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Bitcoin’s Financial Risk Properties in a Global Portfolio

MASTER’S THESIS

By

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Stavanger, 15th June 2018

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Abstract

In this thesis the dynamic between Bitcoin and a selection of various financial assets is analyzed to examine whether Bitcoin offers diversification, hedging and safe haven properties for risk management benefits in a global market portfolio. A dynamic conditional correlation model is used to obtain the co-movement between the assets. Optimizing portfolios by including Bitcoin is done to examine what the inclusion does for the portfolio properties. Lastly, Value at risk (VaR) is estimated to see whether including Bitcoin in a portfolio can lower the VaR.

The conditional correlation coefficients for Bitcoin against the other assets for all sample periods investigated shows correlation coefficients around zero. In addition, the analysis of volatility spillovers between the selected markets implies that there is no significant contagion between the markets and for these reasons, Bitcoin exhibits effective diversification properties. This is also supported by the analysis of MVF and CML. The MVF and CML shows that portfolios including Bitcoin make it possible to obtain the same expected return, but for a lower risk. The VaR analysis shows that a including a Bitcoin weight between 0.0-5.0% lowers the VaR, despite Bitcoin’s high volatility.

Finally, the near zero conditional correlation coefficients between Bitcoin and the other assets also imply that Bitcoin does not exhibit hedging properties. Bitcoin is also considered as a weak safe haven as it is uncorrelated with the other markets during market turmoil.

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1. Introduction

The cryptocurrency market has grown rapidly since Satoshi Nakamoto published a white paper about Bitcoin. The white paper was released shortly after the collapse of Lehman Brothers which initiated the global financial crisis in 2008. From its inception in 2009 until now, Bitcoin has unquestionably been the dominating leader of the cryptocurrency boom by market cap. It has been well known that Bitcoin first had found support among tech nerds and people with limited trust to governments and banks. The price started at a few cents and has grown exponentially in a few years attracting a whole new domain of speculators and investors. However, shortly after CME Group Inc launched Bitcoin futures in December 17th, 2017, Bitcoin peaked at an all-time high of $US 20 089 intraday with a market capitalization of $US 326 billion as seen in figure 1. The launch of Bitcoin futures finally allowed the pessimistic investors who believed that the price would collapse, to enter the market. The rapid decline of the price bottomed at $US 6048 in February 6th and has since that day been in a period of consolidation up until June, relative to the rapid growth in the end of December.

Figure 1: Bitcoin's 12-month performance (CoinMarketCap, 2018)

There is no clear agreement among investors on the long-term price formation of Bitcoin beyond the end point shown in figure 1. However, the inception of Bitcoin in the future market has opened the door to a new set of investors in the crypto market. A lack of clarity on the regulation of cryptocurrencies has been one of many barriers in many countries, but regulators and policy makers across the world have been starting to recognize cryptocurrencies as an asset. Therefore, the traditional financial institutions and hedge funds specialized in trading cryptocurrencies are starting to recognize them as an investment. The literature on Bitcoin's financial properties in a global market portfolio is quite narrow. As a result of this, the characteristics of Bitcoin and its implementation in a global portfolio are investigated throughout this thesis. Moreover, the literature on generalized autoregressive conditional heteroskedasticity, portfolio optimization and value at risk has been adapted to conduct the empirical analysis.

1.1 Thesis Objectives

The main objective of this thesis is to evaluate Bitcoin as an asset in financial market risk context and investigate whether Bitcoin can act as a diversifier, hedge and safe haven in a global market portfolio. Thus, this thesis will give an introduction on Bitcoin and financial risk, in order to understand Bitcoin's technology and the mechanisms that are applied to assess financial risk. The evaluation will be based upon portfolio theory and financial data which is processed through a general autoregressive conditional heteroskedasticity (GARCH) model and value at risk estimations.

2. Essential Features of Bitcoin

In October 2008, a person or collective group using the pseudonym Satoshi Nakamoto published a white paper that described the idea of Bitcoin. Bitcoin can be described as a decentralized digital currency and uses a peer-to-peer (P2P) network in such a way that no central authority can issue new money (Nakamoto, 2008). Therefore, decentralization is one of the most important characteristics of Bitcoin that differentiates it from conventional currencies (e.g. euro and dollar). Transaction management and money issuance are therefore carried out solely by the network through nodes. However, even though Bitcoin is decentralized, it stores the detail of every single transaction that has ever happened in its ledger. Every Bitcoin is associated with a Bitcoin address (public key) and can be sent from one address to another (Dwyer, 2014). The public key also contains a private key that allows the owner of the public key to access the Bitcoin address. Since Bitcoin is highly stringent in storing all transactions in a ledger, anyone can know how much Bitcoins is stored in any address.

However, before a transaction can be made, the nodes must verify the transaction by checking the syntax, structure and the unspent transaction output. If an input to the transaction can not be found in the unspent transaction output database, it is invalid. This might occur through double-spending[1], or because the transaction is trying to allocate Bitcoins that don't exist. The valid transactions are then sent to a "pool" where they are mined (Dwyer, 2014). Once Bitcoins are exchanged or transferred, there is a publicly available database otherwise known as blockchain, which records every trade of the digital currency. Bitcoin manages the double-spending problem by maintaining a universal time-stamped transaction ledger (the blockchain) to remain secure and function to its full potential, without the need of a trusted authority (e.g. a financial institution).

Unlike conventional currencies (fiat currencies) produced by governments, Bitcoins are produced from the mining process and have a limited supply. Bitcoins are created each time a new block is discovered by the miner and there will only be slightly less than 21 million Bitcoins made. It can be divided down to 8 decimal places (0.00000001 = 1 Satoshi), where one Satoshi is the smallest fraction that currently can be sent (Coindesk, 2018). Bitcoin is scarce by design, since there are only going to be a finite number of Bitcoins, comparative to gold. However, Bitcoin holds no intrinsic value, like most modern fiat currencies.

[1] Double-spending is a unique problem to digital currencies where the possibility to create multiple copies of the digital tokens to spend them more than once exists.

Fiat currencies that are created by central banks tend to lose their value over time by inflation. Bitcoin, however, through its tight and finite money supply is deflationary if the Bitcoin economy is growing. This deflationary spiral and the high average volatility make Bitcoin a poor medium of exchange, extremely risky as a standard of deferred payment and certainly not a stable store of value. This thesis is going to look at Bitcoin as a new asset class, and to what degree it can be used in a portfolio as a diversifier or hedge.

2.1 Bitcoin: Mining Technology

Mining is a resource-intensive process of verifying the transactions by adding the next chain to the blockchain. Both transaction fees and the miner's acquisition of newly created coins (block reward) provide incentives for miners. The miners solve complex algorithms to create the new blocks which are added to the blockchain. These blocks include recent transactions and the new Bitcoins that miners are rewarded (Dwyer, 2014). The primary purpose of mining is to build a trustworthy commitment to reach a secure and tamper-resistant distributed ledger.

The ideal average mining rate has been established at 10 minutes per block in the Bitcoin protocol and is regulated by an algorithm which makes the difficulty of mining harder as more miners join. The reward for mining started at 50 Bitcoins per block, but the rate is set to be halved each time 210 000 blocks are mined (Bitcoinminig, 2018). This results in halving the rewards every four years, with the last block halving being estimated to take place in October 2140. After this event, it is expected that a higher transaction fee alone will be a necessary and sufficient incentive for the miners to continue maintaining the shared ledger.

To begin with, Bitcoin miners solved algorithms with processors on their normal computers. Soon miners discovered that graphic cards used for gaming were much better suited. Graphic cards are faster, but they use more electricity and generate a lot of heat. The first commercial Bitcoin mining products included chips that were reprogrammed for mining Bitcoin. These chips were faster but still power hungry. ASIC (Application-specific integrated circuit) was then introduced and designed specifically for Bitcoin mining. AISC technology has made Bitcoin mining even faster while using less power and is therefore preferred by most miners today (Bitcoinmining, 2018).

2.2 Bitcoin: Markets & Trading Processes

Bitcoins can be acquired through mining, simply bought through an entity, or be spent person-to-person regardless of geography. Before any trade can take place, a Bitcoin wallet is needed. A wallet is offered by most cryptocurrency exchanges and has an associated address to it. The wallet behaves just like a bank account where the goal is to be able to keep the funds safely stored, monitor the balance, and to send and receive Bitcoins. There are several types of wallets with the security features being the varying variable. Bitcoin hardware wallets are most secure and web wallets are least secure. A hot wallet refers to any form of Bitcoin wallet that is connected to the internet (Sharma, 2017). These are the most popular wallets and can be connected through a web service, installed on the computer or a mobile phone. Market exchange sites, betting sites and other Bitcoin services frequently require deposits into their online wallets to access their services. The web wallets require the user to fully trust the third party keeping your funds safe. However, some Bitcoin exchanges offer cold storage of Bitcoins in exchange for a fee. Cold storage is a way of storing the Bitcoins offline to keep them safer against hackers.

Unlike other conventional exchanges, the digital currency exchanges operate 24 hours a day every single day of the week. Bitcoin or cryptocurrency exchanges works the same way as one would buy and sell assets through other exchanges.

Another key cornerstone about Bitcoin is that the usual transaction fee is a lot lower than the cost to send money internationally through a bank. The fee will depend upon the bank that is being used. Table 1 shows an example of some various banks.

Financial Institution	Incoming international wire	Outgoing international wire
Bank of America (U.S)	16$	35$*/45$**
Citibank (U.S)	15$	35$
HSBC (U.S)	15$	35$
Wells Fargo (U.S)	16$	45$**
Sparebank 1 SR-Bank (Norway)	13$	38$***

*Table 1: A representation of fees from various banks that are added to international wire, collected from the respective banks in March 6, 2018.*Sent in foregin currency, **sent in U.S. Dollars, ***The sender covers the costs in Norway and abroad.*

The average transaction fee between May 2013 and March 2018 for Bitcoins that were sent between two addresses has been $1.53 US dollars. In December 2017 the price of Bitcoins and the average transaction fee surged to a new all-time high. However, the transaction fees and Bitcoin prices eventually peaked at about $56.7 and $19,476 $US dollars the same month as shown in figure 2.

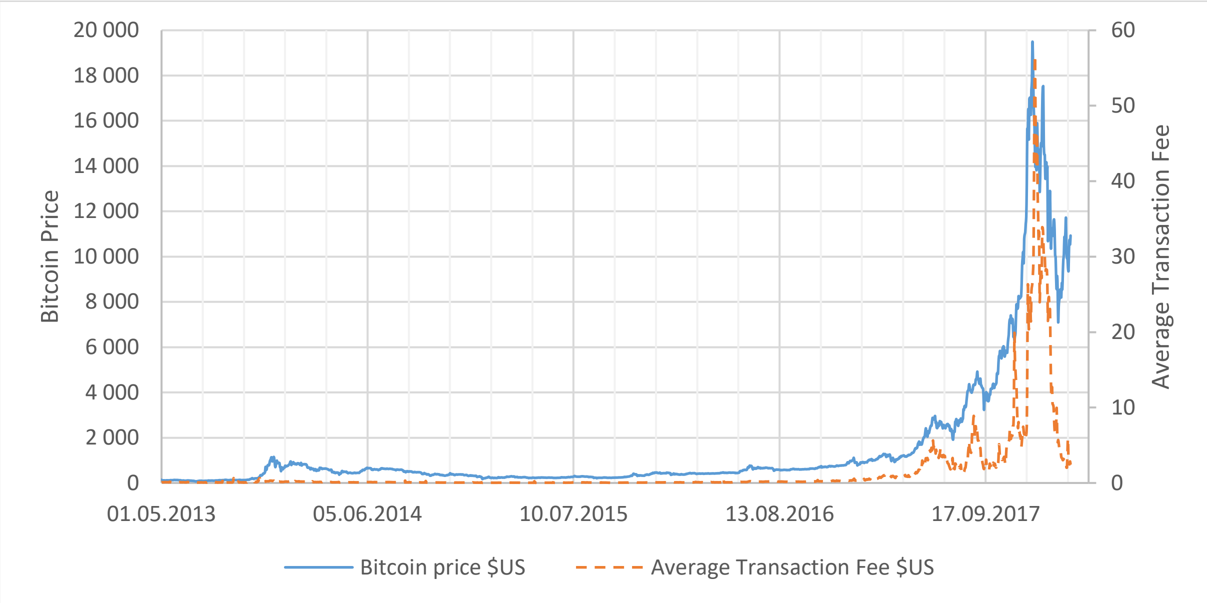

Figure 2: Average transaction fee and Bitcoin price between May 2013 and March 2018

By studying the relationship between the two variables in figure 2 by correlation analysis, it clearly shows that Bitcoin's average transaction fees have kept pace with the increase in Bitcoin's price. Figure 3 shows a plot of these two variables with a correlation coefficient of 0.8475 with a p-value less than 0.001. Hence, evidence suggests that there is a significant positive relationship between Bitcoin's average transaction fee and Bitcoin's price. Therefore, a potential problem with full implementation of Bitcoin as a potentially viable currency is that the cost of transaction could be more than its worth (e.g. paying $10 in transaction costs for a cup of coffee worth $2).

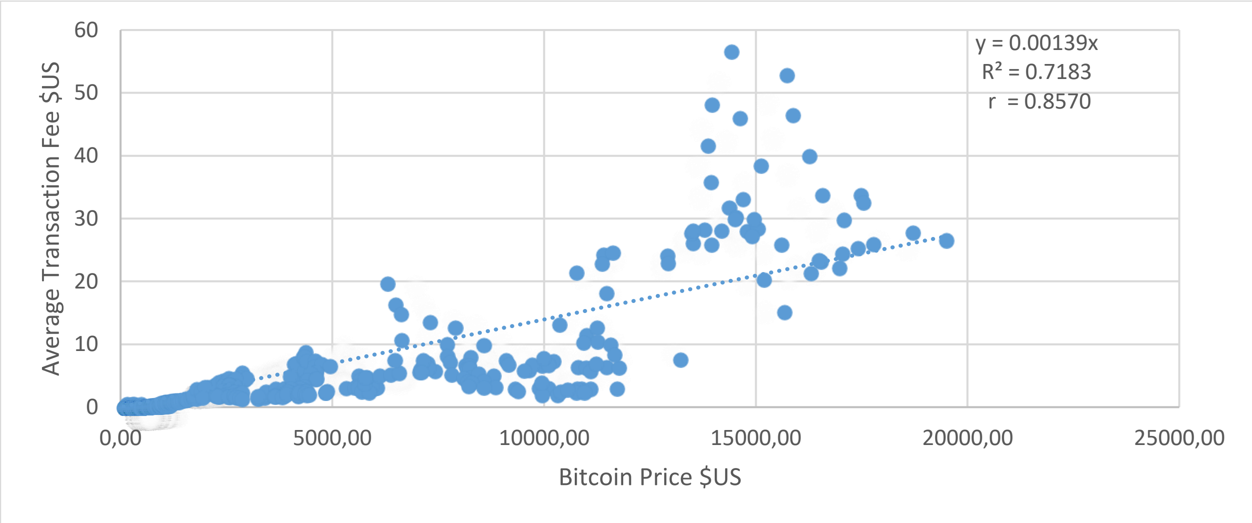

Figure 3: The correlation between the price of Bitcoin and average transaction fee

Since the network on average adds a new block to the ledger every 10 minutes, with one block having a maximum limit of 1 megabyte, there will be an upper limit of how many transactions the network can handle every 10 minutes. This limit was set by Satoshi to prevent denial-of-service[2] (DoS) attacks on the ledger.

If the average size of a Bitcoin transaction is 250 bytes, there can be a maximum of 4000 transactions for each block that is added. Furthermore, 4000 transactions every 10 minutes gives the network a capacity of 6.66 transactions every second. When the demand of transactions exceeds the network capacity, the sender of the transactions with the highest included fee can expect to get their transaction attached to the block first (Nadeem, 2017).

Digital currencies could be the beginning of a new way to exchange value without an intermediary as it allows for cheaper and faster payments. However, since the inception of Bitcoin in 2009 to this day, it is only regarded as an alternative to the conventional currencies as it is still necessary to use conventional currencies to buy Bitcoins. Slow transaction times and high transaction costs doesn't make Bitcoin a well-positioned system that can replace the conventional payment problems.

[2] In a denial-of-service attack, a cyber-attacker attempts to prevent legitimate users of a machine or network resource from accessing the information that is intended to its users. This is typically accomplished by "flooding" the machine or network to make it so busy that the network cannot fulfill its requests (US-CERT, 2013). The nodes in the Bitcoin network is vulnerable to sophisticated DoS attacks, as they can be overloaded such that it cannot process Bitcoin transaction.

3. Financial Risk

There is no widely agreed definition of the concept of risk. However, today in general, risk is used to describe exposure to the possibility of loss, damage, injury, or other adverse or unwelcome circumstances (Aven & Staff, 2015). The uncertainty about future market prices investors are facing is the starting point for every financial model.

The disruptive innovation behind the cryptocurrencies raises both threats and opportunities among a variety of stakeholders. Cryptocurrencies are still very much in its infancy even though Bitcoin has been available since 2009. Bitcoin is a highly volatile asset class that has delivered a rate of return that is quite unusual in a short amount of time. These features create an opportunity for making good profits for short-term investors. Traditional investors, such as institutional investors with a buy-and-hold strategy, have been absent when it comes to diversifying their portfolios with Bitcoins because of its highly volatile and unregulated nature. However, risk management in finance is about designing proper responses to avoid or mitigate bad risk to make sound investment decisions. Before investment decisions are made, investors position themselves for numerous financial risks that can affect the investment. These financial risks may be in the form of interest rate, inflation, social/political/legislative and volatility in the financial market, which could affect all financial securities in the same manner. The latter will be put on focus in the coming sections as volatility and correlation between assets (financial integration) are important components that are needed in evaluating the risk during portfolio optimization and hedging.

3.1 Volatility and Co-movement

Volatility is a fundamental characteristic of financial markets and is widely accepted as a measure of risk. It describes the degree of variation in the returns over a given period for a given security and can be measured by the standard deviation. High volatility means that the return of the underlying asset can change over a larger range of values in either direction over a period of time. Low volatility, on the other hand, implies more predictable changes in the returns.

A sudden increase in stock market volatility can be explained by investor's interpretation of good and bad news. Macroeconomic changes can be one of the influencing variables that cause market returns to fluctuate due to the uncertainty among investors about the future returns of their investments. Mandelbrot (1963) noted that large changes in price of an asset tend to be followed by other large changes, and small changes tend to be followed by small changes (of either sign). The high volatility tends to exhibit persistence for a while before the market returns revert back to mean levels after the initial shock. This is called volatility clustering. It is often observed in time series of financial securities by the positive serial correlation in the absolute value of returns. This phenomenon can therefore be used to model how much the clustering will influence the expectation of volatility in many periods in the future (Engle & Patton, 2001).

Volatility forecasting can be used to manage asset allocation for investors or funds that want to stay within a volatility band (e.g. 5%-10%), or in conjunction with returns as a tradeoff. In general, it is an essential part of risk management and can also be used in financial activities such as derivative pricing, market making (setting a fair bid-ask spread in times of volatility) and hedging. The Autoregressive conditional heteroskedasticity (ARCH) and the generalized autoregressive conditional heteroskedasticity (GARCH) are two of the most widely-used models to analyze and estimate volatility and will be discussed further in section 5.1.1 and 5.1.2.

Co-movement in the financial markets can be described as the tendency of which price and volatility exhibit a high degree of correlation across stock markets. Chen and Trang (2017) shows that common global factors can be a significant source of international stock market fluctuations and that strong co-movements across the international stock market exist. However, the degree of co-movements also depends on how developed and integrated the country is to the global economy. Evidently, the co-movements are stronger between developed countries. Moreover, an increasing economic globalization and international capital flow therefore increase the risk and impact of financial contagion[3] (Dornbusch et al., 2000).

During periods of financial distress, the presence of contagion effect can be identified by an increase in the conditional correlation between indices (Kohn & Pereira, 2017). Both volatility spillover and asset return co-movement are therefore important factors for portfolio allocation and risk management. Choosing a portfolio by combining assets that are less correlated is a widely embraced investment strategy that reduces the risk of volatility spillover.

[3] Financial contagion refers to the spread of market disturbances, mostly on the downside, in which local shocks are transmitted to other financial sectors or even to another country.

3.2 Portfolio Theory

This section explains the theory behind portfolio optimization and the terms diversification, hedging and safe haven. Before introducing Markowitz's portfolio theory, statistical definitions are presented as they are widely used in this section.

3.2.1 Statistical Definitions

The expectation of a discrete random variable x in a sample of n, is defined as

$$E(x) = \sum_{i=1}^{n} x_i p(x_i) \tag{1}$$

Where x_i are the values in the sample with respective probability for them to occur, $p(x_i)$. The expectation indicates the expected value of the variable.

The variance and standard deviation of a discrete random variable x in a sample of n are defined as

$$\sigma_x^2 = E[x_i - E(x)]^2 = \sum_{i=1}^{n} [x_i - E(x)]^2 \, p(x_i) \tag{2}$$

$$\sigma_x = \sqrt{\sigma_x^2} \tag{3}$$

Where x_i are the values in the sample with respective probability for them to occur $p(x_i)$ and expected value $E(x)$. The variance is a measure of spread around the expected value. The standard deviation indicates how much the variable on average differs from the expected value.

The covariance of two variables x and y in a sample of n is defined as

$$\sigma_{xy} = E\big[\big(x - E(x)\big)\big(y - E(y)\big)\big] = \sum_{i=1}^{n} [x_i - E(x)][y_i - E(y)] \, p(x_i, y_i) \tag{4}$$

Where x_i and y_i are the values in the sample with respective probability for them to occur $p(x_i, y_i)$. $E(x)$ and $E(y)$ are expected values of x and y. The covariance is a measure for the linear relationship between x_i and y_i.

The expected return of a portfolio p with n assets is defined as

$$E(r_p) = \sum_{i=0}^{n} w_i r_i \tag{5}$$

Where w_i is weight of asset i, and r_i is expected return of asset i. The expected return is an indication for what the portfolio will bring in terms of profit.

The variance and standard deviation of a portfolio p with n assets are given as

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=i}^{n} w_i w_j \sigma_{ij}\ , \ \ i \neq j \tag{6}$$

$$\sigma_p = \sqrt{\sigma_p^2} \tag{7}$$

where w_i and w_j are weights of asset i and j, σ_i^2 is the variance of asset i and σ_{ij} is the covariance between asset i and j. The standard deviation is an indication for how volatile the portfolio is.

3.2.2 Markowitz Portfolio Optimization

Harry M. Markowitz is regarded as the main creator of the modern portfolio theory is still used by people all over the world. In 1952 Markowitz published the article "Portfolio Selection" where he separated the selection process into two steps. The first step starts with observations, experiences and expectations of future returns of the available assets. The second step is about deciding which assets to include in the portfolio based on the expectations. By going through these steps, Markowitz came up with a portfolio optimization model that optimizes portfolios based on the investor's utility. The utility is used to express an investor's preference towards risk. Investors may be risk averse, risk lovers or something in between, which makes every investor's optimal portfolio unique.

The portfolio optimization model creates optimal portfolios based on correlation between assets. Markowitz (1952) assumed a one-period model where investors hold the same investment through the whole period. The investors base their decisions on expected return and variance that maximize their personal utility. Free access to correct information about return and risk, along with effective markets that absorb information fast and correct is also assumed.

Eventually, investors are considered risk averse, as risk exposure must be compensated for with increased expected return.

The main purpose of the portfolio optimization is to maximize expected return and minimize risk. Risk in portfolios is measured as the standard deviation of the logarithmic returns. The expected return and standard deviation of assets can be calculated as daily, weekly, monthly, yearly or whatever is preferable for the estimation.

Calculating several portfolios consisting of the same assets and plotting the results in a risk-return chart will reveal a pattern. Drawing a line around the outer plots will then reveal the *minimum variance frontier* (MVF) as seen in figure 4.

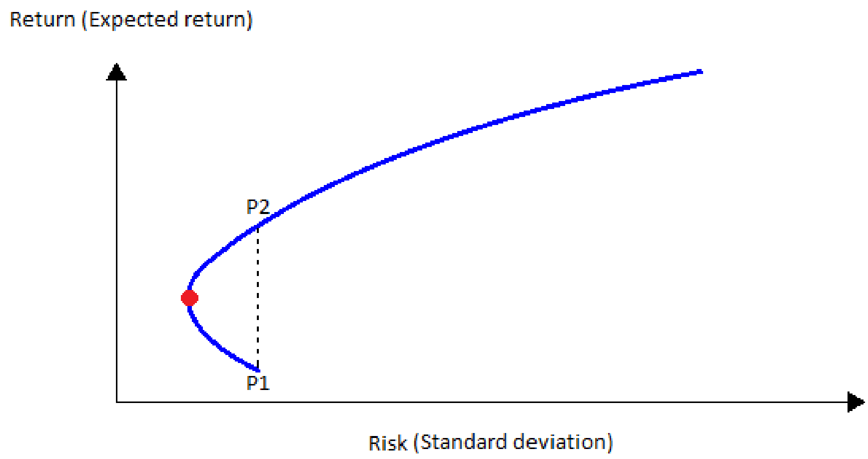

Figure 4: Minimum variance frontier

This line shows the minimum variance that can be reached for each level of expected return (Francis & Kim, 2013). This means that for any portfolios plotted inside the MVF, there is an alternative portfolio that offers the same return for a lower risk along the line. It is also normal to separate the MVF by the red dot, indicating the minimum variance. The part below the red dot is inefficient because the portfolios plotted there offer less return, but for the same amount of risk. This is illustrated with p_1 and p_2. Both portfolios have a standard deviation of 6,00%, but p_2 offers significantly higher return. The part over the red dot is called the *efficient frontier* (Francis & Kim, 2013). Portfolios plotted on this line represents the highest possible returns for a given amount of risk. Efficient frontier is often used together with the *capital market line* (CML).

The CML is the tangent line drawn from the risk-free rate on the expected return-axis to the efficient frontier (Francis & Kim, 2013), as shown in figure 5. All points along the CML have superior risk-return profiles to any portfolio on the efficient frontier. The line's slope indicates how much additional return is gained for taking on more risk. This is often referred to as the *Sharpe ratio* (Sharpe, 1994). The more return is gained per risk unit added, the steeper the line is. The point where the CML meets the efficient frontier is called the market portfolio, and this point represents the entire market.

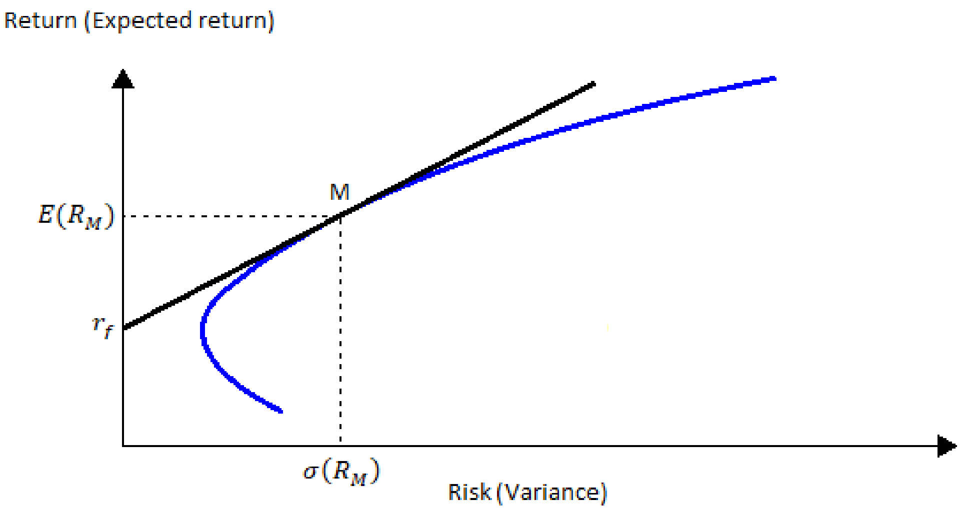

Figure 5: Capital market line

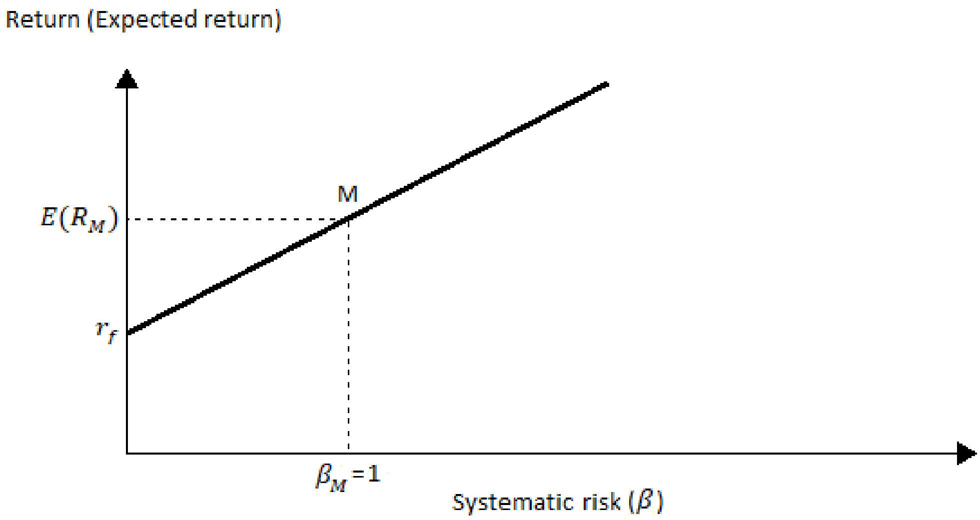

Figure 6: Security market line

These theories lead further to the *capital asset pricing model* (CAPM). CAPM is used to calculate the return that can be expected from a single asset by comparing it to the market, looking at time value and risk (Fama & French, 2004). The CAPM formula is defined as:

$$E(r_i) = r_f + \left(E(r_m) - r_f\right)\frac{\sigma_{i,m}}{\sigma_m^2} \tag{8}$$

where:

$E(r_i)$ = expected return for asset i,

r_f = risk-free rate,

$E(r_m)$ = expected market return,

$\sigma_{i,m}$ = covariance between asset i and market m,

and σ_m^2 = variance of market m.

The time value is represented through the risk-free rate (r_f), while risk is represented through multiplying the asset's relative volatility ($\frac{\sigma_{i,m}}{\sigma_m^2}$) with the risk premium ($E(r_m) - r_f$). The $\frac{\sigma_{i,m}}{\sigma_m^2}$ ratio, often referred to as β_i, explains the relative risk of an asset compared to the risk in the market. A β_i of 2 indicates that the asset i is twice as risky as the market. By plotting the $E(r_i)$ as a function of the β_i, the *Security market line* (SML) can be plotted as shown in figure 6. The SML shows how much return is required for a given level of *systematic risk* (Francis & Kim, 2013). Systematic risk is the opposite of *unsystematic risk* which leads to the next topic; diversification.

3.2.3 Diversification

A fundamental part of Markowitz's portfolio theory is that the risk of a portfolio can be minimized without reducing the expected return if the portfolios consists of several different assets. This is the idea of *diversification*. It is normal to separate portfolio risk into systematic risk and unsystematic risk (Francis & Kim, 2013).

Systematic risk is market risk that exists for all types of assets in terms of economic cycles caused by events that cannot be planned for or avoided[4]. This risk is not diversifiable regardless of how many assets the portfolio consists of. The best way to measure systematic risk is by calculating the beta.

[4] Inflation, market regulations and natural disaster are examples of systematic risk

Unsystematic risk is the same as risk inherent in a company or industry investment. This risk is related to the company itself and may be diversified by including more unrelated assets in a portfolio. Risk inherent in a company may be news on result reports or signed contracts. The idea behind diversification is to not put all your eggs in one basket. In the long run, this good/bad news will even each other out if the portfolio consists of several uncorrelated assets. The effect of diversification is illustrated in the figure below:

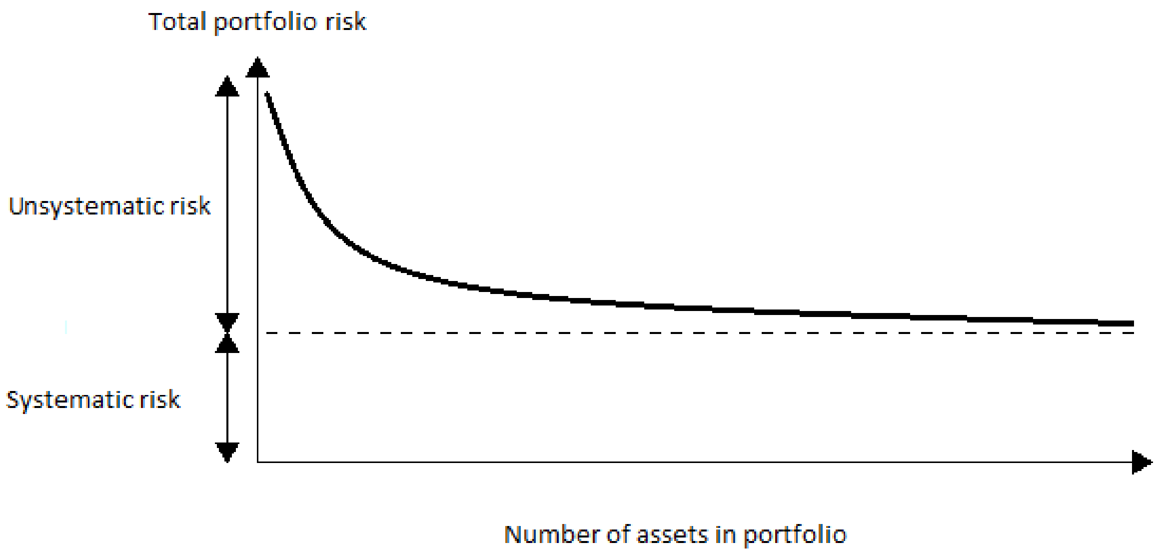

Figure 7: Effect of diversification

3.2.4 Hedging and Safe Haven

While diversification is a great way to remove risk from a portfolio, it is impossible to completely remove all risk. *Hedging* is another method used for removing risk in investments. There are several ways to hedge an investment. A common hedging strategy is to buy derivatives, such as options, futures or forwards (Catlere, 2009). An option gives the holder the right to buy or sell an asset at a specific price. However, the holder is not obligated to do so, which separates options to futures and forwards. The holder of futures and forwards is legally bound to finalize the deal. The main difference between futures and forwards is that futures are usually closed out prior to maturity, while forwards are finalized. This is because futures are mostly used by speculators trying to make profit on price changes, while forwards are mostly used by large distributors of commodities that needs a certain selling price to make profit. By giving the holder alternatives if the price of an asset is very volatile, using derivatives is a great hedging strategy for both investors and companies.

Hedging can also be based on correlation among assets (Catlere, 2009). If two assets are negatively correlated, one can be used as a hedge for the other. Gold and USD is an example for this. Figure 8 shows the correlation between trade weighted USD and gold. The strength of the hedge depends on how strongly negatively correlated the assets are. A correlation coefficient of 0 indicates absolutely no hedge, while a correlation coefficient of -1 indicates a perfect hedge. It is not normal for investors to hedge 100%, usually only parts of the investment is hedged. This is because the overall profit tends to get lower, the more hedge, unless one of the assets are heavily undervalued.

Figure 8: USD/Gold correlation (Marketrealist, 2014)

A *safe haven* is similar to a hedge, but what differs the two terms is the timing of negative correlation. If an asset should be considered as a safe haven, it needs to at least retain its value in times of distress. Baur and McDermontt (2010) define a safe haven as:

"A strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g. in times of falling stock markets"

4. Data

The data used in this thesis consists of daily prices from March 1st 2013 to March 1st 2018. All data are retrieved from EIKON, except Bitcoin prices for the period 01.03.2013 – 16.07.2014. These are retrieved from bitcoincharts.com. Bitcoin prices are given as daily closing price or price at 11:59 p.m. UTC, in USD. These prices are then transformed into logarithmic returns by using the following formula:

$$R_d = \ln \frac{price_t}{price_{t-1}} \tag{9}$$

where $price_t$ is the price on day t, and $price_{t-1}$ is the price the day before.
This is done to make financial comparisons and fit statistical models to the data, which will be discussed further in the coming sections. The daily data of the respective assets used in this thesis have also been modified in order to resolve the problem of weekends and holidays. Bitcoin prices, from the days the stock market and the currency exchanges are closed, have been removed to facilitate the right conditions for the processing of the data (same amount of observations). Prices from the previous day are also filled in where some of the indices lack data because of national holidays.

4.1 Comparison Objects

Since Bitcoin is a complex investment object global assets, currencies and commodities are used for comparison. The investment objects used are listed in the table below:

Cryptocurrency	Indices	Currencies	Commodities
Bitcoin	SP500	Euro	Oil
	DAX	Franc	Gold
	KS11	Yen	
	VGLT		

Table 2: Overview of assets

SP&500, DAX and KS11 were all included because they are considered among the biggest indices for their respective regions America, Europe and Asia. They are all highly attractive for investors all over the world because of great history of stable returns. In addition to this, the Vanguard Long-Term Treasury Bond (VGLT) is also included. This is a long-term treasury bond which is considered as a safer investment, but with lower expected return. The idea behind this inclusion is to add an asset with significantly lower systematic risk.

When it comes to currencies, Euro is naturally included. The Euro is the official currency in 19 European countries. It is also the second most traded currency after USD, while Yen is the third most traded. This is the main reason for the inclusion of those two. The Swiss Franc however, is included because it is considered as a safe haven (Yueh, 2015). Being considered as a safe haven, while also being a currency, makes it interesting for comparison.

Lastly, oil and gold are included because they are global commodities with alternative characteristics to indices and currencies. By including crude oil (West Texas Intermediate), it is also possible to capture movements in prices during the oil price shock in 2014. Gold is also, like Franc, considered as a safe haven (Bauer & McDermott, 2016).

4.2 Stylized Facts

According to Engle & Patton (2001), a good volatility model is characterized by its ability to estimate future movements. To do that it needs to capture as many of the commonly held stylized facts. Cont (2001), mentions that more than half a century of empirical studies on financial time series indicates that a wide range of securities do share some quite non-trivial statistical properties. These properties which are observed throughout financial markets are called stylized empirical facts, with some of the most common ones mentioned below.

- **Absence of autocorrelation**

Asset returns tend to lack any statistical significant autocorrelation, except for very short intraday series (higher frequency of data). The absence of autocorrelation means that the asset can be seen as an open system that continuously reacts to available information. Thereby, estimating the future price movement by past data of the asset is ineffective. This is the evidence for the efficient market hypothesis (Fama, 1970). Nevertheless, there is a whole discipline dedicated to quantitative technical analysis which has been developed in attempts to predict future price movements by studying past price movements.

- **Heavy tails (Leptokurtosis) and negative skewness**

The distribution of returns tends to exhibit leptokurtosis ($k > 3$) which has "fat tails" relative to the normal distribution's tail. However, for less frequent data (e.g. yearly returns) the distribution tends to be more mesokurtic (i.e. similar to kurtosis of a Gaussian distribution with kurtosis = 3). Skewness is used to describe the asymmetry from normal distribution. Skewness of the aggregate stock market returns tends to have negative skewness while firm stocks usually have positive skewness.

- **Gain and loss asymmetry**

One observes that investors are more sensitive to negative than positive information (i.e. the market draw-downs are more intense than the positive increase during an economic expansion).

- **Volatility clustering**

High volatility tends to exhibit persistence for a while before the market returns revert to mean levels after the initial shock. This means that a volatile period is followed by another volatile period as the market digestion may take several periods on big news. Moreover, rejecting the null hypothesis of squared returns being white noise is more likely as the autocorrelation is positive for volatility clustering (Cont, 2001).

- **Leverage effect**

First noted by Black and Cox (1976), his hypothesis postulates a negative correlation between the return of stocks and its volatility. Leverage can to some extend explain this phenomenon. Thus, a negative stock return leads to a lower equity value which increases the financial leverage to equity ratio of the firm and, in addition, creates higher risk for the investors holding the stock. Christie (1982) also supported the conclusion of this work while investigating the variance of equity returns and several explanatory variables. This work concluded that leverage effect and interest rate are positively correlated with stock returns.

- **Co-movement and volatility**

Looking at financial time series across different markets, e.g. rate of return for different stock exchange indices, one can observe big movements in one stock exchange index being matched by a movement in another stock exchange index. This advocates the importance of modelling the cross-correlations between different markets by a multivariate model (Knight & Satchell, 2011).

4.3 Descriptive Statistics

When comparing data, it is often interesting to look at certain key metrics. These key metrics are presented in table 3. By looking at the table, it is clear that Bitcoin differs from the other assets in every category. A mean and standard deviation around ten times higher than the rest indicates a very special kind of asset. The Francs abnormal properties is also notable and will be talked about in more detail in subsection 4.4.

Asset	Mean	Std. dev. dev	Skewness	Kurtosis	Min	1st Quantile	Median	3rd Quantile	Max
Bitcoin	0,442	5,915	-1,099	21,141	-66,395	-1,360	0,284	2,603	48,478
SP&500	0,044	0,756	-0,630	3,694	-4,184	-0,278	0,039	0,444	3,829
DAX	0,035	1,112	-0,369	2,575	-7,067	-0,475	0,058	0,596	4,852
KS11	0,014	0,698	-0,210	1,809	-3,143	-0,328	0,008	0,394	2,912
VGLT	-0,001	0,707	-0,322	0,994	-4,132	-0,459	0,039	0,463	2,416
Euro	-0,005	0,531	0,141	2,370	-2,400	-0,318	0,004	0,288	3,035
Franc	0,000	0,718	10,486	248,897	-2,572	-0,321	-0,020	0,290	17,139
Yen	-0,010	0,618	0,302	3,969	-3,428	-0,340	-0,012	0,302	3,751
Oil	-0,030	2,149	0,160	3,211	-10,794	-1,091	0,000	1,040	11,621
Gold	-0,014	0,970	-0,743	8,084	-8,879	-0,508	-0,007	0,491	4,687

Table 3: Descriptive statistics of assets

To illustrate the variation of the assets, every asset's returns are graphically presented along with a normal distribution for comparison in the figures 9-18. Every asset shows proof of high peaks and heavy tails which indicates that they are not normally distributed. This is expected, as financial data often have these characteristics. A Jarque-Bera test[5] is also performed for confirmation, as shown in table 4.

Asset	Test statistic	P-value	Normal distributed
Bitcoin	24508	0.0000	No
S&P500	826.37	0.0000	No
DAX	389.21	0.0000	No
KS11	187.11	0.0000	No
VGLT	76.037	0.0000	No
Euro	308.99	0.0000	No
Franc	3384600	0.0000	No
Yen	874.65	0.0000	No
Oil	564.82	0.0000	No
Gold	3665	0.0000	No

Table 4: Results from the Jarque Bera test

[5] A Jarque-Bera normality test is performed to detect deviation from the null hypothesis of normality

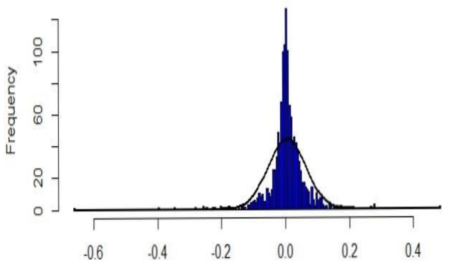

Figure 9: Bitcoin

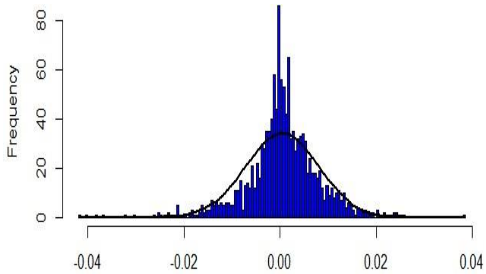

Figure 10: S&P500

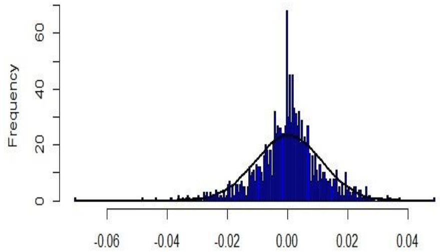

Figure 11: DAX

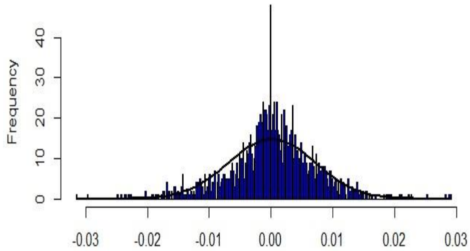

Figure 12: KS11

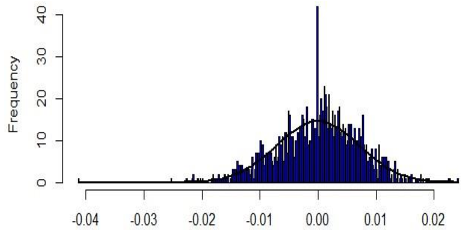

Figure 13: VGLT

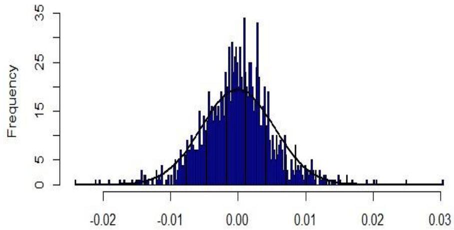

Figure 14: Euro

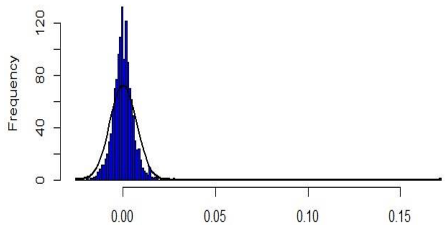

Figure 15: Franc

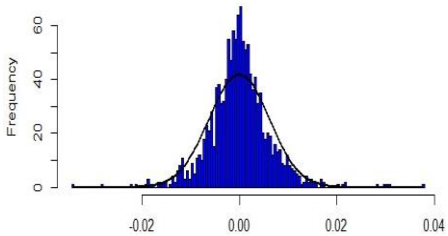

Figure 16: Yen

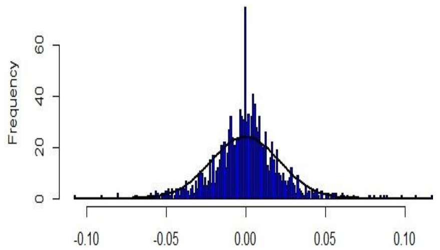

Figure 17: Oil

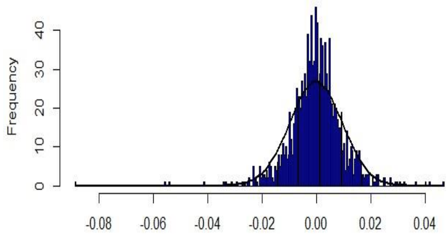

Figure 18: Gold

4.4 Sample period

The sample period in this thesis covers the last five years of trading days. This means that even though Bitcoin prices are constantly in change, prices at 00:00 from days that were not trading days are removed from the sample. This is necessary to make comparisons. It is also decided to cut the beginning of Bitcoins price history and start at 1st of March 2013. This has been decided after looking at the graph for Bitcoin trading volume for data back from 2011:

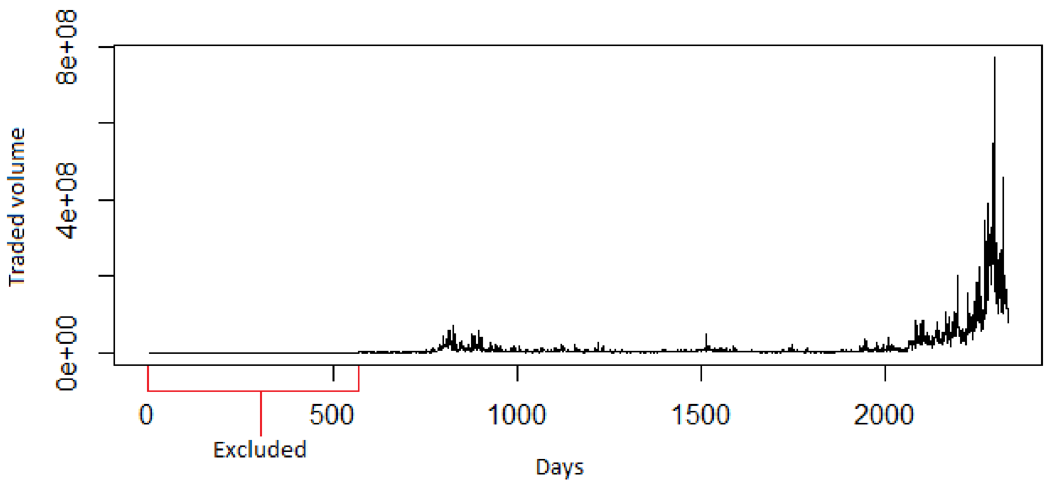

Figure 19: Bitcoin trade volume between 2011 and 2018

The graph shows that it was little to none trading activity in the first period of the original data sample. When fitting statistical models to the data, a period like this may cause inaccurate estimation and that is why it is excluded.

The full period sample from March 2013 to March 2018 is characterized by relatively stable growth rates in the financial markets. For the S&P 500, there were only two short periods of market distress during the stock sell-off in mid-August 2015 and the sell-off late December 2015 which are related to crude oil prices dropping below $30 per barrel. Having a period like this in the sample makes it possible to capture volatility spillover between the various markets if there is a significant increase in the cross-market correlation during the turmoil period. The sample frequencies for detection of a significant increase in the dynamical conditional correlation during turmoil period stretches on a daily basis from August 18th, 2015 to April 20th, 2016. Between these dates, there was one market correction at about 10.22% from the 52-week highs followed by a market pulldown at about 12.22%. Thus, this period is defined as the turmoil period. The stable period before the market turmoil is defined as March 1st, 2013 to August 17th, 2015.

An extreme outlier[6] is also detected in the Franc's distribution and statistic properties for the sample period. The reason for this is that the Swiss central bank abandoned its three-year-old cap at 1.20 francs per euro in January 2015 (Wright, 2015). This resulted in a 25 percent gain against the dollar in a single day and can therefore be detected in the descriptive statistics of franc where the kurtosis is abnormally high.

[6] An outlier is an observation point that deviates significantly from the rest of the observations

5. Methodology

This section of the thesis will present the analysis tools[7] and methods applied to obtain the empirical results.

The first subsection in this section is about the volatility models that are an essential part of this thesis. The coming subsections explains the process of fitting a volatility model to a dataset, and also the math behind the models. A univariate eGARCH model is used for fitting the assets that are used in the analysis of volatility spillover. Furthermore, a multivariate DCC-GARCH model is used to capture the conditional correlation between the assets in the global portfolio.

The second subsection is about how the calculation of MVF and CML is done. It is explained how the data is turned into different portfolios and plotted for comparisons. The last subsection is about VaR and CVaR. This section describes the concept of value of risk and how to perform a simplified Monte Carlo simulation.

5.1 Volatility Models

Before applying a volatility model to a dataset, it is important to check for necessary conditions. A unit root test of the log returns is performed to check whether the transformed time series is stationary. If a unit root exists in a time series it can be detected by an Augmented Dickey-Fuller test (ADF-test), where the null hypothesis states that there is a unit root at some level of confidence.

When the time series is stationary, the next step is to find an optimal mean-model. This is done automatically by using R’s auto.arima function. The function returns the best fitted ARMA model, based on information criterions. These criterions are explained further down, when deciding which GARCH model fits the data best.

Furthermore, a Ljung-Box test is also performed to test the GARCH models for ARCH effects. A significant ARCH effect in the time series identifies the autocorrelation in the squared residuals from the mean-model, meaning the time series does exhibit conditional heteroscedasticity. This implies a time-varying conditional variance (volatility clustering) and can therefore be used to build a model to estimate the volatility. The Ljung-Box test is used to

[7] Microsoft Excel and R are the software used for applying the statistical calculations. Excel is used for calculation regarding MVF and Monte Carlo Simulation, while R is used for the more advanced volatility models. Thereby, all programming codes written in R, are attached to appendix section A.

check the ARMA-model's squared residuals for autocorrelation. The test uses the following formula:

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{p}_k^2}{n-k} \sim X_h^2 \qquad (10)$$

where $\hat{p}_k^2$ is the squared sample autocorrelation at lag k, h is the number of lags, and n is the sample size.

When deciding which ARCH-type model fits the data best, it is normal to rank the models based on information criteria. There are several different information criteria, but the most common are AIC (Akaike information criterion) and BIC (Bayesian information criterion). The main difference between these information criterions is that BIC penalizes additional parameters more than AIC does. While AIC tries to find an unknown high dimensional model, BIC tries to find real models. This means that AIC is more likely to overfit a model than BIC, while BIC is more likely to underfit. The information criterion with the lowest value indicates the best tradeoff between explanatory power and model parsimony.

5.1.1 ARCH

The autoregressive conditional heteroscedastic (ARCH) model was introduced by Engle (1982) to capture volatility persistence in inflation. The ARCH model analyzes the effects unexplained by econometric models that operate under the assumption of uniform variance (homoskedasticity) in the error term. In some circumstances (e.g. volatility clustering) the variance in financial time series is heteroskedastic, making the homoscedastic model not efficient as the estimated standard errors of the coefficients are biased. The ARCH process is mean zero, which allows the conditional variance[8] of the past error terms to change and leaves the unconditional variance constant. ARCH is defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2 \qquad t \in Z \qquad (11)$$

[8] Conditional Variance – The variance in a term is given by the variance(s) of one or more other variables.

5.1.2 Univariate GARCH

The ARCH model was generalized by Bollerslev (1986) into generalized ARCH (GARCH). The conditional variance in the GARCH (p, q) model is parametrized as a distributed lag of past conditional variances (p) and past squared error (q), expressed by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 , \qquad t \in Z \tag{12}$$

Equation (1) in a more compact form:

$$\sigma_t^2 = \omega + \alpha(B)\epsilon_t^2 + \beta(B)\sigma_t^2, \qquad t \in Z \tag{13}$$

Where B is the standard backshift (lag) operator ($B^i\epsilon_t^2 = \epsilon_{t-i}^2 \; and \; B^i\sigma_t^2 = \sigma_{t-i}^2$) for any integer I, and where α and β are polynomials of degrees q and p (Franq & Zakoian, 2010).

To measure the historical volatility of the different asset classes in the global market portfolio a GARCH(1,1) model is introduced:

$$Mean\ equation: r_t = \mu + \epsilon_t \tag{14}$$

$$Variance\ equation: \sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{15}$$

Where $\omega > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$, such that σ_t^2 is less likely to obtain negative values.
r_t = return of asset at time t
μ = average return
ϵ_t = residual returns
ϵ_t is defined as:

$$\epsilon_t = \sigma_t z_t \tag{16}$$

Where z_t is standardized residual returns (a sequence of N(0,1) i.i.d. random variables) and σ_t, is the volatility.
For the variance equation, $\sigma_{(t+1)}^2$, is the one-period ahead forecast of the conditional variance based on the historical data as a function of:
ω = Constant term
ϵ_{t-1}^2 = The information about volatility observed in the previous period (the ARCH term)
σ_{t-1}^2 = Last period forecast variance (the GARCH term)

5.1.3 EGARCH

The ultimate goal of a GARCH model is to capture the various stylized facts of volatility. However, when it comes to the standard ARCH and GARCH models, they are unable to model the asymmetric response of volatility to changes in returns (i.e. the leverage effect observed in asset returns). The GARCH specification in section 5.1.2 assumes that the sign of the errors (rises and falls of the assets) does not give differential impacts since the errors are squared (Knight & Satchell, 2011).

Nelson (1991) introduced an exponential GARCH model (EGARCH) where the logarithm of conditional variance ensures the non-negativity and asymmetric relation without the constraints imposed in the standard GARCH model. The exponential GARCH model with weighted errors $g(\varepsilon_t)$ is given by:

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i g(\varepsilon_{t-i}) + \sum_{j=1}^{p} \gamma_j \ln(h_{t-j}) \qquad (17)$$

Where the parameters ω, α_i and γ_j are not restricted to be non-negative, and that:

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - \mathrm{E}(|\varepsilon_t|)] \qquad (18)$$

Where both ε_t and and $|\varepsilon_t| - \mathrm{E}(|\varepsilon_t|$ is zero-mean i.i.d. random sequences, and

$$\mathrm{E}(|\varepsilon_t| = \sqrt{\frac{2}{\pi}} \qquad (19)$$

for a normal distribution, whereas

$$\mathrm{E}(|\varepsilon_t| = \frac{2\sqrt{\upsilon - 2}\, \Gamma(\upsilon + 1)/2)}{(\upsilon - 1)\, \Gamma(\, \Gamma(\frac{\upsilon}{2})\, \sqrt{\pi}} \qquad (20)$$

for the student-t distribution.

5.1.4 Multivariate DCC-GARCH

A univariate GARCH model explains the persistence and volatility shock on itself. On the other hand, a multivariate GARCH model focuses on analyzing the volatility spillover of a variable on another variable. In order to find contagion between two types of financial markets, one needs to compare the cross-market correlation during the period of crisis with a period of stability prior to the crisis. However, it is only contagion if the cross-market correlation increases significantly during the crisis. Modeling the volatility dynamics and correlation between assets is therefore important in order to avoid co-movement and to determinate optimum weights of a well-diversified portfolio.

The stock market co-movement and volatility spillover effects are two critical factors in financial risk management. A potential pitfall when examining the cross-market co-movement and spillover effects is that the correlation coefficients are biased and inaccurate due to heteroscedasticity (Forbes & Rigobon, 2002). This thesis will therefore focus on a specific multi-variate GARCH method capable of modeling conditional variances and the unbiased conditional correlation coefficients of several series simultaneously.

The Dynamic Conditional Correlation (DCC) first introduced by Engle (2002), allows the correlation to vary with time rather than requiring them to be constant. The idea behind modelling the conditional variances and conditional correlation is that the covariance matrix of a vector of returns, H_t, can be decomposed into the conditional standard deviations, D_t, and a correlation matrix, R_t. Where both R_t and D_t are time-varying in the DCC-GARCH model.
The estimation of Engle's DCC-GARCH model (Celik, 2012):

$$H_t = D_t R_t D_t \tag{21}$$

$D_t = diag\{\sqrt{h_{i,t}}\}$, a diagonal matrix of time varying standard deviations from the univariate model described in the GARCH section:

$$D_t \begin{bmatrix} \sqrt{h_{1t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{nt}} \end{bmatrix}$$

where

$$h_{it} = \omega_i + \sum_{q=1}^{Q_i} \alpha_{iq} \varepsilon_{i,t-q}^2 + \sum_{p=1}^{P_i} \beta_{ip} h_{i,t-p}^2 \tag{22}$$

for any univariate GARCH(p,q) model.
Moreover, R_t, the symmetric conditional correlation matrix of the standardized errors ε_t:

$$\varepsilon_t = D_t^{-1} r_t \sim N(0, R_t) \tag{23}$$

$$R_t = \begin{bmatrix} 1 & q_{12,t} & q_{13,t} & \cdots & q_{1n,t} \\ q_{21,t} & 1 & q_{23,t} & \cdots & q_{2n,t} \\ q_{31,t} & q_{32,t} & 1 & & q_{3n,t} \\ \vdots & \vdots & & \ddots & \vdots \\ q_{n1,t} & q_{n2,t} & q_{n3,t} & \cdots & 1 \end{bmatrix} \tag{24}$$

Two requirements need to be considered when specifying R_t:

1. H_t, needs to be positive definite as it is a covariance matrix. Therefore R_t has to be positive definite ($D_t$is positive definite since the variance in the univariate GARCH models are all positive in the diagonal elements).
2. All the elements in correlation matrix have to be equal or less than one.

In order to meet the two requirements, R_t is decomposed into:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{25}$$

For a DCC(1,1)

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha \varepsilon_{t-1} \varepsilon_{t-1}^T + \beta Q_{t-1} \tag{26}$$

where α and β are non-negative scalars ($\alpha \geq 0$, $\beta \geq 0$) such that $\alpha + \beta < 1$, and Q_0 is positive definite to ensure that H_t is positive definite.
$\bar{Q}$, is the unconditional covariance of the standardized errors $Cov(\varepsilon_t \varepsilon_t^T)$.
Whereas, Q_t^{*-1}, is the inverted diagonal matrix with the square root of the diagonal elements of the matrix Q_t:

$$Q_t^{*-1} = \begin{bmatrix} 1/\sqrt{q_{11t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\sqrt{q_{nnt}} \end{bmatrix} \tag{27}$$

This gives us the correlation structure of the general DCC(Q,P)-GARCH model (Engle, 2002):

$$Q_t = (1 - \sum_{i=1}^{P} \alpha_i - \sum_{j=1}^{Q} \beta_j)\bar{Q} + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-1} \varepsilon_{t-1}^T + \sum_{j=1}^{Q} \beta_j Q_{t-j} \tag{28}$$

5.1.5 Likelihood Ratio Test

To determine the significance of the parameters in the DCC-GARCH model, a likelihood ratio test is used. This test compares the goodness of fit of two statistical models, which is based on the difference in the log-likelihood functions for an unrestricted and restricted model. The unrestricted model generally leads to a higher log-likelihood as it consists of parameters that have been excluded in the regression of the restricted model. The idea behind this test is to check whether the fall in the log-likelihood is large enough to conclude that the dropped variables in the restricted model are important. This is done by calculating the likelihood ratio statistics, which is twice the difference in the log-likelihood (Wooldridge, 2014, page 465):

$$\mathrm{LR} = 2(\mathrm{L_{ur}} - \mathrm{L_r}) \tag{29}$$

Where $\mathrm{L_{ur}}$ is the log-likelihood value for the unrestricted model and $\mathrm{L_r}$ is the log-likelihood value for the restricted model. The distribution that is used for the test is a chi-squared (χ^2) distribution:

$$\mathrm{LR} \sim \chi^2(\mathrm{df} = \mathrm{df_{ur}} - \mathrm{df_r}) \tag{30}$$

Where the number of degrees of freedom is the number of parameters that have been excluded in the regression (i.e. the restricted model). The likelihood ratio statistics can then be used to compute a p-value in the chi-squared distribution to determine the significance of the parameters in the DCC-GARCH model. This can be verified by rejecting the null hypothesis:

$$\mathrm{H_0}: \text{alpha} = \text{beta} = 0 \tag{31}$$

$$\mathrm{H_1}: \text{alpha} \neq 0 \text{ or beta} \neq 0 \tag{32}$$

When the null hypothesis is not rejected, the Constant Conditional Correlation (CCC) model is obtained for the sample. For a more detailed description about (CCC), see Bollerslev (1990). When it is confirmed that alpha or beta are different from zero, the statistical significance of the coefficients in each pair in the DCC model indicates the existence of time-varying dynamic correlations.

5.2 Portfolio Optimization

Bitcoins high volatility and price increase makes it interesting to include in a portfolio. Comparing MVF and CML for two different groups of portfolios, one with Bitcoin included and one without, will give an indication of Bitcoin's diversification properties. If it is possible to obtain the same expected return for a lower risk, it means that some of the unsystematic risk has been reduced.

In order to create portfolios, the returns are used to calculate daily expected return, standard deviation and variance for each asset. These are key metrics used in financial estimation. A covariance matrix is also constructed for capturing the directional relation between the assets. This matrix, together with daily expected returns, is then used to estimate portfolio characteristics.

The portfolios are sorted in four different categories; indices, currencies, commodities and combinations (see appendix B). This is done both for portfolios including Bitcoin, and portfolios without Bitcoin.

Further, Excels Solver function[9] is used to calculate portfolio weights. When choosing what constraints to set for the Solver, it is important to think of which portfolios define the MVF. By targeting lowest possible standard deviation with different constraints on minimum expected return, the MVF can be drawn through the points. Other portfolios, such as even weights and highest expected return with only 5.0-15.0% weight per asset, are also included for giving a better illustration of alternative portfolios. A portfolio with highest Sharpe ratio is included for identifying where the CML is a tangent to the MVF.

When calculating the CML, it is necessary to use a risk-free rate. All calculations are done for daily returns, and therefore a daily risk-free rate is derived from yearly USD Libor interest rates[10]. This is done by first taking the mean of the yearly rates from 2013-2017 and then transforming it by using the following formula:

$$r_{f,d} = (1 + r_{f,y})^{\frac{1}{251}} - 1 \qquad (33)$$

Where:

$r_{f,d}$ and $r_{f,y}$ is the daily and yearly risk-free rate, and 251 is used for yearly trading days.

All portfolios are plotted in a risk-return chart, using the portfolios expected return and standard deviation. The MVF is created by drawing a line through all the outer portfolios in the risk-return chart. The CML is the tangent line drawn from the point on the return axis equaling the risk-free rate and up to where it is a tangent to the MVF. Comparing these charts will reveal differences and show effects of including Bitcoin in a portfolio.

[9] Solver is a Microsoft Excel add-in program that calculates optimal solutions (minimum or maximum) based on constraints.

[10] The Libor interest rate is the world's most widely-used benchmark for short-term interest rates. Since the rate is constantly in change, the average of the last 5 years is used for "risk-free rate" in this thesis (see appendix B).

5.3 Value at Risk

Value at risk (VaR) is a risk measure that explains potential loss from an investment over a given period at a specific confidence level (Angelidis & Degiannakis, 2009). Companies use VaR to calculate internally for business risk and regulatory to report financial risk to government. A VaR model is based on historical data and tries to capture the probability of potential future loss. The model has two main parameters, confidence level and time horizon.

The confidence level describes the probability of potential future loss. Estimation varies from 0.1 to 0.001 quantiles, but 0.01 and 0.05 are the most common. Figure 20 illustrates VaR for a 0.05 quantile. By using this quantile, the VaR model indicates that the potential loss will not be exceeded 95 out of 100 times (95.0% confidence level). The time horizon of a VaR model should be adjusted to the investment, i.e. liquid portfolios should have a short time horizon, while long term investments like funds should have a longer time horizon.

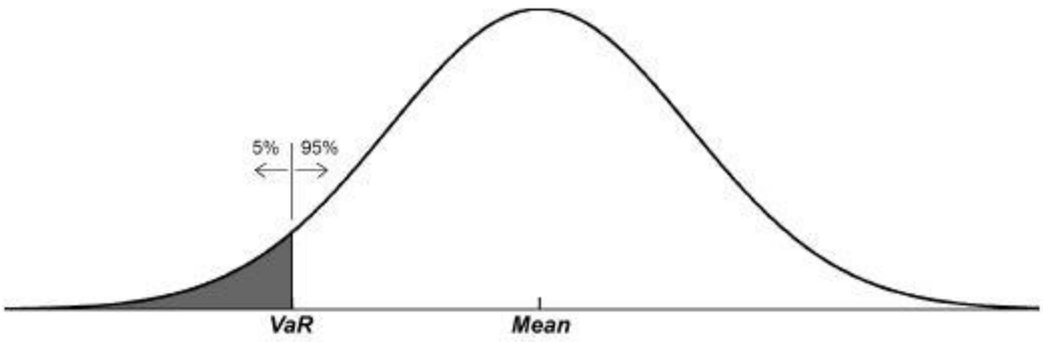

Figure 20: Value at Risk (Investsolver, 2016)

VaR can be calculated in several ways, but there are two main approximations, nonparametric and parametric. Nonparametric approximation is popular because it is very simple and does not need any predictions of the distribution of the data. The idea behind the method is to let the actual data represent the outcome (Dowd, 2002). Parametric approximation, on the other hand, is a method that tries to capture the data with a fitted distribution. The main advantage with parametric approximation is that extra information can be added to the model. This makes it possible to construct precise models for estimation that will give more accurate results. However, parametric approximation is dependent on the model to fit the actual data which may not always be uncomplicated.

Historical simulation is the most common nonparametric approximation. The method is about collecting historical data from a specific time horizon and sort the data by values. VaR is then calculated by looking at the relevant quantiles of the sorted data. This method is relatively easy, but relies on the history to repeat itself. The sample period is also a critical factor, because important periods that are needed for proper predictions may be omitted.

Normal linear VaR is the most common parametric approximation. The VaR model is calculated by assuming the data is normally distributed. Assuming a normal distribution, the only parameters needed are mean and standard deviation. This means that a skewness of zero and a kurtosis of three also are assumed. Only requiring two parameters makes the method easy to use, but the main problem is estimating with the highest confidence levels. Assuming a normal distribution is usually more accurate for central quantiles and more inaccurate for extreme values. This makes estimations with confidence level 99% or more, likely to be unprecise.

VaR can also be calculated by running simulations of possible outcomes. This parametric approximation requires more work, but has the potential to map underlying risk factors and assets more accurately. Monte Carlo Simulation is the most common method when it comes to simulation. The method is often used to price derivatives and solve complex risk management problems. By simulating possible outcomes repeatedly, a distribution of outcomes can be established. This distribution is then used to calculate VaR.

There is also an additional way to look at VaR. Conditional Value at Risk (CVaR), also known as expected shortfall, explains the expected value of the asset loss given that the VaR quantile of the distribution has been surpassed (Angelidis & Degiannakis, 2009). While VaR only describes the probability of a loss of X or more, CVaR describes what loss is expected when it first occurs. This makes CVaR an important value to consider, especially when the tails of the distribution are heavy[11]. CVaR can also be seen as the average loss when VaR is exceeded as shown in figure 21.

[11] Distribution with heavy tails possesses more extreme values than a normal distribution, which means that CVaR will increase significantly.

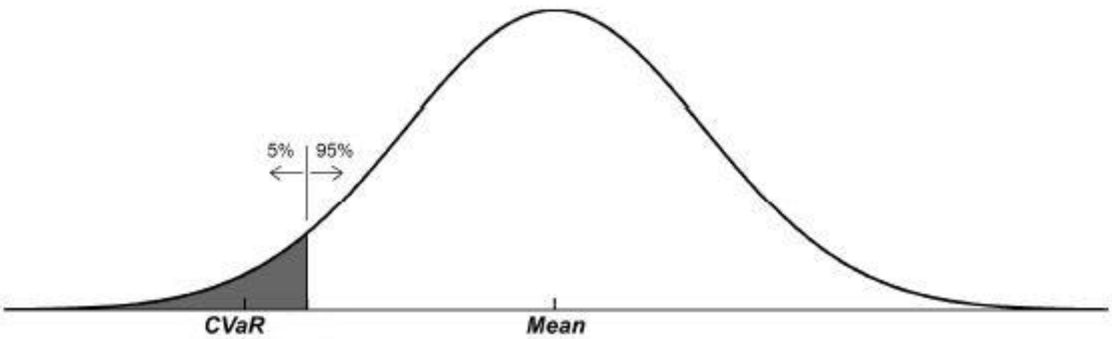

Figure 21: Conditional Value at Risk (Investsolver, 2016)

In this thesis, VaR and CVaR will be calculated both for portfolios with and without Bitcoin by using Monte Carlo Simulation. Calculating VaR for portfolios with various Bitcoin weights is one way to capture Bitcoin's impact on VaR.

When calculating VaR and CVaR for portfolios through Monte Carlo Simulation, the first step is to sort historical data. Sorting data, constructing portfolios and estimating characteristics is already done from calculating the minimum variance frontier. This leads straight to the next step, drawing random numbers and run simulations.

In this simulation, an initial investment of 1 000 000$ is assumed. The investment is then multiplied by random returns over 100 days. Excels NORM.INV and RAND functions are used to randomize the returns multiplied with the investment. These functions used together draw random numbers from a normal distribution with given mean and standard deviation. Since mean and standard deviation are already calculated for each portfolio, the simulation is simplified by assuming that the portfolios returns are normally distributed. In most cases with financial data, it is not recommended to assume a normal distribution for every underlying risk factor, but in this case, it is done for simplicity. The process is repeated 1000 times and the result of the investment on day 100 is then used as to calculate VaR and CVaR. This process is illustrated in the figure below:

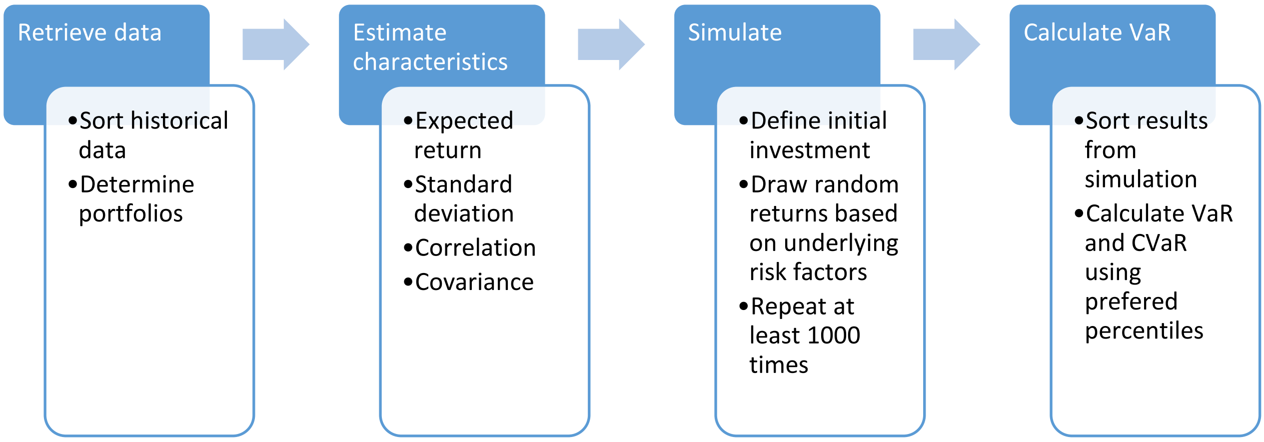

Figure 22: VaR with Monte Carlo simulation

All assets in the portfolios used are evenly weighted except for Bitcoin which varies from zero to ten percent. The simulation is done for five different portfolios, each having the weight of Bitcoin increased by 2.5%. The idea behind this is to see how VaR and CVaR changes, as the Bitcoin weight in the portfolio increases. After the results are sorted, VaR is found by identifying the values corresponding to the used confidence levels. CVaR is then calculated by using the following formula:

$$CVaR = \frac{1}{n}\sum_{i=1}^{n} y_n \tag{34}$$

where n is the number of sorted results equivalent to the chosen percentile and y_n is the value for result number n. All results are given as percentage of initial investment.

6. Empirical Results

This section presents the results from the ARCH-type models, DCC-GARCH, portfolio optimization and VaR analysis that are mentioned in the methodology section. The results presented are of great value, when analyzing Bitcoin's financial risk properties in a global portfolio and as an investment object. DCC-GARCH will show if Bitcoins can be used as a diversifier, hedge and safe haven. Furthermore, the portfolio optimization reveals changes in risk and return, and VaR will capture the risk by simulation.

6.1 ARCH-type

Before deciding what ARCH-type model is most optimal for the returns, the optimal order of a mean equation for the assets and ARCH-effects were investigated in table 5.

Asset	ARMA (p,q)	Test statistic	P-value	ARCH-effects?
Bitcoin	1,1	449.4	0.00	Yes
SP500	1,1	404.81	0.00	Yes
DAX	1,1	228.86	0.00	Yes
KS11	2,2	95.467	0.00	Yes
VGLT	1,1	22.142	0.04	Yes
Euro	1,1	124.93	0.00	Yes
Franc	1,1	1.3478	0.99	No
Yen	1,1	72.982	0.00	Yes
Oil	1,1	570.3	0.00	Yes
Gold	1,1	71.924	0.00	Yes

Table 5: ARMA models used and investigation of ARCH-effects

The results in table 5 show that most of the assets fit to an ARMA (1,1) model, except KS11 with ARMA (2,2). Therefore, one can apply a GARCH model to the assets. The results also show that ARCH-effects are present in all the respective assets, except franc. This could be due to the extreme outlier that was found in subsection 4.4. The ARCH-type models will therefore not be appropriate in modeling franc. With no ARCH-effects, the regression of franc will have little explanatory power.

The Information criteria was obtained from the analysis of GARCH(1,1) and eGARCH(1,1) model for most of the assets. Only, KS11, was tested with a GARCH(2,2) and eGARCH(2,2) model. The information in table 6 is used to determine what ARCH-type should be used to capture the volatility of the assets (goodness of fit). Thereby, these models are analyzed further in subsection 6.2 to investigate if there are signs of any contagion between the respective markets.

Bitcoin				
Distribution	Normal		Student's t	
Volatility Model	GARCH	eGARCH	GARCH	eGARCH
AIC	-3.3115	-3.3289	-3.6018	**-3.6183**
BIC	-3.2876	-3.3011	-3.574	**-3.5865**
S&P500				
Distribution	Normal		Student's t	
Volatility Model	GARCH	eGARCH	GARCH	eGARCH
AIC	-7.1599	-7.2224	-7.239	**-7.2837**
BIC	-7.136	-7.1946	-7.2112	**-7.2519**
DAX				
Distribution	Normal		Student's t	
Volatility Model	GARCH	eGARCH	GARCH	eGARCH
AIC	-6.3185	-6.3553	-6.372	**-6.4173**
BIC	-6.2947	-6.3275	-6.3442	**-6.3855**
KS11				
Distribution	Normal		Student's t	
Volatility Model	GARCH	eGARCH	GARCH	eGARCH
AIC	-7.1381	-7.1661	-7.185	**-7.2104**
BIC	-7.1064	-7.1304	-7.1493	**-7.1707**
Oil				
Distribution	Normal		Student's t	
Volatility Model	GARCH	eGARCH	GARCH	eGARCH
AIC	-5.1486	-5.1856	-5.1859	**-5.2024**
BIC	-5.1288	-5.1618	-5.1621	**-5.1746**
Gold				
Distribution	Normal		Student's t	
Volatility Model	GARCH	eGARCH	GARCH	eGARCH
AIC	-6.5355	-6.5325	-6.6437	**-6.6463**
BIC	-6.5117	-6.5047	-6.612	**-6.6185**

Table 6: Information criteria results from analyzing the ARCH-type models. The optimal models are marked with bold numbers.

The results in table 6 show that the eGARCH models with a student's t distribution for the respective assets analyzed, have the lowest information criteria. This shows that the conditional variance with asymmetric response to market shocks (also discussed in stylized facts) gives a better fit. The recommended Student's t distribution is also in line with the stylized fact that the distribution of returns tends to exhibit leptokurtosis and skewness. These descriptive data were presented in section 4.1.

6.2 DCC-GARCH

The analysis with DCC-GARCH offers two results. Firstly, the results from the off-diagonal elements (equation 24) that would be the conditional correlation coefficients, are retrieved. A DCC-GARCH model for the three sample periods that are presented are used to investigate the effectiveness of Bitcoin as a diversifier, hedge and safe haven by these coefficients. Secondly, one plot the volatility model of Bitcoin against the correlation for each pair to investigate if there are any volatility spillovers between Bitcoin and the financial assets in the global portfolio. This analysis is performed to investigate if there is any indication on contagion, which can be found if the conditional correlation has a strong positive spike when the volatility is increasing sharply. The presence of contagion between Bitcoin and the other assets would imply that Bitcoin would not have the qualities of a good diversifier, hedge or safe haven since the co-movement during times of stress increases.

In appendix A.1, the results of the multivariate ARMA(1,1)-DCC-GARCH(1,1) models for the different sample periods are reported. The results show that the parameters alpha and beta of the DCC models are different from zero. This is also tested with a likelihood ratio test, where table 7 below shows that the alpha or beta are different from zero, and that the sample periods thereby capture the dynamic conditional correlation of the returns and volatility.

Likelihood Ratio Test for the 3 periods analyzed				
	LR	df	P-value	DCC?
Entire Period	2667	20	(0.00)	Yes
Stable Period	795	20	(0.00)	Yes
Market Turmoil	450	20	(0.00)	Yes

Table 7: Likelihood ratio test results

As shown in the results from DCC-GARCH models in appendix A.1, the alphas and betas for all assets are positive and satisfy the inequality alpha + beta < 1, in the full sample. Thus, the dynamic correlation is strictly mean reverting around a constant level. The average sum of alpha and beta for the same period is 0.986, which also indicates a highly persistent volatility. Moreover, the alphas of DAX, FRANC and Oil in the stable period and alphas of VGLT and YEN in the turmoil period are not significant. However, the betas are highly significant for almost all assets. These results in combination with the asymmetry response of Bitcoin implies that Bitcoin as an asset for investment, shares many common characteristics with the other conventional assets in the global portfolio. As seen in the descriptive analysis, the most prominent differences between Bitcoin and the other assets are the heavy tail and high volatility.

Analysis of the conditional correlation coefficients

The conditional correlations of the 10 assets are reported in table 8. The highest correlation coefficient mean value (0.547) is between S&P500 and DAX. The currencies also exhibit high correlation when paired with each other or gold. However, almost all of the various assets in the global portfolio paired with Bitcoin display a very low negative conditional correlation and low positive conditional correlation near zero for the whole sample period. Only KS11 and FRANC are low but positive values.

Conditional Correlation Matrix (A): The entire period: 01/03/2013 to 01/03/2018 - 1303 observations										
	Bitcoin	S&P500	DAX	KS11	VGLT	EURO	FRANC	YEN	OIL	GOLD
Bitcoin	1									
S&P500	-0.098	1								
DAX	-0.015	0.547	1							
KS11	0.011	0.195	0.258	1						
VGLT	-0.021	-0.316	-0.275	-0.059	1					
EURO	-0.018	-0.039	-0.212	-0.057	0.158	1				
FRANC	0.005	-0.122	-0.174	-0.037	0.225	0.527	1			
YEN	-0.003	-0.385	-0.36	-0.162	0.445	0.434	0.398	1		
OIL	-0.023	0.244	0.122	0.096	-0.152	0.079	0.001	-0.035	1	
GOLD	-0.003	-0.098	-0.2	-0.066	0.307	0.357	0.335	0.458	0.131	1

Table 8: Conditional correlation matrix, entire period

The conditional correlations were also analyzed in a stable period as shown in table 8. The period of market turmoil is shown in table 9. The market turmoil period is of particular interest for safe haven categorization as this is the period where an investor usually tends to switch to assets that are uncorrelated with the markets with turbulence.

Conditional Correlation Matrix (B): Stable period: 01/03/2013 to 17/08/2016 - 642 observations										
	Bitcoin	S&P500	DAX	KS11	VGLT	EURO	FRANC	YEN	OIL	GOLD
Bitcoin	1									
S&P500	-0.022	1								
DAX	-0.049	0.547	1							
KS11	-0.031	0.122	0.201	1						
VGLT	-0.012	-0.339	-0.274	-0.047	1					
EURO	-0.012	0.025	-0.188	-0.106	0.168	1				
FRANC	0.02	-0.107	-0.103	-0.052	0.217	0.363	1			
YEN	0.01	-0.385	-0.311	-0.127	0.417	0.391	0.316	1		
OIL	-0.083	0.201	0.062	0.049	-0.111	0.122	-0.006	0.042	1	
GOLD	-0.023	-0.016	-0.099	-0.019	0.201	0.284	0.267	0.332	0.202	1

Table 9: Conditional correlation matrix, stable period

Conditional Correlation Matrix (C): Market turmoil: 18/08/2015 to 28/10/2016 - 314 observations										
	Bitcoin	S&P500	DAX	KS11	VGLT	EURO	FRANC	YEN	OIL	GOLD
Bitcoin	1									
S&P500	-0.032	1								
DAX	0.065	0.575	1							
KS11	-0.015	0.348	0.398	1						
VGLT	0.013	-0.303	-0.352	-0.149	1					
EURO	-0.049	-0.134	-0.239	-0.016	0.08	1				
FRANC	-0.052	-0.155	-0.288	-0.01	0.159	0.835	1			
YEN	-0.053	-0.403	-0.479	-0.258	0.437	0.427	0.448	1		
OIL	0.076	0.405	0.323	0.2	-0.341	0.029	-0.018	-0.192	1	
GOLD	0.001	-0.202	-0.369	-0.182	0.411	0.339	0.371	0.508	-0.016	1

Table 10: Conditional correlation matrix, market turmoil

The co-movement between gold and the respective currencies in the portfolio increase significantly during market turmoil. A stronger co-movement between the indices and between the currencies is anticipated as an investor's liquidity may dry out during market turmoil. In order to satisfy their cash balance or margins, they might sell assets globally, which results in a higher co-movement between the markets. The cross-border financial interdependence and potential herding behavior could also be one of the variables that strengthen this co-movement. As the presence of low co-movement between the assets in a global portfolio is regarded as a diversification technique in periods of market turmoil, there are no signs of any significantly higher co-movement between asset *i* and Bitcoin as the conditional correlation still fluctuates around zero. However, there is notably higher co-movement between SP&500-DAX, SP&500-KS11, SP&500-OIL, and VGLT-GOLD as seen in table 10.

Analysis of volatility spillover

A strong co-movement between assets in different markets during times of high volatility can indicate contagion. The volatility of Bitcoin is therefore plotted against the conditional correlation pairs asset *i* and Bitcoin. The graphical analysis of the pairs vs the volatility, leads to some interesting observations in the figures on the next pages.

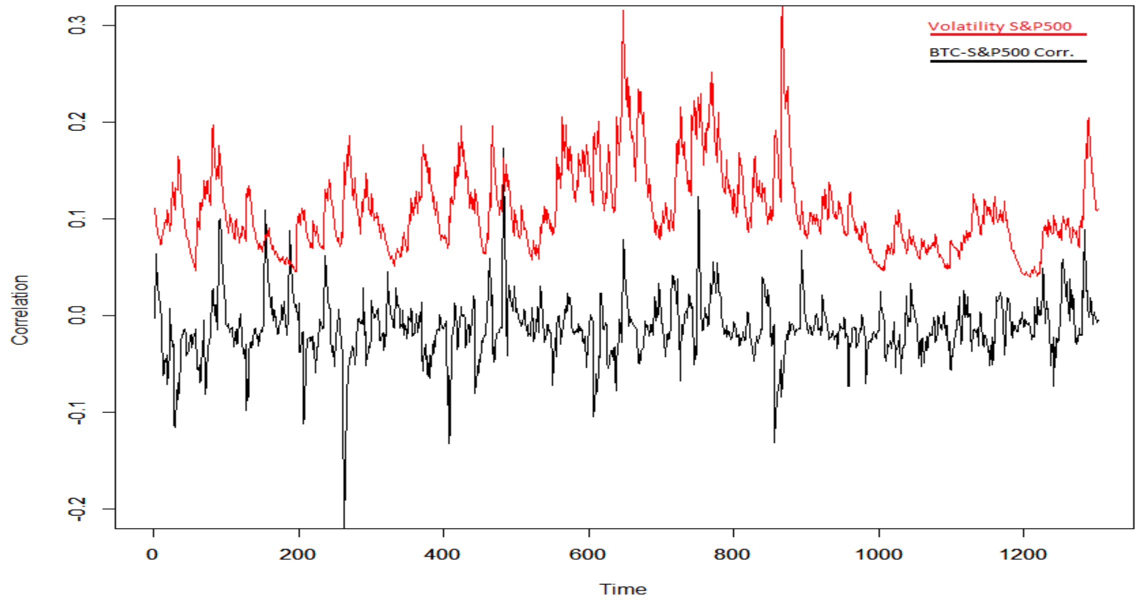

Figure 23: S&P500 volatility and Bitcoin-S&P500 correlation

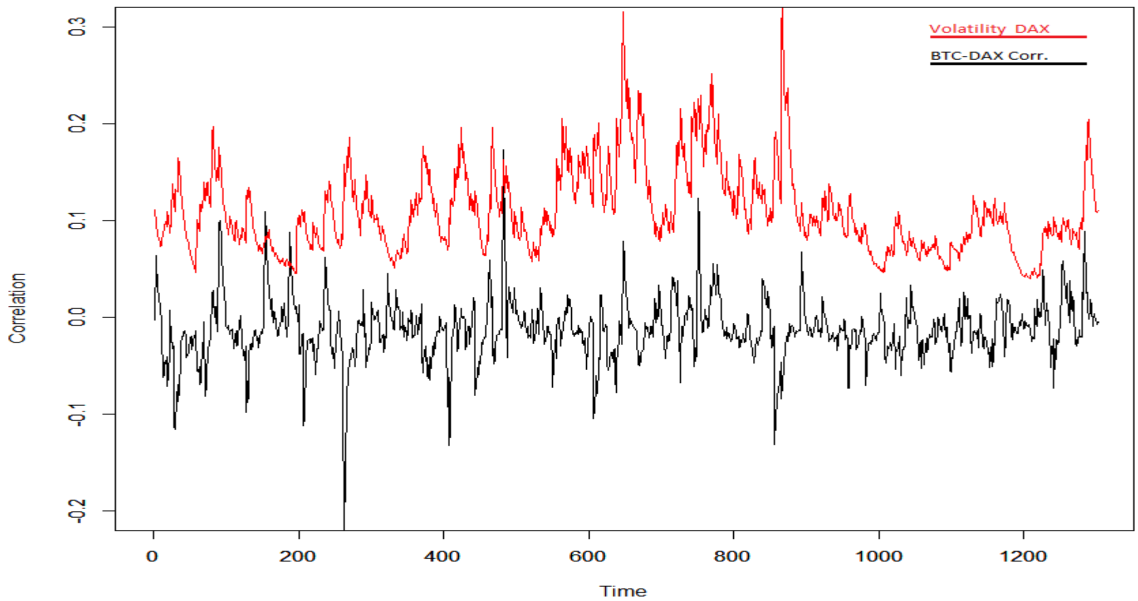

Figure 24: DAX volatility and Bitcoin-DAX correlation

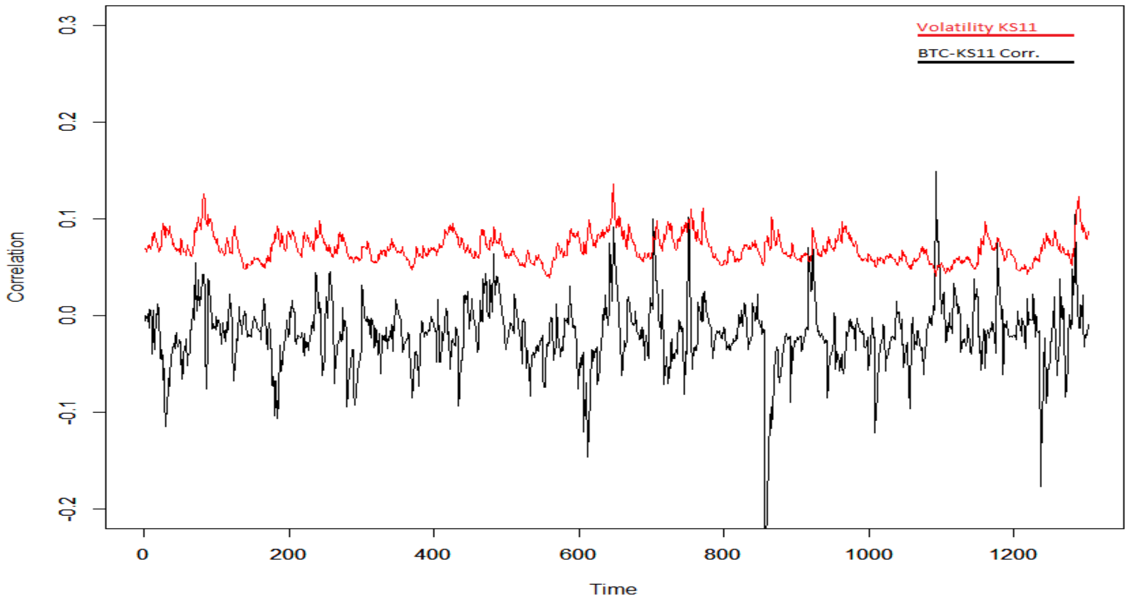

Figure 25: KS11 volatility and Bitcoin-KS11 correlation

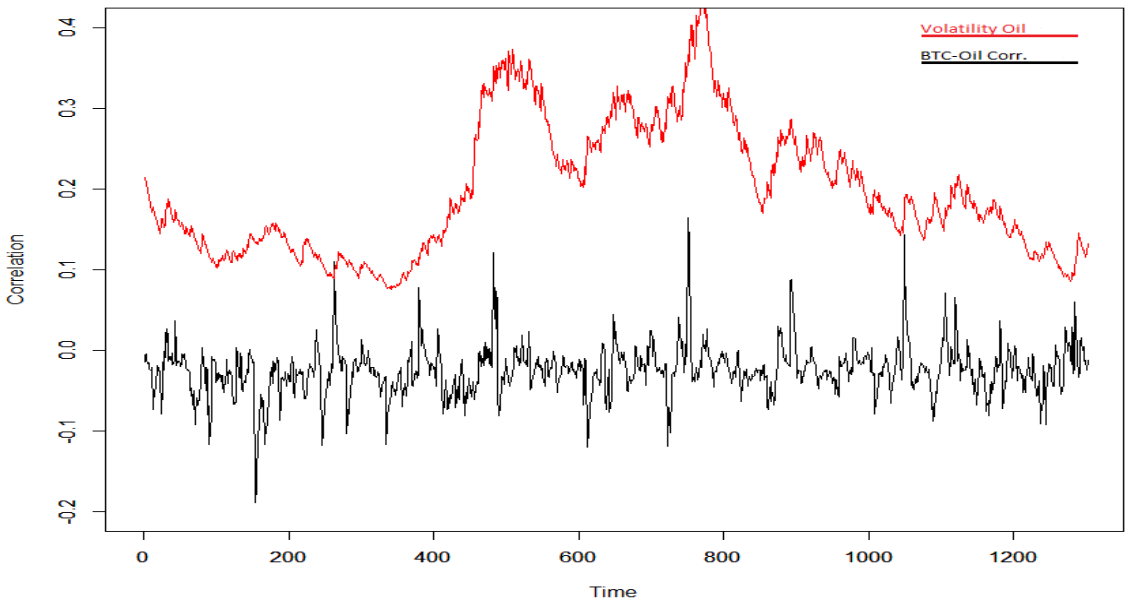

Figure 26: Oil volatility and Bitcoin-Oil correlation

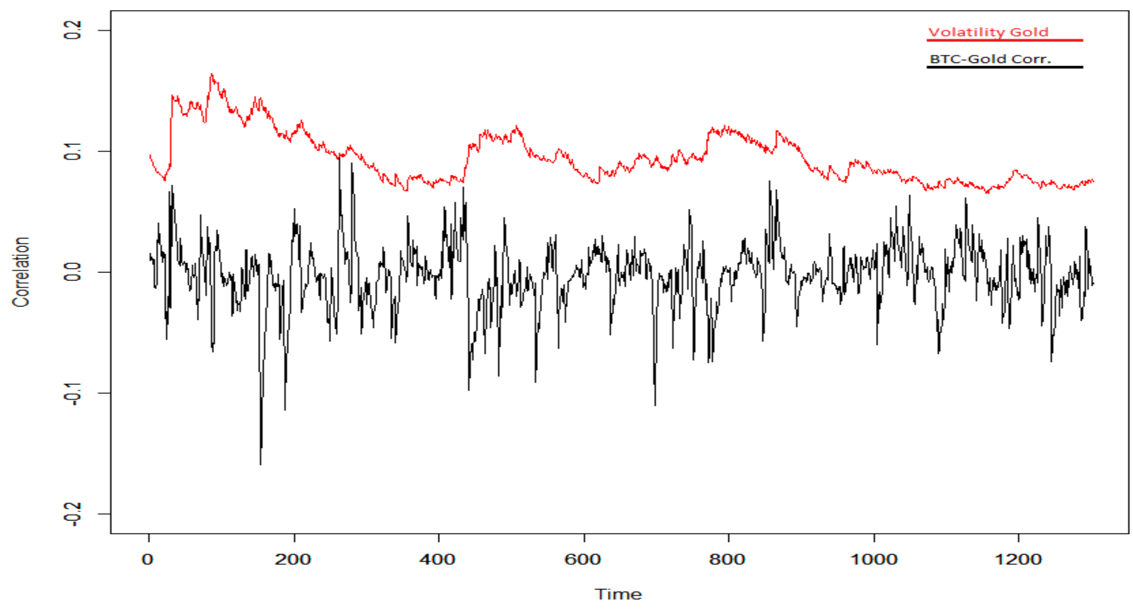

Figure 27: Gold volatility and Bitcoin-Gold correlation

From the perspective of market linkages, a significant volatility spillover between two markets is noticed if there is a strong positive correlation coefficient between the markets after a spike in the volatility of the assets. The graphs for the indices oil and gold show that the correlation coefficient for Bitcoin and asset *i* does not look to increase substantially (about 0.1-0.2 at most). After the spike, the correlations revert quickly back to its mean. The figures shown above suggest that there is no presence of contagion between Bitcoin and the indices, and no contagion between Bitcoin and the commodities.

6.2 Portfolio optimization

After converting the price data to log returns, daily expected return and standard deviation is calculated and plotted in a risk-return chart shown in figure 28. The chart gives a good illustration of how unique Bitcoin is as an investment object. With an expected daily return of 0.4422% and a standard deviation of 0.0591 it clearly stands out. All other assets have an expected return between -0.05% and 0.05% and standard deviation between 0.005 and 0.015, except oil, which has a standard deviation of 0.0212. The three indices have highest expected return as anticipated, with S&P500 having the highest. All the other assets have a negative expected return which makes the portfolio weights dominated by Bitcoin and the indices.

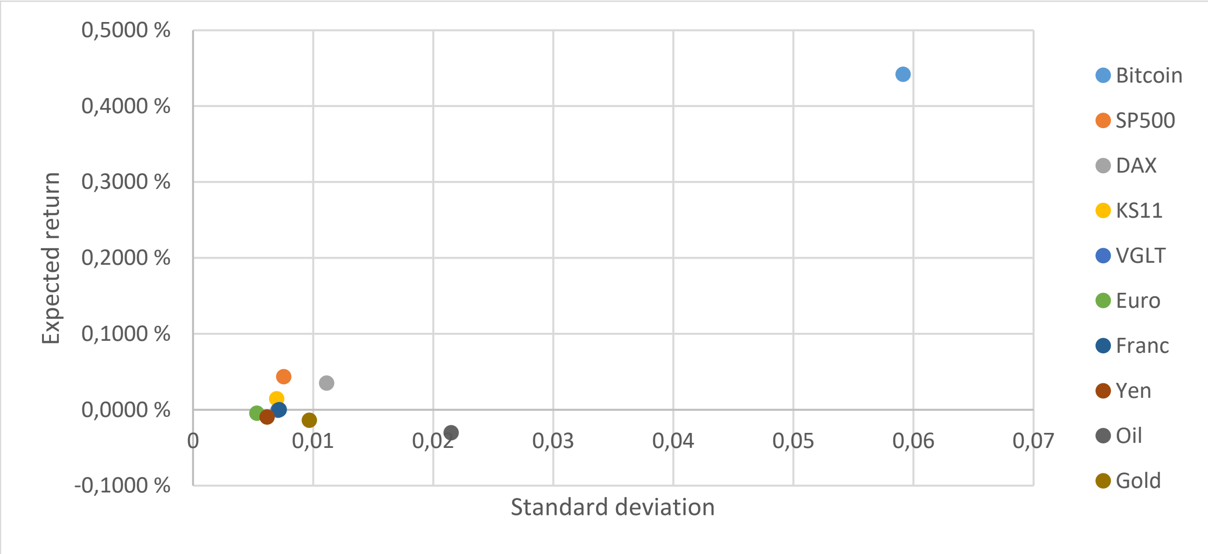

Figure 28: Risk-return chart for assets

The portfolios are sorted (see Appendix B.1 and B.2) and separated for those including and not including Bitcoin. All portfolios are then plotted in a new risk-return chart which also includes the MVF and CML shown in figure 29 and 30. Figure 31 shows the same graph as figure 29, but the axes are scaled for closer comparison to figure 30. It is clearly that the portfolios including Bitcoin offers a much bigger range of investment opportunities. Both charts are made with the same axis range for comparison and there is an additional chart for portfolios including Bitcoin that are multiplied by ten for better illustration. The global minimum-variance portfolios are almost identical, as only 0,30% Bitcoin is included when calculating the minimum variance portfolio, using Solver. A more notable difference is the slope of the CML and thereby the EF. The slope of the CML is, as mentioned, known as the Sharpe Ratio and captures how much additional return is gained for taking on more risk. The CML for the chart with portfolios including Bitcoin are significantly steeper, which indicates more rewarding investment opportunities. This is expected, as Bitcoin has undergone a great increase of value during the sample period.

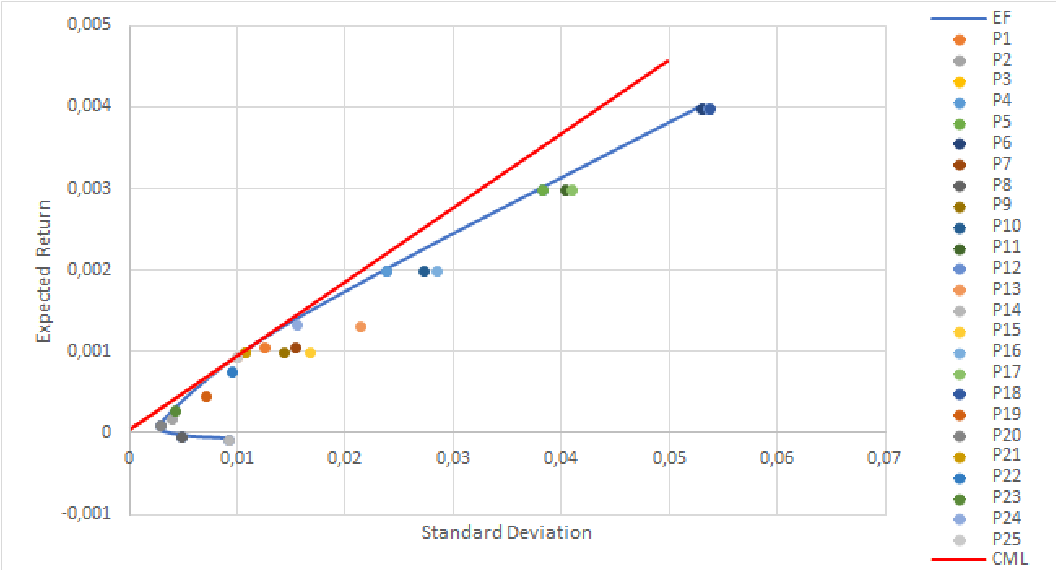

Figure 29: MVF and CML for portfolios not including Bitcoin

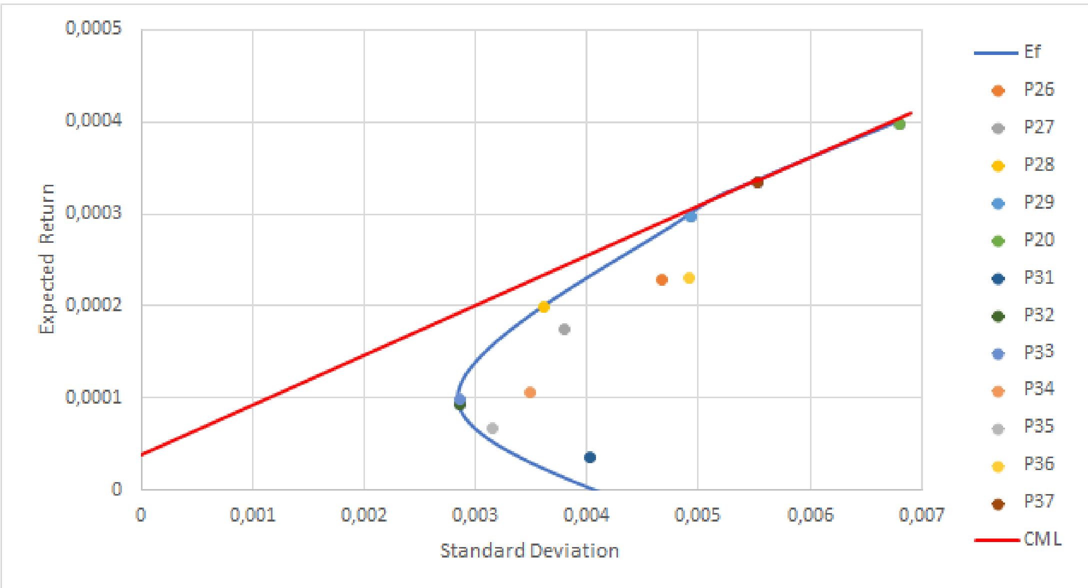

Figure 30: MVF and CML for portfolios including Bitcoin

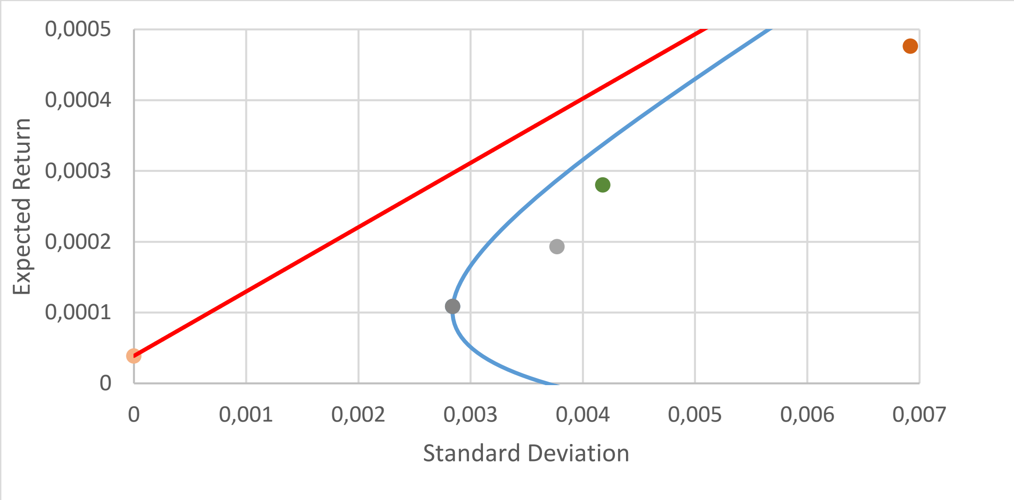

Figure 31: Scaled version of MVF and CML for portfolios not including Bitcoin

6.3 Value at Risk

The Monte Carlo Simulation in this thesis is conducted for examining how VaR and CVaR changes as the portfolios weight of Bitcoin increases from 0-10%. While the Bitcoin weight increases, the rest of the portfolio consists of all other assets described, evenly distributed. An initial investment of 1 000 000$ and timeframe of 100 days are used in the simulation. After running the simulations, the results are sorted and VaR and CVaR are calculated. All calculated VaRs and CVaRs follows the same pattern. They decrease from 0 to 2.5%, but increase gradually after that.

Bitcoin Weights	**0.0 %**	**2.5 %**	**5.0 %**	**7.5 %**	**10.0 %**
VaR 5.0%	6.11 %	5.11 %	5.54 %	6.21 %	6.61 %
CVaR 5.0%	7.67 %	7.01 %	7.39 %	8.29 %	9.18 %
VaR 1.0%	8.78 %	8.15 %	8.58 %	9.45 %	10.93 %
CVaR 1.0%	9.88 %	9.29 %	9.92 %	10.99 %	12.99 %

Table 11: VaR and CVaR results

For the portfolio not including any Bitcoin, there is an estimated 95.0% chance that possible loss will not be lower than 6.11%. If the exceedance occurs, 7.67% is the most likely loss. At 99.0% confidence level, possible loss will not be lower than 8.78% while a 9.88% loss is expected if the limit is exceeded. After 2.5% Bitcoin weight is added to the portfolio, the value at risk decreases. VaR 5.0% decreases to 5.11% and CVaR decreases to 7.01%. VaR 1.0% and CVaR 1.0% also decreases to 8.15% and 9.29%.

This indicates that including Bitcoin in a portfolio is advantageous when considering value at risk. However, the remaining results show that VaR and CVaR increase for 5.0% Bitcoin weight and above. It is estimated a 95.0% chance that possible loss will not be lower than 5.54% for the portfolio with 5.0% Bitcoin weight and if the exceedance occurs, 7.39% percent is the most likely loss. At 99.0% confidence level, possible loss will not be lower than 8.58% while a 9.92% is expected if the limit is exceeded. Further, VaR and CVaR for the portfolios with 7.5% and 10.0% Bitcoin weight continue to increase, which makes sense given how volatile Bitcoin is.

7. Conclusion

By examining the evolution of the time-varying correlation coefficients for Bitcoin against the other assets in the global portfolio, the DCC analysis shows correlation coefficients around zero for all assets. In addition, the analysis of volatility spillovers between the selected markets implies that there is no significant contagion between the markets. No contagion indicates that Bitcoin's price formation is not affected by the other markets studied in this thesis. For these reasons, Bitcoin exhibits effective diversification properties.

The analysis of MVF and CML also implies that Bitcoin can be regarded as a good diversifier, as the portfolios including Bitcoin make it possible to obtain the same expected return, but for a lower risk. The MVF shows a much wider range of investment opportunities and The CML is significantly steeper, which indicates a greater expected return per unit of additional risk. Moreover, the VaR and CVaR estimation indicates that including Bitcoin in a global portfolio makes the potential loss lower, but only for a certain weight. The potential loss and expected shortfall were lower for all significance levels when the portfolio included 2.5% Bitcoin, compared to 0.0% and 5.0%. This means that there is a lower limit for the potential loss in a global portfolio including Bitcoin, than if no Bitcoins were included.

It is also important to consider that a normal distribution is assumed for the underlying risk factors in the Monte Carlo simulation. When the random returns are drawn based on an assumed normal distribution, the extreme values in the end of the heavy tails are omitted. This leads to inaccurate estimation. Since the only purpose of this VaR estimation is to give an indication whether it is advantageous to include Bitcoin in a portfolio for lowering VaR or not, the estimation is included. A suggestion for further research might be to estimate what Bitcoin weight is optimal for lowest possible VaR, since this thesis only concludes with a weight between 0.0-5.0%.

Bitcoin is still in its infancy as an asset and is therefore still subject to structural changes (e.g. regulations and another cryptocurrency with better potential) over time. For this reason, Bitcoin is expected to remain highly volatile in the near future. Moreover, the empirical results indicate that Bitcoin is a good diversifier for the sample periods analyzed. This does not imply that Bitcoin will act as a good diversifier in the future. Investors should actively rebalance the weight of Bitcoin in the portfolio to minimize risk.

The correlation matrices show no sign of Bitcoin being a good hedge as the correlation coefficients obtained are close to zero. Therefore, the global portfolio with its respective assets analyzed also suggests that Bitcoin does not exhibit hedging properties. For the market turmoil period, the results show that the currencies exhibit higher co-movement with gold and are thus more likely to be seen as a safe haven than Bitcoin. By definition, Bitcoin can therefore only be seen as a weak safe haven since it is uncorrelated with the other markets in the sample period with a falling stock market.

To summarize, Bitcoin is clearly an asset that can contribute to a balanced portfolio through its diversification properties. It does not offer any hedging properties, but can be seen as a weak safe haven despite its high volatility. Time will show if Bitcoin will be affected by stricter regulations. If this occurs, further research might be interesting for identifying Bitcoin's potential as an investment in a more stabilized period.

Nomenclature

ADF - Augmented Dickey Fuller

AIC - Akaike Information Criterion

ARMA – Autoregressive Moving Average

ARCH - Autoregressive Conditional Variance

BIC - Bayessian Information Criterion

CAPM - Capital Asset Pricing Model

CCC – Constant Conditional Correlation

CML - Capital Market Line

CVaR – Conditional Value at Risk

DCC – Dynamic Conditional Correlation

DF – Degrees of Freedom

EGARCH - Exponential Generalised Autoregressive Conditional Heteroskedastic

GARCH - Generalised Autoregressive Conditional Heteroskedastic

LR – Likelihood Ratio

Std. dev. dev – Standard Deviation

USD – U.S. dollars

UTC - Coordinated Universal Time

VaR - Value at Risk

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APPENDIX

A - GARCH modeling results

Results of the DCC models for the various sample periods

Estimation results for mean and variance equation ARMA(1,1)-DCC-GARCH(1,1) The entire period: 01/03/2013 to 01/03/2018						
	Cst (w)	AR(1)	MA(1)	Cst (Ω)	Alpha 1	Beta 1
Bitcoin	0.00305	0.960899***	-0.94087***	0.00008*	0.22146***	0.77754***
Std. dev	0.00124	0.08922	0.10880	0.00005	0.03054	0.04320
S&P500	0.00070***	0.93063***	-0.97126***	0.00000	0.22932***	0.74238***
Std. dev	0.00006	0.00931	0.00285	0.00000	0.03775	0.07214
DAX	0.00073***	-0.59834***	0.60883***	0.00000	0.10132***	0.89768***
Std. dev	0.00023	0.22964	0.22566	0.00000	0.02696	0.02475
KS11	0.00036**	0.29095	-0.27820	0.00000***	0.07817***	0.84693***
Std. dev	0.00017	0.24972	0.24785	0.00000	0.00644	0.01375
VGLT	0.00011	0.75596***	-0.78875***	0.00000*	0.02103***	0.97166***
Std. dev	0.00016	0.26095	0.24638	0.00000	0.00175	0.00236
EURO	-0.00007	0.06587	-0.11099	0.00000	0.03432***	0.96380***
Std. dev	0.00011	0.84315	0.83701	0.00000	0.01105	0.01070
FRANC	-0.00011	0.92426***	-0.94049***	0.00000*	0.00387**	0.98632***
Std. dev	0.00010	0.01573	0.01029	0.00000	0.00151	0.00115
YEN	-0.00015	0.90860***	-0.91629***	0.00000	0.05006***	0.94699***
Std. dev	0.00012	0.02202	0.01763	0.00000	0.00715	0.00785
OIL	0.00009	-0.39772	0.36665	0.00000	0.05629*	0.94264***
Std. dev	0.00039	0.67431	0.68289	0.00001	0.03082	0.03119
GOLD	-0.00002	-0.17496	0.12898	0.00000	0.02279***	0.96965***
Std. dev	0.00020	0.21881	0.21795	0.00000	0.00362	0.00487
DCCa1					0.01956***	
Std. dev					0.00480	
DCCb1						0.77630***
Std. dev						0.18051
*, **, and *** indicate rejection of the null hypothesis of associated statistical tests at the 10%, 5%, and 1% levels.						

Table 12: DCC results, entire period

Estimation results for mean and variance equation: ARMA(1,1)-DCC-GARCH(1,1) Stable period: 01/03/2013 to 17/08/2016						
	Cst (w)	AR(1)	MA(1)	Cst (Ω)	Alpha 1	Beta 1
Bitcoin	-0.00065	-0.57237***	0.55634***	0.00023*	0.31789***	0.68111***
Std. dev	0.00108	0.16778	0.17340	0.00012	0.08774	0.09597
S&P500	0.00073***	0.91223***	-0.96875***	0.00001***	0.19742***	0.66044***
Std. dev	0.00012	0.01433	0.00476	0.00000	0.02760	0.04359
DAX	0.00098**	-0.98540***	0.99880***	0.00000	0.11253	0.87657***
Std. dev	0.00039	0.00550	0.00030	0.00001	0.08196	0.08830
KS11	0.00007	0.28046	-0.23382	0.00000***	0.03997***	0.89604***
Std. dev	0.00026	0.26672	0.26721	0.00000	0.00410	0.00888
VGLT	0.00020	0.81593***	-0.83489***	0.00000	0.03003***	0.96463***
Std. dev	0.00024	0.19870	0.19275	0.00000	0.00461	0.00570
EURO	-0.00020	-0.36307	0.28318*	0.00000	0.04566***	0.95334***
Std. dev	0.00015	0.15539	0.15355	0.00000	0.01636	0.01684
FRANC	-0.00024	0.90519***	-0.91717***	0.00000	0.00475	0.98317***
Std. dev	0.00024	0.31784	0.29759	0.00000	0.03118	0.06600
YEN	-0.00030*	-0.16610	0.17180	0.00000	0.05441***	0.94459***
Std. dev	0.00016	0.32618	0.32536	0.00000	0.01520	0.01442
OIL	-0.00038	-0.40355	0.34627	0.00000	0.04671	0.95229*
Std. dev	0.00047	0.91596	0.93543	0.00001	0.02856	0.02987
GOLD	-0.00028	-0.17742	0.09245	0.00000	0.02571***	0.97329***
Std. dev	0.00029	0.23986	0.23690	0.00000	0.00189	0.00605
DCCa1					0.02936***	
Std. dev					0.00928	
DCCb1						0.06906
Std. dev						0.32930
*, **, and *** indicate rejection of the null hypothesis of associated statistical tests at the 10%, 5%, and 1% levels.						

Table 13: DCC results, stable period

Estimation results for mean and variance equation: ARMA(1,1)-DCC-GARCH(1,1) Market turmoil: 18/08/2015 to 28/10/2016						
	Cst (w)	AR(1)	MA(1)	Cst (Ω)	Alpha 1	Beta 1
Bitcoin	0.00241***	0.40173	-0.48015	0.00005	0.19104***	0.80796***
Std. dev	0.00078	0.45313	0.44292	0.00003	0.05675	0.05751
S&P500	0.00038	0.17954	-0.27063	0.00000	0.24940***	0.74172***
Std. dev	0.00033	0.86501	0.83730	0.00001	0.08534	0.08457
DAX	-0.00014*	0.96590***	-1.00000***	0.00001***	0.10554***	0.83784***
Std. dev	0.00008	0.00786	0.00067	0.00000	0.02323	0.02939
KS11	0.00014***	0.94012***	-1.00000***	0.00001**	0.10987***	0.81540***
Std. dev	0.00002	0.01667	0.00041	0.00000	0.02656	0.03677
VGLT	0.00019	0.15762	-0.28150	0.00000	0.00000	0.99899***
Std. dev	0.00037	0.22364	0.20314	0.00000	0.00029	0.00019
EURO	-0.00012	0.79248***	-0.85708***	0.00000	0.00660***	0.99040***
Std. dev	0.00020	0.13783	0.10842	0.00000	0.00193	0.00099
FRANC	-0.00014	0.38589**	-0.33641**	0.00000	0.00609***	0.99195***
Std. dev	0.00033	0.17126	0.16066	0.00000	0.00116	0.00087
YEN	0.00014	0.87194***	-0.90237***	0.00000	0.06464	0.90619***
Std. dev	0.00025	0.14443	0.12793	0.00000	0.06600	0.07282
OIL	0.00070	0.03946	-0.10367	0.00004	0.12728***	0.83323***
Std. dev	0.00147	0.30418	0.30208	0.00002	0.04476	0.04804
GOLD	-0.00008	-0.14850	0.12166	0.00000**	0.02076***	0.95172***
Std. dev	0.00050	0.71947	0.71495	0.00000	0.00270	0.00765
DCCa1					0.01257*	
Std. dev					0.00694	
DCCb1						0.83858***
Std. dev						0.06261
*, **, and *** indicate rejection of the null hypothesis of associated statistical tests at the 10%, 5%, and 1% levels.						

Table 14: DCC results, market turmoil

B - Portfolio optimization

List of the Libor rates used (retrieved from http://www.global-rates.com/interest-rates/libor/american-dollar/usd-libor-interest-rate-12-months.aspx).

Year	Rate
2013	0.84 %
2014	0.58 %
2015	0.63 %
2016	1.17 %
2017	1.69 %
Average	0.98 %

Table 15: Libor rates

B.1 Portfolios including Bitcoin

Indices and VGLT

Portfolio 1 – Even weights									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
20.00 %	20.00 %	20.00 %	20.00 %	20.00 %					
Expected return 0.1069 %									
Standard deviation 0.012355357									
Portfolio 2 – Minimum standard deviation									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
0.38 %	29.90 %	4.26 %	23.64 %	41.83 %					
Expected return 0.0193 %									
Standard deviation 0.003770746									
Portfolio 3 – Minimum standard deviation, expected return minimum 0.1%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
15.03 %	74.07 %	0.84 %	6.78 %	3.27 %					
Expected return 0.1000 %									
Standard deviation 0.010603321									
Portfolio 4 – Minimum standard deviation, expected return minimum 0.2%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
39.24 %	60.76 %	0.00 %	0.00 %	0.00 %					
Expected return 0.2000 %									
Standard deviation 0.023718949									
Portfolio 5 – Minimum standard deviation, expected return minimum 0.3%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
64.33 %	35.67 %	0.00 %	0.00 %	0.00 %					
Expected return 0.3000 %									
Standard deviation 0,038180361									
Portfolio 6 – Minimum standard deviation, expected return minimum 0.4%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
89.42 %	10.58 %	0.00 %	0.00 %	0.00 %					
Expected return 0.4000 %									
Standard deviation 0,052906124									

Table 16: Portfolios including Bitcoin, indices and VGLT

Currencies

Portfolio 7 – Even weights									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
25.00 %					25.00 %	25.00 %	25.00 %		
Expected return 0,1070 %									
Standard deviation 0,015268099									
Portfolio 8 – Minimum standard deviation									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
0.73 %					56.60 %	7.64 %	35.04 %		
Expected return -0,0028 %									
Standard deviation 0,004705367									
Portfolio 9 – Minimum standard deviation, expected return minimum 0.1%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
23.15 %					52.53 %	24.31 %	0.00 %		
Expected return 0,1000%									
Standard deviation 0,014272131									
Portfolio 10 – Minimum standard deviation, expected return minimum 0.2%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
45.47 %					24.02 %	30.50 %	0.00 %		
Expected return 0,2000%									
Standard deviation 0,027114187									
Portfolio 11 – Minimum standard deviation, expected return minimum 0.3%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
67.87 %					32.16 %	0.00 %	0.00 %		
Expected return 0,3000%									
Standard deviation 0,040263747									
Portfolio 12 – Minimum standard deviation, expected return minimum 0.4%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
90.46 %					9.54 %	0.00 %	0.00 %		
Expected return 0,4000%									
Standard deviation 0,053530923									

Table 17: Portfolios including Bitcoin and currencies

Commodities

Portfolio 13 – Even weights									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
33.33 %								33.33 %	33.33 %
Expected return 0,1326 %									
Standard deviation 0,021335751									
Portfolio 14 – Minimum standard deviation									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
1.99 %								14.33 %	83.67 %
Expected return -0,0071 %									
Standard deviation 0,009042681									
Portfolio 15 – Minimum standard deviation, expected return minimum 0.1%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
25.20 %								6.56 %	68.25 %
Expected return 0,1000 %									
Standard deviation 0,016579654									
Portfolio 16 – Minimum standard deviation, expected return minimum 0.2%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
46.89 %								0.00 %	53.11 %
Expected return 0,2000 %									
Standard deviation 0,028355009									
Portfolio 17 – Minimum standard deviation, expected return minimum 0.3%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
68.82 %								0.00 %	31.18 %
Expected return 0,3000 %									
Standard deviation 0,04090522									
Portfolio 18 – Minimum standard deviation, expected return minimum 0.4%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
90.75 %								0.00 %	9.25 %
Expected return 0,4000 %									
Standard deviation 0,053710698									

Table 18: Portfolios including Bitcoin and commodities

Combinations

Portfolio 19 – Even weights									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
10.00 %	10.00 %	10.00 %	10.00 %	10.00 %	10.00 %	10.00 %	10.00 %	10.00 %	10.00 %
Expected return 0,0476 %									
Standard deviation 0,006917361									
Portfolio 20 – Minimum standard deviation									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
0.30 %	18.62 %	6.72 %	15.17 %	15.31 %	20.50 %	1.99 %	21.39 %	0.00 %	0.00 %
Expected return 0,0109 %									
Standard deviation 0,002840627									
Portfolio 21 – Minimum standard deviation, expected return minimum 0.1%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
15.02 %	74.29 %	0.75 %	6.63 %	3.31 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
Expected return 0,1000 %									
Standard deviation 0,010603307									
Portfolio 22 – Maximum expected return, 5.0-15.0% per asset									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
15.00 %	15.00 %	15.00 %	15.00 %	5.00 %	5.00 %	15.00 %	5.00 %	5.00 %	5.00 %
Expected return 0,0774 %									
Standard deviation 0,009463924									
Portfolio 23 – Minimum standard deviation, 5.0-15.0% per asset									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
5.00 %	13.92 %	6.08 %	15.00 %	15.00 %	15.00 %	5.00 %	15.00 %	5.00 %	5.00 %
Expected return 0,0280 %									
Standard deviation 0,004178666									
Portfolio 24 – Maximum expected return, 0.0-25.0% per asset									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
25.00 %	25.00 %	25.00 %	25.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
Expected return 0,1338 %									
Standard deviation 0,015477841									
Portfolio 25 – Highest Sharpe ratio									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
11.47 %	63.50 %	1.60 %	10.76 %	12.67 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
Expected return 0,0804 %									
Standard deviation 0,008396657									

Table 19: Portfolios including all assets

B.2 Portfolios not including Bitcoin

Indices and VGLT

Portfolio 26 – Even weights									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	25.00 %	25.00 %	25.00 %	25.00 %					
Expected return 0,0231 %									
Standard deviation 0,004655889									
Portfolio 27 – Minimum standard deviation									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	30.12 %	4.23 %	23.59 %	42.06 %					
Expected return 0,0177 %									
Standard deviation 0,003777311									
Portfolio 28 – Minimum standard deviation, expected return minimum 0.02%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	36.48 %	3.67 %	21.33 %	38.53 %					
Expected return 0,0200 %									
Standard deviation 0,003820791									
Portfolio 29 – Minimum standard deviation, expected return minimum 0.03%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	64.47 %	1.21 %	11.33 %	22.99 %					
Expected return 0,0300 %									
Standard deviation 0,004917285									
Portfolio 30 – Minimum standard deviation, expected return minimum 0.04%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	64.47 %	1.21 %	11.33 %	22.99 %					
Expected return 0,0400 %									
Standard deviation 0,006785411									

Table 20: Portfolios including indices and VGLT

All portfolios calculated for **Currencies** and **Commodities** without Bitcoin, had a negative expected return and is therefore excluded from the thesis.

Combinations

Portfolio 31 – Even weights									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	11.11 %	11.11 %	11.11 %	11.11 %	11.11 %	11.11 %	11.11 %	11.11 %	11.11 %
Expected return 0,0038 %									
Standard deviation 0,004006609									
Portfolio 32 – Minimum standard deviation									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	18.79 %	6.71 %	15.10 %	15,45 %	20.43 %	2.12 %	21.40 %	0.00 %	0.00 %
Expected return 0,0096 %									
Standard deviation 0,002846101									
Portfolio 33 – Minimum standard deviation, expected return minimum 0.01%									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	19.70 %	6.59 %	15.05 %	15,55 %	19.67 %	2.34 %	21.11 %	0.00 %	0.00 %
Expected return 0,0100 %									
Standard deviation 0,002847152									
Portfolio 34 – Maximum expected return, 5.0-15.0% per asset									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	15.00 %	15.00 %	15.00 %	15.00 %	10.00 %	15.00 %	5.00 %	5.00 %	5.00 %
Expected return 0,0107 %									
Standard deviation 0,003465168									
Portfolio 35 – Minimum standard deviation, 5.0-15.0% per asset									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	15.00 %	7.39 %	15.00 %	15.00 %	15.00 %	7.61 %	15.00 %	5.00 %	5.00 %
Expected return 0,0068 %									
Standard deviation 0,003128552									
Portfolio 36 – Maximum expected return, 0.0-25.0% per asset									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	25.00 %	25.00 %	25.00 %	0.00 %	0.00 %	25.00 %	0.00 %	0.00 %	0.00 %
Expected return 0,0233 %									
Standard deviation 0,004892682									
Portfolio 37 – Highest Sharpe ratio									
Bitcoin	SP&500	DAX	KS11	VGLT	Euro	Franc	Yen	Oil	Gold
	74.76 %	0.31 %	7.39 %	16.89 %	0.00 %	0.66 %	0.00 %	0.00 %	0.00 %
Expected return 0,0336 %									
Standard deviation 0,005520365									

Table 21: Portfolios including all assets except for Bitcoin

C - Codes from R

C.1 Fitting a GARCH model & Statistical tests

```
Install.packages ("tseries")
install.packages ("TSA")
install.packages ("rugarch")
install.packages ("forecast")
library ("tseries")
library ("TSA")
library ("rugarch")
library ("forecast")

#Augmented Dickey Fuller test
adf.test(sp_df, alternative = "stationary")
#Automated ARIMA
SPX <- auto.arima(sp_df, trace=TRUE)
#Example, forcing a significant ARMA order on a variable that has
been given white noise as best fit.
GLD<- arima(gld_df, order=c(1,0,1))
#Ljung-Box test
Box.test(SPX$residuals^2, lag=12, type="Ljung-Box")
#Likelihood Ratio Test P-Value
1-pchisq(2677,20)
1-pchisq(795,20)
1-pchisq(450,20)
#Univariate GARCH(1,1) for each series
SPXspec <- ugarchspec(variance.model = list(model = "sGARCH",
garchOrder = c(1,1)), mean.model = list(armaOrder = c(1,1)),
distribution.model = "std")
SPXfit <- ugarchfit(spec=SPXspec, data = sp_df)
#Univariate eGARCH(1,1) for each series
SPXspec <- ugarchspec(variance.model = list(model = "eGARCH",
garchOrder = c(1,1)), mean.model = list(armaOrder = c(1,1)),
```

```
distribution.model = "std")
SPXfit <- ugarchfit(spec=SPXspec, data = sp_df)
```

C.2 DCC model

```
install.packages('rmgarch', dependencies = TRUE)
library("rmgarch")
library("zoo")
#Data converted from excel
Dat<-
data.frame(Multivariate$Bitcoin,Multivariate$SP500,Multivariate$DAX,
Multivariate$KS11,Multivariate$VGLT,Multivariate$Euro,Multivariate$F
ranc,Multivariate$Yen,Multivariate$Oil,Multivariate$Gold)
#Returns
retSP<-diff(log(Multivariate$SP500))
retDAX<-diff(log(Multivariate$DAX))
#UNRESTRICTED MODEL#
xspec <- ugarchspec(mean.model = list(armaOrder = c(1, 1)),
variance.model = list(garchOrder = c(1,1), model = 'sGARCH'),
distribution.model = 'std')
#RESTRICTED MODEL#
xspec1 <- ugarchspec(mean.model = list(armaOrder = c(1, 1)),
variance.model = list(garchOrder = c(1,1), model = 'sGARCH'),
distribution.model = 'std',fixed.pars=list(alpha1 = 0, beta1 = 0))
#Choosing how many variables to add in the DCC model.10 assets in
this case.
uspec <- multispec(replicate(10, xspec))
uspecx <- multispec(replicate(10, xspec1))
#DCC Order
spec1 <- dccspec(uspec = uspec, dccOrder = c(1, 1), distribution =
'mvnorm')
spec2 <- dccspec(uspec = uspecx, dccOrder = c(1, 1), distribution =
'mvnorm')
```

```
#Return vector
r_t1 <-
cbind(retBTC,retSP,retDAX,retKS11,retVGLT,retEUR,retCFH,retYEN,retOi
l,retGLD)
#Unrestricted DCC model
dcc.fit <- dccfit(spec1, data = r_t1, fit.control=list(scale=TRUE))
#Restricted DCC model
dcc.fit1 <- dccfit(spec2, data = r_t1, fit.control=list(scale=TRUE),
solver = "nlminb")
#Result
print(dcc.fit)
print(dcc.fit1)
#Finding the mean correlation between Bitcoin and asset (S&P500 and
DAX)
#BTC-SPX Example
r1 <- rcor(dcc.fit, type="R")
plot(r1[1,2,], type="l")
r1.x <- zoo(r1[1,2,], order.by=time(r_t1))
dates <- seq(as.Date("01/03/2013", format = "%d/%m/%Y"),by = "days",
length = length(r1.x))
plot(dates,r1.x,type='l', main=paste(colnames(r_t1)[1],"-",
colnames(r_t1)[2], "Conditional Correlation", sep="
"),ylab="Conditional Correlation",  sub=paste("mean:",
round(mr1z,3),"sd:", round(sr1z,3)), xlab="Date")
abline(h=mean(r1.x), lty=2, lwd=1, col="red")
abline(h=(mean(r1.x)+sd(r1.x)), lty=2, lwd=1, col="blue")
abline(h=(mean(r1.x)-sd(r1.x)), lty=2, lwd=1, col="blue")

mr1z <- mean(window(r1.x, start="2013-03-01"))
sr1z <- sd(window(r1.x, start="2013-03-01"))
#BTC-DAX Example
r1.y <-zoo (r1[1,3,], order.by=time(r_t1))
dates <- seq(as.Date("01/03/2013", format = "%d/%m/%Y"),by = "days",
```

```
length = length(r1.y))plot(dates,r1.y,type='l',
main=paste(colnames(r_t1)[1],"-", colnames(r_t1)[3], "Conditional
Correlation", sep=" "), ylab="Conditional Correlation",
sub=paste("mean:", round(mr2z,3),"sd:", round(sr2z,3)),xlab="Date")
abline(h=mean(r1.z), lty=2, lwd=1, col="red")
abline(h=(mean(r1.z)+sd(r1.z)), lty=2, lwd=1, col="blue")
abline(h=(mean(r1.z)-sd(r1.z)), lty=2, lwd=1, col="blue")
mr2z <- mean(window(r1.y, start="2013-01-03"))
sr2z <- sd(window(r1.y, start="2013-01-03"))
```

C.3 Volatility spillover

```
#Price from excel
sp <- Multivariate$SP500
#Converting to returns
sp_df <- diff(log(sp))

#Garch models for the volatility and correlation comparisons for
spillover effects.
Example from S&P500 Only
Gmodelsp <- ugarchspec(variance.model = list(model = "eGARCH",
garchOrder <- c(1, 1)), mean.model = list(armaOrder = c(1, 1),
include.mean <- TRUE), distribution.model = "std")
fitGmodelsp <- ugarchfit(spec = Gmodelsp, data = sp_df)
sigma4=(sigma(fitGmodelsp))
sigmasp <- data.matrix(as.data.frame(sigma4))
#FIGURE OF BTC VS S&P500) for spillover effects.
plot(rcor(dcc.fit)['retBTC','retSP',],ylab="",xlab="Time", type='l',
xlim=c(-0.1, 1300),ylim=c(-0.2, 0.3))
par(new=T)
plot((sigmasp*10),type='l',ylab="Correlation",col='red',xlab="",xlim
=c(-0.1, 1300),ylim=c(-0.2, 0.3))
```