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MASTER'S THESIS

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Abstract

Sufficient still water airgap is important both for fixed and floating platforms. What is a sufficient airgap according to the rules depends on the rule regime under which the platform is designed. All fixed platforms and floating platforms operating at one site for its design life time, platform design will follow the regulations provided by the Petroleum Safety Authority Norway. For floating platforms operating as drilling rigs, there is an opening in the regulations to design platform according to the maritime regulation.

In this master thesis we will focus on a semi-submersible platform. The aim is to identify a proper set of design sea states for the platform regarding airgap assessment. We assume the platform to be operating in the Northern North Sea. NORA10 data for the years from 1957 – 2017 will be made available for the project. Transfer functions for the global motions of the platform are also available. Airgap values will be presented.

Keywords: Air gap, semi submersible, relative wave elevation, significant wave height, peak period, Response Amplitude Operator, extreme value.

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1. Introduction

Sufficient still water airgap is important both for fixed and floating platforms. What is a sufficient airgap according to the rules depends on the rule regime under which the platform is planned to be operating. All fixed platforms and floating platforms operating at one site for its design life time, platform design will follow the regulations provided by the Petroleum Safety Authority Norway. For floating platforms operating as drilling rigs, there is an opening in the regulations to design platform according to the maritime regulation.

In the MSc focus shall be on a given semi-submersible platform. The rigid body transfer functions are made available. The aim of the MSc is to estimate q-probability airgap, $q = 10^{-2}$/year and $q = 10^{-4}$/year, for the worst location under platform deck accounting for joint occurrence of wind-sea and swell sea. A consistent estimation of q-probability airgap requires that a long term analysis is performed. The platform is to be operating in the Northern North Sea. NORA10 data for the years from September 1957 – September 2017 will be made available.

Long term response analysis can be carried out using an all sea state approach or an all storms approach (POT). In this master thesis, focus is to be given to the all storms approach. An important part of this method is to choose a proper threshold defining the selection of storms. An important part of the air-gap assessment is to consider sensitivity of results to selected threshold.

A linear response analysis can be utilized for the short term analyses, but non-linearities in the wave crest heights shall be included in the analyses. A possibility is to utilize the approach proposed by DNVGL.

The sub-tasks are developed as follows:

In the Chapter 2 a brief literature review is made. Articles about the development of the the Peak Over Threshold Method (POT) are reviewed with main emphasis on the methodology used by the authors. Articles with assessment of design parameters made with the POT method are also reviewed.

In the Chapter 3, following the guidelines from the literature review, the Peak Over Threshold Method (POT) used in this master thesis is explained.

In the Chapter 4, it is demonstrated how the transfer function for air-gap variable is determined. Since we are accounting for simultaneous occurrence of wind sea and swell sea propagating in different directions, a JONSWAP type of wave spectrum is used in both cases.

The core of the discussion is made in Chapter 5. Air-gap definitions and terminology are made according to DNV. Short term an long term analysis are made for the relative wave elevation.

Discussion of the results is made in Chapter 6 of various analyses of the airgap variable for $q = 10^{-2}$/year and $q = 10^{-4}$/year.

Conclusions and further work are made in Chapter 7.

2. Literature Review

Estimation of extreme waves is always been an important parameter regarding structural safety for platforms. How to define, estimate and predict this extremes is the objective when acquiring metocean data and establish probabilistic models. The random storm approach is formulated for estimating wave and response extremes corresponding to determined return periods based on the availability of enough amount of data (Haver, 2017b).

In (Jahns & Wheeler, 1972), it is defined a "design wave" like the one that the structure must be able to withstand without damage with a specified risk level and a recurrence interval. This value has a finite probability of being exceeded during the life of the structure. It is also contemplated the range of sea states that contribute to the probability of experience a wave higher than the design wave. Then, the designer must select the design conditions so that this probability is small to be accepted for safety requirements and also for environmental and economic factors. The authors present a method that uses weather data to estimate the distribution of sea states that can be considered severe. "This distribution is then integrated with a random noise statistics to estimate long-term wave probability and recurrence intervals". In the paper is established the importance of the crest height as design parameter. Then, in order to interpret historical storm data, seven probability estimations of crest height are made for seven different, sequential, successive and independent events:

1. A single wave from a given sea state.

2. A sequence of waves from a given sea state.

3. A specific storm with a given sequence of sea states.

4. A random storm.

5. A specified number of storms.

6. All the storms occurring in one year.

7. A sequence of years.

According to the authors this must be read for example in Step 4 as "the probability that a given crest height is exceeded during a random storm".

A point is made on the other nonlinearity of the waves when interpreting crest probability calculations in terms of wave height probability. If the wave height is Rayleigh distributed, then can be considered equal as two times the crest height. This assumption is most of the times not satisfied because of the non linear effects for example in shallow waters or the run up on structures.

A example application is made in a determined location on the Gulf of Mexico with 140 m water depth and a frequency of 0,2 storms per year. Hindcast information is available to rank the 10 most severe storms for the last 50 years. Crest probabilities are estimated for "the biggest wave in one storm, in 1 year and in 20 years". Probabilities for the highest crest elevation (crest height plus tide) are also calculated. Wave height estimations are also made with the assumption of high crest always paired with equally depth through. Some efforts are also pointing to calculate wave forces probabilities only associated to the contribution of crest height. It considers the wave force acting on "a cylindrical pile extending from the bottom to the surface". Conclusions of the paper point towards the facility of the method to be programmed and the importance of the available data of the site. Also the necessity of validation of the empirical correlations made only for analysis carried out for locations in the Gulf of Mexico.

A method for obtain extreme wave-height value statistics by extrapolating historical storm data is described in (Petruaskas & Aagaard, 1970). The method presented allows the flexibility of choice different distribution functions and computerized procedures over manual calculations and graphical extrapolations. First, the input data is the "expected maximum wave height" which will determine the profile of the storm. In the second place, the data must be obtained with the same calculation model and must include all the storms above a specified threshold which is the smallest expected maximum wave height among all the storms available. Finally, all the storms must be of the same type. this means that swell, hurricanes and wind sea waves must not me mixed.

Two types of probability models are use: "one, a conditional probability estimating the expected maximum wave height given that a storm exceeding a threshold and the second, the Poisson distribution estimating the number of storms that exceeded the specified threshold in a given time interval". The main objective of the extrapolation method presented in the paper is to find a conditional distribution that fits the input hindcast data. The authors propose eight possible distributions to accomplish this, the Gumbel distribution and seven different 3-parameters-Weibull distributions. The parameters for the Weibull functions are calculated to be unbiased according to the procedure explained in the paper. The selected distribution is chosen according to how well it fits the available data. In order to accept or reject a fitting, this is measured by the mean square deviation and then the distributions are ranked. The output from the extrapolation method are plots of expected maximum wave height versus return period and the non exceeding probability.

The extrapolation method is defined as reliable when fitting the data to a particular distribution but this is done according to the authors without any theoretical basis. It can be used in other variables as significant wave height or maximum wind speed. The uncertainties of the method fall on the amount of storms used in the extrapolation and if this sample can be considered to represent the whole storm population. Then "the only way to reduce this uncertainty is to increase the number of storms". The ranking procedure selects the best distribution but a methodology is presented to select a most conservative wave height.

Finally, a balance of the method is made, establishing the systematic nature of the method and the inclusion of uncertainties as strengths. The weakness are mainly not having theoretical bases for selecting the distributions and the selection of the "true" distribution is not entirely quantitative.

In (Tromans & Vandersohuren, 1995), the authors develop a method to find a long term distribution of loads for a structure in the northern North Sea. In order to accomplish this, storms are characterized by its most probable extreme wave which allows him to treat storms as the random independent event. A storm is here defined as the evolution of wave height during a period from 12 to 36 hours with a peak and a subsequent decay. The method described in the paper uses "the most probable extreme individual wave of the storm history in stead of the peak significant wave height". The identification of storms is made over 25 years of data for a North Sea location and helps to make the data more manageable. The direction of the storm is defined by the direction of the peak.

The short term variability "should converge to an asymptotic form, conditional only on he most probable value of the extreme individual wave height of the storm". One example is given on the paper. The long time statistics of storms is a probability distribution for the most probable extreme wave. Weibull and the generalized Pareto distributions are chosen by the author. Combining this two by convolution, it is possible to obtain the distribution of the extreme wave height of any random storm. Then, the same procedure is used by the author on the prediction of extreme loads statistics. The method is listed here and will be the same used in this document.

- Hindcast database.
- Identification of storms.
- The probability distribution of the extremes.
- The short time variability.
- The long time statistics of storm characteristics.
- Long term distribution by convolution.

The conclusions of the paper point towards the formulation of a valid and consistent method of analysis of extreme loads and structural responses.

The purpose of the authors in (Haring & Heideman, 1978) is to estimate "rare wave heights and crest heights" in the Gulf of Mexico based on hindcast of 22 severe hurricanes in the gulf since 1900. The main objective is to determine the statistics of events that will occur based in the statistics of events that have occurred in order to have design criteria for fixed platforms. The authors then, defines explicitly the requirements to be able to perform analysis of extremes. These are: an accurate storm sea-state generation model, knowledge of the conditional probabilities of individual wave parameters in a random sea, and statistical analyses. A sector division is made by taking into account how the water depth up to 180 meters can affect the hindcast sea states. Then, the authors applied three different methods to calculate "the annual nonexceedance probability of wave height and crest height for each site and each group of sites defined by the sectors and water depth ranges". The results show that there is no a significant difference in the variation of wave heights or crest heights over three sectors. The methods are sensitive to the assumptions made but the philosophy and procedures employed are applicable to any area where storms are predominant and where fixed platform will be located.

In (Ferreira & Soares, 1998) the paper describes an application of the Peaks Over Threshold (POT) method to significant wave height data of Figueira da Foz, Portugal collected from 1981 to 1990. The method is described as a solition to the extrapolation issuesand is explained as "fitting the generalized Pareto distribution to the peaks of clustered excesses over a threshold and of calculating return values". Cluster is defined as "a group of consecutive exceedances" that are expected to be independent and well defined for high enough thresholds. The threshold chosen by the authors is 6 m and the 25, 50, and 100 year return values are calculated. The authors describes the selection of the threshold as being high enough to ensure independence in the events and that the distribution of the peak excesses to be close to one of the three forms of the generalized Pareto distribution. The estimation of clusters per year is approximated with a Poisson distribution.

The POT method enables the use of much more data and, this is why is allows to carry out the same analysis for increasing thresholds and compare the results. If the approximation is valid, some stability in the results is expected.

The main conclusion of the author when using this method it the fact that there was no problem in choosing the ”right” threshold since the data is very close to being exponential and this facilitated the application of the method. It is also proposed to use the similar POT analysis of wave data from the various ocean areas in order to verify the method.

In (Naess, 1998) the author discusses the use of the Peaks Over Threshold (POT) method for estimating long return period values of environmental loads. This is made with a statistical estimation technique to analyze wind speed data from 44 American weather stations. A initial transformation is made by squaring of the original wind speed data. The events are considered to be independent since there is at least a 4 days difference between reach other. The POT method is based in the generalized Pareto (GP) distribution and the de Haan estimation method is explained by the authors and used to determine the (GP) parameters.

The authors favour the Gumbel distribution over the Weibull distribution for representing the statistics of transformed extreme wind speed data. Weibull distribution is more appropriated for statistical analysis of non transformed data. It is also estimated the 50 year return period values and compared to the corresponding values obtained by other methods methods in previous researches of the author with good results.

3. Peak over threshold (POT) method

The task of selecting a proper design wave height is central in offshore and coastal engineering. It often involves the use of methods and procedures for the statistical analysis of extreme waves, (Mathiesen et al., 1994). The standard procedure for the POT approach when estimating extremes of significant wave height data is:

- Select data for analysis (significant wave height for all storms above threshold)
- Fit a candidate distribution to the observed data
- Compute (extreme) return values from the fitted distribution
- Compute confidence intervals

In this document, the selected data for analysis will be the same, but the distributions and calculations of extremes will be applied to the relative wave elevation of a semi-submersible platform in the northern North Sea.

One of the most fundamental requirements of any extreme value analysis is that the data sample must be statistically independent and identically distributed (Holthuijsen, 2010). A common method of fulfilling this requirement and allowing for the statistical prediction of extreme wave heights and wave induced response is the peak over threshold (POT) extreme value analysis.

The peaks over threshold method, considers storm peaks above some chosen threshold. The selection of a suitable significant wave height threshold value is key to get a important set of data to be analyzed, (Lee & Ng, 2011). Selecting an appropriate threshold value is important due to the fact that the estimation of extreme events may depend on threshold value.

The threshold must be set high enough to ensure independence between samples, and low enough to ensure that the number of samples is sufficient to have a robust statistical analysis. It is important to use a sufficiently long data set to allow for proper threshold limits and still allow for robust fitting (Yang & Copping, 2017).

In the POT approach, a storm is defined as an uninterrupted sequence of events of significant wave height all exceeding a certain value, preceded and followed by a lower value. (Holthuijsen, 2010). In this document, in order to guarantee the independence of peaks, events within the period of 48 hours will be selected as part of the same storm and estimating the optimal threshold will be the subject of study. The criterion is that a sufficient number of storms can be identified in the long-term time record.

A storm consists of a sequence of sea states (steps) with a peak and then falling off all in a t period of time. This process in reality is not stationary, in reality, the storms will be in a continuous development with time. Therefore, we approximate each step to be stationary in 3 hours and then "jump" to the next 3 hour stationary step. This means that the period and standard deviation of the process during the 3 hours are constant. The horizontal blue lines left and right of the red dots that Figure **3-1** shows, are the way to express the approximation.

(Haring & Heideman, 1978) gives the procedure to apply this methodology and it is shortly described and modified to include the non observed storms to explain the procedure used in this document as follows:

Lets define a variable (x) as our quantity of study, it can represent a wave, a maximum individual wave or relative wave elevation of an stochastic process. Let the probability of an arbitrary peak of this variable to be smaller or equal to a value x in a given storm step be expressed like $F_{X|H_sT_p}(x|h,t)$. This is understood as the global maximum of a given sea state and the Rayleigh distribution for a linear problem will be used to represent it, equation (3-1):

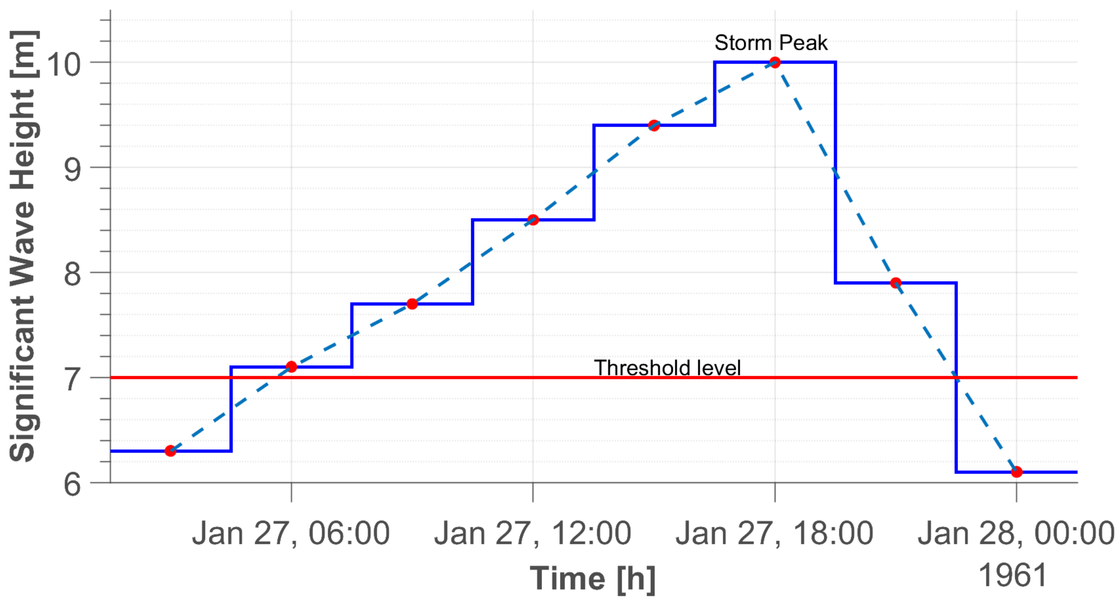

Figure 3-1.: A storm between two successive crossings of the significant wave height through a threshold level.

$$F_{X|H_sT_p}(x|h,t) = 1 - exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right] \tag{3-1}$$

where σ represents the standard deviation of the process with global peaks denoted X.

Assume that the storm step exists for Δt hours, then, we can calculate the probability that the Δt-hour maximum of the variable x has a value lower or equal than x_{step} in that storm step. This is expressed by the equation (3-2).

$$F_{X_{\Delta t}|H_sT_p}(x_{step}) = \left[F_{X|H_sT_p}(x_{step}|h,t)\right]^{N_z} \tag{3-2}$$

where N_z is the number of waves in Δt hours. In our case the storm step account for 3 hours.

Let $F_{X_{storm}|\text{storm i}}(x_{storm})$ be the probability that the storm maximum in a particular storm i is smaller or equal than x_{storm}, equation (3-3)

$$F_{X_{storm}|\text{storm i}}(x_{storm}) = \prod_{i=1}^{M} F_{X_{\Delta t}|H_sT_p}(x_{storm}) \tag{3-3}$$

where M is the number of steps that composes the storm. It is tacitly assumed that the storm maximum of each step are statistically independent.

The most probable storm maximum $\tilde{x}$ corresponds to the peak $\frac{dF_{X_{storm}|\text{storm i}}(x_{storm})}{dx_{storm}} = 0$. Then, the conditional distribution of x_{storm} given $\tilde{x}$ is in our case approximated by the Gumbel distribution and given by the equation (3-4).

$$F_{X_{storm}|\tilde{X}}(x_{storm}|\tilde{x}) = exp\left\{ - exp\left[-\frac{x_{storm} - \tilde{x}}{\tilde{x}\beta} \right] \right\} \tag{3-4}$$

Where the term $\tilde{x}$ represents the most probable largest for the respective storm and β is a parameter.

We have now a exact storm distribution given by the equation (3-3) and an approximate storm distribution given by (3-4). The way to relate this two is to establish a β value such that the variance from the exact distribution is equal to the variance of the approximate distribution. This has to be done for all the storms found above the corespondent threshold, then the mean value of β will be used as the Gumble parameter in the short term variability.

The key idea of this method is at the end to establish a long term distribution of the largest response during a random storm (Haver, 2004). The long term distribution of the largest response is obtained by convolution of the short term variability with the long term distribution, equation (3-5).

$$F_{X_{storm}}(x_{storm}) = \int_{\tilde{X}} F_{X_{storm}|\tilde{X}}(x_{storm}|\tilde{x})\ f_{\tilde{X}_{storm}}(\tilde{x}_{storm}) d\tilde{x} \tag{3-5}$$

where x_{storm} is the quantity being analyzed, in this case the response of the structure during a random storm.

$F_{X_{storm}|\tilde{X}}(x_{storm}|\tilde{x})$ represents the short term variability of the response given the most probable maximum response ($\tilde{x}$) and is approximated with the Gumble distribution.

$f_{\tilde{X}_{storm}}(\tilde{x}_{storm})$ is the long term variability of the most probable largest response $\tilde{x}$. Since it is possible to calculate a $\tilde{x}$ for each observed storm, then, the long term variability of $\tilde{x}$ is can be found by fitting in our case by a Weibull 3-parameter distribution function.

3.1. Storms information

The number of events above the threshold and the number of storms built from those events are shown in the Table **3-1**. It is clear that lowering the threshold implies that the data to be treated will increase, in this case duplicate and even more, then the amount of numerical calculations increase considerably. For this reason, one of the objectives is to determine the optimal threshold to avoid unnecessary calculations with a huge amount of data.

Table 3-1.: Events and storms for each threshold

Threshold	6m	7m	8m	9m	10m
Number of events above threshold	3502	1469	604	244	95
Number of storms	675	366	186	81	39

It is important to clarify that the threshold is applied to the significant wave height for wind sea in the NORA10 data, then swell values are the ones associated to the corresponding wind sea. The Table **3-2** shows the information of the storm example taken from the provided NORA10 file. For the Total Sea, the Significant Wave Height is calculated with the equation (3-6), the Spectral Peak Period and Direction are the same of the wind sea.

$$h_{\text{total}} = \sqrt{h^2_{\text{wind sea}} + h^2_{\text{swell}}} \tag{3-6}$$

The Figures **3-2**, **3-3** and **3-4** show the parameters of the storm used as an example for wind sea, swell and total sea respectively. This information is contained in the Table **3-2**.

Table 3-2.: Storm Example Data. From NORA10.

NORA10 DATA
LATITUDE: 60.79, LONGITUDE: 3.56

				TOTAL SEA					WIND SEA			SWELL		
YEAR	M	D	H	HS	TP	TM	DIRP	DIRM	HS	TP	DIRP	HS	TP	DIRP
1961	1	27	3	6.5	10.2	8.0	197.	189.	6.3	10.2	197.	1.7	13.5	242.
1961	1	27	6	7.3	11.2	8.5	197.	188.	7.1	11.2	197.	1.8	13.5	242.
1961	1	27	9	7.9	11.2	8.8	197.	186.	7.7	11.2	197.	2.1	14.9	242.
1961	1	27	12	8.7	12.3	8.9	182.	181.	8.5	12.3	182.	2.1	14.9	227.
1961	1	27	15	9.6	12.3	9.4	182.	180.	9.4	12.3	182.	2.3	14.9	227.
1961	1	27	18	10.2	13.5	9.9	197.	184.	10.0	13.5	197.	2.1	18.0	272.
1961	1	27	21	9.1	13.5	10.6	212.	198.	7.9	13.5	212.	4.5	16.4	167.
1961	1	28	0	7.4	13.5	9.7	212.	208.	6.1	13.5	212.	4.1	14.9	182.

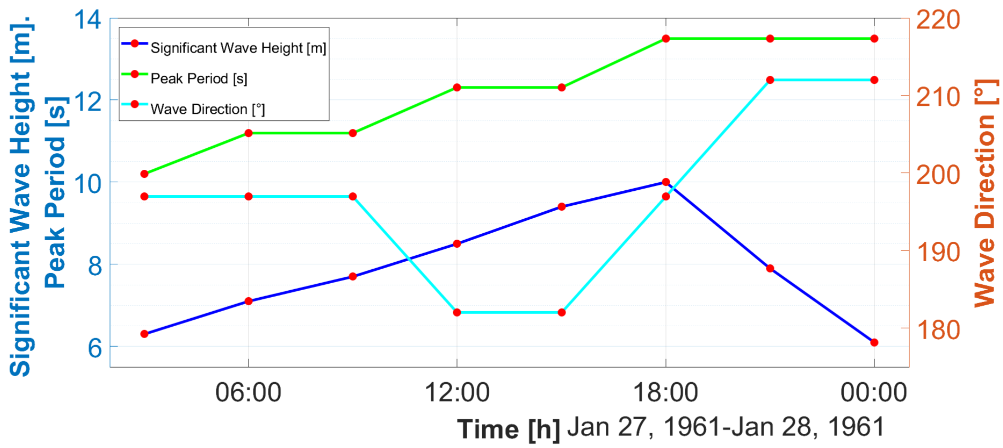

Figure 3-2.: Storm Example. Wind Sea Information.

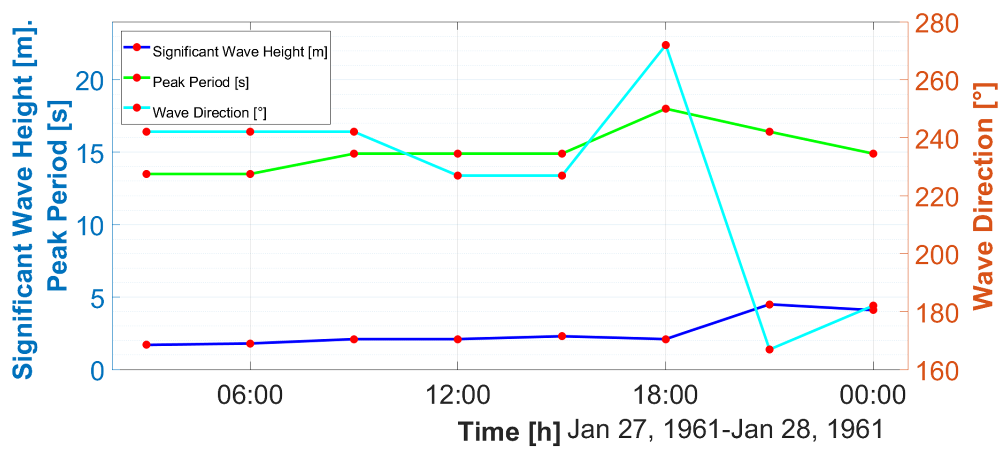

Figure 3-3.: Storm Example. Swell Information.

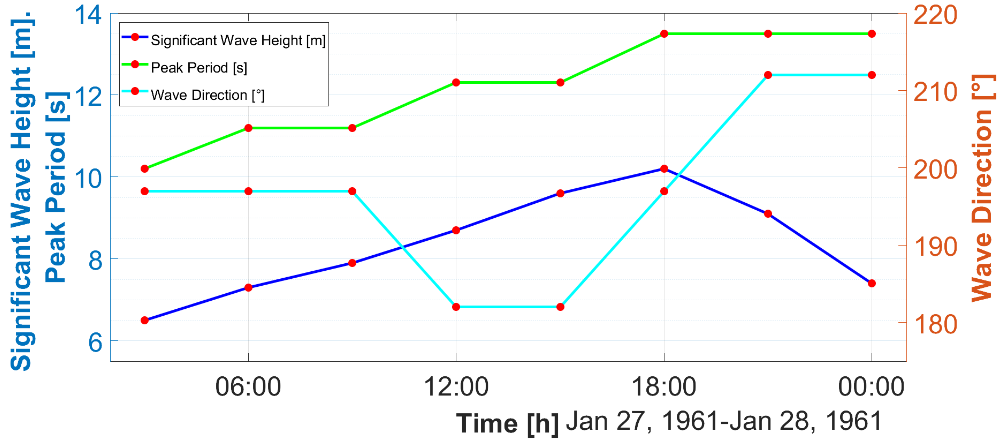

Figure 3-4.: Storm Example. Total Sea Information.

Once the threshold is set, all the events above it are considered to carry out the analysis. Since the location of NORA10 is known, it is very important, for our analysis, to have an idea of the main characteristics of the data we are working with. This means, to determine for example most common peak period or the directions where most of the wind sea and swell come from.

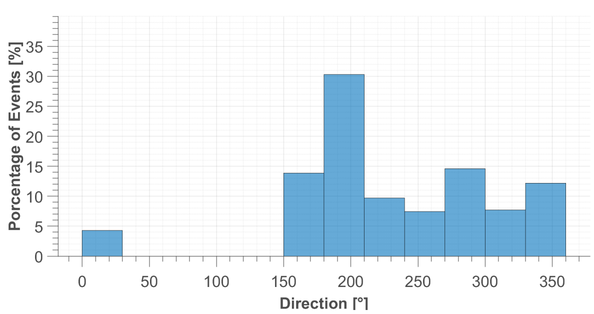

Figure 3-5.: Wind Sea Direction histogram Threshold 6m.

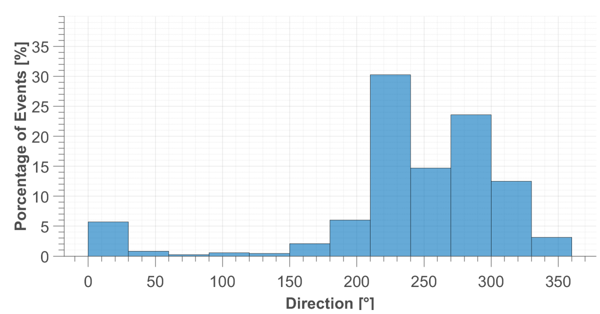

Figure 3-6.: Swell Direction histogram Threshold 6m.

The Figures **3-5** and **3-6** show what is the percentage of events that come from a specific direction. Based on this, we can establish what location on the platform will be facing the worst weather conditions. It is clear that most of the events are coming from areas between South, West and North-West directions. Then apriori , the critical airgap can be found on the edge of the platform facing this directions.

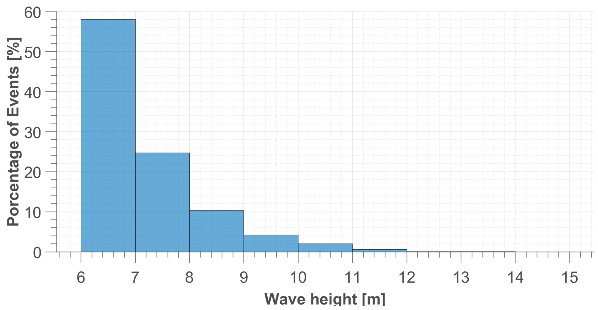

Figure 3-7.: Wind Sea Wave Height histogram Threshold 6m.

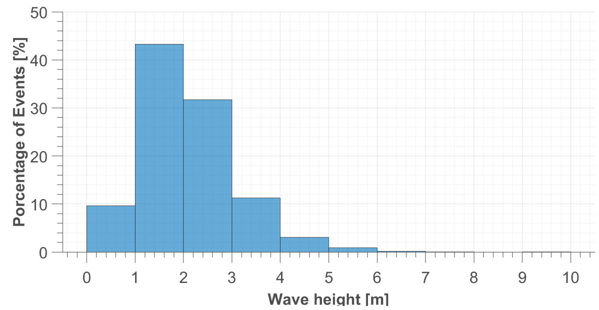

Figure 3-8.: Swell Wave Height histogram Threshold 6m.

The Figures **3-7** and **3-8** show the significant wave height histogram for the events above the 6m threshold. As it was indicated in the Table above, the event number duplicates each time the threshold is lowered. This trend continues as the threshold is lowered.

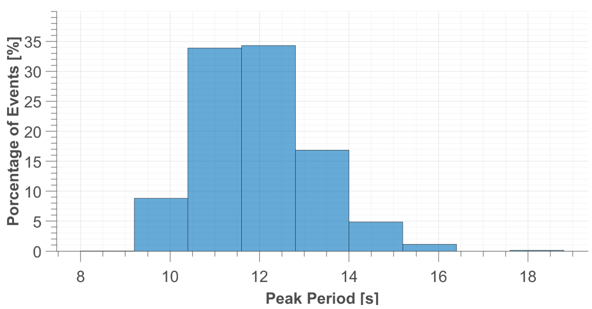

Figure 3-9.: Wind Sea Period histogram Threshold 6m.

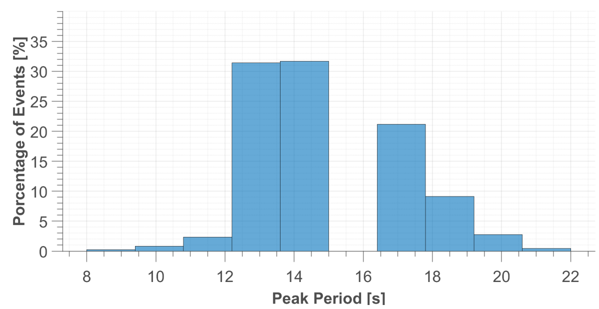

Figure 3-10.: Swell Period histogram Threshold 6m.

The Figures **3-9** and **3-10** show the histogram of peak period for the events above the 6m threshold. Almost the 70 % of the data has a period between 10 and 12 seconds. Then, any floating structure installed in this location, must have a natural period as far as possible of this band, in order to avoid resonance and external forces that can cause undesired forces and damage. This graph helps to understand in a first view which will be the periods that will influence the most the semi submersible motion with the response amplitude operators shown in the next chapter.

The peak period values from the NORA10 file are discretized unevenly, giving a bad resolution of the calculated sea conditions. This is why the Figure **3-10** has no values for periods around 16 s. For the calculations of this documents, no correction was made.

4. Motion of a Point and Response Amplitude Operator

Airgap can be defined as the distance between the underside of the structure's deck (fixed or floating) and the wave crest vertically underneath the considered deck point. (Haver, 2017a). The prediction of minimum airgap in unfavorable environments and the estimation of the probability of having a wave reaching the deck of the offshore structure is a difficult challenge.

In this document, a methodology of calculating the still water airgap will be presented using as an input, a WAMIT file with the semi-submersible displacement RAOs (Response Amplitude Operators) for 6 degrees of freedom and its response to a sea determinate state given by a NORA10 file.

Each displacement RAO consists of a pair of numbers that define the semi submersible response, for one particular degree of freedom and one particular wave direction and period. The two numbers are the amplitude, which relates the amplitude of the semi submersible's motion to the amplitude of the wave, and the phase, which defines the delay of the semi submersible motion relative to the wave time history.

For instance: A surge RAO of 0.5 m/m in a wave of height 4m (and hence wave amplitude 2m) means that the semi submersible surges to and from -1m to +1m from its static position; a pitch RAO of 0.5° per metre in the same wave means that the semi submersible pitches from $-1°$ to $+ 1°$.

The six degrees of freedom of a body are shown in the Figure **4-1**. Our interest will be in determinate the vertical movement of the platform (Z-axis).

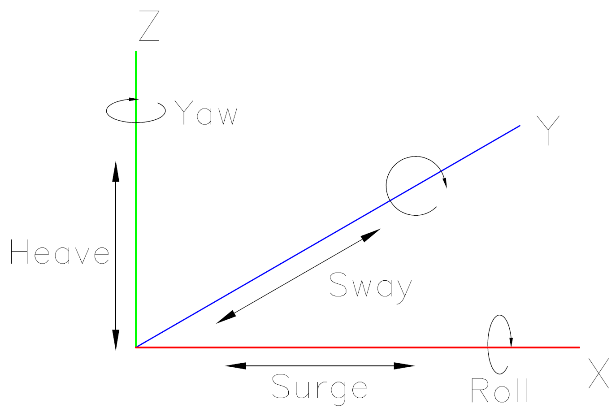

Figure 4-1.: Degrees of freedom

In order to calculate the total vertical motion of a point X,Y in the platform, we must calculate each one of the contributions of the degrees of freedom to the total vertical displacement. Figure **4-2**.

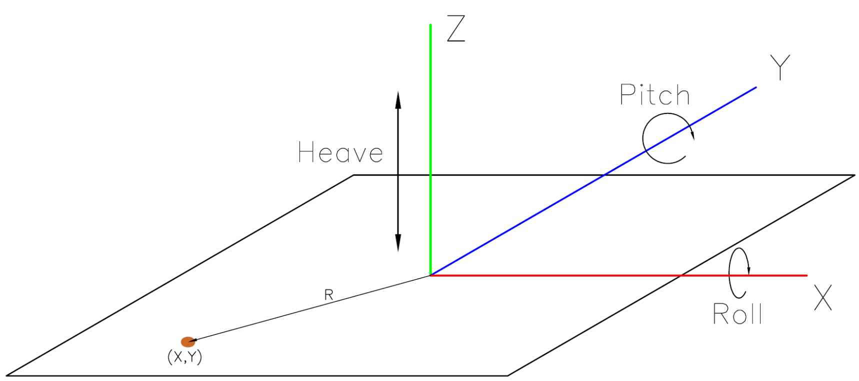

Figure 4-2.: Motion of a point X,Y

4.1. Transfer function $H(\omega, \beta)$

The motion of the semi-submersible is given by a WADAM file which describe responses for bodies as a function of the direction and the period of harmonic waves. Figure **4-3**. The translation motions (heave, sway and surge) are expressed in meter per meter wave amplitude. The roll, pitch and yaw are given in degrees per meter wave amplitude. The responses are normalized with respect to the amplitude of the incident wave. With a transfer function $H(\omega, \beta)$ the corresponding time dependent response variable $R(\omega, \beta, t)$ can be expressed as shown in (DNV, 2011).

$$R(\omega, \beta, t) = A \cdot Re\left[|H(\omega, \beta)|\, e^{i(\omega t + \phi)}\right]$$

$$|H(\omega, \beta)| = \sqrt{H(\omega, \beta)^2_{Re} + iH(\omega, \beta)^2_{Im}}$$

where $|H(\omega, \beta)|$ is the amplitude of the transfer function, A is the amplitude of the incoming wave, ω is the frequency of the incoming wave, β describes the direction of the incoming wave, t denotes time and ϕ is the phase angle between the incident wave and the time varying response. The transfer function and the phase angle are shown in the equations (4-1) and (4-2).

$$H = H(\omega, \beta)_{Re} + iH(\omega, \beta)_{Im} \tag{4-1}$$

$$\phi = atan\frac{H(\omega, \beta)_{Im}}{H(\omega, \beta)_{Re}} \tag{4-2}$$

The incoming wave is expressed as

$$\xi = A \cdot cos(\omega t) \tag{4-3}$$

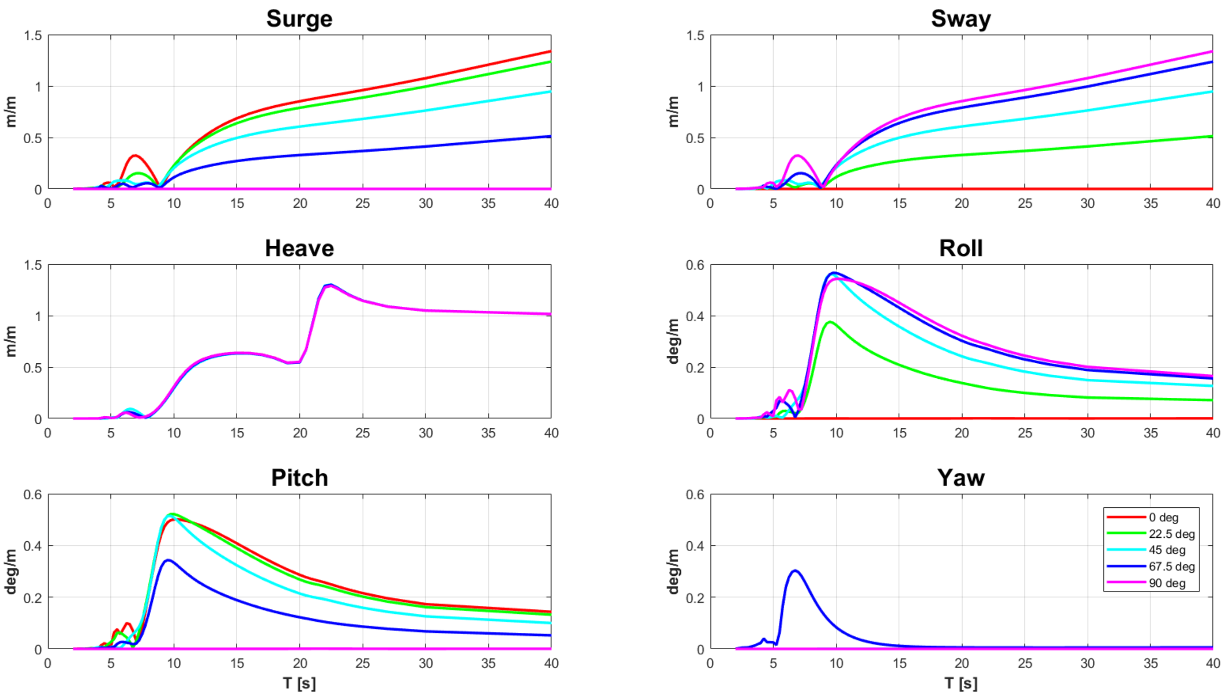

Figure 4-3.: RAO for the semi-submersible. WADAM data.

For simplicity, the calculations of the resultant transfer function $H_{Z_P}(f)$ of a point $P_{(X,Y)}$ in the structure will be done in the complex space. This means that the displacements and rotations will be expressed with a real component (subscript Re) and an imaginary component (subscript Im) as shown in the equation (4-4). The total real component shown in the equation (4-5) is expressed as the sum of the real parts of all the the degrees of freedom j for a determinate period and wave direction. We must do the same for the imaginary component with the equation (4-6).

$$H_{Z_P}(f) = H_{Re_{Z_P}}(f) + iH_{Im_{Z_P}}(f) \tag{4-4}$$

where

$$H_{Re_{Z_P}}(f) = \sum_{j}^{n} H_{Re_{Z_{Pj}}}(f) \tag{4-5}$$

$$H_{Im_{Z_P}}(f) = \sum_{j}^{n} H_{Im_{Z_{Pj}}}(f) \tag{4-6}$$

Here, $H_{Z_P}(f)$ is the transfer function in the point (X, Y), in terms of frequency, and n is the number of degrees of freedom considered. In this case we will only count heave, roll and pitch since the other three degrees of freedom (surge, sway and yaw) do not cause any vertical motion.

The total vertical motion $Z_{P(X,Y)}$ of the (X, Y) point is determined by the equation 4-7.

$$Z_{P(X,Y)}(f) = A\,|H_{Z_P}(f)|\,cos(\omega t + \Phi) \tag{4-7}$$

where $|H_{Z_{P(f)}}|$ is the norm of the resultant transfer function in the point (X, Y) also known as Response Amplitude Operator (RAO). Now, it is necessary to determine the transfer function for the vertical motion $|H_{Z_P}(f)|$ and the total phase angle Φ.

4.2. Degrees of freedom

Heave Motion

Is the pure vertical motion generated by a sinusoidal wave with unit amplitude and given frequency. The heave transfer function $H_{Z_{heave}}(f)$ is expressed as follows

$$H_{Z_{heave}}(f) = H_{Re_{heave}}(f) + iH_{Im_{heave}}(f) \tag{4-8}$$

Roll Motion

Is the angle of rotation about $x - axis$ generated by a sinusoidal wave with unit amplitude and given frequency. The roll transfer function $H_{Z_{roll}}(f)$ is defined as the cross product of the angle of rotation about $x - axis$ and the arm $\vec{R}$ between the origin and point (X,Y).

$$\begin{aligned}
H_{Z_{roll}}(f)\,\hat{k} &= H_{roll}(f)\,\hat{i} \times \vec{R} \\
&= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ H_{roll}(f) & 0 & 0 \\ X & Y & 0 \end{bmatrix} \\
&= (H_{roll}(f) \cdot Y)\,\hat{k} \\
H_{Z_{roll}}(f) &= [H_{Re_{roll}}(f) + iH_{Im_{roll}}(f)] \cdot Y
\end{aligned} \tag{4-9}$$

Pitch Motion

Is the angle of rotation about $y - axis$ generated by a sinusoidal wave with unit amplitude and given frequency. The pitch transfer function $H_{Z_{pitch}}(f)$ is defined as the cross product of the angle of rotation about $y - axis$ and the arm $\vec{R}$ between the origin and point (X,Y).

$$\begin{aligned}
H_{Z_{pitch}}(f)\,\hat{k} &= H_{pitch}(f)\,\hat{j} \times \vec{R} \\
&= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & H_{pitch}(f) & 0 \\ X & Y & 0 \end{bmatrix} \\
&= -(H_{pitch}(f) \cdot X)\,\hat{k} \\
H_{Z_{pitch}}(f) &= -\left[H_{Re_{pitch}}(f) + iH_{Im_{pitch}}(f)\right] \cdot X
\end{aligned} \tag{4-10}$$

4.3. Total motion of a point $P(x, y)$

Adding the equations (4-8), (4-9), (4-10) and separating the real and the imaginary part, we have the transfer function of the vertical movement of a point X,Y due to heave, pitch and roll $H_{Z_{P(X,Y)}}(f)$:

$$
\begin{aligned}
H_{Z_P}(f) = & H_{Z_{heave}}(f) + H_{Z_{roll}}(f) + H_{Z_{pitch}}(f) \\
& = H_{Re_{heave}}(f) + iH_{Im_{heave}}(f) + [H_{Re_{roll}}(f) + iH_{Im_{roll}}(f)] \cdot Y \\
& \quad - \left[H_{Re_{pitch}}(f) + iH_{Im_{pitch}}(f)\right] \cdot X
\end{aligned}
$$

$$
\begin{aligned}
H_{Z_P}(f) = & \left[H_{Re_{heave}}(f) + Y \cdot H_{Re_{roll}}(f) - X \cdot H_{Re_{pitch}}(f)\right] \\
& + i\left[H_{Im_{heave}}(f) + Y \cdot H_{Im_{roll}}(f) - X \cdot H_{Im_{pitch}}(f)\right]
\end{aligned}
\tag{4-11}
$$

Now separating the real part and the imaginary part according to the equations 4-5 and 4-6 we get.

$$
H_{Z_P}(f) = H_{Re_{Z_P}}(f) + iH_{Im_{Z_P}}(f) \tag{4-12}
$$

where the norm $|H_{Z_P}(f)|$ and the phase angle Φ are:

$$
RAO_{Z_P}(f) = |H_{Z_P}(f)| = \sqrt{\left(H_{Re_{Z_P}}(f)\right)^2 + \left(H_{Im_{Z_P}}(f)\right)^2} \tag{4-13}
$$

$$
\Phi = \text{atan}\left[\frac{H_{Im_{Z_P}}(f)}{H_{Re_{Z_P}}(f)}\right] \tag{4-14}
$$

So far we have defined all the terms of the equation (4-7) for the total heave motion of a point P on the deck with respect to its coordinate axis in the center. In the Figure **4-4**, the local axis of the platform $x' - y'$ coincide with the global axis North - East.

- The positive x'-axis is pointing towards Platform East
- The positive y'-axis is pointing to towards Platform North
- The positive z'-axis is pointing upwards

The global axis is used to define the direction of the incoming wave and this will be defined in the next section.

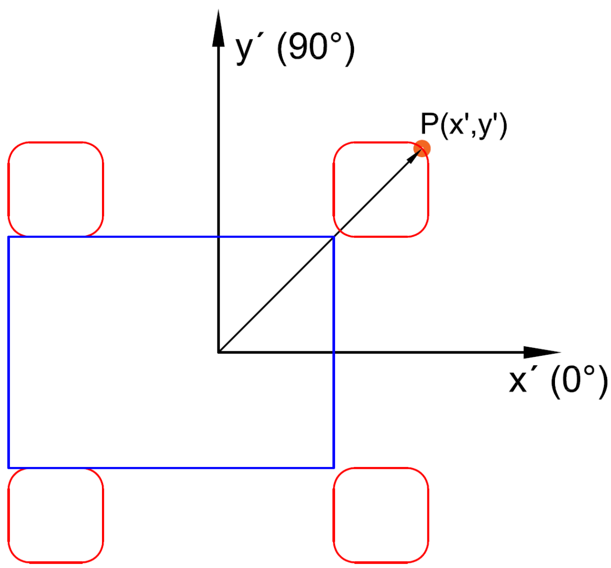

Figure 4-4.: Point P in local coordinates attached to the center of the platform

4.4. Wave directions and orientation of the platform

In the present work, the wave direction and platform orientation are presented as follows:

The WAMIT output has a convention for the wave direction shown in the Figure **4-5**. The angle of the incoming wave is measured positive in counterclockwise direction with respect to the x'-axis of the platform.

NORA10 has a coordinate system where waves with direction 0°, means waves coming from True North, and waves with direction 90° means waves coming from East. This is, positive angle in clockwise direction with respect to the True North. Figure **4-6**. Since the environmental conditions are given from the NORA10 file, then all the wave directions make reference to this coordinate system.

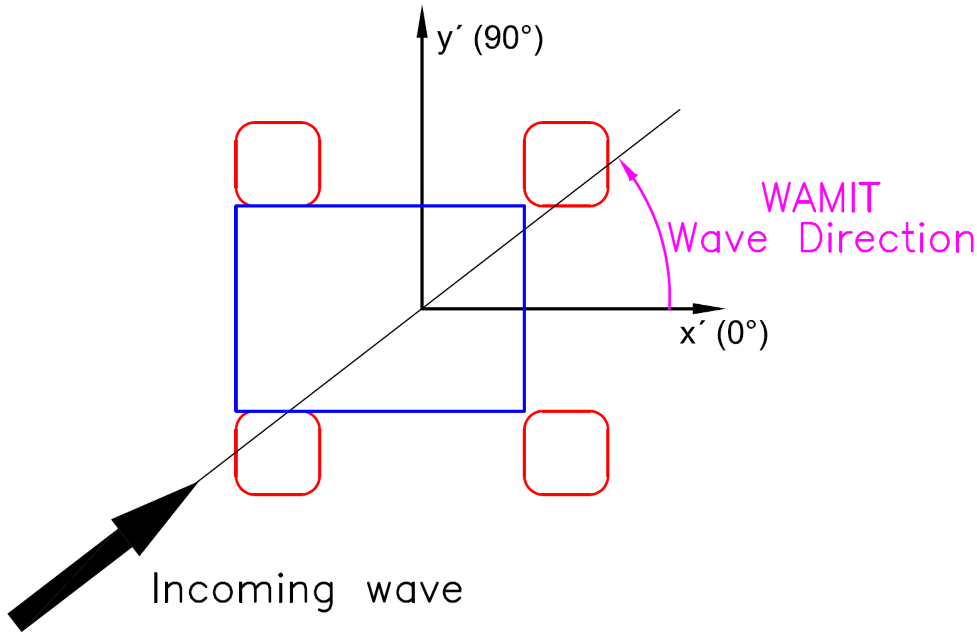

Figure 4-5.: WAMIT convention for wave direction

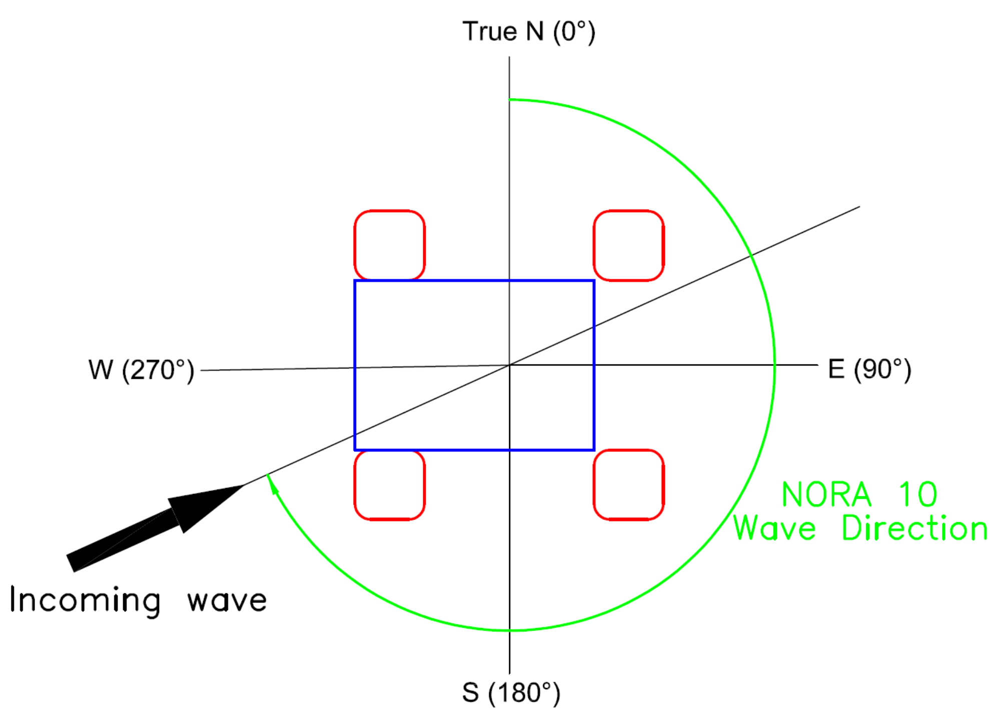

Figure 4-6.: NORA10 convention for wave direction

In this document, the $x-axis$ of the platform will be oriented towards the South direction, heading what are considering the worst sea states in the NORA10 location. This is shown in the Figure **4-7**. If the incoming wave is said has a direction of 225°, then the wave is coming from North-East and traveling towards South-West. This means that a NORA10 wave direction of 225° corresponds to a WAMIT direction of 135°. This equivalences are presented in the Table **4-1**.

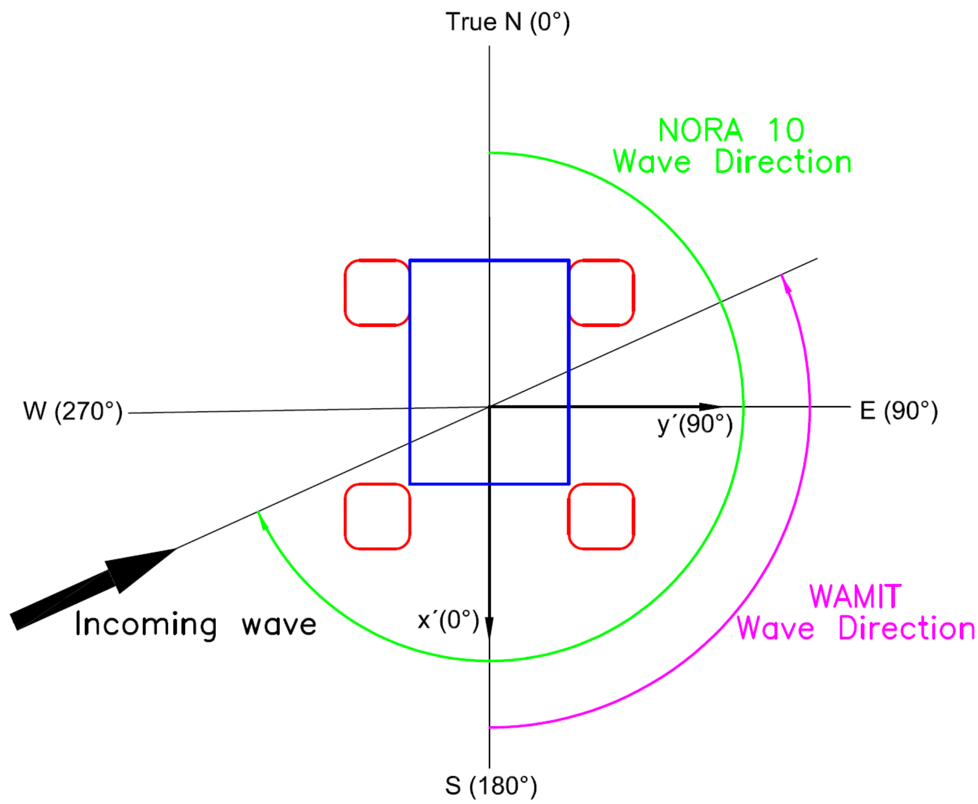

Figure 4-7.: Orientation of the platform with respect to the True North and wave direction comparison.

Table 4-1.: Equivalences between directions

NORA10 [°]	WAMIT [°]
0	0
45	315
90	270
135	225
180	180
225	135
270	90
315	45

5. Airgap Assesment

For the description of airgap terms and airgap calculations, the definitions given in (DNVGL, 2017) are are used.

5.1. Wave elevation $\eta(x, y, t)$

It is common to assume that the sea surface is stationary for a duration of 20 minutes to 3 - 6 hours.(DNVGL, 2010). This sea surface $\Xi(t)$ is characterized by the significant wave height H_s and the spectral peak period T_p and is composed of m corrected stochastic variables, $\Xi(t_i)$; with $i = 1, 2, 3, 4, ..., m$. $\Xi(t)$ will be a Gaussian variable, having a probability density function given by:

$$f_{\Xi}(\xi) = \frac{1}{\sqrt{2\pi}\sigma_{\Xi}} exp\left[-\frac{1}{2}\left(\frac{\xi}{\sigma_{\Xi}}\right)^2\right]$$

We say we have a Gaussian process, (Gudmestad, 2015), where σ_{Ξ} is the standard deviation of the process.

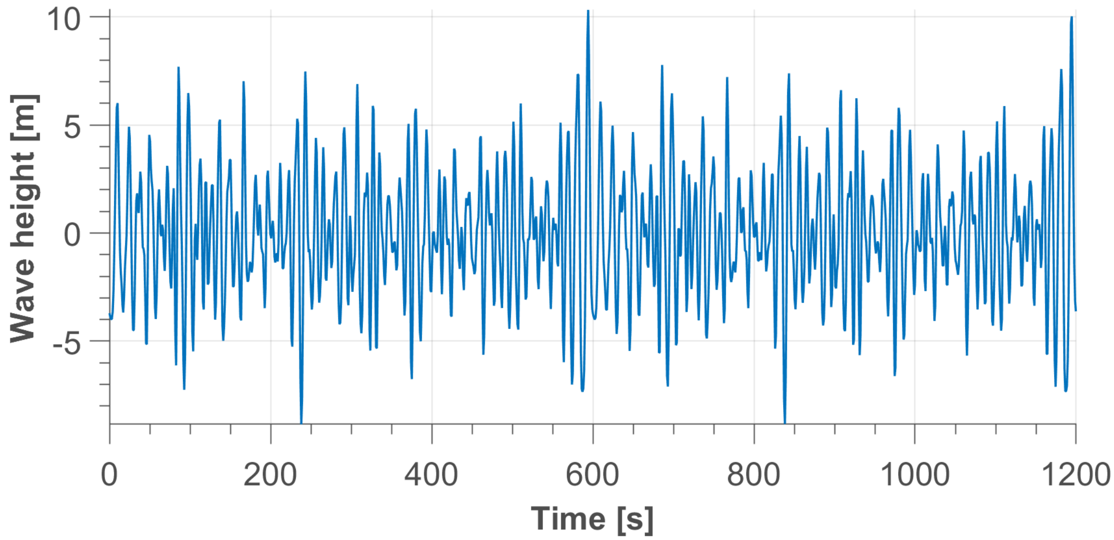

Figure 5-1.: Example of the surface elevation of a Gaussian Process in a 20 minutes window

The random ocean wave is described by an "energy" density spectrum $S(f)$. The wave "energy" spectrum describes the energy content of an ocean wave and its distribution over a frequency range of the random wave (Subrata & Cliakrabarti, 2005). The Pierson-Moskowitz (PM) spectrum and JONSWAP spectrum are frequently applied for wind seas. The following parameters useful to describe the sea state are taken from (DNVGL, 2010):

The spectral moments m_n of general order n are defined as

$$m_n = \int_0^\infty f^n S(f) df \tag{5-1}$$

where f is the wave frequency, and $n = 0, 1, 2, ...$

The variance is defined as follows:

$$\sigma_\Xi^2 = m_0 = \int_0^\infty S(f) df$$

The significant wave height H_s is given by:

$$H_s = 4\sqrt{m_0}$$

The mean zero-up-crossing period T_z can be estimated by:

$$T_z = T_{m02} = \sqrt{\frac{m_0}{m_2}} \tag{5-2}$$

The mean wave period T_1 can be estimated by:

$$T_1 = T_{m01} = \frac{m_0}{m_1}$$

The response of the bodies in harmonic waves RAO is always expressed as a function of the amplitude of the incoming wave. The surface elevation $\eta(x, y, t)$ describes the vertical distance of the wave at the point (X, Y) at the time t. here is defined as:

$$\eta(x, y, t) = A\ cos(\omega t - k_x x - k_y y)$$

where A is the amplitude, (x,y) are the coordinates of the crest, k_x, k_y the wave numbers and ω is the frequency in radians.

The wave behavior can also be expressed with a transfer function in the same way that is shown in the equation (4-1) with a real part and an imaginary part. This is the diffracted wave field, i.e. the transfer function from a undisturbed linear wave process in origo (platform centre) to a disturbed wave at a point under the platform deck. The Figure **5-2** is an example of the surface elevation's RAO of a specific point in the semi submersible.

$$H_{wave}(f) = H_{Re_{wave}}(f) + iH_{Im_{wave}}(f) \tag{5-3}$$

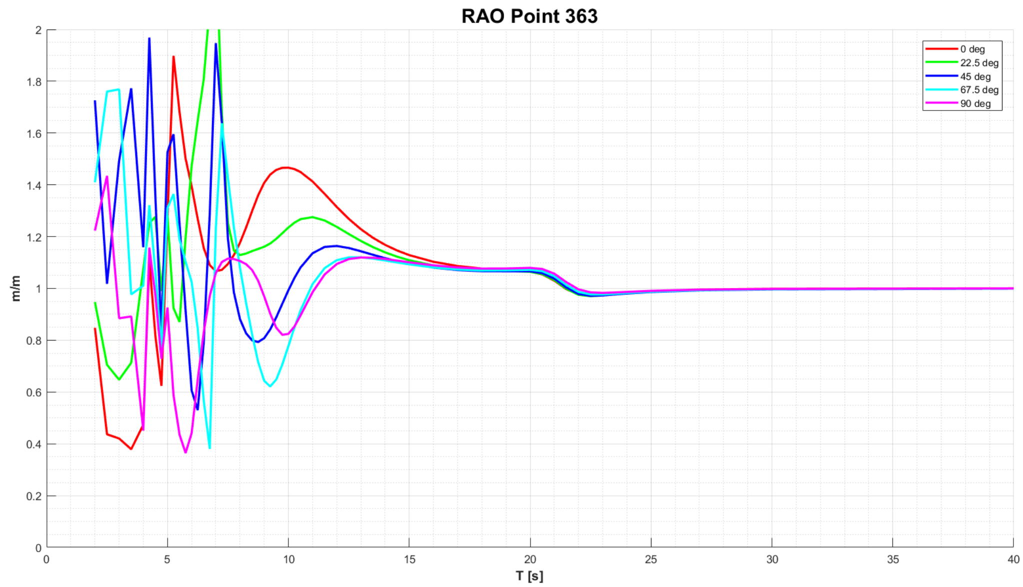

Figure 5-2.: RAO of the surface elevation under the semisubmersible deck at the Point 363 with local coordinates x' = 20.40 m y' = -20.40 m. The RAO values vary with the direction of the incoming wave. From WADAM file.

5.2. Initial airgap $a_0(x, y, t)$

The initial airgap is the vertical distance between the still water level (SWL) and the point of interest in the bottom of the lover deck of the platform when this is balanced.

5.3. Airgap calculation $a(x, y, t)$

The wave actions are the main contributor to the reduction or increase of the airgap. Upwell $R_P(x, y, t)$ is the relative wave elevation, e.g. the distance between the disturbed surface and the semi submersible mean water line. This will be the quantity of interest for this document.

$$R_P(x,y,t) = \eta(x,y,t) - Z_P(x,y,t)$$

Air gap is defined as the distance between the underside of the deck, (located at $a_0(x,y,t) + Z_P(x,y,t)$) and the wave surface $\eta(x,y,t)$. Typical values of a_0 for production platforms are between 18-20 m.

$$\begin{aligned} a(x,y,t) &= a_0(x,y,t) + Z_P(x,y,t) - \eta(x,y,t) \\ &= a_0(x,y,t) - R_P(x,y,t) \end{aligned}$$

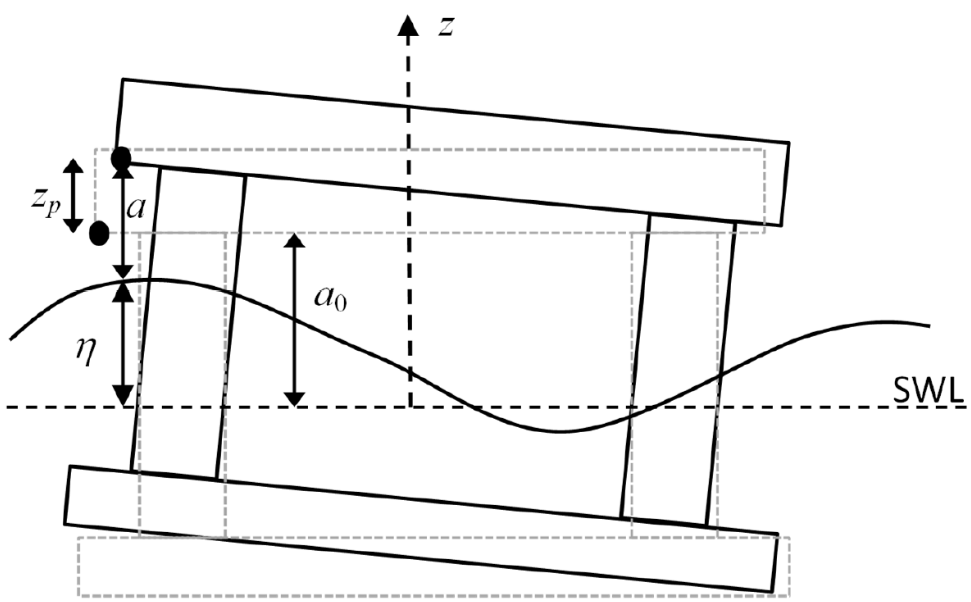

Figure 5-3.: Definition of air gap for a column stabilized unit. Dotted lines indicate position of deck in still water. From (DNVGL, 2017)

Negative air gap $a(x,y,t) < 0$, means that there is impact between the wave surface and the structure.

Following the recommendations given by (DNVGL, 2017), the surface elevation's RAO showed in the Figure **5-2** is "modified by an asymmetry factor α to account for the asymmetry of non-linear waves and the effect of non-linear diffraction". Non linear waves are asymmetric. This means that crests are higher than for a linear sinusoidal wave and troughs are shallower than for a linear sinusoidal wave. The presence of the asymmetry factor attempt to account for this effects.

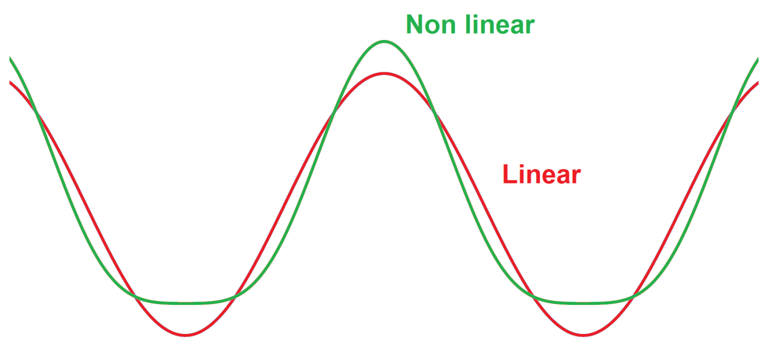

Figure 5-4.: Linear vs Non linear waves

The relative wave elevation is then:

$$R_P(x, y, t) = \alpha\, \eta(x, y, t) - Z_P(x, y, t) \tag{5-4}$$

The asymmetry factor α varies with horizontal position and wave direction but to limit the extend of this document, we will set a fixed value of 1.2 for the wave spectrum of the wind sea, and 1.0 for the wave spectrum of the swell . A more correct way to use this factor is to be determined according to the position of the platform where the relative wave elevation is to be calculated.

The RAO of the response is calculated by writing the equation (5-4) in the Complex space, i.e. using the definition of point motion and surface elevation given by (4-12) and (5-3).

$$RAO_{R_P}(f) = \sqrt{\left[\alpha \cdot H_{Re_{wave}}(f) - H_{Re_{Z_P}}(f)\right]^2 + \left[\alpha \cdot H_{Im_{wave}}(f) - H_{Im_{Z_P}}(f)\right]^2} \tag{5-5}$$

Different RAO values are shown in the Figures from **5-7** to **5-12**.

The sea state will be defined with the JONSWAP spectrum $S_J(\omega)$ given by the equation (5-6) defined in (DNVGL, 2010).

$$S_J(\omega) = A_\gamma \; S_{PM}(\omega) \; \gamma^{exp\left[-0.5\left(\frac{\omega-\omega_p}{\sigma\omega_p}\right)^2\right]} \tag{5-6}$$

where S_{PM} is the Pierson-Moskowitz (PM) spectrum

$$S_{PM} = \frac{5}{16} \; H_s^2 \; \omega_p^4 \; \omega^{-5} \; exp\left[-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right]$$

with $\omega_p = 2\pi/T_p$ is the angular spectral peak frequency, a non-dimensional peak shape parameter $\gamma = 2$ suitable for the location of NORA10, $A_\gamma = 1 - 0.287Ln(\gamma)$ and σ is the spectral width parameter defined by

$$\sigma = \sigma_a \text{ for } \omega \leq \omega_p$$
$$\sigma = \sigma_b \text{ for } \omega > \omega_p$$

Average values for the JONSWAP experiment data are $\sigma_a = 0.07$, $\sigma_b = 0.09$.

The spectrum for each event will be calculated then using the significant wave hight H_s and peak period T_p for wind sea and for swell separately. Figure **5-5**. Then the wind sea and the swell spectrum are added arithmetically to establish a total spectrum. Is then when the standard deviation of the process is calculated.

$$S_J(\omega)_{\text{Total}} = S_J(\omega)_{\text{Wind Sea}} + S_J(\omega)_{\text{Swell}}$$

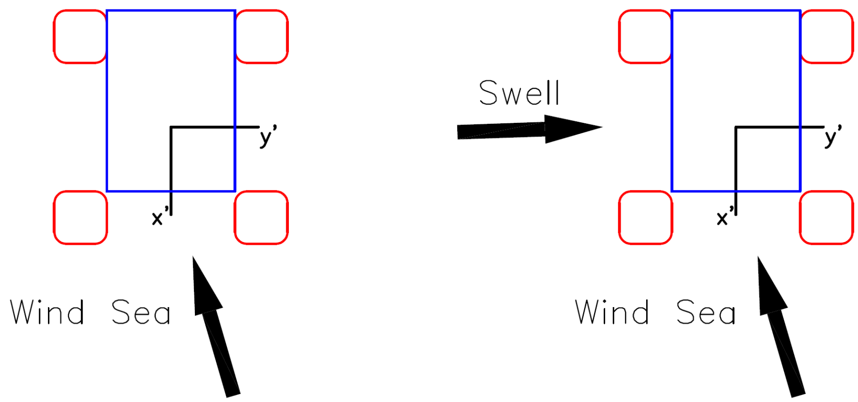

Figure 5-5.: Wind sea (left) and combined sea (right) in the structure. Wind Sea Direction 197°. Swell Direction 272°.

As is shown in before and is well explained in (Haver, 2017b), the surface elevation process for short term periods is modeled as a Gaussian process, then it can be characterized by a wave spectrum as a function of the frequency f, $S_{\Xi\Xi}(f)$. This means that the response process is also Gaussian and described by the response spectrum $s_{R_P R_P}$ which is given in the frequency domain by:

$$s_{R_P R_P}(f) = RAO^2_{R_P}(f)\ s_{\Xi\Xi}(f) \tag{5-7}$$

The Figure **5-6** shows the Spectrum for one case taken from the Table **3-2** where wind sea has $H_s = 10$ m and $T_p = 13.5$ s and Swell $H_s = 2.1$ m and $T_p = 18$ s. Using the transformation $\omega = 2\pi f$, it is possible to express the spectrum in Hertz $[s^{-1}]$ or radians easily. Therefore, it is expected to observe a peak around the value of $1/T_p$ in both cases. The total spectrum is also shown and does not differ much from the wind sea's one.

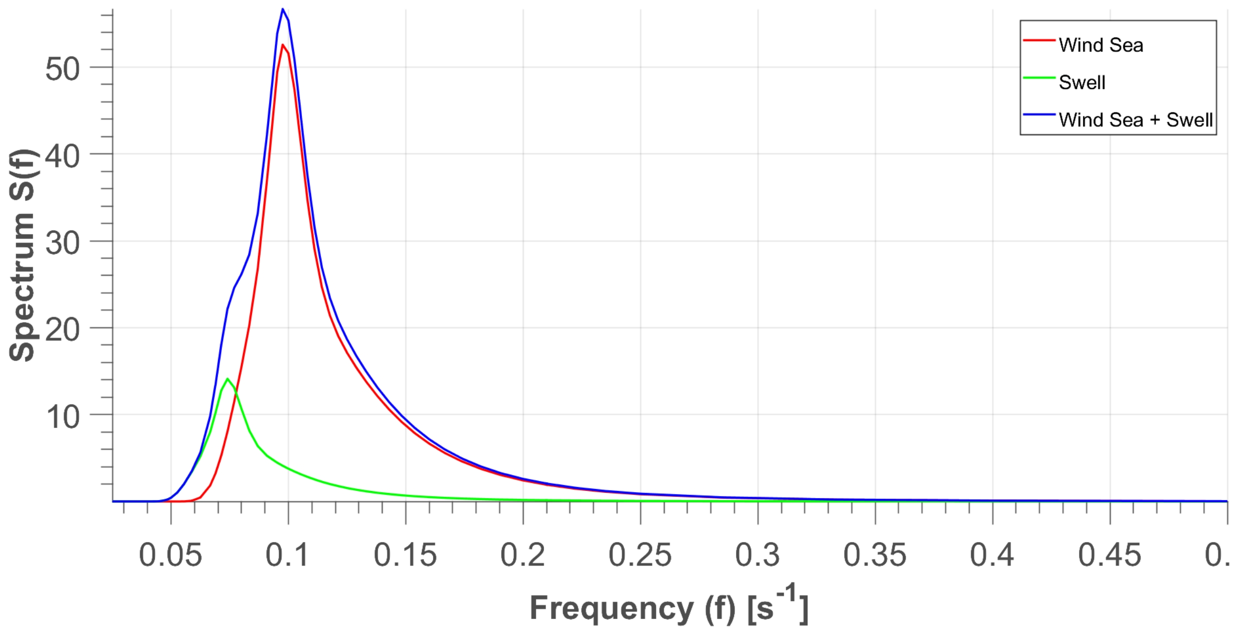

Figure 5-6.: Wave spectrum for 2 different scenarios. Wind sea ($H_s = 10$ m and $T_p = 13.5$ s) and Swell ($H_s = 2.1$ m, $T_p = 18$ s)

As an example, the next paragraphs will be destined to calculate the response amplitude R_P of the Point 363 with local coordinates x' = 20.40 m y' = -20.40 m when wind sea ($H_s = 10$ m, $T_p = 13.5$ s and direction 197°) and swell ($H_s = 2.1$ m, $T_p = 18$ s and direction 272°) act on the platform. This data is taken from the peak of the storm example shown in the Figure **3-1**. The directions are shown in the Figure **5-5**.

In order to do this, we must find in the WAMIT files the RAOs for heave, pitch and roll having as input the wave direction. This information is divided in real and imaginary parts, therefore is easier to calculate the total response in the Complex space.

The Figures **5-7** and **5-8** show the pure heave of the semi submersible. This is, the vertical motion of the center of gravity of the platform. Also in these figures is presented the corrected heave which has the effect of pitch and roll on the vertical motion of the point.

With the help of the equation (5-5), the RAO of the relative wave elevation of the Point 363 for the wind sea (WS) action ans swell action (S) separately can be calculated and expressed as a function of all the frequencies. Figures **5-11** and **5-12**.

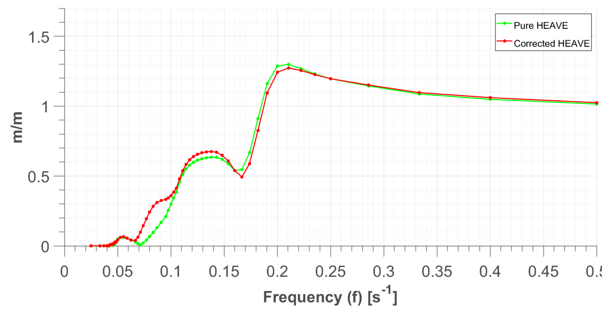

Figure 5-7.: RAO for pure and corrected heave. Wind Sea. Eq. (4-8) and (4-12). Point 363.

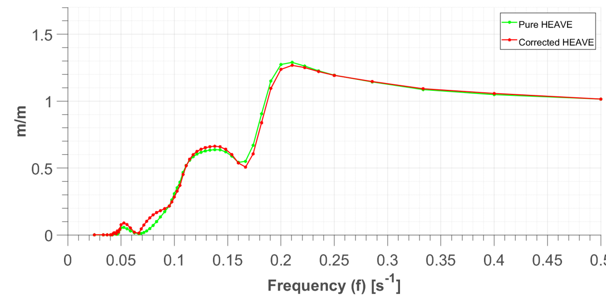

Figure 5-8.: RAO for pure and corrected heave. Swell. Eq. (4-8) and (4-12). Point 363.

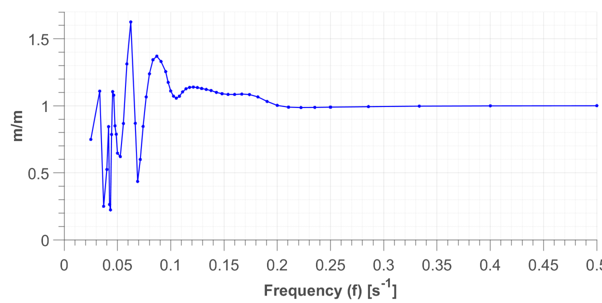

Figure 5-9.: RAO of the diffracted wave elevation under the Point 363 in the platform. Wind Sea.

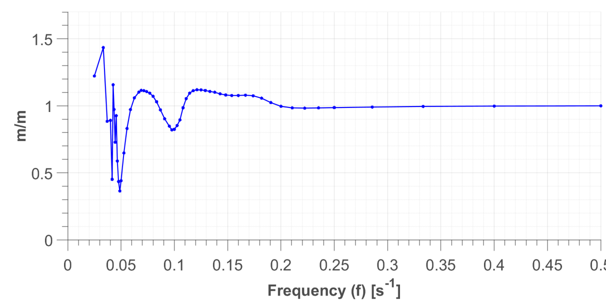

Figure 5-10.: RAO of the diffracted wave elevation under the Point 363 in the platform. Swell.

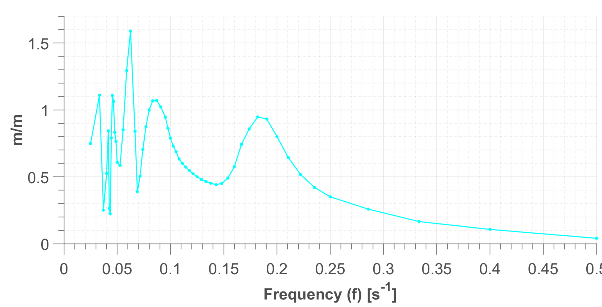

Figure 5-11.: RAO of relative wave elevation. Wind Sea. Eq. (5-5) Point 363.

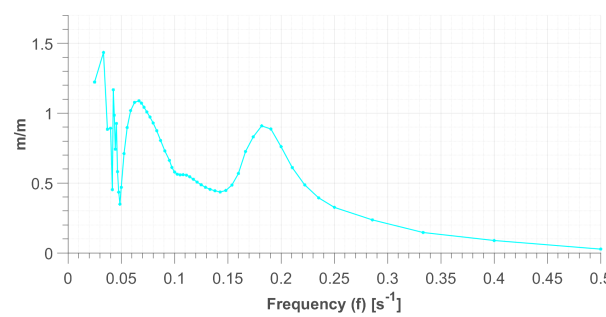

Figure 5-12.: RAO of relative wave elevation. Swell. Eq. (5-5) Point 363.

An additional WAMIT file provides the RAO of the wave elevation under the Semi submersible and around it, Figure **5-13**, discretized in 612 points. The edge of the platform and the position of the columns is also shown. The Figures **5-9** and **5-10** show the disturbed wave elevation η of the Point 363. The equation (5-5) allows to calculate the RAO of the response taking into account the asymmetry factor α.

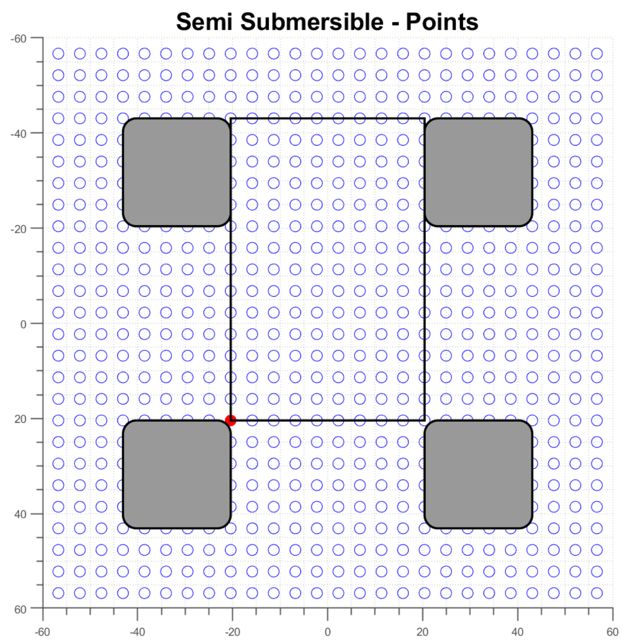

Figure 5-13.: Discretization of the RAO wave elevation in 612 points under the semi submersible and the surroundings. The Point 363 is shown in red.

Once the RAO of the response is calculated with the equation (5-5), it is possible to calculate the response spectrum in the frequency domain with the equation (5-7). The results for each frequency is shown in the Figure **5-14**. Compared with the Figure **5-6**, the energy spectrum is 60% lower for this particular case.

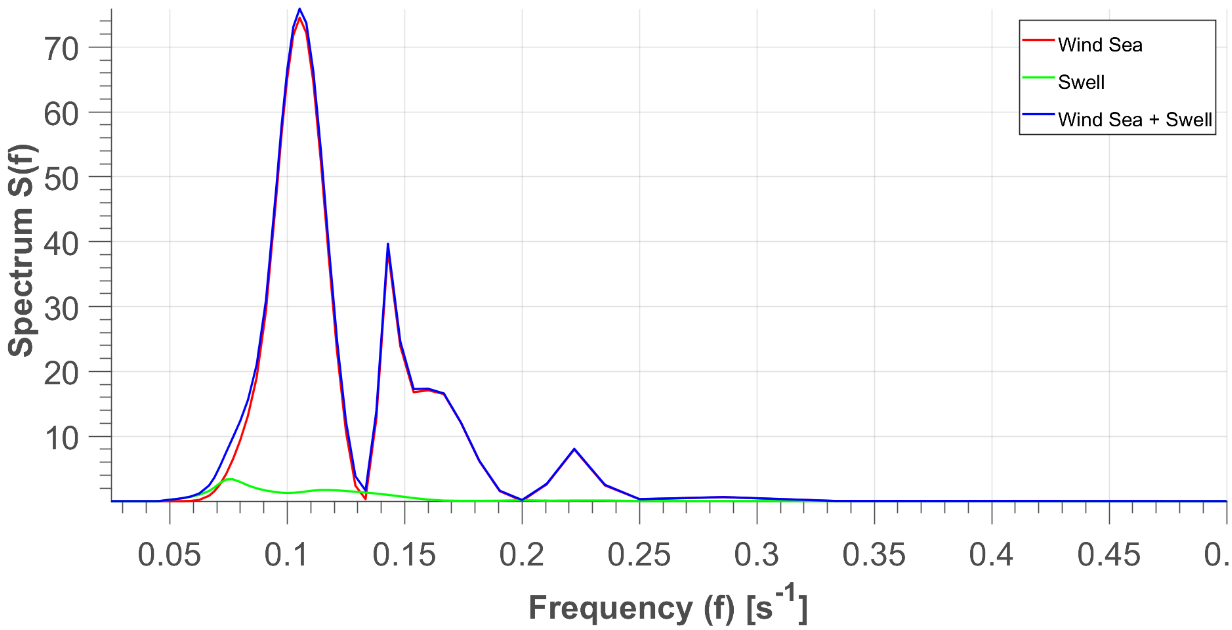

Figure 5-14.: Response spectrum for Wind Sea and Swell. Point 363.

5.4. Distribution function for the 3-hour maximum relative wave elevation

For a linear response quantity, the response process can be modeled as a Gaussian stochastic process. (Haver, 2017b). The global maxima are defined as the largest maximum between adjacent zero-up-crossings.

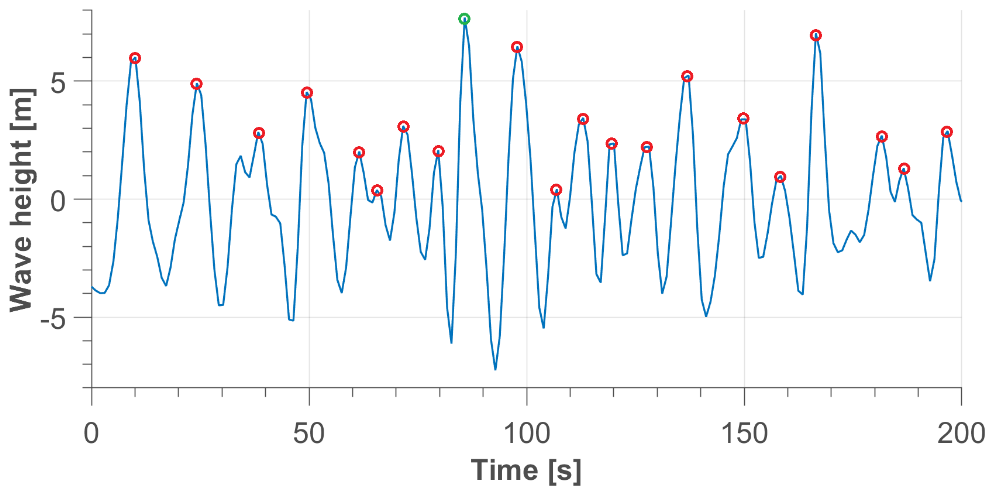

Figure 5-15.: Surface elevation for a 200 seconds window

For each one of the 3-hour events that compose a storm, the global maxima of the relative wave elevation r_P follow a Rayleigh distribution and are shown in red in the Figure **5-15**.

$$F_{R_P}(r_P) = 1 - exp\left[-\frac{1}{2}\left(\frac{r_P}{\sigma_{R_P}}\right)^2\right]$$

where σ_{R_P} is the standard deviation of the process. This is the square root of the variance of the wave spectrum of the response as it is explained in the section 5.1.

The distribution function for the 3-hour maximum relative wave elevation, $F_{R_{P3h}}$, is given in the equation (5-8),

$$F_{R_{P3h}}(r_P) = \left\{1 - exp\left[-\frac{1}{2}\left(\frac{r_P}{\sigma_{R_P}}\right)^2\right]\right\}^{n_{3h}} \tag{5-8}$$

The quantity n_{3h} is known as the expected zero upcrossing period, and it expresses the expected number of crest heights during 3 hours (10.800 seconds) for any sea state. The value of the zero upcrossing wave period T_z is calculated as shown in the equation (5-2).

$$n_{3h} = \frac{10800}{T_z} \tag{5-9}$$

In this example, the calculated variance is 5.2625 then the standard deviation is 2.2904. The number of waves in 3 hours (zero upcrossing period) results 1199.18. The equation (5-8) then must be determined for each 3-hour event (step) in a storm and in this cases results:

$$F_{R_{P3h}}(r_P) = \left\{1 - exp\left[-\frac{1}{2}\left(\frac{r_P}{2.2904}\right)^2\right]\right\}^{1199.18}$$

5.5. Distribution function of the storm maximum response

As it was defined by (Haring & Heideman, 1978), a storm might be composed of M consecutive 3-hours sea states. This sea states are considered independent and identically distributed. Then, the distribution function of the maximum is obtained by multiplying the distributions of all the 3-hour event. The Figure **5-16** shows the 3 hour maximum for the 6 different events that compose the storm of the Figure **3-1** plotted along with the Storm Maximum Response.

$$F_{R_P|storm}(r_P|storm) = \prod_{i=1}^{M}\left\{1 - exp\left[-\frac{1}{2}\left(\frac{r_P}{\sigma_{RP_i}}\right)^2\right]\right\}^{n3h_i} \tag{5-10}$$

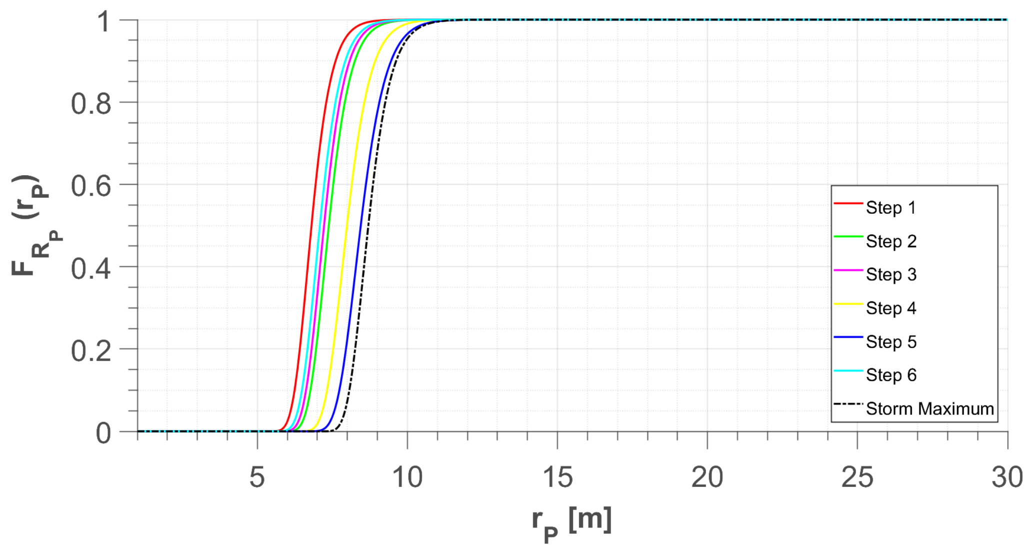

Figure 5-16.: 3 hour maximum for the 6 different events that compose the storm plotted along with the Storm Maximum Response. Point 363.

According to (Haver, 2017b), the exact distribution i.e $F_{R_P|storm}(r_P|storm)$, represented by the equation (5-10) can be approximated to be a Gumbel distribution (5-11) that is read as the conditional distribution of the storm maximum response r_P given the most probable largest storm maximum, $\tilde{x}$ (mpm):

$$F_{R_P|\tilde{X}}(R_P|\tilde{X}) = exp\Bigg\{ - exp\left[- \frac{r_P - \tilde{x}}{\tilde{x}\beta} \right] \Bigg\} \tag{5-11}$$

where $\tilde{x}$ is considered to be the location parameter and β the scale parameter. By definition, the expected value and the variance for the Gumbel distribution are:

$$E\left(R_P|\tilde{X}\right) = \tilde{x} + 0.5772\beta$$
$$Var\left(R_P|\tilde{X}\right) = \frac{\pi^2}{6}(\tilde{x}\beta)^2$$

In order to valid this approximation, the conditions to be fulfilled are:

- The Gumbel distribution should have the same most probable largest storm maximum, $\tilde{x}$, as the exact distribution.
- The variance of the Gumbel distribution should be equal to the variance of the exact distribution.

As is established in (Tromans & Vandersohuren, 1995), when $F_{R_P|storm}(r_P|storm)$ is approximated to be Gumbel distributed, the most probable largest storm maximum, $\tilde{x}$, is obtained when $F_{R_P|storm} = 1/e$. Figure **5-17**.

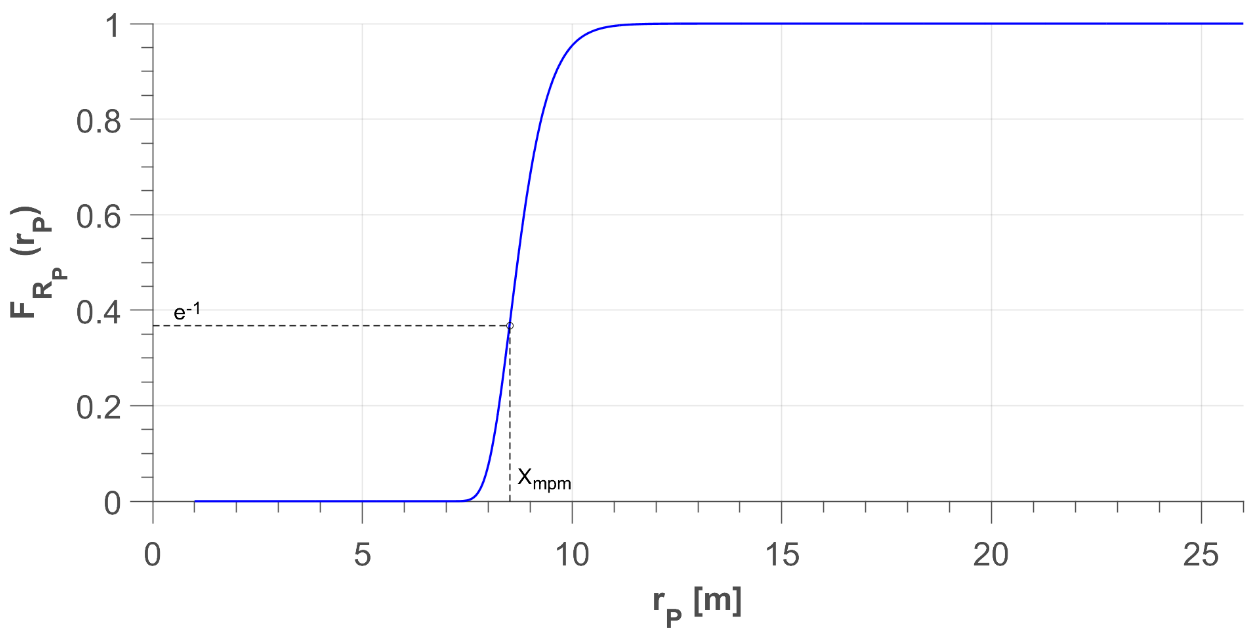

Figure 5-17.: Method to obtain the most probable largest storm maximum. Point 363.

Once the mpm value is obtained, (in this example is 8.52 m), the next step is to calculate the variance of the exact distribution (equation (5-10)). This can be done by calculating first the expected value and then the variance itself defined theoretically by the equations (5-12) and (5-13).

$$E(X) = \int_0^\infty x \cdot f_X(x)dx \tag{5-12}$$

$$Var(X) = \int_0^\infty [x - E(X)]^2 \cdot f_X(x)dx \tag{5-13}$$

Since the calculations made in this paper are done numerically, it is possible to develop a formula to apply the theoretical definition of expected value and variance to the obtained values. This can be done using the relation between Probability Density Function (pdf) and Cumulative Density Function (cdf) where the first is equal to the derivative of the second with respect to the random variable x. We develop then, the equation (5-14).

$$\begin{aligned} f_X(x) &= \frac{dF_X(x)}{dx} \\ f_X(x)dx &= dF_X(x) \end{aligned} \tag{5-14}$$

Then, since we have calculated already the values of the cdf (F_{r_P}), at each interval it is possible to calculate $dF_X(x)_i = F_X(x)_{i+1} - F_X(x)_i$ and then add all together as is shown in the equations (5-15) and (5-16).

$$E(X) = \sum_{i=1}^{n-1} x_i \cdot dF_X(x)_i \tag{5-15}$$

$$Var(X) = \sum_{i=1}^{n-1} [x_i - E(X)]^2 \cdot dF_X(x)_i \tag{5-16}$$

Once the exact variance is calculated, this value can be equated to the variance of the Gumbel distribution. Then, together with the mpm of each storm, the scale parameter β is calculated with the equation (5-17).

$$\begin{aligned} \sigma^2 = Var(R_P|\tilde{X}) &= \frac{\pi^2}{6}\,(\tilde{x}\,\beta)^2 \\ \frac{\pi^2}{6}\,(\tilde{x}\,\beta)^2 &= \sigma^2 \\ \frac{\pi}{\sqrt{6}}\,\tilde{x}\,\beta &= \sigma \\ \beta &= \frac{\sigma \cdot \sqrt{6}}{\pi\,\tilde{x}} \end{aligned} \tag{5-17}$$

The Figure **5-18** shows the comparison plot of the exact distribution and the approximated distribution in a Gumble paper. Notice that the mpm value returns zero in the vertical axis of the Gumble paper for both distributions.

$$
\begin{aligned}
-Ln\left[-Ln\left(F(R_P|\tilde{X})\right)\right] &= 0 \\
-Ln\left(F(R_P|\tilde{X})\right) &= e^0 \\
F(R_P|\tilde{X}) &= 1/e
\end{aligned}
$$

which is the value of the mpm.

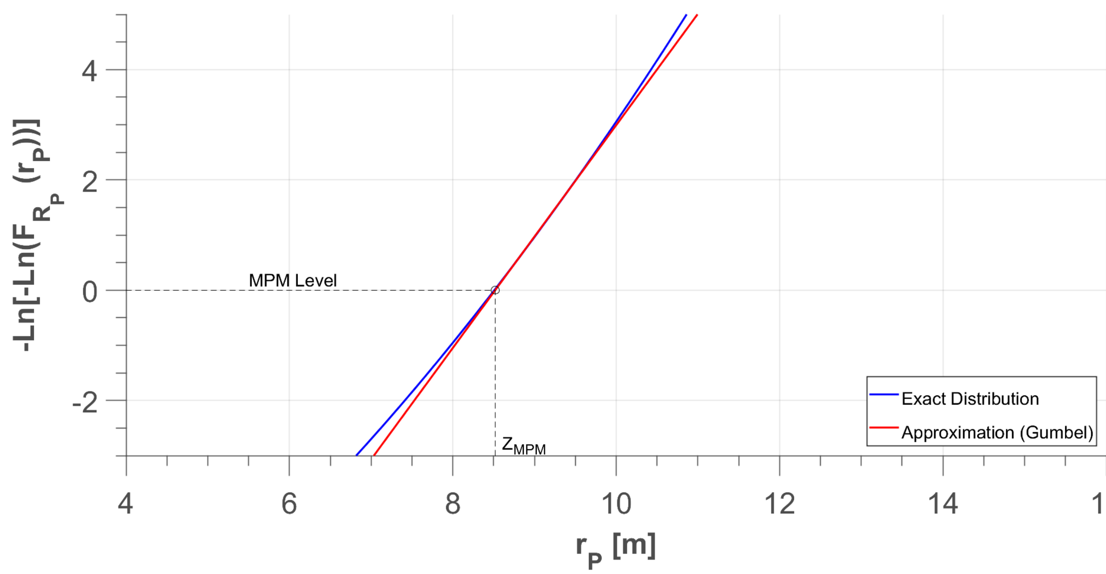

Figure 5-18.: Comparison between the exact and the approximate storm distribution function. Point 363.

The storm maximum response is obtained for all the storms found above the determined threshold. Each one of the storms will have a different value of β. The scatter of the obtained values for all the storms above 10 meters threshold, along with the mean value of β, are shown in the Figure **5-19**.

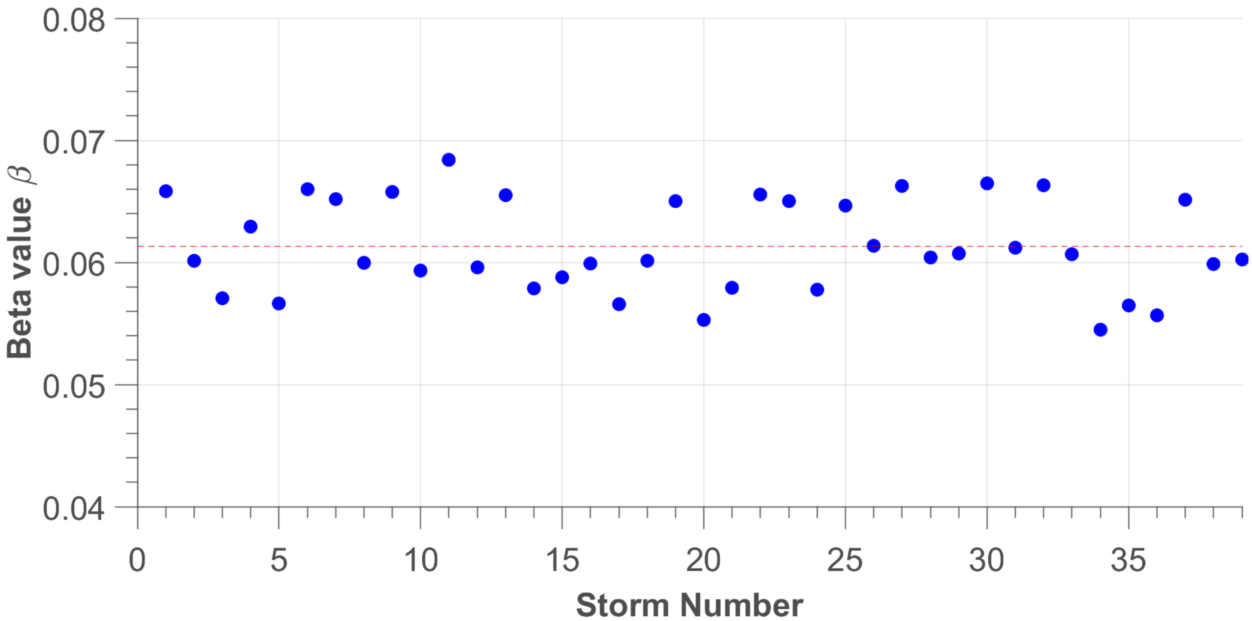

Figure 5-19.: Beta values for all storms above 10m threshold. Point 363.

We can appreciate that the different values of β are well distributed around the mean value. This is the reason why we can use, for the short term analysis, a constant value of beta for all the storms found above the threshold of 10m equal to 0.0613.

Same procedure is executed for the thresholds of 6, 7, 8 and 9m and the results are expressed in the Table **5-1**

Table 5-1.: Values for the Scale parameter β of the Gumbel approximation

Threshold	β
6 m	0.0579
7 m	0.0589
8 m	0.0598
9 m	0.0602
10 m	0.0613

There is a way to determine if the storm maximum response, approximated with a Gumble distribution, is actually representing the maximum value of the 3h-events that compose the storm. This is made by simulation by generating a probability value and then assign it to the 3-hour event distribution function (equation 5-8). Doing this, we can find one possible maximum for each 3h event in the storm. Then, the plot of the highest of this maximum against its probability value is shown in the Figure **5-16** together with the exact and the approximate storm distribution functions. The results of the simulation are as expected close to the exact and approximate distribution that lies one over the other.

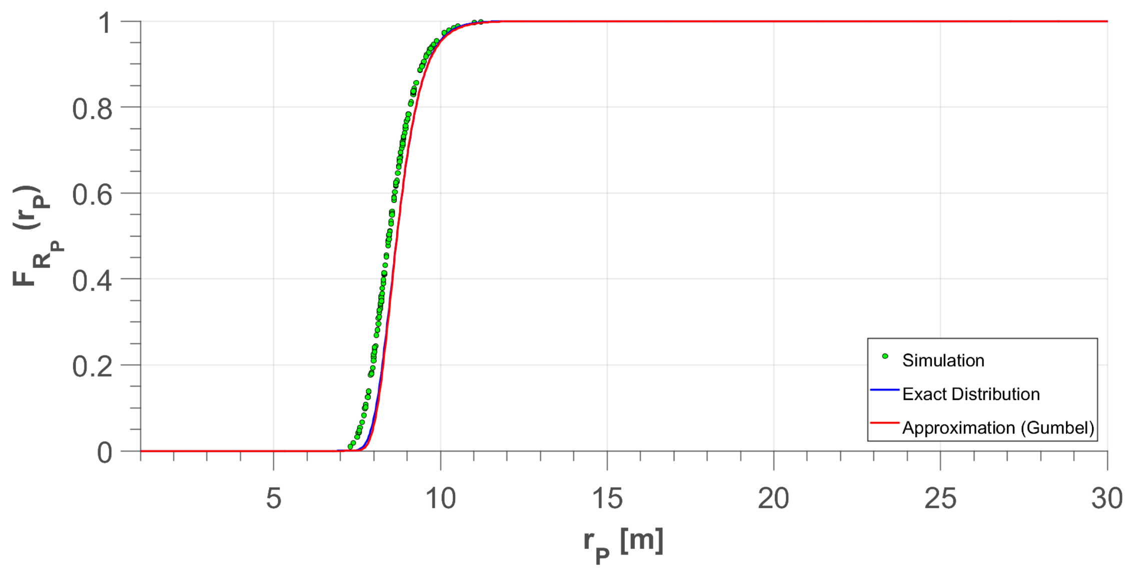

Figure 5-20.: Comparison between the exact and the approximate storm distribution function. Point 363.

As an exercise to check the evolution of the storm distribution when the threshold is lowered, we can use the values from the storm example shown in the Figure **3-1**, and plot the different storm distributions. Then, we can compare the values of the most probable maximum, $\tilde{x}$ (mpm), obtained for each case. This will be a first approach of determining the optimal threshold in the POT methodology.

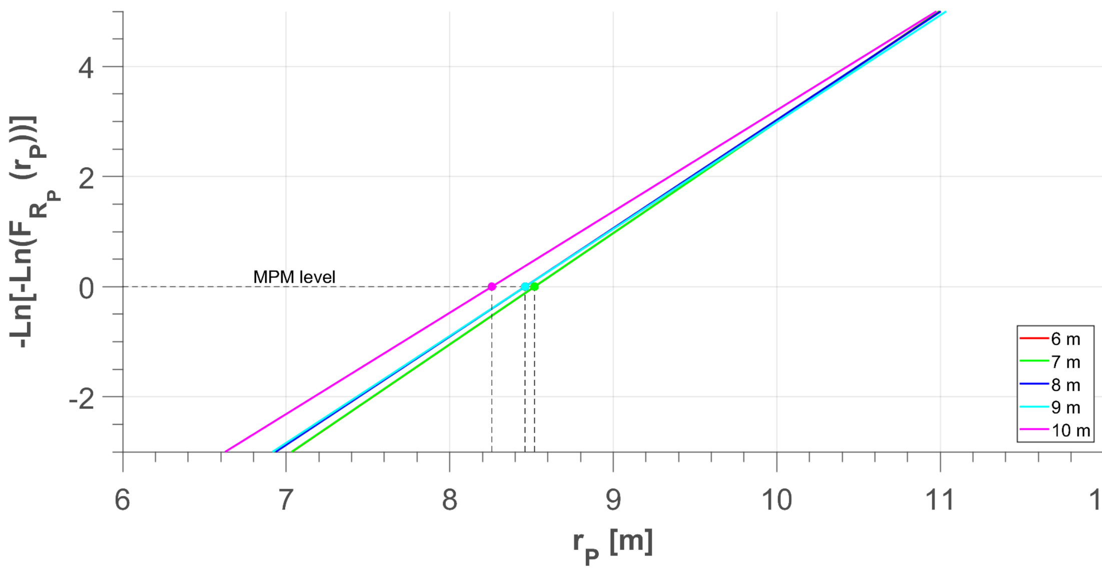

Figure 5-21.: Comparison of storm distribution functions for different thresholds. Point 363.

Table 5-2.: Values of the most probable storm maximum (mpm) for each storm distribution. Storm example.

Threshold	MPM value
6 m	8.52 m
7 m	8.52 m
8 m	8.46 m
9 m	8.46 m
10 m	8.26 m

The Table **5-2** presents the mpm values confirming what is shown in the Figure **5-21**. The storm distribution for the thresholds 6m and 7m are the same. Similar occurs with the thresholds 8m and 9m. The 10m threshold gives a lower value and underestimates the value of the $\tilde{x}$ (mpm). This is easy to understand since in this particular case, the storm is conformed by only one 3 hours event. Also it is important to notice that the most probable value is different of the peak value of the storm.

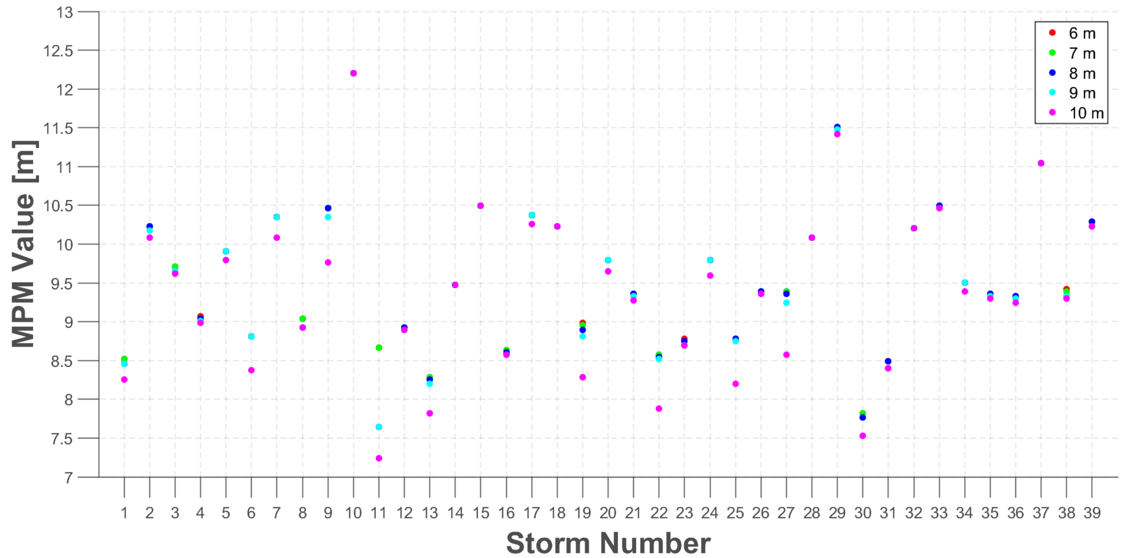

Figure 5-22.: Comparison of $\tilde{x}$ (mpm) values for all the storms above the 10 m threshold and its evolution. Point 363.

The Figure **5-22** help us to understand the evolution of the mpm for all the storms found above 10m when the threshold is lowered. Notice that the case showed above corresponds to the Storm 1. We can confirm how in all the cases, the 10m threshold values are giving the lower values of mpm. Then this threshold is discarded to be set as the optimal one. The values for 8m and 9 m threshold coincide most of the cases but are still lower than the values obtained with the 7m and 6m threshold. This gives a clue that the optimal threshold must be lower or equal to 8 meters.

5.6. Distribution of the most probable maximum

For each storm above the threshold, one most probable largest storm maximum is found. Then, these mpm are chosen to be modeled by a 3 parameter Weibull distribution:

$$F_{\tilde{X}}(\tilde{x}) = 1 - exp\left[-\left(\frac{\tilde{x}-\lambda}{\alpha}\right)^{\beta}\right] \tag{5-18}$$

where α is the scale parameter, β is the shape parameter and λ is the location parameter.

The probability density function $f_{\tilde{X}}(\tilde{x})$ is the derivative of the equation (5-18) with respect to $\tilde{x}$ with constant parameters

$$f_{\tilde{X}}(\tilde{x}) = \frac{\beta}{\alpha}\left(\frac{\tilde{x}-\lambda}{\alpha}\right)^{\beta-1} \cdot exp\left[-\left(\frac{\tilde{x}-\lambda}{\alpha}\right)^{\beta}\right] \tag{5-19}$$

Using the method of moments, the parameters α and β and λ are calculated

5.6.1. Method of moments

This method involves equating the population mean and variance to the corresponding sample mean ($\overline{x}$) and sample variance (σ^2) and solving for the parameters, the results being the moment estimators.(Walpole, Myers, Myers, & Ye, 2014). Since the cumulative density function that is going to be estimate has 3 parameters, it is necessary to include the skewness, (γ_1), of the population and the sample. They are defined in the equations (5-20), (5-21) and (5-22) respectively. It is possible that two storms have the same mpm, then the method of moments must be applied with the non repeated values of mpm as the population.

$$\mu_{\tilde{X}} = \lambda + \alpha\Gamma\left(1+\frac{1}{\beta}\right) \tag{5-20}$$

$$\sigma^2_{\tilde{X}} = \alpha^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right] \tag{5-21}$$

$$\gamma_1 = \frac{\Gamma\left(1+\frac{3}{\beta}\right) - 3\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right) + 2\Gamma^3\left(1+\frac{1}{\beta}\right)}{\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]^{3/2}} \tag{5-22}$$

The estimated values for the hindcast data available are shown in the Table **5-3**

Table 5-3.: Parameters for the 3-parameters Weibull distribution function

Weibull Parameters					
Threshold	6m	7m	8m	9m	10m
α **(scale)**	3,9826	3,4713	2,7137	2,7183	2,8651
β **(shape)**	2,4719	2,3326	2,0211	2,1773	2,4434
λ **(location)**	4,0760	4,8421	5,9518	6,4380	6,7811

In order to indicate the adequacy of the fitted distribution, and if it makes a correct interpretation of the data, we should first calculate the empirical distribution function as shown in the equation (5-23). Let us first order the sample of size n in increasing order $x_1 \leq x_2 \leq \cdots x_k \cdots \leq x_n$. Then, the k-esim element will have a empirical probability calculated as follows:

$$\hat{F}_X(x_k) = \frac{k}{n+1} \tag{5-23}$$

From the equation (5-18), the cumulative distribution function arranged to be plotted in a probability paper is:

$$Ln\left[-Ln\left(1 - F_{\tilde{X}}(\tilde{x})\right)\right] = \beta Ln(\tilde{x} - \lambda) - \beta Ln\alpha$$

The Figure **5-23** shows the sorted data plotted it along with the fitted distribution on a probability paper.

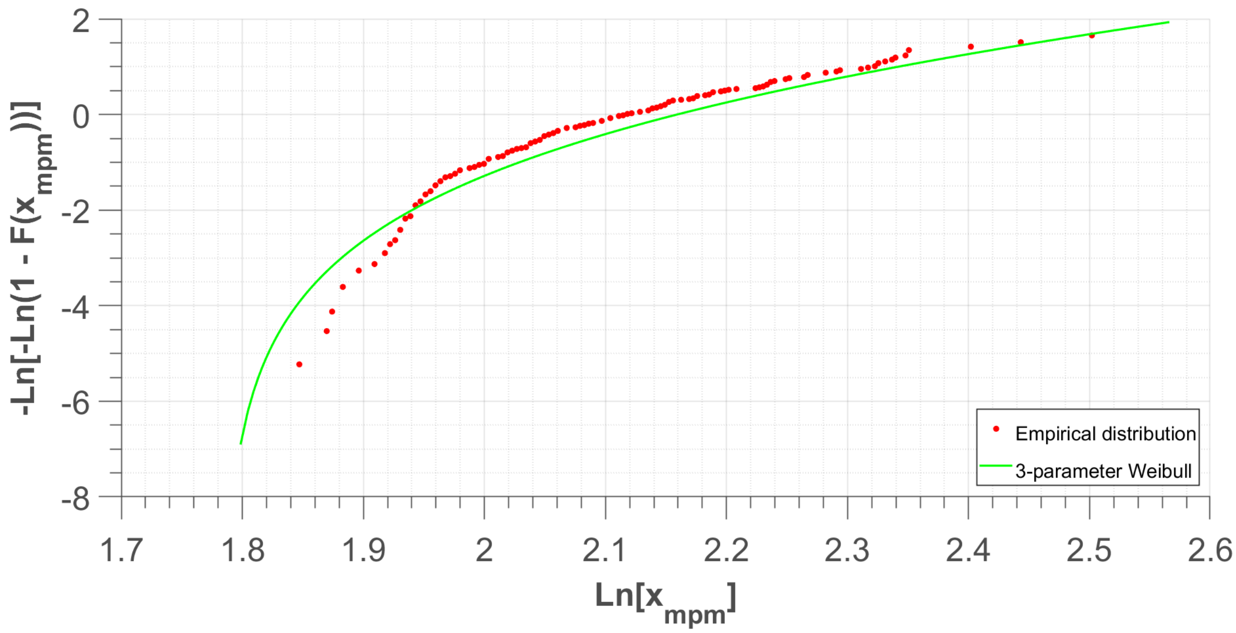

Figure 5-23.: Empirical distribution of mpm vs. fitted distribution. Threshold 8m. Point 363.

As we can see, the fitted Weibull model is good description of the variable mpm mostly in the upper part. For further uses of this function, values of the mpm lower than λ will not be taken into account.

In order to have an idea of the order of magnitude of the relative wave elevation, it is valid at this point to calculate the 100 yr and 10.000 yr most probable maximum values. This will be taken as a reference since only the short term analysis has been carried out and no long term analysis has been done of the relationship between the relative wave elevation (r_P) and the most probable values of each storm ($\tilde{x}$).

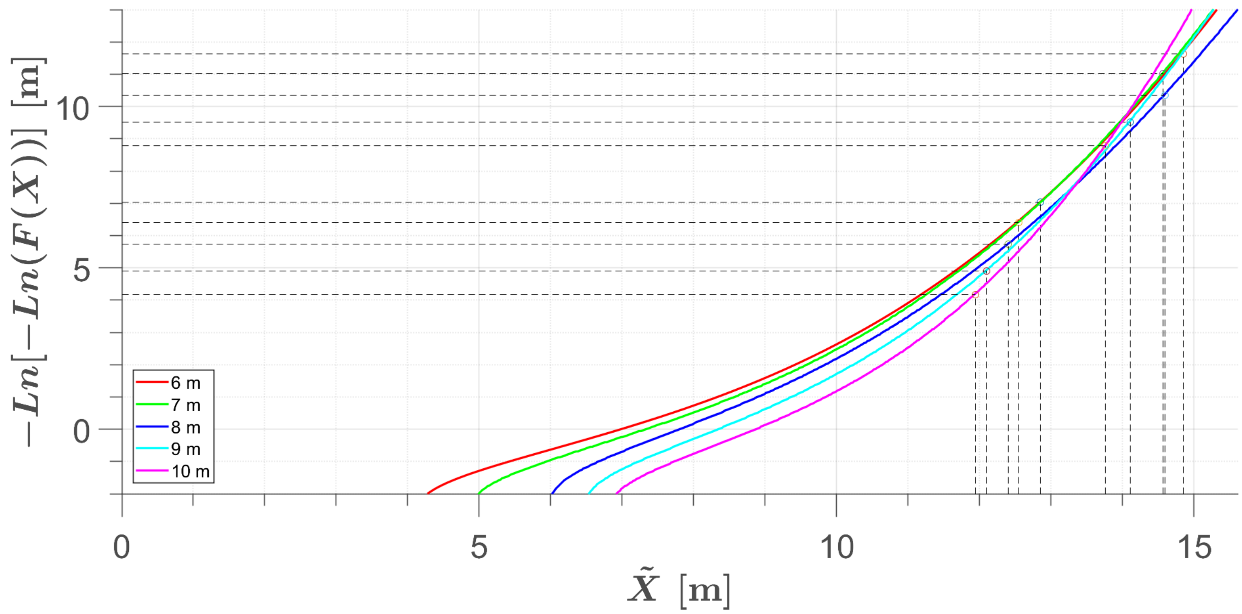

Figure 5-24.: 100 yr and 10.000 yr most probable maximum values. Point 363.

The Figure **5-24** shows the long term distribution of the most probable values for each threshold in a Gumbel plot. The 100 and 10.000 yr values are written in the table **5-4**.

Table 5-4.: Long term distribution of the most probable values

Threshold	6m	7m	8m	9m	10m
100yr	12,85	12,55	12,40	12,09	11,94
10.000yr	14,84	14,56	14,59	14,10	13,75

Once the Long term Analysis will be carried out, these values will most likely tend to increase, then the real values of airgap are defined.

5.7. Long Term Analysis

If the structural response depends both on significant wave height and on the period and also on the previous history of the wave process, the most consistent method for predicting characteristic loads is some sort of a stochastic long term response analysis (Haver, 2017b). The long term distribution of the maximum response is obtained by convolution of the conditional distribution of the maximum given the most probable maximum(i.e. the short term variability) with the long term distribution of the most probable maximum, (5-24)

$$F_{R_P}(r_P) = \int_{\tilde{X}} F_{R_P|\tilde{X}}(r_P|\tilde{x}) \cdot f_{\tilde{X}}(\tilde{x})d\tilde{x} \tag{5-24}$$

where $F_{R_P|\tilde{X}}(r_P|\tilde{x})$ represents the short term variability of the relative wave elevation given the most probable maximum response and $f_{\tilde{X}}(\tilde{x})d\tilde{x}$ represents the long term variability of the most probable largest response. (Sandbakken, Haver, & Larsen, 2017)

This convolution integral is calculated numerically using the equation (5-14) the equation becomes for each value of relative airgap.

$$F_{R_P}(r_P) = \sum_{i=1}^{n-1} F_{R_P|\tilde{X}}(r_P|\tilde{x})_i \cdot dF_{\tilde{X}}(\tilde{x})_i \tag{5-25}$$

The number of storms per year is calculated by taking the average number of storms in 60 years for a determined threshold. These values and the the annual probability of exceedance of 10^{-2} and 10^{-4} are shown in the Table **5-5**. The value with an annual probability of being exceeded can be estimated by (5-26).

$$1 - F_{R_P}(r_{P_q}) = \frac{q}{m_q} \tag{5-26}$$

where m_q is the number storms and q is the annual exceedance probability.

The long term distribution of the storm maximum is shown in the Figure **5-25** and the 100 or 10.000 year airgap are in the Table **5-6**:

Table 5-5.: Exceedence for each threshold

Threshold	6m	7m	8m	9m	10m
Average of storms per year	11.25	6.10	3.10	1.35	0.65
$F_X(x)$ 100yr	0,999111	0,998360	0,996774	0,992592	0,984615
$F_X(x)$ 10.000yr	0,999991	0,999983	0,999967	0,999925	0,999846

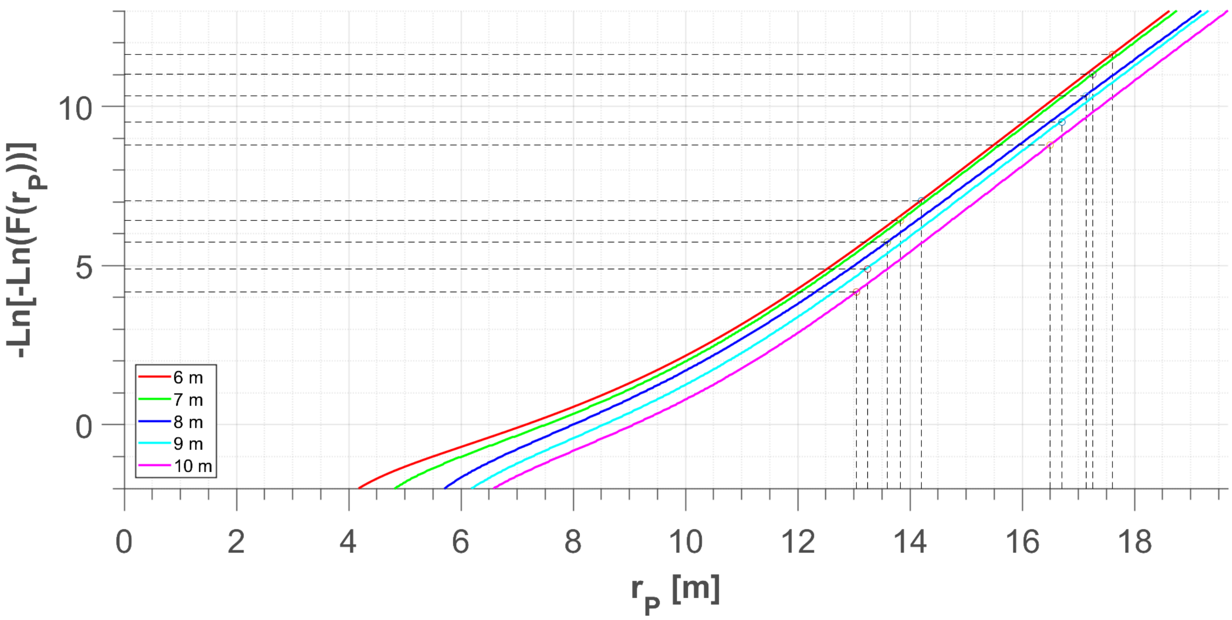

Figure 5-25.: Long Term Airgap Values

Table 5-6.: Relative wave elevation. Point 363.

Threshold	6m	7m	8m	9m	10m
100yr	14,21	13,83	13,59	13,25	13,05
10.000yr	17,60	17,25	17,14	16,70	16,50

There is a difference of around 2 meters between the 100 yr values of the relative wave elevation (r_P) and the most probable maximum ($\tilde{x}$). For the 10.000 yr level this difference is around 4 meters. This indicates that the analysis of the mpm alone cannot be used as a design parameter of airgap in a semi submersible. The 100 and 10.000 year relative wave elevation for all the points is shown in the Chapter 6.

One of the objectives of this document is to determine whether the Swell is decisive in the airgap calculations. For this reason, more simulations were ran with different input parameters. In the first simulation, the swell was discarded and relative airgap calculations were performed for all the points only for the Wind Sea data (wave height, spectral peak period and direction). In the second one, the significant wave height is taken from the Total sea. The Table **5-7** has the information of the simulations.

Table 5-7.: Input data for airgap simulations.

Simulation Number	**Wave Height** H_s	**Spectral Peak Period,** T_P	**Wave Direction**
1	Wind Sea and Swell	Wind Sea and Swell	Wind Sea and Swell
2	Wind Sea	Wind Sea	Wind Sea
3	Total Sea	Wind Sea	Wind Sea

On the next chapter, additional results and analysis are developed.

6. Results

The amount of calculations and variables used to obtain the relative wave elevation on each point is considerably hight. To make an analysis of each one of this variables for each threshold may be extensive. In this chapter we will focus on analyzing the impact, effect and severity of the most probable storm maximum (mpm) for each threshold. At the end of the chapter, the results for relative air-gap in each point will be shown.

6.1. MPM Severity

The most probable storm maximum is the most important parameter in the short term analysis. The it is very useful to relate it for example with a measurable variable such as significant wave height (H_s), spectral peak period (T_P) and wave direction. This can be useful to identify what combination of H_s and T_P can generate a high mpm value.

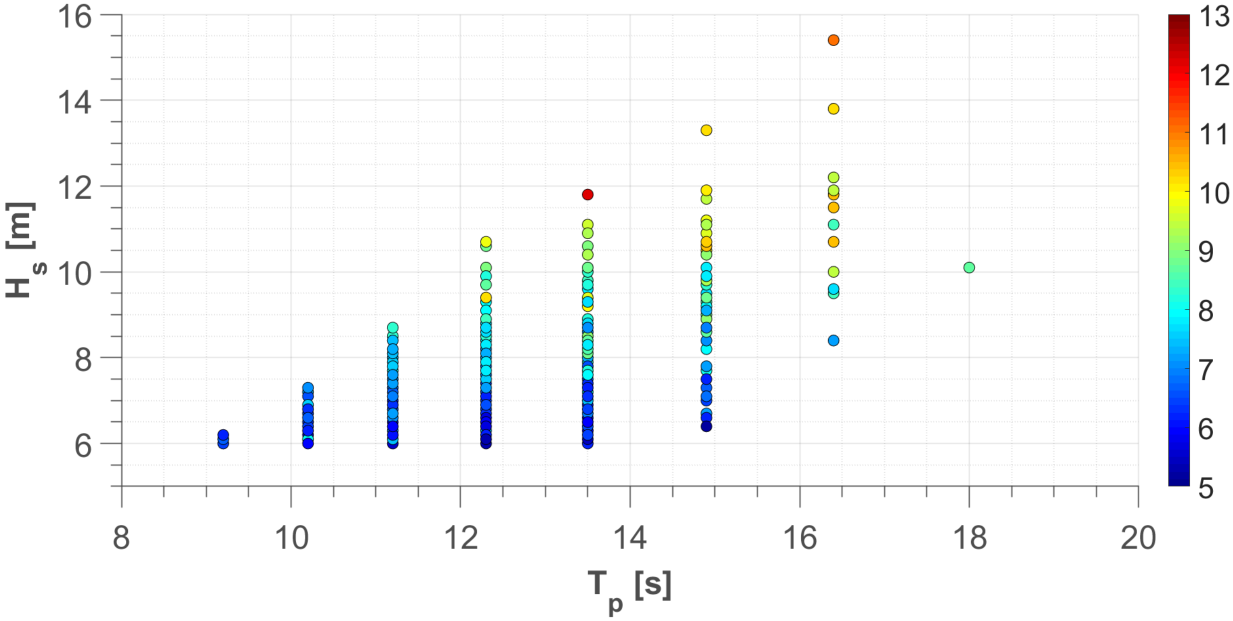

Figure 6-1.: MPM Severity. Point 363.

The Figure **6-1** shows the severity of the mpm for the peak of the storms found above the 6m threshold. It can be noticed how the value of mpm increases with high values of H_s and this means high values of relative wave elevation (r_P), then high air gap.

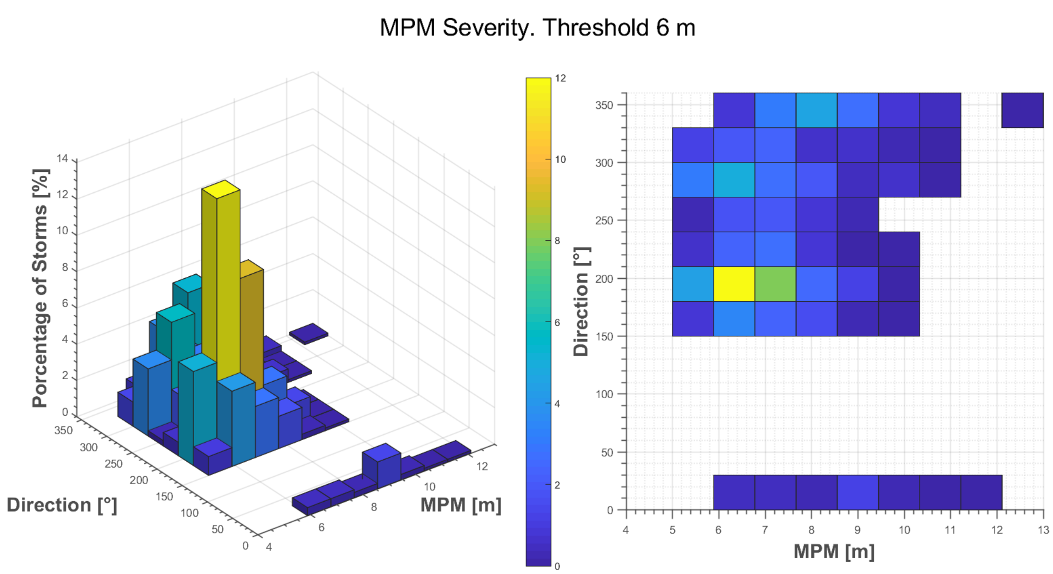

Figure 6-2.: MPM Direction Severity. Point 363.

Regarding the wave direction, the mpm has a correspondent severity. The Figure **6-2** shows the relation between the wind sea direction and the percentage of storms with a specific value of mpm. In the case of 6m threshold, around 10% of the total of the storms have an mpm between 6 and 7 meters and are coming from South.

6.2. Air-gap

In the previous chapter, the air gap analysis was carried out only in one location on the platform. (Point 363). This point was of interest since is located in the corner, far from the center of gravity of the platform, then the effect of roll and pitch should be important. This point is in the corner facing the worst weather conditions from South-West. No conclusion can be made only by the results obtained in this location. It is very necessary to carry out the same analysis for all the points provided by the WAMIT files and then determine the critical point where the relative wave elevation is maximum. This can be used as design criteria.

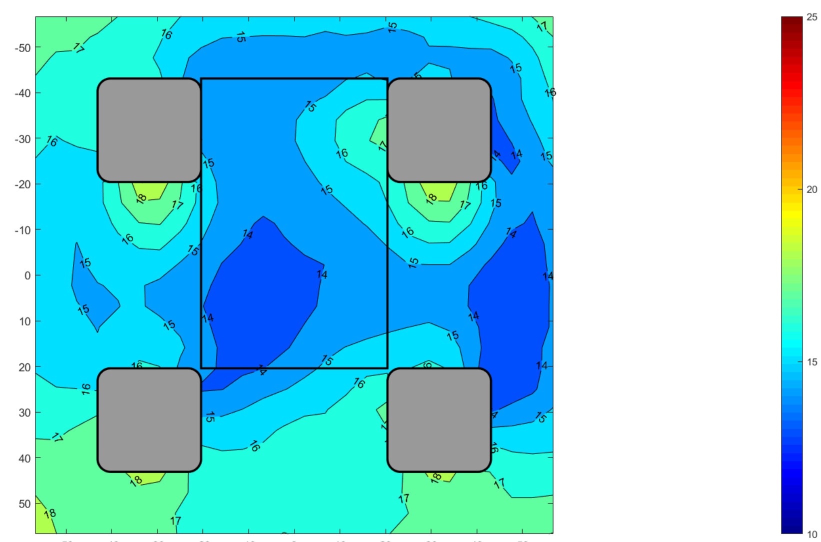

Figure 6-3.: 100 yr relative wave elevation.

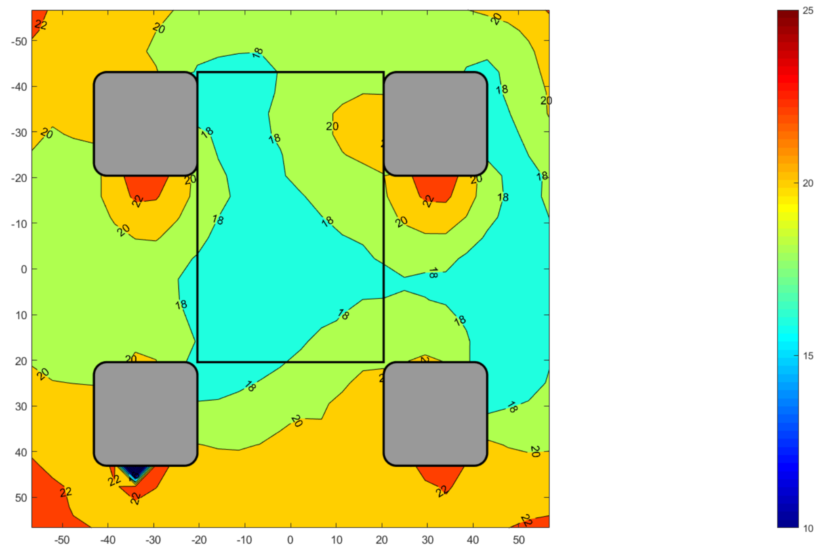

Figure 6-4.: 10.000 yr relative wave elevation.

The Figure **6-3** shows the evolution of the relative wave elevation under and around the semi submersible with a return period of 100 years. With this information, a 100 yr air-gap can be defined since there is information about all the points under and around the platform. Then, the highest relative wave elevation calculated in the edges of the platform can determine the initial air-gap of the platform. Columns are shown in grey.

An interesting phenomena happens close to the platform legs. The calculated relative wave elevation increases, around this area, with the waves coming from South and West which are the main direction for this location as seen before. This implies that the necessary airgap in that location needs to overcome this action and increase considerably. This is why is important to apply the procedure of this document in a region on the edge of the platform far from the columns. The waves smashing the columns and the water run up along the columns hitting the underside the deck is a non linear effect, studied with the use of simulation and CFD analysis.

The Table **6-1** shows the recommended minimum airgap values for each threshold. As it was expected, the values stabilizes when the threshold for the storms is lowered below 8m.

This value was estimated by taking the maximum relative wave elevation from all the points under and around the platform and approximate it to the next integer. The choose of any initial air-gap above the values shown in the table, will be a safe decision for air-gap design.

Table 6-1.: Minimum air-gap for various thresholds [m].

Threshold	6m	7m	8m	9m	10m
100yr	19	19	19	18	18
10.000yr	24	24	24	23	23

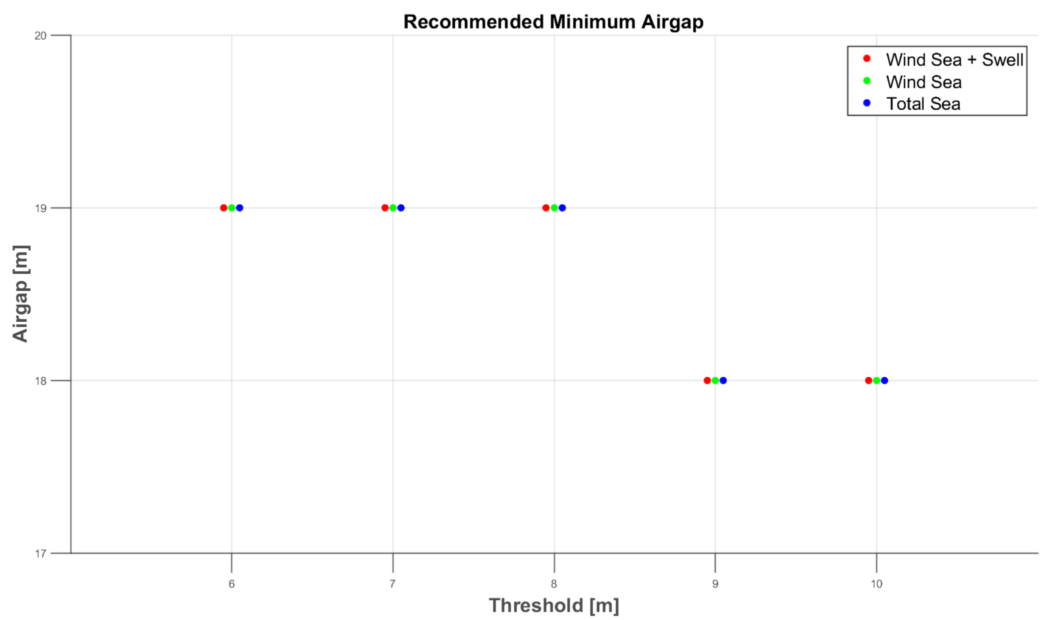

Figure 6-5.: Recommended Minimum Airgap

7. Conclusions

The results of the simulations can be understood in three different aspects.

The most important subject and the core of this document is the airgap calculation. We have shown a procedure to calculate airgap values with a established return period using the POT Methodology. This method reveals that the storm distribution function remains the same when lowering the threshold with a small underestimation of the most probable maximum for the 10 meters threshold storms. The Weibull 3 parameters distribution function is suitable to represent the mpm behavior even thought there are some differences in the lower tail of the function.

The threshold becomes then the value to optimize taking into account the simulation time and the obtained results. A threshold analysis can not be done without comparing results between high an low thresholds, then this makes necessary to establish a star point and an end point bases on the characteristics of the data that it is being used. Here, the significant wave height, and an initial threshold of 10 meters gave an reasonable amount of storms to continue with the procedure. The threshold was finally lowered to 6 meters in a combination of simulation time and stabilization of the obtained results. The final airgap results still show how it is underestimated for the two highest thresholds and then stabilizes as the threshold is lowered. Then, the band between 6 an 8 meters is established as the optimal threshold.

The final airgap results confirm the fact that including the swell information is unnecessary. Even though the calculations are considered more completes including swell, the results shows that its influence is low, it demands more time for the simulation and the results are the same. If still the swell information want to be taken into account, the best way to do it is using the total sea values as it was done in the third simulation.

7.1. Further Work

7.1.1. Hindcast data, Tp correction

Airgap is sensitive to wave period. In the NORA10 data, the peak period values are discrete unevenly giving a bad resolution of the calculated sea conditions. In order to make a correction, the procedure explained in (Haver, 2017b) can be applied. The results may vary but the change will allow to identify easily for instance the most dangerous sea states for airgap calculations.

7.1.2. Asymmetry factor α

According to (DNVGL, 2017), the asymmetry factor should be selected according to the location where the airgap is calculated and the wave direction. This certainly affect the minimum airgap, increasing it and then making the results more conservatives than the ones obtained in this document. This will also have an influence on the non linearities for the wave-columns interaction.

7.1.3. Occurrence of wind

Since the NORA10 information contains data of wind speed and direction, air-gap assessment should be done with this information. The aim will be to develop a joint model that accounts the occurrence of wind, wind-sea and swell sea.

References

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Appendix

A. Histogram of Wind Sea and Swell Direction

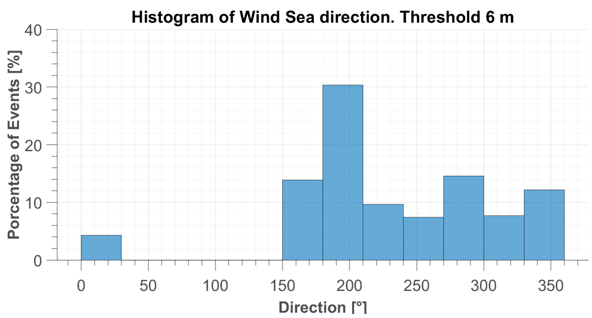

Figure A-1.: Histogram Wind Sea Direction. Threshold 6m.

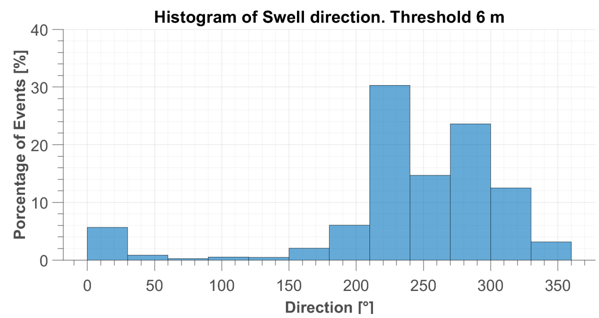

Figure A-2.: Histogram Swell Direction. Threshold 6m.

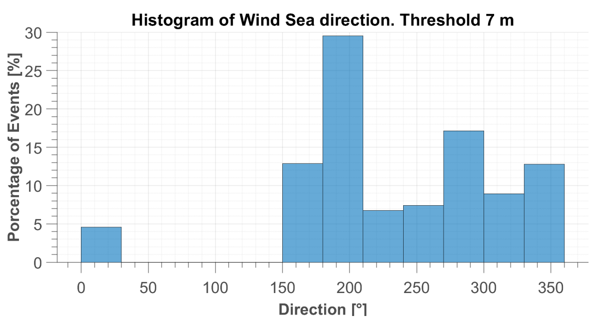

Figure A-3.: Histogram Wind Sea Direction. Threshold 7m.

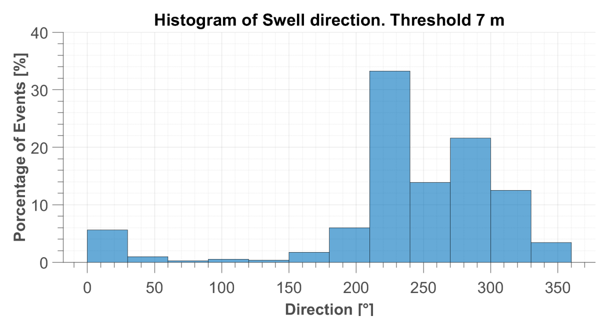

Figure A-4.: Histogram Swell Direction. Threshold 7m.

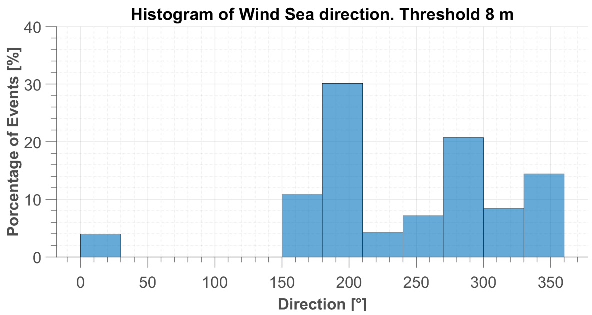

Figure A-5.: Histogram Wind Sea Direction. Threshold 8m.

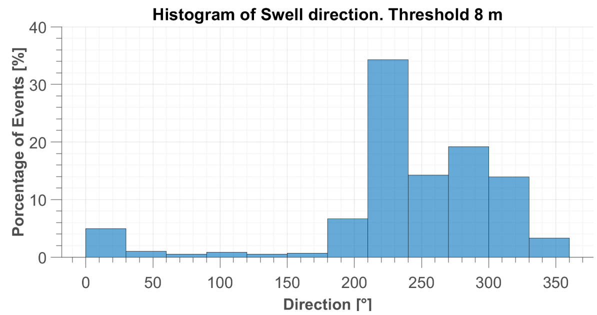

Figure A-6.: Histogram Swell Direction. Threshold 8m.

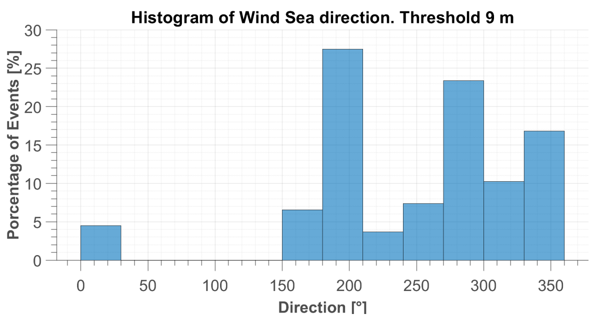

Figure A-7.: Histogram Wind Sea Direction. Threshold 9m.

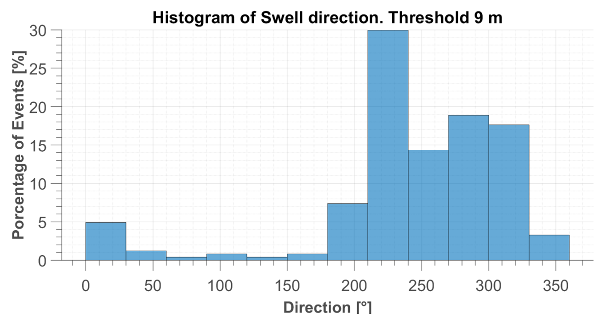

Figure A-8.: Histogram Swell Direction. Threshold 9m.

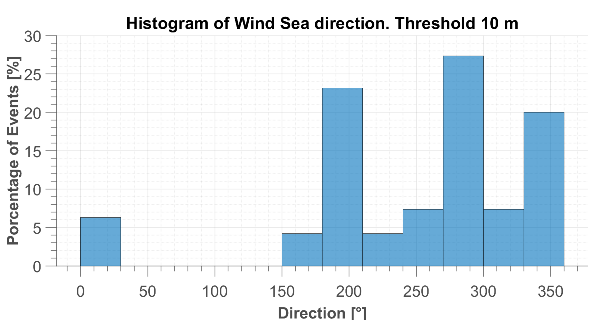

Figure A-9.: Histogram Wind Sea Direction. Threshold 10m.

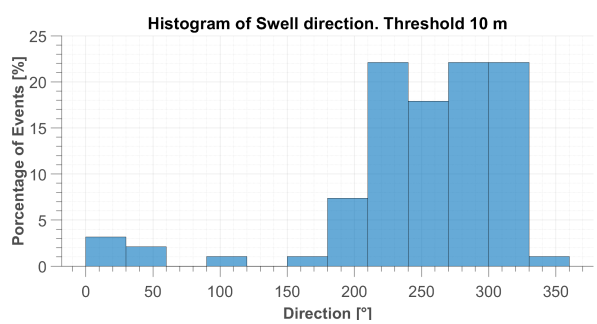

Figure A-10.: Histogram Swell Direction. Threshold 10m.

B. Histogram of Wind Sea and Swell Significant Wave Height

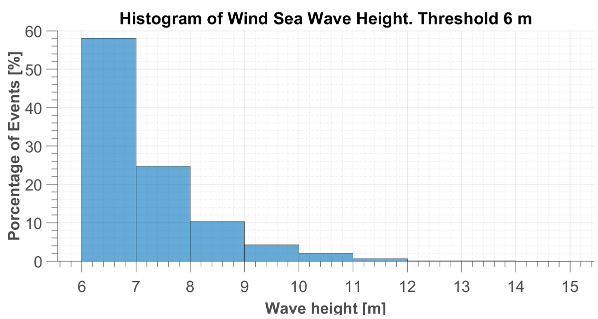

Figure B-1.: Histogram Wind Sea Significant Wave Height. Threshold 6m.

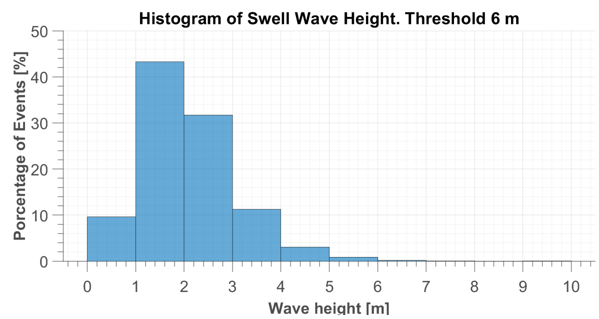

Figure B-2.: Histogram Swell Significant Wave Height. Threshold 6m.

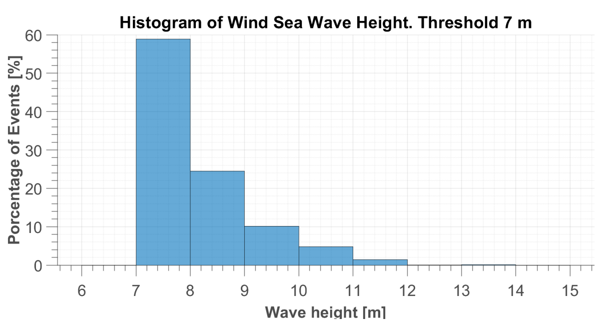

Figure B-3.: Histogram Wind Sea Significant Wave Height. Threshold 7m.

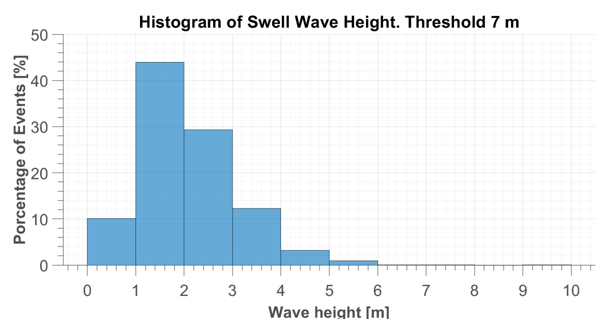

Figure B-4.: Histogram Swell Significant Wave Height. Threshold 7m.

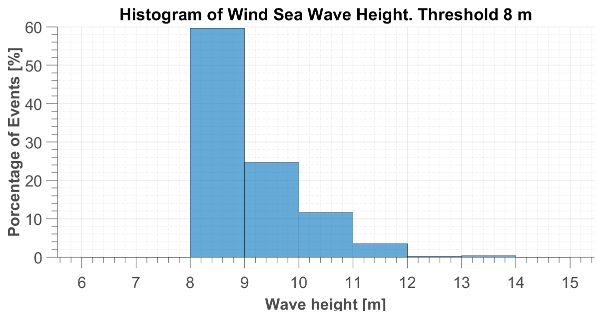

Figure B-5.: Histogram Wind Sea Significant Wave Height. Threshold 8m.

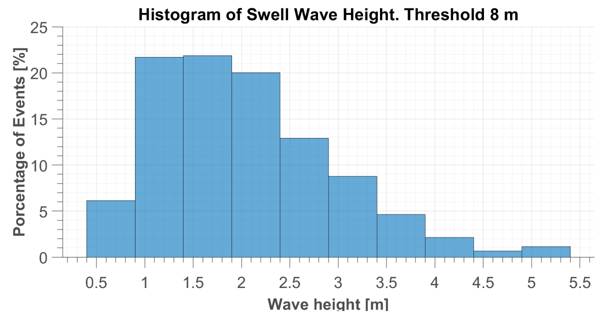

Figure B-6.: Histogram Swell Significant Wave Height. Threshold 8m.

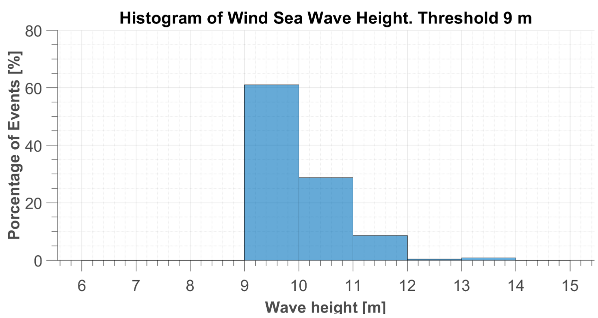

Figure B-7.: Histogram Wind Sea Significant Wave Height. Threshold 9m.

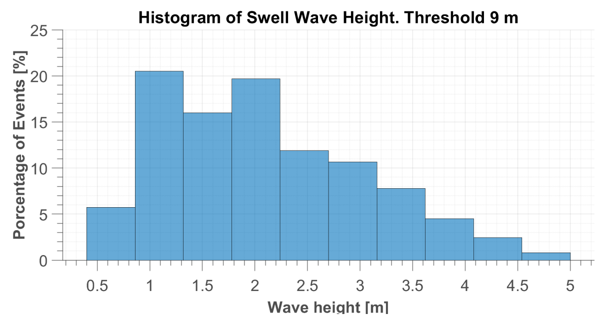

Figure B-8.: Histogram Swell Significant Wave Height. Threshold 9m.

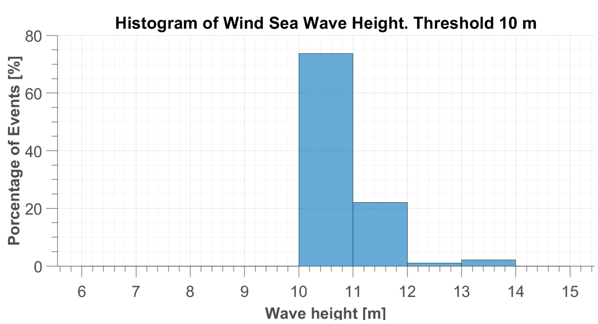

Figure B-9.: Histogram Wind Sea Significant Wave Height. Threshold 10m.

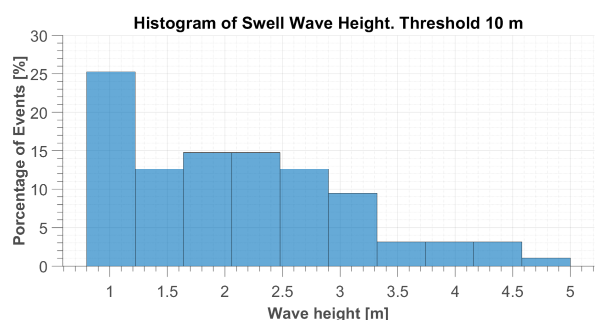

Figure B-10.: Histogram Swell Significant Wave Height. Threshold 10m.

C. Histogram of Wind Sea and Swell Spectral Peak Period

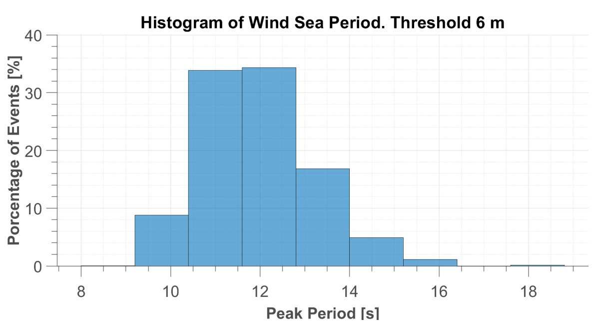

Figure C-1.: Histogram Wind Sea Spectral Peak Period. Threshold 6m.

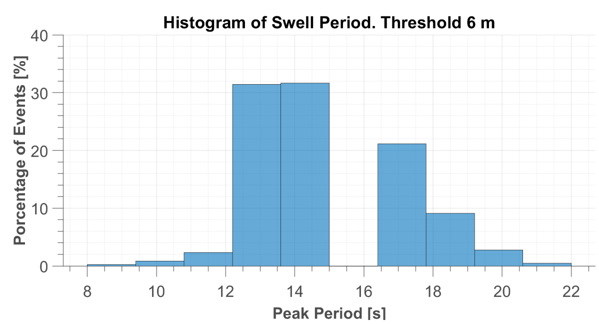

Figure C-2.: Histogram Swell Spectral Peak Period. Threshold 6m.

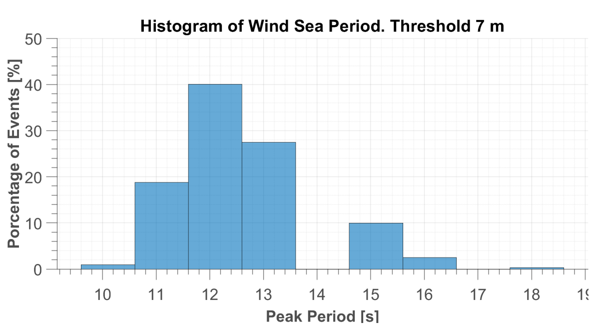

Figure C-3.: Histogram Wind Sea Spectral Peak Period. Threshold 7m.

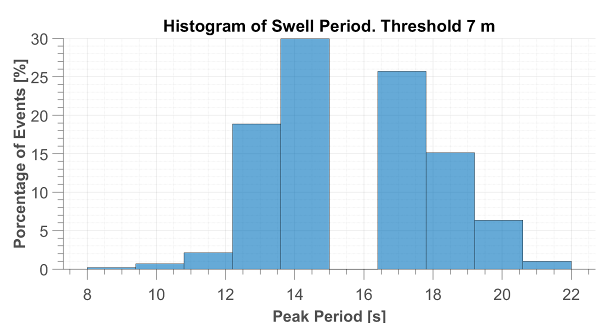

Figure C-4.: Histogram Swell Spectral Peak Period. Threshold 7m.

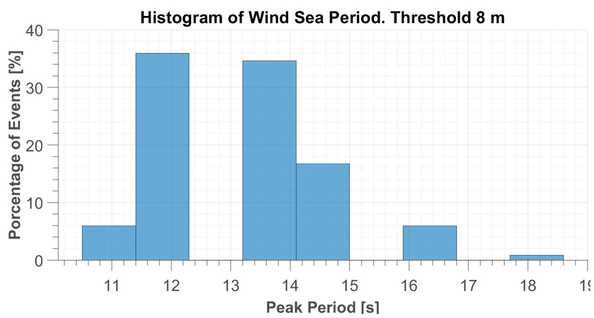

Figure C-5.: Histogram Wind Sea Spectral Peak Period. Threshold 8m.

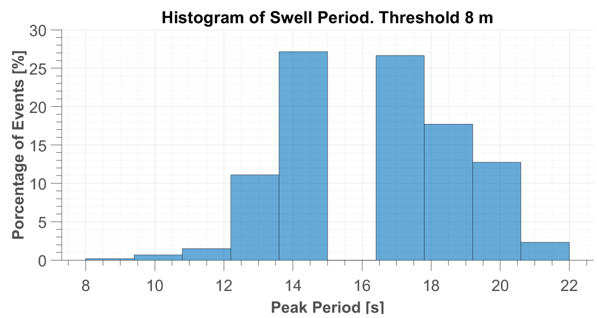

Figure C-6.: Histogram Swell Spectral Peak Period. Threshold 8m.

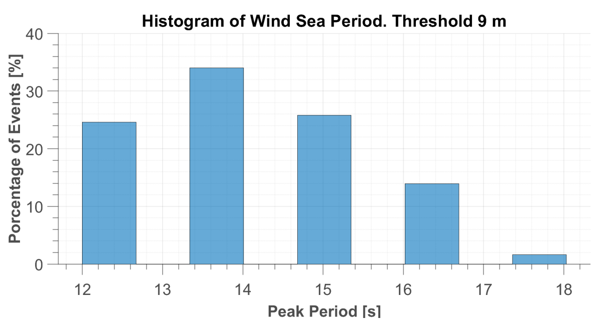

Figure C-7.: Histogram Wind Sea Spectral Peak Period. Threshold 9m.

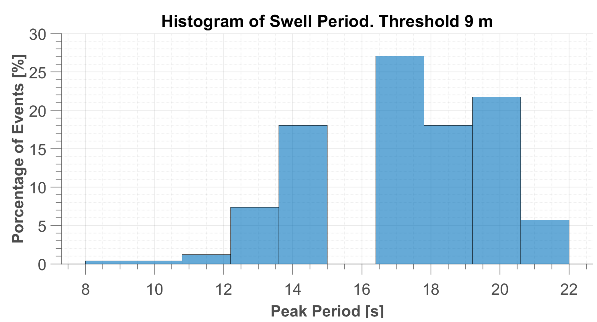

Figure C-8.: Histogram Swell Spectral Peak Period. Threshold 9m.

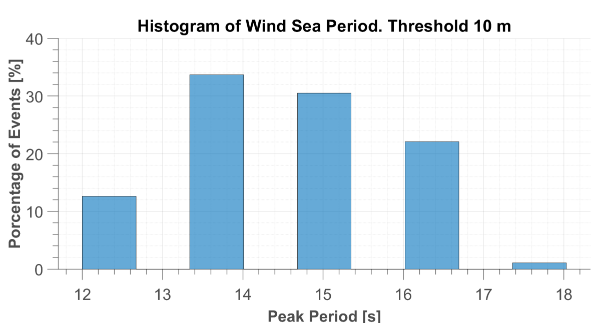

Figure C-9.: Histogram Wind Sea Spectral Peak Period. Threshold 10m.

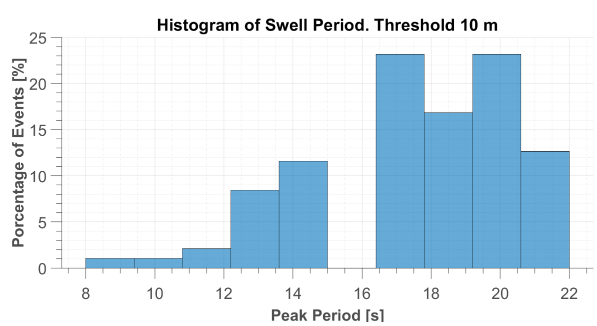

Figure C-10.: Histogram Swell Spectral Peak Period. Threshold 10m.

D. Distribution of the Gumbel Parameter Beta

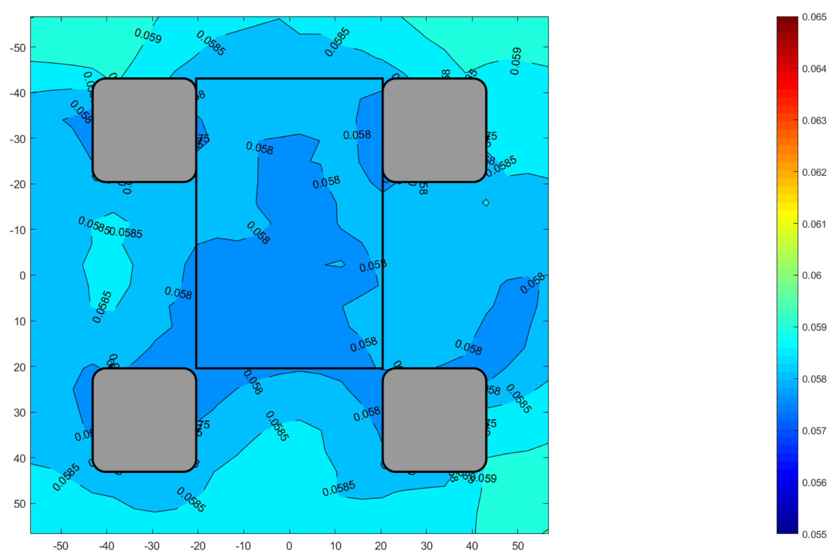

Figure D-1.: Gumbel Parameter Beta. Threshold 6m.

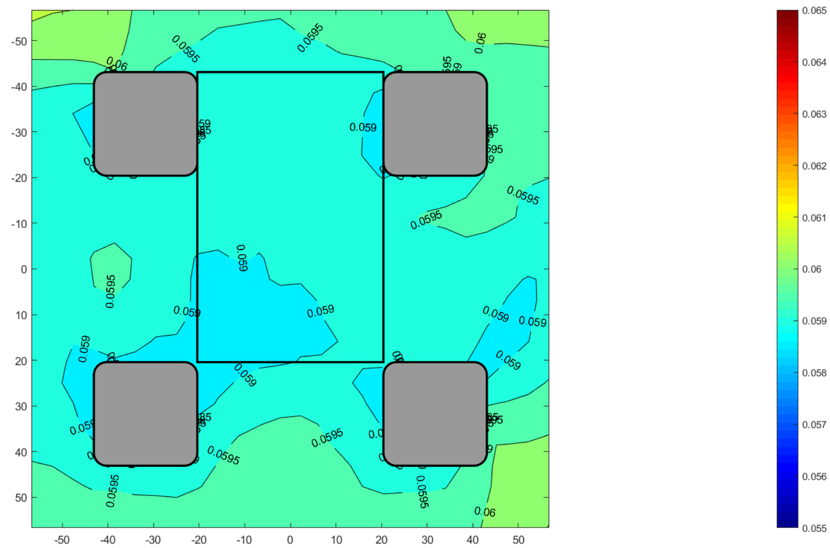

Figure D-2.: Gumbel Parameter Beta. Threshold 7m.

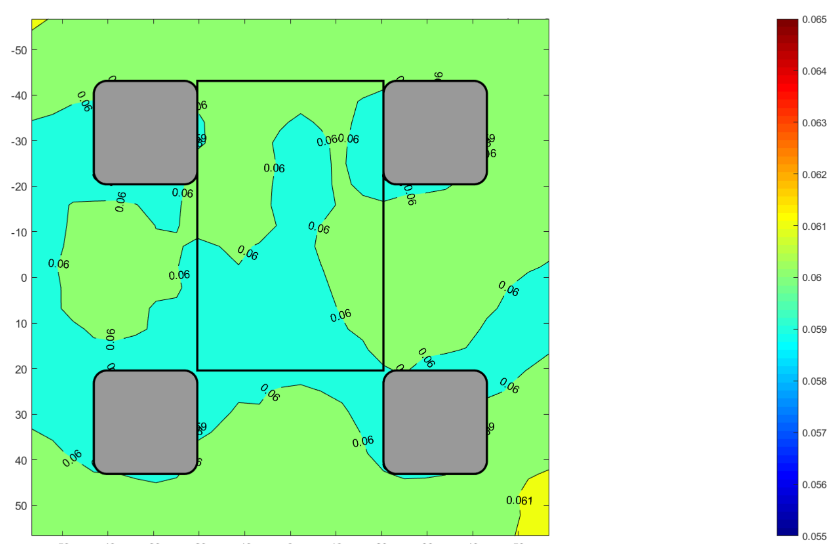

Figure D-3.: Gumbel Parameter Beta. Threshold 8m.

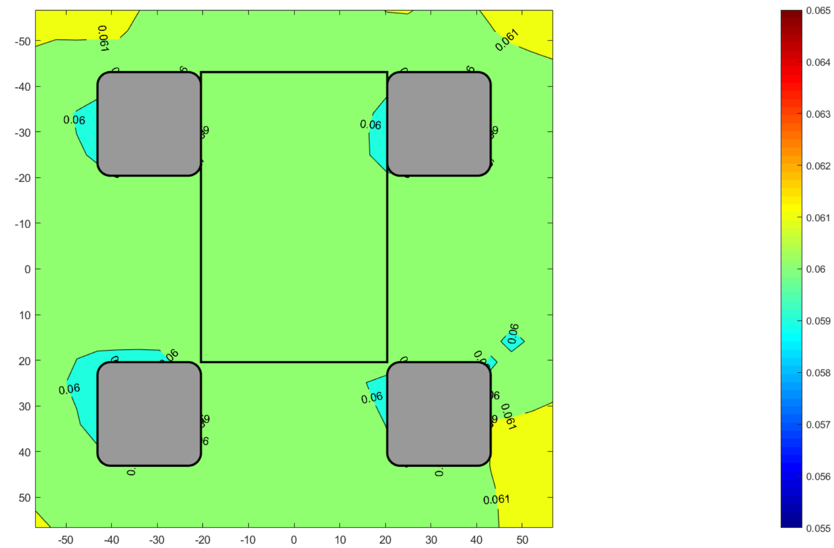

Figure D-4.: Gumbel Parameter Beta. Threshold 9m.

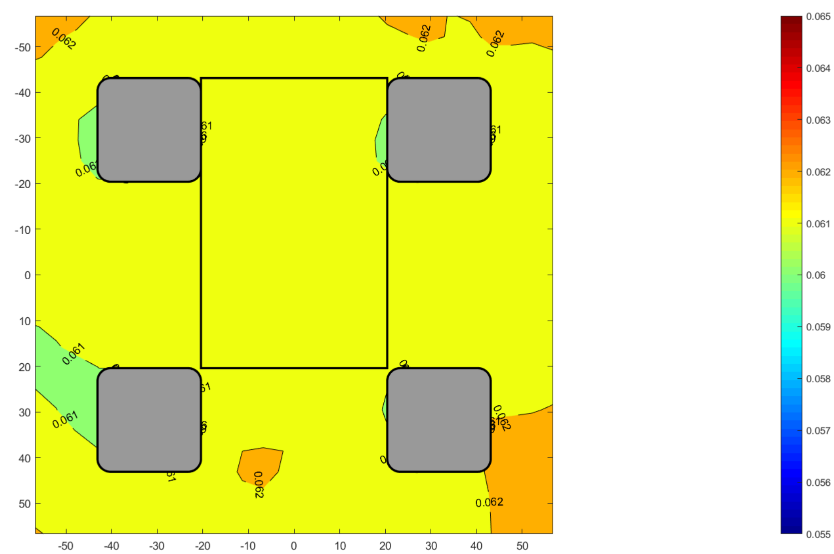

Figure D-5.: Gumbel Parameter Beta. Threshold 10m.

E. 100yr Relative Wave Elevation

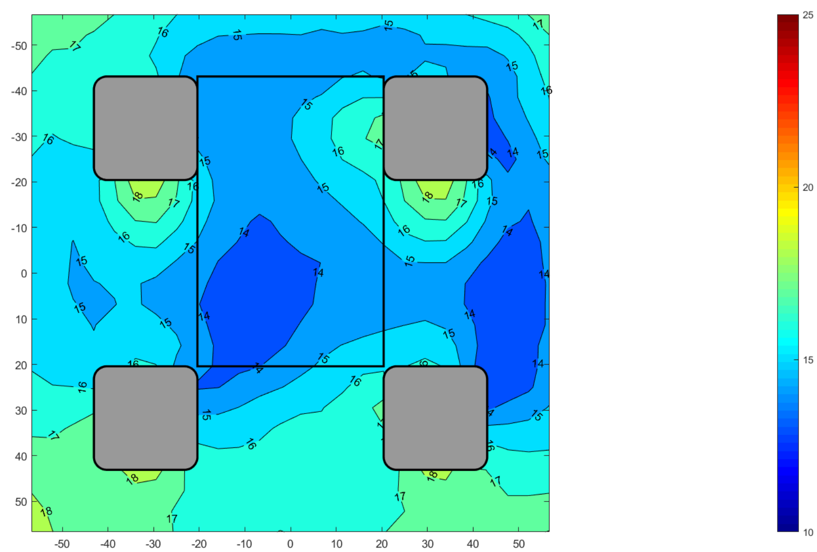

Figure E-1.: 100yr Relative Wave Elevation. Threshold 6m.

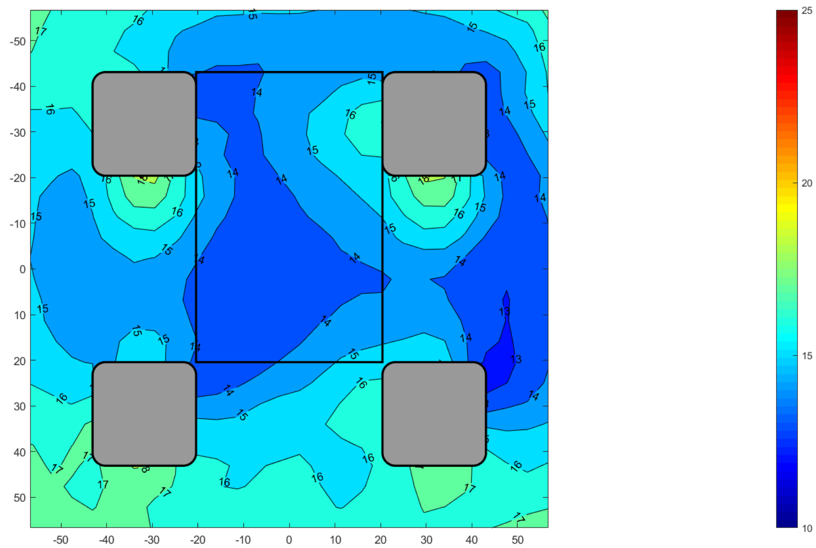

Figure E-2.: 100yr Relative Wave Elevation. Threshold 7m.

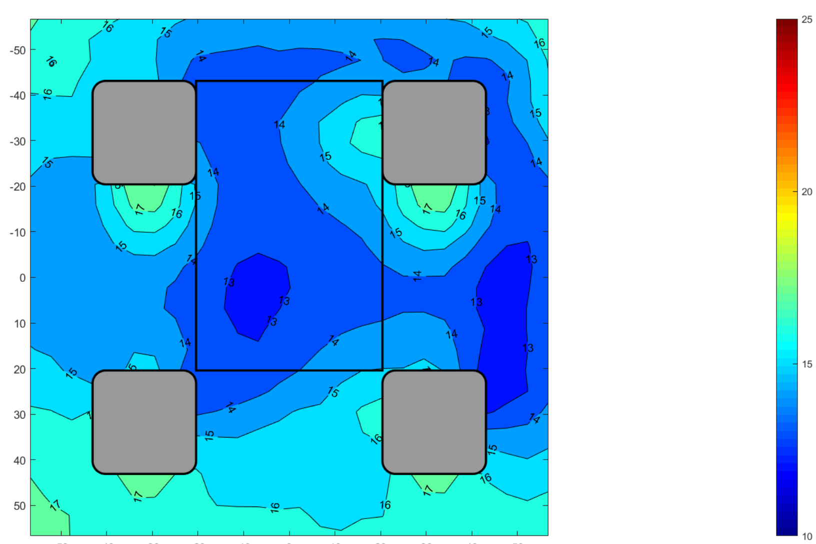

Figure E-3.: 100yr Relative Wave Elevation. Threshold 8m.

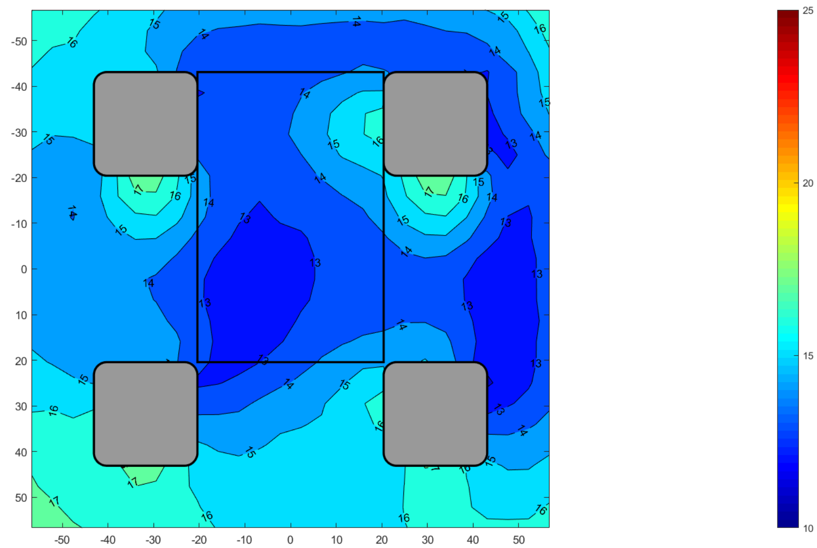

Figure E-4.: 100yr Relative Wave Elevation. Threshold 9m.

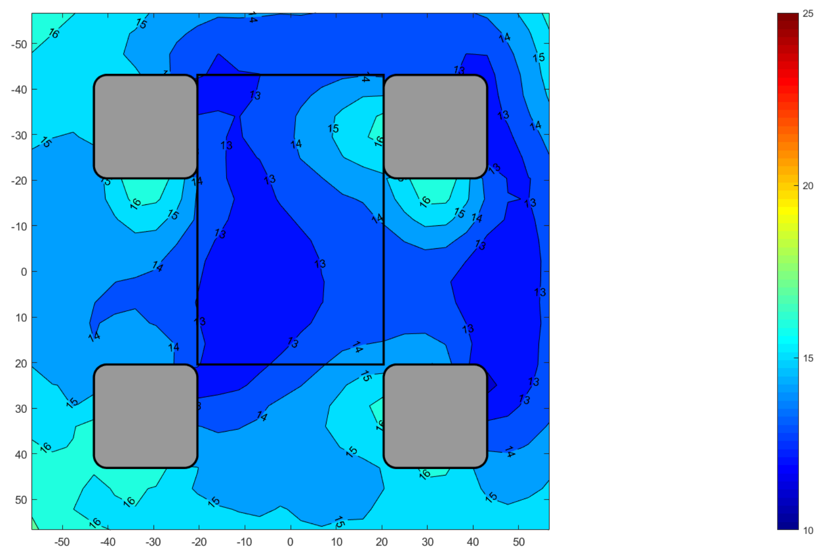

Figure E-5.: 100yr Relative Wave Elevation. Threshold 10m.

F. 10.000yr Relative Wave Elevation

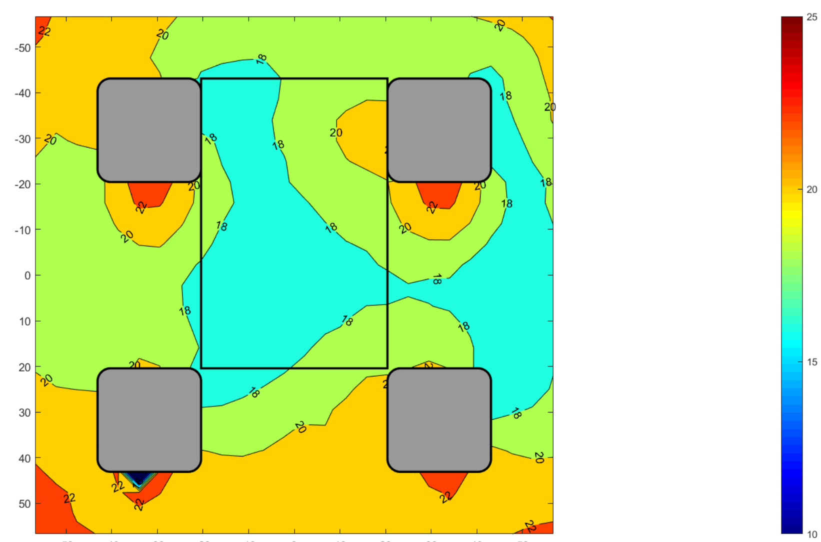

Figure F-1.: 10.000yr Relative Wave Elevation. Threshold 6m.

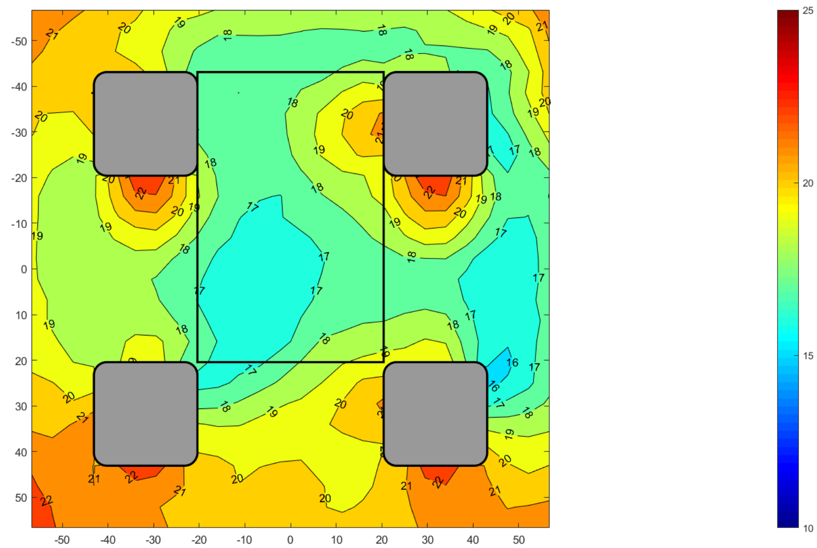

Figure F-2.: 10.000yr Relative Wave Elevation. Threshold 7m.

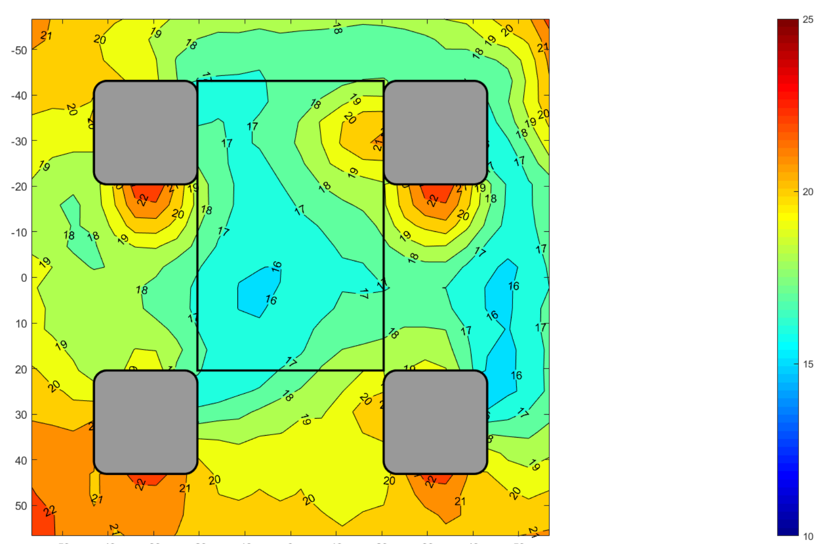

Figure F-3.: 10.000yr Relative Wave Elevation. Threshold 8m.

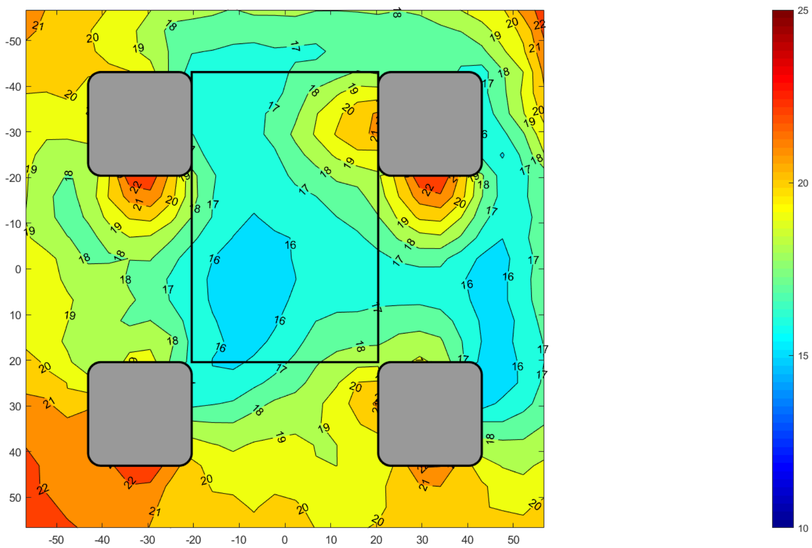

Figure F-4.: 10.000yr Relative Wave Elevation. Threshold 9m.

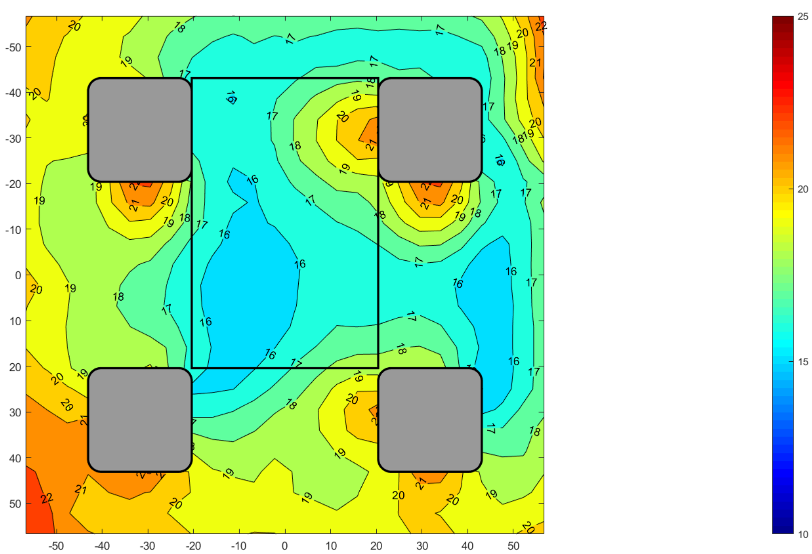

Figure F-5.: 10.000yr Relative Wave Elevation. Threshold 10m.

G. Wind Sea Direction Severity

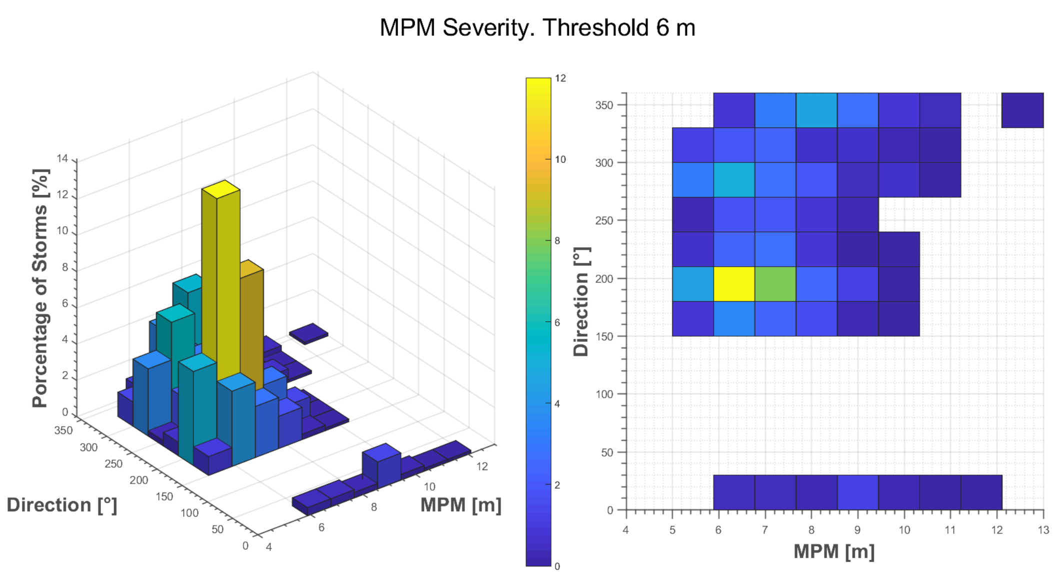

Figure G-1.: MPM distribution according to the wind sea direction. Threshold 6m.

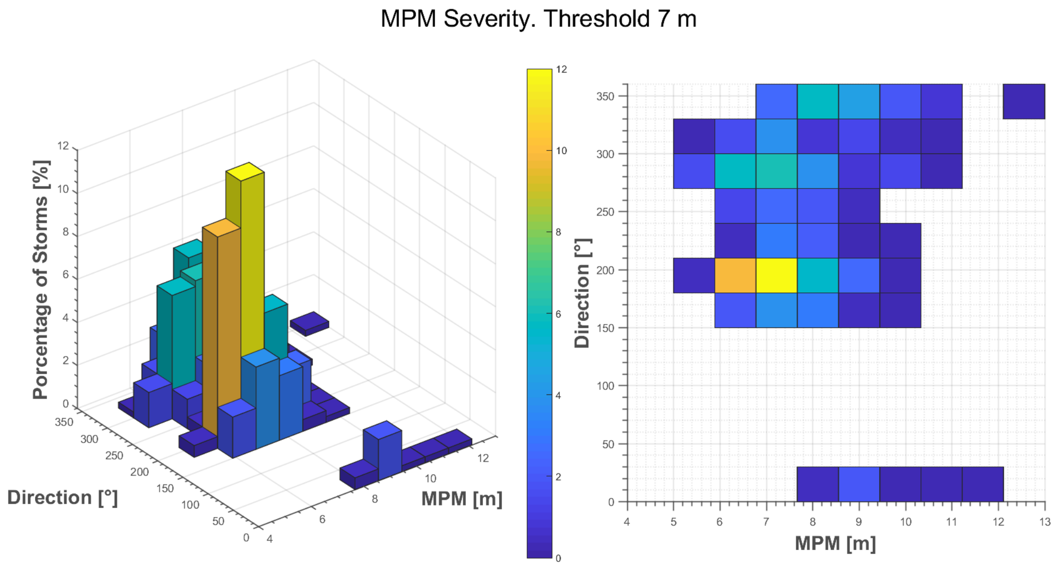

Figure G-2.: MPM distribution according to the wind sea direction. Threshold 7m.

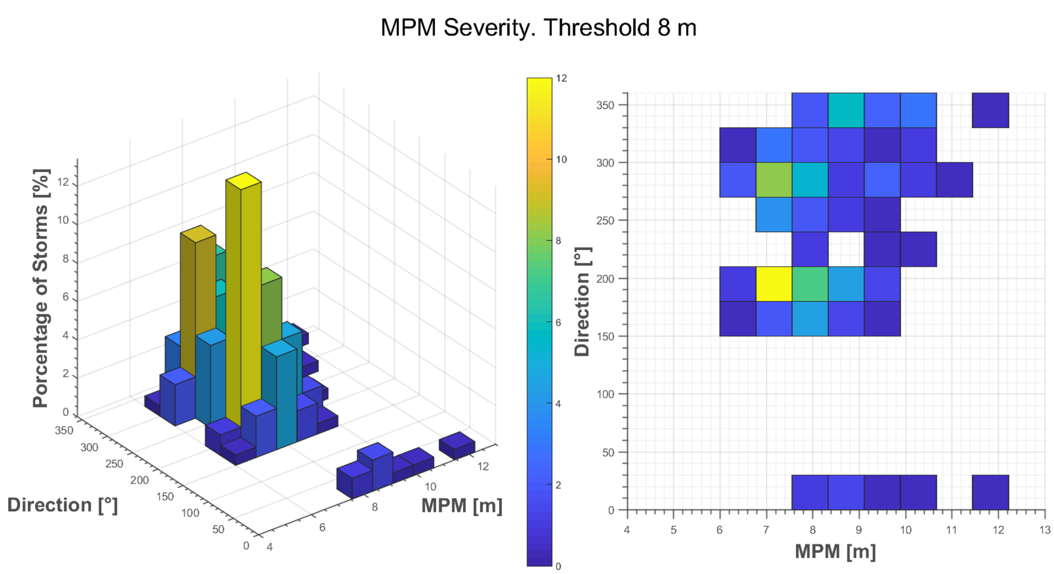

Figure G-3.: MPM distribution according to the wind sea direction. Threshold 8m.

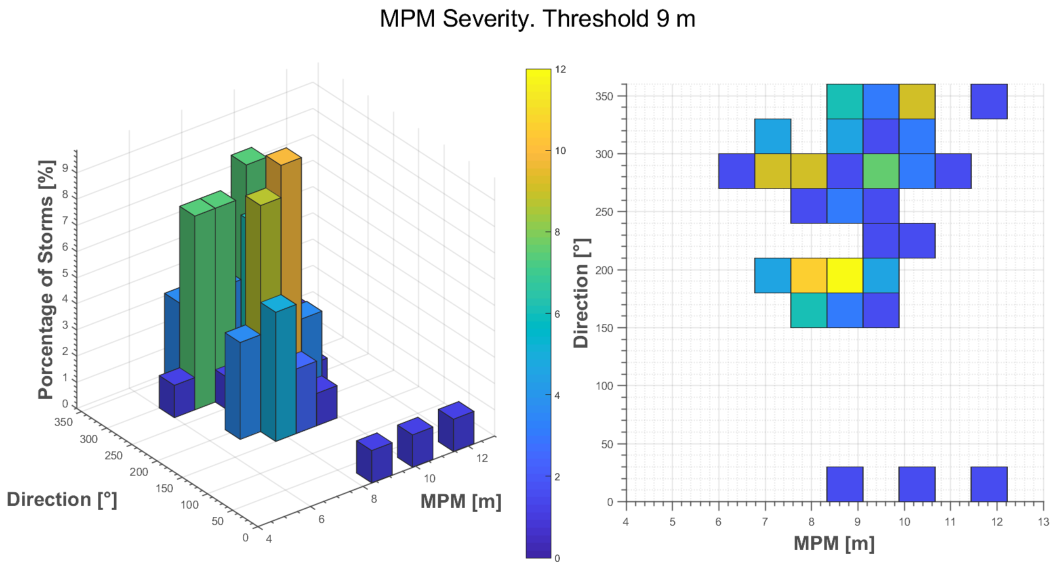

Figure G-4.: MPM distribution according to the wind sea direction. Threshold 9m.

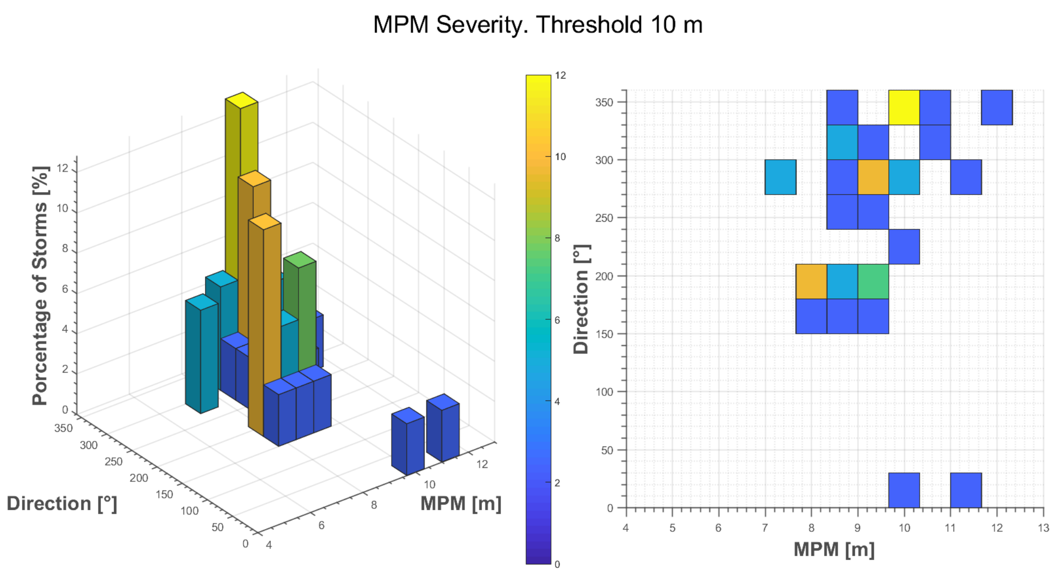

Figure G-5.: MPM distribution according to the wind sea direction. Threshold 10m.

H. MPM Severity

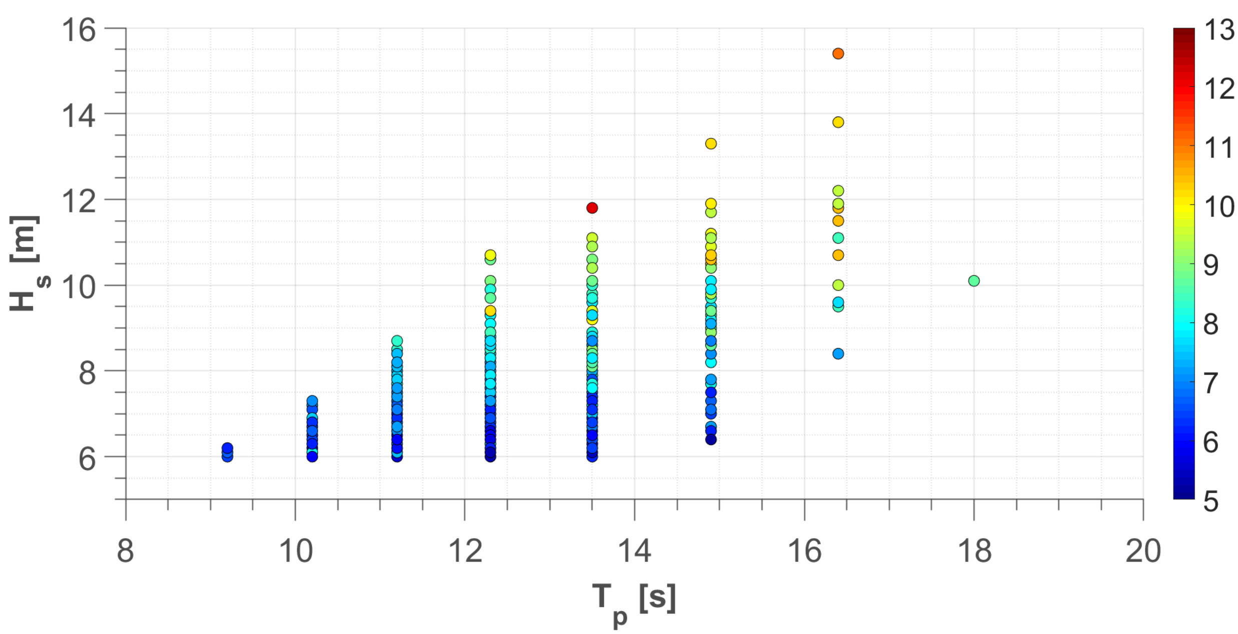

Figure H-1.: MPM severity. Threshold 6m.

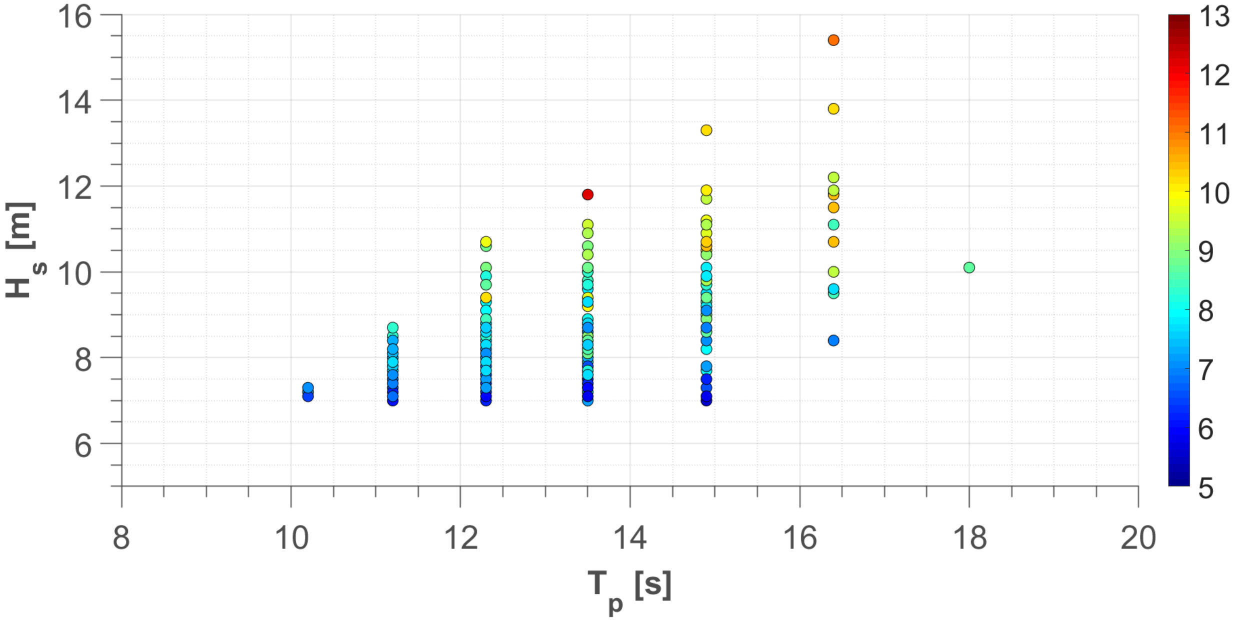

Figure H-2.: MPM severity. Threshold 7m.

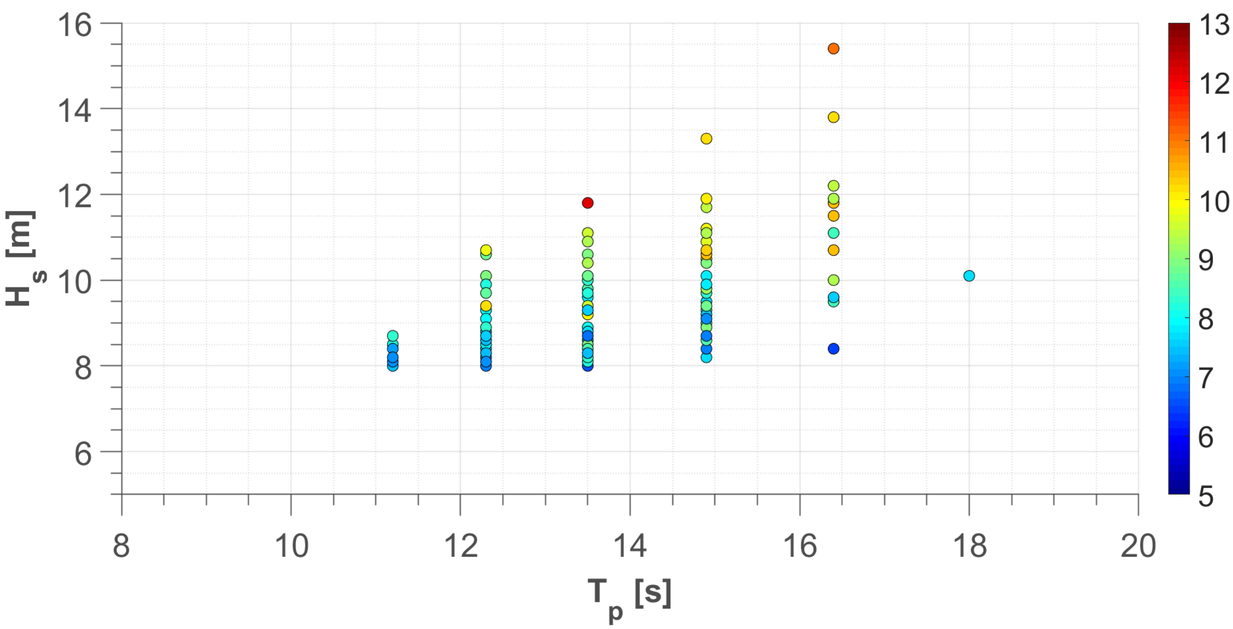

Figure H-3.: MPM severity. Threshold 8m.

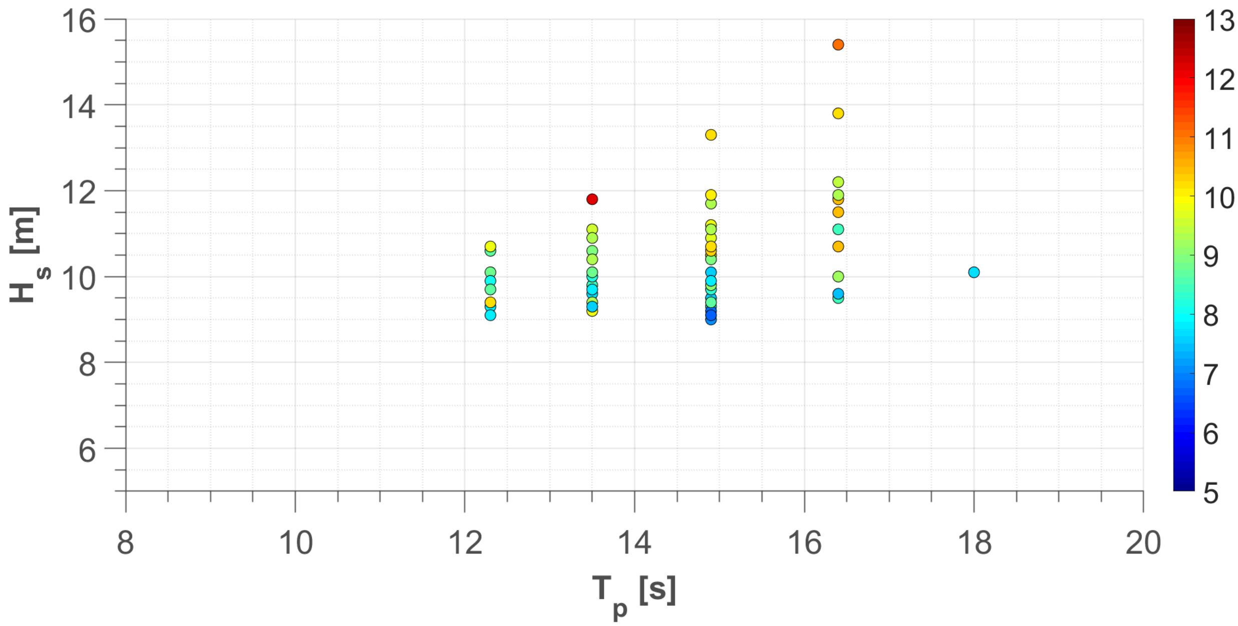

Figure H-4.: MPM severity. Threshold 9m.

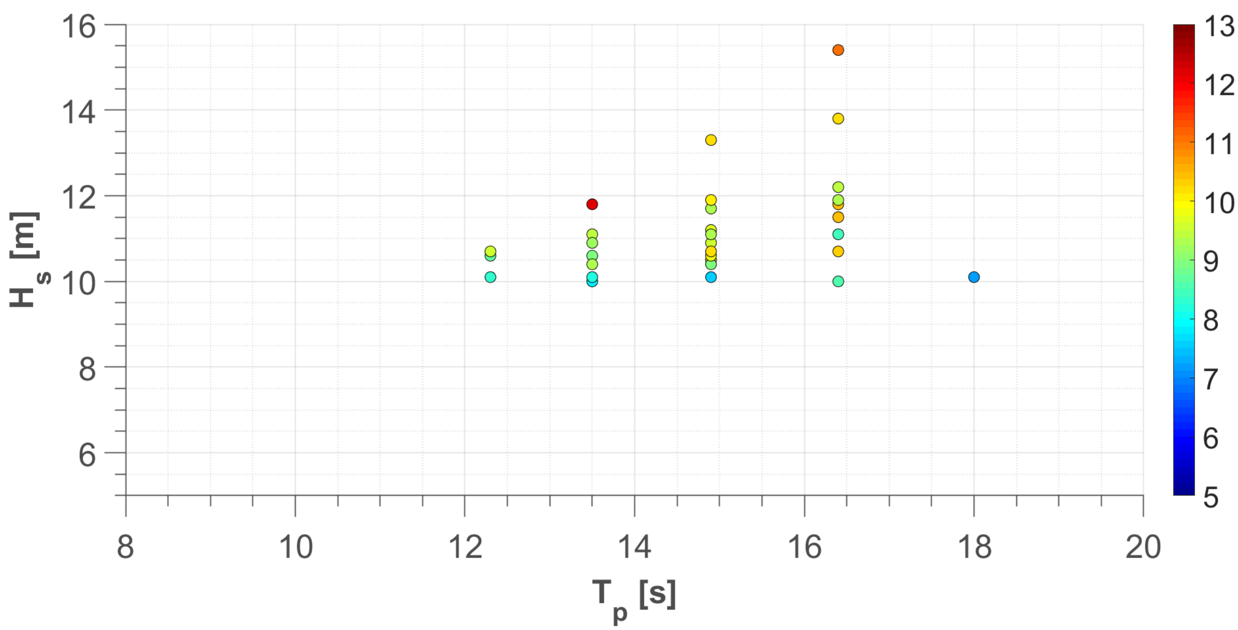

Figure H-5.: MPM severity. Threshold 10m.

I. Master Thesis Proposal - Sverre Haver

Assessment of necessary airgap of semi-submersible accounting for simultaneous occurrence of wind, wind sea and swell

Student: Julio Patino

Background

Sufficient still water airgap is important both for fixed and floating platforms. What is a sufficient airgap according to the rules depends on the rule regime under which the platform is planned to be operating. All fixed platforms and floating platforms operating at one site for its design life time, platform design will follow the regulations provided by the Petroleum Safety Authority Norway. For floating platforms operating as drilling rigs, there is an opening in the regulations to design platform according to the maritime regulation.

In the MSc focus shall be on a given semi-submersible platform. The rigid body transfer functions are made available. The aim of the MSc is to estimate q-probability airgap, $q = 10^{-2}$/year and $q = 10^{-4}$/year, for the worst location under platform deck accounting for joint occurrence of wind, wind-sea and swell sea. A consistent estimation of q-probability airgap requires that a long term analysis is performed. The platform is to be operating in the Northern North Sea. NORA10 data for the years from September 1957 – September 2017 will be made available.

Long term response analysis can be carried out using an all sea state approach or an all storms approach (POT). In this master thesis, focus is to be given to the all storms approach. An important part of this method is to choose a proper threshold defining the selection of storms. An important part of the air-gap assessment is to consider sensitivity of results to selected threshold.

A linear response analysis can be utilized for the short term analyses, but non-linearities in the wave crest heights shall be included in the analyses. A possibility is to utilize the approach proposed by DNVGL. The analysis can at first be done by neglecting wind speed. If time permits, one may consider to include wind in the joint modelling. As a minimum, effect of wind speed shall be discussed for some few important storm cases in order to indicate the error in results due to neglecting wind.

Below a possible division into sub-tasks is given.

1. Demonstrate how a short term air-gap analysis is to be done when accounting for simultaneous occurrence of wind sea and swell sea propagating in different directions. This should include a demonstration how the transfer function for air-gap variable is determined. Use a JONSWAP type of wave spectrum both for wind sea and swell sea.

2. Present and discuss how the all sea states method and the all storms method can be formulated for the given problem. For the all storms method discuss the various approaches planned to be included in this study. Demonstrate how q-probability air-gap can be estimated by the various approaches.

3. For an example problem, for which an "exact" all sea states approach can be done, compare the q-probability values obtained using the various approaches suggested in point above compared to the "exact" all sea states method.

4. Do the long term analyses for the air-gap variable. For one of the approaches do the long term analysis using merely total significant and dominating spectral peak period as sea state characteristics. Do the analysis both for JONSWAP spectrum and Torsethaugen spectrum.

5. Investigate the effect of neglecting the direct effect of wind on the air-gap variable.

6. Discuss the results of various analyses of the airgap variable for $q = 10^{-2}$/year and

$q = 10^{-4}$/year.

7. Present the work in a scientific report and present conclusions regarding your main findings.

The candidate may of course select another scheme as the preferred approach for solving the requested problem. He may also involve other subjects than those mentioned above if found to be important for answering the overall problem; air-gap requirement for semi-submersibles.

The work may show to be more extensive than anticipated. Some topics may therefore be left out after discussion with the supervisor without any negative influence on the grading.

The candidate should in his report give a personal contribution to the solution of the problem formulated in this text. All assumptions and conclusions must be supported by mathematical models and/or references to physical effects in a logical manner. The candidate should apply all available sources to find relevant literature and information on the actual problem.

The report should be well organised and give a clear presentation of the work and all conclusions. It is important that the text is well written and that tables and figures are used to support the verbal presentation. The report should be complete, but still as short as possible.

The final report must contain this text, an acknowledgement, summary, main body, conclusions, suggestions for further work, symbol list, references and appendices. All figures, tables and equations must be identified by numbers. References should be given by author and year in the text, and presented alphabetically in the reference list. The report must be submitted in two copies unless otherwise has been agreed with the supervisor.

The candidate should give a written plan that describes the progress of the work mid-way through the MSc period. The plan can be limited to give a draft table of content for the MSc thesis, status regarding completion for the various chapters and what is consider the main remaining challenges. As an indication such a plan should be available by mid-April. From the report it should be possible to identify the work carried out by the candidate and what has been found in the available literature. It is important to give references to the original source for theories and experimental results. The report must be signed by the candidate, include this text, appear as a paperback, and - if needed - have a separate enclosure (binder, diskette or CD-ROM) with additional material.

Supervisor: Sverre Haver, UIS

J. MATLAB code

```
clc
clear
close all
format long
close all
tic

addpath('.\THESIS')

threshold = [6 7 8 9 10];
point_semi = (1:1:612);

z_rel = linspace(1,30,1000);

%% PRELOCATING VARIABLES

levels = length(threshold); % tt
n_points = length(point_semi); % k

count{levels}    = [];
events{levels}   = [];
F100             = zeros(1,levels);
F10000           = zeros(1,levels);
STORMS{levels}   = [];
ev_yr100         = zeros(1,levels);
```

```
ev_yr10000      = zeros(1,levels);

X   = zeros(1,n_points);
Y   = zeros(1,n_points);

Fxbar_s{levels} = [];
z_mpm{levels}   = [];

mu{levels}                  = [];
var{levels}                 = [];
beta{levels}                = [];
Fxbar_fit{levels}           = [];
z_mpm_sort{levels}          = [];
empirical_prob{levels}      = [];
z_mpm_sort_new{levels}      = [];
empirical_prob_new{levels}  = [];

skewness_z_mpm  = zeros(levels,n_points);
std_z_mpm       = zeros(levels,n_points);
mean_z_mpm      = zeros(levels,n_points);
beta_3par       = zeros(levels,n_points);
alpha_3par      = zeros(levels,n_points);
lambda_3par     = zeros(levels,n_points);

zmpm{levels}        = [];
F_xbar_3p{levels}   = [];
F_xbar_pos100       = zeros(levels,n_points);
zmpm100             = zeros(levels,n_points);
F_xbar_pos10000     = zeros(levels,n_points);
zmpm10000           = zeros(levels,n_points);

Beta = zeros(levels,n_points);

Fxbar_fit_new{levels} = [];
```

```
FX{levels}  = [];
F_pos100    = zeros(levels,n_points);
z100        = zeros(levels,n_points);
F_pos10000  = zeros(levels,n_points);
z10000      = zeros(levels,n_points);

% n_storms = length(count{1,tt}); % ii
% n_events_s = count{1,tt}(1,ii); % j

%%

for tt = 1:length(threshold)

[count{1,tt},events{1,tt},F100(tt),F10000(tt),STORMS{tt}, ...
HS_ws,TP_ws,DIR_ws,HS_s,TP_s,DIR_s,ev_yr100(tt),ev_yr10000(tt)] ...
= storms(threshold(tt));
load('\THESIS\database_wamit') % Created by data_treatment
F2 = 1./flipud(TT); % frequency vector

for k=1:length(point_semi)

X(k)    = WAMIT_XY(point_semi(k),2);
Y(k)    = WAMIT_XY(point_semi(k),3);

for ii = 1:length(count{1,tt})

Ftemp = ones(1,length(z_rel));
for j = 1:count{1,tt}(1,ii)

% WIND SEA SPECTRUM
sea_type = 10;
alpha = 1.2;
[s_zpzp_ws,var_ws] = get_spectrum(STORMS,sea_type, ...
tt,k,ii,j,point_semi,alpha,TT,HEAVE,ROLL,PITCH,X(k),Y(k), ...
WAVE_RAO,F2);
```

```
% SWELL SPECTRUM
sea_type = 13;
alpha = 1.0;
[s_zpzp_swell,var_swell] = get_spectrum(STORMS,sea_type, ...
tt,k,ii,j,point_semi,alpha,TT,HEAVE,ROLL,PITCH,X(k), ...
Y(k),WAVE_RAO,F2);

% TOTAL SPECTRUM
s_zpzp_tot = s_zpzp_ws + s_zpzp_swell;

var_sf  = trapz(F2,s_zpzp_tot); % Variance - Zero Moment
std_sf  = sqrt(var_sf); % Standard Deviation
m2_sf   = trapz(F2,F2.^2.*s_zpzp_tot); % Second Moment

t02     = sqrt(var_sf / m2_sf); % zero-up-crossing period
n3h     = 10800/t02;

Fz3h{1,tt}{1,ii}(j,:)    = ( 1 - exp (-0.5.*( z_rel ./ ...
 std_sf ).^2) ).^n3h;
Ftemp   = Ftemp .* Fz3h{1,tt}{1,ii}(j,:) ;

end

Fxbar_s{1,tt}{ii,k} = Ftemp;
Fz3h_pos = find(1/exp(1) > Fxbar_s{1,tt}{ii,k},1,'last');
z_mpm{1,tt}(ii,k) = z_rel(Fz3h_pos+1);

end

% MEAN
for ii = 1:length(count{1,tt})
sum1 = 0;
for j = 1:length(z_rel)-1
```

```
dF      = Fxbar_s{1,tt}{ii,k}(1,j+1) - Fxbar_s{1,tt}{ii,k}(1,j);
sum1    = sum1 + dF * ( z_rel(j) );

end
mu{1,tt}(ii,k) = sum1;
end

% VARIANCE
for ii = 1:length(count{1,tt})
sum2 = 0;
for j = 1:length(z_rel)-1

dF = Fxbar_s{1,tt}{ii,k}(1,j+1) - Fxbar_s{1,tt}{ii,k}(1,j);
sum2 = sum2 + dF * ( z_rel(j) - mu{1,tt}(ii,k) )^2;

end
var{1,tt}(ii,k) = sum2;
beta{1,tt}(ii,k) = ( sqrt( var{1,tt}(ii,k).*6 ) ) ./  ...
( pi .* z_mpm{1,tt}(ii,k) );
end

% Generate the Aprox Storm Distribution

for ii = 1:length(count{1,tt})

Fxbar_fit{1,tt}{ii,k} = exp( -exp ( - ( z_rel - ...
z_mpm{1,tt}(ii,k) ) ./ ( beta{1,tt}(ii,k).* ...
z_mpm{1,tt}(ii,k) ) ) );

end

%% Empirical

z_mpm_sort{1,tt}(:,k)        = sort(z_mpm{1,tt}(:,k));
empirical_prob{1,tt}(:,k)    = ( 1:1:length(z_mpm{1,tt}(:,k)) ) ./ ...
```

```
( length(z_mpm{1,tt}(:,k))+1 );
empirical_prob{1,tt}(:,k)    = empirical_prob{1,tt}(:,k)';

jj=1;
for i = 2 : length(z_mpm_sort{1,tt}(:,k))

if z_mpm_sort{1,tt}(i-1,k) < z_mpm_sort{1,tt}(i,k)

z_mpm_sort_new{1,tt}(jj,k)       =    z_mpm_sort{1,tt}(i-1,k);
empirical_prob_new{1,tt}(jj,k)   =    empirical_prob{1,tt}(i-1,k);
jj = jj+1;

if i == length(z_mpm_sort{1,tt}(:,k))

z_mpm_sort_new{1,tt}(jj,k)       =    z_mpm_sort{1,tt}(i,k);
empirical_prob_new{1,tt}(jj,k)   =    empirical_prob{1,tt}(i,k);

end

elseif  z_mpm_sort{1,tt}(i-1,k) == z_mpm_sort{1,tt}(i,k)

z_mpm_sort_new{1,tt}(jj,k)       =    z_mpm_sort{1,tt}(i,k);
empirical_prob_new{1,tt}(jj,k)   =    empirical_prob{1,tt}(i-1,k);
end

end

%% 3-parameter Weibull model

skewness_z_mpm(tt,k) = skewness(z_mpm_sort_new{1,tt}((1:jj),k));
std_z_mpm(tt,k)      = std(z_mpm_sort_new{1,tt}((1:jj),k));
mean_z_mpm(tt,k)     = mean(z_mpm_sort_new{1,tt}((1:jj),k));

syms x
fun = (gamma(1+3/x)-3*gamma(1+1/x)*gamma(1+2/x)+2* ...
```

```
(gamma(1+1/x))^3 ) / ( gamma(1+2/x) - (gamma(1+1/x))^2 )^1.5 ...
 == skewness_z_mpm(tt,k);
beta_3p = vpasolve(fun,1.5);

alpha_3p = solve( subs ( x^2*(gamma(1+2/beta_3p) - ...
(gamma(1+1/beta_3p))^2) == std_z_mpm(tt,k)^2, ...
beta_3p,double(beta_3p) ) ,x );

a1 = subs ( x+alpha_3p(2)*gamma(1+1/beta_3p) == ...
 mean_z_mpm(tt,k), alpha_3p(2),double(alpha_3p(2)));
a1 = subs (a1, beta_3p, double(beta_3p) );

lambda_3p = solve( a1 ,x);

beta_3par(tt,k)     = double(beta_3p);
alpha_3par(tt,k)    = double(alpha_3p(2));
lambda_3par(tt,k)   = double(lambda_3p);

%probability paper weibull3p
Fx_3p{1,tt}(k,:)    = linspace(0,1,1000)';
x_3p{1,tt}(k,:)     = lambda_3par(tt,k) + exp(( ...
   log(-log(1-Fx_3p{1,tt}(k,:))) ...
+ beta_3par(tt,k)*log(alpha_3par(tt,k)) ) / beta_3par(tt,k)) ;

zmpm{1,tt}(:,k) = linspace(lambda_3par(tt,k),30,1000);

% write the weibull function
F_xbar_3p{1,tt}(:,k) = 1 - exp ( - ( (zmpm{1,tt}(:,k) - ...
 lambda_3par(tt,k)) ./ ( alpha_3par(tt,k) ) ) .^ beta_3par(tt,k) );

% get the 100 yr value - short term
F_xbar_pos100(tt,k)      = find(F100(tt) > ...
 F_xbar_3p{1,tt}(:,k),1,'last');
zmpm100(tt,k)            = zmpm{1,tt}(F_xbar_pos100(tt,k)+1,k);
```

```
% get the 10.000 yr value - short term
F_xbar_pos10000(tt,k)    = find(F10000(tt) > ...
F_xbar_3p{1,tt}(:,k),1,'last');
zmpm10000(tt,k)          = zmpm{1,tt}(F_xbar_pos10000(tt,k)+1,k);

Beta(tt,k) = mean(beta{1,tt}(:,k));

% Convolution
for i=1:length(z_rel)
sum3 = 0;
for j=1:length(zmpm{1,tt}(:,k))-1

Fxbar_fit_new{1,tt}{1,k}(i,j) = exp( - exp( - ( z_rel(i) - ...
zmpm{1,tt}(j,k) ) ./ ( Beta(tt,k).*zmpm{1,tt}(j,k) ) ) );

dF      = F_xbar_3p{1,tt}(j+1,k) - F_xbar_3p{1,tt}(j,k);
sum3    = sum3 + dF * Fxbar_fit_new{1,tt}{1,k}(i,j);

end

FX{1,tt}(i,k) = sum3;

end

F_pos100(tt,k) = find(F100(tt) > FX{1,tt}(:,k),1,'last');

if F_pos100(tt,k) == length(z_rel)
z100(tt,k) = 0;
else
z100(tt,k) = z_rel(F_pos100(tt,k)+1);
end

F_pos10000(tt,k) = find(F10000(tt) > FX{1,tt}(:,k),1,'last');

if F_pos10000(tt,k) == length(z_rel)
```

```
z10000(tt,k) = 0;
else
z10000(tt,k) = z_rel(F_pos10000(tt,k)+1);
end

threshold(tt)
point_semi(k)
end

end
%%
z100'
z10000'

toc
```