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Quantum dynamical dissociation of quarkonia by wave function decoherence in quark-gluon plasma

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Abstract

In this study, we investigate the real-time evolution of quarkonium bound states in a quark-gluon plasma in an improved QCD based stochastic potential model. This model describes the quarkonium dynamics in terms of a Schrödinger equation with an in-medium potential and two noise terms encoding the residual interaction between the heavy quarks and the medium. The time evolution described by this equation is unitary, since the effective potential term is real-valued. At a glance this is at odds with lattice results, but we explain why it is actually not the case.

We discuss the time evolution of the admixtures of bound states in a static medium and in a boost-invariantly expanding quark-gluon plasma. We draw two conclusions from our results: One is that the outcome of the stochastic potential model is qualitatively consistent with the experimental data in relativistic heavy-ion collisions. The other is that the noise plays an important role in order to describe quarkonium dynamics in medium, in particular it causes decoherence of the quarkonium wave function.

Keywords: Quarkonia, Open quantum system

1. Introduction

Quarkonium (charmonium, bottomonium) is a bound state of a heavy quark pair. In relativistic heavy ion collisions, the measurement of quarkonia gives us an important clue to understanding the properties of hot nuclear matter, i.e. quark-gluon plasma (QGP).

Historically, the yield suppression of quarkonia was predicted using model computations based on static potentials [1]. In the vacuum, the binding force from the potential is approximately linearly rising and thus long ranged. On the other hand, in the QGP, the force becomes shorter ranged because the light quarks and gluons screen the color charges of the heavy quark and antiquark. These static descriptions provide an intuitive explanation to the scenario of the quarkonia suppression as a signal of the QGP formation in heavy-ion collisions.

Recently the quarkonium potential in the medium has been derived perturbatively from thermal field theory using the thermal Wilson loop [2] and has also been calculated by lattice simulations [3]. These results indicate that the potential takes complex values.

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In this paper, we study the dynamics of quarkonia based on the framework of open quantum system. We consider the effect of thermal fluctuations as well as the conventional static thermal effect of the screening in a quantum mechanical potential. We study the effect of the fluctuations, namely decoherence of the quarkonium wave function, in simple one-dimensional numerical simulations.

2. Open quantum system

The concept of open quantum systems is very useful idea when we study the dynamics of a quantum system in a medium, and can be applied to a quarkonium in the QGP.

Generally, the total system composed of a system in contact with a medium has the Hamiltonian

$$H_{\text{total}} = H_{\text{sys}} \otimes I_{\text{med}} + I_{\text{sys}} \otimes H_{\text{med}} + H_{\text{int}}, \tag{1}$$

where H_{int} represents the interaction between them. For the total system, its dynamics is described by the von Neumann equation for the total density matrix.

When we focus on the dynamics of the open system, which is a quarkonium in our case, we integrate out the degrees of freedom which physically correspond to the light quarks and gluons in the QGP. We execute it under the assumptions of weak coupling which is applicable at high temperature and an appropriate separation of timescales between the quarkonium system and the QGP, and then obtain the master equation in Lindblad form for the reduced density matrix ρ_{sys},

$$\frac{d}{dt}\rho_{\text{sys}}(t) = -i\left[H_{\text{sys}}, \rho_{\text{sys}}\right] + \sum_n \left(2L_n\rho_{\text{sys}}L_n^\dagger - L_n^\dagger L_n\rho_{\text{sys}} - \rho_{\text{sys}}L_n^\dagger L_n\right) \tag{2}$$

where L_n is Lindblad operator. The corresponding time evolution is Markovian and exhibits the three basic requirements: (i) the reduced density matrix ρ_{sys} is hermitian ($\rho_{\text{sys}} = \rho_{\text{sys}}^\dagger$), (ii) properly normalized ($\text{Tr}\,\rho_{\text{sys}} = 1$), and (iii) positive ($\langle\alpha|\rho_{\text{sys}}|\alpha\rangle \geq 0$ for any state $|\alpha\rangle$) during its time evolution [4]. Note that the second term corresponds to the residual interaction between the quarkonium and the QGP.

3. Quantum state diffusion method

It is known that the Lindblad master equation (2) can be solved by the stochastic evolution of wave functions, and this procedure is called the stochastic unravelling.

One of these procedures is the quantum state diffusion method [5], via which we solve the Lindblad master equation by an equivalent nonlinear stochastic Schrödinger equation. The nonlinear stochastic Schrödinger equation is equivalent to the Lindblad master equation, because we can reproduce the mixed state of the reduced density matrix by taking an ensemble average of the stochastic wave functions. The quantum state diffusion method has an advantage that it requires less numerical cost than directly solving the master equation in the matrix form. Recently this method is applied to the one heavy quark in QGP [6] and there is ongoing work on implementing it for a quarkonium.

In the following analysis, for simplicity, we consider the recoilless approximation, i.e. the heavy quark is assumed to be so heavy that it does not move during the collision. We ignore these recoil terms which include spatial derivatives. The resulting master equation can be solved by the linear stochastic evolution of wave functions, which we call the stochastic potential model.

4. Stochastic potential model

In this section let us introduce the stochastic potential model [8, 9]. Note that this model is not a phenomenological model, but is perturbatively derived from the Quantum Chromodynamics by integrating out the QGP degrees of freedom and taking the recoilless approximation. For simplicity, we drop the color matrices in the following.

In this model, we consider thermal fluctuations of the quarkonium potential and add the fluctuation term Θ to the Hamiltonian,

$$\begin{aligned} H(\boldsymbol{r}, t) &\equiv -\nabla_{\boldsymbol{r}}^2/M + V(\boldsymbol{r}) + \Theta(\boldsymbol{r}, t), \\ \Theta(\boldsymbol{r}, t) &\equiv \theta(\boldsymbol{R} + \boldsymbol{r}/2, t) - \theta(\boldsymbol{R} - \boldsymbol{r}/2, t). \end{aligned} \tag{3}$$

The fluctuation term Θ consists of two noise sources, which describe the collisions between the heavy quarks and QGP particles. In this term, the first noise represents a random kick to a heavy quark and the second represents one to a heavy antiquark, and the different sign comes from the difference of their charges. The noise θ is introduced as the Gaussian white noise with spatial correlation $D(r)$,

$$\langle\theta(\boldsymbol{x},t)\rangle = 0, \tag{4a}$$

$$\langle\theta(\boldsymbol{x},t)\theta(\boldsymbol{x}',t')\rangle = D(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t'). \tag{4b}$$

By expanding the time evolution operator $e^{-i\Delta t H}$ in terms of a small time scale Δt, we get a linear stochastic Schrödinger equation. Note that time evolution is unitary because the Hamiltonian is hermitian. Once time is discretized, the delta function $\delta(t-t')$ is expressed as $\frac{\delta_{tt'}}{\Delta t}$, and the noise is scaled as $(\Delta t)^{-1/2}$ when we take $\Delta t \to 0$ limit. The expansion of the time evolution operator in terms of Δt is

$$\begin{aligned} e^{-i\Delta t H} &\simeq 1 - i\Delta t H(\boldsymbol{r},t) - \frac{1}{2}(\Delta t\Theta(\boldsymbol{r},t))^2 + O(\Delta t^{3/2}) \\ &\equiv 1 - i\Delta t H_{\text{eff}}(\boldsymbol{r},t), \end{aligned} \tag{5}$$

$$H_{\text{eff}}(\boldsymbol{r},t) \simeq -\nabla_{\boldsymbol{r}}^2/M + V(\boldsymbol{r}) - i\{D(\boldsymbol{0}) - D(\boldsymbol{r})\} + \Theta(\boldsymbol{r},t), \tag{6}$$

and from these equations, the stochastic Schrödinger equation for a quarkonium wave function is obtained,

$$i\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = H_{\text{eff}}(\boldsymbol{r},t)\Psi(\boldsymbol{r},t). \tag{7}$$

Now the apparent imaginary potential appears in the effective Hamiltonian H_{eff} because of the noise correlation, but still the time evolution is unitary. The complex potential obtained by the thermal field theory [2] and the lattice simulation [3] is calculated by the ensemble average of the Schrödinger equation, but in this formalism here it is not related to the physical expectation value of the Hamiltonian, i.e. the energies of the system remained real valued. When we get the physical expectation values $\langle O\rangle = \text{Tr}[\rho_{\text{sys}}O]$, we need the density matrix $\rho_{\text{sys}} = \langle\Psi\Psi^*\rangle$ (note that $\rho_{\text{sys}} \neq \langle\Psi\rangle\langle\Psi^*\rangle$).

Here, we discuss the physical consequences of adding thermal fluctuations to the Hamiltonian. The noises give an additional mechanism of the quarkonia suppression: decoherence. The noise sources in the time evolution operator induce phase rotations of the wave function. When the noise correlation length is much smaller than the distance between a heavy quark and antiquark, the noise brings about a local phase rotation and the quarkonium wave function easily mixes with other excited states. This is the phenomenon of wave function decoherence, which is a new mechanism of quarkonia suppression. This finding leads us to conclude there are two mechanisms contributing to suppression: the Debye screening and the wave function decoherence.

5. Numerical results

We solve the stochastic Schrödinger equation for a quarkonium in one spatial dimension. For simplicity, we set the potential in the Hamiltonian and the spatial correlation of noise as below,

$$V(x) = -\frac{\alpha_{\text{eff}}}{|x|}\exp(-m_D|x|), \qquad D(x) = \gamma\exp\left(-|x|^2/l_{\text{corr}}^2\right). \tag{8}$$

Table 1. Parameters in the model.

	M [GeV]	α_{eff}	m_D	γ	l_{corr}	T_0[GeV]
Bottomonium	4.8	0.3	T	$0.3T$	$1/T$	0.4
Charmonium	1.18	0.3	T	$0.3T$	$1/T$	0.4

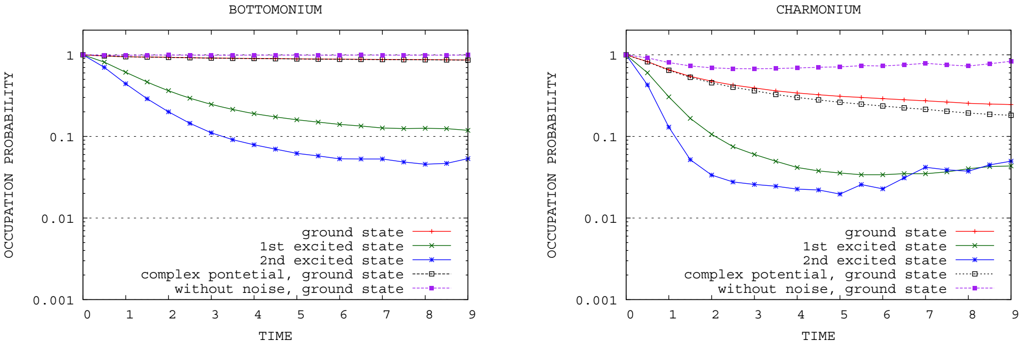

Fig. 1. Time evolution of the occupation probability of quarkonium bound states (the ground, the first excited, and the second excited states) in the stochastic potential model in a Bjorken-expanding QGP. Both the initial states and the projected states are the bound states in the vacuum Cornell potential. The left figure shows the calculation for bottomonium and the right shows one for charmonium. For comparison purposes, we also plot the probability of the ground state from an evolution only with the Debye screened potential, i.e. without noise (dashed purple lines), and the probability of the ground state with the complex potential.

Here the noise correlation length is characterized by "$l_{\rm corr}$". The temperature decreases in time according to Bjorken's solution [10], which is $T(t) = T_0 \left(\frac{t_0}{t_0+t}\right)^{1/3}$, where t_0 denotes the time between the collision and the production of the QGP, and t is the time passed after forming the QGP. The value of each parameter is chosen with reference to the perturbative results (show Table 1).

In figure 1, we present our main results. Starting from three different initial conditions which are the ground state, the first and second excited states in the vacuum potential $V_{\rm vac}(x) = -\frac{\alpha_{\rm eff}}{|x|} + \sigma|x|$, we solve the stochastic Schrödinger equation with in-medium potential. At each time, we calculate the occupation probability of each bound state by projecting it onto the vacuum potential and plot them as a function of time. We confirm that the higher excited states disappear more rapidly because the decoherence is more effective. At later times the occupation probabilities of the charmonium excited states start to increase again, because when the Debye screening length becomes longer due to the temperature decreasing, and the attractive force reaches out to longer distances. Note that the decoherence plays a significant role in time evolution compared with the effects of Debye screening.

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