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Preface

This thesis concludes our master's degree in Industrial Economics at the University of Stavanger. It has been interesting to write about topics which also extends to our bachelor's degree in Petroleum Engineering.

Three excel workbooks have been used for the analysis in this thesis, and they can be made available by request.

We want to express special gratitude to our supervisor, Roy Endré Dahl, for guidance and constructive feedback during our work on this thesis.

Stavanger, June 13th 2019

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Abstract

In this thesis, the historical model is compared to both the normal and student t distributions to find the best risk metric using Value at Risk (VaR). To investigate the diversification effect, six portfolios have been created; two portfolios of energy commodities, two portfolios of other non-energy commodities and two total portfolios that contain all of the assets. One portfolio from each segment gives all assets equal weight (balanced), while the other portfolio from each segment uses the allocation that provides the minimum variance.

The minimum variance allocations have been calculated using a rolling window of 250 days. This was done by programming a Python code. To avoid heavy investment in some assets, an upper limit has been added as a constraint. The energy and commodity portfolios had an upper limit of 20% per asset, while the total portfolios had an upper limit of 15% per asset. The data sample period starts from 01.05.2003 (coal limitation) and ends 01.02.2019.

VaR is calculated using the following five methods:

- Historical VaR.
- Normal VaR with simple volatility (standard deviation) and Exponentially Weighted Moving Average (EWMA).
- Student t VaR with simple volatility (standard deviation) and Exponentially Weighted Moving Average (EWMA).

All the VaR models are calculated using a 95% confidence level and a 99% confidence level.

All the calculations have been backtested with Kupiec and Christoffersen test. This has been done using a rolling window of 1000 days. This makes it possible to determine which of the models performed best overall and if there are some periods where the models performed better.

The best risk metric on 95% confidence level is the normal distribution, while the best risk metric on 99% confidence level is the student t distribution. The preferred volatility model is the EWMA.

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1 Introduction

Value at Risk is the most common risk metric tool for banks and financial institutions. It is recommended by the Basel regulations, where the institutions can choose a suitable probability distribution. The most common models are the normal distribution and the historical method. The normal distribution has often been criticized for underestimating the tail risk, especially after the financial crisis, and a distribution having heavier tails are often more suitable for financial data. A report from (O'Brien et al., 2014) reflects on the difficulties in predicting loss using the historical simulation for calculating VaR, which was widely used by banks before the financial crisis. Earlier studies (Skår, 2017) has also stated that daily returns for commodities are not normally distributed. It will, therefore, be interesting to see how Student t VaR performs compared to the normal and historical model, to see if any of the models can predict the risk accurate.

The energy market has high volatility. It is important to mitigate this risk, and one must find a suitable risk estimation model. In this thesis, the historical model is compared to both the normal and student t distributions to find the best risk metric. Two volatility models will be used; the simple volatility and the Exponentially Weighted Moving Average (EWMA). The thesis includes assets from both energy and non-energy commodity markets, and a total of six portfolios are created to study the diversification effect. The financial crisis in 2008/09 and the oil price drop in 2013/14 is of special interest in this thesis. The financial crisis affected all commodities, while the oil crisis mostly affected the energy market.

1.1 Objective

This thesis is set to find the answer to the following objective questions:

(I) Effect of including other commodity assets into a portfolio consisting of energy assets with regards to diversification.

(II) Comparing the historical model to both the normal distribution and student t distribution to find the best risk metric.

(III) Comparing simple volatility and Exponentially Weighted Moving Average (EWMA) in the models.

1.2 Layout

The thesis includes the following structure to answer the objective questions:

- **Chapter 2 Portfolio Theory:** Introduces basic concepts within portfolio theory.
- **Chapter 3 Risk management:** Introduces different Value at Risk models to measure the risk in portfolios, as well as some theory on backtesting.
- **Chapter 4 Market analysis:** Gives an introduction to the theory of supply and demand, and which factors that are important for the oil and natural gas markets.
- **Chapter 5 Data analysis:** Presents statistical data on assets and portfolios used in the thesis and gives a brief analysis of the price behavior for the different commodity markets used.
- **Chapter 6 Empirical results:** Presents the results from the five VaR models and the backtests.
- **Chapter 7 Conclusion:** Summarizes the thesis and presents a conclusion. Also, some thoughts about future recommendations are presented.

2 Portfolio Theory

2.1 Volatility

Volatility is a measure of the risk. It measures the dispersion of returns for a given security or market index over a given period of time. It does not measure the likelihood of loss, but measure how far the given security moves away from its mean value. Volatility can be measured in different ways. Two methods will be presented in this thesis, the simple volatility and the Exponentially Weighted Moving Average (EWMA). (Chen, 2019a)

2.1.1 Simple volatility

Simple volatility is the variance. It is calculated using the following formula:

$$Var(X) = \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \tag{2.1}$$

Where n is the number of observations/days, and $(X_i - \bar{X})$ is the difference between the return on day i and the average return. The variance is simply the average of these squared returns, meaning that more recent days returns do not influence the variance more than the return last month. This is not the case when using the EWMA. (Grant et al., 2019)

2.1.2 Exponentially Weighted Moving Average (EWMA)

EWMA introduces λ, which is the smoothing parameter, and is always < 1. Instead of giving each squared return equal weight, each squared return is weighted by a multiplier $(1-\lambda)\lambda^n$, n = 0,1,2,...,m. In this case, m is the number of observations/days. For the most recent day, n=0, and that squared return is weighted by $(1-\lambda)$. And the following squared returns are a constant multiplier (λ) of the prior day's weight. This makes the variance more biased towards more recent observations. (Grant et al., 2019)

EWMA can also be calculated using the following recursive formula:

$$\sigma^2_{n,EWMA} = \lambda\sigma^2_{n-1} + (1-\lambda)u^2_{n-1} \tag{2.2}$$

Where, σ^2_{n-1} is the weighted variance of the day before, and u^2_{n-1} is the weighted squared return of the day before.

2.2 Covariance

Covariance is a statistical measure of the linear dependence between the returns of two assets. If the returns move in the same direction, the covariance will be positive, while the opposite is true for a negative covariance. The covariance does not say anything about the strength of the relationship between two assets; the coefficient of correlation describes this strength. The formula for calculating covariance is (Chen, 2018c):

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n-1} \tag{2.3}$$

2.3 Correlation

Similar to covariance, correlation measures how two random variables are related. The formula for correlation is:

$$\rho = Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y} \tag{2.4}$$

The correlation always has the same sign as the covariance because the standard deviation always will be positive. The correlation is always between -1 and +1. This comes from the standardizing procedure of dividing by two standard deviations. Two assets are positively correlated if the correlation is positive, and negatively correlated if the correlation is negative. (Ross et al., 2018)

2.4 Skewness and kurtosis

Skewness and kurtosis are terms that are used in statistics to describe the deviation from a normal distribution (symmetrical bell curve) in a set of data. Skewness can be positive, negative, or zero. A normal distribution has a kurtosis of 3 and a skewness of 0. If the kurtosis is larger than 3, the distribution has heavier tails than the normal distribution, and if the kurtosis is smaller than 3 the distribution has thinner tails. A leptokurtic density has a higher peak than a normal distribution, which is illustrated in figure 2.1. Since the area under the curve must equal 1, it means that the tail of the distribution must be fatter. (Alexander, 2009a) (Chen, 2019c)

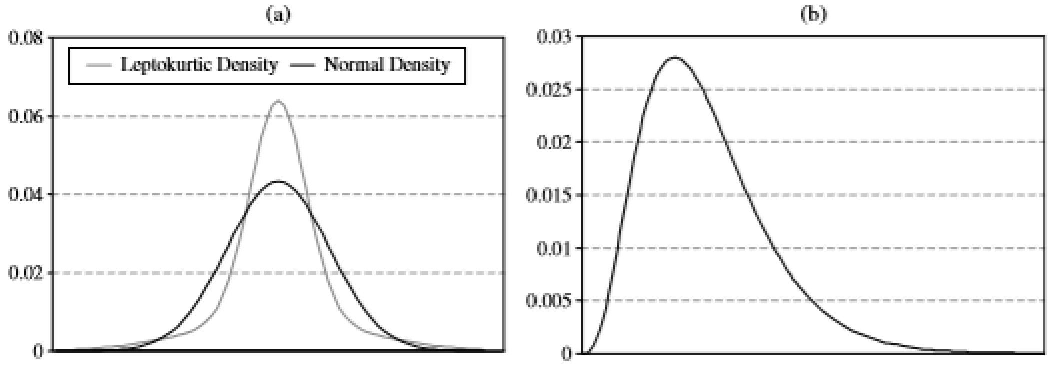


Figure 2.1: (a) A normal density and a leptokurtic density; (b) a positively skewed density (Alexander, 2009b)

2.5 Normal distribution

Normal distribution, or Gaussian distribution, is a bell-shaped probability distribution which is symmetric about the mean. The standard normal distribution has two parameters, the mean, and the standard deviation. The central limit theorem is an important aspect, which states that averages from independent, identically distributed random variables are approximately normally distributed, regardless of the type of distribution from which the variables are sampled.

The normal distribution is the most commonly used probability distribution in stock market analysis and statistical analyses. However, assuming normal distribution in value at risk calculations often underestimates the tail risk because assets return often has negative skewness and kurtosis larger than 3. (Chen, 2019b)

The probability density function (PDF) of the normal distribution is given as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2} \tag{2.5}$$

Where σ is the standard deviation, and μ is the average.

2.6 Student t distribution

A student t distribution is like normal distribution a bell-shaped curve, but it is leptokurtic, meaning it has a heavier tail which gives a larger probability of extreme outputs. The tail heaviness is determined by a parameter called degrees of freedom. Small values of this parameter give heavy tails, and higher values make this distribution converge towards a normal distribution. When the degrees of freedom rise to about 15 the student t distribution converges to a normal distribution with a mean of 0 and a standard deviation of 1. The standard student t distribution has a mean of zero, a variance of 1 and zero skewness. (Hayes, 2019)

The probability density function (PDF) of the student t distribution is given as:

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}} \tag{2.6}$$

Where $\Gamma(x)$ is the gamma function, which is an extension of the factorial function of non-integer values. The degrees of freedom are denoted by ν.

2.7 Portfolio diversification

One can determine the variance of a portfolio consisting of two assets, X and Y. A nominal amount of w is invested in asset X, while $(1-w)$ is invested in asset Y. The portfolio variance of the return is then determined by the following formula:

$$Var(R_T) = w^2\sigma_X^2 + (1-w)^2\sigma_Y^2 + 2w\rho(1-w)\sigma_X\sigma_Y \tag{2.7}$$

One can also determine the variance of a portfolio consisting of n risky assets. The n x 1 vector of portfolio weights are denoted w and we assume these are all non-negative and that they sum to 1. The n x n matrix of variances and covariances of the asset returns is denoted V. This may be written as V = DCD, where D is the n x n diagonal matrix of standard deviations and C is the correlation matrix of the asset returns. The variance of the portfolio return can then be written as (Alexander, 2009b):

$$Var(R_T) = w'Vw = w'DCDw = x'Cx \tag{2.8}$$

Where $x = Dw = (w_1\sigma_1, \ldots, w_n\sigma_n)$ is a vector where each portfolio weight is multiplied by the standard deviation of the corresponding asset return.

If the asset returns are perfectly correlated then $C = 1_n$, the n x n matrix with element 1. In this case, the standard deviation of the portfolio return is the weighted sum of the asset return standard deviations. Most portfolios have not a perfect correlation. In this case, when asset returns have less than perfect correlation, then C has some elements that are less than 1. This means that the vector x has non-negative elements. In this case, $Var(R_T) = x'Cx \leq x'1_nx$. (Alexander, 2009b)

This proves the principle of diversification, meaning that including more assets into the portfolio reduces the risk relative to the risk of the individual positions in the assets. It is not possible to reduce all the risk, due to the systematic risk associated with the market. Empirical studies have shown that holding 30 assets is enough to remove the unsystematic risk, but one is still left with the systematic risk due to the exposure to a general market risk factor. (Alexander, 2009b)

Figure 2.2 below illustrates how the risk changes as two assets with different levels of correlation are combined. As can be observed, the risk is reduced when the correlation is less than one, which illustrates the principle of diversification by combining assets that are not strongly correlated.

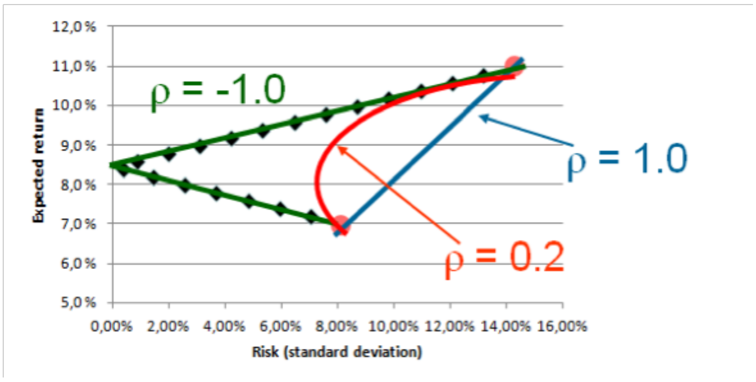


Figure 2.2: Correlation

3 Risk Management

3.1 Risk types

There are two types of risk; systematic and unsystematic risk.

Unsystematic risk can be explained as the uncertainty related to a company or industry investment. An example of unsystematic risk can be when a company invested in loose market shares to a new company entering the market. Unsystematic risk can be reduced through diversification, as discussed in chapter 2.7. (Chen, 2017)

Systematic risk is often referred to as market risk, which is a type of risk that is not possible to diversify away because the risk applies to the whole market. Systematic risk incorporates interest rates, inflation, recessions, wars, and other significant changes. Changes in these domains will have an impact on the whole market. The only possibility to handle systematic risk is to include different asset classes in the portfolio. This should be asset classes that will react differently to factors that will influence most of the market. This can be cash, real estate or fixed income (Fontinelle, 2018)

3.2 Value at Risk

Value at risk (VaR) is a market risk metric measuring the uncertainty of a portfolio's future value, or in other words, it estimates the profits and losses of a portfolio in the future. VaR is commonly used as a representation of possible losses for a (relatively) unlikely scenario. VaR's characteristics as a risk metric are described in "Market risk analysis, value at risk models (page 1)" as (Alexander, 2009b):

- *It corresponds to an amount that could be lost with some chosen probability*
- *It measures the risk of the risk factors as well as the risk factor sensitivities.*
- *It can be compared across different markets and different exposures.*
- *It is a universal metric that applies to all activities and to all types of risk.*
- *It can be measured at any level, from an individual trade or portfolio, up to a single enterprise-wide VaR measure covering all the risks in the firm as a whole.*

- *aggregated (to find the total VaR of larger and larger portfolios) or disaggregated (to isolate component risks corresponding to different types of risk factor) it takes account of dependencies between the constituent assets or portfolios.*

VaR will answer the question: *How much is it possible to lose over a certain time given a certain confidence level?*

Value at risk depends on the following variables:

1. The time horizon (Usually one day)
2. Chosen confidence interval. (Typically, 95%, 99%, 99,9%)

The answer to the question may be: I am 99% (confidence level) certain that my losses will not be bigger than $0.5 million (VaR) the next day (time horizon).(Alexander, 2009b)

VaR can be calculated using different approaches:

- Historical simulation (Non-parametric approach)
- Normal/student t distribution (Parametric approach)
- Monte carlo simulation

Parametric approaches are methods that are based on statistical parameters of the risk factor distribution, and non-parametric are only based on historical data.

3.2.1 Historical Simulation (Non−parametric)

The historical simulation is the simplest method for calculating VaR. It is easy to compute, and there is no need for an underlying probability distribution. In order to calculate VaR by the historical method, the daily returns have to be calculated for the asset/portfolio. These values are then sorted in order from low to high. As an example, 1000 observations of daily returns are gathered and a confidence level of 95% is chosen, then the 5% VaR would be the 50^{th} worst observation. This method has some downsides; It is relying on history to repeat itself, and it is vulnerable to the sample period chosen by the analyst. (Alexander, 2009b)

3.2.2 Normal VaR (parametric)

This method is like the historical method relatively easy to compute. In addition to normal VaR, it is also called the covariance method. A benefit of this method is that after the standard deviation and mean are calculated it is possible to calculate VaR knowing only these two numbers. The method has been criticized for being too optimistic and to underestimate the VaR at high confidence levels due to the fact that the assumption about the normality of the data does not hold. (Alexander, 2009b)

Estimation method:

1. Calculate the daily returns, standard deviation, and correlation for all the assets.
2. Estimate volatility (standard deviation/EWMA) for the portfolio returns.
3. Normal VaR can then be calculated using the following formula:

$$VaR = \sigma Z_\alpha + \mu \tag{3.1}$$

where the parameters are:

- α = chosen significance level
- Z_α = The α quantile of the standard normal distribution
- σ = The standard deviation of the data
- μ = The mean of the data

3.2.3 Student t VaR (parametric)

Financial return distributions are most often leptokurtic. The returns are also often negatively skewed, and volatility tends to cluster. A result of this is that the normal VaR method is too optimistic and does not take fat tails into consideration and is, therefore, underestimating the VaR. A better approach may be to use a student t distribution to describe the data. The student t distribution can be used as a model for financial returns that exhibit excess kurtosis, allowing more realistic calculations of VaR, especially at high confidence levels. For degrees of freedom larger than 2 ($\nu > 2$), the variance of a student t distribution is not 1, but can be estimated by: $Var(T) = \frac{\nu}{\nu-2}$. The α quantile of the student t distribution is denoted by t_α. The α quantile of the standardized student t distribution can be estimated by: $\sqrt{\nu^{-1}(\nu-2)}t_\alpha$, since quantiles translate under monotonic transformations. Student t VaR can then be calculated using the following formula (Alexander, 2009b):

$$VaR_t = \sqrt{\frac{\nu - 2}{\nu}} t_\alpha \sigma + \mu, \qquad \nu > 2 \tag{3.2}$$

Where the parameters are:

- ν = degrees of freedom
- σ = The standard deviation of the data
- μ = The mean of the data
- t_a = the left inverse t distribution

3.3 Backtesting of VaR

It is important to quality assure the VaR models to determine if the models are predicting VaR accurate. Backtesting is a way to measure the accuracy and effectiveness of VaR models. This is done by using different statistical methods to compare predicted losses from VaR calculations with the actual losses realized at the end of a time horizon. (Holton, 2014b)

A very basic backtest can be conducted by counting the numbers of exceedances and compare them to the α-value:

$$I_t(a) = \begin{cases} 1 & \text{if } r_t < -VaR_{t|t-1}(a) \\ 0 & \text{else} \end{cases}$$

The hypothesis test will be:

$$H_o : \frac{\sum I_t(a)}{N} \leqslant \alpha$$

Example: If a significance level of $\alpha = 0.01$ and a time horizon of 1000 days are used, the VaR model is accurate if the exceedance is less than 1% (or 10 days). If the violations are higher than 1% it means that the model is predicting VaR inaccurate, and more advance backtests should be performed to determine the performance of the VaR model.

Volatility in financial data tends to come in clusters. This can have huge impact on the accuracy of the value at risk estimates and must be investigated when backtesting the models. If the results from the backtest shows fewer violations than the α used, and most of these violations come in clusters in a short time period the VaR model might be underestimating the risk and the investor may not capable to handle this risk. It is therefore important to include at least one backtest that examines the clustering effect. To evaluate the VaR models further both Kupiec test (Coverage test) and Christoffersen test (clustering effect) will be used with the exceedance from the equation over as input. These two methods will be described in detail below.

The critical value that is used in the hypothesis test is taken from a Chi-squared distribution with a significance level of 5% and one degree of freedom. This corresponds to 3,84.

3.3.1 Kupiec test for coverage (POF)

The Kupiec test was first introduced in 1995 and it is a variation of the binomial test called the proportion of failures (POF). The likelihood ratio is calculated by the following formula(Holton, 2014b):

$$LR_{POF} = 2ln\left(\frac{N-X}{qN}\right)^{(N-X)}\left(\frac{X}{(1-q)N}\right)^{X} \tag{3.3}$$

where the parameters are:

- X is the number of VaR breaks
- N the number of observations
- q = 1 − significance level

"The likelihood ratio is asymptotically distributed as a chi-square variable with 1 degree of freedom. The VaR model fails the test if this likelihood ratio exceeds a critical value. The critical value depends on the test confidence level." (MathWorks, 2019)

3.3.2 Christoffersen test for independence

Christoffersen test measures the independence from the exceedances from the VaR models. This test will examine if an exceedance of a day will have a correlation with the exceedance of the previous day.

The first step of the Christoffersen test is to compare VaR calculation with the actual return for each day. The days where the daily returns exceed VaR gets the value 1 and 0 if not.

$$I_t = \begin{cases} 1 & \text{if violation occurs} \\ 0 & \text{if no violation occurs} \end{cases}$$

Then n_{00}, n_{10}, n_{01}, n_{11} are given value 1 or 0 based on combination of exceedances between day t and day t-1, and then summarized. This can be illustrated by a 2 x 2 contingency table:

	$I_{t-1} = 0$	$I_{t-1} = 1$
$I_t = 0$	$n_{00} = 1$	$n_{10} = 1$
$I_t = 1$	$n_{01} = 1$	$n_{11} = 1$

π_i are calculated to represent the probability of observing an exceedance conditional on state i on the previous day:

$\pi_0 = \frac{n_{00}}{n_{00}+n_{01}}$
$\pi_1 = \frac{n_{10}}{n_{10}+n_{11}}$
$\pi = \frac{n_{00}+n_{10}}{n_{00}+n_{01}+n_{10}+n_{11}}$

Then the relevant test parameter for independence of exceedances is a likelihood-ratio:

$$LR_{ind} = -2ln\left(\frac{(1-\pi)^{n_{01}+n_{11}}\pi^{n_{00}+n_{10}}}{(1-\pi_0)^{n_{01}}\pi_0^{n_{00}}(1-\pi_1)^{n_{11}}\pi_1^{n_{10}}}\right) \tag{3.4}$$

The null hypothesis H_0 gets rejected if $LR_{ind} > 3{,}84$. (From Chi-squared table with a significance level of 5% and one degree of freedom)

The equation above and the procedure for performing the Christoffersen test are taken from Value-at-Risk, Second Edition – by Glyn A. Holton. (Holton, 2014a)

By comparing the results from the Kupiec test and the Christoffersen test, the performance of the VaR models can be examined and a conclusion can be taken. If the model passes both tests one can assume that the model is good and that it measures the risk accurate. If one of the tests fails the model, it is not good enough and one should improve the model or consider using a different method/approach.

4 Market Analysis

4.1 Supply and demand

Market demand is defined as the alternative quantities consumers in a market are willing to buy as price varies. The slope of the demand curve is negative, due to the law of demand, which states that there is an inverse relationship between quantity demanded and the price. (Tomek, 2014)

Market supply is the quantity producers are willing to produce and sell as prices vary. The slope of the supply curve is positive, and this positive correlation is based on the potential increase in profitability that follows from an increase in price. (Boundless Economics, 2019b)

When demand equals supply, the economy is said to be in equilibrium (see figure 4.1 below) At this point the allocation of goods are at its most efficient, because the goods supplied equals the goods demanded. This point is only reachable in theory, as prices of goods is constantly changing due to constant changes in demand and supply.

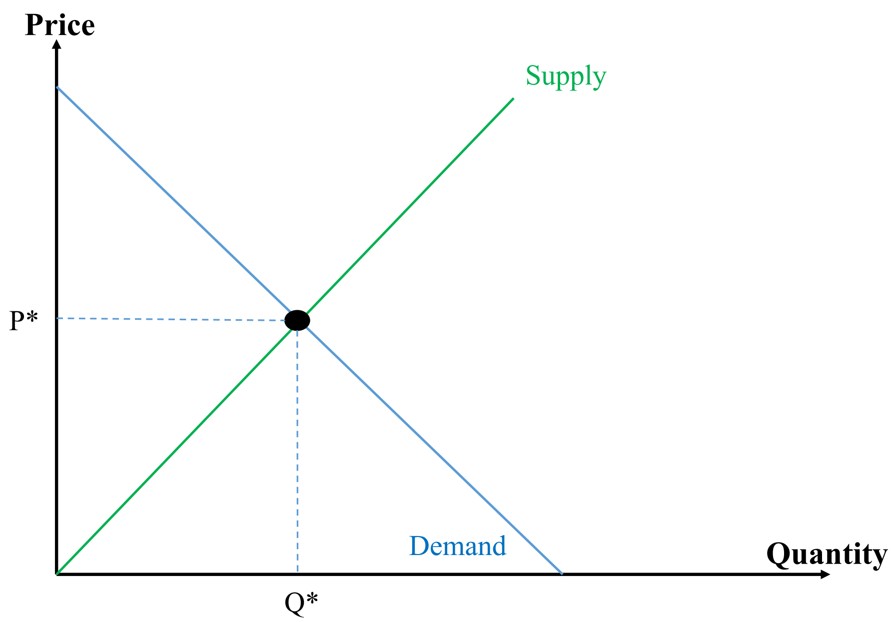


Figure 4.1: Equilibrium price and quantity

An increase in demand will shift the demand curve to the right (D2 on figure 4.2). This means that consumers are willing to buy more of the commodity at the same price or that they are willing to buy the same quantity at a higher price. The opposite will be the case of a decrease in demand (D1 in figure 4.2).

The major factors influencing the level of demand is

(i) Demographic factors

(ii) Economic factors

(iii) Consumer tastes and preferences

Two effects that are important for the demand are the income and substitution effect. The income effect refers to the effect on demand from a change in real income due to the price change. When the price of a commodity goes up, the real income will decrease, and demand for the commodity will decrease given it is a normal good. (Tomek, 2014)

The substitution effect refers to the change in demand for commodities due to the pure change in relative prices. This will affect commodities that are considered substitutes (ex. oil & gas). If a commodity gets relative more expensive, then demand for the other substitutes will increase, while the demand for the commodity itself will go down. (Tomek, 2014)

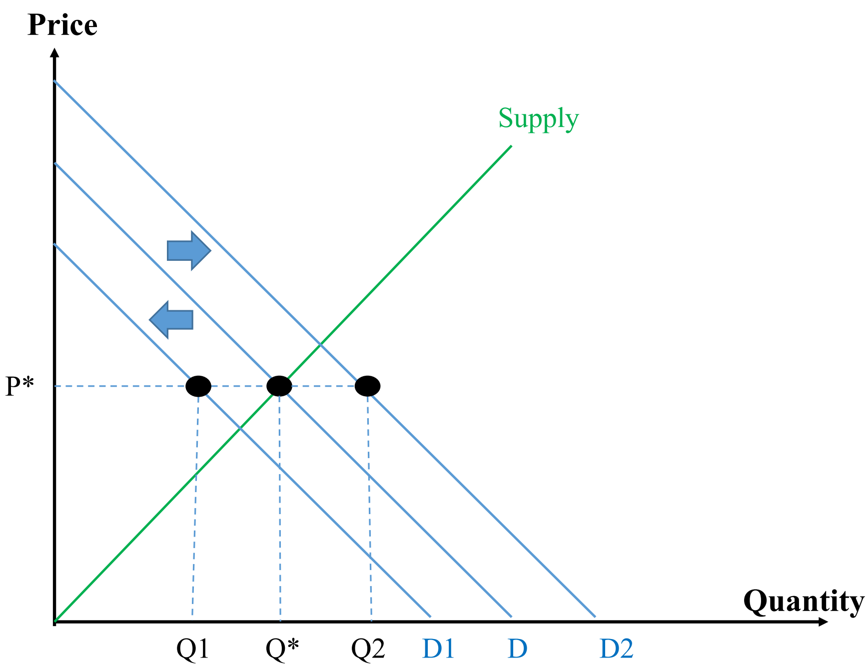


Figure 4.2: Dynamic shifts in demand

A shift in the supply curve to the left is associated with rising production costs and means that less can be produced at any given price (S1 on figure 4.3). This will increase the equilibrium price. The opposite will be the case of a shift in the supply curve to the right (S2 on figure 4.3).

The major factors influencing supply are:

(i) Changes in input prices (factor prices)

(ii) Changes in prices of commodities competing for the same resources

(iii) Changes in prices of joint products (fuel and gas for crude oil)

(iv) Changes in price or output risk

(v) Changes in technology affecting cost of productio

(vi) Changes in institutional factors (market regulations)

In addition, supply can be influenced by random events like the weather.

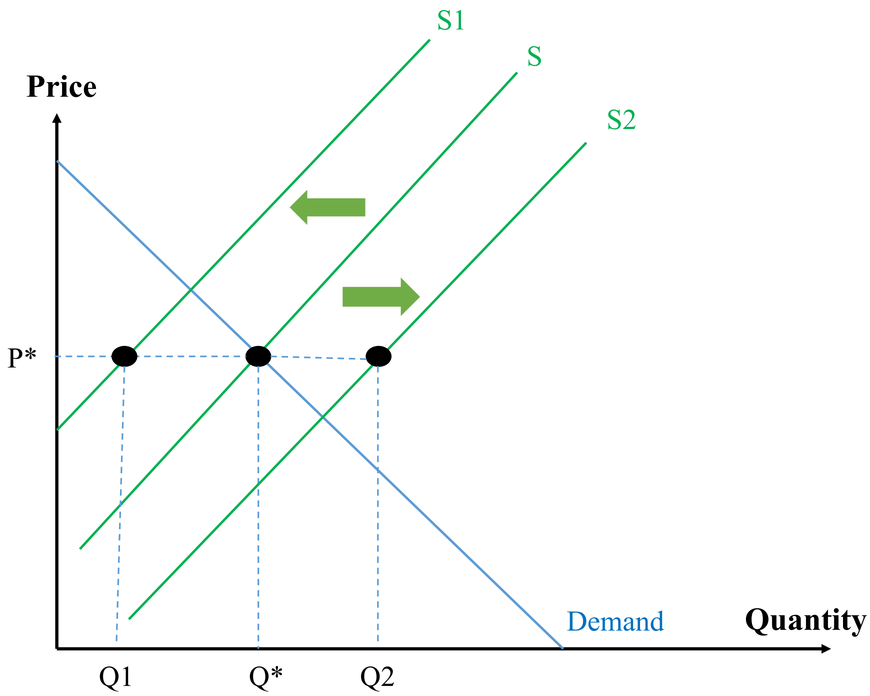


Figure 4.3: Dynamic shifts in supply

In a perfectly competitive market, the firm maximizes profits by producing at a level where marginal cost is equal to marginal revenue (MC=MR, see figure 4.4 below). In the short-term, it is possible for the firm to make a profit in the case where the price is bigger than the average total cost (ATC). In the case where the price is lower than ATC, the firm is making a loss in the market. (Boundless Economics, 2019a)

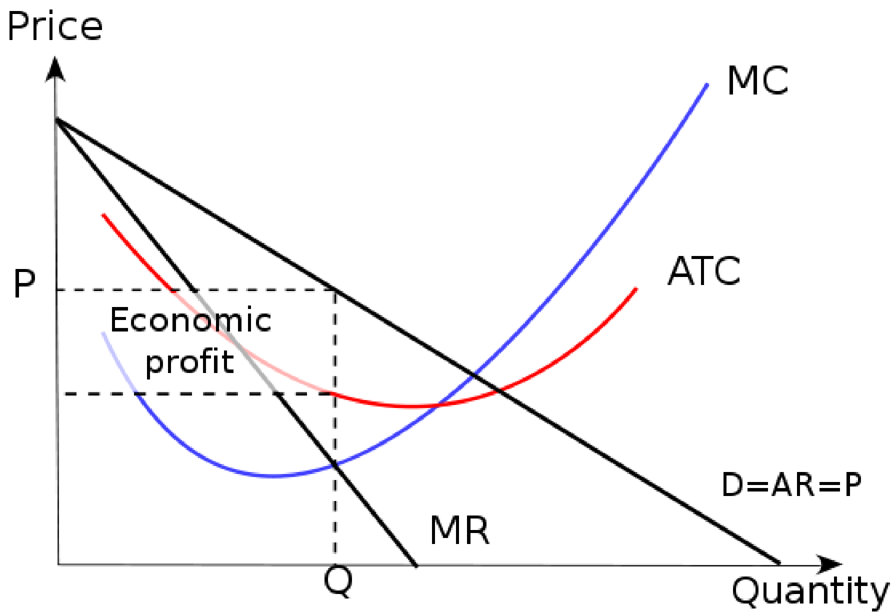


Figure 4.4: Profit maximizing price and quantity in a perfect competitive market

In the long run, if firms are making an economic profit, more firms will enter the market. This will shift the supply curve to the right and this will cause the equilibrium price to go down. This will cause economic profit to decrease until it becomes zero. When the price moves below ATC, the firms are making a loss, which will cause more firms to leave the market, and cause the equilibrium price to go up. In sum, firms engaged in a perfectly competitive market will make zero economic profit in the long run. The long-run equilibrium point in a perfectly competitive market is where the demand curve (D or price) intersects the MC and in the minimum point of the ATC. (Boundless Economics, 2019a)

Perfect competition is representative for many markets, but for the oil market, a market structure called oligopoly is more representative. This market structure consists of two or more firms. The firms in an oligopoly can set prices collectively in a cartel, or under the leadership of one firm, instead of taking prices from the market. This increases the profit margins compared to a perfectly competitive market. OPEC has a huge amount of oil resources, and can price fix to a huge degree. The conditions that enable oligopolies are high entry cost in capital expenditures (CAPEX, typical for the oil industry), legal privilege and a platform that gains value with more customers (social media). Governments try to respond to oligopolies with laws against price fixing, but a cartel like OPEC has no overarching authority, which makes it difficult for the governments to counteract the price fixing. (Chappelow, 2019)

4.2 Price elasticity

Elasticity is a measure of how responsive the supply or demand for a good is when the price changes. The elasticity is defined as the percentage change in quantity when the price of a good is changed with 1%. Elasticity can be defined in three groups (Khan Academy, 2009):

- **Elastic** - Elastic price elasticity means that a change in demand/supply will adjust rapidly when the price changes.
- **Unit elastic** - Unitary elasticity means that a given change in price by percentage leads to an equal percentage change in supplied or demanded amount.
- **Inelastic** - Inelastic price elasticity means that the supply/demand will stay unchanged by a change in price.

The formula for price elasticity is defined as:

$$\varepsilon_p = \frac{\delta Q}{\delta P}\frac{P}{Q} \tag{4.1}$$

4.3 Oil market analysis

In this chapter, the oil market will be described to get a better understanding of the factors that affect the pricing of oil. Crude oil is an important source of energy that people over the whole world depend on to maintain today's living standards. According to the BP statistical review of World Energy 2018, oil is the world's leading fuel, accounting for approximate 34,2% of global energy consumption. (BP, 2018)

The oil prices have the past decades fluctuated significantly with high booms and low busts. The world economy is highly correlated with oil prices, and the oil prices are, therefore, a good measure of the world economy. This chapter will first start by describing the different qualities of oil, and then some of the factors that affect the oil prices.

4.3.1 Benchmarks for oil prices

A crude oil benchmark is used as a reference price for a specific quality of crude oil. In this thesis, the three most common benchmarks for oil prices worldwide are used, which are:

- Brent Blend
- West Texas Intermediate (WTI)
- Dubai Crude

The Brent benchmark is used for pricing of light sweet crude. It refers to crude oil produced from different fields in the North Sea. Brent has good export possibilities since it is produced offshore, and it makes up the majority of the crude oil traded internationally. Brent serves as a direct or indirect reference for about 2/3 of global crude oil sales. The characteristics of Brent is that it is sweet and light, and it has, therefore, good quality. (Chen, 2018b)

WTI refers to oil that is produced from wells in the US, mostly from Texas, Louisiana and North Dakota which are transported through a pipeline to Cushing, Oklahoma. The US has historically been a net importer of oil, and they also had a law that denied export of oil, which was reversed late in 2015. Therefore, the primary market for WTI is in the United States. The characteristics of WTI are sweet and light. (Chen, 2019d)

Dubai crude is the average price of Dubai and Oman crudes. It is used for pricing oil coming from the Persian Gulf and the Middle East. The market for Dubai crude is mainly Asia. Dubai crude is heavier and sourer than Brent and WTI. (Chen, 2018a)

The quality of oil is mainly determined by two factors:

- The density of the oil.
- The sulfur content of the oil.

To measure how light or heavy oil is compared to water, API gravity is used to measure the density of the oil. API stands for American Petroleum Institute. Oil with API gravity higher than ten is lighter than water, and vice versa. Oil density based on API gravity is classified as light, medium, heavy or extra heavy. Light oil is more preferable because it contains more amounts of hydrocarbons that can be converted to gasoline. (Petroleum, 2015)

$$APIGravity = \frac{141,5}{SG} - 131,5 \tag{4.2}$$

- Light – API > 31,1
- Medium – API between 22,3 and 31,1
- Heavy – API < 22,3
- Extra Heavy – API < 10,0

Another characteristic of the quality of the oil is the amount of sulfur it contains. Oil is called sweet if the sulfur content is below 0,5% and sour if the sulfur content is above 0,5%. Sulfur must be removed from the oil before refining which gives higher costs. In additional sulfur is corrosive and this leads to damage on the refineries, which gives higher maintenance costs. (Chen, 2018e)

The table below shows the sulfur content and the specific density of Brent, WTI, and Dubai crude. These factors have an impact on the oil price and can explain some of the price differences between the benchmarks.

Table 4.1: API gravity and Sulfur content of WTI, Brent and Dubai crude

	API Gravity	**Sulfur content**
Brent	38,08	0,37%
WTI	39,6	0,24%
Dubai	31	2%

As the table shows, WTI has lower sulfur content than Brent and lower specific density than Brent. This implies better quality and therefore it is rational to think the price of WTI should be higher than Brent. This was mostly the case until about 2010, which can be observed in figure 4.5. The reason that Brent is being traded premium after 2010 will be described in chapter 4.3.6.

4.3.2 Pricing of crude oil

Crude oil is being traded on the global market and is the most traded commodity in volume. The primary use of oil is gasoline, diesel, and other petrochemicals that are used in products as textiles, fertilizers, plastics, steel, and other consumables. As oil is produced and consumed all over the world, the different benchmarks of crude oil tend to move closely together, although there is a slight difference in quality. Figure 4.5 shows the prices for Brent, WTI, and Dubai crude from 2003.

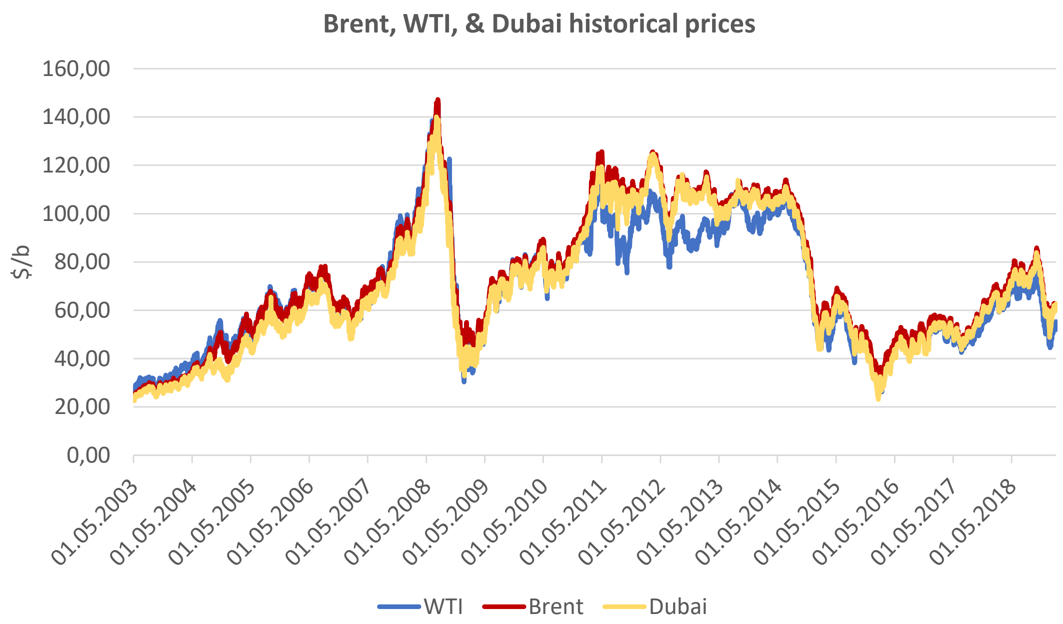


Figure 4.5: Historical oil price: Brent, WTI and Dubai crude

Table 4.2: Correlation matrix between WTI, Brent and Dubai crude(2003-2019)

	WTI	**Brent**	**Dubai**
WTI	1		
Brent	0,98	1	
Dubai	0,97	1	1

The correlation matrix in table 4.2 is calculated based on oil prices from May 2003. As the correlation matrix shows, the prices of these three benchmarks are highly correlated.

4.3.3 Supply and demand

Various oil blends are traded on commodity exchanges between suppliers and buyers. Oil is traded in options, futures, and physical delivery. The major exchanges that oil is traded on are the Intercontinental Exchange (ICE), located in London and on The New York Mercantile Exchange (NYMEX), located in New York City.

Since oil is produced and transported globally all over the world, this leads to many factors that influence supply and demand. Many oil producing countries have bad political systems that have shown to influence the oil price significantly and the oil price is therefore very volatile. Figure 4.6 shows proven oil reserves by 2017 by region. New discoveries and enhanced technology continue to provide new estimates for proven oil reserves.

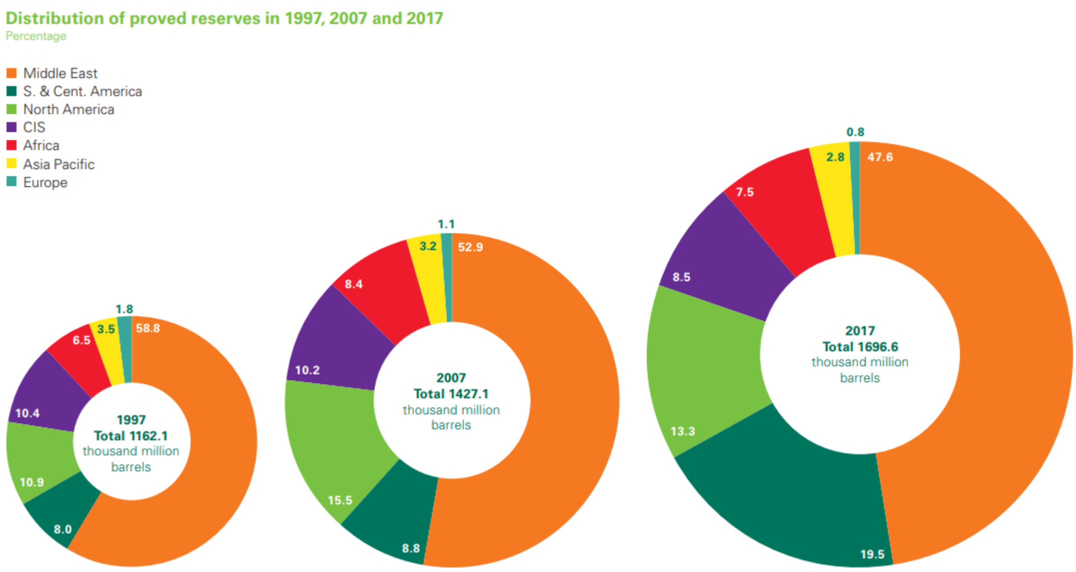


Figure 4.6: Distribution of proven reserves in 1997, 2007 and 2017 (BP, 2018)

The demand for crude oil comes from several sources. Transportation and industry cover most of global oil consumption. Electric power generation, heating, and use of oil in products stand for the rest of the consumption.

The US has the past decades been the largest oil consumer in the world and consumes today about 20 million barrels per day. Figure 4.7 shows a significant growth by countries in the Asia Pacific. The last years China and India have been the most significant contributors to the growth. According to Wood Mackenzie, India contributed to 14% of the global demand growth or 245 000 barrels per day in 2018. (Mackenzie, 2019) Transport fuels; gasoline and diesel, as well as residential LPG (Liquefied Petroleum Gas) are the two main drivers for oil demand growth.

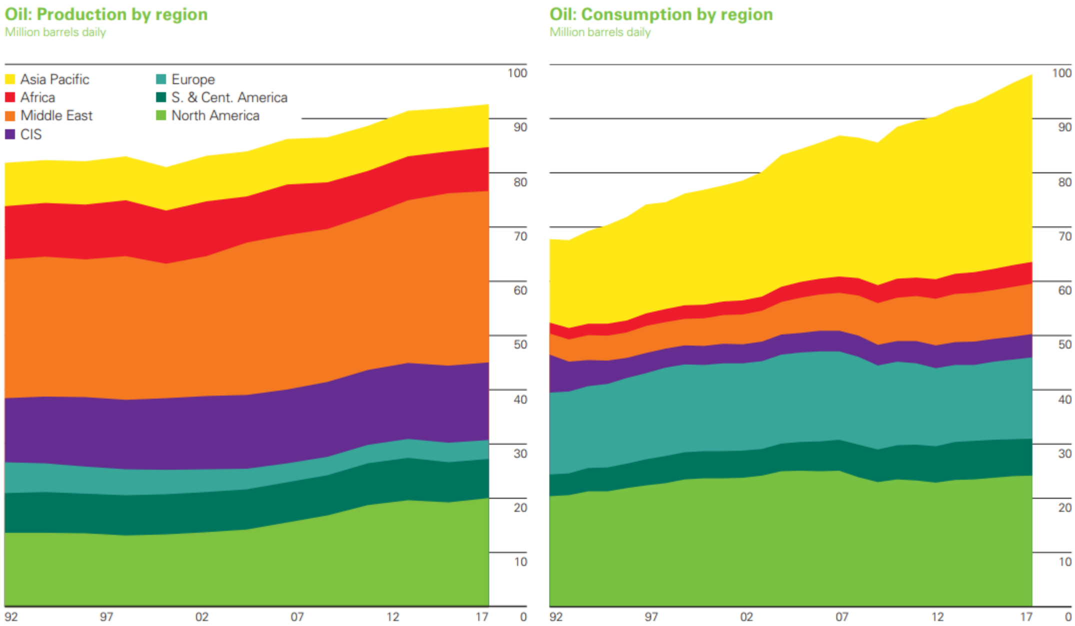


Figure 4.7: Oil production and consumption by region (BP, 2018)

4.3.4 OPEC

OPEC (Organization of Petroleum Exporting Countries) is the largest oil cartel in the world. Their goal is to adjust supply/demand to influence the market to the member countries interest. OPEC were founded in Baghdad, Iraq, in 1960 by Islamic Republic of Iran, Iraq, Kuwait, Saudi Arabia and Venezuela. (OPEC, 2019)

> *These countries were later joined by Qatar (1961), Indonesia (1962), Libya (1962), the United Arab Emirates (1967), Algeria (1969), Nigeria (1971), Ecuador (1973), Gabon (1975), Angola (2007), Equatorial Guinea (2017) and Congo (2018). Ecuador suspended its membership in December 1992, but rejoined OPEC in October 2007. Indonesia suspended its membership in January 2009, reactivated it again in January 2016, but decided to suspend its membership once more at the 171st Meeting of the OPEC Conference on November 30 2016. Gabon terminated its membership in January 1995. However, it rejoined the Organization in July 2016. Qatar terminated its membership on January 1 2019. (OPEC, 2019)*

Currently, the Organization has a total of 14 member countries. (OPEC, 2019)

OPEC produces about 42% (BP, 2018) of global oil production, and they have about 73% (BP, 2018) of the proven oil reserves in the world, which gives OPEC able to adjust the supply of oil and hence the oil price. It has been proven the past decades that if OPEC thinks the price is too high they increase the supply of oil with a production boost and if they think the price is too low they lower the production. Figure 4.8 shows the effect of Saudi Arabia's (OPEC'S largest producer) oil production and percentage change in WTI crude oil price. As the other oil prices are highly correlated with WTI, changes in OPEC production have an impact on the oil prices in general.

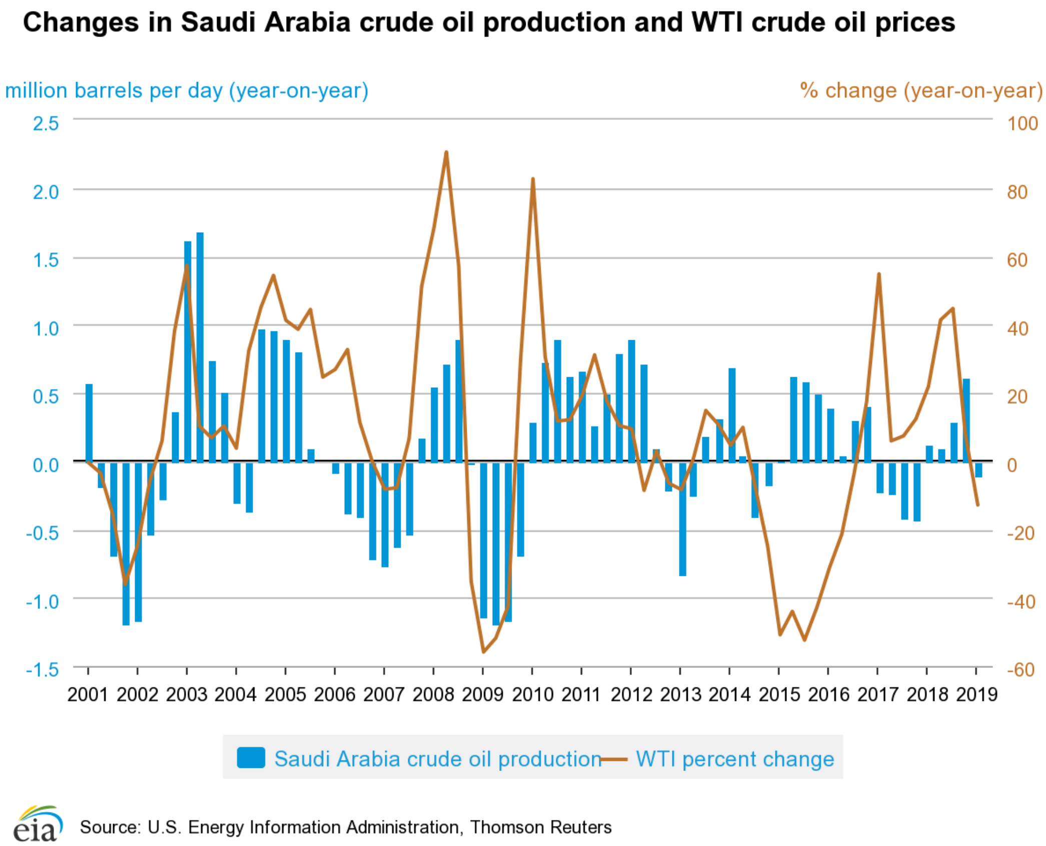


Figure 4.8: Changes in Saudi Arabia crude oil production and WTI crude oil price (Eia.gov, 2019)

4.3.5 Futures market

On the futures market contracts for oil are sold and bought to secure a price in the future for a certain amount of oil. For example, to reduce risk, an airline will buy futures or options contracts today to avoid the fuel cost to rise above a certain level in the future, and an oil producer wants to sell futures or options contracts today to avoid the risk of lower prices in the future. (Eia.gov, 2019)

Most of the transactions on the futures market are done by banks, hedge funds, commodity trading advisors, and other money managers who often do not have interests in trading physical oil. They speculate in the future price with an expectation of earning profit from the price changes. It is therefore common for hedgers to add future contracts into their portfolio to diversify the portfolio risk. They can speculate on the future price by either go long or short and take the opposite positions before the maturity of the contract. The futures market has, therefore, an essential role in the price determination of oil. (Eia.gov, 2019)

4.3.6 Brent vs WTI spread

An interesting observation from figure 4.9 is the spread between the WTI and the Brent prices the past decades. WTI has historically been traded premium to Brent because it has better quality due to lower sulfur content and lower specific density. As the figure shows, this relationship changed in year 2010 and Brent has been traded premium to WTI since then.

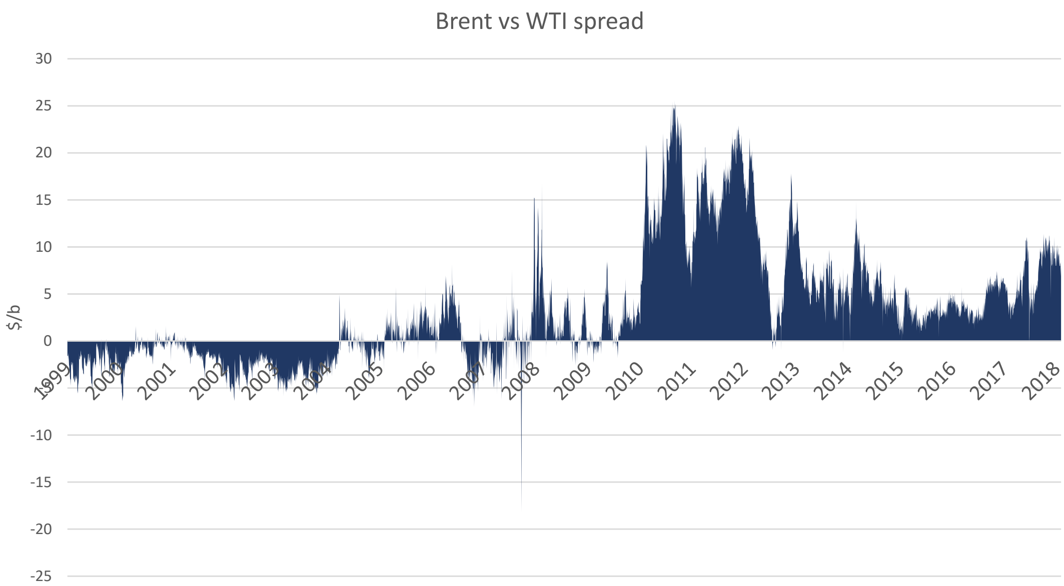


Figure 4.9: Brent vs WTI spread

The reason for this was that the US historically has been a large oil importer. An increase in WTI-price made it attractive for actors to import oil from other markets such as Europe. Therefore the market price between WTI and Brent adjusted itself with a spread approximate equal to the transport cost of Brent oil from Europe to the US. Arbitrage the other way has not been possible due to different circumstances (export ban). The US has had phenomenal technology advance in their fracking technology which made oil producing from tight shale reservoir more economically feasible. This led to a fast increase in the US oil production and combined with bad infrastructure in Cushing, Oklahoma, where all the oil in the US are gathered made the WTI price decrease. At approximately the same time the Arab spring took place in Africa and the Middle East and the oil demand increased, which led to a higher Brent price. The US had a law that blocked oil export which was lifted in 2015. However, in 2016 the US exported only 0,5 barrels of oil and it seems like they will remain a net importer of oil for now. Figure 4.10 shows the US shale oil production from 2010 and the prospect of the production in the future.

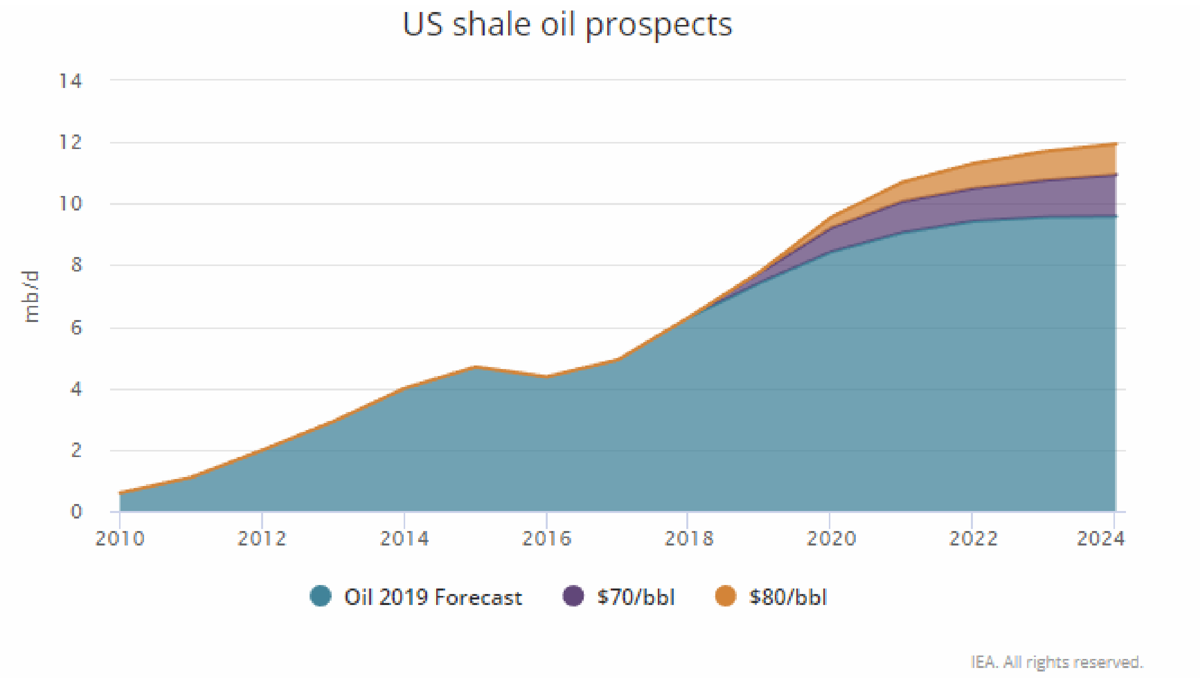


Figure 4.10: US shale oil prospects (IEA, 2019)

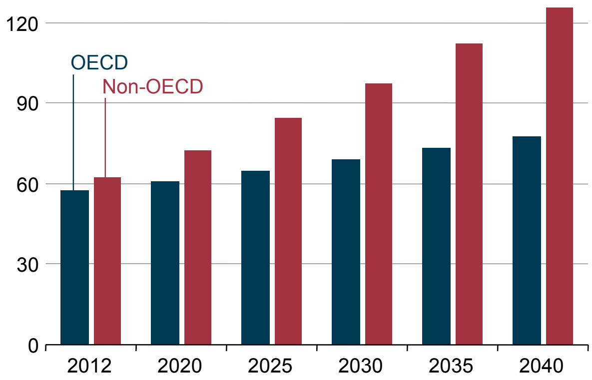
4.4 Natural gas market analysis

In this chapter, the natural gas market will be described. A description of the more regional markets in the US and in the UK will also be presented. According to the International Energy Outlook (IEO) report from 2016, natural gas production is expected to increase to 203 trillion cubic feet (tcf) in 2040, compared to 120 tcf in 2012. Due to good resources and stable production, natural gas is competitive to other energy resources, which is one of the reasons why it stands for the biggest increase in world primary consumption by energy source. (eia.gov, 2016)

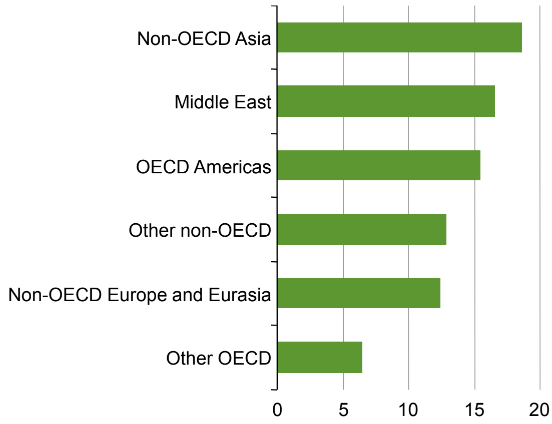
Natural gas remains a key fuel in the electric and industrial sectors. The world consumption is expected to increase 1,7%/year in the industrial sector, compared to 2,2%/year in the electric power sector, between 2012 and 2050. Combined, these two sector stands for 73% of the total increase in the world natural gas consumption. (eia.gov, 2016)

An increase in the consumption of natural gas is expected for all IEO regions. For nations outside the Organization for Economic Cooperation and Development (non-OECD), the demand is expected to increase twice as fast compared to nations in the OECD (see figure 4.11a). Due to economic growth, the biggest increase in natural gas is expected for non-OECD Asia. In the non-OECD nations, the natural gas consumption grows by an average of 2,5%/year from 2012 to 2040, compared to 1,1%/year in the OECD regions in the same period. (eia.gov, 2016)

The supply of natural gas is expected to increase by almost 69% from 2012 to 2040, due to the rising demands. Non-OECD Asia, the Middle East and OECD Americas stand for the most significant increase in production (see figure 4.11b). (eia.gov, 2016)



(a) World natural gas consumption, 2012-2040 in tcf



(b) World increase in natural gas production by country, 2012-2040 in tcf

Figure 4.11: World natural gas consumption and production, 2012-2040 (eia.gov, 2016)

4.4.1 United States

The increases in U.S natural gas production is mainly due to unconventional shale gas resources, which have become more accessible and economical to produce due to advancements in horizontal drilling and hydraulic fracturing. From 2006 to 2013 the production of shale gas has increased from 5% to 40% of total natural gas production. (API, 2014)

Figure 4.12 shows dry natural gas production by type in the U.S. Natural gas production from shale gas and tight oil continues to grow in both share and volume. In 2050 it is expected that about 90% of the natural gas production will come from tight oil and shale gas. The cumulative production from shale gas is 18% higher in the high oil and gas resource and technology case than the reference case, while it is 24% lower in the low oil and gas resource and technology case. (eia.gov, 2019)

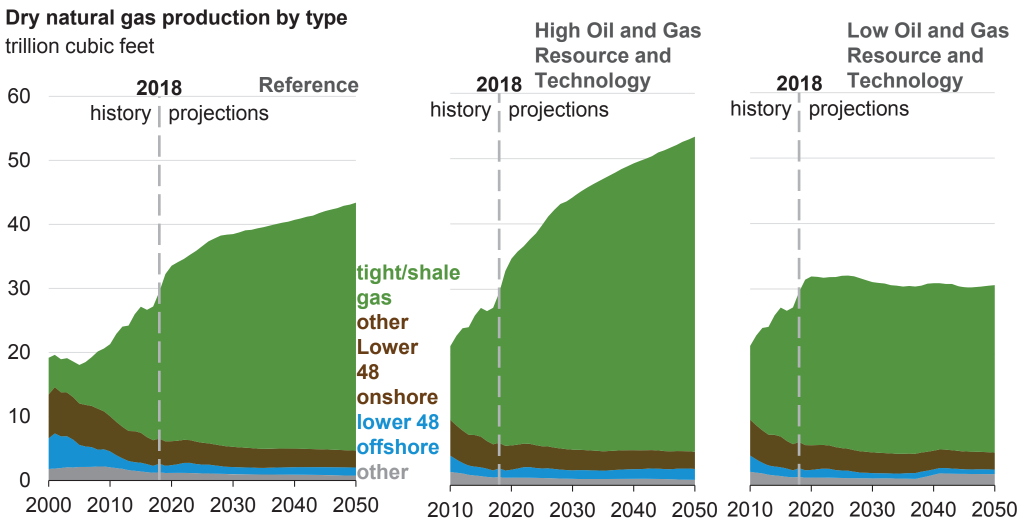


Figure 4.12: Dry natural gas production by type in the U.S, 2000-2050 (eia.gov, 2019)

The natural gas production had an average growth of 4%/year from 2005 to 2015. Between 2018 and 2020, the reference case estimates an average growth of 7%/year. After 2020, this average growth is expected to be less than 1%/year, due to a decrease in both domestic consumption and demand for U.S natural gas exports. This can be observed in figure 4.13. However, production grows at a higher rate than consumption from 2020 in most cases, leading to a growth in U.S exports of natural gas to global markets. (eia.gov, 2019)

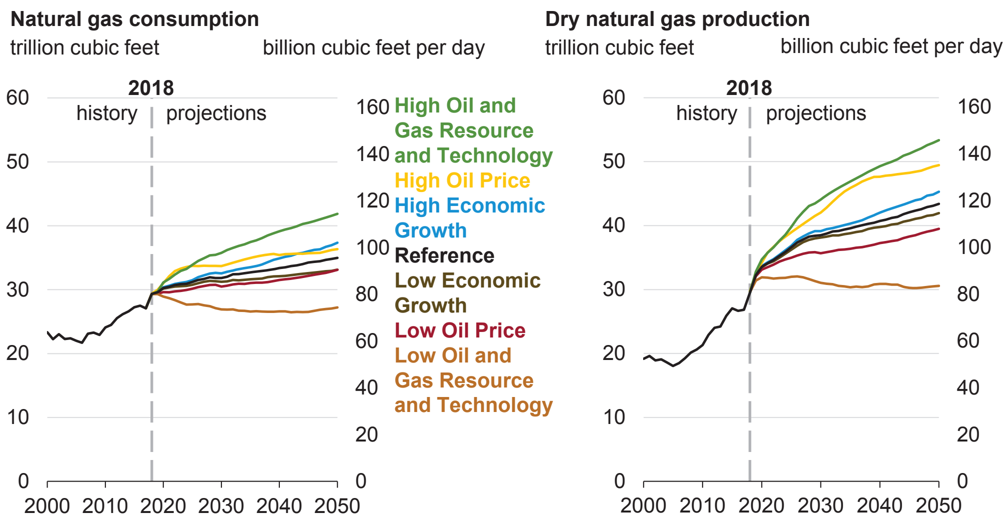


Figure 4.13: Left: Natural gas consumption in the U.S, 2000-2050; Right: Natural gas production in the U.S, 2000-2050 (eia.gov, 2019)

Natural gas use for electricity generation is expected to increase due to low natural gas prices and because more power plants that use coal will be closed down because of environmental regulations. Demand growth in the industrial sector is also expected due to low natural gas prices. (API, 2014)

There has been shifting flows on the U.S interstate pipeline network due to the growth in shale gas production. Marcellus shale production in Pennsylvania and West Virginia is located close to the major east coast consuming markets. This has reduced the need for long-haul pipeline transportation from traditional supply areas like Canada and the Gulf Coast, while at the same time increasing the need for local pipeline infrastructure to support new production. Figure 4.14 shows the major flow patterns in the U.S as of 2012. (API, 2014)

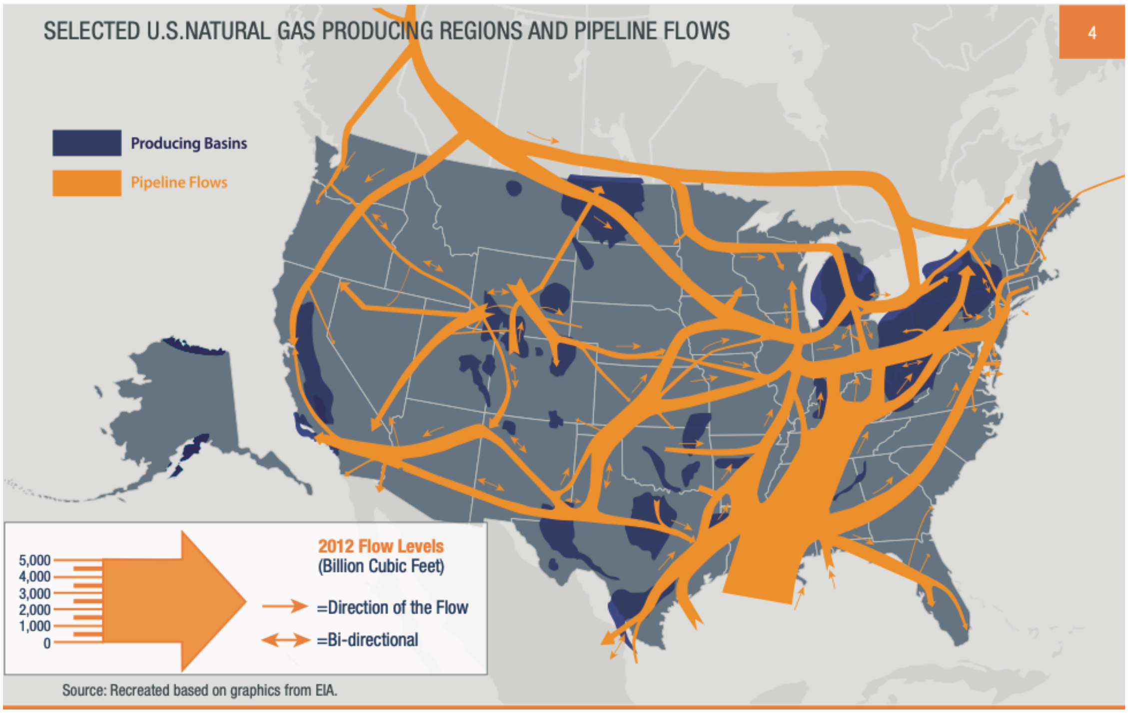


Figure 4.14: Flow pattern in the U.S as of 2012 (API, 2014)

The production increase and the low prices of natural gas are leading to the development of both LNG (Liquefied Natural Gas) and pipeline export projects. This could lead to that U.S could be a net exporter of natural gas, while historically the U.S has been a net importer due to its reliance on Canadian natural gas to meet domestic demand. (API, 2014)

The natural gas production and delivery system are not designed to produce and transport natural gas during periods of peak demands. Large customers and distribution companies , therefore, injecting some of the gas into underground storage located near final customers. Typical gas storage facilities include salt domes, depleted gas reservoirs, and deep aquifers. Figure 4.15 shows an illustration of gas storage in salt caverns. There is also storage in producing areas, which allows producers to maintain constant production and helps balance supply and demand. (API, 2014)

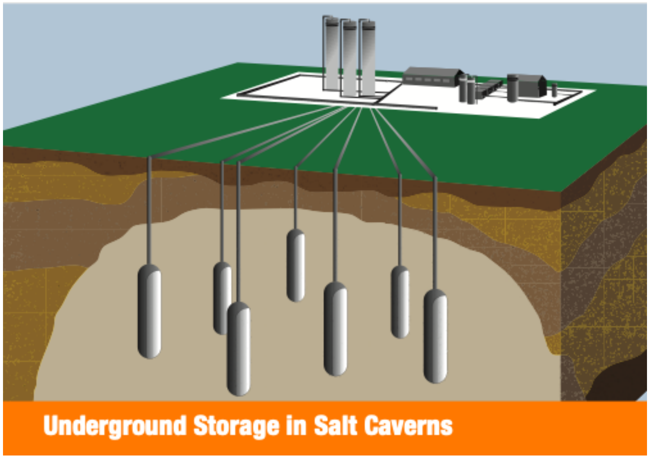


Figure 4.15: Gas storage in salt caverns US (API, 2014)

4.4.2 United Kingdom

The natural gas sector in the UK is fully privatized, including production, transmission, and distribution. As of January 2018, the UK held an estimated 6,2 tcf (Trillion cubic feet) of proven natural gas reserves. Most of the natural gas in the UK comes from offshore fields. In 2016, 66% of total gross natural gas production came from offshore fields. (eia.gov, 2018)

In 2000 the natural gas production in the United Kingdom peaked at 3,8 tcf. After the peak, the production declined again from 2000 to 2014, at about 7% per year. This can be seen in figure 4.16. The years before, and during 2014, there were high oil and gas prices, which increased investments in North Sea assets which have resulted in increased production from 2014. More precise, natural gas production has increased by an average of 5% per year from 2014 to 2016. (eia.gov, 2018)

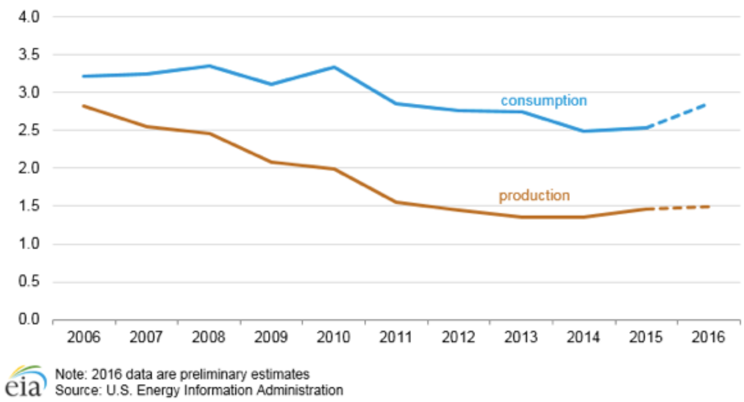


Figure 4.16: Production and consumption in the UK, 2006-2016 in tcf (eia.gov, 2018)

Shale testing is in an early phase in the UK, and the shale geology is more complex compared to the U.S. The two formations which have been given the most attention so far are the Bowland shales and the Weald basin. The fracturing of shale formations has caused some problems. In 2011, two minor earthquakes were triggered by hydraulic fracturing in the Bowland Basin. This caused the UK government to impose a moratorium. Additional requirements for monitoring and control was imposed in 2012 before they allowed shale drilling and fracturing to resume. Companies must receive permission from both the UK government and local council governments to drill or fracture new wells. (eia.gov, 2018)

Natural gas consumption was about 2,9 tcf in 2016, a 13% increase from 2015. 35% of total consumption in 2016 was residential natural gas consumption. Consumption in the public electricity sector increased by more than 45% from 2015 to 2016. This increase is mainly due to declining coal use in the electricity sector. (eia.gov, 2018)

The UK has been a net importer of natural gas since 2004. The UK imported 1,7 tcf of natural gas in 2016. 77% of which came by pipelines and the rest was imported as LNG. 87% of total imports came from Norway and Qatar. In 2016, the UK exported 0,4 tcf of natural gas to the European continent and the Republic of Ireland. (eia.gov, 2018)

The UK has several pipelines that carry natural gas from the UK and Norwegian offshore platforms to coastal landing terminals. Furthermore, the UK has two natural gas pipeline interconnections with the Republic of Ireland, an undersea link from Scotland, and a smaller-capacity link from Northern Ireland. There also exist two pipeline connections with continental Europe, including the Interconnector pipeline. A map of the onshore pipeline system can be seen in figure 4.17. (eia.gov, 2018)

Figure 4.17: Gas pipes UK from (National Grid, 2019)

4.4.3 Henry Hub & NBP

Henry Hub is a natural gas pipeline located in Erath, Louisiana. It is the official delivery location for futures contracts on the New York Merchantile Exchange (NYMEX). The hub connects to four intrastate pipelines and nine interstate pipelines, including the Transcontinental, Acadian and Sabine pipelines. Henry Hub is used as a benchmark for the entire North American natural gas market and parts of the LNG market. (Chen, 2018d)

National Balancing Point (NBP) is a virtual trading location in the UK. The UK gas market is the benchmark for most of the gas traded in Europe thanks to its size. Natural gas in the UK is traded in three main forms (The Switch, 2014):

- Bilateral contracts: Natural gas contracts are negotiated between a seller and a shipper for big quantities of natural gas over longer periods.
- Over the counter (OTC): Most common in the UK. Here, clips of natural gas are traded over over specific periods. These trades are bilateral but are becoming more and more standardized.
- Exchange futures: Regulated by the Financial Services Authority. InterContinental Exchange (ICE) operates the UK gas futures market.

5 Data Analysis

In this chapter the six portfolios will be presented. First, the historical prices of the assets will be presented. Second, descriptive statistics will be presented for both the assets and the portfolios followed by their distribution graphs. In this thesis, we have set a minimum value of the degrees of freedom parameter to 3, and a maximum value of 15. Finally, the rolling volatility models are presented for both the assets and the portfolios.

5.1 Portfolios

All of the daily prices used in this thesis (except Henry Hub) are downloaded from Thomson Reuters Eikon. The daily prices of Henry Hub have been downloaded from eia.gov. Daily prices for assets from the energy and commodity market have been collected. To investigate the diversification effect, six portfolios have been created; two energy portfolios, two portfolios of other non-energy commodities and two total portfolios that contain all of the assets. One portfolio from each segment gives all assets equal weight (balanced), while the other portfolio from each segment uses the allocation that provides the minimum variance (see table 5.1 for an overview of the portfolios and appendix B for overview of the assets). Due to the limitations of the coal data, all of the assets have a starting date of 01.05.2003, and the ending date has been set to 01.02.2019.

Table 5.1: An overview of the six portfolios

1	Energy balanced portfolio
2	Energy minimum variance portfolio
3	Commodity balanced portfolio
4	Commodity minimum variance portfolio
5	Total balanced portfolio
6	Total minimum variance portfolio

5.1.1 Portfolios 1 & 2 - Energy

The energy portfolios consist of three oil benchmarks, two natural gas benchmarks, and two coal indexes. There were not available daily coal prices from the Reuters database, so coal indexes were used instead. The energy portfolio consists of the following assets:

- Crude Oil-WTI Spot Cushing U$/BBL - DS MID PRICE
- Crude Oil BFO M1 Europe FOB $/Bbl
- Crude Oil Dubai 1Mth FOB Asia $/BBL
- Henry Hub Natural Gas Spot Price Daily
- TR Natural Gas NBP UK 1st Fut. Day - SETT. PRICE
- TRAPI2Mc1 (TRPC Coal API2)
- TRAPI4Mc1 (TRPC Coal API4)

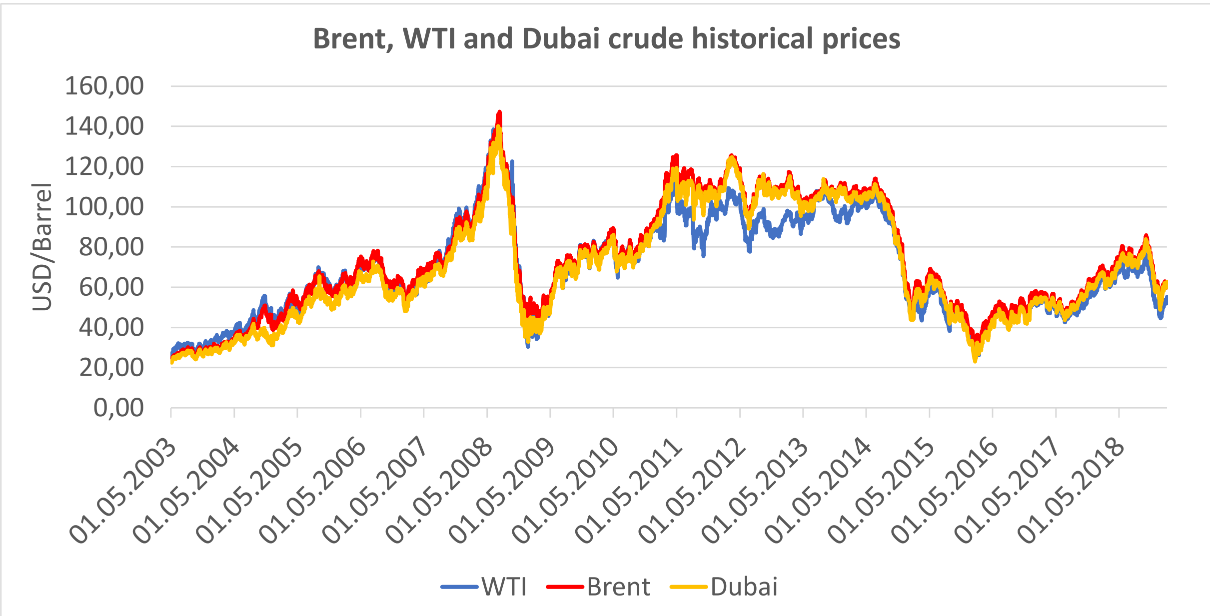


Figure 5.1: Historical oil price from data used in this thesis: Brent, WTI and Dubai

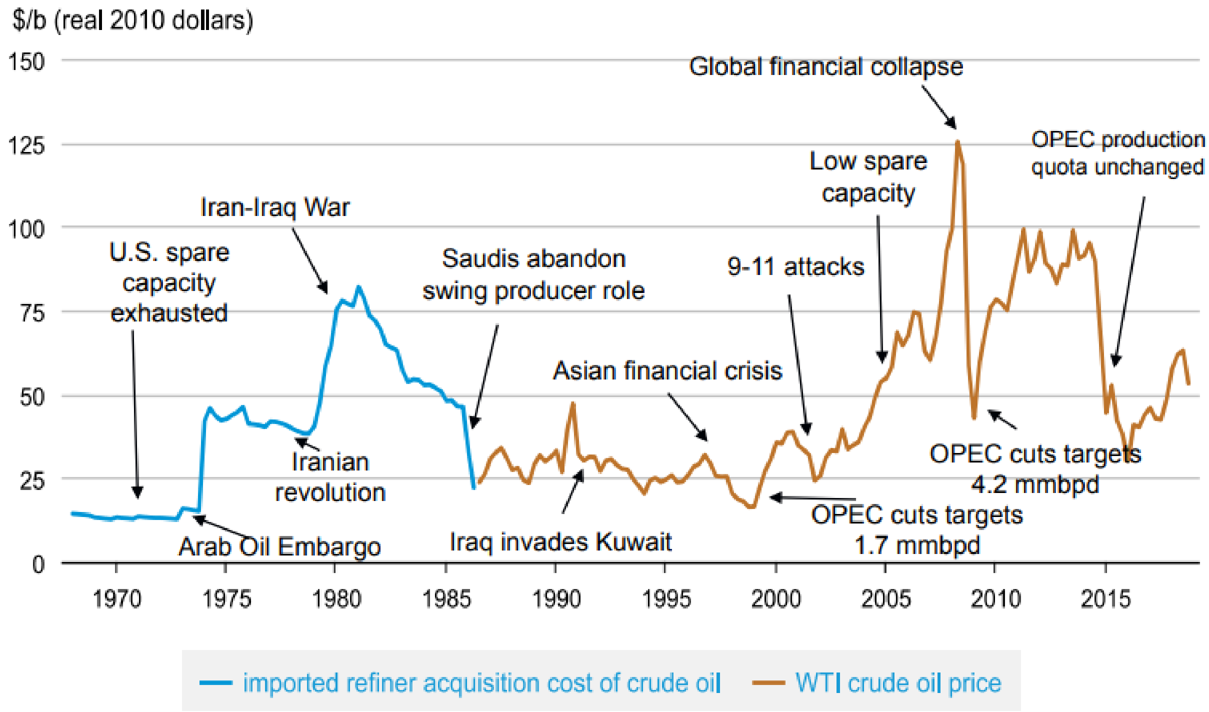


Figure 5.2: Historical events effect on the WTI-price (Eia.gov, 2019)

Figure 5.2 shows historical events that have influenced the WTI price the past decades. Since the Brent blend and Dubai crude are highly correlated with WTI (Table 4.2) these events have an impact on all benchmarks that are used in this thesis.

After the dot com bubble bust in 1995 and the Asian financial crisis in 1997, the oil price declined as a result of a decrease in demand. In 1999 the oil price got a significant increase in price due to OPEC production cut of approximate 1,7 million barrels of oil per day (Cnnmoney, 1999). This is one of several examples of how OPEC's actions can affect the oil price. After the 9/11 attacks in 2001 and the invasion of Iraq in 2003, the oil price increased caused by tension in the market. In the coming years up to 2008, the oil price increased rapidly up to an all-time high of approximate 147 dollars per barrel. The reason for this considerable increase was a very high growing demand for energy, mainly driven by China. OPEC experienced low spare capacity as well, which made them unable to control the market.

This changed quickly in 2008 after the banking firm Lehman brothers went bankruptcy as a result of the crisis in the subprime mortgage market in the United States. This had a dramatic impact on the oil industry and the world economy in general. The oil price went from an all-time high to approximately 33 dollars per barrel in just a couple of months. In the first quarter of 2009, OPEC acted and reduced their production by 4.2 million barrels of oil per day (Agoawike, 2009), which drove the oil price up to approximate 75 dollars per barrel.

In 2015, the oil price dropped to levels that had not been seen since the global recession in 2008. The oil price was cut in half in less than a year to as low as 30 dollars per barrel. The main reason for this burst was an oversupply of oil in the market where the US was the main driver. The US made huge technological improvements with the use of hydraulic fracturing, which led to a production boom of shale oil. OPEC refused at the same time to cut production, and the price of oil decreased. Since 2015 OPEC has reduced production several times, which has stabilized the oil price. (Amadeo, 2019). The political situation in Venezuela and the fact that the US sets an export ban of oil from Iran has also contributed to driving the price up.

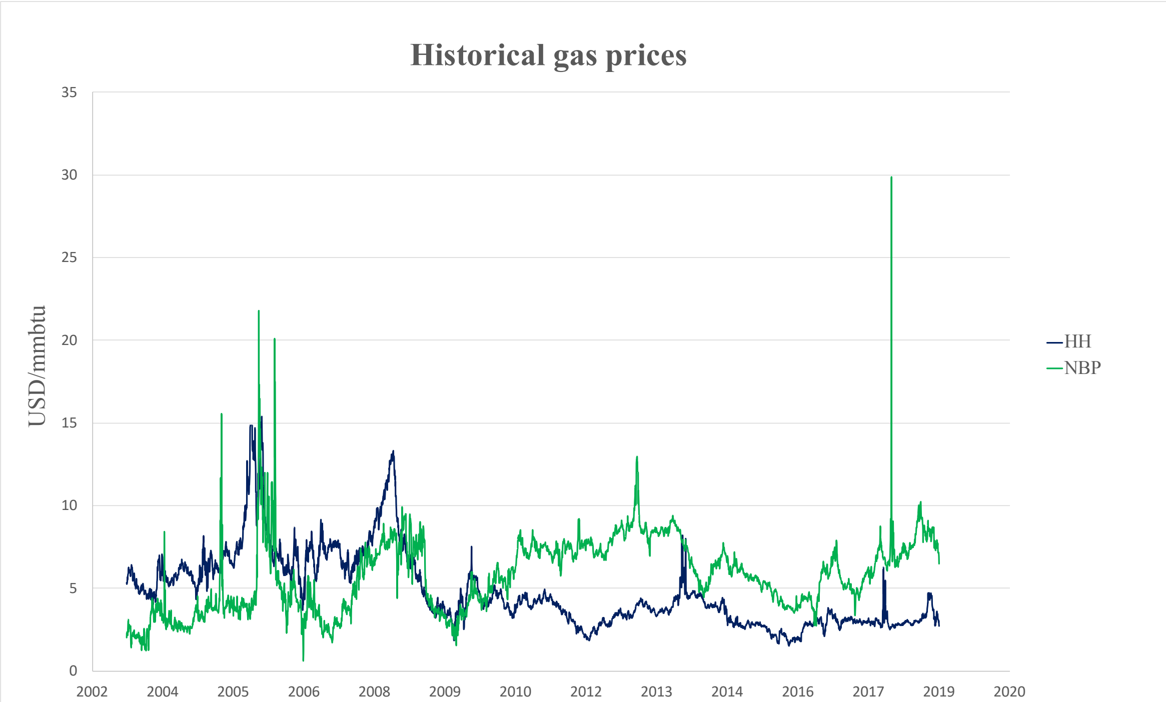


Figure 5.3: Historical gas prices: HH & NBP

Figure 5.3 above shows the historical gas prices from 2003-2019. Since the 1980s, the gas prices have been 60-70% of the oil prices, higher transportation costs being the main reason for the price difference. Traditionally, the gas prices have followed the oil prices with a lag. The higher transportation costs have also resulted in more regional markets for natural gas. As for price determination, OPEC plays a vital role in the oil market, but no such cartel exists in the gas market. Gas prices are quite volatile, and are strongly affected by local conditions like weather which affects the demand.(Winje et al., 2011)

The gas prices fell considerably in 2008 due to the sharp decline in oil prices followed by the financial crisis. The oil prices quickly recovered, but the gas prices continued the decline until summer 2009 and remained low. In the period 2008-2011, gas supplies have risen substantially. This is mainly due to higher production of unconventional shale gas in the U.S and a sharp increase in export capacity for LNG from the Middle East. (Winje et al., 2011)

From figure 5.3 above we can see that until about 2010-2011, HH and NBP followed similar paths. The difference was often reflected by local conditions, like storage and weather conditions. However, since 2010, HH tends to be lower. This is mainly due to the U.S being a highly competitive market and having a greater supply, which keeps the price low. NBP and other hubs in Europe are indexed to crude oil, which has different supply and demand factors affecting the price. (Winje et al., 2011)

Figure 5.4: Historical price of TRAPI2Mc1

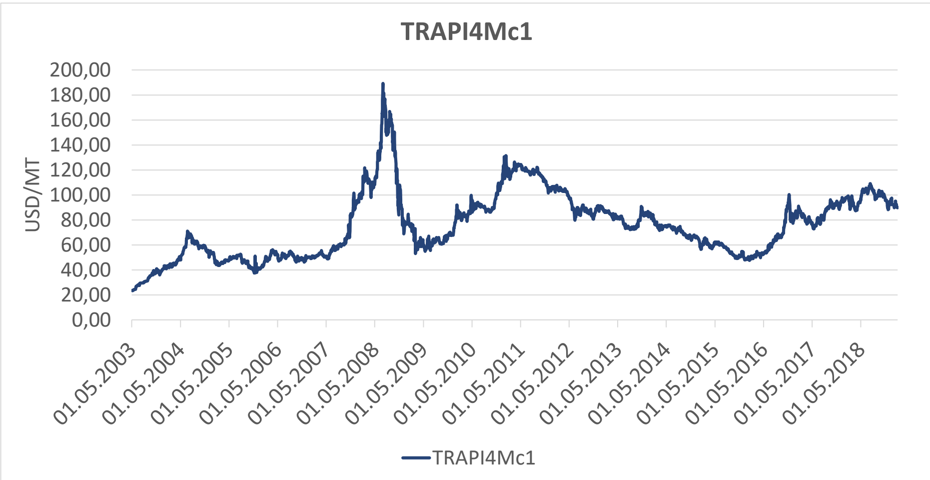


Figure 5.5: Historical price of TRAPI4Mc1

TRAPI2Mc1 refers to TRPC Coal API2 and is the benchmark price reference for coal imported to Northwest Europe. It is calculated as an average of the Argus cif ARA price assessment and the IHS McCloskey NW Europe Steam Coal marker. (Argusmedia, 2019a)

TRAPI4Mc1 refers to TRPC Coal API4 which is the benchmark price reference for coal exported from South Africa's Richards Bay terminal and is used in physical and over-the-counter contracts. It is calculated as an average of the Argus fob Richards Bay price assessment and the IHS McCloskey FOB Richards Bay marker. (Argusmedia, 2019b)

Coal is mainly being used in power generation, manufacturing of cement, iron, and steel. The coal reserves are spread among the Asia Pacific, North America, Europe, and Eurasia. Coal is shipped large distances to be sold in different markets, and therefore, the pricing of coal are correlated and affected by the cost of shipping. The weather has also an impact on the production of coal and thus the pricing of coal. (Mernier, 2010)

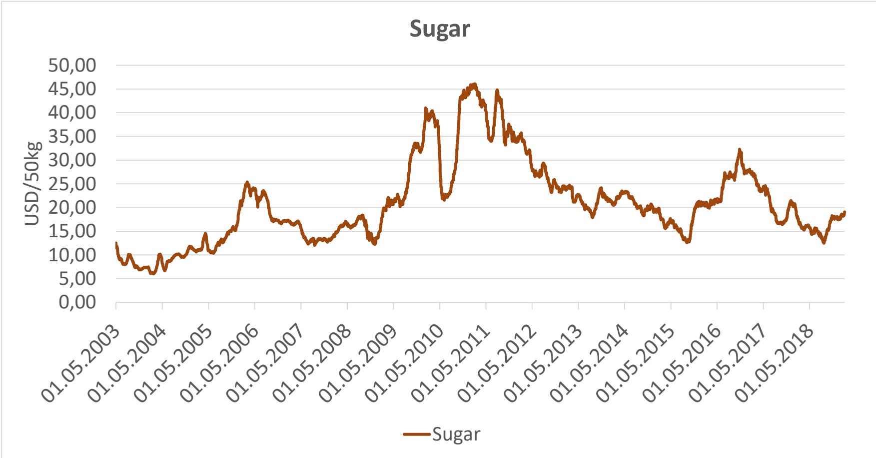
As figure 5.4 and 5.5 shows, there have been three significant booms in the coal price in the last decades. The correlation matrix in table 5.5 shows a high correlation between oil prices and coal prices. When the oil price hit an all-time high of 147 dollars per barrel, coal prices also reached their peaks. As shown in figure 5.4 and figure 5.5, the price for TRAPI2Mc1 in July 2008 was approximate 225 dollars per million tons, and the price for TRAPI4Mc1 in July 2008 was approximate 190 dollars per million tons. The reason for this increase was the increasing demand for energy, which was mainly driven by China. (Mernier, 2010)

The coal prices were, like most commodities affected after the financial crisis in 2008. The coal price fell rapidly to 75 dollars per million tons late in 2008 because of less demand as a result of the collapse in the worldwide economy. The coal price started to recover again after the financial crisis was over and reached about 120 dollars per million tons in the first quarter of 2011 across both the Atlantic and Pacific basins. A combination of oversupply to Chinese and international markets and the slowing of the global economy resulted in a falling coal price again, which ended at approximate 50 dollars per million tons in 2015. Due to an oversupply of coal in the market, the Chinese government introduced a range of new policies to curb oversupply, including restricting mine operating rates to 276 days/year which led to higher coal prices. There is a decreasing demand for coal in Europe because of more renewable energy available, while there is still high demand from countries such as China and India, which keeps the coal price relatively high. (Refinitiv, 2019)

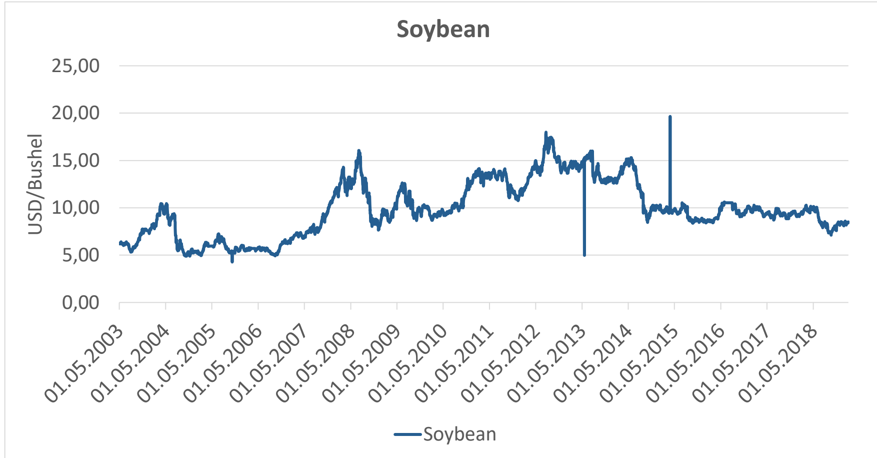
5.1.2 Portfolios 3 & 4 - Other commodities

The commodity portfolios consist of the following assets:

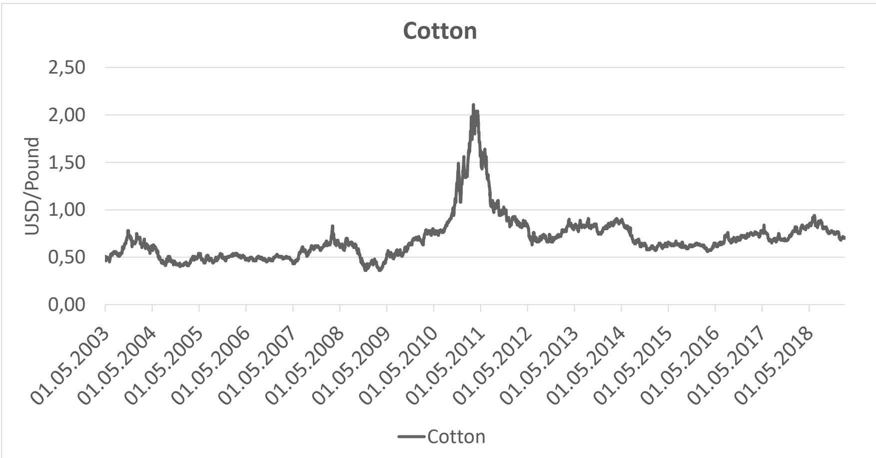
- LME - Aluminium 99.7% Cash U$/MT
- LME - Copper Grade A
- Gold Bullion LBM $/t oz DELAY
- Sugar, Crystal, Sao Paulo U$/50KG
- Cotton,1 1/16Str Low -Midl,Memph $/Lb
- Wheat No.2,Soft Red U$/Bu
- Yellow Soybn US NO.1 Sth Dvprt U$/Bsh
- Colombian Cofee ARAB Ex DC NY Cts/Lb ¨



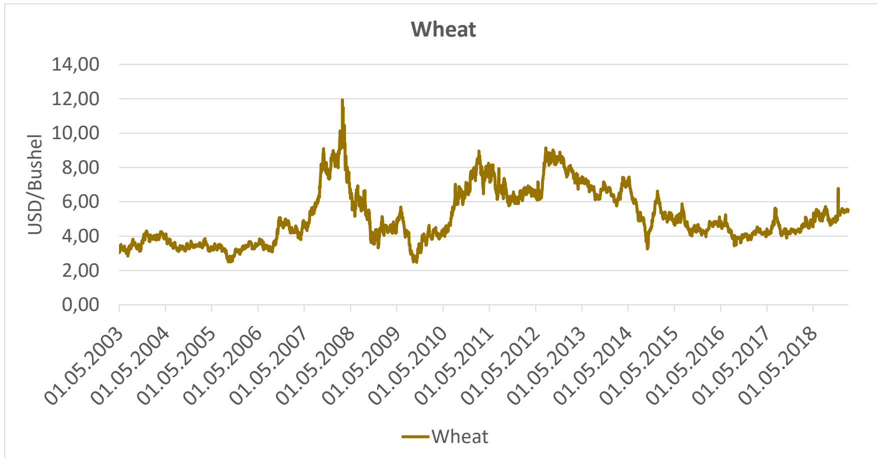
(a) Sugar



(b) Soybean



(c) Cotton



(d) Wheat

Figure 5.6: Historical prices of commodities

Commodities can be classified as perishable and non-perishable commodities. The perishable commodities must be consumed when they are fresh in a relatively short time frame after production. Long storage will reduce the quality of the commodities. For most consumable commodities the supply is dependent on the weather. If there are adverse weather as floods, droughts or cold periods over long periods and in areas where there are significant producers of a specific commodity, an ample supply drop may occur, and the price will spike. Vice versa, the price will drop if there are good weather conditions that lead to higher production, especially for perishable commodities. Further, in this subsection, the most noticeable price fluctuations of the commodities in figure 5.6 will be described to get an overview of factors that influences the price of these commodities. (Tomek, 2014)

The three largest producers of sugar in the world are Brazil (739.300 TMT), India (341.200 TMT) and China (125.500 TMT) (TMT: thousand metric tons). Sugar is the main product of sugarcane. Ethanol is also produced from sugarcane, especially in Brazil, where many automobiles use ethanol as fuel. As Brazil are the largest producer of sugar, the demand for ethanol will, therefore, have an impact on the sugar price. (Sheth, 2011)

The sugar price has the past decade been very volatile, with large fluctuations in price in several periods, which can be observed in figure 5.6. The first significant price spike happened in 2005-2006 when the sugar price more than doubled before it fell rapidly again. Brazil, which is the world most significant producer of sugarcane, produced more ethanol from sugarcane, which leads to less production of sugar and a supply drop occurred, which drove the sugar price up. (Taylor, 2007) After the price had been stable in some years, the sugar price hit a new record high in 2010, of approximate 42 dollar/50kg. Adverse weather in Brazil in 2009/2010, and uncertainty about export from India, while the demand was unchanged drove the price up. (Mason, 2010) After the price had stabilized for a short period it went up to a 30-year high. Australia, which is the world third largest exporter of sugar, had a period with adverse weather which caused a flood that damaged sugar crops in December 2010, and January 2011, which made a significant supply drop in the market. (King, 2011) The latest significant price spike of sugar can be observed in 2016, because of "reduced planting in Brazil in 2015, combined with the onset of El Niño conditions which led to lower supply and higher sugar price." (Spend Matters, 2016) The sugar price is mainly affected by supply drop as a result of adverse weather in production areas, but as mentioned, also the ethanol demand will influence the sugar price.

Wheat is one of the most important agricultural commodities in the world. Most of the produced wheat, about 2/3, goes to food consumption. Wheat has excellent versatility for production, and therefore it is produced evenly over the whole world in different seasons. Wheat production is mainly depended on the weather, and adverse weather periods can lead to very high prices in the short term. (Folger, 2019)

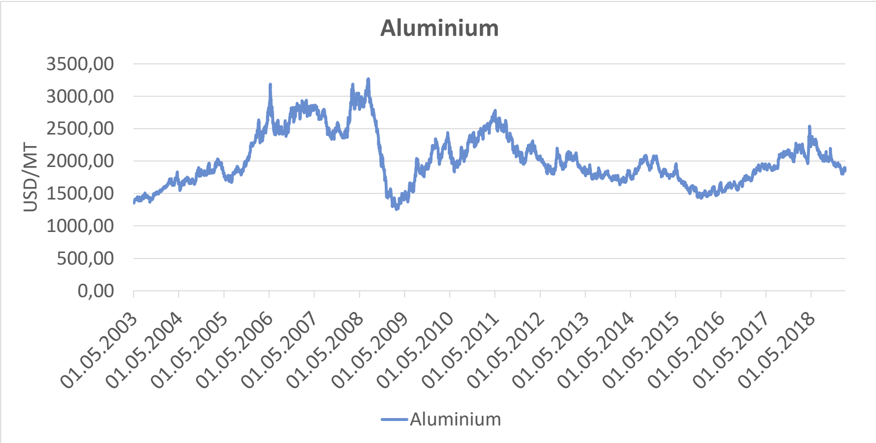
In 2008 most food commodities got a price spike. Figure 5.6 shows that this was also the case for the wheat price, which increased significantly from 2006 to 2008. The reason for this was because of a combination of high oil prices, high inflation, and a poor harvest in Ukraine, Australia, and Argentina. This led to a decrease in wheat production, and the price for wheat increased significantly as can be observed in figure 5.6. After the financial crisis hit later in 2008, the prices decreased drastically again as a result of lower demand. (Fogarty, 2011)

The wheat price also hit high peaks in 2011 and 2013. In 2010 Russia crop was significantly lower because of adverse weather conditions. After a short period with stable market conditions in 2011, the worst drought in a half-century in the US took place in 2012, which made the wheat price increase again. (KRAMER, 2010) (Zabarenko, 2012)

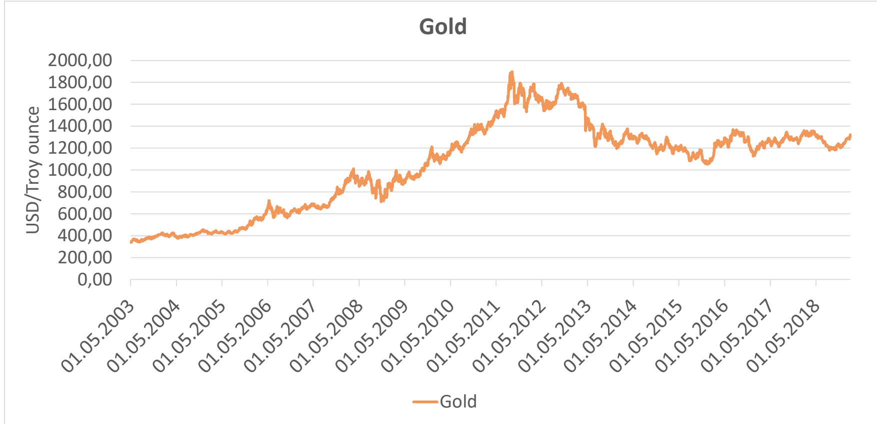
Soybean is affected by a variety of factors. Soybean is traded on the international market, so the dollar price plays a key role in the pricing of this commodity. Due to the growing use of biofuel, the crude oil price will also have an impact on the price. The correlation to corns such as wheat is also significant. As most perishable commodities, soybean is affected by the production level, and a good year and stable production tend to decrease the price. Brazil and the US is the largest producers of soybean, so their production level are of great importance for the price. Soybean experienced a drop in 2008 as all commodities due to the financial crisis. (Adeyanju, 2014)

The cotton price has the past decades been stable with an exception in 2011 as can be observed from figure 5.6. The percentage change from 2010 to 2011 was as high as 150 percent. This large jump can be explained by the demand for textiles rebounded from the global financial crisis and India, the world's second-largest exporter, restricted shipments to help its domestic textile industry. (Meyer, 2011)

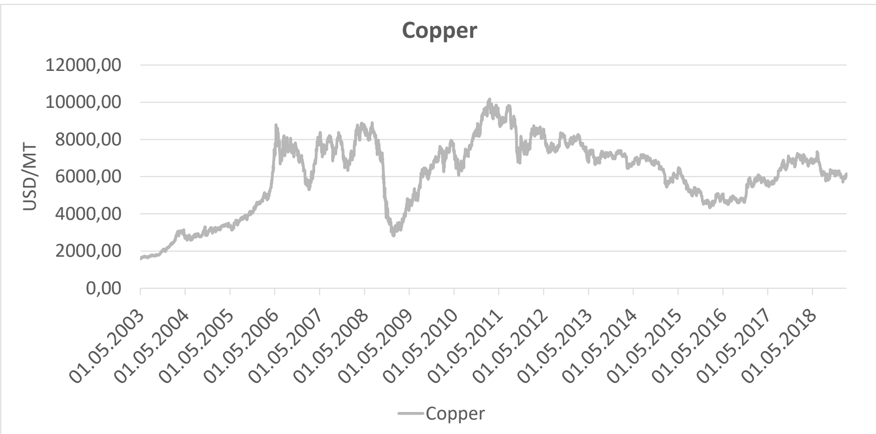
This section has proved that the commodity market can be described as very volatile in the short term because production is depended on weather conditions. In the longer term, farmers will priorities their crops towards the most profitable production and bring the market in equilibrium again.



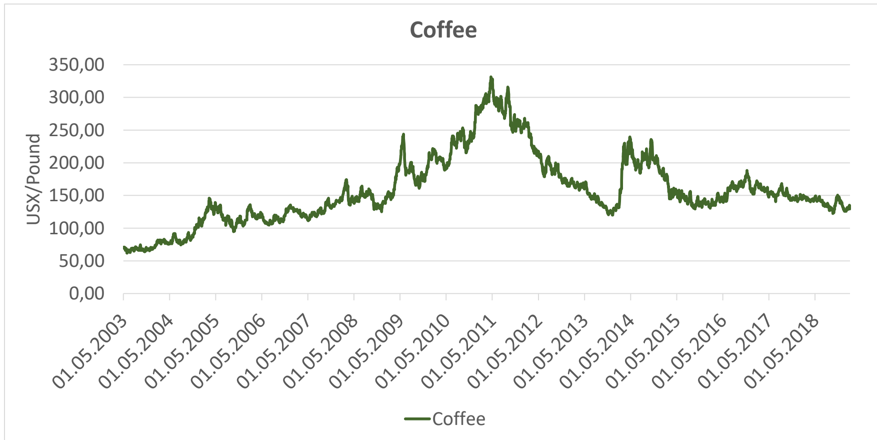
(a) Aluminium



(b) Gold



(c) Copper



(d) Colombian coffee

Figure 5.7: Historical prices of commodities

The copper price is mainly determined by the possibility to extract and transport the product. The housing industry plays a vital role in the demand for copper. During the financial crisis in 2008, the housing market declined, and the copper price dropped. During the 2000s, aluminum substituted copper in cables, electrical equipment, and refrigeration tubes due to the rising copper price. The copper price is also dependent on the weather, which can affect demand, production, and transportation. Copper is mainly found in South America, where worker strikes are frequent, and such a situation can affect the price. Any worries on geopolitical instability can force the price upwards.(Investopedia, 2018)

During the twenty-first century, aluminum had two price spikes (2006 and 2011). The first spike is referred to as the "raw material boom". Numerous factors affect the aluminum price. The first factor is the cost of electricity since primary aluminum is very energy intensive. Aluminum traded on exchanges is physically located in warehouses in seaports, and therefore stock exchange aluminum includes the cost of transporting the metal to its destination. Aluminum is also dependent on the economic conditions in which the metal is associated (construction, automotive, transportation). Also, the prices of other metals such as copper could influence the price and key energy resources such as oil. In recent years, the price has been influenced by the growing export of China, which has increased the supply of this metal significantly. Also, natural events, like extreme weather, could contribute to difficulties in the transport of aluminum, and could cause disturbances in the supply of the energy needed in its production. (Wzorek, 2017)

Mining firms and central banks play a key role in the pricing of gold. Gold is traded by central banks depending on the economic conditions. Many investors tend to invest in gold during economic crisis, as the gold price tends to perform well during economic busts. Interest rates affect the price of gold. Banks pay interest on savings account, and the US government pays interest on Treasury bonds. The price of gold truly reflects the interest rates in these markets. The price is also very dependent on production, as this commodity follows the laws of supply and demand. Government deficits, gross domestic product (GDP) growth, manufacturing data, wage data, and job reports will influence the price. A strong US economy will, for instance, push the gold price down. Based on (Zaky, 2016)

From 1965 until 1989 coffee was a regulated market and the price volatility could be corrected by the use of export quotas. The price has been quite volatile since the start of the free market in 1990. There was a long period of low prices from 1999 to 2004, which is known as the "coffee crisis". This period was the most extended period of low prices ever recorded for this commodity. The production averaged 11.5 million bags in Columbia between 1990/91 and 2012/13, up 1.4 million bags compared to the regulated market period. The production in Columbia experienced some issues regarding coffee leaf rust between 2008/09 and 2011/12, which drove the price down. The country started on a replanting program, and the production has recovered much. In crop year 2012/13 the production was 10.4 million bags, which is on the same level as the free market period. (ICO, 2014)

5.1.3 Portfolios 5 & 6 - Total (Combination of 1 and 2)

These portfolios include all assets from both the energy portfolio and the commodity portfolio and have been selected to obtain diversification by including more assets and assets from different markets.

5.2 Descriptive statistics assets

This section summarizes the assets using statistical properties. The descriptive statistics that are shown in the tables in this chapter are calculated based on the log-returns of each commodity price for the whole sample period.

Table 5.2: Descriptive statistics for assets from energy markets, based on the daily log-returns

	WTI	**Brent**	**Dubai**	**HH**	**NBP**	**TRAPI2Mc1**	**TRAPI4Mc1**
Degrees of freedom	3,53	3,80	3,47	2,42 (3)	1,42 (3)	2,01 (3)	1,84 (3)
Mean	0,0002	0,0002	0,0002	-0,0002	0,0003	0,0002	0,0003
Standard Error	0,0004	0,0003	0,0004	0,0007	0,0012	0,0003	0,0003
Median	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Mode	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Standard Deviation	0,0231	0,0190	0,0235	0,0423	0,0758	0,0173	0,0196
Sample Variance	0,0005	0,0004	0,0006	0,0018	0,0057	0,0003	0,0004
Kurtosis	4,4820	3,4962	4,4449	20,6250	39,8724	41,5056	47,6668
Skewness	0,0088	0,1181	-0,1830	0,5887	-0,4639	-1,7996	-1,0482
Range	0,2924	0,2564	0,3127	1,0010	2,0715	0,4858	0,5958
Minimum	-0,1283	-0,1079	-0,1600	-0,4756	-1,0814	-0,3205	-0,3202
Maximum	0,1641	0,1484	0,1527	0,5254	0,9901	0,1653	0,2757
Sum	0,7526	0,9529	1,0045	-0,6782	1,1636	0,8776	1,3426
Count	4111	4111	4111	4111	4111	4111	4111

Table 5.3: Descriptive statistics for assets from other commodity markets, based on the daily log-returns

	Sugar	**Cotton**	**Wheat**	**Soybean**	**Coffee**	**Aluminium**	**Gold**	**Copper**
Degrees of freedom	3,72	3,62	3,13	2,51 (3)	3,60	5,19	2,94 (3)	3,41
Mean	0,0001	0,0001	0,0001	0,0001	0,0002	0,0001	0,0003	0,0003
Standard Error	0,0002	0,0003	0,0004	0,0005	0,0003	0,0002	0,0002	0,0003
Median	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0004	0,0000
Mode	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Standard Deviation	0,0123	0,0196	0,0262	0,0338	0,0168	0,0141	0,0112	0,0174
Sample Variance	0,0002	0,0004	0,0007	0,0011	0,0003	0,0002	0,0001	0,0003
Kurtosis	3,6879	1,8096	8,4970	663,1862	2,4605	2,4117	5,6434	4,0976
Skewness	-0,2217	-0,0457	-0,2126	-0,3702	0,1113	-0,2172	-0,4801	-0,1254
Range	0,1599	0,1950	0,4858	2,2205	0,2019	0,1465	0,1703	0,2208
Minimum	-0,0807	-0,1040	-0,2467	-1,1132	-0,1041	-0,0826	-0,1016	-0,1036
Maximum	0,0792	0,0910	0,2391	1,1073	0,0978	0,0640	0,0687	0,1173
Sum	0,4205	0,3167	0,6034	0,3123	0,6349	0,3114	1,3483	1,3360
Count	4111	4111	4111	4111	4111	4111	4111	4111

Table 5.1 and 5.2 shows descriptive statistics for the daily returns for all the assets. The degrees of freedom parameter is calculated in Python using Maximum Likelihood Estimation (MLE); the other parameters are computed in Excel. In the case the degrees of freedom parameter is less than 3, it is replaced by 3 in all the calculations.

As expected, all of the daily returns are close to zero. All of the assets from the energy markets have kurtosis larger than three, and also has positive or negative skewness, indicating that this data is not normally distributed. As mentioned in the theory chapter, this means that it is possible that the data has heavy left tails, which will be better described by a student t distribution.

The assets from the commodity markets in table 5.3 show varying kurtosis and skewness. All the assets except coffee have negative skewness. All except cotton, coffee, and aluminum have kurtosis larger than three. It seems like coffee is the asset that is closest to a normal distribution based on the kurtosis and skewness.

5.3 Descriptive statistics portfolios

This section summarizes the data for the six portfolios using statistical properties. The descriptive statistics that are shown in the table are calculated based on the log-returns of each portfolio for the whole sample period.

Table 5.4: Descriptive statistics for the six portfolios based on the daily log-returns

	Total portfolios		**Energy portfolios**		**Commodity portfolios**	
	Minimum variance	**Balanced**	**Minimum variance**	**Balanced**	**Minimum variance**	**Balanced**
Degrees of freedom	4,13	3,43	3,92	2,94 (3)	4,17	3,82
Mean	0,0002	0,0002	0,0002	0,0002	0,0002	0,0002
Standard Error	0,0002	0,0001	0,0003	0,0002	0,0002	0,0001
Median	0,0002	0,0003	0,0001	0,0003	0,0001	0,0002
Mode	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Standard Deviation	0,0105	0,0075	0,0167	0,0129	0,0096	0,0082
Sample Variance	0,0001	0,0001	0,0003	0,0002	0,0001	0,0001
Kurtosis	6,1900	5,9226	9,6492	4,8366	24,4135	31,8092
Skewness	-0,3340	-0,2468	-0,2268	-0,2263	-0,3501	-0,3633
Range	0,1499	0,1277	0,2920	0,2003	0,2714	0,2601
Minimum	-0,0796	-0,0607	-0,1512	-0,1224	-0,1388	-0,1329
Maximum	0,0703	0,0671	0,1407	0,0780	0,1326	0,1272
Sum	0,7133	0,8096	0,7737	0,7453	0,6605	0,8376
Count	4111	4111	4111	4111	4111	4111

Table 5.3 shows descriptive statistics for the daily returns for the energy portfolios, commodity portfolios, and the total portfolios. The degrees of freedom parameter is calculated in Python using MLE, and the other parameters are computed in Excel. In the case the degrees of freedom parameter is less than 3, it is replaced by 3 in all the calculations.

As expected, all of the daily returns are close to zero. All of the portfolios have relatively large kurtosis and negative skewness, which indicates that the data are not normally distributed. As mentioned in the theory chapter, this means that it is possible that the data has heavy left tails, which will be better described by a student t distribution.

5.4 Volatility models

To calculate the risk for an asset or portfolio, the standard deviation is often used as a risk measure. This tells us about the volatility of the asset or the portfolio. The more the returns vary from the expected value, the more volatile the stock or portfolio is. To show how the risk varies, we have plotted the standard deviation for each asset in a rolling window of 250 days in figure 5.8 and 5.9. To compare the volatility for the single assets to the portfolios, the standard deviation for the balanced and the minimum variance portfolios are also plotted with a rolling window of 250 days. The volatility for the minimum variance portfolios are plotted in figure 5.10, while the volatility for the balanced portfolios are plotted in figure C.1. Because the prices of Henry Hub (HH) and Soybean had periods with high standard deviation compared to the other assets, their values are plotted on a separate y-axis, respectively in figure 5.8 and 5.9.

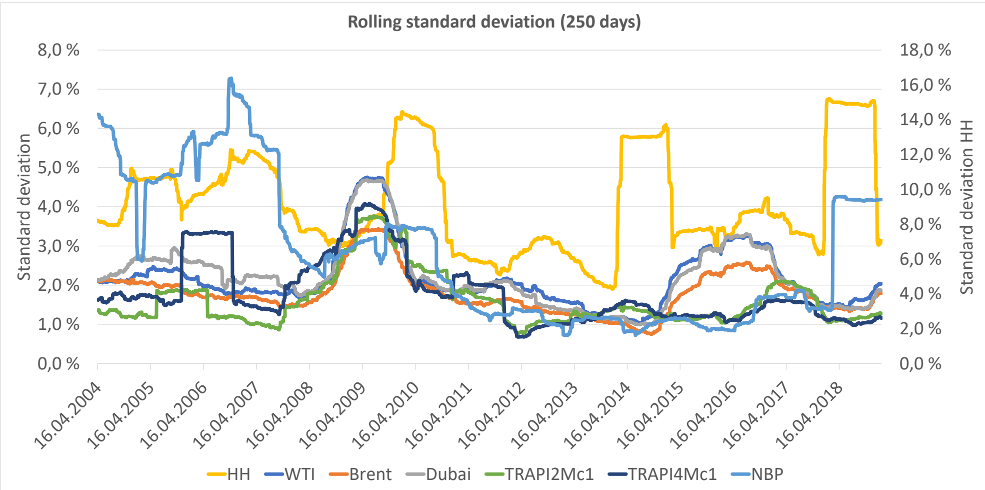


Figure 5.8: Rolling standard deviation for assets from the energy portfolio

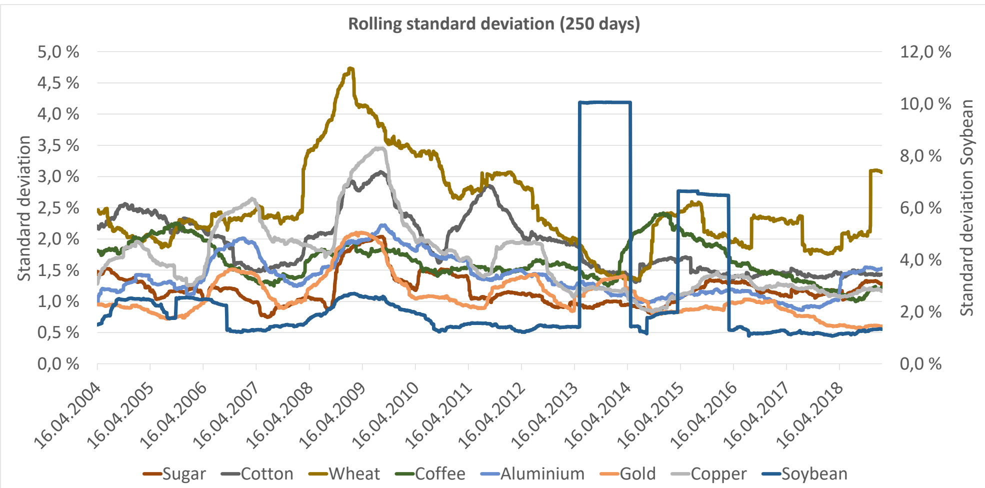


Figure 5.9: Rolling standard deviation for assets from the commodity portfolio

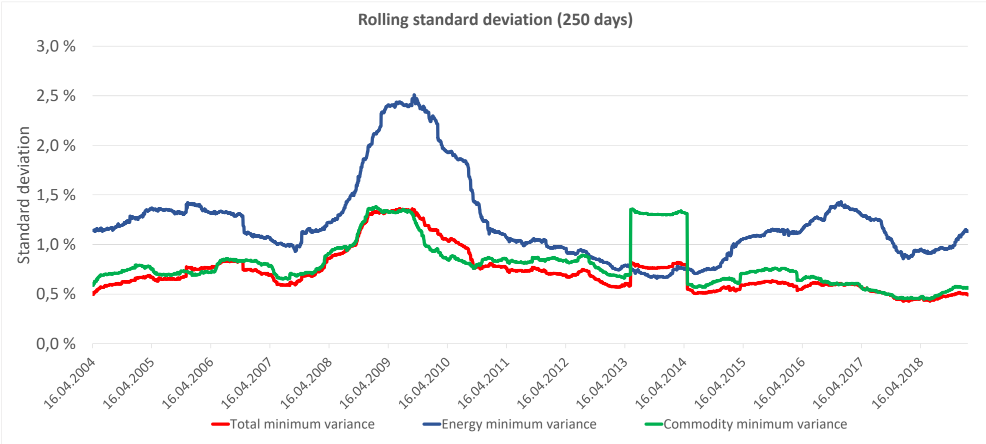


Figure 5.10: Rolling standard deviation for the three minimum variance portfolios

Figure 5.10 presents the rolling standard deviation for the minimum variance portfolios. The energy portfolio is by far the most volatile, and the total portfolio is the least volatile. However, at certain times, the commodity portfolio has slightly lower volatility than the total portfolio. This model also captures volatile periods, such as the financial crisis in 2008-2009, and also the oil crisis in 2015. This model also seems to account for the fact that volatility comes in clusters and busts. The same trend can be seen for the three balanced portfolios in figure C.1 in Appendix C.

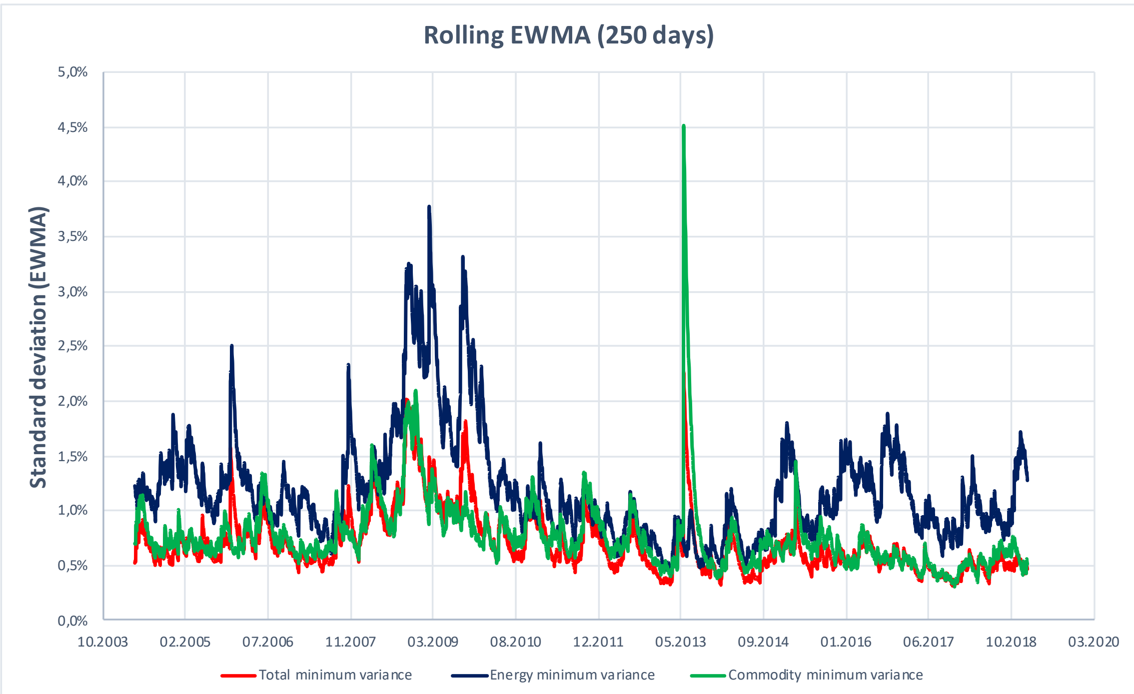


Figure 5.11: Rolling standard deviation (EWMA) for the three minimum variance portfolios

For the EWMA model, λ was set to 0,94 since this is considered a standard value in financial risk management. The returns from the three minimum variance portfolios have been used to calculate a rolling window using the data from the previous 250 days. One can observe from figure 5.11 that the EWMA model seems more volatile than the simple standard deviation. However, one can observe the same trend as with the simple standard deviation. This meaning that the energy portfolio seems to be the most volatile, and the total portfolio has the lowest volatility. However, at specific periods, the commodity portfolio has slightly lower volatility than the total portfolio. The financial crisis in 2008-2009 is quite clear, and also the oil crisis in 2015 is clear in the energy portfolio. From the figure, it seems like the EWMA model reacts to changing market conditions much faster than the simple standard deviation, and the clustering effect is not that evident here. The same trend can be observed for the three balanced portfolios in figure C.2 in Appendix C.

5.5 Diversification effect

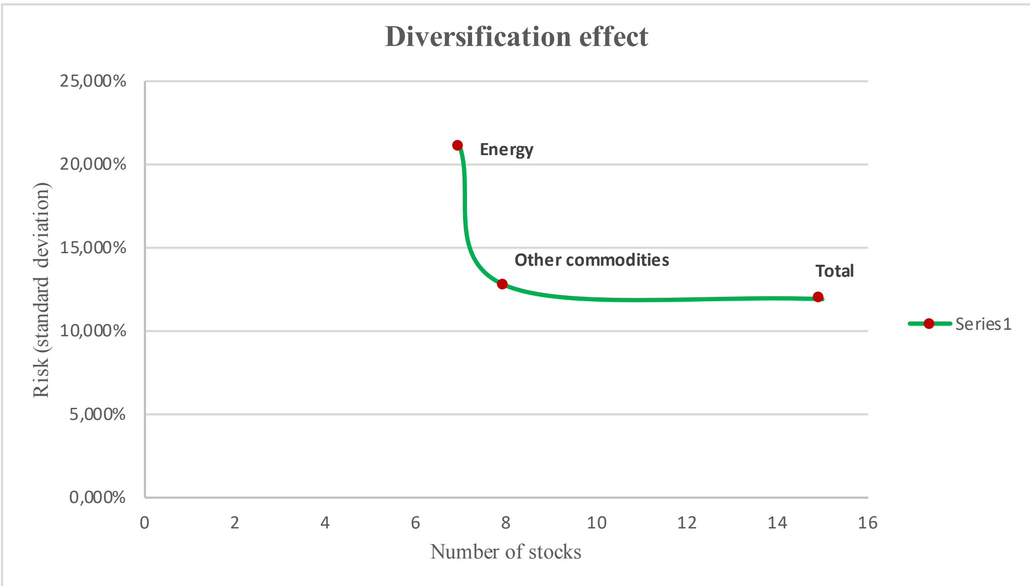


Figure 5.12: Diversification effect based on the three minimum variance portfolios

Figure 5.12 was made in Excel by calculating the minimum variance combination for the three portfolios based on data from the whole period from 2003 to 2019. The solver in Excel was then used to find the minimum variance allocation by finding the allocation which minimizes equation (2.8) from chapter 2.7.

As discussed in section 2.7, including more assets, reduces the unsystematic risk of the portfolio. This can also be observed in figure 5.12 above. The energy portfolio has a relatively high risk, and combining these assets with other commodities with lower risk, reduces the unsystematic risk. It seems from figure 5.9 that the risk of the total portfolio converges to the systematic risk related to the market.

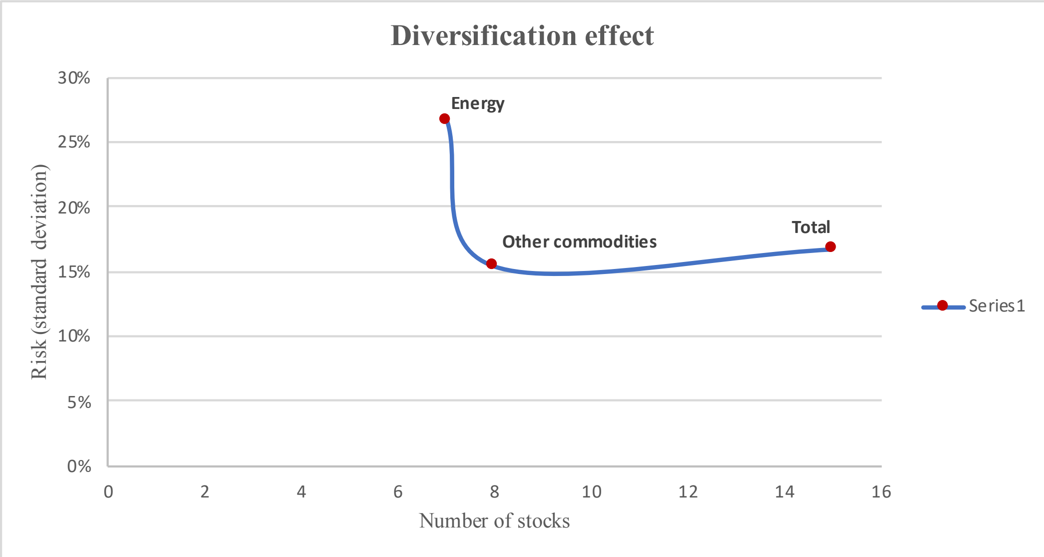


Figure 5.13: Diversification effect based on the three balanced portfolios

Figure 5.13 was made in Excel by giving all assets equal weight for the three balanced portfolios. Then the variance was calculated by using equation (2.8) from chapter 2.7 based on data from the whole period. We can see a similar trend also for the balanced portfolios, that the energy assets are the most volatile, but here the commodities have the lowest volatility. The total portfolio still has a much lower risk than the energy portfolio, so we can clearly see a diversification effect.

5.6 Correlation

Table 5.5: Price correlation based on whole period

	WTI	Brent	Dubai	Henry Hub	NBP	TRAPI2Mc1	TRAPI4Mc1	Sugar	Cotton	Wheat	Soybean	Coffee	Aluminium	Gold	Copper
WTI	1,00														
Brent	0,98	1,00													
Dubai	0,97	1,00	1,00												
Henry Hub	0,15	0,03	-0,01	1,00											
NBP	0,54	0,60	0,60	-0,06	1,00										
TRAPI2Mc1	0,71	0,68	0,67	0,22	0,46	1,00									
TRAPI4Mc1	0,69	0,72	0,72	-0,08	0,58	0,91	1,00								
Sugar	0,46	0,53	0,53	-0,34	0,29	0,31	0,51	1,00							
Cotton	0,46	0,52	0,53	-0,31	0,39	0,44	0,60	0,68	1,00						
Wheat	0,69	0,74	0,75	-0,15	0,52	0,58	0,60	0,38	0,56	1,00					
Soybean	0,70	0,76	0,78	-0,31	0,50	0,51	0,64	0,50	0,56	0,80	1,00				
Coffee	0,56	0,63	0,63	-0,34	0,35	0,44	0,62	0,85	0,72	0,53	0,62	1,00			
Aluminium	0,55	0,50	0,47	0,46	0,21	0,61	0,45	0,24	0,22	0,32	0,07	0,23	1,00		
Gold	0,50	0,63	0,65	-0,63	0,54	0,33	0,62	0,68	0,60	0,62	0,76	0,75	0,02	1,00	
Copper	0,80	0,83	0,83	-0,12	0,49	0,61	0,68	0,65	0,58	0,69	0,61	0,68	0,67	0,68	1,00

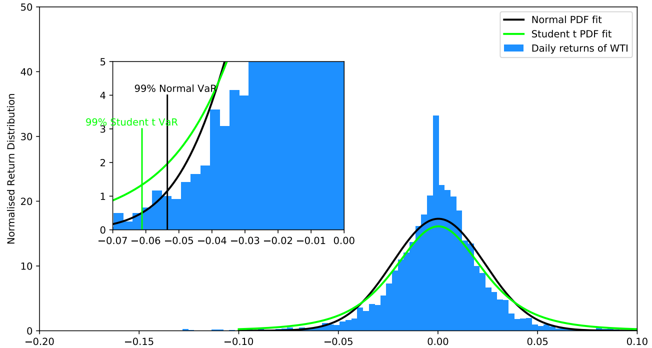
In the above table, the correlation matrix for all assets is presented. As discussed earlier, the oil prices are highly correlated. Brent and Dubai have perfect correlation! Most commodities (except Henry Hub) have a significant correlation to the oil prices.

Henry Hub is the only asset that has a low correlation to the other assets. Henry Hub has, in fact, a negative correlation to most of the other assets, which means Henry Hub is an excellent asset when it comes to diversification and reducing the risk. It can also be mentioned that coffee is highly correlated to gold and sugar. Soybean is highly correlated to wheat and gold, and sugar has a significant correlation to cotton.

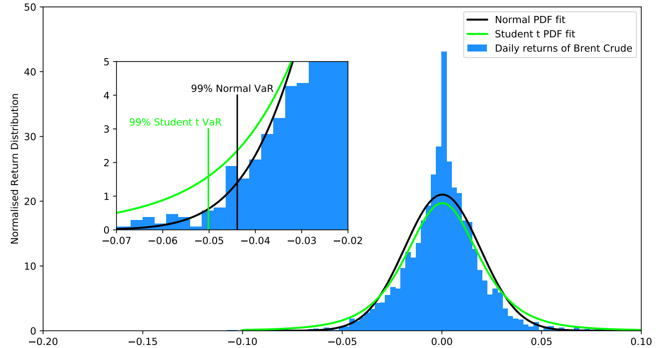
The correlation coefficients shown in table 5.5 is just to give an idea of the price behavior. It is not representative of the VaR calculations used in this thesis, since these calculations are done in a rolling window of 250 days, meaning the correlations are also calculated in a rolling window of 250 days. The correlations for these small periods can differ significantly from the values in this table.

5.7 Probability distributions assets

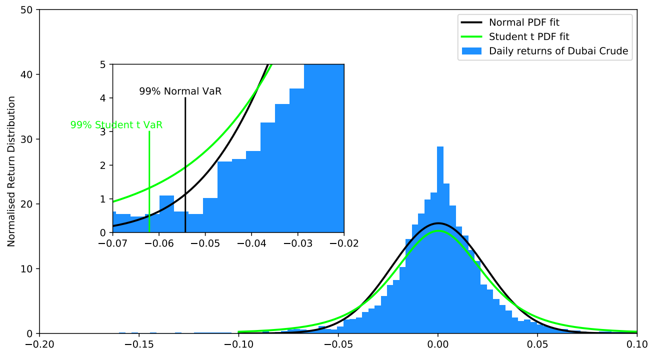
In this section, the daily returns for the whole sample period are plotted in a histogram with overlaying normal and Student t distributions. For each asset, the 99% normal VaR and 99% student t VaR is calculated based on the daily log-returns for the whole period. These figures are made in Python, see appendix A1 for details.



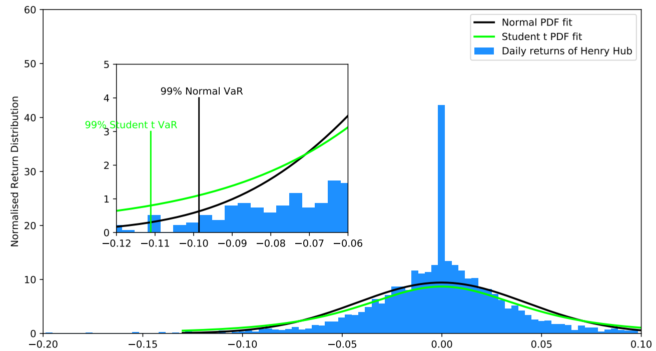
(a) WTI



(b) Brent

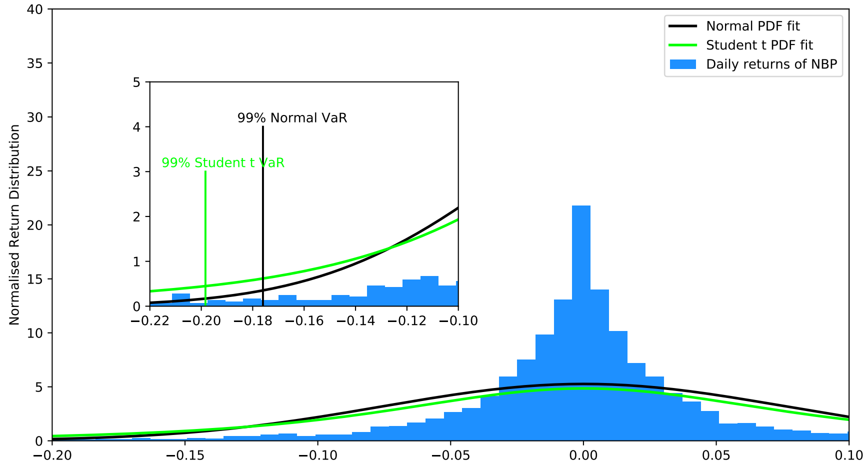


(c) Dubai Crude Oil

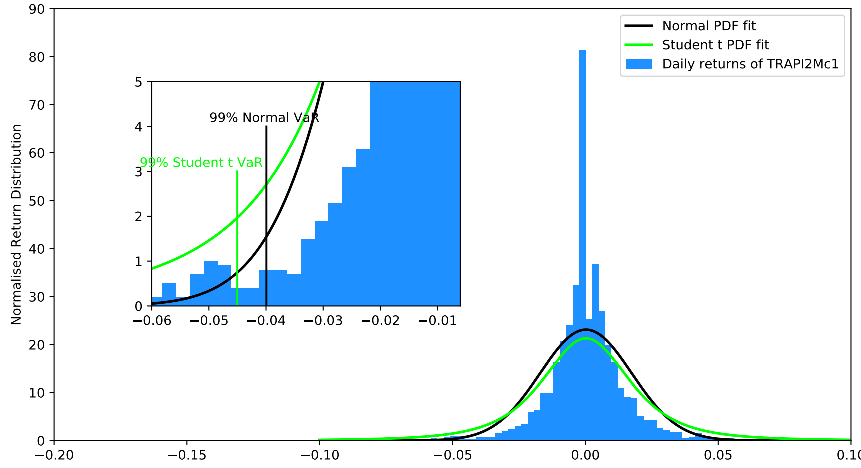


(d) Henry Hub

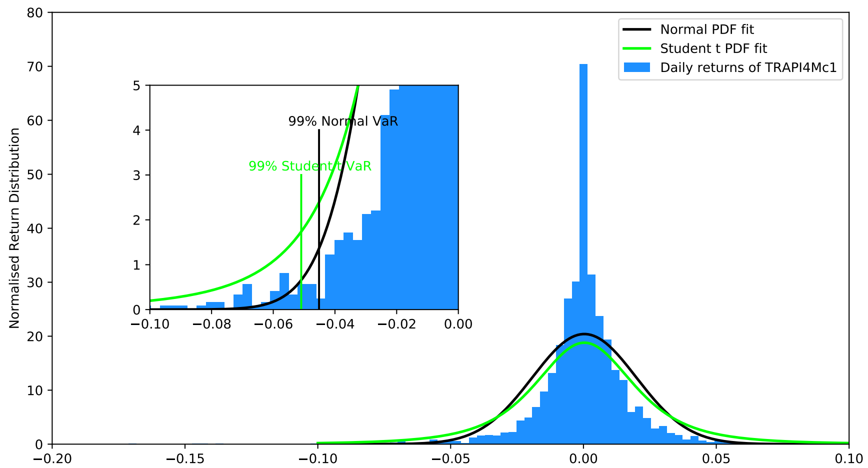
Figure 5.14: Probability distributions



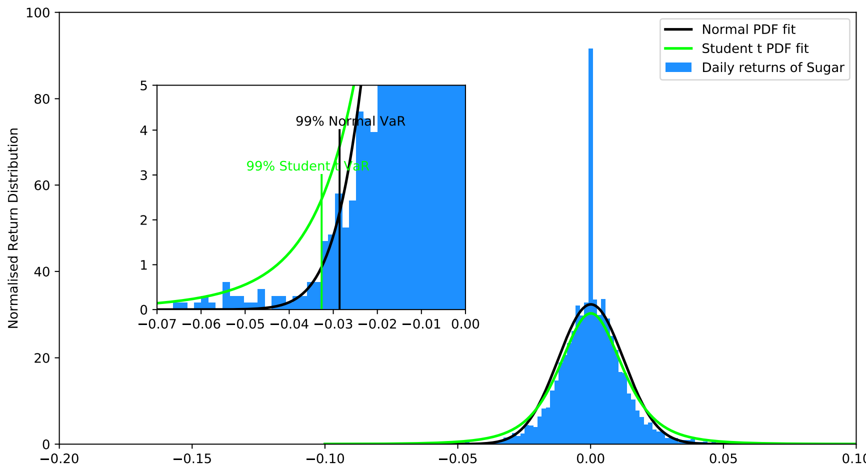
(a) NBP



(b) TRAPI2Mc1

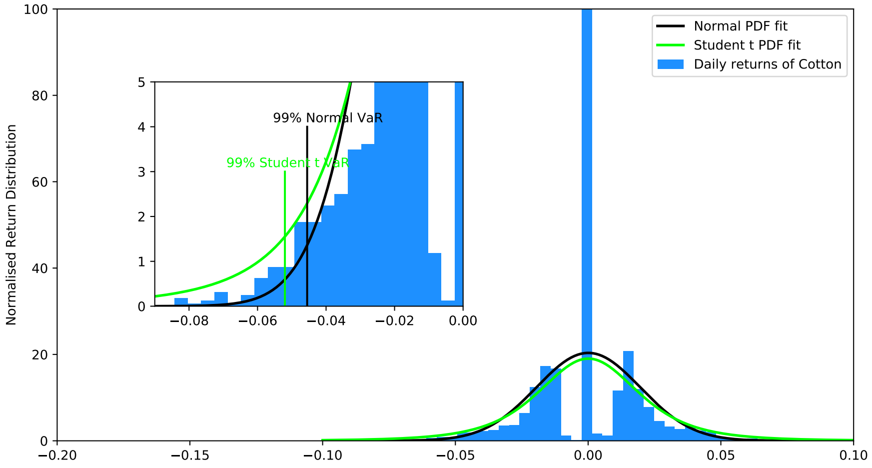


(c) TRAPI4Mc1

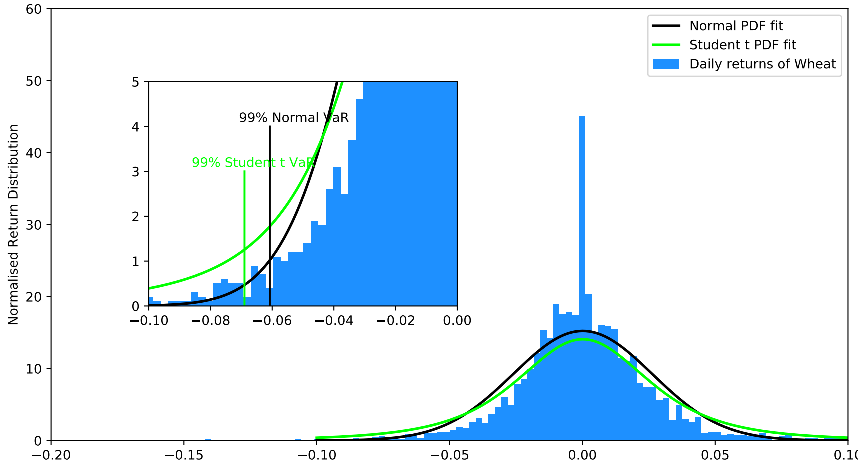


(d) Sugar

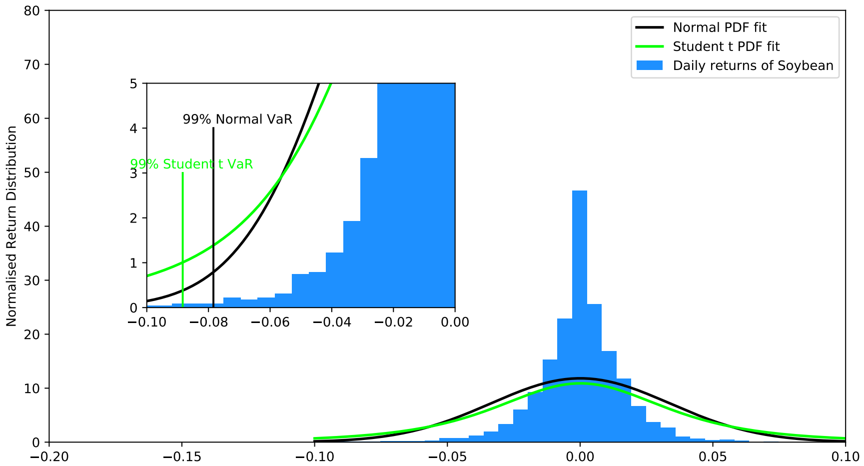
Figure 5.15: Probability distributions



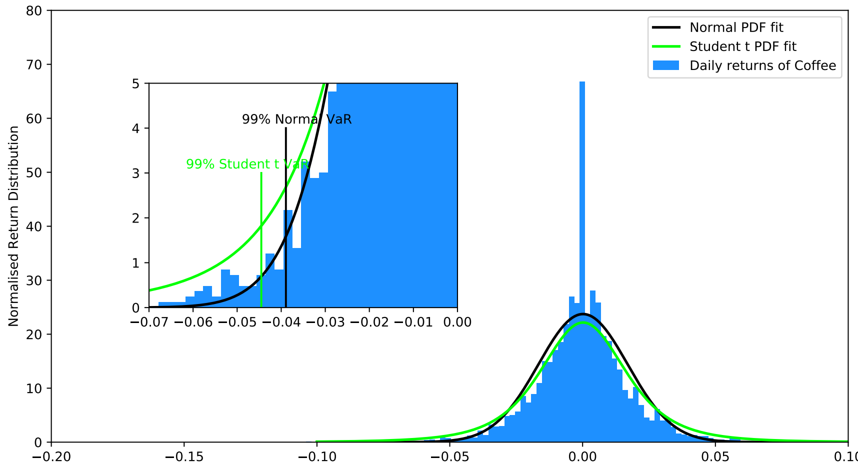
(a) Cotton



(b) Wheat

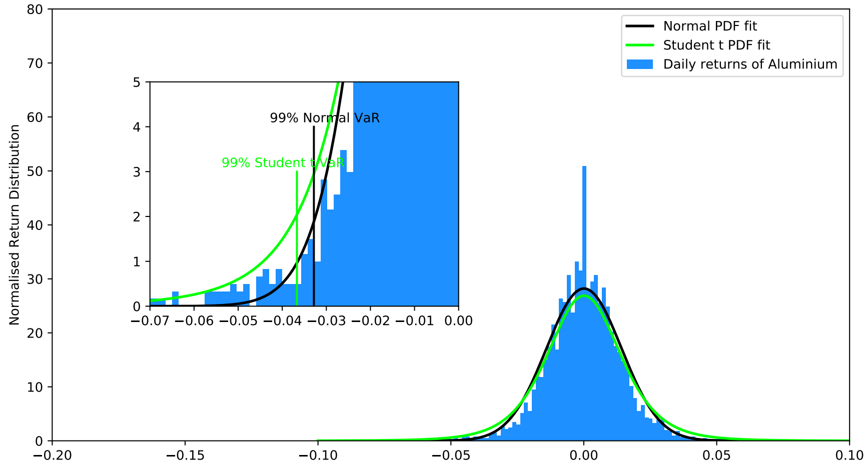


(c) Soybean

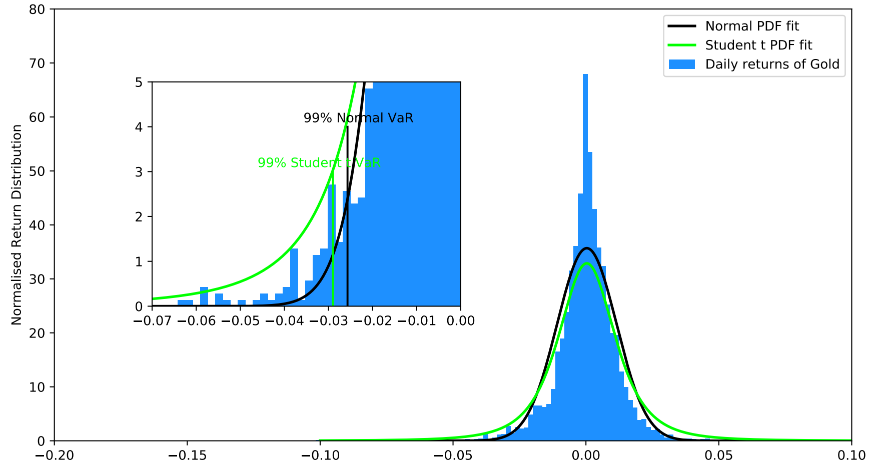


(d) Coffee

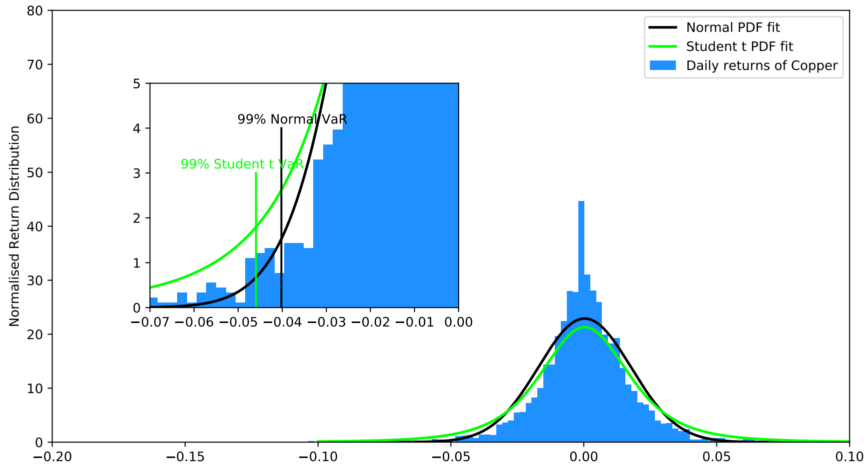
Figure 5.16: Probability distributions



(a) Aluminium



(b) Gold



(c) Copper

Figure 5.17: Probability distributions

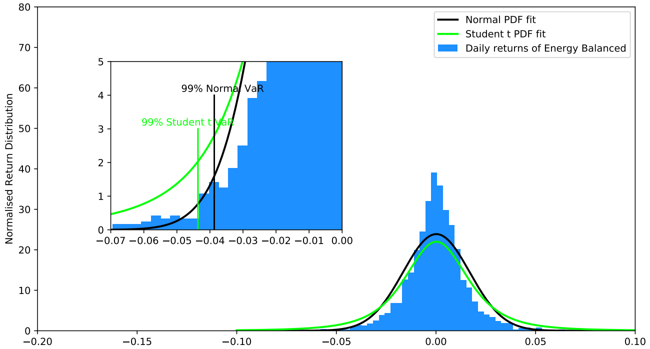
As seen from the figures above these assets have a mean of about zero. They all have high peaks at zero, unlike the probability distributions. One can also observe that the student t-tail fits the data better and therefore, does not underestimate the risk, which is the case for the normal distribution. However, the student t distribution tends to overestimate the risk for several of the assets.

Both student t and the normal distribution has low peaks when it comes to NBP and Henry Hub. One can observe that the 99% VaR estimates are higher for these assets, meaning there is a larger tail risk. This decreases the peak of the distributions, to cover more of the tail risk. This again points to the fact that natural gas is a quite volatile market.

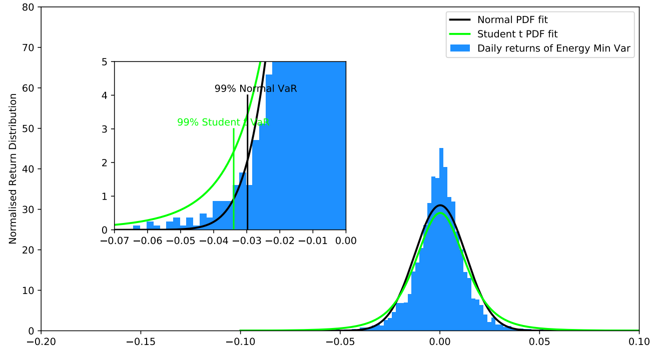
As for cotton, it seems like the histogram misses many data around zero. This is mainly due to the cotton price is relatively stable, and have many days with zero change in price. Also, when a change in price occurs, it comes at a value of at least ± 1%, explaining the lack of data between 0 and 1%.

5.8 Probability distributions portfolios

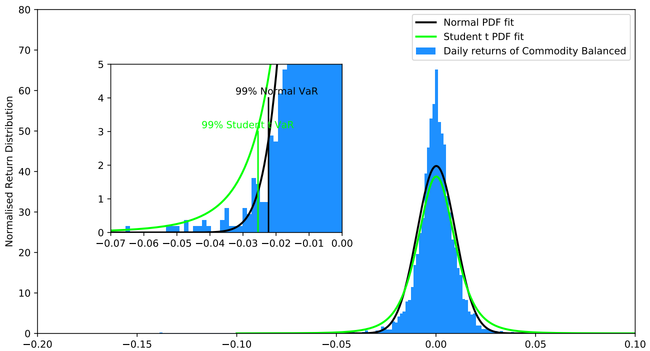
In this section, the daily returns for the whole sample period are plotted in a histogram with overlaying normal and Student t distributions. For each of the six portfolios the 99% normal VaR and 99% student t VaR is calculated based on the daily log-returns for the whole period. These figures are made in Python, see appendix A1 for details.



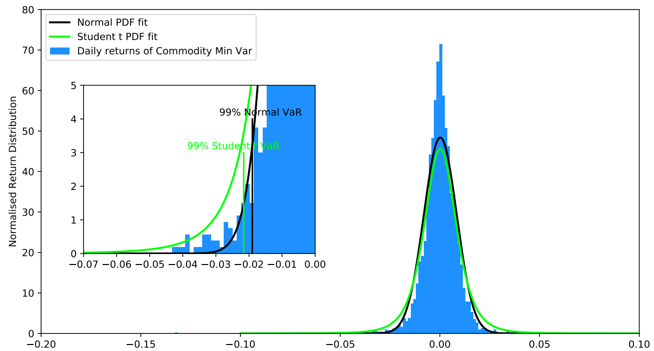
(a) Energy Balanced Distribution



(b) Energy Minimum Variance Distribution



(c) Commodity Balanced Distribution



(d) Commodity Minimum Variance Distribution

Figure 5.18: Probabiliy distributions for energy and commodity portfolios

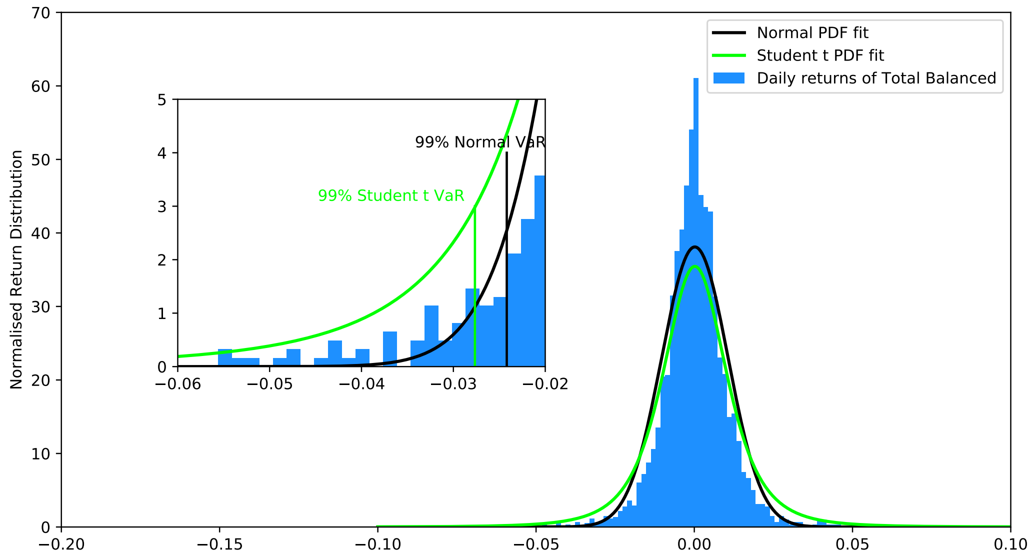


Figure 5.19: Probability distribution for total balanced portfolio

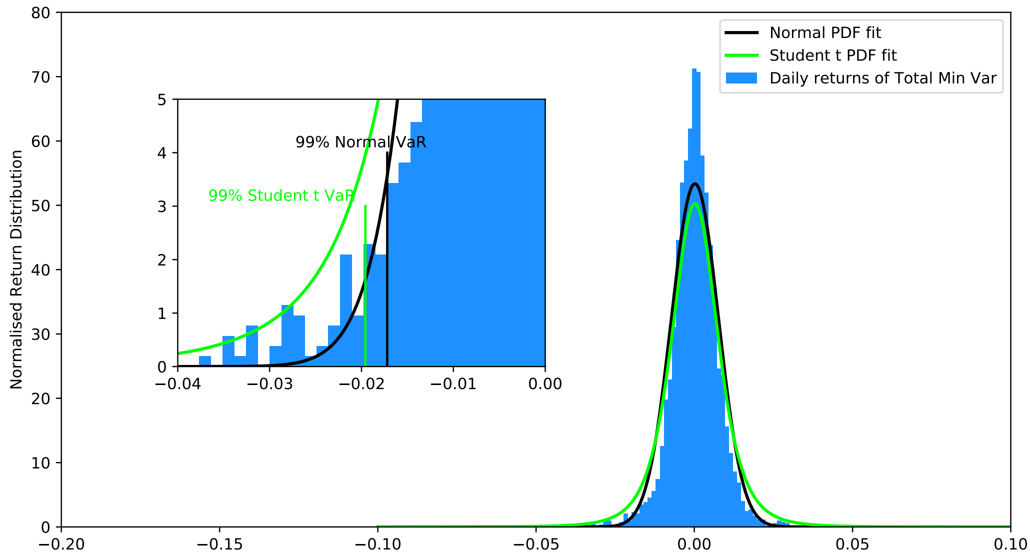


Figure 5.20: Probability distribution for total minimum variance portfolio

As observed in figures 5.18-5.20 all of the portfolios have high peaks at zero, matching poorly with the two distributions. It also seems like the minimum variance portfolios fit better to the distributions than the balanced portfolios.

One can see clearly that the student t distribution fits the data better than the normal distribution in the left tail of all the portfolios except for the balanced energy portfolio where it overestimates the risk. The normal distribution clearly underestimates the risk in all the portfolios.

6 Empirical Results

In this chapter, all of the results from the VaR calculations will be presented. VaR has been calculated for all of the six portfolios presented in chapter 5. All calculations are done using a rolling window of 250 days by three approaches; Historical VaR, normal VaR, and student t VaR. The normal and student t VaR has been calculated using two volatility models: simple volatility and EWMA. All of the calculations are also computed using a 95% confidence level and a 99% confidence level. The backtesting results are also presented in this chapter to test the different VaR models.

6.1 Methodology

All of the VaR calculations were computed in Microsoft Excel. Some of the inputs are calculated using a Python code in the program Visual Studio Code.

To calculate the daily return (profit & loss) of the assets, the logarithmic return is used:

$$P\&L = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{6.1}$$

The P&L results were used to calculate EWMA for each portfolio. As mentioned, λ was set to 0,94 in the calculations, and the EWMA was calculated using a rolling window of 250 days. The function SUMPRODUCT was used in Excel, calculating the weighted squared returns as described in chapter 2.1.2.

The standard deviation for each portfolio was calculated from the P&L data in a rolling window of 250 days. The function STDEV.S in Excel was used for these calculations.

The historical prices, and as well the results from the standard deviation and EWMA was used as input for the estimations of VaR for each of the six portfolios.

To perform the student t VaR calculations in a rolling window of 250 days, estimates of the degrees of freedom is needed. This has been done using a Python code that uses MLE to find a good estimate of the degrees of freedom in a rolling window of 250 days (see section A3 for more details). These values were then loaded into excel.

To quality ensure the results all of the VaR calculations are backtested by two different methods; Kupiec test for coverage and Christoffersen test for independence. In this thesis, the backtests are performed in a rolling window of 1000 days. This approach gives a sufficient amount of data to conclude how the different VaR models perform in different periods.

6.2 Portfolio allocations

In this section, the minimum variance allocations are presented. This was done in Visual Studio Code, using a Python code. The Python code used SLSQP (sequential quadratic programming) (similar to Solver in Excel) to find the minimum variance allocation. See Appendix A2 for details. This function essentially minimizes equation (2.8) from chapter 2.7. The minimum variance allocations were updated every day, using the data from the previous 250 days as an estimate for the variance. The minimum variance allocations will be calculated from 16.04.2004, 250 days after the first observation.

To avoid heavy investment in some assets, an upper limit has been added as a constraint. The energy and commodity portfolio had an upper limit of 20% per asset, while the total portfolio had an upper limit of 15% per asset due to the higher number of assets in this portfolio.

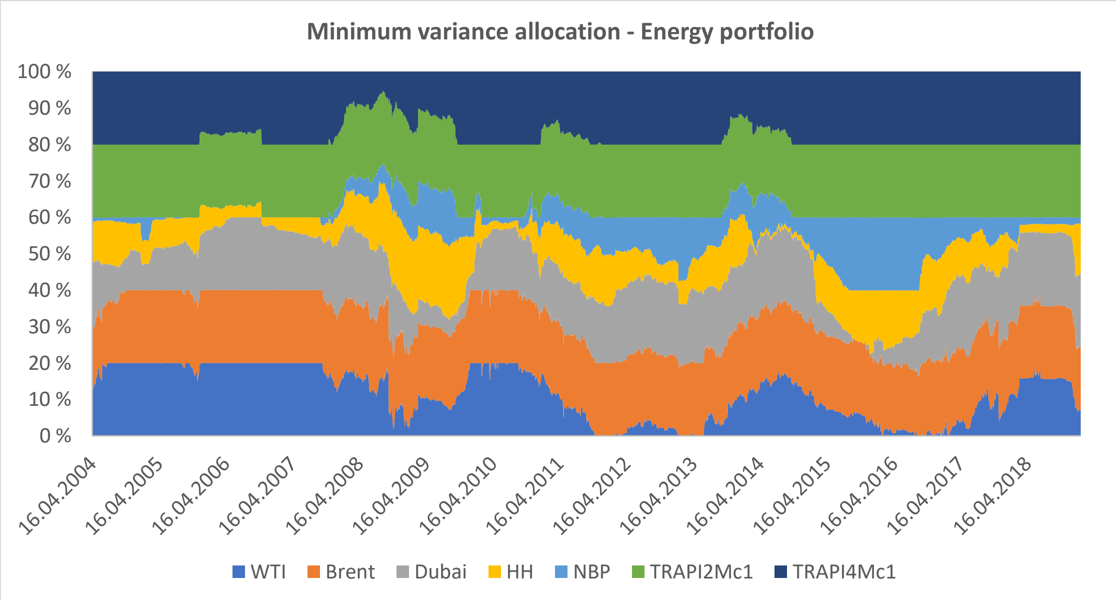


Figure 6.1: Dynamic allocation for the energy portfolio

The coal indexes have the most stable allocation over the whole period and have an average allocation close to 20%, which were the maximum constraint under the calculations. Another observation that can be done from this graph is that the NBP asset allocation is close to zero until 2008. The reason for this is the high volatility in the price until 2008. Oil assets are less invested in during the financial crisis and especially in the oil crisis in 2015. The oil assets have approximately 60% of the allocations at the highest and approximately 22% of the allocations at the lowest during the oil crisis.

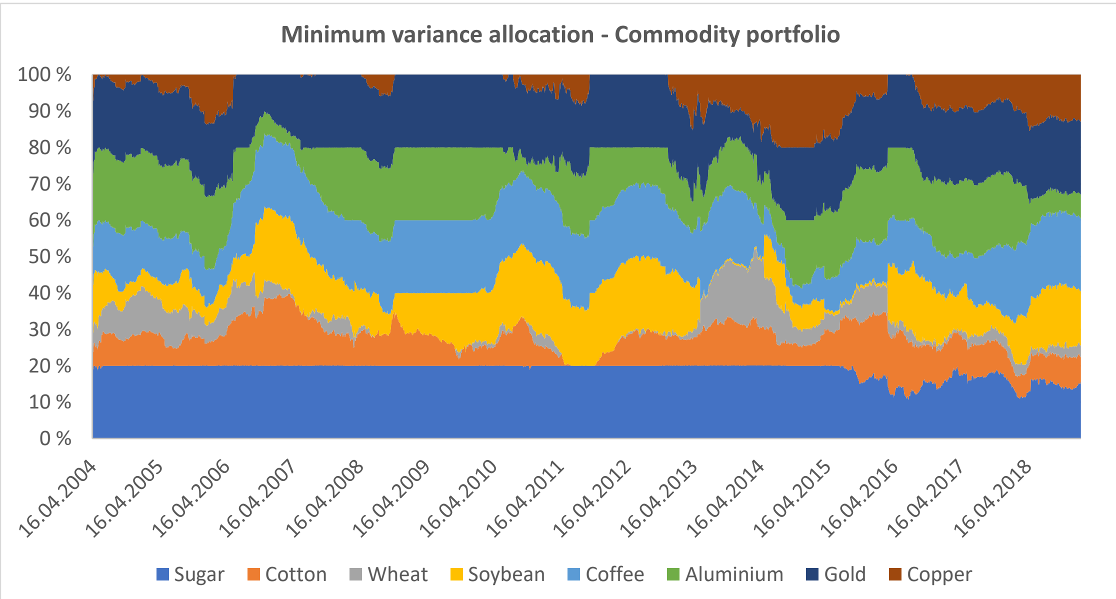


Figure 6.2: Dynamic allocation for the commodity portfolio

Sugar has a very stable allocation where it is at a maximum of 20% until late 2015 where it decreases slightly. This indicates that sugar is a good asset to diversify risk in a portfolio. Gold, coffee, and aluminum also have very high allocations during the whole period.

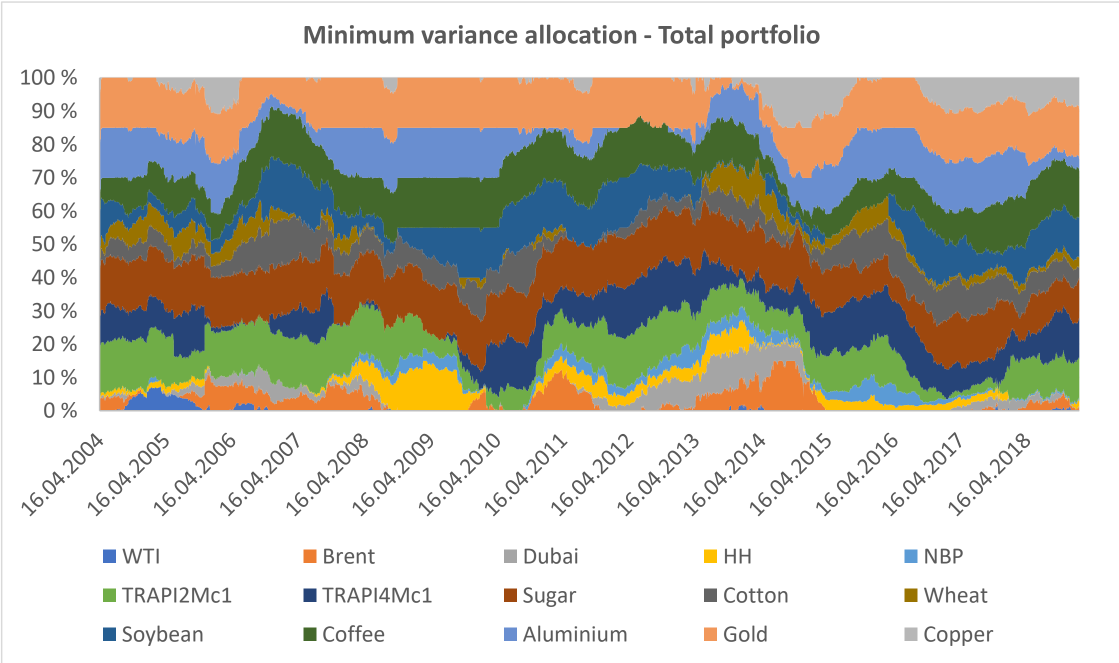


Figure 6.3: Dynamic allocation for the total portfolio

As can be observed from figure 6.3, the oil assets are as expected weighted low under periods such as the financial crisis and the oil crisis in 2014, where the price dropped significantly. Gold, coffee, and sugar are weighted high during the whole period, which indicates that these assets are good to include in a portfolio that contains energy assets in terms of diversification.

6.3 Value at Risk results

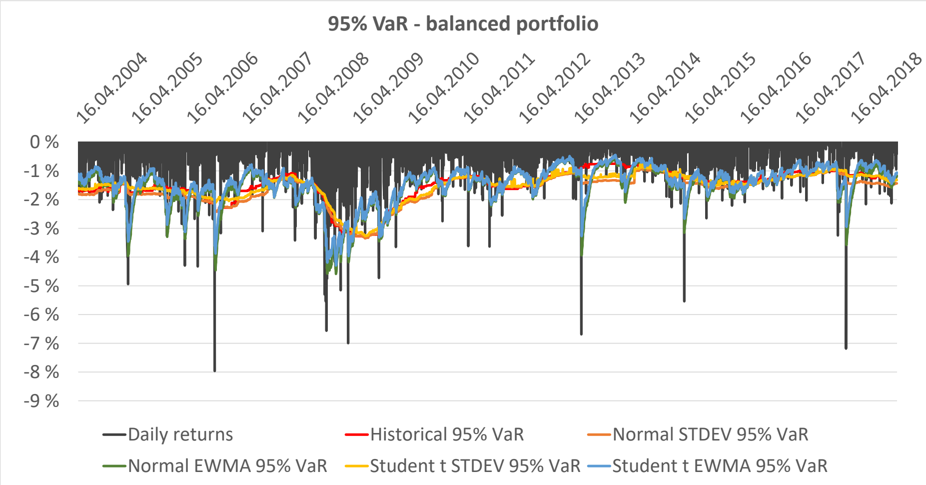
In this section, all of the VaR calculations for the six portfolios are presented. The daily returns are included in the same plot to give an idea of where the VaR models overestimate and underestimate the risk.

The historical model was calculated using the function PERCENTILE.INC in Excel. It requires two inputs, the data range, and the significance level.

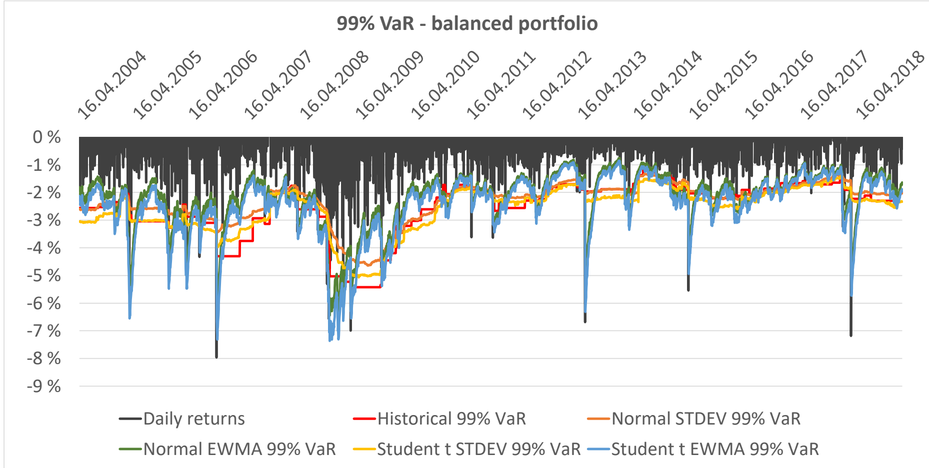
The normal model was calculated using equation (3.1). The function NORM.S.INV was used to calculate the quantile value.

The student t model was calculated using equation (3.2). The degrees of freedom were calculated using a Python code. See Appendix A3. To find the quantiles, the function T.INV in excel was used. The quantile depends on the degrees of freedom, and this function truncates the values, so linear interpolation was used to get a better estimate of the quantile value.

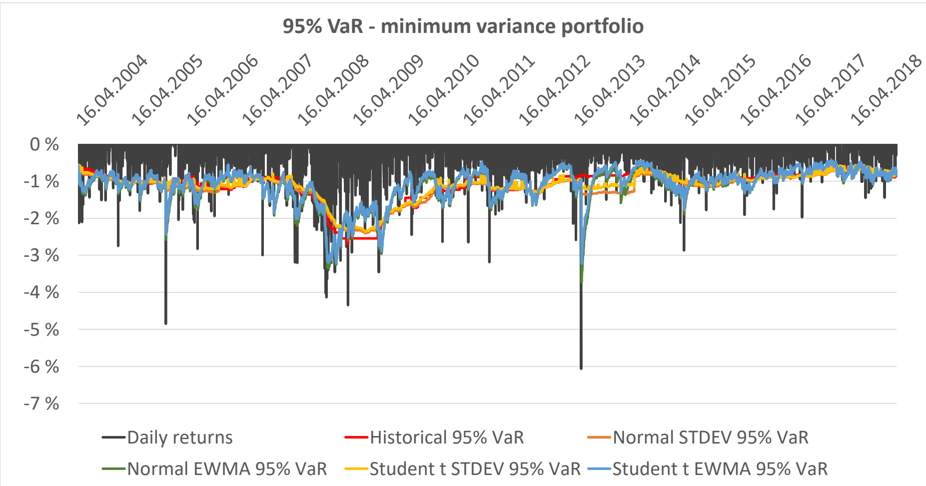
All the calculations were done in a rolling window of 250 days, meaning the models used the data from the previous 250 days to calculate VaR. The first calculation will then be 16.04.2004, which is 250 days after the first observation. To calculate the return on the minimum variance portfolios, the function SUMPRODUCT was used in Excel between the daily returns of the assets and the minimum variance allocations calculated in section 6.2. For simplicity, the days between 01.05.2003 and 16.04.2004 were assumed to have the same allocation as the one calculated on 16.04.2004. To calculate the return on the balanced portfolios, the function SUMPRODUCT was used in Excel between the daily returns of the assets and the equally weighted allocations. The energy portfolio invested 1/7 in each asset, and it was invested 1/8 and 1/15 in each asset for the commodity and total portfolio, respectively.



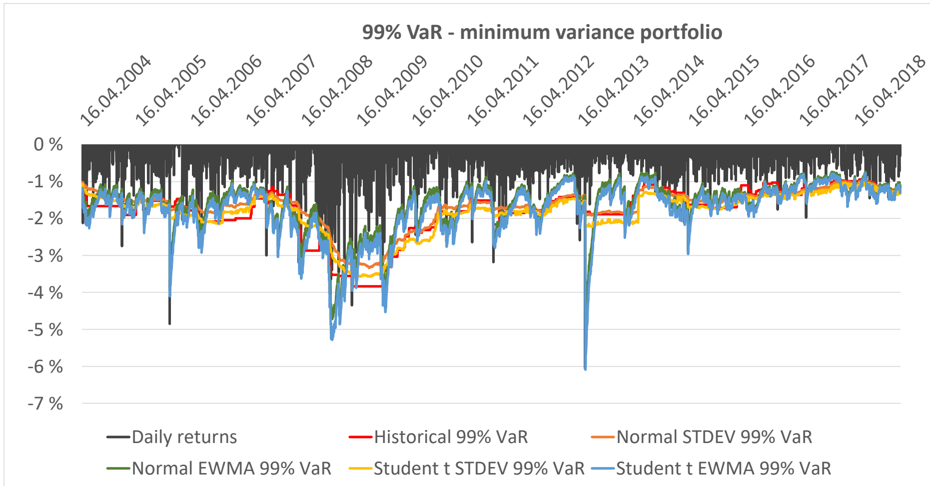
(a) 95% VaR balanced total portfolio



(b) 99% VaR balanced total portfolio

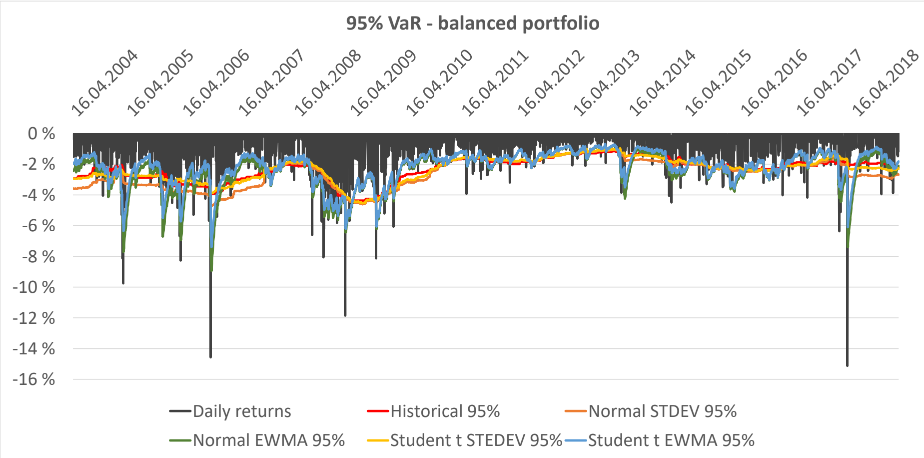


(c) 95% VaR minimum variance total portfolio

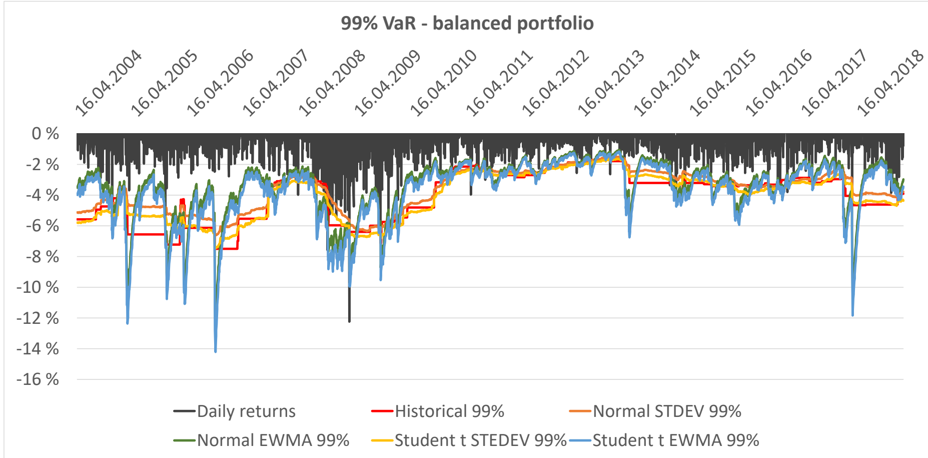


(d) 99% VaR minimum variance total portfolio

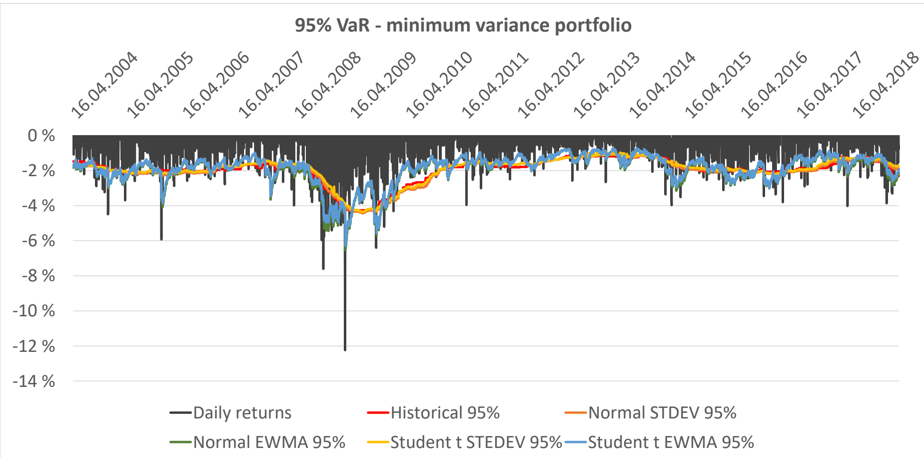
Figure 6.4: VaR calculations for total portfolios



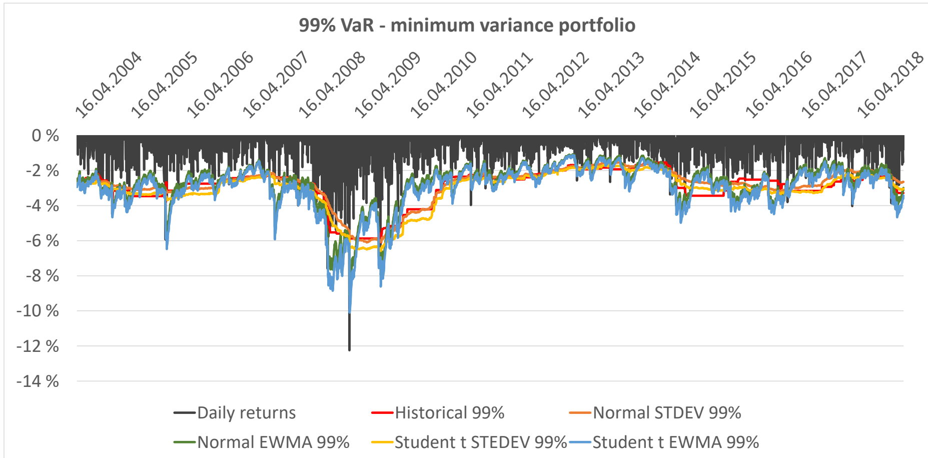
(a) 95% VaR balanced energy portfolio



(b) 99% VaR balanced energy portfolio

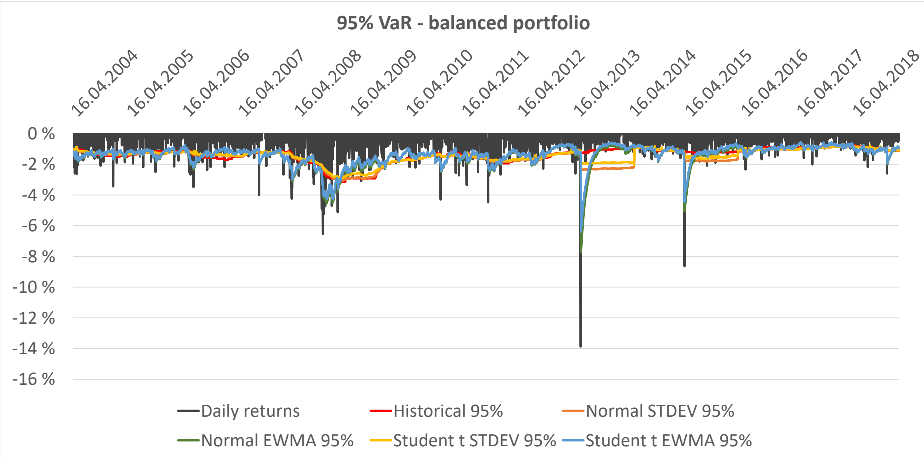


(c) 95% VaR minimum variance energy portfolio

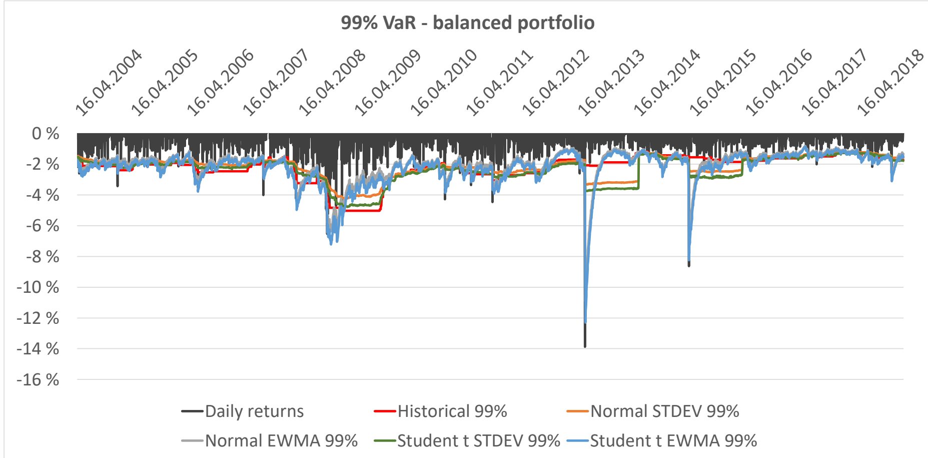


(d) 99% VaR minimum variance energy portfolio

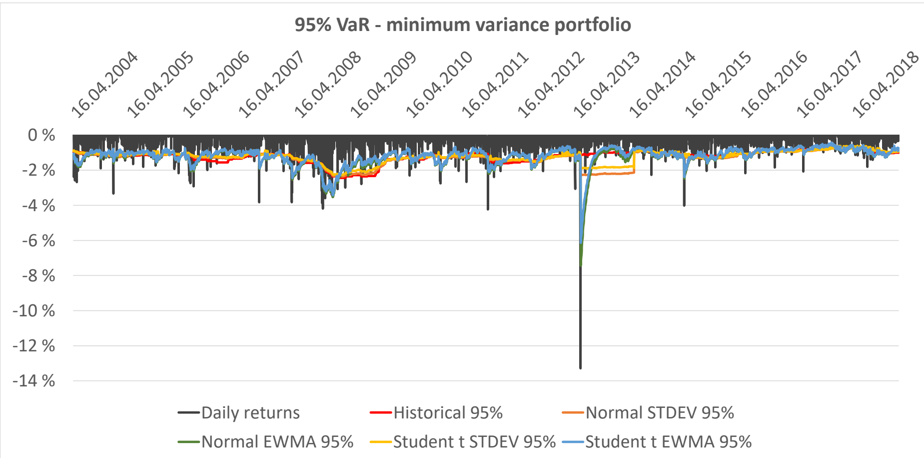
Figure 6.5: VaR calculations for energy portfolios



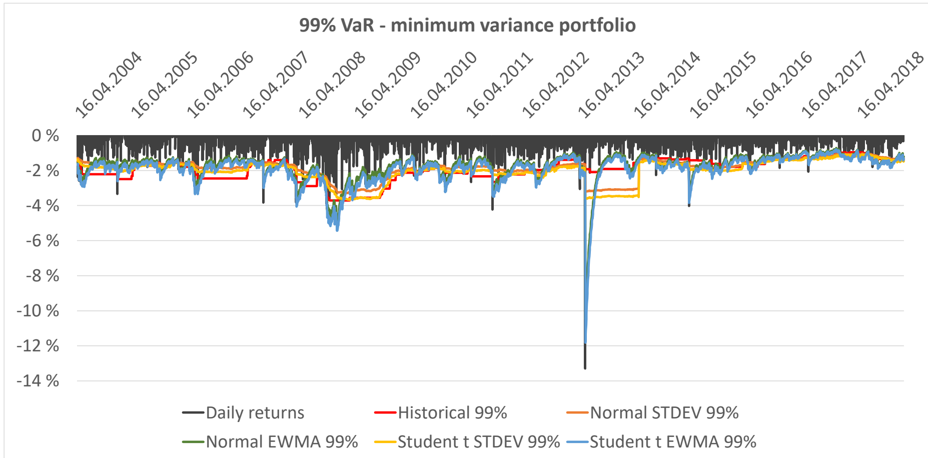
(a) 95% VaR balanced commodity portfolio



(b) 99% VaR balanced commodity portfolio



(c) 95% VaR minimum variance commodity portfolio



(d) 99% VaR minimum variance commodity portfolio

Figure 6.6: VaR calculations for commodity portfolios

6.4 Backtesting results

In this section, the backtesting results for the six portfolios are presented. The five VaR models are tested for coverage by the Kupiec test and independence by the Christoffersen test.

A critical value of 3,84 is used in the backtesting. Chapter 3.3 describes how the calculations are done. All of the calculations use a rolling window of 1000 days, meaning the likelihood ratio is based on data from the previous 1000 days. The tests pass if the values are lower than the significance test (3,84).

In addition, a mixed test is presented for each portfolio. The mixed test shows a continuous line when both Kupiec and Christoffersen are accepted for the different VaR models, to give a better idea of which time periods the different VaR models fail to predict the risk.

6.4.1 Backtesting results for total portfolios

Figures 6.6 and 6.7 below show the Kupiec test, the Christoffersen test, and a mixed test that combines the result from the Kupiec and Christoffersen test for the two total portfolios.

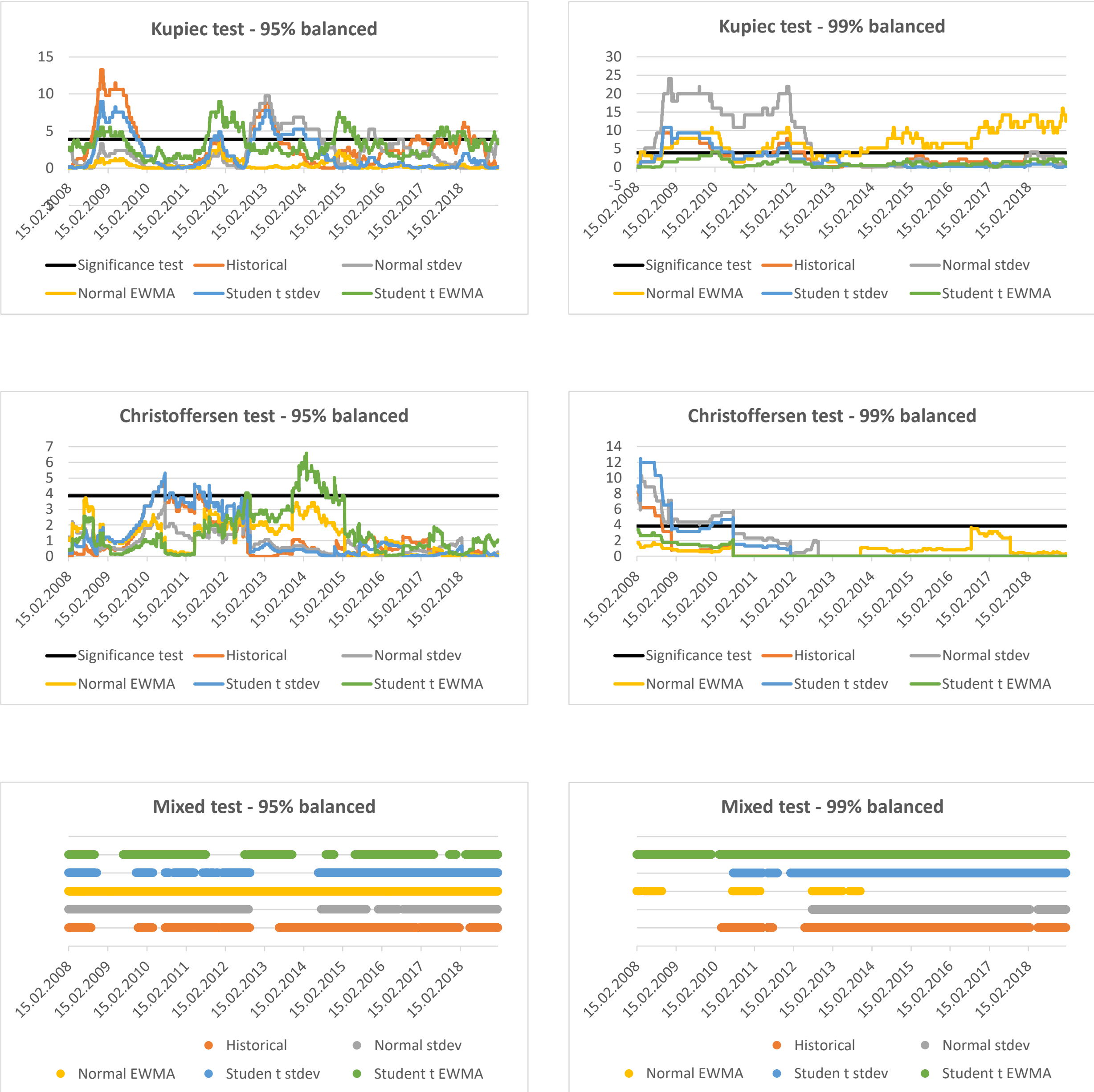


Figure 6.7: Backtesting results for total balanced portfolio. First and second row provides the Kupiec test and Christoffersen test, respectively. The third row provides an overview for when both tests are accepted.

95% total balanced portfolio:

- The historical model performs well on the Kupiec test, except from a peak between the end of 2008 until 2010, and in 2013. This model is accepted by Christoffersen throughout the whole period.
- The normal model with simple volatility also performs well but has a peak between 2013 and 2014. This model is accepted by Christoffersen throughout the whole period.
- The normal model with EWMA is accepted by both Kupiec and Christoffersen throughout the whole period.

- The student t model with simple volatility performs average on the Kupiec test, except from a peak in 2009-2010, and between 2013 and 2014. This model is rejected by Christoffersen between 2010 and 2012.
- The student t model with EWMA performs average on the Kupiec test. It has a peak in 2009, 2012 and 2014-2015, otherwise, it is accepted. This model is accepted by Christoffersen except for a peak between 2014-2015.

99% total balanced portfolio:

- The historical model gets rejected between 2008-2010 by Kupiec, and also has a peak in 2012. Except for a peak in 2008, this model is accepted by Christoffersen.
- The normal model with simple volatility is accepted by both Kupiec and Christoffersen from 2013 and onwards.
- The normal model with EWMA performs poorly and is only accepted by Kupiec a small period in 2008 between 2010-2011, and between 2013-2014. It is accepted by Christoffersen throughout the whole period.
- The student t model with simple volatility is accepted by Kupiec and Christoffersen between 2010-2011 and from 2012 and onwards.
- The student t model with EWMA is accepted by both tests throughout the whole period.

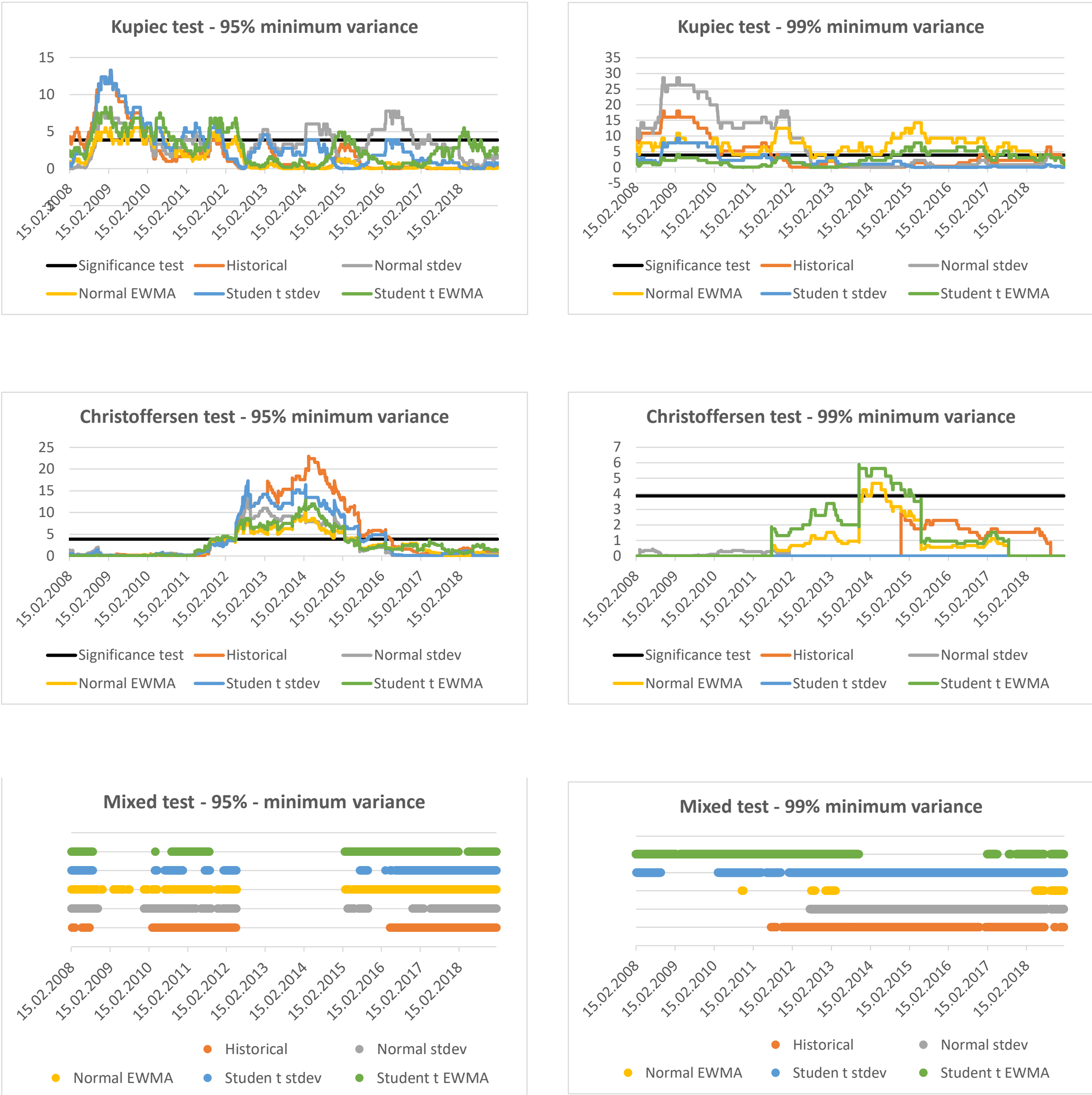


Figure 6.8: Backtesting results for total minimum variance portfolio. First and second row provides the Kupiec test and Christoffersen test, respectively. The third row provides an overview for when both tests are accepted

95% total minimum variance portfolio:

- The historical model performs well in the Kupiec test, except from a peak in 2009-2010. This model gets rejected in the Christoffersen test between 2013 and 2016.
- The normal model with simple volatility has some peaks in the Kupiec test in 2009, 2013, 2014-2015 and 2016-2017. It gets rejected by the Christoffersen test between 2012 and 2015.
- The normal model with EWMA performs well in the Kupiec test, with some minor peaks in 2009-2010. This model gets rejected in the Christoffersen test between 2012 and 2015.

- The student t model with simple volatility gets rejected in the Kupiec test from 2009 to 2010. This model is also rejected by Christoffersen between 2012 and 2015.
- The student t model with EWMA performs poorly in the Kupiec test from 2009-2010. This model is also rejected by Christoffersen between 2012 and 2015.

99% total minimum variance portfolio:

- The historical model gets accepted in both tests from 2011 and onwards.
- The normal model with simple volatility is also accepted in both tests from 2012 and onwards.
- The normal model with EWMA is almost rejected by Kupiec throughout the whole period. Except for a peak in 2014, this model is accepted by the Christoffersen test throughout the whole period.
- The student t model with simple volatility is accepted by both tests in almost the entire period except from a period in 2009-2010, where it is rejected by the Kupiec test.
- The student t model with EWMA performs well. It gets rejected by the Kupiec test between 2015 and 2017, and rejected by Christoffersen between 2014 and 2015.

6.4.2 Backtesting results for energy portfolios

Figures 6.9 and 6.10 below show the Kupiec test, the Christoffersen test, and a mixed test that combines the result from the Kupiec and Christoffersen test for the two energy portfolios.

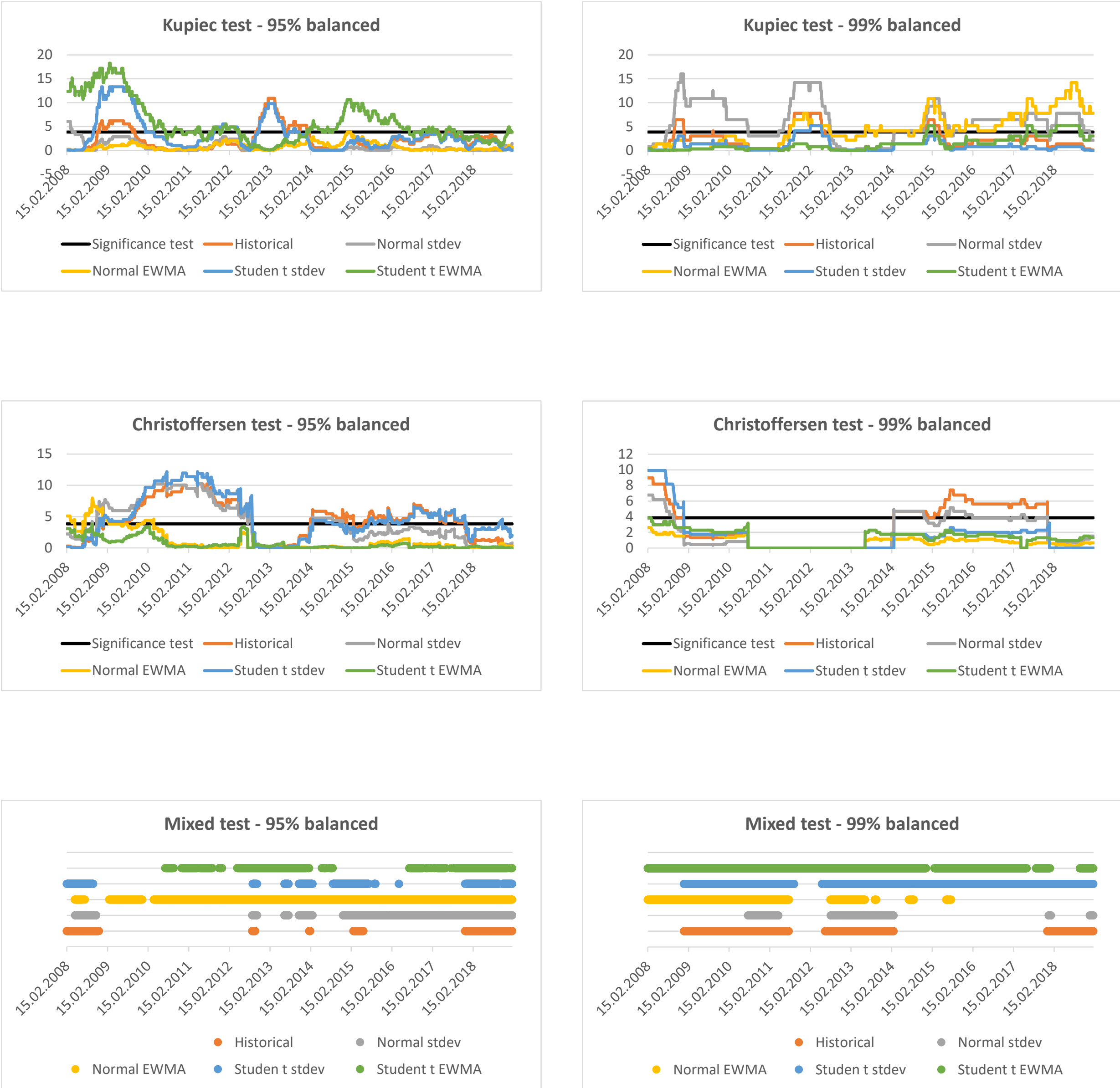


Figure 6.9: Backtesting results for energy balanced portfolio. First and second row provides the Kupiec test and Christoffersen test, respectively. The third row provides an overview for when both tests are accepted

95% energy balanced portfolio:

- The historical model performs well except from a peak in 2009-2010 and 2013-2014 in the Kupiec test. However, it is rejected by the Christoffersen test in the period between 2010 and 2012, and also in the period between 2014 and 2017.
- The normal model with simple volatility is accepted in the Kupiec test throughout the whole period, except from the start of 2008. It is rejected by the Christoffersen test in the period between 2009 and 2012, and also between 2014 and 2015.
- The normal model with EWMA is accepted by Kupiec throughout the whole period. It is however rejected by the Christoffersen test in 2009.

- The student t model with simple volatility also performs well on the Kupiec test except a peak between 2009-2010 and in 2013. It is rejected by the Christoffersen test in the period between 2010 and 2013, and also fails a lot in the period between 2015 and 2018.
- The student t model with EWMA is accepted by Kupiec in the period between 2010 and 2014, with a few peaks here. It is also accepted from 2016 to 2019 by Kupiec. This model is accepted by the Christoffersen test throughout the whole period.

99% energy balanced portfolio:

- The historical model performs well here also, but has a peak between 2009, and also in 2012 and 2015 in the Kupiec test. This model is rejected by the Christoffersen test in 2008, and also between 2015 and 2018.
- The normal model with simple volatility does not perform well in the Kupiec test. It is only accepted in 2011, and in the period between 2013 and 2015. This model is rejected by the Christoffersen test in 2008, and also between 2014 and 2018.
- The normal model with EWMA performs ok. It is accepted in the Kupiec test between 2008 and 2011. It has a peak in 2012, and is rejected from 2015 and onwards. This model is accepted by the Christoffersen test throughout the whole period.
- The student t model with simple volatility performs well and has a minor peak in 2012 in the Kupiec test. Except from a peak in 2008, this model is accepted by the Christoffersen test.
- The student t model with EWMA has a minor peak in 2015, 2017 and 2018 in the Kupiec test. This model is accepted by the Christoffersen test throughout the whole period.

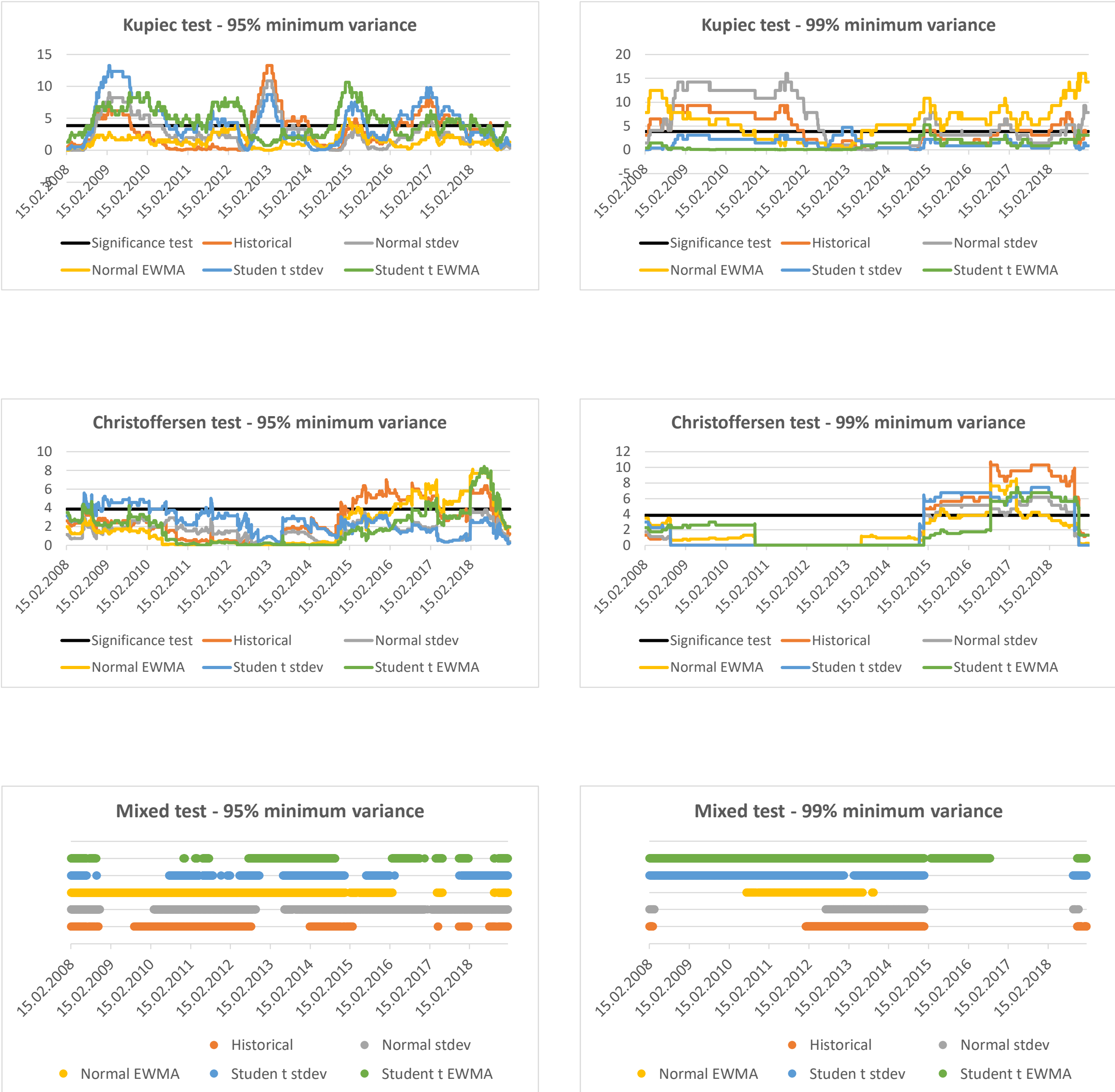


Figure 6.10: Backtesting results for energy minimum variance portfolio. First and second row provides the Kupiec test and Christoffersen test, respectively. The third row provides an overview for when both tests are accepted

95% energy minimum variance portfolio:

- The historical model performs ok with peaks in 2009-2010, 2013, and in 2017 on the Kupiec test. This model is rejected by the Christoffersen test from 2015 and onwards.
- The normal model with simple volatility performs well except from a peak in 2009-2010 and in 2013 on the Kupiec test. This model is accepted by the Christoffersen test throughout the whole period.
- The normal model with EWMA is accepted by Kupiec throughout the whole period. It is rejected by the Christoffersen between 2016 and 2018.

- The student t model with simple volatility has peaks in 2009-2010, 2013, 2015, and between 2016-2018 in the Kupiec test. This model is rejected by Christoffersen between 2008 and 2012.
- The student t model with EWMA does not perform well. It is accepted by Kupiec in 2008, 2011, between 2013 and 2015, and 2016-2017. It is rejected by Christoffersen in 2016 and 2018.

99% energy minimum variance portfolio:

- The historical model is accepted by Kupiec from 2012 and onwards, but this model is rejected by Christoffersen from 2015 and onwards.
- The normal model with simple volatilty is also accepted by Kupiec from 2012 and onwards and rejected by Christoffersen from 2015 and onwards.
- The normal model with EWMA is only accpeted between 2011 and 2014 in the Kupiec test. It is also accepted by Christoffersen in this period.
- The student t model with simple volatility is accepted by Kupiec throughout the whole period (minor peak in 2013). However, it is rejected by Christoffersen between 2015 and 2018.
- The student t model with EWMA is also accepted by Kupiec throughout the whole period (minor peak in 2015). This model is also rejected by Christoffersen between 2017 and 2018.

6.4.3 Backtesting results for commodity portfolios

Figures 6.11 and 6.12 below show the Kupiec test, the Christoffersen test, and a mixed test that combines the result from the Kupiec and Christoffersen test for the two commodity portfolios.

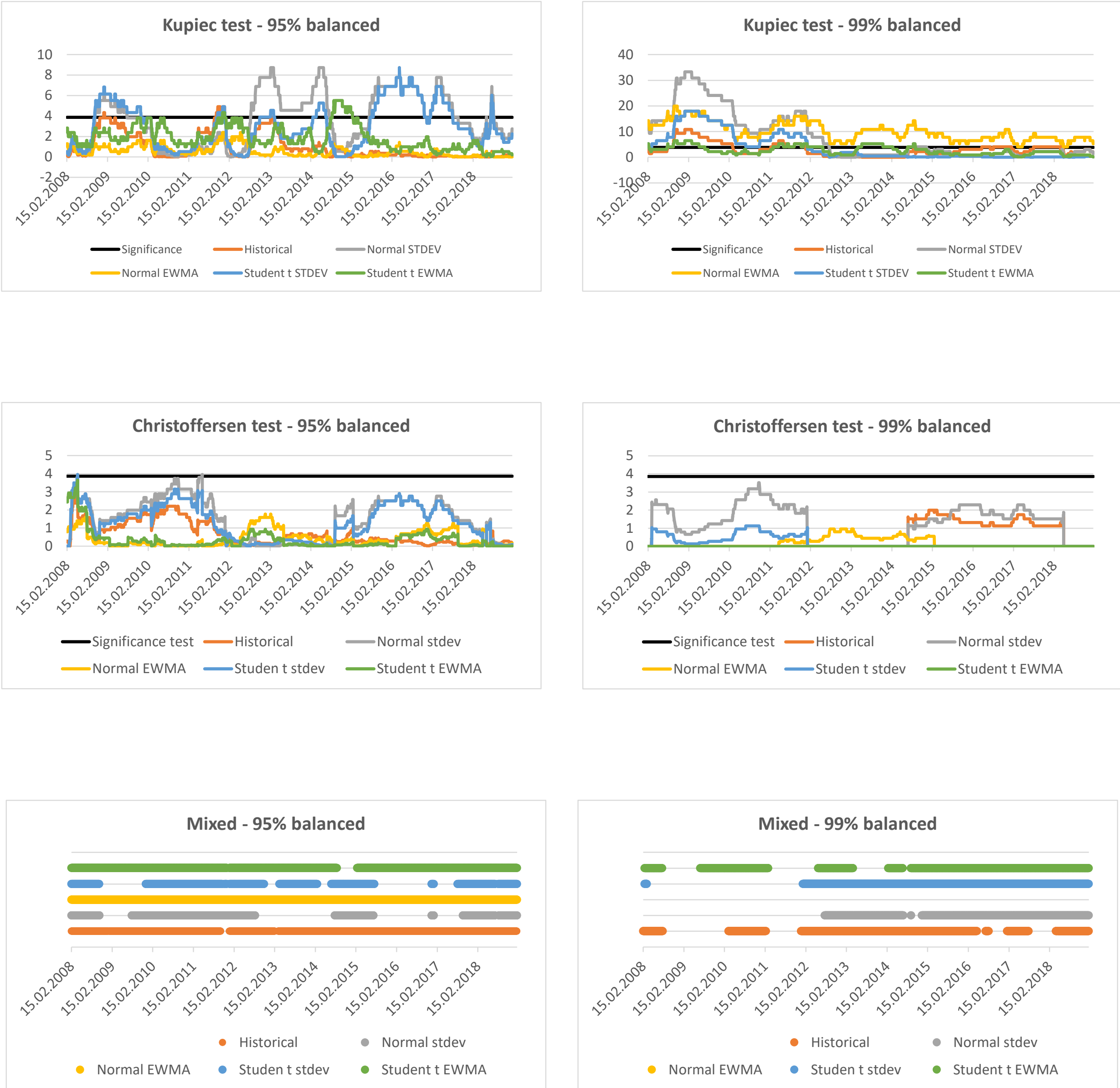


Figure 6.11: Backtesting results for commodity balanced portfolio. First and second row provides the Kupiec test and Christoffersen test, respectively. The third row provides an overview for when both tests are accepted

95% commodity balanced portfolio:

For the commodity portfolio at 95% confidence level the Christoffersen test is accepting all the models at significance test 3,84. Therefore the Kupiec test will determine how the different models perform in different periods.

- The historical method performs well expect from a small period late in 2012 where it is rejected by the Kupiec test.
- The normal model with simple volatility is not performing so good. It has large periods between 2009-2010, 2013-2015 and 2016-2018 where it is rejected by the Kupiec test.
- The normal model with EWMA is accepted by both Kupiec and Christoffersen throughout the whole period.

- The student t model with simple volatility is rejected between 2009-2010 and 2016-2018 by kupiec test and performs poor in this periods. It has also some small periods in 2013 and 2014 where it is rejected by the Kupiec test.

- The student t model with EWMA performs good with only a little period late 2014/early 2015 where it is rejected by the Kupiec test.

99% commodity balanced portfolio:
For the commodity portfolio at a 99% confidence level, the Christoffersen test is accepting all the models at significance test 3,84. Therefore the Kupiec test will determine how the different models perform in different periods.

- The historical method is rejected by the Kupiec test in the period late 2008 to 2010. It is also rejected by Kupiec in small periods in 2011, 2017 and 2018. Apart from this, the method is accepted by both tests and performs well.

- The normal model with simple volatility is rejected by the Kupiec test from 2008 to the middle of 2012. After this, it was accepted by both tests with a little expectation late in 2014.

- The normal EWMA model is rejected for the whole period and performs badly.

- The student t model with simple volatility is rejected by Kupiec from 2008 to 2012. After this, it is accepted by both tests and performs well.

- The student t model with EWMA is rejected by the Kupiec test between the middle of 2008 to the middle of 2009, 2011-2012 and 2013-2014. Except this, it performs well.

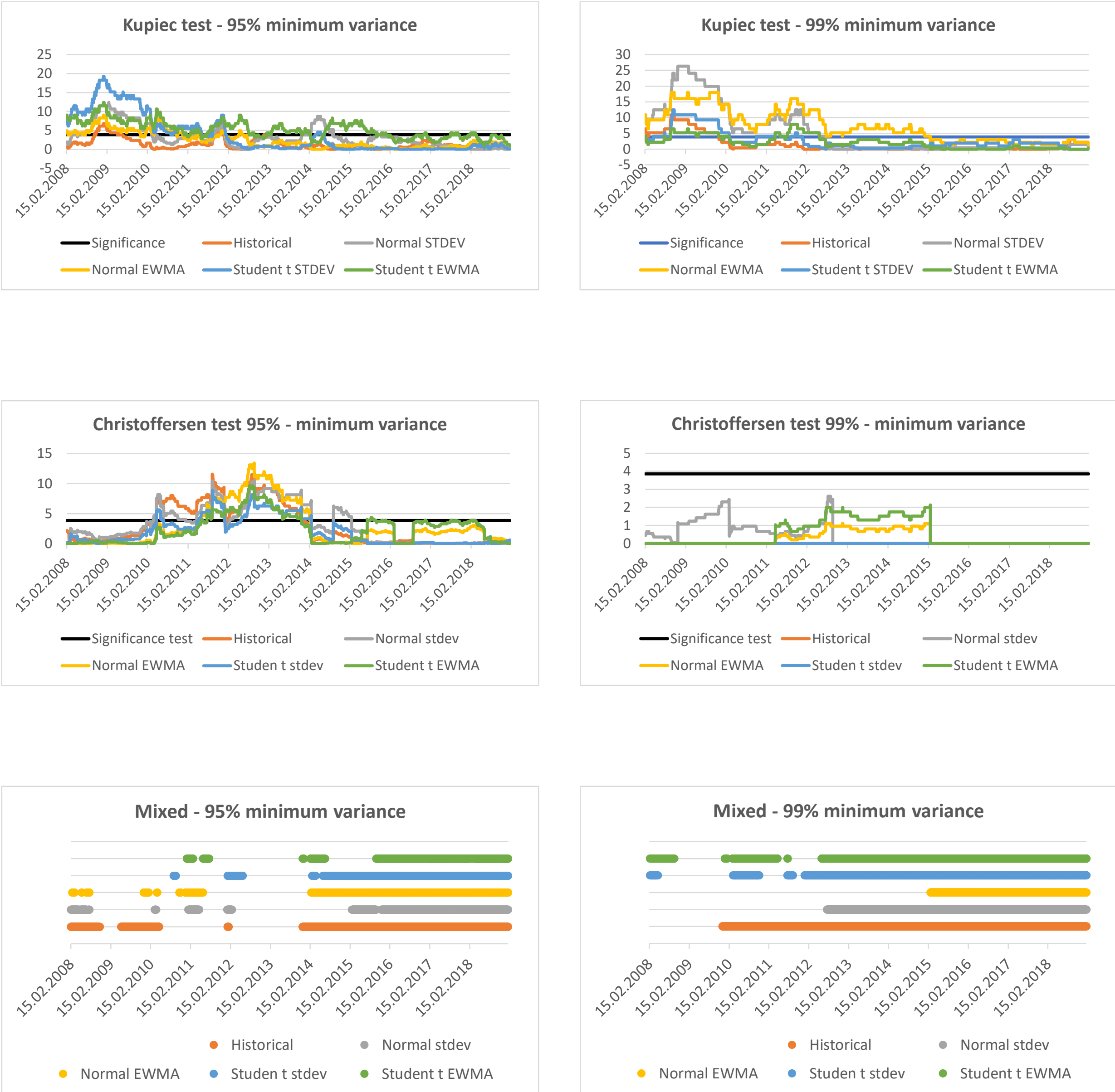


Figure 6.12: Backtesting results for commodity minimum variance portfolio. First and second row provides the Kupiec test and Christoffersen test, respectively. The third row provides an overview for when both tests are accepted

95% commodity minimum variance portfolio:

- The historical model was rejected by the Kupiec test early in 2009. Between 2010 to 2014 it is rejected by Christoffersen with a small exception in 2012. After 2014 it is accepted by both models and performs well.
- The normal model with simple volatility is either rejected by Kupiec or Christoffersen with small exceptions until 2015. After 2015 it is accepted by both tests and performs well.
- The normal model with EWMA is performing similar to the normal model with simple volatility. Both tests accept the model after 2014.

- Student t model with simple volatility is rejected by Kupiec until the middle of 2011, then it is rejected by Christoffersen between 2011-2014. After 2014 it is accepted by both.
- The student t model with EWMA is accepted by both tests between 2011-2012, 2014, and after 2016. In the other periods, it is either rejected by Kupiec or Christoffersen test.

99% commodity minimum variance portfolio:

All the test is accepted by Christoffersen test at 99%, and therefore, the Kupiec test will determine how the models perform.

- The historical model is rejected by Kupiec until late 2009. After this, it is accepted by both tests, and the model is good.
- The normal model with simple volatility is rejected by Kupiec until the middle of 2012, after this it is accepted by both tests and performs well.
- The normal model with EWMA is performing badly until 2015, after this it is accepted by both tests.
- The student t model with simple volatility is rejected by the Kupiec test between the middle of 2008 to 2010 and large parts of 2011. Except this, it is accepted by both tests.
- The student t model with EWMA is rejected by Kupiec from late 2008 to early 2010 and late 2011 to the middle of 2012. Except this, it is accepted by both tests.

6.4.4 Summary backtest

2861 backtests are calculated for each of the five VaR models for all of the portfolios. Table 6.1 below shows in percentage how many of the tests is accepted by Kupiec, Christoffersen, and both (mixed).

All models seem to be having trouble during the financial crisis, where they underestimate the risk. None of the models manage to capture the most extreme events, but the student t model with EWMA captures many of the extreme events on a 99% confidence level, and the normal model with EWMA captures many of the extreme events on 95% confidence level. This can be seen in the VaR calculations in figures 6.4-6.6.

Both the commodity and total portfolios seem to be having some troubles in the period between 2013 and 2014. This might be connected to price volatility for the commodity assets, and increased risk, as observed in figure 5.10. Especially the price drop in soybean followed by the increased standard deviation for this commodity seems to affect the portfolios. From the VaR calculations, it seems like the models overestimate the risk in this period, which causes the Kupiec test to fail them.

The energy portfolios have some trouble during the oil crisis between 2015 and 2018. Most of the prices of energy assets dropped, and this increased uncertainty is not reflected in the models. From the VaR calculations in figure 6.5, it seems like the models overestimate the risk in the period right after the oil crisis, especially on a 99% confidence level. Here the EWMA models react faster to the increased risk which came from the price drop in oil.

Table 6.1: Summary of the level of acceptances in the backtests

		Energy portfolio			Commodity portfolio			Total portfolio		
Confidence level	**Risk metric**	**Kupiec**	**Christoffersen**	**Mixed**	**Kupiec**	**Christoffersen**	**Mixed**	**Kupiec**	**Christoffersen**	**Mixed**
95% balanced	**Historical**	80 %	33 %	21 %	97 %	100 %	97 %	79 %	97 %	75 %
	Normal STDEV	90 %	58 %	48 %	55 %	100 %	55 %	79 %	100 %	79 %
	Normal EWMA	100 %	90 %	90 %	100 %	100 %	100 %	100 %	100 %	100 %
	Student t STDEV	74 %	39 %	29 %	63 %	100 %	63 %	74 %	94 %	67 %
	Student t EWMA	45 %	100 %	45 %	95 %	100 %	95 %	73 %	90 %	64 %
95% minimum variance	**Historical**	64 %	74 %	48 %	93 %	68 %	62 %	81 %	63 %	46 %
	Normal STDEV	79 %	100 %	79 %	73 %	64 %	42 %	63 %	73 %	46 %
	Normal EWMA	99 %	77 %	76 %	75 %	75 %	52 %	88 %	73 %	61 %
	Student t STDEV	51 %	82 %	48 %	65 %	76 %	47 %	69 %	67 %	41 %
	Student t EWMA	46 %	90 %	40 %	38 %	75 %	34 %	69 %	70 %	47 %
99% balanced	**Historical**	79 %	57 %	49 %	65 %	100 %	65 %	76 %	94 %	70 %
	Normal STDEV	27 %	67 %	22 %	55 %	100 %	55 %	59 %	78 %	57 %
	Normal EWMA	42 %	100 %	42 %	0 %	100 %	0 %	20 %	100 %	20 %
	Student t STDEV	94 %	92 %	86 %	65 %	100 %	65 %	79 %	86 %	74 %
	Student t EWMA	89 %	100 %	89 %	70 %	100 %	70 %	98 %	100 %	98 %
99% minimum variance	**Historical**	49 %	65 %	30 %	83 %	100 %	83 %	62 %	100 %	62 %
	Normal STDEV	46 %	67 %	25 %	59 %	100 %	59 %	59 %	100 %	58 %
	Normal EWMA	27 %	78 %	27 %	36 %	100 %	36 %	9 %	94 %	9 %
	Student t STDEV	98 %	66 %	64 %	74 %	100 %	74 %	84 %	100 %	84 %
	Student t EWMA	98 %	80 %	79 %	77 %	100 %	77 %	72 %	87 %	64 %

7 Conclusion

Gold, sugar, and coffee account for a large percentage of the dynamic allocations for the total minimum variance portfolio. These assets are, therefore, useful to include in portfolios to reduce the risk. Gold is generally seen as a good hedging asset, and this is also proving to be true in this thesis.

The energy portfolio contains the riskiest assets; this is seen by the high rolling standard deviation (both simple and EWMA), and the higher VaR values. These assets are also highly correlated, which increases risk. This is also shown in previous studies performed by (Skår, 2017). Combining these assets with other commodities reduces the risk significantly, meaning that including other commodity assets into a portfolio consisting of energy assets reduce the risk, and one gets a more diversified portfolio.

With the application of EWMA and simple volatility, we have estimated daily VaR assuming both the normal distribution and the student t distribution and compared these models to the historical model on two confidence levels.

For the balanced portfolios on 95% confidence level, the normal model with EWMA is by far the best model with the highest number of acceptances for all three portfolios. For the commodity portfolio and total portfolio, the historical model also has a high number of acceptances. The student t model with EWMA is also very good for the commodity portfolio.

For the minimum variance portfolios on 95% confidence level, the normal model with simple volatility is the best model for the energy portfolio. The normal model with EWMA is best for the total portfolio. For the commodity portfolio, the historical model gets the highest number of acceptances.

For the balanced portfolios on 99% confidence level, the student t models get the highest number of acceptances for all three portfolios. The student t model with EWMA is slightly better. The historical model also gets a high number of acceptances for all three portfolios.

For the minimum variance portfolios on 99% confidence level, the student t models get the highest number of acceptances. The student t model with EWMA is the best model for the energy and commodity portfolios, while the student t model with simple volatility is the best model for the total portfolio. The historical model is a good model for the commodity portfolio and performs average on the total portfolio as well.

The normal model with EWMA is, in general, the preferred risk metric on a 95% confidence level, and performs well during the financial crisis. It also performs well during the oil crisis on this confidence level for all portfolios except for the 95% energy minimum variance portfolio. The student t model with EWMA is clearly the one that covers most of the daily returns on a 99% confidence level and performs well during the financial crisis for all except the commodity portfolios. It also performs well during the oil crisis except for the total 95% minimum variance portfolio.

The Maximum Likelihood Estimator (MLE) in Python and the GRG Non-Linear estimation in Excel's Solver may not be correct. This is due to the fact that it is possible that a local maximum point was found in the calculations instead of a global maximum point. In such a case, the calculations are not accurate, but this is always a risk using MLE and GRG.

7.1 Recommendations for future work

For further studies, it is recommended to use Expected Shortfall (ES) for both the normal distribution and the student t distribution and compare this to the VaR models. Other distribution models, such as a delta-gamma approach can also be tested. Several of the models tested in this thesis underestimate the risk, and using extreme value statistics could be better to mitigate the risk. In this thesis a maximum constraint on the dynamic allocations are used, it would be interesting to see if a minimum constraint had an effect, for instance, a minimum constraint on a few % in each asset.

Other more advanced volatility models such as the ARCH, GARCH, and EGARCH would be interesting to compare to the more simpler volatility models presented in this thesis.

Other backtesting methods should be performed. For instance, the binomial Kupiec test or other new methods to compare it to the two methods presented in this thesis. It would be interesting to try using a different time horizon than the rolling window of 1000 days presented here. A rolling window of 500 or 750 days should be tested.

It would also be interesting to make use of the Markowitz problem. This would mean that in addition to having the minimum variance constraint, one also has an expected target level of return.

8 References

Adeyanju, C. (2014). *The Top Factors that Move the Price of Soybeans.* Retrieved from http://www.futuresknowledge.com/news-and-analysis/grains/the-top-factors-that-move-the-price-of-soybeans/

Agoawike, U. A. (2009). *Opec annual report 2009.* Retrieved from https://www.opec.org/opec_web/static_files_project/media/downloads/publications/AR2009.pdf

Alexander, C. (2009a). *Market risk analysis, quantitative methods in finance.*

Alexander, C. (2009b). *Market risk analysis, value at risk models.*

Amadeo, K. (2019). *Oil price forecast 2019 - 2050.* Retrieved from https://www.thebalance.com/oil-price-forecast-3306219

API. (2014). *American petroleum institute - understanding natural gas markets.* Retrieved from https://www.api.org/~/media/Files/Oil-and-Natural-Gas/Natural-Gas/API-Understanding-Natural-Gas-Markets.pdf

Argusmedia. (2019a). *The api 2 coal price assessment.* Retrieved from https://www.argusmedia.com/en/methodology/key-prices/api-2-coal

Argusmedia. (2019b). *The api 4 coal price assessment.* Retrieved from https://www.argusmedia.com/en/methodology/key-prices/api-4-coal

Boundless Economics. (2019a). *Perfect competition.* Retrieved from https://courses.lumenlearning.com/boundless-economics/chapter/perfect-competition/

Boundless Economics. (2019b). *Supply.* Retrieved from https://courses.lumenlearning.com/boundless-economics/chapter/supply/

BP. (2018). *Bp statistical review of world energy 2018.* Retrieved from https://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html

Chappelow, J. (2019). *Oligopoly Definition.* Retrieved from https://www.investopedia.com/terms/o/oligopoly.asp

Chen, J. (2017). *Unsystematic Risk.* Retrieved from https://www.investopedia.com/terms/u/unsystematicrisk.asp

Chen, J. (2018a). *Benchmark oils: Brent blend, wti and dubai.* Retrieved from https://www.investopedia.com/articles/investing/102314/understanding-benchmark-oils-brent-blend-wti-and-dubai.asp

Chen, J. (2018b). *Brent blend.* Retrieved from https://www.investopedia.com/terms/b/

brentblend.asp

Chen, J. (2018c). *Covariance.* Retrieved from https://www.investopedia.com/terms/c/covariance.asp

Chen, J. (2018d). *Henry Hub.* Retrieved from https://www.investopedia.com/terms/h/henry_hub.asp

Chen, J. (2018e). *Sour crude.* Retrieved from https://www.investopedia.com/terms/s/sourcrude.asp

Chen, J. (2019a). *Definition Historical Volatility (HV).* Retrieved from https://www.investopedia.com/terms/h/historicalvolatility.asp

Chen, J. (2019b). *Normal Distribution Definition.* Retrieved from https://www.investopedia.com/terms/n/normaldistribution.asp

Chen, J. (2019c). *Skweness.* Retrieved from https://www.investopedia.com/terms/s/skewness.asp

Chen, J. (2019d). *West Texas Intermediate (WTI).* Retrieved from https://www.investopedia.com/terms/w/wti.asp

Cnnmoney. (1999). *Opec bosses agree to oil cuts.* Retrieved from https://money.cnn.com/1999/03/23/worldbiz/opec/

eia.gov. (2016). *International energy outlook 2016.* Retrieved from https://www.eia.gov/outlooks/ieo/pdf/nat_gas.pdf

eia.gov. (2018). *United kingdom.* Retrieved from https://www.eia.gov/beta/international/analysis.php?iso=GBR

eia.gov. (2019). *Annual energy outlook 2019.* Retrieved from https://www.eia.gov/outlooks/aeo/

Eia.gov. (2019). *What drives crude oil prices.* Retrieved from https://www.eia.gov/finance/markets/crudeoil/

Eia.gov. (2019). *What drives crude oil prices: Financial markets.* Retrieved from https://www.eia.gov/finance/markets/crudeoil/financial_markets.php

Fogarty, D. (2011). *2008 food price crisis – what caused it?* Retrieved from https://www.reuters.com/article/us-climate-crops-price-factbox/factbox-2008-food-price-crisis-what-caused-it-idUSTRE7590IJ20110610

Folger, J. (2019). *Commodities: Wheat.* Retrieved from https://www.investopedia.com/university/commodities/wheat.asp

Fontinelle, A. (2018). *Systematic Risk.* Retrieved from https://www.investopedia.com/terms/s/systematicrisk.asp

Grant, M., et al. (2019). *Exploring the Exponentially Weighted Moving Average.* Retrieved from https://www.investopedia.com/articles/07/ewma.asp

Hayes, A. (2019). *T Distribution Definition.* Retrieved from https://www.investopedia.com/terms/t/tdistribution.asp

Holton, G. A. (2014a). *Backtesting with independence tests.* Retrieved from https://www.value-at-risk.net/backtesting-independence-tests/

Holton, G. A. (2014b). *Kupiec's pf coverage test.* Retrieved from https://www.value-at-risk.net/backtesting-coverage-tests/

ICO. (2014). *International coffee organization (ICO) - world coffee trade (1963 – 2013): A review of the markets, challenges and opportunities facing the sector.* Retrieved from http://www.ico.org/news/icc-111-5-r1e-world-coffee-outlook.pdf

IEA. (2019). *Oil 2019 analysis and forecasts to 2024.* Retrieved from https://www.iea.org/oil2019/

Investopedia. (2018). *What factors affect the price of copper?* Retrieved from https://www.investopedia.com/ask/answers/021715/what-factors-affect-price-copper.asp

Khan Academy. (2009). *Price elasticity of demand and price elasticity of supply.* Retrieved from https://nb.khanacademy.org/economics-finance-domain/microeconomics/elasticity-tutorial/price-elasticity-tutorial/a/price-elasticity-of-demand-and-price-elasticity-of-supply-cnx

King, D. (2011). *Sugar Prices Hit Three-Decade High on Australia's Heavy Weather.* Retrieved from https://tinyurl.com/https-www-aol-com-2011-02-02

KRAMER, A. E. (2010). *Russia, crippled by drought, bans grain exports.* Retrieved from https://www.nytimes.com/2010/08/06/world/europe/06russia.html

Mackenzie, W. (2019). *Woodmac: India's oil demand growth momentum to continue, exceeding china in 2019.* Retrieved from https://www.oilfieldtechnology.com/special-reports/22012019/woodmac-indias-oil-demand-growth-momentum-to-continue-exceeding-china-in-2019/

Mason, R. (2010). *Sugar hits new 30-year peak on tight supply from Brazil and India.* Retrieved from https://www.telegraph.co.uk/finance/markets/8105551/Sugar-hits-new-30-year-peak-on-tight-supply-from-Brazil-and-India.html

MathWorks. (2019). *Mathworks, overview of var backtesting.* Retrieved from https://se.mathworks.com/help/risk/overview-of-var-backtesting.html#bu_mbvb-1

Mernier, A. (2010). *Putting a price on energy: international coal pricing.*

Meyer, G. (2011). *Cotton prices surge to record high amid global shortages.* Retrieved from https://www.ft.com/content/3d876e64-35c9-11e0-b67c-00144feabdc0

National Grid. (2019). *Network route maps.* Retrieved from https://www.nationalgridgas.com/land-and-assets/network-route-maps

OPEC. (2019). *Opec member countires.* Retrieved from https://www.opec.org/opec_web/en/about_us/25.htm

O'Brien, J., et al. (2014). *An Evaluation of Bank VaR Measures for Market Risk During and Before the Financial Crisis.* Retrieved from https://www.federalreserve.gov/pubs/feds/2014/201421/201421pap.pdf

Petroleum. (2015). *petroleum.co.uk.* Retrieved from http://www.petroleum.co.uk/

Refinitiv. (2019). *The coal market outlook in 2019.* Retrieved from https://www.refinitiv.com/perspectives/market-insights/coal-market-outlook-2019/

Ross, S. A., et al. (2018). *Corporate finance 5th edition.*

Sheth, K. (2011). *Top Sugarcane Producing Countries.* Retrieved from https://www.worldatlas.com/articles/top-sugarcane-producing-countries.html

Skår, H. (2017). *A Comparison Study of Value at Risk and Expected Shortfall.*

Spend Matters. (2016). *Sugar prices jump 40% since january.* Retrieved from http://spendmatters.com/2016/09/01/sugar-prices-jump-40-since-january/

Taylor, W. W. K. . R. D. (2007). *2007 Outlook of the U.S. and World Sugar Markets, 2006-2016.* Retrieved from https://bit.ly/2ZiDAwP

The Switch. (2014). *Wholesale gas market natural gas prices in the uk.* Retrieved from https://theswitch.co.uk/guides/gas-market

Tomek, W. G. (2014). *Agricultural product prices.*

Winje, P., et al. (2011). *Increased gas exports, but what about prices?* Retrieved from https://static.norges-bank.no/contentassets/20618af9e93d4ca6841548d1cd17a164/ec_4_2011.pdf

Wzorek, A. (2017). *Mechanik - analysis of the factors influencing the price of aluminium on the world market.* Retrieved from http://www.mechanik.media.pl/pliki/do_pobrania/artykuly/22/2017_07_s0565_eng.pdf

Zabarenko, K. P. . D. (2012). *Grain prices set records as u.s. drought, food worries spread.* Retrieved from https://www.reuters.com/article/us-usa-drought/grain-prices-set-records-as-drought-food-worries-spread-idUSBRE86F1D420120720

Zaky, D. (2016). *Valuewalk - major factors that influence gold price.* Retrieved from https://www.valuewalk.com/2016/11/factors-influence-gold-prices/

A Python codes

A.1 Code for probability distributions

```
1
2
3 import numpy
4 import matplotlib.pyplot as mp
5 from scipy.stats import t,norm
6 import pandas
7
8 returns_excel = pandas.read_excel(r"Totalportfolio.xlsx",
9 sheet_name="Returns",skiprows=5,usecols="C")
10 returns = numpy.array(returns_excel.values)
11 # replacing usecols = " " with another column number to change asset
12
13 mean = numpy.mean(returns) #Average returns
14 stddev = numpy.std(returns) #Standard deviation from returns
15 pdf_norm = norm.pdf(numpy.arange(-0.2, 0.2, 0.0002), mean, stddev)
16
17
18 #Finding parameters for best fitting student t distribution
19 #using MLE. Degrees of freedom (df) is then used to calculate
20 #value at risk for t distribution.
21 t_fit = t.fit(returns)
22 df, stddev_t, mean_t = t_fit
23 pdf_t = t.pdf(numpy.arange(-0.2, 0.2, 0.0002), df, mean, stddev)
24
25 Confidencelevel = 0.99
26
27 student_t_VaR =
28 ((df-2)/df)**(1/2)*stddev*t.ppf(Confidencelevel, df) - mean
29 Normal_VaR = stddev*norm.ppf(Confidencelevel) - mean
30
31 #df replaced with 3 on line 23 & 28 if the df returned from best fit is < 3
32
33 #Shows the VaR calculations
34 print("Normal_VaR: "+"{:.3%}".format(Normal_VaR))
35 print("Student_t_VaR: "+"{:.3%}".format(student_t_VaR))
36
37
38 mp.subplots(figsize=(12, 7))
39 mp.hist(returns, bins=100, density="None", color="dodgerblue"
```

```
, label = "Daily returns of WTI")
mp.plot(numpy.arange(-0.2, 0.2, 0.0002), pdf_norm, "black"
, label="Normal distribution", linewidth=2.0)
mp.plot(numpy.arange(-0.2, 0.2, 0.0002), pdf_t, "lime"
, label="Student_t distribution", linewidth=2.0)
mp.xlim([-0.15, 0.15]) #scaling x-axis of distribution
mp.ylim([0, 40]) #scaling y-axis of distribtion
mp.legend(loc="upper right")
mp.ylabel("Normal and student t distribution")

#Var indicator figure
 minifigure = mp.axes([.18, .50, .3, .3])
mp.hist(returns, bins=100, density="None", color="dodgerblue")
mp.plot(numpy.arange(-0.2, 0.2, 0.0002), pdf_norm, "black", linewidth=2.0)
mp.plot(numpy.arange(-0.2, 0.2, 0.0002), pdf_t, "lime", linewidth=2.0)

# t_VaR indicator
mp.plot([-student_t_VaR, -student_t_VaR], [-2, 5], color="lime")
# N_VaR indicator
mp.plot([-Normal_VaR, -Normal_VaR], [-2, 5], color="black")
mp.text(-Normal_VaR-0.045, 3, "99% Normal VaR", color="black")
mp.text(-student_t_VaR-0.041, 2, "99% Student_t VaR", color="lime")
mp.ylim([0,8])  #scaling y-axis for subfigure displaying VaR values
mp.xlim([-0.1,0.01])  #scaling x-axis for subfigure displaying VaR values
mp.show()
```

Listing A.1: Python code for probability distributions

A.2 Code for dynamic allocations

```
import pandas
import numpy
import scipy.optimize

mylist=[]

prices = pandas.read_excel(r"Prices.xlsx",
sheet_name="Ark2",usecols="b,c,d.......")

ret = numpy.log(prices) - numpy.log(prices.shift(1))
avg_ret = ret.mean()
covariance = ret.cov()

def Formula(w_assets, avg_ret, covariance):
      ret = numpy.sum(avg_ret*w_assets )
      stdev_formula = numpy.sqrt(numpy.dot(w_assets.T,
      numpy.dot(covariance, w_assets)))
      return stdev_formula, ret


def formula1(w_assets, avg_ret, covariance):
      return Formula(w_assets, avg_ret, covariance)[0]

def solver(avg_ret, covariance):
      number = len(avg_ret)
      minimizing = scipy.optimize.minimize(formula1, number*[1./number,],
      args=(avg_ret, covariance),method="SLSQP",
      bounds=tuple((0,0.20) for asset in range(number)),
      constraints=({'type': 'eq', 'fun': lambda x: numpy.sum(x) - 1}))
      return minimizing

def minimizing(avg_ret, covariance):
      optimize = solver(avg_ret, covariance)
      sdp_min, rp_min = Formula(optimize['x'],
      avg_ret, covariance)
      optimizew_assetseigthing = pandas.DataFrame(optimize.x,
      columns=['w_assets1'])
      optimizew_assetseigthing.w_assets1 =
      [round(i*100,2)for i in optimizew_assetseigthing.w_assets1]
      print (optimizew_assetseigthing)

      mylist.append(minimizing(avg_ret, covariance))

df = pandas.DataFrame(mylist)
```

```
df.to_excel("Asset-allocation-output.xlsx")
```

Listing A.2: Python code for dyanamic allocations

A.3 Code for degrees of freedom

```
import numpy
from scipy.stats import t,norm
import pandas
from pandas import ExcelWriter
from pandas import ExcelFile

listOfdf=[]

for i in range(1,3861):

    returns = pandas.read_excel(r"Returns.xlsx",usecols="b",skiprows=i,nrows=249)

    t_fit = t.fit(returns)
    df, mean_t, stddev = t_fit
    print("Degrees of freedom: "+"{:.3}".format(df))

    listOfdf.append(df)
    print()


import xlwt
from tempfile import TemporaryFile
book = xlwt.Workbook()
sheet1 = book.add_sheet('listOfdf')

for i,e in enumerate(listOfdf):
    sheet1.write(i,1,e)

name = "Output.xls"
book.save(name)
book.save(TemporaryFile())
```

Listing A.3: Python code for degrees of freedome

B Full asset description

Table B.1: Overview of assets

Category	Commodity	Asset cateogory description	Instrument description	Name	Currency	Unit
Energy	WTI	Commodity spot	Crude Oil-WTI Spot Cushing U$/BBL - DS MID PRICE	CRUDOIL	USD	BBL
Energy	Brent	Commodity spot	Crude Oil BFO M1 Europe FOB $/Bbl	BFO1MEU	USD	BBL
Energy	Dubai	Commodity spot	Crude Oil Dubai 1Mth FOB Asia U$/BBL	OLDUB1M	USD	BBL
Energy	Natgas HH	Commodity spot	Henry Hub Natural Gas Spot Price Daily	NG.RNGWHHD.D	USD	MMBTU
Energy	Natgas NBP	Commodity spot	TR Natural Gas NBP UK 1st Fut. Day - SETT. PRICE	TRGNBD	USD	MMBTU
Energy	Coal	Futures (1Mth)	API 2 Coal price	TRAPI2Mc1	USD	MT
Energy	Coal	Futures (1Mth)	API 4 Coal price	TRAPI4Mc1	USD	MT
Metals	Aluminium	Commodity spot	LME-Aluminium 99.7% Cash U$/MT	LAHCASH	USD	MT
Metals	Gold	Commodity spot/cash	Gold Bullion LBM $/t oz DELAY	GOLDBLN	USD	Troy ounce
Metals	Copper	Commodity spot	London Metal Exchange (LME) - Copper Grade A	LCPCASH	USD	MT
Food	Sugar	Commodity spot	Sugar, Crystal, Sao Paulo U$/50KG	SUGCSP	USD	50kg
Textile	Cotton	Commodity spot	Cotton,1 1/16Str Low -Midl,Memph $/Lb	COTTONM	USD	Pound
Grains	Wheat	Commodity spot	Wheat No.2,Soft Red U$/Bu	WHEATSF	USD	Bushel
Food	Soybeans	Commodity spot	Yellow Soybn US NO.1 Sth Dvprt U$/Bsh	SOYADSC	USD	Bushel
Food	Colombian coffee	Commodity spot	Colombian Cofee ARAB Ex DC NY Cts/Lb	COFCLAR	USX/pence	Pound

C Rolling volatility

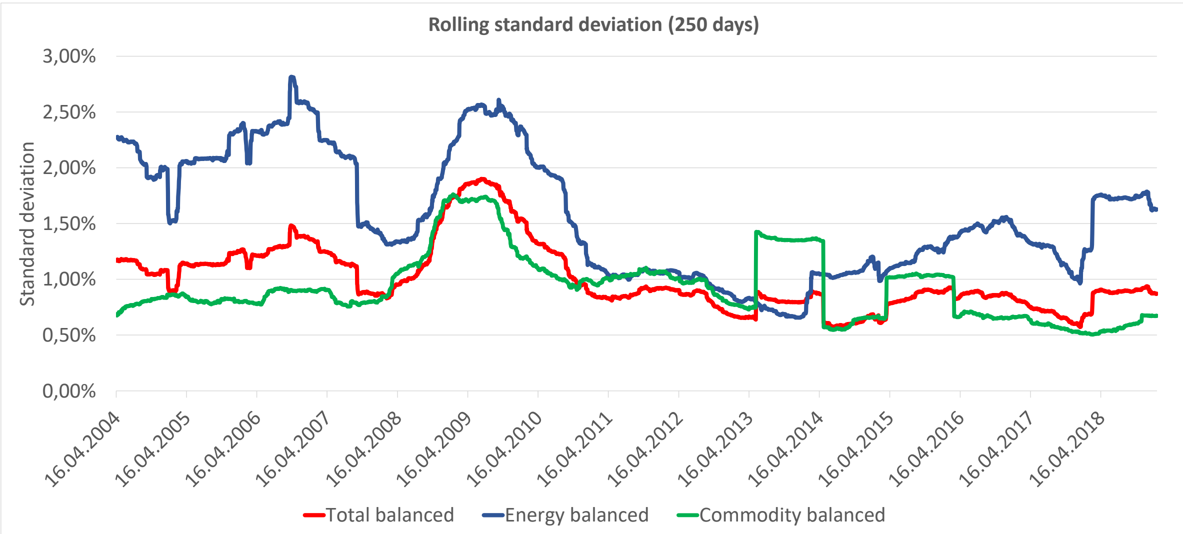


Figure C.1: Rolling standard deviation for the three minimum variance portfolios

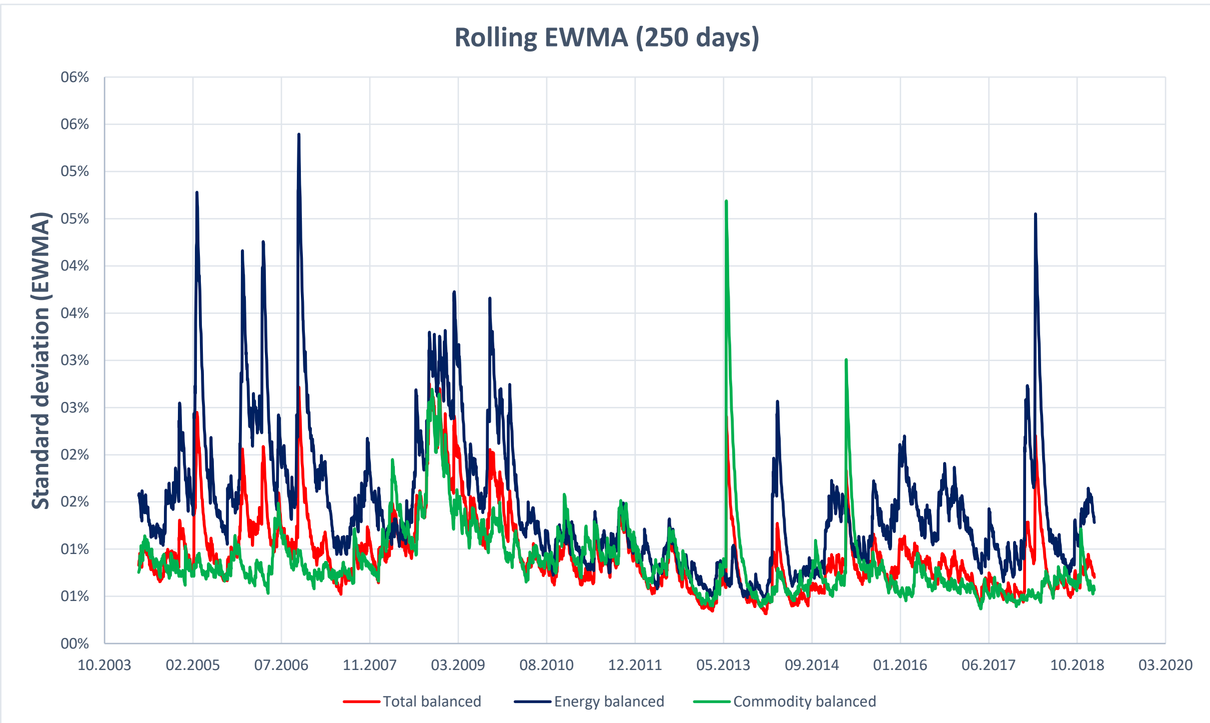


Figure C.2: Standard deviation (EWMA) of returns for the three balanced portfolios