University of Stavanger

Faculty of Science and Technology

MASTER'S THESIS

Study Program/Specialization: Petroleum Engineering / Natural Gas Eng.	Spring Semester, 2019 Open / ~~Restricted access~~
Author: Yasin Amini Shahreza	……………………………………………… (Writer's signature)
Faculty Supervisor: Rune Wiggo Time	External Supervisor: Karl Erik Thoresen (NOV)
Thesis title: Manipulation of the choke pressure during Managed Pressurized Drilling (MPD) operation	
Credits (ECTS): 30	
Key Words: MPD Choke Pressure Pressure Control Automation Automated Drilling	Pages: 52 Stavanger, 14.06.2019

Table of Contents

Table of Contents II

List of Figures IV

List of Tables V

ABSTRACT VI

A c k n o w l e d g m e n t s VII

Nomenclature VIII

1 Chapter 1: An introduction to drilling mechanism and its significance 1

1.1 Pressure Manipulation 1

1.2 Heave Effect 2

1.3 Comprehensive Model (CM) 2

1.4 Scope and Emphasis 5

2 Chapter 2: Process Description 6

2.1 Pressure Constraints 6

2.2 Drilling Equipment 6

2.3 Operational Disturbances 8

2.4 Operational Technology condition and developments 9

3 Chapter3: Applied Theory 10

3.1 Background 10

3.2 Hydraulic Model 10

3.2.1 Free Bottom Hole Model 14

3.2.2 Annulus Model 14

3.2.3 Elastic Drill String 15

3.2.4 Boundary Condition 16

3.2.5 Well Pressure Effect on the Bore Hole 18
3.2.6 Viscous Friction 19
3.3 Model Simplification 21
3.3.1 Implicit to explicit conversion 21
3.4 The implementation of the Finite LTI well model 22
3.4.1 Linear Distributed System 22
3.4.2 Discretization 23
4 Controller Design 25
4.1 Input Parameters 25
4.1.1 Choke parameters and control design 25
4.2 Back Pressure Pump 27
4.3 Normal Operation Control Design 28
4.3.1 PID Controller 29
4.3.2 Model Predictive Controller (MPC) 33
4.3.3 Coupled controller 35
4.4 Intervention Operations 35
4.4.1 Controller Designing Parameters 35
4.4.2 MPC design 38
4.4.3 Constrained MPC design 41
5 Results and Discussions 43
6 Conclusion and Future Work 50
References 51

List of Figures

Figure 1-1 The Overlook of the CM 3
Figure 1-2 The controller pallet of the drilling simulation in CM 4
Figure 2-1 Well Configuration for MPD Operation...7
Figure 3-1 The well study sections showing annulus, Drillpipe nad Freehole 13
Figure 3-2 : Change in the annulus diameter 15
Figure 3-3 Coupling the boundary conditions of all three sections 17
Figure 3-4: Velocity profile in the annulus 20
Figure 4-1 Three types of choke characteristics...26
Figure 4-2 Choke characteristics used in CM 27
Figure 4-3 Step Response of the optimized PID controller 32
Figure 5-1 Input step function on the manual mode 44
Figure 5-2 Step response of 30 bar on the automated mode 45
Figure 5-3 Step response of the 10 bar on the automated mode 46
Figure 5-4 Choke opening graph from 10 bar to 30 bar and reverse 47
Figure 5-5 Heave speed and the estimated bit speed 48

List of Tables

Table 2-1 Control Variables in the MPD operation 8
Table 5-1 The physical properties of the well 43

ABSTRACT

In the present investigation, the disturbance and its effect on the bottom hole pressure has been studied. After digging into the prior works, it’s figured out that the reason of their failure is oversimplification and ignoring some parameters that have considerable impact on the BHP like frictional forces resulting from fluid and drill string movements. Following the weakness of the last studies, the feasibility of applying new hydraulic model has been studying and new assumptions have been presented.

The model is provided by a set of linked PDEs 1. the pressure dynamics of the well annulus during unsteady Couette flow with a pressure gradient; 2. The movement of the elastic drill string coupled with the pressure dynamic through viscous friction and displacement of drilling mud. It is shown how the model can be simplified to a linear system and under which assumption this simplification can be proceeded. By using the Laplace transformer and inserting appropriate boundary conditions, the transfer function is derived from the linear system. The resulting model uses heave disturbance and controlled flow into the wellbore as input, and the measured pressure at the top of the well, as well as the pressure at the bottom of the well as output.

Based on the Hydraulic model developed, for the sake of simplification and faster running time, a lower order of the model with fewer control volumes have been introduced for the Model Predictive Controller (MPC). Two control algorithms for both normal and intervention operations have been evolved by application of PID and Model Predictive Controller (MPD). The Optimization of the PID controller has been done theoretically and validified by MATLAB.

Acknowledgments

I gratefully acknowledge the guidance and suggestions of Karl Erik Thoresen, Senior Development Advisor at National Oilwell Varco (NOV) who was always very helpful, inspiring and considerate since the beginning of my journey. I am greatly indebted to him and without him, this thesis might not have been written.

I also want to extend special thanks to the following individuals: Professor Rune Wiggo Time, my co-advisor who was extremely helpful and supportive throughout the project, Dr Amir Hossein Nikoofard, Research fellow at Norwegian University of Science and Technology (NTNU) for guiding me through the control theory and providing me with Teaching Assistantship.

Last but certainly not least, I would like to thank the University of Stavanger (UiS) for providing good infrastructure, resources and an excellent learning environment.

Nomenclature

A_a = Annulus Cross-Sectional Area[m^2]

A_d = Drill String cross-sectional area[$m2$]

A_f = Free-Hole section cross-sectional area[$m2$]

ΔAa = change in annulus cross-sectional area around the BHA[$m2$]

β = Bulk modulus of the drilling mud

βd= Youngs modulus of the drill string

$\bar{\beta} a$ = Effective bulk modulus in the annulus

C_f = Effective Free Hole Compressibility [1/Pa]

C = Fluid Compressibility [1 /Pa]

Ei = Youngs modulus of the drill string [Pa]

Eo= Youngs modulus of the formation around the well [Pa]

$fa(va, v_d)$ = Viscous drag acting on the mud in the annulus [Pa/m]

$f_f(v_f)$ = Viscous drag acting on the mud in the Free hole section [Pa/m]

$f_d(v_a, v_d)$ = Viscous drag acting on the drill string [Pa/m]

F_a= Forces acting on the mass in a control volume[N]

g = Acceleration of gravity [$m/s2$]

k_a = Linear viscous friction coefficient of mud in annulus w.r.t. mud velocity[$mkg3*s$]

k_f = Linear viscous friction coefficient of mud in free hole [mkg $3*s$]

k_d = Linear viscous friction coefficient of mud in annulus w.r.t. pipe velocity [mkg $3*s$]

K_1 = Hoop-strain coefficient, dimensionless

l_j = Length of the *j*th control volume of the annulus and drill string [m]

l_{jb} = Length of the *j*th control volume of the Free hole section [m]

L = Length of the annulus and drill string [m]

N = Number of control volumes in the annulus and drill string

N_b = Number of control volumes in the Freehole section

p_a = Annulus mud pressure [$P\,a$]

p_b = Freehole mud pressure [$P\,a$]

p_d = Drill string pressure [$P\,a$]

p_c = Mud pressure at the top of the annulus [$P\,a$]

p_r = Mud pressure around the BHA [$P\,a$]

q_c = Volumetric flow into the top of the annulus: $q_c = q_{bpp} - q_{choke}$[$m^3/s$]

q_{bpp} = Volumetric flow through the back-pressure pump [m^3/s]

r_i = Inner radius of the annulus [m]

r_o = Outer radius of the annulus [m]

t = Time, seconds

u_i = Displacement of the inner radius of the annulus [m]

u_o = Displacement of the outer radius of the annulus [m]

v_a = Velocity of the mud in the annulus [m/s^2]

v_f = Velocity of the mud in the Freehole section [m/s^2]

v_{dt} = Velocity of the drill string at the top of the well [m/s^2]

v_d = Velocity of the drill string [m/s_2]

v_r = Velocity of the mud around the BHA [m/s_2]

x = Position in the annulus and drill string, x = 0 is at the BHA

x_f = Position in the Freehole section, x = 0 is at the bottom-hole

α, α_1, α_2, α_3, α_4, α_5 = Parameters dependent on the annulus diameter ratio

μ = Mud viscosity[m/s]

μ_i = Poisson ratio for drill string

μ_o = Poisson ratio for formation

ρ = Mass density of the mud [kg/m_3]

ρ_d = Mass density of the drill string [kg/m_3]

1 Chapter 1: An introduction to drilling mechanism and its significance

1.1 Pressure Manipulation

The growing need for energy production made the oil and gas industry to explore and drill new sources of hydrocarbons. Despite being one of the most disturbing environmental activities, its business attraction makes the oil companies produce more and more, accomplished by lowering the cost of operation. At the same time, the introduction of GPU's and the fast data transition have affected different industries including the oil industry. The main impact of this evolution has been seen at the downstream oil business where almost every process has been automated efficiently while in the upstream, drilling specifically, are still relying on individual's interpretation and skills which is at high risk of mistakes. One of the most important factors in automated drilling is pressure control. In this regard, drilling mud, a Non-Newtonian fluid, is pumped through drill pipes into the well in order to act as the fluid medium to induce a pressure at the BHP following by cooling the drill bit and transporting the cuttings. The pressure should be less than fracture pressure to avoid any damage to the reservoir structure and higher than reservoir pressure to avoid the hydrocarbon inflow.

Conventionally the pressure is controlled by circulating a new mud with required densities whenever necessary to change the pressure. The main impact of this process comes from varying the hydrostatic pressure. Despite the simplicity, the process has a large dead and response time. As an example, suppose a 5 km well with an average cross-section of 0.01 m^2 and the flow rate of 1500 $\frac{L}{min}$.Then the propagation speed will be 2.5 $\frac{m}{s}$, so by simple calculation, it can be seen that it takes almost 34 minutes to upgrade the pressure. It is also not a very flexible and robust way since it mostly relays on a trial and error. This problem has been widely solved by the introduction of Managed Pressurized Drilling (MPD) where the pressure is manipulated by the opening of the choke valve installed at the end of the annuals and a back-pressure pump for a backup flow which is more sophisticated and accurate. MPD also allows the well to be drilled at the narrower drilling window

1.2 Heave Effect

One of the proven challenges in the drilling operation is a situation when the drilling takes place from a floating rig or vessel. As the waves move the vessel up and down, the drilling string can be followed by this motion. Having said this motion cannot be expected to transfer fully to the bit since the drill pipe made from stainless steel having an elastic motion. Inevitably the bit can act like a piston and it will affect BHP. For the active drilling operation, this back and forth movement can be compensated by the draw works, however during the connection when the drill string disconnected from the slips, it will cause a large pressure fluctuation.

An attempt to compensate this fluctuation had been done by Statoil in 2010 (Pavlov, Kaasa, & Imsland, 2010) using a simple hydraulic model. Despite being successful in simulation, this experiment failed practically making it important to analyze the reasons for its failure and try to address them in a new model that satisfies the requirements and assumptions.

1.3 Comprehensive Model (CM)

Comprehensive Mode (CM) is a dynamical simulator developed by NOV in Lab View programming language for visualizing the drilling phenomena. The controller that is developed in this study will be tested and validified against CM and the resulting graphs are got from the real-time tests. In the figure below an overlook of the opening window and its modules has been shown.

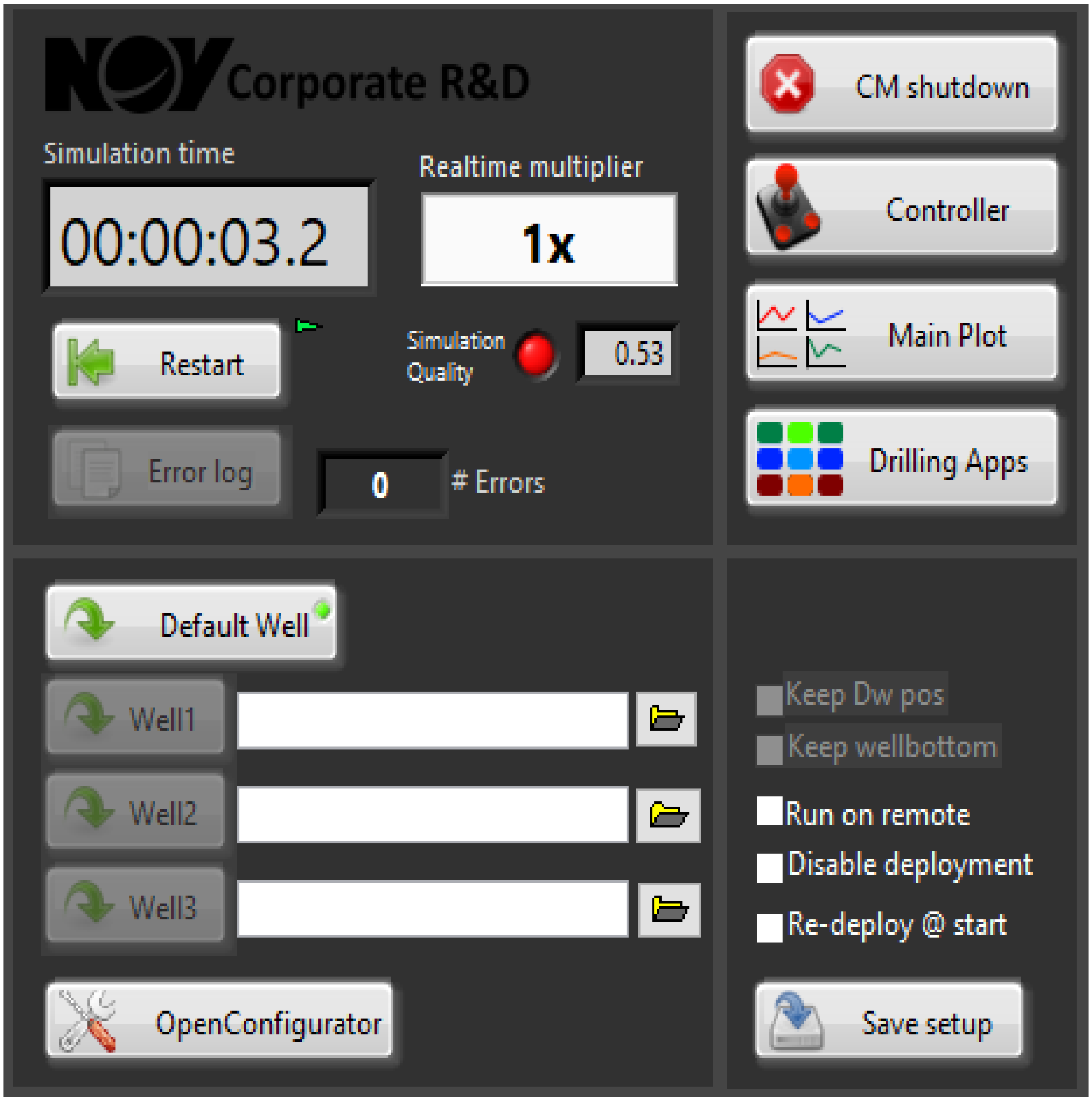


Figure 1-1 The Overlook of the CM

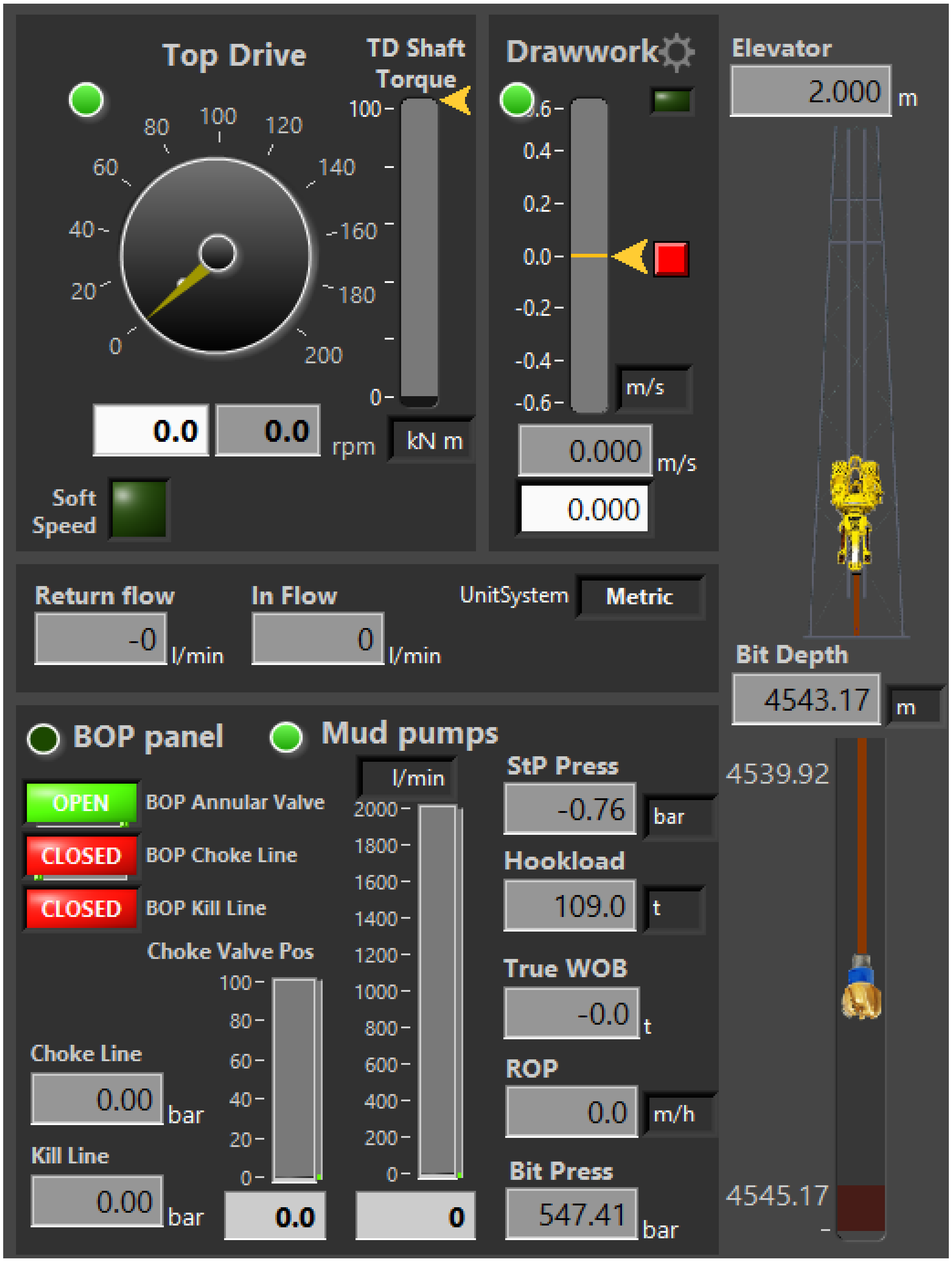


Figure 1-2 The controller pallet of the drilling simulation in CM

1.4 Scope and Emphasis

The Thesis is organized in the following way, In the next chapter, an overview of the processes compromises each part of the study zone of the phenomena are described. In the third chapter, the hydraulic model is developed, and the assumption and simplifications are applied to it. In the fourth chapter, the control design and the algorithm which was used in different scenarios are presented. The results are depicted in the fifth chapter and the reason for the outcome is described and finally, in the sixth chapter, the conclusions and a prospective of the future works will be drawn.

2 Chapter 2: Process Description

2.1 Pressure Constraints

Controlling the BHP is one of the greatest interests at the oil industry since it's so crucial in a way that it can take so much time and money if it exceeds its range from the lower or the upper border. The decisive factors are reservoir fracture pressure(p_{frac}), collapse pressure (p_{col}) and reservoir pore pressure (p_{pore}). So, the pressure of the well (p_{well}) should lie at this interval during the whole operation., i.e.:

$$max\ (p_{coll}\ (t, x), p_{pore}\ (t, x)) < p_{well}(t, x) < pfrac(t, x)$$

Where x is the position along the well and t is the time. The reservoir pore pressure is the function of both time and position. Since the density of the formation and collapse pressure is higher than the drilling fluid in upper parts, the drilling operation will get the well pressure closer to the fracture pressure (p_{frac}). However, in the bottom parts, the well pressure will be closer to the pore pressure (p_{pore}). That's one of the reasons we use casing, to protect the formation to be able to continue.

When drilling into the depleted reservoir, the pressure margin between fracture and pore pressure becomes so small and here is the importance of automated BHP pressure control is highlighted.

2.2 Drilling Equipment

Figure 1 shows a typical setup for the drilling operations. The well pressure can be manipulated by the density of the drilling fluid injected by mud pump through the drill string. The density will have the dominant change in hydrostatic pressure, while the mud rate has the most share in frictional pressure. Consequently, one can manipulate the well pressure by those two parameters. In addition, the drilling crew can also manipulate the annulus pressure by topside choke opening and backpressure pump flow. There is also a check valve at the end of the drill string to avoid backflow, especially during heave motion. Table 2-1 Shows variables that can be used as controlling input to the system.

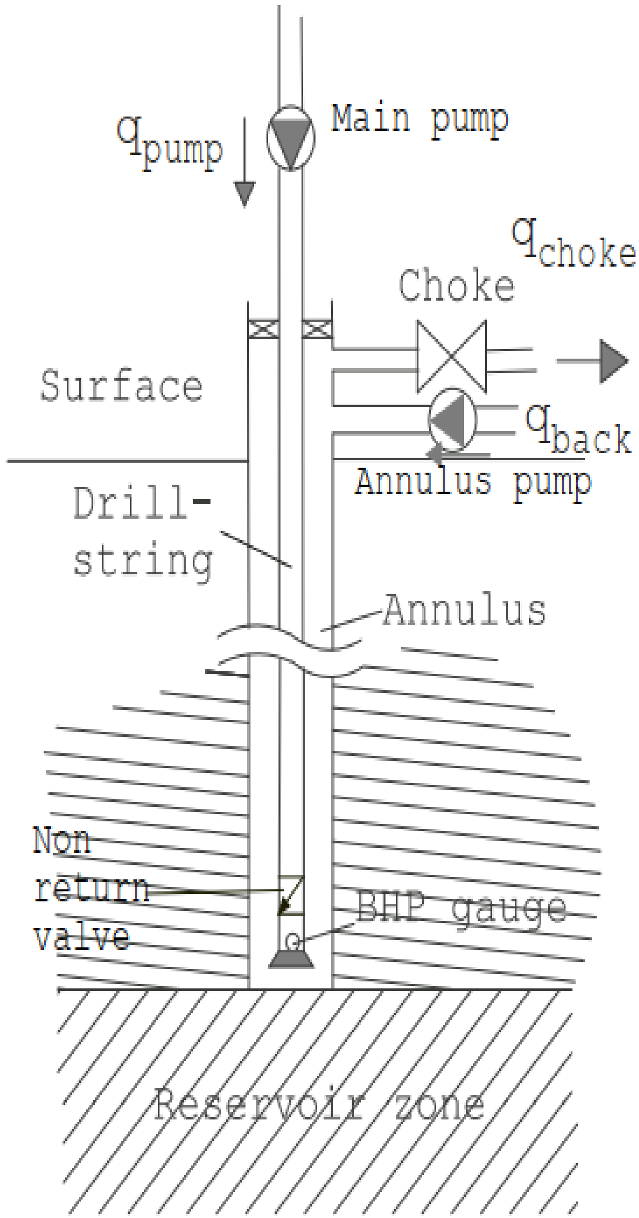


Figure 2-1 Well Configuration for MPD Operation

The more information exists from the downhole, the less uncertainty which results in more accurate control decisions. Despite measuring deferent parameters, some of the items can't be measured. The most important once are flow rate in the annulus and the BHP. Additionally, time delay at the measurements will make the uncertainty band wider and more difficult to react. The information is usually sent to surface by mud-plus-telemetry. The system rate is slow (6bps) and inaccurate which make it impossible to use it for the heave compensation. Recently an alternative system was introduced named drill pipe telemetry having both much faster rate (12Mbs) and more accurate data. (Russell, Hernandez, MacNeill, Reeves, & Hamel, 2008)

Although this recent technology represents a great development in measurements, most wells are still be drilled by conventional drill pipes but as the technology matures and costs are reduced, interest might be seen in using them.

Table 2-1 Control Variables in the MPD operation

Input	Control Variable	Notation, Unit
u_1	Main Pump Rate	$q_{P,}$ [l/min]
u_2	Back Pressure Pump rate	q_{bit}, [l/min]
u_3	Fluid Density	ρ, [SG]
u_4	Topside choke Opening	Cv(z), [%]

2.3 Operational Disturbances

There are some processes that might affect the BHP However the two most important ones are pipe connection procedure and downlink procedure. When the well is drilled further, depends on the desired weight on bit (WOB), a drill string or a drill collar should be used. During this procedure, the pump pressure ramped down to zero unless some technologies are used (Continuous Circulation for instance) which will result in the BHP decrees due to the frictional pressure loss of the annulus. During this procedure, when drilling from floaters, the drill string oscillates sinusoidally which leads to a dramatic pressure fluctuation. For narrow drilling windows thee fluctuations might be challenging in a way resulting in kicks. The remedy for these problems may lay in the pressurizing the annulus by the combination of top choke manifold and backpressure pump.

Another source of disturbance might come from the downlink procedure where mud pulses should be sent downhole to activate the directional drilling tool. As same as the pipe connection fluctuating the pump rate will also change the annular frictional pressure that can result in a change in BHP

2.4 Operational Technology condition and developments

The Items which are listed in table 2-1 are usually manipulated manually. However, some recent investigations have been done to control those controlled variables automatically. By today, only a small number of wells have been drilled using automatic pressure control. (Wylie & Streeter, 1978), (Egeland & Gravdahl, 2002), (Fontenot & Clark, 1974), (G.-O. Kaasa, Stamnes, Imsland, & Aamo, 2011), (Gjerstad, Time, & Bjørkevoll, 2012). Advanced control theories, fast GPU's and drill pipe telemetry advancement can get the drilling process more automatic and operations such as, ramping down/up, pipe connection, trip in/out, hole cleaning and well control incidents can get the most benefits out of it.

3 Chapter3: Applied Theory

3.1 Background

There have been some efforts for the modelling of the disturbance and the approaches of its rejection. For instance, Kassa and his Collogues (Godhavn, Pavlov, Kaasa, & Rolland, 2011) developed a full-scale model to attune the effect of the disturbances on the BHP. Their models worked well theoretically but it failed when they put it into the practice. Mahdianfar's study (H. Mahdianfar, O. M. Aamo, & A. Pavlov, 2012a) was used as the fundamentals of Kassa's model. (Mahdianfar et al., 2012a)represents methods to estimate the BHP during the normal drilling operation. So, the motion of the drill bit was much slower than the normal heave. One another fact that also needed to be considered is the phase shift of the disturbance. As the drill pipes and collars work like a spring due to their elasticity, this phase shift will highly depend on the length of the connections.

A more general model for the hydraulic dynamics was presented in (H. Mahdianfar, O. M. Aamo, & A. J. I. P. V. Pavlov, 2012b) where more realistic dynamics have been considered in it. However, it wasn't created to design the controller. It compromises the Rheology and the Non-Newtonian fluid relations. The results of applying it has been filed in (Landet, Mahdianfar, Pavlov, & Aamo, 2012) and(Mäkinen, Piche, Ellman, & control, 2000). In the end, it was concluded the moving pipe wall will have great influence at the BHP. Consequently, the thicker mud, the higher the chance of system failure. Therefore, it can also be another reason why Kassa's model failed to work. The distributed form of the model was presented at (Burkhardt, 1961), (Aziz, 1979)and also in the discretized form at (G.-O. J. S. R. C. Kaasa, Porsgrunn, Norway, 2007). This also was used to design the controller in (Zhou, Doyle, & Glover, 1996).

3.2 Hydraulic Model

In this section, the focus will be on the methods and assumptions being used to develop a high-fidelity design of the model to perfectly satisfies the unknown parameters such as q_{bit} and BHP. The following configurations and assumptions are just for the heave motion where the mud pump is zero. The solution approach will also be presented and under some consideration, it also can be used during the normal operation.

The MPD configurations are as follows:

- There is no flow in the BHA, so it can be considered that the pressure is constant at the drill pipe. This configuration is actually quite realistic as it was mentioned in the previous chapter that there is a check valve at drill string stopping the backflow
- The well top is sealed.

The Assumptions are also as follows: (Landet, 2011)

- The effect of a geothermal impact is neglected, so the temperature is constant at the whole annulus
- The flow is laminar. i.e. the Reynold number is less than 2300
- The radial velocity is assumed to be zero, so the velocity profile is only along the well. (Longitudinal velocity)
- Asymmetrical flow condition, that is small changes in the diameter are negligible.
- Non-linear corrective acceleration is zero. This assumption shows its importance when the mud velocity is well below the sound velocity
- Mass properties are constant.

In the current hydraulic model analysis, the well will be divided into three sections. Free hole section where the fluid is moving in the distance between the drill bit and the bottom hole, donated by f. In this region, the fictional forces excreted on the control volume coming from the annulus. The second and third section is annulus model and the elastic drill string donated by a and d , in which the resulting PDE's are coupled together through viscous friction term. Additionally, the boundary condition of latter sections used to link with the free hole section. The pressure p and velocity v in the annulus and drill string is the function of both time t and position x . As it's shown at the figure below the boundary condition $x_f = 0$ is at the bottom hole. $x_f = L_f$ is the upper limit of the Free Hole where it's the bottom limit of the annulus boundary condition , $x = 0$. The upper boundary of the annulus has been represented with $x = L$. See figure 3-1

The PDE's for all the sections are derived from the dimensional momentum and mass equation. For the one-dimensional mass balance in a control volume with the length dx , the mass inlet equals to the mass outlet:

$$\frac{\partial \rho}{\partial \mathrm{t}} = -\frac{\partial(\rho v)}{\partial \mathrm{x}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

And by considering the fluid compressibility which is equivalent to Bulk Modules β : $dp = \beta \frac{d\rho}{\rho}$ and replacing in equation 3,1 will give:

$$\frac{\partial p}{\partial t} = -\beta \frac{\partial v}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

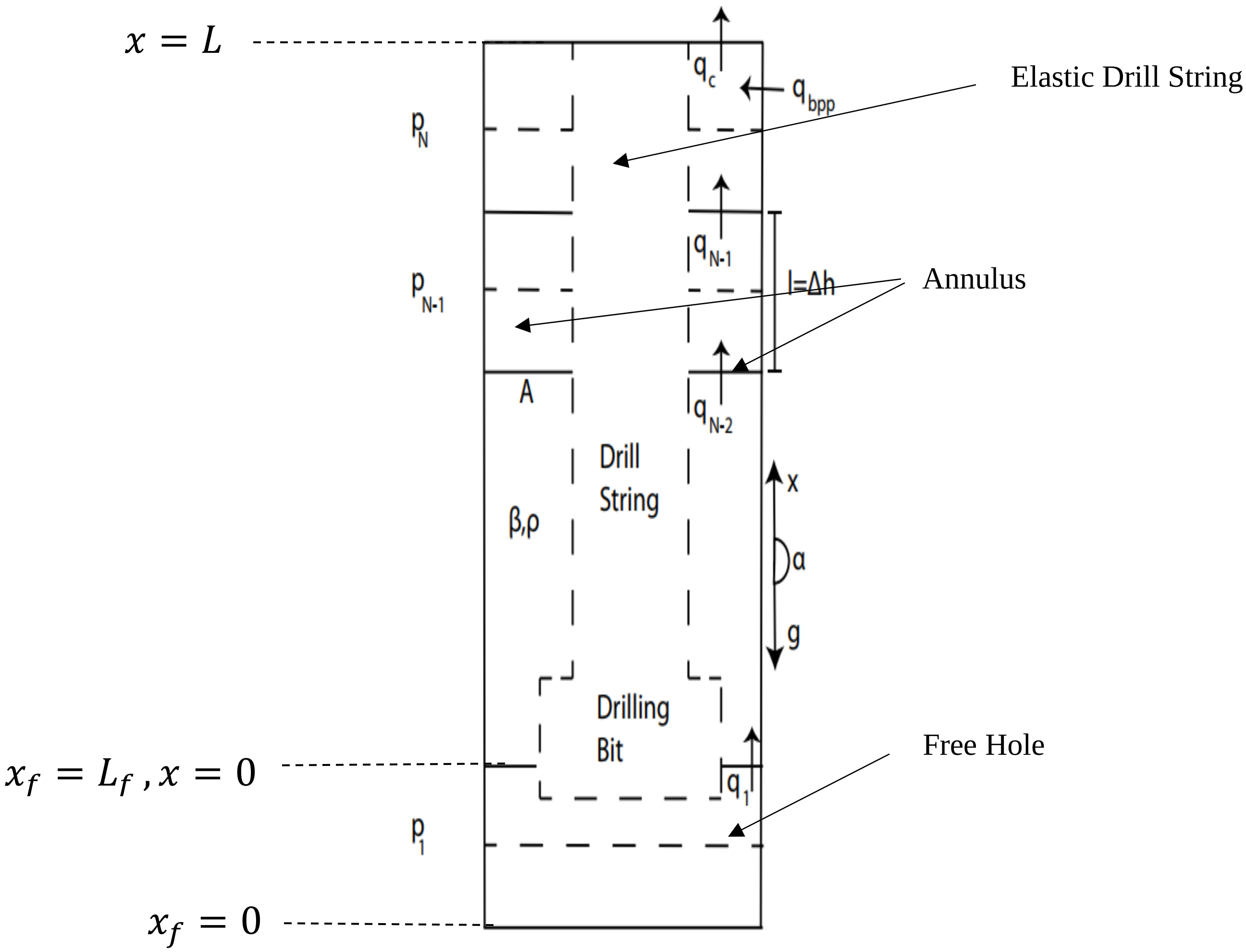


Figure 3-1 The well study sections showing annulus, Drillpipe nad Freehole

Also, by writing the Momentum Balance for the control Volume:

$$\sum F = \frac{d}{dt}(\rho v A\, dx) \quad (3.3)$$

Considering constant density, equation 3.3 leads to:

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x} - F \quad (3.4)$$

And F is the forces apart from the pressure forces. The mass and momentum balance equation will give the unsteady state of the elastic medium.

3.2.1 Free Bottom Hole Model

As described before this section is distance below the bit. By writing the mass and momentum balance:

$$\frac{dp_f}{dt} = -\beta_f \frac{\partial v_f}{\partial x} \qquad (3.5)$$

$$\rho \frac{\partial v_f}{\partial t} = -\frac{\partial p_f}{\partial x} - f_f(v_f) \qquad (3.6)$$

Where β_f is the Bulk Module of the fluid under the Bottom Hole Assembly (BHA). As in most cases, the regular drilling mud compressibility is to the order of 10^{-6}, considering the linear compressibility along the well bore shouldn't affect the calculation in an unacceptable uncertain level. Having said β_f will be derived theoretically at section 3.2.6.f_f is the viscous drag force.

3.2.2 Annulus Model

Like the free-hole model, by writing a mass and momentum balance for the annulus:

$$\frac{dp_a}{dt} = -\beta_a \frac{\partial v_a}{\partial x} \qquad (3.7)$$

$$\rho \frac{\partial v_a}{\partial t} = -\frac{\partial p_a}{\partial x} - f_a\,(v_d, v_a) \qquad (3.8)$$

The f term represents the importance of the friction term in the annulus. It has been shown, it's the function both drilling string velocity and fluid velocity itself. $f_a\,(v_f, v_a)$ will be theorized at section 3.2.6.

As shown in figure 3-2, due to the connections at the annulus and Bottom Hole Assembly, there will be changes in the effective flow area. This area can be nuzzle or diffuser shaped. In these occasions, the control volume is split into two parts, Downstream of the change spot and the upstream. These two regions are donated by + and – respectively.

$$A_a^+ v_a^+ = A_a^- v_a^- + \Delta A_f v_f \quad (3.9)$$

$$P_a^+ = P_a^- \quad (3.10)$$

The Equation above actually says that the resulting flow rate through the connection, considering the density change is negligible, will have a variance equals the change in diameter. Additionally, the pressure can be considered constant if the pressure loss is small enough to be ignored. Equation 3.10

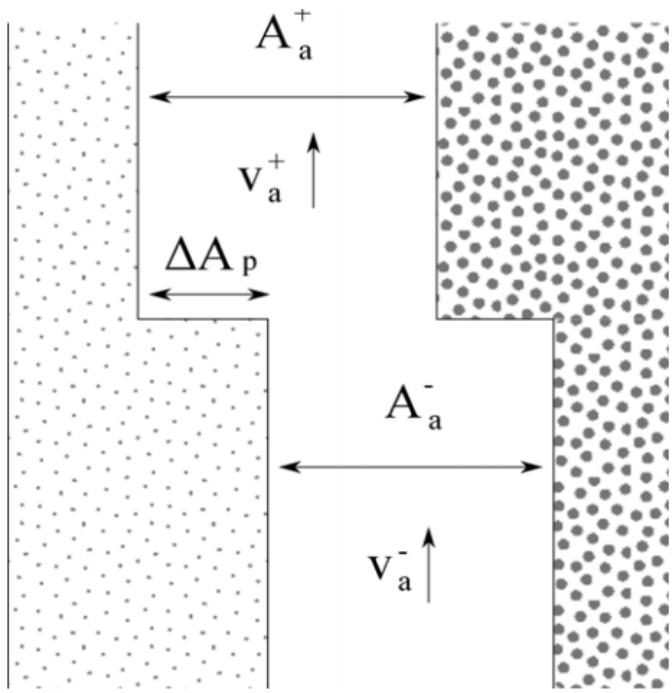


Figure 3-2 : Change in the annulus diameter

3.2.3 Elastic Drill String

Once more time, Mass and Momentum balance reveals to:

$$\frac{dp_d}{dt} = -\beta_d \frac{\partial v_d}{\partial x} \quad (3.11)$$

$$\rho_d \frac{\partial v_d}{\partial t} = -\frac{\partial p_d}{\partial x} + K_d \frac{\partial}{\partial x} P - f_d \left(v_d, v_a\right) \quad \text{(3.12)}$$

Where β_d is the Young Modulus of the pipe. K_d is derived from the Hoop Stress Effect. This effect will be bold when the pipe squeezed as a result of the mud pressure. And finally, as same as before f function is term effected by the mud viscous force on the pipe.

3.2.4 Boundary Condition

On the topside of the well either one of these conditions can be enforced.

$p_a(x = L) = p_c$ The annulus pressure equals to the choke pressure

$A_a v_a(x = 0) = q_{choke} - q_{bpp}$ Flow through choke is equal to the bit (Constant density assumption)

The second condition is what has been used in the Comprehensive Model to measure the drill bit speed. Having said, there are uncertainties in it.

- Due to the drill string movement, the drill string excretes a drag force to the mud and cause to record opposite speed of the drill string at first, but the flow rate caused by the drill bit movement is large enough to compensate it.
- The choke opening must be fully open (100%) to sense the flow rate changes as fast as possible. Otherwise, it will affect the amplitude and phase shift of the recorded data.

For the drill string as well, there will be following boundary conditions:

$v_d(x = L) = v_{ds}$ Movement of the pipe at the top of the well

$A_d P_d(x = L) = F_d$ Topside excreted force on the pipe

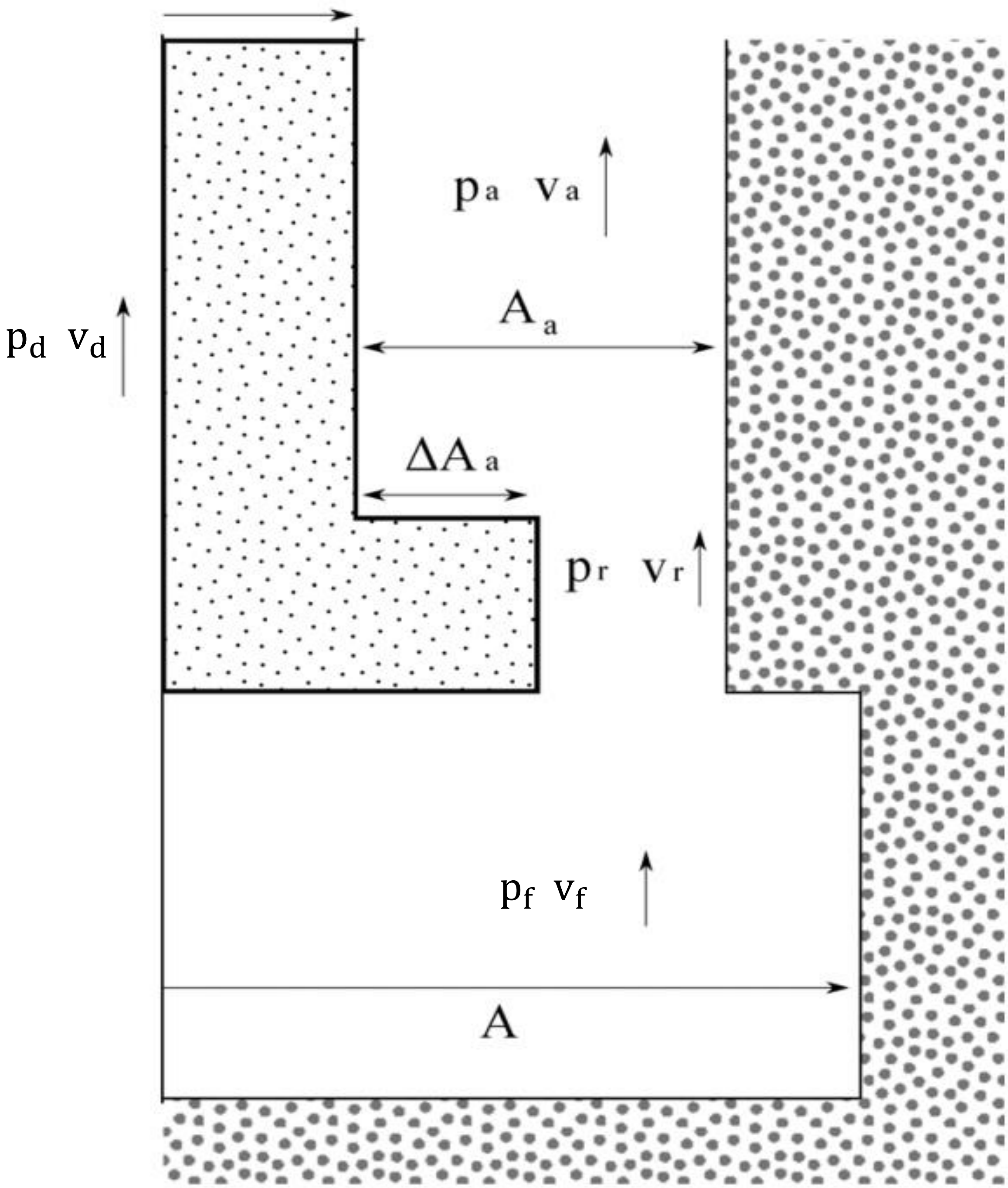


Figure 3-3 Coupling the boundary conditions of all three sections

As mentioned previously, the bottom hole was assumed to be rigid, so the bottom hole boundary condition is $v_f\left(x_{f=0}\right) = 0$

3.2.5 Well Pressure Effect on the Bore Hole

The effective flow area at the annulus is obtained by:

$$A_a = \pi((r_0 + u_0)^2 - (r_i + u_i)^2) \quad (3.13)$$

Where r_0 and r_i are the outer and inner diameter of the annulus respectively which are casing (Or formation) and drill string diameter. u_0 and u_i are also the effected diameter change due to the pressure to the annulus. The later parameters can be found by:

$$u_i = \frac{r_i}{E_i}(1 - \mu_i)p_a \quad (3.14)$$

$$u_o = \frac{r_o}{E_o}(1 + \mu_o)p_a \quad (3.15)$$

Where E and μ are Youngs Modulus and Poisson's Ratio of the pipe and casing (Or formation). The relative expansion of the effective flow area (Annulus Cross section) can be found by:

$$\frac{1}{A_a}\frac{dA_a}{dp_a} = \frac{2{r_o}^2(1+\mu_o)}{E_o({r_0}^2-{r_i}^2)} + \frac{2{r_i}^2(1-\mu_i)}{E_i({r_0}^2-{r_i}^2)} \quad (3.16)$$

So, the effective compressibility (inverse of Bulk Modulus) of the drilling mud will be:

$$C_a = \left|\frac{2{r_o}^2(1+\mu_o)}{E_o({r_0}^2-{r_i}^2)} + \frac{2{r_i}^2(1-\mu_i)}{E_i({r_0}^2-{r_i}^2)} + \frac{1}{\beta}\right| \quad (3.17)$$

Where C is the fluid Compressibility $(:= \beta)$

For the Free-Hole section, the effective compressibility above will be simplified to:

$$C_f = \left|\frac{2\left(1+\mu_f\right)}{E_0} + \frac{1}{\beta}\right| \quad (3.18)$$

3.2.6 Viscous Friction

The calculation of the viscous friction for Non-Newtonian drilling fluid in the well is very challenging. Depending on the drill string velocity, pressure gradient and mud characteristics, the velocity profile can have multiple flow regime for the Bingham Plastic (Obinata & Anderson, 2012) where the PDE's are solved explicitly. The analytical Solution also for unsteady state flow regime in Power Low flow has been solved in (Breyholtz, Nygaard, Godhavn, & Vefring, 2009).

In this analysis below, the fluid has been considered as the Newtonian fluid where the viscous term for the equation presented in 3.2.1, 3.2.2 and 3.2.3 are derived and solved. For Newtonian Fluid under the assumption of Laminar flow (Presented at the assumption), the viscous fiction has a linear relationship with the velocity (See figure 3-4). As shown in (Obinata & Anderson, 2012),

$$f_a(v_a, v_d) = -12\frac{\mu}{(r_o - r_i)^2} v_a + 6\frac{\mu}{(r_o - r_i)^2} v_d \alpha_1 \qquad (3.19)$$

$$\alpha = \frac{r_i}{r_o} \qquad (3.20)$$

$$\alpha_1 = \frac{8\alpha^4 \ln(\alpha)^2 + \alpha_4 \alpha_5 \ln(\alpha) + 2\alpha^2 \alpha_5^2}{-\alpha_2 \alpha_3 \alpha_5} \qquad (3.21)$$

$$\alpha_2 = 2\ln(\alpha) + 1 - \alpha^2 \qquad (3.22)$$

$$\alpha_3 = 2\alpha^2 \ln(\alpha) + 1 - \alpha^2 \qquad (3.23)$$

$$\alpha_4 = 3\alpha^4 + 6\alpha^2 + 1 \qquad (3.24)$$

$$\alpha_5 = (1 - \alpha)^2 \qquad (3.25)$$

In equation 3.19 above μ is the viscosity, by rearranging and substituting equation 3.19 with the following equations,

$$k_a = 12\frac{\mu}{(r_o - r_i)^2} \qquad (3.26)$$

$$k_c = 6\frac{\mu}{(r_o - r_i)^2} \qquad (3.27)$$

$$k_d = \frac{A_d}{A_a} k_c \quad (3.28)$$

$$k_f = \frac{4\mu\pi}{A_f} \quad (3.29)$$

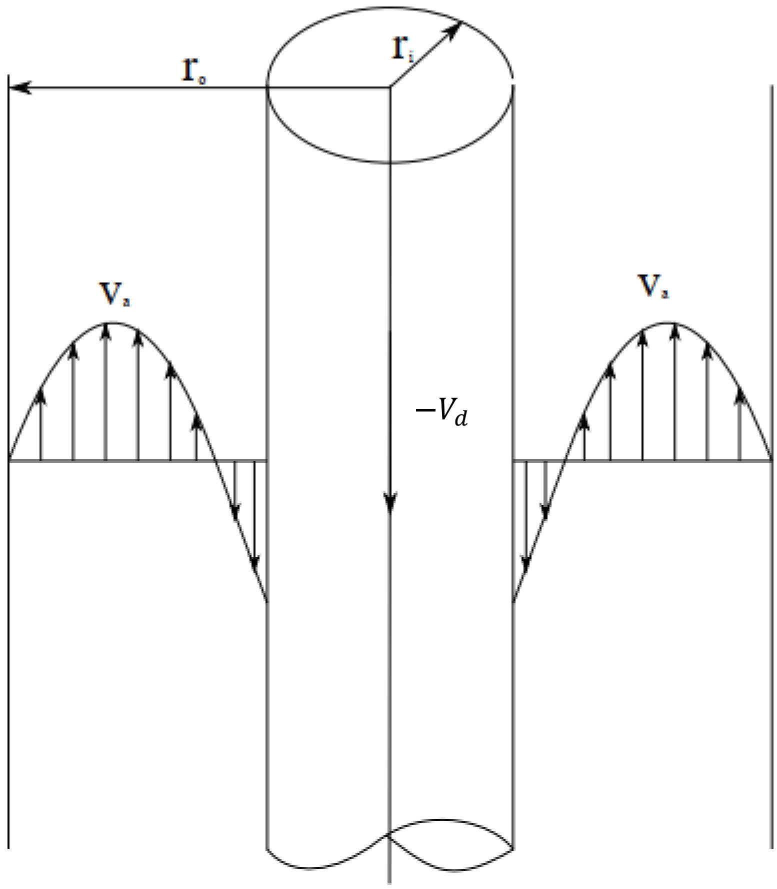


Figure 3-4: Velocity profile in the annulus

In the End, the parameters were defined in equation 3.19 are obtained by:

$$f_a\,(v_d, v_a) = k_a v_a - k_c v_d \quad (3.30)$$

$$f_d\,(v_d, v_a) = k_d(v_d - v_a) \quad (3.31)$$

$$f_f\left(v_f\right) = k_f v_f \quad (3.32)$$

3.3 Model Simplification

The model which was derived in section 3.2 would be complex since it's coupled with boundary condition or Non-Newtonian used as the drilling fluid. In such cases, the analytical solution should be applied implicitly. This fact makes the controller design so difficult. As a result, some simplification is needed to be assumed under specific conditions. In this next section, these methods are discussed.

3.3.1 Implicit to explicit conversion

As mentioned in the last section to ease the controller design, two simplifications are required to find a model where all the equations are explicit,

1. The nozzle pressure effect at the joints considered as negligible.
2. If there is Non-Newtonian fluid, the coupling relations of the drill string and annulus should be considered explicit.

A linear relation for employing the explicit form should be considered. Having said, as it's already assumed in the assumptions, as the flow regime is laminar, the viscous friction having a linear relation with both string and annulus flow velocity. Linear system allows the superposition principle for both time and input where the system behavior can be investigated by the convolution of the step function and input which results in the impulse response of the system.

Finally, the discretization of the model which converts hyperbolics PDE's to the finite number of ODE's will allow for easier implementation in the simulator and the control designs. The ODE's are formed for the finite number of non-zero control volumes. The combination of the linear approximation with the discretization will lead to an LTI system that can be analyzed and designed in the frequency domain.

3.4 The implementation of the Finite LTI well model

3.4.1 Linear Distributed System

By using the simplification presented above, measuring the viscous friction by equation at the part 3.2.3 and considering that the fluid is Newtonian and solving the nozzle pressure effect explicitly, the following governing equations can result:

$$\frac{dp_a}{dt} = -\frac{1}{C_a}\frac{\partial v_a}{\partial x} \quad (3.33)$$

$$\rho\frac{\partial v_a}{\partial t} = -\frac{\partial p_a}{\partial x} - k_a v_a - k_c v_d \quad (3.34)$$

$$\frac{dp_f}{dt} = -\frac{1}{C_f}\frac{\partial v_f}{\partial x} \quad (3.35)$$

$$\rho\frac{\partial v_f}{\partial t} = -\frac{\partial p_f}{\partial x} - k_f v_f \quad (3.36)$$

$$\frac{dp_d}{dt} = -\beta_d\frac{\partial v_d}{\partial x} \quad (3.37)$$

$$\rho_d\frac{\partial v_d}{\partial t} = -\frac{\partial p_d}{\partial x} + K_d\frac{\partial}{\partial x}P - k_d(v_d - v_a) \quad (3.38)$$

The Equations are required to be solved by six integrations which need six boundary conditions. The boundary conditions are as follows:

$$A_a v_a(x=0) = q_{choke} - q_{bpp} \quad (3.39)$$

$$p_a(x=0) = p_f(x_f = L_f) \quad (3.40)$$

$$v_d(x=L) = v_d \quad (3.10)$$

$$p_d(x=0) = p_f(x_f = L_f) + p_f^s(x_f = L_f) \quad (3.41)$$

$$A_f v_f(x_f = L_f) = A_a v_a(x=0) + A_d v_d(x=0) \quad (3.42)$$

$$v_f(x=0) = 0 \quad (3.43)$$

Where q_{choke} *and* q_{bpp} are the system input that and during the heave v_d, the drill string velocity at the wellhead, is the disturbance affected the system. p_f^s is the steady state pressure at $t = 0$, which is equal to the hydrostatic pressure and the topside choke pressure.

As presented in section 3.2.2, if the effective flow area changed due to the area change or at the BHA, the volume should be splitting into downstream and upstream followed by the later approaches.

3.4.2 Discretization

The PDE's above need to be converted to ODE's by finite discretization method to determine the parameters at different positions. Solving it involves dividing the control volume into a finite number of sections and integrating the equations above at each section

Landet.et.Al.(Landet, 2011) understood that dividing the control volume into five discrete will be enough to describe the dynamics of the well in an acceptable accuracy. The experiment has been conducted in Ullrig faucitis in a depth of 2000m and a water-based mud.

The analysis will be also based on Landet. et Al works, so the set of the ODE's is as follows:

$$\dot{p}_a{}^j = \frac{\beta_a{}^j}{l_j}\left(v_a^{j-1} - v_j{}^j\right) \qquad j = 1, \dots, 5 \qquad (3.44)$$

$$\rho \dot{v}_a{}^j = \frac{1}{l_i}\left(p_a{}^j - p_a{}^{j+1}\right) - k_a{}^j v_a{}^j - k_c{}^j v_d{}^j \qquad j = 1, \dots, 4 \qquad (3.45)$$

$$A_a^5 v_5 = q_c, \;\; p_a^0 = p_f^5$$

$$\dot{p}_d{}^j = \frac{\beta_d{}^j}{l_j}\left(v_d^{j-1} - v_d{}^j\right) \qquad j = 1, \dots, 5 \qquad (3.46)$$

$$\rho \dot{v}_d{}^j = \frac{1}{l_i}\left(p_d{}^j - p_d{}^{j+1}\right) + \frac{K_1}{l^j}\left(p_a{}^j - p_a{}^{j+1}\right) - k_p{}^j\left(v_p^j - v_a{}^j\right) \qquad j = 1, \dots, 4 \qquad (3.47)$$

$$v_d^5 = v_h, \;\; p_d^0 = p_f^5$$

$$\dot{p}_f{}^j = \frac{\beta_f{}^j}{l_j}\left(v_f^{j-1} - v_f{}^j\right) \qquad j = 1, \ldots, 5 \quad (3.48)$$

$$\rho \dot{v}_f{}^j = \frac{1}{l_i}\left(p_f{}^j - p_f{}^{j+1}\right) + k_f{}^j v_f{}^j \qquad j = 1, \ldots, 4 \quad (3.49)$$

4 Controller Design

The controller design algorithm and analysis are the most crucial part of this study. As mentioned before, during the normal operation some of the disturbances like the induced heave can be compensated by the draw works. During connections, however, as the string is disconnected from the slips, the pips can move along with waves which result in string-piston like movements leading to a huge pressure fluctuation.

The main reason for the pressure fluctuating is that a material with much higher Bulk Modules (Drill String/bit) replaces a fluid (Drilling Mud) with far smaller Bulk Modules relatively. As presented in the last chapter, this replacement will create a pressure disturbance $\dot{p}$. As mentioned previously another reason for pressure fluctuation lies with the effect of friction forces which is insignificant in comparison to the drill string effect.

The controllability of every process requires knowing the physics and transfer functions of the components involved, including input parameters, process transfer function, disturbance transfer function and the resulting effects. In the following chapter, each of the stages above will be analyzed and solved to design the final controller.

4.1 Input Parameters

These are factors that are being used to control the system which leads to the final disturbance rejection.

4.1.1 Choke parameters and control design

The topside choke manifold is the most crucial tool in MPD as for any opening input, it reacts fast, effective and reliable.

Generally, the choke valve is theorized as follows:

$$q_c = C_v(z)\sqrt{\frac{p_c - p_0}{\rho_{out}}} \quad (4.1)$$

Where q_c is the flow through the choke, p_c is the choke pressure, ρ_{out} is the fluid density at the choke and finally the foremost is $C_v(z)$ known as choke characteristics linking choke opening and the flow through. It is usually represented in the graphs known as the characteristic's curves. Certainly, C_v has a direct relationship with the opening. That is C_v increases by increasing z.

There are three kinds of characteristics curves which are as follows (See Figure 4.1)

1. Quick opening

   It opens quickly, and it can't offer good control for sensitive processes. It is usually used for on-off systems
2. Linear

   It offers equal change of flow per unit of choke stroke. It presents the same controllability through the strokes
3. Equal Percentage

   It produces superior control on the low end of the control while offering less control but higher capacity at the end of the stroke

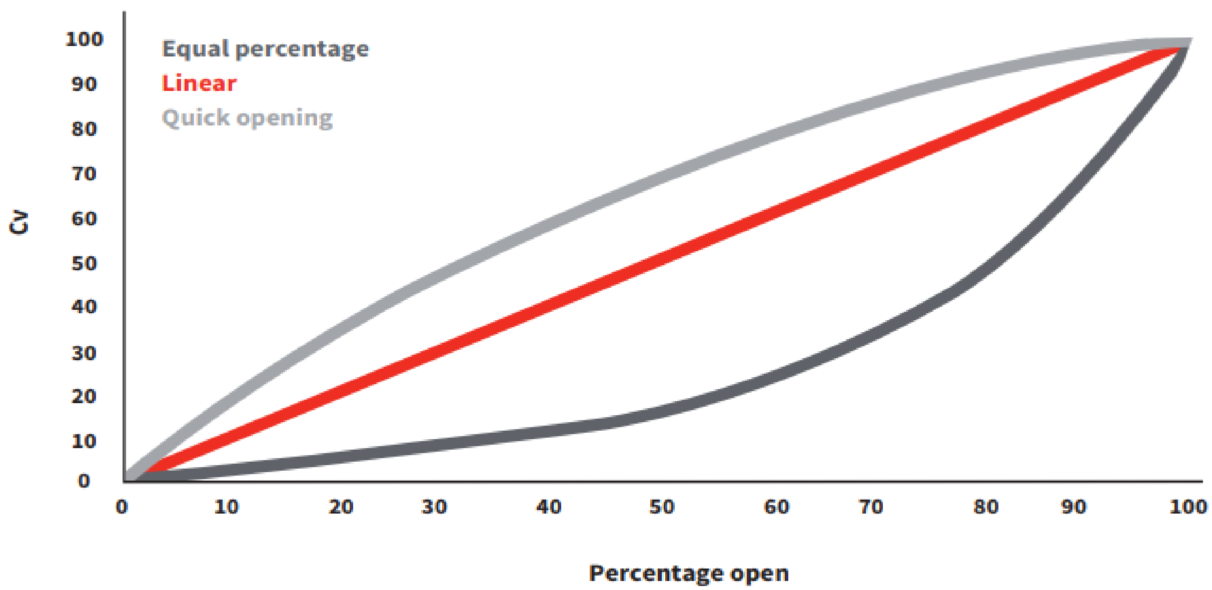


Figure 4-1 Three types of choke characteristics

So, the curve is highly dependent on the type of the manifold which has been used. The type of choke which has been used at CM is cage type where its curve is between the linear and equal percentage type.

In CM that the controller is designed and run, the time lag of the choke manifold opening input is assumed to be zero and also the resistance and robustness of the internal components are also considered to be negligible while in a real scenario an uncertainty margin should be applied to make the design and number as accurate as possible.

The figure below shows the characteristics curve utilized, So the control design will be based on it.

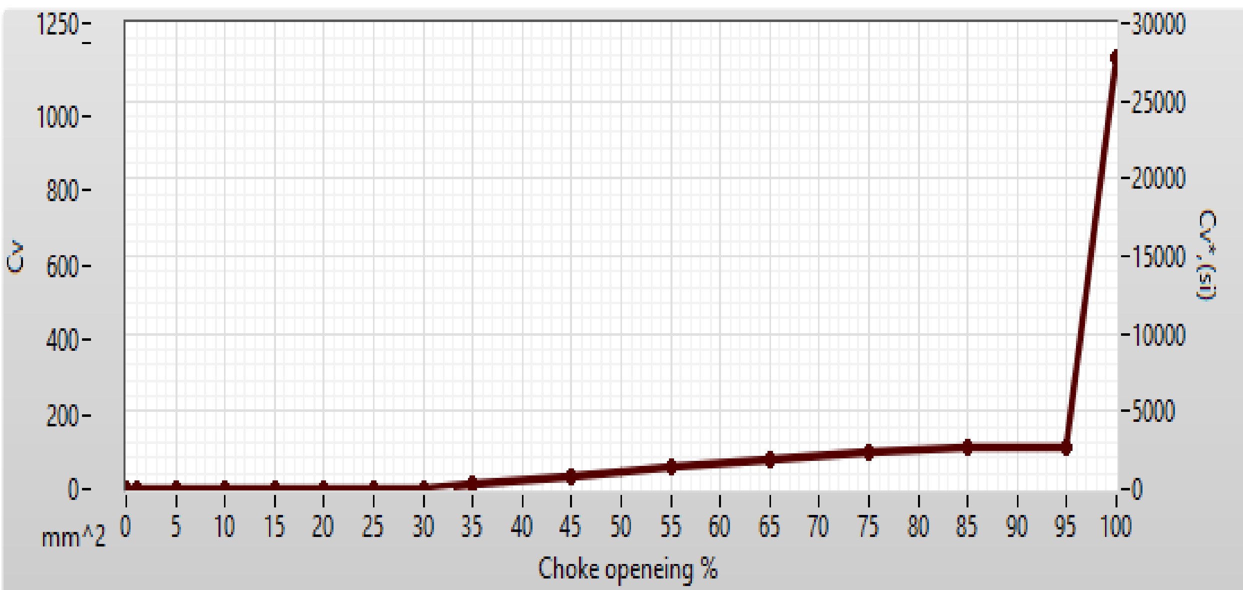


Figure 4-2 Choke characteristics used in CM

4.2 Back Pressure Pump

As discussed earlier the back-pressure pump shows off its' effect at the last control volume (starting from bottom-hole) as following relation in the simplified form of what has been used in the last chapter:

$$\dot{p}_N = \frac{\beta_N}{A_N l_N}(q_N - q_c + q_{bbp}) \quad (4,2)$$

The most typical backpressure pump which has been used in the MPD control is a positive displacement pump. Utilizing these kinds of pump making some problems for the full automated MPD control discussing in the investigation. The most import problems are as follows:

- The piston-cylinder pumps inherently making a lot of noises in the system. Especially as they are close to the sensors used for sensing the elements in MPD, this disturbance which depends on its frequency can affect the PID controller's performance.
- Despite that physically both positive-displacement and centrifugal pumps reaction to the input is much slower than choke (Its reason was presented in the last chapter), for the sake of pumps functionality, centrifugal pumps can react significantly much faster than cylinder-piston ones. The reason for that is because the latter one can pump each stroke of the fluid in each back and forth movement which takes much more time and energy(friction).
- The capacity of advanced centrifugal pumps is much larger than another type. Having said, as it will be seen, the controller design doesn't need that much capacity However it shows its effect when there is a huge ramp down at the circulation where it's also challenging for the choke opening to follow it. So, depending on the pressure range of the well to be controlled, larger capacity would be helpful.

Here again like the choke, the time delay at the transfer function in the CM is supposed to be negligible, this means that the distance that flow needs to travel out of the pump outlet to the flow stream inlet (where the sensors are installed) in a specific period is much smaller than the relative pumped fluid velocity.

4.3 **Normal Operation Control Design**

As mentioned earlier at the normal operation the draw-works will compensate the heave effect for the BHP. However, a reliable system is needed to control the BHP with the minimum offset. This control is done by the choke opening and the back-pressure pump described earlier in the chapter.

During the drilling, it's so often to change the BHP in different geological rheology. In the algorithm defined here two kinds of controllers are used. Proportional-Derivative-Integral (PID) controller and Model Predictive Controller (MPC).

Supposing a step function as an input, the response will be divided into four parts: Dead time, Rise time, settling time and stabilizing time. It will be seen in the next sessions that each of two controllers mentioned above will fit better in the different specific stages of the step response.

4.3.1 PID Controller

A PID controller in choke is given by:

$$z = K_p \left(e_t + \frac{1}{T_i} \int e \ dt + T_d^{\cdot} \ \dot{e} \right) \ldots \tag{4.3}$$

Where z is the process input, e is the difference between choke (p) and desired pressure, r ($e = r - p$). The tuning parameters in PID are K_p, Controller gain, Integral time T_i and the derivative part T_d.

Following proven PID functionality at the industry, application of PID is so common for different processes. As it's known, the derivative part of PID will control the change rate of the parameter hitting the set point. For instance, if the choke pressure increase with a high rate, as its most likely to pass the set point which leads to the overshoot, the derivative part will help the valve to decrease the rate to avoid large undershoot or overshoot which might lead to kick or damage to the formation.

In the practical drilling process, however, due to the many disturbance sources, the derivative part cannot be used since it's so sensitive to the noises. On the other side without using the derivative part, large rates in pressure cannot be controlled and relatively high undershoot or overshoot is inevitable, so in the model, the only proportional and integral part will be used (T_d). Besides, because of the nonlinearity of the choke characteristics ($C_v,$) as shown in the figure 4.2 even linearized controller has considerable overshoot. (more than 2.5 bar (Not satisfying the requirements mentioned in (Godhavn, 2009)).

With all the drawback pointed above, there is one strong advantage that makes the author use it in the model. It's high robustness in keeping the pressure constant for different scenarios like ramping down / up of the mud pump. So, by defining a PID margin, it's possible to get its benefit out of it.

PID margin is an interval where the PID comes into service when the pressure difference between choke pressure and the setpoint is less than determined range. This range can be defined by the operator. The relation between PID, MPC and PID margin will be discussed later when MPC is also described.

For designing the PID controller, the liberalization should be carried out due to the non-linearity relation between choke opening and pressure. So, we have:

$$Q_{out} = C_v(z_0)\sqrt{\frac{p_0}{\rho_{out}}} \quad (4.4)$$

$$p_0 = \rho_{out} \left[\frac{Q_{out}}{C_v(z_0)}\right]^2 \quad (4.5)$$

If a first-order system assumed between the choke pressure and choke opening:

$$\dot{Y} + \frac{1}{\tau} Y = X(t) \quad (4.6)$$

Where Y is Δp and X(t) will be pressure affecting parameters, which is equal to $\frac{\partial p}{\partial z}\Delta z + \frac{\partial p}{\partial q}\Delta q$.By taking the Laplace transformation of the equation 4.6 and rearranging the equation:

$$\Delta p = \frac{\frac{\partial p}{\partial z}\Delta z + \frac{\partial p}{\partial q}\Delta q}{1+\tau s} \quad (4.7)$$

The partial derivatives above can be assumed as constant parameters in short ranges, and they are measured as follows:

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial z}\left(\rho_{out}\left[\frac{Q_{out}}{C_V(z)}\right]^2\right) =$$

$$\frac{-2\rho_{out}Q_{out}^2}{[C_V(z)]^3}\frac{\partial C_v}{\partial z}(z) \quad (4.8)$$

By considering the initial condition at zero as steady state and after integration from 0 to z, the answer results in:

$$-2\ \frac{p_0}{C_V(z_0)} \quad (4.9)$$

The change in the opening will also change the flow rate which again makes the pressure change as follows:

$$\frac{\partial p}{\partial Q_{out}} = \frac{\partial}{\partial Q_{out}}\left(\left[\frac{Q_{out}}{C_v(z_0)}\right]^2\right) = 2\,\frac{Q_{out}}{[C_v(z_0)]^2}$$

$$= -2\ \frac{p_0}{Q_{out}} \quad \text{(4.10)}$$

The time constant $(\tau)$of the process which is crucial in the controller design is theorized below:

$$\tau = \frac{-1}{\frac{\partial \dot{p}}{\partial p}} = \frac{-1}{\frac{\partial}{\partial p}\left[\frac{1}{V_{ann}\beta}\left(Q_{in} - C_v(z)\sqrt{\frac{p}{\rho_{out}}}\right]\right.}$$

$$= \frac{2V_{ann}\beta\sqrt{p\rho_{out}}}{C_v(z)} = 2V_{ann}\,\beta\ \frac{p_0}{Q_{out}} \quad \text{(4.11)}$$

So, by having process time constant and gain and applying them into the PID tool in MATLAB the optimum gains for the controller can be obtained. The controller should have the minimum rise time, over/undershoot. It should also robust enough to make the pressure back to the setpoint pressure with the least oscillation if a disturbance affects the well hydraulic.

The figure below shows the optimized controller's curve in the PID tool.

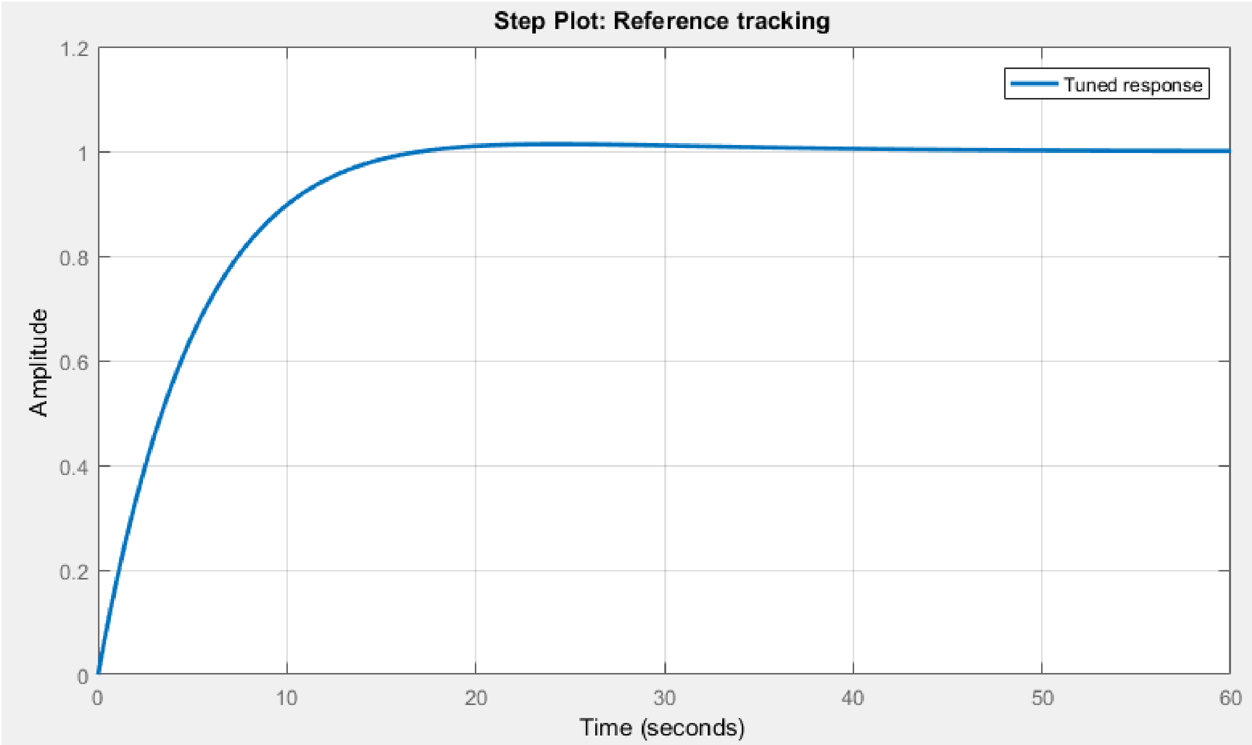


Figure 4-3 Step Response of the optimized PID controller

The optimized PID controller has 1.37 % overshoot which is negligible. Analytically, applying the equations 4.9, 4.10, 4.11 into the PID function will lead to (without feedforward and derivative):

$$\Delta p = \frac{\frac{\partial p}{\partial z}\Delta z + \frac{\partial p}{\partial q}\Delta q}{1+\tau s} \quad (4.12)$$

The Controller output is:

$$\Delta z = K_p\left(1 + \frac{1}{\tau s}\right)e \quad (4.13)$$

Where e is:

$$e = \Delta r - \Delta p \quad (4.14)$$

And by replacing equations 4.12 and 4.13 into equation 4.14

$$e = \Delta r - \frac{\frac{\partial p}{\partial z}K_p\left(1+\frac{1}{T_i s}\right)e + \frac{\partial p}{\partial q}\Delta q}{1+\tau s} \quad \text{(4.15)}$$

$$e = \frac{T_i s(1+\tau s)\Delta r - \frac{\partial p}{\partial q}T_i s \Delta q}{T_i \tau s^2 + \left(1+\frac{\partial p}{\partial z}K_p\right)T_i s + \frac{\partial p}{\partial q}K_p} \quad \text{(4.16)}$$

The equation above is the second order system which can be used to tune the controller by pole placement with different methods like Root-Lucas. If the expression above has poles at the right-hand side of the S-plane, the system is physically unusable and cannot be controlled by a PID controller. When the root of the denominator (pole) placed at the LHS, the optimum result at the end will be as same as figure 4.3 obtained by MATLAB.

4.3.2 Model Predictive Controller (MPC)

As understood in the last section, the PID controller works perfectly fine for applying to the pressure stabilization to maintain constant pressure at the choke. But for the reasons described, it will have significant over/undershoot. The algorithm utilized here is a form of reverse engineering. That is, get the desired(setpoint) pressure, continuously sensing density and flow rate out of the sensors and predicting the final desired choke opening by interpolation and then repeating the procedure again to update the choke value. The interpolation of choke characteristics has been written in Python and it's coupled with CM. So, if the equation 4.4 is rearranged as follows:

$$C_v(z_0) = \frac{1}{Q_{out}}\sqrt{\frac{p_0}{\rho_{out}}} \quad \text{(4.17)}$$

As can be seen in the equation above, all the parameters should be at steady state, so choke flow will be (assuming density difference at the mud pump and choke line is negligible)

$$Q_{out} = Q_{main\ Pump} + Q_{bpp} \quad \text{(4.18)}$$

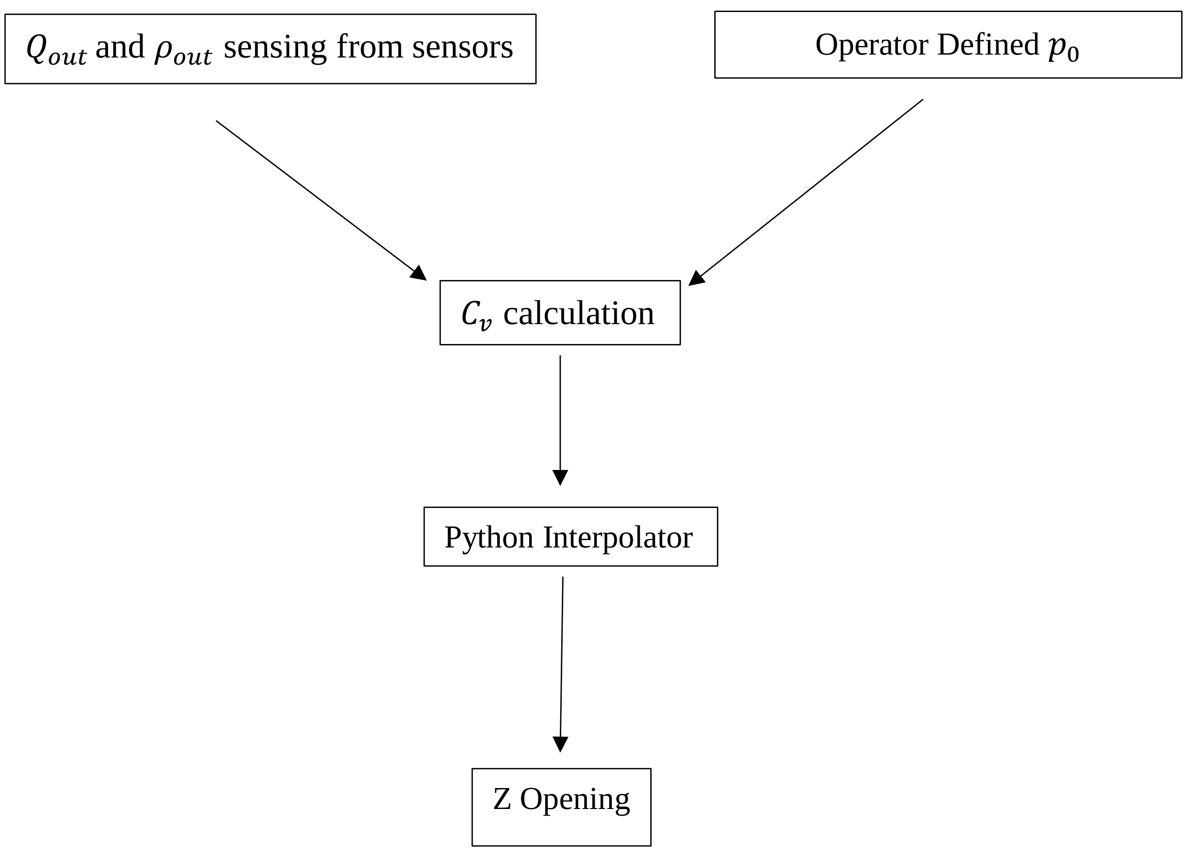
As presented above in this method the z is updating for each iteration, and in each iteration, the sensing elements (flow rate and density) should be at their steady-state mode. It's obvious that z changes at each iteration will change flow rate and will produce a deviation from the steady-state mode, so time delay is needed for each iteration, this time delay is equal to:

$$time\ delay = \frac{\partial t}{\partial q}\ \Delta q_{ss} \qquad (4.19)$$

Where Δq_{ss} is the deviation between sensing and steady state flow rate. $\frac{\partial t}{\partial q}$ is also measured as written blow:

$$\frac{\partial t}{\partial q} = \frac{1}{\frac{\partial q}{\partial t}} = \frac{1}{Cv(z)\frac{\frac{\dot{p}_C}{\rho}}{2\sqrt{\frac{p_C - p_0}{\rho}}}} \qquad (4.20)$$

After sensing element and applying time lag the system is ready to go to the iteration, the block diagram has been shown below:



4.3.3 Coupled controller

The final algorithm to control the BHP pressure via choke pressure will be getting the pressure of the choke via choke opening by MPC controller. The MPC works until the difference between the set point and chokes pressure would be less than the number operator will tune as the PID margin. When the choke pressure enters the pressure margin defined, PID controller comes into service and it will get the pressure until it hits the set point. Because of the nonlinearity of the process, the sooner PID comes to service, the higher overshoot will result on the hand if the PID margin is so small, the rise time becomes large and also it may never hit the set point in the presence of uncertainties, Besides, In case of incidents in the well making the pressure drop, its ability to retrieve the pressure would highly decrease. So, a logical combination between MPC and PID through PID margin should be considered by the operator.

4.4 Intervention Operations

4.4.1 Controller Designing Parameters

As mentioned in the introduction, as long as the drill string is connected to the slips during the drilling, the heave effect on the drill string is compensated by the draw works. However, once they're disconnected from the slips the heave effect is strong enough to make a catastrophe such as kick or mud loss, especially for narrow pressure margins reservoirs.

Generally, the sea waves can be simulated by sine function if the frequency of waves and their amplitudes are known, So the wave speed

$$A_1 \sin(2\pi\omega t) \quad (4.21)$$

Where A_1 is the wave speed in $\frac{m}{s}$, and ω is the wave frequency in Hz. The wave amplitude can be obtained by integrating equation 4,21 over half of the period, So:

$$\int_0^{\frac{T}{2}} A_1 \sin(2\pi\omega t)\, dt \quad (4.22)$$

$$Wave\ amplitude(m) = \frac{A_1}{\pi\omega} \quad (4.23)$$

The sea waves will be transferred to the drill string and bit with the same frequency but the different amplitude and phase-shift. The most challenging part in the designing of a controller for heave compensation is figuring out the amplitude and the phase shift of the drill string and bit. In the following part, the algorithm to measure those in the simulator is presented.

The heave induced movement to the drill bit can be formulated as sine functions as below:

$$A_2 \sin(2\pi\omega t + \varphi) \quad (4.24)$$

As during the connections period, the cables and the sensors of the drill string are disconnected, new technologies like drill pipe telemetry cannot be used as well, So the only way is to focus on the choke manifold and the fluid behavior to sense the bit movement.

It's obvious that once the drill string moves back and forth, the fluid volume equals the volume change of the drill string moves. If a control volume is assumed from the Free-Hole (Chapter 3) to the choke opening, the difference between the steady and unsteady state of the fluid will give the fluid volume affected by the movements. So, by applying mass conservation for the whole section:

$$m_{in} = m_{out} \quad (4.25)$$

$$A\, v_b\, \rho_{BHP} = \left(q_c - q_{bpp}\right) \rho_{choke} \quad (4.26)$$

As the sensors are not in the service and due to the high bulk modulus of the mud (water-based), surface density can be used for ρ_{choke} and the corresponding hydrostatic pressure density can be applied for ρ_{BHP}, by having the average area of the drill string (or bit) , the velocity of the bit will be:

$$v_b = \frac{(q_c - q_{bpp})\, \rho_{choke}}{A\, \rho_{BHP}} \quad (4.27)$$

For measuring of the phase shift, the propagation of the flow should be calculated. This factor is equal to the sound velocity which is measured as written underneath:

$$v = \sqrt{\frac{\beta}{\rho}} \quad (4.28)$$

Where β is the Bulk modules which has been considered constant for sake of simpler calculations. This assumption is true for the incompressible-fluid based mud like water-based mud.

By recording the data for both bit velocity and the heave velocity over one period of the disturbance the phase shift can be found quite straight-forward. For the fast sensing of the motion and decreasing the unrealistic phase shift, some modifications should be considered during the measurement:

- The choke opening should be 100 % opened in order to eliminate the resistance to the flow rate.
- There should be a minimum Back-Pressure pump rate in the choke line to speed up the flow rate change

Some uncertainties should also be noted during the phase shift estimation, including:

- **Friction Effect:** Once the heave is transferred to the drill string, due to the movement of the drill pipes and it's elasticity nature, as shown in the hydraulic model, the frictional shear leads to the movement of the neighboring fluid which results in a velocity profile shown in figure 3.4. However, this effect is not strong enough to have a huge impact on the recording data despite depicting opposite bit speed in the first period of the measurement.
- $\boldsymbol{\frac{\partial q}{\partial t}}$: Flow change rate will have an impact in recording the phase shift, in practical situation, where there still be huge resistance at the choke valve, flow rate change occurs much slower, so at the final consideration, this factor also needs to be considered.

By taking into the account of the descriptions above, the time that it takes for the disturbance propagated into the system and $\frac{\partial t}{\partial q} \times \Delta q_{ss}$ should be subtracted from the resulting recorded graph.

The calculation of the exact phase shift might be challenging, Beside the frequency, there are many side parameters affecting this factor such as Geological Rheology, the length of the drill string and the components which were used in the drill string and etc. so due to the high uncertainties, it might sound a bit impossible to evolve a comprehensive algorithm being able to find the phase shift. Then most of the problems might be solved on a case study basis.

4.4.2 MPC design

Due to the obstacles mentioned in the last section, to some degree, the designing of the controller for the heave disturbance might be based on a case study, depending on the well friction, geological rheology, the type of drilling components and etc. Additionally, the hydraulic dynamics have been used to predict the reaction of the choke opening or back pressure pump has some assumptions that can increase the uncertainties. The method has been used here is a simplified form of the hydraulic model presented in chapter 3. The model is based on the Landet investigation on the pressure propagation at the annulus. It's been understood, due to the Laminar flow regime at the annulus, the impact of the drill string movement is much higher than the effect of the shear force on the drilling mud, So for the sake of simple calculation, the free-hole and drill string section is ignored and the analysis will be based on the annulus section hydraulic diameter. By discretizing the equations 3.44 - 3.49 to five control volumes and replacing fanning friction factor instead, the resulting equation will be as follows:

$$\dot{p}_1 = \frac{\beta_1}{A_1 l_1}(-q_1 - v_d A_d) \quad (4.29)$$

$$\dot{p}_2 = \frac{\beta_2}{A_2 l_2}(q_1 - q_2) \quad (4.30)$$

$$\dot{p}_3 = \frac{\beta_3}{A_3 l_3}(q_2 - q_3) \quad (4.31)$$

$$\dot{p}_4 = \frac{\beta_4}{A_4 l_4}(q_3 - q_4) \quad (4.32)$$

$$\dot{p}_5 = \frac{\beta_5}{A_5 l_5}\left(q_4 - q_c + q_{bbp}\right) \quad (4.33)$$

$$\dot{q}_i = \frac{A_i}{\rho_i l_i}(p_i - p_{i+1}) - \frac{F_i(\, q_i) A_i}{l_i \rho_i} - A_i g \frac{\Delta h_i}{l_i} \quad (4.34)$$

$$q_c = K_c\sqrt{p_c - p_0}G(u) \quad (4.35)$$

As mentioned before, by considering five control volumes shown from 1 to 5, there are also 4 momentum equations ($i = 1 \ldots ,4$). The first control volume ($i = 1$) in the pressure equations refers to the lowest control volume ($p_1 = p_{bit}$) and the last one ($i = 5$) shows the topset one. ($p_5 = p_c$). l_i is the length and Δh_i is the height of each control volume respectively. At the vertical sections, these two parameters are equivalent. The control variables are the back-pressure pump flow rate q_{bbp} and the topside choke opening. K is the choke characteristics corresponding to the both density and choke pressure. Finally, $G(u)$ is the factor relating the physical choke opening and the input signals which is between [0,1].

Since the back-pressure pump rate relates linearly to the pump frequency, it's change can't be fast enough to compensate for the heave-induced pressure. However, as it was seen in previous parts of the current chapter, it can stabilize the pressure quite fast and make it robust by including it in the control algorithm, especially in the case the main mud pump lost. So, the main parameter manipulating the BHP is choke opening.

As it was mention at the assumptions, it's safe to consider the fluid flow at the annulus laminar. The experimental results out of the Ullirg will validate this assumption [18]. The friction factor at each control volume i_{th} is calculated by the following equation:

$$F_i(\, q_i) = \frac{K_{fric}q_i}{A_i} \quad (4.36)$$

Where the friction coefficient (K_{fric}) is constant (Or slowly varying)

The hydraulic model described in equations 4.29 - 4.35 is the nonlinear strict feed-back type where the stochastic disturbance accomplishing. Assuming:

$$a_j = \frac{\beta_j}{A_j l_j} \ , \quad b_j = \frac{A_j}{l_j \rho_j} \ , c_j = \frac{K_{friction}}{\rho_j l_j}$$

And the State-Space model is:

$$\begin{cases} \dot{X} = AX + Bu_a + B_1 + Ed \\ y = CX \end{cases} \quad \text{(4.37)}$$

The parameters above are Matrixes and the goal is to calculate the u_a which is the back-pressure pump and choke rate difference:

$$u_a = q_{bbp} - q_{choke} \quad \text{(4.38)}$$

The Matrixes are:

$$\dot{X} = [p_1 \quad q_1 \quad p_2 \quad q_2 \quad p_3 \quad q_3 \quad p_4 \quad q_4 \quad p_5]^T \quad \text{(4.39)}$$

$$A = \begin{bmatrix} 0 & -a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_1 & -c_1 & -b_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & -a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_2 & -c_2 & -b_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & -a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_3 & -c_3 & -b_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_4 & 0 & -a_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_4 & -c_4 & -a_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_5 & 0 \end{bmatrix} \quad \text{(4.40)}$$

$$B = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad a]^T \quad \text{(4.41)}$$

$$B_1 = -Ag\frac{\Delta h}{l}[0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0] \quad \text{(4.42)}$$

$$E = [-a_1 A_d \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad \text{(4.43)}$$

$$C = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad \text{(4.44)}$$

The disturbance was donated by d in the state-space model. As the sine wave was considered for the heave motion, the induced wave is also a sine wave

The generalized algorithm by Constrained MPC will be derived in the next section.

4.4.3 Constrained MPC design

Evolution of equation 4.37 into the discrete-linear form of time-invariant system for each iteration will lead to:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + \ B_1 + Ed(k) \\ y(k) = Cx(k) \end{cases} \quad \ldots\ldots (4.45)$$

At the interval of the following constraint at all time constants ($k \ \geq 0$)

$$y_{min} \leq y(k) \leq y_{max} \ldots\ldots (4.46)$$

$$u_{min} \leq u(k) \leq u_{max} \ldots\ldots (4.47)$$

By applying a cost function to optimize at each time interval:

$$\min \ \{J = \ \sum_{i=1}^{N} [(u(k+i|k)^T \ Ru(k+i|k) + (y(k+i|k) - r(k+i|k) \ ^T Q(y(k+i|k) - r(k+i|k)]\} \ldots\ldots (4.48)$$

$$y_{min} \leq y_{i+k|k} \leq y_{max} \qquad i+1, \ldots, N$$

$$u_{min} \leq u_{i+k|k} \leq u_{max} \qquad i = i, \ldots, N$$

$$x_{k|k} = x(k)$$

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + \ B_1 + Ed(k) \\ y(k) = Cx(k) \end{cases} \quad \ldots\ldots (4.49)$$

The controller above should be able to give the optimum value for U by solving the quadratic programming (QP) where J, N and r are the cost function, finite horizon and reference trajectory respectively. It’s been assumed that R and Q are the positive matrixes. The velocity of the bit has been known in this study, however, if for any reason the future prediction of disturbance is not known then the disturbances are assumed to be zero.

5 Results and Discussions

The Implementation of the controller to CM was successfully done and the results are as they were expected and described in the last section.

Table 5-1 The physical properties of the well

Control Variable	Notation, Unit
Main Pump Rate	, 1001 [l/min]
Steady-State Pressure	10 [bar]
Step Function	30 [bar]

At this chapter, the simulator is subjected to a step function of the pressure as the input function to evaluate the controller performance. Supposing steady state pressure at 10 bar and the operator or the system is tending to go into 30 bar. The blue, red and green lines in the graphs are the current choke pressure, the set-point pressure and the Non-hydrostatic pressure.

Non-Hydrostatic pressure is the summation of both acceleration and the frictional bottom hole pressure. Its importance bolds up when the main mud pump is running and it can be seen that how much frictional pressure can affect the bottom hole pressure. It means that the BHP is too different for two wells with the same choke pressure in which one is having the main pump running and the other is not.

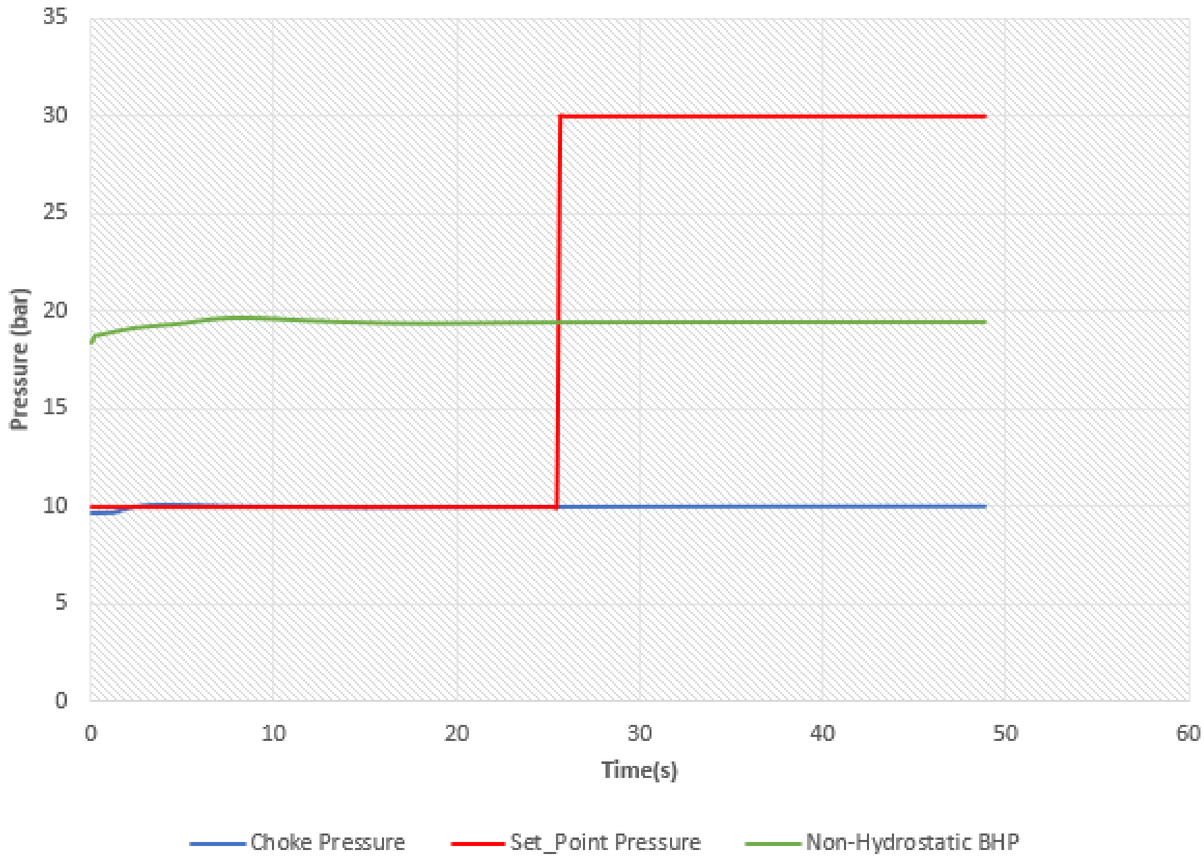


Figure 5-1 Input step function on the manual mode

Figure 5-1 shows choke set point and non-Hydrostatic pressure with blue, red and green respectively. The simulator works on the manual mode, so the pressure should be controlled by the operator and the simulator itself doesn't change it automatically. The most important highlight about the figure above, as described before is the effect of frictional pressure on the BHP making around 10 bar pressure difference.

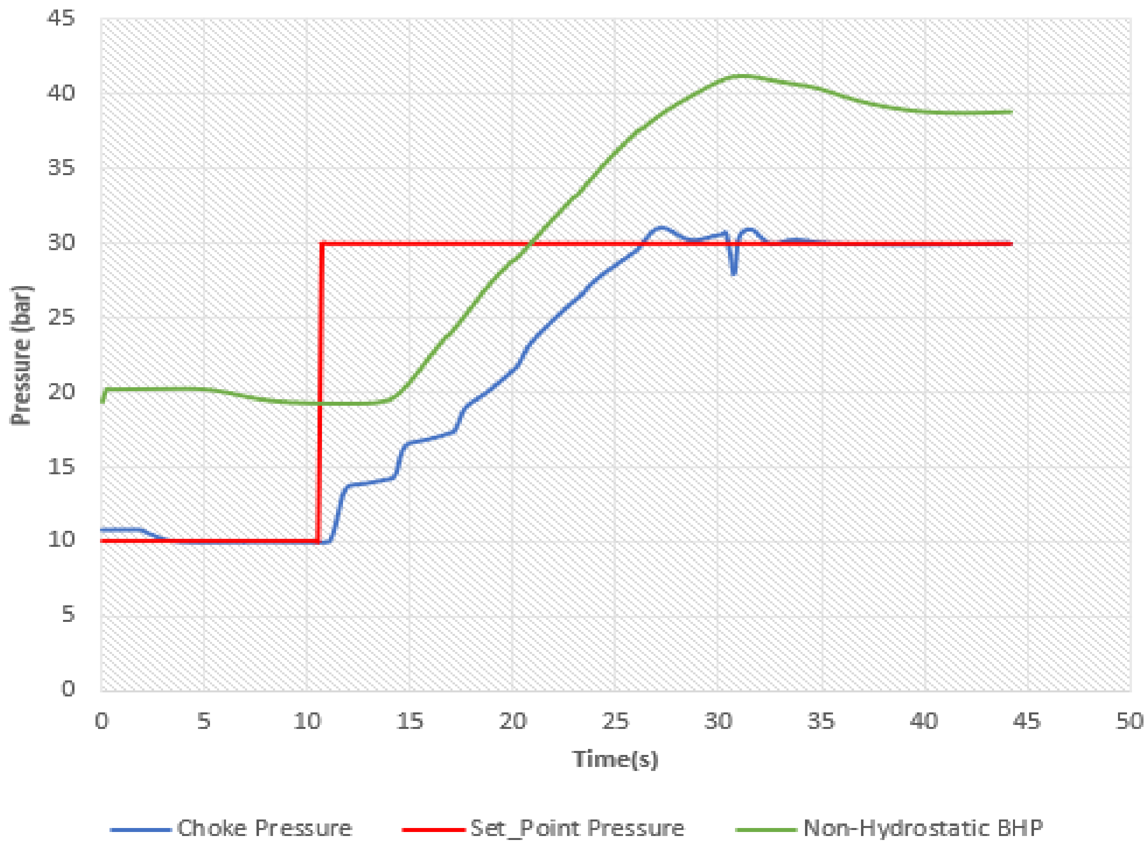


Figure 5-2 Step response of 30 bar on the automated mode

The figure above shows the step response of the simulator when it's in the automatic mode. It can be seen that pressure is building up step by step. As it was described, MPC manipulates the pressure build-up till 1 bar pressure difference of choke and the set point where PID comes into service and the PID overshoot is almost 1.4 % close to the percentage predicted in the last chapter by MATLAB. The raise time can be decreased by faster sampling rate; however, this leads to higher overshoot since the flow rate has not backed to its steady state mode making the estimator has more variance.

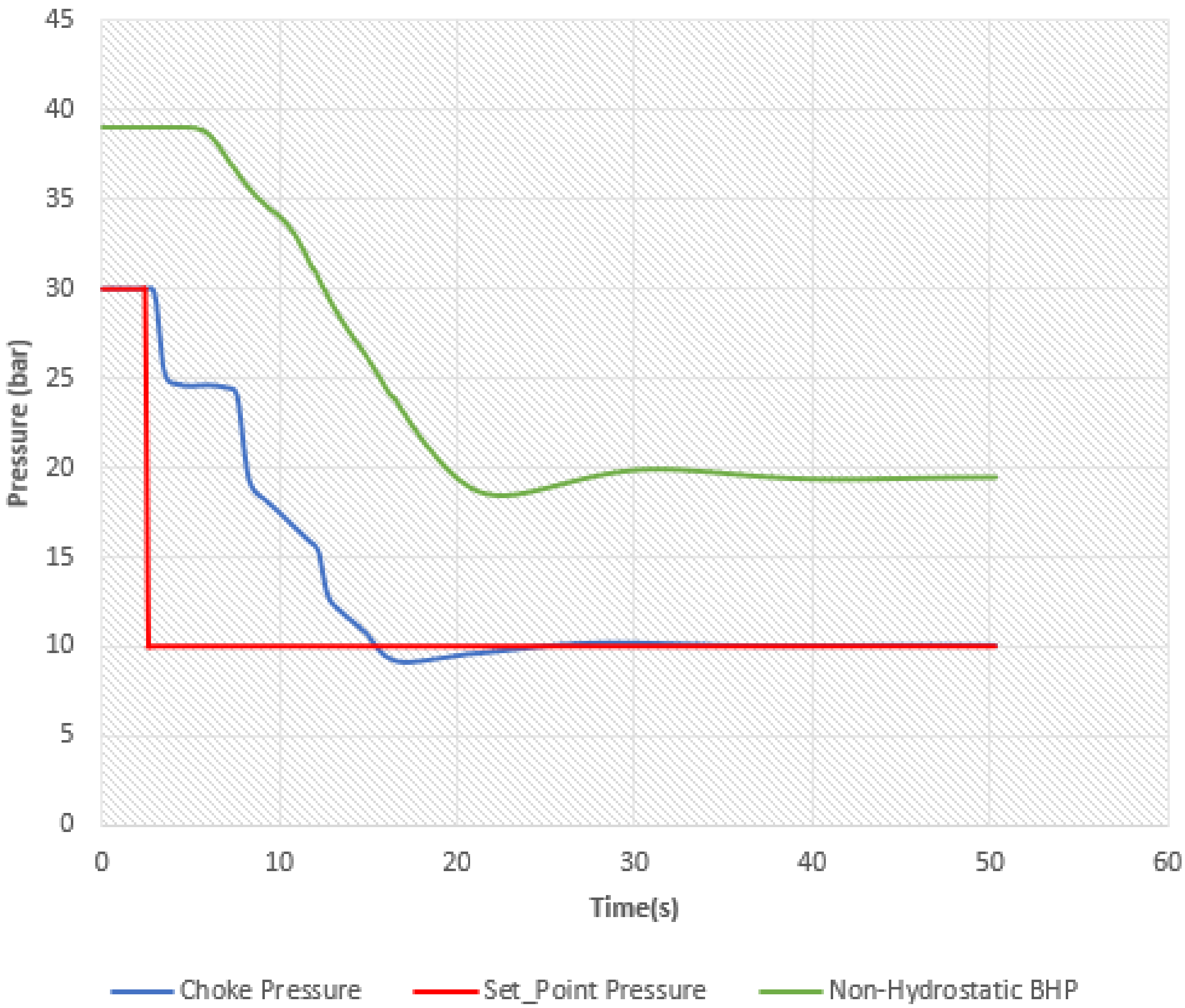


Figure 5-3 Step response of the 10 bar on the automated mode

Figure 5-3 shows the reverse path of what has been shown in figure 5-2. It can be seen that the percentage of undershoot is smaller than overshoot. That's because, in descending pressure, the back-pressure pump is also lowering its rate which makes the time constant of the process shorter getting to the set-point faster. Another aspect worth mentioning is the time lag between the choke and bottom hole which is equal to the time that sounds takes to move along the well in the drilling mud which was calculated in the equation 4.28

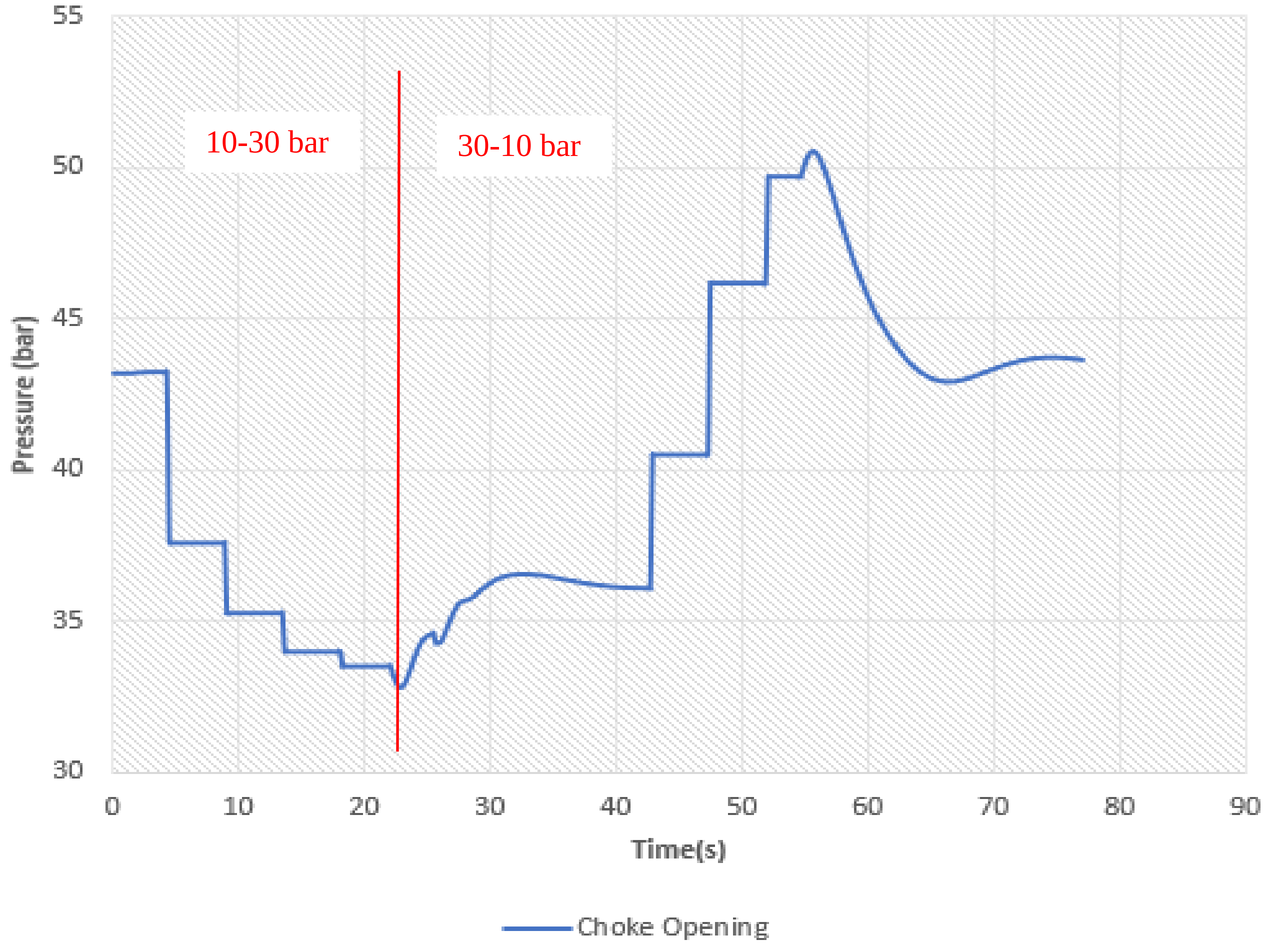


Figure 5-4 Choke opening graph from 10 bar to 30 bar and reverse

The step response for the change in the pressure set point has been visualized in the figure above. First, the pressure is set to be 30 bar starting from 10 (steady-state pressure) and then it brought down to the initial pressure which was 10 bar. As the figure illustrated, the performance of both MPC and PID is distinguishable. For MPC it's been updated for each time resulting in a step function response and then once it goes into the PID margin it shaped close to what was PID optimization had been predicted (Figure number).

The notable fact about the figure is that if the controller design was merely on the choke opening, the response of the system form 10 bar to 30 bar and vice versa would give the same opening. However, since the back pump is also on service and the gains of the back-pressure pump and choke manifold is different, then the hysteresis effect is inevitable as described in the Applied Theory chapter.

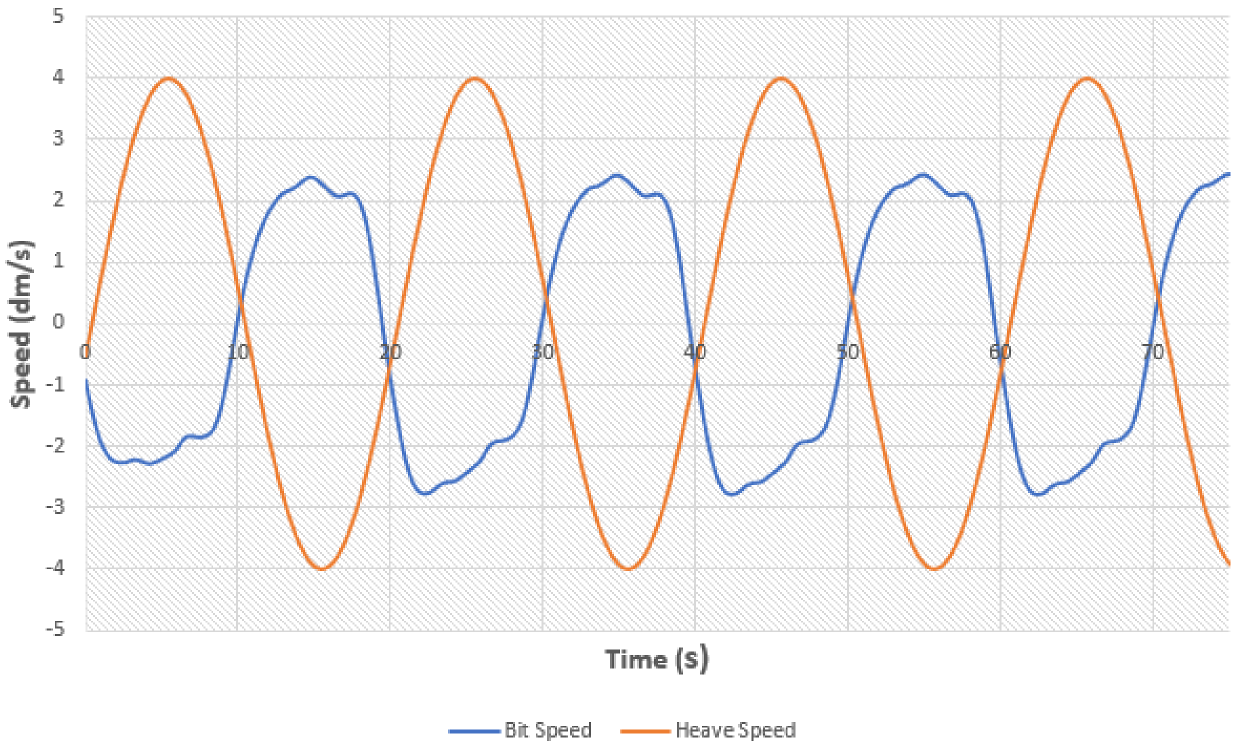


Figure 5-5 Heave speed and the estimated bit speed

As its stated in the methodology, the only approach to sense the bit motion is through the change in the choke flow rate. The phase change and the bit amplitude are crucial factors in designing the controllers. The figure above represents the induced wave and the resulting bit speed. It was assumed that the period is 20 seconds (0.05 HZ) and the heave amplitude is 0.4 $\frac{m}{s}$ (4 $\frac{dm}{s}$). By equation 4-23 the wave amplitude is measured to be 2.55 m. Reminding two import factors of the fluid time lag between bit and choke and the flow rate change that need to be considered in the estimation to have the most accurate result. So, the simulator will give the string time lag of 6.97 seconds which is equal to the 125.48 ° . The bit amplitude also be calculated equals to the 2.43 $\frac{dm}{s}$. So, the bit equation will be (Equation 4-24):

$$0.243\ \sin(0.1\pi t + \ 2.19)$$

As it's been expected, due to the well friction and Young Module of the drill string, the deeper the drill string, the longer the time response and larger phase shift as a result. Also, the amplitude of the bit is also influenced which in most cases is smaller than the heave amplitude. On the other hand, if the well and BHA have large well friction with each other, the frictional force makes the drill string to squeeze too much and being released with huge force which makes the bit speed higher than the heave speed.

6 Conclusion and Future Work

In this work, the earlier efforts on the heave attenuation were presented and it was seen that the Kasse's model failed because the effect of the time lag in the frequency response has not been considered. Also, the importance of frictional based on Rune's work on the BHP was studied. A more comprehensive hydraulic model was suggested and by some simplification and assumptions, the conditions of its application have been described.

After the definition of the hydraulic model, Choke manifold and the back-pressure pump characteristics have been shown and the application, pros and cons of using each of them has been investigated. Based on the properties presented, the operation divided into either normal and intervention operation. For each of those, a specific control algorithm has been designed by using both PID and Model Predictive controller (MPC). I was said that since the string time lag and amplitude are the two most crucial parameters in designing of the MPC for intervention operation, the demand for measuring them is so crucial. As a result, a measurement algorithm based on the flow rate change at the choke line is designed. So, the simulator calculates phase shift and amplitude automatically after one period of the induced wave. Having said, as phase and amplitude of the string depend on many parameters that are unique and out of the human control, it's described that a case study should be done for each length, rheology etc. All the efforts were on defining a well-rounded model where the uncertainties are small enough having not much effect at the result while keeping the calculations as simple as possible.

The focus of this thesis was to keep the choke pressure constant, However It was depicted that the frictional pressure can have a huge effect on BHP which is affectless on topside pressure, and as the interest is to retain BHP constant, the model should be updated through new LTI discretization for the BHP in order to calculate the corresponding choke pressure. The new model should also be able to compensate the change of frictional pressure during the main pump ramping. The final model can be validified by the real well data and an extended Kalman filter will be added as the correction factor.

References

Aziz, K. J. A. S. P. (1979). Petroleum reservoir simulation. *476*.

Breyholtz, O., Nygaard, G., Godhavn, J.-M., & Vefring, E. H. (2009). *Evaluating control designs for co-ordinating pump rates and choke valve during managed pressure drilling operations.* Paper presented at the 2009 IEEE Control Applications,(CCA) & Intelligent Control,(ISIC).

Burkhardt, J. J. J. o. p. t. (1961). Wellbore pressure surges produced by pipe movement. *13*(06), 595-605.

Egeland, O., & Gravdahl, J. T. (2002). *Modeling and simulation for automatic control* (Vol. 76): Marine Cybernetics Trondheim, Norway.

Fontenot, J. E., & Clark, R. J. S. o. P. E. J. (1974). An improved method for calculating swab and surge pressures and circulating pressures in a drilling well. *14*(05), 451-462.

Gjerstad, K., Time, R. W., & Bjørkevoll, K. S. J. J. o. N.-N. F. M. (2012). Simplified explicit flow equations for Bingham plastics in Couette–Poiseuille flow–For dynamic surge and swab modeling. *175*, 55-63.

Godhavn, J.-M. (2009). *Control requirements for high-end automatic MPD operations.* Paper presented at the SPE/IADC Drilling Conference and Exhibition.

Godhavn, J.-M., Pavlov, A., Kaasa, G.-O., & Rolland, N. L. J. I. P. V. (2011). Drilling seeking automatic control solutions. *44*(1), 10842-10850.

Kaasa, G.-O., Stamnes, O., Imsland, L., & Aamo, O. M. (2011). *Intelligent estimation of downhole pressure using simplified hydraulic model.* Paper presented at the IADC/SPE Managed Pressure Drilling and Underbalanced Operations Conference & Exhibition.

Kaasa, G.-O. J. S. R. C., Porsgrunn, Norway. (2007). A simple dynamic model of drilling for control. *66*.

Landet, I. S. (2011). *Modeling and Control for Managed Pressure Drilling from Floaters: Heave Compensation by Automatic Nonlinear Control.* Institutt for teknisk kybernetikk,

Landet, I. S., Mahdianfar, H., Pavlov, A., & Aamo, O. M. (2012). *Modeling for mpd operations with experimental validation.* Paper presented at the IADC/SPE Drilling Conference and Exhibition.

Mahdianfar, H., Aamo, O. M., & Pavlov, A. (2012a). *Attenuation of heave-induced pressure oscillations in offshore drilling systems.* Paper presented at the 2012 American Control Conference (ACC).

Mahdianfar, H., Aamo, O. M., & Pavlov, A. J. I. P. V. (2012b). Suppression of heave-induced pressure fluctuations in mpd. *45*(8), 239-244.

Mäkinen, J., Piche, R., Ellman, A. J. J. o. d. s., measurement,, & control. (2000). Fluid transmission line modeling using a variational method. *122*(1), 153-162.

Obinata, G., & Anderson, B. D. (2012). *Model reduction for control system design*: Springer Science & Business Media.

Pavlov, A., Kaasa, G.-O., & Imsland, L. J. I. P. V. (2010). Experimental disturbance rejection on a full-scale drilling rig. *43*(14), 1338-1343.

Russell, R. B., Hernandez, M., MacNeill, D. W., Reeves, M. E., & Hamel, P. (2008). *'Intelligent'Wired Drill-Pipe System Allows Operators to Take Full Advantage of Latest Downhole Sensor Developments.* Paper presented at the International Petroleum Technology Conference.

Wylie, E. B., & Streeter, V. L. J. N. Y., McGraw-Hill International Book Co., . 401 p. (1978). Fluid transients.

Zhou, K., Doyle, J. C., & Glover, K. (1996). *Robust and optimal control* (Vol. 40): Prentice hall New Jersey.