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Catch, Stock Elasticity, and an Implicit Index of Fishing Effort

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Abstract *Economists are interested in the relationship between fishing effort and stock size and their impact on catch levels. The interest lies in stock elasticity, where it is thought that for pelagic fish species it is close to zero; for demersal fish stocks, closer to one. We statistically model and estimate the relationship between stock size and catch for two species, Northeast Arctic cod and saithe. In doing so we are able to recover estimates of stock elasticity but also estimates of catchability coefficients for different age classes and importantly an implicit index of fishing effort. Data on observed catch and a measure of biomass-at-age are available from the International Council for the Exploration of the Sea. The generated stock data are econometrically problematic, and we use an IV estimator with bootstrapping in estimation. Time-series techniques applied to panel data are used to statistically motivate the estimation, which is carried out within a two-way panel framework.*

Key words Stock elasticity, fishing effort, age class, catchability coefficients.

JEL Codes C72, Q22.

Introduction

Economists have long been interested in the relationship between fishing effort and stock size and their impact on catch levels (Hannesson 1993).[1] The primary interest lies in stock elasticity with respect to catch. Where, on one hand, it is thought that for pelagic fish species stock elasticity is close to zero (Ulltang 1980; Butterworth 1981; Bjørndal 1987). The idea here is that pelagic species form schools and have a lumpy distribution in the sea. In such cases, once a school of fish has been targeted, the actual stock of the species is moot to the harvest/profit process; thus, the stock elasticity should be close to zero (Hannesson 1983; Flaaten 1987). On the other hand, for demersal fish stocks, stock elasticity is thought to be closer to one (Schaefer 1957). The idea here is that demersal species in the fishing area are thought to be somewhat evenly dispersed over the sea floor and well modeled using a uniform spatial distribution. In this case, an increase in stock size will proportionately increase the density of the stock and catch per unit of effort (Sandberg 2006). In application, the actual size of the stock elasticity for either pelagic or demersal species is an empirical question.

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The purpose of this article is to statistically model and estimate the relationship between stock size and catch for two species, Northeast Arctic saithe (*Pollachius virens*) and Northeast Arctic cod (*Gadus morhua*). Both species are demersal, but saithe, a member of the cod family, can occasionally behave like pelagic species in that it congregates in schools. We speculate that cod should show a stock elasticity close to one and saithe less than one. In doing so we are able to recover estimates of stock elasticity but also estimates of catchability coefficients for different age classes and importantly an implicit index of fishing effort.[2] Fishing effort is difficult to model directly because it is a combination of different vessel types, gear types, vintage of capital equipment, and so on.[3] In practice, economists have used proxy variables for fishing effort, typically vessel numbers as a ratio of days at sea, which introduces serious econometric problems of endogeneity and inconsistent coefficients (Gordon 2013). Our empirical research implicitly recovers an index of fishing effort based on actual catch data and estimates of stock size.

Biologists are interested in measures of fishing effort that would be directly proportional to the mortality it generates in the stock (Jennings, Kaiser, and Reynolds 2001). If fishing effort is directly proportional to mortality, catch per unit effort could be used as an index of stock size. But this depends on an assumption of uniform spatial distribution of the fish stock within the fishing area. Under this assumption, a unit of fishing effort would always remove a given fraction of fish in the stock.[4] However, if the distribution were lumpy, a unit of fishing effort would always remove a given quantity of fish and thus an increasing fraction of fish in the stock.[5]

Catchability coefficients by year class are of interest because they are a measure of gear efficiency and related to gear selectivity (May 1984). The probability of a fish being caught at different age levels depends on both biological factors (availability, behaviour, size and shape of the fish, season, environment, other fish species, etc.) and technological factors (gear type, gear position, management skill, etc.) (Jul-Larsen *et al.* 2003). Catchability coefficients are really a composite factor where 'fish catchability' primarily implies changes in fish behaviour (May 1984), whereas 'fishing efficiency' indicates changes in fishing practices or relative fishing power (Neis *et al.* 1999). Estimates of catchability coefficients by age class are recovered that provide information on gear selectivity in fishing the different year classes.

In application we use an interesting data set made available from the International Council for the Exploration of the Sea (ICES).[6] The data generated using Virtual Population Analysis (VPA) is interesting in that it is rich in its coverage of catch, mortality, and stock size information, but mortality and stock size are *generated* variables based on biological assumptions, actual catch levels, and assumed decay functions. Moreover, the generated regressors are endogenous, and a least squares estimator produces inconsistent and inefficient estimates. In application, an instrumental variable and bootstrapping techniques are used to address this issue (Pagan 1984; Zhang and Smith 2011). For the problem at hand we collect data for two species cod and saithe. For these species the data are organized in a panel setting. For cod we have age classes 3–13 for a 35-year period (1977–2011), giving a total cross-section time-series data set of 385 observations. For saithe we have age classes 3–15 for a 35-year period (1977–2011), giving a total cross-section time-series data set of 455 observations. Each data set is balanced in a panel setting. Time-series techniques applied to panel data are used to statistical-

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[2] Our measure of fishing effort is narrowly defined in terms of removing fish from the sea. The activity of fishing, of course, is a complicated one requiring the use of many factor inputs (Squires 1987). As fish stocks are depleted and catch per unit of effort decreases, additional economic inputs would be required to maintain fishing effort.

[3] See Squires (1987) for a detailed examination of fishing effort. Also, see Kirkely, Squires, and Strand (1998).

[4] Of course, the numbers of fish must decline with each unit of fishing effort.

[5] See Gudmundsson (1994) and Fournier and Archibald (1982) for an interesting discussion on using catch-at-age data.

[6] Hannesson (2013) uses ICES data in estimating a recruitment equation.

ly motivate the estimation, which is carried out within a two-way panel framework. We are careful to account for clustered residuals allowing for both heteroskedasticity and autocorrelation within the panels. The estimated equations are validated based on residual analysis and robustness checks. It is important to emphasize that our estimated results for stock elasticity, catchability coefficients, and fishing effort are a function of the data-generating process and environmental design of VPA procedures.

The article is organized as follows. Given the ICES data and the econometric issues that arise, we start by providing a detailed description of the data-generating process and our methodology for dealing with empirical problems. Next, based on early work by Hannesson (1993) a simple model of catch rates and stock size is presented. The model, although straightforward, captures the bioeconomic relationship between catch and stock size and allows direct specification of the econometric panel equation. Then we provide a brief summary description of the data used in empirical work and detail the econometric equation estimated. Following this, we describe the empirical research strategy and present results. The final section concludes.

Data Issues and Empirical Methodology

The VPA method for generating stock data is somewhat involved and worth outlining in detail. It is also important to note econometric modelling issues. In general, the procedure for predicting stock in the previous period relies on biological assumptions and back forecasting based on current and previous period actual catch levels.[7] To make this clear, define catch in each period according to time period t, ($t = 1,2,\cdots,35$), age class a, (for cod $a = 3,4,\cdots,11$ and for saithe $a = 3,4,\cdots,15$), and cohort c, ($c = -9,-8,\cdots,35$). (We define cohort 1 as age class three in time period 1.) So, catch in period t, age class a, and cohort c is written, $C^t_{a,c}$. Of particular importance in VPA methods is the cohort. Using cod as the example, table 1 sets up the panel framework for catch levels by age class and time. (Notice age class 13 in time period 1 is cohort –9.)

Table 1
Catch Levels

	Time Period				
Age	1	2	3	⋯	35
3	$\boldsymbol{C^1_{3,1}}$	$C^2_{3,2}$	$C^3_{3,3}$	⋯	$C^{35}_{3,35}$
4	$C^1_{4,0}$	$\boldsymbol{C^2_{4,1}}$	$C^3_{4,2}$		
5	$C^1_{5,-1}$		$\boldsymbol{C^3_{5,1}}$		
⋮	⋮			⋱	
13	$C^1_{13,-9}$	⋯			$C^{35}_{13,25}$

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[7] VPA uses assumptions on biological parameters and some current stock survey data to predict an initial stock level.

The columns define the catch distribution by age class for a given year. The rows define the catch levels at age for different cohorts over the time period studied. The diagonal elements define catch level of cohorts over time. VPA analysis uses the diagonal elements, cohorts, in order to back forecast stock level. For example, to define stock levels in period 1, age class 3 cohort 1, we need catch levels in period 1 and 2 for cohort 1 or $\hat{S}^1_{3,1} = S^1_{3,1}(C^1_{3,1}, C^2_{4,1})$ or in general $\hat{S}^t_{a,c} = S^t_{a,c}(C^t_{a,c}, C^{t+1}_{a+1,c})$.

The generated stock data is arranged as in table 2. The important point to emphasize in this table is that the diagonal elements; *i.e.*, generated stock values following a cohort are biologically and mathematically linked by VPA procedures, and an econometric equation following the cohort would merely approximate the deterministic VPA decay equations. Note that for any given age class (*i.e.*, a row), stock values are not linked by common catch variables. So, for each element in a row the stock estimate represents an independent (*i.e.*, based on different catch values) draw from the data-generating process. Of course, all stock estimates are subject to the VPA framework of analysis as are all econometric parameters recovered in estimation. Subject to this caveat, our empirical approach is to use the generated stock data based on known catch levels to recover the parameters of interest in our study. To be clear, the data will be organized by age-class panels. For example, panel one defines the data for age class 3 as reported in table 3.

Table 2
Stock Estimates

	Time Period			
Age	1	2	3	⋯
3	$\boldsymbol{S^1_{3,1}(C^1_{3,1}, C^2_{4,1})}$	$S^2_{3,2}(C^2_{3,2}, C^3_{4,2})$	$S^3_{3,3}(C^3_{3,3}, C^4_{4,3})$	
4	$S^1_{4,0}(C^1_{4,0}, C^2_{4,0})$	$\boldsymbol{S^2_{4,1}(C^2_{4,1}, C^3_{5,1})}$	$S^3_{4,2}(C^3_{4,2}, C^4_{5,2})$	
5	$S^1_{5,-1}(C^1_{5,-1}, C^2_{4,-1})$		$\boldsymbol{S^3_{5,1}(C^3_{5,1}, C^4_{6,1})}$	
⋮				

Table 3
Data Structure

Time	Catch	Stock
1	$C^1_{3,1}$	$S^1_{3,1}(C^1_{3,1}, C^2_{4,1})$
2	$C^2_{3,2}$	$S^2_{3,2}(C^2_{3,2}, C^3_{4,2})$
⋮	⋮	⋮
35	$C^{35}_{3,35}$	$S^{35}_{3,35}(C^{35}_{3,35}, C^{36}_{4,35})$

By writing the data as in table 3, it is clear that generated VPA stock estimates must be correlated with the error term in a regression equation of catch on stock because current stock is a function of current catch. To achieve consistent estimates of the econometric equation we need to instrument out current catch in the VPA stock estimate. Table 2 provides us with some insight into a possible instrument. For instance in period 2 age class 4 of table 2, predicted stock is written $S^2_{4,1}(C^2_{4,1}, C^3_{5,1})$, but notice that the argument $C^2_{4,1}$ appears also as an argument in predicting stock in period 1 age class 3 $S^1_{3,1}(C^1_{3,1}, C^2_{4,1})$. Implicitly solving we observe that $C^2_{4,1} = S^{-1}{}^1_{3,1}(C^1_{3,1}, S^1_{3,1})$, which implies correlation between the cohort lagged catch variable and current catch and provides an exogenous instrument to address the correlation problem identified in table 3. (Appendix A derives the inverted function.) Note that lagged stock also appears in the inverted function, but it is not a valid instrument.[8] It might be argued that the cohort lagged catch variable merely reflects common shocks in the VPA stock-generating process, but this ignores the importance of exogenous and independent current shocks to catch levels based on say, current surface weather conditions impacting current fishing effort and independent of VPA stock estimates. Consequently, for each stock estimate we instrument out $C^t_{a,c}$ using $C^{t-1}_{a-1,c}$.

We have identified a valid exogenous instrument in $C^{t-1}_{a-1,c}$ that is structurally correlated with current stock, impacts current catch only through its influence on current stock, and is subject to independent shocks. However, if we view the endogeneity problem as measurement error, an additional instrument is available. The VPA procedure for generating stock is an approximation and subject to measurement error; thus correlation between stock and the regression error term. It is common in empirical practice to use the rank order $(RO^t_{a,c})$ of the stock variable as an instrument to avoid the correlation problem. The argument is that the rank order is correlated with stock but not correlated with measurement error. This is true as long as the measurement error is not strong enough to change the rank order. Accepting the rank order as an exogenous instrument, we proceed to estimation using both $C^{t-1}_{a-1,c}$ and $RO^t_{a,c}$ in the first-stage regression for predicting the instrumental variable for generated stock. To be complete, the instrumental variable for stock is the predicted values from the first-stage regression written generally as $\hat{S}^t_a = \beta C^{t-1}_{a-1} + \gamma C^{t+1}_{a+1} + RO^t_{a,} + \alpha_{a,} + \delta t + \vartheta^t_a$, where α_a is the age-class fixed effect, t dummy outs time shocks, and ϑ^t_a is a random error term. The second-stage estimation (the catch regression) uses the predicted values in place of VPA stock estimates, and bootstrapping techniques are used to approximate efficient standard errors.

The Model

We follow Hannesson (1993) in setting up a simple, direct relationship amongst catch, fishing effort, and stock size. To do this we simply write catch (per unit effort proportional to stock size or:

$$\frac{C}{E} = qS, \tag{1}$$

where q is a catchability coefficient, a parameter expressing the vulnerability of the fish to gear selectivity. If the catch per unit effort is less than proportional to S, we need to modify equation (1) as:

$$\frac{C}{E} = qS^b, \tag{2}$$

where parameter b is the stock elasticity with respect to catch per unit effort.[9]

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[8] The reason for this is that lagged stock is a function of lagged catch by VPA procedures, and this violates the conditions for a valid instrument (see Angrist and Pischke (2009) pp. 64-68). Angrist and Pischke detail the issue, but intuitively lagged catch is the chosen instrument and introducing lagged stock, a function of lagged catch, adds no new information to the IV estimator.

[9] See Steinshamn (2011).

Rewriting equation (2) we are able to write catch as a function of fishing effort and stock size or:

$$C = qES^b. \tag{3}$$

Although simple and straightforward, equation (3) allows for a clear, direct test of stock elasticity where the null of $H_O{:}b = 0$ defines a pelagic stock and an indirect test of a lumpy distribution of fish density, and the null of $H_O{:}b = 1$ defines a demersal stock and an indirect test of a uniform spatial distribution of fish density. In anticipation of the empirical results, we expect saithe to have a measured stock elasticity between zero and one, indicating some schooling behaviour patterns for the species; whereas cod is expected to have a measured stock elasticity statistically near one.

In anticipation of the age-structured time-series data available for estimation, we rewrite equation (3) by age structure and time period, and introduce a random error term identified for each panel and time period as:

$$C_{a,i}^t = q_{a,i} E_i^t {S_{a,i}^t}^b e^{\varepsilon_{a,i}^t}, \tag{4}$$

where $q_{a,i}$ is the catchability coefficient for age class a, species i; E_i^t is an index of fishing effort for species i in year t; ${S_{a,i}^t}^b$ is the age specific biomass for species i, age class a, in year t. Biologically the parameter b is restricted between zero and one, but we maintain this as a testable parameter in estimation. Finally, $\varepsilon_{a,i}^t$ is the idiosyncratic error term for species i, age class a, in year t. The error term captures all additional factors impacting catch levels.

Note that in equation (4) we assume an elasticity of effort equal to one. Although this is a common assumption in biological work (see Schaefer 1954), in application this may not be true, in which case our measure of the implicit index of effort will also include a measure of elasticity. Given the identification demands for sorting out catchability and effort variables in equation (4), we are not able to identify a specific measure for the elasticity of effort.

Data and Empirical Equation

ICES (2012) published catch and biomass-at-age for a number of fish stocks in the North Atlantic. Biomass-at-age estimates are based on virtual population analysis. In this article, we are interested in the Northeast Arctic cod and saithe stocks. There is some question as to the accuracy of the estimates of the ICES data[10] prior to the 1970s, and this, combined with the establishment of the exclusive economic zones for both Norway and the Soviet Union in 1976, set our decision to start data selection in 1977. For Northeast Arctic cod we have 3–13 age classes over a 35-year period (1977–2011), and for saithe we have 3–15 age classes over the same time period.

To provide a flavour for the variation in the data, figures 1a and 1b graph landings and total biomass for the period 1977–2011 for cod and saithe, respectively. Both cod and saithe landings are the total landings for all age classes. For cod, figure 1a, prior to 1990 although a total quota was agreed on between Norway and Soviet Union exceptions were allowed and enforcement inconsistent, resulting in a high level of landings and reduced biomass (Bergland and Pedersen 2000). After 1990 TAC levels were enforced and Norway imposed vessel quotas. Figure 1b shows a somewhat different evolution of catch

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[10] See, Jennings, Kaiser, and Reynolds (2001).

and biomass for saithe over the period. Saithe does not have the high-value status of cod, but the fishery was regulated in a similar manner. From time to time we observe peaks of increased catch level while the stock size declines. This is obvious around 1990 when catches increased for a time while stock size declined. The 1990 incident is probably a result of compensating for the poor cod fishery at that time.

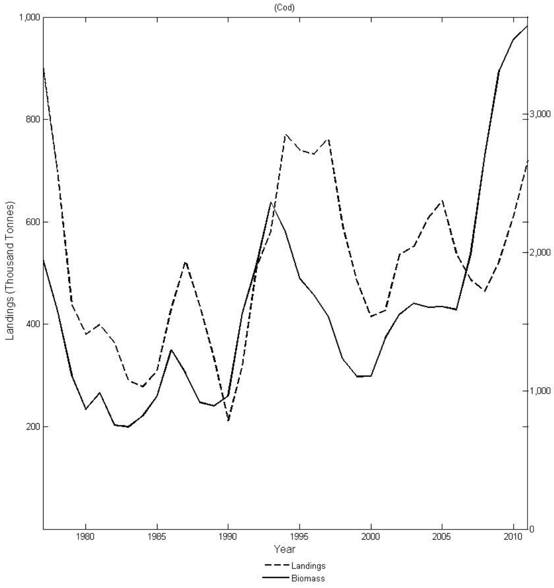

Figure 1a. Cod Landings and Total Biomass, 1977–2011

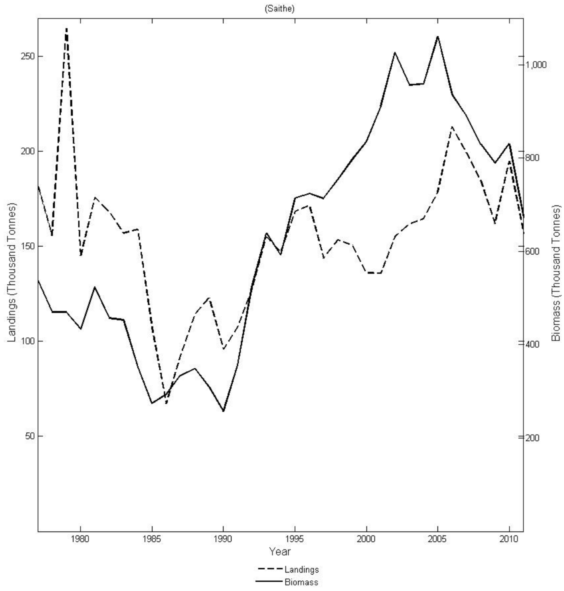

Figure 1b. Saithe Landings and Total Biomass, 1977–2011

Figure 2 offers a different perspective and shows catch at age for both cod and saithe. Here we observe that for cod age classes 5–7 dominate catches in terms of biomass, but for saithe we see that age class 4 is by far the most dominant. Finally, to emphasize the richness of the data, figures 3a and 3b show catches for cod and saithe, respectively, plotted against biomass for four age classes. For cod age class 3 we see low levels of catch at all biomass levels, whereas for age classes 5 and 6 we observe high variation in catch levels at different biomass levels. For age class 12 notice the very low stock values and, again, high correlation between landings and stock levels. For saithe we observe high variation in catch at all age classes and all stock levels, except age class 12.

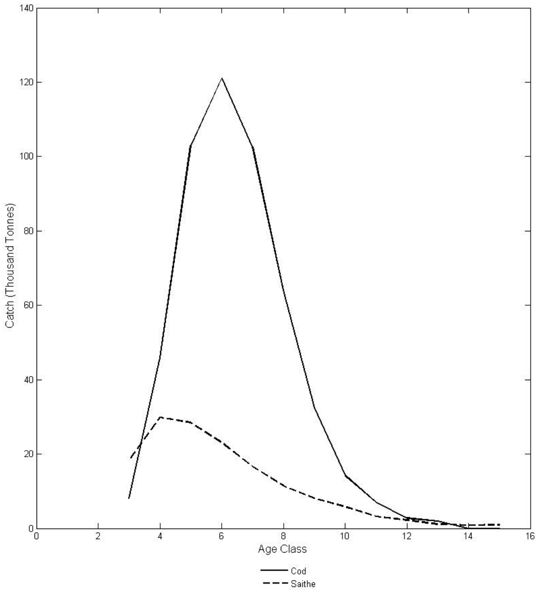

Figure 2. Catch by Age Class: Northeast Arctic Cod and Saithe, Average Values

The panel data structure of the data set is crucial to the econometric identification of equation (4). To address the identification issue write equation (4) taking logs of both sides or:

$$\ln\left(C_{a,i}^{t}\right) = \ln\left(q_{a,i}\right) + \ln\left(E_{i}^{t}\right) + b \cdot \ln\left(S_{a,i}^{t}\right) + \varepsilon_{a,i}^{t}. \tag{5}$$

With cross-section data only, we would not be able to identify fishing effort, whereas time-series data alone would not allow identification of the catchability coefficient by age class. However, the panel structure will allow identification and estimation of both catchability coefficients for each species and an implicit index of fishing effort over time. For estimation we use both a within estimator and for a robustness check a two-way fixed-effect estimator, allowing binary variables on age to recover catchability coefficients and binary variables on year to recover an implicit index of fishing effort. The base period is age class 3 in 1977.

As a practical matter, good estimates of equation (5) require good data variation. With panel data, variation can occur within the panel and between the panels. Table 4 reports this variation for catch and stock size for each species. For both species, with the exception of the within variation in catch for saithe, we measure more variation both within and between for stock size compared to catch; between variation dominates within variation for both species.

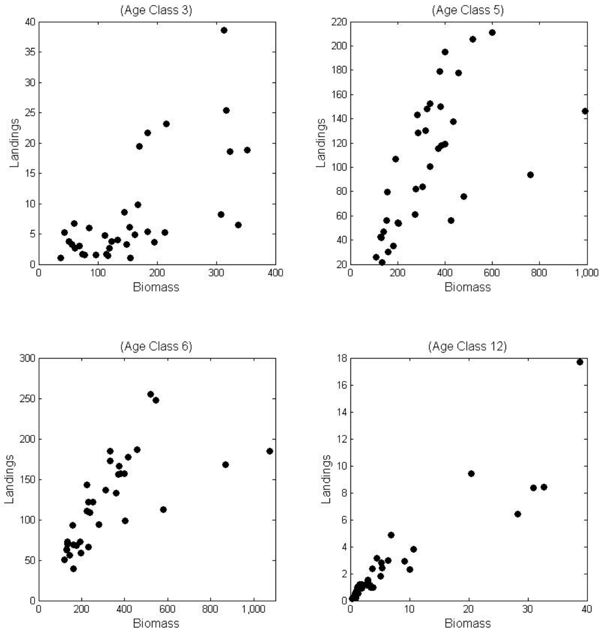

Figure 3a. Cod Landings Plotted Against Biomass (thousand tonnes)

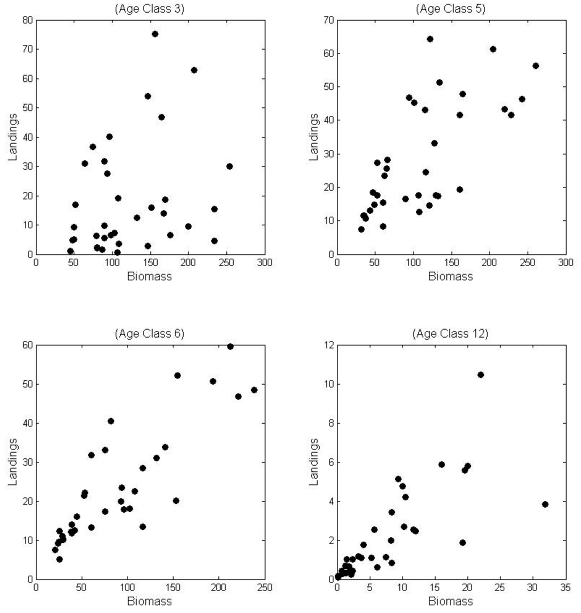

Figure 3b. Saithe Landings Plotted Against Biomass (thousand tonnes)

Table 4

Within and Between Variation[a] for Catch and Stock Size: Cod and Saithe

	Within Variation	Between Variation
Cod		
Catch	0.80	1.66
Stock	0.82	1.75
Saithe		
Catch	1.08	1.48
Stock	1.04	1.62

[a] Standard deviation.

Econometric Results

For both the cod and saithe, our empirical strategy is first to test the statistical validity (*i.e.*, stationarity of the variables) of the empirical specification of equation (5) in a panel setting. Second, we generate and validate instrumental stock variables for both cod and saithe as defined previously. Third, we apply a within estimator to equation (5) that is specified as a two-way, fixed-effect model correcting for clustered residuals and accounting for both heteroskedasticity and autocorrelation within the panels. Finally, we validate the estimated equations by evaluation of the predicted residuals and then report results.

Testing for stationarity in a panel setting uses both variation over time, which is standard in time-series econometrics, and variation across panels. We evaluate the stationary prospects of our model using two statistics for testing unit-root hypotheses in panels. Levin, Lin, and Chu (2002) (hereafter the LLC test) developed procedures that test the null hypothesis that for each age class the variables catch and stock have unit root versus the alternative hypothesis of stationarity. The pooling approach across age classes yields greater power in testing than performing a separate unit-root test for each age class.[11] The second test is a Fisher-type test where individual Dickey-Fuller statistics for each variable separately are combined using meta-analysis to generate a test for stationarity in the panels. The null and alternative hypotheses are as defined for the LLC test. Choi (2001) showed that if the time dimension is large and the number of cross-sectional units is finite, the test statistic follows the inverse normal.

The LLC and Fisher-type test statistics for fishing mortality and biomass for both species are reported in table 5. We observe for each variable and for each statistical test that the null hypothesis of unit root can be rejected. This provides statistical evidence that within the panel data framework all variables have stochastic properties amenable to equilibrium evaluation and provide statistical support for the model as specified in equation (5).

Our strategy is to use both lagged cohort catch and rank order of the stock variable as exogenous variables to build the IV. However, with both past and forward lags in the IV regression, we lose degrees of freedom and age class 3. ICES provide additional catch information outside of our period of analysis that allows us to recover all but 45 observations for cod and 47 observations for saithe. The IV equation is written as:

$$\ln\left(S_{a,i}^{t}\right) = \ln\left(q_{a,i}\right) + \ln\left(E_{i}^{t}\right) + \delta_{1}^{IV} \cdot \ln C_{a-1,i}^{t-1} + \delta_{2}^{IV} \cdot lnRO_{a,i}^{t} + \delta_{3}^{IV} \cdot lnC_{a+1,i}^{t+1} + \vartheta_{a,i}^{t}. \quad (6)$$

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[11] The LLC test procedure is recommended for moderate-sized panels, with perhaps between 10 and 250 individuals and 25 to 250 observations per individual. These requirements are met for the data sets at hand. Furthermore, the LLC test requires that the panels be balanced.

We apply a within estimator with fixed time effects corrected for heteroskedasticity and autocorrelation within the panels to estimate equation (6). A test of the strength of the correlation of the assumed exogenous variables, lagged catch, and rank order on current stock can be tested with a joint F-test. We test the null that $H_0\colon \delta_1^{IV} = 0 \text{ and } \delta_2^{IV} = 0$ with generated F-statistics of 23.80 (0.00) and 103.84 (0.00) for cod and saithe, respectively (p-values in parentheses). These tests provide some statistical validation for using the exogenous instruments in the IV equation. Consequently, based on both the stationarity and IV testing we are statistically confident in the empirical specification of equation (5) and move on to an IV estimation procedure to recover parameters.

Table 5

Panel Stationarity Tests: Cod and Saithe[a]

	LLC[b]	Fisher[c]
Cod		
Catch	–9.18 (0.00)[d]	–7.4 (0.00)
Stock	–7.80 (0.00)	–7.70 (0.00)
Saithe		
Catch	–8.07 (0.00)	–3.11 (0.00)
Stock	–6.32 (0.00)	–2.36 (0.01)

[a] Null hypotheses is that all panels have unit root.

[b] LLC asymptotically normal without trend, lags chosen by AIC and each panel demeaned.

[c] Fisher inverse normal under the assumption that t approaches infinity.

[d] p-values in parentheses.

We apply a within estimator with fixed time effects corrected for heteroskedasticity and autocorrelation within the panels and bootstrap the standards errors reporting results in Appendix B. As a robustness check, we re-estimate the models using a generalized linear estimator with binary variables defining both age and year effects. The results obtained were very similar to the within panel estimator. For the equations presented in Appendix B, the residuals are stationary and show no systematic behaviour. We will present the econometric results in three parts; catchability coefficients, implicit index of fishing effort, and stock elasticities.

Figure 4 graphs out catchability coefficients by age class for both cod and saithe. The coefficients are averaged over the full period of study. For cod we observe catchability coefficients increasing though age class 6 and declining through older age classes. This is consistent with a passive gear fishery and implies gear selectivity in targeting age-specific codfish. For saithe we observe a clear negative trend over age class. This would imply weak age selectivity over all age classes. For saithe this seems reasonable given the many gear types used in the fishery, including the purse seine that targets all year classes. For saithe the profile can be taken as evidence of growth overfishing, where fishing pressure is as heavy on the young fish as the old. These results are consistent with recent work by Diekert (2012) and Diekert *et al*. (2010).

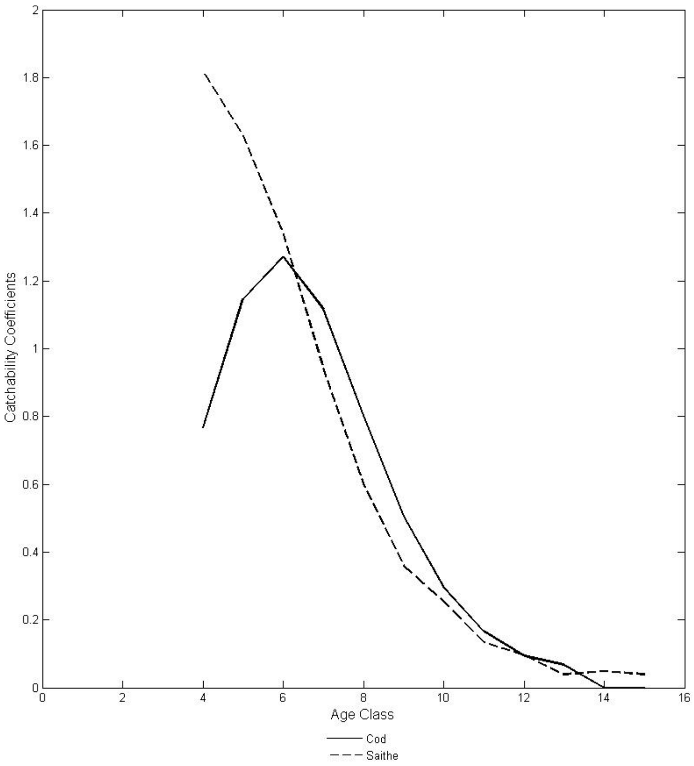

Figure 4. Catchability Coefficients for Cod and Saithe

Figure 5 reports the implicit index of fishing effort over the period 1977–2011 for age class 9 for both cod and saithe. This is an interesting result because it represents the actual index of fishing effort required to reflect actual catch. Again, this is subject to the VPA environment for data generation. For cod, we see a declining trend over time in effort, reflecting both improved fisheries management and improved efficiency in harvesting techniques. For cod notice the large drop in fishing effort in 1990, reflecting a strong effort on the part of regulators to enforce the TAC and address the serious decline in stock levels. Saithe fishing effort seems to have a stable trend over time but is more variable and erratic compared to cod and likely reflects the low value of saithe relative to cod; fishers target saithe when high-valued cod is not available.

Finally, the estimates of stock elasticities are reported in table 6. We report estimates of stock elasticity and tests of the null hypothesis that $b = 0$ and $b = 1$. The first row headed 'Overall' uses the entire period and recovers the stock elasticity. For cod it is easy to reject a stock elasticity of zero, but we cannot reject a stock elasticity equal to one. This result is consistent with expectations for demersal species. On the other hand, for saithe we statistically reject a stock elasticity of either one or zero. The value lies between zero and one, which is what we would expect for a demersal species that sometimes behaves like a pelagic species. To test the robustness of the results, we subdivide the data by year based on figures 1a and 1b. We define four year groups 1977–1986, 1987–1996, 1997–2005, and 2006–2011. We re-estimate the model and introduce specific stock variables for the groups defined and report the results in the bottom half of table 6. For cod, the results are robust over the first three year groups, and we cannot reject a stock elasticity equal to one, but for the period 2006–2011 we measure a stock elasticity of $b=0.79$, statistically less than one. The stock elasticity less than one is associated with a large increase in cod stock biomass during this time period. These results are consistent with other cod studies reporting a stock elasticity different from zero but less than one (Sandberg 2006; Eide *et al*. 2003; Hannesson 1983). On the other hand, for saithe we measure a stock elasticity less than one in both the first and last periods. In both periods, stock biomass is declining, albeit stock levels are considerably larger in the last period, as are catch levels. In summary, a stock elasticity near one for cod is likely the norm and is suggestive of a stock distribution that is dispersed somewhat evenly over the sea floor, but for saithe the stock elasticity varies between zero and one and may be associated with the stock level.

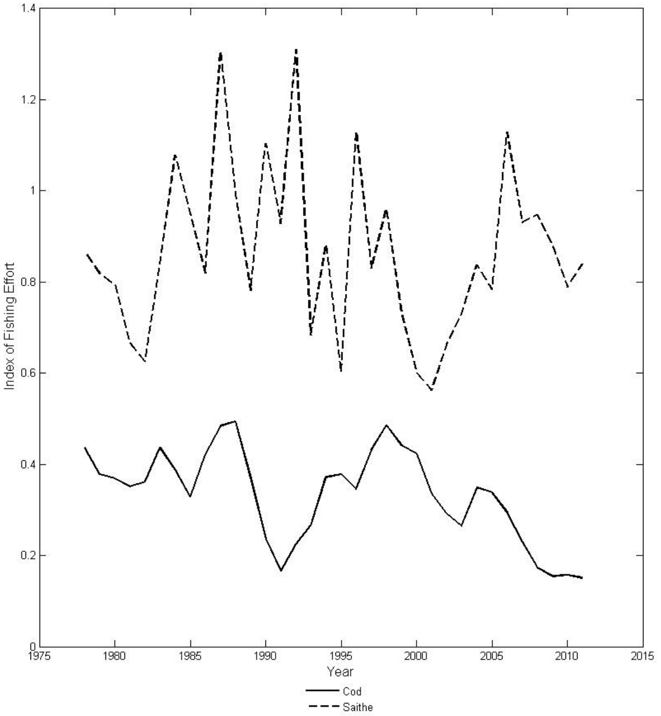

Figure 5. Implicit Index of Fishing Effort for Cod and Saithe, Age Class 9

Table 6
Stock Elasticity: Cod and Saithe

	Cod			Saithe		
	$\hat{b}$	H_o:b=0	H_o:b=1	$\hat{b}$	H_o:b=0	H_o:b=1
Overall	1.03	reject	not-reject	0.90	reject	reject
1977–1986	1.01	reject	not-reject	0.75	reject	reject
1987–1996	1.02	reject	not-reject	1.01	reject	not-reject
1997–2005	1.01	reject	not-reject	0.97	reject	not-reject
2006–2011	0.79	reject	reject	0.80	reject	reject

Conclusion

This article provides a simple but useful statistical evaluation of the relationship between stock size and catch levels. In estimation we recover not only stock elasticity but also catchability coefficients by age class and an implicit index of fishing effort. Data for observed catch levels and biomass-at-age are collected from ICES for both the Northeast Arctic cod and saithe. The results are interesting on four fronts: First, ICES stock data are generated based on current and future catch levels, biological assumptions, and a deterministic decay function. This forces an instrumental variable to allow consistent estimation. We show how the matrix configuration of stock and catch allows us to define the lagged cohort catch level as one exogenous instrument to correct correlation of stock with the error term in the catch equation. We are also aware of measurement error issues in VPA stock estimates, and we use a stock rank order variable as an additional exogenous instrument. Second, stock elasticity estimates provide some support for the general notion that for demersal species, like cod, stock elasticity is statistically close to one. On the other hand, saithe, although demersal, exhibits behaviour somewhat similar to pelagic species, and we found statistical evidence that elasticity may be a factor of the actual level of the fish stock. During 1977–1987 and 2006–2011, periods of low stock levels, we measured a stock elasticity statistically between zero and one. Third, the catchability coefficients show that for cod we observe some selectivity for age classes around 6, but for saithe we see almost no selectivity across age classes. Finally, our index of fishing effort for cod shows a general downward trend over time, which is likely a function of improved fisheries management and fishing efficiency.

The benefit of the statistical approach used in this study is that it does not require detailed vessel-level data but relies only on aggregate age-related information on catch levels and biomass-at-age in estimation. From the estimated model, we are able to recover an implicit index of fishing effort, stock elasticities, and catchability coefficients. A natural next step in this research is to link the fishing effort index to data on the costs of fishing.[12]

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[12] See Ekerhovd (2013).

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Appendix A

For a given cohort (c) in year t, ICES calculates the number that must have been alive the previous year by adding the number caught this year to the number estimated to have died of natural causes over the same period. The standard formulation in population biology is to account for both natural and fishing mortality, which is called the exponential decay equation (Jennings, Kaiser, and Reynolds 2001):

$$N_{a+1,c}^{t+1} = N_{a,c}^{t} \cdot e^{-F_{a,c}^{t}+M}. \tag{A1}$$

Here $N_{a+1,c}^{t+1}$ is the number of individuals alive at time $t+1, N_{a,c}^{t}$ is the number alive at time t, M is the instantaneous rate of natural mortality, F is the instantaneous rate of fishing mortality, and a denotes the age of the fish. (To reduce notation let $Z = F + M$.)

ICES also calculates the numbers caught at time t, $C_{a,c}^{t}$, from fishing mortality and natural mortality at time t using the catch equation:

$$C_{a,c}^{t} = \frac{F_{a,c}^{t}}{Z_{a,c}^{t}} N_{a,c}^{t} \cdot \left(1 - e^{-Z_{a,c}^{t}}\right). \tag{A2}$$

Catch multiplied by the mean weight of individuals and summed over age classes provides an estimate of total yield in biomass to the fishery. To start the simulation, VPA uses catch data and initial base assumptions of Z to work backwards through time to reconstruct previous values of F and stock structure.

Rearrange equation A2 to substitute for $N_{a+1,c}^{t+1}$ from equation A1 to get:

$$C_{a,c}^{t} = \frac{F_{a,c}^{t}}{Z_{a,c}^{t}} N_{a,c}^{t} - \frac{F_{a,c}^{t}}{Z_{a,c}^{t}} \frac{Z_{a+1,c}^{t+1}}{F_{a+1,c}^{t+1}} C_{a+1,c}^{t+1} \left(e^{Z_{a+1,c}^{t+1}} - 1\right). \tag{A3}$$

Solving for $N_{a,c}^{t}$:

$$N_{a,c}^{t} = N_{a,c}^{t}\left(C_{a,c}^{t}, C_{a+1,c}^{t+1}\right) = \frac{Z_{a,c}^{t}}{F_{a,c}^{t}} C_{a,c}^{t} + \frac{Z_{a+1,c}^{t+1}}{F_{a+1,c}^{t+1}} C_{a+1,c}^{t+1}\left(e^{Z_{a+1,c}^{t+1}} - 1\right), \tag{A4}$$

which shows the number of fish alive at the start of the year $(N_{a,c}^{t})$ as a function of the catch in year $t\ (C_{a,c}^{t})$ and $t+1\ (C_{a+1,c}^{t+1})$ for cohort c. The argument $C_{a,c}^{t}$ appears also as an argument in predicting stock in period $t-1$ for age class $a-1$:

$$N_{a-1,c}^{t-1} = \frac{Z_{a-1,c}^{t-1}}{F_{a-1,c}^{t-1}} C_{a-1,c}^{t-1} + \frac{Z_{a,c}^{t}}{F_{a,c}^{t}} C_{a,c}^{t}\left(e^{Z_{a,c}^{t}-1}\right). \tag{A5}$$

Implicitly solving we observe that $C_{a,c}^{t} = {N^{-1}}_{a-1,c}^{t-1}\left(C_{a-1,c}^{t-1}\right)$, thus:

$$C_{a,c}^{t} = \left(N_{a-1,c}^{t-1} - \frac{Z_{a-1,c}^{t-1}}{F_{a-1,c}^{t-1}} C_{a-1,c}^{t-1}\right) \frac{F_{a,c}^{t}}{Z_{a,c}^{t}} \left(1 - e^{-Z_{a,c}^{t}}\right). \tag{A6}$$

implies correlation between the cohort lagged catch variable and current catch and provides an instrument to address the correlation problem. Notice that lagged stock also appears as a right-hand-side argument in equation A6, but because it is a function of lagged catch (equation A5), it is an outcome variable and not a valid instrument (Angrist and Pischke 2009, pp. 64–68).

Appendix B

Table 1

Fixed Effects within Regression, Instrumental Variable for Stock,
Bootstrapped 1,000 times

	Cod			Saithe		
	Coefficient	Std. Error	p-value	Coefficient	Std. Error	p-value
Base	–0.87	0.17	0.00	–1.27	0.321	0.00
IVStock	1.03	0.03	0.00	0.91	0.05	0.00
1978	0.12	0.16	0.45	0.28	0.14	0.04
1979	–0.03	0.11	0.82	0.22	0.15	0.15
1980	–0.05	0.11	0.63	0.19	0.19	0.32
1981	–0.09	0.12	0.39	0.02	0.18	0.92
1982	–0.07	0.10	0.49	–0.05	0.19	0.80
1983	0.12	0.09	0.21	0.25	0.16	0.12
1984	–0.01	0.11	0.99	0.49	0.15	0.00
1985	–0.17	0.15	0.28	0.37	0.17	0.03
1986	0.08	0.11	0.44	0.22	0.24	0.35
1987	0.22	0.10	0.02	0.69	0.19	0.00
1988	0.24	0.11	0.02	0.42	0.40	0.30
1989	–0.05	0.17	0.77	0.18	0.45	0.69
1990	–0.49	0.11	0.00	0.52	0.24	0.03
1991	–0.85	0.17	0.00	0.35	0.31	0.27
1992	–0.55	0.13	0.00	0.69	0.28	0.01
1993	–0.38	0.13	0.00	0.04	0.31	0.89
1994	–0.04	0.14	0.77	0.29	0.22	0.17
1995	–0.03	0.10	0.80	–0.08	0.25	0.74
1996	–0.11	0.15	0.43	0.55	0.25	0.03
1997	0.11	0.12	0.37	0.24	0.18	0.19
1998	0.23	0.12	0.06	0.38	0.30	0.21
1999	0.13	0.10	0.21	0.11	0.17	0.49
2000	0.09	0.09	0.33	–0.09	0.18	0.62
2001	–0.14	0.09	0.13	–0.15	0.19	0.43
2002	–0.28	0.11	0.01	0.01	0.17	0.95
2003	–0.38	0.11	0.00	0.11	0.20	0.59
2004	–0.10	0.09	0.27	0.25	0.23	0.28
2005	–0.14	0.09	0.17	0.18	0.19	0.34
2006	–0.27	0.12	0.02	0.54	0.24	0.02
2007	–0.52	0.11	0.00	0.35	0.21	0.09
2008	–0.80	0.10	0.00	0.37	0.16	0.02
2009	–0.92	0.15	0.00	0.29	0.17	0.08
2010	–0.90	0.19	0.00	0.19	0.18	0.29
2011	–0.94	0.18	0.00	0.25	0.17	0.15
Sigma_u	1.08	–	–	1.47	–	–
Sigma_e	0.29	–	–	0.67	–	–
R^2 between	0.98	–	–	0.81	–	–
R^2 within	0.88	–	–	0.65	–	–

Notes: Cod, number of groups 10, number of time periods 35, number of observations 350.
Saithe, number of groups 13, number of time periods 35, number of observations 455.
Corrected for clustered residuals by panel.