| (I) <br> University of Stavanger <br> The Faculty of Arts and Education MASTERS THESIS |  |
| :---: | :---: |
| Study programme: MGLINT-119H MA thesis <br> In Mathematics Education <br> For International Students | June, 2020... <br> Open |
| Author: KOLACE PILIRANI SILWIMBA | (signatur author) |

Supervisor: Prof. Arne Jakobsen

Title of thesis: Investigating the Introduction of Algebra in Primary Schools in Malawi. A Case Study of Three Teachers in Standard 5.

Keywords: Algebra in Primary School in Malawi, Standard 5, Procedural teaching, Apprenticeship of observation knowledge, nonstandard problems, Count on addition method

Pages: 110

+ Attachment/other: 122............
Stavanger, 12 June, 2020
date/year


## DEDICATION

To my loving daughters, Laurah, Doreen and Esther, my mum, brothers and sisters, with deepest gratitude and veneration.

## ACKNOWLEDGEMENTS

I am sincerely grateful to all the people who kindly supported and guided me in various ways throughout the study. But first of all, I would like to thank God, Almighty, for the Grace to pursue this study, which I had always craved for, and for giving me the good health and strength.

To Professor Kazima-Kishindo, she is special! I run short of vocabulary to describe her. Without her, my study would surely have been impossible. I downheartedly gave up due to financial constraints. Through her effort and hardworking, and of course through the Norwegian Program for Capacity Development in Higher Education and Research for Development (NORHED) sponsored by the Norwegian Agency for Development Cooperation (NORAD), of which she was the coordinator, and through the Norwegian Partnership Programme for Global Academic Cooperation (NORPART) projects, it was made possible for me to access the scholarship.

To Dr Eneya, the Malawian coordinator for NORPART project, I salute him for making the mobility study possible.

I am thankful to the Norwegian government for the sponsorship, and for accepting me to be part of the NORPART students who were offered a one year mobility study scholarship to University of Stavanger, Norway. Without the scholarship, I would not have realised the dream of pursuing a master degree. My gratitude is extended to the Faculty of Education, Chancellor College for a study place, and to all the head teachers and teachers who made this study possible.

I am indebted to Professor Arne Jakobsen of University of Stavanger, my supervisor, and my father during my stay in Norway. I thank him for being so kind to me, and I always felt safe because of his support. He tirelessly provided assistance for me to reshape my senseless writings to become sensible. I have managed to reach this far because of his hardworking and unceasing assistance. From Professor Arne, I have learnt a lot and benefitted a lot both academically and morally. God bless Arne!

My appreciation also goes to the Norwegian Centre for Research Data (NSD) for granting me the permission to go ahead with my research project. I am also thankful to Prof. Janne Fauskanger, Prof. Nina Helgevold, Prof. Reidar Mosvold, Prof. Raymond Bjuland and Mrs. Magdalena Brekke for their various support they rendered to me while in Norway. I do not take their support for granted.


#### Abstract

Students in Malawi perform very poorly in secondary school mathematics, and algebra in particular. This has prompted the government of Malawi to introduce algebra as topics in the upper primary school curriculum. The aim was to equip learners with algebraic knowledge and skills that can give them a better background for later algebra learning in secondary school. However, much as the introduction of algebra in upper primary school seems to be a solution to learners' poor performance in secondary school algebra, there is a need to find out how primary teachers teach this algebra. Therefore, the main goal of this study is to investigate the introduction of algebra in primary schools in Malawi. The study aims at finding out how teachers introduce algebra to primary schools in Malawi, particularly in Standard (grade) 5 where algebra is introduced for the first time as a topic. Using the Mathematical Discourse in Instruction framework, the study was guided by the main research question "How do teachers introduce algebra to Standard 5 learners in Malawi?" The study sample comprised of three Standard 5 mathematics teachers from three primary schools in the rural area of central Malawi. The study is a qualitative research study which used lesson observations, document analysis (lesson plans) and interviews of the teachers as methods of data collection. Video and audio recordings were also used to obtain rich data. The study findings reveal that the three teachers who took part in the study lacked sufficient knowledge on how to introduce algebra to Standard 5 learners. They used a traditional "fruit salad" approach of introducing algebra. The examples and tasks selected by the teachers and provided for learners were of the same type, with only one form of variation. The teachers' word use was found to be non-mathematical, for example, algebraic letters were introduced as names of objects. It was also established that the teachers depend on teachers' guide and learners' books for information about how to teach, so, lack of information in textbooks was a challenge for teachers. Basing on the findings, the study concludes that it might be that the teachers have insufficient knowledge on what resources, examples and tasks to use when introducing algebra to beginners. They also expressed weaknesses on how to involve learners in the lessons so that collaborative learning is promoted, and they showed to have inadequate knowledge on how to explain the algebraic concepts to learners to achieve the object of learning.


Key words: Algebra in Primary school in Malawi, Standard 5, Procedural teaching, Apprenticeship of observation, nonstandard problems, count on addition method

## Table of Contents

Introduction ..... 1
1.1 Background and the concept of algebra ..... 1
1.2 Problem Statement ..... 3
1.3 Purpose of the study ..... 4
1.4 Research Questions ..... 4
1.6 Significance of the study ..... 5
1.7 Chapter summary ..... 5
2 Literature Review and Theoretical Framework ..... 7
2.1 The Teaching of Algebra in Primary Schools. ..... 7
2.2 Use of Resources, Letters, coefficients, symbols and signs ..... 7
2.3 Theoretical Framework ..... 9
2.3.1 Object of learning ..... 11
2.3.2 Exemplification ..... 11
2.3.3 Explanatory Talk ..... 12
2.3.4 Learner participation ..... 12
2.4 The Malawi context ..... 13
2.4 Chapter summary ..... 14
3 Method ..... 17
3.1 Research Design ..... 17
3.2 Pilot Study ..... 17
3.3 Sample ..... 18
3.4 Data Construction ..... 19
3.4.1 Lesson Observations and video Recording of teaching ..... 20
3.4.2 Interviews. ..... 21
3.4.3 Document Analysis. ..... 22
3.5 Instruments Used ..... 22
3.5.1 Lesson Observation Guide ..... 22
3.5.2 Interview Guide ..... 23
3.6 Instrument Administration ..... 23
3.7 Validity and Reliability ..... 24
3.8 Data Analysis ..... 24
3.9 Ethical Considerations ..... 27
3.10 Limitations ..... 27
3.11 Chapter summary ..... 28
4 Analysis of Data and findings ..... 29
4.1 Data from the Teaching: School A ..... 29
4.1.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 29
4.1.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5? ..... 34
4.1.3 Research Question 3: What opportunities are provided for learners' participation? ..... 38
4.1.4 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 38
4.2 Data from the Teaching: School B ..... 39
4.2.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners ..... 39
4.2.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5? ..... 42
4.2.3 Research Question 3: What opportunities are provided for learners' participation? ..... 45
4.2.4 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 46
4.3 Data from the Teaching: School C ..... 47
4.3.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 47
4.3.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5? ..... 50
4.3.3 Research Question 3: What opportunities are provided for learners' participation? ..... 52
4.3.4 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 52
4.4 Data from Lesson plan: Teacher A ..... 53
4.4.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 53
4.4.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5? ..... 54
4.4.3 Research Question 3: What opportunities are provided for learners' participation? ..... 55
4.4.4 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 56
4.5 Data from Lesson plan: Teacher B ..... 56
4.5.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 56
4.5.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5? ..... 57
4.5.3 Research Question 3: What opportunities are provided for learners' participation? ..... 59
4.5.4 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 59
4.6 Data from Interviews: Teacher A ..... 60
4.6.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 61
4.6.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5? ..... 62
4.6.3 Research Question 2b. What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5? ..... 64
4.6.4 Research Question 3: What opportunities are provided for learners' participation? ..... 65
4.6.5 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 65
4.7 Data from Interviews: Teacher B ..... 66
4.7.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 66
4.7.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5? ..... 67
4.7.3 Research Question 2b. What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5? ..... 68
4.7.4 Research Question 3: What opportunities are provided for learners' participation? ..... 69
4.7.5 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 69
4.8 Data from Interviews: Teacher C ..... 70
4.8.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? ..... 70
4.8.2 Research Question 2a. What type of resources, examples and tasks for students do teachers use to introduce algebra in Standard 5? ..... 71
4.8.3 Research Question 2b. What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5? ..... 71
4.5.3 Research Question 3: What opportunities are provided for learners' participation? ..... 71
4.8.4 Research Question 4: How do Standard 5 learners participate in algebra lessons? ..... 72
4.9 Chapter summary ..... 72
5 Discussion of Findings, Conclusion, Implications and Recommendations ..... 75
5.1 Discussions of Findings ..... 75
5.1.1 Teachers' introduction (explain) of the algebraic concepts to Standard 5 learners ..... 75
5.1.2 Types of resources, selection of examples and tasks for introducing algebra to Standard 5 learners as beginners ..... 79
5.1.3 Teachers' arguments on the use of the resources, examples and tasks they use to introduce algebra in Standard 5 ..... 82
5.1.4 Opportunities provided for learner participation ..... 83
5.1.5 How Standard 5 learners participate in algebra lessons ..... 84
5.2 Conclusions from the study ..... 84
5.2.1 Introduction (explanation) of the algebraic concepts to Standard 5 learners ..... 85
5.2.2 Type of resources, examples and tasks teachers use to introduce algebra in standard ..... 85
5.2.3 Teachers' arguments for using the resources, examples and tasks they decide to use to introduce algebra in Standard 5 ..... 86
5.2.4 Opportunities provided for learners' participation ..... 86
5.2.5 Learner participation (involvement) in algebra lessons ..... 87
5.3 Implications ..... 87
5.4 Recommendations for future research ..... 88
References ..... 90
6. Appendices ..... 97
6.1 Permission letter from NSD (Apendix 1) ..... 97
6. 2 Lesson observation guide (Appendix 2) ..... 99
6. 3 Interview guide (Appendix 2) ..... 101
6.4 Consent and information letters (Appendix 4) ..... 102
6.5 Consent and information letter to Teachers (Appendix 5) ..... 105
6.6 Consent and information letter to parents (Appendix 6) ............................................................ 107

## List of Figures

## 1 Elements of the MDI framework (Adler \& Ronda, 2015) <br> 10

## List of Tables

1 Coding of Exemplification........................................................... 26
2 Coding of Explanatory talk.............................................................. 26
3 Coding of Learner Participation..................................................... 27
4 Summary of Findings for the three teachers..................................... 73

## ACRONYMS

CPD: Continued Professional Development
KAT: Knowledge for Algebra Teaching
MIE: Malawi Institute of Education
MTPDS: Malawi Teacher Professional Development Support
MDI: Mathematics Discourse for Instruction
MoE: Ministry of Education
NSD: Norwegian Centre for Research Data
TTC: Teacher Training College
SACMEQ: Southern Africa Consortium for Monitoring Educational Quality in Mathematic

## Introduction

This study aims at investigating how algebra is introduced in primary schools in Malawi ${ }^{1}$, a case study of three teachers in Standard $5^{2}$. This chapter will present the background and context of the study, statement of the problem, purpose of the study, research questions and significance of the study.

### 1.1 Background and the concept of algebra

Algebra is taken as one of the most important area of mathematics of which Carraher and Schliemann (2007), conceives as a key to problem solving, and also as the area of mathematics that develop students' abstract thinking more than other areas (Star et al., 2015). However, algebra is considered as a difficult area of mathematics because of its nature of generalization and abstraction. Students in many countries (Kieran, 2006), do not do well in secondary school algebra. As a solution to the problem, some countries have introduced algebra in the primary school curriculum, believing that this can help students develop an enlarged sense of the equal sign, to represent unknown quantities with letters, to represent relations with variables, and to formulate and solve equations (Wang, 2015). Mastery of principles and representations of algebra by learners in primary school, can lay a better foundation for later algebra learning, and deepen their understanding of the properties of the number systems.

Malawi just like other countries experience problems of students' poor performance in mathematics in general, and algebra in particular. Mean performance reports of students in mathematics in Malawi dates back in 2007, (SACMEQ, 2010). In an attempt to overcome the problem of poor performance in mathematics, particularly in the area of algebra, the government of Malawi through the Ministry of Education (MoE) and Malawi Institute of Education (MIE), introduced algebra as a topic among other mathematics topics in the upper primary school curriculum in 2007, that's starting from Standard 5 (MIE, 2007, pp. 80-82). The aim was to make sure that learners are equipped with knowledge and skills of algebraic concepts that will give them a better background for later algebra learning in secondary school. It is believed that adequate practice with algebraic language as early as Standard 5 can help learners acquire the basic algebraic knowledge needed for secondary school algebra (MIE, 2008 p. 134).

[^0]Various study findings in Malawi have shown that students perform poorly in secondary school algebra in Malawi. For example, the results from the regional standardized tests, the Southern Africa Consortium for Monitoring Educational Quality in Mathematics (SACMEQ, 2010), show that Malawi performs very poorly in mathematics in general, and algebra in particular. Malawi came second from last out of the fourteen participating countries in the SACMEQ test in 2010 (Jakobsen, Kazima, \& Kasoka, 2018). Another report of students' poor performance in mathematics, and algebra is that of an assessment research exercise that was conducted by Malawi Teacher Professional Development Support (MTPDS) in 2010, which investigated the level of mathematics skills that students in Malawi have. It was established that students in Malawi are performing much far below the expected standards (USAID, 2010). Kazima and Jakobsen (2013), through the project, "Improving the quality and capacity of mathematics teacher education in Malawi," also observed that the very low mathematics, and algebra levels, show among other things, that the teaching of mathematics and algebra in primary schools is not being effective. Jakobsen, Kazima and Kasoka (2018) suggests that teacher quality seems to be one of the main causes of ineffective teaching, hence, low performance. With all these findings, very little has been said about investigating how teachers introduce algebra to young learners in Malawi. Apart from introducing algebra in primary schools, there is also a need to investigate how teachers introduce this algebra to learners, especially in standard 5.

Many teachers in primary schools in Malawi teach mathematics and algebra in a procedural way, they start teaching by presenting the examples which are followed by exercises which are taken from the textbooks (Kazima \& Jakobsen, 2013). Wang (2015), also observed that different countries also use a procedural ${ }^{3}$ way of teaching algebra. This might be due to lack of expertise in handling the subject, which Khalid (2009) notices that can hinder effective algebra teaching. Rubenstein and Thompson (2002) observes that classes are one of the few places where learners can freely express themselves mathematically, yet teachers do not provide such opportunities to learners. Instead of making lessons interactive, teachers do not involve learners much, the resources they use sometimes are not relevant for the teaching of the concept being taught, and the examples and tasks presented in most cases do not encourage learners to think critically (Walters, 2014). As a result, many learners do memorize the algebraic concepts passively, without

[^1]understanding them. The questions remain, can the inclusion of algebra in primary schools in Malawi really matter? How is the concept of algebra introduced to learners in primary schools in Malawi? There is more into this that needs to be investigated, hence the purpose of the research.

### 1.2 Problem Statement

Malawi MoE introduced algebra as a topic in primary school curriculum with an aim of improving students' performance in algebra, which Grouws (2007) call it a stumbling block in school mathematics. The introduction of algebra in primary schools in Malawi seeks to address the problems of students poor performance, however, there is need to find out how algebra is introduced to learners in primary schools. Grouws (2007) explains that many teachers present algebra to learners traditionally, as readymade artefacts. They rarely provide learners with opportunities to explore and discover new concepts on their own. Frade, Acioly-Regnier and Jun (2013), explains that teachers need to engage learners in classroom activities which can help them develop ways of thinking about problem solving. One of the reasons for poor performance of learners in algebra might be the way teachers introduce algebra in primary schools. Khalid (2009) explains that effective teaching requires teachers to decide what aspects of a task to highlight, how to organize and orchestrate the work of learners, what questions to ask learners, having varied levels of expertise, and how to support learners without taking over the process of algebraic thinking for them. In this view, investigating how teachers introduce algebra to beginners in Standard 5 in primary schools in Malawi is important.

Tsang, Mohammad and Shahrill (2014) argues that it's the teacher who designs the learning activities, and leads the class in those learning activities, for effective learning to take place. The way teachers teach can bridge the existing learners' knowledge gap or widen it. As Kieran (2006) puts it out:

Little expertise in algebra can lead to the use of the knowledge that primary school teachers themselves got from their secondary schooling. This can make it difficult for them to engage primary school learners in activities that can effectively prepare them for later algebra learning (pp. 27-28).

This study is therefore designed to investigate how teachers introduce algebra in Standard 5 by specifically finding out what resources, examples and tasks teachers use when introducing algebra, how they explain algebraic concepts and how they involve learners to ensure learner participation
in the lessons. Understanding what goes on in the classroom in regard of how algebra is introduced in Standard 5 may help in exploring on more ways of supporting learners' learning of algebra.

### 1.3 Purpose of the study

Literature has shown that much research has been done on the benefits of introducing algebra in primary schools (Cai \& Knuth, 2011; Kieran, 2004; National Council of Teachers of Mathematics, 2000). It has been revealed that when learners are introduced to algebra in elementary (primary) school, they can gain experiences that can better prepare them for formal study of algebra in the later grades (NCTM, 2000). As MIE (2008) specifies, the knowledge which learners get from the teaching of algebra in primary school lays a foundation for later algebra learning in secondary schools. However, this can be possible if and only if algebra is introduced to learners in such a way that learners are given opportunities to acquire the prerequisite knowledge and skills needed for secondary school algebra. Nevertheless, it's the way how teachers introduce algebra to learners in primary schools that can enable learners acquire or not to acquire the necessary knowledge and skills that will make both teachers and learners see the value of learning algebra in primary education. Thus, the main objective of this study is to investigate how algebra is introduced to learners in primary schools in Malawi. Using lesson observations, interviews and document analysis, the study will explore on how teachers introduce algebra to Standard 5 learners as beginners by focusing on examples, resources, explanations, tasks and learner participation during lessons.

### 1.4 Research Questions

The research questions have been developed from the research topic to facilitate the investigation on how teachers introduce algebra in primary schools in Malawi. The main research question for the study is: How do teachers introduce algebra to Standard 5 learners in Malawi? To answer this question, five specific research questions have to be investigated, and these are:

1. How do teachers explain the algebraic concepts to Standard 5 learners?

2 (a). What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5?

2 (b). What argument do teachers give for the resources, examples and tasks they use to introduce algebra in standard 5?
3. What opportunities are provided for learners' participation?
4. How do Standard 5 learners participate in algebra lessons?

### 1.6 Significance of the study.

The research is believed to be relevant to all stakeholders concerned with primary school teaching. This includes the Ministry of Education, the Malawi Institute of Education as curriculum developers, teacher educators, teachers and learners. The Malawi MoE is included because it funds the education activities. It is through MoE that in service trainings can be intensified to help teachers acquire knowledge on various ways of introducing algebra in primary schools in Malawi as suggested by other researchers, as well as my own study. MIE as curriculum developers who will know what sort of examples, tasks and instructions need to be included in the instructional materials so that teachers are guided properly in their teaching. The findings of the study might give a gist to MIE to see if there is need to review the work on algebra in the teachers' guides, or develop supplementary reference materials for algebra to help teachers have access to required information for the teaching of algebra in primary schools in Malawi. Teacher educators are the ones responsible for training primary school teachers, hence they need to know what is happening in the classroom so that they reflect on their teaching approaches and improve where necessary. The study will be helpful to teachers by providing recommendations which teachers should follow if there is need for improvements, and that they should introduce learners to algebra in the way that they understand the basic algebraic concepts. Furthermore, the findings of this study will be disseminated to all concerned stake holders through the provision of copies of the thesis. Should opportunities be there, the research findings will be shared at educational conferences, and at Teacher Training College (TTC) levels. This will be done during Continued Professional Development (CPD) meetings for the benefit of all the mathematics teachers and also through inservice trainings to practicing teachers in primary schools for improvement purposes.

### 1.7 Chapter summary

The chapter has introduced the research study. It has discussed the aim of the research study, the background and context of the study, statement of the problem, and purpose of the study, the research questions and significance of this research study to different stakeholders. Following this chapter is the literature review.

## 2 Literature Review and Theoretical Framework

This chapter gives an overview on research on the teaching of algebra in primary schools focusing on the use of resources and symbols. It also discusses the theoretical framework which underpins this study, describing object of learning, exemplification, explanatory talk and learner participation which are the elements of the Mathematics Discourse for Instruction (MDI) framework.

### 2.1 The Teaching of Algebra in Primary Schools.

Teachers should be aware of the necessary approaches of introducing algebra to beginners in primary school in order to avoid misconceptions which McNeil and Alibali (2005) suggest can create barriers to the learning of formal algebra once learners have internalised them. As a prerequisite for an effective algebra lesson, teachers must be competent in the subject content knowledge and possess good pedagogical skills (Shahril \& Mundia, 2014). Primary school algebra aims at infusing arithmetic activities with algebraic concepts to bring out the algebraic character (Roberts, 2012). As such, teachers must be knowledgeable on how to introduce algebra to primary school learners so that the algebraic foundation is properly laid. For learners to be successful in algebra learning, they need to be exposed to algebraic ideas that would help them make connections between properties of arithmetic and algebra (Walters, 2014). Teachers must understand that algebra is generalized arithmetic and should strive to instil the same in learners. Molina, Rodriquez-Domingo, Canadas and Castro (2017) argues that lack of understanding of algebra as generalised arithmetic can cause difficulties in structuring algebra. Primary school algebra requires learners to demonstrate an understanding of variables as changing quantities represented by letters, understanding given equations as relationships, letters as unknown quantities and also develop a sense of variation (Roberts, 2012). Alternatively, Wang (2015) explains that when teaching algebra in primary school, teachers should focus on the relations, operations, representations, letters and numbers, letters as unknowns, variables and parameters, unclosed literal expressions, equivalence numerical evaluation and refocusing on the meaning of equal sign.

### 2.2 Use of Resources, Letters, coefficients, symbols and signs.

According to Chick and Harris (2007), few teachers are aware of the kinds of algebraic ideas that can be fostered in primary school. Frade et al. (2013) discovered that the Chinese elementary school curriculum encourages learners to use both arithmetic approach and algebraic approach to solve problems. It is believed that this helps learners develop problem solving skills that assists
them during formal algebra learning. A study conducted on how teachers introduce algebra in China and Singapore, revealed the use of the three new ideas for teaching algebra (Cai \& Moyer, 2008), which are through pattern, symbolic and function approaches. In Singapore, for example, Frade et al. (2013) write that elementary mathematics, algebra inclusive, provides a wide variety of experiences such as the use of "model methods" or "pictorial equations" to help children develop algebraic thinking. In contrast, many countries use the traditional fruit salad approach with variables representing names of objects, for example, "a" for apples and "b" for bananas (Knapp, 2016). Kuchemann (1981) argues that the letters introduced through a traditional fruit salad approach are usually treated as labels or names of objects and not as unknowns. Epp (2011) explains that, letters in an algebraic expression, equation or number sentence represent variables or unknowns. Cai and Knuth (2011) asserts that the use of literal symbols in algebra poses challenges in algebra learning. It is argued that learners start viewing literal symbols as labels for objects, i.e., thinking that D stands for David or anything that starts with a letter D. Alternatively, where students think of literal symbols as numbers, they usually believe that the literal symbols stand for specific numbers only. Similarly, McNeil et al. (2010) argue that the use of fruit salad approach strengthens learners' naïve conception that letters in algebraic expressions stand for objects instead of quantities. Knapp (2016) explains that countries like Australia and Fiji had experienced the same fruit salad algebra approach, but upon realizing that the approach does not have the potential to help reinforce the concept that variables represent numbers rather than objects, they changed the teaching approach. They resorted to a teaching approach that encourage algebraic thinking through the use of simple problems about numbers that motivate conjecture and discussion, reasoning, solving and verification (Knapp, 2016).

Leung, Park, Holton and Clarke (2014) explains that in some countries the concept of equivalence in algebra is introduced by using a balance model approach. The balance model approach, as Kieran (2018) points out, has been proven by many researchers as suitable for demonstrating the idea of equality and quantitative sameness. An equal sign should be used as a relational symbol showing equivalence, and the context of non-standard form should be properly explained to learners so that the equality relationship between the quantities on each side of the equation or number sentence is highlighted (Vincent, Bardini, Pierce \& Pearn, 2015). In South Africa, studies show that the introduction of algebra into the primary curriculum was initially by name because teachers did not know how to teach algebra to primary school learners (Roberts, 2012). It is said
that teachers did not even have an apprenticeship ${ }^{4}$ of observation knowledge for teaching algebra in primary schools (Roberts, 2012). There was no meaning of algebra in the work presented to learners. Later, algebra teaching was improved after teachers were provided with a more detailed guidance on how to teach algebra in primary schools (Roberts, 2012).

According to Kullberg, Kempe and Marton (2017), algebraic concepts should not be presented to learners as readymade symbolic entities but should base on problem situations that can lead to symbolization. The activities should be designed to enable learners experience the desired patterns of variation that will help them learn (Lo, 2012). The MDI framework (Adler \& Ronda, 2015) say that in order for the examples to be of high quality (high rank, what they call level 3) examples should provide learners opportunities to experience more than two forms of variation in more than one episode, while with low ranking examples (level 2 and level 1), examples should provide opportunities for learners to experience two forms of variation and one form of variation (respectively). However, there has been limited research on how teachers introduce algebra in primary schools in many countries (Knapp, 2016), despite that research can help teachers to come up with new teaching approaches that can help learners learn algebra better. Therefore, investigating how teachers introduce algebra in primary schools in countries such as Malawi is needed, for this can unveil the strengths and weaknesses that are encountered in introducing algebra to beginners, and that necessary measures can be put in place.

### 2.3 Theoretical Framework

Numerous frameworks have been developed in order to investigate and improve the teaching of algebra, for example, the Kuchemann's levels of understanding letters in algebra (Kuchemann,1981), the Knowledge for Algebra Teaching (KAT) framework (Huang, 2014) and the variation theory of Marton and Tsui (Kullberg et al., 2017), to mention but a few. All the theories and frameworks aim at examining the effectiveness of the teacher in combining content and pedagogy to make learning possible. This study is grounded in the MDI framework, which was developed by Adler and Ronda at the University of the Witwatersrand (Wits-University) in South Africa. The aim of the framework was to provide a nuanced interpretations of mathematics made available to learn (Adler \& Ronda, 2015). The MDI framework has been chosen for this

[^2]study because it is a framework that enables a description of mathematics made available to learn in a lesson, how the lesson goal needs to be mediated, and so exemplified and elaborated (Adler \& Ronda, 2017). The framework provides an ability to stay close to what teachers do in a lesson, how examples are offered, how words are used to name the mathematics being talked about, how explanations are built on what is supposed to be taught and be done, and how learners are invited to participate in the lessons (Adler \& Rhonda, 2015). The figure below shows the elements of the MDI framework:


Figure 1: Constitutive elements of the MDI framework and their interrelations, (copy from: Adler \& Ronda, 2015, p 3)

Algebra as an abstract entity requires to be exemplified and explained properly so that learners understand it. Teaching of algebra is considered to have taken place if there are examples and tasks which are accompanied with explanations. Examples, tasks and explanations are considered as the common practices in algebra teaching and are often used to focus learners' attention towards the object of learning (Adler \& Ronda, 2015). Explanatory talk forms part of active interaction between teacher and learners, and amongst learners themselves. Learner participation relates to what the learners are invited to say, write and reason mathematically (Adler \& Ronda, 2015). The approaching subchapters provide more information about each of the components of the MDI framework presented in figure 1.

### 2.3.1 Object of learning

Learning is about bringing into focus the central idea of teaching, the object of learning (Adler \& Ronda, 2015). The role of the teacher is to bring the object of learning into learners' focus through the use of proper exemplification, explanatory talk and learners' participation (Adler \& Ronda, 2015). The object of learning can be described as the lesson goal that teachers usually announce at the start of the lesson or write on the board, focusing on the content and the capabilities of learners in respect to that content. This is what learners need to know and be able to do. Exemplification, explanatory talk and learner participation stand in between the object and the subject, and are pathways to achieving the object of learning.

### 2.3.2 Exemplification.

Exemplification includes the examples, tasks and representations. This looks at what examples are used, what are the associated tasks, and what representations are used.

## Examples

This is the sequencing of, and variation across a succession of examples as well as teachers' attention to choice and use of examples in their teaching (Adler \& Ronda, 2015). Examples must be selected as particular instances of the general case in focus and for drawing attention to relevant features (Marton \& Pang, 2006). Teachers need careful analysis of the examples before selecting which ones to use. Examples must be selected according to the levels and forms of variation they display. This study will focus on similar (level 1) and contrasting (level 2) examples because of the level of the class of my study, however fusion examples (level 3) will be taken into consideration if used. A set of examples is judged as level 1 if the sequence of examples displays only one form of variation, and as level 2 if at least two forms of variation are displayed. Where there are opportunities for learners to experience more than two forms of variations, a set of examples is judged as level 3.

## Tasks

Tasks can be defined as what learners are asked to do with the various examples presented. Tasks are designed to bring capabilities to the fore (Marton \& Tsui, 2004). Tasks require different actions, at different levels of complexity or cognitive demand, and so in this way can make available different opportunities for algebra learning. According to Ronda and Adler, (2017), tasks must have the potential to engage the learners to make connections among features of mathematical
content. According to MDI framework, Adler and Ronda (2015), there are tasks that require learners to carry out known (K) operations or procedures which are classified as level 1 , tasks that require K and some application (A), and these are classified as level 2, and level 3 tasks with K and/ or A and C/PS (Adler \& Ronda, 2015).

### 2.3.3 Explanatory Talk

The purpose of explanatory talk is naming and legitimating the mathematical issues discussed in examples or tasks (Adler \& Ronda, 2015).

## Naming

Naming is defined as the use of words to refer to other words, symbols, images, procedures or relationships (Adler \& Ronda, 2015, p. 244). Naming is considered as the use of colloquial (nonmathematical) and mathematical words within and across episodes of a lesson. It is categorized in levels. Level 1, talk is colloquial, non-mathematical (NM) whereas level 2, mathematical language is used appropriately, and there is movement between NM and MS, some MA, while level 3 is where there is movement between NM and MA.

## Legitimating

The legitimating criteria is non mathematical (NM), if there is everyday knowledge (E), visual cues $(\mathrm{V})$, assigning authority to the position $(\mathrm{P})$ of the speaker of the statement, the teacher. NM in legitimation is classified as level 0 . Criteria of what counts as mathematical that are particular or localized is (L) and level 1, another criteria is where there is partial generality (PG), and full generality (FG). Level 2 is where the legitimating criteria is beyond NM, L and include PG. Level 3 is where the criteria is FG (Adler \& Ronda, 2015).

### 2.3.4 Learner participation

Learner participation is about doing maths and talking maths. It is concerned with what learners are invited to write and say, apart from doing the tasks assigned to them. When learners are given opportunities to answer yes/ no questions or offer single words to teachers unfinished sentence, its $(\mathrm{Y} / \mathrm{N})$, where learners answer what/ how questions in phrases/ sentences is (P/S), and opportunities for learners to answer why questions, present ideas in discussion, teacher revoices, confirms and asks questions is (D) (Adler \& Ronda, 2015). Learner participation is specifically about whether
learners have opportunities to speak or non-verbally display mathematical reasoning. Learner participation also seeks to find out if learner activity builds towards the learning goal.

### 2.4 The Malawi context

Malawi, a small landlocked country, with an area of $118484 \mathrm{~km}^{2}$ (Masperi \& Hollow, 2008) follows an education system of 8-4-4 pattern comprising of the primary, secondary and tertiary levels. English is the school and official language of communication. The current recommended age for enrolment into Standard 1 is six years. This means that primary school learners are aged between 6 and 13 years old, and Secondary School caters for 14 to 17 years old students. However, classes are of a mixture of different ages in both primary and secondary schools respectively, and in secondary there can be students of more than 17 years. This is because some children enrol in Standard 1 when they are older than six whereas others repeat classes due to various reasons.

Primary education consists of eight years in primary school, divided into three sections: The infant section (Standard 1 and 2), the junior section (Standard 3 and 4) and the senior section (Standard 5, 6, 7 and 8). In Standard 8, learners take the Primary School Leaving Certificate Examinations in order to gain access to secondary school education (Jakobsen, Kazima, \& Kasoka, 2016; Masperi \& Hollow, 2008). Secondary school education lasts for 4 years and is split into two sections, the junior section and the senior section. At the end of secondary school, students take Malawi Schools Certificate of Education national examinations, this is equivalent to the ordinary level (general certificate of education) (Jakobsen, Kazima, \& Kasoka, 2016). The duration for tertiary education depends on the specifics of the chosen courses, but it ranges from two years to four years, and in very rare situations it goes up to five years.

The Numeracy and Mathematics curriculum in Malawi aims at developing learners' critical awareness of the mathematical relationships in social, cultural and economic context (MoE, 2004). However, in the early years, the curriculum aims at enabling learners to count and carry out basic mathematical operations. The Malawi numeracy and mathematics curriculum has six core elements (broad areas). These are: Number operations and relationships, Patterns, functions and algebra, Space and shape, Measurement, Data handling and accounting and business (MIE, 2007). These core elements are for the whole primary curriculum, but the difference is on the depth of coverage in each standard. For instance, standard one learners learn patterns only from the core element "patterns, functions and algebra" (MIE, 2007). Patterns, functions and algebra is the broad
area which covers algebra. This shows that learners are exposed to some elements of algebra right away from Standard 1. However, algebra is introduced as a topic (text book chapter), among other topics in Standard 5 under the core element of Patterns, functions and algebra.

## Introduction of algebra in primary school curriculum in Malawi

The teaching of algebra in Standard 5 in Malawi aims at instilling in learners the skills of writing number sentences, completing number sentences and solving algebraic expressions (MIE, 2007, p. 80). For learners to understand these concepts in earlier grades, Walters (2014) argue that their teachers need a solid understanding of both the content and the instructional strategies, as well as the resources to use in order to support the teaching and learning. Teachers should use the thinking of learners (Walters, 2014), to introduce and teach algebra for learners to understand the new concepts better. As mentioned earlier on, algebra in Malawi is introduced as a topic among several other mathematics topics that are taught in Standard 5. As a result, teachers in most cases present algebra to learners in the same way they do with other arithmetic topics. Walters (2014) recommends that arithmetic should provide learners prerequisite knowledge and skills for the introduction and learning of algebra. Knowing what learners need to learn, and how to link arithmetic to algebra in the introductory stage of algebra, can reduce errors and misconceptions in algebra learning. For example, Ormond (2012) argues that teachers' use of objects to solve the problem $4 b+5 b=9 b$ should not always aim at finding a single answer as $9 b$. Booth (1988) clarifies that the focus of activity in algebra is not on finding a specific numerical value. Teachers should show learners, for instance, that $4 b+5 b$ can also have a solution as $4 b+5 b=7 b+2 b$. Instilling such an idea in learners can help to minimize errors such as $4 a+5 b=9 a b$ (Ormond, 2012). Kilpatrick, Swafford and Findel (2001) explains that elementary school algebra should focus on representing relations and not to be answer-oriented (p. 261).

### 2.4 Chapter summary

This chapter has discussed the teaching of algebra in primary schools globally. It has looked on how algebra is introduced to primary schools in different parts of the world. It has discussed the types of resources and approaches used by different countries to introduce algebra. The chapter has also presented recommendations by different researchers on how to introduce algebra to beginners and what literature has said on the use of operational symbols and equal signs. The
chapter has also discussed the MDI framework and its four elements, thus, exemplification, explanatory talk and learner participation. The next chapter presents the methods part of the study.

## 3 Method

The research methodology part describes the research design, the pilot study, sample, data construction, instruments used for data collection, instrument administration, validity and reliability, data analysis, ethical considerations and limitations of the study.

### 3.1 Research Design

This research study used a descriptive qualitative research design. A descriptive qualitative research design allows the researcher to study the phenomenon (teaching of algebra) as it occurs in the natural settings without any intervention or manipulation of variables (Nassaji, 2015). The goal of this research is to investigate the teaching of algebra in primary schools. Data triangulation was done in order to obtain rich data from different sources that would provide a deeper understanding of individual participant's opinions, perspectives and attitudes. As Noble and Heale (2019) stipulates, triangulation of data sources helps to increase reliability and validity of the research findings by ensuring that fundamental biases that arise from a single observer are overcome. Creswell and Poth (2018) describe triangulation as where researchers make use of multiple and different data sources, methods, investigators and theories to provide corroborating evidence for validating the accuracy of the findings of the study. Different data sources were used to collect information to answer the specific questions of this research study.

### 3.2 Pilot Study

Before data collection was done in the schools involved in the research project, a very small pilot study was conducted. This was done at one school only in one of the districts of the central region of Malawi. One Standard 5 Mathematics teacher volunteered to be observed teaching and later on interviewed, during the consultations. The main aim of this pilot study was to be sure if the questions in the lesson observation and interview guides could make the participants provide responses in line with this study. As Teijlingen and Hundley (2001) explains, a pilot study provides advance warning about where the main research project could fail, where research protocols may not be followed, or whether proposed methods or instruments are inappropriate or too complicated. During the lesson observation, the observation checklist proved to have included the required indicators that would provide information to the specific research questions of the main research project. However, the answers given in the interview with the teacher after the lesson observation, indicated that this teacher was not interested in this pilot study despite being a volunteer. However, there was no alternative way to go about since there are no parallel classes for Standard 5 in this
school, and there is only one mathematics teacher for Standard 5. Due to time constraints, the researcher did not do another pilot study. The researcher went on to collect data for the main research study using the same lesson observation and interview guides, there were no changes.

### 3.3 Sample

This study used convenience sampling because of limitation in the project resources and time. This also made it difficult to achieve a more rigorous probability sampling. The sample was drawn without any underlying probability-based selection method. Only those willing (volunteers) to participate in the study were involved. As Etkan, Musa and Alkassim (2016) explains, convenience sampling is affordable, easy and the subjects are readily available. Three schools, from one district in the central region of Malawi took part in the study. The schools were chosen because they were within reach of the researcher. All the three schools have no parallel classes for Standard 5. From each school, one mathematics teacher volunteered to participate in the study, hence a total number of three mathematics teachers. The number of learners involved in this study were as follows: School A had 63 learners, 29 boys and 34 girls, school B had 87 learners, 46 boys and 41 girls, and School C had 93 learners, 52 boys and 41 girls. In total, 243 learners were present on the days of the observation and took part in the study. However, the study's main focus was on the teachers.

During the researcher's first visit to the three schools, the head teachers for the three schools were briefed about the research project. They were also requested to identify the Standard 5 mathematics teachers who could be the candidates for the research project. After the teachers were identified, the researcher briefed them of the study. The teachers were asked if they would voluntarily participate in the study. They were given the letters of consent and were also told about the lesson observations, the interviews, the collection of their lesson plans as documents to analyse, and about the video recordings that were to take place. The learners were informed about the research project and also about their freedom of participation. The letters of consent to parents were distributed to the parents through the Parent Teachers Association (PTA) meeting. All the learners present on the day of the observations, gave consent and participated in the study. However, one teacher from school C though initially gave consent to the video recordings, on the day of observation, she decided to withdraw her consent to be video recorded. The decision made by this teacher was respected. Moreover, this particular teacher did not hand in the lesson plan to the researcher to be photocopied. When requested about the lesson plan, she said that she did not prepare a lesson plan for a mathematics lesson on that particular day.

### 3.4 Data Construction

The study aimed at answering the following main research question: How do teachers introduce algebra to Standard 5 learners in Malawi? The five research questions guided the collection of data in the following way:

1. How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?

The researcher wanted to find out how teachers introduce the concept of letters to learners (letters as variables that can stand for multiple values), explain the meaning of an equal sign in an equation, (a number sentence), and the meaning of a coefficient.

2(a). What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5?

The researcher wanted to find out what resources teachers use to introduce algebra to Standard 5 learners, for example, the use of representations. The researcher wanted to construct data on how the teachers used the resources they had in their lessons, and how they selected the examples and tasks for the learners.

2(b). What arguments do the teachers give for the use of the resources, examples and tasks to introduce algebra in standard 5?

The research question guided the researcher to conduct interviews with the participants seeking to find out the reasons why they decided to use the resources they used. The researcher also wanted to find out from the participants why they selected the examples they used during the lessons and how they selected the tasks for the learners.
3. What opportunities are provided for learners' participation?

The researcher wanted to find out how the teachers provided learning opportunities for learner participation. This includes the opportunities in terms of structuring practices, student-oriented practices and enhanced activities. In the three categories, the researcher was looking for the opportunities that the teacher gave learners to explore and discover the new concepts and the object of learning. The researcher wanted to find out the type of activities that teachers provided for learners to help them define and understand the object of learning, and how this helped them to make connections of mathematics being learnt to real life situations.

## 4. How do Standard 5 learners participate in algebra lessons?

The researcher wanted to construct data on how learners participated in the lessons. That is whether teachers allowed learners to work together collaboratively, sharing ideas, presenting ideas about the mathematics learnt, and evaluating each other, which can be done through: small group discussions, whole group discussions, and pair work or through activity or verbal or individually.

Data was collected through lesson observations, interviews and document analysis as well as audio visual and audio materials. Lesson observation and interview guides were developed as instruments for collecting data and the focus was on the details of specific questions of study. Lesson plans were photocopied for analysis.

### 3.4.1 Lesson Observations and video Recording of teaching

Observations is one of the major techniques for collecting data in qualitative research (Creswell, 2013, p. 166). During observations, the researcher can be a participant observer, or a nonparticipant observer or a complete observer (Creswell, 2013). In this study, the researcher was a non participant observer who did not participate in the classroom activities, but just observed the participants teaching. Creswell and Poth (2018) explains that observations help the researcher to have first-hand experience with the participants; the researcher gets an insight into the real things that people do, rather than what they say or think they do, and information can be recorded as it occurs. Susuwele-Banda (2005) claims that sometimes what teachers claim to know about their teaching is not what is reflected in the real classroom practices. It is in this view that during observation, the researcher can notice unusual aspects which could otherwise not be noticed if observation was not done.

The main objective for choosing observations as a method of collecting data was to gather firsthand information on how Standard 5 mathematics teachers introduce algebra in Standard 5, being the class where algebra is introduced for the first time as a topic in primary schools in Malawi. During the observations, the focus was on the resources the teachers used, the selection of examples and tasks for the learners, the introduction and explanations of the letters, the equal sign and the coefficient. The researcher was also looking for information on the opportunities for learner participation that the teachers gave learners during the lessons and how the learners were involved to participate in the lessons. All the three teachers from the three schools allowed me to observe them teaching and each teacher was observed once. A lesson observation guide was
designed to focus on the details of the specific questions of the study. However, research question 2 b was not addressed using the observations.

Nevertheless, using observation can be problematic in ways that sometimes participants might not feel comfortable with the researcher in the class; as such, they might behave in a way not as they would normally do (Creswell, 2009).The learners and even the teacher may act differently in a classroom if there is an observer than they would do if there was no observer around them. In addition, if the researcher does not have good observational skills, accurate recording of some important quotes for inclusion in field notes can be a challenge (Creswell, 2013). In this study, these limitations were dealt with by visiting the classes before the actual observation day. This was during the first visit when the researcher briefed the teachers and learners about the research project. This helped the teachers and the learners as well as the researcher to get to know each other. Moreover, the pilot study which was conducted also helped the researcher to have an experience on how the observations should be done. The researcher also video recorded the lessons so that every important quote that was missed out in the field notes written during the observation time, was recaptured.

### 3.4.2 Interviews

This study also used interviews to collect data apart from the lesson observations. Interviews are used to source out information that cannot be directly observed such as from opinions, beliefs, feelings and perceptions (Merriam, 2001).The researcher asks the participants questions and get responses from them. The researchers conducted one-on-one in-depth interviews with participants who have experience with the phenomena under investigation (introduction of algebra in Standard 5). From the different forms of interviews; structured, unstructured, semi-structured and focus group interviews (Cohen, Manion, \& Morrison, 2007), this study used one-on-one semi-structured interviews. As Creswell (2013) asserts, one-on-one semi-structured interviews allow for flexibility of the researcher to diverge from the original question in order to follow a response in more detail.

The same three Standard 5 mathematics teachers who participated in lesson observations were interviewed. In these interviews, the participants were asked to explain more about why they chose to use the resources they used in their lessons, where they got the examples from, how they selected the examples and tasks for the learners, where they got the explanations of algebraic concepts from, the opportunities that they were supposed to provide to learners during the lessons and how
they involved learners in their lessons. However, according to Creswell (2009), during interviews, the presence of the researcher may bias the responses of the participants, they can withhold some information. The limitation was dealt with by building a rapport with the respondents in order to make them feel comfortable. Before the interviews, the researcher assured the respondents that the interviews were merely for study purposes, and not for their professional evaluation. They were assured that their names and details would not be revealed, and that all reporting was going to present participants anonymously.

### 3.4.3 Document Analysis

Creswell (2013) describes documents as written or printed materials that have been produced in some form. Examples of documents that could be analysed in schools may include tests, registers, schemes of work, lesson plans, and progress books. This study used lesson plans as documents to analyse. Lesson plans were chosen because they contained data that could corroborate evidence from lesson observations and interviews. This way of collecting data helps the researcher to get hold of the information in an event where the lesson proceeded differently just because it was under observation. As Bowen (2009) argues, documents are unobtrusive and non-reactive, they are unaffected by the research process. In addition, data contained in lesson plans as documents, can provide supplementary research data to enrich the data collected from lesson observations and interviews (triangulation). Information obtained from the document helped to answer all the specific research questions of this study except for research question number 2 b .

### 3.5 Instruments Used

The main instruments that were used to collect data for this study were lesson observation and interview guides.

### 3.5.1 Lesson Observation Guide

To avoid researcher's bias that might arise from selective recording of information (Baker, 2006), an observation guide was used (see Appendix 2). The observation indicators in the observation guide helped the researcher to focus on the information required to answer the specific research questions, and recording of the notes. The observation guide had indicators on the resources used and how they were used, and on the examples and tasks used. It also focused on the organization of lesson activities; that's how the teacher involved students in a lesson, the opportunities provided for learner participation and the teachers' explanations of the algebraic concepts that are found in the Standard 5 mathematics text books. The algebraic concepts that Standard 5 learners need to
know include the meaning of number sentence, the coefficient, the use of equal sign in a number sentence and the meaning of letters in algebra. The observation guide was developed to suit the aim of the research, and it was designed to answer the specific research questions.

### 3.5.2 Interview Guide

An interview guide for interviewing teachers was developed to find out how algebra is introduced in Standard 5 in Malawi (see Appendix 3). The focus was on what resources teachers used, why they used those resources, where they got the examples used in the lesson, how they selected the examples, how they involved/ involve learners in the lesson/lessons, the opportunities they are supposed to provide to students and where they got the explanations of the algebraic concepts from. The interview guide also sought a follow up on what was observed during the lesson observation. It had questions that could give data that could answer all the specific research questions of this research study.

### 3.6 Instrument Administration

Before the beginning of lesson observations, each participating teacher and parents of children were informed about the study and consent forms to show that they willingly accepted to participate in the study were distributed and later collected. During lesson observation, the lesson observation guide was used as a guide for note taking. It was used to write required information from the lesson plans and also from the class activities as the lesson progressed. The lessons from two of the three schools were video recorded, whereas in one school the teacher did not give consent to be video recorded, but allowed the researcher to observe the teaching. For this teacher, only observation and interviews were used. Video recordings were used for the two other teachers in order to get rich data for analysis. As Baker (2006) asserts, cameras are instrumental extension of our senses that may help researchers to see more and with greater accuracy.

Interviews were conducted on the second visit to the schools after lesson observations were done. The same three teachers who were observed, were interviewed. Before the interviews, the researcher asked the respondents if the voice recorder could be used, and two respondents had no problems. The teacher who had not allowed for video recording also did not give consent to be audio recorded, and the researcher took notes of the responses during the interview. The interviews helped the researcher to obtain some more information which could not have been obtained through lesson observations. The interview guide was used to guide the flow of the interview
questions and also to make sure that the information provided answered the specific questions of the research. The responses for the two respondents were being written down and audio recorded whereas for the third participant, the responses were just written down. The guide was with the researcher, and the participants could just answer the questions orally without necessarily looking at the questions on the interview guide. The voice recorder was used in order to capture information which was missed during writing of responses as the interview was in progress. As soon as the researcher got home, the voice records were transcribed. The files were listened to over and over again in order to make sure that no important data was missed.

### 3.7 Validity and Reliability

This study used triangulation of data sources as a means of ensuring validity. Document analysis (lesson plans), video recordings and audios were used to enrich data collected from lesson observations and interviews. Bowen (2009) describes triangulation as a combination of methodologies or information from different data sources in the study of the same phenomenon. It is further explained that triangulation helps the research to guard against the accusation that a study's findings are simply an artefact of a single method, a single source, or single investigator's bias. This is possible because in triangulation, researchers make use of multiple and different sources, methods, investigators and theories in order to provide corroborating evidence. As the researchers locate the evidence from different sources of data, they are triangulating the information and providing validity to their findings (Bowen, 2009). Observation and interview guides were also used as a way of addressing bias that would have been caused by a single observer and interviewee during observations and interviews.

### 3.8 Data Analysis

This study has followed a qualitative descriptive research design. Data collected in this study has been analysed using framework analysis, the MDI framework. Framework analysis as Parkinson, Eatough, Holmes, Stapley and Midgely (2016) explains, provides flexibility to use the data systematically so as to facilitate greater familiarity and immersion in the data, and eventually a better understanding of the insights and experience of the participants. Before starting formal analysis of data, the video recordings from lesson observation and the audio recordings from the interviews that were captured as part of data collection were transcribed. As one way of getting familiar with the data, Mills and Morton (2013) suggests "immersion" as the first step of data analysis. This is where the researcher spends some time with the research material, reading and re-
reading the transcripts and the field notes. The researcher took some time reading and re-reading the field notes, the transcriptions, and also listening to the videos and audios in trying to immerse herself in the data she collected. The MDI framework was developed for the purpose of analysing classroom (Adler \& Ronda, 2015) and also later to analyse textbooks (Ronda \& Adler, 2017). However, the researcher also used the MDI framework when she analysed lesson plans and interviews, because the teachers mentioned examples that could be used in classroom teaching. Examples mentioned in the lesson plan and during the interviews were related to the teaching of the lesson with the known object of learning, hence it makes sense to evaluate the examples mentioned in the lesson plan and during interviews using the MDI framework as if the examples were used in the real classroom teaching. In addition, what the teachers wrote in the lesson plan was a reflection of what they would do if it was the actual teaching in the classroom, since they used the same lesson plan for their actual teaching. Data from interviews was analysed using the framework because if teachers were to teach the way they responded during the interviews, their teaching would have been analysed using the MDI framework. Data from lesson plan and the interviews provided an enrichment to the data from the classroom. After the transcription of the video and audio recordings, and after being familiar with whole set of data, I proceeded to coding using the MDI framework as mentioned earlier on. Data was organized into chapters to form different units of analysis in accordance with the research questions but basing on the elements of the MDI framework which are object of learning, exemplification, explanatory talk and learner participation (Adler \& Ronda, 2015). Below is how the coding was done:

| Coding of Exemplification |  |
| :--- | :--- |
| Examples | Tasks |
| Similarity: | Use of known operations and procedures, coded |
| Experiencing one form of variation, coded | K, and level 1 |
| S, and level 1 |  |
| Contrasting: | Application of known skills, or decide on |
| Experiencing two forms of variation, coded | operation to use, coded A, and level 2 |
| $\mathbf{C}$, and level 2 |  |
| Fusion: |  |
| Experiencing more than two forms of | connections, coded C/ PS, and level 3 |
| variation, coded F, and level 3 |  |
| Level 0: |  |
| Experiencing no similarity and no | C/ PS $\rightarrow \mathbf{K}$, level 1 |
| contrasting. |  |

Table 1: Coding of exemplification (Adler \& Ronda, 2015)

| Coding for Explanatory Talk |  |
| :--- | :--- |
| Naming | Legitimating |
| Word use is colloquial, non mathematical, | Criteria is non mathematical, coded NM, and level |
| coded NM, and level 1 | 0 |
| Mathematics word used as names only, | Criteria is localized, coded L, and level 1 |
| coded MS, and level 2 where there is NM | Criteria counts as having partial generality, coded |
| and MS, and some MA | PG, and level 2 if criteria is beyond NM, |
| Mathematical language used | L and has PG |
| appropriately, coded MA, and is level 3 | Criteria is of full generality, mathematics |
| when there is movement between NM and | legitimating is proved, coded FG, and level 3 |
| MA |  |

Table 2: Coding of explanatory talk (Adler \& Ronda, 2015)

```
                        Coding of Opportunities for Learner Participation
To speak yes/ no, or single words to teachers' sentence, coded Y/N, and level 1
To speak some phrases and sentences in more than one episode, answering what/ how questions, coded P/S, and level 2
Some discussions in more than one episode, why questions, teacher revoices, coded \(\mathbf{D}\), and level 1
```

Table 3: Coding of Opportunities for Learner Participation (Adler \& Ronda, 2015)

### 3.9 Ethical Considerations

Everything related to this project followed the standards set by the Norwegian Centre for Research Data (NSD). I applied for a permission to NSD to carry out my research project. Permission to carry out the research project was given, refer to the permission letter in the appendix (see Appendix 1). Processing of personal data had been done in accordance with the principles under the General Data Protection Regulation of the NSD. No information that can be used to identify participants has been revealed. Personal names and school names have been replaced by codes or pseudonyms. After analysing the video recordings of the lessons and transcribing the audio recordings of interviews, both the video and audio recordings' files were deleted. Only anonymised text are kept after the research project. Personal data has been treated with confidentiality and in accordance with data protection legislation. In addition, participants were briefed in detail on what the study was about, and were told of their freedom to withdraw if they no longer wished to continue participating in the study.

### 3.10 Limitations

This study had some limitations. The subjects were not a true representation of the population, as such the results might not portray the true picture of what is really happening in Malawi as a country. Out of the many districts, many schools and many teachers that are there in Malawi, only 1 district, 3 schools and 3 teachers were involved. Although the findings of this study are not meant to be generalized to a larger population, dealing with more schools and more teachers could be of great help in finding out more concerning what this study was looking for. However, this is a phenomenological research which seeks construction of meaning from experiences. The researcher's position as a teacher educator might also have had an impact on the study as participating teachers could think that they were under supervision, and changed the way they
normally teach. As Baker (2006) discerns, even if participants agree to observe them, the researcher's presence can make participants behave differently from normal, and as a researcher, one can never be sure that these participants would do the same things if they were not being observed.

### 3.11 Chapter summary

This chapter has given a description of the design that this research study used, how the pilot study was conducted, the sample and how the sample was generated, how data was constructed, the instruments which were used to collect data, how the instruments were administered, the validity and reliability of the methods for collecting data and data analysis. It has also looked at the ethical considerations and limitations of the study. There were 3 participants who took part in the study. All the 3 teachers were observed teaching and then interviewed. Amongst the 3 teachers, 2 teachers had lesson plans which were photocopied for analysis whereas 1 teacher said that she did not write the lesson plan for that particular day. Each participant was observed teaching once, and two of the three participants were video recorded. The participants were interviewed after the lesson observations. The interviews with the 2 teachers who were video recorded were also audio recorded. One teacher did not accept to be video recorded and audio recorded during the teaching and during the interview despite initial arrangements that she would be video recorded and audio recorded. Using the methods outlined in this chapter, data was collected, analysed and the findings are presented in the next chapter.

## 4 Analysis of Data and findings

This chapter presents and analyses data from three teachers, Teacher A, Teacher B, and Teacher C, from School A, School B and School C respectively. Data from each teacher is presented in separate subchapters, one chapter for each teacher. First I present data from the teaching from the three teachers, Teacher A, Teacher B and Teacher C. This data is presented in subchapters 4.1 to 4.3. Then, I present data from the lesson plans from Teacher A and Teacher B only in the subchapters 4.4 and 4.5. Teacher C did not write the lesson plan on the day of lesson observation, as such there is no data from teacher C on lesson plans. Finally, I present data from the interviews for the three teachers in the subchapters $4.6,4.7$ and 4.8 respectively. For each teacher, data presented and analysed is based on the specific research questions of the study. The specific research questions of the study are:

1. How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?

2 (a).What type of resources, examples and tasks do teachers use to introduce algebra in Standard 5?

2(b).What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5?
3. What opportunities are provided for learners' participation?
4. How do Standard 5 learners participate in algebra lessons?

### 4.1 Data from the Teaching: School A

In the sub-chapters below, I present data relevant for each research question on the teaching that took place in school A. Data is on how Teacher A introduced algebra to Standard 5 learners as beginners. School A has only one Standard 5 class, and one mathematics male teacher, who volunteered to take part in the research study. There were 63 learners in total, 29 boys and 34 girls, who participated in the study. The object of learning on the day of observation was "writing number sentences."
4.1.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?
In this sub-chapter, I present data that can help answer the research question on how the teacher introduced and explained the basic algebraic concepts to standard 5 learners. Below, I present data
separately on how Teacher A introduced the concept of number sentence, letters, symbols and coefficient in relation to the object of learning to learners. Data is rated on how the concepts were introduced and explained by the teacher in line with explanatory talk of the MDI framework (Adler \& Ronda, 2015).

## Introduction of algebraic number sentence

Teacher A started the lesson by asking learners two oral questions:
Teacher A: What can be the answer if you add 3 mangoes plus 4 mangoes?

Learner: Seven mangoes.
Teacher A: What about 13 pens plus 2 pens, what can be the answer?
Learner: $\quad 15$ pens.
Teacher A then said to learners: "Today we are going to deal with algebra, and the main work is on writing number sentences," (writes the object of learning, "Writing number sentences" on the chalkboard).Teacher A defined a number sentence orally by repeatedly saying: "A number sentence is a way of expressing the relationship of numbers." He said to the learners: "You said that 3 mangoes plus 4 mangoes, is 7 ." He then wrote $3+4=7$ on the chalkboard. He told the learners that $3+4=7$ is a number sentence, "We have the numbers 3 plus 4 giving us a certain number which is 7 , so, 3 and 4 are related because they add up to 7."

Teacher A took out a chart with number sentences and pasted it on the chalkboard. He asked the learners to read the number sentences on the chart. The number sentences written on the chart were: 2 bottle tops plus 6 bottle tops is equal to 8 bottle tops, 3 leaves plus 3 leaves is equal to 6 leaves and 3 guavas plus 4 guavas is equal 7 guavas. He told learners that these are number sentences. He repeated telling learners the meaning of number sentence.

Kullberg et al. (2017) gives an example claiming that you cannot possibly understand Chinese language by simply listening to different people speaking if you have never heard another language. In a similar manner, it means learners cannot understand what a number sentence is only by looking at a number sentence presented by the teacher since they do not know what not a number sentence is. The characteristics that makes a sentence a number sentence, is not visible.

Algebraic concepts should not be presented to learners as readymade entities (Booth, 1988). Marton and Pang (2013) argue against the view of developing new meanings from experience of sameness by pointing to the examples; this is a number sentence and this is a number sentence. In terms of the MDI framework, pointing to the examples of number sentences and telling learners that this is a number sentence, and this is a number sentence is non mathematical, NM, and level 1 of naming in explanatory talk, there is no focused math talk (Adler \& Ronda, 2015).

## Introduction of the use of letters in algebra

Teacher A told learners that there is a short way of writing the number sentences. He showed learners number sentences on a chart and said:

Teacher A: Now, we have a way of writing those number sentences. Let us look at how we can write those number sentences.

Teacher A: From the first statement which is 2 bottle tops plus 6 bottle tops is equal to 8 bottle tops. This is a mathematical statement, a number sentence whereby we can also write it in short.

Teacher A: Instead of writing 2 bottle tops plus 6 bottle tops is equal to 8 bottle tops, You can simply think of the first letter which is $b$, here we have $b$, and here another b , and here also another b (pointing at the letter b in 2 bottle tops + 6 bottle tops $=8$ bottle tops on the chart).

Teacher A: Which means that to write this number sentence in short you can simply write $2 b+6 b=8 b$ (writing on the chalkboard). Where the letter $b$ represents objects, thus $2 b$ is for 2 bottle tops, $6 b$ is for 6 bottle tops and $8 b$ is for 8 bottle tops. This is how you are supposed to write number sentences in short.

Teacher A called for volunteer learners to come in front and write the following two number sentences in short on the chalkboard: 3 leaves plus 3 leaves is equal to 6 leaves, 3 guavas plus 4 guavas is equal to 7 guavas.

One learner volunteered and wrote the number sentence 3 leaves plus 3 leaves as $3 L+3 L=6 L$ on the chalkboard. Another learner wrote the number sentence 3 guavas plus 4 guavas is equal to 7 guavas in short as $3 g+4 g=7 g$ on the chalkboard.

From what has been explained, Teacher A introduced letters to learners as names or labels of objects. As Kullberg, Kemp and Marton (2017) explains, it is necessary to use representations, where, for example, pictorial images can be used to improve the recognition that letters are variables or specific unknown values in an algebraic number sentence, and not names of objects. Kuchemann (1981) claims that letters are variables or specific unknown values. From the transcript, the discourse can be classified as colloquial, non-mathematical (NM) in terms of word use and legitimating criteria (Adler \& Ronda, 2015), since letters in algebra are not defined as labels. Hence the explanatory talk is of level 1 , on naming and level 0 , legitimation.

## Symbols: Introduction of equal sign and addition symbol

Teacher A modelled the number sentences written on the chart using the objects he brought to class. He read the number sentence from the chart: 3 leaves plus 3 leaves is equal to 6 leaves. Then he counted 3 leaves and put them in one group and counted another 3 leaves and put them in another group. With emphasis on the plus, he combined the two groups and counted the objects to find the total number of the objects altogether, then he stressed on equals as he wrote down the equal sign on the chalk board followed by the answer. The modelling of addition done by the teacher, the stressing on the equals sign and the writing of the answer after the equal signifies that the equal sign was used as a symbol calling for an answer. Pointing to the chart which had the following number sentences; 2 bottle tops +6 bottle tops $=8$ bottle tops; 3 leaves +3 leaves $=6$ leaves; and 3 guavas +4 guavas $=7$ guavas, Teacher A had this to say:

Teacher A: Okay, the first statement goes like 2 bottle tops plus 6 bottle tops is equal to 8 bottle tops, (shows learners the bottle tops) these are bottle tops, so they are saying 2 bottle tops, one, and two (counts them) plus 6 bottle tops one, two, three, four, five, six.

Teacher A: So, we have two bottle tops plus six bottle tops. If we add them, they will give us a certain number which is 8 bottle tops (counts the total number of bottle tops after adding them). One, two, three, four, five, six, seven, eight. So, that one is the first statement which is also showing the relationship of numbers.

Teacher A: So, we have two bottle tops plus six bottle tops. If we add them, they will give us a certain number which is 8 bottle tops (counts the total number of bottle tops after adding them). One, two, three, four, five, six, seven, eight. So, that one is the first statement which is also showing the relationship of numbers.

Teacher A: Yes, anyone to read for us the second statement. (Selects a volunteer learner), yes can you read for us?

Learner: Reads, 3 leaves plus 3 leaves is equal to 6 leaves.
Teacher A: Okay, fine. This statement is 3 leaves plus 3 leaves is equal to 6 leaves. Which means that I can have three leaves (counts) one, two, three, then plus other three leaves (counts) one, two, and three. I have added the leaves, if I count those in my hands there will be six. One, two, three, four, five, six. So three leaves plus three leaves is equal to six leaves.

Teacher A: And we have got 4 guavas plus 3 guavas is equal to 7 guavas. If we add 4 guavas plus 3 guavas it gives us 7 guavas. I have got guavas here (.) one, two, three, four guavas plus one, two, three guavas. Now, counting them together they will give us a certain number which is seven. So it's (counts the guavas) One, two, three, four, five, six, seven. Which means that if you add 4 guavas plus 3 guavas, they will give us a certain number which is seven guavas. Is that clear class?

As we can see from the transcript, Teacher A used the objects he brought to class to model the number sentences that were written on the chart.

Teacher A mentioned addition as plus throughout the lesson, and he wrote the addition symbol in place of the word plus. The addition symbol was used as a symbol for combining groups of objects to come up with a single group of objects. He did not explain about the use of equal sign to learners. Kullberg et al. (2017) explains that when introducing algebraic number sentences, the focus of attention should not primarily be on finding the answer, but instead should be on understanding what a number sentence is by varying the terms and the operations used as a means of discernment.

According to Adler and Ronda (2015), the symbols were not used appropriately (in the context of algebra), hence NM, and level 1 of the explanatory talk.

## Teacher's introduction of coefficients in algebra to learners

Teacher A did not say anything concerning coefficients. There was no mention of the word coefficient in his teaching.

### 4.1.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5?

This sub-chapter provides data from the teaching with a focus on the resources that the teacher used, and how he selected examples and tasks for learners. The resources used were rated on whether they were used to support learners to achieve the object of learning, and the examples were rated on their similarity ( S ) and contrasting (C). The tasks that the teacher gave to learners were rated on whether the tasks required learners to carry out a known (K) operation or apply (A) what is known in relation to the object of learning (Adler \& Ronda, 2015).

## Types of resources used and how they were used

The teacher used a chart with number sentences and objects like fruits. The first letters of the objects were used as letters in algebraic number sentences presented. For example, the teacher told learners that:

Teacher A: Instead of writing 2 bottle tops plus 6 bottle tops is equal to 8 bottle tops, You can simply think of the first letter which is $b$, here we have $b$, and here another $b$, and here also another $b$ (pointing at the letter $b$ in 2 bottle tops + 6 bottle tops $=8$ bottle tops).

Teacher A: Which means that to write this number sentence in short you can simply write $2 b+6 b=8 b$. Where the letter $b$ represents objects, thus $2 b$ is for 2 bottle tops, $6 b$ is for 6 bottle tops and $8 b$ is for 8 bottle tops. This is how you are supposed to write number sentences in short.

From the transcript above, the teacher used a traditional approach of teaching algebra which is often termed as "fruit salad" (Kuchemann, 1981), or use of mnemonic literal symbols where objects are used as letters. Researchers such as Kuchemann (1981) and McNeil and Weinberg
(2010), warn that the use of mnemonic literal symbols or fruit salad approach strengthens learners' naïve conception that literal symbols in algebraic expressions stand for labels instead of quantities.

In the following transcript, Teacher A modelled the addition of the terms in a number sentence using the leaves, the bottle tops and the guavas he brought to class:

Teacher A: And we have got (.) 4 guavas plus 3 guavas is equal to 7 guavas (reading from the chart). If we add 4 guavas plus 3 guavas it gives us 7 guavas. I have got guavas here (.) one, two, three, four guavas plus one, two, three guavas. Now, counting them together they will give us a certain number which is seven. So, it's (counts the guavas) one, two, three, four, five, six, seven. Which means that if you add 4 guavas plus 3 guavas, they will give us a certain number which is seven guavas. Is that clear class?

Teacher A here used objects he brought to class as counters for modelling addition of the terms in a number sentence to find the total number of objects, and a chart to show the learners number sentences. He also used objects to come up with letters that were used in the number sentences. This is contrary to what other researchers like McNeil and Weinberg (2010); Kullberg et al. (2017) recommends, that algebra should not be introduced using objects such as fruits or mnemonic literal symbols to beginners. Adler and Ronda (2015) recommends that for proper exemplification, teacher should provide tasks to enable learners use multiple representations. There was no use of representations, hence exemplification categorized as level 1.

## Teacher's selection of examples.

Teacher A used the examples which he wrote on the chart, and gave this explanation to learners:
Teacher A: From the first statement which is 2 bottle tops plus 6 bottle tops is equal to 8 bottle tops. This is a mathematical statement, a number sentence whereby we can also write it in short.

Teacher A: 2 bottle tops +6 bottle tops $=8$ bottle tops; to write this number sentence in short you can simply write $2 b+6 b=8 b$, where the letter $b$ represents objects, thus $2 b$ is for 2 bottle tops, $6 b$ is for 6 bottle tops and $8 b$ is for 8 bottle tops. This is how you are supposed to write number sentences in short. Can someone come and write in short:

Teacher A: 3 bananas +5 bananas $=8$ bananas on the chalkboard.
Learner: Writes $3 b+5 b=8 b$ on the chalkboard
Teacher A: What about 3 Leaves +3 Leaves $=6$ Leaves?
Learner: $\quad$ Writes $3 L+3 L=6 L$ on the chalkboard
Teacher A: 4 guavas +3 guavas $=7$ guavas
Learner: Writes $4 g+3 g=7 \mathrm{~g}$ on the chalkboard
The teacher used the first letters of the objects as letters in the number sentences he presented as examples. In other words, letters were introduced as names or labels of objects in all the examples that the teacher provided. The meaning of number sentence was not stressed in the examples. Nowhere in the examples were letters explained to learners as variables or unknowns, and the equal sign was explained as a relational symbol.

In the transcript above, Teacher A used the objects for introducing letters and for modelling the addition process in all the examples he presented. All the examples presented to learners were of the same type, there were no variations, and therefore no contrasting examples, hence, providing an opportunity for learners to experience one form of variation, similarity (S). This is what Adler and Ronda (2015) classifies as level 1 examples.

## Tasks selected for learners

Teacher A started the lesson by showing learners number sentences which were written on a chart, and then asked the learners to read the number sentences. Then the teacher wrote on the chalkboard the same number sentences that were on the chart. He then demonstrated to learners how to write the number sentences in short. This was followed by an exercise where learners individually were asked to write number sentences in short:

Teacher A: If there is no question, open your books on page 105, and go to exercise 1A, (Writing number sentences). You do for me number 2 and number 5.Thereare two of them.

Teacher A: Before you start writing let's look at number two. Number two goes like four cups plus five cups equals nine cups, four cups plus five cups equals nine cups (repeats). So you have to write that mathematical statement in
short as we have done in our examples. Then, number 5 goes like if you add 11 mangoes to 13 mangoes you will have 24 mangoes. You have also to write that one in short.

After marking learners work, Teacher A did correction with learners where he explained to learners that he expected them to write the problems in short as:

$$
\begin{aligned}
& \text { 1. } 4 \text { cups plus } 5 \text { cups equals } 9 \text { cups } \\
& \qquad=4 c+5 c=9 c
\end{aligned}
$$

## 2. 11 mangoes plus 13 mangoes equals 24 mangoes <br> $$
=11 m+13 m=24 m
$$

He wrote the problem; the sum of 12 goats and 25 goats is equal to 37 goats on the chalkboard and asked for a volunteer learner to come and write the number sentence in short. One learner volunteered and wrote the solution as:

$$
\begin{aligned}
& \text { 3. The sum of } 12 \text { goats and } 25 \text { goats is equal to } 37 \text { goats } \\
& =12 \text { goats }+25 \text { goats }=37 \text { goats } \\
& =12 \mathrm{~g}+25 \mathrm{~g}=37 \mathrm{~g} \text {. }
\end{aligned}
$$

Research studies have shown that teachers in Malawi often teach mathematics and algebra in particular, using procedural teaching and learning. They present their lessons by following what has been written in the teacher's guide. They start teaching by giving learners examples that are taken from the textbooks, followed by a task for practice known as an exercise which is also taken from the text books (Kazima \& Jakobsen, 2013). The teaching done by Teacher A seems to be an example of this traditional way of teaching mathematics.

The problems in the task for learners were all similar to the examples which the teacher presented to learners. Names of objects in the number sentences were changed into letters, as such, letters in the number sentences were presented as names or labels of the objects. Considering the task given to learners, all the problems in the task for learners were of the same type, and they were all similar to the examples that the teacher had given learners, hence classified as level 1 tasks in Adler \& Ronda (2015). Such task required learners to use known procedures and operations, coded K, and classified as level 1tasks in the MDI framework (Adler \& Ronda, 2015).

### 4.1.3 Research Question 3: What opportunities are provided for learners' participation?

This sub-section presents data on Research Question 3. The focus is on the opportunities that the teacher provided for learners to explore and discover what a number sentence is during the class, opportunities that would have supported learners to understand the object of learning "writing number sentences."

Teacher A provided learners with opportunities to speak short sentences (P/S) in the lesson by asking them to read the number sentences from the chart, both individually and in choir. He provided learners with opportunities to speak Yes/ No and single words (Y/N). He gave learners an exercise to write individually. However, Teacher A first demonstrated the procedure in all the examples. By doing so, he reduced learner's tasks to arithmetic operations and participation to answering Yes/No questions, or completing sentences with single words (Y/N). YN is classified as level 1, in the MDI framework (Adler \& Ronda, 2015).

### 4.1.4 Research Question 4: How do Standard 5 learners participate in algebra lessons?

In this sub-chapter, the researcher presents data on how the teacher involved learners so that they could all participate in the lesson. Teacher A allowed learners to participate individually through verbal and written exercises like where he said, "Remember, we said 3 plus 4 gives us a certain number $\approx 7$." Teacher A also used what questions for example:

Teacher A: What can be the answer if you add 3 mangoes plus 4 mangoes?
Learner: Seven mangoes.
The questions the teacher asked learners throughout the lesson required learners to answer Yes/ No answers, for example the teacher would ask learners, "is it right?" and at other times Teacher A would ask learners questions that required them to speak a single word to finish the teachers unfinished sentence like:

Teacher A: What about 13 pens plus 2 pens, what can be the answer?
Learner: 15 pens.

Teacher A also at times used how questions, For example where he asked questions like: "How can you write the number sentence 3 guavas plus 4 guavas in short?"

In the transcripts above it shows that Teacher A involved learners individually through written exercises and through verbally. He also allowed choral answers. According to MDI framework, the what/ how questions are coded P/S. Therefore, learner participation was Y/N, P/S, but the demonstrations the teacher made unfolded activities and reduced them to level 1 (Adler \& Ronda, 2015).

### 4.2 Data from the Teaching: School B

In the sub-chapters below, I present and analyse data relevant for each research question on the teaching done by Teacher B. School B has only one Standard 5 class, and one mathematics female teacher who volunteered to take part in the research project. There were 87 learners in total in the class, 46 boys and 41 girls, who participated in the study. Data to be presented in the subsections below is on how Teacher B introduced algebra to Standard 5 learners as beginners. The object of learning on the day of observation was "completing number sentences."
4.2.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners
Data obtained from this research question is on how the teacher introduced and explained the basic algebraic concepts to Standard 5 learners. The subchapter presents data on how Teacher B introduced algebraic number sentences, letters, symbols and coefficients to learners in Standard 5. Data collected was rated on how the algebraic concepts were explained by the teacher in line with the MDI framework (Adler \& Ronda, 2015). The object of learning of the lesson on the day of observation was completing number sentences.

## Introduction of algebraic number sentences

Teacher B started her lesson by asking learners to create number sentences basing on what they learnt in the previous lesson. Learners created and wrote the number sentences in short form on the chalkboard. The following was the conversation that the teacher had with the learners:

Teacher B: Can you create any number sentence that you know?
Learner: $\quad 4$ bananas plus 1 banana is equal to 5 bananas
Teacher B: Can someone come and write that number sentence in short on the chalkboard?

Another learner volunteered and wrote the number sentence on the chalkboard as $4 b+b=5 b$

Teacher: (Pointing to the number sentence written by the learner on the chalkboard) so, it's $4 b+1 \approx$

Learners: $\quad \approx b=5 b$.
Teacher B then wrote on the chalkboard the following problem for learners to write in short: 9 birds take away 4 birds equals $\qquad$ . After the learners solved the problem in their exercise books individually, the teacher asked them to exchange their exercise books to mark each other's work. Teacher B then demonstrated to learners how to write 9 birds take away 4 birds equals
$\qquad$ in short form by saying:

Teacher B: When you are writing a number sentence you just write the first letter, in this case, 9 birds will be $9 b, 4$ birds will be $4 b$. What is the meaning of take away?

Leaner: Take away means minus.
Teacher B: It's now straight forward ; ( writes on the chalkboard), $9 b-4 b=\ldots$, so what is the answer?

Learners: $\quad 5 b$ (answer in choir)
Teacher B: $\quad 9 b-4 b=5 b$ (Writes on the chalkboard).
Teacher B did not give learners any explanation of what a number sentence is or teach learners how the terms in a number sentence are related. She emphasized on the filling of missing terms in a number sentence. According to Molina et al. (2017), it is necessary that teachers explain the relationship of numbers or terms in a number sentence to learners in order to avoid errors in algebra. There was no activity that encouraged maths talk. This is non mathematical (NM) way of presenting work, and level 1 explanatory talk (Adler \& Ronda, 2015).

## Introduction of the use of letters in algebra

Teacher B asked learners to solve: 9 birds take away 4 birds equals $\qquad$ , and asked learners to write the number sentence in short. To come up with letters to be used in the number sentences, Teacher B told learners that they should just write the first letter of the name of the object that has been used in the number sentence. For example, 9 birds will be $9 b$, and 4 birds will be $4 b$. The answer will be $9 b-4 b=5 b$.

Letters were explained to learners as first letters of the names of objects such as fruits. This is in contradiction with information from researchers such as Kuchemann (1981), who explains that letters in an equation (number sentence) can only be used as variables. At one time the teacher referred to letter y in $9 y+\ldots=17 y$ as 9 things plus dash equals 17 things, so $y$ was referred to as things. In terms of MDI framework, word use is colloquial, non-mathematical (NM) and ambiguous (letter $y$ referred to as things), so naming is of level 1 (Adler \& Ronda, 2015).

## Symbols: Introduction of equal sign and addition symbol

Teacher B wrote $9 y+\ldots=17 y$ on the chalkboard and asked learners to read the number sentence. Teacher B told learners that $9 y+\ldots=17 y$ is the same as adding $9 y$ to a certain number to get $17 y$. She had to say this:

Teacher B: Let's forget about the letters and concentrate on the numbers only.
Teacher B: Listen (.) let's do together. Check and correct your work. Don't get confused with letters, the focus should be on the numbers only. In the problem $3 n-\ldots=2 n$; what can we subtract from 3 to get 1 .

Teacher B: asks learners to count 3 objects and remove some objects to remain with 2.
Learners: (counts 3 stones and removes 1 stone to remain with 2 stones). In a choir they respond 2

Teacher B: Writes the answer $3 n-n=2 n$ on the chalkboard.
To solve $9 y+\ldots=17 y$, Teacher B instructed learners in vernacular language as follows:

Teacher B: Take out your counters and count 17.Because 17 is the result after adding 9 with another number. (Explains in vernacular language), now, to find the missing number, subtract 9 counters from 17 counters

Learners: (counts 17 stones and removes 9 stones to get 8 ).
Teacher B: What do you get? (...)
Learners: 8
Teacher B: Good

Teacher B: So, $9+8=17$. But don't forget we are adding things of the same group, y things, so it will be $8 \approx$

Learners: $\approx y$.
Teacher B: $\quad$ So it will be written as $9 \mathrm{y}+8 \mathrm{y}=17 \mathrm{y}$ (writes on the chalkboard).
From the transcripts above, we see that Teacher B asked learners to find the missing number by using stones as counters. The emphasis was on what was to be added to 9 to make 17 , and what was to be subtracted from 3 to get 2 . She told learners to forget about the letters. The equal sign was used as a symbol calling for a calculation to put an answer after the equal sign. In all the examples and the exercise that Teacher B gave learners to do, she emphasized on the use of counters to find a single term answer and put it after the equal sign. The addition and subtraction symbols were used as operational symbols. Addition was used to combine two groups of objects to come up with one group, everyday language, NM, and everyday procedures, E, while with subtraction the learners were told to remove objects from the bigger group of objects, everyday language, NM, and everyday procedures, E (Adler \& Ronda, 2015).Therefore NM in naming is level 1 and E is level 0 in legitimating. Kullberg et al. (2017) argues that teachers should not primarily focus their attention on finding the answer when teaching number sentence concept but instead they should aim at helping learners understand the relationships in a number sentence.

## Introduction of coefficients in algebra to learners

Teacher B told learners that if the coefficient is 1 , we don't write the number 1, we just write the letter, as in $3 n-1 n=2 n$, this is written as $3 n-n=2 n$. She also told learners that the coefficient goes together with the letters. However, Teacher B did not clearly explain to learners what a coefficient is in algebraic expressions, such as in number sentences. Telling learners that "coefficients go together with letters," classifies word use as colloquial, non-mathematical (NM), and categorized as level 1 explanations (Adler \& Ronda, 2015).

### 4.2.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5?

This sub-chapter provides data and analyses data on the teaching with a focus on the resources that Teacher B used, and how she selected examples and tasks for learners. The resources that the teacher used were rated on whether they were used to achieve the object of learning or not. The examples that the teacher used were rated in terms of similarity and contrasting, and the tasks that
the teacher selected for learners were rated on whether the tasks required learners to carry out a known (K) operation or apply (A) what is previously known in relation to the object of learning (Adler \& Ronda, 2017).

## Types of resources used and how they were used.

Teacher B used different objects like fruits and stones in the lesson. Objects were used for introducing letters in number sentences and also as counters for modelling addition and subtraction of terms to find the missing terms in number sentences. The first letters of the name of objects were used as letters in the number sentences presented. For example, Teacher B told learners that: "When you are writing a number sentence, you just write the first letter, in this case, 9 birds will be $9 b$, and 4 birds will be $4 b$. " During feedback on the problem $18 s+\ldots=22 s$, the teacher said that 18 s means 18 stones. Teacher B then asked learners to count 18 stones, using count on addition ${ }^{5}$ method, she explained to learners how to find the missing term in the number sentence. Learners were instructed to keep on adding one stone at a time to the group of 18 stones until they had 22 stones. Learners found that the missing number was 4 . Teacher B also used the resources to explain the simplification of like and unlike terms. For example, the teacher said to learners: "Remember when we are adding things, we add things that are alike. Leaves with other leaves, stones with other stones, bananas with bananas. We cannot add leaves with stones, or we cannot add bananas with leaves."

From what has been explained above, Teacher B used objects like stones and number cards in her lesson. The objects were used to introduce letters in algebraic number sentences. The first letters of the objects were considered as letters in the number sentences. In this case, Teacher B taught learners that objects are letters in number sentences. This contradicts what other researchers like McNeil and Weinberg (2010); Kullberg et al. (2017) recommend, that algebra should not be introduced using objects such as fruits or mnemonic literal symbols to beginners. It is argued that this may promote learners' naïve conception that literal symbols in algebraic expressions stand for labels instead of quantities. The objects were also used as counters to model addition of terms in an algebraic number sentence. This shows that Teacher B taught algebra just like arithmetic. As

[^3]part of exemplification, the teacher did not use representations in the tasks during the examples, hence level 1of exemplification in the MDI framework (Adler \& Ronda, 2015)

## Teacher's selection of examples

Teacher B used stones and number cards in the examples she presented in her lesson. In all the examples used, the first letters of the names of the objects were presented as letters in the number sentences. On one occasion, Teacher B asked learners a question which led to learners coming up with number sentences which were later used as examples. This was the conversation that Teacher $B$ had with her learners:

Teacher B: It's time for mathematics. Can I have someone to create any number sentence?

Learner: 4 bananas plus 1 banana is equal to 5 bananas
Teacher B: Can you clap hands for him? (Leaners clap hands)
Teacher B: Can someone come and write that number sentence in short on the chalkboard?

Learner: $\quad$ Writes $4 b+1 b=5 b$ on the chalkboard.
Teacher B then gave learners number cards with algebraic terms on them to arrange them in order so as to complete a number sentence that she wrote on the chalkboard. The number sentence that was completed by arranging the number cards with the algebraic terms in order was $9 y+\ldots=$ $17 y$. However, the teacher told learners to ignore the letters when solving the problem. Teacher B continuously reminded learners to forget about the letters. In her lesson, she said:

Teacher B: Let us forget about the letters and concentrate on the numbers only. We already know that we have y things which we are adding and they are things of the same group, they are alike. So, our focus should be on the numbers only.

The learners identified 8 y as the missing term, and the teacher completed writing the number sentence on the chalkboard as $9 \mathrm{y}+8 \mathrm{y}=17 \mathrm{y}$. Finally, Teacher B wrote $3 n-\ldots=2 n$ on the chalk board for learners to do individually. She then showed learners how they were supposed to write
the answer by writing the answer on the chalkboard, then asked the learners to exchange their exercise books and marked each other's work.

All the examples that Teacher B gave learners were leading to a single term solution. The examples presented to learners were similar in the sense that they had missing terms in them for learners to fill in. However, the examples were contrasting because they had different operations that were used. The first two examples were addition problems whereas the last example was a subtraction one. However, the teacher demonstrated and explained to learners what they were supposed to do in vernacular language. This act unfolded the set-up of the examples and reduced the use of examples to level 1, though with two forms of variation. Therefore, Teacher B's examples have been classified as level 1 (Adler \& Ronda, 2015).

## Tasks selected for learners

The teacher asked learners to create a number sentence, which one of the learners created as 4 bananas plus 1 banana is equal to 5 bananas. Teacher B then asked learners to write the number sentence in short on the chalkboard, and one learner wrote it as $4 b+b=5 b$. Then Teacher B distributed number cards to learners for them to arrange so that they find the missing term in a number sentence $9 y+\ldots=17 y$. Then the teacher asked learners to work out $3 n-\ldots=2 n$ and later wrote the answer on the chalkboard for the learners to check their work. Finally, Teacher B gave learners the following exercise to do individually:
(a) $-\quad-3 m=2 m$
(b) $4 t+3 t=$ $\qquad$
(c) $18 s+\ldots s=22 s$

Teacher B explained to learners how to complete the number sentences (filling in the missing terms). The tasks given to the learners required them to use known operations (K), and level 1, the MDI framework (Adler \& Ronda, 2017).

### 4.2.3 Research Question 3: What opportunities are provided for learners' participation?

The teacher grouped learners and provided them with number cards and asked them to make number sentences from the number cards given to them by arranging them. Through presentation of their work, during feedback, learners were given a chance to speak some phrases. She also asked
learners some oral questions, for example, learners were asked to create a number sentence, then the teacher asked the learners to read the number sentence aloud in choir. This is an example of the situation in class where learners could speak some sentences. On some occasions she asked learners what questions, giving them a chance to speak single words. The teacher gave learners individual work for them to practice finding the missing terms, after marking, learners were given a chance to speak by saying what the answers were. However, all the opportunities that the teacher provided for the learners to participate did not allow leaners to explore on the object of learning. There were no how and why questions that would have provided learners with opportunities to explore and discover what makes sense of a number sentence, and then filling in of the missing terms in a number sentence.

Teacher B provided learners with opportunities to speak single words, and this is what is classified as (Y/N), level 1, in the MDI framework (Adler \& Ronda, 2015), and opportunities to speak short expressions P/S (phrases and sentences) classified as level 2 (Adler \& Ronda, 2015). Hence learner participation was $\mathrm{Y} / \mathrm{N}, \mathrm{P} / \mathrm{S}$ and level 2.

### 4.2.4 Research Question 4: How do Standard 5 learners participate in algebra lessons?

In this sub-chapter, I present data on how the teacher involved learners so that they could all participate in the lesson. Learners worked in small groups, pair work and individually. The questions that Teacher B asked learners at other times required them to finish the teachers incomplete sentences by using single words, for example, "What is missing in the number sentence $3 n-\ldots=2 n$ ?" By answering " $n$ " learners were given a chance to speak a single word. Teacher B also used what questions, for example:

Teacher B: The second one is $18 s+\ldots=22 s$. If we add a certain number to 18 we are going to get 22. What is this number?

Learner: 4
Teacher B: What should we write on the blank space?
Sometimes she used questions that required learners to answer Yes / No, for example;
Teacher B: What is the meaning of take away?
Learner: Take away means minus.

Teacher B: Is that true?
In the transcripts above, Teacher B used questions that required learners to answer Yes / No and complete teacher's sentences by a single word or term, and questions where learners said some phrases and sentences. MDI framework classifies the yes /no and single word answers as Y/N, and what questions where learners are required to say phrases and sentences as P/S. Allowing learners to work in small groups, pair work and individually is a clear indication that Therefore, learner participation was Y/N, P/S, and level 2 (Adler \& Ronda, 2015).

### 4.3 Data from the Teaching: School C

In the sub-chapters below, I present and analyse data for each research question on the teaching done by Teacher C from School C. School C has only one Standard 5 class and one mathematics female teacher, who volunteered to take part in the research project. School C had in total 93 learners present in the Standard 5 class on the day of lesson observation. There were 52 boys and 41 girls. The focus of this chapter is on how Teacher C introduced algebra to Standard 5 learners as beginners. The object of learning on the day of observation was "Writing number sentences."
4.3.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?
In this sub-chapter, I present data that can help answer the research question on how Teacher C introduced and explained the basic algebraic concepts to standard 5 learners. Below, I present data separately on how the teacher introduced the concept of number sentence, letters, symbols and coefficient in relation to the object of learning to learners. Data is rated on how the concepts were introduced and explained by the teacher in line with explanatory talk of the MDI framework (Adler \& Ronda, 2015).

## Introduction of algebraic number sentences

Teacher C started the lesson by stating the object of learning of the day. She said, "In mathematics today, we are going to learn about algebra, writing number sentences." Then, Teacher C wrote the following number sentences on the chalk board:

3 bananas plus 5 bananas is equal to 8 bananas.
2 oranges plus 1 orange is equal to 3 oranges.

She asked volunteer learners to read the number sentences written on the chalkboard. Two learners read the number sentences written on the chalkboard, one learner read each sentence. Teacher C told learners that they were going to write number sentences in short. She explained to learners how to write number sentences in short using the examples of number sentences written on the chalkboard. Thus, 3 bananas plus 5 bananas is equal to 8 bananas, which in short form, Teacher C wrote as $3 b+5 b=8 b ; 2$ oranges plus 1 orange is equal to 3 oranges which was written as $2 o+$ $o=3 o$. Teacher C advised learners to use capital letter $O$ when writing number sentences in short form because small letter $o$ looked like a zero. So, the number sentence was written as $2 O+O=$ 30 .

In all her explanations, Teacher C did not explain what a number sentence was to learners. She concentrated on writing number sentences in short without explaining to learners what a number sentence was. In terms of explanatory talk, there was no math talk, and this is coded NM, and is classified as level 1 in the MDI framework (2015).

## Introduction of the use of letters in algebra

Teacher C told learners that algebra uses alphabet letters which can be the first letters of the objects. She presented different objects to learners and asked them to name the objects. The objects shown to learners were bananas, potatoes, oranges and tomatoes. Teacher C , alone, in front of the learners modelled number sentences using the objects. She wrote the number sentences that she modelled on the chalkboard, for example:

> 3 bananas plus 5 bananas is equal to 8 bananas 2 oranges plus 1 orange is equal to 3 oranges

> 3 potatoes plus 2 potatoes is equal to 5 potatoes.

Teacher C asked the learners to identify the first letters of the names of the objects used in the modelled number sentences. For example, in a number sentence, 3 bananas plus 5 bananas is equal to 8 bananas, learners were asked to identify the first letter of the objects used and students said " $b$ ". Teacher C told learners that the letter $b$ in bananas can be used for writing algebraic expressions in short. She then wrote the number sentence as $3 b+5 b=8 b$. She further told learners that the letter $b$ in the number sentence stands for bananas, instead of writing the whole word banana, they should just write $b$. She did the same with other number sentences where learners
identified the first letters of the objects used as $O$ in 2 oranges plus 1 orange is equal to 3 oranges, and $p$ in 3 potatoes plus 4 potatoes is equal to 7 potatoes. So, the number sentences were written as $2 O+O=3 O$ and $3 p+4 p=7 \mathrm{p}$ respectively.

Teacher C introduced letters to learners as names or labels of objects. As Kullberg et al. (2017) explains, with reference to variation theory, it is necessary to use iconic representations, where pictorial images can be used to improve the recognition that letters are variables or specific unknown values in an algebraic number sentence, and not as names of objects. Kuchemann (1981) claims that letters are variables or specific unknown values. In the transcript, the discourse can be classified as colloquial, non-mathematical (NM) (Adler \& Ronda, 2015), since letters in algebra are not defined as labels. Hence the explanatory talk is of Level 1.

## Symbols: Introduction of equal sign and addition symbol

Teacher C modelled the number sentences written on the chalkboard using the objects she brought to the class. She asked the learners to read the number sentences from the chalkboard, thus, 3 bananas plus 5 bananas is equal to 8 bananas, and then counted 3 bananas and put them in one group and counted another 5 bananas and put them in another group. Emphasizing on the plus symbol, she combined the two groups and counted the objects to find the total number of the objects altogether, and then wrote the answer as $3 b+5 b=8 b$ on the chalkboard. While stressing on equals, Teacher C wrote the answer after the equal sign. The equal sign was used as a symbol calling for an answer.

From what has been explained above, Teacher C combined the groups of objects she modelled to come up with a single group of objects to illustrate addition process. An equal sign was used as a symbol calling for an answer. She did not explain about the use of equal sign to learners. Kullberg et al. (2017) explains that when introducing algebraic number sentences, the focus of attention should not only be on finding the answer but also on assisting learners to understand relationships in a number sentence. Teacher C used addition and equals as used in everyday language, NM, and everyday procedures, E (Adler \& Ronda, 2015).Therefore NM in naming is level 1 and E is level 0 in legitimating.

## Introduction of coefficients in algebra to learners

Teacher C told learners that:

Teacher C: Where there is one object used, as in a number sentence 2 oranges plus 1 orange is equal to 3 oranges, 1 orange is a coefficient and 1 is not written, instead you just write the letter so it becomes $2 \mathrm{O}+\mathrm{O}=3 \mathrm{O}$.

From the explanation given by the teacher in the transcript above, concerning coefficients, it seems that to Teacher C, coefficient means 1 object. Word use is colloquial, NM and explanatory talk I terms of naming is level 1.

### 4.3.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5?

This sub-chapter provides data on the teaching with a focus on the resources that Teacher C used and how she used them, and how she selected the examples and tasks for the learners. The resources used were coded using the MDI framework (Adler \& Ronda, 2017), on whether they were used to achieve the object of learning or not. The examples were rated in terms of similarity $(\mathrm{S})$ and contrasting (C) and the tasks that the teacher gave to students, were rated on whether the tasks required learners to carry out a known (K) operation or apply (A) what is known in relation to the object of learning (Adler \& Ronda, 2017).

## Types of resources used and how they were used.

The teacher used different objects like bananas, potatoes, tomatoes and oranges which she brought to class. Teacher C said that the first letters of the names of objects are letters in algebraic number sentences. Teacher C also used the objects for modelling number sentences. For instance, to illustrate the meaning of 2 oranges plus 1 orange, Teacher $C$ counted 2 oranges and put them in one group, then counted 1 orange and put it in another group. Then she combined the two groups to make a single group of 3 oranges.

From the explanation above, Teacher C used a traditional approach of teaching algebra known as "fruit salad" (Kuchemann, 1981), where letters are treated as names of objects. She also used objects as counters. The use of resources in the examples, as well as in the tasks selected for learners should help learners to understand and achieve the object of learning (Adler and Ronda, 2015). Researchers like Kullberg et al. (2017) recommends, that algebra should be introduced using multiple representations. The use of objects as resources is categorized to level 1 in exemplification.

## Teacher's Selection of Examples

Teacher C used examples from the teachers' guide and from the learners' book. She wrote the examples on the chalkboard. These were the examples that she used:

3 bananas plus 5 bananas equals 8 bananas which in short was written as $3 b+5 b=8 b$
2 Oranges plus 1Orange equals 3 Oranges and it was written as $2 O+O=3 O$ in short form.

According to the MDI framework (Adler \& Ronda, 2015), the examples that Teacher C used provided opportunities for learners to experience one form of variation in terms of similarity (S), and so of level 1

## Teacher's Selection of Tasks for Learners.

Teacher C asked learners to read the number sentences written on the chalkboard and asked them to write the number sentences in short on the chalkboard. She at other times invited learners to write the number sentences on the chalkboard. Finally, Teacher C gave learners a task in form of an exercise to do individually. The teacher told learners to do exercise 1A on page 105, of the learners' book. The task was on writing number sentences in short. The problems were similar to the examples that the teacher had given learners. The task that the teacher had given learners to do individually consisted of the following problems:

8 girls plus 4 girls equals 12 girls
3 mangoes plus 3 mangoes equals 6 mangoes
2 bottle tops plus 6 bottle tops equals 8 bottle tops
From observation, the task given to learners had all the problems in standard context form. This contradicts recommendations by Vincent et al. (2015), who asserts that instead of giving learners tasks that are of standard context only, teachers should give learners tasks that include some nonstandard problems ${ }^{6}$, so that the concept of equality relationship between quantities on each side of an equation is emphasized. The problems were very similar to the examples presented by the

[^4]teacher. Using the MDI framework (Adler \& Ronda, 2015), the tasks that the teacher gave learners could only provide learners with an opportunity to practice known procedures and operations (K), therefore, classified as level 1 tasks.

### 4.3.3 Research Question 3: What opportunities are provided for learners' participation?

This sub-chapter presents data on the opportunities that the teacher provided for learners to explore and discover the meaning of a number sentence and other algebraic concepts. Teacher C provided learners with opportunities to speak in the lesson by asking them to read the number sentences she wrote on the chalkboard, both individually and in choir. She also gave learners an exercise to write individually. Teacher C demonstrated the procedures in all the examples, therefore, she reduced learner's tasks to arithmetic operations, and participation to answering Yes/No, for example, "Are we together class?" This is what is coded Y/N, and classified as level 1 , in the MDI framework (Adler \& Ronda, 2015). She also provided learners with opportunities to speak short expressions P/S (phrases and sentences) in more than one episode by asking what and how questions, and this is classified as level 2 (Adler \& Ronda, 2015). Teacher C did not ask learners why questions which would have provided learners with opportunities to explore and discover the object of learning and the meaning of algebraic concepts. Therefore, learner participation was Y/N, P/S and level 2.

### 4.3.4 Research Question 4: How do Standard 5 learners participate in algebra lessons?

This sub-chapter is concerned with how the teacher carried out class activities (methods used) so that all learners were involved in the teaching and learning process.

Teacher C involved learners individually by giving them a task in form of an exercise to do. She also asked learners to read number sentences written on the chalkboard, as such, learners were invited to participate through speaking in choir. Teacher C allowed learners to participate individually through verbal and written exercises. The questions the teacher asked during the lesson required learners to answer Yes/ No answers (choral responding), for example the teacher would ask learners, "are we together class?" and at other times the what and how questions, For example, the teacher could ask learners, "How can you write 3 mangoes plus 3 mangoes in short?", learners individually would answer $3 m+3 m$. The teacher could ask learners to identify the first letter of the objects that were being used for modelling number sentences, for example, she could say: "what is the first letter in potatoes?"

Teacher C used questions that enabled learners to participate individually and in choir. She also used the what/ how questions where learners could say some phrases and sentences. Therefore, learner participation was $\mathrm{Y} / \mathrm{N}, \mathrm{P} / \mathrm{S}$ and this is classified as level 2 (Adler \& Ronda, 2015).

### 4.4 Data from Lesson plan: Teacher A

In this chapter, I present data from analysis of the lesson plan for Teacher A, which was analysed in line with each research question. The object of learning which was written in the lesson plan was "Writing number sentences."
4.4.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard
5 learners? 5 learners?
In this sub-chapter, I present data from the analysed lesson plan on how the teacher explained the concept of number sentence in algebra to learners, the use of letters, symbols and coefficient. Data was rated on how the concepts were explained by the teacher in a lesson plan in relation to the object of learning.

## Introduction of number sentence in algebra.

In the lesson plan, Teacher A indicated that he intended to discuss with learners the meaning of number sentence as, "a way of expressing the relationship of numbers." He indicated in the lesson plan that he would show learners a chart with the following mathematical statements:

```
2 bottle tops +6 bottle tops \(=8\) bottle tops
3 leaves +3 leaves \(=6\) leaves
3 guavas +4 guavas \(=7\) guavas
```

Teacher A mentioned in the lesson plan that he would ask learners to read the number sentences aloud. The lesson plan showed that he had planned to discuss with learner how to write the number sentences in short. If Teacher A was to discuss the meaning of number sentence with the learners in the real classroom situation, then, the explanatory talk would have been classified as level 3, and coded D according to MDI framework (2015).

## Use of letters in algebra

Teacher A did not mention anything about the use of letters in algebraic expression in the lesson plan.

## Symbols: Use of equal sign and addition symbol

Teacher A did not mention anything concerning the use of equal sign and addition symbol in a number sentence in the lesson plan. What was seen in his lesson plan were examples of writing number sentences in short. One of the number sentences that he wrote in his lesson plan as a task for learners was 4 cups +5 cups $=9$ cups, which was written in short in the learners activities column as $4 c+5 c=9 c$, as an expected answer.

## Explanations on the meaning of coefficient

Teacher A did not write anything about the meaning of coefficient in the lesson plan.

### 4.4.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers

 use to introduce algebra in standard 5?This sub-chapter provides data from lesson plan for Teacher A on the resources that the teacher planned to use and how he selected the examples and tasks for learners. The resources he had planned to use were rated on whether they were planned to achieve the object of learning. The examples were rated on similar and contrasting, and the tasks that the teacher planned for students in the lesson plan were rated on whether they would require learners to carry out known (K) operations or apply (A) what is known in relation to the object of learning (Adler \& Ronda, 2015).

## Resources used and how they were used

The lesson plan indicated that Teacher A planned to use a chart with number sentences, bottle tops, leaves and learners books as resources. Teacher A indicated in his lesson plan that he would show learners a chart with number sentences and then ask them to read aloud the number sentences written on the chart, and then discuss with them how to write the number sentences in short. The number sentences which the teacher indicated in the lesson plan that they were written on the chart were:

2 bottle tops +6 bottle tops $=8$ bottle tops

$$
3 \text { leaves }+3 \text { leaves }=6 \text { leaves } .
$$

From what was written in the lesson plan, it shows that Teacher A planned to use the chart with number sentences to show learners what number sentences are, ask them to read the number sentences, and then discuss with learners how to write the number sentences on the chart in short.

He did not mention anything on how he was to use the bottle tops, the leaves and the learners' books in his lesson plan.

## Teacher's selection of examples

Teacher A's lesson plan showed that he used examples from the learners' books. The examples that he planned to discuss with learners were from exercise 1A, page 105 (MIE, 2007). The lesson plan did not have any additional examples written in it apart from those which the teacher planned to write on the chart.

## Tasks selected for learners

In his lesson plan, Teacher A had included two oral questions he planned to ask learners:

$$
\begin{aligned}
& 3 \text { mangoes }+4 \text { mangoes }= \\
& 13 \text { pens }+2 \text { pens }=
\end{aligned}
$$

He also planned to ask learners to do exercise 1A, numbers 2 and 5 on page 105 (MIE, 2007). In his conclusion in the lesson plan, the teacher wrote the number sentence, "The sum of 12 goats and 25 goats is 37 goats" which he mentioned in his lesson plan that he would ask one volunteer learner to write it in short on the chalkboard.

The lesson plan showed tasks of the same type where learners were required to use known operation (K). Therefore, the lesson plan contained level 1 tasks according to the MDI framework (Adler \& Ronda, 2015)

### 4.4.3 Research Question 3: What opportunities are provided for learners' participation?

In the lesson plan, Teacher A planned to ask learners to read number sentences on the chart, giving learners an opportunity to speak. He also planned to ask to learners oral questions, this would enable learners to speak some sentences (P/S). He also planned to give learners individual work to do, thereby providing an opportunity for students to accept responsibility of their own work. The teacher did not plan for any activity that would enable learners to explore the meaning of number sentence. There were no examples of situations where the teacher planned to ask learners questions that would enable learners to speak yes/ no, but there were opportunities for learners to say some sentences through reading of number sentences and answering of oral questions. Opportunities for learners to answer questions and speak some phrases/sentences are coded $\mathrm{P} / \mathrm{S}$ in the MDI
framework, implying level 2 (Adler \& Ronda, 2015). Teacher A did not have any how/ what and why questions in his lesson plan yet he indicated in the lesson plan that he would discuss with learners the meaning of number sentence. He did not show how he would conduct the discussions. In the MDI framework, discussions are coded D. Teacher A, in his lesson plan, showed an example of $\mathrm{P} / \mathrm{S}$ and D as opportunities for learner participation, which is level 3 in the MDI framework (Adler \& Rhonda, 2015)

### 4.4.4 Research Question 4: How do Standard 5 learners participate in algebra lessons?

Teacher A had planned for learner involvement through individual work. In his lesson plan he had written that he would ask learners to open their books at page 105 and do exercise 1A, numbers 2 and 5 (MIE, 2007). He wrote in his lesson plan that he would ask learners following oral questions:

$$
\begin{aligned}
& 3 \text { mangoes }+4 \text { mangoes }= \\
& 13 \text { pens }+2 \text { pens }=
\end{aligned}
$$

He also planned to invite a volunteer learner to write in short on the chalkboard the number sentence: "The sum of 12 goats and 25 goats is 37 goats."

He planned to discuss with learners the meaning of number sentence and how to write number sentences in short. Involving learners in discussion is coded D in the MDI framework and is level 3 (Adler \& Rhonda, 2015).

### 4.5 Data from Lesson plan: Teacher B

The sub-chapter presents data from the lesson plan for Teacher B. Below, I present data on each research question separately. The object of learning which was written in the lesson plan was "Completing number sentences."
4.5.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?
I present data on what the teacher wrote in her lesson plan on the meaning of number sentence, on how letters, symbols and coefficients were to be introduced and explained to learners. Data presented is rated on how the algebraic concepts were explained by the teacher in a lesson plan in relation to the object of learning.

## Teacher's explanation of number sentence in algebra.

Teacher B did not write anything about the meaning of number sentence in her lesson plan. She had written tasks for learners to do both in the introduction and developmental steps. The problems she wrote in her lesson plan involved completing number sentences, where there were gaps in the number sentences for learners to fill in. Some of the problems presented in the lesson plan were:

$$
\begin{aligned}
& 9 y+\ldots=7 y \\
& \ldots+5 b=14 b \\
& 3 n-\ldots=2 n
\end{aligned}
$$

The focus was on filling the blank spaces in order to complete the number sentences.

## Use of letters in algebra

Teacher B did not mention anything about the use of letters in a number sentences in her lesson plan.

## Symbols: Use of equal sign and addition symbol

Teacher B did not mention anything about the use of the equal sign and addition symbol in a number sentence as well as in algebra, in her lesson plan.

## Explanations on the meaning of coefficients

Teacher B did not write anything about the coefficients in her lesson plan.
4.5.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in standard 5 ?
I present and analyse data on the resources, examples and tasks that Teacher B planned to use in her lesson. Data on the resources is on the type of resources that the teacher planned to use in the lesson, how she planned to use them so that they would help learners understand the object of learning. The examples were rated on similarity ( S ) and contrast (C) and the tasks that the teacher planned for learners in the lesson plan were rated on whether they required learners to carry out a known (K) operation or apply (A) what is known in relation to the object of learning (Adler \& Ronda, 2017) and (Adler \& Ronda, 2015).

## Resources used and how they were used

Teacher B indicated in her lesson plan that she would use number cards and counters in her lesson. In step 1 of the developmental steps of her lesson plan, Teacher B wrote "provide the number cards and counters to learners then let learners solve _ $+5 b=14 b$ in groups." She did not elaborate in the lesson plan what learners were actually going to do with the resources she provided.

## Teacher's selection of examples

Teacher B had the following problems in her lesson plan which she planned to solve together with learners as examples. She mentioned in her lesson plan that she would ask learners to solve first, then discuss later together. The examples that were written in the lesson plan for learners to solve were:

9 birds take away 4 birds

$$
\begin{aligned}
& 9 y+\ldots=17 y \\
& 9 b+\ldots=14 b \\
& 3 n-\ldots=2 n
\end{aligned}
$$

Teacher B planned to use addition and subtraction problems as examples in the lesson plan. Therefore, the examples that the teacher planned to use showed two forms of variation, contrasting (C) and classified as level 2 examples according to the MDI framework (Adler \& Ronda, 2017; Adler \& Ronda, 2015).

## Tasks selected for learners

Teacher B in the lesson plan stated that she planned to ask learners to solve "9 birds take away 4 birds $=$ $\qquad$ " in pairs in the introduction part of the lesson plan. Then in step 1, she wrote that she would ask learners to solve _ $+5 b=9 b$ in groups. In step 2, Teacher B planned to ask learners to solve $3 n-\ldots=2 n$ individually. Finally, the teacher planned to ask the learners to do exercise 2 on page 106 in their textbook (MIE, 2007).

From what the teacher presented in her lesson plan, the tasks required learners to use known operations (K), and make some applications (A). Therefore, the tasks were of level 2 according to the MDI framework and (Adler \& Ronda, 2015).

### 4.5.3 Research Question 3: What opportunities are provided for learners' participation?

This part presents data and analysis of data on the opportunities that the teacher had planned to provide for learners so that they explore and discover the meaning of a number sentence and how to complete number sentences. It also presents data on the activities that the teacher planned for learners in order to provide opportunities for them to understand the meaning of letters, coefficients, and equal sign and addition symbols in a number sentence.

In her lesson plan, Teacher B planned to give learners work to do in pairs and in groups. She had planned that learners would work with others in small groupings where she would provide them with number cards and counters to solve the problem _ $+5 b=14 \mathrm{~b}$. This would enable learners speak some mathematical sentences, which in real classroom situation would have been coded (P/S) according to the MDI framework (Adler \& Ronda, 2015). However, she did not mention where the problem would be written, whether on the chalkboard or on the number cards. By planning for individual work, she provided an opportunity for learners to accept responsibility of their own work. The teacher did not plan for any activity in her lesson plan that would have helped learners to explore the meaning of number sentence. The lesson plan did not show anywhere that the teacher planned to ask learners questions that would have enabled them to speak yes/ no. Teacher B did not have any how/what and why questions in her lesson plan. The teacher indicated in the lesson plan that she would discuss with learners the answers to the problems in the task that was planned for learners to do. According to MDI framework, discussions in a mathematics lesson are coded D. Therefore, Teacher B planned for the activities that showed P/S and D learner participation, which then imply that the work in the lesson plan can be classified as level 3 (Adler \& Ronda, 2015).

### 4.5.4 Research Question 4: How do Standard 5 learners participate in algebra lessons?

Teacher B planned to use pair work, group work and individual work. She also had planned to involve learners verbally during class discussion and to give learners individual work from, exercise 2 on page 106 in learners book (MIE, 2007). She planned to discuss with learners the answers to the problems in the task that was given to them as an exercise. Involving learners in
group work, and discussion is coded D in the MDI framework and is level 3 (Adler \& Ronda, 2015).

### 4.6 Data from Interviews: Teacher A

The interviews were conducted after lesson observations. They were conducted to collect data on the resources that Teacher A use when introducing algebra to Standard 5 learners, a class where algebra is introduced for the first time as a topic in primary schools in Malawi. The interviewer sought to find out why Teacher A chooses to use the resources he mentioned, how he selects examples and tasks for the learners. Teacher A was also asked to explain where he gets the examples and tasks as well as the explanations about the algebraic concepts that he gives learners from. The interviews were set to answer four research questions of this study, and an interview guide with questions guided by the research questions was used. Some of the questions asked to the respondent were guided by the teaching as observed by the researcher during the lesson observations. The interviews were semi-structured, as such, some questions asked were follow up questions and modified questions depending on the responses given by Teacher A. The following were the research questions which were addressed through interviews:

1. How do teachers introduce (explain) the algebraic concepts to Standard 5 learners.

2 (a). What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5?

2 (b). What argument do teachers give for the resources, examples and tasks they use to introduce algebra in standard 5?
4. How do Standard 5 learners participate in algebra lessons?

On research question 1, the researcher wanted to find out where Teacher A got information on how to explain the algebraic concepts to learners during the teaching from. On research questions 2 a and $2 b$, the main focus was on the reasons that the teacher had for choosing the resources, the examples and tasks that he used to introduce algebra during his lessons. Finally, research question 4 aimed at finding out from the teacher how he involves learners in class activities. Data in the sub-chapters below is presented according to the research questions that were answered through interviews.

The researcher used an interview guide during interviews, see an interview guide (Appendix 2), and in some cases follow up questions were used. These were the modified questions based on what was observed during the teaching and also on the responses given by Teacher A. Teacher A was allowed to respond in the language he felt comfortable, and he chose to respond to the questions in English throughout the interviews.
4.6.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?
On this research question, the researcher wanted to find out from Teacher A how he introduces algebraic concepts when teaching algebra to learners in standard 5 for the first time. This was part of the conversation between the researcher and Teacher A:

Interviewer: How do you introduce the algebraic concepts to learners? Algebraic concepts such as letters, symbols like plus, equals as well as coefficients which are introduced in Standard 5 in number sentences. How do you explain these concepts to learners when you are teaching algebra for the first time?

Teacher C: Okay. So, we normally explain to them that we can have maybe a group of similar objects like stones whereby we can have a number of stones, maybe seven stones whereby stones can also be represented by a letter s whereby we can try to add or subtract those stones. For example, seven stones minus six stones. It'll give us a certain number whereby we can have a number with a letter together. So normally we explain the concept of algebra to learners basing on those things like coefficients. We can include the coefficients even letters as well. That's all I can say.

From the transcript, it shows that Teacher A use objects when introducing the concept of letters in algebra and models the addition process using the objects. He gave an example of using stones to introduce letter $S$ where he said the letter $S$ stand for stones. He further said that when introducing letters to learners for the first time, he thinks of which letters to use since some of the letters can be used while others cannot be used. He further explained:

Teacher A: We can use a small letter $b$ or we can use a small letter $x$ whereby if we can use letter $x$ it should be a small letter instead of using a capital letter because
a capital letter $X$ is a symbol for multiplication and this can confuse learners.

According to the explanations given by Teacher A, to him letters are names of objects. The teacher's use of objects to introduce algebra contradicts with what other researchers like Kuchemann (1981) asserts, that though encouraged by some algebra texts, letters should not be introduced as names of objects in algebra to minimise learners' misconception of "letter as object". The explanations on how Teacher A introduces equal sign, addition symbol and coefficients are not clear in the transcript. Therefore, it is difficult to interpret what he meant in his responses. It indicates that Teacher A might not have much knowledge about the algebraic concepts and how to introduce them to learners. When asked where he gets the explanations from by the interviewer, this is what he said:

Interviewer: Do you have anywhere you get these explanations from?
Teacher A: Okay, most of the times we get those explanations from their books, or sometimes you just think of how I can explain the concept to learners so that those learners can easily understand. But first we go by the book, then if there is any other option to add, we add according to how the situation is to make the learners well understand.

Teacher A responded that he gets the explanations from the textbooks, or sometimes he just thinks of how he can explain the concepts to learners so that they easily understand them. This entails that Teacher A depends on the Standard 5 textbooks (MIE, 2007) for information to teach. When he does not get enough information from the textbooks, especially when the information is not much detailed, Teacher A said that he just thinks of what to teach learners. Explaining that letters are names of objects in the teaching would be non-mathematical (NM) in terms of MDI framework, and of level 1 word use (Adler \& Ronda, 2015).

### 4.6.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5?

Teacher A was asked about the resources that he uses when teaching algebra for the first time to Standard 5 learners, where the first lesson, for example is on writing number sentences. This is what was said during the interviews:

Interviewer: What resources do you use when introducing algebra in Standard 5, where the first lesson is on number sentences?

Teacher A: OK, so when we're introducing the topic of algebra in Standard 5, we normally use resources like stones, seeds. We can talk of any seeds like maize seeds, bean seeds even other seeds that we can use simply to introduce number sentence in algebra. Even leaves we can also include those things.

From the conversation in the transcript above, Teacher A explained that he uses objects like stones, leaves and seeds such as maize seeds and bean seeds when introducing algebra to beginners in Standard 5. The first letter of an object becomes a letter in an algebraic number sentence or expression. When asked where he gets the examples and tasks from, he said he normally gets them from the resources he use, for example if using stones he can use letter $s$ and when using bananas he can use letter $b$. This is what Teacher A said:

Interviewer: What type of examples do you use to introduce algebra, especially when you are introducing letters?

Teacher A: Some of the examples which I use when I am introducing algebra are examples with letters. Mainly for those letters, I also consider some letters they can be used while other letters cannot be used. If they are used, we use capital letters, for example, letter $B$; we can use a small letter $b$, or we can use a small letter $x$ if using letter $x$ in an example instead of using a capital letter because a capital letter $X$ is a symbol for multiplication and this can confuse learners. So we use small letters like $a, b, s$ like that.

From Teacher A explanations, he uses objects when introducing algebra. He said that selection of the examples and tasks depends on the objects available for use as teaching and learning resources. He said he chooses the examples where letters can be used as names of objects like bananas, stones. Teacher A argued learners to always use small letter $x$ in a number sentence to avoid confusing it with multiplication sign. He said if $x$ is used as a capital letter it can be taken as a multiplication symbol because multiplication symbols are always written in capital letters. If this was what he had done in class during the teaching, it would have been classified as non-mathematical, NM in MDI framework (Adler \& Ronda, 2015).

### 4.6.3 Research Question 2 b . What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5?

When asked why he chooses to use resources like stones, seeds and leaves, Teacher A said:
Interviewer: Why do you decide to use those resources?
Teacher A: Those resources mainly are used because they are locally available. We can easily find them.

Teacher A: And, in most cases they are also the resources which are of the same category. Normally if they are bananas, If they are fruits like bananas, it means that we can use bananas, which are locally available. If we can take resources like stones they are also locally available of which they can also be many whereby we can split to learners at least each of the learner can also have those sources. So, they are adequate as well as locally available.

Teacher A said that he chooses to use the objects such as fruits, seeds, leaves and stones as mentioned earlier on, when introducing algebra to Standard 5 learners because:

Those objects are locally available

They are of the same category

That they can be collected in abundance and be adequately split to all the learners.
He said that the selection of examples depends on the available objects, and how familiar they are to learners. For example, Teacher A said that he can use fruits like bananas, where 5 bananas can be represented as $5 b$ in an example because learners are familiar with bananas.

In his explanations, Teacher A showed that he introduces algebra in a traditional "fruit salad" approach, he looks for locally available, familiar and objects that are adequate to use. This is an implication that Teacher A does not have an idea that "fruit salad" approach to algebra teaching is not recommended by other researchers like Kuchemann, (1981). Kuchemann (1981) said that introducing letters in algebra using objects can lead to the misconception of the "letter as object." Using objects and not representations would have been non mathematical, NM, and would have been classified as level 1 (Adler \& Ronda, 2015).

### 4.6.4 Research Question 3: What opportunities are provided for learners' participation?

Teacher A was asked about the learning opportunities he provides to learners during the lessons so that the object of learning is achieved. Teacher A said:

Interviewer: What learning opportunities do you provide for learners so that they discover the meaning of new concepts they are learning?

Teacher A: As I have said already, the learners will be able to count how many learners are there altogether, in this case learners are being involved.

From what Teacher A said, concerning opportunities for learner participation, it draws an implication that he is not sure of the learning opportunities that he can provide for learner participation in his algebra lessons for the learners to learn new concepts.

### 4.6.5 Research Question 4: How do Standard 5 learners participate in algebra lessons?

Teacher A said that he involves learners to participate in the lesson by putting learners in small groups to represent learners in the classroom situation. This was what he said:

Interviewer: How do you involve learners when teaching algebra?
Teacher A: Okay (...) how $\approx$
Interviewer: $\approx$ yeah, if you are introducing algebra for the first time, and introducing algebraic concepts such as letters, algebraic number sentences, how do you involve learners in class activities?

Teacher A: Okay, when I am introducing the concept of algebra, I involve learners in the way that I can group a number of learners whereby learners is representing those learners in the classroom situation. So I can then put maybe a number of learners whereby they can be four and the other group can be five. Whereby we can count together with learners how many learners are there? The learners can respond by saying five. So you can write five, and its five what? Are they bananas or fruits or what? And those learners can respond by saying, are learners. So instead of writing the full learners we can just write the $L$ which is representing learners. So we include learners to explain the concept of algebra by also making some small groups whereby we can add those learners together, or we can subtract
so those letters will represent the learners, we can use L to represent learners, whereby in that situation I think and I hope that in that way we are also including those learners to explain the concept of algebra.

From the transcript, Teacher A's explanation on learner involvement in the lesson, showed that he uses learners as resources in his lessons. His response to the question can be interpreted that he did not understand what to involve learners in a lesson means, despite the modification of the question by the researcher. If this was teaching in classroom, this would have been coded NM, and level 1 of naming in the MDI framework (Adler \& Ronda, 2015).

### 4.7 Data from Interviews: Teacher B

The interview was conducted to answer all the four research questions of this study, guided by the interview guide. The interview was used to collect data on the resources that Teacher B use when introducing algebra to Standard 5 learners as beginners, the reasons why she uses the resources mentioned, how she selects examples and tasks for learners, where she gets the examples and tasks from, and where she gets the explanations about algebraic concepts from.

### 4.7.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?

The researcher asked Teacher B:

Interviewer: How do you explain algebraic concepts to Standard 5 learners, a class where algebraic concepts are introduced for the first time?

Teacher B: (Um...) it is somehow difficult to explain for the first time, but with resources, with example from themselves and emphasize explanations, it is not all that difficult, and with resources they understand better.

When Teacher B was asked where she gets the explanations from, this is what she said:

Teacher B: As for me, the explanations are from my past experience, what I remember from my secondary school education. If I forget, I ask my fellow teachers to assist me. There are no explanations about algebraic concepts in the teacher's guide for Standard 5.

Teacher B said that she uses resources to introduce and explain the algebraic concepts to learners. However, in her response she did not clearly explain how the resources are used. She also said
that she depends on the knowledge of algebra which she got from her secondary school education. This is because the Standard 5 mathematics teacher's guide and learners' book do not provide explanations on the algebraic concepts. She further said that she consults fellow teachers for assistance when she fails to remember a certain concept. From what Teacher B explained concerning the source of information on the algebraic concepts, it might be that Teacher A does not have much knowledge for teaching algebraic concepts to learners. She complained in her explanations during the interview that the teacher's guide does not contain detailed information that would help her teach the topic of algebra effectively. However, the use of objects would have been classified as level one under exemplification (Adler \& Ronda, 2015), if this was the actual teaching in class.

### 4.7.2 Research Question 2a. What type of resources, examples and tasks for learners do teachers

 use to introduce algebra in Standard 5?The interviewer and Teacher B had this to discuss on the resources:

Interviewer: What resources do you use when you are introducing algebra in the Standard 5?

Teacher B: I use stones, leaves and sticks, as the resources that are familiar.
Teacher B said that she uses stones, leaves and sticks when introducing algebra to learners in Standard 5. She further said that she makes sure that she uses objects of the same kind, for example, if she chooses to use bananas, she makes sure that she uses bananas in the example, no mixing of objects. On selection of examples and tasks for learners, Teacher B said the following:

Interviewer: What examples do you use to introduce algebra, especially when you are introducing letters?

Teacher B: (sighs...) usually, for example, if we're talking of five bananas, I just have to make sure that I'm dealing with bananas only. Yeah.

Teacher B: It's like we don't mix like bananas and mangoes so that learners should get used to understand that when we add, we add things of the same group, when we're subtracting, we subtract things of the same group. On their level, they understand better when using those kind of examples.

In the transcript, Teacher B said that she uses objects of the same kind when giving examples to learners. This can be said that if it was in class, she would have used S , and level 1 examples (Adler \& Ronda, 2015). She said that she picks some of the examples and tasks from the text books, sometimes she creates the examples, while other examples come from learners themselves. She gave an example of a situation where she asked learners to create number sentences during her lesson.

### 4.7.3 Research Question 2b. What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5?

On this research question, the researcher wanted to find out the reasons for using the resources, the examples and tasks that Teacher B mentioned she use. This is what was said during the interview:

Interviewer: Why do you decide to choose those resources?
Teacher B: Yeah, I think I've already said that those resources, are a kind of objects that learners are familiar with. So even if we introduce them with the numbers, it wouldn't be very new to them.

On the question why she uses the resources she mentioned, Teacher B said that she uses objects that are familiar to learners to introduce the concept of letters. According to the transcript, to her, letters are names of objects. When selecting examples, she looks for familiar objects to use as letters in the examples. To her, letters in algebra represent familiar objects, and that learners can understand the concept of letters better if they are introduced to them using familiar objects.

When asked about the reasons why she selects the examples and the tasks that she uses in her algebra lessons, this was her response:

Interviewer: Why do you select the examples and tasks which you give learners in your algebra lessons? Examples using resources that are familiar to them, examples using resources that are from the same class?

Teacher B: You know, the word algebra itself is somehow/ maybe very strange to them. So, with those examples that are familiar to them, the objects that are familiar to them it's like they are learning in a real environment. They understand better because they're already familiar with the numbers. Now
with just the other examples, like five boys, five girls, it's like we are mixing now the letters and the numbers. So if we use the familiar examples, it won't seem to them like the strange algebra. It will be just like a continuation of just numbers and letters.

Interviewer: Where do you get the examples from?
Teacher B: Some examples, and most of them can be better if they come from the learners themselves. Yeah. As you observed in the lesson I was writing, they were formulating their own number sentences after they understood what a number sentence is. So, usually I use those examples from learners themselves. Then I also create mine and use some from the books.

From the transcript, Teacher B said that she uses familiar examples and tasks. If this was done in class, it would have been said that she uses examples that provide learners with opportunities to experience one form of variation in terms of similarity (S) (Adler \& Ronda, 2017; Adler \& Ronda, 2015).

### 4.7.4 Research Question 3: What opportunities are provided for learners' participation? Teacher B said:

Interviewer: What learning opportunities do you provide for learners so that they discover the meaning of new concepts they are learning?

Teacher B: I sometimes ask them to mark each other's work, now that is peer assessment.

From what Teacher B said, if this was to be done in class, the learning opportunities for learners would be classified as level 3, MDI framework (Adler \& Ronda, 2015).

### 4.7.5 Research Question 4: How do Standard 5 learners participate in algebra lessons?

Teacher B had this to say on this research question:
Interviewer: How do you involve learners when introducing algebra?
Teacher B: I use different methods. As you saw in the lesson, there was individual work there. There was also some kind of discussion in pairs

Teacher B: There were also groups when they were solving their number sentences from the number cards. They were doing it in groups. So, we used different methods. When I give them the homework, that means when they go home, that is the research now. If they have problems, they have to ask from other people at home.

From the transcription, we can see that Teacher B said that she involves learners in algebra lessons by giving them individual work to do. Teacher B further explained that she allows learners to work in pairs and in groups, through this they share and discuss ideas. By giving learners opportunities in class to discuss, is coded D. If Teacher B was to use discussions in her lesson, the opportunities for learner participation would be of level 3 in the MDI framework (Adler \& Ronda, 2015).

### 4.8 Data from Interviews: Teacher C

The interviews with Teacher C were not audio recorded. The participant had initially given consent to be recorded, but changed her mind. However, Teacher C allowed to be interviewed without being audio recorded, and allowed for the researcher to take notes during the interview. Hence, the responses given by Teacher C were being written down by the researcher. Data collected was on the resources that Teacher C use when introducing algebra to standard 5 learners as beginners, the reasons why she uses the resources mentioned, how she selects the examples and tasks for learners, where she gets the examples and tasks from, and where she gets the explanations about algebraic concepts from.

### 4.8.1 Research question 1: How do teachers introduce (explain) the algebraic concepts to Standard 5 learners? <br> Teacher C was asked to say how she "explains algebraic concepts to Standard 5 learners, a class

 where algebraic concepts are introduced for the first time?" Teacher C responded that she gives learners number sentences with missing numbers, for example $\ldots+5 b=14 b$, and asks them to identify the missing number. However, Teacher C did not explain much on how the giving of learners the number cards with missing items would help learners understand the basic algebraic concepts. The researcher, unfortunately, did not ask the teacher a follow up question to give her an opportunity to explain further. She did not mention about any algebraic concept that can be introduced through identification of a missing number, this raises questions whether this teacher knew what the researcher meant by algebraic concepts. Therefore, explanatory talk on thealgebraic concepts would have been rated non mathematical, NM, and level 1 if it was real teaching.

### 4.8.2 Research Question 2a. What type of resources, examples and tasks for students do teachers use to introduce algebra in Standard 5?

The interviewer asked Teacher C about the resources that she uses when introducing algebra to Standard 5 learners being a class where algebra is introduced as a topic for the first time. She said, she uses objects like oranges, tomatoes, potatoes, and books. When Teacher C was asked how she selects the examples and tasks for learners, she said that she does not select the examples, but she uses the examples that have been given in the textbooks, thus in the learners' book and the teacher's guide. When asked an additional question by the researcher to find out if she creates some examples basing on the real life situations, Teacher $C$ said that she does not create own examples for the learners. On the selection of tasks, she said she gives learners tasks following the activities and instructions that have been suggested in the teacher's guide. She said that she normally gives learners exercises from the learners' book. From her explanations, Teacher C seems to rely much on the prescribed textbooks for information and gives learners exercises that are found in the textbooks for practice.

The way she explained during the interviews, if it was teaching in a classroom situation, the level of examples and tasks would be of level 1 (Adler \& Ronda, 2015).

### 4.8.3 Research Question 2b. What argument do teachers give for the resources, examples and tasks they use to introduce algebra in Standard 5?

The interviewer asked Teacher C to explain why she chooses to use the resources she mentioned in chapter 4.8.2, in her lessons. Teacher C responded that she uses the resources which she mentioned in chapter 4.8.2 for easy communication to the learners, so that they have knowledge of algebra, and for learners to visualize what the teacher is talking about. On the part of examples and tasks, she said she picks the examples and exercises from Standard 5 textbooks, she does not create examples. However, it is difficult to interpret what she meant by saying the resources are for easy communication, so that learners have the knowledge of algebra.

### 4.5.3 Research Question 3: What opportunities are provided for learners' participation?

When asked about the opportunities of learning that Teacher C provides for learners so that they all participate in the lessons, she responded that she asks learners to model number sentences. Teacher C said that by doing so, she provides opportunities for hands-on activities. This is what
she said, "Learners will practice modelling number sentences using objects and they will visualize what they are doing." However, the modelling that Teacher C talked about as observed in her lesson during the teaching is about modelling addition of numbers just as in arithmetic. If this was in a lesson, it would have raised questions whether the modelling done provided opportunities for learners to learn new algebraic concepts. Therefore, it would be classified level 1, NM, of learner participation of the MDI framework, (2015).

### 4.8.4 Research Question 4: How do Standard 5 learners participate in algebra lessons?

The interviewer asked Teacher $C$ to explain how she involves learners in her lessons. She said that she gives learners some work to do both individually and in groups. For example, she said that she can provide learners with bananas or tomatoes or any other objects in their groups to model the number sentences. In her explanation during the interviews, she gave an example of asking learners to model 3 bananas plus 5 bananas is equal to 8 bananas in their groups. This implies that Teacher C uses group work in her lessons.

Effective group work encourages learners to speak and share ideas through discussion. If the use of discussion was in the class, learner participation would be classified as level 3 in the MDI framework (Adler \& Ronda, 2015).

### 4.9 Chapter summary

This chapter has presented data and findings from the teaching, the lesson plans and the interviews. Data from the teaching has been presented for all the three teachers and schools which participated in the research study. Data collected from the teaching and from the lesson plans was expected to be similar since the lesson plans which were collected for data analysis were the ones the teachers used to guide their teaching during lesson. However, this was not the case. Both Teachers A and B who had lesson plans on the day of lesson observations, wrote undetailed lesson plans which lacked some information as observed during lessons. They did not explain in their lesson plans how they would introduce the algebraic concepts to learners and how they would use the resources they mentioned in their lesson plans. However, both Teacher A and Teacher B used the objects they mentioned in the lesson plans for letter generation, and also as counters during the lessons. Teacher A and Teacher B showed in their lesson plans that they would discuss with learners the meaning of number sentence through the examples they planned to present to learners, but did not elaborate in the lesson plan on how the discussions would be conducted to ensure effective learner
involvement. During the teaching, Teacher A showed learners examples of number sentences by saying "these are number sentences" while pointing to the number sentences on a chart, and then told learners the definition of a number sentence. Teacher B, most of the times switched to vernacular language in trying to instruct learners on what to do. Both Teacher A and Teacher B planned for lessons that according to the MDI framework would be categorised as level 3 of learner participation (Adler \& Ronda, 2015). For instance, they mentioned in the lesson plans that learners would be engaged in discussions. In the contrary, during the actual teaching, there was low levels of learner involvement in the lesson, and the opportunities for learner participation was level 2 according to the MDI framework. There were no discussions throughout in the lessons as mentioned in the lesson plans. Both Teachers A and B demonstrated how to write number sentences to learners. Teacher C said that she did not have a lesson plan on the day of lesson observation. As such, her lesson was observed without a lesson plan. The table below summarises the findings from the teaching, lesson plans and interviews for the three teachers on exemplification, explanatory talk and learner participation:

| Data source |  | Examples | Tasks | Explanatory talk | Learner participation |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Data from the teaching | Teacher A | S, Level 1 | K, Level 1 | NM, Level 1 | Y/N, P/S, <br> Level 2 |
|  | Teacher B | $\mathrm{C} \rightarrow \mathrm{S}$, Level 1 | K, Level 1 | NM, Level 1 | Y/N, P/S, Level 2 |
|  | Teacher C | S, Level 1 | K, level 1 | NM, Level 1 | Y/N, P/S, Level 2 |
| Data from lesson plans | Teacher A | S, Level 1 | K, level 1 | D, level 3 | P/S, Level 2 |
|  | Teacher B | C, Level 2 | C, level 2 | NM, Level 1 | D, Level 3 |
| Data from the interviews | Teacher A | S, Level 1 | K, Level 1 | NM, Level 1 | Y/N, Level 1 |
|  | Teacher B | S, Level 1 | K, Level 1 | NM, Level 1 | Y/N, P/S, <br> Level 2 |
|  | Teacher C | S, Level 1 | K, Level 1 | NM, Level 1 | Y/N, P/S, <br> Level 1 |

Table 4: Summary of findings for the three teachers

The next chapter presents the discussion of the findings, conclusion, implications and recommendations.

## 5 Discussion of Findings, Conclusion, Implications and Recommendations

This chapter presents discussion of the research findings, conclusion, implications of the findings and recommendations for future study

### 5.1 Discussions of Findings

According to the findings of this study, data from the teaching shows that all the three teachers used level 1 examples and tasks (exemplification), their explanations were of level 1, while learner participation was level 2. Data from the lesson plans for the two teachers, Teacher A and Teacher B show that Exemplification for both teachers was level 1. Teacher A had a lesson plan with level 2 explanations while Teacher B's lesson plan showed level 1 explanations. The lesson plan for Teacher A showed level 2 of learner participation and Teacher B's lesson plan showed level 3 of learner participation. On data from the interviews, the examples and tasks mentioned by the three teachers were of level1 and their responses on explanatory talk were also of level 1 . Teacher A responses in the interviews on learner participation were of level 1, whereas the responses for Teacher B and Teacher C were of level 2.

### 5.1.1 Teachers' introduction (explain) of the algebraic concepts to Standard 5 learners

The findings of the study have revealed that all the three teachers who took part in the study did not explain the algebraic concepts to learners effectively. For instance, Teacher A showed learners examples of number sentences and defined a number sentence for learners. The focus was on writing number sentences in short, and completing number sentences without explaining much to learners what an algebraic number sentence is, being the object of learning in all the three classes observed. They explained to learners that letters were names of objects. The approach they used for explaining the meaning of letters to learners (the fruit salad approach), was not adequate enough for learners to understand what letters are in a number sentence. The coefficients, the equal sign and addition symbols in a number sentence were not correctly explained. For example, one teacher explained to learners that 1 is the coefficient. This can be interpreted that the teachers lacked sufficient pedagogical knowledge on how to introduce and explain the algebraic concepts to learners. This was evidenced during the interview when the teachers were asked where they get information that they use to introduce and explain the algebraic concepts to learners from. All the three teachers said that they get information about how to teach algebra mainly from the teachers' guide, and sometimes they apply what they learnt during their secondary school education where the teachers' guides do not provide much information. This is in line with findings from Kazima
and Jakobsen (2013) who reported that many teachers in Malawi follow the textbooks when teaching. Teachers had problems with the teaching of algebra because the teacher's guide seems not to provide adequate information. Teacher A and Teacher B during the interviews said that the textbooks did not have enough information about the algebraic concepts. Teacher A claimed to use knowledge from secondary school education, and where possible she asks for information from friends. Teacher A said that he just thinks of what to teach. He said that sometimes he can just think of how to teach algebra so that learners understand. Teacher C did not complain about inadequate information in the teacher's guide. Despite the explanations they gave about the algebraic concepts to the researcher during the interviews, they did not explain the algebraic concepts to the learners during the lessons when they were teaching. This was what was observed:

## Introduction of algebraic number sentence

Number sentences were presented to learners as readymade entities. Teacher A showed learners examples of number sentences, and told the learners that those were the examples of number sentences, and defined a number sentence for learners. Later, he demonstrated how to write number sentences in short. Teacher C modelled the number sentences using objects and wrote the number sentences on the chalkboard. She then told learners that those were examples of number sentences. Teacher B did not say anything about the meaning of a number sentence. This is in contradiction with what Booth (1988) advises, that algebraic concepts should not be presented to learners as readymade entities. Teachers should teach or introduce new concepts to learners by allowing them to explore the new concepts through meaningful discussions, where they can question their thinking and make sense of what they are learning. For example, instead of teachers showing learners examples of number sentences, they were expected to allow learners to explore and discover what is composed of an algebraic number sentence and the relationships in a number sentence. The study findings reveal that what the teachers did opposes what Kullberg et al. (2017) recommends, that the introduction of the algebraic concepts to learners should base on problem situations that can lead to symbolization. Marton and Pang (2013) argue against the view of developing new meanings from experience of sameness by just pointing to the examples of number sentences and saying; this is a number sentence and this is a number sentence, as done by the teachers that participated in this study. This entails that learners cannot understand what a number sentence is, only by looking at a number sentence presented by the teacher, since they do not know
what not a number sentence is, as the characteristics that makes an algebraic number sentence are not explained or made visible to learners.

In the number sentence, $3 g+4 g=7 g$, Teacher A told learners that 3 and 4 are related, ignoring the letters. Teacher B also told learners to forget about the letters in a number sentence and concentrate on the numbers only whenever working with an algebraic number sentence. This is an evidence that these teachers lacked some knowledge on the use of letters in an algebraic number sentence and how they can introduce algebraic number sentences to learners. This type of teaching algebra contradicts the findings by other researchers like Roberts (2012), who asserts that primary school algebra should aim at infusing arithmetic activities with algebraic concepts to bring out the algebraic character. Ignoring the letters in an algebraic number sentence reduces the algebraic activities to arithmetic teaching. An algebraic number sentence is an algebraic equation which has coefficients, variables and operational symbols that need to be explained to learners. Molina et al. (2017) argues that lack of understanding of algebra as generalized arithmetic can cause difficulties in structuring algebra. Learners can hardly understand the idea of relationships, and also of the varying quantities if they don't understand the concept of generalization. According to Molina et al. (2017), it is necessary that teachers explain the relationships of the terms in a number sentence to learners in order to avoid errors in algebra. Adler and Ronda (2015), classifies this type of teaching, where explanations are non-mathematical, as level 1.

## Introduction of the use of letters in algebra

All the three teachers introduced letters to learners as names or labels of objects. The teachers told learners to identify and pick the first letters of the names of objects and use them as letters in algebraic number sentences, which by some researchers is described as "fruit salad" algebra (McNeil \& Weinberg, 2010; Knapp, 2016; Cai \& Knuth 2011; Kuchemman, 1981). This study finding supports the research findings of Knapp (2016), that many countries introduce letters in algebra using the traditional fruit salad approach. For example, "a" for apples and "b" for bananas. Although the method remains popular in teaching the rules of simplification and collecting like terms together in algebra, some researchers argue that problems occur in the later algebra learning when more complex concepts are introduced to learners (McNeil \& Weinberg, 2010). In view of this, it is important that teachers introduce algebra in a way that will enhance learners' capabilities to build on their prerequisite knowledge when more advanced topics are reached. Kuchemann
(1981), argues that introducing letters to learners through a traditional fruit salad approach can instil in learners the idea that letters are labels or names of objects. As Kullberg et al. (2017) explains, it is necessary to use representations, where concrete objects and pictorial images can be used to improve the recognition that letters are variables in an algebraic number sentence, and not names of objects. At some time, Teacher B referred to the letter $y$ in a number sentence $9 y+\ldots=$ $17 y$ as things. This, according to Adler and Ronda (2015), is non-mathematical use of wording and is classified as level 1 of naming in the MDI framework. The letter $y$ in the number sentence $9 y+$ $\ldots=17 y$ represent a certain value that is not yet known, and not things as the teacher explained. In addition, the algebraic sentence is a bit complicated as it has two unknowns, the $y$ and the gap. This is an advanced equation which Standard 5 learners might find difficult to understand as beginners. Teachers should give learners meaningful experiences and explanations in algebra learning that would enable them to learn new concepts, for example use of through multiple representations. Use of representations in algebra learning can help learners develop an understanding that letters in algebra are unknowns, constant values, and also varying quantities.

## Introduction of equal sign and addition symbol

The findings on how teachers use the addition symbol when introducing algebra revealed that teachers use an addition symbol as an operation symbol for combining groups of objects to come up with a single group of objects, called an answer. Teachers used the addition symbol in algebra the way it is used in arithmetic , thus, using it to conjoin the terms in an algebraic number sentence, similar to what Wang (2015) discovered in his research. Teachers should observe the important differences between the use of the basic operations in the context of arithmetic and the use of the same in the context of algebra. Booth (1988), explains that the focus of activity in algebra learning is not on finding a specific numerical value. Teachers should show in their teaching that in algebra, for example, $4 b+5 b=9 b$ can also has a solution as $4 b+5 b=7 b+2 b$. Such use of addition symbol can help in minimizing errors that arise due to the use of an addition symbol as a symbol for conjoining terms (Ormond, 2012). Ormond (2012) further claims that learners who use an addition symbol in algebra, the way they use it in arithmetic can find $9 a b$ as a solution to $4 a+5 b$, which is not true with algebra.

Although it was discovered that the teachers did not explain to learners how the equal sign is used in a number sentence, from observations, teachers used an equal sign as a symbol calling for an
answer. This contradicts the recommendation that when introducing algebraic concepts to learners, the focus of attention should not primarily be on finding the answer but instead it should be on helping learners understand the algebraic concepts (Kullberg et al. 2017). An equal sign should be understood as a relational symbol showing equivalence. Teachers through activity should show learners the equality relationship between the quantities on each side of the equation or algebraic number sentence. Researchers like Kieran (2018) and Leung et al. (2014) explain that in some countries the concept of equivalence in algebra is introduced by using a balanced model approach so that the idea of equality and quantitative sameness is demonstrated.

## Introduction of coefficients in algebra to learners

The concept of coefficient was not explicitly explained to learners by all the teachers. Teacher B told learners that 1 object is the coefficient, while the other teacher said the coefficients go together with the letters. This is what the Adler and Ronda (2015) call colloquial word use and they classify it as level 1 word use. During the interviews, when the teachers were asked to explain where they get information about the algebraic concepts (including coefficients) from, the teachers said that they usually get information from the teachers' guides. This finding concurs with what Kazima and Jakobsen (2013) found, that primary school teachers in Malawi teach mathematics, algebra in particular by following what has been written in the textbooks. However, the teachers complained that there is no information about the algebraic concepts in the standard 5 textbooks, as such, they use knowledge that they got from their secondary school education. In this view, it can be suggested that teachers did not manage to introduce and explain the concept of coefficients to learners appropriately due to lack of enough information on coefficients. The findings are in support of what Kieran (2006) found, that little expertise in algebra can lead to the use of the knowledge that primary school teachers themselves got from their secondary schooling. This is low level of explaining concepts to learners as classified by Adler and Ronda (2015).

### 5.1.2 Types of resources, selection of examples and tasks for introducing algebra to Standard 5 learners as beginners.

The findings of this research study show that all the teachers used objects in introducing algebra to standard 5 learners. Although there was use of a chart and number cards by some teachers, the concepts that were being communicated to learners carried the sense of objects. For example, Teacher A wrote on the chart 2 bottle tops plus 6 bottle tops equals to 8 bottle tops, and then modelled the number sentence using bottle tops. The number sentences on the chart were about
the objects that the teacher brought to class. All the examples that the two teachers used in their lessons were of the same type and allowed learners to experience one form of variation, (S). The use of similar examples with no pattern of variation as Lo (2012) explains, does not provide opportunities for learners to learn. Only Teacher B used examples with two forms of variation, however learners were instructed in vernacular language on what to do at each and every step. According to Adler and Ronda (2015), this reduced her examples to level 1. The tasks that the teachers gave the learners were all similar to the examples. The teachers presented their lessons in a very traditional and instrumental way of teaching, where the examples are first presented, then followed by tasks which are exercises, and very similar to the examples are given to learners for practice.

## Types of resources used and how they were used

The teachers used objects such as fruits, leaves, bottle tops and stones. All the teachers asked learners to identify the first letters of the names of objects and then used them as letters in algebraic number sentences that they presented. They introduced letters to learners as names or labels of objects. These findings support the research findings of Knapp (2016), that many countries use the traditional "fruit salad" approach with letters representing names of objects to introduce algebra. Kuchemann (1981) and McNeil and Weinberg (2010), warn that teachers should not introduce letters in algebra as names or labels of objects or mnemonic literal symbols because this can strengthen learners' naïve conception that letters are objects and not quantities. Roberts (2012), explain that teachers should use other alternatives of introducing algebra other than fruit salad approach. For example, teachers should use multiple representations and patterns to allow learners visualize and recognize that different letters can be used to represent the unknown value. It was also discovered that teachers used the objects as counters. The findings on the use of objects during the teaching do not give an impression that learners understood the meaning of the object of learning. As such, the findings are in contrast with what Adler and Ronda (2015) explains, that teachers should use the resources in such a way that learners understand and achieve the object of learning (Adler \& Ronda, 2015).

## Teacher's selection of examples

From what was discovered, the teachers used examples from the teachers' guides as well as from the learners' books. Some were worked examples others were taken from the suggested exercises
in the learners' books. The study findings show that teachers used objects such as fruits, stones, bottle tops, potatoes and leaves in all the examples they presented. The use of letters-as-labels in the introduction of algebra as the findings of McNeil and Weinberg (2010) suggest, hinder the interpretation of what letters are in algebraic expressions. Teacher A and Teacher C Presented similar examples which showed one form of variation. Teacher B had contrasting examples in terms of the operations used in the examples, however she instructed learners in vernacular language at each and every step. This study finding contradicts the findings that examples must be selected as particular instances of the general case in focus and for drawing attention to relevant features (Marton \& Pang, 2006). Explaining and demonstrating to learners how to carry out the operations in contrasting examples step by step and in vernacular language, as done by Teacher B, reduces the examples to level 1 as classified by Adler and Ronda (2015). This means that all the examples presented to learners by all the teachers were of level 1of the MDI framework, Adler and Ronda (2015).

Teacher A and Teacher B in their lesson plans, mentioned that they were to discuss with learners the algebraic concepts. However, during lesson observations, the researcher observed that teachers did not ask learners questions that would have directed them to proper discussions as claimed by the teachers. I suggest that it might be that the teachers have insufficient knowledge on how to conduct discussions so that learners can actively participate. Lack of why questions during the teaching limits learners reasoning. The level 1 examples presented to learners during the lesson, lack of explicit explanations on the algebraic concepts in the examples given to learners and the use of examples from the Standard 5 textbooks by the teachers, demonstrates that the teachers lacked some knowledge on how to introduce algebra Standard 5 learners as beginners.

## Teacher's selection of tasks for learners.

All the tasks that teachers gave learners reinforced the use of objects as letters. This finding contradicts the recommendation by McNeil and Weinberg (2010) and Kuchemann (1981) which discourages the use of fruit salad as an approach to algebra teaching. The exercises given by all the three teachers were taken from learners' books, pages 105 and 106 (MIE, 2007), and this implies that the textbooks contain the "fruit salad" approach to algebra. All the teachers started by giving learners examples and then tasks consisting of the problems similar to the examples that the teacher presented. This is a type of procedural learning where all the problems given to learners
as a task, are often of standard context. These findings contradict the recommendation by Vincent et al. (2015), who asserts that instead of giving learners tasks that are of standard context only, teachers should give learners tasks that include some nonstandard contexts, so that the concept of equality relationship between quantities on each side of an equation is emphasized. The study's findings also support the findings of Kazima and Jakobsen (2013), that many teachers in Malawi often teach mathematics, and algebra in particular, using procedural teaching and learning. They present their lessons by following what has been written in the teacher's guide. From what was discovered, the problems in the tasks were all similar to the examples presented by the teachers, which the MDI framework (Adler \& Ronda, 2015), categorise as level 1 tasks since they only require learners to use known (K) procedures and operations.

### 5.1.3 Teachers' arguments on the use of the resources, examples and tasks they use to introduce algebra in Standard 5.

The teachers said that they choose to use objects such as fruits, seeds, leaves and stones when introducing algebra to Standard 5 learners because those objects are familiar, locally available, of the same category, and that they can be collected in abundance and be adequately split to learners. Teachers said that the selection of examples depended on the availability and familiarity of the objects to learners. For example, the teachers said that they can use fruits like bananas, where 5 bananas can be represented as $5 b$ in an example because learners are familiar with bananas. The claims that using familiar objects such as fruits, "fruit salad" in lessons enable learners to understand algebra contradict the information by kuchemman (1981) and McNeil and Weinberg (2010), which says the use of objects in the introduction of algebra to beginners, hinders the interpretation that letters are variables. If learners are not given opportunities to understand that letters are variables during the preparatory stage, they will find problems in solving later secondary school algebra, since to them letters will remain objects. According to the MDI framework, teachers are supposed to use resources, select examples and tasks that can enhance learners' understanding of the object of learning (Adler \& Ronda, 2015). Teachers need to make sure that everything done in the lesson should aim at achieving the object of learning. This means that resources, examples and tasks selected for learners should help learners understand the object of learning. Teachers' use of familiar objects such as fruits to introduce algebra as opposed to Kuchemann (1981), is an indication that probably they don't know that "fruit salad" approach to algebra teaching is disputed by other researchers. On the part of examples, the responses that
teachers gave during interviews, indicate that they rely on the examples and tasks from the books. This promotes instrumental understanding in learners, where learners just follow the demonstrations and procedures done by the teachers without questioning.

### 5.1.4 Opportunities provided for learner participation

During the interviews, and in their lesson plans, teachers indicated that they would provide different learning opportunities to learners. The actual teaching in the classroom, as observed showed teachers providing low levels of learner participation in contrary to what they said during the interviews, and what was mentioned in their lesson plans. The learning opportunities that the teachers mentioned but differently, include the opportunities for hands-on activities, for learners to be responsible for their own work, for peer assessment, opportunities to speak short expressions, to speak yes or no and single words. However, all the opportunities that the teacher provided for the learners to participate in the lessons did not allow leaners to explore on the object of learning. There were no how and why questions that would have provided learners with opportunities to explore and discover the object of learning and the meaning of some algebraic concepts that are included in Standard 5 algebra.

From what the teachers said during the interviews and in their lesson plans concerning opportunities for learner participation, it shows that they are not sure of what learning opportunities they can provide learners with to ensure learner participation in algebra lessons. For example, what teachers planned in their lesson plans and what they said during interviews did not match with the actual teaching in the classroom. Their lesson plans indicated that there would be discussions during the lessons with learners, however there were no clear discussions observed during the lessons, the teachers explained everything to learners. The way the lessons were conducted did not encourage learners to discuss and understand the mathematics they were learning. There were no how and why questions that teachers asked learners in order to promote critical thinking and reasoning in learners. The MDI framework (Adler \& Ronda, 2015) encourages high level discussions in a mathematics lessons where teachers should probe learners' critical thinking by asking them why and how questions. The low level questions which were asked during the teaching, required learners to follow the procedures done by the teacher and to answer yes or no. Such questions do not promote learners' reasoning and understanding (Adler \& Ronda, 2015).

### 5.1.5 How Standard 5 learners participate in algebra lessons

The study findings showed that all the teachers during the interviews said that they involve learners in the lessons through individual work, pair work, group work and verbally. During lesson observations, only Teacher B used pair work and group work in her lesson. However, it's only effective group work that encourages learners to speak and share ideas through discussion. According from what was observed during the teaching, there were on clear discussions as learners were not given activities to explore on the mathematics they were learning. Teachers' responses on how they involve learners in algebra lessons, during the interviews, were different from how they engaged learners in the real classroom activities.

Teacher A's explanations on learner involvement in the lesson, showed that he uses learners as resources in his lessons. His response to the question showed that he did not understand what to involve learners in a lesson means, despite the modification of the question by the researcher. During lesson observation, he did much of the demonstrations and talked throughout the lesson with few chances for learners to speak.

From what was discovered, there was low involvement of learners in the lessons. They planned for level 3 learner participation according to the MDI framework (Adler \& Ronda, 2015), but their teaching was level 2.

### 5.2 Conclusions from the study

This study aimed at finding out how teachers introduce algebra to Standard 5 learners in Malawi. In order to establish how teachers introduce algebra in primary schools in Malawi, the study was designed to answer the following specific research questions:

1. How do teachers introduce (explain) the algebraic concepts to Standard 5 learners?

2(a).What type of resources, examples and tasks for learners do teachers use to introduce algebra in Standard 5?

2(b).What argument do teachers give for the resources, examples and tasks they use to introduce algebra in standard 5?
3. What opportunities are provided for learners' participation?
4. How do Standard 5 learners participate in algebra lessons?

This sub chapter is giving a summary of findings by drawing conclusions from these research questions.

### 5.2.1 Introduction (explanation) of the algebraic concepts to Standard 5 learners

The study findings reveal that teachers used low levels of explanations. They introduced some of the concepts as readymade entities. They did not indulge learners in activities that would have helped learners to explore, discover and understand the algebraic concepts. Some of the algebraic concepts, for example, the coefficient were wrongly defined for learners. Letters in algebra were interpreted as names or labels of objects and not as variables. When the teachers were asked where they got the information from, they said that they got the information from the Standard 5 textbooks, and Teacher B said she also uses secondary school knowledge.

From the study findings, the researcher concludes that teachers seem to have insufficient knowledge on how to introduce the algebraic concepts to Standard 5 learners as beginners. Probably, they themselves have little understanding of the algebraic concepts since they said that they rely on information from the textbooks. It has also been noted that the textbooks do not contain much information on algebraic concepts, the little information that is found in the textbook is about the traditional way of teaching algebra, the "fruit salad" approach. This was observed from the examples and tasks that the teachers presented in their lessons, which they claimed to have taken from the teachers' guide and learners' books. In addition, low levels of explanations of the algebraic concepts to learners by the teachers can be an indication that textbooks do not contain enough information to guide teachers in introduce and teach algebra in Standard 5 effectively.

### 5.2.2 Type of resources, examples and tasks teachers use to introduce algebra in standard

From the findings, teachers used objects like fruits, leaves, bottle tops and stones when introducing algebra. This makes the researcher to conclude that some teachers in primary schools have little knowledge of what resources to use when introducing learners to algebra. They use objects which reinforces the idea of the like and unlike terms during simplification of algebraic expressions. Literature has disputed the use of letters-as- objects, "the fruit salad" and advocates for the use of representations. Teachers used the name of fruits, bottle tops leaves and stones they had brought to class as letters in algebraic number sentences and as counters. The use of objects as counters makes the researcher to conclude that teachers introduce algebraic number sentences as arithmetic number sentences.

The examples presented during the lesson were all similar, they required learners to experience one form of variation. All the examples that the teachers gave learners were leading to a single term solution. The tasks that the teacher gave learners allowed learners to use known procedures and operations. Teachers started teaching by giving learners examples that were taken from the textbooks, followed by a task for practice known as an exercise. This is can be considered as low level of teaching that hardly enhances mathematical reasoning in learners. The researcher also observes that the three teachers followed what is written in the teachers' guides when teaching. This is in line with what other researchers have reported that some teachers follow textbooks when teaching.

### 5.2.3 Teachers' arguments for using the resources, examples and tasks they decide to use to introduce algebra in Standard 5

The findings of this study has revealed that the teachers consider availability, familiarity and also quantity of the resources before they decide on what examples to use in an algebraic lesson. By saying that the selection of examples depends on the objects available and familiar to learners, it is totally worrisome. What determines the resources to use in the lesson is the content or the nature of the lesson. In this view, the researcher concludes that teachers lack adequate knowledge on choosing suitable teaching and learning resources for introducing algebra.

Teachers said they take the examples from the teachers' guides and learners' books, and that when selecting the examples they choose those examples which can allow them to use familiar objects. This argument raises questions to the researcher. The introduction of algebra does not need the use of familiar objects, but rather the selection of resources and examples that can enhance understanding of the new concept being taught. The low level tasks given to learners promotes instrumental understanding which is currently considered as a big challenge in mathematics, and algebra learning in particular. Learning is said to have taken place if learners are provided with examples that can allow them to experience different forms of variation, and also tasks that can help them think critically and make some applications.

### 5.2.4 Opportunities provided for learners' participation

From the study findings, the teachers claimed to have provided different learning opportunities to the learners. The teachers mentioned the provision for hands-on activities, opportunities to speak short sentences, short expressions, to write an exercise individually and then discuss with learners. Although the teachers managed to provide level 2 opportunities for learner participation, there are
other learning opportunities that were mentioned during the interviews and in the lesson plans but they were not done during the teaching. Teachers mentioned that they would discuss the algebraic concepts, the examples and tasks with learners, but this was not observed during the lessons. What was observed during the lessons was that there were low level discussions. Activities for learner participation that the teachers provide to learners should demand high levels of discussions to help learners achieve the object of learning in a lesson. The how and why questions when used properly in a lesson, provide learners with opportunities to explore and discover the object of learning and help them see the meaning of the mathematics they are learning. With all this, the researcher concludes that the teachers do not use discussion method to the fullest in order to enhance high levels of learner participation in a lesson.

### 5.2.5 Learner participation (involvement) in algebra lessons

The study findings show that the teachers used the questions that required learners to answer yes or no during the lessons, they also used "what" questions so that learners can finish up the number sentences or fill in the missing terms. They also asked learners the "how" questions which required learners to write the number sentences in short. From the researcher's observation, teachers involved learners in activities that required learners to practice what the teachers had already demonstrated to them. The researcher noticed that teachers asked learners low order questions where learners needed to show understanding by just practicing what teachers had already done in the examples.

### 5.3 Implications

Although the study used a small sample, it has discovered very important issues regarding how teachers introduce algebra to Standard 5 learners in primary schools in Malawi. These are learners in primary schools who encounter algebra as a topic for the first time. The teachers' knowledge on how to introduce algebra to beginners in primary schools in Malawi is very important. We cannot claim to have laid a foundation for algebra by introducing algebra in primary schools if teachers have limited or no knowledge on how to introduce and teach algebra to learners in primary schools. Its only good teaching approaches that can yield to success in algebra. For example, proper explanations of the algebraic concepts, proper choice of resources, examples and tasks to use in order to enhance learners' understanding of the new concepts, provision of adequate learning opportunities for learners to explore, discover and speak the mathematics they are learning, and good learner involvement in the lessons. If teachers find challenges in explaining the algebraic
concepts to learners, and do not know the suitable resources, examples and tasks to use in the lessons during the introduction of algebra, it means that the purpose of introducing algebra in Malawian primary schools cannot be achieved. In addition, teachers should provide learning opportunities and fully engage learners in activities that can promote critical thinking and mathematical reasoning. It has been established that the teachers who participated in the study had some challenges to introduce algebra to learners appropriately because of inadequate knowledge. The low levels of exemplification, the low levels of explanatory talk and low levels of learner participation as classified by Adler and Ronda (2015) implies that the teachers have difficulties in introducing algebra to learners. The struggle that the teachers encountered in introducing algebra to learners might mean that the teachers have inadequate pedagogical content knowledge which is acquired through pre service and in service trainings. It might also imply that, teachers hold on to the old traditional knowledge of introducing algebra, the "fruit salad" which according to research, many countries have abandoned it.

Knowledge is dynamic, therefore any changes in terms of approaches to teaching of algebra must be communicated to implementers through in service trainings. Although teachers' guides are just meant to guide teachers in the teaching, it has been established that the three teachers rely on textbooks such as teachers' guides for information to teach. If there is little information about concepts and how to teach them, these teachers are likely to face difficulties to teach learners the way the material is supposed to be taught. This implies that if possible, much information on algebra should be included in the primary school teachers' guides or else supplementary reference materials should be developed to help teachers find required information on how to introduce algebra to learners. There is no subject specification in primary school education, this means that, teachers who are not good enough in mathematics, and algebra in particular, struggle to teach. They can hardly introduce and teach algebra as required.

### 5.4 Recommendations for future research

The major conclusion of this study is that the three teachers had difficulties in introducing algebra to Standard 5 learners. Their explanations about the algebraic concepts to learners were of low standards as classified in Adler and Ronda (2015). They used objects instead of representations in the introduction of algebra. There was no variation in the examples and tasks presented to learners. As such, there is need to:

- Investigate how teacher education prepare student teachers on the teaching of algebra in primary schools in Malawi.
- Investigate how teachers introduce algebra in Standard 5 in Malawi on a larger scale so that the findings of the study can be generalised.
- Investigate how teachers teach algebra in upper primary schools in Malawi.
- Investigate how teachers teach algebra in Standard 8 as a transition class to secondary school education.


## References

Adler, J., \& Ronda, E. (2017). Mathematics discourse in instruction (MDI). A discursive resource as boundary object across practices: In G. Kaiser (Ed.), Proceedings of the 13th International Congress on Mathematical Education (pp. 125-143). Hamburg, Germany: Springer.

Adler, J., \& Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. African Journal of Research in Mathematics, Science and Technology Education, 19(3), 237-254. http://doi.org/10:1080/10288457.2015.1089677

Baker, L. (2006). Observation: A complex research method. Library Trends, 55(1), pp.171-189.
Booth, L. (1988). Children's difficulties in beginning algebra. In A.F. Coxford \& A.P. Shulte (Eds.), The Ideas of Algebra, K-12. 1988 Yearbook. Reston, VA: NCTM.

Bowen, G. (2009). Document analysis as a qualitative research method. Qualitative research journal, 9(2), pp. 27-40.

Cai, J., \& Knuth, E. (2011). Early Algebraization: A Global Dialogue from Multiple Perspectives. Berlin, Germany: Springer.

Cai, J., \& Moyer, J. (2008). Developing algebraic thinking in earlier grades: Some insights from international comparative studies. In C. Greenes \& R. Rubenstein (Eds.), Algebra and Algebraic Thinking in School Mathematics, 70th Yearbook of the National Council of Teachers of Mathematics, (pp. 169-180). Reston, VA: NCTM.

Carraher, D., \& Schliemann, A. (2007). Early algebra and algebraic reasoning. In F. Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics, (pp. 669-705). Charlotte, NC: Information Age.

Chick, H., \& Harris, K. (2007). Grade 5/6 teachers' perceptions of algebra in the primary School curriculum. In J. H. Woo, H. C.Lew, K. S. Park, \& D. Y. Seo (Eds.), Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, Vol. 2, (pp. 121-128). Seoul, Korea: PME.

Cohen, L., Manion, L. \&Morrison, K. (2007) Research Methods in Education (Sixth ed.). London, UK: Routledge, Taylor \& Francis Group.

Creswell, J., \& Poth, C. (2018). Qualitative Inquiry and Research Design: Choosing Among Five Approaches, (4th ed.). Thousand Oaks, CA: Sage Publications, Inc.

Creswell, J. (2013). Research design: Qualitative, Quantitative, and Mixed Methods Approaches ( $3^{\text {rd }}$ ed.). Thousand Oaks, CA: Sage Publications, Inc.

Creswell, J. (2009). Research design: Qualitative, Quantitative, and Mixed Methods Approaches ( $3^{\text {rd }}$ ed.). Thousand Oaks, CA: Sage Publications, Inc.

Epp, S. (2011). Variables in mathematics education. In: P. Blackburn, H. van Ditmarsch, M. Manzano, F. Soler-Toscano, (Eds.) Tools for Teaching Logic. TICTTL 2011. Lecture Notes in Computer Science, vol. 6680, pp. 54-61. Heidelberg, Berlin: Springer.

Etikan, I., Musa, S., \& Alkassim, R. (2016). Comparison of convenience sampling and purposive sampling. American Journal of Theoretical and Applied Statistics, 5(1), 1-4. doi:10.11648/j.ajtas.20160501.11

Frade, C., Acioly-Reigner, N., \& Jun, L. (2013). Beyond deficit models of learning mathematics: Social-cultural directions for change and research. In M.A. (Ken) Clements, A. Bishop, C. Keitel, J. Kilpatrick, F. Leung (Eds.), Third International Handbook of Mathematics Education, pp. 101-144. New York, Springer

Grouws, D. (2007). Handbook of research on mathematics teaching and learning: A Project of the National Council of Teachers of Mathematics. New York, NY: MacMillan Publishing Company.

Huang, R. (2014). Prospective mathematics teachers' knowledge of algebra. A Comparative Study in China and the United States of America, Middle Tennessee State University, USA: Springer.

Jakobsen, A. Kazima, M., \& Kasoka, D. (2018). Assessing prospective teachers' development of MKT through their teacher education: a Malawian case.1: Nordic Research in Mathematics Education. Papers of NORMA 17. The Eight Nordic Conference on Mathematics Education, pp. 219-227, Sweden: Stockholm.

Kazima, M., Jakobsen, A. \& Kasoka, D. (2016). "Use of mathematical tasks of teaching and the corresponding LMT measures in the Malawi context," The Mathematics Enthusiast, 13(1), pp. 171-186

Kazima, M. \& Jakobsen, A. (2013). Moving from quantity to quality in mathematics education in Malawi. Proceedings of the $4^{\text {th }}$ Africa Regional Congress of the International Commission of Mathematical Education (AFRICME) 11-14 June, 2013, (pp.178-189). Maseru, Lesotho: Lesotho College of Education.

Kazima, M., \& Mussa, C. (2011). Equity and quality issues in mathematics education in Malawi schools. In B. Atweh, M. Graven \& P. Valero (Eds.), Managing Equity and Quality in Mathematics Education (pp. 163-176). New York, NY: Springer.

Khalid, M. (2009). Mathematical thinking in Brunei curriculum: Implementation Issues and Challenge Retrieved from: http://earchives.criced.tsukuba.ac.jp/data/doc/pdf/2009/02/Madihah_Khalid.pdf

Knapp, A. (2016). Basic Algebra. Digital (2 ${ }^{\text {nd }}$ ed.). East Setauket, NY: Project Euclid.
Kieran, C. (2018).Teaching and learning algebraic thinking with 5 to 12-year-olds: The Global Evolution of an Emerging field of research and practice, Cham, Switzerland: Springer.

Kieran, C. (2006). Research on the learning and teaching of algebra: A broadening of sources of meaning. In A. Gutierrez \& P. Boero (Eds.), Handbook of Research on the Psychology of Mathematics Education, Past, Present and Future. Rotterdam, Canada: Sense publication. https://doi.org/10.1163/9789087901127_003

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? The Mathematics Educator, 8(1), 139-151.

Kilpatrick, J., Swafford, J., \& Findell, B. (2001). Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academy Press.

Kuchemann, D. (1981). Algebra. In K. M. Hart (Ed.), Children's Understanding of Mathematics 11-16, (pp. 102-119), London: Murray.

Kullberg, A., Kempe, U. R., \& Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? ZDM Mathematics Education 4(49), 559 - 569 .

Leung, F., Park, Holton, D., Clarke, D. (2014). How is algebra taught around the world? In F. Leung, K. Park, D. Holton, D Clarke (Eds.), Algebra Teaching Around the World (pp. 1_ 15). Rotterdam, Canada: Sense Publishers.

Lo, M (2012). Variation Theory and the Improvement of Teaching and Learning. Goteborg, Sweden: Acta Universitatis Gothoburgensis.

Marton, F., \& Pang, M. F. (2013). Meanings are acquired from experiencing differences against a background of sameness, rather than from experiencing sameness against a background of difference: Putting a conjecture to test by embedding it into a pedagogical tool. Frontline Learning Research, 1(1), 24-41.

Marton, F., \& Pang, M. (2006). On some necessary conditions of learning. The Journal of the Learning Sciences, 15(2), 193-220.

Marton, F., \& Tsui, A. (2004). Classroom Discourse and the Space of Learning. Mahwah, NJ: Lawrence Erlbaum.

Malawi Institute of Education (2008). Mathematics Teachers' Guide for Standard 7. Domasi, Malawi: MIE.

Malawi Institute of Education (2007). Mathematics Teachers' Guide for Standard 5. Domasi, Malawi: MIE.

Malawi Institute of Education (2007). Mathematics Learners' Book for standard 5. Domasi, Malawi: MIE.

Malawi Institute of Education (2007). Mathematics Learners' Book for standard 1. Domasi, Malawi: MIE.

Masperi, P., \& Hollow, D. (2008). The Potential Role of Portable Interactive Learning Technology within Basic Education in Malawi. Institute for statistics country data, UNESCO.

McNeil, N., \& Alibali, M. (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. Child Development, 76(4), 883899. doi:10.1111/j.1467-8624.2005.00884

McNeil, N., Weinberg, A., Hattikudur, S., Stephens, A., Asquith, P., Knuth, E., \& Alibali, M. (2010). A is for apple: Mnemonic symbols hinder the interpretation of algebraic expressions. Journal of Educational Psychology, 102(3), 625-634. https://doiorg.ezproxy.uis.no/10.1037/a0019105

Merriam, S. (2001). Qualitative research and case study applications in education. San Francisco, CA: Jossey-Bass Publishers.

Mills, D. \& Morton, M. (2013). Ethnography in Education, Los Angeles: Sage Publications, Inc.
Ministry of Education (2004). Malawi Primary Education Curriculum Assessment Framework, Domasi, Malawi: Malawi Institute of Education.

Molina, M., Rodríguez-Domingo, S., Canadas, M. \& Castro, E. (2017). Secondary school students' errors in the translation of algebraic statements. International Journal of Science and Mathematics Education, 15(6), 1137-1156.

National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

Nassaji, H. (2015).Qualitative and descriptive research: Data type versus data analysis. Language Teaching Research, 19(2), 129-132.

Noble, H., \& Heale. R. (2019). Triangulation in research, with examples. Evidence-Based Nursing, 22(3). Pp. 67-68. http://dx.doi.org/10.1136/ebnurs-2019-103145

Ormond, C. (2012). Developing "algebraic thinking": Two key ways to establish some early algebraic ideas in primary classrooms. Australian Primary Mathematics Classroom, 17(4), 13-21.

Parkinson, S., Eatough, V., Holmes, J., Stapley, E. \& Midgely, N. (2016). Framework analysis: A worked example of a study exploring young people's experiences of depression. Qualitative Research in Psychology, 13(2), 109-129.

Roberts, N. (2012). Patterns, functions and algebra in the South African primary mathematics Curriculum: Towards more detail for South African teachers. In S. Nieuwouldt, D. Laubscher \& H. Dreyer, Proceedings of the Eighteenth National Congress of the Association for Mathematics Education of South Africa, (pp. 302-319), Potchefstroom, South Africa: North West University.

Rubenstein, R., \& Thompson, D. (2001). Learning mathematical symbolism: Challenges and Instructional Strategies. Mathematics Teacher, 94(4), 265-271.

SACMEQ (2010). SACMEQ III project results: Pupil achievement levels in reading and mathematics. Retrieved from: http://www.sacmeq.org/downloads/sacmeqIII

Shahrill, M., \& Mundia, L. (2014). The use of low-order and higher-order questions in mathematics teaching: Video analyses case study. Journal of Studies in Education, 4(2), 15-34.

Star, J., Caronongan, P., Foegen, A., Furgeson, J., Keating, B., Larson, M.,... Zbiek, R. (2015). Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students. Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Educational Sciences, U.S. Department of Education. Retrieved from the NCEE website: http://whatworks.ed.gov.

Susuwele-Banda, W. (2005).Classroom assessment in Malawi: Teachers' Perceptions and Practices in Mathematics, Virginia: Blacksburg.

Teijlingen, E. \& Hundley, V. (2001). The importance of pilot studies. Social Research Update, 35(35), 1-4.

Tsang, H., Mohammad M., \& Shahrill, M. (2014). The leadership styles of secondary mathematics teachers in Brunei Darussalam. In F. Uslu (Ed.), Abstracts and Proceedings of the International Conference on Social Sciences and Humanities (pp. 923-931). Istanbul, Turkey: International Organization Center of Academic Research.

USAID (2010). Evaluation of the Malawi Teacher Professional Development Support (MTPDS): Final Evaluation Report, Lilongwe, Malawi: US Agency for International Development.

Vincent, J., Bardini, C., Pierce, R., Pearn, C. (2015). Misuse of the equals sign: An entrenched practice from early primary years to tertiary mathematics. Australian Senior Mathematics Journal, 29 (2) pp. 31-39.

Walters, K. (2014). Professional development strategies to support student success in algebra 1: Promoting Student Success in Algebra 1. American Institutes for Research. Retrieved from: https://www2.ed.gov/programs/dropout/professionaldevelop0914.pdf

Wang, X. (2015). The literature review of algebra learning: Focusing on the Contributions to Students' Difficulties. Creative Education, 6(2), 144-153.

## 6. Appendices

### 6.1 Permission letter from NSD (Apendix 1)

# NS norsk senter for forskingcsoata 

## NSD sin vurdering

## Prosjekttittel

Investigating the teaching of Algebra in primary school

## Referansenummer

131227

## Registrert

22.08.2019 av Kolace Silwimba - k.silwimba@stud.uis.no

## Behandlingsansvarlig institusjon

Universitetet i Stavanger / Fakultet for utdanningsvitenskap og humaniora / Institutt for grunnskolelærerutdanning, idrett og spesialpedagogikk

## Prosjektansvarlig (vitenskapelig ansatt/veileder eller stipendiat)

Arne Jakobsen, arne.jakobsen@uis.no, tlf: $\square$

## Type prosjekt

Studentprosjekt, masterstudium

## Kontaktinformasjon, student

Kolace Silwimba, kolacesilwimba@yahoo.com, tlf:

## Prosjektperiode

01.08.2019-01.08.2020

## Status

13.09.2019 - Vurdert

## Vurdering (1)

### 13.09.2019 - Vurdert

Our assessment is that the processing of personal data in this project will comply with data protection legislation, so long as it is carried out in accordance with what is documented in the Notification Form and attachments, dated 13.09.2019, as well as in correspondence with NSD. Everything is in place for the processing to begin.

## NOTIFY CHANGES

If you intend to make changes to the processing of personal data in this project it may be necessary to notify NSD. This is done by updating the information registered in the Notification Form. On our website we explain which changes must be notified. Wait until you receive an answer from us before you carry out the changes.

## TYPE OF DATA AND DURATION

The project will be processing general categories of personal data until 01.08.2020.

## LEGAL BASIS

The project will gain consent from data subjects to process their personal data. We find that consent will meet the necessary requirements under art. 4 (11) and 7 , in that it will be a freely given, specific, informed and unambiguous statement or action, which will be documented and can be withdrawn. The legal basis for processing personal data is therefore consent given by the data subject, cf. the General Data Protection Regulation art. 6.1 a).

## PRINCIPLES RELATING TO PROCESSING PERSONAL DATA

NSD finds that the planned processing of personal data will be in accordance with the principles under the General Data Protection Regulation regarding:

- lawfulness, fairness and transparency (art. 5.1 a), in that data subjects will receive sufficient information about the processing and will give their consent - purpose limitation (art. 5.1 b ), in that personal data will be collected for specified, explicit and legitimate purposes, and will not be processed for new, incompatible purposes
- data minimisation (art. 5.1 c ), in that only personal data which are adequate, relevant and necessary for the purpose of the project will be processed - storage limitation (art. 5.1 e ), in that personal data will not be stored for longer than is necessary to fulfil the project's purpose


## THE RIGHTS OF DATA SUBJECTS

Data subjects will have the following rights in this project: transparency (art. 12), information (art. 13), access (art. 15), rectification (art. 16), erasure (art. 17), restriction of processing (art. 18), notification (art. 19), data portability (art. 20). These rights apply so long as the data subject can be identified in the collected data.

NSD finds that the information that will be given to data subjects about the processing of their personal data will meet the legal requirements for form and content, cf. art. 12.1 and art. 13.

We remind you that if a data subject contacts you about their rights, the data controller has a duty to reply within a month.

## FOLLOW YOUR INSTITUTION'S GUIDELINES

NSD presupposes that the project will meet the requirements of accuracy (art. 5.1 d ), integrity and confidentiality (art. 5.1 f ) and security (art. 32) when processing personal data.

To ensure that these requirements are met you must follow your institution's internal guidelines and/or consult with your institution (i.e. the institution responsible for the

## 6. 2 Lesson observation guide (Appendix 2)

Date $\qquad$ Name of school $\qquad$

Class $\qquad$ Time: From $\qquad$ To: $\qquad$

Number of students: Boys $\qquad$ Girls $\qquad$ Total $\qquad$
Unit/ Topic $\qquad$

| INDICATORS | COMMENTS |
| :--- | :--- |
| -The type of resources used in the lesson: |  |
| Objects |  |
| Representations |  |
| -How resources are used (in the examples |  |
| given and during learners' activities): |  |
| -Learners use the resources to discover the |  |
| meaning of new algebraic concepts. |  |
| -Selection of examples and tasks for learners: |  |
| Examples |  |
| Tasks |  |
| -How algebraic concepts are introduced and |  |
| explained to learners: |  |
| Number sentences |  |
| Letters |  |
| Coefficients |  |
| Operation symbols (addition and subtraction) |  |


|  |  |
| :--- | :--- |
| -How learners are involved in the lessons and |  |
| what opportunities for learning are provided: |  |
| Individual |  |
| Pairs |  |
| Groups |  |
| Verbal |  |
| Activity |  |

## 6. 3 Interview guide (Appendix 2)

Name of Teacher

1. How do you explain algebraic concepts to Standard 5 learners, a class where algebraic concepts
are introduced for the first time?
2. Where do you get the explanations from?
3. What resources do you use when introducing algebra in standard 5?
$\qquad$
$\qquad$
4. Is there any reason for using such resources?
$\qquad$
$\qquad$
5. What type of examples and tasks do you give your learners?
6. Why do you decide on selecting the examples you give learners?
7. Where do you get the examples and tasks from?
8. What learning opportunities do you provide for learners so that they discover the meaning of new concepts they are learning?
9. How are learners involved in class activities?

# Are you interested in taking part in the research project Investigating the teaching of Algebra in primary school? <br> <br> Information to the Headteacher 

 <br> <br> Information to the Headteacher}

This is an inquiry about participation of volunteer mathematics teachers from your school in a research project. The main purpose is to investigate the teaching of algebra in Malawi primary schools. In this letter, I will give you information about the purpose of the project and why I am asking you for permission to approach mathematics teachers teaching standard 5 (grade 5).

## Purpose of the project

This project is part of my master studies at the University of Stavanger, Norway, where I will be writing my master thesis in Mathematics Education. The purpose of my master thesis project is to investigate the teaching of algebra in Malawi primary school. I will narrow this to study how this is done in standard 5 (grade 5), and my study is guided by the following research questions:

1. What type of resources, examples and tasks do teachers use to introduce algebra in standard 5?
2. How do teachers introduce (explain) the algebraic concepts to standard 5 learners.
3. How do standard 5 learners participate in algebra lessons?

In particular, I am interested in studying:
a) The resources that teachers use to introduce algebra.
b) The introduction of letters and symbols.
c) The type of examples and tasks the teacher gives learners.
d) The activities that learners are involved in during lessons.
e) How teachers explain the algebraic concepts to learners (the letters, symbols and coefficient).

Who is responsible for the research project?
The master thesis is written under the supervision of Professor Arne Jakobsen, Department of Education and Sports Science, at the University of Stavanger, Norway, who is responsible for the project. If any questions, he can be contacted at email: arne.jakobsen@uis.no, or by phone:

Why am I asking for permission to collect data at your school?
I have selected a convenient sample of 3 schools in Bembeke zone in the Dedza district in Malawi. This district was selected due to convenient access. I would like to seek permission
from you as the head teacher to carry out research at your school. I would like you to identify standard 5 (grade 5) mathematics teachers who can voluntarily take part in the research.

## What does participation involve for your school?

During one week, I wish to observe and video record regular teaching in the mathematics teachers' classroom related to introduction of algebra. I will also take observational notes during teaching. After I am done with observing teaching, I also want to do an interview with the teachers that volunteer that will be audio recorded. I will also ask children and their parents for consent to video record the teaching with their children present in class.

## Participation is voluntary

Participation in the project is voluntary. If you allow me to involve teachers and children at your school, all information gathered about them will be made anonymous. There will be no negative consequences for them or your school, and they can withdraw at any time from the project.

## Personal privacy of participating teachers and children - how we will store and use their personal data

I am only interested in the teaching taking place, guided by the research question above, and no information that can be used to identify teachers, children - or your school - will be revealed as personal names and school names will be replaced by codes/pseudonyms. After video of the teaching is analysed, and audio recording of interviews are transcribed, both video and audio files will be deleted. I will process your personal data confidentially and in accordance with data protection legislation (the General Data Protection Regulation and Personal Data Act).

- In addition to me as researcher, only Professor Arne Jakobsen at the University of Stavanger will have access to the data.
- Video and audio recording will be stored on an encrypted separate hard disk that is locked away when not used, until is deleted at the end of the project. The list of names, contact details and respective codes used in transcript will be stored separately from the rest of the collected data,

Part of transcripts and findings from classroom teaching will be part of my master thesis, but it will not be possible to identify schools or teachers from the data made public.

What will happen to your personal data at the end of the research project?
The project is scheduled to end 1.8.2020. All video and audio recordings will be deleted latest at this date, and only anonymised text will be kept after that.

What gives us the right to process teachers and children's personal data?
We will process teachers and children personal data based on their consent (parents' consent for children).

Based on an agreement with University of Stavanger, NSD - The Norwegian Centre for Research Data AS has assessed that the processing of personal data in this project is in accordance with data protection legislation.

## Where can I find out more?

If you have questions about the project, contact:

- University of Stavanger, Norway, via Professor Arne Jakobsen.
- NSD - The Norwegian Centre for Research Data AS, by email:
(personverntjenester@nsd.no) or by telephone: +4755582117.

> Yours sincerely,

Project Leader
Arne Jakobsen
Professor

Student<br>Kolace P. Silwimba

## Consent form

I have received and understood information about the project investigating the introduction of Algebra in primary school and have been given the opportunity to ask questions. I give consent:
$\square$ to the researchers to contact mathematics teachers at my school and ask for volunteers to conduct research as outline above.

[^5]
# Are you interested in taking part in the research project Investigating the teaching of Algebra in primary school? <br> <br> Information to teachers 

 <br> <br> Information to teachers}

This is an inquiry about participation in a research project where the main purpose is to investigate the teaching of algebra in Malawi primary schools. In this letter, I will give you information about the purpose of the project and what your participation will involve.

## Purpose of the project

This project is part of my master studies at the University of Stavanger, Norway, where I will be writing my master thesis in Mathematics Education. The purpose of my master thesis project is to investigate the teaching of algebra in Malawi primary school. I will narrow this to study how this is done in standard 5 (grade 5), and my study is guided by the following research questions:

1. What type of resources, examples and tasks do teachers use to introduce algebra in standard 5.
2. How do teachers introduce (explain) the algebraic concepts to standard 5 learners.
3. How do standard 5 learners participate in algebra lessons.

In particular, I am interested in studying:
a) The resources that teachers use to introduce algebra.
b) The introduction of letters and symbols.
c) The type of examples and tasks the teacher gives learners.
d) The activities that learners are involved in during lessons.
e) How teachers explain the algebraic concepts to learners (the letters, symbols and coefficient).

Who is responsible for the research project?
The master thesis is written under the supervision of Professor Arne Jakobsen, Department of Education and Sports Science, at the University of Stavanger, Norway, who is responsible for the project. If any questions, he can be contacted at email: arne.jakobsen@uis.no, or by phone: +

## Why are you being asked to participate?

I have selected a convenient sample of 3 schools in Bembeke zone in the Dedza district in Malawi. This district was selected due to convenient access. I have first contacted head teacher at
your school and have been granted permission to asked for volunteers mathematics teacher in standard 5 (grade 5), and you are one of the standard 5 (grade 5) teachers I am contacting. What does participation involve for you?

During one week, I wish to observe and video record regular teaching in you classroom related to introduction of algebra. I will also take observational notes during teaching. After I am done with observing teaching, I also want to do an interview with you that will be audio recorded. I will also ask children and their parents for consent to video record your class.

## Participation is voluntary

Participation in the project is voluntary. If you chose to participate, you can withdraw your consent at any time without giving a reason. All information gathered will be made anonymous. There will be no negative consequences for you if you chose not to participate or later decide to withdraw.
Your personal privacy - how we will store and use your personal data
I am only interested in the teaching taking place, guided by the research question above, and no information that can be used to identity you as a teacher - or your school - will be revealed - as personal names and school names will be replaced by codes/pseudonyms. After video of your teaching is analysed, and audio recording of interviews are transcribed, both video and audio files will be deleted. I will process your personal data confidentially and in accordance with data protection legislation (the General Data Protection Regulation and Personal Data Act).

- In addition to me as researcher, only Professor Arne Jakobsen at the University of Stavanger will have access to the data.
- Video and audio recording will be stored on an encrypted separate hard disk that is locked away when not used, until is deleted at the end of the project. The list of names, contact details and respective codes used in transcript will be stored separately from the rest of the collected data,

Part of transcripts and findings from classroom teaching will be part of my master thesis, but it will not be possible to identify schools or teachers from the data made public.

## What will happen to your personal data at the end of the research project?

The project is scheduled to end 1.8.2020. All video and audio recordings will be deleted latest at this date, and only anonymised text will be kept after that.

## Your rights

So long as you can be identified in the collected data, you have the right to:

- access the personal data that is being processed about you
- request that your personal data is deleted
- request that incorrect personal data about you is corrected/rectified
- receive a copy of your personal data (data portability), and
- send a complaint to the Data Protection Officer or The Norwegian Data Protection Authority regarding the processing of your personal data


## What gives us the right to process your personal data?

We will process your personal data based on your consent.

Based on an agreement with University of Stavanger, NSD - The Norwegian Centre for Research Data AS has assessed that the processing of personal data in this project is in accordance with data protection legislation.

## Where can I find out more?

If you have questions about the project, or want to exercise your rights, contact:

- University of Stavanger, Norway, via Professor Arne Jakobsen.
- NSD - The Norwegian Centre for Research Data AS, by email:
(personverntjenester@nsd.no) or by telephone: +4755582117.

Yours sincerely,

Project Leader
Arne Jakobsen
Professor

Student<br>Kolace P. Silwimba

## Profer

## Consent form

I have received and understood information about the project Investigating the teaching of Algebra in primary school and have been given the opportunity to ask questions. I give consent:to be observed during my teaching of algebrato be video recorded during my teachingto be interviewed after my teaching $\square$ for my lesson plan for the teaching to be analysed.

I give consent for my personal data to be processed until the end date of the project, approx. 1.8.2020
(Signed by participant, date)

### 6.6 Consent and information letter to parents (Appendix 6)

# Are you interested in taking part in the research project <br> Investigating the teaching of Algebra in primary school? 

## Information to parents

This is an inquiry about participation in a research project where the main purpose is to investigate the teaching of algebra in Malawi primary schools. The focus of the project is the teacher, but video recording of the teaching might capture your child. In this letter, I will give you information about the purpose of the project and what your participation will involve for your child.

## Purpose of the project

This project is part of my master studies at the University of Stavanger, Norway, where I will be writing my master thesis in Mathematics Education. The purpose of my master thesis project is to investigate the teaching of algebra in Malawi primary school. I will narrow this to study how this is done in standard 5 (grade 5), and my study is guided by the following research questions:

1. What type of resources, examples and tasks do teachers use to introduce algebra in standard 5.
2. How do teachers introduce (explain) the algebraic concepts to standard 5 learners.
3. How do standard 5 learners participate in algebra lessons.

In particular, I am interested in studying:
a) The resources that teachers use to introduce algebra.
b) The introduction of letters and symbols.
c) The type of examples and tasks the teacher gives learners.
d) The activities that learners are involved in during lessons.
e) How teachers explain the algebraic concepts to learners (the letters, symbols and coefficient).

Who is responsible for the research project?
The master thesis is written under the supervision of Professor Arne Jakobsen, Department of Education and Sports Science, at the University of Stavanger, Norway, who is responsible for the project. If any questions, he can be contacted at email: arne.jakobsen@uis.no, or by phone:

Why are you being asked to participate?
I have selected a convenient sample of 3 schools in Bembeke zone in the Dedza district in Malawi. This district was selected due to convenient access. I have first contacted head teacher at your school and the mathematics teacher in your child's class has volunteered to participate in my project.

## What does participation involve for your child?

During one week, I wish to observe and video record regular teaching in your child's mathematics class. The camera will be placed in the back in the classroom and most of the time, focusing on the teacher, but your child might get captured sometimes, and the teacher might use the name of your child. Your child identity will remain anonymous and data about your child will not be used in my study.

## Participation is voluntary

Participation in the project is voluntary. If your child does not want to be part of this, or that you as parents don't give permission that your child is part of the class being video recorded, your child will be seated behind the camera. There will be no negative consequences for your child if he/she does not want to be part of the recording, or you as a parent don't allow your child to be part of the recording.

## Your personal privacy - how we will store and use your personal data

I am only interested in the teaching taking place, guided by the research question above, and no information that can be used to identity your child - or the school - will be revealed - as personal names and school names will be replaced by codes/pseudonyms. After video of the teaching is analysed video files of teaching will be deleted. I will process your personal data confidentially and in accordance with data protection legislation (the General Data Protection Regulation and Personal Data Act).

- In addition to me as researcher, only Professor Arne Jakobsen at the University of Stavanger will have access to the data.
- Video recording will be stored on an encrypted separate hard disk that is locked away when not used, until is deleted at the end of the project. The list of names, contact details and respective codes used in transcript will be stored separately from the rest of the collected data,

Part of transcripts and findings from classroom teaching will be part of my master thesis, but it will not be possible to identify schools, children or teachers from the data made public.

## What will happen to your personal data at the end of the research project?

The project is scheduled to end 1.8.2020. All video recordings will be deleted latest at this date, and only anonymised text will be kept after that.

## Your rights

So long as you can be identified in the collected data, you have the right to:

- access the personal data that is being processed about you
- request that your personal data is deleted
- request that incorrect personal data about you is corrected/rectified
- receive a copy of your personal data (data portability), and
- send a complaint to the Data Protection Officer or The Norwegian Data Protection Authority regarding the processing of your personal data


## What gives us the right to process your personal data?

We will process your personal data based on your consent.
Based on an agreement with University of Stavanger, NSD - The Norwegian Centre for Research Data AS has assessed that the processing of personal data in this project is in accordance with data protection legislation.

## Where can I find out more?

If you have questions about the project, or want to exercise your rights, contact:

- University of Stavanger, Norway, via Professor Arne Jakobsen.
- NSD - The Norwegian Centre for Research Data AS, by email:
(personverntjenester@nsd.no) or by telephone: +4755582117.

Yours sincerely,

Project Leader
Arne Jakobsen
Professor

Student
Kolace P. Silwimba

## Consent form

I have received and understood information about the project investigating the introduction of Algebra in primary school and have been given the opportunity to ask questions. I give consent:to be observed during my teaching of algebrato be video recorded during my teachingto be interviewed after my teachingfor my lesson plan for the teaching to be analysed.
I give consent for my personal data to be processed until the end date of the project, approx. 1.8.2020
(Signed by participant, date)


[^0]:    ${ }^{1}$ Primary school education in Malawi comprises of eight years referred to as Standard 1 to Standard 8.
    ${ }^{2}$ Standard 5 in Malawi is the same as Grade 5.

[^1]:    ${ }^{3}$ Procedural teaching, teaching starting by giving learners examples from the textbooks, followed by exercises similar to the examples for learners to practice.

[^2]:    ${ }^{4}$ Apprenticeship of observation is a phenomena whereby teachers tend to apply in their teaching the knowledge and skills they got through observing their teachers teaching them as school children.

[^3]:    ${ }^{5}$ Count on addition method is where you start counting on from the bigger number between the two numbers being added in an equation.

[^4]:    ${ }^{6}$ Non-standard problems are differently phrased questions that would help learners to recognize familiar algebraic steps in unfamiliar contexts.

[^5]:    (Signed by participant, date)

