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Relationship between problem solving and motivation in mathematics: A qualitative study on how problem solving influences pupils' motivation in mathematics.

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FOREWORD

A two-year master's degree in mathematics didactics is now over. This has been an exciting and educational journey for me, where I have been inspired by various topics in mathematics, including problem solving. I chose to do research within this topic, and this master's thesis is a result of my research which I hope can inspire others to further work on problem solving in school. After nine years of experience as a teacher in lower secondary school, I have now realized that I have worked far too little with problem solving tasks with my pupils, and I am motivated to have more focus on this from now on.

Working on my master's thesis has been interesting, inspiring and not least educational, but at the same time it has been demanding and challenging to keep up the motivation through my writing. Through the more strenuous periods, it has been very helpful with all the support I have received, and I would like to start with thanking my family for encouraging me and for helping me stay motivated till the end. Special thanks to my fiancé who has showed interest in my study and who has made it possible for me to work long hours the last couple of months, by taking care of our dogs and sending nutritious lunch packages with me.

I would also like to thank the teacher and the class that I was allowed to observe and interview over a period of two weeks. Without you it would not have been possible for me to carry out this master's thesis. Also huge thanks to my former colleague, Harald Kristiansen, who undertook the big job of proofreading my thesis before handing it in. I really appreciate you doing this for me! Furthermore, I would like to thank the municipality of Stavanger and the administration at Gautesete lower secondary school for having arranged for me to be able to do this study.

Last, but not least, I would like to thank my supervisor at the University of Stavanger, Raymond Bjuland. You inspired me to do my research within problem solving, and through my work with this master's thesis you have given me guidance and concrete feedback, something which has made me succeed in achieving a good result in my research.

Pontus Thente

Stavanger, June 2019

ABSTRACT

The school's youth is our future. Therefore, it is important that school encourages the pupils to use their full potential. To do this, teachers need to make their pupils like to be at school, and to see schoolwork as something useful for their future. An important task for a teacher is to motivate their pupils in such way that they become willing to learn.

This is a qualitative case study that examines how problem solving tasks can influence pupils' motivation for mathematics, through observing the challenges and opportunities that pupils meet when dealing with this kind of tasks. The study has been conducted in a ninth class at a lower secondary school in western Norway and is based on systematic data collection of both video and audio recordings of the teaching. The empirical material consists of semi-structured interviews with the teacher and the pupils, as well as observation in the classroom.

The result of this study shows that variation and challenges in the lessons are important for obtaining more motivation among pupils. Problem solving tasks give the pupils great opportunities to discuss strategies and to reason together to find solutions. In this master's thesis I have come to the conclusion that through trying and failing, and then experiencing control when solving different problems together, pupils' motivation to learn more may be increased.

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1 INTRODUCTION

The Norwegian curriculum in mathematics will be updated and renewed in 2020. In this the pupils will work more with methods and ways of thinking so that they get a greater understanding of the subject. Two of the major areas are exploration and problem solving. Exploration is about pupils looking for patterns and finding connections. Pupils should place more emphasis on the strategies and procedures than on the solutions. Problem solving is about pupils developing a solution method for a problem they do not know beforehand. Algorithmic thinking is important in the process of developing strategies and procedures, and involves being able to break down a problem in sub-problems that can be solved systematically (UDIR, 2018). Fauskanger and Bjuland (2018), have cited from the steering document that problem solving is often seen as a means of achieving deep learning. Furthermore, they say that it is a long history that discusses how problem solving gives pupils opportunities to explore maths in the classroom, and that problem solving is important for the pupil to seek knowledge and understanding. This is because teachers and pupils work closely together and co-operate in learning processes that engage the pupils' learning.

In a mathematics lesson, we find many different types of pupils. We have those who like to work with mathematics and who therefore often have an inner motivation for the subject. We also have pupils who are not particularly interested in mathematics but still master the subject. In addition, we have the last category, the pupils who struggle with the subject, some of whom may have the motivation to learn more, while most within this category are not particularly active. It is often these pupils who are not so active in the mathematics lessons the teachers, therefore, have to work a little extra with get them more motivated for the subject. This concerns both if they have a good mastery and if they are at a low level. Is there any opportunity to get those who do not like maths to do more during the lessons? Is there a way for pupils who perform at a high level to become even more motivated, and is it possible to get the pupils who do not do so much to increase their efforts?

There are, of course, many different reasons why pupils behave differently towards mathematics. In some cases, there are pupils who have the same competence as other pupils, but they will still often perform at a lower level. In such cases, motivation will be one of the determining factors for the pupils' efforts, and next for the pupils' achievement.

According to Wæge and Nosrati (2018), motivation within the subject mathematics can be absolutely decisive for which activities the pupils participate in, in addition to how much time and energy they choose to use. If a pupil does not have enough motivation, he or she can choose not to participate in the activities during the lessons. This applies both to discussions and when to solve his or her own tasks, and they may get the feeling that everything they will have to do is just difficult and boring. On the other hand, motivated pupils can feel completely absorbed when they work with an activity, and they can feel great joy and lose the sense of time, place and themselves.

Stylianides and Stylianides (2014), write about how achievements affect pupils' experiences to master. By this they mean both successes and failures. Success contributes to improvement, and mistakes (especially repeated failures) reduce this factor. In addition, unintentional errors, which are later resolved by a certain effort, can improve self-confidence and increase self-motivation. We find that even the most difficult obstacles can be strengthened by endurance. This is something that sets a mark on this master's thesis. Is it the case that pupils who are able to solve mathematical problems feel more motivated? And is it true that those pupils who make repeated mistakes, but finally manage to achieve something, manage to gain greater confidence?

This study will investigate how pupils can be motivated to learn more mathematics by being engaged in problem solving tasks. It is also an issue to identify how the teacher can introduce her pupils to problem solving, hopefully to see increased pupil motivation as they manage to solve the tasks. This study builds on George Pólya’s crucial work on what problem solving is, and how different motivation theories might influence the pupils to keep on working with the problems.

1.1 OUTLINE OF THE MASTER THESIS

In this thesis, I have looked at how problem solving in the mathematics lessons develops the motivation for the subject of mathematics among pupils by observing how the pupils solve different problems and by studying if obtaining motivation is important for pupils' school performance.

In this context, I will focus specifically on how the pupils with different quantity of work input are developed with the help of problem solving tasks.
I have also looked at how the teacher helps pupils master the tasks.

Based on the subject renewal, that problem solving will become part of the new competence goals in order to teach pupils to develop strategies and procedures, I have chosen to look at how this can be linked to pupils' motivation. With this in mind and on the basis of these factors, I have chosen to make the starting point from these two research questions:
How might problem solving influence pupils' motivation to learn more mathematics?
In what way can a teacher influence the pupils' motivation to learn more mathematics by introducing them to problem solving?

Through this master's study, I have been exposed to problem solving in a way that inspired me to see how problem solving can help me as teacher to give my pupils more motivation, so that they will be willing to learn even more maths. As my research questions state, I want to figure out how problem solving can motivate pupils. David Gay (1992) mentions in his book "Solving Problem using Elementary Mathematics", that solving problems can be quite a journey. You will probably first get frustrated, but once you have found out and learned how you should approach the problems, problem solving becomes something that is funny, instructive and inspiring.

I will start this thesis by defining important key terms such as problem solving and motivation. Then I will present theory. Then, the research method and some methodological considerations will be described, and I will present some selected data that I will analyse and discuss. Finally, there will be a summary where I will attempt to answer the research question.

2 THEORETICAL BACKGROUND

In this theoretical framework of the study I present theory that is related to my research questions. I have divided the chapter into four parts, where in the first part I have chosen to include how problem solving has been used in a historical perspective. The second part deals with how problem solving can be involved in mathematics lessons, while the third part defines problem solving. In addition, I have in this part focused on various problem solving strategies based on Pólya's problem solving model (2004), Borgersen´s (1994) extension of Pólya's model and Schoenfeld’s (1985) framework for analysing problem solving behaviour. The fourth part deals with the definition of motivation, and various theoretical frameworks within motivation that can help me answer my research questions. Therefore I have to look at how motivation and achievement belong together. I have also chosen to include descending motivation to see if there is a link between them. Finally, I have chosen to include a lot about how we can get more self-sustaining pupils, since I want to find out how problem solving can motivate the pupils to become willing to learn more mathematics.

2.1 HISTORICAL BACKGROUND

Since ancient times, mathematicians have been dealing with problem solving, to learn, but also to teach others more about the subject. Already in ancient Egypt, a practical handbook was made that dealing with 85 mathematical problems, this handbook got the name Rhindpapyrus. It describes the Egyptians´ brilliant method of multiplication and division based on the same principles as the total system used in computers today. The Egyptians calculated solutions of linear equations and second-degree equations, but only expressed in words. We also find the method called Regula Falsi (guess and adjust). The Egyptians also had geometric problems, among other things, showing that they knew of some of the Pythagorean problems. The Pythagorean doctrine was derived from the Babylonians, but it was Pythagoras who first came up with a demonstration of why it was this way, and made a proof. The Babylonians described the process of geometric solutions (Burton, 2011).

One of the most famous expressions in the history of mathematics is HEUREKA! It was Archimedes screaming when he ran naked through the streets. King Hiero II had ordered a new king's crown, but was not quite sure if the blacksmith had laid the crown with all the gold that the king had given him and instead exchanged it with silver. Archimedes then took on the task of finding out if the crown was made only of gold. One day he sat in the bath, and then he discovered that the water flowed over the edge, more water the more of his body he led down into the tub (Katz, 2009). That's how he found out Archimedes´ principle. Heureka, who in Greek means “I have found it”, has become an expression of science when making a new discovery.

Euler is seen as one of the greatest mathematicians of all time. His interests covered almost every aspect of mathematics, from geometry to calculation to trigonometry to algebra to number theory, as well as optics, astronomy, cartography, mechanics, weights and measures, and even music theory. In 1735, Euler solved an irreconcilable mathematical and logical problem, known as Königsberg Problem's seven bridges, which had confused scholars for many years. This he did by proving that there was no solution. If Königsberg had had fewer bridges, on the other hand, with an even number of bridges that led to each piece of land, a solution would have been possible (Boyer & Merzbach, 2011). These are just some of the various mathematical events that have occurred in history that have given rise to a problem-solving mind. This master project has a focus on how the teacher can motivate their pupils so that they get an increased interest and become willing to learn more mathematics by working on problem solving in their learning.

Kilpatrick (2009), writes that in 1980, NCTM published its document “Agenda for Action”. Here it was suggested that problem solving should be the focus of school mathematics and that basic skills should be redefined to extend beyond calculation only. First there was a positive reaction, but eventually it became a setback, and there were complaints about the mathematics. Concepts such as "fuzzy math", "whole math" and "new-new math" were used, and mathematics was called "parrot mathematics." The reason for this was that many pupils failed to learn basic facts. Therefore, measures were taken to inform the pupils about the abstract structures of mathematics so that they could better understand what the school mathematics was about. Furthermore, Kilpatrick (2009) writes that today the reforms are more educational with a wider purpose. The curriculum is more directed towards pupils so that they become more active in content and make their work more meaningful and get pupils more engaged. According to Lam (2009), since the 1980s, there has been a worldwide pressure for problem solving to be the central focus of the school mathematics´ curriculum.

In Singapore, the goal is to develop the pupils' ability to solve math problems. Figure 1 shows how the Singaporean curriculum framework in mathematics highlights problem solving abilities of five interrelated components: concepts, skills, processes, attitudes and metacognition.

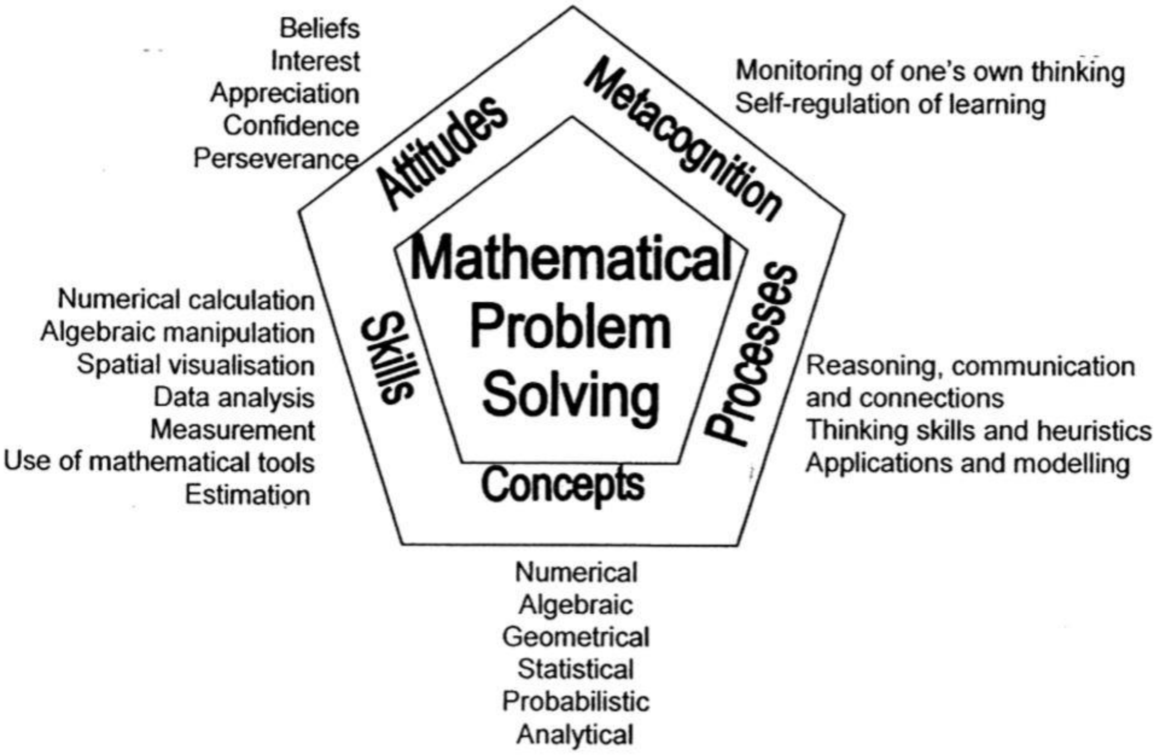

Figure 1: Framework of the Singapore school mathematics curriculum (Lam, 2009, p. 242)

Solving mathematical problems and gaining interest in mathematics have been discussed by educators around the world. In China, teachers have looked at the development by linking the mathematical content to real-life situations. Using practical activities and getting pupils to see the math in real-life situations has helped pupils see the relevance of mathematics.
In addition, it strengthens the curiosity of the pupils in the subject (Lam, 2009). Curiosity is important as it makes our pupils want to explore to try new things and to learn something new. The Norwegian curriculum we use today in mathematics says that, mathematical competence involves using problem solving and modelling to analyse and transform a mathematical problem, solve it and evaluate the validity of the problem. This also has a linguistic aspect, such as communicating, discussing and reasoning about ideas. In most mathematical activities, tools and techniques are used. Both being able to use and evaluate various tools and know how to define them are important parts of the subject.

Mathematics in school helps to develop the mathematical competence that society and the individual needs. To achieve this, the pupils must have the ability to work both practically and theoretically. The teaching alternates between exploratory, playful, creative and problem solving activities and finishing training (UDIR, 2013). With increased awareness of individual learning pathways, the teacher's task has become somewhat more situational, which means that the teacher basically needs problem solving.

2.2 Introduction of problem solving in mathematics lessons

Lam (2009), writes that a problem solving task can be used as an illustration, and the teacher can use this activity as a motivation for starting a new chapter in the class to arouse the pupils' curiosity in the use of mathematics in everyday life. The teacher can also use this activity to supplement the regular classroom lessons.

One of the most important aspects of psychology is what increases pupils' curiosity in mathematics. What is curiosity then? Curiosity is not just that the pupils are thinking about practicing a performance or an event. Curiosity is important because it enables our pupils to explore to try new things and learn something new. According to Schmitt and Lahroodi (2008), curiosity is a motivation and a desire to know. This desire arises and maintains the pupil's attention and interest to know. Curiosity is a characteristic feature that is often observed by our pupils. The importance of curiosity cannot be exaggerated. Curiosity can lead pupils to explore new ideas in mathematics. Fulcher (2008), mentions that some researchers have maintained the importance of curiosity as an important link to a person's lifelong learning. Curiosity is needed to make the pupils want to know the subject without having bad feelings of wanting to know it. The desire may be to know an object or to know a proposal. Curiosity then requires attention to be given to the subject, and it requires a motivational original desire to know the topic. Problem solving tasks can in this way get the pupils to pursue their curiosity and through it go into different mathematical areas through new angles.

In Norway's steering documents, the Norwegian Ministry of Education and Research (NOU 2015:8, 2015) writes about problem solving and motivation. In this inquiry it is mentioned, that critical thinking and problem solving can often be seen in connection with creativity and innovation. Problem solving is about being able to reason and analyse, identify relevant issues and to be able to use relevant strategies for complex problem solving. These documents mention that problem solving is important today and that it will have an increased importance in the future, as we increasingly have to cope with various issues and problems both in our work, society and in our private life. By working on problem solving, pupils will have to learn to analyse and estimate which knowledge and methods are relevant to use. In addition, they must learn that they are not always able to find a solution. The pupils must therefore be able to create strategies, ask questions, try out, gain experience and increased knowledge that provide the basis for new questions. To achieve this, pupils need to gain experience where they have to solve problems and deal with situations where it is not clear which strategies and methods they can use to reach a solution. Therefore, pupils need to use more competences together, both metacognition, creativity, innovation, critical thinking and problem solving, and they must reflect on the purpose of what they learn, what they have learned, and how they learn. Furthermore NOU 2015:8 (2015), writes about depth learning and how pupils develop an understanding of concepts, concept systems, methods and contexts within a subject area. Depth learning means that pupils use their ability to analyse, solve problems and reflect on their own learning to construct a lasting understanding.

In the future, it will be especially important to have a vocational education or education at a higher level, in order to get a job. It therefore becomes especially important for schools to facilitate learning processes that contribute to an increased understanding, as this can lead to a stronger motivation for the pupils, experience more control and relevance in school life, so that they are able to complete and pass the training.

You can also read about metacognition in NOU 2015:8 (2015) that is about being able to reflect on your own thinking and learning. Pupils must learn to reflect on what they are learning, why and how. They must actively and purposefully use thinking and learning strategies to promote their own learning. By developing metacognition and self-regulation, pupils learn to engage in the learning process in a way that promotes in-depth learning, which in turn can lead to the motivation to learn in school.

2.3 Problem solving

Piggott (2004) mentions that problem solving includes assessing it as a skill to be learned or as a tool to learn through. Problem solving is, among other things, an instrument we use to reach the educational goal: mathematical understanding. A problem-oriented work method is close to what we mean by a concrete and obvious work on the school's tasks. The problem at best is a bridge between a real world of everyday activities and an abstract mathematical reality. Furthermore, Piggott (2004), says that mathematical thinking is linked to certain mathematical skills that pupils must learn to solve the problem effectively. The tasks must be engaging issues which can develop and use problem solving strategies, as well as encourage mathematical thinking.

Problem solving tasks that deal with group work, exploration, mathematical communication, and which have the goal to improve pupils' attitudes, gaining a growing understanding of mathematics as a discipline, as well as developing conceptual structures that support mathematical understanding and thinking are basically important.

Piggott (2004), utilises this model to illustrate how problem solving and mathematical thinking relate to facts, skills and algorithms.

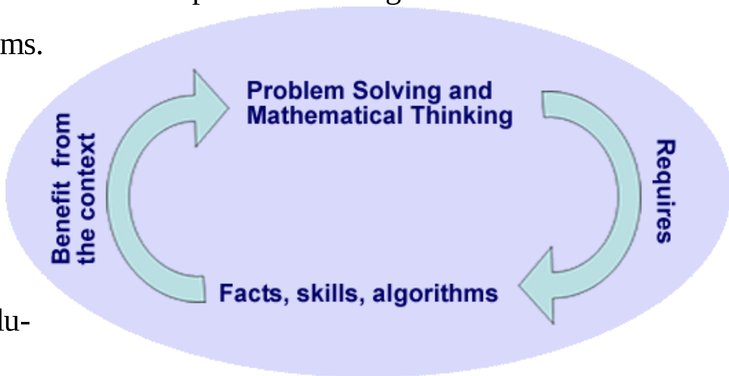

Figure 2: Relation between Problems solving and Mathematical thinking, towards Facts, skills and algotithms, Piggott (2004)

Problem solving can be understood as the management of strategies that allow to define or describe a problem, to determine possible consequences, to choose possible solutions, to choose strategy, to test the categories, to assess consequences and to review the steps taken if necessary. Wilson, Fernandez, and Hadaway (1993), mention four reasons why problem solving is important.

The first one says that problem solving is a big part of mathematics. There is no mathematics without problem solving, then we remove large parts of the mathematics from the pupils. A second reason why problem solving is important for mathematics is that the subject is used in other subjects and in our everyday life. Communication and mathematical understanding are represented daily. The third reason that is the main basis for this master's thesis is that there is a special motivation to solve mathematical problems. To include problem solving in school mathematics can stimulate pupils' interest and enthusiasm. The last reason is that problem solving can be funny. Many of us solve math problems for recreation.

With the new curriculum, problem solving must be a part of the subject, and it is therefore important that teachers choose to incorporate a plan where problem solving takes part in the teaching. According to NOU 2015:8 (2015), mathematical problems are meant both as problems from everyday life and society where mathematics can be used and abstract mathematical problems and questions. This master thesis will focus very much on George Pólya's problem solving model. Pólya (2004) defines having a mathematical problem as seeking, consciously, for an appropriate action to achieve a clear goal, which is not immediately achievable. That problem solving is based on cognitive processes that result in finding a way out of trouble, a way around an obstacle, reaching an object that could not immediately be achieved. Schoenfeld (1985), describes a mathematical problem for each pupil as a task that the pupil does not quite know which method to solve, which captures the pupil's interest and commitment, until it arrives at a solution. Based on the pre-interview with the teacher, Pólya's problems solving model, is the method the teacher in this study is known to use, therefore she has chosen to have this as an implement for her pupils. I will describe this model in a following subchapter.

Pólya explains that a "problem" is a question that a person does not know in advance. This causes a problem, something that is personal, as not everyone has the same experience or knowledge. Solving a problem is taking knowledge in a varied game, mathematically or not, as well as seeking new relationships among them. To achieve this, it must be a matter that the person is interested in, that he is a little provoked and the desire to solve it. Then again something that is personal, and it depends a lot on how it is presented to us, the task must capture your interest, but also challenge you in a way that makes it a problem. It must be a task that a person is willing to spend time and forces on. The result of all this, when a person has reached a solution, it could lead to a good sense of joy.

2.3.1 George Pólya's problem solving model as an alternative solution

The following model is made by George Pólya, consisting of a set of four steps and questions that lead the quest and exploration of a problem that may have alternative solutions. That means, the model shows how to effectively tackle a problem and how to learn from experience.

The George Pólya model's four phases

George Pólya (2004) writes in his book "How to solve it" about his model with the various phases that establish four stages to solve a problem. They are general solution strategies and decision-making processes used to solve problems, based on past experience with similar issues that cause significant learning. These strategies indicate ways or possible approaches to follow to reach a solution. The model itself is described in part 1 of the book "In the classroom". And with the help of this chapter I have taken out the questions that Pólya describes and made a list for each of the four steps.

Understand the problem

This phase involves understanding both the text and the situation that presents the problem, being ready to distinguish between different types of information provided in the statement and understand what to do with the information that is provided. This seems so obvious that it is often not even mentioned, but the pupils are often hindered in their efforts to solve problems simply because they do not fully understand it, or even parts of it. Therefore, it is wise to read the problem itself slowly and carefully. As support one can try to answer these questions:

- Do you understand everything you're being told?
- Can you rethink the problem in your own words?
- Can you distinguish what the different data are?
- Do you know what you want to come up with?
- Is there enough information?
- Is this problem similar to another you've solved before?

Design a plan

This is the basic part of breaking up problems. It is when the problem is understood and one is aware of what is necessary to reach the goal. You must plan which actions to perform to find the answer. It is therefore necessary to address the problems and find out how to use the data given in the problem. What can be calculated on which operations should be used, and in which order everything should happen. Various things that can be helpful are:

- Use the Sample and Error method (presumption and test presumption).
- Use a variable
- Search for a pattern
- Make a list
- Solve a similar problem that is easier.
- Make a figure.
- Make a chart
- Use direct justification
- Use indirect reasoning
- Use the properties of the numbers.
- Work backwards
- Use cases
- Solve an equation
- Search for a formula
- Use a model
- Use dimension analysis
- Identify sub-targets.
- Use coordinates
- Use symmetry

Perform the plan

The third phase consists of initiating each of the steps designed in the planning. Within problem solving, it is important to remember that the thought process itself is not entirely linear. There are always ongoing jumps between the plan's design and the assumption in practice. The plan gives a general line, and it must be ensured that the details fit well in that line, so it is necessary to examine the details one after the other, patiently until everything is clear.
If the pupil has been caused to design a plan, he or she will perform it with satisfaction.
If this does not happen, it must be insisted that the pupil verifies and checks to be sure of the accuracy of each step. It is during this phase that the execution process is controlled and carried out. Within this phase, the following will be taken into account:

- When performing the plan, you must check each step.
- Before you do anything, you should think, what has been achieved with this?
- It must follow every mathematical operation to explain, clarify what is done and what is not done.
- Don't be afraid to start again. It usually happens that a new start or a new strategy leads to success.
- Check and confirm each step.

Review of the process, Looking Back

In the final phase, it is practical to conduct a review of the process, to analyse whether the way the resolution is reached is correct. It is necessary to contrast the result obtained to know if it really gives a valid answer to the situation, to reflect on whether this solution could have been reached in other ways, by other bases. Some questions that are good to ask during this phase:

- Is it your right solution?
- Does your answer meet what is established in the problem?
- Is there an easier solution?
- Can you see how to extend the solution to a general case?

The purpose of the model is that the person should take advantage of something that Pólya calls productive thinking, which is to examine and rebuild their thinking methods, systematically eliminate obstacles and establish effective mental habits. It does not mean that after using these four steps you are guaranteed you have reached the correct answer. One of the reasons is that problem solving is a complex and rich process and is not limited to following step-by-step instructions that will lead to a solution as if it were an algorithm. However, it can help and guide the pupils in the process of solving the problem.

By using Pólya's model, one gets an understanding of mathematical situations in four basic steps that lead to a solution of problems. Especially the mental operations are usually useful in this process (Pólya, 2004). The George Pólya model is based on a thorough examination of solution methods. It presses a new aspect of mathematics that gives pupils a process of invention, but does not solve the problems. It is a guide on how pupils can tackle the problems. It thus provides ways pupils can use to solve the various problems, and in addition, it can help to arrange thoughts in an intuitive manner when an unsolved problem arises.

The Pólya model consists of general resolution strategies and decision rules used to solve problems, based on past experience with similar problems. By asking pupils to adapt their knowledge, the teachers can make them become more curious and get them to use approaches to task they have not tried before and help them solve them by asking stimulating questions. This model focuses on solving mathematical problems, so it becomes important to distinguish between "exercise" and "problem". The creativity of problem solving depends largely on the person's mental state. To solve an exercise, one uses a routine procedure that leads to the answer. To solve a problem, one must reflect and even have to perform original steps that one had not tried before to find the answer.

Doing exercises is very valuable in math learning, as it helps us learn concepts, attributes, and procedures, but also many other things that we can seek when facing the problem of solving problems.

However, Pólya (2004) emphasizes that when solving a problem, the interest and attitude are the main factors for the pupils. If it is uninteresting, it is not possible to move on. Therefore, it is important for the teacher to encourage their pupils and awaken their curiosity.

2.3.2 Different approaches on Pólya’s problem solving model

Schoenfeld (1985), describes in his preface that he was very interested in Pólya's problem solving model, and posed the questions: What does it mean to "think mathematically?" and “How can we help pupils to do it?” With these two questions in mind he laid out a framework that is based on Pólya's problem solving model. He wanted to investigate the extent to which problem solving best achieves and stores knowledge (heuristics), when pupils solve mathematical problems and to develop a mind-set that demonstrates consistent features of mathematical practice. The theory that Schoenfeld’s presses is a framework for analysing the pupils' complex problem-solving behaviour. His research showed that there was more than just one application of the Pólya’s problem solving model, considering that the problem-solving process is a dialogue between the problem solver's prior knowledge, his attempts and thoughts along the way, and that there are other factors that can affect a successful problem-solving.

Schoenfeld presents four aspects:

1. Cognitive Resources - The various facts and procedures that are available at one's disposal.
2. Heuristics - As he describes as 'rules of thumb' to make progress in difficult situations.
3. Control - has to do with the effectiveness of how individuals exploit their available knowledge and how they monitor their own thinking, that is, metacognition.
4. Belief systems - one's perspectives and attitudes regarding the nature of a discipline and how to work with it.

These four aspects that Schoenfeld puts forward allow the problem solver to have a framework that he can focus on when difficulties arise. As Pólya’s model focuses more on concrete actions, Schoenfeld's model looks more at the abstract conditions in the process.

Schoenfeld divides the problem-solving process into 6 steps:

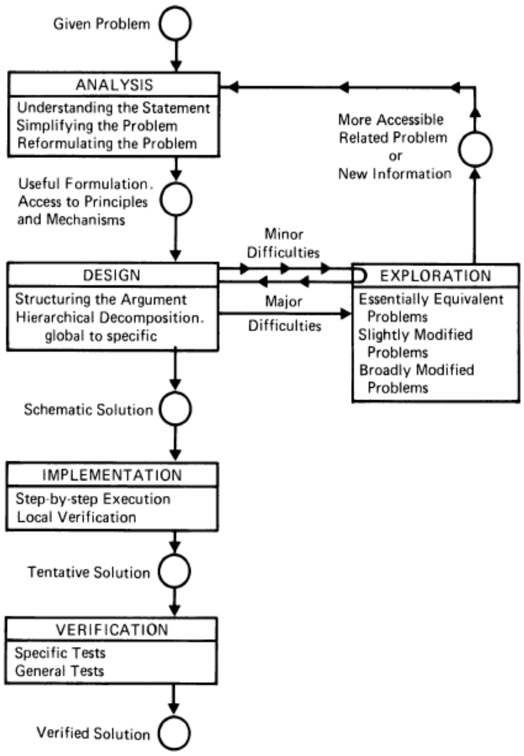

Figure 3: Schematic outline of the problem-solving strategy (Schoenfeld, 1985, p. 110)

1. Read the given problem.
2. Analyse, in order to get a sense of the problem: what is given, what do they ask about what is the goal of the problem, what do facts say and are the goals plausible, which main principles or mechanisms seem relevant or appropriate to bear, in which mathematical context the problem fits in, and so on. This step is important to simplify the problem.
3. Exploration, this is the heuristic heart of the process. It is in this part that most problem solving heuristics come into play.
4. Design, this part is something that goes into the different steps and is in a way a "master control". The function of the design is to ensure that you spend time on what is profitable. This should have a rough outline of good quality and can be changed as new challenges arise.

The parts about the analysis, design and exploration are in a cyclical nature, and the problem solver can either choose after the exploration to go back to create a new plan or return to the section of analysis.

5. Implementation, is according to Schoenfeld the last part. In this part you perform the plan and do a local check to verify.
6. Verification, this is the most important part and is something, according to Schoenfeld, pupils often forget. It is important that they control their solutions, check if there are more solutions and maybe notice connections to other subjects.

Another approach to Pólya's problem solving model has been made by Borgersen (1994). He has expanded Pólya's four-step model to seven steps, and also emphasized collaboration in small groups.

Borgersen´s seven steps:

1. Analyse and define the problem. What is the situation, and what is the problem? Do we understand the meaning of the words? How do we understand (define) the situation and the problem? Here it is important that the pupil tries to link the problem to previous experiences in the subject of mathematics, and uses his earlier knowledge. Therefore, it is important that pupils identify the problem.
2. Create a drawing or a model. This step encourages the pupils to find or create a model of the situation. For example, they can make a drawing or draw a model. This is very important at all levels of problem solving and can often be part of the analysis to get started and find patterns to investigate.
3. Qualified guessing by trying and failing. Guessing, making measurements, and guessing again can help the pupil improve and gain a better understanding of the problem. If the guess is based on earlier knowledge and experience, Borgersen calls it qualified guess.
4. Find a hypothesis. Here the pupil should try to formulate a general solution. This is still a guesswork and requires to be tested out. The answer means that we either reword or reject the hypothesis.
5. Develop a proof. If we succeed in developing a proof from a hypothesis, we get a sentence. Some use algebraic evidence while others may apply geometric. Many times there may be a mixture of these. Sometimes we can't make a proof. The reason may be that the hypothesis was wrong or that the task is too difficult. Other times it is that the pupil only needs more time.
6. Reflect on solution and the process of solution. At this stage, the pupil should reflect and interpret their solution. This can lead to other possible ways to solve the problem or other answers.
7. Generalize the problem and formulate new problems. By working on this model, the pupil will encounter new questions and problems, which he or she may be encouraged to investigate. In this last part, therefore, the pupil should combine ideas and try to create new problems with the help of the information they have been given. They may also attempt to make a generalization so that the answer they have received to the problem can be applied to other similar problems.

Borgersen, mentions that it is important to emphasize that these seven steps do not necessarily describe a linear process, but rather cyclic processes where the problem solver must jump back and forth between the steps, and even if the pupils fail to prove the theorem, they can give them a good understanding so that they can use them in other problems or in real-life situations.

2.4 MOTIVATION

Pupils in school have a different degree of knowledge. This also applies to the degree of motivation for a subject, where some are more motivated for one subject and others for another. The word motivation comes from the Greek word "*movere*" and means moving (Deci & Ryan, 2002). The definition of motivation is, according to Bandura (1997), activation to take action, where the level of motivation is reflected in pattern of action, intensity and endurance. Deci and Ryan (2002), define it as the process of providing energy and direction to behaviour, and claim that motivational studies are about exploring the energy level and direction of behaviour. This means that the claim of energy affects how much effort a pupil will use, and the actual direction towards what makes sense to the pupil. Therefore, one can say that the action is directed towards a goal that fulfils the go-ahead spirit, or the desire a pupil has.

Skaalvik and Skaalvik (2015), clarify this by saying that the motivation a pupil has for his or her school work is significant for how long he or she endures, and how much effort the pupil in question gives, in addition to whether he or she seek help when they are standing in a task. Imsen (2014) defines motivation as what causes activity in the individual, what maintain the activity, and what gives it purpose and meaning. Wæge and Nosrati (2018), write that motivation for people is not constant, it is constantly changing. Motivation is influenced by the situation and the various factors such as values, experiences, expectations and needs. Wæge and Nosrati (2018) suggest that the teacher's role and classroom culture have a great influence for pupils' motivation in the work with mathematics. Motivation is not something that can be observed directly, but it can be considered a condition, a feeling or an experience that causes activity in the individual. This condition controls the activity in specific directions, and it keeps it alive.

2.4.1 Theoretical directions on learning and motivation

There are many different theories about learning and motivation, and there are some similarities and many differences between several of the theories. For example, teachers may have a common base of departure for their pupils to learn as much as possible, but at the same time they may disagree on what is important knowledge and how the pupils will be motivated and acquire it (Lyngsnes & Rismark, 2016). Three major theories about learning and motivation are the behavioural, the cognitive and the socio-cultural theory, and these theories have different views of motivation and learning.

2.4.2 Behavioural view of motivation

American Burrhus Frederic Skinner was the pioneer of the behavioural learning theory (Lyngsnes & Rismark, 2016). As behaviourism was originally formed, the focus was on what is possible to observe directly, that is, pupils' behaviour and their actions. This led us to only observe the visible impact (stimuli) and visible reaction (the response), which were shown by the individual. Through proper stimulation one can get an individual to learn almost anything. Humans are passive and impressionable and can be controlled from the outside in the direction of the learning goals set up in advance (Imsen, 2014).

"The Hedonistic Principle" is a fundamental principle of the behavioural theory, and implies that man at all times seeks what is comfortable and pleasant and tries to avoid punishment and what causes discomfort (Lyngsnes & Rismark, 2016). Thus, reward and punishment, according to Skinner's theory, can be the main reason for the amount of engagement pupils make to perform various actions. Because the same type of reward or punishment is likely to repeat itself, pupils perform the action in a specific way based on this (Imsen, 2014).

To summarise, one can say that the behavioural learning theory emphasizes external behaviour and simple forms of learning, and the motivation itself is the external reward that the pupils will seek. In behaviourism, man has a passive form of learning, and the view of knowledge is that it is finished knowledge that is transferred to the individual. The teacher's task is to facilitate tasks and to reward the pupils for performing them.

2.4.3 Cognitive view of motivation

The background for the cognitive theory is based on the emphasis on how our thoughts contribute to creating motivation and action. In contrast to behaviourism, within the cognitive learning theory, the inner, cognitive processes are the starting point for understanding learning. The cognitive theory assumes that man is curious and investigative. There are none of us people who like to walk around wondering what is going on or not understanding a problem or why things are the way they are. We would rather organize the whole world in categories and concepts, see them in relation to each other again and look for relationships and patterns (Imsen, 2014).

Jean Piaget has been of great importance for the development of the cognitive theory, and he claims that one understands all new things one is facing, from what one already knows from before and from the experiences one has from before. He uses the term "schemas" when talking about the experiences we have, and when one is to understand new information, one interprets it from the schemas that one already has. This is called assimilation. When new information arrives and there is a lack of balance between what you previously know and the new experiences, you do not have an appropriate schema and must therefore change existing schemas or develop a new one. The new knowledge thus replaces or expands earlier understandings. This is called accommodation (Lyngsnes & Rismark, 2016). Thus, learning occurs as an interaction between the child and the environment. The child examines and interprets the information that the environment provides, where it turns out that the environment has more information, and so it continues (Imsen, 2014).

In summary, it can be said that within the cognitive theory, emphasis is placed on internal memory processes and complex forms of learning. The very driving force in learning lies in finding structure and pattern, which is an inner motivation. Man, experiences being both active and passive and they "save" their knowledge. The view of knowledge is that it is finished knowledge that is characterized by the individual's processing, and the teacher's task is to structure, explain and stimulate good learning strategies (Imsen, 2014).

2.4.4 Social development theory´s view of motivation

Russian Lev Vygotsky has developed many of the ideas in the socio-cultural learning theory. He believed that how motivated children are and how much they learn, depends on the people in the child's environment, and that the child's knowledge, ideas, attitudes and values are developed in collaboration with others (Lyngsnes & Rismark, 2016). The most important tool for thinking and learning within the socio-cultural learning theory is therefore the language, as it is through the language in which one can ask questions and express ideas.

The current level of development is what Vygotsky calls the knowledge that one pupil has right now. On this level, the pupils are able to solve tasks and problems on their own without any help, but the pupil does not learn anything new here and does not develop. On this level pupils may get a feeling of mastery for a while, but sooner or later they will get bored. He or she, however, has a potential for development in the extension of the current level of development. This is one of some major themes in the social development theory, and Vygotsky has named it "the zone of proximal development". This zone is the area between what the pupil is able to solve alone and what the pupil can understand and manage with help the from others (Lyngsnes & Rismark, 2016).

This leads us to the second major theme of the theory, the more knowledgeable other. When we talk about what pupils can manage to do with some help from others, we refer to anyone who has better understanding or a higher ability level than the learner, for example the teacher, other pupils or even computers. Through small hints, showing similar examples or asking leading questions, teachers, or other people in the pupils' environment who know more than the pupils, can help them understand just a little more about the problem than they knew before (Lyngsnes & Rismark, 2016).

In the social development theory learning is distributed, which means that in a group the knowledge is always spread. No one knows everything but everyone knows something, and through sharing knowledge pupils will be more motivated (Lyngsnes & Rismark, 2016). The knowledgeable other can, in fact, function as "teachers" or at least as assistants for the teacher. When working with tasks in the classroom, the teacher can put together smaller groups within the class so that the pupils can discuss, ask questions and spread their knowledge to their classmates. In this way everyone in the group can understand a little bit more and also be more motivated to be creative when thinking of how to solve the tasks.

In the zone of proximal development, pupils will achieve a sense of mastery when actually solving the tasks that they find just a little too hard. It is only through some help from the more knowledgeable others; they will be motivated to try to solve similar, or even harder, tasks on their own (Lyngsnes & Rismark, 2016).

2.4.5 Theory of inner and outer motivation / self-determination theory

As a teacher one would like the pupils to be motivated and to study because they want to themselves or have an interest in becoming educated. One would like for the pupils to either have inner motivation or autonomous external motivation (Skaalvik & Skaalvik, 2015). "The term autonomy, or self-determination, literally refers to regulation by the self, and for an act to be autonomous it must be endorsed by the self, fully identified with and owned" (Ryan & Deci, 2006). In other words, autonomy involves acting out of own interests and values (Deci & Ryan, 2002).

Figure 4 shows a cyclic model that contains three phases of self-regulated learning, and “the model assumes significant correlations between variables within a particular self-regulated learning phase, and it assumes potentially causal influences of self-regulated learning processes across phases” (Zimmerman, 2008, p. 178). The forethought phase involves several motives to self-regulating, such as self-efficacy beliefs, outcome expectations, task interest or value, and goal orientation. In addition, this phase contains two important self-regulatory processes, which is goal setting and strategic planning. Through setting goals and planning strategically pupils can achieve better results and thus also get a sense of mastery, which again will lead to motivation. Zimmerman and Bandura (1994), present an example of a research that supports this model, and this research has shown that “pupils’ level of self-efficacy about their writing performance was positively correlated with the grade goals they set for themselves as well as the grades they actually received in a writing course”. Furthermore, we can see that the performance phase, which includes strategic processes and self-recorded outcomes, is directly related to the self-reflection phase where the pupils get a reaction, a feeling of satisfaction. Finally, the pupils’ self-reflection phase with feelings of satisfaction with their performance is predictable of two sources of motivation from the forethought phase: self-efficacy and task interest or value (Zimmerman, 2008). From the research that has been done, one can clearly see that if pupils use self-regulated learning processes, it can enhance their motivation to continue additional cycles of learning.

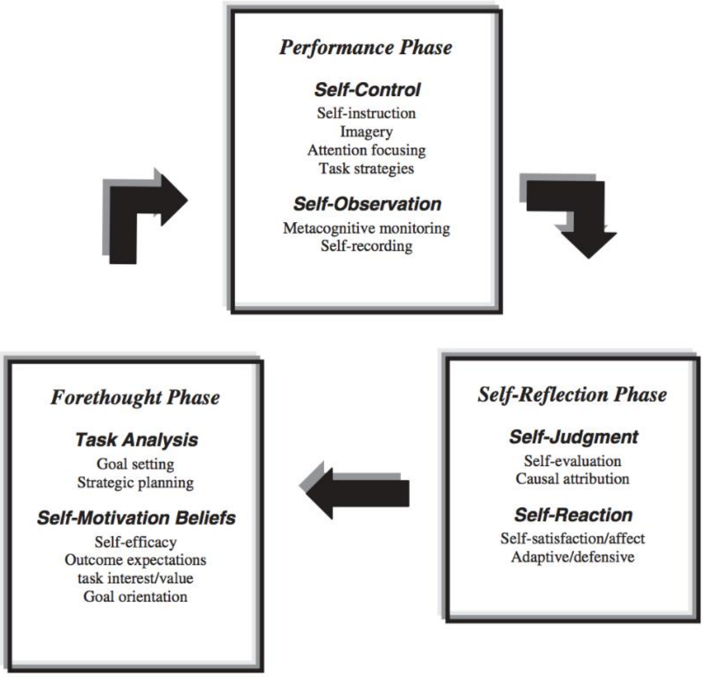

Figure 4: Phases and subprocesses of self-regulation (Zimmerman & Campillo, 2003)

Skaalvik and Skaalvik (2015), write in their book about motivation for learning about different motivation theories, and among these theories they present the theory of inner and outer motivation – also called self-determination theory. In this theory, it is claimed that pupils promote inner motivation and autonomous external motivation when the activity and the environment satisfy the three basic psychological needs of the pupils:

1. The need for autonomy or self-determination - The feeling of doing something voluntarily or having a choice.
2. The need for expertise - mastery and coping expectation, academic self-assessment and an expectation for success
3. The need for belonging - Positive social relations, a sense of being respected and included, but also a sense of security and trust

For a teacher to be able to satisfy these needs, he or she must be an autonomous supportive teacher. According to Skaalvik and Skaalvik (2015, p. 69 my translation), there are many things teachers can do to be an autonomous supportive teacher for their pupils. One should:

- Give the pupils good reasons for the choices that are made
- Listen to the pupils and let them express their views
- Provide pupils with choices whenever possible
- Provide as few directives as possible
- Take the pupils' questions, experiences and wishes seriously
- Encourage pupils to take the initiative

These are things that will promote the inner and outer autonomous motivation among the pupils, because it can lead them to experiencing mastery and at the same time give them a feeling that the teacher cares about them and their opinions and views. In this theory, about self-determination, it is important for the pupils to feel like they have choices, and that their actions are being approved by the self rather than being controlled.

2.4.6 The connection between motivation and mastery

Perceived self-efficacy is concerned with people's beliefs in their ability to influence events that affect their lives. This core belief is the foundation of human motivation, performance accomplishments, and emotional well-being (Bandura, 1997). Pupils' self-efficacy in school refers to their expectation of being able to perform specific tasks. It is not about how good the pupils feel in general or in a specific area, but whether they think they will handle the tasks they are facing at any given time. When pupils are facing a task or challenge, they will ask themselves: "Will I manage this task?" How the learners answer them on this issue will be very important for the pupils’ motivation and will influence their choice, commitment, effort and perseverance when the pupils try to solve the challenge they are facing (Skaalvik & Skaalvik, 2015).

Skaalvik and Skaalvik (2015), present in their book about motivation for learning, four different sources to expectations of mastery. These sources tell something about what helps determine if pupils want to try to solve the problem or task they are facing, or not:

1. Past experience with coping with similar tasks: The feeling of accomplishment increases expectations of mastering again, while experiences of failing will later weaken expectations of managing. This is the most important source to expectations of mastery.
2. Observing that others are able to solve the tasks: Seeing others manage to solve a task can strengthen the belief that one can manage it oneself. However, this applies only when others, who otherwise are perceived as similar to oneself, manage the tasks.
3. Encouragement and trust from significant others: Parents, teachers, or others who give encouragement can be perceived as signals that they have confidence in what the pupil will do, which can strengthen their own faith in themselves.
4. Physiological reactions: a specific situation or challenge, for example, in math or other subjects, can awaken unpleasant experiences in similar situations. Examples of physiological reactions can be sweat or breathing problems.

Skaalvik and Skaalvik (2015), are also talking about "learning self-efficacy" in their book. This concept is about the pupils' expectations of mastering gradually becoming attached. This means, pupils who are unable to solve mathematical tasks at school will lose their expectations of managing tasks of the same type later. Then, the lessons of failure will result in the pupil losing the expectation of learning math at a more general level. Likewise, pupils who experience to master tasks that he or she receives at school will have an expectation to cope with the same type of tasks later. Thus, the previous experiences will help motivate the pupil to make more efforts and have greater perseverance in order to solve similar tasks. Most likely he or she will manage to solve the tasks and experience mastery again. This connection between the experience of mastery and motivation is shown in a figure 5 below:

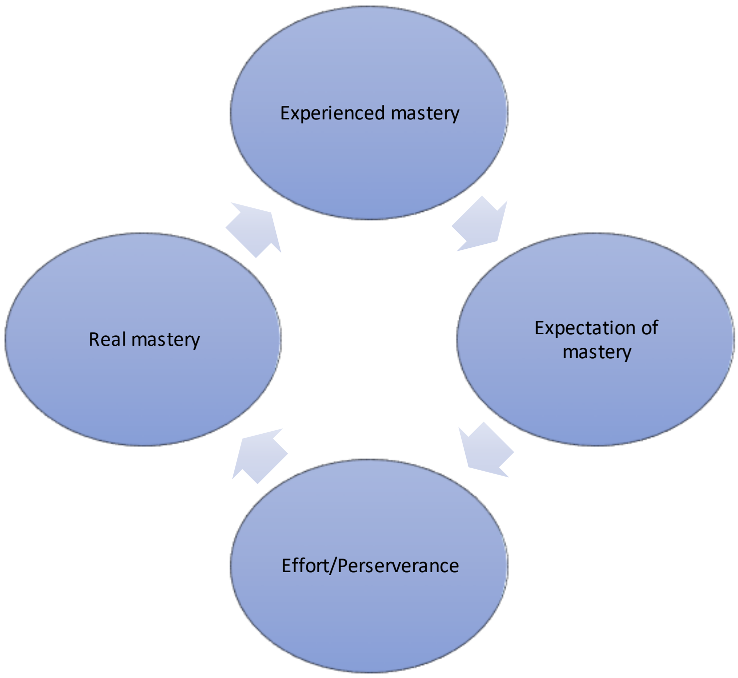

Figure 5: Connection between the experience of mastery and motivation (Skaalvik & Skaalvik, 2015).

3 METHOD

The study takes part in a collaboration with another student in the master's programme. We have conducted the actual data collection together, but we had different orientations and research questions regarding what we have chosen to investigate. The study itself is a qualitative case study that builds on the challenges and opportunities pupils in the ninth class meet when dealing with problem solving tasks, and how this promotes the pupils' motivation.
In this chapter I will explain my role as a researcher, and it will be discussed successively throughout the study, I will try to justify the choice of methods. Furthermore, I will explain the collection of data, try and discuss strengths and weaknesses with the overall data material. The selection will be accounted for, and so will the basis on which I made this choice. Ethical reflections, validity and reliability will be explained and discussed. At the end of the chapter I will describe how the analysis of data is performed, and the topic around it will be explained and discussed.

3.1 RESEARCH DESIGN

I have previously described in the thesis the background for the research focus and research questions (see section 1.1 Outline of the master thesis). I have used a qualitative approach with the help of a case study, to collect my data. The empirical material consists of semi-structured interviews with both the teacher and pupils, as well as observations in the classroom where I have noticed what the teacher does and how the pupils have worked with various problem solving tasks. The reason for this is the qualitative research focus on finding patterns and creating an understanding of what is to be investigated (Kvale & Brinkmann, 2015). Video observations and field notes from experiences in the classroom have given me insight into how the teacher introduced her pupils for problem solving tasks. The interviews are what will provide the most of the basis for my findings, since it is in these interviews I get input on how the pupils think and how they have been influenced by the work method. In a semi-structured interview I also have the opportunity to ask follow-up questions that arise during the interview. By using me through interviews I can better understand and explore how the teacher and pupils think about the working method. As I want to find out the pupils' thoughts on how the working method motivates them or not, interviewing becomes the most effective method for my research (Kvale & Brinkmann, 2015). I can certainly have used more of the observations to strengthen the validity of my findings, but when I have to limit the task, the focus is mostly on the interviews.

This is a case study which is often used within qualitative research when dealing with detailed analyses of a particular case (Postholm & Jacobsen, 2018). The disadvantage of a case study is that the research is time consuming, and for this reason one has to limit the number of informants in the survey. In addition, I do not get an answer as to whether this is something that represents other groups, which means that a generalization is not possible. One way for the results to be more valid is to compare those with other representative schools with the same research focus. It is only then that one can say whether this is a result that applies to a larger population (Kvale & Brinkmann, 2015).

3.2 Data Collection

The research project is a qualitative case study focusing on how pupils in the 9th class can work with problem solving tasks. My goal is to look at how working with problem solving contributes to influencing the pupils' motivation for the subject mathematics. The project period was two weeks, consisting of six mathematical lessons of 45 minutes each. During this period we have observed a class (9th grade) in lower secondary school, where the goal has been to notice how the pupils solve, develop and work with problem solving tasks. We also observed the teacher with a special emphasis on how she introduced problem solving to her pupils.

Figure 6 shows how an overview of how the data collection was collected and preformed.

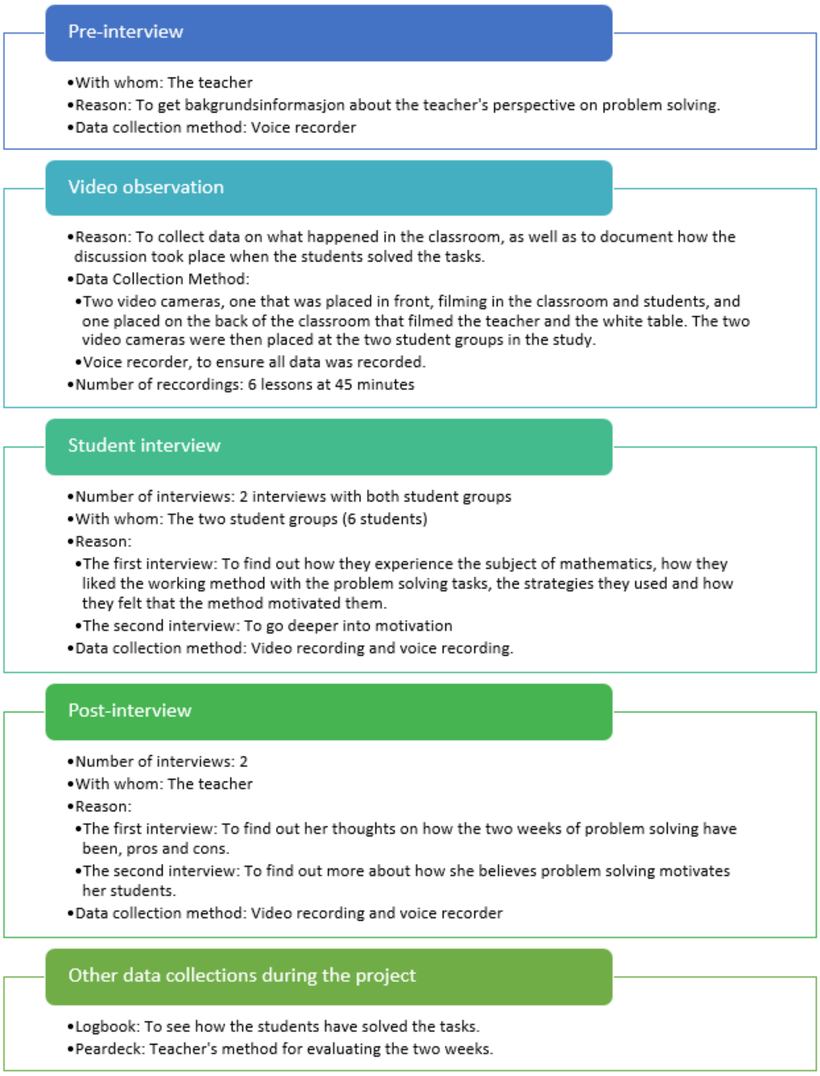

Figure 6: Overview of the data collection

The study is based on systematic data collection of video and audio recordings of the teaching. To get some background information from the teacher, the data collection started with a pre-interview (Attachment 6) two weeks before the two projects weeks started. In this interview, the teacher told us about experiences and thoughts about problem solving. For this purpose, this interview was only audio recorded. Each lesson during the project was filmed with two cameras, one that was placed in front, filming the classroom and the pupils, and one placed in the back of the classroom filming the teacher and the white board. In addition to these two cameras, it was taken audio recordings of the teacher using a microphone that she had attached to her sweater. During the project, the pupils have worked together in groups and as individuals. They have also conducted a log to show what they were doing. These logs are used in my study to see how the pupils solved the various problem solving tasks. The log books have been scanned and loaded in a memory stick.

To document what the pupils did during the group work, we used one video camera on each of the two main groups. In addition to this, we used a sound recorder to make sure we did not lose anything the pupils said. I also moved around between the groups and took some field notes, most to see what the other groups had for thoughts and ideas.

After the two weeks of problem solving, we had group interviews with the pupils from the two small groups in focus in our study (Attachment 4), to find out more about their thoughts and how they approached the different problems. We also had a post-interview (Attachment 7) with the teacher to find out what her thoughts and experience during this project had been, but also if she had anything else she wanted to share. The interview with the teacher and the interview with the pupils were also filmed and recorded with a camera and sound recorder. After the interview with the teacher, she had an evaluation with her pupils in Pear Deck, a Google Suite program that turns presentations into discussions in class with a variety of interactive and formative assessment questions. With Pear Deck, the pupils engage with the interactive activity on their own screen and know that their response will be seen by the teacher, but anonymously to the rest of the class (GoogleSuite, 2018). With the help of these answers I got an even larger material and could see more about how each pupil had been motivated and which feelings they had about the learning method itself. To get even more data targeted to my research questions, I had an extra interview with the teacher (Attachment 8) and the six pupils (Attachment 5), as a supplement to the previous interviews. These interviews had more focus on motivation.

3.2.1 The Selection

During the pre-interview I learnt more about the teacher. The teacher is a woman who has worked as a teacher for several years and has a master's degree in mathematics education. The teacher is very interested in problem solving and has experiences both from her own studies and from having worked as a teacher. During her years as a teacher, she has tried to use problem solving as a start of the week and to awaken the mathematical thinking of her pupils. But the current class has unfortunately not had this opportunity. Today she works as a teacher in the 9th grade at a lower secondary school in western Norway. According to her, the class is a typical Norwegian class, with pupils in most levels of knowledge in mathematics. By this she means pupils who have much experience with mathematics and those who do not have so much experience. They have a quite high work ethic, and they do as they are told. The class consists of 26 pupils (14-15 years). During this project, the teacher divided the pupils into six small groups of three pupils and two groups of four pupils. I and the other master student have chosen to focus on two of these groups. We wanted one group where the pupils had good skills and one group where the pupils were at a basic level. These two groups were selected after the conversation with the teacher. I have chosen to call the groups for group A and group B. Group A consists of two girls and one boy. According to the teacher, these are all pupils who perform at a high level, while group B consists of two boys and one girl, and as we wished, the pupils in this group did not have much experience in math. For the six other groups I have only taken some field notes.

3.2.2 Interviews and observation

The observation of the teaching and the interviews with the teacher and the pupils are two different data collection methods that emphasize different aspects of the reality. Systematic data collection using observation as a method requires that the observation have a focus. In research the focus usually is decided by the research question, and for observations the research question usually is associated with questions like: “What are we going to look for, and where should we look for it?” Observation as a data collection method is used in both the inductive and deductive data collection strategies. The more inductive the observation is, and the less the research question involves a strong and clear focus, the more we call it for an open observation, which is what we have done in our project.

Data collection using interview, or dialog, as a method, is primarily supposed to illuminate the research question (Kvale & Brinkmann, 2015). This means that the conversations will be targeted, i.e. aimed at getting a particular type of information. When collecting data through interviews, we can use both individual interviews and group interviews as strategies. In this project we have used both of these two types of interviews. To conduct conversations individually with people has the great advantage that the interviewee does not need to take into account how he or she appears to others. One is pulled out of a social context and can thus answer openly and honestly on questions. The best thing about conducting interviews in groups is that it not only brings out the individuals´ isolated opinions, but also how different concepts are discussed and elaborated.

During observation, the focus is on human actions, but on the basis of the observation you make you do not get an understanding of the reasons why people do what they do. Group interview is a good way to find out how views in a group are set up against each other, but this method is less suitable for bringing forward individuals' opinions. An individual interview, on the other hand, would be a strong method of bringing forward individuals' opinions, but it is a weak method to use to find out what people do, because not all do what they always mean. Of these methods, none of them will alone show reality. A research ideal is therefore a combination of methods, as we have done in our project, where reality is attempted described through several different types of data. In this way, these data can complement each other as well (Postholm & Jacobsen, 2011).

In the interviews both with the pupils and the teacher, we used a semi-structured interview. The pupils sat in the groups, and we let one and one answer the questions we had. We had two selected groups that I call for group A and group B. In order to find answers to my research question, I have in the interviews with the pupils been careful to ask relatively guiding questions about problem solving and motivation, to find out how their motivation is influenced by working with problem solving. In the same way, the interviews were conducted with the teacher. For the pupils to be anonymous, I have given them fictitious names. Group A, I have chosen in this master study to call them Ella, Lotte and Linus. Group B where it is a bit more mixed and more at a basic level of mathematics, consists of a boy with low skills in mathematics whom I have named Tor, in addition to a girl and a boy who are more at a normal basic level. These has got the names Jenny and Jan. The female informant in the project is called teacher in the analysis section. In Attachments 4 and 5, I have listed the questions that gave the pattern for the semi-structured interviews with the pupils, attachments 6, 7 and 8 are for the teacher.

3.2.3 Transcripts

According to Roth and Bautista (2011) most transcripts within mathematics reflect situations such as teaching, interviewing or problem solving. In addition to words, an extension with events and content is required. To transcribe a video with words only, flattens the observed situation into language. Reducing everything to words is called logo centrism. A transcript of this type requires a little more than playing a video and to notice of the words that are heard. In general, we use a digital video file and transcribe the words that are heard directly in a word processing program. Characteristic for this kind of transcription is that no temporality to the participants is included, both in terms of chat and physical appearance. Verbal transcripts within mathematics, such as this form, are most often used to find out what and how pupils think, how they solve tasks or how they construct their mathematical mental structures.

In my project I have used this type of transcription combined with a second form. This second type of transcript includes all sounds made, breaks, hesitation, respiration, and prosodic information. In order to understand languages, opinions are not important, but the way words are used. A speech consists of three parts: locution (saying something), illocution (the way it is being said) and per locution (the effect) (Roth & Bautista, 2011). I have used a little bit of both of these types of transcription. I included every word that were said by both the pupils and the teacher, and some acts as well. To keep the informants anonymous, the pupils were given fictional names, and the teacher is just called teacher in the transcripts (see method part 3.2.1).

In my analysis, I have looked at the various video recordings and sound recordings that are relevant to my research question. These have been transcribed and translated into English. One issue that I had to be aware of was that the inclusion of an interview will never be entirely accurate, as transcription loses information such as gestures, mood and tone.
The transcriptions of sound recordings is an extensive and time-consuming work, but are extremely important because they render the conversations so that the essence of the interviews will be conveyed (Thagaard, 2013). The transcripts that will follow in this project are not written in dialect and were first written in Norwegian “bokmål”. While translating the transcripts into English, I have taken necessary steps to maintain the proximity to the language. Therefore, the words and sentences might not be exactly the same as what the teacher or the pupils actually said. With that said, the transcript will still have the same main content as the original one.

3.2.4 Ethical aspects

As I did my research on young people, I had to consider that there were a few ethical issues. Children and adolescents have special protection requirements, and research methods must therefore be adapted. As a researcher, I have a major responsibility for maintaining the integrity of children, and all research must therefore be treated confidentially. In this publication, I must therefore make sure that those involved in the research remain anonymous (NESH, 2016). All information and data are thus treated confidentially, the participants' names and other identifiable information that I have on my own computer and external hard drive will therefore be coded with fictive names and pseudonyms for the participants, despite the fact the participants will be produced as I perceive them (Thagaard, 2013). When the project ends, all data, video and audio recordings will be deleted, but I will keep the anonymous transcripts and the report itself. The logbooks will be returned to the pupils, and the teacher can now make use of the best they want. I will not comment on pupils' logbooks or evaluate and mark them.

A piece of information on what the project is about, was distributed to the immediate superior and the teacher. As I did my project on children and adolescents between 13 and 15 years, I needed consent from their parents. In this letter (attachment 1 and 2 written in Norwegian bokmål) of information it was important for me to clarify what the project is about and what I wished to investigate, that the participation is voluntary and that it was possible to withdraw from the project whenever desired (NESH, 2016). With my information letter, I tried to give the participants in the project a little better picture of what the results can be and what the purpose of the research is and how I will analyse and interpret the data.
It is required that all empirical surveys that involve the processing of personal data must be reported to the Data Inspectorate (Jacobsen, 2015). Therefore, the project was also sent to Norwegian Centre for Research Data (NSD) to be approved in accordance with the guidelines (see attachment 9).

3.2.5 Methodical selections and criticism

When you have research questions in which you want to know why and how, a case study is a good choice. In this master thesis, I have chosen to look at how working with problem solving tasks can make the pupils more motivated.
I will try to figure this out by asking the following questions:

1. How might problem solving influence pupils' motivation to learn more mathematics?

2. In what way can a teacher influence the pupils' motivation to learn more mathematics by introducing them to problem solving?

This kind of questions pose many follow-up questions that I must answer to manage to come to a reasonable conclusion. In order to find an answer to this, I used observation as a method to view the pupils if they started a discussion and if they were able to solve the task or not. If they accomplished it, how did the pupils act? Were they satisfied and went on to the next task, or did they try to expand their responses or see if there were other solutions? If they did not accomplish the task, what did they do then? Did they ask the teacher for help, or did they try to find answers elsewhere? Did this make them want to learn more or did they give up? Observation is therefore not enough, and here is the reason why I must use the semi-structured interviews where I can examine more thoroughly and ask the pupils more how they felt and how the teacher experienced the problem solving activities. This is still not enough for me to be able to draw a conclusion if my research gives a definitive answer to my research questions. In order to achieve this, I needed to do the research on a larger selection and for a longer period.

3.3 PRESENTATION OF PROBLEM SOLVING TASKS

Research highlights the importance of pupils working on cognitively demanding tasks that promote reasoning and problem solving. Wæge and Nosrati (2018), use the word LIST to describe the types of tasks we have chosen: LIST (Lav Inngangsterskel og Stor Takhøyde), in English we say **Low Threshold High Ceiling (LTHC)** activities. These types of tasks are tasks that should be suitable for all pupils, regardless of which level they are at in mathematics and have many opportunities for the participants to do much more challenging math. Wæge and Nosrati (2018), describe the tasks as follows: Imagine a room. You can easily enter the room without any problems. Once inside the room there are many opportunities to do some activities. Many of these activities are unproblematic, while others are more challenging. The only limit set on your choice of activity is the room's space and its height.

Schoenfeld and Sloane (2016), mention that we must stimulate the pupils' imagination by giving them a lot of experience by thoughtful examples and that we should try to convince them by including frequent and realistic applications. It is important that pupils understand that what they learn is not meaningless, but important and useful. With this in mind the pupils therefore were given 12 problem solving tasks. These were to be solved together in the groups that the teacher had placed them in. All of the pupils had to do the first two tasks; therefore, these two tasks were in one way an introduction to problem solving. The tasks are described in the two sub-sections which follow.

3.3.1 The mobile phone task

This task Berry (2019) presented at the SAARMSTE conference, but we have adapted it to our survey, see attachment 1 task 1. The purpose of this task is to place mathematics as a collective intellectuality, place pupils as a supportive contributor to the ideas of each other and ask the pupils about their work and thinking in the classroom. Berry (2019), in a figure (Figure 7) in his presentation shows how to establish a mathematical goal that looks at the relationship between a task that promotes reasoning and problem solving and a task that builds a processual flow from conceptual understanding and how they are interconnected with meaningful mathematical discourses. The big box about discourse refers to four practices around discourse. The task we have chosen from Berry's presentation about the mobile phone can be linked to this figure. Pupils need to take advantage of problem solving to find an answer. The different strategies the pupils make use of, are entirely up to themselves. The idea of the task is that it should promote a discussion in the classroom. The task has questions that provoke pupils to think about what they actually want to find out when they see the first picture. The task also stimulates the support of a productive struggle to find the answer. This is achived by the pupils being able to use different mathematical representations. Exercising these representations increases the task of mathematical competence, in which the pupils participate. The task gives the teachers the

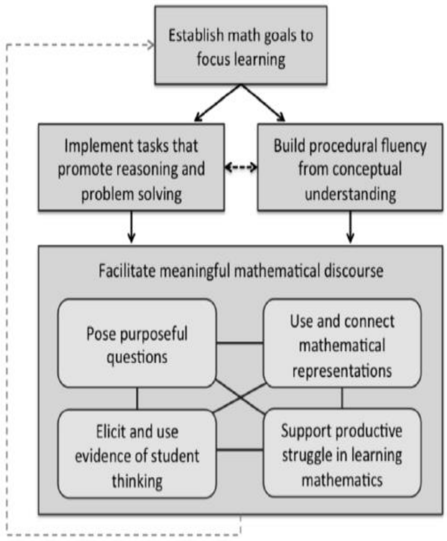

Figure 7: Effective Teaching Practices (Berry, 2019)

opportunity to understand the pupils' thinking, and see which thoughts are promoted with different skills. These do not necessarily need to be correct, as long as they can participate in the discourse.

3.3.2 The ice-cream cones task

This task is from Wæge and Nosrati (2018), (see attachment 1, task 2) and is a task that can be used at different grade levels. The idea of the task is that the teacher should go around to see when the pupils work, observe and come up with different hints to support them in the process. The task does not say how the order of the balls will be placed, or whether it has any significance for the answer. This is something that the pupils have to come up with themselves. This means that the task can have different solutions and make the pupils experience a large degree of autonomy. According to Wæge and Nosrati (2018), the task gives rise to four issues, depending on whether each taste can be selected several times, Yes / No, and if the order has something to say, Yes / No. The task is an LTHC activity, as it has a low entry threshold, which means that all pupils, regardless of their marks and level of knowledge, are able to work with it. The task also has a high ceiling as it allows pupils to develop a generalization of the problem. Wæge and Nosrati (2018), write that the task can be used in group work and that in this way it helps to satisfy the pupils' need for belonging. This leads to the fact that the inner motivation of the pupils is improved.

4 ANALYSIS

This chapter is divided into three parts, the first being a background description of how the teacher works with the pupils in problem solving, how the teacher introduced Pólya's working model for problem solving to her pupils and how the pupils approached the various tasks. This part of the chapter gives the reader a better survey of how the two obligatory tasks were dealt with.

The second part of the analysis is based on the second pupil interview with the two groups A and B. Here I have asked questions about how the pupils have experienced these two weeks, to find their thoughts on motivation and problem solving. To do this, I have put the questions into various blocks that relate to each other. The analysis itself first looks at group A's responses and afterwards group B, and I have commented on these answers with focus on research. I have then tried to find comparisons between the groups.

The third part of the analysis is based on the two post-interviews with the teacher. Here, too, I have chosen to put the questions into blocks to analyse the teacher's answers with the help of research. This part of the analysis is important to find out to which degree the teacher feels that the pupils' motivation has been affected.

4.1 BACKGROUND FROM THE CLASSROOM

In the first lesson that the teacher had with her pupils, she introduced them to Pólya's problem solving model, (see theory part 2.3.1). The pupils were given a notebook to use as a logbook, and they called the book "Abelboka[1]". The teacher introduced the Abelboka in the following way:

Teacher: In this book you can write only with ballpoint pen and with colour, and you are not allowed to erase anything. If you have a great need to remove something, then you should put a line over it, so that you still can see what's underneath.
What is the reason for that?
Unknown boy: To see how we think.
Unknown girl: To see what we answer regardless of whether it is wrong or not.

[1] The name Abelboka is the teacher's own ideas and descendants of the Norwegian mathematician Niels Henrik Abel (Ore, 2018)

Teacher: Yes, because the wrong answers are equal to the right ones, and because we should see what you think, and thoughts we cannot erase. We think new thoughts, and we cannot erase those we have already thought. Therefore, we can write new suggestions, but we cannot erase what we have thought.

Panaoura, Philippou, and Christou (2003), say that one of the main reasons why pupils may have difficulties in solving problems, is that they lack the ability to regulate the cognitive processes that take part in problem solving. In order to have a successful process in problem solving, several metacognitive processes are needed. Furthermore, they say that pupils who have high levels of metacognitive abilities perform better in problem solving tasks, as they take the time to control the facts and that they can break down complex problems to easier steps. Another important part of making use of their cognitive skills is to get answers and to clarify their thoughts. One way to improve on metacognition is to practice it. Therefore, Abelboka becomes something that can help pupils make their thoughts clear, and practice to find out how to take advantage of their metacognitive skills. This development can be affected when pupils learn, for example, new strategies and are tested on them. I will not analyse how the pupils have thought in their logbooks, as I do not need it in my research. This is just to show how the teacher and her pupils were working with the different tasks.

After the distribution of Abelboka, the teacher asked her pupils: “What is problem solving? What do you think when you hear the word problem solving?” She got different responses from her pupils; some of them were alike, and I have only selected a few of them: #Find the answer to a problem, #Find out about the problem, #that once you have figured out the problem, the answer is part of the problem.

Teacher: But what is a problem?
Answers from various pupils: #something that is difficult, #it does not have to be difficult, but it is when you are stuck and do not quite know how to find a solution. #a problem is something you can find an answer to, but you don't really know where to start. #something you want to find out that you don't really know the answer to yet. #a problem can also be when two answers contradict each other.
Teacher: Problem solving is when you get some tasks that you do not quite know how to solve. Perhaps you have not solved that kind of tasks before, or something similar has been solved, but not exactly like those tasks. One does not know exactly how to approach to these

tasks. Also, maybe one must use new methods. You said a problem is something that must be difficult. Then, is what is difficult for one difficult for all the others, too? Or can it vary?
Unknown pupil: We are good at different things.

4.1.1 Teacher introducing her problem solving model.

Teacher: Here, on the first page (showing the blank page), we are supposed to write something important, and that is simply like a recipe for how to proceed when we solve mathematical problems.

The teacher wrote her problem solving model, which is exactly like Pólya's, but she added a fifth step, see figure 8. The teacher carefully explained how to work between these four steps and then the booklet of problem solving tasks was distributed.

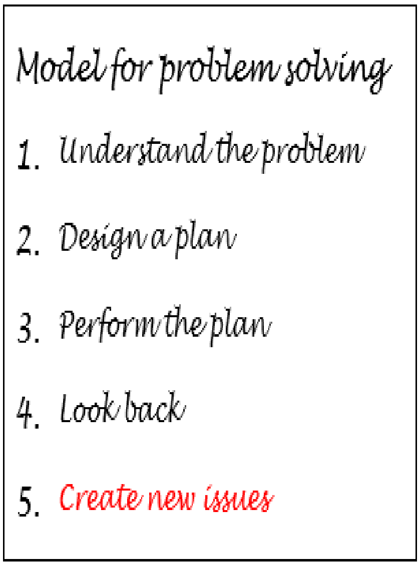

Figure 8: The teacher's problem solving model, translated into English.

Discussion on step 1:

Teacher: How many here have experienced that when they work with a test or a task, that they do not fully understand what the question says? What are they really asking for? Or if you are in a conversation or in a discussion with someone, what is it we really are discussing, what is the problem? And we don't get very far if we don't understand what the problem is. Many times, we also believe that we have solved a task, but then we find out that this was not the case. This is because we have really not understood the problem. So, what do we have to do then?
Linus: Read the task.
Teacher: Yes, read the task information carefully and, for example, put a line under things that we think are important or difficult. For some of these tasks you should solve, you may not be able to move on from step 1.
Ella: First I read through, and if there is something I do not understand, I try to see what they are asking about. Then I usually filter which information is relevant to the question and write it down. Then I look at the question again.

What Ella describes is actually step 2 in Schoenfeld's problem solving model (se theory part 2.3.2). To get an understanding of what is to be solved and to simplify it by gathering the information needed, is very important before one goes further in the process.

Step 2:

Teacher: This step here is a step that many people forget to do. You can use this on the mathematics exam and otherwise when you work with math. Because once you have realized what the problem is, what do you have to do then? Have you seen "Olsenbanden"[2]? This plan should not be a ten-page thesis. Is there anything I can't do, it's there anything I need to find out more about, which information do I have, what is omitted from the information? Do you see that they go into each other like this (pointing to steps 1 and step 2)?

The teacher talks about different strategies that can be good for pupils to use when planning, such as using different colours, using passers and rulers and creating patterns. While doing this, she points to the different steps in her problem solving model. This is something Bjuland, Luiza Cestari, and Borgersen (2008), calls "*pointing movements*". These authors describe various types of pointing. If a person repeatedly points to the same object, it is defined as repeated pointing. If the person points to different objects, it is called "*consecutive pointing*", and if a person holds his finger for more than 3 seconds on an object, it is called a "*held-point*". The teacher used the last alternative, like a memory marker, when she went through step 2. In addition, she used what Bjuland et al. (2008), call for "*sliding point*", to hold a point and move the hand between two semiotic representations (linear and circular), this took advantage of the teacher, when she went between the different steps.

Step 3:

At this step the teacher takes up "Olsenbanden" again.

Teacher: When to implement the plan, does it always go as they have thought? No, not always, it does not always go as they had planned, and such is the case here as well. We make good plans, but it is not always the case that you have planned for all situations. If this is not what you have thought, what do we do then?

Unknown pupil: You go back again.

The teacher explains between the different steps. "Have I understood correctly, or do I have to make a new plan?"

By mentioning Olsenbanden, the teacher may have caught up on pupils who are familiar with this series, and these pupils became even more curious of what she was talking about.

[2] Olsenbanden, popular Norwegian series of 14 feature films (1969–1999) (Svendsen, 2018)

Entering step 4.

Teacher: After a test, how many of you have received a feedback saying, “is this logical?” For example, you have calculated how much pocket money a 13 years old boy will have, and the answer you have found is that he received NOK 3,800 for pocket money a week. Is this logical? Or that a litre of juice costs 50 øre[3]. If this does not seem so logical, then you need to look at whether the answer is realistic. You also have to look back (pointing to the different steps again).

Step 5, the extra step:

This step is sometimes used because it is a little funny, so we write it in a different colour. Create new issues. The teacher further explains that problem solving tasks are not such which should only go from a, b, c and then one is done. So, when you are done you can create new problems with the help of the information. Or maybe you can create a rule where other things within a problem can be solved in the same way. Then you can use it, too. This step is based on two of Borgersen´s (1994), steps: 5. Develop a proof, and 7. Generalize the problem and formulate new problems.

Tips

* Read carefully and be sure you understand the text
* Draw
* Divide the problem into several minor problems
* Make a model, with things you have around you. E.g. colours.
* Look for patterns
* Trying and failing
* Use equations
* Create graphs

Figure 9: Tips from the teacher to the pupils, translated into English.

The teacher completed her introduction by asking her pupils to write various tips on the next page, see figure 9. These should be helpful when using the problem solving model. We also find these tips from the teacher included in the three different problem solving models from the theory part 2.3.1 and 2.3.2.

4.1.2 Pupils' work on the mobile phone task

The first task is the task about the mobile phone see method part 3.3.1 about the mobile phone task. This was a compulsory task that all pupils should solve. The task was read aloud by a pupil, (task 1 on appendix 1) and discussed in plenary between the teacher and her pupils.

Teacher: What is the time on this picture?

Unknown Pupil: A quarter to one.

Teacher: Do we see how much it is charged?

[3] Øre, is a coin and weight designation used in the Nordic countries, which corresponds to 1/100 of a krone. Since May 1, 2012, øre ceased to be a valid means of payment. Øre is still used as calculating coins (Sterri & Paulsen, 2016).

Unknown pupil: 13 percent
The pupils were able to read through the questions to the task and write down their thoughts. The idea was that the pupils only were supposed to make some ideas, so that when they came to the groups, they would be ready to discuss the task further.

After the pupils had written down their own thoughts to the task, they went to their respective groups. Now they would use the various steps (Polya's four steps) that the teacher had written on the board. Thus, the task was first discussed in the different groups in order to end in a new class discussion. In this way, the task became an introduction to how the challenges in problem solving might look.

What follows is an episode when Group A discusses the mobile phone task.
Ella: I wrote that it is equal time, but that it still was dependent on how it has loaded. Equal difference in time.
Lotte: No, because you see the first time it took seven minutes, while next time it was nine minutes, then it was nine, and then eight.
Ella: I meant the two which were alike
Linus: Do not charge at night since there is a fire hazard.
Ella: Should we suggest an estimate that it is about one percent? For the most part it was about the same length of time.
Lotte: That we take one percent every minute like that?
Ella: Yes, or should we say that it is a little more than one percent, because sometimes it was a bit more.
Lotte: Yes, for two of them it's over.
Ella: Okay, let's try to understand it then.
The pupils in group A did not fully follow the task information, they tried to solve the task directly. But then they found that they had to follow the model that the teacher had presented.
The second part of the task was: ***What do you want to find out when you see this?***
Everyone in the group agreed that they would like to find out when it was fully charged.
Ella: Are we going to make a plan how to get there then?
Linus: First, we need to find out how fast it charges.
Ella: Actually, we should have had more screenshots. Then it would have been easier
Linus: Or fewer.
Ella: Just two pictures, or?

Linus: Yes, then we had something to relate to, because here we see that it changes between each, so the percentage varies in relation to time.
Lotte: During 33 minutes, it has increased up 37 percent, so it sounds faster one time. At least, it goes up a little more than one percent for every minute.

The pupils discuss that in the pictures there are different percentage increase. We can see in Figure 9 how they thought when they solved the task.

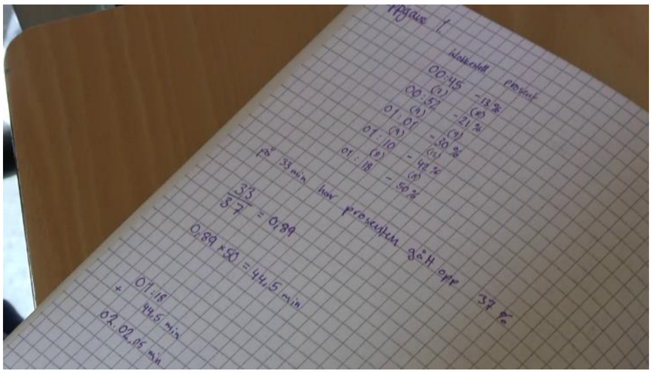

Figure 10: Screenshot from Lotte's logbook.

Ella: If we take an example with 1.2
Lotte: But it does seem uneven
Linus: If we take the average, type number variation width and all that.
Ella: Yes, but the average speed will be 1.12 if we look at the whole period all the time, but should we count on 1.2, or should we count on 1.12 if it gets bigger?
Lotte: I don't quite understand what you asked for.
Linus: So, the average speed of each percent is slightly more than one percent, then one percent per minute.
Lotte: Yes, but we have to go up 50 percent, and then it will take at least 50 minutes.
Ella calculates a little and says: I find that it will take 44.6 minutes to go up 50 percent. The reason is that I divided 33 percent by 37, so I found out how many minutes passed per percent. Then I found out that it will be about 44.6 minutes.

The pupils in group A continue to discuss what they have come up with and jump back and forth between how they should argue that the point of time of when the mobile phone is fully charged is based on the fact that the average it takes the first 50 percent must be the same for the next 50 percent. Both Lotte and Ella mention that this is logical. It must take a similar period of time throughout the process. They agree that they have used all the information they were given in the task and are satisfied with their answer.

Second lesson, the teacher asks pupils about task 1.

Teacher: What did you do in task 1, the one with the mobile phone and the charging? Did you come up with some problems, or was it pure reasoning solving this?

Ella: We felt we didn't get enough information to get a one hundred precise answer as we know for sure how quickly the battery percentage moved. It was uneven, but we got it pretty good considering the information we had been given.

Teacher: What you say with a 100 percent precise answer is a sure answer. Was this something someone else also discovered, that it was a little difficult to find one answer that is right?

Unknown pupil: Yes.

Teacher: Was it okay, or was it unfamiliar?

Unknown pupil: It was a little unfamiliar.

Teacher: Did it make the task better?

Unknown pupil: Yes, but we got a similar answer.

Teacher: Have you had such tasks before, which does not provide an answer?

Ella: I usually meet for chess all the time. There are endless possibilities there all the time (talking about chess), many possibilities can be bad, and many can be good.

The teacher tells that mathematics is a practical subject, and that even though we calculate in our books, it is about everyday life and everything around us. Therefore, there is not always an answer.

The sixth lesson, the last lesson in the project. Revision of task 1.

In the last lesson of these two weeks, lesson six, the teacher returns to task 1, to discuss how the pupils thought when they saw the task.

The teacher reads the task out loud in the classroom.

Teacher: Task 1, what do you see?

Answers from different pupils, at the places where I have set a ¤ sign, the teacher asks the pupil: “How do you see it?” #The phone has little power, #It is late, #She has Telia[4], #The phone is in saving mode ¤, It's yellow, # She has Bluetooth on, # She is not on the phone ¤, There is a lock up there, #The phone is in the charger ¤, You see it because of the lighting, #

[4] Telia Company AB is a Nordic telecom operator

She has location services on ¤, with that arrow. # She has full internet ¤ On the thing that can be reminded of sound. #Little coverage.
Teacher: What do you want to find out when you see this? It's an interesting question this one. Do you have the information for what you want to find out?
Answers from different pupils: # When is the phone fully charged, #The password for the mobile phone, #How long it takes to get the mobile phone up to 100 percent.
Teacher: The same as we said last time?
Pupil: Yes, almost. There is a slight difference because there is a difference between what the time is when it is fully charged and how long it takes to get it charged.
Continuing on what they will find out: #If the battery is as it should be, or if it is damaged? #How much does the different services e.g. the Bluetooth affect the charge? #Why she is up so late at night? #What day it is?
Teacher: Here are many questions that I have not thought of, which make it so much funnier when we can share it with one another.

A long discussion continues the lesson, and the pupils make suggestions about how they have solved the task. Among other things, it is mentioned that they can never get a 100 percent safe answer as the pupil phone differs all the time. For example, it is mentioned that saving mode is up to 80 percent. Then maybe the mobile phone loads slower. We do not know how many programs are in the background either, and similar things were said.

Teacher: In what way do you think this task is different from the tasks that you get for the exam, for example?
Pupil responses: #It gives us a challenge, #More answers that may be right, ask how you do it.
Teacher: Did you spend more time on these kinds of tasks, compared to what you do on the ones in the mathematics book?
Unknown pupil: Yes, much more time.
Teacher: Was this something you think was okay? Did you think about the time spent?
Unknown pupil: Seems it was okay, as there were not so many tasks we should do, and then we got much more time on each of them.
Ella: The degree of difficulty was so different that I spent more time on the new kind of task. Some were very simple, and others in a category one had to think long for.

As we can see in the various statements from the pupils, this kind of task awakens many strange thoughts in them. But this is good, as this is something that makes them start thinking more in the problem solver's world.

The teacher was clever at making use of everyone's discoveries to present, which info the task gave them and how it could affect the result, regardless of whether the discoveries were completely hopeless or if they were actually usable. The pupils came up with a number of answers, and most were about the same. They had used a lot of averages to arrive at an answer. What everyone agreed on was that they did not have a 100 percent response to this task. How did it feel? Many thought it was strange, since mathematics must have an answer, or? Others thought it was pretty good because it made them think in a way they weren't used to, that everything actually didn't have a 100 percent precise answer.

4.1.3 Pupils' work on the ice-cream cone task

The second task was also a mandatory task that all pupils had to solve. This is about Hanne who is going to buy an ice-cream cone, and she can choose from four different flavours. She wants only two ice balls. How is she going to choose these balls? (See method part 3.3.2 The ice-cream cone task).

The pupils started and had many different strategies on how they could come up with this answer.

Group A has discussed the task and concluded that it must be square numbers that determine how many choices Hanna has, but then this happens:

Lotte: I said it can be square numbers, but it's actually triangular numbers.

Ella: Is this triangular number? Yes, I understand. There is a difference between vanilla and chocolate. There is no difference between vanilla / chocolate and chocolate / vanilla.

Lotte: Yes, see you have four colours. At first you got this one, it can be stand alone. Same with the next one, it can also be with itself, but it can also be together with the orange. So, if we put the line here, then we also know that the orange will go here as well.

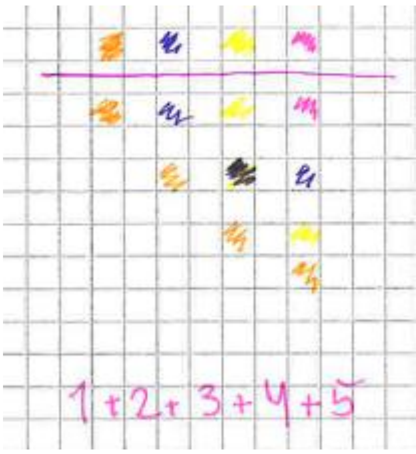

Figure 11: Scanned from Lotte's logbook, describes triangular numbers

Furthermore, Lotte tells her way of solving the task for Ella and Linus, and they agree that there are triangular numbers.
Lotte: I realized this when I was on the bus.
Teacher: So what do you think of this task afterwards?
Lotte: I always come up with solutions afterwards.

The teacher asks them to think about their answer before they agree it is the right one.
The group reads the thesis again. Then Ella says: Is there maybe any difference if I take chocolate / vanilla, and vanilla / chocolate? I say it in different order.
Lotte: In that case, it seems to work as we started, we can take the green ice-cream and then put on orange. No, I don't think that's what they mean.
Ella: We can say we thought so, too.
Linus: Yes.
Lotte: Yes, we have two identical combinations, but they are the opposite of each other. But it is as if we have two numbers first, one takes one, later the other. The answer is the same.
(Describes the commutative law, without knowing it).

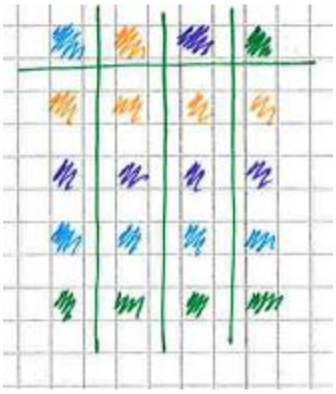

Figure 12: Scanned from Lotte's logbook, describes square numbers

If you have four flavours, you can have ten different combinations. But this is when the order of combinations has nothing to say. If the order counts, there will be 16 combinations, but then they use square numbers.
Teacher: How will this be if you have more flavours?
Lotte: It will be the same way, but you just add one flavour. For example if you got five then you just add one. For triangular numbers you will have 15 instead of ten and square numbers give you 25 instead of 16.
Teacher: Do you manage to make a rule do you think? An algebraic expression. Do you take the challenge?

The pupils started to try this out, but they never find an answer. Even though the teacher gave them some hints, they still couldn't find what they were looking for. They were very close and had some thoughts about using the previous triangular number. By using their formula book, they found a formula that was actually the right one, but they didn't trust themselves to be totally sure. This is something that is very hard and needs a lot of practice, but still, it challenges the pupils in a mathematical way and will evolve their thinking. This latest part of the task put the pupils in Borgersen´s last three steps of his problem-solving model, to generalize, reflect and to develop a proof for their findings. During the actual process, they used Schoenfeld's cycle between analysis, design and exploration, as they had to go back to make a new plan or return to the analysis section.

In group B, they disagreed slightly on how to solve the task. They drew four circles and tied them together. After that they started to count.
Jan: one, two, three and the same four. Also, the next can be with one, two, and three. And that can the next one, too. So finally we have 13.
Jenny: Yes, but if you have one, two, three and four, then you have one, two and three. This will be seven. After that one, two and the last one is alone. So then we have ten.
Jenny found this by drawing lines back and forth between the four circles she put in the logbook.

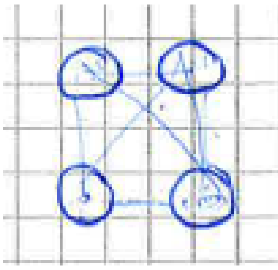

Figure 13: Jenny's illustration of ice-cream cones task

Jan does not quite follow Jenny's plan.
Jan: It can go with three, and itself, that is four.
Jenny: Yes, and it can go with two pieces and itself. Then we have seven. And this with one and itself, nine. Also the last with itself, then we got ten.

Actually, Jenny and Jan discuss the same thing, but they talk in slightly different themes and do not quite get what the other one thinks. A commognitive conflict occurs, something that Sfard (2008), defines as a communication that can be both to communicate with oneself and with others. It occurs when one tries to communicate but refers to various things that are different and incompatible and do not share a common ground.
The group agrees that the answer is ten and goes on to other tasks.
This task was not discussed in front of all the pupils, but it was one of the tasks the pupils discussed in the groups. One of the most common ways in which pupils proceeded to solve this task was to use a table where they structured the different flavours. Group A used colour codes on the different ice-cream flavours. The teacher walked around among the groups and gave them different hints and asked if they were happy with their answers, or maybe there were other solutions. We could see this in group A when they started looking at square numbers and triangular numbers and the teacher tried to get them to generalize the answer. What would happen if Hanne wanted three flavours instead, or what if there were five different flavours?

The teacher's support in this task meant that both group A and group B progressed but perhaps not completely reached their goal. But both groups completed the task with a good feeling and could move on to other tasks. With the help of the group work on this task, the pupils had been involved in mastering, collaborating and in this way increased their inner motivation. Also Tor, even though he did not say so much, had got notes in his logbook written that the answer was ten different ways to choose ice-cream. The teacher has had a big part in this when she has guided and observed the pupils' strategies. This has made them produce a meaningful discussion (Wæge & Nosrati, 2018).

4.2 INTERVIEW WITH PUPILS ABOUT THEIR THOUGHTS

To analyse how the pupils have experienced these two weeks, I have divided the second pupil interview into three categories which are composed of questions that relate to each other. The first block looks at questions related to the relationship pupils have to mathematics.
The second block looks at how pupils experienced problem solving as a working method.
And the third block looks at how pupils experienced mastering during this work on problem solving tasks.

4.2.1 The relationship pupils have to mathematics

I wanted to know how the pupils' point of views were in relation to mathematics. Which relationship do the pupils in groups A and B have to mathematics, how do they really like the subject? I knew from the pre-interview with the teacher that Group A consisted of pupils who had a good understanding of mathematics, and group B consisted of pupils who were more at middle and basic levels. I started the interview by asking the pupils what level they themselves thought they were at in mathematics. In group A, I got the same short answer from the three pupils and that was the words "High level". The pupils in Group A thought that they all had a high academic level. The same question Group B got, but the answers were a little different. Tor, said that he had a low level in mathematics: "I can't do math, almost never get anything right, so I mean I have a low level". Jenny was a little more in the defensive position and thought she was better than she performed: "I think I have a fairly decent level of that upper medium, but I have not got the marks I want, feel that I am unfortunate with tests and practice". Jan, also said that he was at an average level: "I like mathematics and think I perform well at a medium level".

How do these pupils really feel for the subject itself? Below are two questions that I chose to use to examine the pupils´ answers more thoroughly.

Do you like mathematics as a subject?

This question gave similar answers that I already had received. In this question each of the pupil in group A answered that they like mathematics. But in group B the answers varied. Tor answered: "No, I don't like math, I don't get it", Jenny said: "I like math a little, but think it is difficult, and as I said, I think I can achieve more than I perform, so therefore I like it and do not like it. Jan was also a little mixed in his feelings about math. "As I said, I like it somehow, but not always. Seems that it can be a little difficult sometimes”.

Group B is very uncertain whether they like or do not like mathematics. Why is it like this? Do your feelings towards a subject have any influence on how you perform? Is it in a way that when you are good at something, then you also enjoy it? As group B pupils say, they are not so interested in mathematics. Is this because they don't understand it? According to Tobias (1994), prior knowledge has long been considered the most important factor affecting learning and pupil presentations, and interest research has shown that people work harder and learn more with tasks related to their interests than others. Holm (2012) mentions that uncertainties associated with a school subject often result in pupils trying to avoid or escape activities that affect the subject. In a learning situation, motivation and well-being become two aspects of the subject. If one fails all the time, it will inhibit motivation, and the well-being will also deteriorate. This also means that the attitude to the subject becomes repellent. To control their emotions and anxiety, several pupils with mathematical difficulties describe a clear strategy to support the mathematics lessons, linking lack of mastery with lack of effort. In this way you can protect yourself and your own image (Holm, 2012). The earlier experiences that the pupils have from the subject mathematics can be both positive and negative. The positive experiences can help the pupils manage to believe more in themselves and therefore put a little greater effort into new challenges. If the pupil, on the other hand, has a negative attitude and a poor self-image of the mathematics subject, with the feeling of being stupid and that he does understand anything, this can lead to the pupil giving up. The earlier a pupil gets these feelings and thoughts, the greater the influence it has on the subject of mathematics. If, on the other hand, the pupil manages to turn this around to be able to reflect on his or her learning through being activated and regulating his earlier knowledge and thoughts, this can lead to something positive about the pupil's learning and mastering (Sjøvoll, 2008).

What do you like, or what do you not like about mathematic?

On this question group A answered like this:

Ella: I like the sense of mastery I get. I do not like it when I get tasks that are too easy. This makes the math boring. It also gives you challenges and encourages the systematic thinking.

Linus: Math is okay, and I find it convenient to be able to know how to use it. I don't like that sometimes the same tasks come again and again. I do not like it when the teacher speaks during the whole lesson.

Lotte: I like to feel the mastery when I get things done. I don't like doing tasks that are very similar, for this is not challenging. When things go too slow and we go through the easy things over and over instead of understanding the rest of the chapter, I am bored.

Mathematics is easy for the pupils in this group, and as I mentioned in the previous question, Holm (2012) says that earlier experience in a subject has both positive and negative effects on it. And if a pupil has good experiences, he will put a little more effort into new challenges. These pupils in group A all have high competence in mathematics, and they all seems to be interested in the subject. A challenge is to keep them motivated, as they can easily become quite bored if the tasks deal with similar topics and have too little variation. Deci and Ryan (1991) write that pupils like tasks that interest them, whether or not they lead to the achievement of rewards and other goals. If the tasks are of interest, this increases the motivation for the pupils.

Group B answered the same question in the following way:

Tor: I almost like nothing about mathematics. I don't think I'm going to use mathematics so much in the future. I think it's boring. I don't like it to be so difficult ALL THE TIME.

Jenny: I think mathematics is a difficult subject, and I manage to do something, but not everything, and I don't think things are nice when I don't get it. I do not like mathematics to be so difficult and that it must have such advanced questions etc. Math is fun as long as it is practical, if it is not, I think it is very boring.

Jan: I like math when I understand something and when I understand what I'm working with, but I don't like math when I don't understand what we are working with or when I don't feel like I obtain mastery, and I do not like that there is so much you need to know and to count.

There are lot of aspects pupils have to remember in the subject of mathematics. Mathematics contains of many formulas and algorithms that pupils need to learn, and this can cause pupils to become annoyed when they do not remember how to use them, especially these pupils who have lower self-esteem in mathematics. If the task becomes too complicated, we risk that pupils with lower marks lose their self-confidence and give up. Pupils who have great reason to fail because they do not have sufficient knowledge within the area, can experience anxiety. The pupil can thus get a lower effort or in some cases get a small boost and try a little extra to avoid failure (Pantziara & Philippou, 2015).

This means that a pupil who is afraid of failure can both be perceived as a pupil who withdraws or as a pupil who works motivated towards a goal as it would rather create success. A pupil's motivation can be influenced by its self-image of earlier experiences that the pupil has been exposed to in mathematics. Lunde (2009) says that motivation and goal orientation relate to each other, and if a pupil experiences not mastering mathematics, it will thus reduce his self-image. This causes the pupil to lose faith that mathematics is something it can achieve. A negative attitude to the subject and low motivation can in the worst case contribute to reducing learning or, at worst, stop the whole learning process. Therefore, the goal must be to protect a reduced self-image, more than trying to achieve the academic goals.

4.2.2 How pupils experienced problem solving as a working method

Because I want to know how the pupils look at motivation in relation to problem solving, I have in this section chosen questions that are related to the pupils' thoughts on how they have experienced these two weeks with problem solving tasks. The first question that I will discuss and analyse is:

When you were working on problem solving, how did you like it?

Answers from group A:

Ella: I think it was really funny, because I felt I got it and that I had to use the brain a lot, and there was a lot of logical and systematic thinking. I liked to solve big, difficult tasks instead of working on small, easy tasks, because I like variety, and I had to use more of my brain.

Linus: I think problem solving was pretty nice because it was a little different from what we usually do. Working in groups was very funny. I thought it was quite nice to work in groups and make up our minds about the problems we didn't normally have.

Lotte: I like it because you have to think for yourself “or in groups” how to solve the task, because you have to interpret the problem to find out how to find a solution. I enjoyed working with others who thought the same in groups. It was nice to work with a type of tasks I haven't worked much with before.

Answers from group B:
Tor: There were practical tasks, and challenging. I liked that we got into groups and discussed. I enjoyed working with other pupils and listening to their opinions and then we discussed the tasks together and came up with the right answer.
Jenny: I liked it very much! It was different and exciting tasks, and I liked that we got to work in groups and I also liked that it was a bit out of the ordinary, and that there was a little variation in the math lesson compared to what we usually work with. It was a little better than regular lessons because we did something else. It was also exciting and new, and I got a feeling of mastery when I thought I managed it. But when I didn't get it, I was depressed.
Jan: I thought it was funny, I liked it because it was something different from what we normally do, and it was nice to work in groups. I also think it was nice because we had to think in a completely new way.

In both groups, the pupils were very positive about working with problem solving. Pupils found that problem solving is interesting, especially because they were allowed to work in groups and collaborate. That they had to think outside the box was something that looked interesting to these pupils, also that there was a little unusual variation in the lesson.
As Ella says, it was fun with great variety and that she had to use her brain more. This is something Wæge and Nosrati (2018) clearly propose that, problem solving tasks give pupils a sense of autonomy, which means that they find it more fun to work with mathematics. Pupils get the experience that they learn more when they think for themselves and have to decide for themselves which tasks they will work with and how they will solve the tasks. What makes me particularly happy, is that those pupils who also have poorer mathematical skills, liked problem solving. Common to all the pupils is that they very much like to get variety in mathematics lessons, not just having the same regular traditional mathematics teaching.

Was it just because the lessons were different the pupils liked to work with problem solving, or did they actually think they could learn from it? To find out more about this, I had this question:

Do you think that problem solving can help you improve in math?
In group A, the students answered as follows:
Ella: Yes, I think so, because it makes you think in different ways. It was a good way to collaborate with other pupils, and I think that if I work more with problem solving, then I will probably improve in solving text tasks.

Linus: Yes, because it helps you find new ways of thinking and because there is good cooperation.
Lotte: I think it can help me because it makes me think differently, and working with math in various ways can help you see what you are good at and what you need to work more with. Problem solving may imply many ways of calculating and is therefore a good way to work with math.

Pupils who develop a conscious relationship with their own learning, learn about learning, and think about how they learn, are better equipped to solve problems in a reflective way, alone and with others. By creating a broad understanding, pupils gradually develop knowledge, skills, strategies, attitudes, motivation and ability to interact with the environment. If the pupils also obtain increased motivation by encouraging the teaching to their interest, this means that the pupils engage more in a way that promotes understanding (NOU 2015:8, 2015).

Group B answered the question with the following statements:
Tor: Do not know, maybe, 50/50, but then we must have more time to solve the tasks.
Jenny: Yes, maybe yes, maybe a little, if I start thinking a little more about the questions.
Jan: Don't know, maybe I can think a little bit about it. Yes, I thought so, because this means you have to use the methods that you don't use too often.

The answers I received from group B on this question are not very complementary. Therefore, I chose to combine it with the answers from the previous question, about how they felt working with problem solving. Here they answered that they liked it by working in groups and having a collaboration. On this question they answered that thought it could help them develop their math, but as Tor said, they only need more time. Problem solving can help you improve in math because it expands the horizon for methods we can use when we solve tasks. Collaborating with problem solving has many benefits, pupils are more involved when working with problem solving, and pupils with lower math skills feel more confident because they help and support each other by getting collaborative support. The development of mathematical communication and logical reasoning also helps pupils see a broader selection of problem solving strategies and alternative solutions (Zsoldos-Marchis, 2015).

The pupil gets a better feeling for mathematic because of the cooperation in the groups. They also get the feeling that there are some advantages working with problem solving. You need to use more math than you usually do. Nunokawa (2005), says that by using realistic real-life situations, this can reduce mathematical anxiety, and because of the interest and knowledge the pupils have in the situation, they will therefore try to explore such tasks closer.

What did you like / didn’t you like about working with problem solving?

This question is because there are both advantages and disadvantages to a new working method. So, it becomes important for my analysis to know what the pupils think. Group A said the following:

Ella: You really have to use your head, and you have to see things from different angles to find a logical answer. I like that I got to work with someone. It made me see the problems from a different perspective.

Linus: I liked the fact that with problem solving it was a bit different math than just working with tasks in the book. What I liked best about working with problem solving, was that I heard how the others thought when we tried to figure out the answer.

Lotte: I liked that we worked in groups and to listen to the other's methods that might work better than mine. I think it was very good that there was variation in the difficulty of the tasks. I also liked the fact that the tasks required you to think outside the box.

Group B has previously mentioned that they do not like the subject of mathematics. On the question of how they liked problem solving, they responded like this:

Tor: Challenging, and I liked working best together with the groups, and if I make mistakes I can talk to those in the group and understand better. That we got plenty of time and that the tasks were quite practical.

Jenny: I liked working with problem solving more than most of the topics we have. What I liked best, was to do something a little bit out of the ordinary, that I got a sense of achievement and that there are less tasks, but they are harder and challenge me.

Jan: Collaborate, use different methods to arrive at the answer and that it was something new and that I managed the task and challenged myself. That there was some variation in the lessons. I liked the fact that there were different tasks, where you had to use more common sense and I also enjoyed working with others in groups. That I had someone there if I needed help, or the feeling of being able to help.

Working with problem solving means that pupils need to think in different ways, by having to use everything they know and have learned from their normal math lessons. One very good aspect is that the pupils get to hear and discuss different strategies to solve the problems, and this can help them develop in mathematics by seeing how different pupils have approached the problem. Both if you are at a high level and at a basic level, the pupils answer that they like the challenge. Problem solving has advantages that are important for becoming independent in the subject mathematics. Problem solving shows pupils a more accurate math, than just solving tasks that are ritualized. Problem solving tasks give rise to mathematical minds, as you encounter unfamiliar situations, analyse contributing factors, create a plan, and finally execute the plan (Schoenfeld, 1985).

4.2.3 How do the pupils experience mastering by working with problem solving tasks?

The idea of how it feels when the pupils work with problem solving tasks is important to me in this study. Is it a fact that the pupils got to experience control? How did they really feel? As I mentioned in the theory section 2.4.5, a part of the teacher's role is to get pupils to strive for a desire to be educated. This is something that is influenced by the pupil's mastery and their self-determination. Therefore it is important to let the pupils become more autonomous. As Deci and Ryan (2002) describe, autonomy is influenced by interest and values. In this part of the analysis I have therefore chosen to deal with the questions about how the pupils have felt mastery and defeat.

Did you ever get the feeling that you would, should or needed to learn more math?

Group A answered like this:

Ella: I felt like I wanted and that I should learn more math when I couldn't solve some of the problems. But I have always felt that I want to learn more math. It has not changed. Now I want it even more, like when we found some problems impossible to solve, I thought it would be good if I could have worked more with text tasks.

Linus: I felt I could do what I needed, and I feel that I know what I have to know. I always want to learn more math, but I didn't get that impression when I was working with problem solving, because I managed to do everything. If I get a mistake on a task or test, I will often find solutions the next time because then I know it's something I have to learn.

Lotte: I occasionally experienced that I want to learn more about mathematics, for example when I came across something new or that I did not understand so well. If I have time, then I try to tackle the problem and try to get a better understanding. When we did the tasks, I felt that knowing how to use mathematics is quite practical and that I want to learn more because of it.

In group A the pupils have good motivation for mathematics, and they always feel that they want to learn more. If they couldn't solve the task, they wanted to get information so that they could solve it. When I observed the pupils, I did not notice that they wanted to try something else, they just continued with the same strategy until the teacher gave some hints. Such hints are important, and if we want pupils to continue to grow in motivation, we need to show them different ways of solving the various problems.

In group B, pupils are aware that they need to learn more mathematics, but from the answers they give, they do not have the stamina to get it done.
Tor: I feel that I must get better in mathematics, but I do not feel that I want to work with mathematics at all. I feel that I have to learn more, just to get better marks. I don't think I need to be able to do and know everything we learn in school. So no, I feel I can and know what I need to know.
Jenny: I do not feel that I need all the advanced things we learn when I get older, but it is not so funny that I both get bad marks and feel a little stupid. Yes, I feel I should learn more mathematics. It will help me with my marks in the future, and I should just look a little more at math videos and really dare to ask for help. I am a little split, but both yes and no. Clearly, it would have been funny to handle all the tasks, but I feel that I am not sufficiently interested in mathematics.
Jan: I feel that I am trying to understand mathematics, but of course there are some tasks and subjects that are easier for me to understand than others. After every test we have, I know that I can do better than my marks show. But, yes, I should learn more math, maybe just get better at asking when I need help. Thought a few times that I might be able to do a little more, but it went pretty well anyway.

Tor has an attitude that is not particularly good for the subject. He doesn't see the benefits of learning math. For him it is enough to learn to get better marks. In the theory section 2.4.2, I write about behaviourism, about the behaviour of the pupils and their actions. As Tor says, he knows he has to learn math to get better marks, but he won't learn more when he feels he knows what he needs. Marks can be an amplifier and can serve as a reward for some, but not necessarily for all! Therefore, it is difficult to motivate pupils as a whole through one and the same "reward". Accordingly the teacher has to find tasks or ways of working that can help Tor find joy in mathematics. Jenny says she has no interest in learning math, as she does not see the utility of it. At the same time, she knows that she needs to work with mathematics to get better marks. She also has the desire to solve the task and not feeling stupid. So Jenny has a lack of inner and outer autonomous motivation, as she does not seem to like the subject of mathematics, nor does she see the benefit from learning math. Lack of motivation and that Jenny has an experience of not mastering the tasks, cause her self-efficacy and expectation of mastering mathematics to weaken. As Figure 5 shows in the theory section 2.4.6 on the relationship between motivation and mastery, this will create a vicious circle if the teacher does not help Jenny to get a sense of mastery. Negative expectation of coping leads to poor efforts and little perseverance, which in turn will lead to Jenny easier giving in to try. As Jan says, he can do something better than others, and he thinks he gets worse marks on test than his knowledge should perform. He also says that he would like to be better in mathematics, and as theory says about self- efficacy this can have a positive effect on his learning. If he manages to master and handle his expectations, this can lead to success in the subject. This is important for the pupil's motivation and will influence his choice, commitment, effort and perseverance. Self-esteem has to do with the pupils´ belief in what they can, and will influence events that affect their lives and are the basis for experiencing motivation.

On the last question I had two follow-up questions. I added these in order to get the pupils to become a little more explanatory in their answers about how it felt to master or not master a task.

If you did not solve the tasks, how did you feel?

Group A felt:

Ella: If I managed to solve the task I felt a great sense of mastery. If I didn’t make it, I thought I could try after I finished the others. I feel that we understood all the tasks, but at some it was almost like we didn't understand them, and then I either got more motivation to solve it, or I skipped it before going back later.

Linus: It was a little silly that I unable to solve all the tasks, but I felt that it was quite normal since you can't handle everything. I was a little annoyed when I didn't understood the tasks because we had spent so much time on them.

Lotte: If I don't understand a task, I want to do it even more because I like to get things done. Therefore, I become more motivated, but if there are many tasks I do not get, I can lose my motivation and concentration.

Frustration and annoyance is something that most of these pupils feel when they are struggling with a task. In group A the pupils actually gained motivation when they didn’t manage to solve the task directly. According to Akay and Boz (2010), research has shown that when the pupils experience problems, they become more motivated and keen to find answers to these. This explains why there is a strong positive relationship between attitude towards mathematics and mathematical success, leading to math being accepted as a link for success or failure. Therefore, it is important to improve pupils' attitudes to mathematics and their confidence. Another thing the pupils mentioned was that they got a bit frustrated and felt stupid. But as Ella says, they did not think of it for a while and then came back to it later. Lotte mentioned that if there had been more tasks which they failed to solve, she would probably lose motivation and her concentration. According to Holm (2012), a good school performance in mathematics is related to the fact that the pupils are willing to do an effort. Pupils who have experienced negative things with mastery, doubt whether they will master upcoming challenges and therefore have a tendency to reduce their efforts. On the other hand, those pupils who are able to experience control that are associated with their own effort during the mathematics lessons, will get stronger motivation for the will to learn more within the subject.

When the pupils in group B didn't manage to solve the tasks, they felt this way:
Tor: I was a bit annoyed, but mostly I didn't feel anything special. I didn't feel that we didn't get the tasks, it was the time I thought most about, that we didn't get enough time.
Jenny: I didn't feel anything special if I didn't manage to solve a task, maybe a bit annoyed and that I needed to take a break from the task and come back again after a while.
Jan: I think it was stupid, and I got a little headache, but I tried not to give up. I do not know, I felt irritated and stupid. I felt a little stupid or empty in my head, a bit torn and abandoned since it looked like there was no solution.

Also in group B, the pupils had a feeling of being a little annoyed. But both Tor and Jenny thought that they really didn't have any strong feelings. Jan, on the other hand, felt abandoned when he did not get it. At the same time he would not give up, but became a little stressed as there was no answer to all the tasks. One important factor for Tor was that he thought there was little time pressure and that this was what influenced him the most.

Mathematics can be time-consuming. Mathematics is manipulated primarily by limiting available time for each decision or choice. Caviola, Carey, Mammarella, and Szucs (2017), say that several studies have shown that time pressure disturbs the decision-making process by changing the strategy. The presence of a time limit in any math or problem solving situation can affect performance: The presence of time limits can either encourage pupils' engagement with the task or increase the choice of the wrong strategies for the task. Time pressure is therefore a factor that influences what strategy people choose to handle a particular mathematical problem. In addition, Caviola et al. (2017), mention that previous problem solving studies indicate that time constraints inhibit creative thinking but recent research suggests that time constraints can sometimes prove beneficial. It may be important if attention is directed to the performance or outcome of performance: These situations of the attention system can affect the results. The pressure not only provides a reduction in leading resources, but can change the pupils' motivational state, leading to failure or success.

So what did the pupils do? I asked them two follow-up questions about what they did if they failed to solve a task, and how the feeling was after this.

a) **What did you do when you did not manage to solve the tasks, and did it help you? (QA)**
b) **If yes / no, how did it feel after this? (QB)**

The pupils in group A answered like this:
Ella: (QA) It was just one of the tasks we didn't get, and then I asked how to do the task. When I did not solve the task, I asked for help from the others in the group to get a different viewpoint on the task. It helped, because sometimes we came up with different answers, and then we could try to understand each other and discuss what was most correct.
(QB) Yes. It felt like I understood the math, I was happy and got a total feeling of mastery. It also felt good to hear other ways of solving it, because then I could use it in another context as long as this was a better way.
Linus: (QA) I asked the others in my group or the teacher, and it made me understand the tasks. Tried even more, and if we still did not make it, we went on to the next task.
(QB) It seemed quite okay to ask someone else and to hear what others were thinking. I was happy when we managed to solve the problem after we did not understand it.
Lotte: (QA) When I have an answer to a task, I double check if my answer is correct before I move on to the next task. If I did not know the answer I asked the others in the group. Tried even more, and if we did not manage to solve it, we went on to the next task. I started on the next task, and when I was finished, I went back to the task I struggled with. Then we asked the teacher and got small hints from her. She never gave us answers, but it helped us know what to look for.
(QB) If you have to find the answer all by yourself, it's more challenging, and therefore more fun. I was proud when we got the tasks done.

By working in groups, the pupils have the opportunity to ask each other when they are unable to solve the tasks. This is something that all pupils used when they had struggled with the various problems. The discussion in the group was important. But of course, they received help from the teacher, but her task in those two weeks was not to give answers, she just guided them. As Lotte mentions, they took advantage of the teacher's tips, and in that way they could get back on track. According to Archambeault (1993), problem solving works best in interactive situations, where the teacher is given the opportunity to guide the pupils when examining possible problem solving strategies. This can be difficult for the teacher as he or she must be able to ask good questions and guide pupils through the tough mathematical world. Pupils will usually engage in these tasks, so it is the teacher's role to provide appropriate support without removing pupils' ability to learn. The way the teacher can do this, is to ask questions without giving answers. In addition, the teacher must listen carefully and take an assessment on where the pupil's surrender lies and make the pupils talk about their strategies. In this way, the pupils find the answer themselves. The pupils also mention that they often just went to another task, and went back after solving that one.

Group B

Tor: (QA) We talked about it in the group many times, and it helped a little sometimes. If we answered wrong and got to know how the task was solved, I felt that I learned more from the task, and it helped to go on with the other tasks.

(QB) I felt good after realizing it.

Jenny: (QA) I got angry, but tried to find another solution. Then I tried again, occasionally, I asked those in my group what they thought, and we often came up with an answer. Otherwise we took a break and came back later. "Thinking outside the box".

(QB) After I found the answer I was happy, then I got a sense of mastery.

Jan: (QA) I tried and read the task again until I realized what I had to do. I also tried to think of other ways to come up with the answer. I either asked the teacher or those I worked with or tried to look around the question a little. Otherwise I read in the book and found helpful answers and calculation methods. Sometimes we just jumped to the next task unless the teacher was present. Discussed with my group. Went through the task again and discussed with the others in the group, and it helped

(QB) Yes, it felt good, relieved.

During the observation of the six lessons, I noticed that group B almost never asked the teacher when they needed help. It happened once in a while, or when the teacher came up and talked to them and asked how the work was going, they asked for help. Are they perhaps afraid to ask for help, or do they not care? One reason why pupils do not ask the teacher so much, and rather ask a group member, can be that the pupils feel more equal to their classmates than with teachers. If the information becomes too difficult, they lose motivation and then the teacher must intervene and lead them on the right track. They also say that mathematics is too difficult and they can't really understand why the need to know all these things. My own observations say so, and Tor mentions that when they got tips from the teacher or when they managed to learn how to solve the task, he got happy and felt that he had learned something. This made him move eager with other tasks. All the pupils became happier when they managed to solve the tasks. The pupils were satisfied when they managed to solve the tasks. "One gets a sense of mastery, it makes one happy when one manages a task. It becomes more fun. Knowing how others think makes me better in mathematics", were answers they gave.

The pupils asked questions to the teacher and their group and explained what in the task they did not understand. To ask the rest of the group and get their opinion, was sometimes helpful. Their answers give me the feeling that when they actually managed to get something out of the task their motivation increased, so they were ready to move on to the next task.
As Skaalvik and Skaalvik (2015), mention about coping, there are both positive and negative consequences. If a pupil masters a task, he or she will recognize this type later and remember how this type of task was approached. On the other hand, failing to experience mastery can cause the pupil to not look at similar tasks at all. Furthermore, they say that observing what other pupils do, as the pupils in this study have been allowed to do in the group work, can cause the pupils to feel that they can also cope with the task. By encouraging each other or by getting support from the teacher, the pupils obtain a better faith in themselves.

Bottge, Heinrichs, Chan, and Serlin (2001), recommend that teaching for pupils with difficulties in mathematics should contain meaningful problems that give pupils training opportunities where they can use past experience. Such teaching will help pupils connect their new knowledge to future activities. The teaching also should help pupils to use their ideas in groups, where they feel confidence and affiliation. In addition, the teaching should be at such a level that the expectations of the pupils are high. By using such a teaching, this will, according to Bottge et al. (2001) motivate the pupils to want to work more with the subject. These authors also write that if there is too much confusion concerning the tasks, this can mean that the pupils get a low performance. In such cases, it is important that the pupils explain the tasks as they are able to understand the context. Unfortunately, can problems with word many times according to Deci and Chandler (1986), indicate that pupils´ motivation is weakened. This is especially among the pupils who do not read very well. This may mean that school spends time on something that is not beneficial to all the pupils. Therefore, it becomes important that when pupils strive to be effective, they need to know how to do it, and they will be happy and motivated to know that they have done well.

Deci and Chandler (1986), also tell us how positive feedback is something that can easily be used to get an increased inner motivation among the pupils. So when the pupil does something right in a task, it is good to note this, also in the tasks where the pupil may not have got the correct answer. The positive comment that is said by the teacher is something that strengthens the pupil's motivation. This can also have the effect that the pupil only does what the teacher wants it to do. This is why there are no errors by negative feedbacks. In case one should give these negative feedback, it is important that they are not evaluative, as the pupil usually has very little control over this, and it knows very well that it has been wrong. The negative response can therefore be given in a way that points to the error when a problem is solved and guides the pupil on a new track.

As a final question, I asked pupils if there was anything else they had to say about these two weeks.

Was there anything I did not ask them about which they felt they had benefited from? Was there anything they felt motivated them to work with problem solving?

In Group A, the pupils clearly express that there is a relation between problem solving and motivation:

Ella: I would like to have problem solving again since then I really have to use my head instead of always doing the same kind of tasks. After solving the tasks I feel that the motivation goes up even if I do not think about it. I also feel that when you have finished a task that was difficult, you get a good feeling that also helps with the motivation.

Linus: I thought it was good that we worked in groups, because then it did not feel so boring, and then I was able to keep my motivation up.

Lotte: The motivation changes because one works in a different way.

The responses from group B show that there is a link between problem solving and motivation also among these pupils:
Tor: I am never motivated to work with mathematics. But the motivation changed when I had to work with it. I feel that it was not something that helped me improve my math skills with problem solving, but my motivation changed a little, perhaps with more and better teaching and smaller math tests instead of one or some big, maybe that might be good.
Jenny: At school, I do not think we have to learn so many different things which we feel we will not benefit from when we get older. We would rather have small chapters where we learn what is most important for each. So in problem solving, I learned that we do not need to get answers for all questions, because there is not always an absolute and final answer. So I hope math can be more like the kind we did in the problem solving because then math for once can be funny.
Jan: I think that mathematics is unnecessarily difficult, but I thought problem solving was better than what we usually do. I went out with more motivation than I had before we started, because I got motivational progress when I finally found the solution to the problems.

Skaalvik and Skaalvik (2015), write that there is a connection between how the pupil feels mastery and its motivation. The pupils usually have an attitude and expectation of what they can and cannot do, and therefore they choose tasks based on the level they feel they are at. High-level pupils are often more motivated for math as a subject, but they can easily find that math becomes boring as it is repetitive and predictable with similar tasks all the time. In general, too much of the same thing and no focus on stimulation can lead to pupils becoming mentally tired and bored. Problem solving is therefore another approach for pupils to use their mathematical skills. As the pupils in group A say, it was much more exciting to work with new and different tasks, where they had to use mathematics in other ways.

Difficult tasks can affect motivation and learning both positively and negatively. Baker, D'Mello, Rodrigo, and Graesser (2010), say that pupils who prefer to be fed with difficult tasks usually do not have so much trouble if they fail, but the pupils who prefer slightly easier tasks do everything to avoid mistakes and to get a negative feeling. Furthermore, they mention that tasks that are too simple can easily become too boring. In contrast, the tasks that pupils perceive to be too difficult can lead to anxiety. As for group B, they say that math is difficult and they can't see what they can use it for. But from the answers I can also see that these pupils have experienced some kind of motivation when they have worked with problem solving. These pupils usually do not see how mathematics can be useful to them. Problem solving gives them the hope that they can actually take advantage of the things they have learned and that everything really does not have to have an answer, but it is what you have done along the way and how to find the answer that is important. As Jan says, he has more motivation for mathematics now than he had before the two weeks began.

Pupils who unfortunately do not feel motivation or mastery in the subject such as Tor, mentions that he does not like mathematics at all. This can lead to what Skaalvik and Skaalvik (2015), say about pupils who fail to solve mathematical tasks at school. These do not have the belief to succeed later. The pupils lose the expectation of learning mathematics at a more general level. To avoid this, the teacher must add mastery to each pupil, so that the foundation for continuing education is good experiences. This can be achieved with the LTHC tasks because these problem solving tasks do not necessarily have answers that you find in an answer book. These can help motivate pupils at all levels, and contribute to that everyone can experience mastering. Coping expectation leads to motivation, and increased motivation can come from working with tasks as problem solving. Another important thing is that by using these LTHC tasks and by working together in groups, pupils observe that other pupils at their own level are able to solve a task. This causes pupils to see that others master the same thing that they are struggling with. When the pupils experience mastering, this can lead them to cope with similar tasks later (Skaalvik & Skaalvik, 2015).

4.3 INTERVIEW WITH TEACHER

With the teacher we first had a pre-interview, and then a post-interview. The focus of this analysis is mostly on questions from the post-interview, in addition to the second post-interview I had with her.

In the pre-interview with the teacher, we learned that she works in a class that she experiences as a normal class, as most of the pupils are at a medium level. There are also some who are on a basic and some on a high level.

4.3.1 Teacher's thoughts

What are your experiences after this two-week problem solving period?

Teacher: All in all, my impression is that the pupils have liked it well. I see that in the Pear Deck there is someone who writes no and they also compare against the tasks in the book. They say that they are not more motivated because they must work with the same tasks in the

book afterwards. But when I talked to these, the majority with the exception of one said, that they thought this kind of task was funnier to work with and that it was more motivating. So it was a bit interesting.
(This pupil who said no, has not been able to take part in the whole project. The pupil was not included in the briefing, and missed a lot of lessons on the way).

Do you think they have had learning outcomes for these two weeks?
Teacher: Yes, I certainly think so, not necessarily such a clean professional, but some have had that too because they have been actively searching for information to solve the tasks. But I think this is the part we often ignore: How do we solve tasks? What kind of toolkit do we have to solve these tasks? How do we proceed? How important it is to read the assignments carefully over and over again. To place the information, what is it we are going to use, what should we not use? These are the tasks that are used for exams, I mean they meet similar tasks in part two on exams and on tests. It is clear then they know a little what type of procedure they are going to use, but they still have to find information and use the information and put it back in the context, and they get the training here.

A regular mathematics exam in Norway consists of two parts. Part 1 does not allow pupils to use any aids, but in Part 2 they are allowed to use all aids that are non-communicative. As the teacher mentioned, her pupil gained the experience of how to approach difficult tasks by working with problem solving. To use a problem solving model also when dealing with normal tasks can be good for the pupils when they are arranging the information that's needed to solve the task. As Pólya's model illustrates, it is important to understand the problem. Have you solved tasks similar to this kind before? With training, we expose the pupils to a larger base of knowledge. Beyond this, the pupils use different strategies. The Ministry of Education and Research (2015), says that learning strategies involve being able to recognize and formulate mathematical problems, represent them in different ways, develop a solution strategy, and evaluate how reasonable the solution is. This is important for progressing when something is difficult and is a part of developing endurance. The pupils' motivation to learn, the willingness to achieve goals and experience of autonomy and relevance will also affect pupils' learning. In addition, the pupils' expectations of their own control have significance for motivation, effort, perseverance and the goals they set, and are therefore related to competence in learning. The pupils' motivation for learning and expectations of mastering are influenced by different conditions, including previous experiences of mastering or failing, the knowledge the pupils have in a subject, and the support they receive in the learning environment.

How do you motivate your pupils to solve tasks?

Teacher: By basing choices in the teaching of the pupils, let the pupils gain insight into the purpose of the choices for the method that is taken, and let them have the opportunity to influence and come up with feedback in relation to the teaching. I believe it is a crucial factor to motivate pupils. At the same time, customary teaching and constructive feedback are important in helping pupils to keep their motivation up.

By giving the pupils alternative approaches, one can also motivate them to try again. Alternatively, one can ask questions that help the pupils look at the tasks in a different perspective. Suggest to use new strategies can also help pupils look at the task with new eyes. They get a new start with a new approach.

To make pupils motivated to solve tasks, is about being able to appeal to each pupil's inner motivation. We all want to experience mastery, and we all have an inherent desire for knowledge. They can provide usual tasks that both contribute to an increased sense of mastery, while at the same time requiring the pupils to develop professionally.

Questions from the teacher can help pupils put their thoughts into words, or even make them understand why they are not moving forward with the task. In (Opplæringslova, 1998), is said that pupils must get training to see connections. It requires scheduled and easy-to-use tasks over time. Pupils should be able to use their knowledge in new situations, and this requires them to get a survey of incoherent facts. By allowing pupils to use their knowledge from theoretical and practical situations from before, this can help build up the pupils’ motivation and get them engaged. According to (NOU 2015:8, 2015), this means that the pupils must be able to see mathematics as sensible, useful and valuable, and having faith that it is possible to become competent in mathematics, and that efforts contribute to learning.

Personal interest can help strengthen the inner motivation, so if the teacher is able to make the pupils feel that they are the ones who achieve the results, and to strengthen this impression with positive feedback, this can support the pupils´ experiences of competence and give a positive effect on their motivation (Ryan & Deci, 2017).

4.3.2 Pupils' development according to the teacher

Who do you think has benefited most from this?

Teacher: I noticed one group with pupils who are not so academically strong. They did not have much progress and should have had more help from a teacher to get started. So it is clear that the method has its limitations when I walk around as I do, and then when I want to focus on all the pupils, I also focus more on some groups compared to others. I noticed that this was a group that has gone a little under my radar, so maybe if they had got a little more help, they would probably have had a little more benefit from this type of teaching. But my impression is not necessarily that it is the academic level that they have which is decisive for what academic advantage they experience from this working method. I saw that some of the groups at a basic level of mathematics managed to discuss and ask questions, so when they start to figure out how it works, they will benefit from this type of teaching, and this is not necessarily done in two weeks. This was an introduction and is something that must be worked on over time to be good at, and here we come to what goes into the basic skills in mathematics, the oral skills. This is something that one can become even better at and practice and they obtain much more when they work like that in small groups. And if they make advance, they also become better at improving each other.

As the teacher started her answer to this question, she said that it is hard to manage to give all her pupils the attention they deserve. This project had to focus on groups, and she gave them a bit extra during these two weeks. Due to difficulties in seeing everyone, this group had been overlooked, and in that way might not have received all the benefits that problem solving could have given them. But this does not mean that they have not received any impulses from the activity. Problem solving in groups can lead to positive collaboration, as this facilitation from the teaching can contribute to an increased sense of togetherness in a pupil group, which in turn leads to increased internal motivation (Skaalvik & Skaalvik, 2015). Working in groups is a good way of getting pupils listen to each other. They need someone to ask questions which they have not thought of themselves, in addition to justifying and explaining to others.

The three following questions deal with how the teacher believes that motivation has an effect on her pupils at the different skill levels.

Do you see that pupils who have a lower skill level in mathematics, get or do not get motivation when dealing with problem solving? How?

Teacher: Problem solving motivates pupils regardless of the academic level. However, when one looks at pupils who work at lower skill levels in mathematics, one sees that this group of pupils expresses that they appreciate this type of working method because they feel that the tasks given are more real compared to the mathematics they are going to work with later in life. The distance between classroom mathematics and everyday mathematics is reduced. Similarly, it is seen that these are tasks where the pupils have a certain freedom of choice. They choose the tasks that appeal to them the most. At the same time this is another way of working where they not only work in groups, but rely on the others in the group to move on. Likewise, the others in the group are also dependent on the individual pupil. This can both contribute to motivation and sense of mastery.

An example the teacher told me was from one of the groups where a pupil who usually works at the basic level, in combination with customary, simplified tasks, worked with the other pupils in the group. In the beginning, he was unable to keep up with the same pace as the others in the group, but fortunately he dared to ask the others why they did what they did, or meant what they meant. Wæge and Nosrati (2018) describe it, when pupils are included in such a task where they are able to question each other, then they may find that this pupil´s questions are crucial to progress in their work. Without questions and demands for justice, they would not be able to continue. Participating in mathematical discussions and conversations can lead to pupils experiencing mathematics to be more important.

Do you see that pupils who have average skills in mathematics, get or do not get motivation when they work with problem solving? How?

Teacher: As I said about those pupils with lower skills, pupils who work with tasks that reflect the medium level of competence in mathematics may find that, through problem solving and working with other pupils, they will be able to solve tasks that are at a higher level of competence.

Both weak pupils and those at an average level thus benefited from the method, but could one really expect the same from the last category, the pupils at the highest level? For these pupils one might assume that problem solving would also seem simple, they therefore would not particularly benefit from the method. Excited about the answer, I asked the question:

Do you see that pupils who have high skills in mathematics get or do not get motivation when dealing with problem solving? How?

Teacher: Pupils who have a high level of competence in mathematics will appreciate problem solving, both as a working method, and but also because problem solving as a method gives them room to wonder and develop their skills and mathematical level in a different way than traditional mathematics teaching can. The pupils are given the opportunity to choose tasks themselves, which also allows them to choose tasks they had not usually encountered in a traditional mathematics book. Some of these pupils may, especially in the beginning, experience a greater degree of frustration. One of the frustration factors is that they are going to work in groups, and that the tasks require something different from them than they traditionally experience with "textbook tasks".

As the teacher says, the pupils get a selection of tasks that they can choose from when they work this way. In the theory section 2.4.5, I wrote that this is something that is influenced by autonomy. The fact that pupils can choose for themselves based on their own interests, can help them to develop both their inner and outer motivation. See also figure 4 which shows how self-regulating processes contribute to coping and motivation, by the fact that the pupils are allowed to do math which is influenced by their interest in the tasks.

4.3.3 Teacher's evaluation of the working method

This last part of the analysis describes and analyses how the teacher evaluated the working method.

Which benefits do you see with this kind of work?

Teacher: The biggest advantage is that they get to see the usefulness of mathematics, they get to use the mathematics, they begin to talk, and they are able to use each other. To conclude, the math becomes more practical, as it should be. Not least the pleasure of working with mathematics in itself, and getting a pleasure from it, is important. Then you have managed to awaken the inner motivation to continue working with mathematics. This is probably the biggest advantage.

The teacher is very positive about working with problem solving and finds many good answers that are the result of the working method. But it is equally important to me to know if there are disadvantages so I ask this question:

You have mentioned several advantages with problem solving as a method in mathematics. Are there absolutely no disadvantages?

Teacher: It is quite clear that this is most advantageous. There are no direct drawbacks to working with problem solving, as I see it. I would rather call it challenges. Challenges in dealing with problem solving are quite clear. The way of working requires more from the teacher. He or she must be confident enough in his role to dare to be wondering with the pupils. The disadvantage of this in a school perspective is that one has a time pressure on what one should do and how much the pupils should have gone through in subject matter. I tried to explain to the boys today, because they said they thought it was so wonderful when no one said they had to do so many tasks and that they could use the time they needed, and they could work as long as they wanted with a task. This was something I really thought they would be frustrated with, but they really did appreciate it, but it has been in this way through mathematical history. The mathematics that I work with now has not been the result of people working with a time pressure. It has come about because people have worked with problems, thought about them, discussed with others, put them aside to take them back again and think further. This is a process that takes time, and I feel that here we allow the pupils to work in line with what they can discover in mathematics and that is a positive thing, but the backside is then that it takes a lot of time and unfortunately more time than we have in school.

Borgersen (1994), states that if the pupil does not manage the problem solving task, it may be advisable to put it aside to find new methods and gain a deeper understanding. Next time, the pupil will be able to master the problem. This is a normal process according to Borgersen (1994), and it is therefore important that the pupils try to ensure that this is part of the process and not give up without just trying again. If you then succeed, you will experience the good things with mathematics, and you can then appreciate the task and enjoy it. It is important that the pupils also appreciate the actual process through the entire problem solving.

Do you think they are dependent on a teacher, or do you think they did this without your help and hints?

Both, and I see there are many pupils who have managed to do a lot, without the involvement of me. It is especially those who are happy to ask the questions themselves, who are well advanced in the process. I believe that what they need assistance with, is both helping them to see which different strategies they can use and the moral support through the process, "I think you can do this." It is so important to say, but then it is clear that sometimes these are the questions. I see that I should have worked a little more with the tasks before they got them.

During these two weeks, I have observed the teacher as she walks around helping her pupils. I got to see when she asked guiding questions to them. As she says, this helped the students, but in some cases they managed to solve the different problems alone with the help of the group. The next question is about the teacher's belief of how her own knowledge and strategies influence her pupils' strategies.

What do you think of the guiding questions that you give? How is it to hold back what you want to answer? You have a way to solve a task, with the strategy that you use. But when you see that the pupils are on a completely different track, how do you ask the questions then?

Teacher: You just have to change your mind then. I think it's exciting to see when they solve things in another way than how I thought it was. Oh yes, you thought that way, so it's interesting, you learn more about both your pupils, and you get more strategies and more solutions and approaches when you work.

I can tell about the blond hair task which I began to think about What is this? When I went around talking to different pupils and different groups working on this task, I got their questions. And their questions together made me manage to solve the task, and this I explained to the class afterwards. If it wasn't for their questions, I could never do the job.

How did they react?

Teacher: Well, they smiled. They are a little reserved, but it is clear that it must have been nice for them to hear. Because when one points out how important it is to ask these questions, and dare to wonder about things, dare to wonder in general in questions like, why in this wide world this is here, what is the point of that, how does this coincide?

According to Wæge and Nosrati (2018), in a mathematical discussion it is not just about the pupil telling how he or she has thought. The teacher must see the connections between the different methods and the mathematical ideas that make up the learning goal of the lesson. This will help the teacher know what to listen to and let all the pupils share all their ideas. Wæge and Nosrati (2018), call this "open strategy sharing". At other times, the teacher chooses to lead the discussion towards specific learning goals, to bring out specific concepts or representations, a "targeted discussion".

4.4 SUMMARY OF THE ANALYSIS

The analyses from the interviews have shown both the pupils' and the teacher's thoughts on how work with problem solving tasks has influenced the pupils' motivation.
There are both disadvantages and advantages of the working method.
If the pupils do not have enough motivation to work with the subject, the likelihood is great that they are not motivated to look at the tasks from a wondering perspective either. When pupils dare to work with the problems for a long time, and have no problem to put them aside, and then return to them to try again, they will profit from the working method. In this phase, the teacher is required to be present and to guide and encourage so that the pupils do not give up. In school, a teacher often has eight to ten small groups at the same time. It goes without saying that this can be difficult, and you risk that some groups do not get the guidance they need to move on.

Challenges from a pupil perspective are that it can be difficult to work with a specific task over time. The pupils often want to finish with a task, but in the problem solving it is all about developing new problems. You find yourself wondering about the subject, and in this way you get the opportunity to develop professionally.

Problem solving is also based on allowing the pupils to have a certain freedom of choice, something which makes it difficult to associate this kind with a specific subject area. In addition, when one learns that this is a working method that requires teachers to set aside time, which contrasts with a curriculum with many specific goals, and little time to achieve the goals, one understands why many teachers struggle to create space to make easier problem solving in teaching.

Other factors may include knowledge about problem solving in teaching, lack of experience among multiple teachers, and some textbooks support a false picture of what problem solving is. An example of this is the textbook “Factor”, which after each chapter has a separate page with problem solving tasks that the school in the project used. In practice, these tasks are essentially text tasks related to the chapter. But by using similar problem solving tasks used in this project, LTHC tasks, through all school years, you finally have a greater opportunity to allow pupils to truly explore math.

5 Discussion

This has been a qualitative study that has been based on a case study. Therefore it becomes difficult to assert that this is something that works universally. The findings that I have made I have tried to connect to the theoretical data, which means that they can be included in further research.

5.1 About the tasks

The tasks that have been used in this study, are tasks that should be suitable for all pupils, regardless of their level of math, and have many opportunities for the participants to do much more challenging math. The content of the task can be quite simple, but the level the pupils have to think on can be demanding and very sophisticated. Therefore, this type of assignment fits the new Norwegian curriculum in mathematics which is planned for 2020, as it gives pupils the opportunity to think mathematically.

In addition to this, with LTHC tasks we can start the whole class with the same activity. The tasks also create a classroom discussion that allows each pupil to feel involved. They also get to hear what other pupils in the class think and how they have solved the tasks.

The tasks meet all the pupils at their level, even those who usually do not like mathematics or are able to express themselves with mathematics. With these tasks one can focus more on what they can and not on what they cannot. Clever pupils can be stimulated and attempted to elaborate a little more and perhaps make them ready to generalize the various problems.

To master and be competent in mathematics is often related to intelligence. This means that many pupils perceive their achievements at school as a measure of intrinsic value (Holm, 2012). Thus, many pupils struggling with mathematics and having difficulties in the subject can feel stupid at school. This is something that affects the pupil's self-acceptance, and because of this, many pupils choose to focus entirely on whether the answer is correct or wrong, more than they work with the learning process towards the goal. The experiences and the calm the pupils receive from their different school goals, affect their self-assessment and expectations of mastering.

According to the Education Act (Opplæringslova), all pupils should receive adapted training. This applies to both pupils who have difficulties and those pupils who have talent in the various subjects. Alternatively, children are entitled to the teaching being adapted to their abilities, facilities and conditions. That means, teachers must give them satisfactory challenges so that they can benefit from the teaching. There is no easy way to do this, but LTHC activities can do it. In addition, the curriculum mentions that pupils should cooperate and be able to express themselves orally discussing mathematics. The pupils who struggle in mathematics will have challenges that make them able to experience mastering. Here too, the LTHC gives the activities this opportunity. Pupils are allowed to do mathematics at their own level. Tasks should be adapted to the pupils' skills and the level of knowledge they find themselves at. All pupils are required to see the connections and the structure in the subject and learn how to think mathematically, by being activated and challenged in different ways (Lunde, 2010). In order to let pupils experience mastery, the teacher must praise the pupils, and they must be recognized for this. When the pupils see that they are able to improve further with the tasks, they will experience encouragement and it will be a kind of reward that positively influences the pupils´ motivation. In addition to this, working in groups can help the pupils get more information, more ways to see the problem in several ways and further experience is exchanged.

Separately, we may not be able to solve a problem, but with the help of each other we get puzzle out pieces that enable us to move on. An example of this is for example the teacher who was stuck on one task, but when she walked around and heard what the various pupils had to say, she finally managed to solve it (See analysis section 4.3.3).

5.2 THE CLASSROOM

Teachers want their pupils to be able to solve problems, both in mathematics and in real life. If pupils fail to solve problems with the mathematics they have learned, what will they use it for? This means that pupils must have a deeper conceptual understanding of mathematics, if not they will only be able to solve tasks that follow specific routines. As problem solvers, one needs a good understanding, as problems usually create an imbalance and confusion among the pupils. By getting trained in problem solving, the pupils will therefore be able to create strategies on how to tackle the various problematic tasks that they approach. If pupils gain understanding, this will improve their problem solving ability.

The pupils in this study had to put themselves in a mental state where they needed to understand how to use the various mathematical skills they possess. They have developed and grown within mathematics as the problem solving tasks have forced them to think on a more demanding level. In section 4.2.2 the pupils answered that they were pleased that they got variation in their teaching. Pupils from both groups enjoyed working with problem solving tasks because it made them think in a new way. Like Jenny says, she got excited and got a feeling of mastery when she thought she managed to solve the tasks. She also mentions that the problem solving tasks were a lot harder and that they challenged her in a new way, that the textbook tasks normally do not do.

Burns (1996), mentions that by using manipulative learning methods, that is, specific methods that can make pupils see task in a more practical way, and not only in the traditional learning method, the teacher can provide more motivation and increase the pupils want to learn. Manipulative learning methods help to make abstract ideas more concrete. It increases the pupil's self-confidence by giving them a way to confirm their reasoning. This makes the learning of mathematics more interesting and enjoyable, because it gives the pupil the option to work with a problem in his or her own way. Solving problems with this approach gives pupils a clear advantage and helps them see mathematics as a topic that must be understood, not memorized.

The pupils mention in section 4.2.3 that their motivation have changed after working with problem solving tasks. For example, Ella explained how she got good feelings when she solved tasks that she experienced as difficult or challenging for her, and that this led to increased motivation. Linus added that he felt that it was great to work with these tasks in groups, because discussing the tasks with classmates helped him to keep up the motivation. Another pupil Jan, who does not really like the subject of mathematics, says that he felt that working with problem solving tasks “was better than what we usually do”, and that he got more motivation in the subject after finding solution to the problems he was introduced to during the teaching these two weeks. These responses from the pupils indicate the effect that variation in the teaching, as well as the experience of mastering the tasks that the pupils are given, have for their motivation. According to Bandura’s (1997) theory about self-efficacy, the pupils’ motivation will increase when their expectation of mastering is experienced through actual managing to solve the problems that they are facing.

Allowing pupils to develop mathematical problem solving strategies contributes to so much more than increased motivation. Motivation caused by problem solving tasks contribute to other important skills. Pupils can learn how to take leadership, and have to collaborate, they get a more flexible mindset and adaptability, and in addition, they can learn to take their own initiative and gain ownership of their results. Pupils receive effective oral and written communication, they have access to analysing information, and curiosity and imagination are created during the work with the tasks. In this way, the pupils become much more creative in their thinking, and that increases the interest in mathematics a little more. Problem solving is thus something that is highly recommended, as it gives a great variety to school's everyday life.

After having worked with problem solving for two weeks and having had the opportunity to see that the pupils become more active and how they deal with the tasks, speak, ask questions, and use the concepts, have made me feel that that the project has been a success. Just to see how well the pupils thought it was to present their answers on the board in the last lesson, was so motivating. These two weeks have been very motivating to me for my future job as a teacher, but probably for the pupils and their teachers as well.

6 Conclusion

This research project has shown that there can be a link between working with problem solving tasks in mathematics and motivation. The pupils who have taken part in the project have in different ways experienced motivation, something which suggests that working with problem solving tasks can have a positive effect on pupils' motivation. Based on theory and with the help of the answers from interviews and what I have seen in my observations, I have made this model (see Figure 14). The model describes how problem solving might influence pupils' motivation to learn more mathematics.

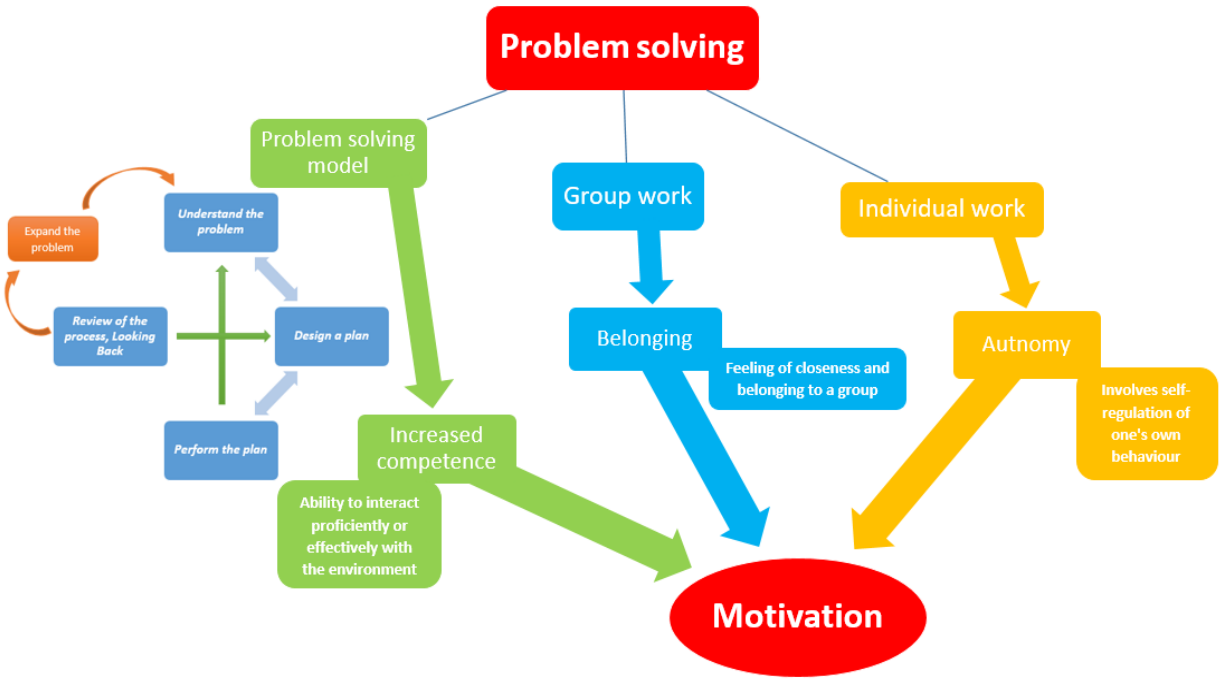

Figure 14: How problem solving, and motivation can be related

The model is divided into three tracks, all of which can be linked to what Skaalvik and Skaalvik (2015) mention about the theory of self-determination.

The first track shows how to work with problem solving models such as Pólya's (2004), Schoenfeld´s (1985) and Borgersen´s (1994) models, and lets us see how pupils get to experience mastering by means of increased competence. This first track enables pupils to critically evaluate themselves, their thoughts, and others, and create the expectations of success. Pupils can therefore, through problem solving, be trained to judge their own and others' work. When pupils work with problem solving tasks, they are subjected to a process where they continuously can try and fail. They experience control when they interact in a knowledgeable and efficient way with the environment. This can lead to increased competence, something which in turn may lead to increased motivation.

The second track in my model is group work. When pupils work with problem solving tasks, they have the opportunity to work and discuss together in groups, and when they are allowed to do this, the sense of feeling affiliation is strongly strengthened. According to Lyngsnes and Rismark (2016), Vygotsky and the social development theory, say that learning is distributed and by working in groups the pupils will learn from each other. It is also mentioned that combined with listening to how others solve tasks, has a positive effect on the learning environment, and experiencing a good learning environment can further lead to increased motivation among the pupils (Skaalvik & Skaalvik, 2015). Thus, being part of a group is something that can have a positive effect on the pupils learning because of the exchange of mathematical reasoning and strategies developed among them.

The third track leads us to see how pupils can work alone with problem solving tasks. When the pupil chooses a task to work with, this choice can be influenced by their interest, and according to Deci (1991), the motivation of the pupils is influenced by the interest they have for the tasks. Thus, if the pupil is in a position to choose, the pupil's feeling of autonomy might strengthen, and he the feeling that he is the one who is responsible for his own learning. By encouraging pupils to take such initiatives, the motivation can be increased.

This master thesis has shown that problem solving can have a positive effect on how pupils' motivation can be changed by using problem solving as a working method. But since this is a case study, I cannot make a general conclusion on this, it is something that needs to be more investigated, and one needs a much larger basis for being completely safe. But what I can say with certainty is that the pupils in this 9th class have noticed an effect, and they were all positive for having taken part and tried a new method of learning math.

6.1 Implications for further research

My research project has looked at the relationship between problem solving and motivation in mathematics. However, as this is a study that extends over a shorter period of time, it may be interesting to look at how motivation to learn more math among pupils can be improved by the help of problem solving tasks over an extended period. An example of this could have been what the teacher in this study has worked with earlier, having the problem of the week.

What we do not know, is if this method is something that will increase interest among the pupils on the subject of mathematics. Therefore, by examining this on a larger population, by using more schools and more years in a research, this may perhaps lead to a more general conclusion.

6.2 Implications for teachers

Having done this research work, I myself have noticed that using mathematical problem solving tasks is something I must try to use more in my mathematics lessons. My own observations about the working method have been very positive, and my opinion is that the project has been successful. Certainly, I have not managed to generalize the results, nor have I been able to come up with a wider conclusion, than that the pupils in the research project have experienced control, motivation and developed strategies that they can use in mathematics.
My opinion is that problem solving is something that teachers should use more in their teaching, and I hope that when they read my master's thesis, they may want to try it for themselves. Especially now when the curriculum is about to be changed and problem solving is to become a more central part of mathematics, it becomes important that more teachers try the method.

I hope my master's thesis has helped the reader to understand how to use problem solving in school and to see how it can create a positive learning environment in the classroom, with good discussions among the pupils.

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ATTACHMENT 1 - CONSENT FORM LETTER TO THE IMMEDIATE SUPERIOR (NORWEGIAN)

Informasjonsskriv

Vil du la barna dine delta i et forskningsprosjekt om problemløsning i matematikkfaget?

Dette er et spørsmål til deg/dere som foresatte til barn på XX ungdomsskole om å la barna delta i et forskningsprosjekt hvor formålet er å undersøke problemløsningsstrategiene til elevene, samt om problemløsning skaper motivasjon i matematikkfaget. I dette skrivet gir vi deg informasjon om målene for prosjektet og hva deltakelse vil innebære.

Vi er to masterstudenter fra Universitetet i Stavanger som ønsker å samle inn data til våre masteroppgaver. Formålet med prosjektet er å tilegne seg kunnskaper og erfaringer om læring og undervisning i matematikk. Arbeidet vil dreie seg om bruk av problemløsningsoppgaver, der vi ønsker å se på hvilke strategier elevene bruker, hva lærerens rolle betyr for arbeidet, og se hvordan problemløsningsoppgaver kan føre til motivasjon i faget.

Vi kontaktet en praksisskole, og tok videre kontakt med en lærer som jobber på denne skolen. Dermed er utvalget denne læreren og hennes klasse. Vi ønsker å observere elevene og læreren i matematikktimene over to uker i januar/februar. Det er også ønskelig å samle inn to gruppeintervju av elever, samt to lærerintervju, i tillegg til aktuelle arbeidsbøker. Det vil bli gjort video- og lydopptak fra undervisningen og intervjuene, i tillegg til feltnotater. Hvis dere ønsker å se intervjuguidene på forhånd, ta kontakt med Pontus eller Linda.

Det er frivillig å delta i prosjektet, og eventuell deltakelse innebærer at dere når som helst kan trekke samtykke tilbake uten å oppgi noen grunn. Alle opplysninger vil da bli anonymisert. Det vil ikke ha noen negative konsekvenser for dere eller barna deres hvis dere ikke vil delta eller senere velger å trekke dere.

Vi vil bare bruke opplysningene om barna dine/læreren til formålene vi har fortalt i dette skrivet. Veileder i tillegg til oss to studenter har tilgang til opplysningene. Vi behandler observasjoner og kommentarer konfidensielt og i samsvar med personvernregelverket. Dette vil vi være bevisste på gjennom hele prosessen, altså i innsamling, bearbeidelse, analyse og presentasjon av data. Alle navn vil anonymiseres med fiktive navn. Skolen vil også være anonym. Deltakerne vil derfor ikke kunne gjenkjennes i publikasjonen. Video- og lydopptak vil bli oppbevart på en sikker måte på egen datamaskin til bruk i prosjektet. Prosjektet skal avsluttes 31.12.19. Da vil alle opptak bli slettet/destruert. Vi behandler opplysninger om deg basert på ditt samtykke. Prosjektet er meldt til NSD – Norsk senter for forskningsdata AS. Alle involverte parter fra UiS er underlagt taushetsplikt, og data vil bli behandlet deretter.

Så lenge du kan identifiseres i datamaterialet, har du rett til:

- innsyn i hvilke personopplysninger som er registrert om deg,
- å få rettet personopplysninger om deg,
- få slettet personopplysninger om deg,
- få utlevert en kopi av dine personopplysninger (dataportabilitet), og
- å sende klage til personvernombudet eller Datatilsynet om behandlingen av dine personopplysninger.

Hvis du har spørsmål til studien, eller ønsker å benytte deg av dine rettigheter, ta kontakt med Pontus Thente på telefon ******** eller Linda Århun på telefon ********, som er ansvarlige for dette prosjektet sammen med veileder Raymond Bjuland, telefon ********, ved Universitetet i Stavanger. Du kan også ta kontakt med NSD på epost personverntjenester@nsd.no, telefon ********.

Med vennlig hilsen

Veileder Raymond Bjuland   Studenter Pontus Thente og Linda Århun

Svarslipp:

Jeg har mottatt og forstått informasjon om forskningsprosjektet, og har fått anledning til å stille spørsmål. Jeg samtykker til at deltakere i prosjektet fra UiS observerer og eventuelt intervjuer vårt barn.

Underskrift at foresatt(e):………………………………………………………………………

Dato:………….

Jeg godtar også at det blir samlet inn data som beskrevet ovenfor

Ja   Nei   (sett ring)

ATTACHMENT 2 - CONSENT FORM LETTER TO THE TEACHER (NORWEGIAN)

Informasjonsskriv

Vil du delta i et forskningsprosjekt om problemløsning i matematikkfaget?

Dette er et spørsmål til deg som lærer til elever på XX ungdomsskole om å delta i et forskningsprosjekt hvor formålet er å undersøke problemløsningsstrategiene til elevene, samt om problemløsning skaper motivasjon i matematikkfaget. I dette skrivet gir vi deg informasjon om målene for prosjektet og hva deltakelse vil innebære.

Vi er to masterstudenter fra Universitetet i Stavanger som ønsker å samle inn data til våre masteroppgaver. Formålet med prosjektet er å tilegne seg kunnskaper og erfaringer om læring og undervisning i matematikk. Arbeidet vil dreie seg om bruk av problemløsningsoppgaver, der vi ønsker å se på hvilke strategier elevene bruker, hva lærerens rolle betyr for arbeidet, og se hvordan problemløsningsoppgaver kan føre til motivasjon i faget.

Vi ønsker å observere dine elever og deg som lærer i matematikktimene over to uker i januar/februar. Det er også ønskelig å samle inn to gruppeintervju av elever, samt to lærerintervju, i tillegg til aktuelle arbeidsbøker. Det vil bli gjort video- og lydopptak fra undervisningen og intervjuene, i tillegg til feltnotater. Hvis du ønsker å se intervjuguidene på forhånd, ta kontakt med Pontus eller Linda.

Det er frivillig å delta i prosjektet, og eventuell deltakelse innebærer at du når som helst kan trekke samtykke tilbake uten å oppgi noen grunn. Alle opplysninger vil bli anonymisert. Det vil ikke ha noen negative konsekvenser for deg eller elevene hvis du ikke vil delta eller senere velger å trekke deg.

Vi vil bare bruke opplysningene om elevene/læreren til formålene vi har fortalt i dette skrivet. Veileder i tillegg til oss to studenter har tilgang til opplysningene. Vi behandler observasjoner og kommentarer konfidensielt og i samsvar med personvernregelverket. Dette vil vi være bevisste på gjennom hele prosessen, altså i innsamling, bearbeidelse, analyse og presentasjon av data. Alle navn vil anonymiseres med fiktive navn. Skolen vil også være anonym. Deltakerne vil derfor ikke kunne gjenkjennes i publikasjonen. Video- og lydopptak vil bli oppbevart på en sikker måte på egen datamaskin til bruk i prosjektet. Prosjektet skal avsluttes 31.12.19. Da vil alle opptak bli slettet/destruert. Vi behandler opplysninger om deg basert på ditt samtykke. Prosjektet er meldt til NSD – Norsk senter for forskningsdata AS. Alle involverte parter fra UiS er underlagt taushetsplikt, og data vil bli behandlet deretter.

Så lenge du kan identifiseres i datamaterialet, har du rett til:

- innsyn i hvilke personopplysninger som er registrert om deg,
- å få rettet personopplysninger om deg,
- få slettet personopplysninger om deg,
- få utlevert en kopi av dine personopplysninger (dataportabilitet), og
- å sende klage til personvernombudet eller Datatilsynet om behandlingen av dine personopplysninger.

Hvis du har spørsmål til studien, eller ønsker å benytte deg av dine rettigheter, ta kontakt med Pontus Thente på telefon ******** eller Linda Årthun på telefon ********, som er ansvarlige for dette prosjektet sammen med veileder Raymond Bjuland, telefon ********, ved Universitetet i Stavanger. Du kan også ta kontakt med NSD på epost personverntjenester@nsd.no, telefon ********.

Med vennlig hilsen

Veileder Raymond Bjuland   Studenter Pontus Thente og Linda Årthun

Svarslipp:

Jeg har mottatt og forstått informasjon om forskningsprosjektet, og har fått anledning til å stille spørsmål. Jeg samtykker til at deltakere i prosjektet fra UiS observerer og eventuelt intervjuer vårt barn.

Underskrift lærer: ………………………………………………………………………

Dato: ………….

Jeg godtar også at det blir samlet inn data som beskrevet ovenfor

Ja   Nei   (sett ring)

ATTACHMENT 3 – THE PROBLEM SOLVING TASK (NORWEGIAN)

Problemløsningsoppgaver

Alle skal gjøre oppgave 1 og 2. Deretter kan dere velge oppgaver sammen på gruppen. Skriv med **PENN**, ikke blyant. Dere skal ikke viske ut noe som skrives ned. Begrunn hvorfor dere på gruppa velger den enkelte oppgaven.

1. Mobiloppgave

En kveld var mobilen til Lise nesten utladet. Da plugget Lise Iphonen i laderen og tok flere skjermbilder av prosenten av oppladet batteri i forhold til tiden. Det var kun skjermbilder som mobilen ble brukt til disse minuttene.

Viser til følgende bilde:

a. Hva ser du?
b. Hva har du lyst å finne ut av når du ser dette?
c. Når tror du (begrunn gjetning) at mobilen vil være fulladet?
d. Hvilken annen informasjon trenger du for å finne ut dette?

Se nå på disse bildene:

e. Ut fra opplysningene du ser på bildene – kan du nå forsøke å finne en løsning på problemet: Når vil mobilen være fulladet?
(Michael Fenton´s Reason and Wonder Web Site http://reasonandwonder.com/charge/ i Berry, 2019)

2. Kuleis

Hanne skal kjøpe kuleis og kan velge mellom fire ulike smaker.

Hun vil ha to iskuler.

a) Hvor mange måter kan hun velge isen sin på?

b) Hva om det er flere smaker å velge mellom?

(Wæge & Nosrati, 2018).

3. Produkt

a. Finn det største mulige produktet som kan lages av to tall som har sum 41.

b. Hvordan vet du at dette er riktig svar?

(Tone ark)

4. Ektepar

Fire ektepar som møtes ofte har navnene Alice, Barbara, Christa, Edith, Al, Frank, Fred og Ernest. Kan du finne ut hvem som er gift med hvem når Al og Edith er søsken, Edith og Fred var forlovet en gang, men det ble slutt da Edith møtte sin nåværende mann, Christa har en søster, men mannen hennes er enebarn og Alice er gift med Ernest.
Kan det være andre muligheter enn det dere kom frem til?

(Tone ark)

5. Blondt hår problemet

Etter mange år uten å ha sett hverandre, møtes to venner igjen, med navnene Hypatia og Pythagoras. Begge har likt matematikk godt. Her er samtalen mellom dem:

Pythagoras: *Er du gift? Har du noen barn? Hvor mange? Hvor gamle er de?*

Hypatia: *Ja, jeg er gift! Jeg har tre barn og produktet av deres alder er 36.*

Pythagoras: *(etter noe tenking) Jeg kan ikke finne det ut. Jeg har ikke nok ledetråder.*

Hypatia: *Ok. Hva hvis jeg sier at summen av alderen deres er det samme som tallet på din adresse?*

Pythagoras: *(etter noe tenking) Jeg kan fortsatt ikke finne ut alderen deres. Jeg trenger et annet hint.*

Hypatia. *Jeg forteller deg også at det eldste barnet har blondt hår.*

Pythagoras: aha! Nå kan jeg, uten tvil, si hva alderen på dine barn er.

a) Hva er alderen på Hypatias barn? (Alderen deres kan bare være naturlige tall).

b) Hvordan kan du vite at dette er det eneste riktige svaret?

(Stylianides & Stylianides, 2014).

6. Bordpartner

Tone inviterte 17 venner til middag og hun ga hver gjest et kort med et tall fra 2 til 18 og beholdt nummer 1 selv. Da alle hadde satt seg, viste det seg at summen av tallene på kortene til hvert par ble et kvadrattall. Hvilket tall hadde Tones bordkavaler (bord-partner).

(Tone ark)

7. Trekant

I en trekant ABC er $\angle A=22°$ og $\angle B=100°$. Punkt D på AC er slik at AD=AB. Hva er $\angle DBC$?

(Abelkonkurransen 2014).

8. Terninger

Du kaster tre vanlige sekssidede terninger. Hva er sannsynligheten for at du får ett oddetall og to partall (Abelkonkurransen 2012).

9. Ståltråder

Du har to ståltråder som begge er 24 cm.

a) Lag to ulike rektangler av disse ståltrådene. Vil disse ha samme areal?

b) Kan du lage flere figurer av samme areal?

c) Hvilken figur må du lage for å få størst areal?

Tema, N. s.120 (1995)

10. Kvadrat

Du har et kvadrat ABCD, og et punkt P i det indre av kvadratet.

a) Vis at hvis trekant PCD er likesidet, så er $\angle ABP=15°$.

b) Undersøk om det omvendte gjelder: Hvis $\angle ABP=15°$, så er trekant PCD likesidet.

(Bjuland, R. (2017, september 4.). Powerpoint: Oppstart Problemløsning).

11. Skoletur

730 elever og lærere fra en skole skal på tur. Hver buss kan ha 50 passasjerer. Hvor mange busser trenger skolen?

Ahlström, R. et al. (1996)

12. Kvadratisk ark

Et kvadratisk ark klippes til seks rektangler. Hvis vi regner ut omkretsen på hvert av disse rektanglene og adderer disse sammen, så blir summen 120 cm.

Hvor stor er arealet av kvadratet?

Kilder

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ATTACHMENT 4 - INTERVIEW GUIDE WITH PUPILS 1 (NORWEGIAN)

Guide til gruppeintervju med elever

- Hva synes dere om faget matematikk?
- Hvilket tema liker dere best i faget?
- Hvordan liker dere best å arbeide med matematikk?
    - Lærer underviser på tavla
    - Gjøre oppgaver alene
    - Jobbe sammen med medelever
- Hva er problemløsning for dere?
- Liker dere å arbeide på denne måten som du har gjort disse to ukene?
- Hva likte dere best?
- Hva gjør dere om dere ikke forstår oppgaven?
    - Får du mer motivasjon å vilje løse oppgaven etter dette?
- Hva gjør dere om dere ikke klarer å løse oppgaven?
    - Gjør dette at du vil lære deg mer matematikk?
    - Hvis du får til oppgaven:
        - Blir du motivert til å utvikle og å generalisere ditt svar?
        - Blir du motivert til å prøve på vanskeligere oppgaver?
- Er det greit at vi gir dere en problemløsningsoppgave nå?
    - Hva tenkte dere da dere løste denne oppgaven?
    - Hvordan kom dere frem til dette svaret?
- Blir dere motivert til å utforske mer innenfor matematikken når du arbeider med problemløsning?
    - På hvilken måte?
    - Hvordan motiverer du deg til å bli bedre i matematikk?
        - Er problemløsning en metode som hjelper deg med dette?

ATTACHMENT 5 - INTERVIEW GUIDE WITH PUPILS 2 (NORWEGIAN)

Guide for ekstra gruppeintervju med elever

<u>Husk å be elevene om å forklare seg hvordan de tenker og hva de mener.</u>

- Hvilken nivå mener du selv, at du er på i matematikk?
    - Eventuelt følgespørsmål: Hvilken karakter har du i matematikk?
- Liker du matematikk som fag?
- Hva liker du/liker du ikke med matematikk?
- Når du arbeidet med problemløsning hvordan likte du det?
    - Få dem å forklare ikke bare et enkelt svar.
- Hvis du ikke fikk til oppgavene hvordan følte du da?
    - For å få bedre svar kan disse to stilles ved behov:
    - Hva gjorde du når du ikke fikk til en oppgave, hjalp det deg?
    - Hvis det ja/nei, hvordan føltes det etter dette?
- Opplevde du noen gang at du følte at du ville, bør eller må lære deg mer matematikk? I så fall når og hvordan?
- Føler du at problemløsing kan hjelpe deg å bli bedre i matematikk?
- Hva likte du / liker du best med å arbeide med problemløsning?

Ekstra spørsmål:

- Er det noe jeg glemt å spør deg om som du vil dele med deg av disse to ukene med problemløsning?
- Prøve å få de å snakke om motivasjon.

ATTACHMENT 6 - PRE INTERVIEW GUIDE WITH TEACHER (NORWEGIAN)

Guide for pre intervjuet med læreren

- Hva legger du i ordet problemløsning?
- Arbeider dere mye med problemløsning i klasserommet?
- Har du tidligere arbeidet med problemløsning?
    - Hvordan gikk det?
    - Noe spesielt du la vekt på?
- Har du erfaringer du ønsker å formidle angående elevers arbeid med problemløsning?
    - Ser du utvikling i forhold til elevers strategier?
    - Ser du at elevene blir motiverte?
- Hvordan ser du på problemløsning som en mulighet for å lære?
- Hvordan veileder du elevene under løsningsprosessen?
- Hva tenker du om problemløsning i smågrupper?
    - Ser du læringseffekt?

ATTACHMENT 7 - POST INTERVIEW GUIDE WITH TEACHER 1(NORWEGIAN)

Guide for post intervjuet med læreren

- Hva tenker du etter gjennomføring av denne toukersperioden?
- Tror du elevene har hatt læringsutbytte av opplegget?
- Er det noe du sitter igjen med som utfordringer med å arbeide slik?
- Hvilke fordeler ser du med denne arbeidsformen?
- Hva tenker du nå om elevers strategier?
- Hva tenker du om utvalgte episoder av elevene?

Attachment 8 - Post Interview guide with teacher 2(Norwegian)

Guide for ekstra post intervjue med læreren

- For hvem mener du at problemløsning er for?
- Ser du at elever som har lavere ferdighetsnivå i matematikk, får eller ikke får motivasjon når det gjelder problemløsning?
    - Hvordan?
- Ser du at elever som har mellomfag i matematikk, får eller ikke får motivasjon når de jobber med problemløsing?
    - Hvordan?
- Ser du at elever som har sterke ferdigheter i matematikk får eller ikke får motivasjon når man arbeider med problemløsning?
    - Hvordan?
- Hvordan motiverer du elevene til å løse oppgaver?
- Er det noen ulemper med å jobbe med problemløsning?

ATTACHMENT 9 - NSD

NSD NORSK SENTER FOR FORSKNINGSDATA

NSD sin vurdering

Prosjekttittel

Problemløsning

Referansenummer

387018

Registrert

05.12.2018 av Pontus Thente - p.thente@stud.uis.no

Behandlingsansvarlig institusjon

Universitetet i Stavanger / Fakultet for utdanningsvitenskap og humaniora / Institutt for grunnskolelærerutdanning, idrett og spesialpedagogikk

Prosjektansvarlig (vitenskapelig ansatt/veileder eller stipendiat)

Raymond Bjuland, raymond.bjuland@uis.no, tlf:

Type prosjekt

Studentprosjekt, masterstudium

Kontaktinformasjon, student

Pontus Thente, pontusthente@hotmail.com, tlf:

Prosjektperiode

01.01.2019 - 31.12.2019

Status

27.01.2019 - Vurdert

Vurdering (1)

27.01.2019 - Vurdert

Det er vår vurdering at behandlingen av personopplysninger i prosjektet vil være i samsvar med personvernlovgivningen så fremt den gjennomføres i tråd med det som er dokumentert i meldeskjemaet med vedlegg, samt i meldingsdialogen mellom innmelder og NSD, den 27.01.19. Behandlingen kan starte.

MELD ENDRINGER

Dersom behandlingen av personopplysninger endrer seg, kan det være nødvendig å melde dette til NSD ved å oppdatere meldeskjemaet. På våre nettsider informerer vi om hvilke endringer som må meldes. Vent på svar før endringer gjennomføres.

TYPE OPPLYSNINGER OG VARIGHET

Prosjektet vil behandle alminnelige kategorier av personopplysninger frem til 31.12.19.

LOVLIG GRUNNLAG

Prosjektet vil innhente samtykke fra de registrerte til behandlingen av personopplysninger. Vår vurdering er at prosjektet legger opp til et samtykke i samsvar med kravene i art. 4 og 7, ved at det er en frivillig, spesifikk, informert og utvetydig bekreftelse som kan dokumenteres, og som den registrerte kan trekke tilbake. Lovlig grunnlag for behandlingen vil dermed være den registrertes samtykke, jf.
personvernforordningen art. 6 nr. 1 bokstav a.

PERSONVERNPRINSIPPER

NSD finner at den planlagte behandlingen av personopplysninger vil følge prinsippene i personvernforordningen om:

- lovlighet, rettferdighet og åpenhet (art. 5.1 a), ved at de registrerte får tilfredsstillende informasjon om og samtykker til behandlingen
- formålsbegrensning (art. 5.1 b), ved at personopplysninger samles inn for spesifikke, uttrykkelig angitte og berettigede formål, og ikke behandles til nye, uforenlige formål
- dataminimering (art. 5.1 c), ved at det kun behandles opplysninger som er adekvate, relevante og nødvendige for formålet med prosjektet
- lagringsbegrensning (art. 5.1 e), ved at personopplysningene ikke lagres lengre enn nødvendig for å oppfylle formålet

DE REGISTRERTES RETTIGHETER

De registrerte vil ha følgende rettigheter i prosjektet: åpenhet (art. 12), informasjon (art. 13), innsyn (art. 15), retting (art. 16), sletting (art. 17), begrensning (art. 18), underretning (art. 19), dataportabilitet (art. 20). Rettighetene etter art. 15-20 gjelder så lenge den registrerte er mulig å identifisere i datamaterialet.
NSD vurderer at informasjonen om behandlingen som de registrerte vil motta oppfyller lovens krav til form og innhold, jf. art. 12.1 og art. 13.
Vi minner om at hvis en registrert tar kontakt om sine rettigheter, har behandlingsansvarlig institusjon plikt til å svare innen en måned.

FØLG DIN INSTITUSJONS RETNINGSLINJER

NSD legger til grunn at behandlingen oppfyller kravene i personvernforordningen om riktighet (art. 5.1 d), integritet og konfidensialitet (art. 5.1. f) og sikkerhet (art. 32).
For å forsikre dere om at kravene oppfylles, må dere følge interne retningslinjer og/eller rådføre dere med behandlingsansvarlig institusjon.

OPPFØLGING AV PROSJEKTET

NSD vil følge opp behandlingen ved planlagt avslutning for å avklare status for behandlingen av opplysningene.
Lykke til med prosjektet!

Kontaktperson hos NSD: spesialrådgiver Kjersti Haugstvedt
Tlf. Personverntjenester: 55 58 21 17 (tast 1)