the shoe is set at a depth where it can withstand fracture in the event of a kick. This criterion also applies to the intermediate and production casing, where it is very likely to encounter a hydrocarbon interval. These casing shoe depths shall be selected such that they can withstand the pressure of a defined volume of kick<sup>[13]</sup>.

Aadnøy developed a relationship between the pore and fracture pressure to determine the maximum length of an open hole section<sup>[13]</sup>. He presented the bottom to up principle based on the mud weight in the casing seat selection<sup>[13]</sup>. This principle works by starting the casing seat selection from the bottom of the Well to the top, that is, from production casing to conductor. This method of casing seat selection, allows a minimum number of casing pipes to be used and is relevant to deep-water Well development solution<sup>[13]</sup>. Figure 3 is an illustrating example, a mud weight of 1.7sg is used at 3000m depth.



**Figure 3:** Demonstrates the selection of casing shoe depth, using bottom-up principle based on mud weight, source:<sup>[13]</sup>

Among hole stability, formation pressures and kick margin, the other factors which may affect the shoe depth design are; regulatory requirement, encountering drilling problems like differential sticking and the immediate need for an interval isolation.

### 2.2.1 Kick Tolerance/Kick margin<sup>[13]</sup>

This refers to the maximum volume of kick that an open hole section can take during shut-in and kill circulation without fracturing the shoe of the previous casing. The kick tolerance concept is used to define the levels of well Integrity, the full well integrity, and the reduced well integrity.

The kick tolerance is calculated periodically when the mud weight changes or prior to casing off a new open hole section. An example can be 4m<sup>3</sup> kick volume in 8<sup>1</sup>/<sub>2</sub> in, hole.

#### **2.4 Well Integrity**

*'Well integrity is the application of technical, operational and organizational solutions to reduce the risk of uncontrolled release of formation fluids and well fluids throughout the life cycle of a well'*<sup>[16]</sup>.

Well design must ensure that the well will contain its fluids without incidents during the lifetime of a well. It must also ensure that the well can withstand abnormal incidents such as lost circulation of mud and high-pressure kick which can lead to serious Well control issues<sup>[13]</sup>.

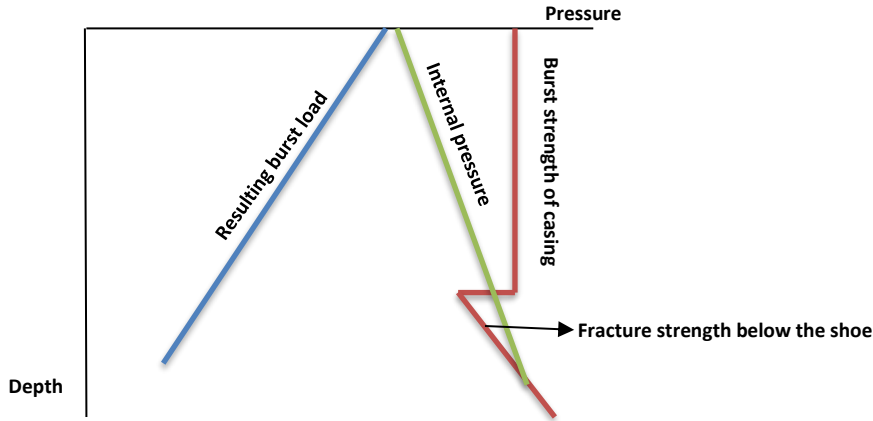
High-pressure kick can be caused by using too low mud weight, wrong pore pressure estimation, loss of mud return, swab, and surge (causes fracture that may lead to kick)<sup>[13]</sup>. One of the consequences of taking in a kick is that it can rupture the casing especially if the well is shut in. A kick taken in an open well gives low loading on the casing initially<sup>[13]</sup>. However, when the well is shut in, and with a full or partial kick filling the well, substantial pressure may develop in the in the well. Usually, the formation may break down first. In demonstrating the reliability based casing method, we assume that the whole well is filled with kick fluid at shut-in.

Lost circulation of mud implies drilling mud leak into the formation resulting in a reduction of the hydrostatic head and can lead to an increased collapse loading on the casing.

Furthermore, well integrity can be categorized into three classes, the full well integrity, the reduced well integrity with the weak point at the shoe and the reduced well integrity with the weak point at the well head.

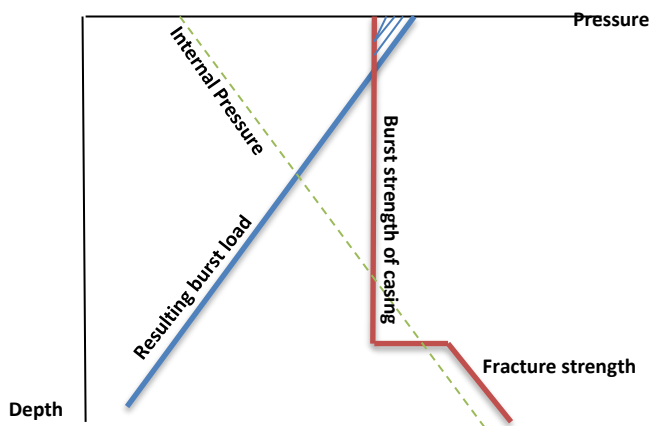
In the full well integrity scenario, the worst case is always assumed by supposing a gas kick is taken into the casing. The formation pressure acts at the bottom, therefore, the pressure at the top is the formation pressure minus the hydrostatic column of gas. The casing at the top is exposed to a significant high pressure because of a light fluid in the well. In this scenario, the hole, the formation at the shoe and top of casing can withstand the kick without fracturing at any point. This is a perfect illustration of the casing burst strength exceeding the load as shown in [Figure 6](#).

In the case of the reduced well integrity with the open hole below the casing shoe being weak, the fracture pressure below the casing shoe is less than the pressure in the casing, making it likely for the open hole below the shoe to fracture. In the industry, this is a preferable challenge option compared to the fracture of the wellhead, only if the flow from the casing shoe is controlled well and timely. This scenario is also known as the underground blowout<sup>[13]</sup> as can be seen in [Figure 4](#). This is why we perform kick tolerances in case we have reduced well integrity. We want to ensure that we do not end up in an underground blowout situation by checking that we can manage the kick size that we can detect.



**Figure 4:** Shows the resulting burst load and the reduced well integrity, source [13].

In the third and the last scenario, which is the reduced well integrity with the weak point at the wellhead, the casing is too weak to withstand the pressure that can occur in kick situation. The fracture pressure at the shoe may likely be higher than the casing internal pressure but it is certainly higher than the casing burst strength. Consequently, if the well is shut in, the casing just beneath the wellhead will burst. This situation is known as the full blowout implying the complete loss of well integrity at the surface. It is an unwanted scenario in the industry because it exposes the environment, personnel, and equipment to harm from the outflow of reservoir/well fluids<sup>[13]</sup>. This scenario is illustrated in Figure 5.



**Figure 5:** Illustrating the burst load and the reduced well integrity with the casing top as weak point<sup>[13]</sup>.

## 2.5 Major loads

Casings are subjected to loads during their service lives. As already stated It is critical that the casings can withstand the loads they will be subjected to, throughout the well's lifetime. The following major loads; burst, collapse and tensile load and minor loads, bending load are taken into consideration in casing design.

### 2.5.1 Burst<sup>[13]</sup>

This is the pressure which accumulates in the casing as a result of kick during drilling, or production, tubular leakage in the production phase. It can also be as a result of temperature increase in the casing annuli, for example, during production startup. These cases lead to an internal pressure exceeding the outside pressure of the casing and threatening to rupture the casing. Burst pressure is highest at the top in the case of kick taken during drilling. On the other hand, it is highest at the bottom in the case of tubing leak during production. During drilling, burst scenario can happen in a pre-installation of a casing or post installation of a casing scenario. For a pre-installation case, one can have an open hole with a casing yet to be installed or a case of running the casing and pumping cement. In contrast, a post-installation scenario assumes that the casing is installed, the cement behind the casing is set with water being the only mobile phase there and only fluid outside the casing.

In Figure 6 below, burst pressure is the internal pressure minus the external pressure.

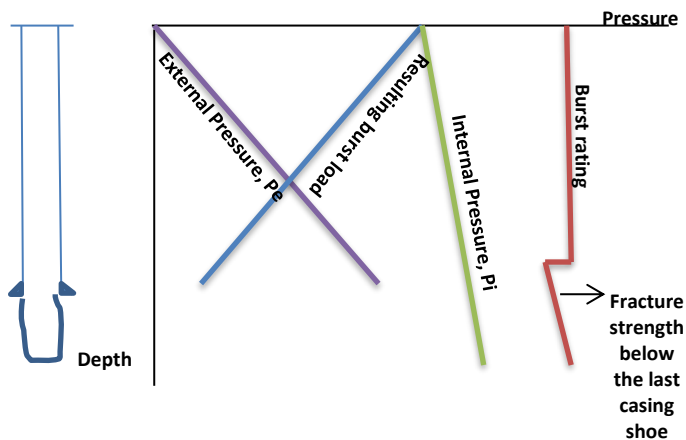


Figure 6: Burst pressures with gas kick taken with full Well integrity, source <sup>[13]</sup>.

### Tubular burst pressure calculation

The basic burst pressure equation is given as:

$$P_{burst} = P_{external} - P_{internal} \quad \text{Equation 3}$$

The API burst formula, based on the Barlow's formula for thin wall pipe is given as<sup>[13]</sup>

$$Pb = Tol \frac{2 \times \sigma_y \times t}{D} \quad \text{Equation 4}$$

Where Tolerance is given as;  $Tol = \frac{1}{SF}$ . According to API the safety factor is 8/7 and the tolerance is 87.5%. This API tolerance factor is a derating factor which supposes that 12.5% of the thickness of the casing is reduced by wear or corrosion effects. In this study, we will assume Tol to be 1.  $\sigma_y$  represents the casing yield strength, t represents the wall thickness and D is the outer diameter of the casing.

### **2.5.2 Collapse**<sup>[13]</sup>

Collapse load can be generated from cementing operations, mud losses during casing installations, temperature induced B-annulus pressure, dropped mud level inside an installed casing, and other major external loads which significantly exceeds the casing internal pressure. An occurrence of this load type can cause elliptical or non-circular deformation of the tubular from circular shape preventing access through the tubular using a tool. This type of deformation is more of a geometric failure than a materials failure<sup>[13]</sup>. For instance, during cementing, if one have a heavy cement on the outside of the casing, the largest load will occur at the bottom. A remedy could be to have a denser brine inside the casing.

#### **Collapse load calculation**

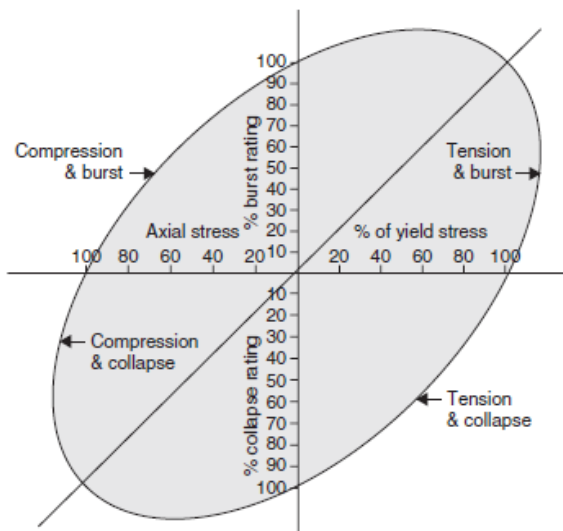
The load is given as the external pressure minus the internal pressure. The API Bulletin 5C3<sup>[18]</sup> contains the different formulas which can be used for calculating the Collapse load on a casing. These include the yield collapse, plastic collapse, transitional collapse and the elastic Collapse. The diameter-wall thickness ratio is used to determine what collapse equation to use. These equations can be assessed in API TR 5C3<sup>[18]</sup>. The collapse resistance of steel mostly depends on the diameter-wall thickness ratio even if casings may have the same elastic properties<sup>[13]</sup>.

### **2.5.3 Tensile Load**

This is the load exerted by the casing weight upon itself. Therefore, a designer must ensure that each casing joint can carry the weight of the strings below its point. This implies that the casings at the top must be strong enough to carry the casings below it. A tensile failure will cause the casing to part. This load is considered during installation<sup>[13]</sup>.

#### 2.5.4 Biaxial loading

Biaxial loading refers to the effect of a load on another imposed load on the casing. For instance, an axial tensile load will decrease the casing's resistance to collapse and at the same time increase its burst resistance. In contrast, an axial compression load will increase the collapse resistance and reduce the burst resistance. The major stresses acting on a casing string are the axial stress, the hoop (tangential or circumferential) stress and the radial stress<sup>[13]</sup>. These stresses are also known as the principal stresses. The casing tensile strength is a function of axial stress while the burst and collapse are a function of circumferential stress<sup>[13]</sup>. The axial and circumferential stress control the physics of failure for casings. As for radial stress, its effect on the casing is too small to be significant. The ellipse of plasticity is a plot of the relationship between the hoop stress, the axial stress, and the yield strength of the material. It is used to derate the resistance (burst or collapse resistance) of a casing for a bi-axial load, that is, for a combined load. The elliptic plot is also known as the bi-axial plot based on the maximum distortional energy or the Von Mises theory. The elliptic curve defines the failure envelope, where a position of stress within the envelope is interpreted as safe and outside the envelope is interpreted as failure. One limitation with the ellipse of plasticity is that one must have a knowledge of the yield strength and behavior of failure of a casing before it can be used for casing selection. Figure 7 shows the ellipse of plasticity.



**Figure 7:** The Ellipse of Plasticity, same as illustrated in<sup>[13]</sup>.

#### 2.5.5 Bending load

Bending stresses occur when installing the casing through a dogleg or when buckling of the casing occurs in a deviated well. The bending effect will decrease the collapse resistance and



tensile load strength<sup>[13]</sup>. During the casing design, the sections of the hole having sharp bends are considered.

### 2.5.5 Torsion and Shear

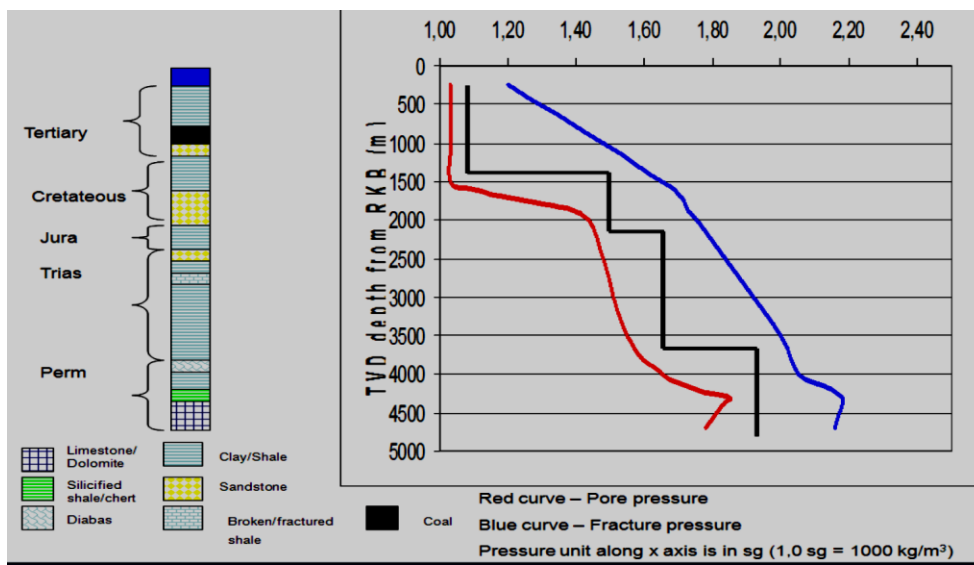
This load is only evaluated for a steerable casing or liner used for drilling or rotated during cementing.

### 2.6 Mechanisms generating loads

The burst, collapse, and tensile loads originate from various operations which include drilling, running casings, cementing, production, accidental situations like kick, lost circulation of mud, annular pressure buildup (APB), also known as temperature-driven load and tubular leakage<sup>[14]</sup>.

#### i. Kick

Kick is the uncontrolled inflow of formation fluid into the well and occurs when the mud weight falls below the pore pressure during drilling. It is one of the mechanisms that generate load in the well. High-pressure kick can also be caused by wrong pore pressure estimation, loss of mud return, swab, low mud weight and insufficient well refill during trip out<sup>[13]</sup>. One of the consequences of taking in a kick is that it builds up high pressure in the well which can rupture the casing especially if the well is shut in. A kick taken in an open well initially gives low loading on the casing<sup>[13]</sup>. On the other hand, if the well is shut in, and with a fully or partially kick filled well, substantial pressure may develop in the top most part of the well<sup>[13]</sup>. Figure 8 diagram below shows a normal drilling condition with planned mud weight in between the pore and fracture pressure in different hole sections.



**Figure 8:** Shows the curves of pore pressure, fracture pressure, and mud weight in different hole sections. The lithology column shows what formations that will be drilled through<sup>[19]</sup>.

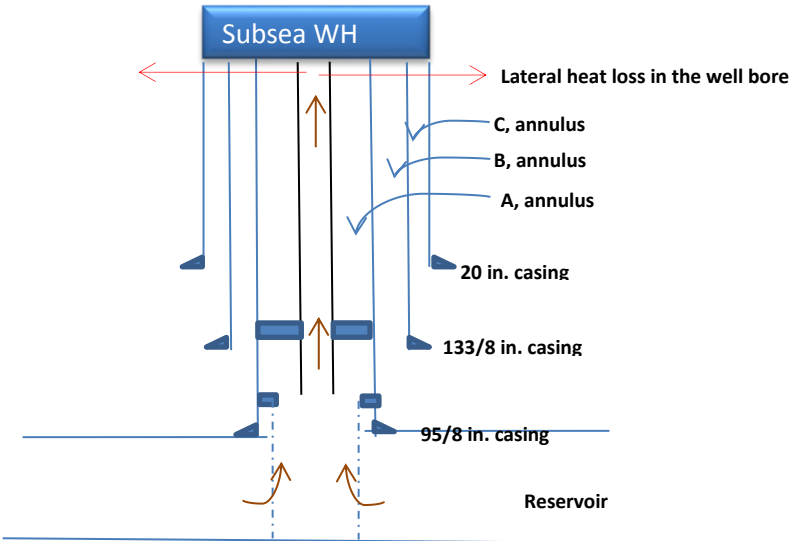
ii. Lost circulation of mud

Loss of mud return can occur during drilling and is characterized by reduced mud level in the well. It can be caused by drilling with a high equivalent circulation density (ECD) that is, exceeding the fracture pressure during drilling<sup>[13]</sup>. Figure 8 above describes the mud weight and fracture pressure curves. In addition, surges and drilling into a naturally fractured formation can lead to lost circulation of mud. The mud seeps into the rock can pressurize the formation behind the casing enough to collapse it<sup>[13]</sup>. Also, casing collapse can occur with a reduced mud level in the casing, resulting in a higher mud level and pressure outside the casing.

iii. Annular Pressure Buildup, APB

APB, also known as the Annular Fluid Expansion, AFE refers to the pressure generated in the casing-tubing annuli and/or casing annuli due to temperatures increase in the well during reservoir fluid production<sup>[15]</sup>. The two main effects responsible for APB are the temperature-induced volumetric enlargement of the annular fluid and casing displacement<sup>[15]</sup>. Production of the hydrocarbon fluids is accompanied by increasing temperatures in the well, particularly noticed in the top most part of the well. The well temperature approaches the reservoir temperature. As a result of this temperature increase, the completions fluid and/or drilling fluid in the casing annuli heats up and expands. The completions fluid is placed in the casing-tubing annulus (A-annulus) to protect the casing from corrosion, to reduce the differential pressure across the tubing during production and to isolate the unproductive zone from the producing zone. In the other annuli (B and C annuli), the drilling fluid is trapped behind the casing during the well construction, that is, casing installation and cementing<sup>[15]</sup>.

An expansion of these annuli fluids can cause high pressure to build up in these closed up annuli, this substantial pressure can in turn cause casing collapse and burst for inner and outer casings respectively<sup>[15]</sup>. Therefore, a casing is designed considering APB in addition to other pressure loads that it may encounter during its service life<sup>[15]</sup>. In addition to appropriate casing selection, other management or mitigation strategies are employed to prevent tubular collapse during production<sup>[15]</sup>. Figure 9 below shows the direction of heat loss from the well to the annuli.



**Figure 9:** A Production well with a subsea wellhead showing A, B and C annulus and the direction of heat loss in the wellbore.

iv. Cementing<sup>[13]</sup>

The operation of cementing the casings in place involves cementing the whole length of the conductor pipe and surface casing from bottom to surface/seabed, while the intermediate and production casings are cemented 200meter from the bottom. As for the liner, its entire length is cemented to the formation, providing a seal-off of the production interval. During the cementing operation, the cement slurry followed by a displacing fluid is pumped down the well to fill in the casing external annuli. Proper care is always taken to meet the requirements during cement pumping because the high-density cement slurry can burst the casing at the top or it can collapse the casing at the bottom. This is because, for the fluid in the outer casing annuli, sea-water has a lower density and therefore a less exerting pressure compared to the cement density. Also, at the bottom, the cement density on the outer casing annuli surpasses the internal pressure of the displacing fluid.

v. Tubing leak criteria<sup>[13]</sup>

In the tubing leak criteria, the production tubing is assumed to leak at its top causing the producing fluid (oil or gas) in it to flow into the A annulus containing the packer fluid. This will create a maximum burst load above the production packer<sup>[13]</sup> which can rupture the casing and/or collapse the production tubing.

## 2.7 Design example of burst load calculation, 1D load model

Consider the design of a production casing having the information listed below. We consider a burst scenario where a kick is taken from the planned depth of the next section. It is assumed that the well is closed and that the kick fluid fills the whole well. We also assume that the wellhead is located at the surface and there is no external pressure.

### Burst Scenario

9<sup>5</sup>/<sub>8</sub>in. 47lbs/ft. L80

Weight: 68.73kg/m

Burst Strength: 473bar / 7330psi

Collapse: 328bar

Tension: 483\*10<sup>3</sup> daN

Mud weight: 1.60sg (inside and outside of the casing)

Inner diameter of

casing: 220.5mm

Setting depth: 2909mMD, 2350mTVD

Depth next hole: 3341mMD, 2655mTVD

Cemented interval: 2131-2909mMD, 1800-2350mTVD

Production packer: 2414m MD, 2000mTVD

Gas density: 0,18sg

Pore pressure: 1.60sg

Acceptable SF 1.1

The burst load and safety factor can be calculated thus:

$$P_{b@ surface} = P_{internal} - P_{external}$$
$$P_{b@ surface} = (\rho_{pore}hTVD - \rho_{gas}gh) - P_{external}$$

The burst load is then given by the following since external pressure is zero.

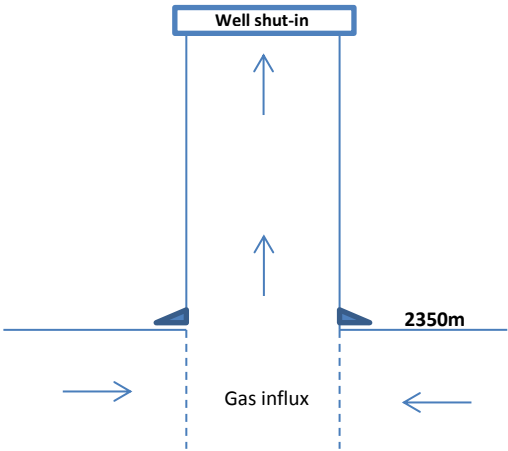
$$P_{pore} = 1.6 \times 0.098 \times 2655 = 416bar \quad P_g = 0.18 \times 0.098 \times 2655 = 46.8bar$$

$$P_{b@ surface} = 416 - 46.83 = 369.2bar$$

The safety factor, SF can be calculated by dividing the casing strength by the load. If the SF is larger than required, then it is acceptable.

$$SF = \frac{Strength}{Load} = \frac{651}{369.2} = 1.76 > 1.1$$

The schematic diagram of the well is represented below:



**Figure 10:** Represents the well schematics.

This approach is used in the working stress design approach which will be discussed in chapter 3. The burst load model used here is a one-dimensional load model where only one load in one direction is considered.

In a **triaxial load model**, however, the three tubular stresses (axial, hoop and radial stresses) as well the torque and bending stresses are considered in the model. These stresses occur simultaneously in a casing string and may be united to give a single equivalent stress, the Von Mises stress. The **Von Mises**, maximum distortion energy theory considers the three main stresses, the shear stress and the torque if present. It says that a critical yield limit exists in the casing, regardless of direction. Mathematically, presented thus;

$$\sigma_{von} = \sqrt{0.5[(\sigma_{\theta} - \sigma_r)^2 + (\sigma_{\theta} - \sigma_a)^2 + (\sigma_a - \sigma_r)^2]} = \sigma_y \quad \text{Equation 5}$$

For a casing uniaxially tested, its yield strength,  $\sigma_y$  is given by the Von mises Equivalent equation in Equation 5<sup>[13]</sup>. The basis of tubular design in the industry today, is the triaxial design check, using the Hencky Von Mises stress theorem to compare to the minimum yield to find an early yielding based on 1.25 design factor, DF according to NORSOK design criteria<sup>[16]</sup>. Note that the DF must be greater than 1.25.

The design factor equation of a ratio of allowable stress to applied stress is given as:

$$DF = \frac{\sqrt{2\sigma_y}}{\sqrt{(\sigma_a - \sigma_r)^2 + (\sigma_a - \sigma_t)^2 + (\sigma_r - \sigma_t)^2}} \quad \text{Equation 6}$$

If the Lamé equation for a thick-walled cylinder is combined with the Von Mises equation, a geometry factor can be developed. This geometry factor can be used to develop the equations for axial, radial and hoop stresses. Furthermore, an ellipse of plasticity with burst and collapse model can be created from the equations<sup>[13]</sup>.

## **2.8 Specifications and Strength of Casing with respect to burst load model**<sup>[18]</sup>

Strength is the measure of a material's ability to resist distortion when a load is imposed upon it. Generally, the strength of a tubular is considered based on the maximum load it can handle before a noticeable failure occurs. Therefore,  $P_b$  in Equation 4 refers to maximum burst load that can be taken by a tubular before an apparent failure and it is a function of casing yield strength, outer diameter, and wall thickness. All casing specifications are available in the API Standard 5CT as previously mentioned. The casing burst strength can be calculated with the following casing properties using Barlow's formula for a thin walled cylinder loaded internally with pressure: outer diameter(OD), wall thickness (t) and the yield strength ( $\sigma_y$ ). The equation is given as:

$$P_b = \frac{2\sigma_y t}{D}$$

$$\sigma_y = \frac{P_b D}{2t} \quad \text{Equation 4}$$

**OD:** the casings outer diameter ranges from 4.5-inch to 36-inch. This information can be found in the API 5C3 standard<sup>[18]</sup>, Drilling data handbook or Manufacturer's catalog.

**Yield strength:** is also referred to as the grade of the casing. A casing grade is specified by a letter and a number, like N80, P110. The number denotes the minimum yield strength of the casing. The minimum yield strength is the tensile stress required to give a total elongation of 0.5% of the length of casing (0.6% of the length for P-110)<sup>[14]</sup>.

The Barlow's equation for tubular yield is a historical API limit state equation. It is a one-dimensional approximate formulation of the Von Mises yield state and approximate representation for the hoop stress in the pipe. The equation does not account for any axial stress in the pipe like the Von Mises triaxial equation or the Lamé equation<sup>[12]</sup>.

## **2.9 Limit State Equations for Burst Strength**<sup>[5, 18]</sup>

The following equations in addition to Barlow's can be used to calculate the casing internal resistance to burst pressure. They all yield a fairly accurate result. These models will be used in our simulation to investigate the effect of models on the casing burst strength. These equations

represent the models for the strength estimation. The input parameters are the casing yield, casing minimum tensile strength, outer diameter, wall thickness, inner diameter, and the correction factor for pipe deformation,  $k_{dr}$ .

- **Barlow's Equation**

$$P_b = \frac{2 \times t_{nom} \sigma_y}{D} \quad \text{Equation 4}$$

- **Von Mises Equation:**

$$P_b = \frac{\sigma_y \times (D^2 - d^2)}{\sqrt{3} \times D^2} \quad \text{Equation 7}$$

- **Klever-Stewart Equation:**

$$P_b = \frac{2 \times K_{dr} \times \sigma_u \times t_{min}}{(D - t_{min})} \quad \text{Equation 8}$$

This model accounts for tubular inherent deformation and hardening up to rupture.

- **Paslay Equation:**

$$P_b = \frac{2 \times \sigma_y \times t_{min}}{(D - t_{min})} \quad \text{Equation 9}$$

- **Moore Equation:**

$$P_b = \frac{\sigma_y \times (D^2 - d^2)}{(D^2 + d^2)} \quad \text{Equation 10}$$

- **Nadai Equation:**

$$P_b = \frac{2 \times \sigma_u}{\sqrt{3}} \times \ln \left[ \frac{D}{D-2t} \right] \quad \text{Equation 11}$$

Where  $P_b$  is the internal burst pressure,  $K_{dr}$  is the correction factor for pipe deformation,  $t_{min}$  is the minimum tubular wall thickness without crack deformations,  $t_{nom}$  is the specified or nominal tubular wall thickness,  $D$  is the tubular outside diameter,  $d$  is the tubular inside diameter,  $\sigma_u$  is the ultimate tensile strength obtained from uniaxial tensile strength tests and  $\sigma_y$  is the tensile strength of the tubular. The Nadai and Klever-Stewart equations are based on the tubular ultimate tensile strengths. As can be seen, the Paslay equation is a special case of the Klever-Stewart model, except that it uses the yield strength and it excludes the correction factor for pipe deformation. Note that if information about the ultimate tensile strength is lacking, the minimum tensile strength can be used in its place<sup>[11]</sup>.

## **2.10 Management guidelines, Government regulation, and NORSOK**

Satisfying government regulation/ management guidelines is one of the key steps in casing design/ selection. The NORSOK standard D-010 is used as a basis for design in this study. NORSOK D-010 is a standard developed by the Norwegian Petroleum industry to ensure cost effectiveness, added value, and safety in Petroleum industry drilling and production operations. The standard proposes that: *'the casing shall be designed to withstand all planned or expected loads and stresses including those induced during potential well control situations'*.

This means that the casing shall be designed with respect to realistic load conditions during the lifetime of the well.

## **2.11 Factors that degrade casing strength**

The casing strength shall be corrected for additional loads and effects such as temperature effects, casing wear, biaxial stress, bending in deviated hole section, corrosion, reservoir compaction/plastic formation and pressure during completion, workover, kill operation. These factors are the common causes of casing failure.

### **1. Temperature effects**

An amount of heat is transferred from the reservoir to the well and up the well during production and well testing. This results in a higher/increasing temperature profile which decreases the strength of casing<sup>[13]</sup>. According to Aadnoy[13] wells with a temperature greater than 100°C will have their casing strength derated due to effects of temperature. The derating is done according to manufacturer's specification or as described in API Bulletin 5C3 (1990)<sup>[13]</sup>. The high temperature causes expansion of the fluids behind the production tubing and behind other casings (A, B and C



annuli). If the annuli are sealed the expansion will not take place and pressure will increase instead. Lack of access to bleed off the casing annuli can lead to collapse or rupture of one the casings<sup>[13]</sup>.

## 2. Casing wear

Casing strength is derated for casing wear which occurs as a result of further drilling by drill string rotation through the casing. This wear reduces the thickness of the casing, thus reducing its strength and increasing the chances of corrosion. In high temperatures and high pressure well, casing wear reduces the burst and collapse strength more significantly<sup>[13]</sup>. For instance, all 9<sup>5</sup>/<sub>8</sub> casings are prone to wear. The wear can be critical if you reuse the casing when making sidetracks (for multilateral wells or slot recovery). The 9<sup>5</sup>/<sub>8</sub> casing is also a critical casing since it will be a permanent second barrier element. In general, casing wear reduced the 9<sup>5</sup>/<sub>8</sub> casing burst and collapse strength.

## 3. Corrosion

This is the physical degradation of the tubular as a result of its electrochemical reaction with its corrosive environment, for example, Co<sub>2</sub>, H<sub>2</sub>S, O<sub>2</sub> etc. The degradation of the tubular leads to casing thickness and strength reduction, thus warranting casing strength deration. Aadnoy<sup>[13]</sup> uses ten percent (10%) as a derating factor. The failure due to corrosion may be long-term material failure or short-term material failure caused by hydrogen embrittlement. Appropriate measures are taken to ensure that the tubular performs in a corrosive environment throughout the well's life. These include increasing the tubular's thickness, also known as giving a corrosion allowance, and proper material selection by choosing a sour serviced casing.

# Chapter 3

---

## 3.1 Methods of Casing design

Different methods can be used to design the casing. These methods are the Working Stress Design, WSD, the Limit State Method, LSM, and the Reliability Based Design, RBD. The LSM is also known as the Limit State Analysis, LSA or the Load and Resistance Factor Design, LRFD, Load and Strength Analysis, LSA.

## 3.2 Working Stress Design, WSD

The working stress design, WSD is the standard method used for the analysis of a structure's reliability. The WSD works by estimating the worst-case load that can be imposed on a structure in its lifetime and applying a safety factor on that load to be on the safe side. The product of the highest occurring load and safety factor is known as the minimum strength of the material and is used to define the allowable or working stress. WSD is also called the Allowable Stress Design. The minimum strength or the allowable or working stress is based on deterministic minimum features<sup>[8]</sup>. The minimum strength of a material is selected in the manner that the material is used without going beyond its elastic limit for WSD. The loads can be obtained from a historical data of a field, company guidelines or government regulations. The worst-case scenario is based on first defining the maximum load that can possibly occur. This load is then multiplied by a safety factor, SF. The casing is selected after applying this safety factor. [Figure 11](#) shows the WSD approach. There is another sister design known as the limit state methodology (load and strength analysis).

## 3.3 Limit State Design, LSD

*'Limit States are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs.'*<sup>[10]</sup>

In LSA, a limit state of a structure's usefulness is defined to avoid exceeding it. However, exceeding this limit state does not imply deformation of the material but implies material's unreliability if used<sup>[20]</sup>. The **load** applied to a material and **strength** of the material are the criteria for the LSA.

Often, WSD is used interchangeably with LSM. However, the two approaches differ. In Working Stress Design, WSD we calculate the load  $L(\tilde{x})$ , then we multiply by the SF, giving  $L(\tilde{x}) \times SF \leq SF$  required. In limit state design, however, we include a limit state equation for the strength. For instance, we have the equation presented thus:  $L(\tilde{x}) \times L_f \leq R(\tilde{x}) \times S_f$ . The  $L_f$  is the safety factor for load and  $S_f$  is the safety factor for strength. One SF for load and strength is not practicable due to the difference in their inherent uncertainties, therefore an individual factor design is defined by both load and strength denoting  $L_f$  and  $S_f$  respectively and is used to obtain the SF given by  $\frac{L_f}{S_f} \leq \frac{S(\tilde{x})}{L(\tilde{x})}$  and  $SF \leq \frac{S(\tilde{x})}{L(\tilde{x})}$

where ( $\tilde{x}$ ) represents the deterministic parameters used in the design verification equation, L represents load effect and S represents the strength <sup>[11]</sup>.

More differences between the WSD and LSM are listed in Table 1 <sup>[20]</sup>. The Limit State Analysis, LSA or Limit State Method, LSM, or Limit State Design, LSD is also the same as the Load and Resistance Factor Design, LRFD. There are different types of LSD, like the ultimate LSD and serviceability LSD. However, the types of LSD are not the focus of this study.

**Table 1:** Differences between the Working Stress Design, WSD and the Load and Resistance Factor Design, LRFD.

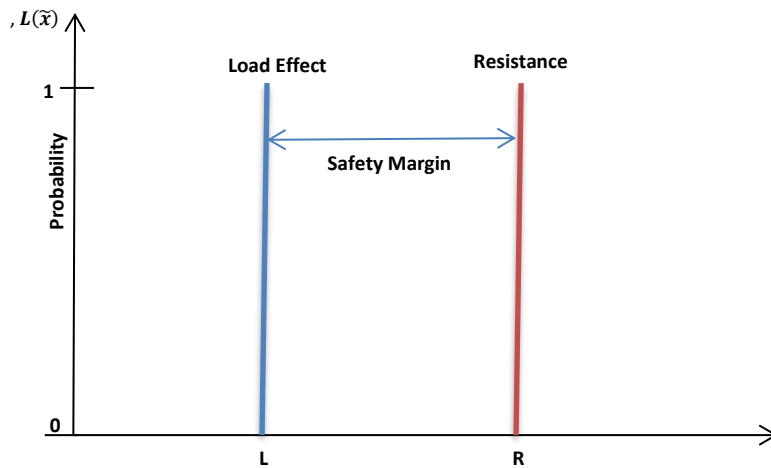
	<b>WSD</b>	<b>LSM</b>
1.	One Safety Factor is used.	Two Safety Factors are used, load factors and resistance factors.
2.	Design criterion is that Strength>Load and stress in the material< allowable stress Allowable stress= minimum strength	Design criterion may be $\frac{L_f}{S_f} \leq \frac{S(\tilde{x})}{L(\tilde{x})}$ $SF \leq \frac{S(\tilde{x})}{L(\tilde{x})}$
3.	Approach is deterministic	Approach may be deterministic or probabilistic
4.	Material strength is undervalued and the loads used are working loads.	Material Strength is undervalued and the loads used are working loads plus extra safety. Loads are overestimated.
5.	WSD fulfills the serviceability criteria. It is also not economical	The main concern is material's serviceability and safety before failure.
6.	Method is simple	Method can be simple or complex
7.	Information about the safety margin for load is not available.	Information about the safety margin for loads is available, particularly the info about the excess load which the tubular can bear beyond the working load without collapse.
8.	Focuses more on the load calculation	Focuses on the load and strength calculation.

WSD and LSM share a similarity in ensuring that the casing strength exceeds the Load imposed on it during its service life.

One aim of a casing design is to select the casing setting depth based on the strength>load criteria. Once that is achieved, the size, weight, and connections are selected to be installed to shoe depth<sup>[11]</sup>. The final casing design should be comprised of: size, weight, grade, connection type, material specification, QA/QC obligations and the operational procedures and Safety<sup>[11]</sup>.

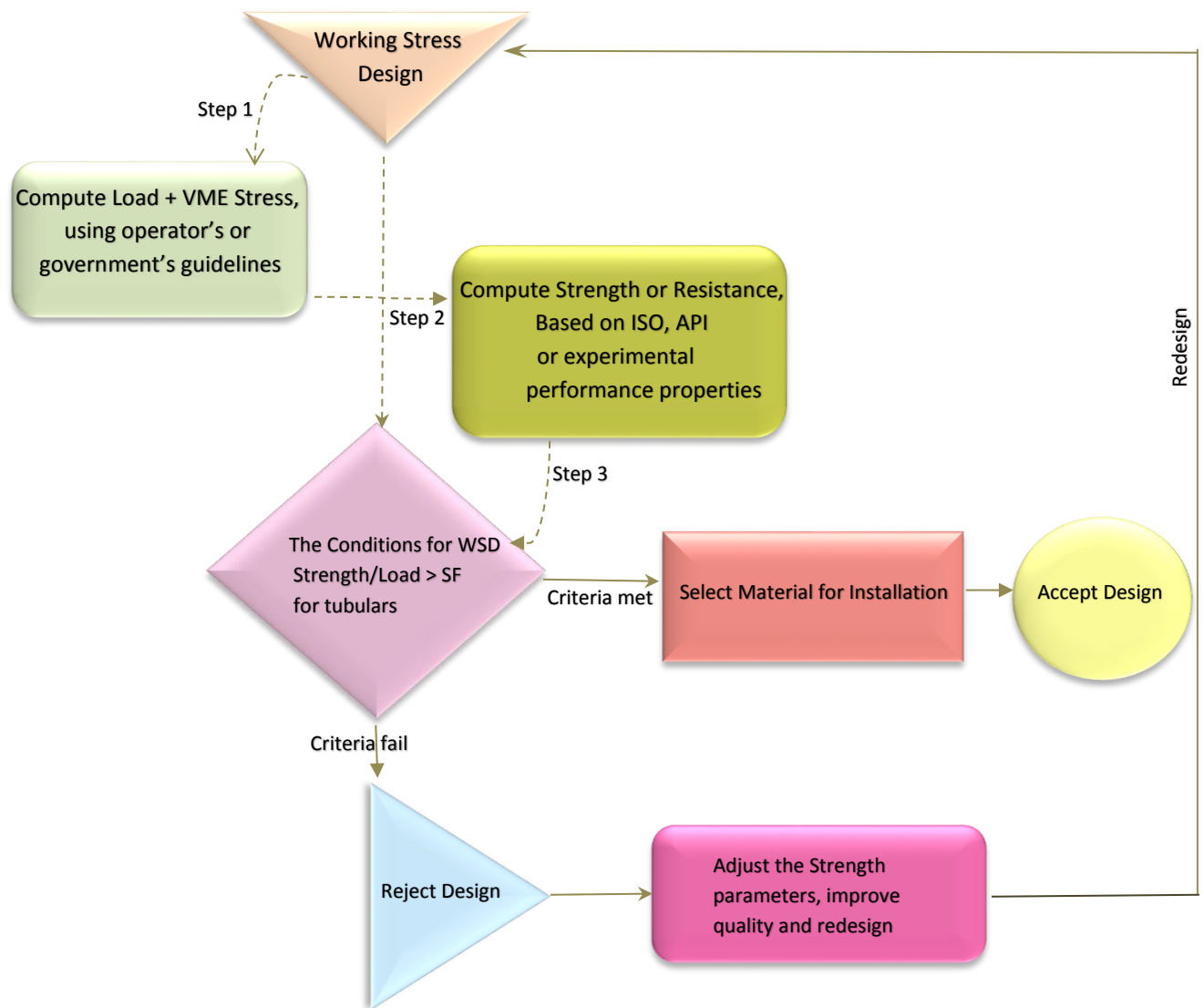
The disadvantages with the WSD approach is discussed in detail in the next section. Nevertheless, WSD is very simple, have clear assumptions and clear presentation of results. All these make up its advantages<sup>[21]</sup>. In WSD, materials are designed such that their limit states are not exceeded, thereby making them fit for their purpose.

As previously mentioned, this design method is full of uncertainties which come from the uncertainties in the load, strength failure mode and consequences<sup>[11]</sup>. One of the reasons for this is that the load occurs in future and there is no standard for load calculation<sup>[11]</sup>. No standards, neither API nor ISO explains how loads will be calculated or mentions the important loads to consider during the design<sup>[22]</sup>. Another reason is that it requires the acceptable limit of SF and the basis of design (burst or collapse) to be defined before starting design work. The strength uncertainty arises from the manufacturing process and it reflects in the strength defining parameters: OD, Wall thickness, Yield, etc<sup>[9]</sup>.



**Figure 11:** Illustrates the WSD with deterministic load effect and deterministic resistance<sup>[8]</sup>.

### 3.4 Working Stress Design steps<sup>[11]</sup>;



**Figure 12:** Working stress design process modified after <sup>[8]</sup>.

Figure 12 illustrates the steps involved in WSD for casing design. The first step is to recognize and calculate all possible loads. The second step is to select a casing and check its strength data. the third step is to calculate the strength-load ratio and check that it satisfies the required safety factor. In the fourth step, if the criterion is met, the selected casing is accepted. If not so, a stronger casing must be considered. We redesign until WSD condition is satisfied.

### 3.5 Designing Safety factors, SF

The designing safety factor which is also the triaxial stress design factor is given by

$$SF = \frac{\sigma_y}{\sigma_{vme}} \quad \text{Equation 12}$$

It is also the strength-load ratio in any one-dimensional load scenario<sup>[13]</sup>, mathematically represented thus:

$$SF = \frac{\text{strength or rating}}{\text{load}} \quad \text{Equation 13}$$

The function of the safety factor in working stress design is to take into account the uncertainties in the strength and load calculation and to isolate the applied load from the resistance of the material to that load<sup>[11]</sup>. The safety factors have been established based on experience and /or tradition even though the evidence of establishing it is hardly documented<sup>[11]</sup>. Most companies, including government bodies, have their safety factors or range of safety factors corresponding to different load combinations which vary amongst them. For example, some safety factors are shown below in Table 2.

**Table 2:** Common safety factors, modified after <sup>[11]</sup>

Pipe body	Minimum safety factors for CSG
VME	1.25
Axial	1.3-1.6
Burst(MIYP)	1.0-1.25
Collapse	1.0-1.1

### 3.6 Working Stress Design example

Considering the design of a production casing having the information below. We consider a burst scenario where a kick is taken from the planned depth of the next section. It is assumed that the well is closed and that the kick fluid fills the whole well. We also assume that the wellhead is located at the surface and there is no external pressure.

#### Burst Scenario

9<sup>5</sup>/<sub>8</sub>in. 47lbs/ft. L80

Weight: 68.73kg/m

Burst Strength: 473bar / 7330psi  
 Collapse: 328bar  
 Tension:  $483 \times 10^3$  daN  
 Mud weight: 1.60sg (inside and outside of the casing)  
 Inner dia of casing: 220.5mm  
 Setting depth: 2909mMD, 2350mTVD  
 Depth next hole: 3341mMD, 2655mTVD  
 Cemented interval: 2131-2909mMD, 1800-2350mTVD  
 Production packer: 2414m MD, 2000mTVD  
 Gas density: 0,18sg  
 Pore pressure: 1.60sg  
 SF 1.12

The burst load and safety factor can be calculated thus:

$$P_{b@surface} = P_{internal} - P_{external}$$

$$P_{b@surface} = (\rho_{pore} hTVD - \rho_{gas} gh) - P_{external}$$

We assume zero external pressure and calculate the burst pressure.

$$P_{pore} = 1.6 \times 0.098 \times 2655 = 416bar \quad P_g = 0.18 \times 0.098 \times 2655 = 46.8bar$$

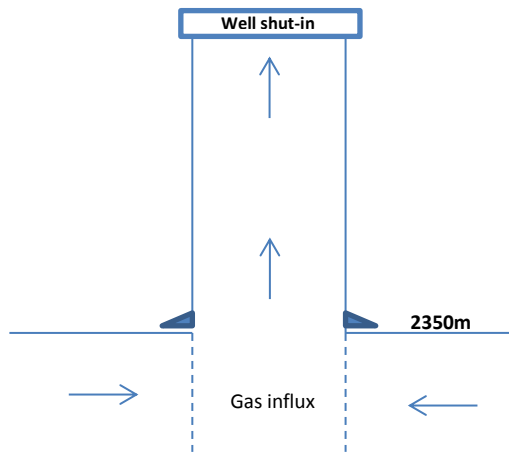
$$P_{b@surface} = 416 - 46.83 = 369.2bar$$

Pb at the surface is the maximum differential pressure. Applying the safety factor of 1.12 will give the casing burst design pressure;

$$Casing\ pipe\ design\ pressure = Maximum\ load \times SF$$

$$413.5\ bar = 369.2 \times 1.12$$

We note that the casing design pressure is smaller than the 473bar that was the specified burst pressure for the chosen casing. Therefore, the criteria for the design is satisfied because the 473bar burst rated casing should withstand the expected load of 413.5bar without failure. The schematic diagram of the well is represented in Figure 13 below. Furthermore, all load scenarios must be considered. For instance, one also has to check the collapse and tensile failure as well as the triaxial design.



**Figure 13:** Represents the well schematics.

### 3.7 Merits of Deterministic Approach<sup>[8, 20]</sup>

- The assumptions in this approach are clear.
- It is simple and straight forward.
- It is not time-consuming.
- This approach gives a result that is easy to understand.

### 3.8 Limitations of Deterministic Approach<sup>[8]</sup>

- The deterministic approach to casing design is too rigid and overestimates the design as shown in [Figure 11](#).
- It does not consider the uncertainties associated with the design.
- It cannot quantify the risks present in the design.
- It does not model the range of possible outcomes.
- It is impossible to estimate the probability of occurrence for any outcome.
- It requires a basis of design such as the safety factor to be defined before the starting design work.



# Chapter 4

---

## 4.1 Reliability

Reliability describes the probability of a material under stated conditions, to perform a required function without failure for a stated period of time<sup>[23]</sup>. It also describes a component or system's ability to operate for a defined time interval<sup>[23]</sup>. Roughly, reliability means availability or can be known as the probability of success.

Reliability engineering is concerned with the measurement, mitigation, and management of uncertainties and failure risks in engineering design<sup>[6]</sup>. Reliability application is vital to a system or component's cost effectiveness as it closely deals with the safety of a system and for a robust failure management<sup>[6]</sup>. In fact, it is applied in engineering to reduce the cost of repair and maintenance of a material, improve safety, increase profit, and add value, competitiveness, and reputation.

Reliability based design, RBD is the probabilistic approach applied in design to measure the uncertainties, occurrence frequency, and variations in parameters associated with the performance and failure of a material<sup>[11, 24]</sup>. RBD have the ability to quantify the risk inherent in a design<sup>[8]</sup>. In RBD approach, an important initial step is to define the necessary limit state models.

An example of a basic RBD approach is presented thus; consider the limit state equations used for casing design and the criterion, strength>load. Also, assume that the strength distribution (resistance distribution) is denoted as  $S(x)$ , the load distribution denoted as  $L(x)$ , the reliability denoted as  $R(x)$ , where  $x$  represents the random variables that determine the strength and load effect. The limit state function for the casing can be presented thus;

$$R(x) = S(x) - L(x) \quad \text{Equation 14}$$

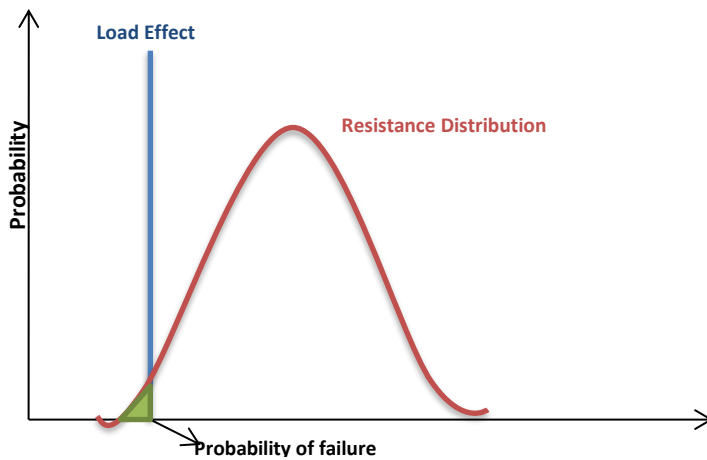
Equation 14 is termed the  $R$  function. From Equation 14, with known  $S(x)$  and  $L(x)$ , the probability of failure,  $P_f$  can be determined using the equation;

$$P_f = P[(S(\tilde{x}) - L(\tilde{x})) < 0] \quad \text{Equation 15}$$

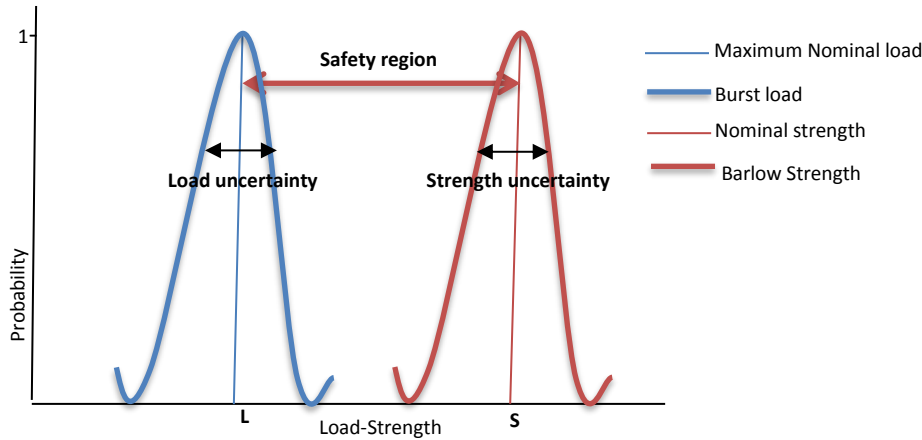
$$P_f = P[R(\tilde{x}) < 0] \quad \text{Equation 16}$$

The  $P_f$  is used to quantify the probability that the load will surpass the strength (load>strength). In Figure 14, the  $P_f$  is shown where the load effect intersects with the strength distribution.  $P_f$  is recognized as the intersection or overlap between the lower tail of the load deterministic line or distribution curve and the upper tail of the strength distribution curve. P.V Suryanarayana et. al.<sup>[8]</sup> recommends  $P_f$  of  $10^{-6}$  to  $10^{-5}$  for a high magnitude load with a low probability of occurrence (i.e. high consequence failures), and  $P_f$  of  $10^{-3}$  to  $10^{-2}$  for low consequence failures. The reliability and probability of failure of a design are complementary. Thus, the reliability of a design can be determined from Equation 17 when the  $P_f$  is known.

$$R(x) = 1 - P_f \text{ Equation 17}$$



**Figure 14:** Shows the RBD with deterministic load effect, Resistance distribution and the probability of failure at the overlap<sup>[8]</sup>.



**Figure 15:** Illustrating RBD with Distribution Load- distribution Strength relationship, there is no overlap<sup>[3]</sup>.

In Figure 15, there is no overlap between the load and strength distributions which indicates no probability of failure.

#### **4.2 Application of Reliability-Based Design in casing Design**

In the application of RBD in casing design, the uncertainties, occurrence frequency, and the variabilities in the parameters that define the load and strength are considered explicitly<sup>[11]</sup>. All variables are considered to be random with a specified distribution. That is to say, that every variable in the limit state equation is considered random variable and defined using a type of distribution. The uncertainties surrounding the burst scenario shall be explored in this thesis. This approach will also be used to quantify the probability of failure for a casing design. The RBD approach to casing design will be estimated by Monte Carlo simulations and are based on limit state equations.

##### **4.2.1 The variable parameters in the strength model**

In a strength model, for example, the Barlow' equation, the variables generating the uncertainties (or having uncertainties) are; the tubular yield strength, the outer diameter, and the wall thickness. As a result, these variables are considered random and are defined using a type of input distribution. To ensure that  $S(x)$  is corrected to include the real data, the Barlow formula with fixed input values (predicted values) for the model parameter is multiplied with a distribution that takes the real data into account. Therefore, this distribution represents the difference between the model and the real data, and it is presented thus,  $\frac{S_{real}}{S_{predicted}} = N(\mu, \sigma)$ . Values for  $N(\mu, \sigma)$  can be found in ISO or the API TR 5C3<sup>[5]</sup>.

It is important to select an appropriate distribution type to represent the available statistical data. One common distribution type most applicable in representing the strength model are the normal distribution where  $\sigma$  and  $\mu$  denote the standard deviation and mean for the normal distribution, N. The  $\sigma$  and  $\mu$  of a casing grade which differs from the nominal value are obtained empirically, making data gathering a challenge for this design method. The mean of a parameter is the average parameter of several tested casings while the standard deviation of a parameter expresses the dispersion of the data from its mean. The uncertainties in these parameters can also arise from manufacturing and testing error.

$S(x)$  is statistically modeled to obtain a resultant distribution and value which is risk consistent.

$$S(x) = P_b = \frac{2 \times t_{nom} \sigma_y}{D} \quad \text{Equation 18}$$

#### **4.2.2 The variable parameters in the load model**

In the load model, the parameters with uncertainties include; the reservoir pressure, the fluid density, the amount of influx, the frequency of the kick etc.<sup>[11]</sup>. Similarly, due to the variability in these variables, they are considered random variables and are represented using input distributions. Here, it is also important to select the right distribution type to match the right data. In comparison to the strength distribution, the load distribution is more difficult to quantify. That is, it is more challenging to gather the data for the load parameters because it is not as measurable as the strength parameters. For instance, the kick density cannot be measured until occurrence. In the WSD, the technique is to assume the lightest kick density which fully fills the well and exerts the highest load on the casing top (below the wellhead). In RBD, these uncertainties are evaluated leading to questions that require answers such as: What is the probability that the kick will occur? If a 100% probability of occurrence is assumed as in WSD, will the kick fully or partially fill the casing? What load will this kick exert on the casing if its density is high or low? What kick density is best to use? These questions are not easy to answer. Naturally, the occurrence of the kick cannot be a 100%, because its occurrence is dependent on factors such as the environment, well temperature and pressure, personnel experience or the operational technique. Due to these unanswered questions, it is challenging to model the load effect stochastically and it may lead to assuming extreme values for the load parameters. Therefore, it is highly recommended and sensible to use worst-case values to deterministically model the load effect, if the load parameters are unknown. This is to say that, it is prudent to assume a 100% probability of kick occurrence to achieve a robust design. If we assume and design for a less than 100% probability of kick occurrence, and it happens that a kick occurs in a

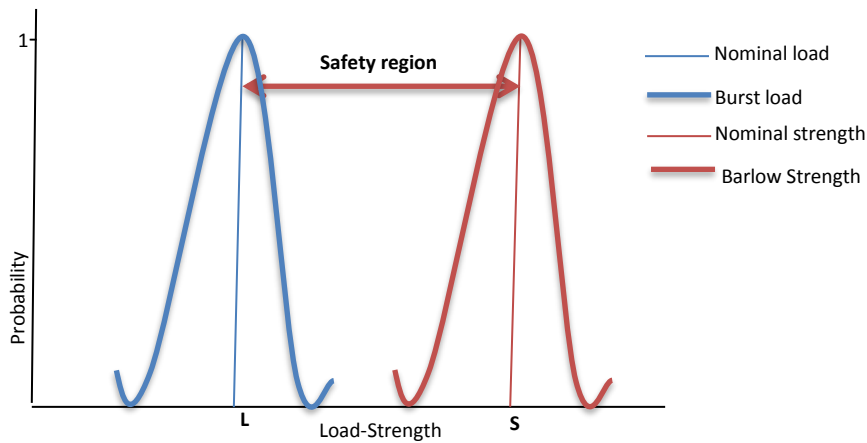
real case, a failure is bound to occur. This is because the design is not robust, considering fully all likelihood of kick occurrence. For the resistance parameter with more information, it is easy and best to represent it using a distribution. In this way, the  $P_f$  obtained will be a realistic representation of the worst-case design<sup>[11]</sup>. Figure 14 illustrates this technique.

### 4.3 Calculation methods for RBD.

Several methods and software can be used to calculate the reliability of a design. Some of the already mentioned software include the Oracle crystal ball. Statistical methods such as the First Order Reliability Method, FORM, the Second Order Reliability Method, SORM, the First Order Second Moment method, FOSM and the Advanced First Order Second Moment method, AFOSM can be used<sup>[25]</sup>. Monte Carlo simulation can also be used.

The FORM has two methods these are the FOSM and the AFOSM. The FORM and SORM are simple and practical to use but have more error due to approximations when compared to MCS. MCS gives a more exact result if a sufficient number of iterations is used.

### 4.4 Reliability Based Design versus Working Stress Design



**Figure 16:** Illustrating RBD with Distribution Load- distribution Strength relationship, there is no overlap <sup>[11]</sup>. This figure is also a comparison of WSD to RBD, demonstrating the strictness of the WSD with respect to quantifying the uncertainties.

**Table 3:** The difference between the WSD and RBD<sup>[22]</sup>.

	WSD	RBD
1.	It does not employ the random selection of variables.	It employs a random selection of variables to simulate in a defined number of times to give a result.

2.	It does not allow numerous simulation of the variables. Assumptions are clear and simple.	Assumptions may be complex. It uses the technique of randomly generating variables.
3.	It defines parameters in a deterministic way in the statistical computation	It defines the parameters using distributions
4.	Safety factor, SF is used to account for uncertainties in design	Failure probabilities, mean, P90-P10 are used to quantify the uncertainties in design
5.	The safety factor is used as the safety margin.	Probability of failure is used as the safety margin

#### 4.5 Reliability Based Design versus Limit State Method

The similarity between the LSM and RBD is that both methods are based on using limit state equations. LSM form the basis for the RBD application.

**Table 4:** The differences between the Load and Resistance Factor Design, LRFD and the Reliability Based Design, RBD

	LSM	RBD
1.	Two Safety Factors are used, load factors and resistance factors.	Safety factors are not used here, but a probability of failure.
2.	It does not employ the random selection of variables.	It employs a random selection of variables to simulate in a defined number of times to give results.
3.	It does not allow numerous simulation of the variables.	Here, for example, Monte Carlo simulation have to be applied.
4.	It defines parameters in a deterministic way in the statistical computation	It defines the parameters using distributions
5.	The calculation method selected for LSM may approximate values too much.	Approximations are reduced here.

#### 4.6 Merits of the Reliability-Based Design approach are<sup>[9]</sup>:

Some of the advantages of probabilistic approach related to tubular design are listed below<sup>[9]</sup>:

- RBD is the most robust and economical design when the statistics (such as the mean and standard deviation) of the input random variables are well defined.
- It gives an excellent understanding of the uncertainties and the risk involved in a design. Hence, it will give a clearer and more realistic casing design methodology.
- It is also a good tool used to reduce the impact of uncertainties in the overall project.
- It provides a means to include unwanted or unexpected events in the model.
- It provides an overview of possible outcomes.
- It makes it possible to apply sensitivity analysis to identify which parameters that are most dominating the result with respect to uncertainty.

- RBD application contributes to meeting overall project design cost and performance targets.

#### **4.7 Limitations of Reliability Based Design with respect to casing design**

- It is complex and the assumptions are also complex compared the Working Stress Design, WSD.
- It may be difficult to obtain the right data for statistical calculation.
- The accuracy of the result depends on a good limit state model, the exactness of the input data as well as a large amount of measured data in order to give the true results. These measured data can be used to calibrate the limit state model.
- RBD approach alone may not be sufficient when designing casings for survival loads and may need to be combined with the WSD approach<sup>[11]</sup>. It is recommended that a probabilistic approach is used where a deterministic load is at its extreme magnitude such that it is compared to a stochastic strength<sup>[8]</sup>.
- A selected method (FORM) for the RBD may be sensitive to the variations in the statistics of the random variables. When we fit a distribution to real data we make an approximation. therefore, it is important that the distribution selected must represent the mean and the variation of the real data in a good manner<sup>[8]</sup>.
- It is a challenge to pick the values (mean and standard deviation) for the uncertain parameters for calculations with RBD method. Access to API TR 5C3 (ISO TR 10400), manufacturer's reports or direct material and dimensional measurements may not be sufficient for data collection.
- It can be a challenge to define the acceptable tolerance limit. For instance, if we obtain a  $10^{-3}$  probability of a failure, is this acceptable or not for the design? What acceptable limit will the industry accept?

#### **4.8 Historical Development of Reliability-Based Design in casing design**

RBD approach to tubular design is an emerging technology, although some people have done some notable work on it. The first application of the statistical approach to tubular design for collapse calculation was by Clinedinst in 1977 in his work *Analysis of API Collapse Test Data and Development of New Collapse Resistance Formulas*<sup>[26]</sup>. Also, RBD approach to casing design has been addressed by K C. Gulati et. al. (1994)<sup>[9]</sup>, Raney, et al (1997)<sup>[22]</sup>, and Payne. et. al. (1990)<sup>[3]</sup>.

Payne. et. al<sup>[3]</sup>. applied the probabilistic reliability methods used in civil engineering to tubular design. They developed a new method that accounts for the variation of field loadings and tubular performance by defining the load effect and strength with distributions to quantify the probability of failure<sup>[3]</sup>.

Bradley<sup>[3, 27]</sup> investigated the probability of failure of worn casing under internal pressure. He used data created by API to develop the probability distribution of failure for single joints of 7-in., 26lbm/ft, N-80 casing under various internal pressure loads. After his evaluation, he concluded that as the length of the casing string increased, the probability of failure for the string increased when he included a weak joint in the system.

James B. Raney et. al.<sup>[22]</sup> compared the reliability based procedures for the design of production tubing with the traditional Working Stress design method, WSD. Their study also included the Load and Resistance Factor Design, LRFD. They also demonstrated the methodologies for the three casing design approaches, WSD, LRFD and the Quantitative Risk Assessment, QRA. They conclude that QRA is the best approach to designing tubing to a desired reliability.

Gulati et. al.<sup>[9]</sup> described the methodology of a reliability based design of drilling casing and tubing in the load and strength factor design format. They based their approach on the fundamental principles of limit state design. They described the stochastic modeling of the load and strength variable and the calibration of design check equations. In their calibration analysis, they used field and laboratory data of load and casing strength to determine the uncertainties in the design variables which they represented by appropriate statistical distributions.

Suryanarayana et. al.<sup>[8]</sup> and Payne et. al.<sup>[3]</sup> highly recommend a probabilistic representation of strength data especially for HPHT wells. API TR 5C3 and ISO TR 10400(2007) introduces the limit state equations to make the application of RBD possible. The limit state equations for the burst and collapse pressures are available in the two documents.

A most recent work on the application of RBD on survival design in Deepwater and high-temperature/high-pressure wells by Suryanarayana et. al.<sup>[8]</sup> categorized the tubular loads into operational and survival loads. The operational loads are defined as high frequency, low magnitude loads. On the other hand, the survival loads, they refer to as the low probability, high magnitude loads with harmful consequences to life and environment if they occur. They defined the survival loads as the Worst case discharge loads (Collapse) and the well Containment loads (Burst)<sup>[8]</sup>. The limitation of WSD affects the quantification of such high magnitude loads. Thus, it is problematic to obtain satisfactory design using WSD for such loads<sup>[8]</sup>. The only option left with a designer using the WSD approach to representing high magnitude load is to modify the load to a realistic, workable value. However, this approach will fail to give a reliable and robust design. Some questions will be left unanswered such as what happens if the high magnitude load occurs?<sup>[8]</sup> A Proper account of the magnitude of possible loads to be encountered is important in the design of structures especially if its failure will affect life and environment<sup>[8]</sup>. This can be achieved by understanding the uncertainties in the design.

Low magnitude loads or the operating loads do not pose as many problems compared to survival or high magnitude loads. In the industry, a low magnitude, high probability load is considered



acceptable. Structural designs today employ the use of RBD to establish and ensure the safety of life and environment when working with high magnitude loads<sup>[8]</sup>. It is highly recommended to employ RBD in tubular design especially when dealing with high magnitude loads. In this reliability-based application, a stochastic load effect is compared to a stochastic resistance to achieve a quantitative and robust design.

As can be seen in [Figure 14](#) which shows a deterministic load effect compared to stochastic resistance distribution. It is important to be aware that the load effect can also be represented using asymmetric shape<sup>[8]</sup>.

#### **4.9 Risk or Unexpected Events**

Most of the time it is difficult to know what events that may occur unexpectedly during a project or design. This, therefore, becomes part of the uncertainty to be understood. One key purpose of RBD (probabilistic approach) is its use in the quantification, understanding, and management of the probability of unexpected events taking place. It is entirely up to the modeler to specify what unexpected events to include in the model and how to include them. Unexpected events can be known from experience based on previous events that have occurred in a project<sup>[21]</sup>. For instance, in casing burst load design where a gas kick is assumed to occur, the type of gas and its effect on the magnitude of the kick is unknown. The amount of gas fill in the casing is also unknown as well as factors that will affect the casing burst strength during gas influx. An occurrence of an unexpected event can lead to loss of life, asset, investment, integrity and poses danger to the environment. As a result, it is important to understand its probability of occurrence in order to reduce or completely eliminate its impact.

# Chapter 5

---

## 5.1 Monte Carlo Simulation

Monte Carlo simulations method is used for modeling the probabilistic outcome of a process and is an exact method for calculating reliability<sup>[28]</sup>. It is a well-known tool amongst the engineers for analyzing uncertainties to aid proper decision making<sup>[28]</sup>. The method has been applied in the petroleum industry to solve a wide range of problems. These include estimation of cost related to drilling and completion operations<sup>[29]</sup>, well forecasting<sup>[30]</sup>, cost and time estimation of P&A operation<sup>[21]</sup>, reserve estimation, Stimations decisions<sup>[28]</sup>, Field development analysis etc...

## 5.2 Why use a Monte Carlo Simulation for RBD casing burst strength design?

The important strengths of the MCS application in RBD include:

- MCS is used because it provides an understanding of the uncertainties in the design. Also, it addresses the true risk events thereby enabling an accurate risk-based decision. It gives a consistent probability of failure. In addition, it allows for any presence of offset in the design to be analyzed.
- Above all, it allows for a proper understanding of the sensitivity of the variables to changes<sup>[30]</sup>. It is easy to make changes to the model and variables. It is also easy to compare results.
- MCS is simple, available and an excellent analytical tool. It is easy to include complex mathematical functions such as logs, power, control statements in the model using MCS. Its results are valid and widely accepted by professionals.
- MCS has less possibility of making mistakes compared to any other analytical method.
- There is no need of rounding off input variables using MCS because it has a wide range of application in stochastic and discrete distributions<sup>[28]</sup>.
- MSC can handle the presence of correlations and dependencies<sup>[28]</sup>. It is easy to model if documented.
- Results from the simulation can be verified by investigating the model's behavior<sup>[28]</sup>.
- MCS is also applied even if the modeler is not interested in understanding the uncertainty in a decision/design attribute and only want to compute an expected value.
- Probabilistic strength model approach is used to justify a low-price design having a low cost of failure and an acceptable high probability of failure<sup>[5]</sup>.
- It can be applied to create an extremely reliable design when the consequences of failure have high cost and safety implication.

## 5.3 Monte Carlo Simulation Work Steps

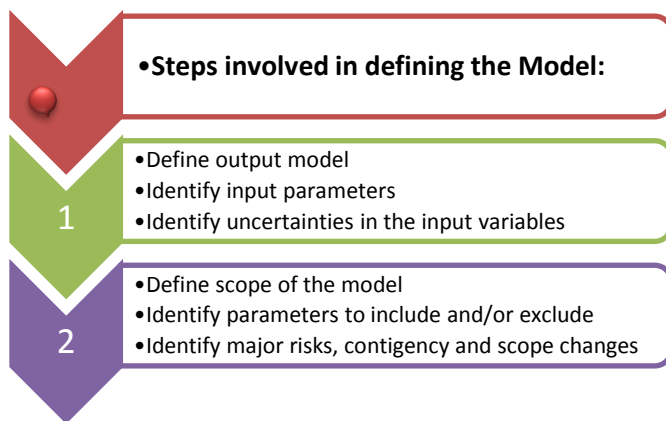
MSC technique is based on the following work steps<sup>[30]</sup>:

- a. Defining an appropriate model

- b. Data gathering
- c. Defining input distribution
- d. Sampling the input distribution, and an intermediate step which is the computation of model result.
- e. Interpreting and using the result

**5.3.1 a. Defining an appropriate model;**

The first step requires the modeler to clearly define the output of the model. The definition of the model output will further lead to identifying the input models and variables. It is very important to get this step right as it can be sometimes complex<sup>[30]</sup>. The modeler also defines the uncertainties in the input variables and includes them using probability distributions<sup>[28]</sup>. The next step is to define the scope of the model. To establish the scope of analysis requires defining variables/parameters to include or exclude from the model. It also requires an understanding and consideration of possible events that are likely to occur. The possible events can be major risks, contingency, and scope changes<sup>[30]</sup>. The steps involved in defining a model are described in Figure 17.



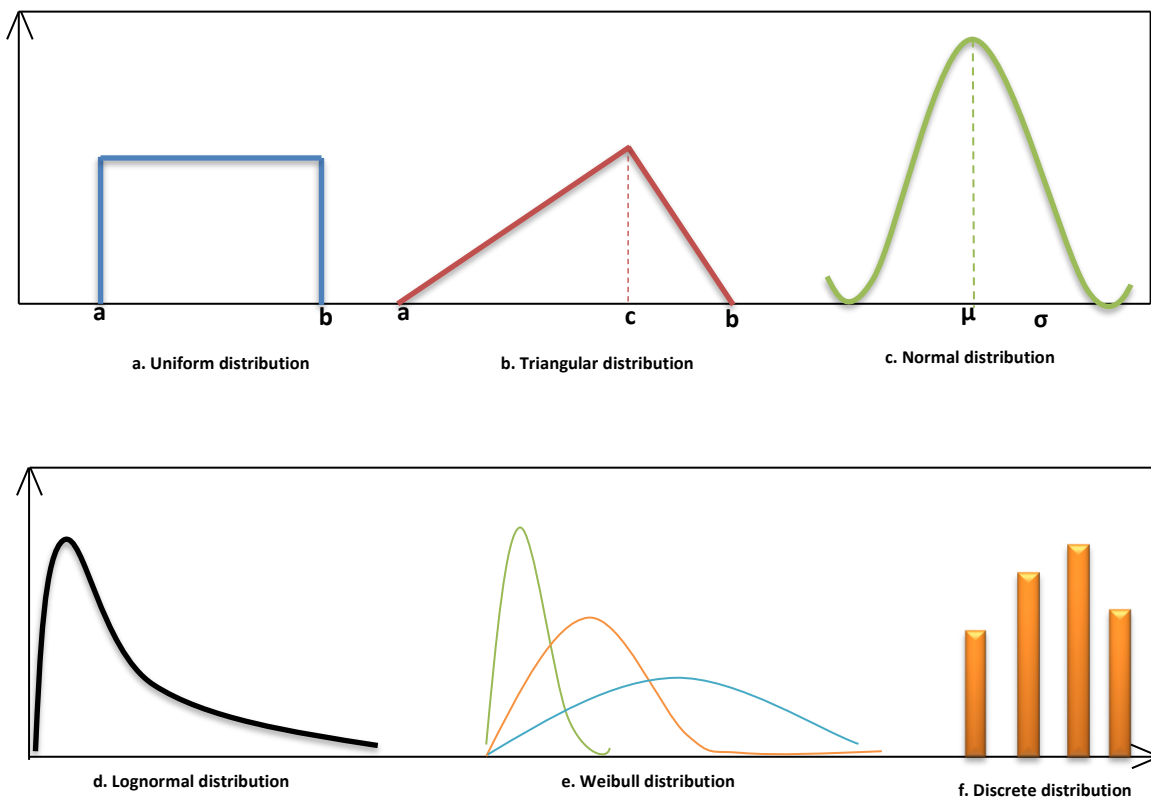
**Figure 17:** Shows the steps involved in defining a model.

**b. Data gathering:**

The information required for sampling must be collected and compiled prior to selecting the type of input distribution. The more precise an input data is, the more reliable result one can obtain<sup>[21]</sup>. However, acquiring the right data can be challenging. Data sources can be from experiments, experience, skilled opinion, company and/or Government database. It is important that the set of collected data be large enough to accurately portray a full range of performance, outcomes and to correspond with any degree of distribution shape<sup>[30]</sup>. Having a large set of data will also reduce or eliminate the effects of small sample size. Particularly, based on the

presumption that the accurate input data of the model is not known, therefore, enough data will be needed to quantify the uncertainty<sup>[30]</sup>. Data sets can be presented using tables or graphs so that they are easily understood for use<sup>[30]</sup>. The data should be sorted and any outliers thoroughly investigated and understood before considering removal. An example of input data can be cost, time, pressure, price etc...

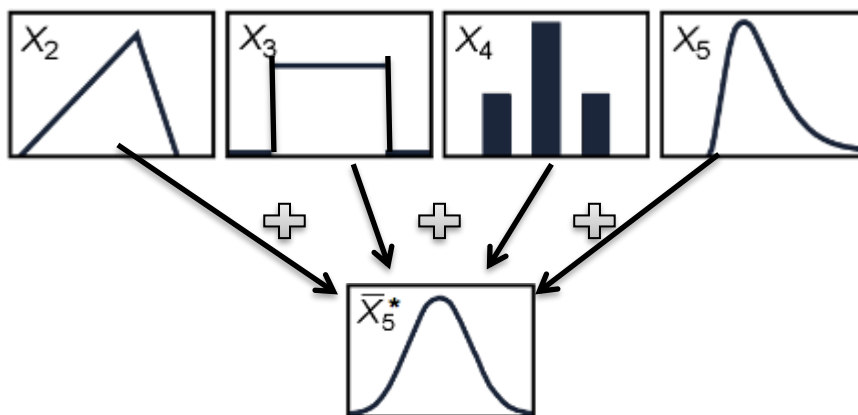
- c. **Defining Input Distributions:** The data sets can be sampled using different types of input distributions. These include Uniform, Triangular, Normal, Lognormal, Weibull, Discrete and PERT (Project Evaluation and Review Technique) distribution<sup>[29]</sup>. The different distribution shapes are shown in Figure 18 below.



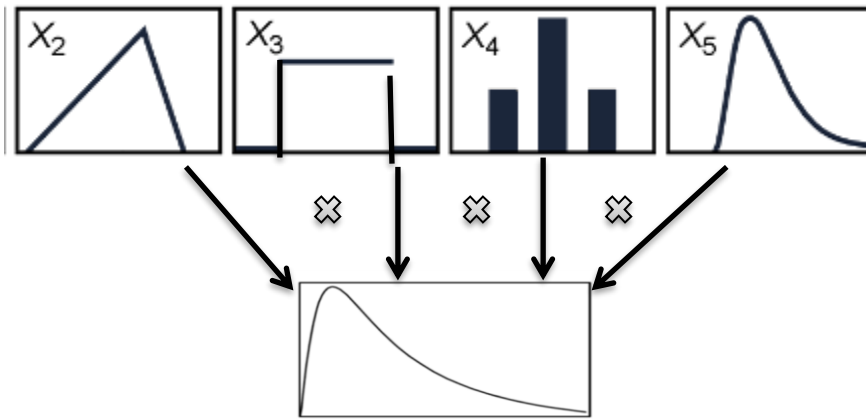
**Figure 18:** Shows the distribution types where (a) and (b) represent the minimum and maximum values respectively for the uniform distribution, (a) denotes the minimum value, (b) denotes most likely value and (c) denotes the maximum value for the triangular distribution.  $\mu$  and  $\sigma$  represent the mean and standard deviation of the normal distribution respectively<sup>[29]</sup>.

In Figure 18, a, b and c represents the minimum, most likely and maximum values for the triangular distribution and  $\sigma$  and  $\mu$  standard deviation and mean for the normal distribution respectively. The a and b in the uniform distribution represents the minimum and maximum values. It is up to the

modeler to choose the distribution parameters: minimum value, maximum value, most likely value, mean and standard deviation [30]. The two main choices for the input distribution are the distribution shape and the distribution parameters. The uniform or triangle distribution shapes are usually the preferable distribution shapes. In SPE 89984 paper[30], it is argued that one should not focus too much on the choice of distribution type but more on its values. This was argued with reference to the central limit theorem. For instance, if we have a large collection of probability distributions, these distributions can be represented using different shapes and ranges. The next step is to sample the different distribution shapes and ranges individually and sum up the resulting values. This sampling is carried out multiple times and a resultant distribution shape is obtained from the summed up individual distributions. Accordingly, the central limit theorem says that for the large collection of distributions sampled individually, that the resulting distribution will have a mean close to the sum of the mean of the individual distributions, a variance close to the sum of the variances of the individual distributions, and the resulting shape is close to normal distribution[30]. A more accurate resulting distribution is obtained as the number of probability distributions sampled increases. Therefore, the resultant distribution of the sampled variables depends solely on the mean and standard deviation of those sampled variables and rarely on their individual distribution shapes[30]. Also, obtaining the product rather than the sum of the distributions of the sampled variables gives a resultant distribution that has a lognormal shape. This type of result is best applicable to forecasting, which is obtained from the product of different unknowns[30]. Finally, it is recommended to spend less time in choosing the type distribution shape but focus more in selecting an appropriate mean and standard deviation for the input distribution[30].



**Figure 19:** Illustrating the addition of the input distributions[30].



**Figure 20:** Illustrating the multiplication of the input distributions<sup>[30]</sup>.

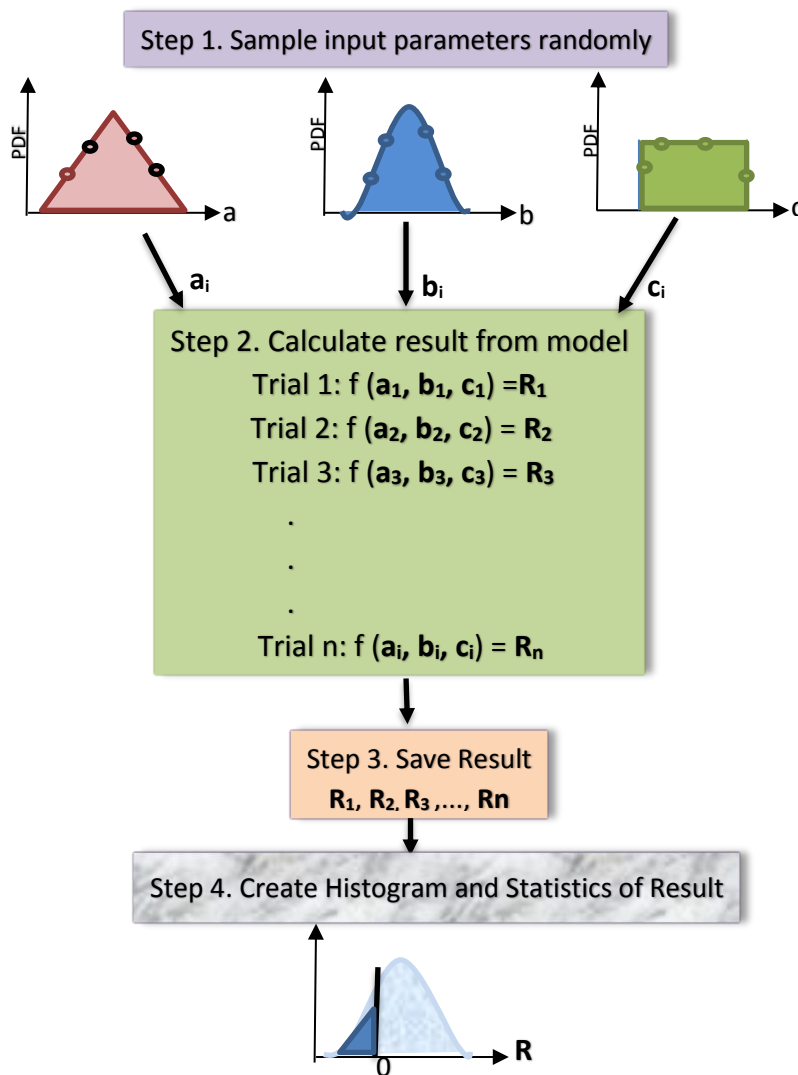
**d. Sampling the input distribution**

The variables (uncertain) to be sampled represent real life unknown parameters, that is to say, that any result obtained from the simulation of the variables represents the real life unknown parameters<sup>[28]</sup>. One major goal of MCS is to ensure that the input data are sampled such that the results obtained represent real life parameters. The steps of the MCS simulation procedure is illustrated in Figure 21. As earlier mentioned, the deterministic modeling samples by a single point estimate, whereas probabilistic modeling samples a full range of variables, thereby examining the probabilistic outcome of a process. The Monte Carlo sampling technique is discussed in the following steps. Firstly, we represent the input variables (with the uncertainty) in the model with any type of probability distribution. MCS converts the probability distribution curves to an equivalent cumulative distribution curve with an interval of 0-1. As shown in Figure 22 below, it describes the first step of the MCS sampling and how the input variables are generated. Here, the software selects a random number between 0 and 1 and enters it into the vertical axis of the cumulative distribution function, CDF curve to obtain a value of the CDF inverse. This inverse value becomes the input value to be used to calculate the output result as shown in Figure 22. MCS picks a sample randomly from the individual input CDFs (the inverse values), applies them in the model, runs the simulation (calculates) to generate the output result and finally saves the result for future analysis.

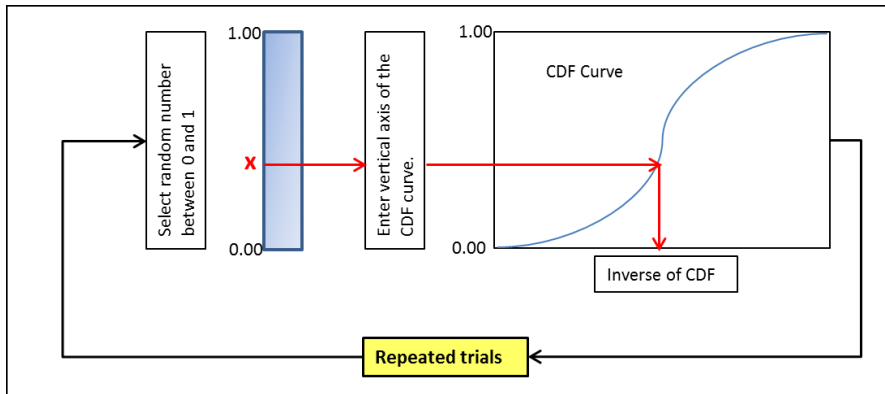
The software runs thousands of simulations to produce the output result which will be saved for later analysis. The saved output results are represented using a histogram, which is later converted to the probability distribution of relative frequency. The process of running simulations a number of times is called iteration<sup>[28]</sup>. The number of iterations is sufficient when the repeated Monte Carlo simulation runs result in an output distribution that does not change substantially. It is worth mentioning that increasing the number of iterations, yields a more exact

estimate. Moreover, the process of sampling input distributions independently and adding them up to give one result is called a Monte Carlo trial. Most significantly, the result obtained from the sampling can be affected by the manner in which the random numbers are chosen from the input distributions<sup>[30]</sup>. Thus, giving rise to a correlation which is defined as the “relationship or common influence between two input quantities that cause their joint distribution to deviate from statistical independence”<sup>[30]</sup>. Also, correlation is a real physical part approximated by the model and needs to be indicated in the sampling method if present<sup>[30]</sup>. Another method of sampling is the Latin hypercube sampling and this method of sampling is more effective compared to the Monte Carlo sampling.

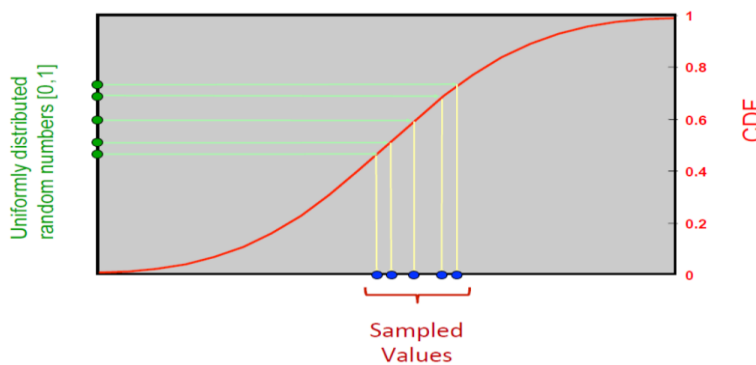
The CDF adds up the probability of all outcomes.



**Figure 21:** The schematic procedure of the Monte Carlo Simulation. Modified after<sup>[28]</sup>.



**Figure 22:** Shows the Monte Carlo technique of sampling, how the input variables are generated [28].



**Figure 23:** Monte Carlo Sampling[31].

**e. Computation of Model result:**

MCS uses the model output results to develop a histogram, which it converts to a probability distribution from which it develops the statistics of the results such as the mean, mode, variance, standard deviation, P90 and P10 values. This step refers to the third step in Figure 21. The probability of failure  $P_f$  is also calculated from the simulation.

**f. Presentation and Interpretation of the result:**

The result from a Monte Carlo simulation is usually presented in a set of distribution curves, PDF, and CDF curves. These distribution curves are usually verified before use[30]. Furthermore, as part of the results, these parameters are also presented; mean (average), variance, standard deviation, P10, P90, and P50 (median). These terms are defined below[29].

**i. Mean:**

The samples mean or an average of a set of  $n$  measurements  $x_1, x_2, x_3 \dots x_n$  is the sum of these numbers divided by  $n$ [32]. The mean,  $\bar{x}$  is mathematically represented thus:



$$x' = \frac{\sum_{i=1}^n x_i}{n}$$

ii. **Standard deviation, SD:**

This parameter expresses the spread of a set of data. In other words, it quantifies their dispersion from their mean. A low SD shows that the data points are close to the average or expected value of the set. On the other hand, a high SD shows a spread of the data points from the mean of the data set. The SD is related to the variance by a square root of the variance<sup>[32]</sup>. It is mathematically represented thus:

$$S_x = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - x')^2}$$

iii. **Variance:**

This parameter is simply the average of squared differences between the sample data points and their mean<sup>[32]</sup>. That is the square of the standard deviation. Also, it quantifies the spread of a set of data and is mathematically represented thus:

$$Variance = \frac{1}{n} \sum_{i=1}^n (x_i - x')^2$$

iv. **Sample median or P50 or 50<sup>th</sup> percentile:**

This parameter is obtained by arranging the data in an ascending order, that is, from smallest to largest value, then picking out the middle value. Note that this value does not always coincide with the mean value<sup>[32]</sup>. It also represents the 50% of the outcomes smaller than an estimate.

v. **P10**

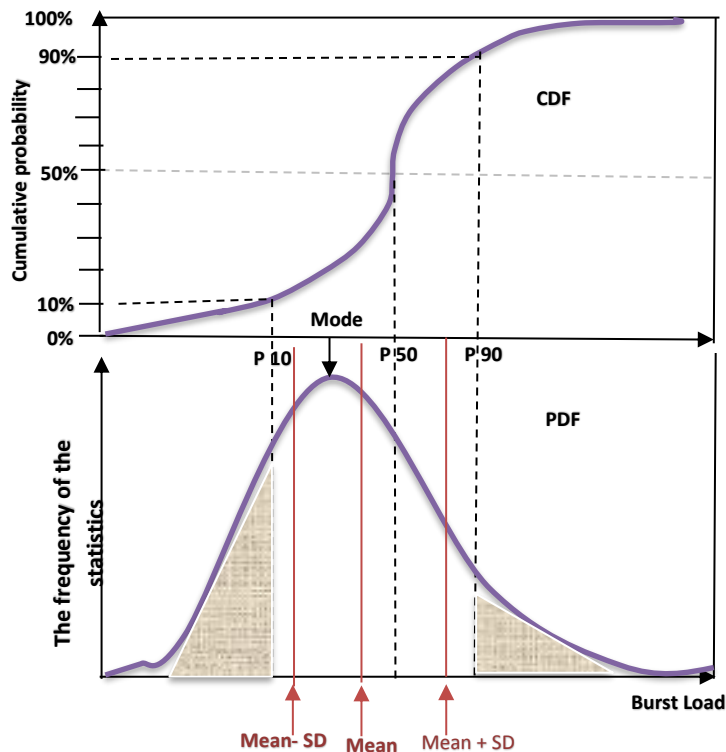
P10 value is a value such that 10% of the sets of data are lower than it. We pick the P10 value from the arranged data, in the increasing order<sup>[32]</sup>.

vi. **P90**

P90 value is a value such that 90% of the data sets are lower than it. This value is also obtained from the data arranged in the increasing order.

The data range between P10 to P90 can be used to estimate data spread. P90 and P10 are high and low estimates respectively. 80% of the outcomes will be between these.

vii. **Mode** is the most frequently occurring data.



**Figure 24:** Illustrating the statistical terms used for result presentation<sup>[30]</sup>.

#### 5.4 Using the result

The result obtained from the simulation is used for decision making in the design process or for further analysis to enhance understanding.

##### 5.4.1 Sensitivity analysis:

The sensitivity analysis is an analysis used to determine the dependency of a given output model on input parameters. This analysis is performed by changing variables in the model, simulating and comparing with a base case where all input variables are varied. By this means, one can easily identify variables which have a most significant effect on the result or uncertainty related to the result<sup>[33]</sup>. One simple technique of this analysis is to change a variable at a time while keeping the other variables unchanged at a base case, then simulate and analyze new result<sup>[33]</sup>. A second more complex technique is to simultaneously change all variables with regards to their distribution types and considering their common characteristics<sup>[34]</sup>. The types of sensitivity analysis include local sensitivity and perturbation analysis, the multidirectional sensitivity analysis and the global sensitivity analysis<sup>[34]</sup>. The local sensitivity method is best applicable on steady state model. The multidirectional sensitivity measure involves the simultaneous investigation of a group of parameters and their effect on the system output. The Global

sensitivity analysis is also used to evaluate the impact of individual or group of variables on the model output. It differs from the multidirectional sensitivity analysis in that it measures the relationship between varied groups of variables. The global sensitivity analysis is also referred to as the uncertainty importance measure because it is good at recognizing the important variables whose uncertainty influences the output uncertainty most<sup>[34]</sup>.

#### **The important uses of Sensitivity Analysis:**

This analysis is used to:

- provide insight into an event driver<sup>[33]</sup>.
- provide knowledge about the variables which have an effect on the expected outcome<sup>[28]</sup>.
- Identify parameters for further study or parameters to overlook<sup>[28]</sup>.
- used to concentrate on crucial data<sup>[33]</sup>.
- Used for performance evaluation via perturbation analysis<sup>[34]</sup>.

Furthermore, the industry application of sensitivity analysis is often presented using graphics such as the Tornado diagram<sup>[29]</sup>. The Tornado diagram is used to present the result of a single-variable sensitivity analysis<sup>[28]</sup>. The single variable sensitivity analysis is synonymous with the one-at-a-time analysis.

The sensitivity analysis in MCS can involve changing all the input parameters simultaneously to obtain a good realistic scenario of uncertainty in the output parameter as in the global sensitivity analysis method. To measure the effect of the uncertainty present in the input parameter on the output variable, one can develop a correlation coefficient or regression between the input and output variables. The correlation coefficient between an input and output variable must be either positive or negative. According to Bratvold and Begg<sup>[28]</sup>, input variables whose uncertainty does not affect the decision characteristic have no need to be further studied<sup>[28]</sup>.

#### **5.5 Some of the assumptions in Monte Carlo Simulation of the casing design:**

In our Monte Carlo simulations, it is assumed that the variables are independent. Put differently, a draw from one distribution will not affect a draw from another distribution. In this case, the variables are not correlated. On the other hand, if a draw from a distribution affects a draw from another distribution, it is known as the effect of correlation.

#### **5.6 Dependent variable and Correlations:**

As already stated, it is assumed in this study that all input probability distributions in the model are independent. It is important for the reader and the modeler to be aware that it is possible to have the variables values that depend on other values. It is also common to have dependent variables in oil and gas application<sup>[28]</sup>. For example, porosity and permeability are correlated variables, cost and rig rate are correlated variables. Correlations may be positive or negative. Positive correlations increase in the same order, for e.g. Original Oil in Place, OOIP and Technical

Recovery Factor, TRF, that is when one variable is high the other is also high. On the other hand, negative correlations imply that when one variable is high, the other is low<sup>[28]</sup>. A good example is the petroleum education, with good students and bad work market.

It is important to include dependencies in the model to avoid impossible combinations of variables being generated in the sampling<sup>[28]</sup>. In addition, an understanding of correlations will help a model distinguish between positive and negative correlation. Furthermore, the kind of the dependency either positive or negative will affect the distribution of the output variable. Positive dependencies expand the output distribution while the negative dependencies narrow it.

### **5.7 How to calculate Probability of Failure**

In reliability based design, (RBD) the probability of failure is calculated. Using the Monte Carlo simulation, the modeler first specifies the number of simulations. In each Monte Carlo trial, a load and a strength are calculated. A variable is used to register if the load exceeds the strength. This variable also counts the number of times this case occurs. At the end of the simulation, the percentage is found by dividing this variable by the number of Monte Carlo simulations and multiplying by 100 to get the result in percentage.

# Chapter 6

---

## **6.1 The Case Study: An application of Monte Carlo simulation for casing reliability design**

### **Burst load design**

Consider the burst load design of a production casing in a deep water having the information listed below. We consider a burst scenario where a kick is taken from the planned depth of the next section. It is assumed that the well is closed and that the kick fluid fills the whole well. We also assume that the wellhead is located subsea and there is an external pressure.

9<sup>5</sup>/<sub>8</sub>in. 47lbs/ft. L80

Weight: 68.73kg/m

Burst Strength: 473bar / 7330psi

Collapse: 328bar

Tension: 483\*10<sup>3</sup> daN

Mud weight: 1.60sg (inside and outside of the casing)

Inner diameter of

Casing: 8.681in., 220.5mm

Casing nominal

wall thickness: 0.472in.

Setting depth of

Previous casing: 3800mMD, 3800mTVD

Well depth: 4300mMD, 4000mTVD

Cemented interval:3700-4000mMD, 3600-3800mTVD

Sea water depth 500m

Gas density: 0.1-0.3sg

Oil density: 0.7-0.9sg

Pore pressure: 1.60sg

Sea water density: 1.03sg

The objective is to calculate the burst load and the probability of failure,  $P_f$  using RBD approach by applying Monte Carlo simulations.

### **6.1.1 Defining a model:**

The first step is to define the appropriate model and parameters for the simulation. The casing design model for the burst scenario is the same as that used in the traditional method and these

are the load limit state equation,  $L(\dot{x})$  and strength limit state equation,  $S(\dot{x})$ . However, the uncertain parameters are randomized by defining them with a probability distribution. The following steps are implemented when developing a simple probabilistic model for ductile burst<sup>[5]</sup>:

Step 1: ascertain the mode of failure, in this case, it is burst.

Step 2: define the behavior of failure and using limit state function express the failure mathematically,  $g(\vec{x})$ , such that a failure is implied when  $g(\vec{x}) < 0$ .

Step 3: define the uncertainty model for the input variables,  $\vec{x}$  in the limit state, one can develop the deterministic models into statistical analyses.

#### a. Defining the Load Model:

$$P_{b@surface} = P_{internal} - P_{external}$$

$$P_{internal} = (\rho_{pore} \times g \times TVD - \rho_{gas} \times g \times h) \quad \text{Equation 19}$$

$$P_{external} = (\rho_{sea-water} \times g \times H_{sea}) \quad \text{Equation 20}$$

$$P_{b@surface} = (\rho_{pore} \times g \times TVD - \rho_{gas} \times g \times h) - (\rho_{sea-water} \times g \times H_{sea}) \quad \text{Equation 21}$$

The **input parameters** for the load model shown in Equation 21 are the pore pressure,  $\rho_{pore}$ , gas density  $\rho_{gas}$ , sea-water density,  $\rho_{sea-water}$ , the true vertical depth, TVD from RKB, the height of the kick,  $h$ , height of sea-water,  $H_{sea}$  and the acceleration due to gravity,  $g$ .

The variable parameters in the load model are the gas density and the pore pressure. Firstly, the gas density has been selected because it is an assumed parameter in the conventional design, where a worst-case scenario is supposed to occur. A worst-case scenario will be a casing totally filled with gas. It is assumed that a gas kick occurs during the drilling of the next open hole section, the well is shut-in and pressure increases in the well from bottom to top. A pressure differential is created between the inner and outer casing liable to rupture it. As previously mentioned, there are uncertainties surrounding this parameter as this may not be the case with regards to how full the well is with the gas. The well may be fully or partially filled with the influx gas and this will have an impact on the average fluid density in the well. This means that the magnitude of the pressure build-up is uncertain. In our simulation, varying the fluid density will mimic the effect of different influx types and the extent of well fill with the kick. Secondly, the pore pressure is a parameter which is usually uncertain due to geological uncertainties and errors from measurement tools. It can also be uncertain due to production and injection effect. The goal of the simulation is to explore these uncertainties.

The burst pressure is maximum at the top. Therefore, in this simulation focus shall be on the maximum burst pressure. Also, the external pressure or the seawater gradient at the top has a

value. Other loads such as collapse and tension are not included in this simulation. In this work, the imperfections in the casing are considered using the Klever-Stewart's casing strength model.

**b. The Strength models<sup>[5, 18]</sup>;**

The following strength models are used in the simulation, with Barlow's equation considered as the base case strength model.

- **Barlow's Equation:**

$$P_b = \frac{2 \times t_{nom} \sigma_y}{D} \quad \text{Equation 4}$$

- **Von Mises Equation:**

$$P_b = \frac{\sigma_y \times (D^2 - d^2)}{\sqrt{3} \times D^2} \quad \text{Equation 7}$$

- **Klever-Stewart Equation:**

$$P_b = \frac{2 \times K_{dr} \times \sigma_u \times t_{min}}{(D - t_{min})} \quad \text{Equation 8}$$

This model accounts for tubular inherent deformation and hardening up to rupture<sup>[18]</sup>.

- **Paslay Equation:**

$$P_b = \frac{2 \times \sigma_y \times t_{min}}{(D - t_{min})} \quad \text{Equation 9}$$

- **Moore Equation:**

$$P_b = \frac{\sigma_y \times (D^2 - d^2)}{(D^2 + d^2)} \quad \text{Equation 10}$$

- **Nadai Equation:**

$$P_b = \frac{2 \times \sigma_u}{\sqrt{3}} \times \ln \left[ \frac{D}{D - 2t} \right] \quad \text{Equation 11}$$

Where  $P_b$  is the internal burst pressure,  $K_{dr}$  is the correction factor for pipe deformation,  $t_{min}$  is the minimum tubular wall thickness without crack deformations,  $t_{nom}$  is the specified or nominal tubular

wall thickness,  $D$  is the tubular outside diameter,  $d$  is the tubular inside diameter,  $\sigma_u$  is the casing ultimate tensile strength and  $\sigma_y$  is the yield strength of the tubular.

### 6.1.2 Data gathering:

The second step is to collect and organize data, such as pore pressure, sea water depth, cement density, cemented region, target depth, casing strength, expected influx types and densities etc. The sources of the data used in this work are can be found in *Modern Well design*<sup>[13]</sup>, *Drilling data handbook*,<sup>[17]</sup> and the *API TR 5C3*<sup>[18]</sup>. The tables below summarize the values of the input data.

**Table 5:** Well data, source:<sup>[13]</sup>.

<b>Well data, source:<sup>[13]</sup></b>	
<b>Parameters</b>	<b>Values in units</b>
Well type	Subsea
Well depth	4000m
Sea water depth	500m
Pore pressure	1.6sg
Gas density	0.1-0.3sg
Oil density	0.6-0.9sg
Number simulations	1000000

We chose two ways of specifying the uncertainty when comparing the different strength models, we used the data shown in [Table 6](#). In this case, we need to specify the uncertainty distribution for each input parameter in the strength model. This is presented in simulation case 3 in chapter 7. However, even if our input data uncertainty is perfectly known we will still have uncertainty in the prediction since we do not have a perfect limit state model. However, when evaluating the burst load scenario using Barlow’s limit state equation, we chose to specify the input data like thickness, yield, and OD as single values: then we took the resulting single value burst strength and multiplied with a normal distribution that took into account the deviation of the model from real measured burst data. This was based on some values found in *ISO TR 10400*<sup>[5]</sup>.



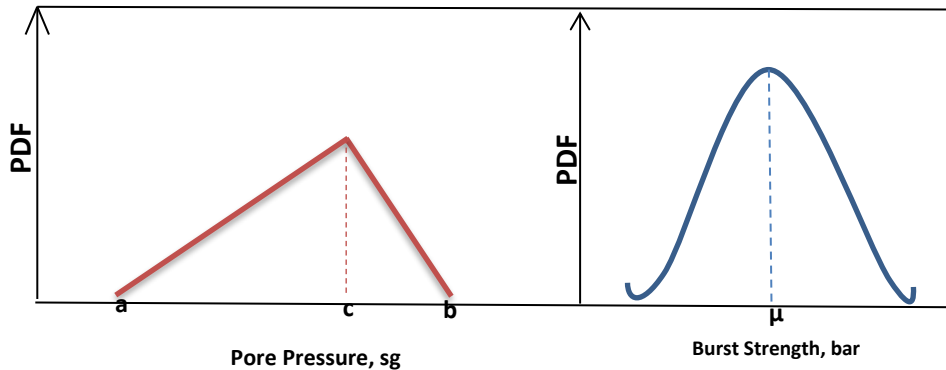
We used N (1.092, 0.052). This approach can be a way of incorporating measurements revealing the real burst pressure and it will also take into account the imperfectness of the limit state model. This approach is used in cases 1, 2a. and 2b (chapter 7). In ISO TR 10400 standard<sup>[5]</sup>, it can be seen that different calibration distributions had to be used for the different models. However, in general, it might be a challenge for an engineer to know what kind of data to use for the RBD calculations but there must be some kind of adjustments or corrections taking into account measurement. In SPE 178907<sup>[8]</sup>, they state that strength parameter data must be collected from API TR 5C3(ISO), manufacturer’s report or direct material and dimensional measurements. We have assumed the mean value of the casing minimum tensile strength to be 100,000psi and the standard deviation to be 2751psi as seen in Table 6. These values were assumed based on the values of the mean and standard deviation of the casing yield strength.

**Table 6:** The Casing properties and parameters for simulation (Input data).

<b>Strength Parameters, of 9.625 in. 47 lb./ft. L80 Casing</b>					
<b>Variables</b>	<b>Nominal values</b>	<b>Data source</b>	<b>Distribution type</b>	<b>Mean</b>	<b>STD</b>
Casing yield strength, $\sigma_y$ , psi	80,000	API TR 5C3,[11]	Normal	87,000	2751
Casing minimum tensile strength, $\sigma_u$ , psi	95,000	Drilling data handbook, the mean and STD are assumed.	Normal	100,000	2751
Outer diameter, OD, in.	9.625	API TR 5C3, [11]	Normal	9.635	0.003
Inner diameter, ID, in.	8.681	API TR 5C3	Normal	8.667	0.003
Average Thickness, t, in.	0.472	API TR 5C3, [11]	Normal	0.479	0.003
Internal pressure for thin walled tubular, psi	7.846	API TR 5C3, [11]		8.650	279
Equation Error correction factor	1.000	API TR 5C3, [11]	Normal	1.092	0,052

### 6.1.3 The input distributions

The third step is to define the input parameters and the form of probability distributions. It is worth emphasizing the point that the choice of distribution shape is not critical for casing loads. The triangle distribution type will be used to represent our data for the Load and the normal distribution is used to represent the Strength parameters.



**Figure 25:** Triangle and Normal distributions respectively<sup>[29]</sup>.

The distribution parameters shall be the minimum value, most likely value, and maximum value for the triangular distribution and the mean and standard deviation for the normal distribution.

### 6.1.4 Sampling the input distributions

Monte Carlo sampling is the process of running several Monte Carlo simulations to produce the output. The output includes the  $P_f$ , mean, standard deviation, P10, P90, and P50. The results are presented in the next chapter.

#### Finding the probability of failure

For each Monte Carlo simulation, a load and a strength value are calculated. A variable counts the number of times the load is larger than the strength. After the MC simulation is finished, the value of this variable is divided by the number of MC simulation and then multiplied by 100 to obtain the percentage failure.

### 6.1.5 Presentation of the result

The final step is to interpret and use the result for its intended purpose. The results from the MC simulation is presented in the form of probability distribution curves, PDF and cumulative density curves, CDF for load effect and strength.

# Chapter 7

---

## 7.1 Presentation and Discussion of Results

### Case 1: Base case

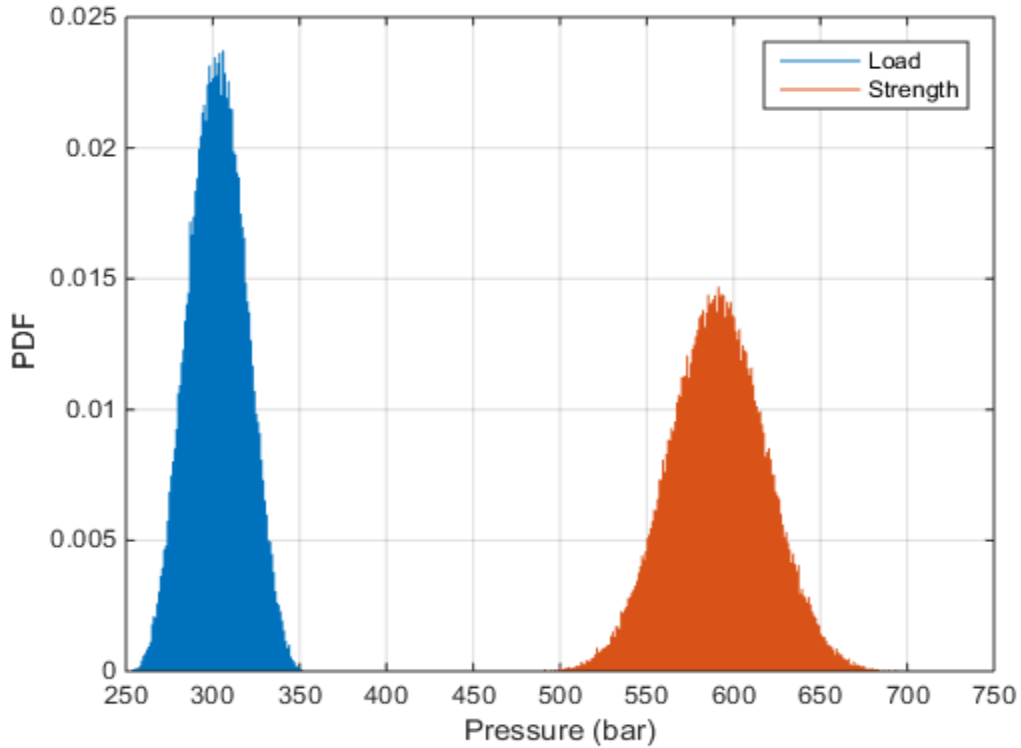
In this case, we will assume the same geometry and parameters as described in Chapter 6. In this burst scenario, we assume that the well is filled with an oil type of kick. The aim is to demonstrate a non-worst case of kick scenario with heavy fluid in the well. A heavy fluid in the well will exert less pressure at the wellhead compared to the pressure from a lighter well fluid. The uncertainties in the input data are given in Table 7. The fluid density will be varied in the next case.  $P_{pore}$  is the pore pressure,  $\rho_{gas}$  is the gas density,  $\rho_{oil}$  is the oil density, T denotes the triangular distribution presented as T (a, b, c), and N denotes the normal distribution presented as N ( $\mu$ ,  $\sigma$ ), where a is the maximum value, b is most likely value, c is the minimum value,  $\mu$  is the mean and  $\sigma$  is the standard deviation.

The simulation run time is 26 seconds and the result from the simulation is shown in [Figure 26](#) and [Figure 27](#).

**Table 7:** Input distributions for Case 1 stochastic simulations

Case 1 data:	Nominal Values	Distribution type	Minimum value	Most likely value	Maximum value	Definition of code
Pore pressure, in sg	1.6	Triangular	1.6-0.05	1.6	1.6+ 0.05	$ppore=T(pp-0.05, pp, pp+0.05,1)$
Oil density, Sg	0.7-0.9	Triangular	0.7	0.8	0.9	$\rho_{oil}=T(0.7,0.8,0.9,1)$
Seawater density, sg	1.03	Deterministic	-	-	-	
<b>Strength model</b>	<b>Barlow's model</b>	-	-	-	-	
Casing yield strength, psi	80000	Deterministic	-	-	-	
OD, in.	9.625	Deterministic	-	-	-	
Wall thickness, t, in.	0.472	Deterministic	-	-	-	
Burst pressure, $P_b$ , bar	$\mu=1.092$ $\sigma=0.052$	Normal				$P_b=(2*YieldStrength*t)/OD*N(1.092,0.052)$

**Case 1 (base case) PDF plot**



**Figure 26:** Shows the PDF plot of case 1, with oil density. There is no overlap between the curves and therefore no chance of failure.

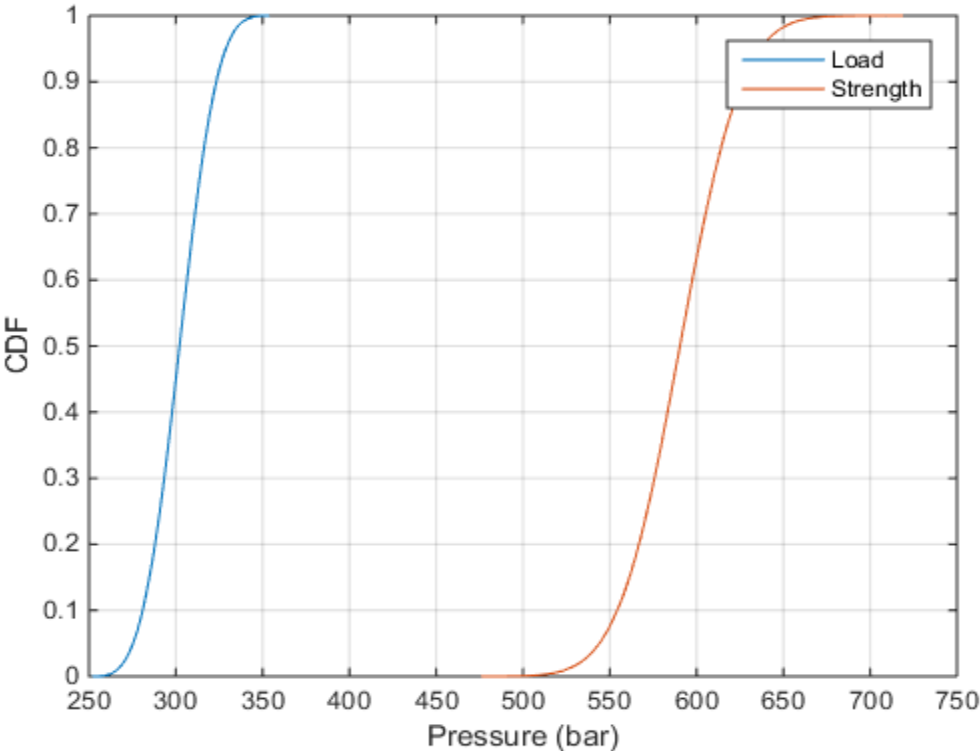
In Figure 26, the  $P_f$  is zero,0. The other statistical data are listed in the table below.

**Table 8:** The statistical quantities from Case 1 simulation.

Statistical quantities	Load, bar	Casing strength, bar
Average	302.6952	590.9035
P50	302.6790	590.8287
P10	281.4223	554.9464
P90	323.9773	626.9133
P90-P10	42.555	71.9669

There is no probability for casing failure in this case. Therefore, the casing can withstand the magnitude of the load without failure. From the data above, one can evaluate the uncertainty. The difference between the P90 and the P10 can be used to measure the spread of uncertainty in the strength and load.

**Case 1, (base case) CDF plot**



**Figure 27:** Showing CDF plot of case 1, with the oil density.

**Case 2 shows different cases of casing failure with varying degrees of Pf**

In this case, we assume a burst scenario with a gas filled well presenting, two simulations where the gas density is lower in simulation case 2b compared to case 2a. The purpose of using the variations of gas density here is to demonstrate a worst case of kick that can occur which the casing may withstand. This worst kick scenario gives the highest-pressure value at the wellhead.

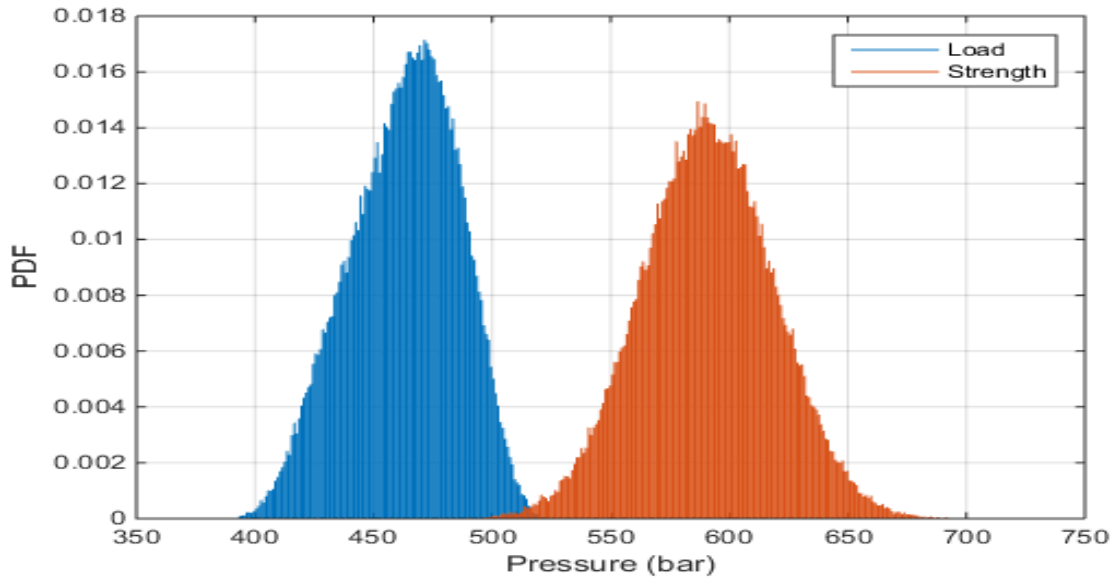
**Case 2a.**

The uncertainties in the input data for case 2a simulation are given in Table 9. The highlighted row is the focus of study in the case 2a. The gas density is varied here and in the next case(2b). Here, the  $P_{pore}$  is the pore pressure,  $\rho_{gas}$  is the gas density, T denotes the triangular distribution and N denotes the normal distribution.

The run time is 29 seconds and the result from the simulation are shown in [Figure 28](#) and [Figure 29](#).

**Table 9:** Summarizes the input data and distributions for case 2a stochastic simulations.

Case 2a data:	Nominal Values	Distribution type	Minimum Value	Most likely value	Maximum value	Definition of code
Pore pressure, in sg	1.6	Triangular	1.6-0.05	1.6	1.6+ 0.05	$p_{pore}=T (pp-0.05, pp,pp+0.05,1)$
Gas density, Sg	0.2-0.5	Triangular	0.2	0.3	0.5	$\rho_{gas}=T (0.2,0.3,0.5,1)$
Sea water density, sg	1.03	Deterministic	-	-	-	
<b>Strength model</b>	<b>Barlow's model</b>	-	-	-	-	
Casing yield strength, psi	80000	Deterministic	-	-	-	
OD, in.	9.625	Deterministic	-	-	-	
Wall thickness, t, in.	0.472	Deterministic	-	-	-	
Burst pressure, $P_b$ , bar	$\mu=1.092$ $\sigma=0.052$	Normal				$P_b=(2*YieldStrength *t)/OD*N (1.092,0.052)$



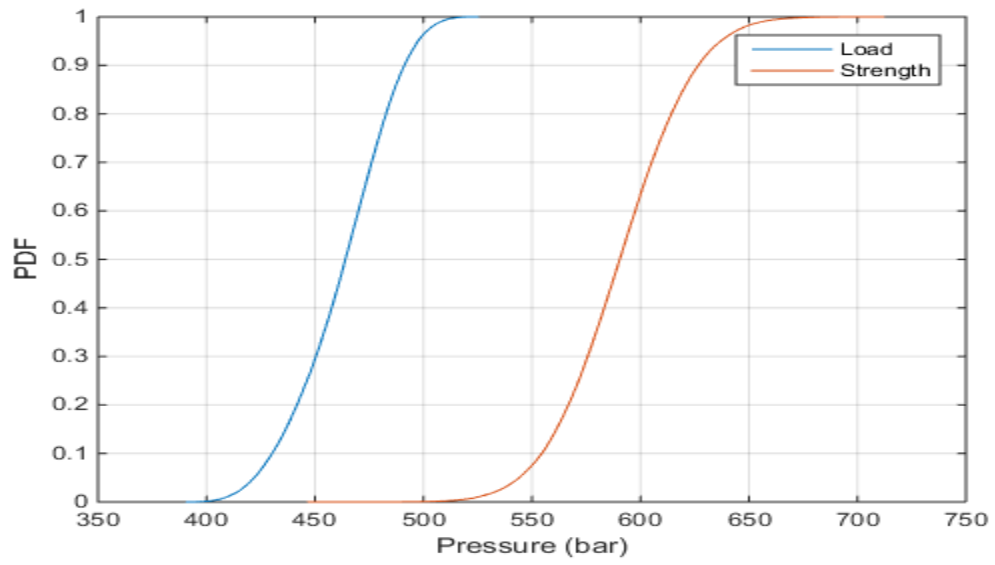
**Figure 28:** PDF plot of case 2a, with gas-filled well. Here, the overlap between the tail end of the load curve and the start line of the strength curve indicates a failure, with a value for the probability of failure.

Figure 28 shows a probable casing failure with a  $P_f$  of 0.008% and the statistical quantities are summarized in Table 10.

**Table 10:** The statistical quantities from case 2a. simulation

Statistical quantities	Load Effect, bar	Casing strength, bar
Average	463.0044	590.8804
P50	464.8027	590.8999
P10	431.2360	554.6951
P90	491.6005	626.8476
P90-P10	60.3645	72.1525
$P_f$	0.008%	

Having a value of  $P_f$ , 0.008% implies that there is a certain risk for burst in the presence of the load. The P90-P10 values for load and strength give the spread of uncertainties.



**Figure 29:** The CDF plot of case 2, with low gas density (0.3-0.5sg).



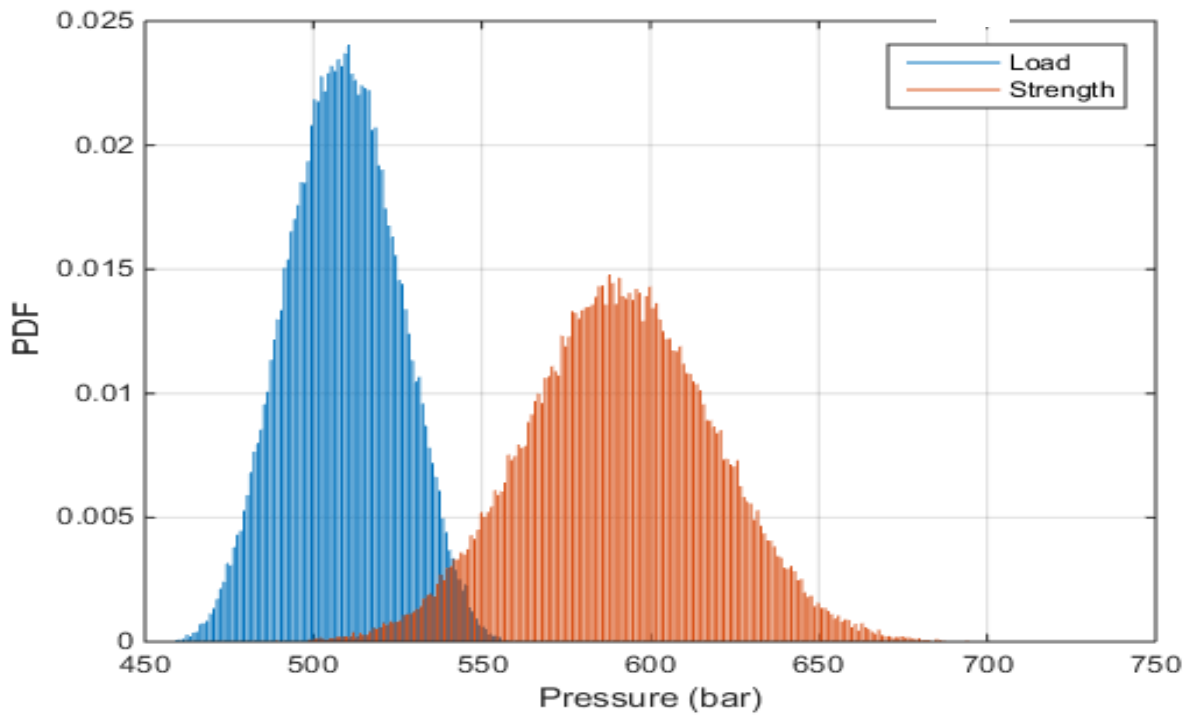
**Case 2b.**

In this case, the gas density is further reduced to give a possible maximum pressure at the wellhead. The highlighted row is also the focus of study in this case. The uncertainties in the input data for case 2b simulation are given in Table 11. Ppore is the pore pressure in sg,  $\rho_{gas}$  is the gas density in sg, T denotes the triangular distribution and N denotes the normal distribution.

The results of the simulation are presented in [Figure 30](#) and [Figure 31](#).

**Table 11:** input data and distributions for case 2b stochastic simulation

Case 2b data:	Nominal Values	Distribution type	Minimum Value	Most likely value	Maximum value	Definition of code
Pore pressure, in sg	1.6	Triangular	1.6-0.05	1.6	1.6+ 0.05	ppore=T(pp-0.05, pp,pp+0.05,1)
Gas density, Sg	0.1-0.3	Triangular	0.1	0.2	0.3	$\rho_{gas}=T(0.1,0.2,0.3,1)$
Sea water density, sg	1.03	Deterministic	-	-	-	
<b>Strength model</b>	<b>Barlow's model</b>	-	-	-	-	
Casing yield strength, psi	80000	Deterministic	-	-	-	
OD, in.	9.625	Deterministic	-	-	-	
Wall thickness, t, in.	0.472	Deterministic	-	-	-	
Burst pressure, P <sub>b</sub> , bar	$\mu=1.092$ $\sigma=0.052$	Normal				$P_b=(2*YieldStrength*t)/OD*N(1.092,0.052)$

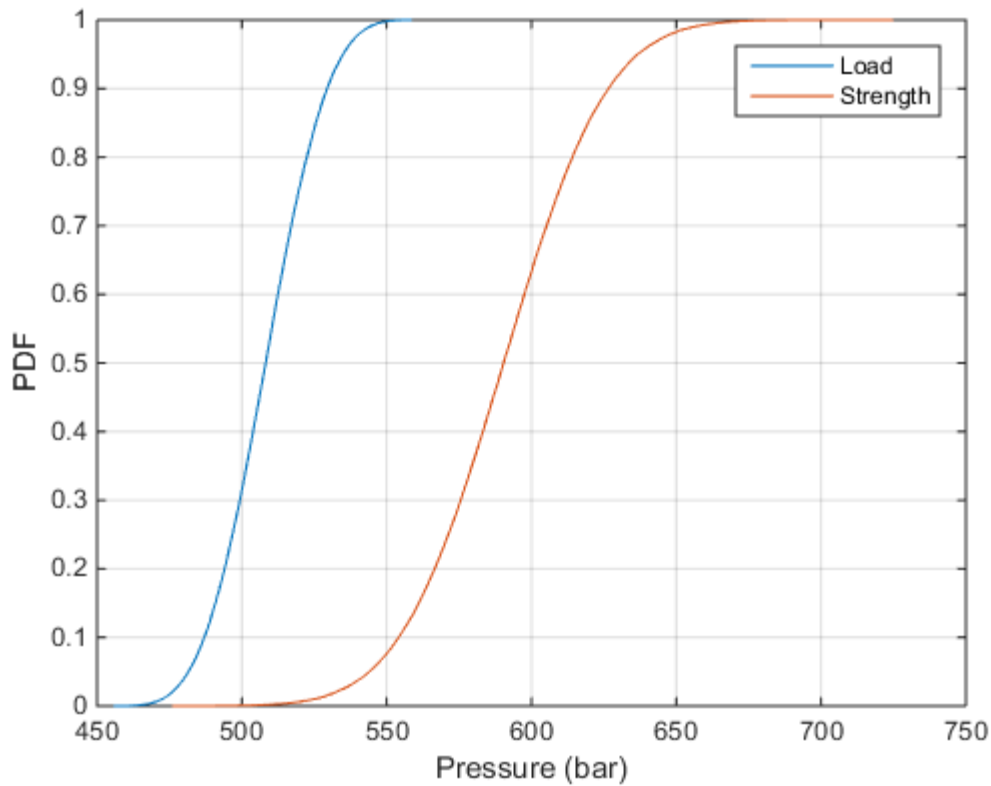


**Figure 30:** Shows a higher probability of failure compared to the previous plot of case 2a.

The probability of failure here is 0.5170%.

**Table 12:** The statistical quantities from case2b. simulation

Statistical quantities	Load Effect, bar	Casing strength, bar
Average	508.7155	590.7936
P50	508.6912	590.7696
P10	487.5093	554.7651
P90	530.0174	626.8756
P90-P10	42.5081	72.1105
Pf	0.5170%.	



**Figure 31:** shows the CDF curve for case 2b. Taking a close look at the two curves, one will notice an overlap between them at their tail ends. The gas density range is 0.1-0.3sg.

**Table 13:** Comparison of the Probabilities of failure of the different cases.

<b>Comparison of the Probabilities of failure of the different cases.</b>	
<b>Cases</b>	<b>P<sub>f</sub></b>
1	0%
2a	0.008%
2b	0.5170%

According to the comparison result of the cases in Table 13, we can conclude that case 2b with the lowest gas density have the highest value for the probability of failure. This observation establishes a well-known fact which is, the lower the gas density becomes, the higher the value of probability for failure obtained.

### **Case 3; Simulation of the strength models**

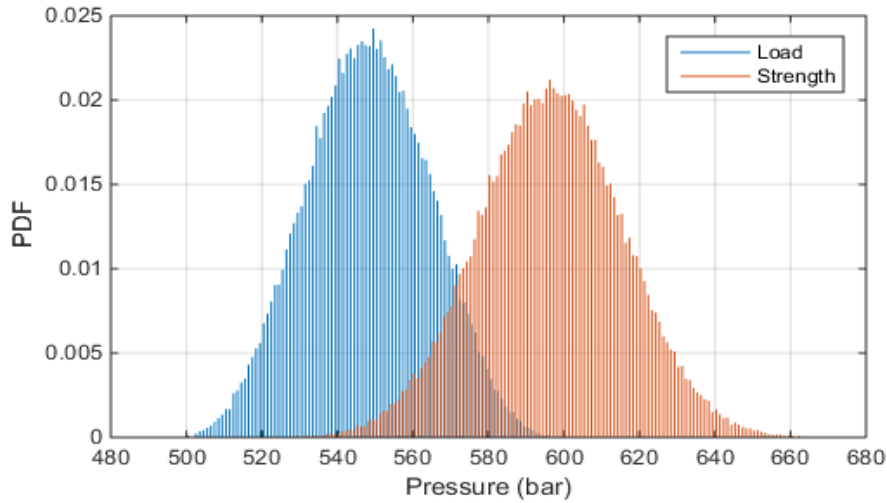
In this case, we will assume the same geometry and parameters as described in Chapter 6. The goal of this simulation is to investigate the effects of different strength models on the probability of failure in casing burst design. We begin by considering a burst scenario, where we assume that the well is filled with a gas type of kick. A pore pressure of 1.7sg is used to achieve a significant probability of failure. Each simulation is run with one strength model chosen at a time. In other words, the strength models are varied while the input data for the load and strength models are fixed. We perform the Monte Carlo simulation by using input distribution for each parameter in the load and strength model. The six different strength models used in this simulation case are the Barlow's, Von mises's, Klever Stewart's, Paslay's, Moore's and Nadai's strength models. In Klever Stewart's and Nadai's model, we use the tensile strength (ultimate strength) in place of the yield strength of the casing. We compare the results obtained from the yield strength with the results obtained from the minimum tensile strength. These results include the probability of failure,  $P_f$ , the mean and the P90-P10. However, in the ISO TR 10400<sup>[5]</sup> standard where a similar model comparison study was carried out, the minimum tensile strength is used in all the strength models, replacing the yield strength for Barlow's and Von Mises' models<sup>[5]</sup>. In Table 14, the highlighted row is the focus of study. The uncertainties in the input data for case 3 simulation are given in Table 14.  $P_{pore}$  represent the pore pressure,  $\rho_{gas}$  is the gas density, T denotes the trianglerand distribution presented as T(a,b,c), and N denotes the normal distribution presented as N( $\mu$ , $\sigma$ ). where a is the maximum value, b is most likely value, c is the minimum value,  $\mu$  is the mean and  $\sigma$  is the standard deviation.

The results of the simulation are presented in Figures 32 to 43, with their corresponding tables. Table 21 summarizes the comparison of the  $P_f$ , mean, and P90-P10 values of the different strength models.

**Table 14:** Input data and distributions for Case 3, strength model’s comparison

Case 3 data:	Nominal Values	Distribution type	Minimum value	Most likely value	Maximum Value	Definition of code
Pore pressure, in sg	1.7	Triangular	1.7-0.05	1.7	1.7+ 0.05	ppore=T (pp-0.05, pp,pp+0.05,1)
Gas density, Sg	0.1-0.3	Triangular	0.1	0.2	0.3	$\rho_{gas}=T$ (0.1,0.2,0.3,1)
<b>Strength model, P<sub>b</sub>, bar</b>	<b>Six different models</b>	-	<b>Mean, <math>\mu</math></b>	<b>Standard deviation, <math>\sigma</math></b>	-	
Casing yield strength, psi	80000	Normal	87000	2751	-	=N (87000,2751)
Casing ultimate tensile strength, psi	95000	Normal	100000	2751		=N(100000, 2751)
OD, in.	9.625	Normal	9.635	0.003	-	=N (9.635,0.003)
ID, in.	8.681	Normal	8.677	0.003	-	=N (8.677, 0.003)
Wall thickness, t, in.	0.472	Normal	0.479	0.003	-	=N (0.479,0.003)

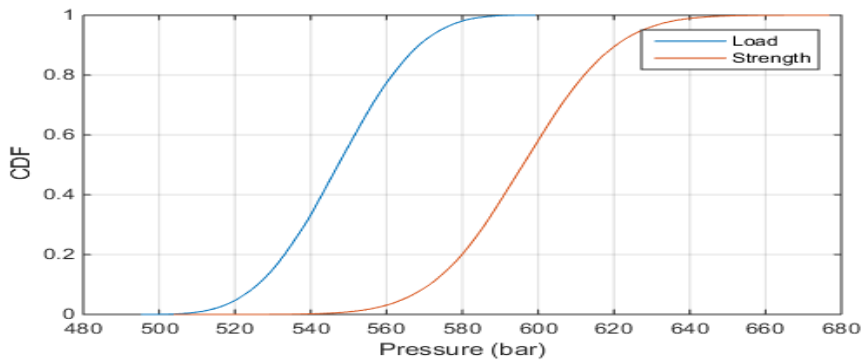
**Barlow's model PDF, CDF, and Statistical quantities;**



**Figure 32:** PDF plot from Barlow' strength model simulation.

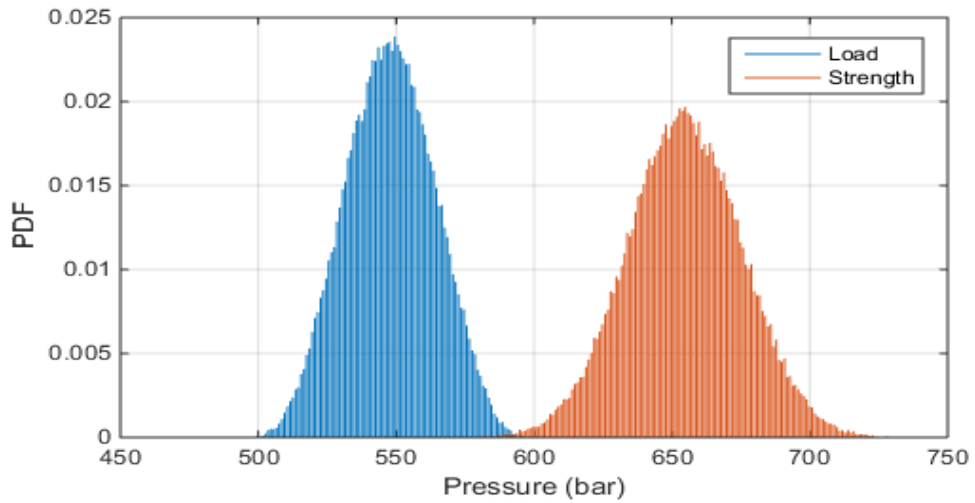
**Table 15:** The statistical quantities for Barlow's formula

Statistical quantities	Load Effect, bar	Casing strength, bar
Average	548.0053	596.6161
P50	548.0114	596.5259
P10	526.7545	572.1292
P90	569.1794	621.2472
P90-P10	42.4249	49.118
Pf, %	2.6180%	



**Figure 33:** CDF plot from Barlow' strength model.

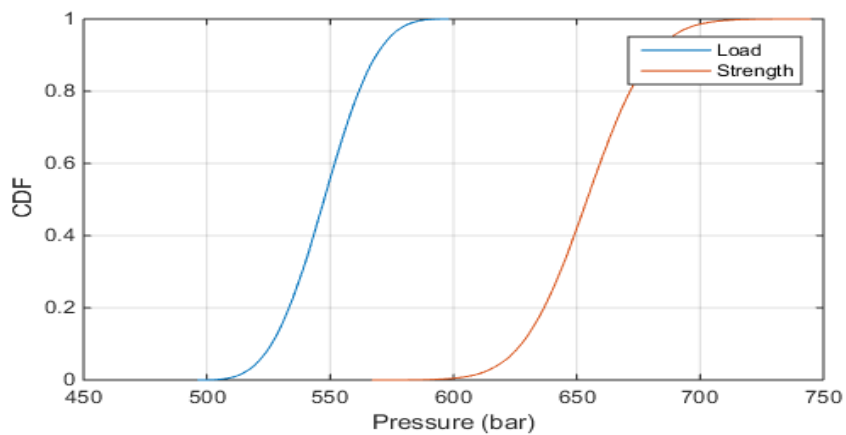
**Von Mises' model; PDF, CDF and Statistical quantities**



**Figure 34:** PDF plot from the Von Mises' strength model simulation.

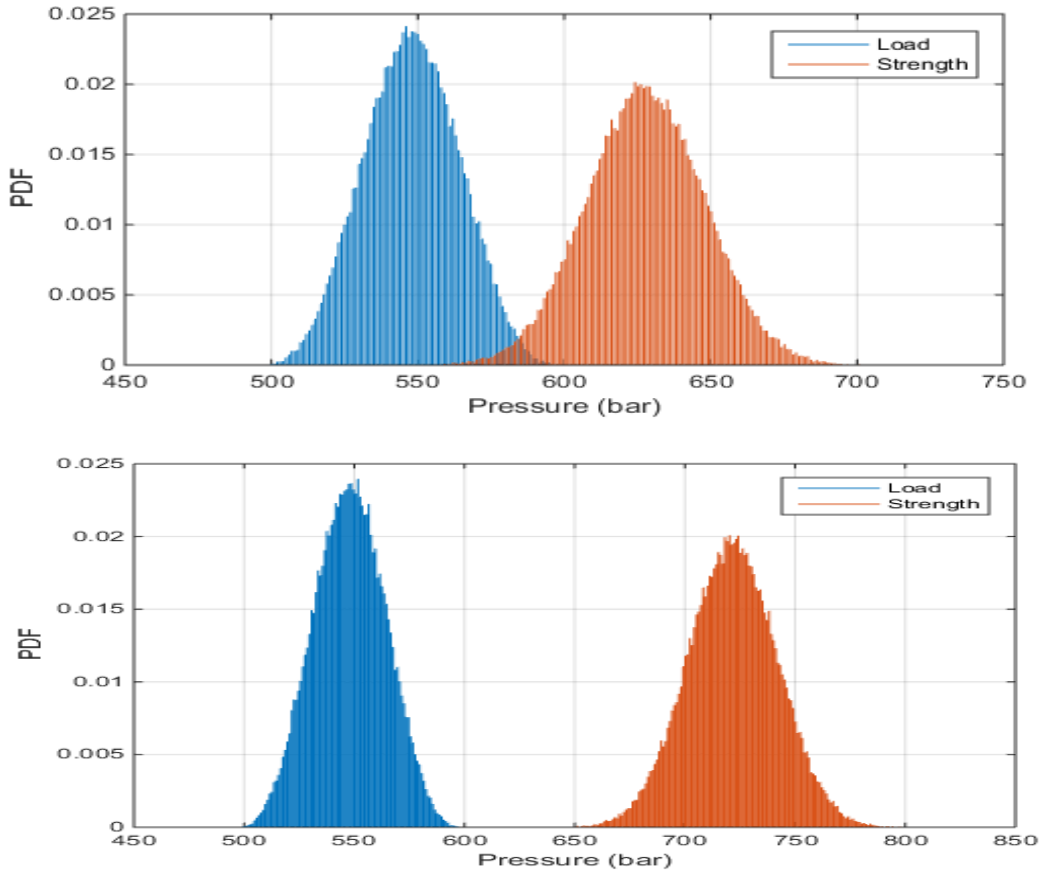
**Table 16:** The statistical quantities for Von Mises' formula.

Statistical quantities	Load, bar	Casing strength, bar
Average	547.9782	654.7405
P50	547.9584	654.8052
P10	526.6659	628.1294
P90	569.3462	681.1605
P90-P10	42.6803	53.0311
Pf, %	1.0e-03	



**Figure 35:** CDF plot from the Von Mises' strength model.

**Klever Stewart's model: PDF, CDF, and Statistical quantities**



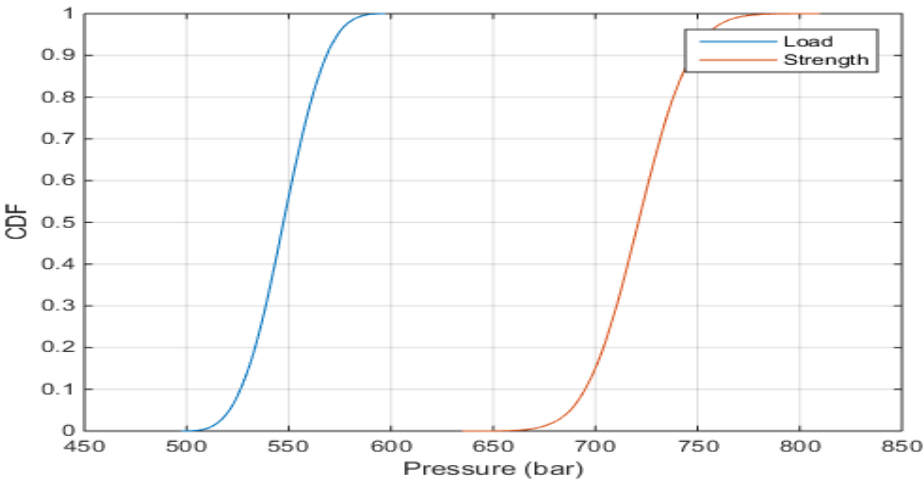
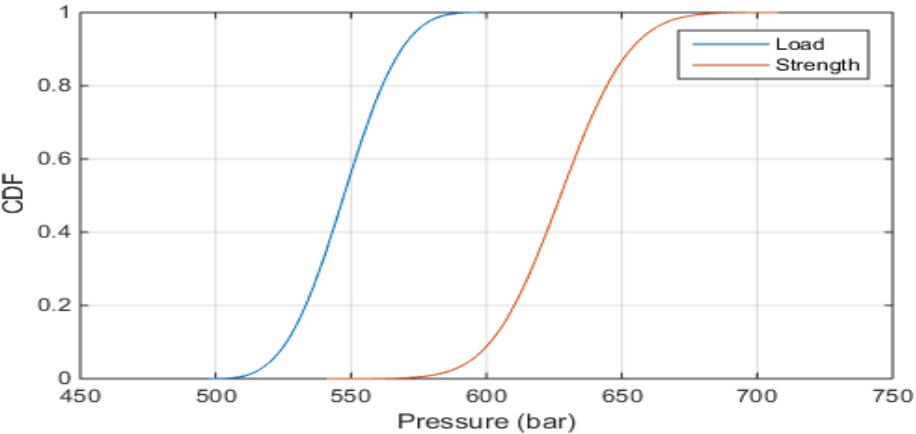
**Figure 36:** PDF plot from the Klever Stewart's strength model simulation. The yield strength is used in the top plot, while the ultimate tensile strength is used in the bottom plot.

**Table 17:** The statistical quantities for Klever Stewart's formula.

Statistical quantities	Load, bar	Casing strength, bar (Yield strength used)	Casing strength, bar (Ultimate tensile strength used)
Average	547.7749	627.8091	721.6238
P50	547.8058	627.7200	721.6215
P10	526.5347	601.8159	695.5327
P90	569.0786	653.7683	747.7836
P90-P10	42.5439	51.9524	52.2509
Pf, %	0.075%		0

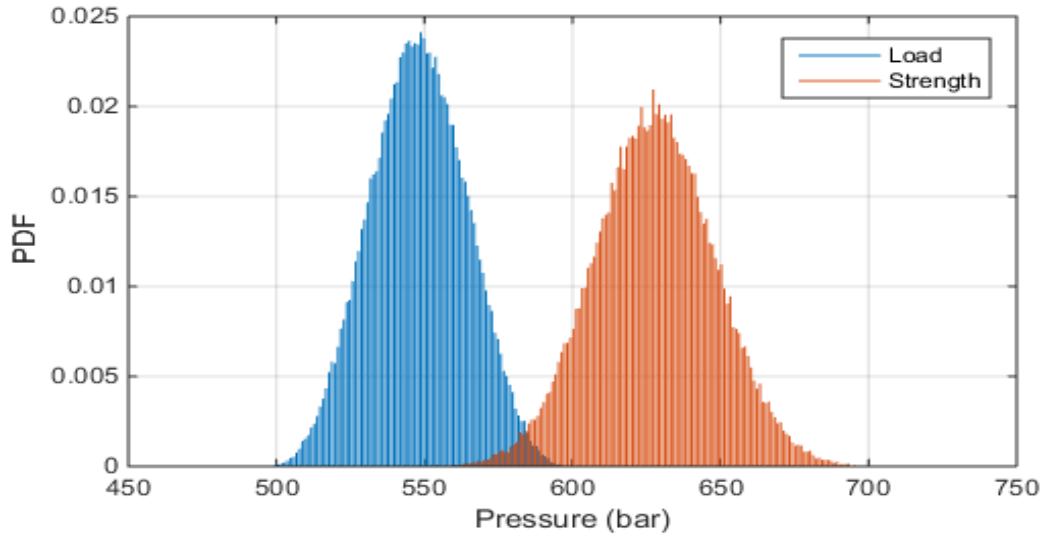


The two different probabilities of failures,  $P_f$  obtained when the yield strength and the ultimate tensile strength are used in the model shows that correct input data is crucial. There is no point in a statistical analysis if the input data does not represent the physics.



**Figure 37:** CDF plot from the Klever Stewart's strength model simulation. The yield strength is used in the top plot, while the minimum tensile strength is used in the bottom plot.

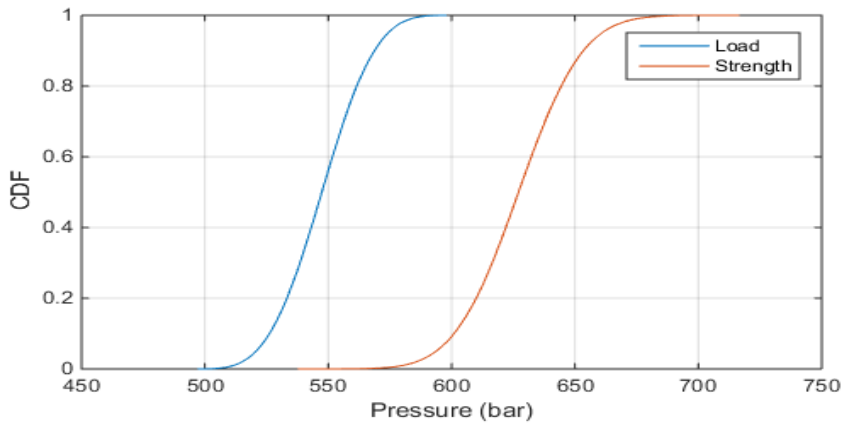
**Paslay's model; PDF, CDF and statistical quantities**



**Figure 38:** PDF plot from the Paslay's strength model simulation.

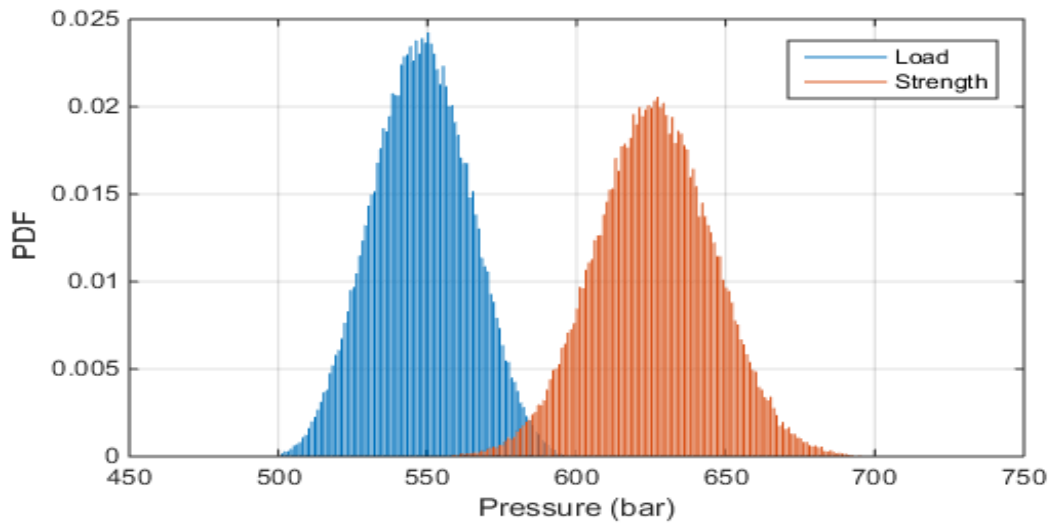
**Table 18:** The statistical quantities for Paslay's formula.

Statistical quantities	Load Effect, bar	Casing strength, bar
Average	547.8743	627.6626
P50	547.8815	627.6521
P10	526.6506	601.5845
P90	569.1015	653.7367
P90-P10	42.4509	52.1522
Pf, %	0.0860	



**Figure 39:** CDF plot from the Paslay's strength model simulation.

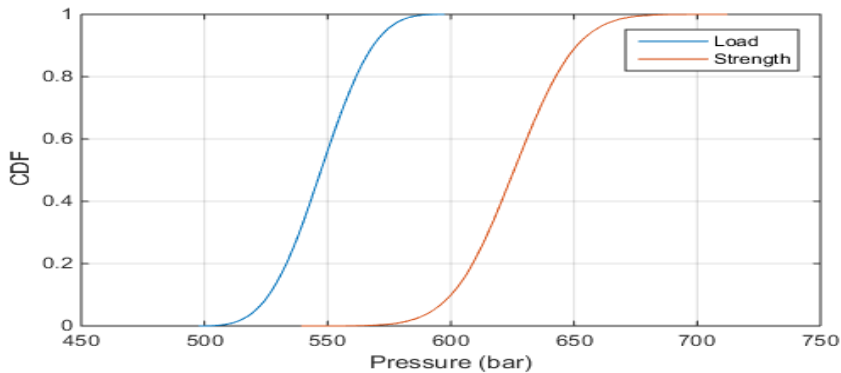
**Moore's model; PDF, CDF and statistical quantities**



**Figure 40:** PDF plot from Moore's strength model simulation.

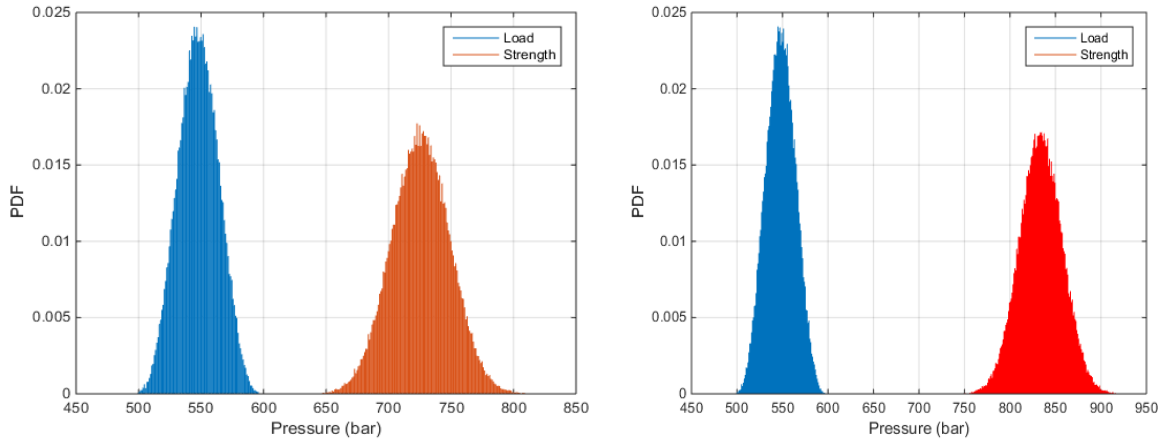
**Table 19:** The statistical quantities for Moore's formula.

Statistical quantities	Load Effect, bar	Casing strength, bar
Average	547.9385	626.1140
P50	547.9288	626.0759
P10	526.6933	600.6848
P90	569.2476	651.6184
P90-P10	42.5543	50.9336
Pf, %	0.0940	



**Figure 41:** CDF plot from Moore's strength model simulation.

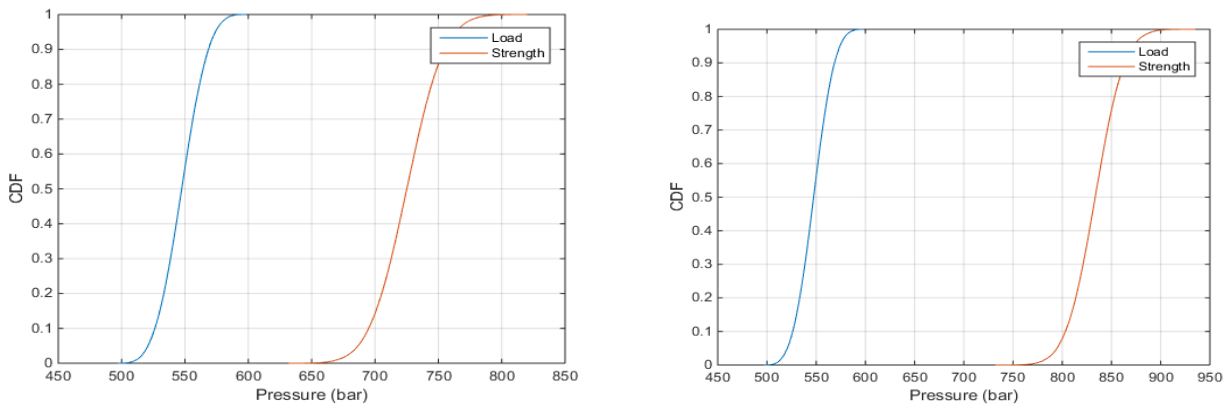
**Nadai's model PDF, CDF, and statistical quantities with yield and tensile strength;**



**Figure 42:** PDF plot from the Nadai's strength model simulation. The yield strength is used in the left plot, while the minimum tensile strength is used in the right plot.

**Table 20:** The statistical quantities for Nadai's formula.

Statistical quantities	Load, bar	Casing strength, bar (Yield strength used)	Casing strength, bar (Minimum tensile strength used)
Average	547.9377	725.7790	833.9677
P50	547.9671	725.7892	833.9244
P10	526.6855	695.7096	803.6265
P90	569.1931	755.9114	864.2773
P90-P10	42.5076	60.2018	60.6508
Pf, %	0		0



**Figure 43:** CDF plot from the Nadai's strength model simulation. The yield strength is used in the left plot, while the minimum tensile strength is used in the right plot

**Table 21:** Summarizes the result of the simulation with different strength models.

<b>Result from the simulation with different strength models</b>						
<b>Model/ input Data</b>	<b>Probabilities of failure, P<sub>f</sub> %</b>		<b>Mean(strength) Bar</b>		<b>P90-P10 (strength) Bar</b>	
	<b>Yield Strength</b>	<b>Minimum Tensile Strength</b>	<b>Yield Strength</b>	<b>Minimum Tensile Strength</b>	<b>Yield Strength</b>	<b>Minimum Tensile Strength</b>
Barlow	2.6180		596.6161		49.118	
Von mises	0.001		654.7405		53.0311	
Klever Stewart	0.075	0	627.8091	721.6238	51.9524	52.2509
Paslay	0.0860		627.6626		52.1522	
Moore	0.0940		626.1140		50.9336	
Nadai	0	0	725.7790	833.9677	60.2018	60.6508

According to ISO<sup>[5]</sup>, these different strength models can be relied on to give a fairly accurate burst pressures. Also, the accuracy of these models is dependent on being calibrated with experimental data. Based on our simulation, the probabilities for failure for the Klever Stewart, Paslay and Moore are close. Similarly, their mean and spread fall within the same range. However, their results differ from the results from other models, Barlow, Von Mises, and Nadai. Barlow gives the highest value for P<sub>f</sub>. It is observed that Nadai predicted quite large burst pressures compared to the other models. Another observation is that the high tensile pressures give high-pressure results. Therefore, one needs to be aware that the values used for the strength (yield or ultimate) will be important for the result.

## **7.2 Application of Sensitivity Analysis**

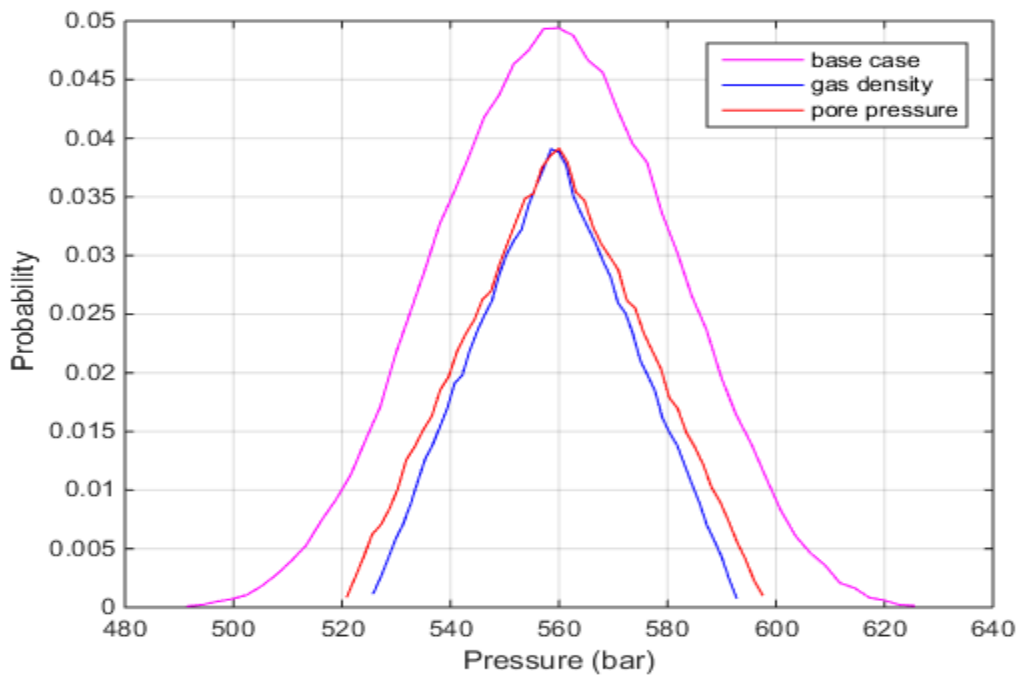
### **Application of the sensitivity analysis on the Load Model**

This sensitivity analysis is focused on the load model, where the uncertain input variables are the gas density and pore pressure. The uncertainties in these variables affect the result. The purpose of this study is to evaluate the dominating parameter in load model with respect to uncertainty. A good understanding of this study will give insight on how to reduce the uncertainty. We start by categorizing the analysis in cases, where the gas density and pore pressure are both varied in one analysis for the base case. In the second case, we analyze the degree of uncertainty of the gas density by varying the gas density while all other parameters in

the load model are fixed. In the third case, we analyze the pore pressure uncertainty by varying the pore pressure and keeping all other parameters in the load model fixed. The input data for the analysis can be seen in Table 22. Ppore represents the pore pressure. The result of this analysis is presented in Figure 44. The pore pressure is fixed at 1.6sg in the analysis of gas density uncertainty and the gas density is fixed at 0.2sg during the analysis of the pore pressure uncertainty.

**Table 22:** Data for the sensitivity analysis on load model

Case	Description
Base case	Vary pore pressure by T (1.5,1.6,1.7). Vary gas density by T (0.1,0.2,0.3)
Gas density uncertainty, sg	Pore pressure=1.6 Vary gas density by T (0.1,0.2,0.3)
Pore pressure uncertainty, sg	Vary pore pressure by T (1.5,1.6,1.7) Gas density= 0.2



**Figure 44:** Shows the effect of the parameters on the surface pressure.

The result from the sensitivity analysis in Figure 44 shows that the pore pressure has a more dominating influence on the load model compared to the gas density. Also, the pore pressure has more uncertainty compared to the gas density, although the difference is not large. Therefore, one can conclude that the dominant factor is the pore pressure in this case. An increase in the pore pressure increases the load exerted just below the wellhead. The pore pressure uncertainty can be reduced by minimizing measurement errors and geological uncertainty. Furthermore, the mean and standard deviation for each distribution curve in Figure 44 can be found in Table 23.

**Table 23:** Standard deviation for each curve in Figure 44 (load sensitivity analysis)

Case	Standard deviation	Mean	Range=max value-min value
Base case	21.3465	559.2824	136.6461=626.8241 - 490.1780
Gas density uncertainty	16.0082	559.2458	78.1599= 598.2935 - 520.1336
Pore pressure uncertainty	14.0170	559.1424	68.3271= 593.4082 - 525.0811

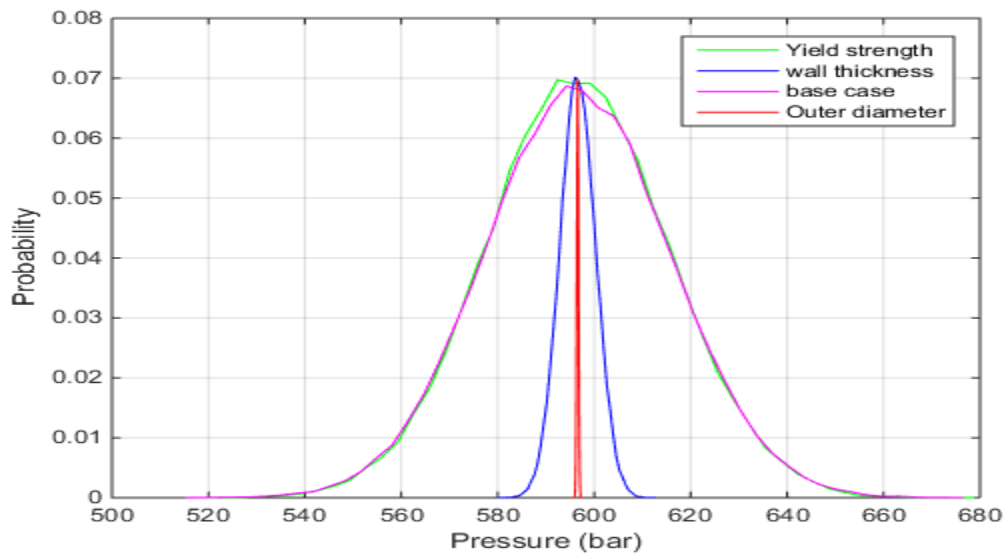
This is a simple example used to demonstrate how to perform the sensitivity analysis and how it can be graphically represented.

### **Application of the sensitivity analysis on the Strength Model**

Here, we focus the Sensitivity analysis on the strength model, where the uncertain variables are the outer diameter, wall thickness and the yield strength of the casing. In order to understand the dominating parameter in the strength model, we vary one parameter at a time while fixing the other parameters as in Table 24. The result from the analysis is presented in Figure 45.

**Table 24:** Summarizes the steps for the strength sensitivity analysis

Case	Description
Base case	Vary OD by N (9.635, 0.003) Vary thickness by N (0.479, 0.003) Vary Yield strength by (87000, 2751)
Outer diameter uncertainty	Vary OD by N (9.635, 0.003) t= 0.479 Yield Strength= 87,000
Wall thickness uncertainty	OD= 9.635 Vary thickness by N (0.479, 0.003) Yield Strength= 87,000
Yield strength uncertainty	OD= 9.635 t= 0.479 Vary Yield strength by (87000, 2751)



**Figure 45:** Shows the effect of the parameters in Strength.



The result shows that the outer diameter has the least uncertainty while the yield strength has the most uncertainty. Therefore, the dominating parameter in the strength model with respect to uncertainty is the yield strength. In fact, the result in Figure 45 indicates that the yield strength uncertainty contributes significantly to the uncertainty in the strength model. The yield strength is estimated by tensile testing of a casing. The uncertainties can be minimized by reducing tensile testing uncertainties or errors during testing. The mean and standard deviation for each parameter in the strength model is summarized in Table 25.

**Table 25:** Standard deviation for each curve in Figure 45 (strength sensitivity analysis)

Case	Standard deviation	Mean
Base case	19.1857	596.5160
Outer diameter uncertainty	0.1861	596.5751
Wall thickness uncertainty	3.7400	596.5788
Yield strength uncertainty	18.8266	596.5438

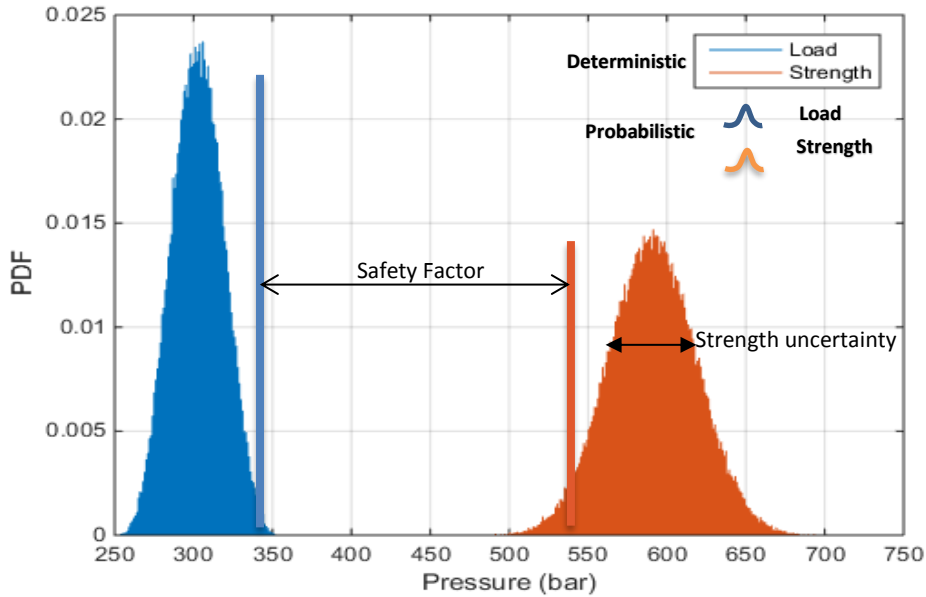
### 7.3 Compare Working Stress Design to Reliability based Approach, RBD

Here, we compare the probabilistic data of the case 1 with its deterministic values. The purpose of doing this is to demonstrate the strictness or approximation of the Working Stress design method, WSD in casing design. In WSD, the maximum pressure is assumed using the lowest density of the fluid. This maximum pressure is then multiplied by the safety factor to obtain a value for the casing design strength. In casing selection, all casings having a nominal strength higher than this value would be acceptable. The safety factor does not provide any information about the reliability or risk related to the design. For instance, reducing the value of the safety factor leads to higher risk in design, but the higher risk cannot be quantified. However, RBD quantifies risk, reliability, and failure using the probability value. As previously discussed, it considers all uncertainties and risk in the design.

Figure 46 shows the RBD approach versus the WSD approach. There is no probability for casing failure in this case. In Figure 47, RBD approach shows a probability of casing failure. In contrast,

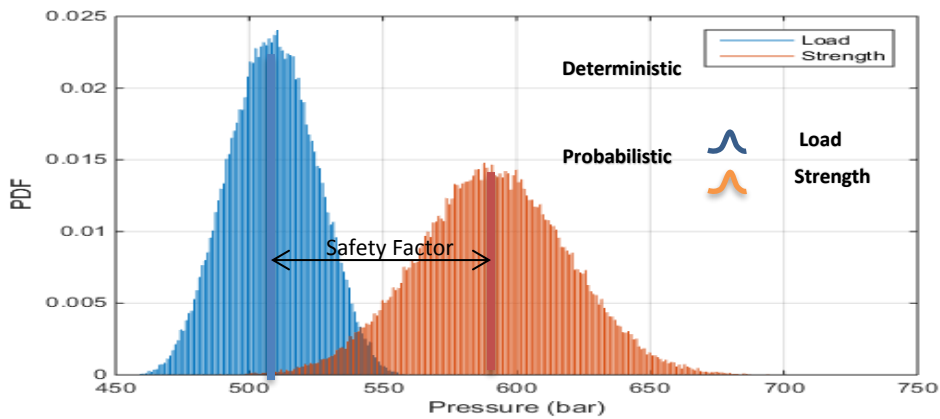
the WSD gives no information about the risk. To demonstrate this, the mean values for the strength and load are 590 bars and 500 bars respectively. The SF would be  $\frac{590}{500} = 1.18$ . This value for the safety factor would be acceptable.

**Case 1 (base case) result**



**Figure 46:** Comparison between the probabilistic Pressures and Deterministic Pressures of Case1 load and strength. Oil density =0.7-0.9sg, Pore pressure = 1.6sg

**Case 2b. result**



**Figure 47:** Comparison between the probabilistic Pressures and Deterministic Pressures of Case2b load and strength. The overlap indicates casing failure. Gas density= 0.1-0.3sg, Pore pressure=1.6sg

#### **7.4 Limitations / Drawbacks of Monte Carlo Simulation in application of Reliability Based Design**

H.S Williamson et. al[30]. addressed some pitfalls with the Monte Carlo simulation, MCS. The popularity of MCS application in proffering solutions to complex problems in the oil & gas industry is growing tremendously. However, this technique can be quite complex and require thoroughness in its application. The drawbacks of the MCS in relation to the RBD approach are listed below:

- MCS requires support from a large set of experimental data, (the input data) to quantify the uncertainty especially due to the assumption that the accurate input data is not known.
- If the limit state equation and load calculations are very complex, MCS simulation can become time-consuming.
- The number of simulations must be high in order to get reliable results.

# Chapter 8

---

## **8.1 Conclusion and Recommendations**

### **8.1.1 Conclusion**

The objective of this study was to demonstrate how to perform the reliability based design (RBD) for casing and obtain results for further analysis or decision making. We also addressed the advantages and limitations of the reliability based design relative to the classical Working Stress Design for a casing design. The burst scenario was used to demonstrate this method. The reliability method accounted for variation in field loads and casing strength to measure the relationship between the load and probability of failure. We demonstrated this method by developing the probability distributions for loads and strengths, then performing Monte Carlo simulations. The probability of failure is then quantified from the load and strength distributions as well as the mean and spread(P90-P10). More so, we evaluated the two casing design approaches with regards to their merits and limitations; to determine the best in terms of understanding and reducing the uncertainty in the design. Based on this study, the conclusion is that the RBD is the most robust and economical design when the statistics (such as the mean and standard deviation) of the input random variables are well defined. It also gives an excellent understanding of the uncertainties and the risk involved in a design. The reliability-based approach provides a theoretical basis for taking the uncertainties into account in a complete decision scheme and to assess a tubular's ability to stay safe and functional during their lifecycle<sup>[4]</sup>. We also believe that this design method gives a more accurate design and is less conservative (resistant to change or strict) compared to the classical Working Stress Design, WSD.

Therefore, a probabilistic estimate of load and strength accounts for the uncertainties in the design, that is, the input variables and the result. It made it possible to create a relationship between the load and a probability of failure. In this way, the approach provides a likely burst or failure pressure instead of a worst possible burst pressure given by WSD.

A probabilistic approach in Well design and development is a new technology. Based on this study, the author highly recommends its application in casing design as the number of costs and technically demanding wells are increasing. Mainly because it addresses the uncertainties in the design of a process or material (for e.g. Casing). It also gives a better understanding of casing performance properties. It provides a proper risk-based decision making for optimal design.

During this study, some observations were made regarding the casing RBD method that is important to mention. Firstly, limit state equations are important for the design and they must be calibrated with experimental data. Secondly, a certain number of Monte Carlo simulations are required to obtain an accurate result. In general,  $10^{-x}$  reliability requires a  $10^{x+2}$  number of

Monte Carlo simulations<sup>[11]</sup>. If Monte Carlo number of simulation is low, it affects the  $P_f$  obtained such that it varies at different Monte Carlo runs. Thirdly, it is important to assume 100% that the kick will occur to avoid underestimating the load design.

The challenges encountered in this study mostly had to do with data gathering. It might be a challenge for an engineer to know what kind of data to use for the RBD calculations. In this study, the standard deviation,  $\sigma$  and mean,  $\mu$  of the casing was used in the simulation. Also, the  $\sigma$  and  $\mu$  of a casing grade differ from the nominal values which are obtained empirically, making data gathering a challenge for this design method. In SPE 178907<sup>[8]</sup>, they state that strength parameter data must be collected from API TR 5C3(ISO), manufacturer's report or direct material and dimensional measurements. The API TR 5C3 may be too complex to study prior to a casing design and may not have all data required for RBD analysis. In our simulation, we solved the problem of incomplete data by assuming the mean value of the casing minimum tensile strength to be 100,000psi and the standard deviation to be 2751psi as seen in Table 6. These values were assumed based on the values of the mean and standard deviation of the casing yield strength. In addition, RBD approach requires mathematical understanding and may be too complex for an engineer to apply.

The probability of failure is used as a safety margin to determine casing failure. Its function is similar to the function of the design factor used in the traditional casing design except that it gives a realistic representation of the design. The author recommends regulatory bodies to look into defining acceptable failure probabilities for design, categorizing it differently for high consequence and low consequence failures. Such that one can choose a desired target probability of failure before design. Based on the result obtained in cases 1,2a and 2b, it establishes the known fact that lower gas density gives a higher probability of failure.

For ultra-deep, high-pressure-high-temperature areas and new fields, where safety factors are yet to be established, the application of RBD in casing design is highly recommended to achieve a robust design. In high pressure-high temperature areas, the magnitude of pressure resulting from a kick is expected to be high. Therefore, it is important to reduce the uncertainties in pore pressure estimation in order to reduce the uncertainties in surface pressure.

The methodology presented in this study allows the designer to understand the unknown and risk inherent in the design.

A sensitivity analysis was conducted to determine the input parameters that contribute most to the variability in the output. The pore pressure contributes most to the uncertainty in the load model, although the difference to the gas density is not much. This understanding will help to reduce the pore pressure uncertainties by minimizing geological uncertainties. In the strength model sensitivity

analysis, the result shows that the outer diameter has the least uncertainty while the yield strength has the most uncertainty. Therefore, the dominating parameter in the strength model with respect to uncertainty is the yield strength. The yield strength is estimated by tensile testing of a casing. The strength uncertainties can be minimized by reducing tensile testing uncertainties, reducing errors during testing by improving inspection and quality assurance and control. This will lead to a reduced probability of failure.

### **8.1.2      Recommendations**

1. In order to improve the reliability based design for casing, adequate and quality statistical data for all casings to be used in a well construction should be provided to develop load and strength distributions.
2. The reliability-based casing design presented here can be extended to collapse, biaxial and triaxial analysis by use of Monte Carlo simulation or other random variable generation methods.
3. Factors derating casing can be correctly quantified so that they can also be accurately included in the RBD design process.
4. More studies should be directed toward establishing acceptable failure probabilities for design. It is recommended that the regulatory bodies establish the acceptable failure probabilities.
5. During this study, it was clear that it takes time to get familiar with the RBD methodology and associated standards. To adopt this approach, it will require proper education and provision of possible simplified steps in documentation on how to use it.

## **REFERENCES**

1. Hand, M. *Day rates for drilling rigs to fall further in 2016: ABN AMRO*. Seatrade Maritime News, 28th January 2016.
2. *Alibaba.com*. [cited 13-02-2017; Available from: [www.alibaba.com/showroom/casing.html](http://www.alibaba.com/showroom/casing.html)].
3. Mike L. Payne, J.D.S., *Application of Probabilistic Reliability Methods to Tubular Designs*. Society of Petroleum Engineering Conference, December 1990. 5(04).
4. Emilio Bastidas-Arteaga, A.-H.S., *Reliability Analysis Methods*. February 2015, HAL.
5. ISO, *ISO/TR 10400*, in *Petroleum and natural gas industries Equations and calculations for the properties of the casing, tubing, drill pipe and line pipe used as casing or tubing*. 2007, ISO: Switzerland. p. 21-28, 95,154.
6. Saleh J.H, a.M.K., *Reliability Engineering and System Safety*. Highlights from the Early (and pre-) History of Reliability Engineering. Vol. 91. February 2006.
7. Blade Energy Partners, L., *Reliability of Annular Pressure Buildup (APB) Mitigation Techniques*, in *RPSEA Final Technical Report*, D.U.B.S. (UB), Editor. Feb 2016: United States of America. p. 25.
8. P.V Suryanarayana, D.B.L., Blade Energy Partners, *A Reliability-Based Approach for Survival Design in Deepwater and High Pressure/High Temperature Wells*, in *IADC/SPE Drilling Conference and Exhibition*. 2016, Society of Petroleum Engineers: Fort Worth, Texas. SPE-178907-MS
9. K.C. Gulati, M.R.D.C., McKenna, Mobil Producing Nigeria; M.A. Maes, University of Calgary; and R.C Johnson, P.R Brand, D. B. Lewis, Lynda Riekels, and R.E Maute, Mobil R&D Corp., *Reliability-based Design and Application of Drilling Tubulars*, in *Offshore Technology Conference*. 1994, Offshore Technology Conference: Houston, Texas.
10. O'Conner, P., *Practical Reliability Engineering*. 2012.
11. Bernt S. Aadnoy, I.C., Stefan Z. Miska, Robert F. Mitchell, and Michael L. Payne, *Advanced Drilling and Well Technology*. 2009, United States of America: Society of Petroleum Engineers. 1-81.
12. American Petroleum Institute, *Technical Report on Equations and Calculations or Casing ,Tubing and Line Pipe used as Casing or Tubing; and Performance Properties Tables for Casing and Tubing*, in *ISO 10400: 2007(Modified), Petroleum, petrochemical and natural gas industries- Equations and calculations for the properties of casing, tubing, drill pipe and line pipe used as casing or tubing*. December 2008, API publishing services: Washington DC, United States of America.
13. Aadnoy, B.S., *Modern Well Design*. 2010, Netherlands: CRC Press/Balkema.
14. Prassl, W.F., *Drilling Engineering*. p. 161.
15. Elisa Lage Modesto Alcofra, A.O.N., Antonio Carlos Vieira Martins Lage, *Annular Pressure Build-up in oil wells*, in *15th Brazilian Congress of Thermal Sciences and Engineering*. 2014, ABCM: Belem, PA, Brazil.

16. Standards Norway, *NORSOK Standard D-010, Well integrity in drilling and well operations*. June 2013, Standards Norway: Norway.
17. Institut Francais du Petrole, *Drilling Data Handbook*. Seventh Editions ed. 1999, Paris: EDITIONS TECHNIP.
18. American Petroleum Institute, *Bulletin on Formulas and Calculations for Casing, Tubing, Drill Pipe, and Pipeline Properties*. 1994, API: Washington D.C.
19. Fjelde, K.K., *Directional drilling and Flow Well Engineering*, in *Pet 505*, U.o. Stavanger, Editor. Autumn 2015.
20. Jain, D.D.K. *RCC design*. November 2016 [11th May 2017]; Available from: <https://www.slideshare.net/DineshJain25/comparison-between-working-stress-method-and-limit-state-method-of-design-of-rcc-structures>.
21. F.Moeinikia, K.K.F., A Saasen, T.Vralstad, O. Arild, *A Probabilistic Methodology to evaluate the cost efficiency of Rigless Technology for Subsea Multiwell Abandonment*, in *IADC/SPE Drilling Conference and Exhibition*. 2014, SPE: Fort Worth, Texas U.S.A. p. 2,.
22. James B. Raney, M.A.M., P. V Suryanarayana,, *A Comparison of Deterministic and Reliability-Based Design Methodologies for Production Tubing*, in *SPE Applied Technology Workshop on Risk Based Design of Well Casing and Tubing*. 1997/1998, Society of Petroleum Engineers: The Woodlands, Texas.
23. Institute of Electrical and Electronics Engineers, *A Compilation of IEEE Standard Computer Glossaries*, in *IEEE Standard Computer Dictionary*. 1990: New York, NY.
24. Fjelde, K.K., *Monte Carlo Simulations and Applications*, in *Pet 530*, University of Stavanger, Editor. 2016: Stavanger, Norway.
25. Du, X., *Probabilistic Engineering Design*, in *Reliability Analysis Method*. September 2005: Missouri, Rolla.
26. Clinedinst, W.O., *Analysis of API collapse test data and development of new collapse resistance formulas*. 1977.
27. Bradley, W.B., *The Effect of Casing Wear on the Burst Strength of Casing: Part 2- Statistical Burst Strength of Worn and Unworn Casing Strings*, in *Annual Petroleum Mechanical Engineering Conference*. September 1975, ASME: Tulsa.
28. Bratvold B. Reidar, S.B., *Making Good Decisions*. December 2009, Society of Petroleum Engineers: United States of America. p. 215.
29. Kullawan, K., *Risk based cost and Duration Estimation of Well Operations*, in *Petroleum Engineering*. Spring 2012, University of Stavanger: UiS.
30. H.S Williamson, S., S.J Sawaryn, SPE, J.W. Morrison, *BP Monte Carlo Techniques Applied to Well Forecasting: Some Pitfalls*, in *SPE Annual Technical Conference and Exhibition*. September 2006, Society of Petroleum Engineers: Houston.
31. *Geo530, Decision Analysis and Geostatics* Spring 2014.
32. Gouri K., B., Richard A. Johnson, John Wiley & Sons, *Statistical Concepts and Methods*. 1977.
33. A. Saltelli, K.C., E.M. Scott, *Sensitivity Analysis*. 1 ed. March 16, 2009.
34. Zhu Xiaoyan, K.W., *Important Measures in Reliability, Risk and Optimization*. April 2012, John Wiley & Sons, Incorporated. p. 474.



## Appendix a: The Matlab Codes

### The function file

```
function f = trianglerand(xstart,mostlik,xstop,N)
% TRIANGLERAND Random numbers from a triangle distribution.
% R = trianglerand(min,mostlikely,max,N) returns a vector of N draws from a
% triangular distribution starting at min, maxpoint at mostlikely and endpoint at
max.
%
% Copyright 2003 RF - Rogaland Research
% Author: Øystein Arild
a = mostlik-xstart;
b = xstop-xstart;
h1 = 2/a;
m1 = h1/a;
A1 = a/b;
p = A1;
f_ = (rand(N,1) < p);
ind1 = find(f_==1);
ind2 = find(f_==0);
N1 = length(ind1);
if (a == b)
u = rand(N,1);
f = sqrt(2*m1*u)/m1;
else
u = rand(N1,1);
f1 = sqrt(2*m1*u)/m1;
h2 = 2/(b-a);
m2 = -h2/(b-a);
beq=h2;
u = rand(N-N1,1);
f2 = a+(-beq+sqrt(beq*beq+2*m2*u))/m2;
f(ind1) = f1;
f(ind2) = f2;
f = f';
end
f = f + xstart;
end
```

## Case 1 codes

main.m

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500;% in meters, also relative to RKB
pp = 1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
YieldStrength = 5517 % This is the yield strength of casing in bar
OD = 9.625 % this is the casing outer diameter in inches
t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
% OD = normrnd(9.635, 0.003);
% t= normrnd(0.479, 0.003);
% YieldStrength= normrnd(6000, 189.72);% versus the 87000psi and
2751psi, mean and standard deviation.
% ID = normrnd(8.681,0.003);% in inch,
% Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.7,0.8,0.9,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability
for
% the event to happen. When the gas density is equaled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
```

```

% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('PDF')

% Commands for picking out statistic quantites
% Example:
average = mean(psurf(1,:))
percentile50=median(psurf(1,:))
percentile10=prctile(psurf(1,:),10)
percentile90=prctile(psurf(1,:),90)

% average = mean(pburststrength(1,:))
% percentile50=median(pburststrength(1,:))
% percentile10=prctile(pburststrength(1,:),10)
% percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Case 2a codes

```
clc
clear
N = 100000; % Number of Montecarlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500; % in meters, also relative to RKB
pp = 1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
YieldStrength = 5517 % This is the yield strength of casing in bar
OD = 9.625 % this is the casing outer diameter in inches
t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
% OD = normrnd(9.635, 0.003);
% t= normrnd(0.479, 0.003);
% YieldStrength= normrnd(6000, 189.72);% versus the 87000psi and
2751psi, mean and standard deviation.
% ID = normrnd(8.681,0.003);% in inch,
% Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.2,0.3,0.5,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen.when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```

```

% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Case 2b codes

```
clc
clear
N = 100000; % Number of Montecarlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500; % in meters, also relative to RKB
pp = 1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
YieldStrength = 5517 % This is the yield strength of casing in bar
OD = 9.625 % this is the casing outer diameter in inches
t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
% OD = normrnd(9.635, 0.003);
% t= normrnd(0.479, 0.003);
% YieldStrength= normrnd(6000, 189.72); % versus the 87000psi and
2751psi, mean and standard deviation.
% ID = normrnd(8.681,0.003); % in inch,
% Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```



```

% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% Example:
average = mean(psurf(1,:))
percentile50=median(psurf(1,:))
percentile10=prctile(psurf(1,:),10)
percentile90=prctile(psurf(1,:),90)

% average = mean(pburststrength(1,:))
% percentile50=median(pburststrength(1,:))
% percentile10=prctile(pburststrength(1,:),10)
% percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

### Case 3 codes

#### Simulating with Barlow's Model

```
clc
clear
N = 100000; % Number of Montecarlosimulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500;% in meters, also relative to RKB
pp = 1.7 %1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
% YieldStrength = 5517 % This is the yield strength of casing in bar
% OD = 9.625 % this is the casing outer diameter in inches
% t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
OD = normrnd(9.635, 0.003);
t= normrnd(0.479, 0.003);
YieldStrength= normrnd(6000, 189.72);% versus the 87000psi and 2751psi,
mean and standard deviation.
ID = normrnd(8.677,0.003);% in inch,
Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probablity for
% the event to happen.when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
```

```

% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% % Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Simulating with Von Mises' Model

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500; % in meters, also relative to RKB
pp = 1.7 %1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
% YieldStrength = 5517 % This is the yield strength of casing in bar
% OD = 9.625 % this is the casing outer diameter in inches
% t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
OD = normrnd(9.635, 0.003);
t= normrnd(0.479, 0.003);
YieldStrength= normrnd(6000, 189.72); % versus the 87000psi and 2751psi,
mean and standard deviation.
ID = normrnd(8.677,0.003); % in inch,
Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); % Von
mises strength equation
% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```

```

% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% % Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```



## Simulating with Klever Stewart's Model

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500;% in meters, also relative to RKB
pp = 1.7 %1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
% YieldStrength = 5517 % This is the yield strength of casing in bar
% OD = 9.625 % this is the casing outer diameter in inches
% t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
OD = normrnd(9.635, 0.003);
t= normrnd(0.479, 0.003);
YieldStrength= normrnd(6000, 189.72);% versus the 87000psi and 2751psi,
mean and standard deviation.
Ultimateyield strength= normrnd (6896,189.72);
ID = normrnd(8.677,0.003);% in inch,
Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen.when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```

```

% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% % Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Simulating with Paslay's Model

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500; % in meters, also relative to RKB
pp = 1.7 %1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
% YieldStrength = 5517 % This is the yield strength of casing in bar
% OD = 9.625 % this is the casing outer diameter in inches
% t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
OD = normrnd(9.635, 0.003);
t= normrnd(0.479, 0.003);
YieldStrength= normrnd(6000, 189.72); % versus the 87000psi and 2751psi,
mean and standard deviation.
ID = normrnd(8.677,0.003); % in inch,
Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```

```

    pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% % Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Simulating with Moore's Model

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500; % in meters, also relative to RKB
pp = 1.7 %1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
% YieldStrength = 5517 % This is the yield strength of casing in bar
% OD = 9.625 % this is the casing outer diameter in inches
% t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
OD = normrnd(9.635, 0.003);
t= normrnd(0.479, 0.003);
YieldStrength= normrnd(6000, 189.72); % versus the 87000psi and 2751psi,
mean and standard deviation.
ID = normrnd(8.677,0.003); % in inch,
Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```

```

% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
    pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
% pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3));
% Nadai strength equation

% Below is an alternative code where we calculate the burst pressure
% and included that there is only a certain probability for the event
to
% happen. In this case 10 % change

% on = (rand<0.9);

% if (on==1)
%     ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%     gasdensity = trianglerand(0.2,0.3,0.4,1);
%     psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
% else
%     psurf(1,j)=1;
% end

% here we count number of times the strength pressure is exceeded
if(psurf(1,j)>pburststrength(1,j))
    counter=counter+1;
end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```



```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% % Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Simulating with Nadai's Model

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500;% in meters, also relative to RKB
pp = 1.7 %1.6 % Assume most likely value of pore pressure
deltappore = 0.05 % Uncertainty band to be used in Triangle distribution
% YieldStrength = 5517 % This is the yield strength of casing in bar
% OD = 9.625 % this is the casing outer diameter in inches
% t = 0.472 % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);
pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop
% Codes for strength models simulation, defined in the loop
OD = normrnd(9.635, 0.003);
t= normrnd(0.479, 0.003);
YieldStrength= normrnd(6000, 189.72);% versus the 87000psi and 2751psi,
mean and standard deviation.
Ultimateyield strength= normrnd (6896,189.72);
ID = normrnd(8.677,0.003);% in inch,
Kdr = 1 % the correction factor for pipe deformation

ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
gasdensity = trianglerand(0.1,0.2,0.3,1); % Better to assume variability
in
% the average kick fluid density in well than assuming a probability for
% the event to happen.when the gas density is equalled to the range of
0.7-0.8sg,
% then it is the oil density

psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
psurf(1,j)=psurf(1,j)-0.0981*1.03*seawaterdepth;
psurf(1,j)=psurf(1,j);

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

% CODE USED FOR STRENGTH MODEL COMPARISON
% pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(sqrt(3)*OD^2); %
Von mises strength equation
% pburststrength(1,j)= (2*Kdr*YieldStrength*t)/(OD-t); % Klever-Stewart
strength equation
```

```

% pburststrength(1,j)= (2*YieldStrength*t)/(OD-t); % Paslay strength
equation
% pburststrength(1,j)= (YieldStrength*(OD^2-ID^2))/(OD^2+ID^2); % Moore
strength equation
    pburststrength(1,j)= (2*YieldStrength*log((OD)/(OD-2*t)))/(sqrt(3)); %
Nadai strength equation

    % Below is an alternative code where we calculate the burst pressure
    % and included that there is only a certain probability for the event
to
    % happen. In this case 10 % change

%   on = (rand<0.9);

%   if (on==1)
%       ppore=trianglerand(pp-0.05,pp,pp+0.05,1);
%       gasdensity = trianglerand(0.2,0.3,0.4,1);
%       psurf(1,j)=ppore*welldepth*0.0981-0.0981*welldepth*gasdensity;
%   else
%       psurf(1,j)=1;
%   end

% here we count number of times the strength pressure is exceeded
    if(psurf(1,j)>pburststrength(1,j))
        counter=counter+1;
    end

end % End of MonteCarlo loop

prob=counter/N*100; % percentage for having load pressure > strength

% Plot probability density functions
e=min(pburststrength(1,:));
f=max(pburststrength(1,:));
s=[e:1:f];
[c,d]=hist(pburststrength(1,:),s);

h=min(psurf(1,:));
f=max(psurf(1,:));
w=[h:1:f];

[a,b]=hist(psurf(1,:),w);

plot(b,a/N,d,c/N);
legend('Load','Strength')
xlabel('Pressure (bar)')
ylabel('PDF')

```

```

% CODE FOR PLOTTING CUMULATIVE DISTRIBUTIONS

% plot(b,cumsum(a)/sum(a),d,cumsum(c)/sum(c));
% legend('Load','Strength')
% xlabel('Pressure (bar)')
% ylabel('CDF')

% Commands for picking out statistic quantites
% % Example:
% average = mean(psurf(1,:))
% percentile50=median(psurf(1,:))
% percentile10=prctile(psurf(1,:),10)
% percentile90=prctile(psurf(1,:),90)

average = mean(pburststrength(1,:))
percentile50=median(pburststrength(1,:))
percentile10=prctile(pburststrength(1,:),10)
percentile90=prctile(pburststrength(1,:),90)

% Note the difference P90-P10 can be used as
% a measure of the uncertainty in the results.

```

## Codes for the Load sensitivity analysis

```
clc
clear
N = 100000; % Number of Monte Carlo simulations

well depth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawater depth = 500;% in meters, also relative to RKB
deltappore = 0.05; % Uncertainty band to be used in Triangle distribution
YieldStrength = 5517; % This is the yield strength of casing in bar
OD = 9.625; % this is the casing outer diameter in inches
t = 0.472; % this is the minimum or average casing wall thickness in
inches

psurf=zeros(1,N);

pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% inner pressure larger that burst strength.

for j = 1:N % Start of MonteCarlo loop

    %ppore=trianglerand(1.5,1.6,1.7,1);
    ppore = 1.6;

    gasdensity = trianglerand(0.1,0.2,0.3,1);
    %gasdensity = 0.2;

    psurf(1,j)=ppore*welldepth*0.0981-0.0981*(welldepth-
seawaterdepth)*gasdensity;
    P2(j) = psurf(j);% This code is used to develop individual plots for
the base case, variable porepressure case and the variable gas density
case
% P(j)= psurf(j);
% P1(j)=psurf(j);
end

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

prob=counter/N*100; % percentage for having load pressure > strength
% M=mean(P)% Code used for calculating mean
% M1= mean(P1)
M2=mean(P2)

% S=std(P)% Code used for calculating standard deviation
% S1=std(P1)
S2=std(P2)
```

```

K=min(P2)% code used to calculate minimum and maximum values, where
Range=Max-Min
K=max(P2)

% P(min)=min(P)
% P1(min)=min(P1)
% P2(min)=min(P2)

% Plot probability density functions1,:)); % for sensitivity analysis

% After developing your P, P1, and P2 curves individually, (develop and
save from workspace individually).
% To produce a combined plot, open the folder you saved P, P1 and P2 to,
Select all, P, P1 and P2,
% right click and select load.
% Then copy this part to command window and run.

[a,b] = hist(P,50);
[a1,b1] = hist(P1,50);
[a2,b2] = hist(P2,50);

bar(b,a/100000, 'g');
hold on
bar(b1,a1/100000, 'r');
bar(b2,a2/100000, 'b');
hold off
xlabel('Pressure (bar)'); ylabel('Probability')

```

## Codes for Strength sensitivity analysis

```
clc
clear
N = 100000; % Number of Montecarlo simulations

wellddepth = 4000; % This is planned TD of next section to be drilled,
relative RKB in meters
seawaterdepth = 500; % in metres, also relative to RKB
deltappore = 0.05; % Uncertainty band to be used in Triangle distribtion

psurf=zeros(1,N);

pburststrength = zeros(1,N);

counter=0; % Variable that is used to calculate the percentage of having
% innerpressure larger that burststrength.

for j = 1:N % Start of MonteCarlo loop

% OD = normrnd(9.635, 0.003);
OD= 9.635;

% t= normrnd(0.479, 0.003);
t= 0.479;

YieldStrength= normrnd(6000, 189.72);
% YieldStrength= 6000;

pburststrength(1,j)=(2*YieldStrength*t)/OD; % In psi, Barlow Equation
% P(j)= pburststrength(j); % simulating the basecase
% P1(j)= pburststrength(j); % simulating the outer diameter uncertainty
% P2(j)= pburststrength(j); % simulating wall thickness uncertainty
P3(j)= pburststrength(j); % simulating Yield strength uncertainty
end

% pburststrength(1,j)=(2*YieldStrength*t)/OD*normrnd(1.092,0.052); % In
psi, Barlow Equation

prob=counter/N*100; % percentage for having load pressure > strength
% M=mean(P) % Code used for calculating mean
% M1= mean(P1)
% M2=mean(P2)
% M3=mean(P3)

% S=std(P) % Code used for calculating standard deviation
% S1=std(P1)
% S2=std(P2)
```

```

% S3=std(P3)
% K=min(P2)% code used to calculate minmum and maximum values, where
Range=Max-Min
% K=max(P2)

% P(min)=min(P)
% P1(min)=min(P1)
% P2(min)=min(P2)

% Plot probability density functions1,:); % for sensitivity analysis

% After developing your P,P1,and P2 curves individually, (develop and
save from workspace individually).
% To produce a combined plot, open the folder you saved P,P1 and P2 to,
Select all, P,P1 and P2,
% right click and select load.
% Then copy this part to command window and run.

[a,b] = hist(P,50);
[a1,b1] = hist(P1,50);
[a2,b2] = hist(P2,50);
[a3,b3] = hist(P3,50);

bar(b,a/100000, 'm');
hold on
bar(b1,a1/100000, 'r');
bar(b2,a2/100000, 'b');
bar(b3,a3/100000, 'g');
hold off
xlabel('Pressure (bar)'); ylabel('Probability')
legend ('base case','Outer diameter','wall thickness','Yield strength')

```