Does Early Introduction of Algebra in Schools Make any Difference? A Causal-Comparative Study of Algebra Skills of Upper Secondary School Students in Norway and Nepal.

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Does Early Introduction of Algebra in Schools Make any Difference? A Causal-Comparative Study of Algebra Skills of Upper Secondary School Students in Norway and Nepal.

Abstract

The main goal of this causal-comparative study is to investigate if the introduction of algebra in an early stage in schools enhance students' understanding of basic high school algebra. Algebra with variables appears in fourth-grade in Nepal, while in Norway, it is not part of the curriculum before seventh-grade. Findings of this study are based on students' performance on an open-ended algebra test conducted among 813 students from different grades in schools in Norway and Nepal. Results indicate that the Norwegian students achieved significantly lower than their Nepalese counterparts. Norwegian students' achievement is also significantly lower than the tenth-graders in Nepal, while there was no significant difference between Norwegian eleventh-graders and Nepalese ninth-graders. Thus, the results of this study suggest that an early introduction of algebra in Nepal has a significant role in students' achievement. These findings agree with the past studies that suggest weak achievement of Norwegian students in algebra. Based on the findings, the study concludes that it is reasonable to argue for early introduction of algebra in Norwegian schools, but further research is needed to explore its pedagogical implications and how effectively it can be incorporated in current teaching-learning environment in Norway.

Key words: Algebra in Norway, Algebra in Nepal, Early Algebra, Early Introduction of Algebra

Preface

This master's thesis is written in an article-format with a supplementary thesis framing. The quantitative study done for this thesis investigates the relationship between the early introduction of algebra in the school curriculum and students' achievement in school algebra. The article presents the study, findings, and interpretations in general while the thesis framing discusses the methodological approaches adopted, and the considerations made in detail. I intend to send the article to Nordic Studies in Mathematics Education (NOMAD) for publication, and therefore, the article is written in the format suggested by NOMAD. *Author guidelines* of the journal is included in the appendix.

This study was done independently and without financial support from any individual or organization. I collected the data myself, both in Norway and Nepal, with the support from the teachers of the participating schools. It is perhaps worthwhile to mention here that my teaching experience in both environments helped me ease this rather tedious process.

I am gratefully indebted to Prof. Arne Jakobsen for his expert supervision and constant encouragement, without which this study would not have been possible. Thank you, Arne! Wherever you were on the planet (may it be Malawi, South Africa, China or Norway), you were always willing to read my drafts and give your feedback. Your "researcher perspectives" have polished this work!

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Stavanger, June 2019

Ramesh Gautam

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Acronyms

CDC-MoE: Curriculum Development Center, Ministry of Education, Nepal
CDC-Nepal: Curriculum Development Center, Nepal
CEROD: Centre for Educational Research and Social Development
MoE-Nepal: Ministry of Education, Nepal
NEP09: Grade 9, Nepal
NEP10: Grade 10, Nepal
NEP11: Grade 11, Nepal
NEPPUB09: Grade 9, public school, Nepal
NEPPUB10: Grade 10, public school, Nepal
NEPPUB11: Grade 11, public school, Nepal
NEPPVT09: Grade 09, private school, Nepal
NEPPVT10: Grade 10, private school, Nepal
NEPPVT11: Grade 11, private school, Nepal
NOR11: Grade 11, Norway
Udir: The Norwegian Directorate for Education and Training
UNDP: United Nations Development Program
UNESCO-UIS: The United Nations Educational, Scientific and Cultural Organization, Institute for Statistics

Part I :

Thesis Framing

This thesis framing (which is called *overbygning* in Norwegian) provides a brief outline of the study and discusses in detail the relationship between the *research question* and the *methods for construction and analysis of data*.

1 Introduction

1.1 Background

As a mathematics teacher, I have always been intrigued about the learning theories, international comparative studies and other recent research activities in mathematics education. Results of these studies, at times, initiate discussions and expect greater involvements of mathematics teachers to evaluate the curriculum of school mathematics. The interest in conducting a comparative study in algebra emerged more conspicuously when I was taking my Post Graduate Certificate in Education four years ago. The exposure to multicultural and multilinguistic teaching-learning environments (Bhutanese, Nepalese and Norwegian) in addition to myself having multicultural background might have added to the quest in this field.

Several studies conclude that school students in developed countries like the USA, Sweden, and Norway show weak achievement in algebra (Phan, 2008). Exploring the causes of this weakness and interpreting them sensibly have been important for research in school mathematics. Analyses from the Trends in International Mathematics and Science Study (TIMSS) and Programme of International Student Assessment (PISA) surveys show that these countries prioritize daily-life mathematics like statistics than formal mathematics like algebra in contrast to the countries in East-Asia and East-Europe (Grønmo, Bergem, Kjærnsli, Lie & Turmo, 2004; Grønmo et al., 2012). Grønmo et al. (2012) argue that one of the explanations for Norwegian students' weak achievement is that algebra is traditionally introduced relatively late in schools.

Nepal has not participated in extensive international surveys like TIMSS and PISA and thus, the comparative reports regarding students' achievement in mathematics (and algebra) are not available. However, as mentioned in the Primary Education Curriculum prepared by the Curriculum Development Centre (hereafter CDC-Nepal) under the Ministry of Education of the Government of Nepal, the introduction of algebra in school mathematics occurs as early as in 4th grade (see Table 1). So as the students complete the secondary school and start at the upper secondary level, it can be assumed that they are better prepared to encounter bigger challenges in mathematics than their Norwegian counterparts who are only introduced to algebra with variables in 7th grade (see Table 1).

1.2 Research Goal

This study investigates if the introduction of algebra in early school years helps the students enhance the understanding of basic school algebra in the later grades. Grønmo et al. (2012) argue that the countries that achieve similar or below the

Norwegian average in 8th-grade mathematics in PISA 2011 are typically developing countries with completely different resource situation than Norway.

Nepalese education system has been encountering several difficulties like language problems, issues of multiculturalism, availability of technology, lack of technical skills and lack of teacher training and professional development programs (Panthi & Belbase, 2017). Despite these adversities, have Nepal achieved anything significant through prioritizing mathematics in schools and introducing algebra in the early stage of children's schooling? If so, what are those implications of those achievements and how could those be assessed? More specifically, the following research question will be answered:

Does the introduction of algebra in the early stage in schools as practiced in Nepal enhance the students' understanding of basic high school algebra compared to when it is introduced relatively late as in Norway?

To answer this question, a brief review of literature of past studies shall be done. This will be followed by the description of the procedure for the data collection, organisation and analysis. And finally, the results will be presented and interpreted.

1.3 Outline of the Thesis Framing

This thesis framing is a part of an article-based master thesis. The article presents a comprehensive literature review, procedure of the study, results and interpretation. This framing supplements the article with background of the study, methodological choices and assumptions made and interpretation of the results that are relevant but could not be included in the article due to limitation of format and structure of the article. As a requirement for this thesis framing, *The Handbook for Master Thesis* prepared by the Faculty recommends to opt for one of the following topics for in-depth discussion:

- Relation between research question and the theoretical perspectives of the study
- Relation between research question and methods for construction and analyses of data
- Relation between research question and the results of the study
- Presentation and discussion of analyses and results that forms the background for the results presented in the article but not possible to include in the article due to limitations.

The quantitative data for this study are collected in two different countries with completely different education system, teaching-learning environment and resource situation. There are methodological considerations that need wider discussions and interpretation. Owing to the word constraints and other limitations, these discussions can not be included in the article. Therefore, I have opted to discuss the relationship between the *research question* and the *methods for construction and analysis of data* in detail in this thesis framing.

2 Literature Review

A brief account of historical development of algebra (sub-section 2.1) and its entry into school mathematics (sub-section 2.2) will be presented here. The contexts of school algebra will be discussed based on the Norwegian (sub-section 2.2.1) and the Nepalese (sub-section 2.2.2) curricula and a short discussion about early algebra and early introduction of algebra in schools (sub-section 2.3) will be presented. More elaborated literature review is presented in the article.

2.1 Algebra: Development and Entry into School Mathematics

Algebra is one of the major topics of the school mathematics curriculum around the world though variations exist both in the content and the time it is introduced in the schools (Leung, Park, Holton & Clarke, 2014; Kanbir, Clements & Ellerton, 2018). Espeland (2017) notes that algebra might be a calculation with letters instead of numbers for many high school students. Understanding these letters, that we call variables today, and their operations form basics of school algebra. Usiskin (1995) conseders that students start learning algebra when they first meet these variables. Carraher, Schliemann & Brizuela (2000) opine that, for many students, algebra is merely memorizing the arbitrary rules and learning to manipulate x's and y's without understanding the fundamental rules in algebra.

Historically, the conceptual basis for algebra existed as early as the period of Babylonian (c. 1700 – 689 BCE) and Greek (c. 800 – 146 BCE) mathematics (Radford, 1996; Katz, 2008; Boyer & Merzbach, 2011). In fact, the Babylonians had great accomplishments in algebra but were hindered by their lack of algebraic symbols and notation (Evans, 2014). According to Boyer & Merzbach (2011), algebra, as it is understood today, got introduced to Europe after a book named *Hisob al-jabr wa'l muqabalah*, or *The Book of Calculation by Completion and Balancing*, written by Arabic scholar al-Khwarizmi (c. 780 – 850 CE). In the 12th century, it was translated into Latin and called *Liber Algebrae et Almucabola* (Evans, 2014). Girolamo Cardano's (1501 – 1576) *Ars Magna*, or *The Great Art* (1545) is considered as the first algebraic work in Europe to advance beyond Islamic algebra (Evans, 2014).

The first secondary school at which algebra was part of the mathematics curriculum was the Royal Mathematical School within Christ's Hospital, in central London, England (Ellerton & Clements, 2017). Established in 1673, the school had "the specific mission of preparing boys aged between 12 and 16 to become mathematically-competent apprentices in the Royal Navy or the merchant marine" (Kanbir et al., 2018, p. 18).

Mathematicians and mathematics educators have never agreed unanimously upon what school algebra should be like and the past three centuries have seen the controversy over what school algebra should embrace (Kanbir et al., 2018). Moreover, the developments in teaching algebra in schools is not well documented. da Ponte & Guimarães (2014) claim that "the history of the teaching of algebra is largely unwritten" (p. 459). Kanbir et al. (2018) consider that a comprehensive history of school algebra focussing on the global perspectives is not yet published.

2.2 School Algebra

When it comes to school mathematics, Katz & Barton (2007) mention that a typical secondary school algebra incorporates a wide variety of topics. Some of these topics include arithmetic of signed numbers, solutions of linear equations, quadratic equations, and systems of linear and/or quadratic equations, and the manipulation of polynomials, including factoring and rules of exponents to name a few. As such, modern algebra is much more than what was understood in the eighteenth century.

2.2.1 The Norwegian Context: Algebra in School Curriculum

The present curriculum (K06), which is termed as "The Knowledge Promotion" (Norwegian: kunnskapsløftet), was introduced at all levels in Norwegian schools in 2006 (Udir, 2018). The curriculum encompasses 10-year compulsory school and a voluntary upper secondary education and training (Espeland, 2017). Students can opt between *Specialization in General Studies* and *Vocational Education Program* in Upper Secondary Schools. The new curriculum, K06, mentions specific competence aims to be achieved at grade 2, grade 4, grade 7, grade 10 and for each year in the upper secondary school. Algebra appears first in 7th grade. There are two mathematics courses in the first year of the upper-secondary school: 1T-Mathematics and 1P-Mathematics. The 1T course is more rigorous and theoretically oriented and forms a basis for specialization within physical sciences and engineering in higher studies.

Following are the competence goals set for 1T-Mathematics:

- Calculate with powers with rational exponents and numbers in scientific notation, algebraic expressions, formulas, expressions with brackets and alphanumerical rational and square expressions, and use quadratic equations to factor algebraic expressions

- Solve equations, inequalities, and systems of equations of the first and second order and simple equations with exponential and logarithmic functions, using algebra and digital aids

- Convert a practical problem into an equation, an inequality or an equation system, solve it and assess the validity of the solution

(Udir, 2006)

2.2.2 The Nepalese Context: Algebra in School Curriculum

In Nepal, the school level curriculum is governed by the Curriculum Development Center (CDC), a government organ under the Ministry of Education, Science and Technology (MOE-Nepal, 2019). The proposed School Sector Development Plan (2016/17 - 2022/23) targets higher achievements in subjects like English, Mathematics and Science (MOE-Nepal, 2016). The seven-year target for grade 5 Mathematics is to reach 60% in 2022/23 from baseline 48% in 2015/16.

Fourth-grade students in Nepal are expected "to solve simple problems of algebraic expressions and equations using algebra skills" (CDC-Nepal, 2009). Thus basic algebra appears as early as in fourth-grade in Nepal. The students have to take a compulsory mathematics course until tenth-grade. Students in ninth- and tenth-grade have an opportunity to opt additional mathematics. In the eleventh grade, students opt for different specialization courses. Based on the specialization program they are enrolled in, the students can opt mathematics for physical sciences, mathematics for economics and mathematics for social sciences. According to CDC-Nepal (2017a), following are the competence goals after eleventh-grade for the students specializing in physical sciences:

- Define functions and illustrate them graphically: inverse function, composite function, functions of special type (Identity, constant, absolute value, greatest integer), Algebraic (linear, quadratic and cubic), Trigonometric, exponential logarithmic functions
- Sketch the curves: periodicity of a function, symmetry (about x-axis, y-axis and origin) of elementary functions, monotonicity of a function, Sketching graphs of polynomial trigonometric, exponential, logarithmic functions
- Define polynomial equations, establish fundamental theorem of algebra and quadratic equation, and find relation between roots and coefficients of a quadratic polynomials

Table 1 summarizes the competence goals in school algebra at different levels in Nepal and Norway.

<table>
<tr><th>After class</th><th>Nepal</th><th>Norway</th></tr>
<tr><td>1, 2, 3</td><td>No algebra</td><td rowspan="3">No algebra</td></tr>
<tr><td>4</td><td>• Solve simple equations of addition, subtraction, multiplication and division with box notation solved through inspection method, hit and trail (using variables too)
• Add and subtract like terms (without using negative terms)</td></tr>
<tr><td>6</td><td>• Solve problems related with the values, addition and subtraction of algebraic expressions
• Multiply and divide algebraic expressions (monomials, binomials by monomials)
• Solve linear equations and problems related to them
• Deal with the laws of inequalities</td></tr>
<tr><td>7</td><td>• Classify polynomials
• Multiply and divide trinomials by binomials
• Interpret $(a \pm b)^2$ geometrically and apply</td><td>• Set up and solve simple equations
• Solve and calculate with parentheses in addition, subtraction and multiplication of numbers</td></tr>
<tr><td>10</td><td>• Find LCM and HCF of algebraic expressions by the methods of factorization (at most up to trinomials)
• Solve radical surd using four fundamental operations
• Simplify the indices and solve equations of indices
• Solve problems involving algebraic fractions
• Solve word problems of linear simultaneous with two unknowns and quadratic simultaneous equations</td><td>• Process, factorise and simplify algebraic expressions
• Associate expressions with practical situations, calculate with formulas, parentheses and fractional expressions and use quadratic expressions
• Solve equations and inequalities of first degree and equation system with two unknowns and use this to solve practical and theoretical problems</td></tr>
<tr><td>11</td><td>• Define functions and illustrate them graphically: inverse function, composite function, functions of special type (Identity, constant, absolute value, greatest integer), Algebraic (linear, quadratic and cubic), Trigonometric, exponential logarithmic functions
• Sketch the curves: periodicity of a function, symmetry (about x-axis, y-axis and origin) of elementary functions, monotonicity of a function, Sketching graphs of polynomial trigonometric, exponential, logarithmic functions
• Define polynomial equations, establish fundamental theorem of algebra and quadratic equation, and find relation between roots and coefficients of a quadratic polynomials</td><td>• Calculate with powers with rational exponents and numbers in scientific notation, algebraic expressions, formulas, expressions with parentheses and alphanumerical rational and square expressions, and use quadratic equations to factor algebraic expressions
• Solve equations, inequalities, and systems of equations of the first and second order and simple equations with exponential and logarithmic functions, using algebra and digital aids
• Convert a practical problem into an equation, an inequality or an equation system, solve it and assess the validity of the solution</td></tr>
<tr><td></td><td colspan="2">Nepal: Class 10 - Optional Mathematics</td></tr>
<tr><td>Optional Mathematics – Class 10 - Nepal</td><td colspan="2">Functions
• Solve slgebraic and trigonometric functions (with graphs) : $y = mx; y = ax^3, a \neq 0; y = sinA; y = cosA; y = tanA, (-2\pi \leq A \leq 2\pi)$
• Solve composite functions, inverse functions and use arrow diagrams
Polynomials
• Use short division method, remainder and factor theorems and their applications (to solve equations up to 3rd degrees)
Sequence and Series
• Define arithmetic sequence and series and find the sum of first n natural numbers, both odd and even
• Define geometric sequence and series and find the sum of finite geometric series
Linear Programming
• Solve linear inequalities (find inequality from graph too)
• Find maxima and minima
Quadratic Equations and Graph
• Graph quadratic and cubic functions and to solve quadratic equations graphically
• Solve simultaneous linear and quadratic equations (graphically and by substitution)</td></tr>
</table>

Table 1: Comparison of competence goals in algebra at the end of different classes in Nepal and Norway (Adopted from Udir (2006); CDC-Nepal (2009, 2012, 2014, 2017b,a)).

2.3 Early Algebra and Early Introduction of Algebra in Schools

Research studies in school algebra have revealed many drawbacks coming from the arithmetic way of thinking among the students of 12 - 15 years when they first meet algebra in high schools (Kieran, Pang, Schifter & Ng, 2016, p. 3). In order to overcome these drawbacks, some researchers proposed proposed to introduce what they termed as *Early Algebra*. The focus of Early Algebra is on the 6- to 12-year olds in contrast to the traditional teaching of algebra that starts when the children are 12-year old (Kieran et al., 2016). The main areas of focus in Early Algebra until the early 2000s included:

1. Generalizing related to patterning activity,
2. Generalizing related to properties of operations and numerical structure,
3. Representing relationships among quantities, and
4. Introducing alphanumeric notation

(Kieran et al., 2016, p. 5)

One of the famous tasks in Early Algebra is a box model that is built on the earlier work of Davis (1964). For example: What is the value of Δ in $18 + 27 = \Delta + 29$?. Carpenter et al. (2003) argue that the questions like this are very effective to reflect on the important properties of the operations. This may also be attributed to the fact that the children learn algebra better if they have a sound knowledge of arithmetic. Mathematics educators have long believed that arithmetic should precede algebra as it provides the foundations for algebra (Warren & Cooper, 2005).

Despite the approaches aimed to make algebra learning easy, many students find algebra difficult. These difficulties may be due to "developmental constraints and the inherent abstractness of algebra, concluding that even adolescents were not ready to learn algebra" (Carraher et al., 2000, p. 137). Further, Filloy & Rojano (1989) claim that students are engaging in algebra only if they can understand and use the syntax of algebra and solve equations with variables on both sides of the equals sign. Bodanskii (1991) observed that the fourth graders who are taught the algebraic notation and equations from grade 1 could solve the algebra problems and equations better than the seventh graders who received five years of arithmetic instruction starting algebra in grade six only.

3 Method

3.1 Research Design

In this section, a detailed discussion will be made regarding research design, sampling method, procedure for data collection, data analysis and assumptions for the use of ANOVA, ethical considerations and limitations of the study.

This research used a quantitative causal-comparative design. The goal of this research is to investigate and compare two naturally occurred phenomena; one situation is the effect of the early introduction of algebra in schools (in Nepal), and another is the effect of the late introduction of algebra in schools (in Norway). If the researcher can not manipulate particular independent variables, the causal-comparative research should be used (Salkind, 2010). In this case, the mathematics curricula adopted by two the countries (Norway and Nepal) are the independent variables that cannot be manipulated, while the algebra skills that the students acquire after studying the course based on the curriculum of the respective countries is the dependent variable.

3.2 Pilot Study

In order to find if the test was appropriate for the groups of the students the test was aimed for, a small pilot study was first carried with the 1T-mathematics students ($N = 18$) in a different school in Norway. The main aim of the pilot study was to examine if the stipulated time was enough for the test, the percentage of the students capable of answering all the questions, and the degree of hardness or the softness of the tasks. According to Van Teijlingen and Hundley (2001),

> One of the advantages of a pilot study is that it might give advance warning about where the main research project could fail, where research protocols may not be followed, or whether proposed methods or instruments are inappropriate or too complicated. (p. 1)

The answer-sheets were collected, evaluated and coded using the same scoring guide that would be used later during the data analysis. The participants of the pilot study were expected to complete the test within 45 minutes, but they were informed that they could get more time if needed. At least 3 participants used about an hour. The time for the final test was thus set to 60 minutes. The number of participants who responded to question number 4c ($N = 12$) and 4d ($N = 11$) was lower than for other questions. None of these respondents had solved the problems correctly; only 3 of them had given partially correct answers. Thus, the contribution to the overall average percentage score by these questions was just 4% each. Owing to the low response rate and average percentage score from these questions, question number 4d was eliminated from the final test while question number 4c was kept.

3.3 Sample

All the participants for the project are selected as per a convenience sampling. Due to limitation in the project resources, it was not possible to achieve a full probabilistic sampling. A total of 111 students participated in this research project in Norway. These participants are enrolled in the first grade of a public secondary school and have opted theoretical mathematics course (1T). The mathematics students school are organized in different blocks consisting of four different groups in this school. A block consisting of a maximum number of students was selected for the project. A fifth group was included to acquire a targeted sample size ($N > 100$).

According to National Education Accounts Report prepared by UNESCO Institute for Statistics (UNESCO-UIS), about 30 % of Nepalese students attend private schools (UNESCO-UIS, 2016). The corresponding percentage for the Norwegian students attending upper secondary schools is about 22 % (Statistics-Norway, 2018). Therefore, both private and public schools in Nepal were included in sampling and the same number of students ($N = 111$ each) enrolled in the first grade of the public and the private upper secondary schools participated. Like their Norwegian counterparts, these students study a rigorous course in mathematics assigned for the students opting science path in the upper secondary education. The students of the public school that participated in this project are further grouped as "General Science Students" and "Engineering Science Students" while those of the private school followed a "General Science Course." No any other demarcation was made apart from matching participants' class and study path (1T that forms a foundation to higher-level mathematics for students opting science path in Norway and mathematics for science stream in Nepal). The science stream students in Nepal are selected so that their mathematics standard would be comparable with the 1T-mathematics students in Norway. Apart from these two groups of students from Nepal, the same test was also run with the students of class nine (public: $N = 106$ and private: $N = 130$) and ten (public: $N = 131$ and private: $N = 113$) of both the private and the public schools. The sampling, both in Norway and in Nepal, is a convenience sampling as more rigorous probability sampling could be very difficult to achieve within the targeted school environments.

The students were informed about the project and were invited to participate voluntarily. All the interested students in the selected groups got the opportunity to participate. The participants were contacted in their regular teaching sessions by their mathematics teachers who were informed about the project. A total of 5 students in Norway who were present during the mathematics period on the day the students took the test opted not to participate. They were provided with an alternative assignment by their mathematics teachers. All the students who were present on the test day in Nepal participated.

3.4 Data Collection

3.4.1 Content of the Test

The data collection was done through an open-ended algebra test that consisted of 17 tasks. 7 of the tasks were adopted from Kunnskap, Utdanning og Læring - Knowledge, Education and Learning (KUL) project organized by University of Agder and financed by the Norwegian Research Council (Espeland, 2017). Other 8 tasks were taken from the past exams prepared by Norwegian Education Directorate (Udir) for the students who have opted theoretical mathematics (1T) in upper secondary school in Norway. The tasks prepared by KUL-project focus especially on basic algebra knowledge such as numbers and letters, text usage and equalities. The tasks adapted from Udir cover the following objectives in ”numbers and algebra” in the curriculum in mathematics subjects (MAT-04):

- Calculate with powers with rational exponents and numbers in scientific notation, algebraic expressions, formulas, expressions with brackets and alphanumerical rational and square expressions, and use quadratic equations to factor algebraic expressions
- Forming expressions and solving equations of first and second order and simple equations with exponential functions
- Converting a practical problem into an equation and solve the math problem without digital tools

3.4.2 Procedure

Data collection was done through a written algebra test that contained open-ended tasks. The test was done without any aid (calculator, computer, etc.) but the students were allowed to use the rough papers. The students were asked to show the necessary steps and procedures they used to solve the problems. Apart from this, the data collection also included the students' age (see Figure 1), sex (see Figure 2) and their score in mathematics in tenth-grade (see Figure 3) before they started at the upper secondary school. Though the variables *age* and *sex* are not used for further analysis, they give important information about how representative the sample was with respect to the age and the sex of the participants.

Figure 1: Distribution of age of the students in different groups. NEPPUB09, NEPPUB10 and NEPPUB11 = 9th, 10th and 11th-graders in public school in Nepal. NEPPVT09, NEPPVT10 and NEPPVT11 = 9th, 10th and 11th-graders in private school in Nepal and NOR11 = 11th-graders in Norway.

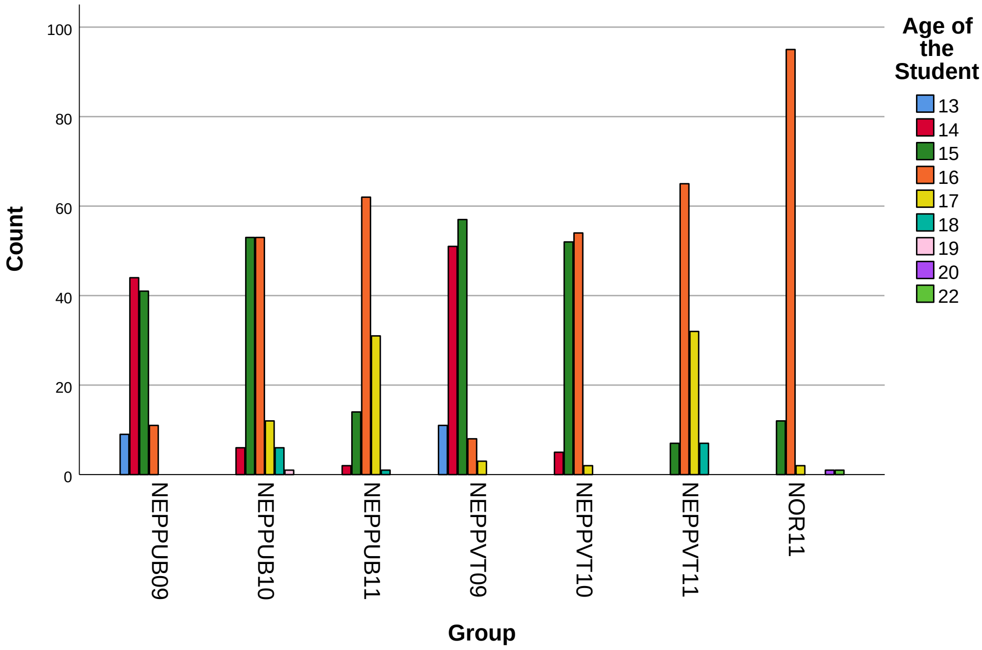


Figure 2: Distribution of sex of the students in different groups. NEPPUB09, NEPPUB10 and NEPPUB11 = 9th, 10th and 11th-graders in public school in Nepal. NEPPVT09, NEPPVT10 and NEPPVT11 = 9th, 10th and 11th-graders in private school in Nepal NOR11 = 11th-graders in Norway.

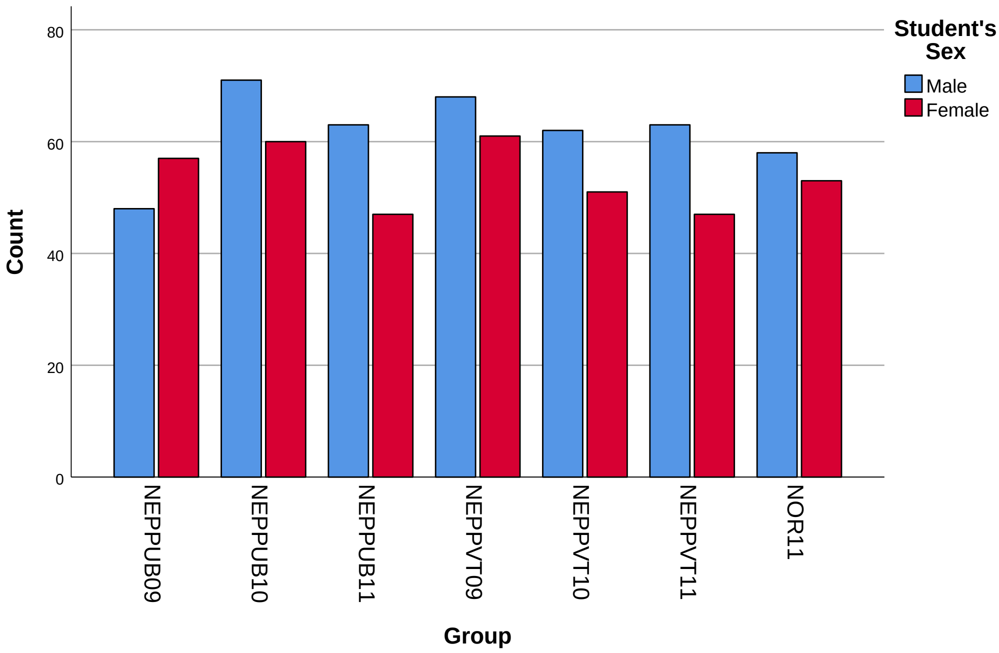
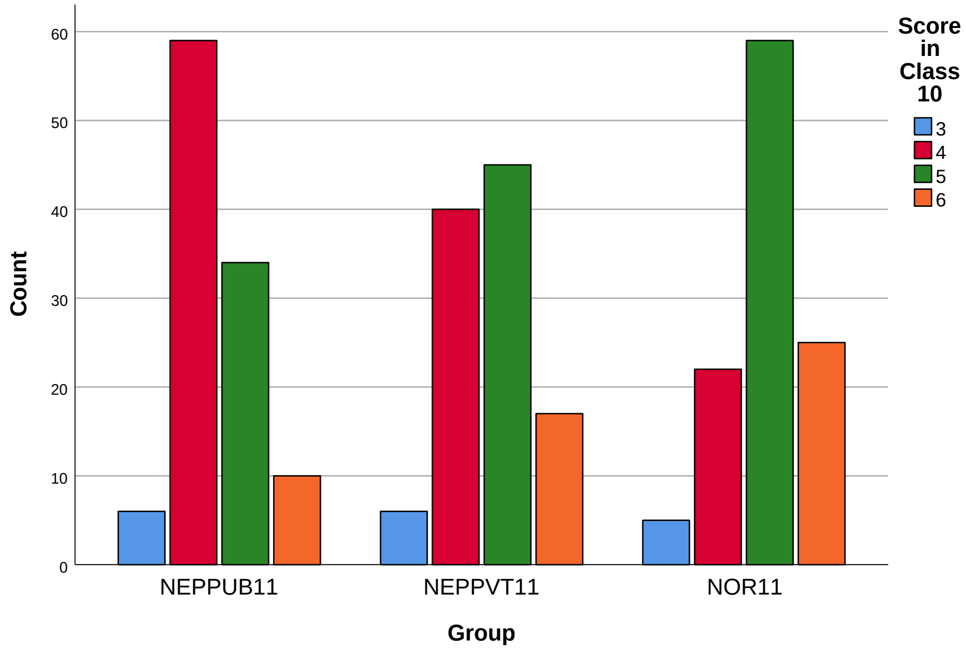


Figure 3: Distribution of score of tenth grade of the students in different groups. NEPPUB09, NEPPUB10 and NEPPUB11 = 9th, 10th and 11th-graders in public school in Nepal. NEPPVT09, NEPPVT10 and NEPPVT11 = 9th, 10th and 11th-graders in private school in Nepal NOR11 = 11th-graders in Norway.



3.5 Data Analysis

3.5.1 Coding

The participants are divided into seven different groups as: 9th-graders in public school in Nepal (NEPPUB09), 10th-graders in public school in Nepal (NEPPUB10), 11th-graders in public schools in Nepal (NEPPUB11), 9th-graders in private school in Nepal (NEPPVT09), 10th-graders in private school in Nepal (NEPPVT10), 11th-graders in private school in Nepal (NEPPVT11) and 11th-graders in Norway (NOR11). Furthermore, grade-wise analyses are also done grouping the participants in their respective grades as: 9th-graders in Nepal (NEP09), 10th-graders in Nepal (NEP10), 11th-graders in Nepal (NEP11) and 11th-graders in Norway (NOR11).

The answer sheets produced by the participants (N = 813, in total) were evaluated, graded and coded for analyzing quantitatively in SPSS. As the tasks were open-ended algebra problems, the coding followed the evaluation guidelines prepared by Udir for grading the 1T-mathematics examinations with a slight modification. Udir follows a grading scale from 1 to 6 where 1 and 2 represent "low achievement," 3 and 4 represent "average achievement" and 5 and 6 represent "high achievement." Since an overall assessment is done while evaluating the answer sheets of the tests and examinations in schools, 0 is not awarded. But since, each sub-question is specifically and quantitatively graded for this research, 0 is awarded for "completely wrong" answer. In addition to this, a code of 99 is given to "not attempted" task. Thus, the coding of the open-ended tasks followed the following scale:

- High achievement: 3
- Average achievement: 2
- Low achievement: 1
- Wrong answer: 0
- No answer: 99

Other parameters were also coded accordingly. Student's age is a numeric scale value and was mentioned as it was. Student's sex was given a value 1 for boys and 2 for girls. The students' grades in mathematics in tenth-grade followed a scale of 1-6 and reflected what they achieved. The percentage or the letter-grades of the Nepalese students were converted to corresponding number-grade practiced in the Norwegian system.

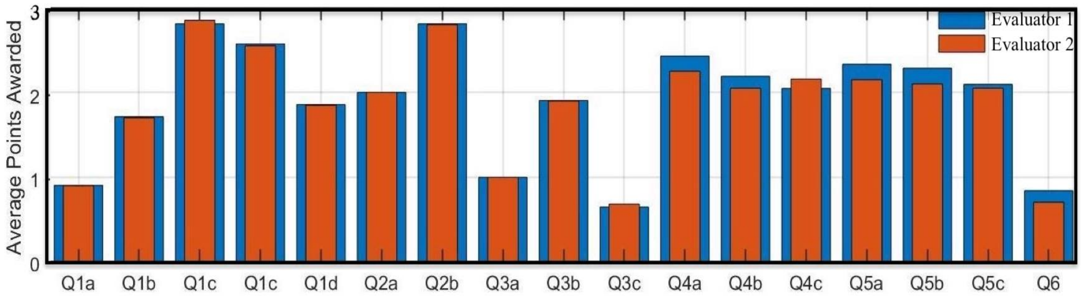
3.5.2 Routine for Coding

Evaluating 813 answer-sheets was a tedious job, but a good routine was established that both made the task less cumbersome and guaranteed the uniformity. Scoring guide with answer keys, possible errors, students' misconceptions, and weaknesses were prepared and followed throughout the process. To avoid the evaluation biases, all the answer sheets were coded minimizing the time-gap between the subsequent evaluations.

3.5.3 Inter-Rater Reliability

Though I did the evaluation and coding of the answer-sheets myself, 21 answer-sheets were coded together with a research fellow based in Denmark. Before the coding began, the same scoring guide was shared and the evaluation procedure was discussed. After the coding was done individually, in order to evaluate whether the established coding system was reliable, inter-rater reliability was determined with the intraclass correlation coefficient in SPSS. Intraclass Correlation Coefficient of 0.993 of the average measures with a lower bound of 0.984 and an upper bound of 0.997 at 95 % confidence interval suggested an excellent agreement between the evaluators (Cicchetti, 1994). Despite the excellent agreement for the established coding system, the codebook and the sensor guidelines were reviewed for possible flaws and anomalies that resulted in slight differences in the average points awarded in some of the questions. The codebook was updated suggesting to award 1 point if the information is correctly presented by drawing a correct figure for question number 6 that constitutes a text-problem for which the solution is comparatively longer.

Figure 4: Average points awarded per sub-question by two evaluators

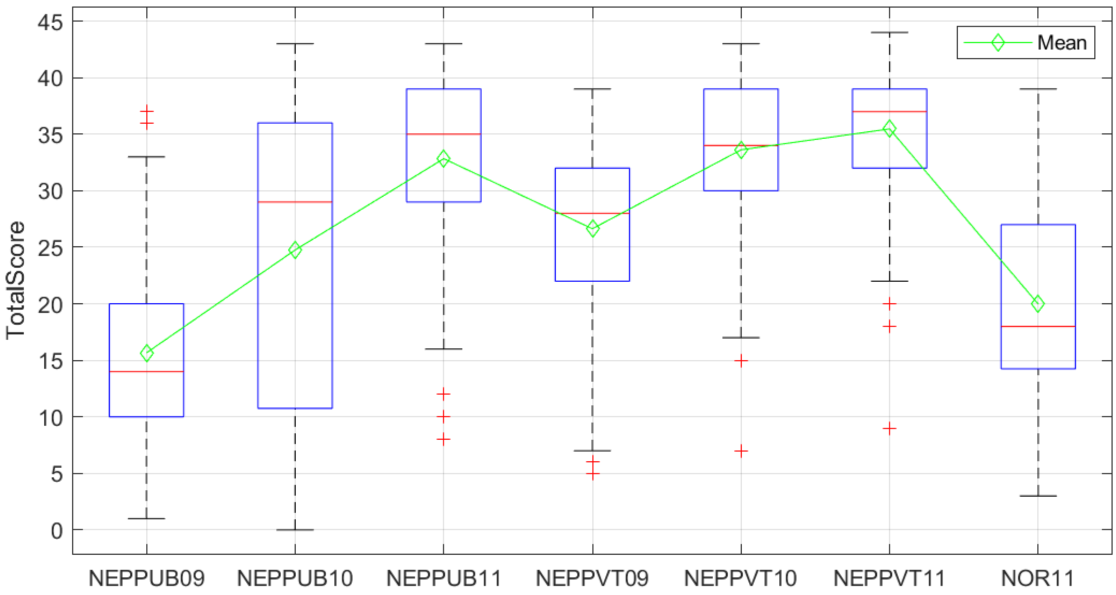


3.5.4 Statistical Analyses and Interpretation

Descriptive analyses, reliability testing (Inter-Rater Reliability, see section 3.5.3) and inferential statistical measures were conducted using SPSS, version 25. Initial data organisation was also done using Excel. Some graphics are produced using MATLAB for better visualization. Table 5 shows descriptive statistics of the mean total score of different groups, while the box and whisker diagram in Figure 5 displays the distribution of data based on minimum, first quartile, median, third quartile, maximum and mean of total score and other parameters of different groups discussed in section 4.1. Similarly, Figure 6 displays the distribution of data based on minimum, first quartile, median, third quartile, maximum and mean of total score and other parameters of different grades as per the number of years algebra being taught.

Statistical significance of the difference between the mean score of different groups was analysed by using one-way ANOVA. ANOVA, which is analysis of variance, is a method that allows us to compare the mean score of a continuous (or ordinal with many scale points) variable between a number of groups (Muijs, 2010).

Figure 5: Box and whisker diagram representing mean total score and mean scores of other parameters of different groups.



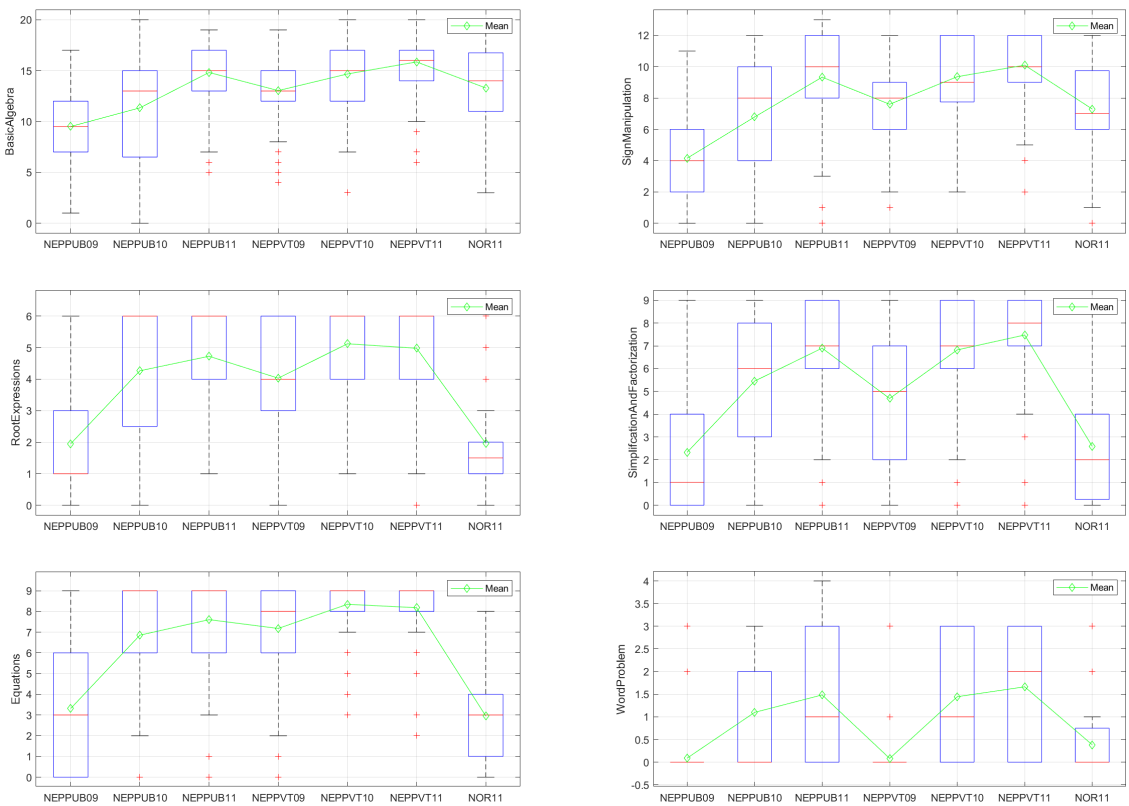
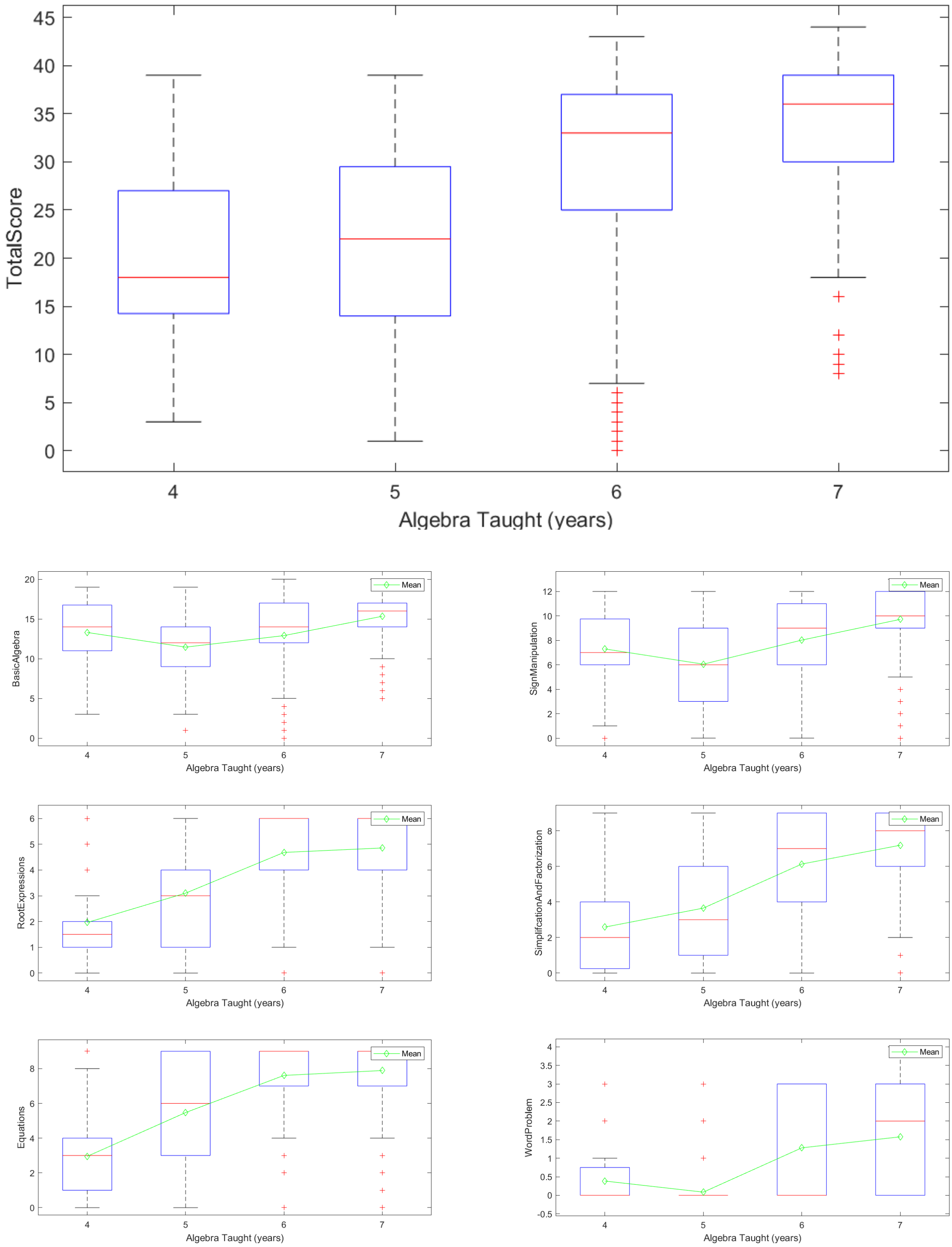


Figure 6: Box and whisker diagram representing mean total score and mean scores of other parameters of different grades as per number of years algebra being taught. 4 = NOR11, 5 = NEP09, 6 = NEP10, 7 = NEP11.



Assumptions and Tests for One-Way ANOVA

Certain assumptions are to be met when utilizing t-test or ANOVA. The model assumptions for ANOVA include independence, normality and homogeniety of variances (Glass & Hopkins, 1996).

Independence

Owing to the nature of this study, random sampling was not possible (refer section 3.3) but the selection of a participant was independent upon the selection of another participant. The participants had equal opportunity to answer the problems posed. The most important issue of independence is that "observations within or between groups are not paired, dependent, correlated, or associated in any way" (Glass & Hopkins, 1996, p. 295). It is assumed that the issue of independence is fulfilled in this study.

Normality

Parametric tests assume that score in each group is normally distributed. Since, this study explores the overall performance of the students in different groups, we are interested in the total score the students received in the test. Normality for ANOVA is normally tested with Shapiro-Wilk statistic (W), z-test for skewness and z-test for kurtosis using the residuals of the data (total score in this case). As seen in table 2, the Shapiro-Wilk statistics show that the total scores of the groups are not normally distributed except for NOR11.

Table 2: Shapiro-Wilk test for normality

	Groups	Tests of Normality					
		Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Residual for Total Score	NEPPUB09	0.138	106	0.000	0.951	106	0.001
	NEPPUB10	0.171	129	0.000	0.839	129	0.000
	NEPPUB11	0.136	111	0.000	0.910	111	0.000
	NEPPVT09	0.095	130	0.005	0.955	130	0.000
	NEPPVT10	0.097	113	0.011	0.937	113	0.000
	NEPPVT11	0.135	111	0.000	0.904	111	0.000
	NOR11	0.106	111	0.004	0.970	111	0.015

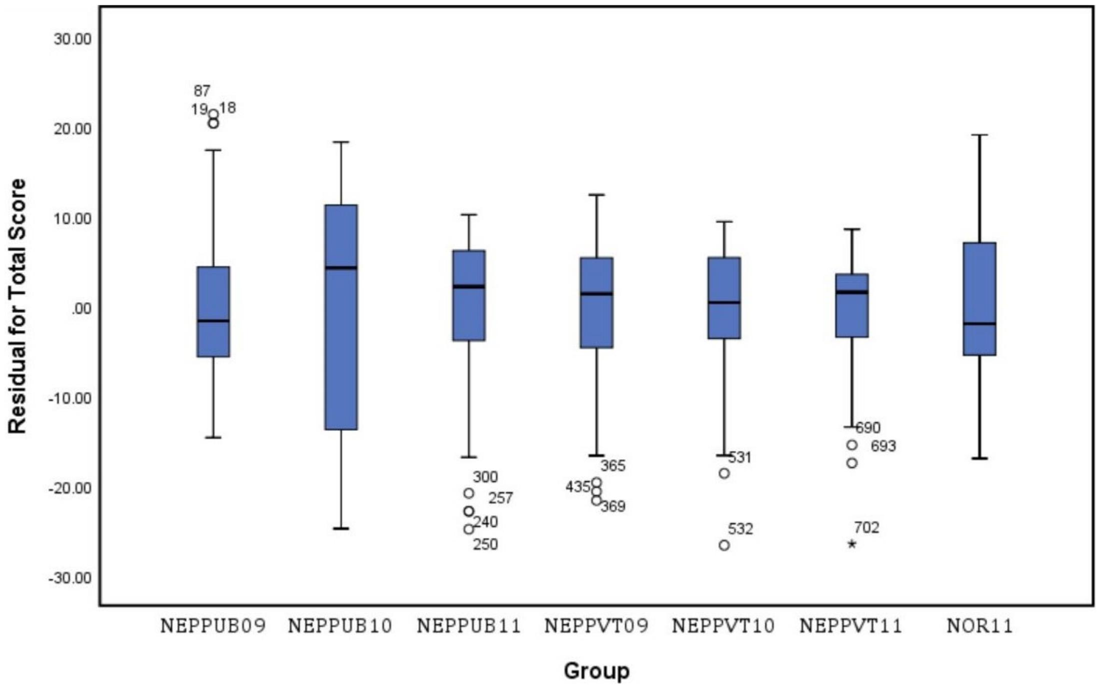
The skewness analyses (Table 3) show that the distribution is fairly symmetric for NOR11, moderately skewed for NEPPUB10, NEPPVT10, NEPPUB09 and NEPPVT09 and highly skewed for NEPPUB11 and NEPPVT11.

Table 3: Skewness and kurtosis values for the distribution of residual of total score among different groups

	NEPPUB09	NEPPUB10	NEPPUB11	NEPPVT09	NEPPVT10	NEPPVT11	NOR11	TOTAL
Skewness	0.694	-0.760	-1.102	-0.762	-0.891	-1.102	0.400	-0.623
Std. Error	0.235	0.213	0.229	0.212	0.227	0.229	0.229	0.086
Kurtosis	-0.112	-0.942	0.972	0.394	0.801	0.972	-0.419	-0.542
Std. Error	0.465	0.423	0.455	0.422	0.451	0.455	0.445	0.171

The box and whisker diagram (Figure 7) shows some extreme values for NEPPUB09, NEPPUB11, NEPPVT09, NEPPVT10 and NEPPVT11. SPSS interprets some of these values as outliers. These values have affected the normality curves as shown in Figure 9. If these values are adjusted in the dataset, the corresponding normality curves would be improved. All the extreme values are rechecked, both in the dataset and the answer sheets of the students and it is confirmed that they are not resulted from data entry errors or grading flaws. These are the real scores of the students and therefore, they are not adjusted. Normal Q-Q plots of residual for total score in Figure 8 show that the residual for total score align approximately on the reference line. However, scores of the 10th-graders in public school in Nepal (NEPPUB10) are more scattered than others.

Figure 7: Box and whisker diagram showing the distribution of residual for total score among different groups



According to Glass & Hopkins (1996), the consequences of violating the normality assumptions are rather minimal, especially when the research is conducted with equal sample sizes in all the groups. Since, the number of participants in different groups of eleventh graders in Nepal and Norway was equal ($N_{NOR11} = N_{NEPPUB11} = N_{NEPPVT11} = 111$), ANOVA is robust to these groups. These are the main groups in focus. From a study of robustness of ANOVA on non-normality discussed in literature from 1930

Figure 8: Normal QQ-plot of residual for total score of different groups.

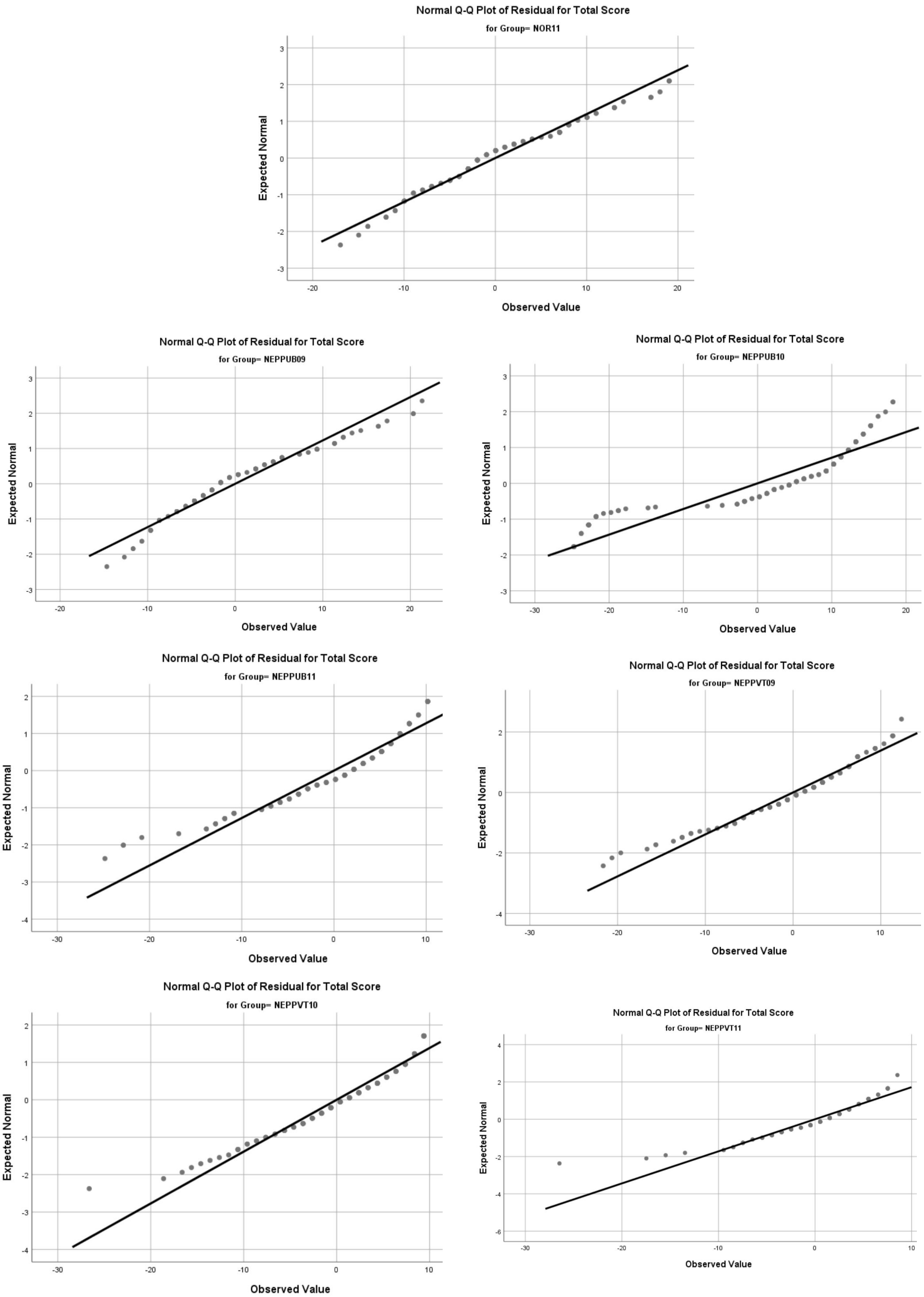
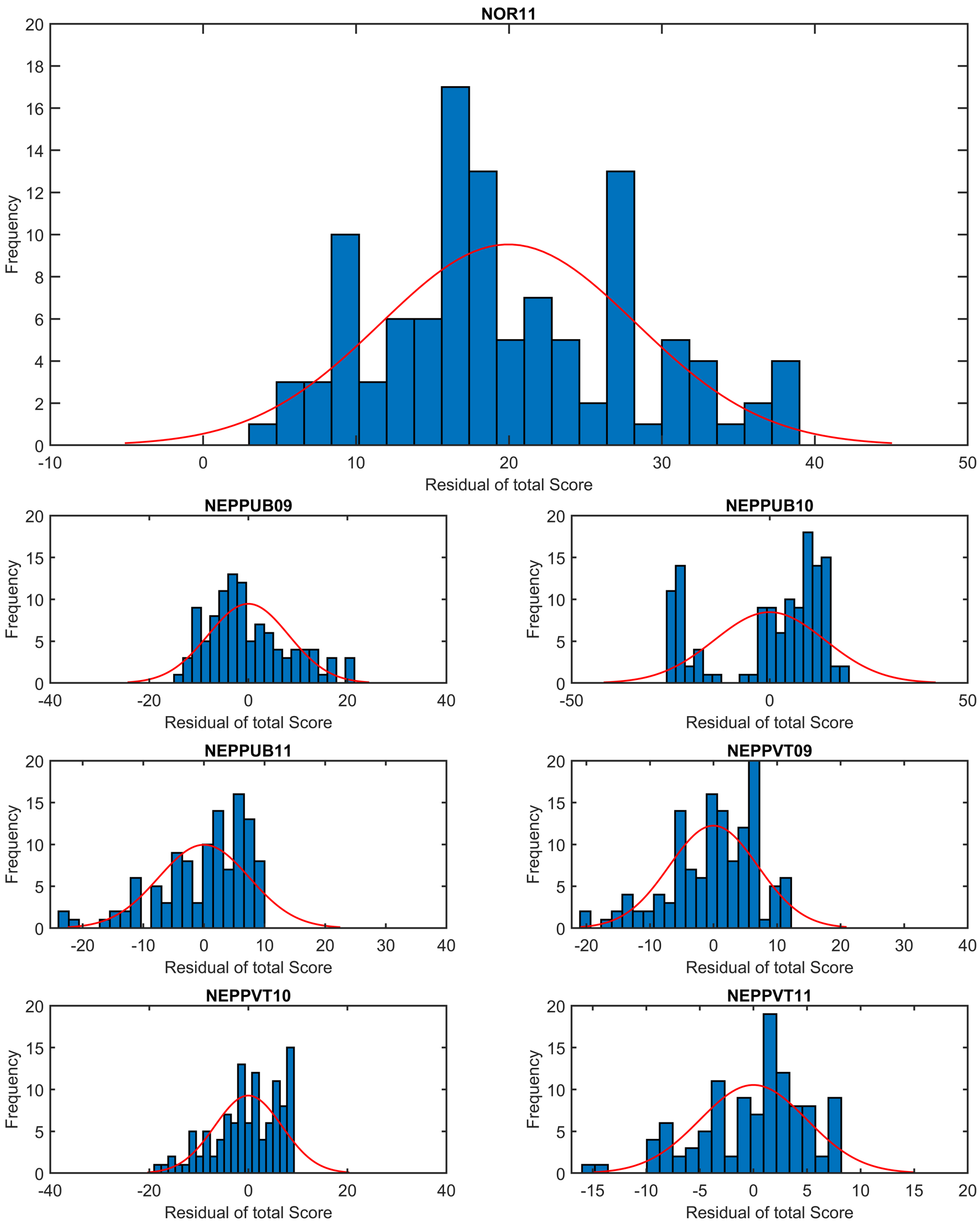


Figure 9: Histogram showing the normality curves for the residual of total score without extreme values adjusted.



through 2017, Blanca, Alarcón, Arnau, Bono & Bendayan (2017) summarize that ANOVA is robust to departures from normality when: a) the departure is moderate; b) the populations have the same distributional shape; and c) the sample sizes are large and equal. Though it is a bit ambiguous to interpret these three assumptions, the normality requirements are considered to be met for the dataset based on the combination of the normality tests, skewness analysis, the graphs produced and these assumptions.

Homogeneity of Variances

Homogeneity of variances deals with with-in group differences (Glass & Hopkins, 1996). SPSS utilizes Levene statistic for the test of homogeneity of variances. Levene statistic (Table 4) for the total score of different groups shows that the variances are significantly different for different groups. But like normality, when sample sizes are equal among the groups, ANOVA is robust to heterogeneous variances. It is, thus, assumed that the issue with non-homogeneity of variances does not have dramatic effect in this study.

Table 4: Levene statistic for the test of homogeneity of variances of the total score of different groups.

Test of Homogeneity of Variances					
		Levene Statistic	df1	df2	Sig.
Total score	Based on Mean	25.88	6	804	.00
	Based on Median	15.84	6	804	.00
	Based on Median and with adjusted df	15.84	6	533.83	.00
	Based on trimmed mean	24.16	6	804	.00

Why not Data Transformation or Non-parametric Analyses?

When the data fail to establish strong agreement with the ANOVA assumptions, it is often recommended to do data transformation or to follow a non-parametric analysis. However, according to Blanca et al. (2017), there is no additional benefits of data transformations over the good control of Type 1 error achieved by F-test. Furthermore, they note that the results are often difficult to interpret when data transformations are adopted. Therefore, this option was not considered appropriate for the analysis here. Non-parametric procedure like Kruskal-Wallis test are widely used in educational research (Muijs, 2010). This test converts quantitative continuous data into rank-ordered data and in doing so, there is a risk that important information is lost (Blanca et al., 2017). Since the data for this study were constructed from a quantitative test that yielded the real continuous scores for students' achievement in algebra (and not only opinion regarding something), non-parametric option is not deemed reliable for this analysis.

3.6 Validity and Reliability

All the research activities have weaknesses and thus the issues of validity. In order to minimize the validity issues of my research, some of the tasks used are based on a past project while the other tasks are adopted from the past examinations in mathematics prepared by Udir (see sub-section 3.4.1). In addition to this, to ensure that the project runs as planned, a pilot study was carried and analyzed. As guided by the result of the pilot project, necessary changes were made in the final test (see section 3.2).

3.7 Ethical Considerations

The Government regulates the research activities in Norway through the Norwegian Center for Research Data (NSD). For this research, the data were collected in Norway and Nepal but since the responsible institution for the project was based in Norway and therefore, everything related to this project (also in Nepal) was regulated by the norms set by NSD. In addition to this, the Nepalese regulations that govern research activities were followed.

NSD was contacted regarding the procedures of data collection. Since the project did not collect any personal information of the informants and that everything was aimed to be kept anonymous, it was not necessary to seek any written permission from NSD. The project, from the start to the end (and beyond) would strictly abide by the research ethics and the guidelines mentioned by NESH (2016). The participants were informed about the project both verbally and in writing and that their participation was entirely voluntary. The participants were guaranteed that they might withdraw their participation at any time.

3.8 Limitations of the Study

The study has some limitations, as with all other studies. The study used a set of tasks prepared for measuring algebra skills of the Norwegian students and used them both in Norway and Nepal. Nepalese students might be used to with a different perspective. Though the test results give an impression of a general scenario, it is difficult to ensure that a set of some tasks measure in-depth knowledge of anything. In addition, there are often parameters other than those accounted in this study that may impact students' algebra skills. Comparing students in two different countries with entirely different education system, pedagogical practices, classroom environment and resource availability pose a serious challenge.

4 Results

Mean total score obtained by the students is considered as a measure of achievement in algebra. The tasks were further classified so that they reflect students' achievements in different categories like *Basic Algebra Skills, Sign Manipulation, Root Expressions, Simplification and Factorization, Equations, and Word Problems* (this is discussed in detail in the article). Total score is the function of these parameters. Differences in mean scores between the seven groups (and four classes) were analysed using ANOVA.

4.1 Total Score as a Measure of Achievement

Table 5 displays the distribution of students in different groups and the total score they received in the test. Except for two blank answer-sheets from 10th grade students of a public school, all other participants have responded to the test achieving total scores ranging from .00 to 44.00. Mean of the total score is distributed as: NEPPUB09 ($M = 15.66, SD = 8.13$) , NEPPUB10 ($M = 24.76, SD = 13.97$), NEPPUB11 ($M = 32.84, SD = 7.83$), NEPPVT09 ($M = 24.64, SD = 7.21$), NEPPVT10 ($M = 33.62, SD = 7.22$), NEPPVT11 ($M = 35.47, SD = 5.81$) and NOR11 ($M = 19.96, SD = 8.86$). The 11th-grade students of the private school in Nepal have the highest score, while the 9th-grade students of the public school in Nepal have scored lowest. The Norwegian students have scored much lower than their Nepalese counterparts (11th graders in Nepal). This score is just a bit higher than the 9th-graders of the public school in Nepal. When the scores are analysed grade-wise, it is seen that there was a slight tendency for a lower mean grade score among Norwegian students than 9th-graders in Nepal ($M = 21.71, SD = 9.38$).

Table 5: Descriptive statistical measures of total score of different groups.

Total Score										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	106	0	15.66	8.13	.79	1.00	37.00	.69	-.11
NEPPUB10	131	129	2	24.76	13.97	1.23	.00	43.00	-.76	-.94
NEPPUB11	111	111	0	32.84	7.83	.74	8.00	43.00	-1.10	.97
NEPPVT09	130	130	0	26.64	7.21	.63	5.00	39.00	-.76	.39
NEPPVT10	113	113	0	33.62	7.22	.68	7.00	43.00	-.89	.80
NEPPVT11	111	111	0	35.47	5.81	.55	9.00	44.00	-1.41	3.45
NOR11	111	111	0	19.96	8.36	.79	3.00	39.00	.40	-.42
Total	813	811	2	27.02	11.06	.39	.00	44.00	-.62	-1.22

The one-way analysis of variance (ANOVA) shows that there was a significant difference between the mean total scores at the $p < .05$ level for different groups ($F(6, 804) = 78.10$, $p < .001$). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 is significantly different than the mean scores of other groups. NOR11 had significantly weaker achievement than NEPPUB10, NEPPUB11, NEPPVT09, NEPPVT10 and NEPPVT11 but a significantly better achievement than NEPPUB09. When analysed grade-wise, one-way ANOVA shows that there was a significant difference between the mean scores at the $p < .05$ level for different grades ($F(3, 807) = 87.57$, $p < .001$). Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 is significantly lower than NEP11 ($M = 34.13, SD = 7.00$) and NEP10 ($M = 28.90, SD = 12.14$), but not significantly different ($p = .39$) from NEP09 ($M = 21.71, SD = 9.38$).

Apart from this general analysis, the article presents the analyses of the test achievement based on the categories mentioned earlier (refer Results section in the article). Multi-comparison graph in Figure 10 shows the significant differences between the average total score of different groups. Figure 11 shows the significant differences between the average total score of different classes as per the number of years algebra being taught.

Figure 10: Multi-comparison graph representing the total score and the scores of other categories of different groups.

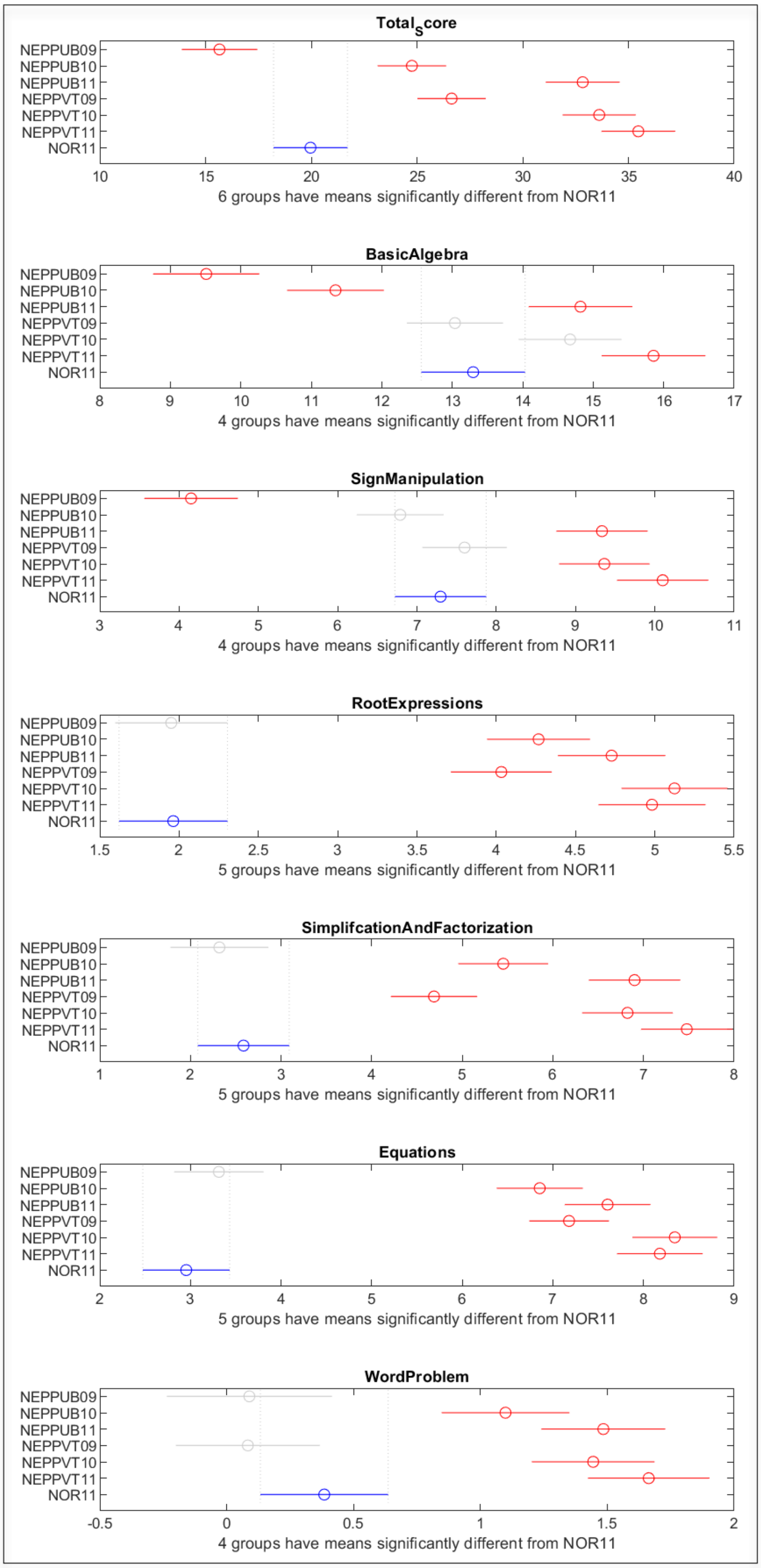
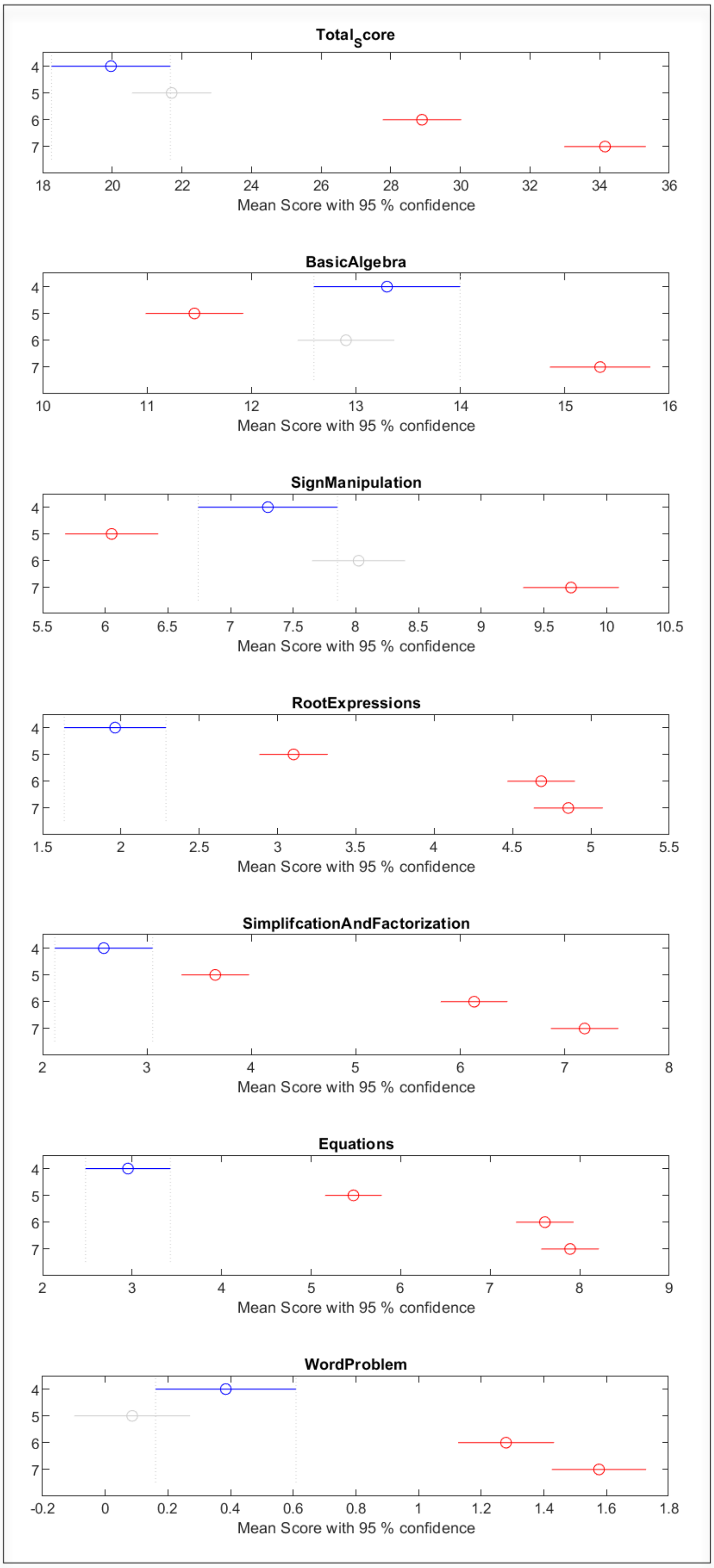


Figure 11: Multi-comparison graph representing the total score and the scores of other categories of different grades as per number of years algebra being taught. 4 = NOR11, 5 = NEP09, 6 = NEP10, 7 = NEP11.



4.2 Number of Years Algebra Studied and Achievement

The analysis of the total score and other parameters used to evaluate achievements in algebra showed that the students who have studied algebra from early grades have scored significantly higher as discussed in section 4.1. A Pearson correlation coefficient was computed to assess the relationship between the number of years students studied algebra and their achievement (total score). There was a positive correlation between years of algebra ($M = 5.71, SD = 1.01$) and total score ($M = 27.02, SD = 11.06$) , r = .49, p < .001. A simple linear regression was calculated to predict total score based on number of years algebra is studied. A significant regression equation was found (F(1, 809) = 248.36, p < .001), with an R^2 of .24.

4.3 Students' Perception of the Test

From Table 6, it can be seen that 38.53 % of the participants considered the test to be moderate when evaluated in a scoring range of 1 - 5, where 1 is "very difficult" and 5 is "very easy". For 14.59 % of the participants, the test was very easy, while 2.87 % of them found it very difficult. Group-wise analysis gives a different picture. For majority of Norwegian students, the test was either difficult (58.49 %) or moderate (30.19 %). 10.38 % of them considered it to be very difficult, while just 1 (0.94 %) out of 111 students considered it to be easy. On contrary to this, majority of Nepalese students considered the test to be either moderate, easy or too easy.

Table 6: Students' Perception of the Test.

		Very difficult		Difficult		Moderate		Easy		Too easy		Total
		N	%	N	%	N	%	N	%	N	%	N
Group	NEPPUB09	6	5.66	15	14.15	70	66.04	12	11.32	3	2.83	106
	NEPPUB10	3	2.31	17	13.08	54	41.54	45	34.62	11	8.46	130
	NEPPUB11	0	0.00	0	0.00	31	27.93	61	54.95	19	17.12	111
	NEPPVT09	1	7.94	6	4.76	74	58.73	34	26.98	11	8.73	126
	NEPPVT10	1	0.88	0	0.00	29	25.66	46	40.71	37	32.74	113
	NEPPVT11	1	0.91	0	0.00	19	17.27	54	49.09	36	32.73	110
	NOR11	11	10.38	62	58.49	32	30.19	1	0.94	0	0.00	106
Total		23	2.87	100	12.47	309	38.53	253	31.55	117	14.59	802

In order to access the relationship between students' perception of the test and their mean total score, Pearson correlation coefficient was computed. There was a positive correlation between students' perception ($M = 3.43$ (= between moderate and easy) , $SD = 1.01$) and total score ($M = 27.09, SD = 11.06$), r = .62, p < .001. A simple linear regression was calculated to predict total score based on students' perception. The result of the regression analysis was: (F(1, 798) = 502.03, p < .001), with an R^2 of .39.

Nepalese students were also asked which class, they felt, would the test be appropriate for. Majority of the respondents ($n = 671$) considered that it would be appropriate for class 6 (11.8 %), or class 7 (19.2 %), or class 8 (25. 1 %), or class 9 (11.8 %) or class 10 (5.0 %). To find the relationship between students' perception of which class the test was appropriate for and their mean total score, Pearson correlation analysis was run. The analysis showed a weak negative correlation between students' perception about the class the test was appropriate for ($M = 7.58, SD = 1.62$) and total score ($M = 28.35, SD = 11.09$), r = -.42, p < .001. A simple linear regression was calculated to predict total score based on students' perception. The result of the regression analysis was: (F(1, 667) = 145.75, p < .001), with an R^2 of .18.

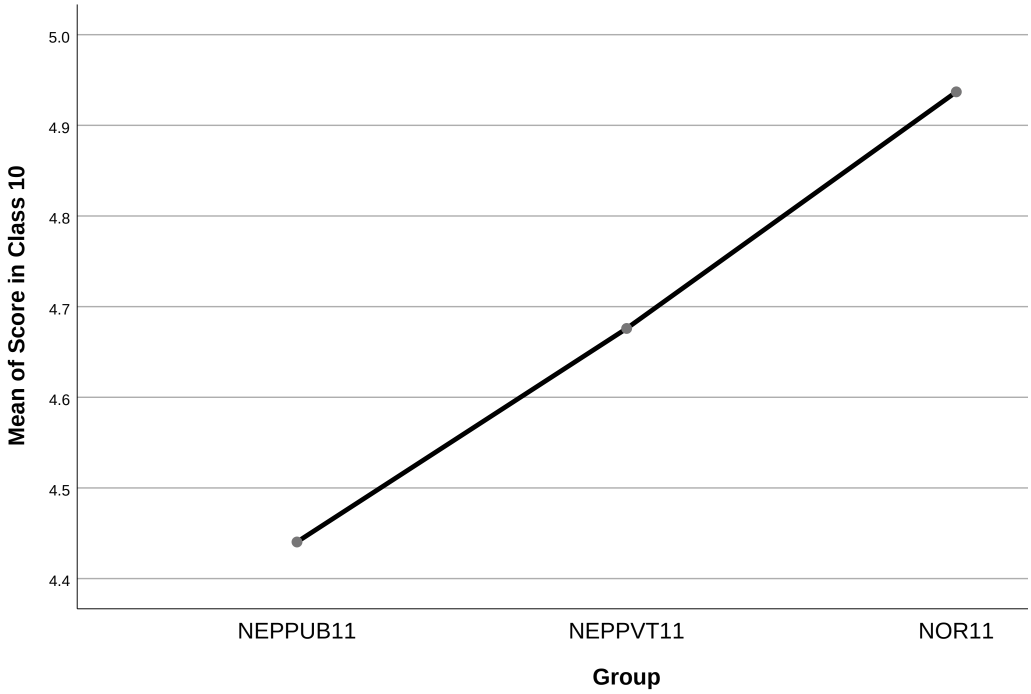
5 Discussion

Norwegian students' weak achievement in algebra is not a new phenomenon. Analysing the data from TIMSS 2011, Grønmo, Borge and Rosén (2013) conclude that algebra and geometry are less prioritized in schools in Norway. They argue that the students' weak achievement in algebra should be the consequence of this drawback. The weakness of the students in algebra may not be totally explained by the fact that they receive fewer algebra hours, but maybe it has a connection with what we think about algebra in schools in Norway (Naalsund, 2012). The functional approach of algebra has not been in focus in curriculum, and therefore, the Norwegian students first encounter algebra as "generalised arithmetic with an emphasis on the transformational activity" (Espeland, 2017, p. 48). Warren and Cooper (2005) opine that functional thinking of algebra helps in developing an understanding of the relationships between the operations.

Analysing the data from TIMSS Advanced 2008, Pedersen (2015) reveals that the Norwegian students "perform weakly on items that place high demands on symbol manipulation; these are usually purely mathematical items with expressions and formulas given in the text" (p. 89). The Norwegian students were stronger in the items that required text comprehension, application, and modeling of mathematical concepts (Pedersen, 2015). My study does not support this conclusion either. Norwegian students have achieved a mean of 0.79 (12.67 %) of the maximum possible score of 3 in word-problem. The weak performance of the Norwegian students in areas that were more demanding might well be attributed to their weak achievement in basic algebra and sign manipulation compared to that of their Nepalese counterparts. However, one should be a bit more cautious in interpreting the result of the parameter "word-problem". In this study, this was a single task that required the students to make use of their knowledge about triangles in geometry and apply it to formulate and solve an algebraic equation.

Interestingly, Figure 12 shows that the Norwegian students had the highest mean score in grade ten before they started at the upper secondary school.

Figure 12: Eleventh graders' mean score in mathematics in grade ten.



This study has clearly shown that there is a significantly positive correlation between the number of years the students studied algebra and their performance in the algebra test. The test score kept isolated as a measure of achievement, may not explain why some students perform better than other students. The students' learning activities and thus, their achievement may be well influenced by among others parental guidance (Cai, Moyer & Wang, 1999), priorities set by the specific education system (Cai et al., 2011), effective classroom management (Hiebert & Grouws, 2007), socio-cultural activities (Radford, 2008), and students' own motivation in learning in term of needs and goals (Wæge, 2009). This study did not collect any quantitative data associated with these factors, but the interpretations shall be made based on the priorities set by the Norwegian and Nepalese curriculum in algebra.

Low achievement of the Norwegian students in this test should be a worrying concern as the test included about 60 % of the tasks from a syllabus they were taught and prepared for a term test. Other 40 % of the tasks were also very familiar problems in basic algebra. On the other hand, the Nepalese students got the information about the test a day before or on the same day the test was held. Moreover, they were completely unaware of research studies as they had never participated in a similar study before. The availability of resources in classrooms in Norway and Nepal cannot be compared (see section 1.1). Schools in Nepal are plagued mainly by among other poor school environments, weak classroom management, and absence of child-centered interactive teaching approaches (Bhatta, 2008). There were a total of 65 registered students in the same classroom in the public school that participated in this study. The situation of the private school was a bit better, but still, 45 students were taught in the same classroom. The pedagogical approach adopted was largely teacher-centered blackboard teaching in both the schools. On the other hand, the Norwegian schools are fully funded by the Government, compulsory free education for all is enshrined in law and the quality in education is assured by the Government (Braathe & Otterstad, 2014).

Given this reality, it would be reasonable to assume that the higher achievement of the Nepalese students can be attributed to their early encounter with school algebra. The Nepalese students start learning algebra at fourth-grade where they are introduced to addition and subtraction of like terms, simple equations of addition, subtraction, multiplication and division with box notation solved through inspection, hit and trail and using variables (CDC-Nepal, 2009). The Norwegian students meet similar competence goals of solve and calculating with parentheses in addition, subtraction and multiplication of numbers and setting up and solving simple equations only when they are introduced to algebra in seventh-grade (Udir, 2006). By this time, the Nepalese students have already started to classify the polynomials, multiply and divide trinomials by binomials and make geometric interpretation of $(a \pm b)^2$ and apply it (CDC-Nepal, 2012). So, the Norwegian students have received four years of algebra teaching before they started at eleventh-grade while their Nepalese counterparts have received seven years of teaching. This might also explain what the students perceived about the test. As the Nepalese students receive more years of algebra teaching, their confidence might be higher when they sit for a test. But this assumption should carefully interpreted. The dataset revealed that at least three Nepalese 1oth-graders who totally skipped the test have also answered that the test was easy. This might show students' inability in reflecting what they have learned. Since the focus of this study is not on perception, attitude and reflection, no further discussion shall be made.

Though the notion of *Early Algebra* has been widely discussed for the past two decades (Cai, 1998; Cai et al., 2011; Carpenter et al., 2003; Kaput, 1998; Kaput et al., 2017; Kieran et al., 2016), *Early Introduction of Algebra in Schools* has not seen similar emphasis in mathematics education research. Further research is needed to explore the implications of including algebra in curriculum in early classes.

6 Conclusion

The analyses of the test results have suggested that the introduction of algebra in an early stage in Nepalese schools has significance in promoting students' understanding of algebra compared to the Norwegian students. The findings raise a question if most of the western countries that postpone the introduction of algebra until adolescence (Carraher, Schliemann, Brizuela & Earnest, 2006) should introduce algebra earlier in the mathematics curriculum. Many researchers, however, believe that young children are incapable of learning algebra because they lack cognitive ability to handle concepts like variables and functions (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; MacGregor, 2001). At the same time, during the past thirty years, we have seen increased interest and focus on the development of algebraic thinking in the early stage . Davis (1985) argued that algebra should begin in Grade 2 or 3. Despite encountering several pedagogical, managerial, technological and socio-economic constraints, the Nepalese students who participated in this study have demonstrated significantly better performance in the test than their Norwegian counterparts who, in contrast, enjoy profound benefits in schools. The findings should, however, be interpreted in light of the methodological considerations of the study discussed in sub-section 3.5.4. In addition to this, it is very difficult to compare the achievements of the students in two different countries with very different education system, teaching-learning environment and resource availability. Owing to the nature of this study, it might be difficult to generalize the results to a bigger population (Salkind, 2010). The issue of generalization is common to causal-comparative studies, but future work should consider the extent to which these findings can be generalized. Based on the findings, the study concludes that it is reasonable to argue for early introduction of algebra in Norwegian schools, but further research is needed to explore its pedagogical implications and how effectively it can be incorporated in current teaching-learning environment in Norway.

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Part II :

Article

Does Early Introduction of Algebra in Schools Make any Difference? A Causal-Comparative Study of Algebra Skills of Upper Secondary School Students in Norway and Nepal.

RAMESH GAUTAM [1]

Abstract

The main goal of this causal-comparative study is to investigate if the introduction of algebra in an early stage in schools enhance students' understanding of basic high school algebra. Algebra with variables appears in fourth-grade in Nepal, while in Norway, it is not part of the curriculum before seventh-grade. Findings of this study are based on students' performance on an open-ended algebra test conducted among 813 students from different grades in schools in Norway and Nepal. Results indicate that the Norwegian students achieved significantly lower than their Nepalese counterparts. Norwegian students' achievement is also significantly lower than the tenth-graders in Nepal, while there was no significant difference between Norwegian eleventh-graders and Nepalese ninth-graders. Thus, the results of this study suggest that an early introduction of algebra in Nepal has a significant role in students' achievement. These findings agree with the past studies that suggest weak achievement of Norwegian students in algebra. Based on the findings, the study concludes that it is reasonable to argue for early introduction of algebra in Norwegian schools, but further research is needed to explore its pedagogical implications and how effectively it can be incorporated in current teaching-learning environment in Norway.

1 Background and Purpose of the Study

Several studies conclude that school students in developed countries like the USA, Sweden, and Norway show weak achievement in algebra (Grønmo et al., 2012; Phan, 2008). Exploring the causes of this weakness and interpreting them sensibly have been crucial for educational research about students' performance in mathematics. Analyses from the Trends in International Mathematics and Science Study (TIMSS) and Programme of International Student Assessment (PISA) surveys show that these countries prioritize daily-life mathematics like statistics than formal mathematics like algebra in contrast to the countries in East-Asia and East-Europe (Grønmo, Bergem, Kjærnsli, Lie & Turmo, 2004; Grønmo et al., 2012). Grønmo et al. (2012) argue that one of the explanations for Norwegian students' weak achievement is that algebra is traditionally introduced rel-

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atively late in schools. It can be argued that owing to the weak algebra background, students often encounter problems understanding the basics of algebra when they start at the upper secondary schools.

Nepal has not participated in extensive international surveys like TIMSS and PISA, and thus, the comparative reports regarding students' achievement in mathematics are not available. Feasibility study on Nepal's participation in international assessment submitted to Education Review Office (ERO) in 2016 by Centre for Educational Research and Social Development (CERSOD) mentioned that TIMSS and National Curriculum Framework (NCF) have around 90 % similar content (CERSOD, 2016). Therefore, CERSOD (2016) opines that it would not be large gap and content mismatch while adopting TIMSS test items by Nepalese students' assessment at the end of eight grade. As mentioned in the Primary Education Curriculum prepared by the Curriculum Development Centre (hereafter CDC-Nepal) under the Ministry of Education of the Government of Nepal (hereafter MoE-Nepal), the introduction of algebra in school mathematics occurs as early as in fourth grade (CDC-MoE, 2006). As the students complete the secondary school and start at the upper secondary level, it can be assumed that they are better prepared to encounter more significant challenges in mathematics than their Norwegian counterparts who are not introduced to algebra with variables before seventh grade.

In Norway, the mathematics curriculum has been revised several times, but algebra has never been adequately prioritized despite the calls of many researchers (Grønmo et al., 2004, 2012; Pedersen, 2015). No significant change has been proposed in algebra in the new curriculum proposed by The Norwegian Directorate for Education and Training (hereafter Udir) to be adopted by 2020 (Udir, 2018). The high achievements of the students of East-Asian and East-European countries have been accounted for their focus on algebra in the curriculum. Grønmo et al. (2012) points out that the countries that achieve a similar or below the Norwegian average in eighth grade mathematics in PISA 2011 are typically the developing countries with completely different resource situation than Norway.

According to the United Nations Development Program (UNDP), Nepal ranks 149 in Human Development with a score of 0.574 in 2018. Only 34.6% of the population has at least some secondary education, and an overall education index is 0.502. Nepal invests 3.7% of its GDP in education. On the other hand, Norway has been ranked number one consecutively for eight years as of 2018. 96.3% of the population has at least some secondary education, and the overall education index is 0.915. Norwegian government invests 7.7% of its GDP in education (UNDP, 2018).

The Nepalese education system has been encountering limitations owing to poverty, low investment in education from the government, and poor student enrolment in schools. In addition to these limitations, Panthi and Belbase (2017) mention several other difficulties like language problems, issues of multiculturalism, availability of technology, lack of technical skills and lack of teacher training and professional development programs in Nepalese schools. Despite these adversities, have Nepal achieved anything significant through prioritizing mathematics in schools and by introducing algebra in the early stage of children's schooling?

Several studies have focused primarily on different teaching and learning approaches in algebra like *problem-solving* (Polya, 2004; Schoenfeld, 2009), *generalization* (Bell, 1995; Usiskin, 1988) and *early algebra* (Carpenter et al., 2003, 2005; Kieran et al., 2016). In a wider study, Cai, Ng and Moyer (2011) found that the Chinese and Singaporean elementary school students are capable of using abstract strategies efficiently to solve algebraic problems. They argue that this is possible because "the Chinese and Singaporean curricula provide concrete examples of promising ways to integrate arithmetic and algebraic ideas in the earlier grades" (Cai et al., 2011, p. 36). Though algebra is traditionally introduced late in the school curriculum, in some countries in Europe and North America, the discussion of integrating algebraic ideas into mathematics in the earlier grades started in the 70s (Cai & Knuth, 2011). The goal of this study is to investigate if the introduction of algebra at an early stage in school mathematics has any implication in shaping the algebraic understanding among the students. If so, what are those implications, and how could those be assessed? More specifically, the following research question will be answered:

Does the introduction of algebra in the early stage in schools as practiced in Nepal enhance the students' understanding of basic high school algebra compared to when it is introduced relatively late as in Norway?

To answer this research question, I pose the following hypotheses:

Research hypothesis, H1:

There is a significant difference between the algebra skills of the Norwegian and Nepalese students and this difference can be attributed to the early introduction of algebra in school mathematics in Nepal.

Null hypothesis, H0:

There isn't any significant difference between the algebra skills of Norwegian and Nepalese students and early introduction of algebra in school mathematics has no significant implication in algebra skills.

To explore this, a brief review of literature of past studies shall be done. This will be followed by the description of the procedure for the data collection and analysis. Finally, the results will be presented and interpreted.

2 Literature Review

2.1 Algebra: Development and Entry into School Mathematics

Algebra is one of the major topics of the school mathematics curriculum around the world though variations exist both in the content and the time it is introduced in the schools (Leung, Park, Holton & Clarke, 2014; Kanbir, Clements & Ellerton, 2018). Today many high school students view algebra as a calculation with letters instead of numbers (Espeland, 2017). Understanding these letters, that we call unknowns or variables today, and their operations form the basis of school algebra. Usiskin (1995) argues that students are considered to be learning algebra when they first meet variables in mathematics. Carraher, Schliemann and Brizuela (2000) opine that, for many students, algebra is merely memorizing the arbitrary rules and learning to manipulate x's and y's without understanding the conceptual basics associated with it.

According to a famous mathematician, Colin Maclaurin (1698 – 1746),

> Algebra is a general method of computation by certain signs and symbols which have been contrived for this purpose and found convenient. It is called universal arithmetic and proceeds by operations and rules similar to those in common arithmetic, founded upon the same principles. (Katz & Barton, 2007, p. 185)

Another mathematician, Leonard Euler (1707 – 1783), defined it as "Algebra is the science which teaches how to determine unknown quantities by means of those that are known" (Katz & Barton, 2007, p. 185).

Though these 18th-century definitions of algebra primarily focus on signs and symbols, rules and operations and determination of the unknowns by making use of the known quantities, the school algebra, as assigned in the curriculum, is much more than this (Katz & Barton, 2007). Historically, the conceptual basis for algebra existed as early as the period of Babylonian (c. 1700 – 689 BCE) and Greek (c. 800 – 146 BCE) mathematics (Boyer & Merzbach, 2011; Katz, 2008; Radford, 1996). The Babylonians had significant accomplishments in algebra but were hindered by their lack of algebraic symbols and notation (Evans, 2014). According to Boyer and Merzbach (2011), algebra, as it is understood today, got introduced to Europe after a book named *Hisob al-jabr wa'l muqabalah*, or *The Book of Calculation by Completion and Balancing*, written by Arabic scholar al-Khwarizmi (c. 780 – 850 CE). In the 12th century, it was translated into Latin and was called *Liber Algebrae et Almucabola* (Evans, 2014). Girolamo Cardano's (1501 – 1576) *Ars Magna*, or *The Great Art* (1545) is considered as the first algebraic work in Europe to advance beyond Islamic algebra (Evans, 2014).

The historical development of algebra can be recognized in three stages:

1. The rhetorical or early stage, in which everything is written out fully in words;
2. A syncopated or intermediate stage, in which some abbreviations are adopted; and
3. A symbolic or final stage. (Boyer & Merzbach, 2011, p. 162)

Besides these three stages of expressing algebraic ideas, Kartz and Barton (2007) argue that four conceptual stages have happened alongside these changes in expressions. The conceptual stages are the geometric stage, where most of the concepts of algebra are geometric; the static equation-solving stage, where the goal is to find numbers satisfying certain relationships; the dynamic function stage, where motion seems to be an underlying idea; and finally the abstract stage, where a structure is the goal.

Though *Hisob al-jabr wa'l muqabalah* did not make use of any syncopation or negative numbers, Boyer and Merzbach (2011) opine that it comes closer to the elementary algebra of today than the earlier works of either Diophantus (c. 200 – 284 CE) or Brahmagupta (c. 598 – 668 CE).

Ellerton and Clements (2017) argue that the first secondary school at which algebra was part of the mathematics curriculum was the Royal Mathematical School within Christ's Hospital, in central London, England. Established in 1673, the school had "the specific mission of preparing boys aged between 12 and 16 to become mathematically-competent apprentices in the Royal Navy or the merchant marine" (Kanbir et al., 2018, p. 18).

Mathematicians and Mathematics educators have never agreed unanimously upon what school algebra should be like and the past three centuries have seen the controversy over what school algebra should embrace (Kanbir et al., 2018). Moreover, the developments in teaching algebra in schools is not well documented. da Ponte & Guimarães (2014) claim that "the history of the teaching of algebra is largely unwritten" (p. 459). Kanbir et al. (2018) consider that a comprehensive history of school algebra focussing on the global perspectives is not yet published.

2.2 School Algebra

When it comes to school mathematics, Katz and Barton (2007) mention that a typical secondary school algebra incorporates a wide variety of topics. Some of these topics include arithmetic of signed numbers, solutions of linear equations, quadratic equations, and systems of linear and/or quadratic equations, and the manipulation of polynomials, including factoring and rules of exponents to name a few. As such, modern algebra is much more than what was understood in the eighteenth century.

2.2.1 Purpose of Algebra in School Mathematics

Debates and discussions about the purpose of school algebra have persisted for a long time and they are still ongoing. In a paper published in *Journal of Education* in 1915, Hedges (1915) argued that the general purpose of including algebra into the elementary schools would be to lessen the strangeness the students might encounter in high school mathematics. He further mentions that the specific purpose of elementary school algebra would be to have the insight into the elementary principles of algebra that the students would be introduced to in high schools.

Usiskin (1988) argues that the *purposes of algebra* are determined by or are related to *conceptions of algebra* and proposes following four conceptions:

1. Algebra as generalized arithmetic,
2. Algebra as a study of procedures for solving certain kinds of problems,
3. Algebra as the study of relationships among quantities, and
4. Algebra as the study of structures

Bell (1995) suggests that the algebraic language should be learned through the activities "defined by the three main modes: (a) generalizing, (b) forming and solving equations, and (c) working with functions and formulae" (p.50). For Bednarz, Kieran and Lee (1996), the perspectives on the introduction and development of algebra are important to determine the direction of school algebra and thus its purposes. They propose the following perspectives:

1. Historical perspectives in the development of algebra,
2. A generalization perspective on the introduction of algebra,
3. A problem-solving perspective on the introduction of algebra,
4. A modeling perspective on the introduction of algebra, and
5. A functional perspective on the introduction of algebra

In addition to these perspectives and purposes, Kanbir et al. (2018) focus that the purpose of algebra is also to empower the students with the knowledge that is a must for higher mathematical and scientific studies.

2.2.2 The Norwegian Context: Algebra in School Curriculum

The present curriculum (K06), which is termed as "The Knowledge Promotion" (Norwegian: kunnskapsløftet), was introduced at all levels in Norwegian schools in 2006 (Udir, 2018). The curriculum encompasses 10-year compulsory school and a voluntary upper secondary education and training (Espeland, 2017). Students can opt between *Specialization in General Studies* and *Vocational Education Program* in upper secondary schools. The new curriculum, K06, mentions specific competence aims to be achieved at grade 2, grade 4, grade 7, grade 10, and for each year in the upper secondary school. Algebra appears first in grade 7. There are two mathematics courses in the first year of the upper secondary school: 1T-Mathematics and 1P-Mathematics. The 1T course is more rigorous and theoretically oriented and forms a basis for specialization within physical sciences and engineering in higher studies.
Following are the competence goals set for 1T-Mathematics:

- Calculate with powers with rational exponents and numbers in scientific notation, algebraic expressions, formulas, expressions with brackets and alphanumerical rational and square expressions, and use quadratic equations to factor algebraic expressions.

- Solve equations, inequalities, and systems of equations of the first and second order and simple equations with exponential and logarithmic functions, using algebra and digital aids.

- Convert a practical problem into an equation, an inequality, or an equation system, solve it, and assess the validity of the solution.

(Udir, 2006)

2.2.3 The Nepalese Context: Algebra in School Curriculum

In Nepal, the school level curriculum is governed by the Curriculum Development Center (CDC), a government organ under the Ministry of Education, Science and Technology (MOE-Nepal, 2019). The proposed School Sector Development Plan (2016/17 - 2022/23) targets higher achievements in subjects like English, Mathematics and Science (MOE-Nepal, 2016). The seven-year target for grade 5 Mathematics is to reach 60% in 2022/23 from baseline 48% in 2015/16.

Fourth-grade students in Nepal are expected "to solve simple problems of algebraic expressions and equations using algebra skills" (CDC-Nepal, 2009, p. 11). Thus basic algebra appears as early as in fourth-grade in Nepal. The students have to take a compulsory mathematics course until tenth-grade. Students in ninth- and tenth-grade have an opportunity to opt additional mathematics. In the eleventh grade, students opt for different specialization courses. Based on the specialization program they are enrolled in, the students can opt mathematics for physical sciences, mathematics for economics and mathematics for social sciences. Following are the competence goals after eleventh-grade for the students specializing in physical sciences:

- Define functions and illustrate them graphically: inverse function, composite function, functions of a special type (identity, constant, absolute value, greatest integer), algebraic (linear, quadratic and cubic), trigonometric, exponential logarithmic functions.
- Sketch the curves: periodicity of a function, symmetry (about x-axis, y-axis and origin) of elementary functions, monotonicity of a function, sketching graphs of polynomial trigonometric, exponential, logarithmic functions.
- Define polynomial equations, establish fundamental theorem of algebra and quadratic equation and find relation between roots and coefficients of quadratic polynomials.

(CDC-Nepal, 2017, pp. 42 - 45)

2.3 Algebra: Is it a Problem for Students?

Algebra is widely considered difficult to learn, and many students encounter problems coping with it (Bell, 1995). It is not only considered difficult but "the problem is that many students who try hard to understand the fundamental principles of algebra, fail to do so" (Kanbir et al., 2018, p. 1). Students' problems in algebra are no longer confined within the four walls of the schools and/or the school research environments, but have gained adequate media coverages in the recent years (Hacker, 2012; Strauss, 2012; NRK, 2016; Barshay, 2019). Unfortunately, these coverages focus basically on the problems and suggest no considerable solution. Bigger international studies like TIMSS and PISA have subsequently published alarming reports regarding students' achievements in algebra in most of the developed countries (Grønmo et al., 2004; Phan, 2008; Grønmo et al., 2012; Pedersen, 2015).

The problems and difficulties encountered by the students are not limited to a specific geographic location but have become general trends (Kanbir et al., 2018). The challenges posed by the students' difficulties are, according to Harvey, Waits and Demana (1995), "so intimidating to some teachers" (p. 75) that the teachers opt to "route some of the students into other mathematics courses" (p. 75) instead of supporting the students to overcome these difficulties.

Several factors may have contributed to making algebra difficult or at least develop an attitude that algebra is difficult. The abstractness of algebra is one of those major factors (Chazan, 2012; Grønmo et al., 2012). Just two decades ago, algebra was largely seen as "abstract mathematics suitable only for students who were developmentally ready and college intending" (p. 20) in the United States (Chazan, 2012). Most of the students encounter problems learning algebra as they have a weak background in arithmetic (Grønmo et al., 2012). Often, there exists a cognitive gap between arithmetic and algebra. Herscovics and Linchevski (1994) opine that this gap "can be characterized as the students' inability to operate spontaneously with or on the unknown"(p. 1). Students might encounter difficulties since the arithmetic way of thinking that they are used to have several shortcomings (Kieran et al., 2016). Teachers' weak relational understandings of algebra might have resulted in students not experiencing mathematically-strong teaching of the subject (Kanbir et al., 2018). In addition to these, Grønmo et al. (2012) argue that introducing algebra relatively late in schools and prioritizing daily mathematics instead of pure and formal mathematics like arithmetic and algebra have attributed to weak algebra knowledge.

2.4 Early Algebra and Early Introduction of Algebra in Schools

Schliemann et al. (2003) notes that the early research about algebraic reasoning highlighted shortcomings such as students' limited interpretations of the equals sign, misconceptions about letters used for representing variables, refusal to accept an algebraic expressions as an answer, and difficulty in solving equations with variables on both sides of the equals sign. "Many researchers originally attributed such findings to developmental constraints and the inherent abstractness of algebra" (Schliemann et al., 2003, p. 127). Research studies in school algebra have revealed many drawbacks coming from the arithmetic way of thinking among the students of 12 - 15 years when they first meet algebra in high schools Kieran, Pang, Schifter & Ng (2016). In order to overcome these drawbacks, some researchers proposed what they termed as *Early Algebra*. The focus on early algebra is on the 6- to 12-year olds in contrast to the traditional teaching of algebra that starts when the children are 12-year old (Kieran et al., 2016). The main areas of focus in Early Algebra until the early 2000s included:

1. Generalizing related to patterning activity,
2. Generalizing related to properties of operations and numerical structure,
3. Representing relationships among quantities, and

4. Introducing alphanumeric notation

(Kieran et al., 2016, p. 5)

One of the famous tasks in Early Algebra is a box model that is built on the earlier work of Davis (1964). For example: What is the value of Δ in $18 + 27 = \Delta + 29$? Schliemann et al. (2003) opine that the example like this one convinced researchers and mathematics educators how algebra can be introduced to fifth-graders. Carpenter et al. (2003) argue that the questions like this are very effective to reflect on the important properties of the operations. This may also be attributed to the fact that the children learn algebra better if they have a sound knowledge of arithmetic. Mathematics educators have long believed that arithmetic should precede algebra as it provides the foundations for algebra (Warren & Cooper, 2005).

Despite the approaches aimed to make algebra learning easy, many students find algebra difficult. These difficulties may be due to "developmental constraints and the inherent abstractness of algebra, concluding that even adolescents were not ready to learn algebra" (Carraher et al., 2000, p. 137). Further, Filloy and Rojano (1989) claim that students are engaging in algebra only if they can understand and use the syntax of algebra and solve equations with variables on both sides of the equal sign. Bodanskii (1991) observed that the fourth graders who are taught the algebraic notation and equations from grade one could solve the algebra problems and equations better than the seventh graders who received five years of arithmetic instruction starting algebra in grade six only.

3 Method

3.1 Research Design and Sample

This research used a quantitative causal-comparative design to investigate and compare students' achievement in algebra test in Nepal and Norway. The mathematics curricula adopted by two the countries are considered to be the independent variables that cannot be manipulated, while the algebra skills that the students acquire after studying the course based on the curriculum of the respective countries is the dependent variable.

In order to find if the test was appropriate for the groups of the students the test was aimed for, a small pilot study was first carried with the 1T-mathematics students ($N = 18$) in a different school in Norway. 1T is a more rigorous mathematics course in upper secondary school in Norway and forms a foundation to higher-level mathematics for students opting science path later. The main aim of the pilot study was to examine if the stipulated time was enough for the test, the percentage of the students capable of answering all the questions, and the degree of hardness or the softness of the tasks. The answer-sheets were collected, evaluated and coded using the same scoring guide (see subsection 3.3.1) that would be used later during the data analysis. The participants of the pilot study were expected to complete the test within 45 minutes, but they were

informed that they could get more time if needed. At least 3 participants used about an hour. Necessary changes were made in the tasks included after analyzing the result of the pilot study.
The 111 students who participated in this research project in Norway were the students studying 1T mathematics course and attend a public upper secondary school. The mathematics students were organized in different blocks consisting of four different groups in this school. A block consisting of a maximum number of students was selected for the project. A fifth group was included to acquire a targeted sample size ($N > 100$).

According to National Education Accounts Report prepared by UNESCO Institute for Statistics (UNESCO-UIS), about 30 % of Nepalese students attend private schools (UNESCO-UIS, 2016). The corresponding percentage for the Norwegian students attending upper secondary schools is about 22 % (Statistics-Norway, 2018). Therefore, both private and public schools in Nepal are included in sampling and the same number of students ($N = 111$ each) attending public and private upper secondary schools (class 11) were selected. Like their Norwegian counterparts, these students study a rigorous course in mathematics assigned for the students opting science path in the upper secondary education. The students of the public school that participated in this project were further grouped as "General Science Students" and "Engineering Science Students" while those of the private school followed a "General Science Course." No any other demarcation was made apart from matching participants' class and study path (1T in Norway and mathematics for science stream in Nepal). The science stream students in Nepal were selected so that their mathematics standard would be comparable with the 1T-mathematics students in Norway. Apart from these two groups of students from Nepal, the same test was also run with the students of class nine (public: $N = 106$ and private: $N = 130$) and ten (public: $N = 131$ and private: $N = 113$) of both the private and the public schools. The sampling, both in Norway and in Nepal, was a convenience sampling as more rigorous probability sampling could be very difficult to achieve within the targeted school environments.

3.2 Data collection

The data collection was done through an open-ended algebra test that consisted of 17 tasks. 7 of the tasks were adopted from Kunnskap, Utdanning og Læring - Knowledge, Education and Learning (KUL) project organized by University of Agder and financed by the Norwegian Research Council (Espeland, 2017). Other 8 tasks were taken from the past exams prepared by Norwegian Education Directorate (Udir) for the students who have opted theoretical mathematics (1T) in upper secondary school in Norway. The tasks prepared by KUL-project focus especially on basic algebra knowledge such as numbers and letters, text usage and equalities. The tasks adapted from Udir cover the objectives aimed at measuring basic algebra skills, sign manipulation, root expressions, simplification, factorization, equations and word-problem included in "numbers and algebra" in the curriculum in mathematics subjects (MAT-04).

The students were informed about the project and were invited to participate voluntarily. All the interested students in the selected groups got the opportunity to participate. The participants were contacted in their regular teaching sessions by their mathematics teachers who were informed about the project. The test was done without any aid (calculator, computer, etc.) but the students were allowed to use the rough papers. The students were asked to show the necessary steps and procedures they used to solve the problems. Apart from this, the data collection also included the students' age, sex and their grade (score) in mathematics in tenth-grade before they started at the upper secondary school.

3.3 Data Analysis

3.3.1 Coding

The participants were divided into seven different groups as: 9th-graders in public school in Nepal (NEPPUB09), 10th-graders in public school in Nepal (NEPPUB10), 11th-graders in public schools in Nepal (NEPPUB11), 9th-graders in private school in Nepal (NEPPVT09), 10th-graders in private school in Nepal (NEPPVT10), 11th-graders in private school in Nepal (NEPPVT11) and 11th-graders in Norway (NOR11). Furthermore, grade-wise analyses were also done grouping the participants in their respective grades as: 9th-graders in Nepal (NEP09), 10th-graders in Nepal (NEP10), 11th-graders in Nepal (NEP11) and 11th-graders in Norway (NOR11).

The answer sheets produced by the participants ($N = 813$, in total) were evaluated, graded and coded for analysing quantitatively in SPSS. The coding of the open-ended tasks followed a scale from 0 to 3. High achievement was awarded 3, average achievement was awarded 2, low achievement was awarded 1 and a code of 99 was given to "not attempted" task. Other parameters were also coded accordingly. Student's age is a numeric scale value and was mentioned as it was. Student's sex was given a value 1 for boys and 2 for girls. The students' grades in mathematics in tenth-grade followed a scale of 1-6 and reflected what they achieved. The percentage or the letter-grades of the Nepalese students were converted to corresponding number-grade practised in the Norwegian system.

Evaluating 813 answer-sheets was a tedious job, but a good routine was established that both made the task less cumbersome and guaranteed the uniformity. Sensor guidelines with answer keys, possible errors, students' misconceptions, and weaknesses were prepared and followed throughout the process. To avoid the evaluation biases, all the answer sheets were coded minimizing the time-gap between the subsequent evaluations.

3.3.2 Statistical Analyses and Interpretation

Descriptive analyses, reliability testing (Inter-Rater Reliability, see sub-section 3.4.1) and inferential statistical measures were conducted using SPSS, version 25. Initial data organisation was also done using Excel. Some graphics are produced using MATLAB for better visualization. Statistical significance of the difference between the mean score of different groups was analysed by using one-way analysis of variance (ANOVA). The results of ANOVA are presented in sub-section 4.1 and summarized graphically in Figure 1. All pairwise comparisons among means of different groups were done using Tukey HSD test. These results are presented sub-section 4.1 and summarized graphically in Figure 2.

3.4 Validity, Reliability and Ethical Considerations

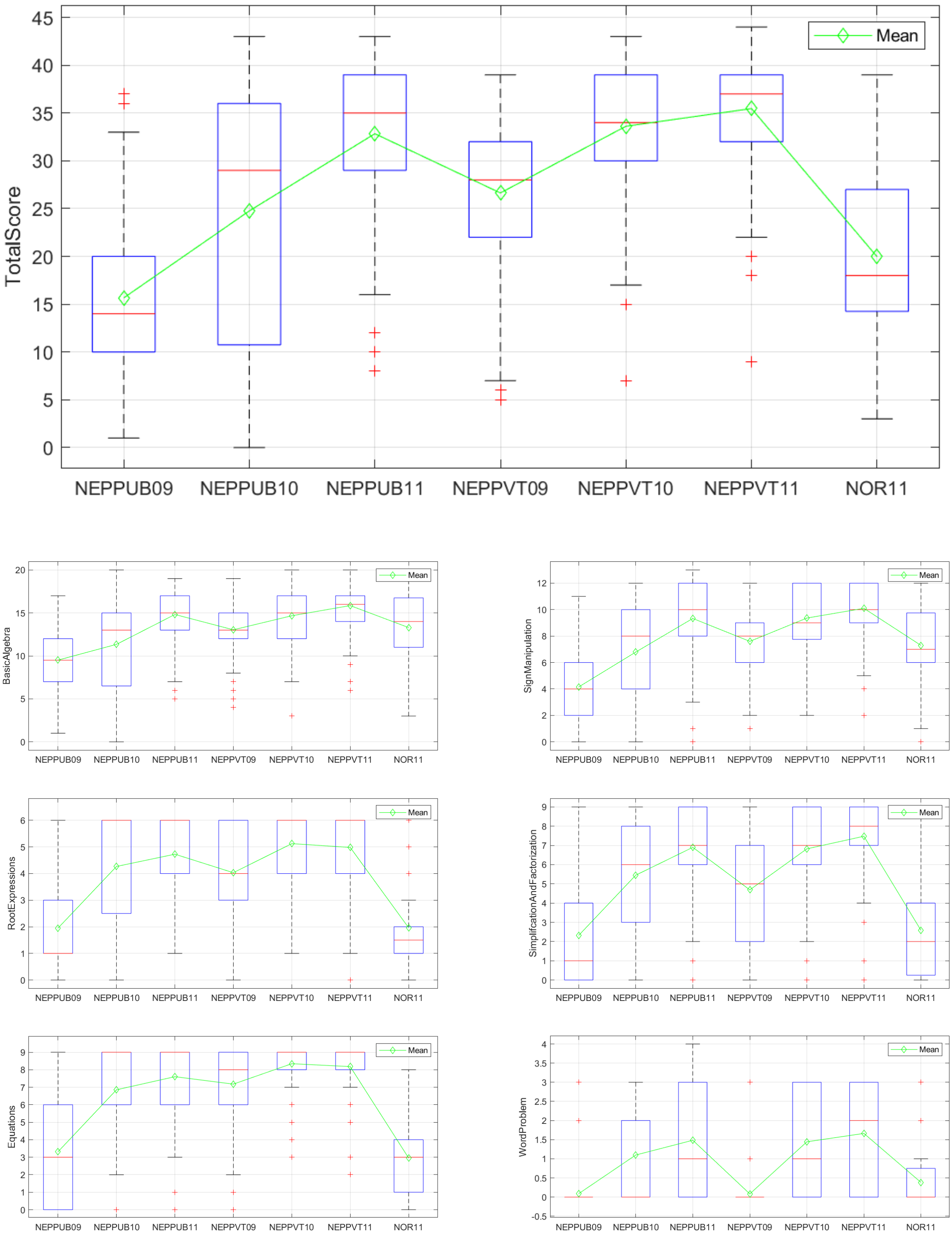
3.4.1 Inter-Rater Reliability

Though I did the evaluation and coding myself, 21 answer-sheets were coded together with a research fellow based in Aarhus University, Denmark. Before the coding began, the scoring guide was shared and the evaluation procedure was discussed. After the coding was done individually, in order to evaluate whether the established coding system was reliable, inter-rater reliability was determined with the intraclass correlation coefficient in SPSS. Intraclass Correlation Coefficient of 0.993 of the average measures with a lower bound of 0.984 and an upper bound of 0.997 at 95 % confidence interval suggested an excellent agreement between the evaluators (Cicchetti, 1994). Despite the excellent agreement for the established coding system, the scoring guide was reviewed for possible flaws and anomalies that resulted in slight differences in the average points awarded in some of the questions.

3.4.2 Assumptions and Tests for One-Way ANOVA

The model assumptions that include independence, normality and homogeneity of variances are to be met when utilizing t-test or ANOVA (Glass & Hopkins, 1996). Owing to the nature of this study, random sampling was not possible (refer sub-section 3.1) but the selection of a participant was independent upon the selection of another participant. The participants had equal opportunity to answer the problems posed. The most important issue of independence is that "observations within or between groups are not paired, dependent, correlated, or associated in any way" (Glass & Hopkins, 1996, p. 295). As this assumption is met, the issue of independence is assumed to be fulfilled in this study. According to Glass & Hopkins (1996), the consequences of violating the normality assumptions are rather minimal, especially when the research is conducted with equal sample sizes in all the groups. Since, $N_{NOR11} = N_{NEPPUB11} = N_{NEPPVT11} = 111$, ANOVA is robust to these groups. These are the main groups in focus. Since, there was not big difference between the sample size of different groups, the normality requirements are considered to be met for the dataset. But like normality, when sample sizes are equal among the groups, ANOVA is robust to heterogeneous variances. It is, thus, assumed that the issue with non-homogeneity of variances does not have dramatic effect in this study.

Figure 1: Box and whisker diagram representing mean total score and mean scores of other parameters of different groups.



3.4.3 Ethical Considerations

Norwegian Center for Research Data (NSD), which regulates research activities in Norway, was contacted regarding the procedures of data collection. Since the project did not collect any personal information of the informants and that everything was aimed to be kept anonymous, it was not necessary to seek any written permission from NSD. In addition to this, the Nepalese regulations that govern research activities were followed. The project, from the start to the end (and beyond) would strictly abide by the research ethics and the guidelines mentioned by NESH (2016). The participants were informed about the project both verbally and in writing and that their participation was entirely voluntary. The participants were guaranteed that they might withdraw their participation at any time.

4 Results

Mean total score obtained by the students in the test is considered as a measure of achievement in algebra. The tasks were further classified so that they reflect students' achievements in different categories like *Basic Algebra Skills, Sign Manipulation, Root Expressions, Simplification and Factorization, Equations, and Word Problems.* Total score is the function of these parameters. Differences in mean scores between the seven groups (and four classes) were analysed using ANOVA.

4.1 Total Score as a Measure of Achievement

Table 1 displays the distribution of students in different groups and the total score they received in the test. Except for two blank answer-sheets from 10th grade students of a public school in Nepal, all other participants have responded to the test achieving total scores ranging from .00 to 44.00 (44.00 was the maximum possible score). Mean of the total score is distributed as: NEPPUB09 ($M = 15.66, SD = 8.13$) , NEPPUB10 ($M = 24.76, SD = 13.97$), NEPPUB11 ($M = 32.84, SD = 7.83$), NEPPVT09 ($M = 24.64, SD = 7.21$), NEPPVT10 ($M = 33.62, SD = 7.22$), NEPPVT11 ($M = 35.47, SD = 5.81$) and NOR11 ($M = 19.96, SD = 8.86$). The 11th-grade students of the private school in Nepal (NEPPVT11) have the highest score, while the 9th-grade students of the public school in Nepal (NEPPUB09) have scored lowest. The Norwegian students (NOR11) have scored lower than their Nepalese counterparts (NEP11). This score is a bit higher than the 9th-graders of the public school in Nepal (NEPPUB09). When the scores are analysed grade-wise, it is seen that there was a slight tendency for a lower mean grade score among Norwegian students than 9th-graders in Nepal (NEP09) when public and private schools are combined ($M = 21.71, SD = 9.38$).

The one-way ANOVA shows that there was a significant difference between the mean total scores at the $p < .05$ level for different groups ($F(6, 804) = 78.10$, $p < .001$). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 is significantly different than the mean scores of other groups. NOR11 had significantly weaker achievement than NEPPUB10, NEPPUB11, NEPPVT09, NEPPVT10 and NEPPVT11 but a significantly better achievement than NEPPUB09. When analysed grade-wise, one-way ANOVA shows that there was a significant difference between the mean scores at the p < .05 level for different grades (F(3, 807) = 87.57, p < .001). Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 is significantly lower than NEP11 ($M = 34.13, SD = 7.00$) and NEP10 ($M = 28.90, SD = 12.14$), but not significantly different (p = .39) from NEP09 ($M = 21.71, SD = 9.38$).

Table 1: Descriptive statistical measures of total score of different groups.

Total Score										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	106	0	15.66	8.13	.79	1.00	37.00	.69	-.11
NEPPUB10	131	129	2	24.76	13.97	1.23	.00	43.00	-.76	-.94
NEPPUB11	111	111	0	32.84	7.83	.74	8.00	43.00	-1.10	.97
NEPPVT09	130	130	0	26.64	7.21	.63	5.00	39.00	-.76	.39
NEPPVT10	113	113	0	33.62	7.22	.68	7.00	43.00	-.89	.80
NEPPVT11	111	111	0	35.47	5.81	.55	9.00	44.00	-1.41	3.45
NOR11	111	111	0	19.96	8.36	.79	3.00	39.00	.40	-.42
Total	813	811	2	27.02	11.06	.39	.00	44.00	-.62	-1.22

4.1.1 Basic Algebra Skills

Descriptive data for the score in Basic Algebra Skills are presented in Table 2. The one-way ANOVA showed that there was a significant difference between the mean scores of Basic Algebra Skills at the p < .05 level for different groups (F(6, 803) = 38.62, p < .001). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 ($M = 13.30, SD = 3.73$) is significantly higher than the mean scores of NEPPUB09 ($M = 9.51, SD = 3.44$) and NEPPUB10 ($M = 11.34, SD = 5.81$), while it is significantly lower than NEPPUB11 ($M = 14.82, SD = 3.13$) and NEPPVT11 ($M = 15.86, SD = 2.61$). No significant difference was found between NOR11 and NEPPVT09 ($M = 13.04, SD = 2.90$) and NEPPVT10 ($M = 14.67, SD = 3.18$). Grade-wise analysis showed that there was a significant difference between the mean scores at the p < .05 level for different grades (F(3, 806) = 37.56, p < .001).

Table 2: Descriptive statistical measures of the scores in Basic Algebra Skills of different groups.

Basic Algebra Skills										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	106	0	9.51	3.44	.33	1.00	17.00	.11	-.63
NEPPUB10	131	128	3	11.34	5.81	.51	.00	20.00	-.81	-.73
NEPPUB11	111	111	0	14.82	3.13	.30	5.00	19.00	-.86	.45
NEPPVT09	130	130	0	13.04	2.90	.25	4.00	19.00	-.65	.27
NEPPVT10	113	113	0	14.67	3.18	.30	3.00	20.00	-.64	.61
NEPPVT11	111	111	0	15.86	2.61	.25	6.00	20.00	-1.15	1.99
NOR11	111	111	0	13.30	3.73	.35	3.00	19.00	-.62	-.10
Total	813	810	3	13.20	4.21	.15	.00	20.00	-.95	.63

Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 was significantly higher than NEP09 ($M = 11.45, SD = 3.61$) and significantly lower than NEP11 ($M = 15.34, SD = 2.92$), but not significantly different (p = .82) from NEP10 ($M = 12.90, SD = 5.04$).

4.1.2 Sign Manipulation

There was a significant difference between the mean scores of Sign Manipulation at the $p < .05$ level for different groups (F(6, 798) = 53.01, $p < .001$). Post hoc comparisons using the Tukey HSD test showed that the mean score for NOR11 ($M = 7.30, SD = 2.79$) is significantly higher than the mean score of NEPPUB09 ($M = 4.15, SD = 2.70$), while it is significantly lower than NEPPUB11 ($M = 9.33, SD = 2.93$), NEPPVT10 ($M = 9.36, SD = 2.54$) and NEPPVT11 ($M = 10.10, SD = 2.15$). No significant difference was found between NOR11 and NEPVT09 ($M = 7.60, SD = 2.66$) and NEPPUB10 ($M = 6.79, SD = 4.04$). When analysed grade-wise, one-way ANOVA showed that there was a significant difference between the mean scores at the $p < .05$ level for different grades (F(3, 801) = 53.45, $p < .001$). Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 was significantly higher than NEP09 ($M = 6.05, SD = 3.18$) and significantly lower than NEP11 ($M = 7.30, SD = 2.87$), but not significantly different (p = .19) from NEP10 ($M = 8.02, SD = 3.64$). With just 8 missing responses, the response rate for the category is very high (99.02 %) as seen in Table 3.

Table 3: Descriptive statistical measures of score in Sign Manipulation of different groups

Sign Manipulation										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	106	0	4.15	2.70	.26	.00	11.00	.63	-.09
NEPPUB10	131	123	8	6.79	4.04	.36	.00	12.00	-.58	-.98
NEPPUB11	111	111	0	9.33	2.93	.28	.00	13.00	-1.25	1.34
NEPPVT09	130	130	0	7.60	2.66	.23	1.00	12.00	-.37	-.35
NEPPVT10	113	113	0	9.36	2.54	.24	2.00	12.00	-.69	-.41
NEPPVT11	111	111	0	10.10	2.15	.20	2.00	12.00	-1.51	2.78
NOR11	111	111	0	7.30	2.87	.27	.00	12.00	-.24	-.57
Total	813	805	8	7.81	3.43	.12	.00	13.00	-.61	-.54

4.1.3 Root Expressions

The one-way ANOVA showed that there was a significant difference between the mean scores of Root Expressions at the $p < .05$ level for different groups (F(6, 781) = 67.77, $p < .001$). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 ($M = 1.96, SD = 1.76$) is significantly lower than the mean scores of NEPPVT09 ($M = 4.03, SD = 1.73$), NEPPUB10 ($M = 4.27, SD = 2.19$), NEPPVT10 ($M = 5.12, SD = 1.38$), NEPPUB11 ($M = 4.73, SD = 1.57$) and NEPPVT11 ($M = 4.98, SD = 1.55$), while no any significant difference was found between NOR11 and NEPUB09 ($M = 1.95, SD = 1.58$). When analysed grade-wise, one-way ANOVA showed that there was a significant difference between the mean scores at the $p < .05$ level for different grades [F(3, 784) = 91.27, $p < .001$]. Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 was significantly lower than all other classes, i.e. NEP09 ($M = 3.10, SD = 1.96$), NEP10 ($M = 4.68, SD = 1.89$) and NEP11 ($4.86, SD = 1.57$). Descriptive data for the score in Root Expressions are presented in Table 4.

Table 4: Descriptive statistical measures of score in Root Expression of different groups.

Root Expressions										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	101	5	1.9505	1.57719	.15694	.00	6.00	.79	-.16
NEPPUB10	131	120	11	4.2667	2.19140	.20005	.00	6.00	-.90	-.66
NEPPUB11	111	110	1	4.7273	1.57334	.15001	1.00	6.00	-.91	-.39
NEPPVT09	130	125	5	4.0320	1.73175	.15489	.00	6.00	-.47	-.73
NEPPVT10	113	113	0	5.1239	1.37017	.12889	1.00	6.00	-1.42	1.04
NEPPVT11	111	111	0	4.9820	1.55495	.14759	.00	6.00	-1.37	.78
NOR11	111	108	3	1.9630	1.76108	.16946	.00	6.00	1.03	.17
Total	813	788	25	3.9048	2.09430	.07461	.00	6.00	-.46	-1.2

4.1.4 Simplification and Factorization

The one-way ANOVA showed that there was a significant difference between the mean scores of Simplification and Factorization at the p < .05 level for different groups (F(6, 776) = 71.74, p < .001). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 ($M = 2.59, SD = 2.59$) is significantly lower than the mean scores of NEPPVT09 ($M = 4.69, SD = 2.86$), NEPPUB10 ($M = 5.45, SD = 3.12$), NEPPVT10 ($M = 6.82, SD = 2.32$), NEPPUB11 ($M = 6.90, SD = 2.31$) and NEPPVT11 ($M = 7.48, SD = 1.94$), while no any significant difference (p = .99) was found between NOR11 and NEPUB09 ($M = 2.32, SD = 2.46$). When analysed grade-wise, one-way ANOVA showed that there was a significant difference between the mean scores at the p < .05 level for different grades (F(3, 779) = 112.19, p < .001). Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 was significantly lower than all other classes, i.e. NEP09 ($M = 3.65, SD = 2.93$), NEP10 ($M = 6.13, SD = 2.83$) and NEP11 ($7.19, SD = 2.15$). Table 5 presents the descriptive data for simplification and factorization.

Table 5: Descriptive statistical measures of score in Simplification and Factorization of different groups.

Simplification and Factorization										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	97	9	2.32	2.46	.25	.00	9.00	1.04	.01
NEPPUB10	131	115	16	5.45	3.12	.29	.00	9.00	-.53	-1.05
NEPPUB11	111	111	0	6.90	2.31	.22	.00	9.00	.23	.46
NEPPVT09	130	125	5	4.69	2.86	.26	.00	9.00	.01	-1.28
NEPPVT10	113	113	0	6.82	2.32	.22	.00	9.00	-.94	-.04
NEPPVT11	111	111	0	7.48	1.94	.18	.00	9.00	-1.80	3.65
NOR11	111	111	0	2.59	2.59	.25	.00	9.00	.98	.06
Total	813	783	30	5.23	3.17	.11	.00	9.00	-.33	-1.32

4.1.5 Equations

The descriptive data for the score in equations are presented in Table 6. The one-way ANOVA showed that there was a significant difference between the mean scores of Equations at the $p < .05$ level for different groups ($F(6, 777) = 96.15$, $p < .001$). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 ($M = 2.95, SD = 2.78$) is significantly lower than the mean scores of NEPPVT09 ($M = 7.18, SD = 2.32$), NEPPUB10 ($M = 6.85, SD = 3.08$), NEPPVT10 ($M = 8.35, SD = 1.33$), NEPPUB11 ($M = 7.60, SD = 2.22$) and NEPPVT11 ($M = 8.18, SD = 1.43$), while no any significant difference ($p = .93$) was found between NOR11 and NEPUB09 ($M = 3.31, SD = 3.01$). When analysed grade-wise, one-way ANOVA showed that there was a significant difference between the mean scores at the $p < .05$ level for different grades ($F(3, 780) = 110.21$, $p < .001$). Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 was significantly lower than all other classes, i.e. NEP09 ($M = 5.47, SD = 3.27$), NEP10 ($M = 7.61, SD = 2.47$) and NEP11 ($7.89, SD = 1.89$).

Table 6: Descriptive statistical measures of score in Equations of different groups.

Equations										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	102	4	3.31	3.01	.30	.00	9.00	.41	-1.01
NEPPUB10	131	110	21	6.86	3.08	.29	.00	9.00	-1.34	.50
NEPPUB11	111	111	0	7.60	2.22	.21	.00	9.00	-1.63	1.90
NEPPVT09	130	129	1	7.18	2.32	.20	.00	9.00	-1.29	.96
NEPPVT10	113	113	0	8.35	1.33	.13	3.00	9.00	-2.24	4.69
NEPPVT11	111	111	0	8.18	1.43	.14	2.00	9.00	-1.88	3.51
NOR11	111	108	3	2.95	2.78	.27	.00	9.00	.84	-.31
Total	813	784	29	6.42	3.14	.11	.00	9.00	-.91	-.58

4.1.6 Word Problem

The one-way ANOVA showed that there was a significant difference between the mean scores of Word Problem at the $p < .05$ level for different groups ($F(6, 600) = 28.71$, p < .001). Post hoc comparisons using the Tukey HSD test indicated that the mean score for NOR11 ($M = 0.38, SD = 0.79$) is significantly lower than the mean scores of NEPPUB10 ($M = 1.10, SD = 1.28$), NEPPVT10 ($M = 1.44, SD = 1.45$), NEPPUB11 ($M = 1.46, SD = 1.31$) and NEPPVT11 ($M = 1.66, SD = 1.39$), while no any significant difference was found between NOR11 and NEPUB09 ($M = 0.09, SD = 0.48$) and NEPPVT09 ($M = 0.08, SD = 0.40$). When analysed grade-wise, one-way ANOVA showed that there was a significant difference between the mean scores at the p < .05 level for different grades (F(3, 780) = 110.21, p < .001). Corresponding post hoc comparisons using the Tukey HSD test indicated that the mean score of NOR11 was significantly lower than NEP10 ($M = 1.28, SD = 1.38$) and NEP11 ($1.57, SD = 1.35$), while no any significant difference (p = .23) was found between NOR11 and NEP09 ($M = 0.09, SD = 0.44$). Table 7 presents the descriptive data for word problem.

Table 7: Descriptive statistical measures of score in Word Problem of different groups.

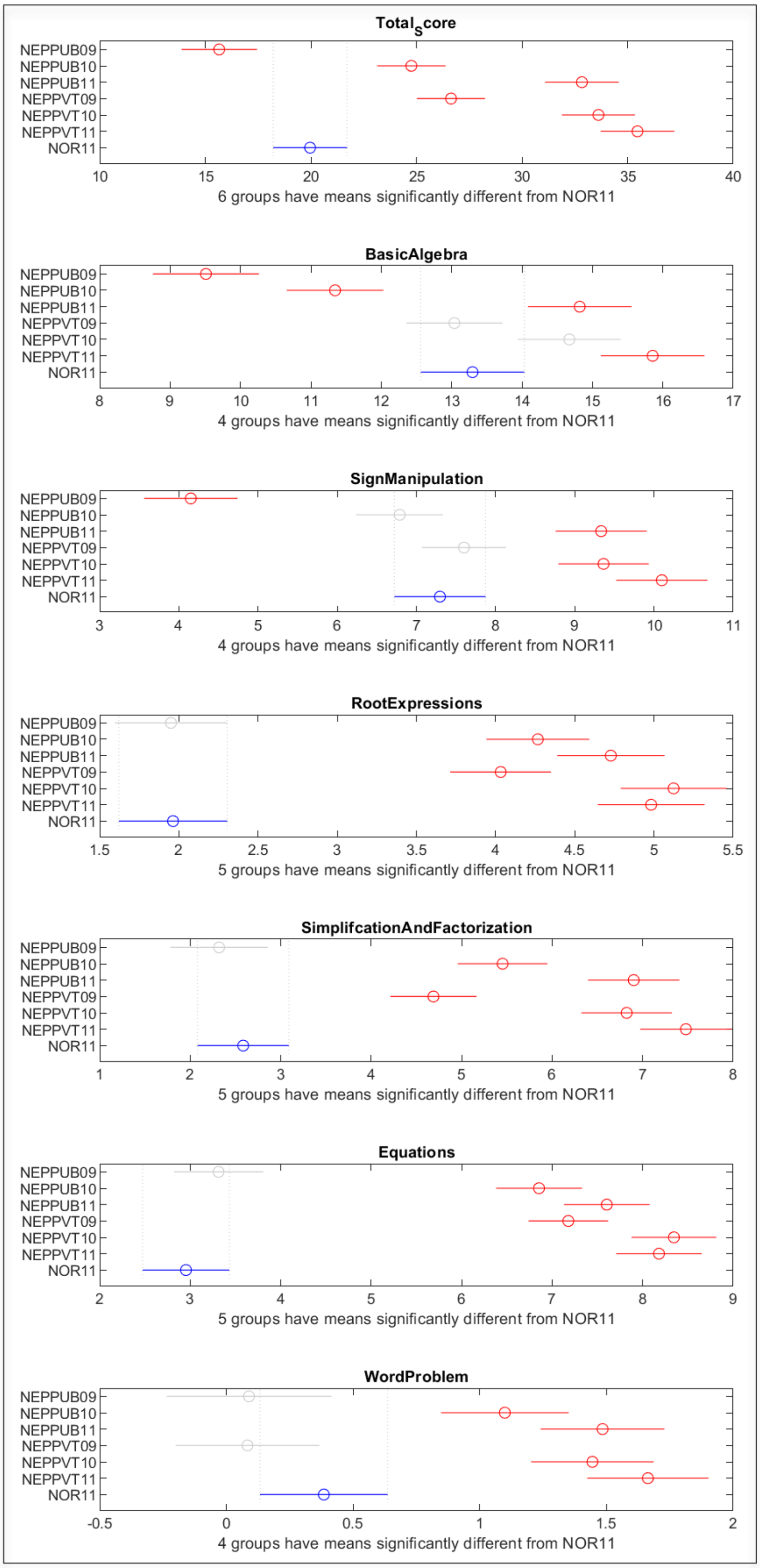
Word problem										
Group	Total N	Valid N	Missing N	Mean	SD	Std. Error	Min.	Max.	Skewness	Kurtosis
NEPPUB09	106	56	50	.09	.48	.06	0	3	5.46	29.96
NEPPUB10	131	91	40	1.10	1.28	.13	0	3	.52	-1.50
NEPPUB11	111	97	14	1.48	1.33	.14	0	4	.05	-1.74
NEPPVT09	130	72	58	.08	.40	.05	0	3	6.02	40.39
NEPPVT10	113	99	14	1.44	1.45	.15	0	3	0.06	-1.97
NEPPVT11	111	101	10	1.66	1.34	.14	0	3	-.22	-1.84
NOR11	111	91	20	.38	.79	.08	0	3	2.29	4.74
Total	813	607	206	.99	1.30	.05	0	3	.73	-1.29

Multi-comparison graph in Figure 2 shows the significant differences between the average total score of different groups. The significant differences of the average score of different parameters discussed in sub-section 4.1 among different groups are also represented.

4.2 Number of Years Algebra Studied and Achievement

The analysis of the total score and other parameters used to evaluate achievements in algebra showed that the students who have studied algebra from early grades have scored significantly higher as discussed in section 4.1. A Pearson correlation coefficient was computed to assess the relationship between the number of years students studied algebra and their achievement (total score). There was a positive correlation between years of algebra ($M = 5.71, SD = 1.01$) and total score ($M = 27.02, SD = 1.06$), r = .49, p < .001. A simple linear regression was calculated to predict total score based on number of years algebra is studied. A significant regression equation was found (F(1, 809) = 248.36, p < .001), with an R^2 of .24.

Figure 2: Multi-comparison graph representing the total score and the scores of other categories of different groups.



5 Discussion

5.1 Interpretation of the Test Achievements

The general impression one can draw from this study is that the Norwegian upper secondary school students (NOR11, M = 19.96) achieve significantly lower than their Nepalese counterparts (NEP11, M = 34.13) on the algebra test. The score of the eleventh grade Norwegian students is also significantly lower than that of grade ten students (NEP10, M = 28.90) in Nepal, while it is not significantly different from that of grade 9 students (NEP09, M = 21.71) in Nepal. The total score as a function of parameters like *Basic Algebra Skills, Sign Manipulation, Root Expressions, Simplification and Factorization, Equations, and Word Problems* gives an extended image of students' achievement. The Norwegian students achieve comparatively better in basic algebra skills with a mean of 13.3 (65.5 %) of the maximum possible score of 20.00 for this parameter. Compared to other parameters, the Norwegian students also demonstrate a better understanding of sign manipulation with a mean of 7.3 (56.15 %) of the maximum possible score of 13 for this parameter. As we move towards more demanding topics like root manipulation, simplification and factorizations, and equations, Norwegian students' achievement falls drastically. All other five study groups except the ninth graders in Nepal (NEPPUB09) achieve significantly higher than the Norwegian students in these three parameters.

Norwegian students' weak achievement in algebra is not a new phenomenon. Analysing the data from TIMSS 2011, Grønmo, Borge and Rosén (2013) conclude that algebra and geometry are less prioritized in schools in Norway. They argue that the students' weak achievement in algebra should be the consequence of this drawback. The weakness of the students in algebra may not be totally explained by the fact that they receive fewer algebra hours, but maybe it has a connection with what we think about algebra in schools in Norway (Naalsund, 2012). The functional approach of algebra has not been in focus in curriculum, and therefore, the Norwegian students first encounter algebra as "generalised arithmetic with an emphasis on the transformational activity" (Espeland, 2017, p. 48). Warren and Cooper (2005) opine that functional thinking of algebra helps in developing an understanding of the relationships between the operations.

Analysing the data from TIMSS Advanced 2008, Pedersen (2015) reveals that the Norwegian students "perform weakly on items that place high demands on symbol manipulation; these are usually purely mathematical items with expressions and formulas given in the text" (p. 89). The Norwegian students were stronger in the items that required text comprehension, application, and modeling of mathematical concepts (Pedersen, 2015). This study does not support this conclusion. Norwegian students have achieved a mean of 0.79 (12.67 %) of the maximum possible score of 3 in word-problem. The weak performance of the Norwegian students in areas that were more demanding might well be attributed to their weak achievement in basic algebra and sign manipulation compared to that of their Nepalese counterparts. However, one should be a bit more cautious in interpreting the result of the parameter "word-problem". In this study, this was a single task that required the students to make use of their knowledge about triangles in geometry and apply it to formulate and solve an algebraic equation.

5.2 Early Introduction of Algebra and its Implications

This study has clearly shown that there is a significantly positive correlation between the number of years the students studied algebra and their performance in the algebra test. The test score kept isolated as a measure of achievement, may not explain why some students perform better than other students. The students' learning activities and thus, their achievement may be well influenced by among others parental guidance (Cai, Moyer & Wang, 1999), priorities set by the specific education system (Cai et al., 2011), effective classroom management (Hiebert & Grouws, 2007), socio-cultural activities (Radford, 2008), and students' own motivation in learning in term of needs and goals (Wæge, 2009). This study did not collect any quantitative data associated with these factors, but the interpretations shall be made based on the priorities set by the Norwegian and Nepalese curriculum in algebra.

Low achievement of the Norwegian students in this test should be a worrying concern as the test included about 60 % of the tasks from a syllabus they were taught and prepared for. The other 40 % of the tasks were also very familiar problems in basic algebra. On the other hand, the Nepalese students got the information about the test a day before or on the same day the test was held. Moreover, they were completely unaware of research studies as they had never participated in a similar study before. The availability of resources in classrooms in Norway and Nepal cannot be compared (see section 1). Schools in Nepal are plagued mainly by among other poor school environments, weak classroom management, and absence of child-centered interactive teaching approaches (Bhatta, 2008). There were a total of 65 registered students in the same classroom in the public school that participated in this study. The situation of the private school was a bit better, but still, 45 students were taught in the same classroom. The pedagogical approach adopted was largely teacher-centered blackboard teaching in both the schools. On the other hand, the Norwegian schools are fully funded by the Government, compulsory free education for all is enshrined in law and the quality in education is assured by the Government (Braathe & Otterstad, 2014).

Given this reality, it would be reasonable to assume that the higher achievement of the Nepalese students can be attributed to their early encounter with school algebra. The Nepalese students start learning algebra at fourth-grade when they are introduced to addition and subtraction of like terms, simple equations of addition, subtraction, multiplication and division with box notation solved through inspection, hit and trail and using variables (CDC-Nepal, 2009). The Norwegian students meet similar competence goals of solve and calculating with parentheses in addition, subtraction and multiplication of numbers and setting up and solving simple equations only when they are introduced to algebra in seventh-grade (Udir, 2006). By this time, the Nepalese students have already started to classify the polynomials, multiply and divide trinomials by binomials and make geometric interpretation of $(a \pm b)^2$ and apply it (CDC-Nepal, 2012). Hence, the Norwegian students have received four years of algebra teaching before they started at eleventh-grade while their Nepalese counterparts have received seven years of teaching. Unlike the Norwegian context, in several countries like China, Singapore and South Korea, students begin the formal study of algebra much earlier (Cai et al., 2005, 2011). The successful introduction of algebra to younger children in former Soviet Union has dragged the attention of researchers (Schliemann et al., 2003).

Though the notion of *Early Algebra* has been widely discussed for the past two decades (Cai, 1998; Cai et al., 2011; Carpenter et al., 2003; Kaput, 1998; Kaput et al., 2017; Kieran et al., 2016), *Early Introduction of Algebra in Schools* has not seen similar emphasis in mathematics education research. Further research is needed to explore the implications of including algebra in curriculum in early classes.

6 Concluding Remarks

The analyses of the test results have suggested that the introduction of algebra in an early stage in Nepalese schools has significance in promoting students' understanding of algebra compared to the Norwegian students. The findings raise a question if most of the western countries that postpone the introduction of algebra until adolescence (Carraher, Schliemann, Brizuela & Earnest, 2006) should introduce algebra earlier in the mathematics curriculum. Many researchers, however, believe that young children are incapable of learning algebra because they lack cognitive ability to handle concepts like variables and functions (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; MacGregor, 2001). At the same time, during the past thirty years, we have seen increased interest and focus on the development of algebraic thinking in the early stage . Davis (1985) argued that algebra should begin in Grade 2 or 3. Despite encountering several pedagogical, managerial, technological and socio-economic constraints, the Nepalese students who participated in this study have demonstrated significantly better performance in the test than their Norwegian counterparts who, in contrast, enjoy profound benefits in schools. The findings should, however, be interpreted in light of the methodological considerations of the study discussed in sub-section 3.4.2. In addition to this, it is very difficult to compare the achievements of the students in two different countries with very different education system, teaching-learning environment and resource availability. Owing to the nature of this study, it might be difficult to generalize the results to a bigger population (Salkind, 2010). The issue of generalization is common to causal-comparative studies, but future work should consider the extent to which these findings can be generalized. Based on the findings, the study concludes that it is reasonable to argue for early introduction of algebra in Norwegian schools, but further research is needed to explore its pedagogical implications and how effectively it can be incorporated in current teaching-learning environment in Norway.

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Appendices

Appendix-1: Algebra Test for the Pilot Study

Algebraprøve
for prosjektet

Does Early Introduction of Algebra in Schools Make any Difference? A Comparative Study of Algebra Skills of High School Students in Norway and Nepal.

Dette er et pilotprosjekt som er en del av en sammenligningsstudie som vil undersøke om tidlig innføring av algebra på skolen hjelper å styrke elevenes algebraferdigheter. Samme prøve skal kjøres i Norge og Nepal med elevene på VG-1 som har valgt teoretisk matte (1T i Norge og *maths for Science stream* i Nepal).

Det skal ikke samles inn identifiserbare personopplysninger som navn, personnummer, fødselsdato, skole, bokommune, bilde, lydopptak, telefonnummer osv og derfor der deltakere helt anonyme. Deltakelsen er frivillig og deltakere kan trekke fra undersøkelsen (i dette tilfelle denne prøven) når som helst.

Samtykke

Jeg er informert om følgende

- ☐ Dette er en del av en sammenligningsstudie son skal kjøres i Norge og Nepal.
- ☐ Min deltakelse i dette pilotprosjektet er frivillig og hvis jeg vil kan jeg trekke meg når som helst.
- ☐ Det skal ikke samles inn identifiserbare personopplysninger som navn, personnummer, fødselsdato, skole, bokommune, bilde, lydopptak, telefonnummer osv og derfor deltar jeg helt anonymt.

Vennligst fyll inn følgende før du tar prøven

Alder:_______

Kjønn: _______

Din karakter i matte på 10. trinn: ______

Testen skal gjøres uten kalkulator, men kladdark kan brukes. Skriv svarene på svararkene du har fått. Vis utregningene.

---

1. **Skriv enklere dersom det er mulig:**
   a) $2y \cdot y^2$
   b) $a - 3a + 2a$
   c) $5a - 2(7 - a) + 6$
   d) $2x(x-2) - (x-2)(2x+1)$ **(Udir, V15)**
   e) $\sqrt{15} \cdot \sqrt{5} - \sqrt{48}$ **(Udir, V18)**

2. **Finn verdien av uttrykkene:**
   a) $a + b - c$, når, $a = 1,\ b = 2,\ c = 3$
   b) $3b^2 - abc$, når, $a = 3,\ b = -1,\ c = 5$

3. **Alltid sant, aldri sant eller kan være sant**

   a) $\boldsymbol{a + b \cdot 2 = 2b + a}$ Dette

   ☐ er alltid sant

   ☐ er aldri sant

   ☐ kan være sant, når ____________________

   b) $\boldsymbol{x + y + z = x + p + z}$ Dette

   ☐ er alltid sant

   ☐ er aldri sant

   ☐ kan være sant, nemlig når _______________

   c) $\boldsymbol{\frac{2x+1}{2x+1+5} = \frac{1}{6}}$ Dette

   ☐ er alltid sant

   ☐ er aldri sant,

   ☐ kan være sant, nemlig når _________________

4. Skriv så enkelt som mulig:

a) $\dfrac{2x^2 - 2}{x^2 - 2x + 1}$ **(Udir, H16)**

b) $\dfrac{\sqrt{x} + \sqrt{x} + \sqrt{x}}{\sqrt{x} \cdot \sqrt{x} \cdot \sqrt{x}}$ **(Udir, H17)**

c) $\dfrac{1}{x} + \dfrac{x-5}{x-1} - \dfrac{2x-6}{x^2 - x}$ **(Udir, V17)**

d) $\dfrac{x + 2 + \frac{1}{x}}{\frac{x}{3} - \frac{1}{3x}}$ **(Udir, V18)**

5. Løs likningene:

a) $\frac{x+1}{x+4} = \frac{4}{5}$

b) $x^2 + 6x = 16$ **(Udir, V11)**

c) $2^3 \cdot 2^x = 2^{2x}$ **(Udir, H16)**

6. Om en rettvinklet trekant får du vite: **(Udir, V17)**

- Lengden av den korteste siden er 20
- Differansen mellom lengdene av de to andre sidene er 2.

Hvor lang er den lengste siden i denne trekanten?

Appendix-2: Algebra Test Run in Norway

Algebraprøve

for prosjektet

Does Early Introduction of Algebra in Schools Make any Difference? A Comparative Study of Algebra Skills of High School Students in Norway and Nepal.

Denne testen er en del av en sammenligningsstudie som vil undersøke om tidlig innføring av algebra på skolen hjelper å styrke elevenes algebraferdigheter. Samme prøve skal kjøres i Norge og Nepal med elevene på VG-1 som har valgt teoretisk matte (1T i Norge og *maths for Science stream* i Nepal).

Det skal ikke samles inn identifiserbare personopplysninger som navn, personnummer, fødselsdato, skole, bokommune, bilde, lydopptak, telefonnummer osv og derfor der deltakere helt anonyme. Deltakelsen er frivillig og deltakere kan trekke fra undersøkelsen (i dette tilfelle denne prøven) når som helst. Elevene som deltar skal bli med i en trekning av et gavekort på 200 NOK.

Samtykke

Jeg er informert om følgende

- ☐ Dette er en del av en sammenligningsstudie son skal kjøres i Norge og Nepal.
- ☐ Min deltakelse i dette pilotprosjektet er frivillig og hvis jeg vil kan jeg trekke meg når som helst.
- ☐ Det skal ikke samles inn identifiserbare personopplysninger som navn, personnummer, fødselsdato, skole, bokommune, bilde, lydopptak, telefonnummer osv og derfor deltar jeg helt anonymt.

Vennligst fyll inn følgende før du tar prøven

Alder:_______

Kjønn: _______

Din karakter i matte på 10. trinn: ______

Testen skal gjøres uten kalkulator, men kladdark kan brukes. Skriv svarene på dette heftet. Hvis du trenger mer plass skriv på baksida. Husk å skrive oppgavenummer. Vis utregningene.

Oppgaver for elever	**For sensor KODE**

1. Skriv enklere dersom det er mulig:

a) $2y \cdot y^2$

b) $a - 3a + 2a$

c) $5a - 2(7 - a) + 6$

d) $2x(x-2)-(x-2)(2x+1)$

e) $\sqrt{15} \cdot \sqrt{5} - \sqrt{48}$

2. Finn verdiene av uttrykkene:

a) $a + b - c$, når, $a = 1$, $b = 2$, $c = 3$

b) $3b^2 - abc$, når, $a = 3$, $b = -1$, $c = 5$

3. Alltid sant, aldri sant eller kan være sant

a) $\boldsymbol{a + b \cdot 2 = 2b + a}$ Dette

☐ er alltid sant

☐ er aldri sant

☐ kan være sant, når ____________________

b) $\boldsymbol{x + y + z = x + p + z}$ Dette

☐ er alltid sant

☐ er aldri sant

☐ kan være sant, nemlig når _______________

c) $\boldsymbol{\frac{2x+1}{2x+1+5} = \frac{1}{6}}$ Dette

☐ er alltid sant

☐ er aldri sant,

☐ kan være sant, nemlig når _________________

4. Skriv så enkelt som mulig:

a) $\dfrac{2x^2 - 2}{x^2 - 2x + 1}$

b) $\dfrac{\sqrt{x}+\sqrt{x}+\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}\cdot\sqrt{x}}$

c) $\dfrac{1}{x}+\dfrac{x-5}{x-1}-\dfrac{2x-6}{x^2-x}$

5. Løs likningene:

a) $\frac{x+1}{x+4} = \frac{4}{5}$

b) $x^2 + 6x = 16$

c) $2^3 \cdot 2^x = 2^{2x}$

6. Om en rettvinklet trekant får du vite:

- Lengden av den korteste siden er 20
- Differansen mellom lengdene av de to andre sidene er 2.

Hvor lang er den lengste siden i denne trekanten?

Til slutt:

Hva synes du om prøven? Var den for lett? Passe? For vanskelig? Kryss

For lett	Lett	Passe	Vanskelig	For vanskelig

For sensor

Rotuttrykk	Potenser	Bokstavuttrykk	Parantes
Å faktorisere	Kvadrat setninger	Likninger	Praktisk problem

Appendix-3: Algebra Test Run in Nepal

Algebra test

for project

Does Early Introduction of Algebra in Schools Make any Difference? A Comparative Study of Algebra Skills of High School Students in Norway and Nepal.

This test is part of a comparative study that will investigate whether early introduction of algebra at school helps to enhance the students' algebra skills. The same test will be conducted in Norway and Nepal with the students at VG-1 (grade XI) who have chosen theoretical mathematics (1T in Norway and Maths for Science Stream in Nepal).

No identifiable personal data such as name, social security number, date of birth, school, municipality, audio recording, telephone number, etc., will be collected and therefore, the participation is anonymous. Participation is voluntary, and participants may withdraw from the survey (in this case this test) at any time.

Consent / सहमति

I am informed about the following

मैले यो जानकारी पाएको छु कि

- ☐ This is part of a comparative study to be conducted in Norway and Nepal.
  यो परीक्षा नर्वे र नेपालमा गरिने एउटा तुलनात्मक अध्ययनको एक अंश हो।
- ☐ My participation in this project is voluntary and, if I want, I can withdraw at any time.
  यस अध्ययनमा मेरो सहभागिता पूर्णतया स्वैच्छिक हो र मैले चाहेको खण्डमा जेतिसुकै बेला आफ्नो सहभागिता फिर्ता लिन सक्ने छु।
- ☐ No identifiable personal data such as name, social security number, date of birth, school, municipality, audio recording, telephone number etc. shall be collected and therefore I participate completely anonymously.
  यस अध्ययनकालागि व्यक्तिगत रूपमा पहिचान खुल्न सक्ने अभिलेख जस्तै उमेर, जन्ममिति, विद्यालय, नगरपालिका, श्रव्यरेकर्डिङ, टेलिफोन नम्बर आदि सङ्कलन गरिने छैन र मेरो सहभागिता पूर्णतया गोप्य (बेनामी) रहने छ।

Please, fill the following before you take this test:

कृपया, परीक्षा लिनुपूर्व तलको विवरण भर्नुहोस्:

Class / कक्षा: ______

Age / उमेर :_______

Sex / लिङ्ग: _______

Your marks in mathematics in SEE / एसएसईमा गणित विषयमा तपाईंको अङ्क: ______

The test shall be done without calculator, but the rough papers may be used. Write the answers in THIS sheet in the space provided. If you need more place, write on the last blank sheet. Remember to write the question number. Show the steps in your calculation.

परीक्षा क्यालकुलेटर बिना गर्नुपर्ने छ तर रफ पेपर प्रयोग गर्न सकिनेछ । तपाईंले प्राप्त गर्नुभएको पानामा निर्धारित ठाउँमा जवाफहरू लेख्नुहोस्। ठाउँ अपुग भए अन्तिमको खाली पानामा लेख्नुहोस्। तपाईंले प्रयोग गरेको विधि पनि देखाउनुहोस्।

Tasks for the students विद्यार्थीकालागि प्रश्न	**For examiner CODE**

1. **Simplify if possible.** सम्भव भए सरल गर्नुहोस् :

   a) $2y \cdot y^2$

   b) $a - 3a + 2a$

   c) $5a - 2(7 - a) + 6$

d) $2x(x-2)-(x-2)(2x+1)$

e) $\sqrt{15}\cdot\sqrt{5}-\sqrt{48}$

2. Find the values of the expressions. मान निकाल्नुहोस्:

a) $a + b - c,$ when जब, $a = 1,\ b = 2,\ c = 3$

b) $3b^2 - abc,$ when जब, $a = 3,\ b = -1,\ c = 5$

3. **Always true, never true or can be true / जहिल्यै सहि, कहिल्यै सहि हुन्न, सहि हुन सक्छ**

a) $a + b \cdot 2 = 2b + a$ This / यो

- ☐ is always true / जहिल्यै सहि हुन्छ
- ☐ is never true / कहिल्यै सहि हुन्न
- ☐ can be true, when / सहि हुन सक्छ, जब ________________

b) $x + y + z = x + p + z$ This / यो

- ☐ is always true / जहिल्यै सहि हुन्छ
- ☐ is never true / कहिल्यै सहि हुन्न
- ☐ can be true, when / सहि हुन सक्छ, जब ____________

c) $\frac{2x+1}{2x+1+5} = \frac{1}{6}$ This / यो

- ☐ is always true / जहिल्यै सहि हुन्छ
- ☐ is never true / कहिल्यै सहि हुन्न,
- ☐ can be true, when / सहि हुन सक्छ, जब ______________

4. **Simplify / सरल गर्नुहोस्:**

a) $\frac{2x^2 - 2}{x^2 - 2x + 1}$

b) $\dfrac{\sqrt{x}+\sqrt{x}+\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}\cdot\sqrt{x}}$

c) $\dfrac{1}{x}+\dfrac{x-5}{x-1}-\dfrac{2x-6}{x^2-x}$

5. **Solve the equations / समीकरणहरू हाल गर्नुहोस्:**

a) $\frac{x+1}{x+4} = \frac{4}{5}$

b) $x^2 + 6x = 16$

c) $2^3 \cdot 2^x = 2^{2x}$

6. In a right-angled triangle,

- The length of the shortest side is 20
- The difference between the lengths of the two other sides is 2.

How long is the longest side in this triangle?

एउटा समकोण त्रिभूजमा,

- सबैभन्दा छोटो भूजाको लम्बाई 20 छ।
- अरू दुई भूजाहरूको लम्बाईको भिन्नता 2 छ।

यो त्रिभूजको सबैभन्दा लामो भूजाको लम्बाई कति हुन्छ ?

Before you complete / र अन्त्यमा:

1. **What do you think about the test? Select one below:**
   परीक्षा तपाईंलाई कस्तो लाग्यो ? तलबाट एउटा छान्नुहोस् :

Too easy अत्यन्तै सरल	**Easy** सरल	**Moderate** मध्यम	**Difficult** कठिन	**Very difficult** अत्यन्तै कठिन

2. **Which class do you think would this test be appropriate for if you think the test is too easy/easy for you?**
   यदि तपाईंलाई यो परीक्षा धेरै सजिलो / सजिलो लाग्यो भने तपाईंलाई यो परीक्षा कुन कक्षालाई उचित हुन्छ जस्तो लाग्छ ?

Thank you for your participation.

😊 😊

For sensor

Sign manipulation	Letter expressions	Bracket use	Root manipulation
Factorization	Quadratic statements	Equalities	Equations

Appendix-4: Scoring Guide

Scoring Guide

<table>
<tr><th>Task</th><th colspan="2">Solution</th><th>Attention</th><th>Code</th><th>Remarks</th></tr>
<tr><td rowspan="3">1a</td><td colspan="2">$2y \cdot y^2$
$= 2y^3$</td><td></td><td>1</td><td rowspan="3">Code accordingly if other representation form mentioned</td></tr>
<tr><td colspan="2">Wrong answer</td><td></td><td>0</td></tr>
<tr><td colspan="2">No answer</td><td></td><td>99</td></tr>
<tr><td rowspan="4">1b</td><td colspan="2">$a - 3a + 2a$
$= 0$</td><td></td><td>2</td><td rowspan="4"></td></tr>
<tr><td>$= -2a + 2a$
$= 0$</td><td>$= -2a + 2a$
$= 0$</td><td></td><td>1
2</td></tr>
<tr><td colspan="2">Wrong procedure, wrong answer</td><td></td><td>0</td></tr>
<tr><td colspan="2">No answer</td><td></td><td>99</td></tr>
<tr><td rowspan="3">1c</td><td colspan="2">$5a - 2(7 - a) + 6$
$= 5a - 14 + 2a + 6$
$= 5a + 2a - 14 + 6$
$= 7a - 8$</td><td>Correct addition & (-) sign distribution
Assembling the like terms
Executing the operations correctly</td><td>1
2
3</td><td rowspan="3"></td></tr>
<tr><td colspan="2">Wrong procedure, wrong answer</td><td></td><td>0</td></tr>
<tr><td colspan="2">No answer</td><td></td><td>99</td></tr>
<tr><td rowspan="4">1d</td><td colspan="2">$2x(x-2) - (x-2)(2x+1)$
Method 1:
$= (x-2)[2x - (2x+1)]$
$= (x-2)(2x - 2x - 1)$
$= (x-2)(-1)$
$= 2 - x \ or \ -x + 2$</td><td>Factor out the common term
(-) sign distribution

Correct multiplication by (-1)</td><td>1
2

3</td><td></td></tr>
<tr><td colspan="2">**Method 2:**
$= 2x^2 - 4x - (2x^2 + x - 4x - 2)$
$= 2x^2 - 4x - 2x^2 - x + 4x + 2$
$= -x + 2$
$= 2 - x$</td><td>Correct multiplication & (-) sign dist.
Correct (-) sign dist.
Correct operation execution</td><td>1
2
3</td><td></td></tr>
<tr><td colspan="2">Wrong procedure, wrong answer</td><td></td><td>0</td><td></td></tr>
<tr><td colspan="2">No answer</td><td></td><td>99</td><td></td></tr>
<tr><td>2a</td><td colspan="2">$a + b - c \ \ when \ a = 1 \ b = 2 \ c = 3$
$= 1 + 2 - 3$
$= 0$
Wrong Answer
No answer</td><td></td><td>1
2
0
99</td><td>Code accordingly if other representation forms mentioned</td></tr>
<tr><td rowspan="3">2b</td><td colspan="2">$3b^2 - abc, \quad when \ a = 3 \ b = -1 \ c = 5$
$3 * (-1)^2 - (3 * -1 * 5)$
$3 + 15$
18</td><td>Correct substitution
Correct sign manipulation
Correct operation</td><td>1
2
3</td><td rowspan="3"></td></tr>
<tr><td colspan="2">Wrong Answer</td><td></td><td>0</td></tr>
<tr><td colspan="2">No answer</td><td></td><td>99</td></tr>
<tr><td rowspan="3">3a</td><td colspan="2">Always True</td><td></td><td>1</td><td rowspan="3"></td></tr>
<tr><td colspan="2">Wrong answer</td><td></td><td>0</td></tr>
<tr><td colspan="2">No answer</td><td></td><td>99</td></tr>
</table>

3b	Can be true Can be true when y=p Wrong Answer		1 2 0	
	No answer		99	
3c	Can be true Can be true when x=0		1 2	
	Wrong Answer		0	
	No answer		99	
4a	$\frac{2(x^2-1)}{(x-1)^2}$ $\frac{2(x+1)(x-1)}{(x-1)(x-1)}$ $\frac{2(x+1)}{x-1}$ *or* $\frac{2x+2}{x-1}$ Wrong Answer No answer	For recognising the quadradic form of the denominator For correct decomposition of the numerator For completely correct answer	1 2 3 0 99	
4b	$\frac{3\sqrt{x}}{(\sqrt{x})^3}$ $= \frac{3\sqrt{x}}{x\sqrt{x}}$ *or* $\frac{3}{(\sqrt{x})^2}$ $= \frac{3}{x}$ Wrong Answer No answer	For correct manipulation of either numerator or denominator For completely correct answer	1 2 3 0 99	
4c	$= \frac{(x-1)+x(x-5)}{x(x-1)} - \frac{2x-6}{x^2-x}$ $= \frac{x^2-4x-1}{x(x-1)} - \frac{2x-6}{x^2-x}$ $= \frac{x^2-4x-1-(2x-6)}{x(x-1)}$ $= \frac{x^2-6x+5}{x(x-1)}$ $= \frac{(x-5)(x-1)}{x(x-1)} = \frac{x-5}{x}$ Wrong Answer Not attempted	For identifying correct common factor between any two initial terms For a correct manipulation after identification of common factor For identifying common factor between all terms (2) and completing one manipulation correctly (1) Completion of one more manipulation For finding roots of the numerator and getting to the final form	1 2 2 3 0 99	Superior Understanding!
4d	$= \frac{\frac{x^2+2x+1}{x}}{\frac{x^2-1}{3x}}$	For identifying correct common factors in both numerator and denominator and doing correct operation	1	

	$= \frac{3(x^2+2x+1)}{x^2-1}$ $= \frac{3(x+1)(x+1)}{(x+1)(x-1)} = \frac{3(x+1)}{x-1}$ Wrong Answer Not attempted	For Inverting and cancelling correctly For identifying each quadradic forms (1*2) and cancelling correctly	2 3 0 99	Included in the pilot study. Omitted in the (main) study
5a	$= 5(x+1) = (x+4)$ $= 5x - x = 4 - 1$ $= x = \frac{3}{4}$	For getting to this form either by cross multiplication or my division with a common term For correct collection of like terms either by using subtraction or by manipulation of (- and + signs) For correct Answers	1 2 3	
5b	$= x^2 + 6x - 16 = 0$ $x^2 + 8x - 2x - 16 = 0$ $\Rightarrow x = \frac{3}{4}$ Wrong answer No answer	For making the equation in the form $ax^2 + bx + c = 0$ For identifying correct roots	1 2 3 0 99	
5c	$= 2^{3x+3} \quad = 2^{2x}$ $3x + 3 = 2x$ $x = 3$ Wrong answer No answer	For using product rule For equating exponents For correct answer	1 2 3 0 99	
6	$20^2 + x^2 = (x+2)^2$ $or\ 20^2 + (x-2)^2 = (x)^2$ $400 + x^2 = x^2 + 4x + 4$ $x = 99$ Longest side = x+2 = 101	Setting up correct equation Correct Evaluation of the variable Finding the longest side correctly	1 2 3	
	Wrong procedure, wrong answer		0	
	No answer		99	

Appendix-5: Author Guidelines for NOMAD

Nordic Studies in Mathematics Education

NORDISK MATEMATIKK DIDAKTIKK

Information for authors and reviewers

Here authors and reviewers can find both general information about the journal and specific information regarding submission and the review process of NOMAD.

About the journal

The journal *Nordic Studies in Mathematics Education* – Nordisk Matematikkdidaktikk, NOMAD – is a journal publishing results from research in mathematics education. It addresses all that are interested in following the progress in this field in the Nordic and Baltic countries, Denmark, Estonia, Finland, Iceland, Latvia, Lithuania, Norway and Sweden. The main objectives of the journal are to stimulate, support and foster Nordic and Baltic researchers and post-graduate students in mathematics education and thereby contribute to the development of mathematics teaching and learning in theory and practice at all levels of the educational system. NOMAD publishes articles based on theoretical analysis as well as on empirical studies, reports on results from research projects and research-based developmental projects.

An article in NOMAD should be of high quality and contribute to the development of the field in the Nordic and Baltic region. The article should have a clear theoretical basis or originate from clearly defined assumptions. Most manuscripts submitted to the journal are not immediately accepted for publishing but authors will be requested to do minor changes or substantial rewriting based on the reviewers' recommendations. NOMAD depends on people volunteering to do the important work of reviewing without any economical compensation.

Submission and the review process

An author submitting a manuscript to NOMAD will normally receive notice of reception within two weeks. The manuscript will be assigned to a responsible editor, who will take care of all further communication with the corresponding author regarding the article (for practicalities, se section *Technical issues for authors* below). The responsible editor will make a first evaluation of the article to see that it complies with the guidelines for NOMAD. If the outcome of this evaluation is positive, the article will be sent to reviewers. Each manuscript will be reviewed by at least two persons. Each reviewer recommends to the responsible editor one of five possible outcomes: *Publish*, *Publish after minor revision*, *May be published after major revision*, *Re-write and re-submit the manuscript*, and *Should not be published in NOMAD.* Based on the reports from the reviewers the responsible editor produces a final review report with a decision, which is sent to the author(s) and to the reviewers. Further details about the possible outcomes are given below.

1. *Publish*
The submitted paper is of very high quality and should be published. The editors, in cooperation with the author, make possible changes during the process of preparing the final manuscript. Very few articles are placed in this category.

2. *Publish after minor revision*
The submitted paper is very good, but certain changes should be made before it can be published. The reviewer may know of certain studies, which should be included among the references made, or may be of the opinion that some of the ideas in the discussion should be developed and made more explicit. A new review of the manuscript is not needed. When the author has made the necessary changes, the responsible editor will prepare the manuscript for publication. Papers that need a considerable revision should not be placed in this category.

3. *May be published after major revision*
The submitted paper is good, but some parts of the paper need further work. When the author has revised the paper, it will be sent to a new review. If possible, one of the reviewers from the first review will look at the paper again and assess to what extent the shortcomings have been dealt with.

4. *Rewrite and resubmit the manuscript*
The submitted paper is of interest to NOMAD. It has a good basis, but considerable revision or development is needed. A paper placed in this category will, when resubmitted, be subject to review as if it were a new manuscript. Normally, in addition to appointing new reviewers, one of the reviewers from the first review round will be asked again, to assess to what extent the author has incorporated previous suggestions for changes. If the paper is resubmitted within six months after the first review round the submission date of the original version will be maintained.

5. *Should not be published in NOMAD*
The submitted paper is not suitable for NOMAD. Reasons could be unsuitable design, insufficient empirical base, difficulties in relating the work to relevant literature or that the manuscript does not add sufficiently to the development of the field. This category should be used if the reviewers think that even after substantial re-writing it is unlikely that the paper can be brought to the level of quality needed for publication. Authors should not be encouraged to carry out a revision when there is little hope that the changes will result in a manuscript that could be published. The author will of course have the right to resubmit a revised version of a rejected manuscript on a later occasion.

Comments from the reviewers
The aim of the review process is to increase the quality of the manuscripts published in NOMAD, as well as providing support for authors to reach an appropriate level of quality.

In addition to give recommendations to the responsible editor regarding publishing (categories 1–5 above), the reviewers are asked to write one or two pages of comments to the author, including reasons for the recommendation and providing suggestions for changes. The reviewers should point to strengths and weaknesses of the paper and write their comments in such a way that they can be copied and sent to the author. Comments could include to what extent the issues raised in the article are seen to be of interest to the readers of NOMAD, if the author in a clear and explicit way has built on existing research in the field, if the research design is appropriate, and if the conclusions are well substantiated. If recommending revision, reviewers are asked to express their suggestions for improvement as clear as possible. The more explicit and detailed the reviewers' comments are, the more help and support the author and the editors will have from them. Especially, remember that one of the aims of NOMAD is to foster Nordic and Baltic researchers and post-graduate students in mathematics education. If the reviewers find it necessary to write comments that only the responsible editor (and not the authors) is supposed to read, the reviewers are asked to do so in a separate note.

Technical issues for authors

Articles published in Nomad should be original and not be published elsewhere. By submitting an article authors give their consent to both printed and electronic publication. If the article is accepted the author receives 5 copies of the issue in which the article is published.

Language

Articles submitted to NOMAD should be written in Danish, Norwegian, Swedish or English. The first page of the article should include an abstract of about 100 words and the last page should contain an abstract in English, if the article is written in a Scandinavian language. A separate page should be supplied, comprising all authors´ names and addresses and a short biography for each author (academic title or position, affiliation, main research interests).

Length, typesetting and naming of files

Articles should contain at most 40,000 characters, including spaces, but not the list of references. Only in exceptional cases will longer texts be accepted. Manuscripts should be submitted in digital form (Word or similar), be doubled spaced with Times 12–14 points, and use wide margins to allow for editorial notes. Headings, diagrams, figures and illustrations should be included in the manuscript. The manuscript has to be submitted in two electronic copies: one full version, and one anonymized version in which the author's name and references to the author's work have been removed. The anonymized version will be used in the review process, which is double blind (both reviewers and authors are anonymous). Manuscript files should be named Author_nomad.xxx and Author_nomadanonym.xxx, and sent to

E-mail: nomad-editors@listserv.gu.se

Citations and references

NOMAD follows, with a few exceptions, the guidelines presented in The American Psychological Association (2010), *Publication manual of the American Psychological Association* (6th ed.). Washington: Author. Basic guidelines for the most common types of references are presented below.

References in the text should state the author's name and publication year within brackets; (Winsløw, 2004). If direct citations are made, the reference should also include page number; (Winsløw, 2004, p. 62). References should be listed in alphabetical order at the end of the article.

A book reference should use the following style:

Blomhøj, M. (2016). *Fagdidaktik i matematik*. Frederiksberg: Frydenlund.

Articles or chapters in anthologies are referred to in the following style:

Grønmo, L. S. (2004). Are girls and boys to be taught differently? In B. Clark et al. (Eds.), *International perspectives on learning and teaching mathematics* (pp. 223–236). Gothenburg: National Centre for Mathematics Education.

A journal reference should use the following style:

Hannula, M. S. (2005). Shared cognitive intimacy and self-defence: two socio-emotional processes in problem solving. *Nordic Studies in Mathematics Education*, 10 (1), 25–41.

Plagiarism
For all figures, tables and longer text passages published elsewhere, the author is requested to obtain permission for both the print and online format. All material without evidence of such permission is assumed to belong to the author(s). No data, text or theories by others should be presented as if they were the author's own ("plagiarism"). This includes presenting previously published work by the author/-s as new ("self-plagiarism") without this being stated clearly.

Illustrations
The preferred file format for photos are tiff or jpeg, and for figures eps. Illustration files should be named in the following way:

Author_figure1.xxx where xxx stands for the file format extension.

Printed copies of illustrations should be sent to:

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