Universitetet i Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

MASTER'S THESIS

Study programme/specialisation: **Petroleum Engineering – Well Engineering and Drilling Technology**	Spring semester, 2018 Open / ~~Confidential~~
Author: **Marcelo Landivar Argandona**	…………………………………………… (signature of author)

Programme coordinator: **Jan Aage Aasen**

Supervisor(s):

Title of master's thesis:

The Physics of Buoyancy, Effective Tension and Pinching-off Effect

Credits: 30

Keywords: Buoyancy, Effective Tension, Pinching-off Effect, Failure Modes	Number of pages: 79 Stavanger, 15.06.2018 date/year

Preface

The following document has been written with the purpose to conclude my studies of the Master degree in the Well Engineering program by June 2018. The University of Stavanger has represented a step more in my academic formation and the experience to study the master in Norway, a country with such experience in the area, has been exceptional.

The thesis was originally thought to adjust any misconception related to the Buoyancy concept as the way it is developed. The misunderstanding of the concept leads to some differences that can be adapted to a particular case when analyzing tension in risers or tubular. The equation and the application are understood to be right; however, for the particular case when there is not an area to apply pressure at the bottom the results should change, which is not the case, for particular assumptions made in the analysis that can be debatable as it was presented in the Master courses. It was suggested the following pinching-off effect was part of the topic since it has been proposed previously as a result of the effective tension concept, and the concept can shed light on the experiment. The Pinching-off effect experiment has been the object of many theses and reproduced several times but with inconclusive results.

The master thesis has been a new challenge for me, but I am glad to have accomplished one step more in my life. The experience of studying in a different country and know a completely different culture as well as another way to understand the industry have been a gratifying experience for me. For me as a student, the thesis has been absolutely engaging despite it involved a lot of mechanics. Grateful to have produced this work with the help of many people supporting and providing ideas.

Marcelo Landivar

Summary

The thesis addresses at the beginning the understanding of the development of forces that result in buoyancy, and the concept of effective forces. Then, it continues more in-depth with an application of effective forces to an experiment named the pinching-off effect, and the following explanation on the unsolved fracture of the rod in the experiment.

Having two different approaches that interpret the origin of buoyant forces, the thesis describes the limitation of the Archimedes' principle when the body under analysis is not in contact with fluids in the whole surfaces. It remarks the absence of an upward force in cases when even if there is fluid displacement, no vertical pressure difference will create the upthrust. Then, the first section of the thesis illustrates the misunderstandings of buoyancy through experiments performed that support the conception of buoyancy following the piston force approach.

In addition, the petroleum industry manages several applications of the principle. Thus, the comprehension of the difference between real forces and effective forces in design and analysis performed for wellbore involves the awareness of how the forces behave in the wellbore regarding fluids and tubular. The thesis dedicates a section to the effective tension and its incidence in cases of sub-sea operations or well engineering where it is shown that there is zero buoyancy although the analyzed tubular is submerged in the well. This happens when the cross-sectional area of steel at the outside bottom of a pipe is isolated. Further, the present document reinforces the concept of the analogy of the total stress state and the deviatoric stress to real and effective forces, respectively.

The "Pinching-off effect experiment" was an experiment conducted by Bridgman around a century ago applying high pressure to different tubular materials in a pressure vessel and studying the results. The experiment was labeled as a paradox and had provoked a series of disagreements in the scientific setting regarding the reasons for the fracture of the rod.

Different theories are exposed, the Poisson effect and the material type plays an important role in the shape and development of the fracture. Different aspects notices in the experiment where included in the analysis to define the criteria and mode of failure. The initial and final fracture shape are addressed as well; however, it is an issue that requires a separate study to analyze the fracture shape post-yield. The conclusion from the theoretical analysis is supported through the previous simulation and laboratory work performed.

Nomenclature

A, A_e = Outer area

A_i = Inner area

b = buoyancy factor

D = well depth

E = Young's modulus

F = force

F_B = Buoyancy force

F_{net} = Net Force

g = Gravitational force

G = Shear modulus

I = Stress invariants

J = Deviatoric stress invariants

P = pressure

R, R_e = Outer radius

r, r_i = inner radius

w = unit pipe weight

W = total weight

L = pipe length

T = Tension

T_e = Effective tension

T_{TW} = True wall tension

W_a = Apparent weight

W_t = True weight

W_f = Fluid weight

U_0 = Total strain energy density

ν = Poisson's ratio

V_{disp} = Displaced volume

σ = Stress tensor

$\sigma_1, \sigma_2, \sigma_3$ = Principal stresses

σ_{dev} = Deviatoric stress

σ_e = Effective stress

σ_{hyd} = Hydrostatic stress

σ_y = Yield limit

σ_{VM} = Von Mises stress

ϵ = strain

$\epsilon_z, \epsilon_y, \epsilon_z$= strain

ρ = density

ρ_i, ρ_o = Density inside/outside

τ = Shear stress

θ = Hole inclination

φ = Angle

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1 Introduction

The buoyancy term has been used in the scientific setting for centuries. It has been the subject of several applications to different branches and further studies to understand the variants where it can be applied. The Archimedes' proposals in his original studies are not in discussion, the topic of discussion is the cases where Archimedes did not consider analyzing. Currently, in most of the different areas where the buoyancy is applied, there are two different interpretations of the origins of buoyant forces.

The Volumetric concept is based on the idea that Archimedes' calculation of the buoyant force is the way these forces are created, through volume displacement. On the other hand, the generalized law of Archimedes explains pressure forces acting on the ends of objects when submerged. This subject will be addressed logically and impartially to clarify a situation that seemed to be resolved. However, the petroleum industry, mainly, mixes the concepts of the calculation, and the analysis of the forces including buoyant or upthrust forces. The clarification is particularly needed in scenarios regarding stuck drill pipe, cemented casing and landed subsea riser where the tubular is connected to the wellbore or other solid surfaces to avoid incorrect analysis due to interpretation. The further explanation of the concept was conducted due to the difference in the results of effective tension in risers between the two approaches meaning that one of the approaches leads to a misunderstanding or misconception of the buoyant forces and the effect of pressure in tension for submerged tubular.

Previous work on the Pinching-off effect was performed in the University of Stavanger in 2013 and 2014 by R. Morten and L. Fossli, respectively. The experiment consists of applying high hydrostatic pressure to a rod confined in a pressure chamber until it breaks. The rupture of the rod is parted differently depending on the type of material. Moreover, the fracture can be perceived as a tensile fracture even though it does not experience an axial force due to its free-end condition. Suggestions about the effective tension concept can be applied for horizontal pipes was analyzed in this experiment. Additionally, the experiment was conducted to validate the observations made by Bridgman and proposed an exposed a possible solution to the so-called “Bridgman Paradox”. As it was expected, the experiment resulted in the pinch-off effect; however, the theories proposed of effective tension and resulting tensile fracture did not fix with the results. Some other observations were made, and the values of rupture pressure and tensile loading did not coincide to assure it was a tensile fracture. Friction was also considered in the study to explain the difference between the results.

In addition, simulations were performed, and the stress concentrations were different from expected. The appearance of a shear stress changed the suggestions previously made and open the issue to more general discussion. More than a century since the experiment was originally conducted, and a general disagreement on the explanation of the resulting fracture, and the inaccuracy to understand the shape of the fracture and the mode of failure of the rod persist. The proposed study will complement the previous work done and cover part of the issues related to the Bridgman experiment.

1.1. Outline of the Thesis

Chapter 2. Analysis of Buoyancy The second chapter presents the physics of buoyancy and describes the cases where there is a limitation on the application of the normal Archimedes' law. It introduces the appropriate approach to the interpretation of buoyant forces without limiting to the physical context of the case. Moreover, it includes a section regarding effective tension and the initial problem to analyze. It bears to mention a part regarding Buoyancy in Petroleum engineering.

Chapter 3. Tubular Mechanics/Bridgman Paradox. The third chapter describes the fracture mechanics of pipes and the basic concepts of tubular failure. It analyzes the Bridgman experiment “The pinching-off effect” and concludes with the reasons for the resulting fracture. It analyses the fracture shape of the experiment through different materials and estimates the most probable criterion of failure.

Chapter 4. Conclusions and Recommendations. The conclusions obtained from the respective research and studies will be addressed and further work will be proposed to continue the understanding of the Bridgman experiment and fracturing of materials.

1.2. Objectives

- Conduct a literature review of the worked conducted previously on the issue.
- Analyze the physics of buoyancy concept and Archimedes' law limitation.
- Clarify the misunderstandings and differences in the true forces and effective forces.
- Evaluate the pipe mechanics and its relation with the Archimedes' law.
- Revise the concept of effective tension and buoyancy in the oil industry.
- Proposed a generalized mode of failure for the Bridgman paradox.
- Analyze the different failure criteria to define the rod breakage to different materials.
- Proposed the conclusions on the stresses regarding the rupture and the final shape of the of the fracture in the rod.

2 Analysis of Buoyancy

2.1. Introduction to Buoyancy

Buoyancy is a basic term in physics which describe the effects of fluids on objects when these are immersed. This term in modern physics is known as upthrust force because the fluid where an object is immersed develops an upward force on it. The concept of buoyancy is based on the celebrated Archimedes' Law that is simply explained as buoyancy equals the weight of the displaced fluids. This terse definition suggests that is a simple and straightforward physic concept, but the explanation of how buoyancy is developed is more complex. Buoyancy has been the reason for extended studies about the effects and action of fluids on solid objects. This chapter will analyze the physics of buoyancy and how it has surged a misunderstanding on how buoyant forces are generated.

2.1.1. Literature and Technical review

Morten Reve wrote a master thesis in 2013 about buoyancy in pipe and risers based on Charles Sparks work. Sparks (2004) deal with cases involving the axial forces of pipes and risers in marine operations, and he is an advocate of the Volumetric conception of buoyancy understanding as well. He states that bodies immersed in fluid perceive buoyancy equal to the weight of the displaced fluid, as Archimedes law, and he adds that all immersed bodies experience buoyancy because it is applied at the centroid of the body at the mid height. Considering the buoyancy is also exposed due to the hydrostatic lateral forces that provokes a thrust in the bodies immersed in fluid, Sparks explains that the difference of lateral forces are due to small and imperceptible curvatures provoking a lateral thrust of the body.

Moreover, Sparks adds that as long as there are fluid lateral forces there is buoyancy due to the lateral thrust mentioned before. Nevertheless, the theory is inconsistent, and it cannot be calculated through Archimedes' law in all the situations. As reported by Sparks, Archimedes' law has exceptions that define the situation where the weight of the displaced fluid cannot apply. Apparently, Archimedes' law is limited whenever a pressure field is not closed. Bodies without a defined centroid have no pressure field and result invalid to obtain the buoyant force applying the Archimedes' concept. The cited work was supporting the concept of buoyancy as volumetric, ignoring the fact that hydrostatic pressure difference is only calculated vertically.

The controversy of the volumetric conception is introduced when exposing the case where there is no fluid underneath a particular body that by logic would suggest there is no force acting upwards underneath the body; therefore, there is no thrust. The situation can be consistently explained through a diagram of forces which coincides with the piston-area approach and supported through a series of experiments that exposes the inconsistency of the volumetric school in punctual cases. Besides, the issue about the discrepancy of concepts is presented further through a comparison of tension results resulting in unequal results. As stated in Reve's thesis, the concepts give the same result at the surface or at the top; however, at the bottom, the difference arises because one considers a force equal to the weight of the displaced fluid, volumetric school, and the other does not consider any applied force, piston-force school. Furthermore, Reve adds that the volumetric concept gives the same answer as the effective tension concept by Sparks because effectively effective tension is based on the axial forces acting on the tubular without the effects of fluid pressure.

The concept of "effective tension" is further discussed as part of the buoyancy issue, and there is an intent to determine when was developed. The pressure area for an uniform vertical tubular is equal to the external pressure times the external sectional area on the bottom of the pipe. The effective tension concept is divided into three parts. The true tension (T_{true}), or the tension at the tubular wall, depends on pressure and the loads imposed to the tubular. Then, the weight of the internal fluid ($-P_iA_i$), and the subtraction of the compressive axial force of the displaced external fluid ($-P_eA_e$). The effective tension is independent of pressure, it depends on the apparent weight or changes in the true tension.

In order to identify the valid approach to determine not only how the buoyant forces are developed, but the right calculation of the tension and stresses in a tubular or body immersed in fluids; it is necessary to become acquainted with Hubert & Rubey's (1961) work. It introduces the worldwide accepted theory that the total stress induced to a body is equal to the hydrostatic stress plus the deviatoric stress. It is necessary to remark that the piston-force approach is analogous to the total stress since it depends on the physical context and considers all the possible forces acting on the body. On the other hand, the effective forces are the force a body is induced by neglecting the fluid pressure effects after the real force is determined. It is comparable to the deviatoric load by subtracting the hydrostatic stress to the total stress. It becomes logical when it is compared to the effective tension. It is a concept to analyze buckling effects in tubular since it is related to the total stress without considering the stability criterion given by the average hydrostatic stress.

$$\text{Total stress} = \text{Hydrostatic stress} + \text{Deviatoric stress}$$

$$\sigma_{Total} = \sigma_{hyd} + \sigma_{dev}$$

The stability criterion or stability force:

$$T_s = \frac{P_o r_o^2 - P_i r_i^2}{r_o^2 - r_i^2}$$

The result of the two conceptions have noticeable differences when calculating the effective tension, used to avoid the tubular or riser to be under compression. It is important to mention that the oil business considers tension to be positive and compression negative. The case of study is when the tubular is attached to the floor, and it does not perceives fluids underneath it, illustration in Figure 2.1). At first glance, when analyzing the two risers, it is possible to realize that the results in both cases cannot be the same since one of the risers is perceiving a force at the bottom due to the fluids, and the other one is seated at the sea-floor. The particular case would be exposed afterwards in the thesis, however, it is important to shed light on the issue because there is no doubt that the equation itself and the general application are right.

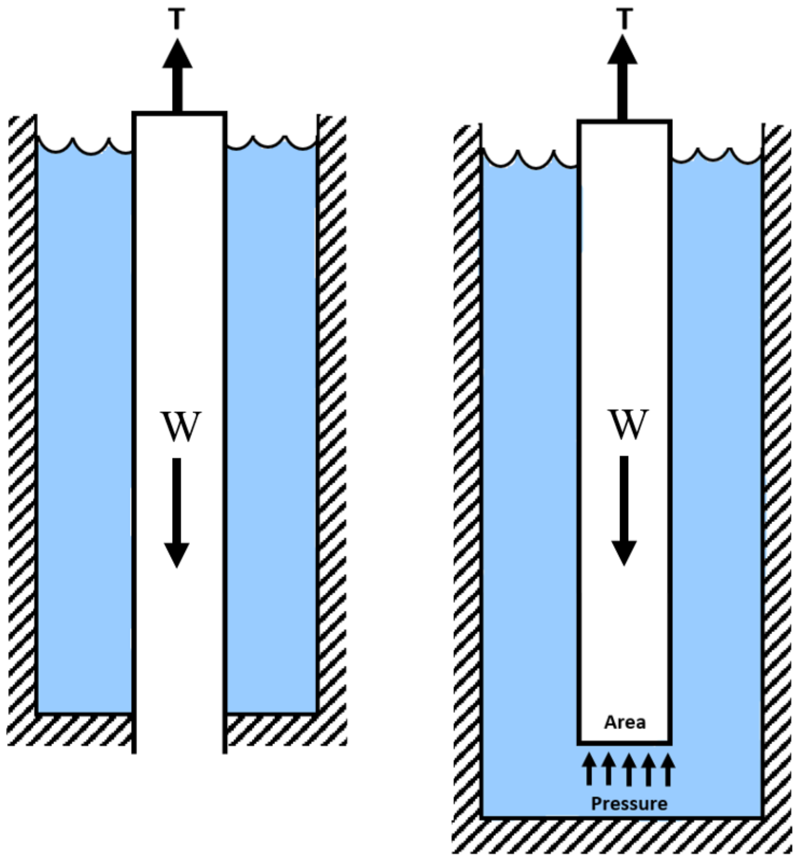

Figure 2.1: Buoyancy effect in a riser with and without projected area

As illustrated in the Figure 2.2, the results are equal at the top of the tubular and vary at the bottom because of this difference of concepts. The results are discussed in a paper published by a professor in the University of Stavanger, Arnfinn Nergaard, he clearly exposes that the two different risers give the same result of effective tension. Nevertheless, the analysis of forces would say something different because one is considering a force that should not be accounted. In the diagram, the line with the letter W represents the weight of the string/riser in the air, in which the tension is decreasing as the bottom is reached. The maximum tension is at the top since the top supports the weight of the whole string, and at the bottom is the minimum since there is no more weight underneath. In this situation, the tubular is always in tension.

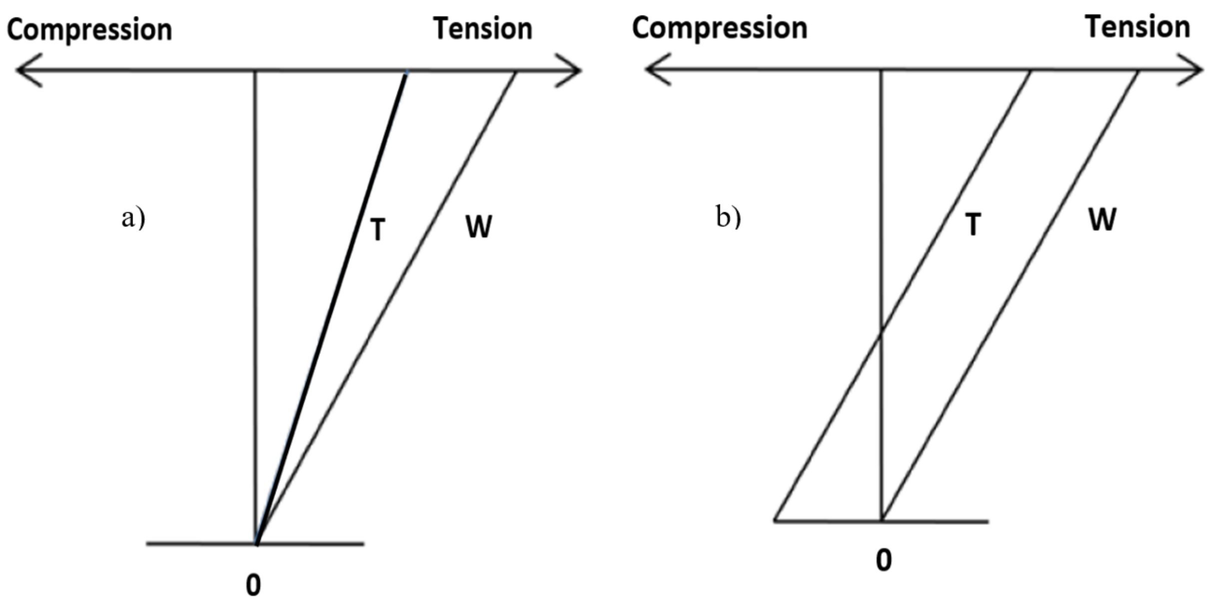

Figure 2.2: Tension force difference according to a) Volumetric and b) Piston-force Concept

The letter T in the diagram represents the tension calculated in two different approaches that should give the same result, however, it reflects two different results in each diagram. The part a) in the figure represents how the case is analyzed by some advocates of the volumetric school stating that the forces underneath the riser are always there as long as the riser is submerged in water without considering the fact the riser is attached to the bottom.

The resulting effective tension (T) shows the tubular to be in tension in its whole extension since it acknowledges a buoyant force at the bottom of the pipe. The buoyant force produces a upward force that sets the pipe in tension by reducing the weight seated in the bottom surface. The effective tension does not consider the stability extra force given by the pressure effects of the hydrostatic force since it cannot cause failure of pipes or risers. The effective tension is considered an important part when determining buckling or straightening of pipe strings because it just considers the deviatoric stress and not the whole stress as it occurs in the True tension.

The b) represents the piston-force view of the case that does not contemplate an upward force in the previous case stated. Hence, there is a difference in the forces at the bottom face. Considering that there is not an additional upward force, the tubular will be subjected to compression at the bottom providing an undesired situation in the riser status. The issue does not happen in the analysis of the effective tension; the source of the difference is in the conception of how the forces are originated, and it affects the final result. Moreover, the importance of the issue results in increasing the possibility of failure of pipes or risers because of erroneous assumptions in the forces.

The document concludes that buoyancy is an external force acting on a body as a result of the pressure difference between the lower end and top end of an object submerged into fluid citing the case of absence of fluids beneath an immersed fluid. The explanations of the issue can be somehow confusing by advocates of the volumetric school as Nergaard or Sparks; nonetheless, it does not mean they are wrong in their concepts. Even though the effective tension concept is valid to calculate the top tension for tubular and risers to avoid buckling, the inconsistency of the theory about buoyancy according to Sparks can create a misunderstanding of the real forces acting on a body and the real tension to be applied to the riser/pipe.

2.2. Archimedes' Law

Buoyancy was studied and mentioned for the first time hundred years ago by Archimedes through his famous work on 'On Floating Bodies.' In 'The Work of Archimedes' by Heath (2002), he compiles the original studies, including the previous section mentioned, that is important to be analyzed for a better understanding and interpretation of buoyancy.

The proposition 3 to 7 are relevant for the buoyancy analysis where Archimedes describes and proves the result of solids immersed in fluids lighter, heavier and equal density. The proposition 3 is the first to mention about floating bodies and the equilibrium it is reached when the densities between the fluid and the body are the same. "Proposition 3: Of solids those which size for size, are of equal weight with a fluid will, if let down into the fluid, be immersed so that they do not project above the surface but do not sink lower". This is the first case when the densities are equal so that the solid will neither stand above the surface nor sink further.

The next three propositions are about solids lighter than the fluid, "Proposition 4: A solid lighter than a fluid will, if immersed in it, not be completely submerged, but part of it will project above the surface". The previous proposition signals that lighter solids than the fluids, in condition of rest, will project above the fluid surface as a signal that floats because of this difference of density. It continues with "Proposition 5: Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced" which explains that the part of the light solid will submerge until both, the body and the fluid, reach a weight equilibrium. The last proposition about lighter solids, "Proposition 6: If a solid lighter than a fluid be forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced". This is a crucial motion by Archimedes because it is the first time he mentions an upward force, and it is the base of the Archimedes' law. Archimedes explains that a body is driven up by a force that is equal to the displaced volume. He clearly stated that the volume displaced is the relevant part that is different as the volume of the submerged object.

The following proposition is about heavier solids, "Proposition 7: A solid heavier than a fluid will, if placed in it, descend to the bottom of the fluid, and the solid will, when weighed in the fluid, be lighter than its true weight by the weight of the fluid displaced". Archimedes signals that a solid will sunk until it equals the weight of the fluid and reaches an equilibrium again. These propositions describe the difference of density as the main factor for bodies to float, and it is the basis buoyancy nowadays.

It is interesting to notice that Archimedes mentions in his writings about fluids exerting a force in a solid body, and the effect of fluids on bodies consists on the equilibrium of forces so that the bodies can be at rest again. It is the case of the second postulate in the book On Floating Bodies I where Archimedes is describing a force that acts perpendicular to the surface of a body that is the definition of pressure and produces a thrust. The postulate says the following "Let it be granted that bodies which are forced upwards in a fluid are forced upwards along the perpendicular [to the surface] which passes through their center of gravity."

2.3. Modern understanding of Buoyancy

Archimedes' principle has myriad types of applications which proves that his original hypothesis about floating bodies is correct. However, the understanding of buoyancy goes beyond than the final result of the upthrust or even the apparent weight; it is referred to the development of the effect of upthrust and the physics that lead to a body float. There are two main interpretations of how the effect of upthrust is developed, Piston force conception and Volumetric conception.

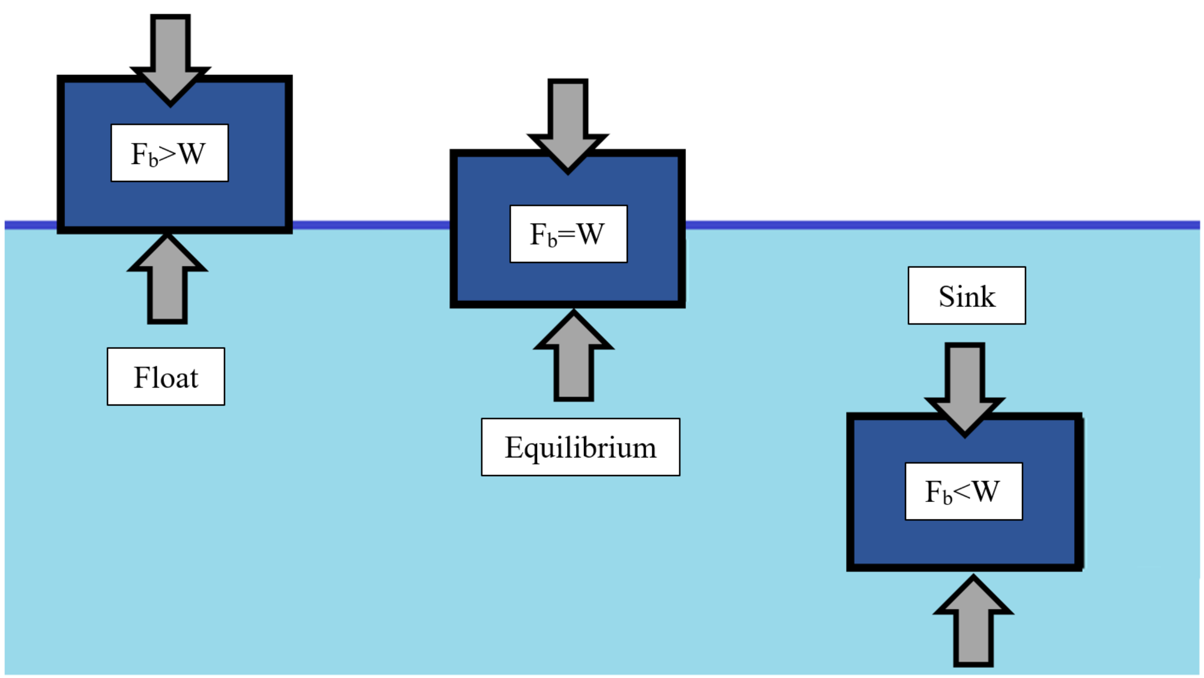

Figure 2.3: Buoyancy mechanics

2.3.1. Piston Force conception

Following the fundamental Newton's laws of motion and in combination with the superposition principle, it is possible to describe most of the physical systems regarding the motion of bodies and the buoyancy mechanics as illustrated in the Figure 2.3. According to Newton, for every action; there is an equal and opposite reaction which is basically how the buoyancy can be explained according to the Piston Force conception.

When a body is submerged into a fluid, the forces acting on this object are the weight itself of the body, and the fluid exerting forces all around it as shown in the Figure 2.4. The lateral forces acting on the object are annulled because are equal and opposite in all directions of the body. As it is known, the force exerted by a fluid is expressed as a pressure and the pressure depends on depth; therefore, the difference between the forces or pressure acting above and below, and the weight of the body will result in the rising or sinking of it. The buoyant forces are understood as a concentrated force acting on the cross-sectional sections of a body in the opposite direction of gravity (Kaarstad & Aadnoy, 2011).

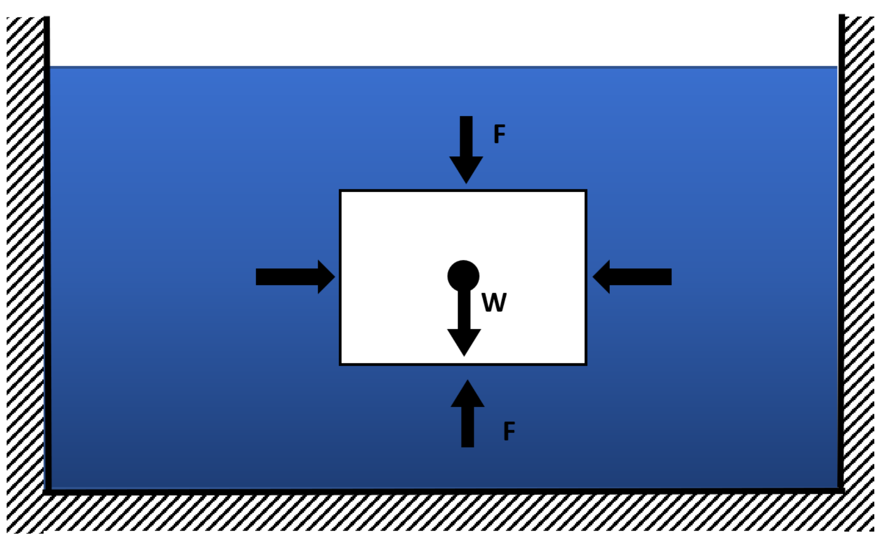

Figure 2.4: Distribution of forces in buoyancy

The calculation of the buoyancy force according to the piston force conception is given by the exerted force of the internal fluid pressure on a submerged body. The internal fluid pressure causes an internal stress on the mentioned body times the differential area in which the fluid is in contact with it. The Gauss or divergence theorem is used to determine the outward flux resulting from the addition of the body into the fluid. It simplifies the integral, and it results in the weight of the volume of the submerged part of the body.

$$F_b = \oint \sigma dA = \int div\sigma dV = -\rho_f g \int dV = -\rho_f g V$$

The piston force school have gone through different experimentations to support its conception on the understanding of buoyancy. An experiment that clearly demonstrates the point of view of the piston force school was achieved by Goins. Goins remarks in his study (1980) that he concurs that buoyancy is an upward force that makes a submerged object to float, but he disagrees with how it is thought that these forces are developed. Goins performed a simple experiment to explain that buoyancy is a force-area through a tank with two cylinders of the same weight.

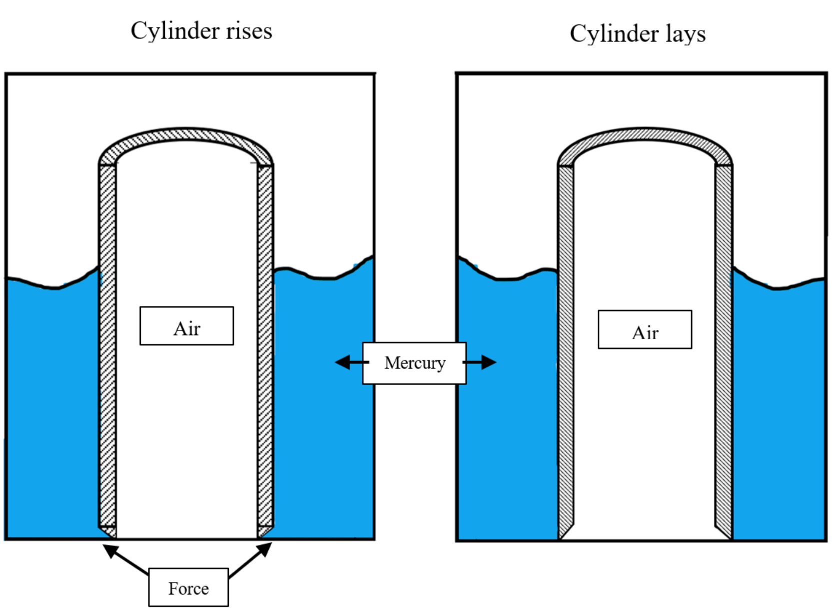

Figure 2.5: Goins' experiment

One cylinder had an internal bevel and the other one an external one (Figure 2.5). The tank was filled with mercury, and then, the two cylinders were observed when the fluid contacts them at the same level so that the displaced fluid would be the same. The cylinder with an external bevel was lifted because it has an area or end where the hydrostatic pressure can act and upthrust the cylinder. On the other case, the cylinder with an internal bevel has no area where the hydrostatic can act; hence, it was laid intact on the bottom of the tank. Goins concluded that the buoyant forces can only act when there is a cross-sectional area exposed where the fluid can exert a pressure vertically.

Another experiment performed by G. Jones and P. Gordon (1979) in which they concluded that the buoyancy force is an upward hydrostatic force in the bottom of the object and stated they could remove buoyancy just by limiting the seepage or amount of liquid in contact with the bottom face of the body of study. The experiment consists of a cork that will normally float in water due to its density. It was glued to a coin-shaped thin layer of aluminum of 0.0005 inches and then polished to obtain a layer of only 4 micro-inches. The aluminum piece was used to obtain a smooth and flat end. The experiment included a larger piece of aluminum as well that was set in the bottom of a tank and where the cork with the aluminum end was seated. After, water was poured into the tank, and the cork stayed submerged for a certain time until it floated (Jones and Gordon, 1979).

2.3.2. Volumetric conception

The volumetric conception simplifies the understanding of buoyancy to a more general form. It describes that a body partially or totally immersed in a fluid experiences an upward force equal to the weight of fluid displaced as it is illustrated in Figure 2.6. Additionally, the understanding of “buoyancy” as an upward force makes it conceive as contrary to gravity creating the concept of apparent weight that is not other than the buoyant effect.

The physical explanation of the volumetric conception is that the volume of liquid displaced transform to a solid object will generate a weight (mass times gravity) equal to the upthrust.

This conception considers a net force on the object equal to zero when it reaches a total equilibrium. Therefore, the weight of the object minus the upthrust equal or zero when the object is at rest.

$$F_b = \rho_f g V_{disp}$$

In equilibrium:

$$mg = \rho_f g V_{disp}$$

Then,

$$F_{net} = mg - \rho_f g V_{disp}$$

It indicates that the object in order to move upwards or downwards, there must be a difference of weights until it attains an equilibrium. In other words, the solid will sink when it has a larger weight than the fluid displaced weight; on the contrary, when the fluid produces a larger weight, it will maintain the object floating. So that the Archimedes' law through this conception has a number of limitations which will be analyzed in the next section.

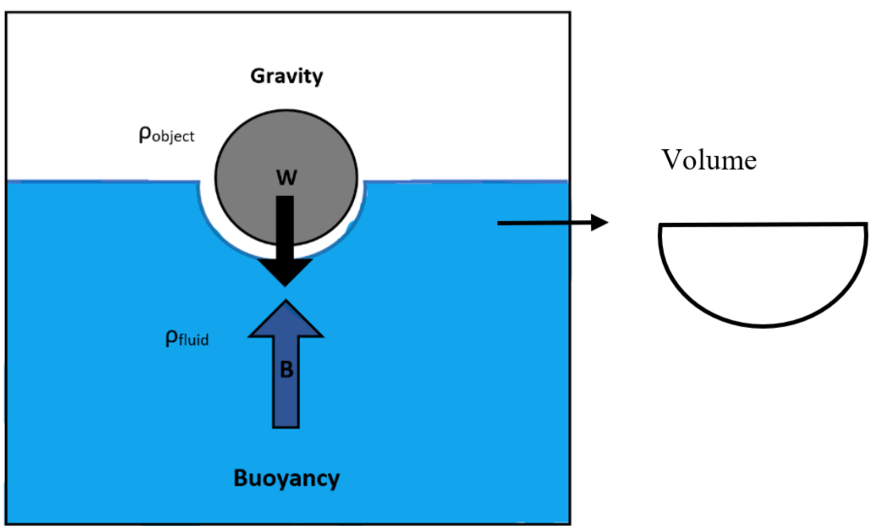

Figure 2.6: Buoyancy according to Volumetric school

Kjetil Lund in his thesis (2014) explains how it is possible to determine that there is no pressure difference over a vertical cylinder. The experiment consists of a barely light rod glued to the bottom of a tank so that there is not a possibility to have fluid underneath the bar. However, this cylinder will be kept perpendicular to the floor. Lund stated that the rod stands vertically and does not buckle because there is not a pressure difference along the height of the rod, but there exists buoyancy. He, then, added that the experiment assures the pressure underneath the bar is zero, and the pressure difference over the diameter is zero.

Another experimentation performed supporting the volumetric school was using a force-sensing scale at the bottom of a container and measuring the forces in the container in three different stages as illustrated in the Figure 2.7 (Erlend, 2004). The first stage is a rectangular body submerged into a container full of a liquid that has a certain density lower than the body. The body is suspended through a cable with a tension T equal to the apparent weight or the buoyed weight of the box. A scale is set on the bottom, in the exact direction where the box is set, reads a force corresponding to the hydrostatic pressure of the column of fluid. In this stage, it is assured the box is under the buoyancy effects.

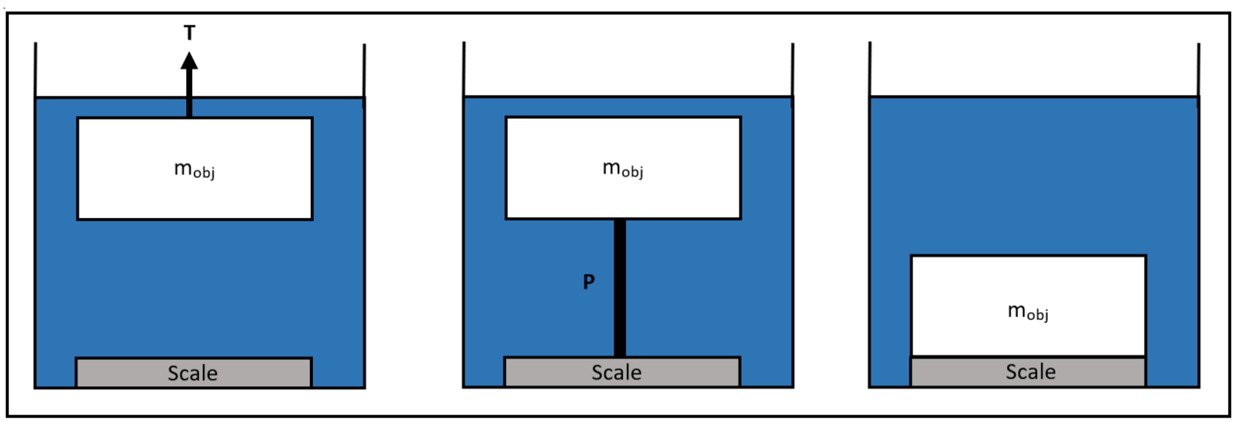

Figure 2.7: Volumetric school experiment using a force-sensing scale

In the following stage, the block is still submerged in the container but connected to a small pin laying on the scale. The pin would reflect the apparent weight of the body in the scale. The results obtained in the documents signals that the scale will read a force FA, and the apparent weight is determined from the reading subtracting the hydrostatic pressure from the reading FA. For the final stage, the pin is removed, and the box is glued to the scale to assure there is no fluid beneath the block. Afterward, the reading of the scale is reviewed, and the result stated in the report was the block's weight plus the weight of the column of fluid above it. Even though the document mentions the weight of the block plus the column of fluid, the calculations are made shows the apparent weight plus the fluid weight. The paper concludes stating that the apparent weight is always present regardless the body of study is resting on the bottom or floating. The author added that Archimedes never mentions a buoyant force for materials denser than the fluid where the object is immersed into.

2.3.3. Analysis

The buoyant forces are determined by the hydrostatic pressure difference between the top and the bottom part of the analyzed body. The resultant hydrostatic pressure times the area where it acts gives the buoyancy. As Archimedes stated, the resultant upthrust force is equal to the weight of the volume of fluid displaced because the depth difference in the hydrostatic pressure is the top and bottom of the object which is the height of the object times the area of action gives the total volume of the body. The weight of the volume displaced is given by the density and gravity force. Therefore, the fluid displacement is not causing buoyancy itself because a pressure must be acting at the bottom and vertically against a surface to create the buoyant force.

$$\rho_f g V = \rho_f g h A$$

Furthermore, the reason that the density is the fundamental factor in both cases is that the weight of the displaced fluid is granted by the density of the fluid. It happens for the pressure as well, the range of the pressure-force is given by the density. It is the reason a denser object than the fluid will tend to sink; and the contrary for a lighter object, it will tend to float.

The modern concept of buoyancy says that it is a force that makes objects float like in the Archimedes' principle; however, Archimedes principle remains true with a remarkable precision. What has changed over the time is the conception of how it is developed. Additionally, it is important to remark that Archimedes mentioned or considered the displaced fluid as the upward force and not the volume of the object itself. The reason is that if you keep adding force to the object (pushing it downwards), the force will be reflected into the fluid and result in more displacing which will be equal to the lifting force as the proposition 6 in Archimedes' work mentions.

The issue about the misunderstanding on how the buoyancy is developed comes due to the wrong interpretation can produce a difference in the tension calculation, as in Figure 2.2. The inclusion of buoyancy in cases where there is no fluid underneath a tension calculation for riser can produce an outcome where the riser is expected to be under tension; but in reality, it undergoes compression. The tension analyses are important in tubular inside the well since it can show the possible instability or buckling of the cylinder at the bottom when the riser/pipe is under compression.

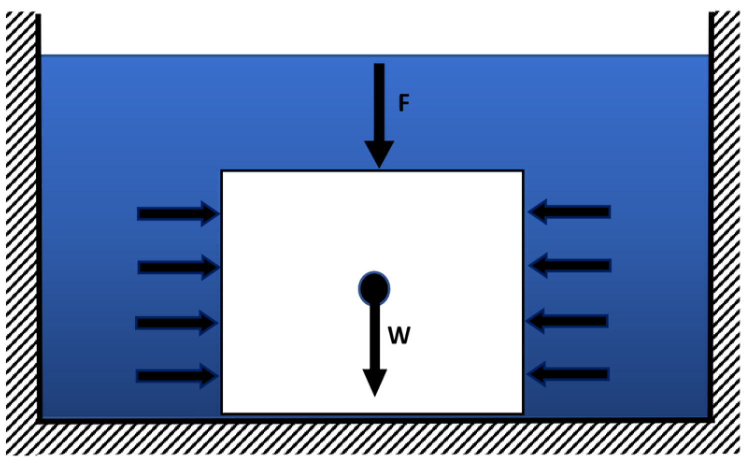

Figure 2.8: Body seated on the bottom of a tank

As the Goins' experiment demonstrated, the difference of concept or misunderstanding arises when the hydrostatic pressure does not find an area of contact to lift the object hence the key factor is the disposition of forces. Using an analysis of forces in the previous case, illustrated in the Figure 2.8, a body is seated on a surface without liquid beneath it. We realized that the forces involved are the weight of the body pointing downwards and the pressure by the fluid. The forces applied to the surface of a body by a static fluid are exerted only in the normal direction. The liquid exerts a pressure on all the fraction of the body in contact with the fluid. The components of the pressure-force that are exerted laterally to the cylinder are canceled out each other. It leaves one axial force pointing downwards that is the force exerted by the hydrostatic pressure. As has been noted the right analysis of where the forces are acting can obtain the same results as Archimedes and it adjusts to all different scenarios whether the forces are acting against the fluid or against the floor.

$$\sum F = -w_{obj} + P_{bot}A_{bot} - P_{top}A_{top}$$

When the body is floating in the water:

$$\sum F = -(m_{obj}a) + \rho_{fl} g d_{bot} A_{bot} - \rho_{fl} g d_{top} A_{top}$$

$$A_{bot} = A_{top} = A \qquad d_{bot} = d_{top} = d$$

$$\sum F = Ah(\rho_{fl} - \rho_{obj}) = w - F_b$$

To summarize, the piston-force conception can explain how the buoyant forces are developed following the physic principles. It covers all the possible scenarios without exception, and it has practical experiments as Goins demonstrated that the weight of the displaced fluid is valid only when there is a projected area available. This does not discredit or defy the Archimedes' principle; on the contrary, it strongly agrees with all the propositions stated by Archimedes and suggests the analysis of pressures and forces for explaining its effect.

2.3.4. Buoyancy force in Petroleum Engineering

Fluids are an integral part of oil wells and its inclusion in well design is not only necessary but essential in form of hydrostatic pressure and buoyancy. Buoyancy has been an important contribution to the industry since Klinkenberg (1951) and Lubinski (1962) published their theory about neutral point and fictitious forces respectively. However, the misunderstanding regarding buoyant forces as volumetric forces or pressure forces in the vertical faces of a body complicate the application of the previously mentioned theories. The issue with these theories is that both are basic for the understanding of the distribution of stresses, pipe failure, and buckling.

Buoyancy has different applications in oil wells; one of the most basic is the apparent weight (w) based on Archimedes' law, applied to all types of wellbores. It is relevant to determine the hook load for a deviated well calculated applying the projected height principle (Aadnoy & Kaarstad, 2006) multiplied by the unit weight of the pipe and the buoyancy factor (β), the last two terms are the mentioned apparent weight.

The buoyancy factor in an oil well can change several times because it depends, as Archimedes stated, in the weight of the displaced fluid. A heavier fluid in the annular will decrease the buoyancy factor that means the upward force is greater consequently the apparent weight will decrease. On the contrary, a heavier fluid inside the drillstring will have the contrary effect in the apparent weight. In case the fluid inside and outside is the same, the equation can be simplified to the suspended weight in mud over the suspended weight in air. This reasoning was first applied by Lubinski (1962) in his paper "Helical buckling of tubing sealed in packers" where he mentions that in presence of fluids the weight per unit (w) must consider the weight of the fluids inside the pipe and the weight displaced outside the pipe. It applies the concept of Archimedes to determine the weight affected by the forces provoked by fluids (apparent weight).

The equation given by Lubinski considering fluids inside and outside,

$$w = w_s + w_i - w_o$$

The weight of the string calculated nowadays is based in the previous equation based on the concept of buoyancy factor:

$$w = \beta w_s$$

$$\beta = \frac{w}{w_s} = \frac{w_s + w_i - w_o}{w_s}$$

Expressed in densities and radius results in:

$$\beta = 1 - \frac{\rho_{fl}}{\rho_s} = 1 - \frac{\rho_o r_o^2 - \rho_i r_i^2}{\rho_{pipe}(r_o^2 - r_i^2)}$$

In addition, Lubinski (1962) establishes that the buoyed weight is directly proportional to the force of the fluids acting at the end of the pipe. Then, it is the equivalent to calculate the force-area of the fluids inside and outside the string. There is a slight error because of the average values in each of the different tools and tool joints diameters. Despite this minimum error, it is a straightforward calculation.

A more debatable issue in the last years has been the axial tension calculation due to the two different approaches. Depending on the approach, the axial tension can give different results that arise into a debate. There are exceptions that can apply when calculating tension for drilling risers attached to the floor or conductor casings all the way up to the platform where the area of contact of the tubular ends is closed. This only occurs in a determined case as cited before, and it is possible to observe in the Figure 2.1 previously introduced. The problem for these cases is that the buoyant force is a contrary force that unbalanced the gravity forces and changes the real weight and the forces involved. Therefore, when a body is resting on the floor, there is a normal force that acts contrary to the gravity and stresses the body. An upthrust force acting on the body will change the failure analysis of a pipe and will not be valid.

The issue about the misunderstanding on how the buoyancy is developed is produced by a wrong interpretation that creates a difference in results when calculating tension. The inclusion of buoyancy in cases where there is no fluid underneath a tension calculation for riser can produce an outcome where the riser is expected to be under tension but in reality, it undergoes compression.

According to Sparks, the disagreement in the aforementioned cases is due to the Archimedes' concept of buoyancy which has limitations when analyzing not closed pressure fields. He identifies a closed pressure field as an enclosed or defined volume with a centroid. He adds that those cases can be analyzed just by artificially closing the pressure field and considering all the resultant forces. It is the case of a body attached to a bottom surface, as in Figure 2.1 at the left, which has not a centroid or has not a 'closed pressure field'. However, when 'closing the pressure fields,' it results in the analysis of forces as made by the piston-force approach.

Both conceptions makes a distinction between the "real force" and the "effective force". The definition of both concepts are the same for both schools, the real forces is defines as the forces acting in the axial direction evenly distributed along the pipe and depends on the physical context. The real forces are calculated contemplating all the possible forces involved in the axial direction. The effective force is a stability force (also named as buckling force) added to the true force that is reflected at the bottom face of the pipe. The effective force removes the effects of the fluids from the real axial force (Samuel and Kumar, 2012).

The difference with the advocates to the volumetric conception is the application of the buoyancy method that results in the calculation of the effective stress. According to the volumetric school, the buoyant force applied to the string is a compressive force distributed all along the string because it is a volumetric force (Sparks, 2007). It is opposite to the concept of the other school that considers the compression only reflected at the bottom and its effect is reduced as it moves upwards until the string is again in tension. The concept of the volumetric schools states that it is not possible to buckle the string due to the effect of fluids since the compressive force is applied to the whole surface of the tubular. Even though the concept is right, it is not possible to buckle the string merely applying hydrostatic pressure at the lower end of the string; buckling can occur due to fluids acting on the bottom end of the string when a mechanical force is applied to the string.

Lubinski was the first to study about the effect of fluids in the stability of the string. He introduced the difference between the real and the effective forces even though in his paper of 1962 he was referring to those concepts as an 'actually existing force' or non-fictitious force (F_a) and a 'fictitious force' (F_f). Lubinski was working on a packer application regarding helical buckling when he stated the concept of fictitious forces for the first time. He stated that in the case of a tubing string freely suspended in a well under a compressive force P, the compressive force will set the bottom of the string into compression. However, when the same tubing string is sealed by a packer, producing a mechanical force; an internal pressure (P_i) applied to the same string will buckle the string more severely due to a compressive fictitious force. The other case considers the same tubing string sealed with a packer but exposed to an outside pressure (P_o) that subjects the pipe to a compression, yet the tubing string remains straight (Lubinski, 1962). Both cases are illustrated in the Figure 2.9.

$$F_f = A_p(P_i - P_o)$$
$$F_a = (A_p - A_i)P_i - (A_p - A_o)P_o$$

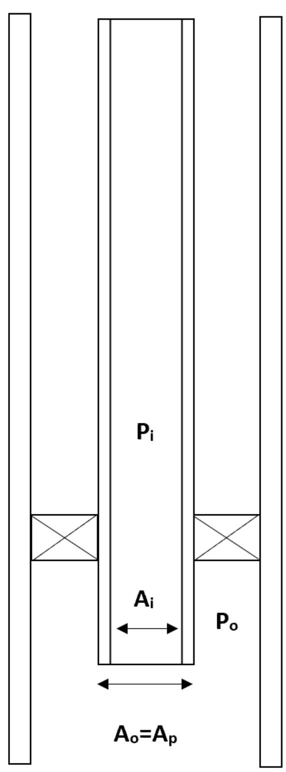

Figure 2.9: Fictitious forces according to Lubinski

Lubinski considered the compressive forces as a positive force and the tensile forces as negative in his work. Therefore, the positive values of the fictitious force means the string is under compression, and it will buckle because of the additional compression. While the negative values will mean that the string remains straight because it is experiencing an additional tension. Furthermore, Lubinski named those forces as fictitious forces because in reality these forces do not exist. He suggested these forces cannot be considered to determined the actual stresses the string is experiencing. However, the fictitious forces can be applied when dealing with cases regarding straightening or preventing buckling of the tubular when accounting for fluids.

Lubinski equation was strictly developed for determining buckling in a packer application (mechanical force involved) even though it has been the base for further application regarding stability for tubulars. Having said that, Pattillo and Randall based their studies regarding the proper determination of neutral point in a drill or tubing string to predict buckling according to Lubinski's work. The effective tension equation was determined from the classical differential equation for lateral deflection in a tube. Patillo and Randall (1980) determined the modern effective tension for tubular and defined it as a force pulling the tube to keep it straight. The equation of effective tension consisted of the real tension or axial tension (T), a straightening force (P_o), buckling force of the inside pressure (P_i). They also explained that the external pressure was equal to the surface pressure applied (P_{so}) in the annular plus the hydrostatic pressure in the outside (γ_{fo}) while the internal pressure was the hydrostatic pressure in the inside (γ_{fi}) of the string plus the internal pressure applied from surface (P_{si}).

$$\frac{d^2}{dx^2}(EI)\frac{d^2y}{dx^2} - \frac{d}{dx}\left[[T + (P_{so} + \gamma_{fo}x)A_o - (P_{si} + \gamma_{fi}x)A_i]\frac{dy}{dx}\right] = 0$$

$$T_e = T + (P_{so} + \gamma_{fo}x)A_o - (P_{si} + \gamma_{fi}x)A_i$$

$$T_e = T + P_oA_o - P_iA_i$$

As Lubinski was stated before, the stability criterion identify where the buckling will occur. In case of the effective forces, the neutral point can be defined as the point where the effective tension term vanishes (Patillo & Randal, 1980). The stability criterion applied (T_s) Furthermore, the concept of effective forces presented by Sparks was introduced previously by Patillo & Randall (1980) as cited in Spark's paper of 1984. However, it is noticeable that both works were inspired in Lubinski application of stability of tubular when a packer was set with the difference of considering tension and compression with opposite sign.

Introducing the stability term:

$$T = T_s$$
$$T_s = P_i A_i - P_o A_o$$
$$T_s = (\gamma_{fo} x) A_o - (P_{si} + \gamma_{fi} x) A_i$$

The work of Lubinski can be stated in the following formulas modified to the actual application in the petroleum industry where the tension is positive and the compression is negative. The real forces considers the weight per unit of steel of the pipe (w_s) and the vertical length coordinate (z),in which z=0 is equal to the bottom of the string. It also accounts for an additional load at the end of the string (Q), and the internal and external pressure. The fictitious force is stated as it was originally by Lubinski but changing the signs of tension and compression.

$$F_R(z) = Q + w_s z + A_i P_i(0) - A_e P_e(0)$$
$$F_E(z) = F_R(z) - A_i P_i(z) + A_e P_e(z)$$

The axial forces calculation leads to another concept called neutral point mentioned the first time by Klinkenberg (1951) that is where the forces are in equilibrium between compression and tension. It is necessary to mention Lubinski's concept of fictitious forces one more time when dealing with pipe stability. When a compressive force is applied at the lower end of a pipe string, the lower section of the string will be subjected to compression that will decrease as it moves upwards along the string until the compression becomes zero. The point where the string is neither in compression nor in tension is the neutral point explained by Klinkenberg.

Furthermore, the transition between tension to compression is a case of pipe stability when the fluids are considered in combination with an applied mechanical force. According to Lubinski, the neutral point in the presence of fluids is not the transition between tension and compression; it rather determines the section of the pipestring that will buckle into a helix, below the neutral point, and the section that will remain straight, above the neutral point. Lubinski applied the concept to determine the shortening and the elongation of the string due to theses forces.

An example can relate a pipe hung up in air, its neutral point will be in the lower end. However, when the pipe is submerged in a fluid; a compressive force will move the neutral point upwards. The compressive force is the result of a mechanical force where in the case of the fluids can increased or decreased the mechanical force. The distribution of the stresses is modified when a mechanical force (compressive or tensile) or an additional load (WOB) acts on the tubular. Consequently, it is essential to know the stress distribution and the correct axial tension when determining the neutral point, which means make the distinction between the real and effective forces. The hydrostatic pressure contacts a submerged tubular all along its surface in the inside and the outside developing radial and tangential stress. The magnitude of the stresses depend on the pressure difference between the internal and external pressure in the pipe. These are the actual forces that stress the pipe. On the other hand, there is also a concentrated force that set the lower end of the pipe into compression.

Sparks (1984/2007) contributed to the industry with the concept of effective tension for risers and other tubulars derived from previous work on axial effective forces studied on the previous years. The equation presented next is just the same as introduced in Lubinski (1962) or more specific about tension in tubular was G. Morgan (1977) and further explained by Patillo & Randall (1980).

$$T_e = T_{tr} - A_i P_i + A_e P_e$$

The concept was introduced with a different application, and named as effective tension. It was developed to determine the tension in risers and pipes. Sparks work is suggested to be inspired on the previous work of Lubinski, Patillo, Randall and Morgan. However, Sparks never cites Lubinski as the precursor of the effective forces, probably because it is regarding a different application. Morgan, on the other hand, is the first to demonstrate how the different loads and forces concentrate along a tubular (Morgan, 1977). Morgan mentions that changes in tension must be accounted for all the different loads imposed to the tubular. He insisted that the reactions to the pressure-area forces are transmitted to the lower end of the pipe that produce changes in tension. Patillo & Randall (1980) go further and explain the effective force and how it affects the determination of the neutral point. Yet Lubinski deserves some credit for the basis of the concept of fictitious forces that was introduced by him years before these authors.

Sparks also had some disagreement with some concepts Lubinski applies regarding fictitious forces and buckling. Sparks asserts that the concept of the lower end of a pipe can be buckled when a fictitious force adds an excessive compressive force is not possible. He explains that if the apparent weight is positive the pipe will never buckle which is true because it means the effective tension is positive and the whole tubular is in tension (Sparks, 2007).

The first issue is to remark that fluid pressure cannot produce buckling; however, the mechanical force intensified by the effects of the pressure fluid can produce a more serious buckling than the anticipated. The second issue with the previous analysis is that the apparent weight acknowledges the string or a section of the tubular as a whole unit even though there are cases where a string can have a section in tension and the other in compression. Indeed, the Klinkenberg's concept of neutral point signals the transition between compression and tension a tubing string can experience. Moreover, Lubinski set a condition for the buckling to happen; there must be a mechanical force linked to the fictitious force.

Additionally, Sparks states that the concept of lateral pressure forces canceling out because they are equal and opposite is erroneous. He supports the conception that the buoyancy is a volumetric force distributed equally all along the pipe. He adds that the lateral pressures provide a net thrust to the string that increases with the pipe deflection and considers the thrust a destabilizing force. He considers a restoring force preventing buckling resulting from the lateral component of compressive axial loads ($P_e A_e$). When the axial compressive loads in the string are less than the compressive loads from the displaced fluid, the string will not buckle due to the restoring force mentioned before (Sparks, 2007).

The issue of the lateral forces in submerged objects has been covered by several papers in the past concluding that the effective forces consider the effect of the difference between lateral forces and the flowing fluid. The analysis of a perfect straight pipe in static conditions considering only the changes in cross-sectional area is simple to anticipate and analyze. It is a straightforward analysis of forces without further complications. Nevertheless, the pipes are never completely straight. There is in some cases an imperceptible curvature that in case of a short string is not significant, yet a drillstring commonly have thousands of feet of distance that makes the difference sensitive. Then, the outer area becomes greater than the inner area that produces an excess of pressure force on the outside of the curvature (see Figure 2.10). An additional lateral force comes from the flowing fluid that produces a centripetal force towards the outer curvature side of the pipe. However, the difference of forces in the curvature is considered when applying the effective forces even for deviated and horizontal wells.

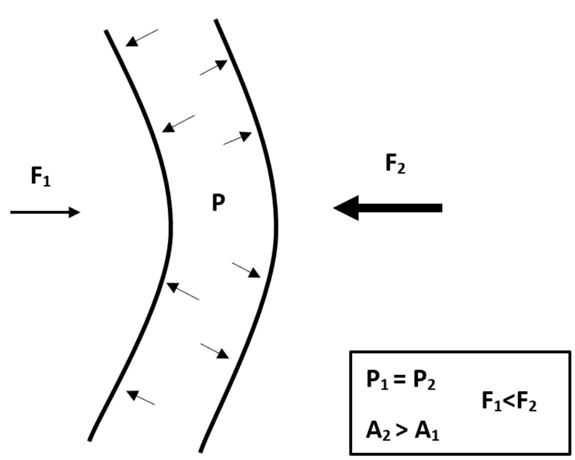

Figure 2.10: Lateral load of a submerged pipe section

The force balance in a drillstring when a load is applied involves the change in the tubing force along the well depth ($\frac{dF}{ds}$), the buoyant weight (w_{bp}), the load on the pipe due to external forces (w_{ef}), and the contact and drag forces acting in the tubular (w_c). The force balance obtained is complicated to interpret; therefore, it can be simplified by transforming it to the axial force (F_a). The positive and negative superscript sign refers to the downstream and upstream flow, respectively. Rearranging the terms, the final equation shows that the effective forces account the internal and external fluid forces since it considers the changes in a sectional area, the stream thrust, and a difference between internal and external loads (Mitchell, 2009).

$$\frac{dF}{ds} + w_{bp} + \Delta W_{ef} + w_c = 0$$
$$F_a^+ - P_i A_i^+ + P_o A_o^+ = F_a^- - P_i A_i^- P_o A_o^-$$
$$F_e^+ = F_e^-$$

Sparks made an important analysis when describing the lateral forces effect. In Mitchell's paper of 2009 called 'Fluid Momentum Balance Defines the Effective Force', Mitchell does the analysis for dynamic conditions when fluids act on the string walls. He concluded that the effective forces are the final result of the association of pipe forces and fluid forces. In addition, Patillo (1980) is cited in Mitchell's document who states that the pipe equilibrium problem is a complex combination of different terms that at the end are equivalent to the effective force. It has been proved that the effective force considers the combination of lateral forces, the action of fluids on the pipe wall, the centripetal force caused by the flowing fluids, and pipe displacement and curvature.

Mitchell also includes a new term for dynamic conditions. The stream thrust that is applied by comparing the local pressure caused by the fluids on the string and the effect of the fluids stream. The values observed are relatively small for well fluids in the final pressure results. The magnitude of the variable becomes important only for really low-density fluids at low pressure and high velocities (Mitchell, 2009).

That being said, fluid statics signals that thrust can only be exerted vertically from the bottom cross-sectional area of the pipe for this case. Under those circumstances, the destabilizing force (buckling) can only be exerted at the lower end in form of a compressive force as Lubinski asserts through his work regarding fictitious forces. The compressive force would be maximum in the lower end, and it will buckle the pipe in the whole lower section under compression. The compression is the maximum at the bottom because the self-weight of the pipe is the minimum, and a compressive mechanical force is the maximum. The fluids themselves will not produce buckling, yet the mechanical force causes the buckling. The fluid pressure can magnify or diminish the additional mechanical force that will produce the compression or tension of the bottom section of the pipe specially is the pipe can experience free motion. The force is not distributed equally because it decreases across the length of the pipe until it reaches the transition to tension (Lubinski, 1962).

Another basic concept to consider is the calculation of the real forces or real tension through the pressure-area approach throughout a free diagram body to simplify the understanding of the calculation (Figure reffig:Area-force). The axial stress is calculated by the forces applied to the different sections of the string. The bottom force (F_{bot}) is a compressive force that can be associated with a fluid pressure in the upward direction plus the additional weight (Q) will depend on the physical case. The external fluid will cause a compression while the internal fluid produces tension on the string for the real forces according to the pressure-area approach. The differential force in the sections of area change (ΔF_A)can be calculated as the difference of cross-sectional areas times the correspondent hydrostatic pressure. The weight in air (W) of each string section is calculated applying the projected height principle shown in the Figure 2.11.

It bears mentioning the surface pressure (P_{pump}) is part of the axial forces calculation for flowing wells adding the effect of the fluids in the end sections depending if the adding pressure is internal or external. The Archimedes' principle is not violated when considering pumping of fluids even though it means displacing of more fluids. This application only reinforces the concept that the buoyancy is originated by forces created by static fluids. However, the effective tension calculation is affected by additional pressure into the system.

The surface internal pressure added to the system affects the stress experienced by the pipe in all directions affecting the final the failure design of the drillstring. The additional internal pressure increases the tension without affecting the final weight of the pipe, which is still given by the apparent weight according to Archimedes, in a range equal to the internal hydrostatic force. The internal pressure would be equal to the hydrostatic pressure plus the internal surface pressure pumped (P_{pump-i}) into the well.

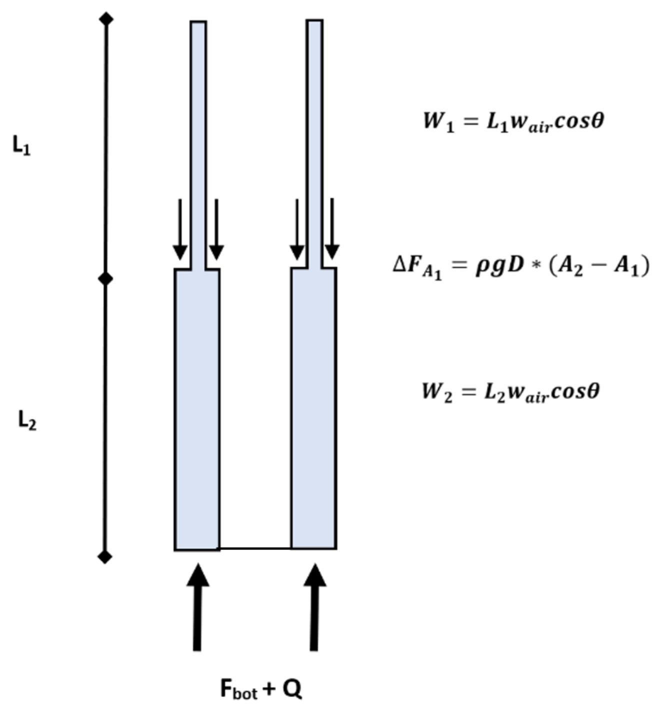

Figure 2.11: Area-Force distribution

On the other hand, the external surface pressure (P_{pump-o}) produces an additional lift in the string apart from the buoyancy thrust. This additional lift force is equal to the surface annulus pressure multiplied by the external area of tubular, and the pumping pressure suggests the case is a live well. Hence, the pressure added to the system is the snub force needed to enter the well for those conditions. It does not affect the buoyancy factor as long as the density of the fluid is the same. However, it reduces the hook load in a proportion equivalent to the force produced by the external surface pressure applied. In case of the pressure applied internal and externally, there is not going to be any modification in the axial stress as well as the deviatoric stress will remain the same as the obtained only applying internal surface pressure. The hook load could be modified depending on the density difference of fluids inside and outside the drillstring (Aadnoy and Kaarstad, 2006). The surface pressure effects are demonstrated in the following equations.

Surface pressure applied inside the string:

$$P_{dp} = \rho g D + \Delta P_{pump}$$
$$P_{annulus} = \rho g D$$

Surface pressure applied outside the string:

$$P_{dp} = \rho g D$$
$$P_{annulus} = \rho g D + \Delta P_{pump}$$

Surface pressure applied inside and outside the string:

$$P_{dp} = \rho g D + \Delta P_{pump}$$
$$P_{annulus} = \rho g D + \Delta P_{pump}$$

The buoyancy force in all three cases will be the same:

$$F_b = (A_o \rho_o - A_i \rho_i) D$$

3 Tubular Mechanics/Bridgman Paradox

The science that studies the effects of bodies or objects under the action of forces and displacements is called mechanics. It is ruled and understood through Newton laws that describe the relationship between a body and the forces imposed on it, and the correspondent motion due to those forces. When we go more in deep about mechanics, we might probably start with an essential question that is how solids can bear external forces or even its own weight.

The understanding of the capacity of structures or materials to bear loads started through Robert Hooke who basically stated the same as Newton that every force has an equal opposite reaction. He explained that forces cannot disappear, there is a logical response to an applied force. Therefore, he concluded that the amount of deformation of a structure or material is directly proportional to the deforming force applied.

He also stated that under small deformations the object will return to its original shape, the elastic behavior. However, it can change in case of unbalanced forces when the condition is unfulfilled due to considerable deformations, then the structure could fail. Hooke tried to demonstrate that all type of materials changes its shape when a force is applied, and the material reacts or pushes back due to this change. The change of shape may occur by stretching, compressing, bending, or other forces that cause a deflection which varies from one material to another. The deflection of materials in response to a load is normal until the deformation becomes large, the material does not push back again, and results in plastic behavior or even failure (Gordon, 2002).

Focusing on tubular strings, it is easy to realize that they are an essential part of the upstream activities of the oil industry in drilling as much as in production operations. Well designing is essentially based on the adequate selection of pipes depending on its resistance to deform in front of external forces, and the engineering work is to effectively design the string to withstand those forces inflicted on the string. The same way as Hooke stated the tubular is exposed to a number of loads or forces of a different kind, then the pipe deforms to a certain degree and depending on the amount of deformation it will cause the tubular to fail or not. Once the deformation reaches the maximum, the tubular will fracture because the molecules that form the tubular are pulled apart. However, there are many scenarios that can occur and result in other forces that must be considered inside the wellbore, which difficult even more the prediction of strains and deal with those type of forces is part of the well design.

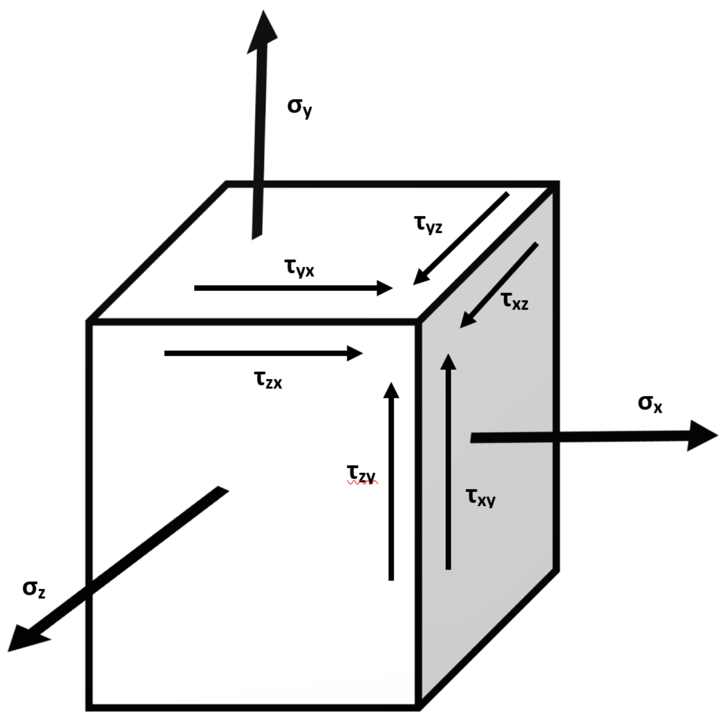

Figure 3.1: Stress state of a body

The pipes, in the case of drilling, are designed to resist the loads imposed from the string itself and the compression it is subjected when drilling; but also, to endure the hydrostatic loads, the wear caused by the formation, the temperature expansion, and many more. It leads to the concept of mechanics of materials that studies how different materials behave through different forces imposed on it and its deformation. It has a vast importance in the design of a drillstring system to analyze the response of the pipe to different external forces and its internal reaction. The right application of mechanics material will ensure an adequate strength and stability of the string. The tubular mechanics is not different than analyzing the behavior of a pipe subjected to diverse forces that follow Newton laws, which is essential to describe any physical event occurring to an object (Aadnoy & Looyeh, 2011).

Tubular mechanics is the most crucial applied science in petroleum engineering to perform a failure analysis. It focuses on the of cylindrical shape objects against various forces and constraints from different directions, and the characteristics of the material of the object. The two key elements in tubular mechanics are stress that is related to the internal forces of a material to resist or balance the external forces and strain that is relative to deformation of the material itself when subjected to various forces. These two terms are the basis of the tubular mechanics that counteract the external forces and prevent the tubular to fail. The following chapter will be dedicated to the last two concepts and failure of tubular.

3.1. Stress

According to physics and engineering, stress is an internal force that describes how much the molecules that form a material are being stretched or contract over a unit of area, and it is produced as a reaction to an external force. The area where it acts could be at any point of a material, at the surface as well as at any plane through the material. The units are the same as pressure (Psi or Pa), but it is important to keep in mind that pressure is exerted in all directions in a fluid.

The stress is the measure of the molecules of a material reacting to external forces where they are being pushed together if a compressive force is applied or pulled apart in case of a tensile force. The stresses can be interpreted externally as a direct reaction to the external force, for instance, an external pressure acts against a surface creating an external stress directly related to the force applied. It can be interpreted as an internal phenomenon. The external stress causes deformation and the deformation is causing additional forces to the material creating internal stress on the material. One is the reaction to the other as physics explain. In more simple terms, stress is a measure that allows predicting when a material could fail or the actual state of affairs according to the forces the material is subjected.

$$\sigma = \frac{F}{A}$$

There are two types of stresses. The normal stresses as the name mentions, it originates from forces perpendicular to the area of action and the shear stresses that occurred through forces acting parallel the cross-sectional area. The stresses are ruled by the Newton's law like most of the elements in physics.

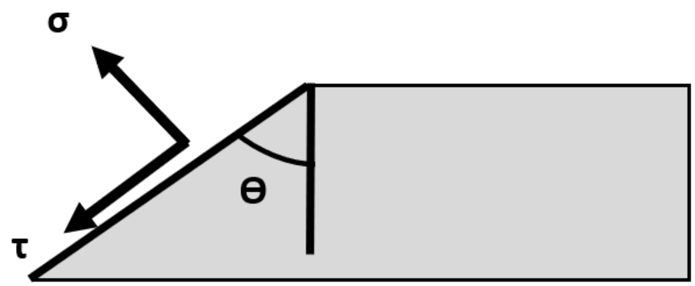

Figure 3.2: Normal and shear stress plane

Stresses are 3-Dimensional because a stress coming from one side is balanced by other coming from the opposite side so that the bodies can be in equilibrium as illustrated in Figure 3.1. The stresses can dictate the failure mechanism a body is subjected to since a normal stress can result in tensile or compressive failure, and the shear stress in shear failure, where the material is opened axially, slipped along a plane or tear apart parallel to the crack.

When discussing failure, it refers to the response of a body when it is carrying loads and it reaches its limit. The failure will depend on many factors governed by the properties of the material is being loaded. If the material is permanently and unwanted deformed (plastic deformation), it is a failure called general yielding. If the material is fractured, it can be involved in two types of failure. The brittle failure when it occurs without a noticeable deformation and it is almost instantaneous. The ductile failure that is characterized by a plastic deformation following the fracture. The permanent deformation occurs when the material changes from the elastic, the material returns to its original shape after the stress is removed, to the plastic behavior. The transition between these two behaviors is called yield stress.

3.1.1. Cylindrical Stresses

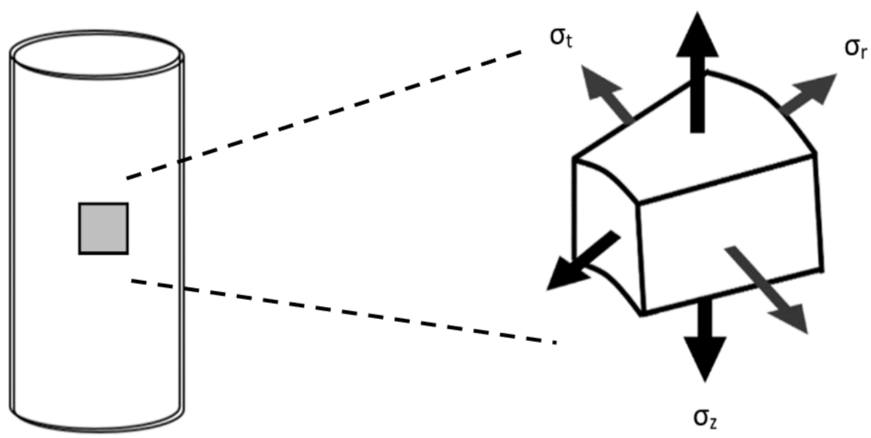

Figure 3.3: Stress in a tubing

The pipe design is based on the evaluation of hydrostatic pressure and mechanical forces induced stresses and determine the pipe limits. The stresses applied to a tubular can be seen in the Figure 3.3. The pipe is subjected to a radial (σ_r) and a tangential (σ_t) stress on the total circular section, and an axial stress (σ_z) vertically. The radial stress is the stress across the tubular in the radial direction; the tangential stress, known as hoop stress as well, is the stress along the circumference; and the axial stress is the vertical stress along the wall of the tubular. The Lame equations define the magnitude of these stresses in a tubular in which the deviatoric forces are the responsible of the correspondent failure (Aadnoy, 2006). The magnitude of the cylindrical stresses is defined by the Lame equations. The Lame equations are the following and were defined for thick-walled cylinders..

$$\sigma_r = \frac{P_1 a^2 - P_2 b^2}{b^2 - a^2} - \frac{a^2 b^2}{r^2(b^2 - a^2)}(P_1 - P_2)$$

$$\sigma_\theta = \frac{P_1a^2-P_2b^2}{b^2-a^2} + \frac{a^2b^2}{r^2(b^2-a^2)}(P_1 - P_2)$$

The axial stress for a closed-ended cylinder is calculated by means of the equilibrium. It is expressed as the difference of the axial force caused for the internal and external forces, and :

$$\sigma_z = \frac{P_1a^2-P_2b^2}{b^2-a^2} + \frac{w_sLcos\theta}{\pi(b^2-a^2)}$$

The average hydrostatic stress is equal to the average cross-sectional stress given by the tangential and radial stress. It can be related to the fictitious forces stated by Lubinski that predicts the possible buckling of the tubing string. In addition, the actual effective force equation is based on the initial Lubinski work in 1962 on the fictitious forces for a different application. Therefore, we can conclude that the effective force is the same as the stability force or buckling force, and the actual force is equivalent to the total force. Then, the deviatoric stress is the subtraction of the real and effective forces.

The total load in a tubular is the addition of the hydrostatic force and the deviatoric force. The importance of the deviatoric stress is that it reflects the shear stress level that is the cause of failure of many materials. Ductile materials, such as tubular steel, are very strong for hydrostatic forces; hence, they fail for distortion or shear mechanism. However, brittle material like rocks is prone to fail by the hydrostatic load. The deviatoric forces are governed by the forces in the axial direction varying according to fluid densities in and out of the pipe. The result will be the force difference between the top and the bottom section analyzed which is equal to the effective force.

To briefly sum up, the piston method equals the total stress on a pipe (real force) where the hydrostatic force is equal to the average hydrostatic stress, and the deviatoric stress is the same as the subtraction of the piston method minus the hydrostatic average stress. The Archimedes' principle can define the deviatoric forces since the result is the same as the buoyancy force, but it must be appropriately used.

The total stress formula is based on the Hubert & Rubey's work:

$$\sigma_{Total} = \sigma_{hydrostatic} + \sigma_{deviatoric}$$

The average hydrostatic stress:

$$\sigma_{hydrostatic} = \frac{P_o r_o^2 - P_i r_i^2}{r_o^2 - r_i^2}$$

The total stress based on the total forces affecting the tubingstring:

$$\sigma_{total} = \rho_{steel} g A d_{bot} - \rho_{fl} g A d_{bot} + \rho_{fl} g A d_{top}$$

3.2. Strain

Deformation is the change of shape when stresses are applied to a particular material, the description of this phenomenon in physics is explained through the concept of strain. It defines deformation as the relative particle displacement a material experienced with respect of its initial shape. The displacement is measured in the different axis one with respect to the other. The following equation is the engineering strain where the l_f is the final length and the l_o is the original length of the body.

$$e = \frac{\Delta L}{L} = \frac{l_f - l_o}{l_o}$$

The deformation of the body is of interest in the failure analysis because it produces a change in the stresses that affect the final analysis. The strain, unlike stress, can be measured through the initial and the final measures of the body. Additionally, failure of materials does not involve only stresses; strain is also part of the analysis since there is a direct relation of the maximum strain a material can stand and the appearance of cracks. In that sense, there is a concept called critical strain. The critical strain is the point when the cracks initiate to appear in the material, and it is a property of the material itself. The material failure is generally described as the point when the release of the strain energy density, related to the increment of stresses, is sufficient to break the atomic bonds of the material.

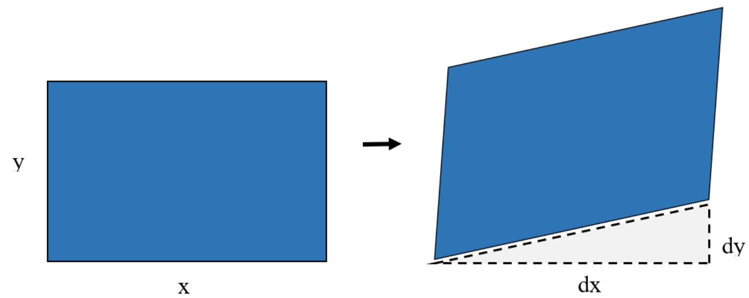

Figure 3.4: Body deformation

3.2.1. Stress-strain relation

The stress-strain relation is the amount of deformation a material is exposed when a certain stress is applied, and it varies from one material to another. The relation between stress and strain can be linear for an elastic behavior and non-linear for a plastic behavior. Therefore, it depends on the amount of deformation the material can bear, and it becomes important the concept of Poisson effect that will be mentioned in the following point. For the cylindrical stresses, the relation of stress and strain equations are the following.

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r + \sigma_\theta)]$$

$$\varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_z + \sigma_\theta)]$$

$$\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)]$$

3.3. Poisson's ratio

Another factor to consider when talking about failure and the coming analysis of the Bridgman paradox is the Poisson ratio. When a material is stressed by an external force, the deformation produced by the force does not produce a strain (deformation) in one direction. The material deformation is always a three-dimensional effect. The Poisson's ratio is then the ratio of the relative contraction of the material over the relative expansion in the transverse and axial direction.

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

The Poisson effect is essential to understand the internal stresses occurred in a body. The distortion of the body due to the compression in one direction and the expansion in the perpendicular direction originates a change in the stresses that can lead to an unexpected failure. It causes to be even more complexed to define the failure mechanism a material has experienced.

3.4. Modes of Failure

It is considered a failure of material when it is permanently deformed, and its behavior situated in the inelastic zone and results in yielding or fracture. The plastic behavior of materials can be described by a yield criterion that identifies when the material reaches the inelastic behavior, the existence of a flow rule that governs the relation between the strain and stress in the inelastic material behavior, and a hardening rule that anticipate the changes in the yield surface (Boresi, Schmidt, & Sidebottom, 1993). For the simplicity, it is only considered essential the mode of failure through the identification of the yield criterion.

Unfortunately, there is not a single accurate failure criterion that adjusts to all types of failure. Even though it can adjust with certain degree according to the type of material, brittle or ductile; it can be interpreted according to the material strength for compressive, tensile and shear stresses. For ductile materials that mostly fail for distortion of the material, the deviatoric stress is strongly related to the shear stress level the material is subjected. For certain brittle materials, the yield behavior increases with a hydrostatic compressive stress that increases its ability to yield and have different compressive and tensile strength (uneven materials). It is a different case for brittle materials with equal compressive and tensile strength (even material) (Norton, 1996).

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3.4.1. Failure criteria

There is a clear relationship between the mode of failure and the critical parameter the material must meet in order to fail. The critical parameters involved in the failure of engineering materials are several, and it can be the case that more than one parameter is necessary to mention to describe the occurrence of a failure. The critical parameters are stress, strain, displacement, load, the existence of flaws, among others. Most of the time one main critical parameter is considered to simplify the analysis even though there has been met more than one. For the materials to fail, it is necessary that the stresses the material is exposed to meet one of the critical parameters according to a mode of failure. Therefore, failure criteria have been developed to predict the occurrence of yielding or failure. However, the complexity to meet one failure criterion that adjusts to each mode of failure and the characteristics of the different materials has occasioned to develop a number of failure criteria that will be briefly described following.

Maximum principle stress criterion is a theory postulated for Rankine. It states that failure occurs in an engineering material when the first principle stress (highest stress) at a point reaches the limiting compressive or tensile yield value, determined by an axial test. This criterion only considers the maximum stress applied to the body regardless of any other stress the material is exposed. For the maximum principal stress criterion to be valid, the main stress must be equal to the yield stress (σ_y), or the shear stress in the material must be equal to the maximum normal stress and the shear yield stress (τ_y) must be at least equal to the tensile or compressive yield stress.

$$\sigma_1 = \pm\sigma_y$$

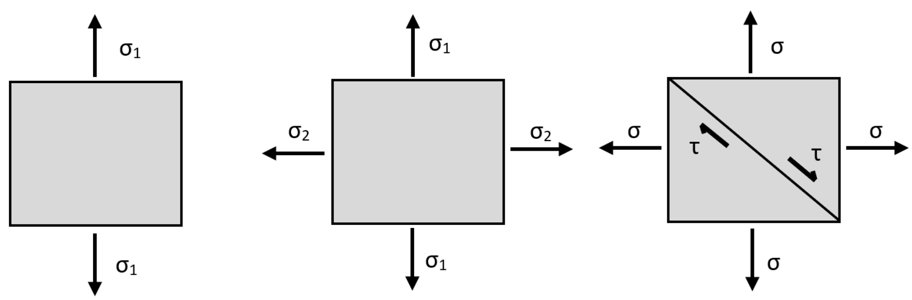

Figure 3.5: Maximum principal stress

Brittle materials can have the same strength for shear than for tension hence this criterion can be applied to predict the failure. Nevertheless, the shear yield stress for ductile materials is normally lower than the yield stress so that these materials normally yield by pure shear. As Chen and Han (1988) mentioned in their work, the maximum principal stress is often occupied to predict failure of brittle material in conjunction with another criterion (Boresi, et al., 1993).

Maximum principal strain criterion was established by St. Venant. He stated that the failure of a particular engineering material did not depend on the stress it is subjected to; yet the yielding occurred when the maximum principal strain reached the critical value of strain of said material. This criterion does not predict accurately yielding of ductile materials. However, it may represent a more accurate alternative to the maximum principal stress in case of brittle materials. The equation for yielding in an uniaxial case is the following.

$$\epsilon_1 = \epsilon_y \qquad where \qquad \epsilon_y = \sigma_y/E$$

Under biaxial and triaxial stress for an isotropic engineering material the equation can vary. The equation is expressed in terms of stress despite the theory explains yielding in terms of strain. The equation for biaxial and triaxial stress is given by the following where ϵ_1 represents the maximum strain. In addition, the inclusion of the Poisson's ratio marks an important difference with the previous criterion that affects the yielding limit of the material.

Triaxial stress:

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3)$$

Biaxial stress:

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

Maximum Strain Energy density criterion rejects the idea of stress or strain provoking the failure. In 1985, Beltrami proposed the theory that failure occurs due to the energy accumulation within the material. He named the stored energy as strain energy and explained that yield is initiated when the strain energy density equals the critical strain energy for any stress state. For instance, when the strain energy density (U_o) reaches the strain energy density for yielding (U_{oy}); the material will fail. This criterion does not fit properly to the ductile material behavior; however, it can be a good compliment for the maximum stress criterion to predict failure of certain types of brittle material (Boresi, et al., 1993).

U_{oy} for an uniaxial loading the critical strain energy is described as:

$$U_{oy} = \frac{\sigma_y^2}{2E}$$

The U_o equals U_{oy} under triaxial loading:

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3) = \sigma_y^2$$

Under biaxial stress it can be reduced to:

$$\sigma_1^2 + \sigma_2^2 - 2\nu\sigma_1\sigma_2 = \sigma_y^2$$

For a state of pure shear without considering the Poisson's ratio:

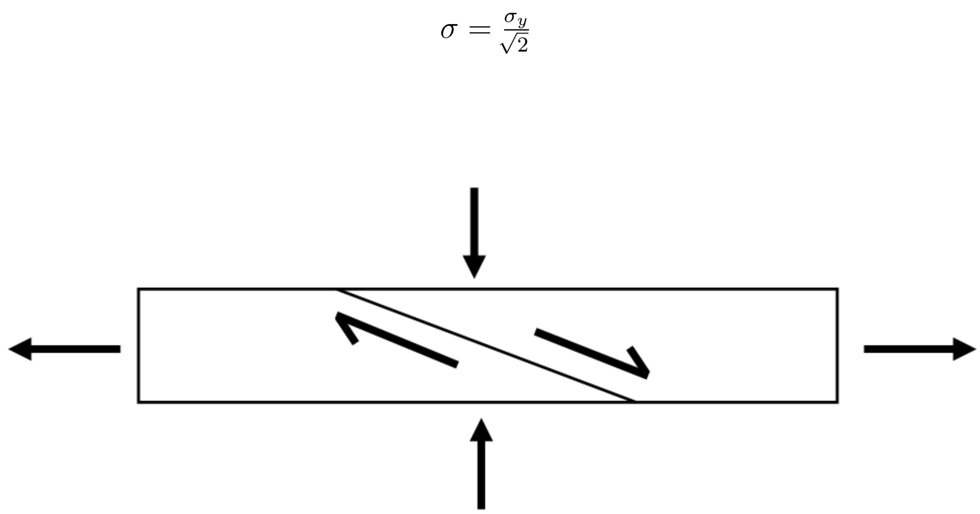

Figure 3.6: The Strain Energy Density criterion for pure shear stress

Maximum shear stress criterion or Tresca criterion indicates that an engineering material will start to yield when the maximum shear stress in the material at a point equals the shear yield stress in uniaxial tension or compression. For a multistress state the maximum shear stress would be half the maximum minus the minimum normal stress and the shear yield stress would be half of the normal yield stress.

$$\tau_{max} = \sigma_{max} - \sigma_{min}/2$$

$$\tau_y = \sigma_y/2$$

The Tresca criterion evidence a shearing-type failure which fits with the failure mode of certain ductile materials. The nature of deformation processes for ductile materials consists in a critical shear stress that initiate the deformation and distorts the material when exceed the cohesive strength.

Distortional density criterion or Von Mises criterion has as a based the maximum strain energy; yet the theory states that the distortional energy causes failure of materials rather than the whole stored energy. The total energy accumulated (U) can be separated into energy of volume change (U_v) and energy of distortion (U_d). It is based to Hubert &

Rubey's that affirms that the total stress state is divided in the hydrostatic stress, involved in the change of volume of materials, and the deviatoric stress, related to the distortion of materials.

$$U = U_v + U_d$$

The energy causing change of volume is given by:

$$U_v = (\tfrac{1-2\nu}{6E})[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$$

The distortional energy for a triaxial state is given by the next equation:

$$U_d = (\tfrac{1+\nu}{3E})[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$$

It adjusts very accurately to the ductile behavior of certain metals to determines the failure with better approximation than Tresca criterion. Determining the yielding initiation under pure-shear stress is more accurate applying Von Mises criterion than Tresca criterion with a difference greater than 15 % for the Von Mises criterion. Normally, the ductile materials can bear a considerable deformation under elastic behavior before initiate the plastic condition; hence, Von Mises shows a fair approximation to this type of material behavior. Tresca criterion provides more conservative results and its application is simple, nevertheless, Von Mises considers characteristics as plastic flow and strain hardening that makes it a more useful option in the present time.

The Maximum Distortion Energy criterion can be manipulated to a different form called the Octahedral Shear Stress criterion.

The Von Mises criterion:

$$\sigma_{VM} = \sqrt{(1/2)[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]}$$

The Octahedral Shear Stress criterion:

$$\tau_{oct} = (1/3)\sqrt{[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]}$$

It results in the following comparison:

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_{VM}$$

Von Mises criterion states that the yielding occurs when the distortional strain energy density of the engineering material at any stress state is equivalent at least to the distortional strain energy density at yield in uniaxial tension or compression. The distortional energy can be expressed in terms of the second deviatoric invariant (J_2) even though the distortional energy is not part of the invariant since it depends on the properties of the material. However, both are directly proportional in value for isotropic engineering materials. If we consider that the second invariant is the distortion of the body without changing the volume we obtain the following comparison (Boresi, et al., 1993).

At yield in uniaxial tension or compression the second invariant is equal to:

$$J_2 = \frac{1}{3}\sigma_y^2$$

The Von Mises criterion for a triaxial state is comparable with the second deviatoric invariant:

$$\sigma_{VM} = \sqrt{(1/2)[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]} = \sqrt{3J_2}$$

As mentioned before the Von Mises criterion shows better results for ductile materials under a state of pure-shear. A pure shear stress state occurs when the first and second principal stress have opposite signs, and the third principal stress is equal to zero ($\sigma_1 = -\sigma_2, \sigma_3 = 0$). In that case, the maximum shear (τ_{max}) for a biaxial stress is equal to the difference of the principal stresses, which is equal to the Von Mises stress of the material. The shear stress at yield is equivalent to the maximum shear stress since both are equivalent to the Von Mises. Combining both concepts is possible to obtain the shear stress at yield according to Von Mises criterion.

$$3\sigma_{VM}^2 = \sigma_y^2$$

where:

$$\tau_{max} = |\sigma_1 - \sigma_2|/2 = \sigma_{VM} \qquad \text{and} \qquad \sigma_{VM} = \tau_y$$

The resulting shear stress at yielding is given by the combination of the two previous equations.

$$\tau_y = \frac{\sigma_y}{\sqrt{3}} = 0{,}577\sigma_y$$

The classical failure criterion can be compared when they are plotted dimensionless in two-spaces as is illustrated in the Figure 3.7. The classical criterion of failure limits are represented by one of the geometrical figure. It is possible to compare the scope of each of them by clustering all in the same coordinate system.

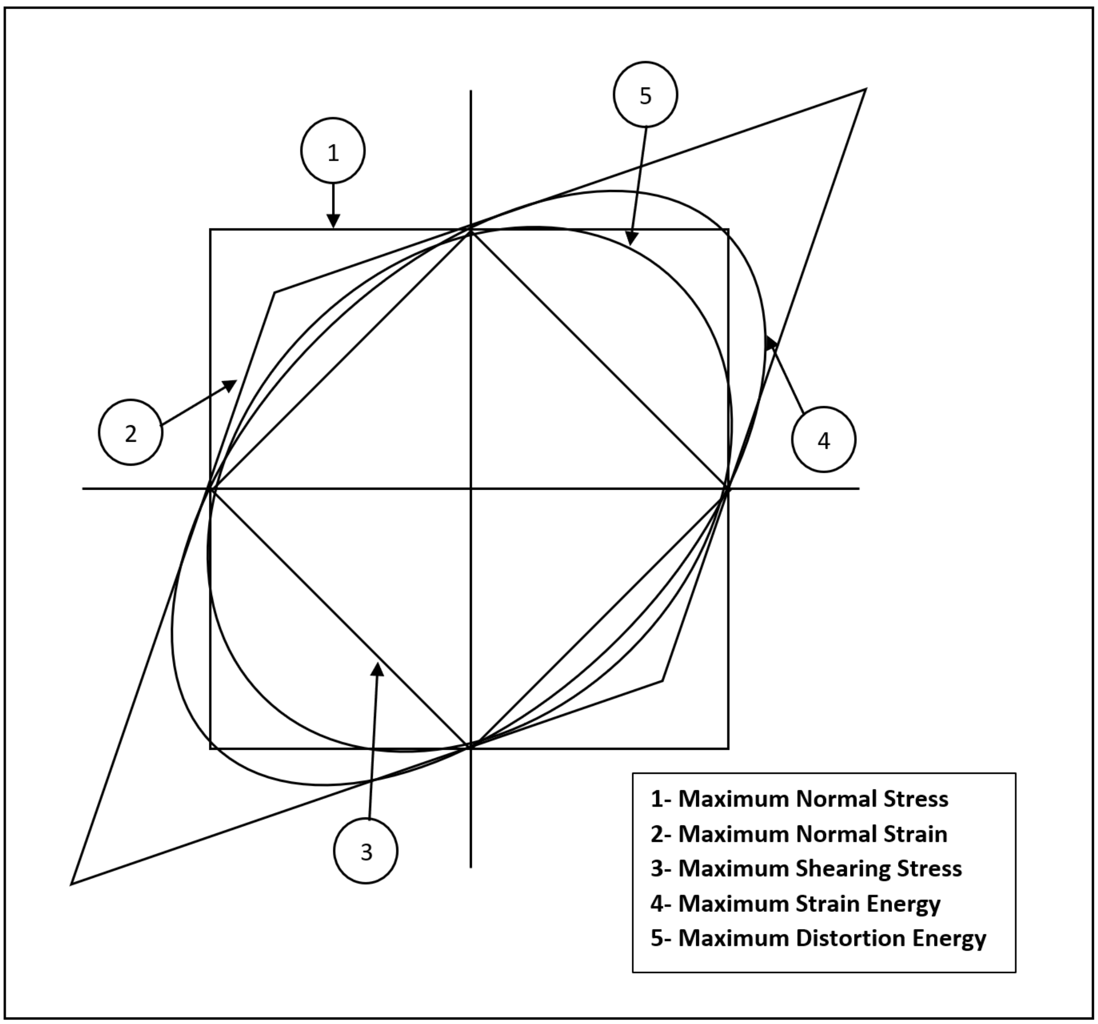

Figure 3.7: Comparison of the classical failure criterion in two-spaces

The normal stress theory is represented by a square, and it is more conveniently applied for certain brittle materials such as iron cast. The maximum normal strain criterion may not be fairly accurate; but it can be applied for failure of confined mass, materials containing cracks or discontinuities It may be represented by an elongated rhombus inclined 45°. The maximum shearing stress is better applied for certain ductile material such as copper or aluminum. It is illustrated as a rhombus. The maximum Strain Energy may be illustrated as a an inclined oval. Finally, the Maximum Distortion Energy criterion is represented as a elongated circle, and it mostly used nowadays for determining yielding initiation for most of ductile materials (Boresi, et al., 1993).

There has been developed another yield criteria besides the classical failure theories to predict yielding of determined engineering materials that exhibit an inelastic behavior. They will be briefly described following.

Mohr-Coulomb failure criterion is the result of the Tresca criterion considering the hydrostatic compressive stress effects as a variable that increases the resistance certain materials to fail (Boresi, et al., 1993). These materials are characterized for having a different compressive and tensile strength. The criterion includes two materials properties, cohesive strength (τ_o) and internal friction (ϕ), to the stress status of the material to determine the shear stress to fail. The Mohr-Coulomb criterion is based as well on the second deviatoric stress and it considers the failure as exclusively a shear failure (Adnoy and Looyeh, 2011).

$$\tau = \tau_o + \sigma \tan\phi$$

$$\tau_o = (\tfrac{\sigma_T}{2})\sqrt{\tfrac{\sigma_C}{\sigma_T}}$$

$$\phi = (\tfrac{\pi}{2}) - 2\tan^{-1}(\sqrt{\tfrac{\sigma_T}{\sigma_C}})$$

The failure points (σ, τ) can be obtained graphically using the Mohr's circle and solved theoretically using the following equations.

$$\tau = 0,5(\sigma_1 - \sigma_3)\cos\phi$$

$$\tau = 0,5(\sigma_1 - \sigma_3)\cos\phi$$

Druker-Prager failure criterion modifies the Von Mises criterion considering the hydrostatic stress as part of the analysis for materials exposed to high levels of stress. Druker and Prager assume that the shear stress reaches a critical value following the next equation where it takes into consideration the first invariant (I_1) and the deviatoric stress (J_2) to evaluate the failure condition and adds β and α that are coefficients related to the cohesive strength and the angle of internal friction (Aadnoy and Looyeh, 2011).

$$\alpha I_1 + \sqrt{J_2} - \beta = 0$$

Griffith failure criterion, as explained by Jaeger and Cook, is limited to materials fail in tension considering the presence of flaws or microcracks to presence of an existing microcrack. There must be sufficient strain energy density release to propagate the cracks through the tensile and compressive stresses. (Adnoy and Looyeh, 2011). The relationship between the triaxial compressive stress and the uniaxial tensile stress according to Griffith criterion is the following.

$$(\sigma_1 - \sigma_3)^2 = -8\sigma_t(\sigma_1 + \sigma_3)$$

Considering the onset of the cracking the tensile failure is determined by the following equation where the uniaxial tensile stress (σ_t) at failure is determined using parameters as k, depends on the application plane, a that is equal to one half of the initial crack, the Young modulus (E) and the crack surface energy (e).

$$\sigma_t = \sqrt{\frac{keE}{a}}$$

3.4.2. Ductile Deformation Mechanics

The ductile deformation mechanism is the process of distortion of a ductile material until it fails. The ductile deformation refers to the ability of certain materials to accumulate strain and permanently deform. It exists more than one mechanism that leads to ductile deformation. One of the mechanism involves the alteration of the crystal structure of the material. When a particular ductile material undergoes a stress or elevated temperature, defects or irregularities occurs within the crystal structure of the material called dislocation. At the same time, the different dislocation produces a local stress such as the screw dislocation that produces shear stress. The motion of these defects provokes the distortion of the ductile body and accumulation of permanent strain. The two-ductile deformation mechanism concerning crystal structure are crystal plasticity and diffusional mass transfer.

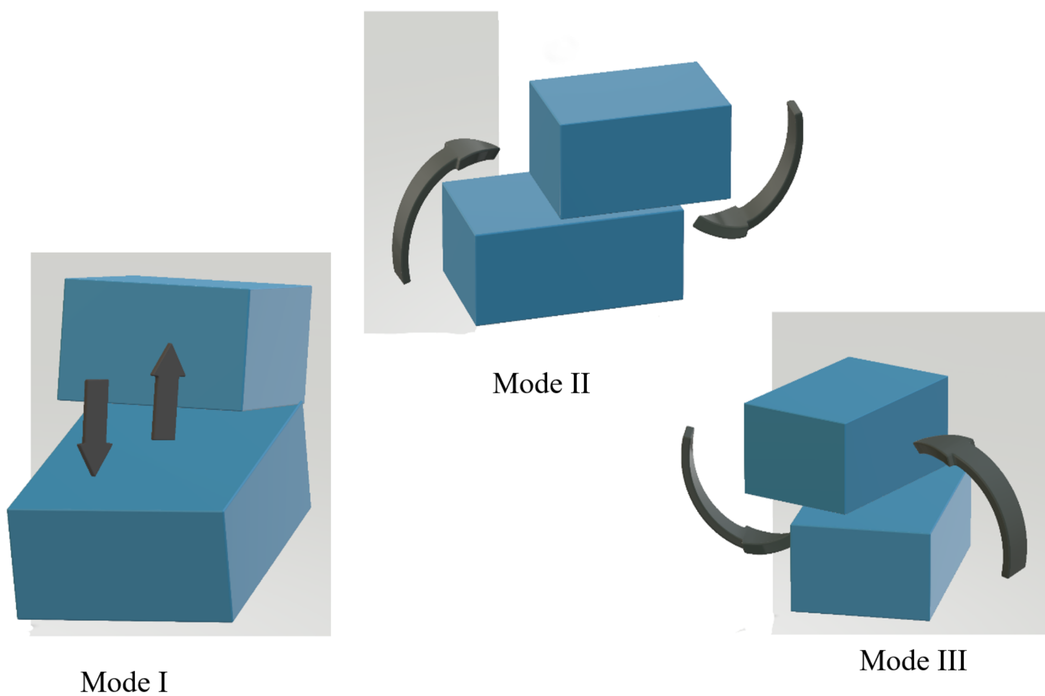

Figure 3.8: Fracture mechanics

However, there is another mechanism that does not involve crystal defects that is Cataclastic flow. It is a mechanism where the change of shape of the material depends on micro-fracturing and frictional sliding of the microscopic components. in a microscopical scale.

3.4.3. Fracture Mechanics/Brittle failure

Brittle deformation unlike plastic deformation takes place after chemical bonds are broken without the following restoration to the original shape. The result of brittle deformation depends on the rheology of the material (ability to deform). It may occur as cracks arises until the material is parted in at least two parts. The cracks can develop with or without displacement. Brittle fracture particularly can be originated by cracks or flaws present in the material as well (Van der Plujim and Marshak, 2006).

The fracture mechanics indicates the manner a fracture will propagate and depends on many factors related to post-yield behavior than define the final shape. According to fracture mechanics, the crack propagation can occur in three different ways. Mode l is the opening mode that consists on a tensile stress normal to the plane of the crack. Mode ll is the sliding mode that consist in a shear stress moving normal to the crack tip. Mode lll is the tearing mode that a shear stress moves parallel to the plane of the crack. The illustration of the fracture mechanics can be observed in the Figure 3.8. Generally, the cracking propagation modes are combined and the three are superposed to give the final fracture shape.

3.5. The pinching-off effect

3.5.1. Introduction to pinching-off experiment

The Bridgman rod compression experiment under high hydrostatic load to what he called a “Paradox” was analyzed. In this experiment (see Figure 3.9) a rod, unsupported at the pipe ends, is inserted into a pressure vessel an adjusted with seals near to the ends of the rod. Bridgman ran several experiments to rods of different material to observe the behavior under high confining hydrostatic pressure. He made various observations with the different rod materials. When ductile materials were utilized, the rod was parted as a typical tensile fracture, necking was observed previous the breakage. For brittle rods, the fractured was observed to be close to the middle of the rod, and the rupture was perpendicular to the rod axis in the same direction the external pressure was exerted. The rod was suddenly parted and ejected from the ends of the pressure vessel (Bridgman, 1964).

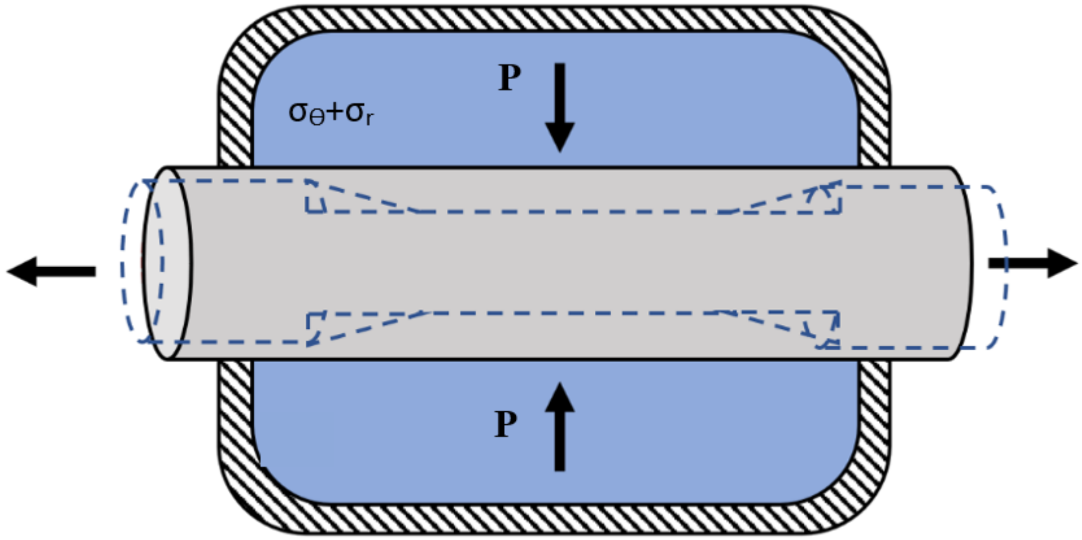

Figure 3.9: Bridgman Experiment

Bridgman found paradoxical the fact that the rod was parted as in a tensile failure when no axial load was applied and applied forces were compressive. Additionally, Bridgman argued in the brittle case that it was not possible to explain the fracture of the rod through a stress fracture criterion because of the absence of stress concentration at the contact point with the seal rings. The rod was supposed to be exposed to friction around the seals and increasing stress in that area; however, the stress concentration was somewhere near the middle of the rod (Bridgman, 1964).

3.5.2. Literature review about The pinching-off experiment/Bridgman Paradox

Kjetill Fossli (2014) previously worked on the topic regarding the 'Pinching-off effect' experiment as he continued with the work developed by Morten Reve on his thesis the year previous. The pinching-off effect was one of the many experiments performed by the physicist Percy Bridgman regarding the behavior of materials under high pressure and failure criteria of materials. He performed this particular study to indicate the lack of agreement in the failure criteria outlining that the current criteria are merely an approximation of the yield point of materials. However, there are several factors not considered whenever a rupture analysis is performed. He suggested that there should be a criteria applicable independent of the stress applied.

In addition, Bridgman insisted that the conditions for a material to fracture are not always met. The three conditions for the rupture that he discussed are the maximum principal stress, maximum shear stress, and the maximum strain. The maximum principal stress refers to the maximum stress a material experiences has to exceed a particular limit value. The maximum shear stress occurs when the difference between the highest and lowest stress in a material surpass a limit value. The third one refers to the maximum elongation a material can have before fracturing.

The pinching-off effect consists of a rod that passes completely through a pressure vessel sealed at both ends using rings. Hydrostatic pressure is applied using a high-pressure pump all around the rod inside the vessel; hence, the rod is radially compressed. The experiment resulted in the rod parted somewhere by the middle of the rod as in a tensile fracture even though there is none longitudinal stress recorded in the experiment. He performed the experiment multiple times using different materials resulting fractures of different shapes. Despite Bridgman discriminates the materials between ductile and brittle, he most of the times refers to the diverse materials according to its toughness. Moreover, Bridgman remarks that the different toughness of the material (he explicitly refers to softer or harder materials) change the final shape of the fracture.

The peculiar about the experiment despite the fact there is no axial stress, there is a resultant tensile stress when the rod presents a necking down. However, he did not consider it as the cause of the fracture. Bridgman concluded after the experiment that the maximum principal stresses and the maximum shear stress did not apply to determined the rupture. According to him, the principal stresses should be a compressive stress equal to the hydrostatic pressure in all directions and the minimum principal stress a small compressive stress due to friction. Therefore, the rupture pressure should be a hydrostatic pressure equal to the tensile strength. However, it is no the case in the experiment since the results exceed the tensile limit in a range between 25 to 50 percent. The closest approximation is for brittle materials, thus, he coincides that there are factors ignored in the current failure criteria. The multiple questions derived from the experiment resulted in future research on the pinching-off effect. The experiment is also called "Bridgman's Paradox". The oldest document that I notice this name was given to the experiment was in 1958 by James M. Clearly in his book "Hydraulic Fracture Theory".

The Bridgman Paradox has provoked different theories on how the rod is fractured under the previous conditions when the rod has open ends, and it is validated that there is no axial load on it. Fossli and Reve's work was conditioned by the idea that the breakage can be described using effective tension. According to Reve, there is an analogy of the effective tension and the rod subjected to high pressure in the vessel. This assumption is based on the fact that the results of rupture pressure are similar to the tensile strength of the rod including a friction originated in the seal rings.

Moreover, an explanation of the Bridgman paradox is given by the professor Arnfinn Nergaard that considers the rupture as a result of tension. He understands the fracture of the rod in three steps. Step 1 consists of a bar inside a pressurized chamber where the ends of the bar protruding from the chamber. The pressurized chamber has seals in each side where the rod protrudes, and the resulting tension is either zero or the differential pressure inside the chamber times the area ($T=\Delta pA$). In step 2, the pressure is reversed. There is no pressure inside the pressure chamber and there is pressure in the same range outside. The solution is either a compressive force ($C=\Delta pA$) acting in the ends of the bar corresponding to the piston effect or zero. The final step 3, the pressure outside the chamber as in step 2 is switched to a negative pressure (suction) producing a tensile force. Professor Nergaard suggests that the tension is in the same range as the compressive force in step 2. Either zero or the pressure-area force ($T=\Delta pA$). It supports the idea that the fracture of the rod is the result of the effective tension producing a tensile fracture. It is suggested by Fossli that the effective tension concept can work for horizontal pipes; hence, the Bridgman experiment was conducted to validate the theory.

In Fossli's document, it is included in a series of theories that could explain the fracture of the rod. The exposed solutions were the following. First, the **material/rod goes plastic under high pressure**. When Bridgman did the experiment, Kahlbaum Roth claimed that in the final result the rod might undergo a plastic behavior under high pressure; yet the experiments showed the contrary, the material was more rigid. The material experiences a hardening effect due to overstrain. When high pressures are applied, the material is deformed generating new dislocations. These dislocations reduce the spaces in the crystal structure and making the material more rigid. Second, **the piston force through cracks** where water seeps the rod through small cracks in the surface all around. The small cracks produce a stress concentration on the weakest points and create an internal pressure in the rod which will break it as in a tensile fracture.

There are two theories proposed regarding the **Poisson effect** as the source of the fracture. One suggests that the packs at each side of the pressure vessel create a high static friction. The seal packs put the rod in a rigid state and produce a friction in the rod at each seal. The friction acts contrary to the elongation of the rod that increases the breakage pressure, and the deformation and elongation produce an internal tensile force that causes the rod to squeeze along the axis. In the second theory regarding the **Poisson effect (II)**, the rod shrinks in diameter due to the radial compression caused by the hydraulic pressure. The Poisson effect on the rod causes the rod to stretch axially. Apparently, it creates a fictitious tension force in the axial direction. As it continues increasing, the strain and axial force become excessively high that the rod is parted at Von Mises stress. One last explanation considers the **effective tension** as the source of breakage. The rod is parted when the pressure is strong enough to cause the rupture due to the high tensile stress. The pressure exerted on the curved surface of the rod creates a tensile force equal to the pressure times the cross-sectional area (T=ΔpA).

Among Fossli's remarks after performing the same experiment as Bridgman did, he identified that the rod presented crazes before it is parted. The rod was fractured normal to the direction of the applied pressure at 75 % of the tensile limit of the material. Additionally, another experiment was conducted where the rod was loaded axially. The strains measured in the first experiment were equivalent to the values of the latter, yet it just demonstrated the critical strain is consistent with the material even if the fracture is a different type. The tension experiment shows different loading results as the breakage pressure in the Bridgman experiment by far. This difference is notorious by a range of 15 to 20 MPa that cannot be attributed to the friction due to the large difference. Nevertheless, it can tell the rod is not parted due to a tensile fracture even though the fracture surface looks similar in both experiments.

In addition, a bending experiment was conducted to check the similarities. The fracture surface was completely different from both previous experiments. The compressive fracture was completely different from the observed in the Bridgman experiment and no similarities were found. Moreover, it was noted that tension and compression zones appear in each of the parted sides. The three experiments resulted in fractures perpendicular to the longitudinal axis, yet the shape of the fracture was different. After the experiments were conducted, the reason the rods were fractured or the precise mechanism was inconclusive. It clearly showed that there are similarities to the tensile fracture, yet the range of forces applied for rupture has no relation. The thesis concludes that there is various fracture mechanism involved in the breakage, and further work is required.

3.5.3. Analysis of the Experiment

The experiment consists of a solid rod compressed radially through hydrostatic pressure in a closed chamber. The radial and tangential stress applied provokes the stretching of the rod.

The rod is deformed circumferentially due to a compressive force following by an axial elongation. This effect is described through the Poisson effect where material tends to expand in a direction perpendicular to the direction of compression. Hence, the rod will expand axially as the radial compression continues. Indeed, the force can be analyzed in one dimension, yet it does not mean the problem should be considered in one direction due to the fact that deformation always occurs in three dimensions. Furthermore, there is no axial force imposed to the rod with open ends so that there is no axial force reported in the experiment. Then, the tensile fracture appears to be paradoxical and not possible.

The value of the radial and tangential stress is equal to -P, which means the rod is under compression, and the third cylindrical stress, the axial stress, a value of zero. The high confining pressure and the absence of axial load provoke an excessive deformation in the case of a material with large Poisson ratio due to a high deviatoric stress. The resultant deviatoric stress causes the shear failure through splitting according to the distortion theory for ductile materials. The crack propagation was at 45° when maximum stress in the rod was reached that coincides with Bridgman's observation.

In the case of a brittle material, according to Bridgman's experiment report the fracture is created at a right angle with almost non-deformation. Nonetheless, it has been observed that the shape of the fractures has a zig-zag appearance that initiates at a 45-degrees-angle that is hampered by the tangential stress that provokes that final shape as professor Aadnoy explains about the experiments performed in the University of Stavanger. In the macroscale, the cracks appear that are a tensile crack and agrees with the observation made by Bridgman. Nevertheless, the fracture initiates at 45° as a shear failure in the microscale, but it is bound straight toward the rod axis direction. This phenomenon occurs as well in the wellbore fractures where the circumferential stress limits the shape of the fracture. Moreover, the pictures of the experiment conducted by Kjetil Lund in 2014 on a PMMA rod shows a clear sliding off of one side of the parted rod to the other as an in-plane shear mechanism.

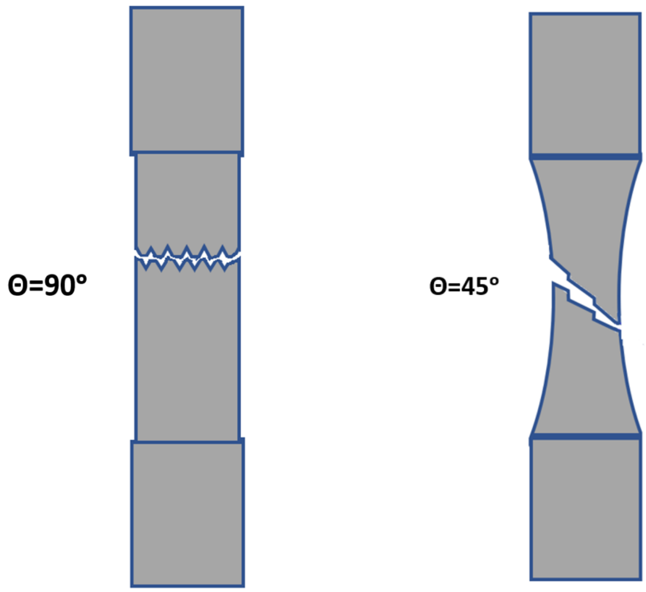

Figure 3.10: Brittle and Ductile failure of the rod

Failure of a material cannot be easily determined by one mode of failure since most of the time is a compound of various failure modes, yet there is one that stands among the others. Effectively, it can happen that for ductile materials with a large capacity to deform will undergo yielding before fracturing. The final shape of a failure depends on myriad factors; the appearance of the failure can rely on factors as the presence of flaws, stresses applied, strength to different stresses, brittleness, and toughness of the material that can modify the final shape of the fracture. Typically, the failure criteria and fracture angles are analyzed according to the response of the material to different load conditions.

A particular failure criterion is used corresponding to the type of material and failure problem. Then, the materials can be divided into yield-dominant failure and fracture-dominant failure. It can be interpreted as a yield-dominant failure for ductile materials state and fracture-dominant for brittle materials state (Boresi, Schmidt, & Sidebottom, 1993). Nevertheless, there is not a clear transition between brittle and ductile materials. Even though materials are classified as both, the classical failure criteria are limited and cannot correspond to all types of failure problems and fracture shapes. Based on the energy release to fracture a material, the failure shape and mode of failure can depend on Poisson's ratio and stress triaxiality as well.

Commonly, the maximum principal stress is the failure criterion is used for brittle materials and the fractures angles are perpendicular to an applied force or 0°, being either the hoop stress or the axial stress the principal stresses of failure. For brittle materials with fractures angles from 0° to 45° the Mohr-Coulomb criterion is suggested for the analysis. For ductile materials, the more adequate failure criterion differs depending on the plastic behavior of certain engineering materials that can adjust to either Tresca or Von Mises criterion. For Tresca criterion, the fracture angle for the maximum shear stress is 45°. The Von Misses criterion, which adjusts for most of the ductile materials, the fracture angles are ranging from 30° and above. All the cases considering uniaxial tensile stress (Yuan et al., 2014).

However, the conventional failure criterion does not apply properly when the case is that the maximum compressive stress is far larger than the maximum tensile stress, which can happen in brittle elements. This uncertainty is due to the variation in resistance to the diverse types of loads the materials can have. The difference in strength of materials in brittle materials is called even and uneven material. Furthermore, the shear strength capacity for brittle materials differentiates from the ductile materials because normally the latter element has a maximum shear strength that is half of the tensile strength. For brittle materials, the shear strength can be greater.

Additionally, there has been studied about the influence of the Poisson's effect and the triaxial stresses to predict the fracture shape. Limited research has shown some useful guidelines that demonstrate a relationship between the fracture shape and changes in Poisson's ratio through the modification of the composition of the material. The strain energy density theory demonstrates the influence of Poisson's ratio in the change of volume and shape of isotropic engineering materials under a multi-stress state for a failure material application (Yuan et al., 2014). Moreover, it is fair to mention that when dealing with a case regarding Poisson's ratio and maximum deformation; it is crucial to consider the critical strain of the material. The critical strain is the point when the cracks initiate to appear in the material, and it is a property of the material itself. The material failure is generally described as the point when the release of the strain energy density, related to the increment of stresses, is sufficient to break the atomic bonds of the material.

The principal stresses are expressed in terms of cylindrical stresses since we are analyzing tubular. The pressure P is the only force applied to the tubular inside the chamber, thus, the hoop stress and the radial stress are obtained based on that solely force applied. According to Lame's equations for thick-walled cylinders, there must be an internal and external radius; but it is possible to adapt it for a rod such as this case. The external radius (b=r) and internal radius (a=0) because the rod does not have an internal radius and the external will be equal to the radius of the rod. The external pressure ($P_2 = P$) and the internal pressure ($P_1 = P$) because it is the only force to consider. The internal pressure equal to P can be debatable since it is possible to argue that there will not be internal pressure because it does not have an internal radius. Nonetheless, the rod is exposed to a pressure internally, and for this particular case is assumed to be equal to the hydrostatic pressure P.

$$\sigma_r = \frac{0-Pr^2}{r^2-0} - \frac{0}{r^2(r^2-0)}(P-P) = -P$$

$$\sigma_\theta = \frac{0-Pr^2}{r^2-0} + \frac{0}{r^2(r^2-0)}(P-P) = -P$$

Average constant stress along the the diameter of the rod:

$$\frac{\sigma_\theta+\sigma_r}{2} = -P$$

As stated before, there is no external axial stress applied to the rod; however, it experiences an internal axial stress as a reaction to the deformation caused totally by the Poisson's effect due to the increasing external pressure. The internal axial load produced should be enough or almost in the same range to produce the yielding of the rod that coincides with the maximum shear stress according to Von Mises theory for ductile materials. Von Mises theory states that the distortional strain energy in uniaxial tension should give yielding as the distortional strain energy density in a multi-stress state. The shear failure results in a maximum shear stress equal to 0.577 times yield stress. It corresponds to the analysis if it is done through the axial internal axial stress caused by the action of the rod deformation knowing that the result of the Von Misses stress, in this case, is equal to P (pressure applied).

The axial strain can by expressed through the cylindrical stresses in the stress-strain (Boresi, et al., 1993) relation given by the following equation:

$$\varepsilon_z = \tfrac{1}{E}[\sigma_{r(external)} - \nu(\sigma_r + \sigma_\theta)]$$

$$\varepsilon_z E = -\nu(\sigma_r + \sigma_\theta)$$

Internal axial stress resulting from the rod deformation with Poisson ratios in the range of 0.25 to 0.30 for a ductile material.

$$\sigma_{z(internal)} = -2\nu P$$

$$\sigma_{z(internal)} = -0{,}6P \qquad \sigma_{z(internal)} = -0{,}5P$$

The Von Mises stress gives a result equal to P and it mentions that the maximum shear stress obtained is the following.

$$\sigma_{VME} = \sqrt{(1/2)[(\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2]} = \mathrm{P}$$

$$\tau_y = 0{,}577\sigma_y = 0{,}577P$$

The results showed in the master thesis of Lund about Bridgman's experiment gives a breakage pressure for the polyoxymethylene (POM) of 95 and 100 MPa for the black and white POM rod respectively (Fossli, 2014). According to the ASTM D638, the Standard Classification for Polyoxymethylene, the POM should have a tensile strength at 73°F (23 °C) of 9.500 PSI (65 MPa), then D732 provides a shear strength at 73 °F of 8000 PSI (55 MPa), and a compressive strength of 9760 PSI (67.3 MPa) according to ASTM695 and ISO 604. Applying the data obtained by Lund, the Pressure applied at rupture 95 to 100 MPa gives a shear stress ranging from 54.8 to 57.7 MPa and an internal axial stress between 50 to 60 MPa. The pressure applied at rupture results in a shear stress enough to fracture the rod according to the theory. Moreover, it confirms that the fracture is a shear failure and not a tensile failure.

There are some other interesting results when applying the other different classical criteria of failure. According to the results, it is possible to observe that there is a criterion that adjust better to the tendency of failure for a particular material. In the previous case for a ductile material, the Von Mises criterion gives a correct approximation to the real data.

Maximum Normal criterion has a maximum principal stress the radial and hoop stress ($\sigma_\theta = \sigma_r$)in the same range according to the assumption stated previously.

$$\sigma_y = \sigma_{max} = P$$

Considering a shear failure according to the criterion the shear is equal to the maximum principal stress, and therefore, the shear is equal to P that is between the range of 95 to 100 MPa according to the data for this case.

$$\sigma = \sigma_{max} = \tau_y = P$$

Maximum Principal Strain criterion considers the Poison's ratio, and the same value taken in the first example will be applied (ν =0.25 or 0.30).

$$\epsilon_1 = \epsilon_y = \tfrac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\sigma_y = P - 0{,}25P = 0{,}75P \qquad \sigma_y = P - 0{,}3P = 0{,}7P$$

Maximum shear stress criterion gives as a result the half of the maximum normal stress that is equal to half of the hydrostatic pressure.

$$\tau_{max} = \tfrac{P}{2} = 0{,}5P$$

The maximum strain energy criterion for a biaxial stress case shows the following results (ν =0.25 or 0.30).

$$\sigma_y^2 = P^2 + P^2 - 2\nu(P^2)$$

$$\sigma_y^2 = 2P^2 - 0{,}5P^2 = 1{,}5P^2 \qquad \sigma_y^2 = 2P^2 - 0{,}6P^2 = 1{,}4P^2$$

$$\sigma_y = 1{,}22P \qquad \sigma_y = 1{,}18P$$

For Pure shear stress the results is:

$$\sigma = \frac{\sigma_y}{\sqrt{2}}$$

$$\sigma = 0{,}86P \qquad \sigma = 0{,}83P$$

Mohr-Coulomb failure criteria for the ductile material case has the following results:

$$\tau_o = (\frac{\sigma_T}{2})\sqrt{\frac{\sigma_C}{\sigma_T}} = 33MPa \qquad \phi = (\frac{\pi}{2}) - 2\tan^{-1}(\sqrt{\frac{\sigma_T}{\sigma_C}}) = 0{,}0151rad$$

$$\tau = \tau_o + \sigma\tan\phi = 34{,}5MPa$$

It bears to mention that for the results can be modified according to the data given for the specific material. Therefore, this data is specifically obtained for the guidelines of ISO and ASTM of the materials. However, it does not necessarily match the data from the material use in the original experiment.

As mentioned before, the analysis for brittle materials is performed through other criteria of failure. The ability to yield for some brittle materials can be increased throughout the application of hydrostatic compressive forces. For brittle materials that has a different compressive and tensile strength (uneven materials), the Mohr-Coulomb yield criterion can relate better to the failure of the material. In case of brittle materials with equal compressive and tensile strength (even material), the maximum principal stress criterion can obtain more valid results (Boresi, et al., 1993).

The Poisson ratio for the PMMA (Poly-methyl methacrylate or acrylic glass), a brittle material, is in the range of 0.35 to 0.40. Using those values of Poisson ratio, it will have an internal axial stress around 0.7P to 0.8P.

$$\sigma_{z(internal)} = -0{,}7P \qquad \sigma_{z(internal)} = -0{,}8$$

The PMMA results to be an uneven material; therefore, applying the Mohr Coulomb to know the shear stress the uneven material will break. The maximum principal stress criterion would show a yield stress equal to the applied pressure (P).

In the same methodology as the ductile case, the breakage of the rod can be explained mechanically through the Poisson's effect that creates the internal axial stress and produces a shear stress. Logically, it is noticeable that one effect is linked to another that leads to failure. External force, external stress, Poisson's effect, internal stress, yield limit, strain limit, the ensemble of all the elements as one reaction of another causes failure. It is assumed that yielding is occurred as pure shear stress when the compressive stress produces the reactive internal axial stress.

The mechanical properties for PMMA or Acrylic according to the ASTM D638 and D732 are tensile strength 10900 PSI (75 MPa), compressive strength 18000 PSI (124 MPa) and shear strength 9000 PSI (62 MPa) both measured at 73 °F or 23 °C. Lund (2014) obtained a rupture pressure for the PMMA of 75 MPa.

$$\tau_o = (\tfrac{\sigma_T}{2})\sqrt{\tfrac{\sigma_C}{\sigma_T}} = 48{,}218MPa \qquad \phi = (\tfrac{\pi}{2}) - 2\tan^{-1}(\sqrt{\tfrac{\sigma_T}{\sigma_C}}) = 0{,}2488rad$$

$$\tau = \tau_o + \sigma \tan\phi = 67MPa$$

$$\sigma_{z(internal)} = 57{,}5MPa \qquad \sigma_{z(internal)} = 60MPa$$

The axial stress resulted from the deformation was in the range of 57.5 to 60 MPa below the tensile strength, the shear stress resulted from the external pressure was 67 MPa enough to overcome the shear yield stress of 62 MPa. It proves the PMMA rod fails due to shear and the resulted axial stress is below the limit; thus, it cannot be a tensile fracture.

The results of the PMMA, as the brittle material, with the other failure criterion will be shown next as was previously exposed for the ductile material.

Maximum Normal stress criterion:

$$\sigma_y = P$$

$$\sigma = \sigma_{max} = \tau_y = P$$

The shear stress resulting form the maximum normal stress criterion has a value around 62 MPa, which is the maximum pressure applied.

Maximum Principal Strain criterion (ν =0.35 or 0.40).

$$\epsilon_y = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$
$$\sigma_y = P - 0{,}35P = 0{,}65P \qquad \sigma_y = P - 0{,}4P = 0{,}6P$$

Maximum shear stress criterion results in:

$$\tau_{max} = \frac{P}{2} = 0{,}5P$$

The maximum strain energy criterion for a biaxial stress case (ν =0.35 or 0.40).

$$\sigma_y^2 = 2P^2 - 0{,}7P^2 = 1{,}3P^2 \qquad \sigma_y^2 = 2P^2 - 0{,}8P^2 = 1{,}2P^2$$
$$\sigma_y = 1{,}14P \qquad \sigma_y = 1{,}09P$$

For Pure shear stress the results is:

$$\sigma = \frac{\sigma_y}{\sqrt{2}}$$
$$\sigma = 0{,}81P \qquad \sigma = 0{,}77P$$

The Von Mises stress criterion or Maximum Distortion energy criterion will result as in the ductile equal to the pressure applied in the pressure chamber until the rupture occurs equal to P.

$$\sigma_{VME} = \sqrt{(1/2)[(\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2]}$$

$$\tau_y = 0{,}577\sigma_y \qquad \sigma_y = 0{,}577P$$

Failure Criterion			
POM-C		**PMMA**	
Criterion	**Result**	**Criterion**	**Result**
Maximum Normal Stress	σ_y=95-100MPa τ_y=95-100MPa	Maximum Normal Stress	σ_y=75MPa τ_y=75MPa
Maximum Normal Strain	σ_y=70-75MPa	Maximum Normal Strain	σ_y=45-48MPa
Maximum Shear Stress	τ_{max}=47-50MPa	Maximum Shear Stress	τ_{max}=37MPa
Maximum Strain Energy	σ_y=83-86MPa	Maximum Strain Energy	σ_y=57-61MPa
Von Mises Stress	σ_y=95-100MPa τ_y=54-57MPa	Von Mises Stress	σ_y=75MPa τ_y=43MPa
Mohr-Coulomb failure criterion	$\tau = 34{,}5MPa$	Mohr-Coulomb failure criterion	$\tau = 67MPa$

Table 3.1: Results of Failure Criteria for both types of Material

All the different failure criteria for each material of analysis are presented in the table 3.1 to be able to compare the results and remark that the actual criteria of failure are merely an approximation to yielding more than an exact description of the failure. The results are adjusted more conveniently depending on the material and its properties. Somehow, it has not been possible to find the exact physical variables to describe the failure of a material and express it into an equation. It is important to mention that the yield point on a particular material is not always close to the fracture point, it can vary depending on the plastic behavior of the material. In addition, the presence of imperfections or the heterogeneity of the surface of the material can affect the determination of fracture point, mode of failure, and the shape of fracture.

The experiment shows there is not axial stress at all due to the no-end condition of the tubular. However, the experiment conducted shows the formation of crazing proper of tensile fractures and usually located perpendicular to the maximum principal stress. The crazes appeared even though there was not sign of irregularities or fissures before. The use of seal rings in each side of the pressure chamber where the rod protrudes allows to maintain the pressure inside the chamber and fix the rod adding rigidity and avoiding twisting of the rod. The applied pressure originates two forces in the opposite direction that lead one side of the bar into compression and the other into tension. The two opposite forces create a concentration stress that leads to a high shear stress.

The analysis of the experiment reflects that the mechanism of fracture is in-plane shearing from the compressive side and opening from the tensile side. Additionally, it produces a static friction contrary to the stretching of the rod causing an additional stress in the other direction or increasing the pressure before fracturing. The external force applied will cause a deviatoric stress that will contribute to the increasing shear stress until the rod is parted in two. The shape of the fracture will depend on the toughness and brittleness of the cylinder. The composition of the material/rod defines the toughness, brittleness and the Poisson ratio as well. Additionally, the final shape of the fracture can be influenced for the post-yield behavior.

Considering the circle yield envelope as it reflects the Von Mises stress criterion (Figure 3.11) is possible to confirm the error aforementioned by Bridgman in the range of 25 % to 50 % with respect to the values of tensile yield.

$$x = \frac{F_E}{\sigma_y A_s}$$

$$y = \frac{\sqrt{3} A_o (P_i - P_o)}{A_s \sigma_y}$$

The radius of the circle will be equal to the Von Mises stress (σ_{VM}). The axis x and y will be defined by the following equation (Aasen, Ostvold, & Aadnoy, 2017). For this particular experiment, it is considered the external and internal pressure equal to the hydrostatic pressure applied inside the chamber ($P = P_o = P_i$). The effective force is equal to the external pressure applied because there is not an axial force in the rod for this case ($F_E = P$).

$$x = \frac{P}{\sigma_y} = \frac{65MPa}{95MPa} = 0{,}68$$
$$y = \frac{\sqrt{3}(0)}{\sigma_y} = 0$$

The results, based on the ductile material POM-C, reflect a difference with the tensile strength in the order of 32 % in the range Bridgman stated. This error includes the pack-off friction due to the seals added to the chamber but without reason it should reach the magnitude of 30 % of the rupture pressure. Nonetheless, The shear stress also represents the limit of yielding in the yield circle envelope for Von Mises criterion at $\tau_y = \sigma_y / \sqrt{3}$ (Boresi, et al., 1993). If the shear stress is considered in the failure analysis, the shear stress magnitude at rupture is around 54MPa to 58MPa. The ratio of the values of shear strength and the shear yielding results in an approximate 0.98, and the error becomes smaller in the range of the 2 % to 3 %. The difference can be accounted as the friction that produce a larger pressure to produce the rupture.

$$\tau = \frac{\tau_V M}{\tau_y} = \frac{54MPa}{55MPa} = 0{,}98$$

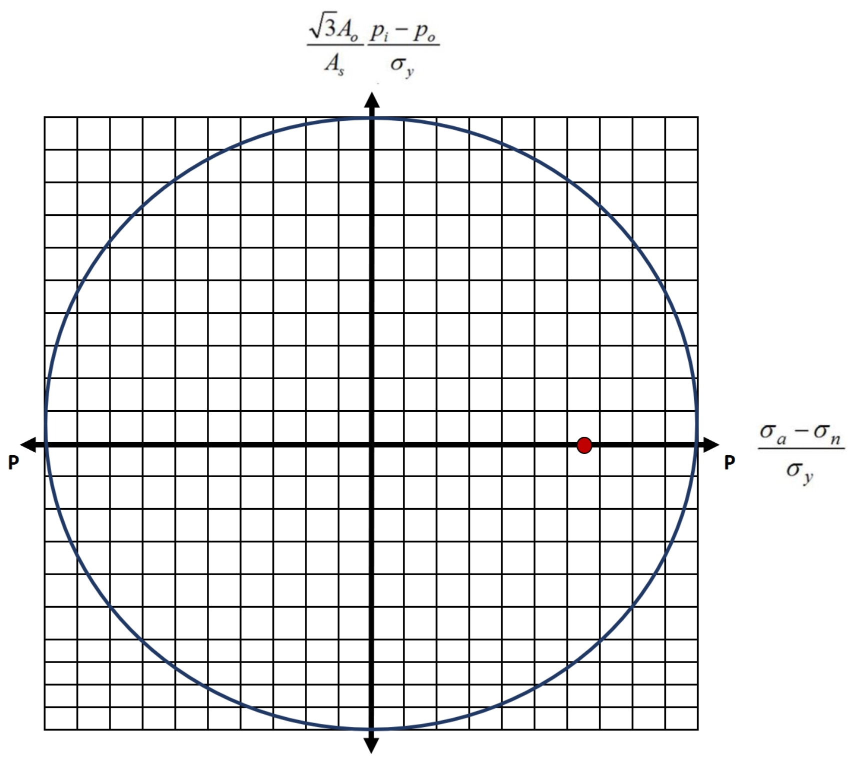

Figure 3.11: Circle of yield for Pinching-off effect - POM-C

4 Conclusions and Recommendations

4.1. Conclusions

The thesis has validated and proved not only by the physical analysis but also by different sources as a force developed by a difference of hydrostatic pressure. The hydrostatic pressure depends on the depth or height of the column of fluids. Then, it can only consider the fluid at the ends of the body of analysis. The buoyant force is an upward force that acts on the opposite side than gravity. It is necessary the presence of fluids to cause the upthrust effect.

The thesis does not reject the approach made by Archimedes hundreds of years before that is completely legitimate and calculates accurately the buoyant effect. On the contrary, it validates the Archimedes' law; however, it does not coincide with the concept of the volumetric approach that considers buoyancy as a volumetric force that acts all around the body and insists in a lateral thrust to consider buoyancy in all circumstances. Based on the classic mechanics and the analysis of forces according to Newton, it is demonstrated the buoyancy as an upward force resulting from pressure difference at the top and the bottom of the object. It coincides with the approach of the Piston-force conception that is valid for every single case.

The interpretation of real forces and effective forces can be a cause of misunderstanding due to a confusion regarding cases such as the listed in the thesis. When it is referred to the real axial forces, then it considers every physical aspect and forces surrounding the affected material including lateral forces that provokes deformation (elongation or contraction). Therefore, the real axial force allows being acquainted of the total stress state of the material that is distributed along the height of the material/body and the magnitude is dependent on the point to be analyzed. Having said that, the inclusion of additional forces that are not present there lead to an obvious difference of results as is the case of the riser attached to the seafloor affecting the result of effective forces that considers the real forces to determine for instance risk of buckling. On the other hand, it bears to clarify that the effective forces can be considered as a reflection of the deviatoric stress; hence, it does consider the forces that can lead to failure. However, it occurs when the forces involved produced a mechanical force due to a restriction of movement in the material.

From that point, it can be remarked that the Pinching-off effect experiment accounting for the hydrostatic force to produce the breakage of the rod can be related to an application of effective forces. There is a general agreement that there is not real axial stress in the rod; nevertheless, the deviatoric stress produced to distort the rod when it breaks relates it to the effective forces.

The study of the experiment concludes that there is no friction at the seal, it is estimated that Von Mises theory states failure equal to the pressure of fracture. The fracture is identified to be originated due to Poisson effect and the inclusion of the pack seals on the sides of the pressure vessel.

The cylinder of the experiment is pressurized radially in two axis direction that cause at the same time a deformation. However, deformation is carried out in three-dimensions; thus, the rod will be compressed in diameter and elongated axially. The rods present a series of crazes axially around the fracture zone. The crazes are originated due to axial loading that it is proved there is none in the experiment. Nevertheless, the strain caused by the external force will cause deformation of the material, and the subsequent internal stress due to Poisson effect. The internal stress is a reactive axial stress that together with the high strain will cause the distortion and shear failure of the tubular. In addition, the internal axial stress can correspond to the appearance of crazes.

Shear is the cause of fracture. The pressure vessel contains two ring seals that keep the rod rigid and avoid it from moving; but causes an additional stress by static friction opposed to the ejection force of the rod through the protrudes of the pressure chamber. The bar is parted somewhere in the middle of the rod where the stress is the maximum with a shear stress equal to 0.577 the pressure in the case of a ductile material with a Poisson ratio ranging 0.25 to 0.30. The results vary depending on the type of the material; but according to the different results of the failure criterion it is more sensible to the Poisson ratio. The mechanism of fracture is complex, as it usually is for compressive shear failure; it is a combination of in-plane shear, normally for compressive forces; and opening, more associated to tensile forces.

The shape of the fracture varies depending on the material composition. The toughness and brittleness of the cylinder defines the initial shape of the fracture as well as the fracture mechanism. The ductile material as steel shows a 45-degrees fracture as a sign of shearing including the necking of the rod. The brittle material is more variable but essentially a right-angle breakage in a macro-perspective. However, it has been noticed by professor Aadnoy a microscopical angle of 45-degrees in the brittle fracture at the initiation of the fracture. The confining pressure by the hoop stress affects the normal crack propagation and form of the fracture. The post-yield shape of the fracture has more variables to consider. The composition of the rod defines the toughness, brittleness and the Poisson ratio as well. These factors can be associated not only to the shape of the fracture but also to the mode of failure.

4.2. Recommendations

- Further work is recommended in the experimental area considering different types of materials from brittle to ductile and observe how the shape varies. The size of the rods should be considered in order to obtain larger values and clearer results to perform the inspection of the fracture.

- Conduct the Bridgman experiment to several types of materials varying the Poisson ratio and analyze the resulting shape of the fracture and the variation of the rupture pressure. Include the variation of the composition of a number of rods in such a way that the Poisson ratio varies. Verify the influence of the Poisson ratio in the fracture shape, the mode of failure and the fracture mechanism.

- Conduct further studies about the appearance of crazes and the relation with the internal axial stress created by the deformation. Using the Scanning Electron Microscope (SEM) determine the precise moment of appearance of crazes and the following developed of the crazes as the pressure increases.
- Review the effect of the seal rings of the pressure chamber and the stress concentration. Conduct the experiment in a manner to reduce the effect/friction of the seals keeping the rod fix as in the previous experiments. Consider the scenario of twisting of the rod when apply the compressive pressure and review the results of the fracture.
- Analyze several shear fractures of different materials and compare with the obtained in the previous experiments to verify any similarities. Focus on the shape of the crack propagation and the effect of confining pressure in the crack propagation. Verify the shape in the macro and micro perspective
- Conduct a study of the post-yield behavior of the fracture in the Bridgman experiment to determine the reasons for the final shape of the fracture. Determine the different variables affecting in the post-yield behavior.

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