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Abstract

Kick tolerance is an evaluation of how large kick sizes an open hole section can withstand without threatening the formation integrity at the shoe. If a certain kick size cannot be handled safely, the planned open hole section needs to be shortened, and the casing design has to be altered.

Three different flow models were used to simulate a kick situation: single bubble, transient flow, and analytical. The single bubble model and the transient flow model, based on the drift flux model, describe the kick circulation in time while the analytical model is a static model where the kick is situated at the bottom. These are all based on gas kicks in water-based mud. The simulations were performed with MATLAB, and to generate the different results, length and size of the open hole, BHA length and kick size are varied.

First, a transient model based on the single bubble concept was considered, and a Monte Carlo simulation framework was implemented. This modification was done along with extensive coding in order to further develop the previously written code. The purpose of the simulations was to see what kick location would give the largest casing shoe pressure: kick at bottom hole assembly (BHA) or kick expanding whilst travelling upwards towards the shoe. The results showed that in most cases, the maximum casing shoe pressure would occur when kick is located at the BHA. However, when simulations were performed with a long hole section, short BHA and large kick volume, gas expansion caused the maximum casing shoe pressure.

Secondly, the three models were simulated, and results were compared. The output was the fracture pressure and casing shoe pressure in the form of distributions. From the many Monte Carlo simulations of each case, the result was a failure probability based on the number of counts where the casing shoe pressure exceeded the fracture pressure at the shoe. The main objective of these simulations was to see how the three models compared by looking at the failure probabilities.

When comparing the models, the results show that the single bubble model provides the most conservative results and the transient flow model the least conservative. It would be reasonable to first calculate the casing shoe pressure to identify kick location at maximum pressure. Then, if the maximum pressure is caused by gas expansion, a transient model should be used to account for this. If not, the analytical model could be deemed appropriate. When calculating failure probability, the single bubble model might be too conservative due to the fact that it assumes the gas kick to be a single slug.

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Abbreviations

BHA	Bottom Hole Assembly
BHP	Bottom Hole Pressure
BOP	Blow Out Preventer
CPU	Central Processing Unit
DP	Drill Pipe
FIT	Formation Integrity Test
ID	Inner Diameter
LOT	Leak-Off Test
MCS	Monte Carlo Simulation
OBM	Oil-based Mud
OD	Outer Diameter
PDF	Probability Density Function
SICP	Shut-in Casing Pressure
SIDPP	Shut-in Drill Pipe Pressure
SM	Safety Margin
TVD	True Vertical Depth
WBM	Water-based Mud
WOW	Wait on Weather

Symbols

sg	Specific gravity
K	Kelvin
bbl	Barrel
μ	Expected value
σ	Standard deviation
ρ_{gas}	Density of gas
ρ_{mud}	Density of drilling fluid
g	Gravitational constant
A_{BHA}	Cross-sectional area outside bottom hole assembly
A_{DP}	Cross-sectional area outside drill pipe
P_{shoe}	Casing shoe pressure
P_{bh}	Bottom hole pressure
P_{fr}	Fracture pressure at the shoe
D	Depth of well
D_{shoe}	Depth of casing shoe
H_{kick}	Height of kick in the well
Q	Flow rate
N	Number of Monte Carlo simulations

1 Introduction

1.1 Background

When constructing a well, there are several elements that need to be estimated in order to plan how the well should be built. One of these is the length of each casing section. It is optimal to make each section as long as possible to save time and cost. One of the defining factors when designing casing and hole sections is the formations ability to handle a certain kick size when we drill down to planned depth. The weakest formation point is usually just below the last set casing shoe. If formation cannot handle it, the planned hole section needs to be shortened which again impacts the length of the casing. This evaluation is called kick tolerances, which represents an evaluation of which kick sizes that can be safely circulated out of the well without threatening the formation integrity.

Often, the worst case scenario is used to design the well. Some industries, e.g. construction, have been moving more towards using probabilistic methods to calculate failure.
To save cost and time, more research into using this method should be considered. Reliability based casing design is an example of this method, where probability is used to look at casing strengths vs. survival loads [1]. The working method in this thesis can be considered a continuation of the reliability based casing design as the main principles are the same.

1.2 Statement of the problem and objective

This thesis is a continuation of the work done in OMAE2017-61391 "Probabilistic Flow Modelling Approach for Kick Tolerance Calculations" [2]. The paper compared simulation results of two flow models in a well: the transient flow model and the analytical model.

Two things that were not considered in the previously mentioned paper, which could be interesting to look into:

- In wells with very long open hole sections, will the highest casing shoe pressure always occur when the kick passes bottom hole assembly (BHA), or is it possible that gas expansion gives the highest casing shoe pressure such that the maximum pressure is achieved when the kick reaches the shoe?
- Comparing the two models with the single bubble model. How similar or different would the output be?

All simulations performed were used to further look at how Monte Carlo simulations can be used in kick tolerance calculations:

- Is the use of Monte Carlo simulations for probabilistic calculation of kick tolerances a viable approach?

1.3 Structure of thesis

The thesis starts with presenting relevant theory, and continues with the simulation results obtained, a discussion and conclusion, and the codes used are attached in the appendix.

Chapter 2-5 makes up the theory:

- Chapter 2 gives a short introduction to well design and what lies behind the decisions as to how a well should be built.
- Chapter 3 dives into kick and kick tolerances. This is the most extensive theory chapter which is important to be able to understand the purpose and method for the simulations done.
- Chapter 4 presents the three models used to calculate kick tolerances. The single bubble model is weighted, as this is the main focus in the simulations. However, the two other models are also presented with their basic principles, advantages, and limitations.
- Chapter 5 explains the concept of Monte Carlo simulations. It contains information about what Monte Carlo simulations are, how they work, and their potential use in the petroleum industry.

Chapter 6 presents the data and results obtained. It contains results about where the maximum casing shoe pressure will be found when using the single bubble model. And it also shows failure probability and mean casing shoe pressure values for different scenarios and for all three models.

Conclusion and recommendations for future work are provided in chapter 7.

Lastly, the reference list and appendix is provided. Appendix A includes all codes that were used to create the results obtained in this thesis. The codes were originally developed in the work related to paper OMAE2017-61391 “Probabilistic Flow Modelling Approach for Kick Tolerance Calculations” [2]. Appendix B contains additional data obtained during simulations with the single bubble model.

2 Well Design

A well consists of a hole of various sizes with corresponding casings which are cemented together with the formation. The hole and casing sizes are largest at seabed and they decrease in size as the well gets deeper. There are some common sets of casing and hole sizes which are widely used. Table 2-1 below gives an overview of these.

Casing type	**Hole size (diameter)**	**Casing size (diameter)**
Conductor	36 in	30 in
Surface casing	26 in	20 in
Intermediate casing	17 ½ in	13 ⅜ in
Production casing	12 ¼ in	9 ⅝ in
Production liner	8 ½ in	7 in

Table 2-1 Common hole sizes and corresponding casings

To determine where the casing shoe of each section should be set, a pore pressure and formation strength prognosis is used. An example of this prognosis is shown in Figure 2-1. This figure shows pressure in specific gravity (sg) plotted against true vertical depth (TVD). The idea is to use the drilling fluid density to create a hydrostatic pressure. The hydrostatic pressure is used to keep pressure inside the well above the pore pressure to avoid unwanted influx, whilst still being below the fracture pressure to avoid fracturing the formation.

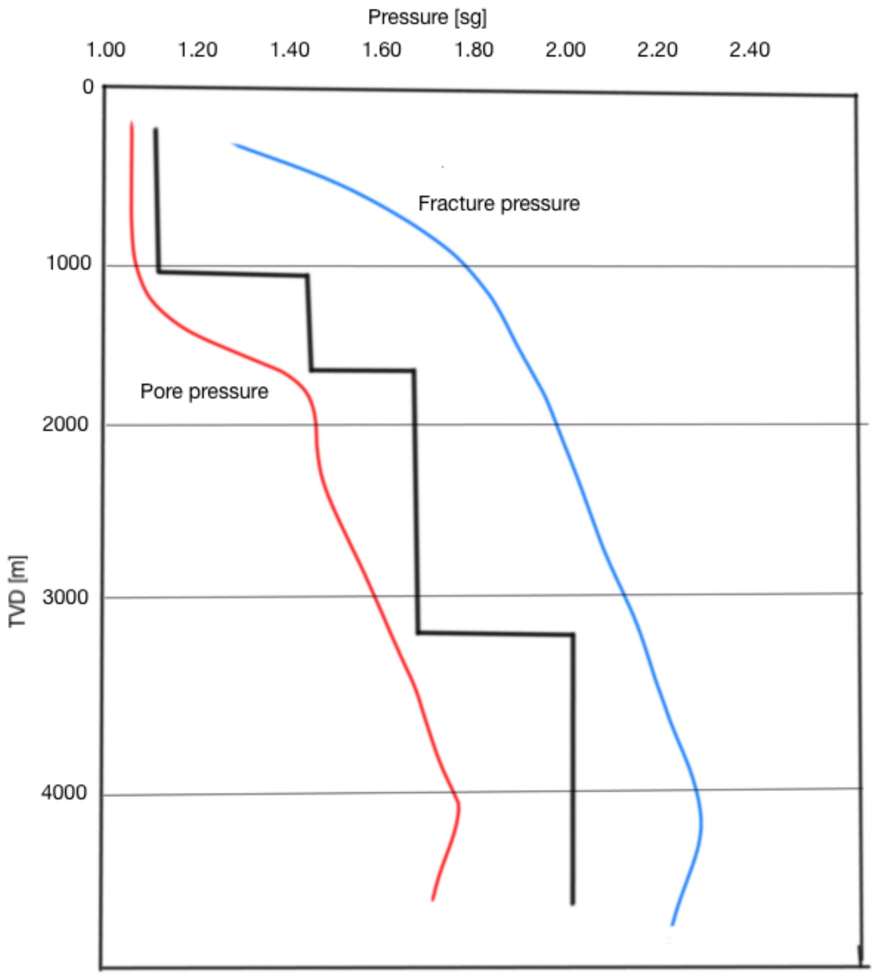

Figure 2-1 A simplified pore pressure plot

In the plot, a vertical straight line is drawn from seabed depth down until it reaches the pore pressure limit. To avoid going below the pore pressure, a casing shoe needs to be set in order to switch to a heavier drilling fluid. A horizontal line can be drawn right towards the fracture pressure curve to move away from pore pressure curve. Then we can repeat the first step by again drawing a vertical line downwards towards the pore pressure curve. This is continued downwards until target depth is reached. The result is a zig-zag line in the pore pressure plot which suggests casing shoe depths at the depth of each horizontal line section.

After a casing shoe for one section is set, pressure testing is done before drilling the next section of open hole. Usually, either a formation integrity test (FIT) or a leak-off test (LOT) is performed. A LOT is performed by pressurizing the shoe and formation until formation fracture occurs. The leak-off pressure is noted, and is used to calculate LOT at the shoe. A FIT is performed by increasing the bottom hole pressure (BHP) to designed pressure. Surface pressure is then increased until the required pressure is reached, and then the test is ceased. It is not continued until fracture, unlike the LOT, because it is only necessary to verify that the next section can be drilled and that the formation at the shoe can handle the pressures that will occur in a kick situation.

To save cost and time, it is most feasible to have sections as long as possible. How far it is possible to drill before setting a new casing shoe, is determined by the loads which can occur during the drilling process. The casing can be subjected to axial, burst and collapse loads. Burst loads can occur when casing is filled with gas, and can cause the casing to rupture. Collapse loads can occur during mud loss scenarios due to a reduction in hydrostatic pressure in the well. Axial loads will occur due to the weight of the casing string, and will increase if the section is extended. Mechanical friction will come in addition. An overview of different load scenarios can be found in “Modern Well Design” by Bernt S. Aadnøy [3].

Special care must be devoted to kick scenarios that can occur. During circulation of a kick, the pressure in the well will increase, and this can threaten the weakest part of the formation. This can e.g. lead to underground blowouts. The maximum well pressure at the weakest part of the formation will occur when kick is at the bottom or when it passes the shoe. Hence if a certain kick size cannot be handled safely, the planned hole section needs to be shortened [4]. This analysis is often termed kick tolerance evaluations, and will be explained in more depth in the next chapter.

3 Well Control and Kick Tolerances

3.1 Kick

Kick is the term used for unwanted fluids entering the well during well operations e.g. drilling or completion. It is likely to occur when the pore pressure in the formation surrounding the well is higher than the pressure in the well. The influx fluid could be water, oil, gas, or a mixture of these.

3.1.1 Causes of kicks

(1) Insufficient mud weight

The fluid column of drilling mud is used to create a hydrostatic pressure inside which is larger than the pore pressure, in order to prevent a kick. To determine the correct mud weight, the formation pressure needs to be predicted. However, during drilling it is possible that the mud weight in reality is lower than planned due to either pressure and temperature effects or wrongful weighting, which could cause a lower pressure inside the well [5].

(2) Uncertainty in pore pressure

The pore pressure can be underestimated when drilling an exploration well in a new area, due to the fact that the pressure is only a calculated estimate and there are no comparable pressure data from the area. Pore pressure is used to determine the density of drilling fluid to be used, and when the pore pressure is in fact higher than calculated, the mud will not create enough of a hydrostatic pressure to keep fluids from entering the well.

(3) Swabbing effect

Swab effects cause a decrease in the bottom hole pressure, which again can lead to unwanted influx [2].

(4) Lost circulation

If for some reason drilling fluids are lost to the formation, a drop in the annulus mud level will occur. This will lead to a drop in the bottom hole pressure, which can cause an influx from the outside permeable formation.

(5) Insufficient refill of well while tripping

Insufficient hole fill is one of the most common causes to kicks. It is usually a result of human error where the hole is not filled properly or failing to notice that it is not properly filled [5].

3.1.2 How kick is detected

There are a few signs which can lead to early detection of a kick:

(1) Increase in pit level

An increase in pit volume at the surface is a good indicator of a kick being taken. This increase will happen because formation fluids entering the well will be added to the existing well fluids.

(2) Reduced drill pipe weight

(3) Pump pressure changes

The pump pressure decreases due to a reduction in hydrostatic pressure in the annulus.

(4) Gas, oil, water-cut mud

(5) Unexpected increase in drilling rate

This can occur if the drill bit encounters a porous formation which can contain formation fluids. This can be considered as the earliest sign of a kick [5].

3.1.3 WBM vs. OBM

Kick detection is different for oil-based mud (OBM) and water-based mud (WBM).

Water-based mud:
Gas kick is easily detected in WBM due to the fact that it will not be dissolved in the drilling fluid. The gas kick is expected to reach the surface faster in WBM than it would in OBM since free gas has slippage relative to liquid. In addition, the kick will migrate upwards after the well has been shut in, and the well pressures will continue to increase. It will come to a stop when the kick is located just below the blow out preventer (BOP). The kick distribution will change during the migration [6].

Oil-based mud:
If the well is under high pressure, a gas kick will dissolve in an oil-based drilling fluid. This can lead to the kick not being detected as early as it would in a WBM. When the kick is dissolved, it will not migrate upwards. The kick will boil out rapidly in the upper parts of the well, and it needs immediate action due to the large expansion [6].

3.1.4 Process of well control after kick detection

If a kick is not controlled, the disastrous result could be a blowout. After the kick has been detected, action needs to be taken in order to safely circulate the kick out of the well.

The common procedure can be compiled into five steps [6]:

(1) Stop pumps and rotation

(2) Close the BOP

Wait for the pressure inside the well to build up. This will continue as long as formation fluids are entering the wellbore. The kick will start to slowly migrate upwards.

(3) Monitor the shut-in pressures

One can note the shut-in drill pipe pressure (SIDPP) and the shut-in casing pressure (SICP) and use this information to estimate the pore pressure and get an idea about the kick size.

(4) Open choke and circulate the kick out through choke line to separator/flare

There are two main methods to circulate a kick out of a well:
The first is the driller's method and the second is the wait and weight method.

Driller's Method:
This method first circulates the kick out of the well, and then proceeds to circulate kill mud. Previously used mud is circulated through the well in order to remove the kick from the wellbore. Then, kill mud is pumped down and circulated through the well. After this process, the well will be balanced and the pressure under control.

The kill sheet and choke pressure development is showed in Figure 3-1 and Figure 3-2.

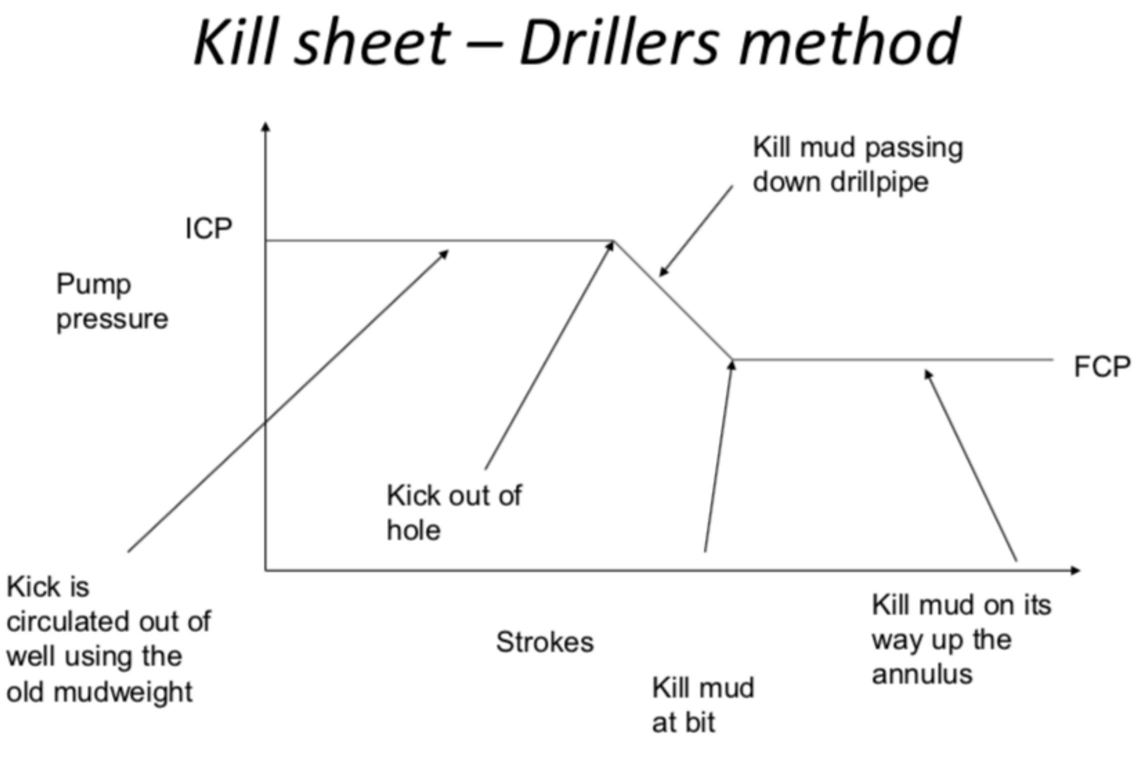

Figure 3-1 Kill sheet for the driller's method [6]

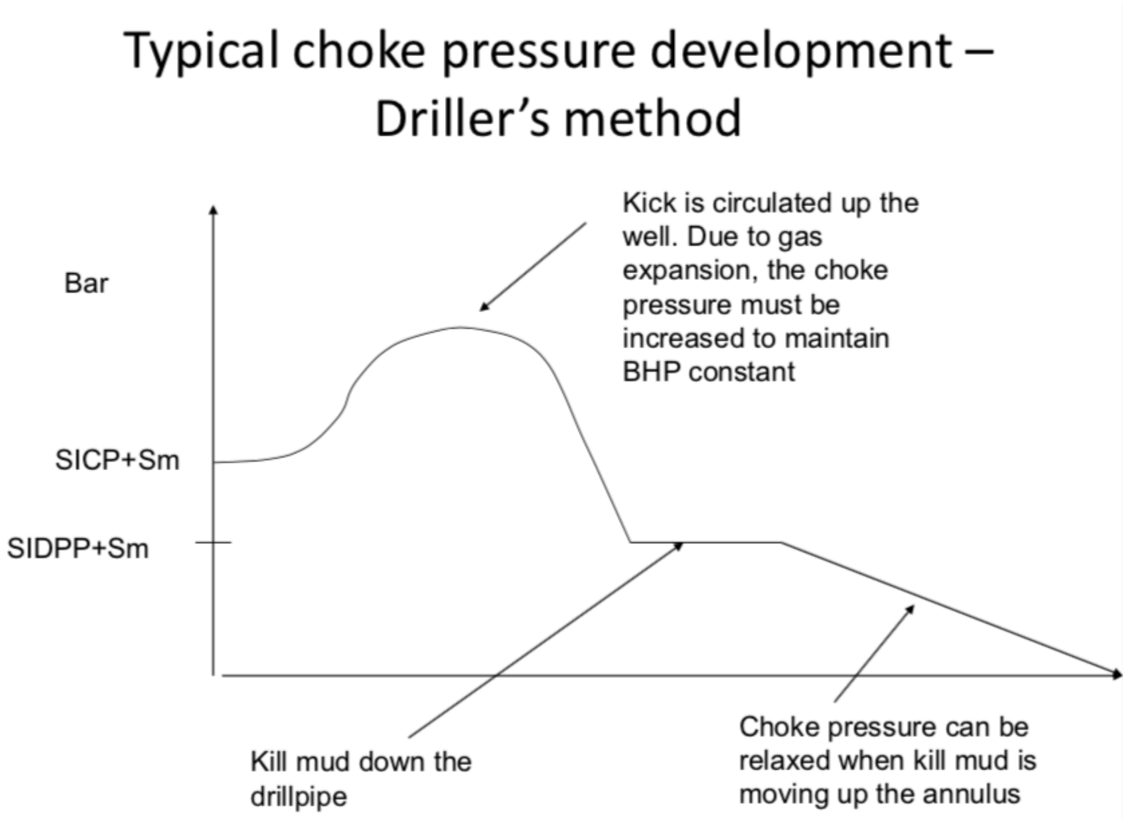

Figure 3-2 Choke pressure development for the driller's method [6]

<u>Wait and Weight:</u>

This method circulates the kick out whilst circulating the kill mud. While the well is shut in, density of the mud is increased to kill mud weight, and pumped down. This means that the well will be killed with one single circulation, unlike driller's method which takes two circulations [5].

The kill sheet and choke pressure is showed in Figure 3-3 and Figure 3-4.

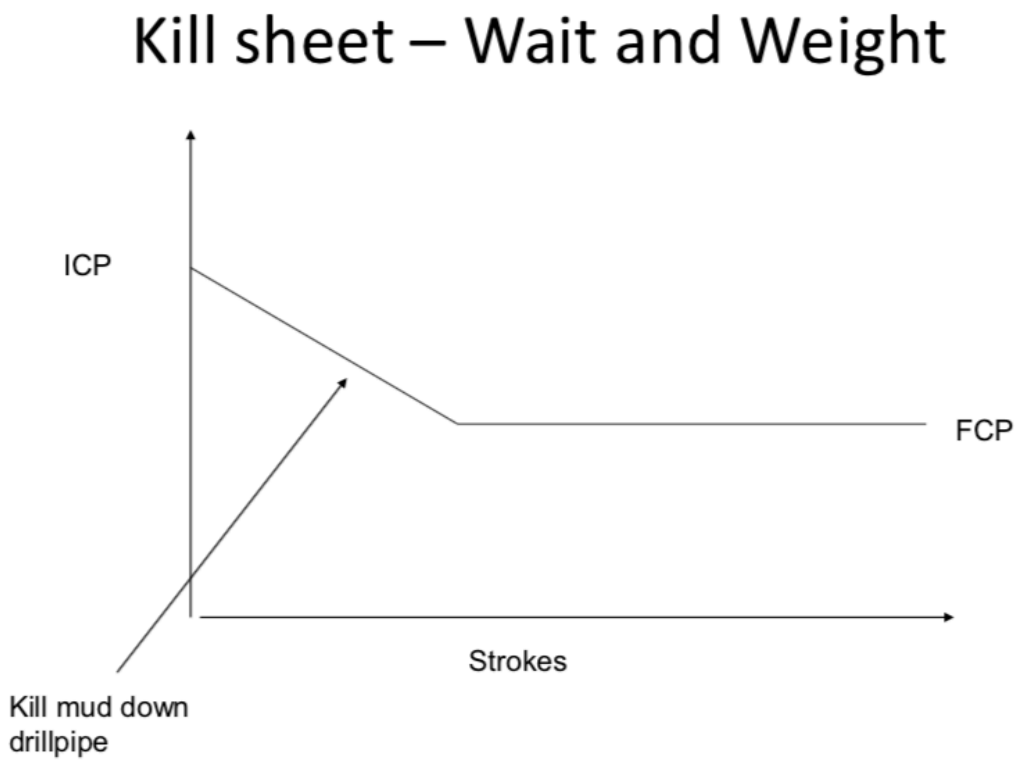

Figure 3-3 Kill sheet for the wait and weight method [6]

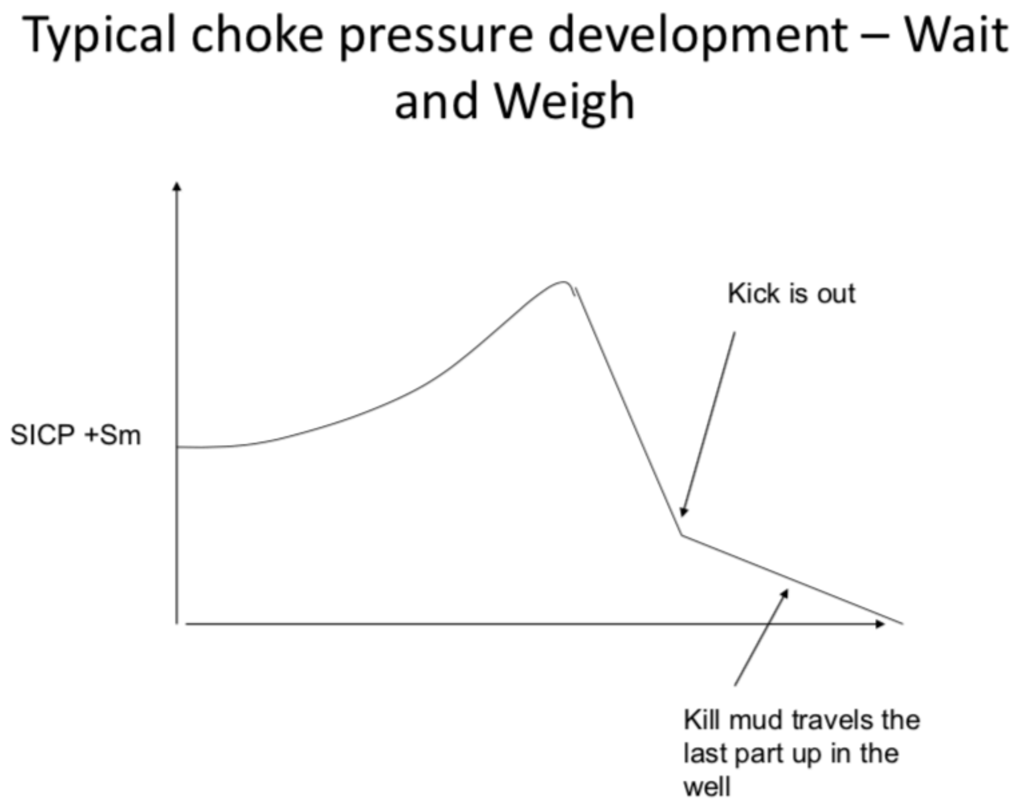

Figure 3-4 Choke pressure development for the wait and weight method [6]

(5) Bullheading

As a last resort, the kick can be bullheaded. This involves forcing the formation fluids back into the formation by using a heavy weighted mud. Bullheading can be an alternative in situations where the kick volume is substantial enough that it can break the formation on its way up the well, or when H_2S is present [6].

3.2 Pressure development in a well during kick circulation

3.2.1 Casing shoe pressure development

When using a transient model to calculate the casing shoe pressure, the pressure development can look something like the example in Figure 3-5. To explain the pressure development, this example is used. The pressure starts at approximately 565 bar, which is the initial pressure before the kick is present. As the kick enters the well at the bottom, it starts by taking up the volume behind the BHA. This will dramatically increase the casing shoe pressure due to the small cross-sectional area around the BHA. The pressure will continue to increase until the first peak at approximately 581 bar. As the kick continues to migrate upwards in the well, and it passes the BHA, the pressure will decrease due to a decrease in the kick height. This height decrease is caused by an increase in the volume as the drill pipe has a smaller cross-sectional are than the BHA. After a few hundred seconds, the pressure will again start to increase. This build up is a consequence of the gas expanding as it moves towards the shoe. The pressure continues to increase until it reaches the second peak, which occurs when the kick is located directly below the casing shoe. The casing shoe pressure will rapidly decrease towards the initial pressure when the entire kick has passed the shoe.

To calculate the casing shoe pressure, Eq. (3.1) below is used if the kick is below the shoe, and considered as a single slug [2]:

$$P_{cas} = P_{bh} - (D - H_{kick} - D_{shoe}) * g * \rho_{mud} - H_{kick} * g * \rho_{gas} \tag{3.1}$$

Where P_{cas} is the casing shoe pressure, P_{bh} is the bottom hole pressure, D is the total depth of the well and D_{shoe} is the shoe depth, g is the gravitational constant, ρ_{mud} and ρ_{gas} are the mud and gas densities, and H_{kick} is the height of the kick. This equation is based on basic physic, where the casing shoe pressure is equal to the bottom hole pressure, minus the hydrostatic pressure caused by the mud and the kick below the shoe.

When designing a well, it is necessary to know what the casing shoe pressure development will look like if a kick occurs. The value of the maximum casing shoe pressure and also the location of the kick when this occurs is essential knowledge e.g. during drilling.

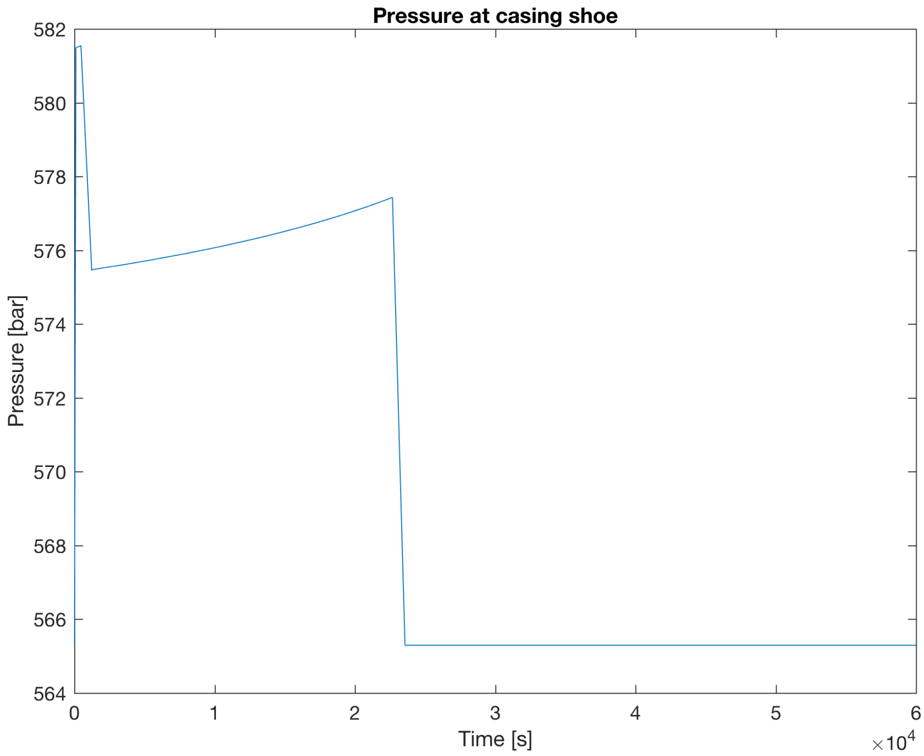

Figure 3-5 Casing shoe pressure vs. Time

When circulating out a kick it is desired to keep the BHP constant, and this can be done by using the choke line. The choke is used to keep the BHP at pore pressure plus a safety margin (SM) [2]. This BHP development is shown in Figure 3-6.

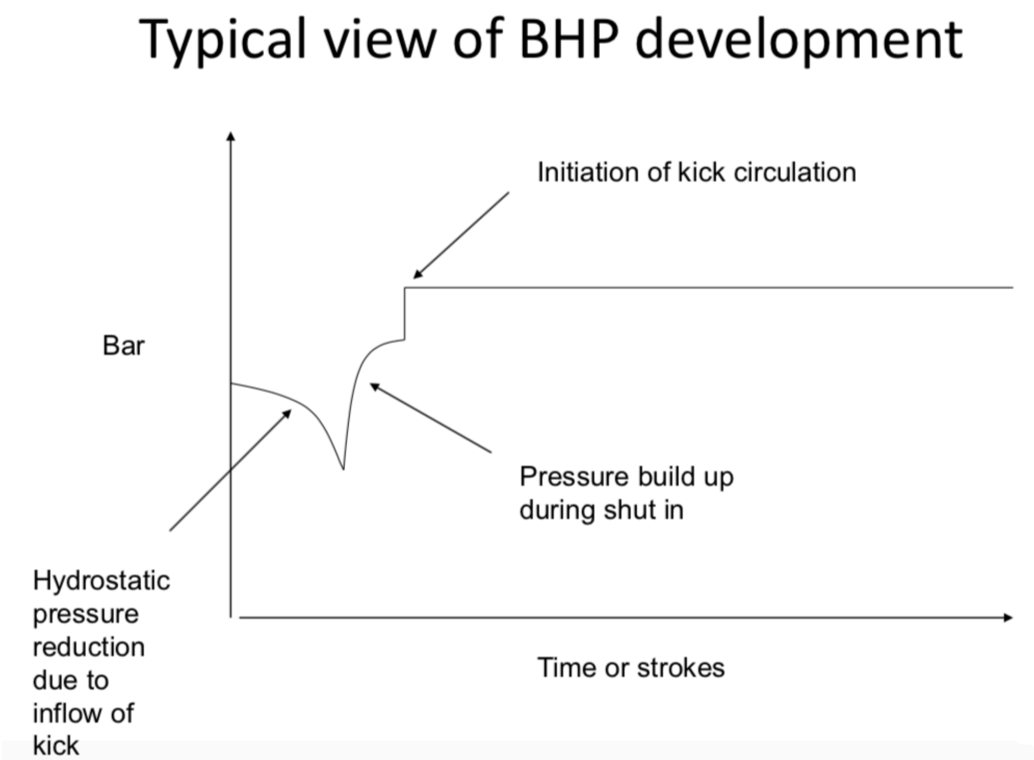

Figure 3-6 BHP development during a kick situation [6]

3.2.2 Simulation case

This section contains several plots to explain what happens in a well when a kick occurs. The example case used is a simulation performed with the single bubble kick simulator. More about the flow model used is covered in chapter 4.

Figure 3-7 shows the change in kick height. The initial height is zero, because no gas has entered the well. The height quickly increases to 100 m when the kick is situated behind the BHA, and the annular cross-sectional area is relatively small. As the kick passes the BHA, the area increases which leads to a decrease in the kick height. The kick height then increases exponentially due to the kick expanding whilst migrating upwards. The height decreases again as the kick gradually exits from the top of the well.

Figure 3-8 shows also shows the change in kick height, but here expressed by the top and bottom position of the kick.

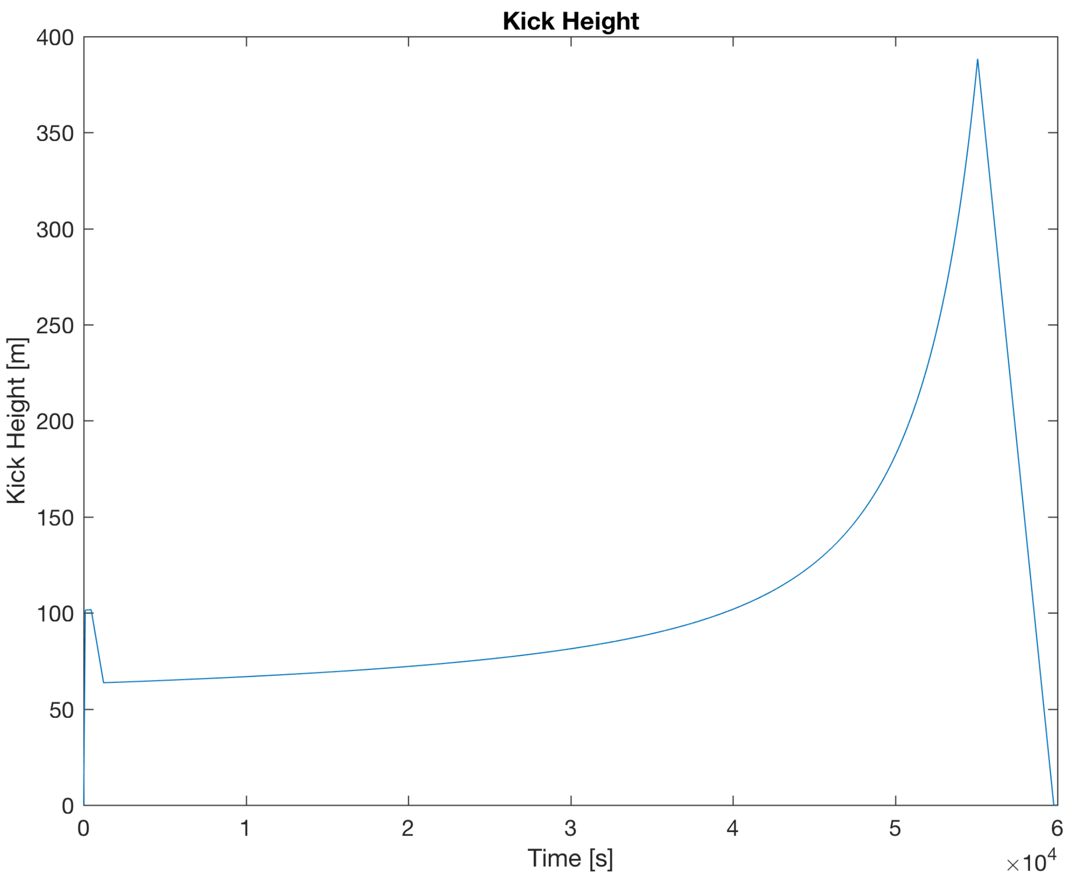

Figure 3-7 Kick height during kick circulation

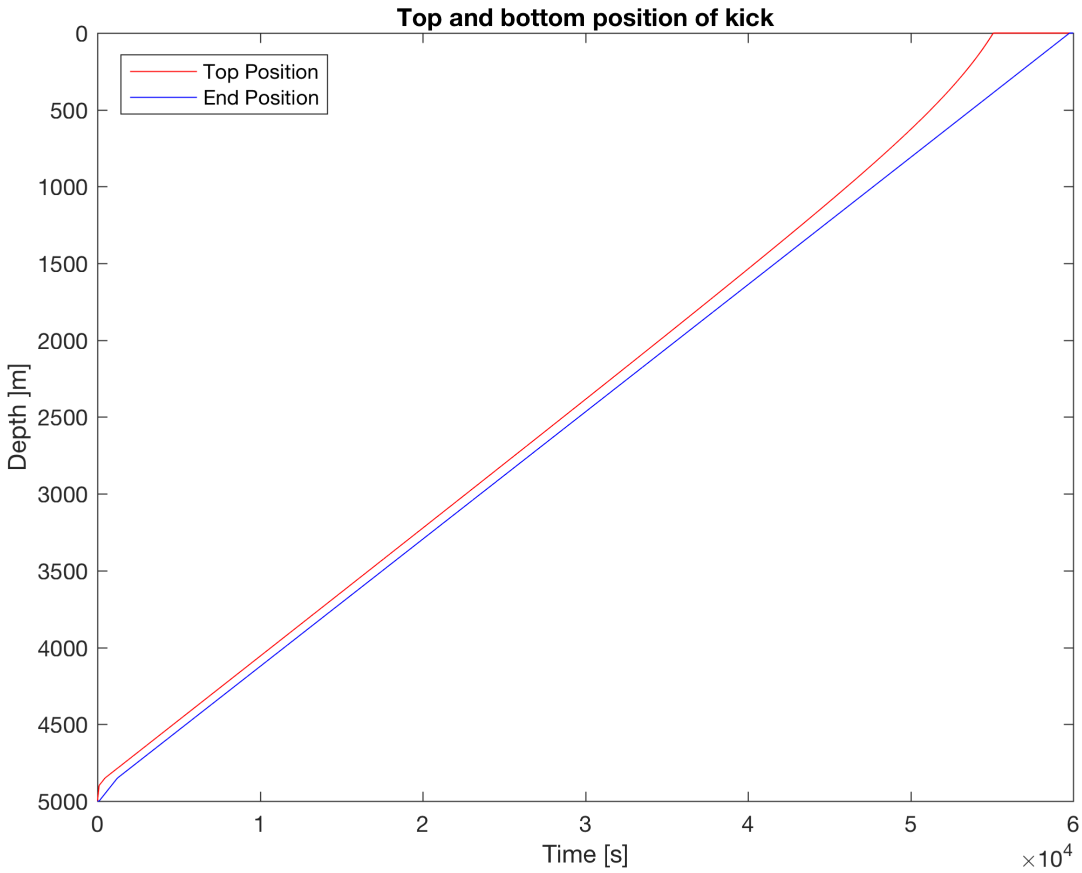

Figure 3-8 Top and bottom position of the kick

Figure 3-9 below describes the choke pressure development during the kick circulation. Because the bottom hole pressure needs the choke line in order to stay constant, the choke pressure increases as the gas expands while travelling upwards.

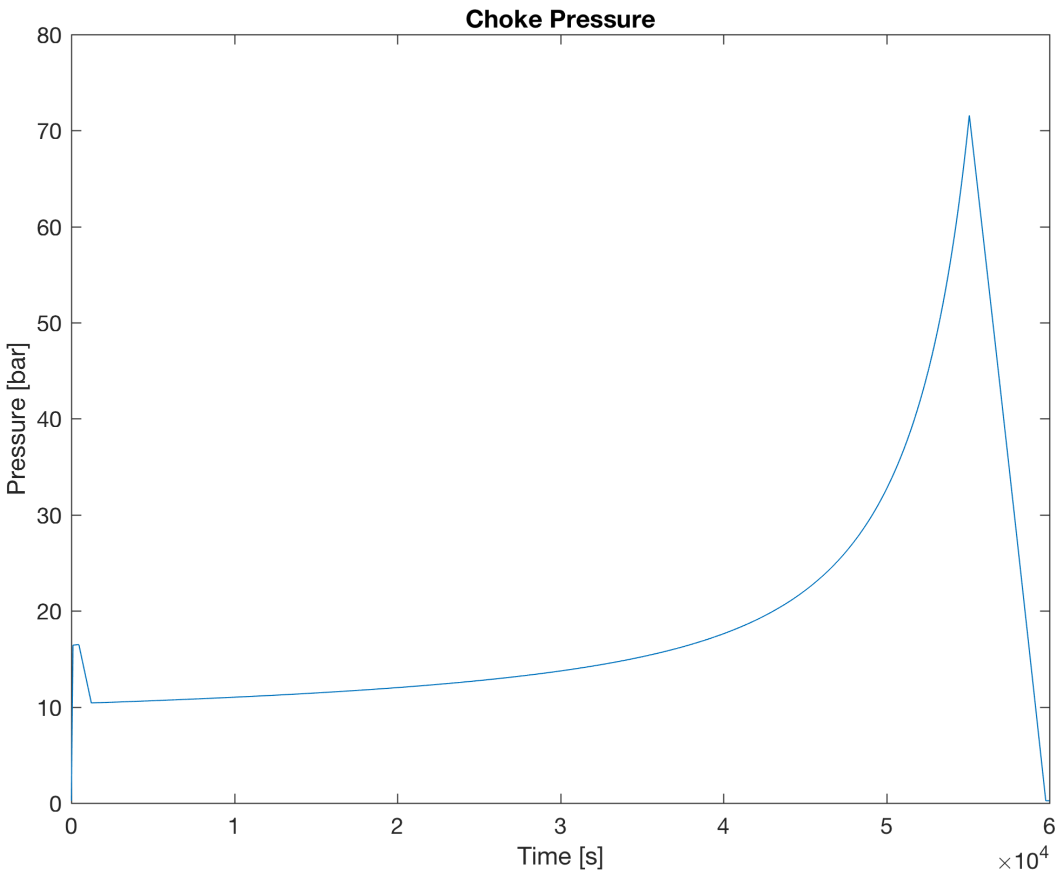

Figure 3-9 Choke pressure development

3.3 Kick tolerance & well design

Santos et al. defined kick tolerance like so: "*Kick tolerance can be understood as the capability of the wellbore to withstand the state of pressure generated during well control operations (well closure and subsequent gas kick circulation process) without fracturing the weakest formation.*" [7, p. 1].

In other words, kick tolerance is the volume of gas which can be safely circulated out of the well without damaging the formation at the weakest point. Formation at the last set casing shoe is usually considered as the weakest point.

When constructing a well, one of the most important decisions is to choose the casing setting depths. There are many factors which affects this decision like: fractured zones, shallow gas, lithology and pore pressure. Kick tolerances will also be an important evaluation that will have an impact on the casing design. Implementing kick tolerance design can make the drilling process safer and more economical, and is therefore important to consider [7].

When using kick tolerances to see how deep the next casing shoe can be set, there are three main data inputs needed: pore pressure at the bottom of the well, the fracture pressure at the shoe, and the kick size that can be circulated out of the well safely [7].

To avoid fracturing the formation during circulation of a kick, Eq. (3.2) must be valid [2, p. 3]:

$$P_{fr} \geq P_{shoe} + SM \tag{3.2}$$

Where P_{fr} is the formation fracture pressure at the shoe, P_{shoe} is the casing pressure at the shoe, and SM is the safety margin.

There are two different methods to determine the maximum casing shoe depths. The first one takes into account the shut-in tolerance, which is the result when calculating the tolerance at well shut-in. The other one calculates the kick tolerance when circulating the well. This method predominantly uses a kick simulator when calculating the casing shoe pressure [8].
More about the models that make up the kick simulators will follow in the next chapter.

Usually, companies have their own set standard as to how large kick volumes the formation should be able to handle to continue drilling. What the value of this number is varies from company to company as there is no regulation as to the volume of kick that needs to be handled at each shoe.

Figure 3-10 below shows what the result of a kick tolerance evaluation might look like. This plot shows which kick sizes that can be handled for each of the pore pressure values (1.75, 1.77, 1.78 sg). When the casing shoe pressure exceeds the fracture pressure, the formation might fracture, and the kick size cannot be considered manageable. The pressure values in this plot shows that when the pore pressure is high (1.78 sg curve), the kick volume that can be safely circulated out of the well is smaller than that of a lower pore pressure.

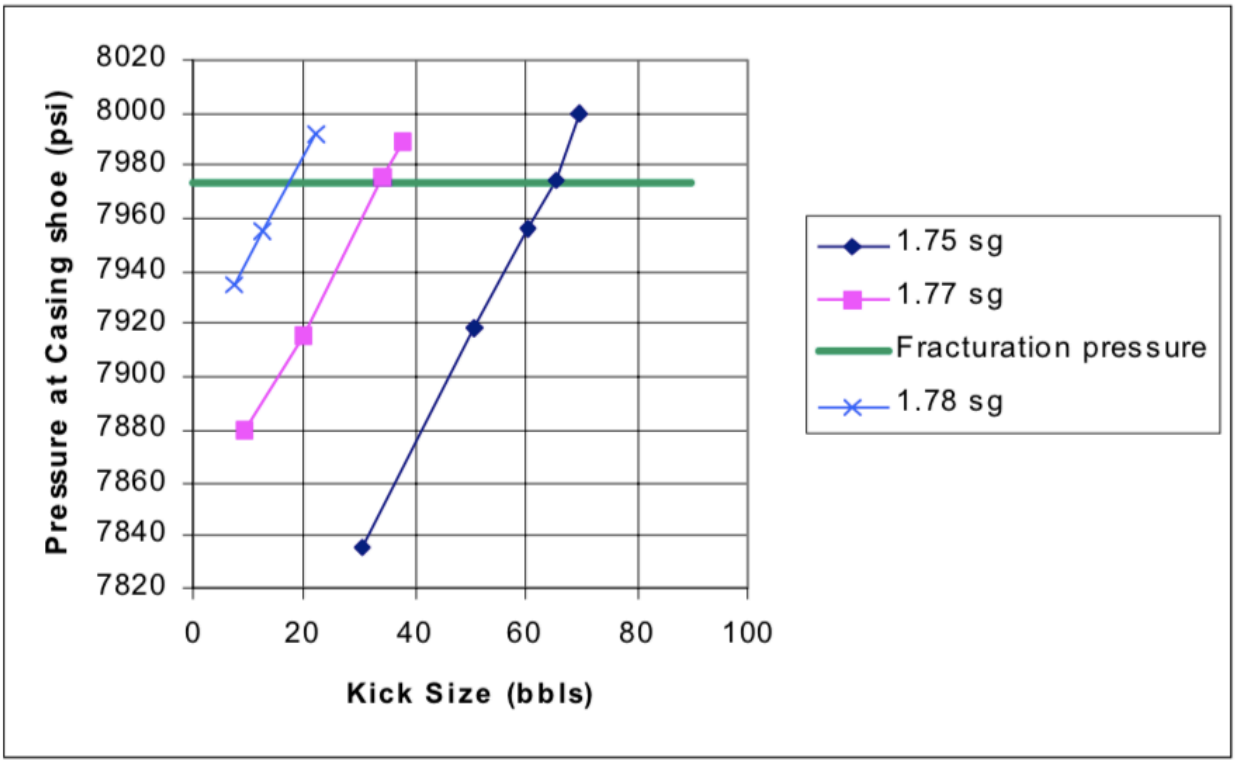

Figure 3-10 Casing shoe pressure vs. Kick size [9, p. 10]

4 Models for Calculating Kick Tolerances

Different models can be used to simulate how the pressure changes in a well when a kick is encountered. They differ due to the assumptions made to make up each model. In this thesis, two of the models used are transient, and one is not. Only two of the models include a gas fraction which describes the relationship between mud and kick in the cross sectional area. These factors can have a great impact on the result, and therefore choosing a model is an important part of the simulation process.

In this thesis, the three models that are considered are as follows: single bubble model, transient flow model, and analytical model. They are described in more detail below. The single bubble model is described in more depth compared to the others due to the extensive work that has been done with the script, and also due to this model being the main focus as the other two models are included in order to compare them against the single bubble model.

4.1 Assumptions

Well geometry

All three models are based on the same simplified well geometry. It consists of a wellbore with a constant inner diameter, a BHA at the bottom of the well, and a drill string from the top of the well down to the BHA. Figure 4-1 and Figure 4-2 show what the wellbore looks like. The depths and lengths of the components can be varied depending on which case is to be looked at. The well is considered to be completely vertical.

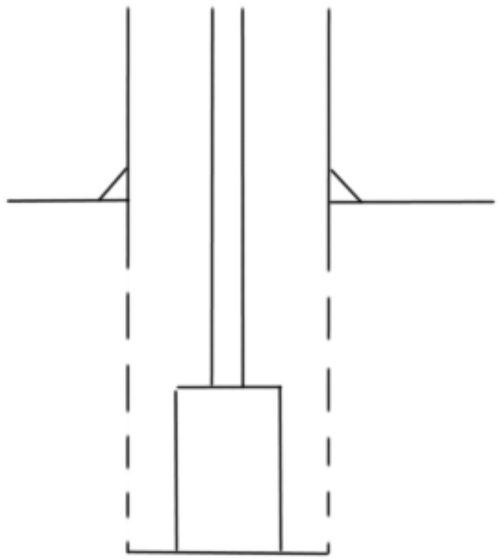

Figure 4-1 Wellbore sketch

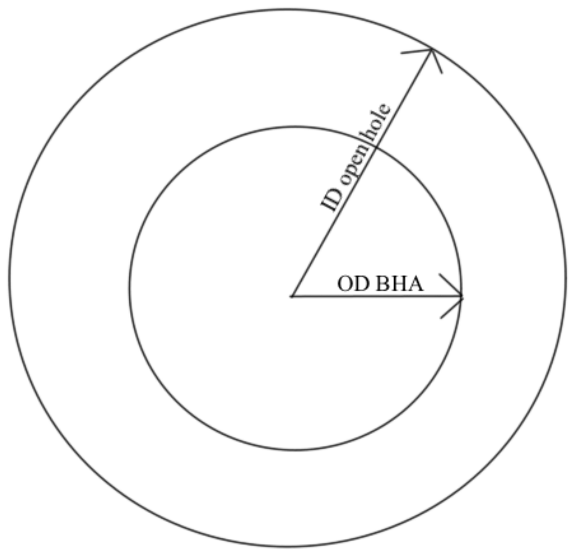

Figure 4-2 Cross-sectional area around BHA

Gas

For the work in this thesis, the kick is considered to be made up of methane gas only, which would correspond to the worst case scenario in the simulations. The density is calculated by using the real gas law, and it will change depending on the location of the kick.

Mud

All three models are constructed for the use of water-based mud.

The transient models can be modified in order for it to be possible to simulate with oil-based mud. The reason for the need of modification is that the kick will dissolve partially or completely in oil-based mud. Also the kinetics should be taken into account to include that the dissolution process takes time [10].

Temperature

The temperatures at the bottom and top of the well are set as constant. A linear temperature gradient is created from these two boundary temperatures.

4.2 Single bubble model

The most significant assumption to this model is based on the kick being a single bubble, meaning the gas fraction is equal to one and that the gas takes up the entire cross-sectional area of the wellbore. In reality, a gas fraction of one is unlikely to happen as the distribution between gas and mud depends on gas slippage [2].

The script calculates depending on the kick location:

1) **Kick is below the casing shoe**

   a) Kick is only around BHA

   b) Kick is in transition zone between BHA and drill pipe (DP), or only around DP

   The kick height dramatically changes due to a change in the kicks cross-sectional area from BHA to drill pipe.

2) **Kick is passing the shoe**

   This is the transition zone where kick will move from being entirely below the shoe to being above the shoe. This will lead to a decrease in casing shoe pressure.

3) **Kick is entirely above the shoe.**

   Casing shoe pressure is no longer affected by the kick.

The equations valid for each of these situations are stated below, along with definitions of each parameter used in the equations which can be found in Table 4-1:

Parameter	**Definition**
$V_{kick\_initial}$	Volume of kick just after the entire kick has entered the well [m^3]
V_{kick}	Volume of kick, time dependent [m^3]
D_{well}	Depth of well [m]
D_{shoe}	Depth of shoe [m]
D_{kick_top}	Depth of the kicks top position [m]
D_{kick_bottom}	Depth of the kicks bottom position [m]
L_{BHA}	Length of BHA [m]
D_{BHA}	Depth of BHA [m] (Depth corresponds to top of BHA)
A_{BHA}	Cross-sectional area around BHA [m^2]
A_{DP}	Cross-sectional area around drill pipe [m^2]
$L_{openhole}$	Length of well from top to shoe [m]
L_{cas}	Length of casing section [m]
$H_{kick,BHA}$	Kick height around BHA [m]
$H_{no\_kick,BHA}$	Mud height around BHA [m]

$H_{kick,openhole}$	Kick height around DP, below shoe [m]
$H_{no\_kick,openhole}$	Mud height around DP, below shoe [m]
$H_{kick,cas}$	Kick height around DP, above shoe [m]
$H_{no\_kick,cas}$	Mud height around DP, above shoe [m]
$H_{abovekick}$	Total mud height above kick [m]
$H_{underkick}$	Total mud height below kick [m]
ρ_{gas}	Density of gas bubble, time dependent [kg/m^3]
ρ_{mud}	Density of mud, constant [kg/m^3]
t	Time [s]
t_{influx}	Time just after the entire kick has entered the well [s]
Q	Mud rate [m^3/s]

Table 4-1 Definition of parameters

1. **Kick is below the casing shoe**

   a) **$V_{kick} < L_{BHA}*A_{BHA}$**

   $H_{kick,openhole} = H_{kick,cas} = 0$

   $H_{no\_kick,openhole} = L_{openhole}$, $H_{no\_kick,cas} = L_{cas}$

   $H_{kick,BHA} = V_{kick}/A_{BHA}$

   $H_{no\_kick,BHA} = L_{BHA}- H_{kick,BHA}$

   b) **$D_{shoe} <= D_{kick\_top}$**

   $V_{kick} = P_{bh}*V_{kick\_initial}*Z_b*T_b/Z_{bhp}*T_{bh}*P_b$

   $H_{kick,cas} = 0$, $H_{no\_kick,cas} = L_{cas}$

   1) If: $H_{kick,BHA} = 0$, no kick around BHA

      $H_{kick,BHA} = 0$, $H_{no\_kick,BHA} = L_{BHA}$

      $H_{kick,openhole} = V_{kick}/A_{DP}$

   2) If: $H_{kick,BHA} > 0$, transition zone between BHA/DP

      $H_{no\_kick,BHA} = Q*t-t_{influx}/A_{BHA}$

      $H_{kick,BHA} = D_{well} - D_{BHA} - H_{no\_kick,BHA}$

$V_{kick,BHA} = H_{kick,BHA} * A_{BHA}$

$V_{kick,openhole} = V_{kick} - V_{kick,BHA}$

$H_{kick,openhole} = V_{kick,openhole} / A_{DP}$

$H_{no\_kick,openhole} = L_{openhole} - H_{kick,openhole}$

$H_{kick\_total} = H_{kick,BHA} + H_{kick,openhole}$

$H_{abovekick} = D_{well} - H_{kick\_total} - H_{underkick}$

$P_{cas} = P_{bh} - (H_{abovekick} - D_{shoe} + H_{underkick})*g*\rho_{mud} - H_{kick\_total}*g*\rho_{gas}$

2. **Kick is passing the shoe**

$D_{kick\_top} < D_{shoe}$ & $D_{kick\_bottom} > D_{shoe}$

$H_{kick,BHA} = 0$, $H_{no\_kick,BHA} = L_{BHA}$

$H_{no\_kick,openhole} = L_{openhole} - H_{kick,openhole}$

$H_{no\_kick,cas} = L_{cas} - H_{kick,cas}$

$P_{cas} = P_{bh} - H_{underkick}*g*\rho_{mud} - H_{kick,openhole}*g*\rho_{gas}$

3. **Kick is entirely above casing shoe**

$D_{kick\_bottom} < D_{shoe}$

$H_{kick,BHA} = 0$, $H_{no\_kick,BHA} = L_{BHA}$

$H_{kick,openhole} = 0$, $H_{no\_kick,openhole} = L_{openhole}$

$H_{kick,cas} = V_{kick}/A_{DP}$, $H_{no\_kick,cas} = L_{cas} - H_{kick,cas}$

$P_{cas} = P_{bh} - \rho_{mud} *g*L_{cas}$

4.3 Transient flow model

The transient flow model is built on the transient drift flux model, which again is based on the conservation of both mass and momentum for a one-dimensional two-phase flow.

The model takes gas slip into account. This principle is based on the fact that gas has a higher velocity than the liquid, which in this case is mud. This will lead to changes in the gas distribution as the kick moves upwards in the wellbore.

This model is based on the usage of water-based mud, where the kick will not be dissolved in the mud. Because this is a transient model, it would be possible to simulate with oil-based mud by implementing changes in the simulation script. More information about WBM vs. OBM can be found in section 3.1.3.

Since this is a transient model, the casing shoe pressure increases when the kick travels towards the shoe due to gas expansion. This is one of the great advantages of this model combined with the fact that the gas is distributed.

The mud density varies with time, and is a function of temperature and pressure.

Because this is a transient model and variables like gas fraction, fluid densities and pressure changes with time, the CPU times will be higher than for a non-transient model. This is probably the largest disadvantage of using this model.

For further details on this model, see OMAE2017-61391 [2, p. 5].

4.4 Analytical model

The analytical model is a static model, which means its input parameters do not depend on time. It assumes that the kick is situated at the bottom. This makes the model unable to be used in cases where the highest casing shoe pressure will occur due to gas expansion. This is a great disadvantage to the model, but will not impact the result as long as there is reason to believe that the largest casing shoe pressure occurs when the kick passes the BHA. This disadvantage will become more prominent in cases of long open hole sections as this would give the gas more time to expand towards the shoe.

The gas fraction is determined with a numerical value or distribution. This means that area where the kick is located contains both gas and mud. This gas fraction will depend on how the kick is taken in the well. By adding the gas fraction as a distribution, the uncertainty of how the kick is taken will be taken into account. More about the use of distributions will follow in the next chapter. This is a more realistic approach than using a gas fraction equal to one like the single bubble model does.

The analytical model has the shortest CPU time of the three models. This is one of the major advantages to using this model, as simulations will be much quicker to perform compared to the other two models.

For a more detailed description of this model, see OMAE2017-61391 [2, p. 4].

5 Monte Carlo Simulation Technique

5.1 Basic statistical concept

Monte Carlo Simulation (MCS) is a statistical tool used to include uncertainty of input values, to be able to predict the outcome in terms of both value and its probability distribution. It is a useful tool to get a probability function which represents the desired output value in a satisfactory manner [11]. In simple terms, it is done by taking input variables in terms of probability density functions, calculating output, and repeating this process a set number of times (number of Monte Carlo simulations). The result will be a probability density function of the output.

One simple example of the use of Monte Carlo simulations is when predicting the travel time of a bus. There are several uncertainties to the travel time such as: amount of stops due to passengers getting off, stops due to passengers stepping on, acceleration and braking times, and also traffic at the time. Travel time can be calculated in this case by making a distribution for each input parameter, and using Monte Carlo simulations to combine these resulting in one distribution which shows the probability of each travel time occurring. All columns add up to 100% probability, which means that all likely outcomes are represented in the distribution. A suggestion as to what this simulation result might look is presented in Figure 5-1 below.

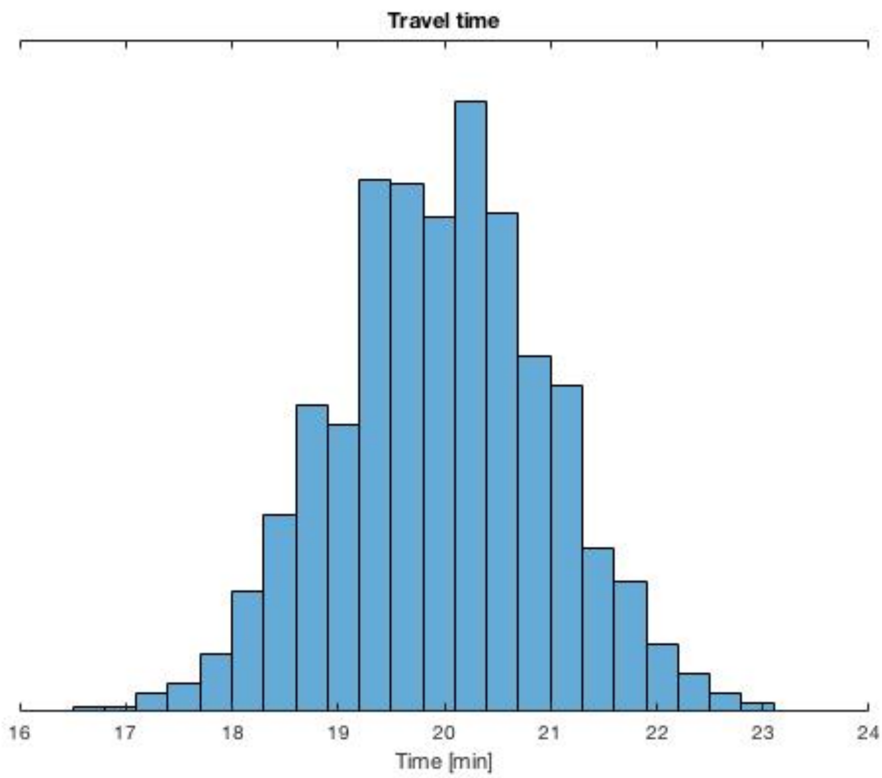

Figure 5-1 Example: Distribution of travel time

5.2 Monte Carlo simulation process

Figure 5-2 simply describes the process of using Monte Carlo simulations. On the left hand side, the uncertainties of each input are included with a probability distribution for each of the input variables. In the Monte Carlo simulation process, where a predetermined model is applied, these uncertainties will be combined by running the simulation a set number of times, and results in a probability distribution function of the output argument. This result is shown on the right hand side of Figure 5-2.

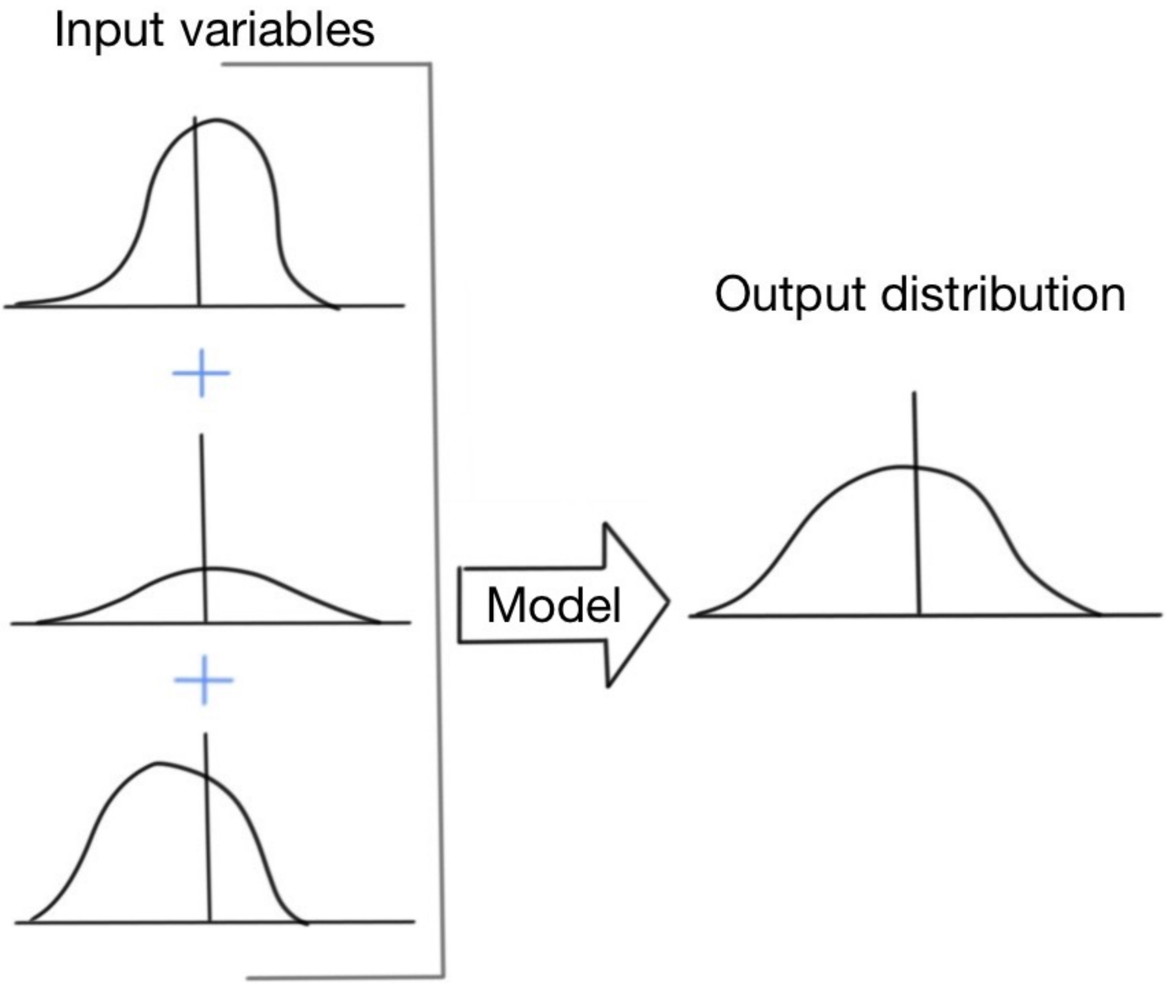

Figure 5-2 Simplification of the MCS process

5.2.1 The five steps of MCS process

The process of Monte Carlo simulations can be divided into five main steps [12]:

5.2.1.1 Define the model

First, a mathematical model has to be defined in order to relate output with the input variables. This could be a set of equations where the output is calculated using the input variables, both constants and distributions.

E.g., when calculating the casing shoe pressure where the kick is below the shoe and a single slug, Eq. (*3.1*) is needed.

Secondly, one has to decide what the output of the simulation should be. This is usually clear as this is the unknown value one wants to identify. Examples of outputs can be cost or time of a well operation, which has been done in [12]. Another output example could be to compare well pressure vs. casing strength, which has been done in [13].

Thirdly, input parameters to the model has to be decided. The inputs can include both single numbers and parameters with an uncertainty distribution.

Events can also be included as input arguments. Examples of events during drilling could be kick or wait on weather (WOW). Each event has a probability distribution attached to it, and will have a certain impact on the output.

5.2.1.2 Data gathering for approximating input arguments

Choosing the right data for estimating each input argument and the uncertainty of each input is important. The data set should include several data points such that it gives a representative sample. Also, data points which seems unlikely or out of place should be investigated, and perhaps be excluded from the data set [12].

Pore pressure is an example of an input argument which needs to be approximated when drilling a new well with no or few existing wells in close proximity. There is no way of precisely measuring the pore pressure before drilling down to the planned depth. To estimate the pore pressure before the well is drilled, data gathered from previously drilled wells in the same area or in the same field can be used.

5.2.1.3 Define input distributions

Input parameters, which have uncertainties will have an impact on the output, needs to be defined properly in order to be suitable for use in Monte Carlo simulations. The distribution shape and parameters quantifying the distribution has to be determined. Distribution shape depends on what type of input it is.

The distribution shapes determine how each input argument is distributed. Figure 5-3 shows some of the most common shapes:

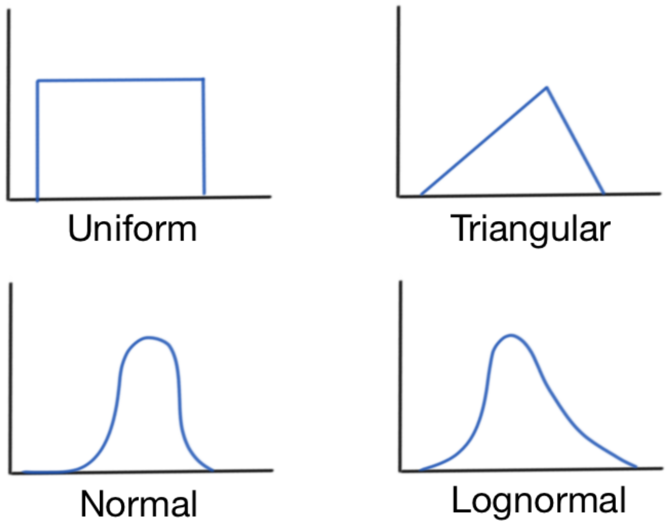

Figure 5-3 Common input distribution shapes in MCS

For cost and time estimation, the use of uniform or triangular distributions is acknowledged. Although they might appear to be overly simplified, they can be quite accurate and the best choice in some situations [14].

In addition to the distribution shape, the parameters of the distribution have to be determined. Which distribution parameters are needed depends on which type of distribution is chosen.

For a normal distribution, these parameters are mean value (μ) and its standard deviation (σ). The mean value equals to the median value for this distribution as the shape is symmetrical.

The equation for calculating the normal distribution is given in Eq. (5.1):

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{5.1}$$

Figure 5-4 shows three examples of normal distributions with varying mean value and standard deviation.

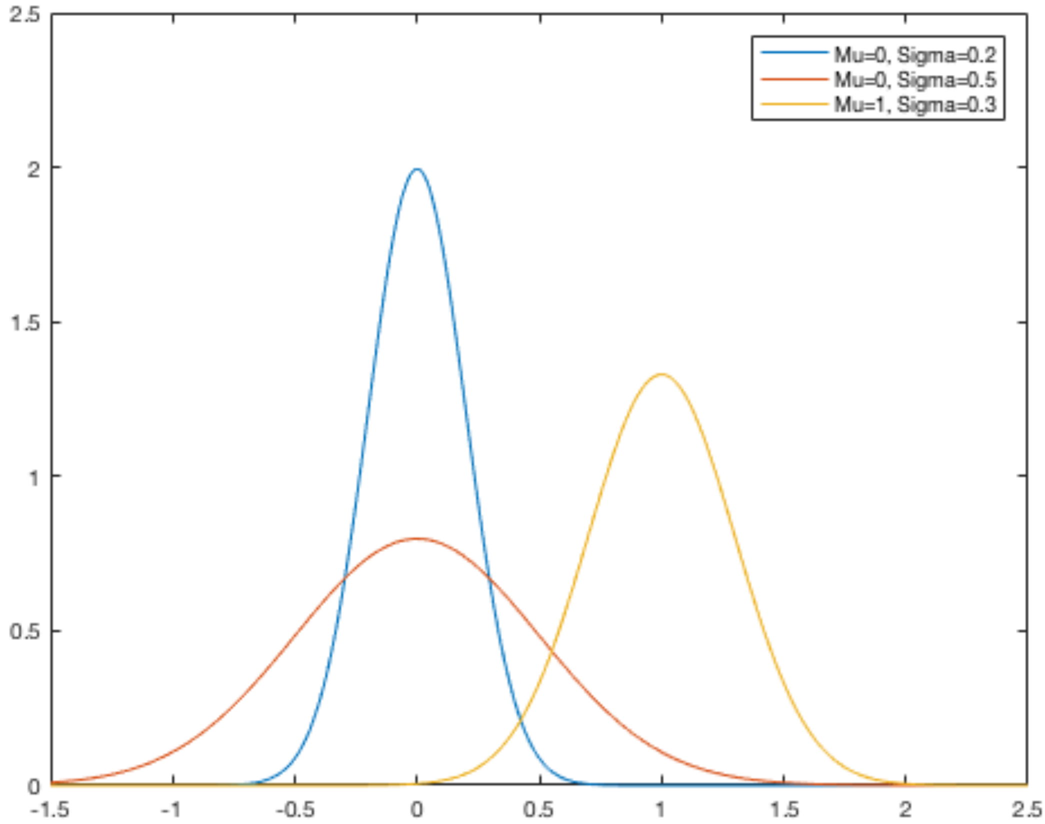

Figure 5-4 Normal distribution with varying characteristics

For a triangular distribution, the distribution parameters are minimum, maximum, and most likely value. Figure 5-5 shows an arbitrary triangular distribution, and how the shape is directly affected by its parameters. To specify the input distribution parameters properly and in accordance with available knowledge is crucial to be able to have any trust in the simulation results. In [12], Williamson et al. points out that choosing the input parameters properly is more important than choosing the right distribution shape.

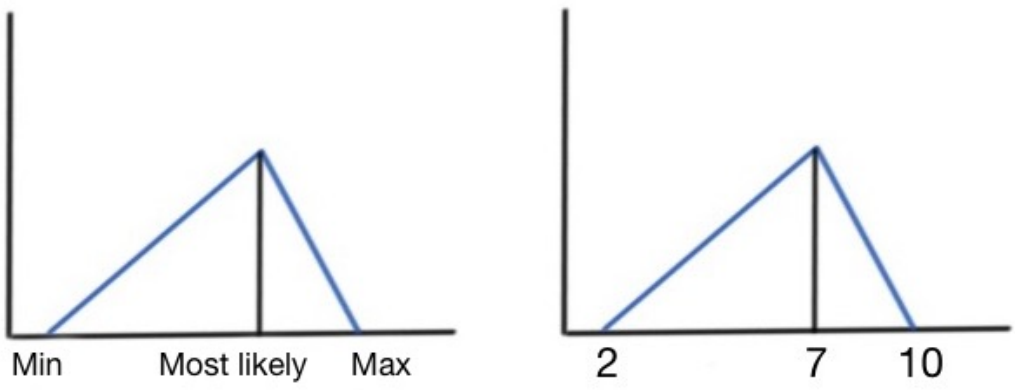

Figure 5-5 Example: Triangular distribution

5.2.1.4 Sample input distributions

Before using the input distributions in a Monte Carlo simulation, their quality should be evaluated. Numbers from the input distributions must be handled with care, as they are randomly generated numbers and can have a great impact on the final result. If input variables are correlated, this will have to be dealt with in the Monte Carlo simulation process and this will affect the results [12].

5.2.1.5 Interpret and use the results

The output of the Monte Carlo simulation will be one or more probability density functions, depending on the number of outputs the simulation was created for. The result for a single output value, like travel time in the previously used example, is a probability-distribution curve. Like the input distribution, the output distribution should also be quality checked before it is used to draw any conclusions or make any decisions [12].

One aspect is to evaluate if the number of simulations chosen is sufficient. Figure 5-6 to Figure 5-9 shows a Monte Carlo simulation with the same input values. The input distribution shape of two of the input parameters is triangular. The only parameter which is changed in the four simulations is the number of Monte Carlo simulations, termed N. As demonstrated in the four figures below, the greater number of simulations, the smoother the probability curve will be. Having a sufficient number of simulations will lead to a more stable result, which means that a repetition of the simulation will generally give the same result. How many Monte Carlo simulations is necessary, depends on how accurate the result needs to be, e.g. what failure probability can be tolerated. If a 0.1% failure probability is tolerated, a variation of e.g. 0.5% for each simulation would not be sufficient.

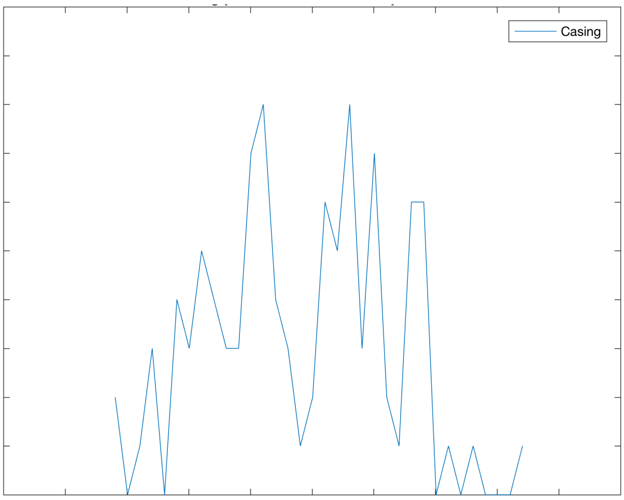

Figure 5-6 N=100

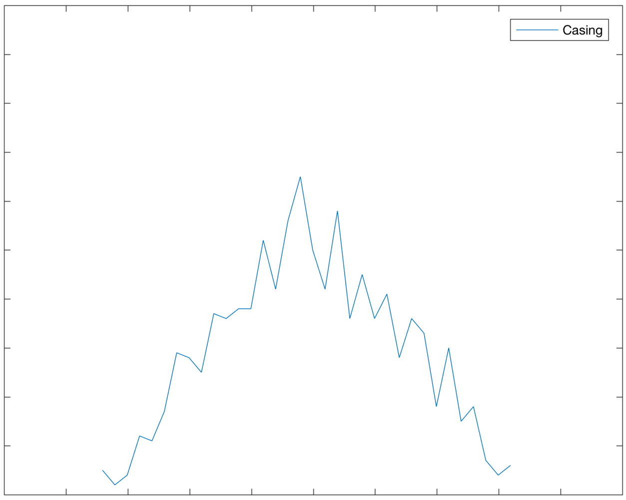

Figure 5-7 N=1,000

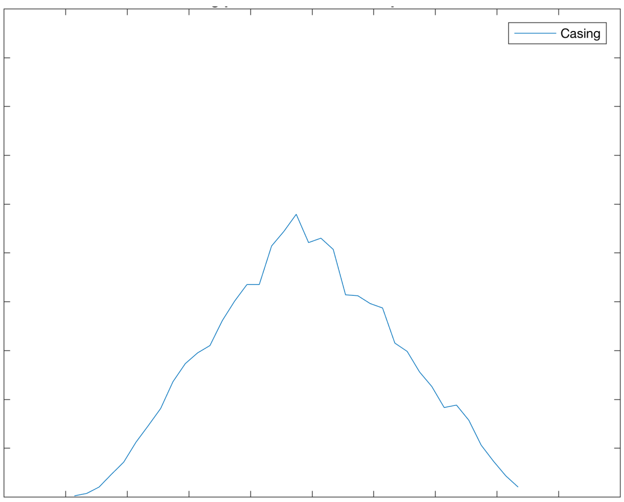

Figure 5-8 N=10,000

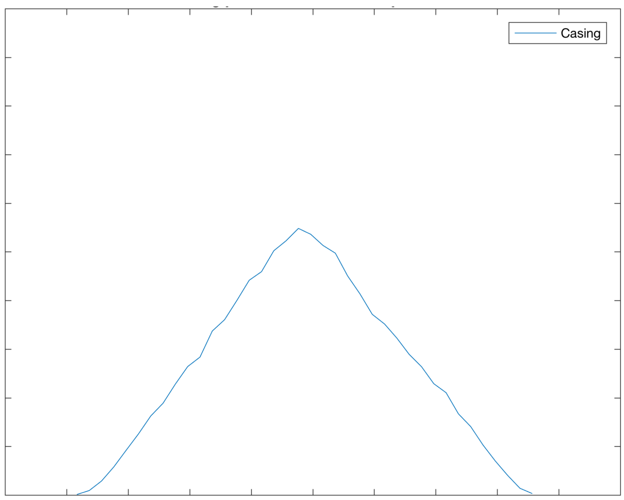

Figure 5-9 N=100,000

5.3 Applications of Monte Carlo for well engineering

Monte Carlo simulations can be used to predict several aspects of a well. It can be used to estimate cost and time of well construction, oil production rates, and also time management of projects [12].

An example of use of Monte Carlo simulations for probabilistic time and cost for P&A operations is shown in [15]. It can also be used for reliability based casing design where the loads which can occur in a well are compared with the casing strength in a probabilistic matter [1], [13].

Monte Carlo simulations can also be used to estimate the probability of a kick occurring [16]. It has also been used to give probabilistic estimates of potential blowout rates and volumes for environmental risk assessment [17].

One specific situation where Monte Carlo simulations can be useful is when examining if casing pressure will exceed the fracture pressure of the rock when drilling a new section. The risk of this happening increases if a kick occurs during drilling. To figure out if the casing can handle the kick, a Monte Carlo simulation can be done to see what the casing shoe pressure will be expressed as a probability density function. An appropriate model must be chosen for estimating the maximum casing shoe pressure that can occur. This is because there is more than one way, and more than one set of equations to calculate this pressure. The chosen model will depend on several uncertain input parameters. In addition, a probability distribution must be created for the fracture pressure. This is done by choosing a distribution shape and its parameters, as explained earlier in this chapter.

By adding the two functions to the same plot, where pressure is projected on the x-axis and probability on the y-axis, the overlapping area will represent the failure probability.

Figure 5-10, Figure 5-11, and Figure 5-12 below demonstrates this concept. All three examples are simulated with the same pore pressure input, but the parameters for the fracture pressure distribution are varied. Example 1 and 3 have the same mean value but different standard deviation, and example 1 and 2 has different mean values, but the same standard deviation. The shaded areas highlight the overlapping space, previously described as the failure probability. A significant increase in failure probability can be seen both when the mean value of the fracture pressure is decreased, and when the standard deviation is increased. It is worth noting that both the casing shoe pressure and the fracture pressure must be located in the shaded area during one realization or draw in the loop representing the Monte Carlo simulation process. Hence, the shaded area can in some sense give a false visual impression of how large the probability of failure really is. During the Monte Carlo simulation process it will be counted how many times the casing shoe pressure exceeds the fracture pressure. This number is then divided by the total number of Monte Carlo simulations and multiplied by 100 to provide a percentage value of the failure probability. It is important to ensure that a sufficient number of Monte Carlo simulations is chosen so that the failure probability doesn't vary too much when repeating the Monte Carlo simulation process.

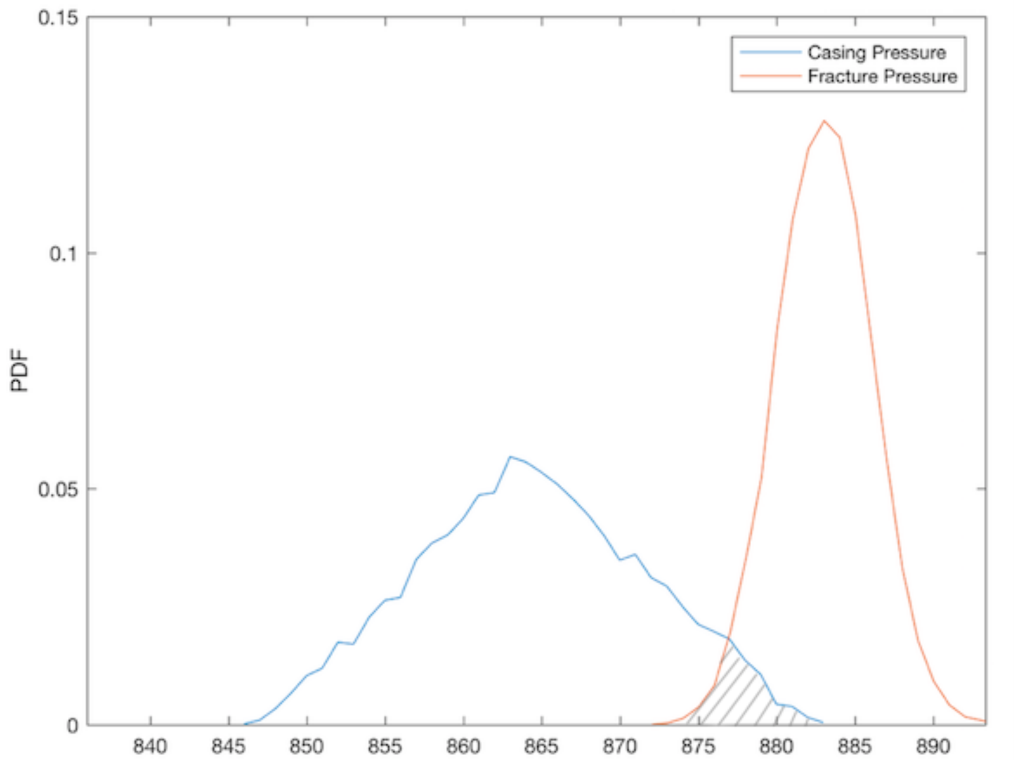

Figure 5-10 Example 1: 0.43% chance of failure

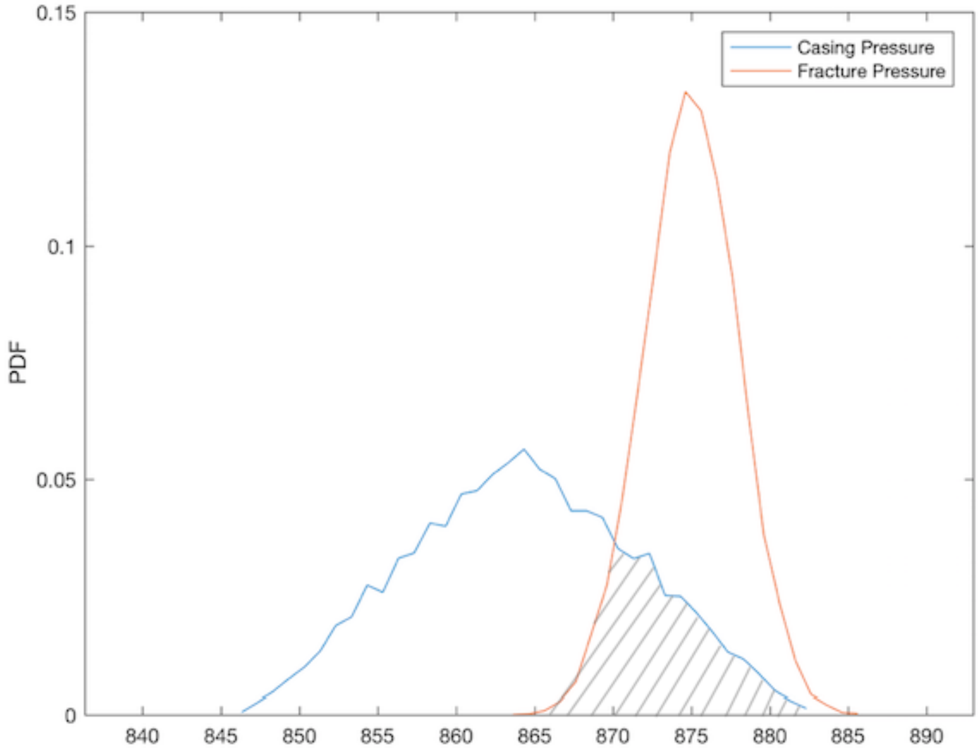

Figure 5-11 Example 2: 8.95% chance of failure

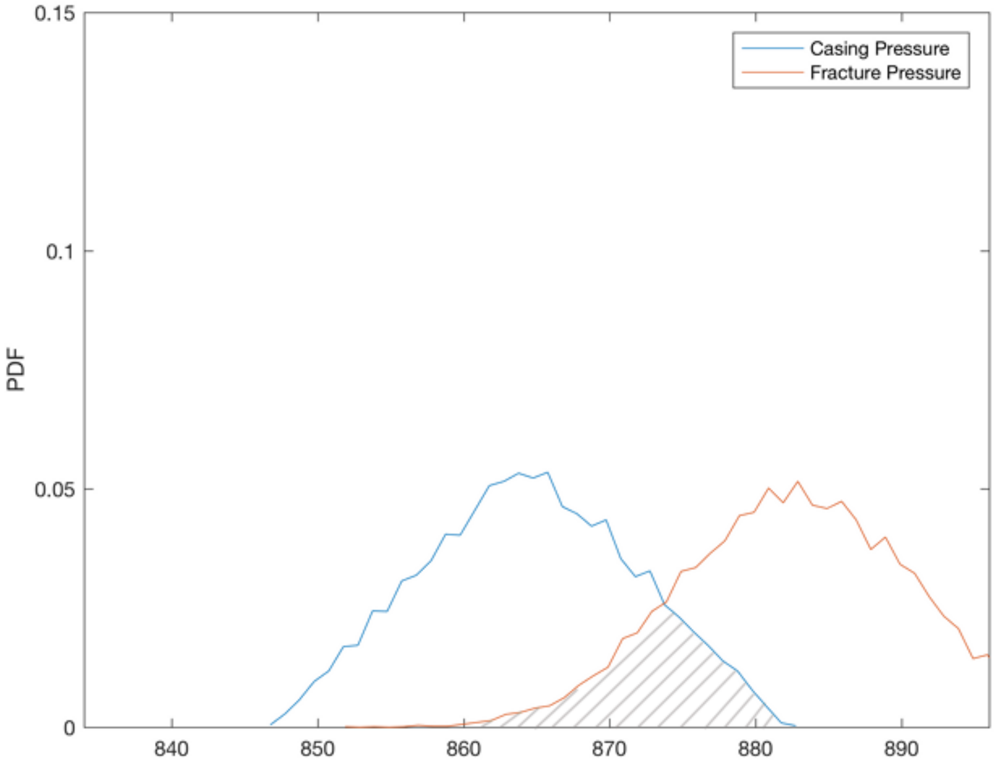

Figure 5-12 Example 3: 4.39% chance of failure

This example of application of Monte Carlo simulations for probabilistic kick tolerance evaluations has been discussed in [2]. The work to be presented in this thesis is a continuation of that work.

6 A Probabilistic Modelling Approach for Evaluating Kick Tolerances

The work done for this thesis can divided into two main parts, and is described in 6.1 and 6.2.

Chapter 6.1 uses the single bubble model to analyze how different parameters, like kick size and BHA length, affect the casing shoe pressure, and also where the kick is located when the highest casing shoe pressure occurs.

Chapter 6.2 implements Monte Carlo loops in the three models presented in chapter 4, and investigates how the well and kick parameters affect the casing shoe pressure in a probabilistic manner. The purpose of this part is to compare the three models, to see how similar the casing shoe pressures are, and how this will affect the probability of failure due to a kick.

Chapter 6.3 sums up the overall results related to the number of Monte Carlo simulations, and simulation times for the different models.

The simulation methods and the way of presenting the results is greatly inspired by the work done in [2].

6.1 Single bubble model – Identifying maximum casing shoe pressure

To perform simulations using the single bubble model, an existing script built on the model was provided by the supervisor of this thesis. The base script was mainly developed by Dalila Gomes, UiS, and then revised for the purpose of this thesis. Changes had to be made in order to make the script more robust, and to make it easier to change values of input parameters. Some additional lines were added to generate plots, and bugs related to generalizing of the script were fixed. The MATLAB code used is provided in Appendix A.1.1.

6.1.1 Purpose of simulation

The purpose of simulating with this single bubble script was to see what kick location would give the highest casing shoe pressure at the shoe. The analytical model does not take into account the gas expansion aspect of casing pressure. Therefore, the most interesting result would be to see if it was possible that gas expansion in some cases would cause the maximum pressure. The simulations with single bubble model were done with various conditions to see if any of them would in fact give maximum casing shoe pressure when the kick expands towards the shoe.

6.1.2 Method of simulation

The pore pressure is given, mud density remains constant, and friction is not accounted for. Simulating with the single bubble model was done as early as in 1968 by J. L. Leblanc and R. L.

Lewis in [18], which was one of the first times this had been done with the assumptions mentioned above.

Most parameters stayed fixed during the simulations, except these five: Shoe depth, BHA length, inner diameter (ID) of open hole and outer diameter (OD) of BHA, and lastly the kick volume. The value of each parameter is shown in Table 6-1 below.

Parameter	**Value**
Fixed parameters	
Bottom hole pressure (pore pressure + SM)	942 bar (932 + 10 bar)
Well depth	5000 m
OD drill string	5 in
Mud density	1920 kg/m^3
Mud pump rate	350 L/min
Influx time for kick	100 s
Temperature at bottom of well	373 K
Temperature at the surface	323 K
Varied parameters	
Shoe depth	3000 m/4500 m
BHA length	50 m/100 m/150 m
ID open hole & OD BHA	12.25 in & 8.5 in/8.5 in & 6.5 in
Kick volume	2 m^3/4 m^3/8 m^3/12 m^3

Table 6-1 Overview of parameters used in maximum casing shoe pressure simulations

The kick is taken during circulation, as mud pump rate is taken into account. Temperature at the top and bottom is set, and a linear temperature gradient is created from these two values. Friction is not taken into account. The single bubble assumes that the cross-sectional area at kick depth is entirely taken up by gas, so that the gas fraction is equal to one. The model is based on the use of water-based mud only, which means that the kick will not be dissolve in the mud. The kick is assumed to consist of methane gas only, which will give the worst case scenario. Appendix A.2.1. contains the code for calculating the density of the gas bubble at any time. The influx time is set to 100 seconds, which means that the kick enters the well gradually and the entire kick is in the well at time equal to 100 seconds.

6.1.3 Simulation results

The results are compounded into four tables. The four tables show results for long and short hole section, and 12.25 in open hole and 8.5 in open hole. Because the purpose of the simulation was to see which kick location that would give the highest casing shoe pressure, the results are presented with the location of the kick which gave maximum casing shoe pressure. They can be found in Table 6-2 to Table 6-5 below. The pressure values can be found in Appendix B.1.

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	BHA	BHA	Shoe	Shoe
	100m	BHA	BHA	BHA	BHA
	150m	BHA	BHA	BHA	BHA

Table 6-2 Long hole section & 12.25" open hole

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	BHA	Shoe	Shoe	Shoe
	100m	BHA	BHA	Shoe	Shoe
	150m	BHA	BHA	BHA	Shoe

Table 6-3 Long hole section & 8.5" open hole

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	BHA	BHA	BHA	BHA
	100m	BHA	BHA	BHA	BHA
	150m	BHA	BHA	BHA	BHA

Table 6-4 Short hole section & 12.25" open hole

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	BHA	BHA	BHA	*
	100m	BHA	BHA	BHA	*
	150m	BHA	BHA	BHA	*

Table 6-5 Short hole section & 8.5" open hole

Table 6-5 remark: For 12 m^3 kick, no result has been presented. This is due to the fact that the kick size is larger than the volume of open hole in the well below the casing shoe.

As can be read from the tables, there are certain situations where gas expansion will cause the highest casing shoe pressure. Short hole sections gave maximum pressure when kick is at BHA, as could be expected. In cases of long hole section, large kick and short BHA, the results indicate that gas expansion caused the highest casing shoe pressures. This indicates that it is not sufficient to assume the highest pressure always occurs when kick is located at BHA. This is especially important to note when long hole sections and large kick sizes are considered.

6.2 Comparing kick tolerance calculation models

When comparing the three models presented in chapter 4, previously written scripts were used. For the single bubble model, the revised code used in 6.1 was further revised in order to implement a Monte Carlo simulation loop. For the two other models, scripts with Monte Carlo framework already implemented were provided by the supervisor of this thesis. These two scripts have been further revised to suit the purpose of this thesis. All codes were created by Dalila Gomes and Kjell Kåre Fjelde, among others, for the purpose of work presented in OMAE2017-61391 [2]. In addition, some supporting MATLAB scripts were provided in order for the flow modelling scripts to run. All codes used in these simulations can be found in Appendix A: A.1.2 to A.1.4 and also A.2.2.

6.2.1 Implementing Monte Carlo simulations in single bubble model

In order to compare failure probabilities of single bubble, transient flow, and analytical model, the single bubble script needed adaptation. A large portion of the script needed to be wrapped in a Monte Carlo loop to be able to calculate the casing shoe pressure and fracture pressure a set number of times. In addition, several lines needed to be added to the script e.g. new vectors for saving pressure values for each Monte Carlo loop and a new plot.

The input values to be varied for each Monte Carlo loop were set to be the pore pressure at the bottom of the well, the safety margin and the fracture pressure at the shoe. The first two had a triangular distribution, while the fracture pressure had a normal distribution.

6.2.2 Purpose of simulation

The purpose of these simulations was to compare the different flow models to see if, and how much, they differ with regards to casing shoe pressure. A Monte Carlo loop is implemented into each of the scripts containing the models. The reason for using Monte Carlo to simulate, is to take into account the uncertainty in the fracture pressure at the shoe, the pore pressure at the bottom, and also the safety margin due to choke operation irregularity.

6.2.3 Method of simulation

These simulations were carried out in a way similar to what was done in 6.1. They were performed for two main cases: A long hole section and a short hole section. All simulations were done for the 13 3/8" casing shoe and 12 1/4" hole size. In addition, three different BHA lengths and four different kick sizes was considered.

Table 6-6 and Table 6-7 below give an overview of many of the fixed and the varying parameters used in the simulations.

Parameter	**Value**
Pore pressure	T(915,932,950) bar
Safety margin	T(8,10,12) bar
Well depth	5000 m
Mud density	1920 kg/m^3
ID open hole	12.25 in
OD BHA	8.5 in
OD drill string	5 in

Table 6-6 Fixed parameters for all three flow models

Fixed parameters						
	Long hole section			Short hole section		
	Single bubble	Transient flow	Analytical	Single bubble	Transient flow	Analytical
Fracture pressure	N(603,3)	N(603,3)	N(603,3)	N(883,3)	N(883,3)	N(883,3)
Gas fraction	N/A	N/A	T(0.3,0.5, 0.7)	N/A	N/A	T(0.3,0.5, 0.7)
Mud pump rate	350 L/min	10 kg/s	N/A	350 L/min	10 kg/s	N/A
Influx time for kick	100 s	100 s	N/A	100 s	100 s	N/A
Temperature at surface	323 K	323 K	N/A	323 K	323 K	N/A
Temperature at bottom	373 K	373 K	N/A	373 K	373 K	N/A
Number of Monte Carlo simulations	100,000	100,000	1,000,000	100,000	100,000	1,000,000
Variable parameters						
	Long hole section			**Short hole section**		
	Single bubble	**Transient flow**	**Analytical**	**Single bubble**	**Transient flow**	**Analytical**
Shoe depth	3000 m	3000 m	3000 m	4500 m	4500 m	4500 m
BHA length	50/100/ 150 m	50/100/ 150 m	50/100/ 150 m	50/100/ 150 m	50/100/ 150 m	50/100/ 150 m
Kick volume	2/4/8/12 m^3	2/4/8/12 m^3	2/4/8/12 m^3	2/4/8/12 m^3	2/4/8/12 m^3	2/4/8/12 m^3

Table 6-7 Varying parameters for all three flow models

6.2.4 Simulation results

As a way to compare the models, the failure probability in percent is used. These values will not have any use in themselves as they greatly depend on how fracture pressure distribution is chosen. However, they can be used to investigate the impact changing BHA length, kick size and hole geometry has, with regards to the different models.

The results can be found in Table 6-8 to Table 6-13 below, expressed by the failure probabilities in percent.

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	0.000	0.007	4.192	41.271
	100m	0.000	0.085	8.214	44.498
	150m	0.000	0.088	15.843	59.258

Table 6-8 Single bubble with long hole section

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	0.000	0.056	7.209	41.938
	100m	0.000	0.434	14.661	56.741
	150m	0.000	0.457	24.305	70.833

Table 6-9 Single bubble with short hole section

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	0.000	0.000	0.005	2.510
	100m	0.000	0.000	0.013	3.655
	150m	0.000	0.000	0.048	5.993

Table 6-10 Transient flow model with long hole section

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	0.000	0.001	1.316	34.297
	100m	0.000	0.005	4.693	40.354
	150m	0.000	0.009	6.065	48.153

Table 6-11 Transient flow model with short hole section

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	0.0000	0.0007	1.893	24.460
	100m	0.0000	0.0059	3.338	30.604
	150m	0.0000	0.0276	5.466	37.328

Table 6-12 Analytical model with long hole section

		Kick size			
		2 m^3	4 m^3	8 m^3	12 m^3
BHA length	50m	0.0000	0.017	4.685	33.971
	100m	0.0003	0.063	7.255	40.442
	150m	0.0004	0.190	10.481	46.722

Table 6-13 Analytical model with short hole section

Over all, the transient flow model gives lowest failure probability, unlike the single bubble model which gives the highest probability. This can be observed for all cases simulated here, and suggests that the model chosen for simulating will have a great impact on the failure probability result.

It can be observed that an increase in the kick volume has a greater impact on the failure probability than an increase in BHA length increase has. Nevertheless, the BHA length is not negligible, and greatly affects the results.

Mean casing shoe pressure values for all three models are presented in Figure 6-1 to Figure 6-4. The way of presenting these numbers are inspired by Figure 30-31 in [2].

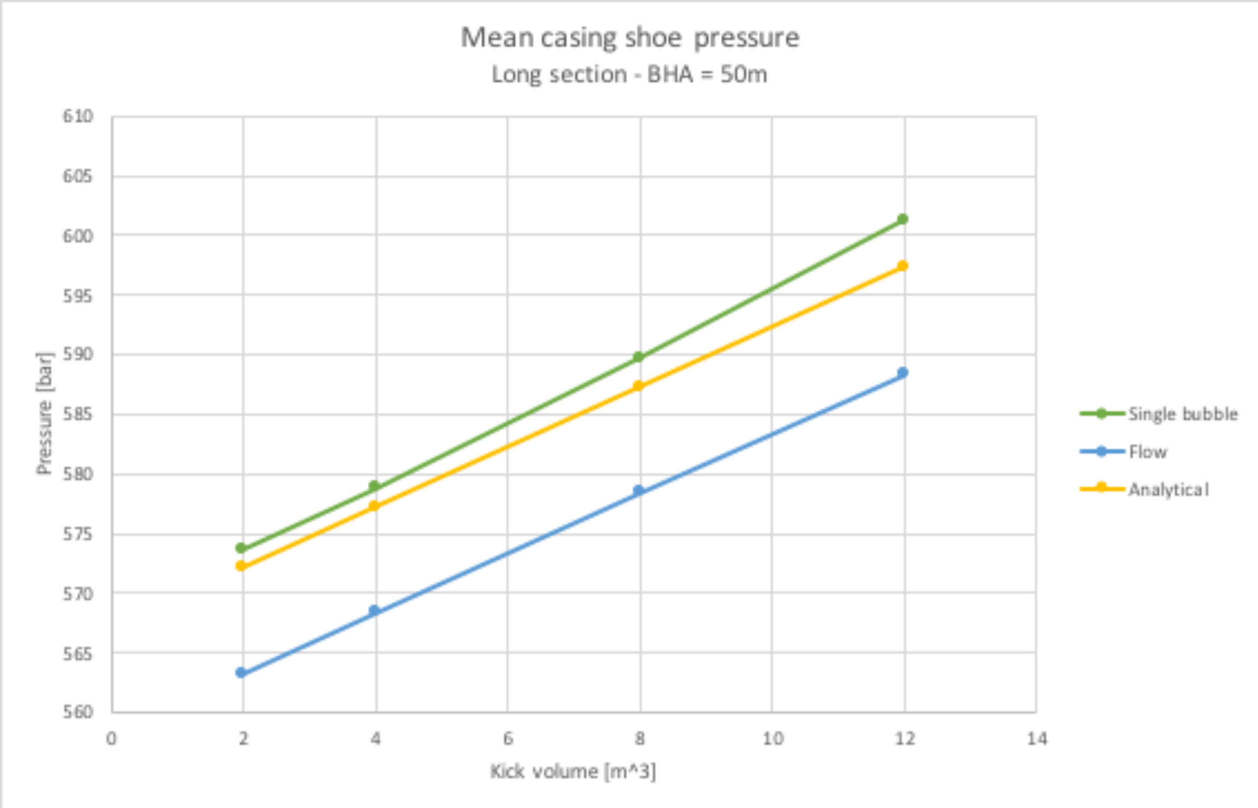

Figure 6-1 Mean values of maximum casing shoe pressure: Long hole section & 50 m BHA

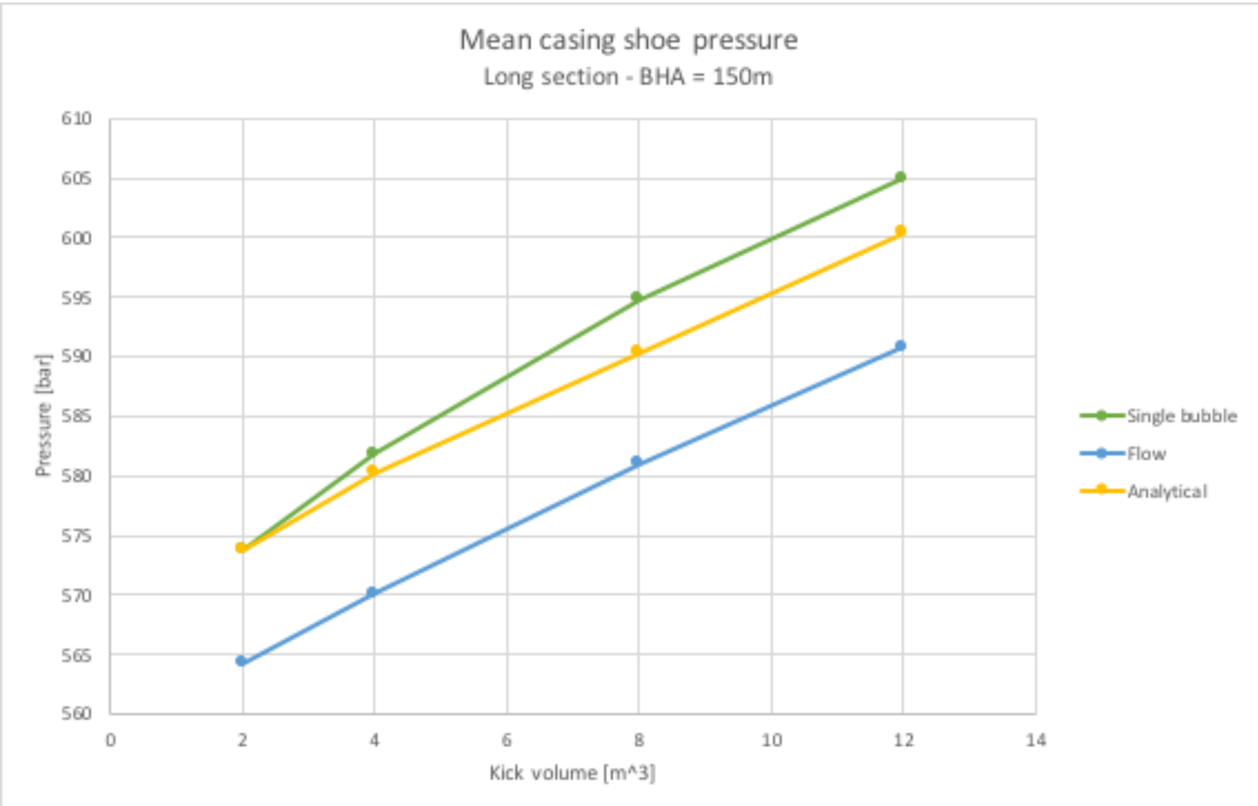

Figure 6-2 Mean values of maximum casing shoe pressure: Long hole section & 150 m BHA

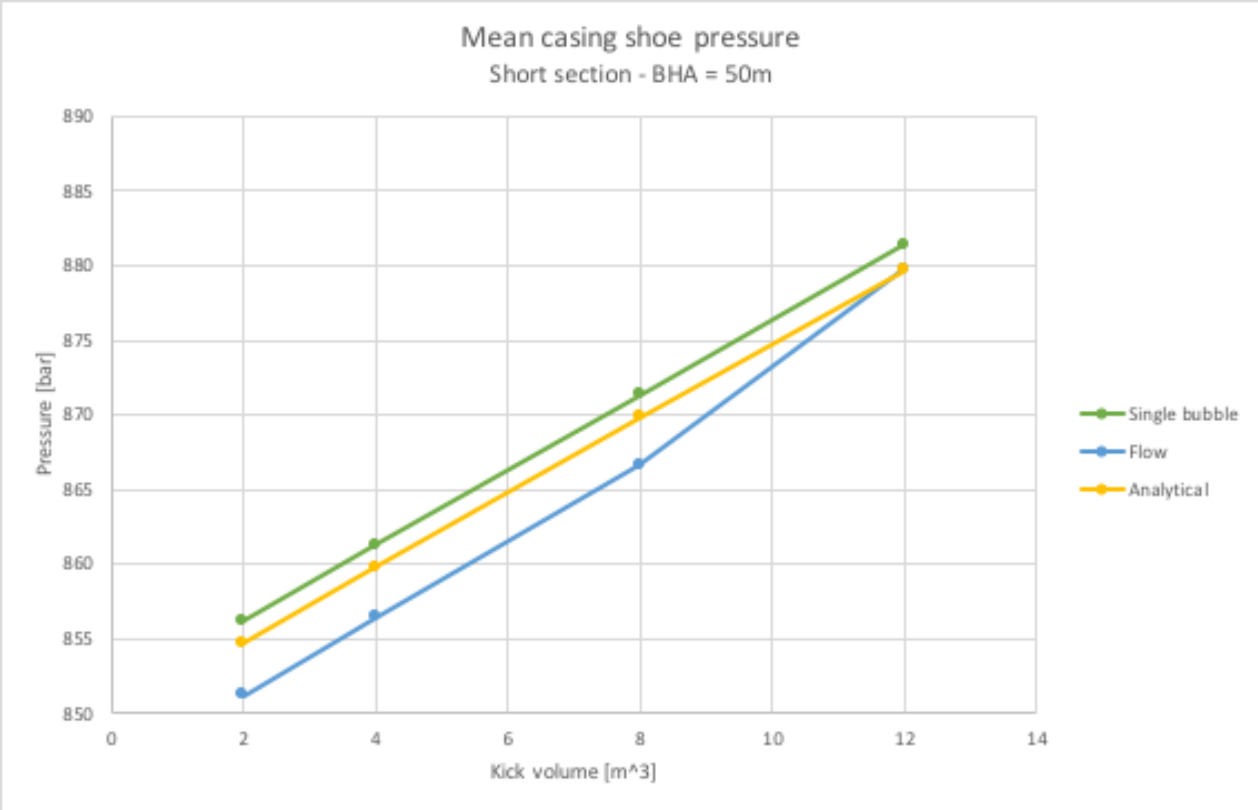

Figure 6-3 Mean values of maximum casing shoe pressure: Short hole section & 50 m BHA

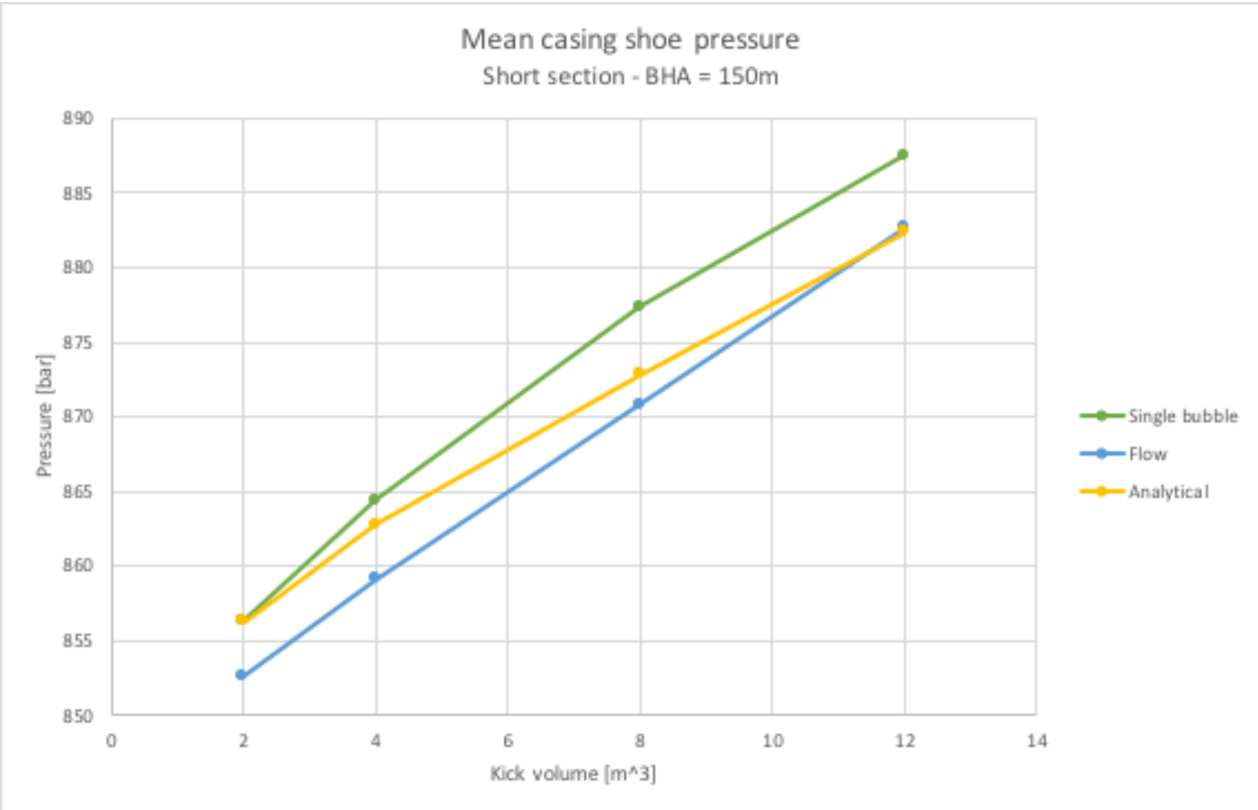

Figure 6-4 Mean values of maximum casing shoe pressure: Short hole section & 150 m BHA

When considering the long hole section, Figure 6-1 and Figure 6-2, it is apparent that the transient flow model is less conservative than the other two models. The analytical and the single bubble model are quite similar when the kick volume is small, and the difference between them increases slightly as the kick size is increased. The reason that the analytical starts to deviate and give lower pressure when the kick size increases, is the gas distribution parameter will make the kick enter the region above BHA and the kick is shortened.

When the short hole section is considered, Figure 6-3 and Figure 6-4, the difference between the models is smaller than for the long section. This could be due to the fact that a long hole section is an exaggerated case of well design, and would emphasize the differences in the model assumptions more than the short hole section will do. For instance, the gas will be allowed to expand more before reaching the shoe and gas slippage in the transient model will also have more time to distribute in the kick.

6.2.5 Comparing single bubble and transient flow

From the results of simulating with the single bubble model in 6.1, it could be observed that gas expansion caused the largest casing shoe pressure in the case of long 12.25” hole section, 50m BHA and both $8m^3$ and $12m^3$ kick. To compare the transient flow model with the single bubble model, the casing shoe pressure can be looked at to see if they would behave similarly when it comes to when the maximum casing shoe pressure would occur. To do this, the transient flow model was used to calculate the casing shoe pressure under the same conditions, with no Monte Carlo loop and with a constant BHP. The results can be found in Table 6-14 below.

Model	Maximum P_{shoe}, kick at BHA [bar]	Maximum P_{shoe}, due to kick expansion [bar]
8 m^3 kick		
Single bubble	588.5	589.3
Transient flow	579.5	579.6
12 m^3 kick		
Single bubble	598.5	601.0
Transient flow	590.0	590.3

Table 6-14 Maximum casing shoe pressures compared

As can be read from this table, they both give maximum casing shoe pressure as the gas travels upwards and expands. In the transient flow model, the pressure difference is quite small, whilst it is more apparent in the single bubble result. However, in both cases, there are minor differences.

6.3 Monte Carlo findings

An important part of simulating with a Monte Carlo framework is to investigate how many Monte Carlo simulations is needed to get a reliable and stable result. The larger number of Monte Carlo simulations, the more trustworthy the result would be. However, the Central Processing Unit (CPU) time would also increase along with the number of Monte Carlo loops, and possibly restrict how large this number could be. E.g. simulating with the scripts provided in Appendix A.1, the CPU time would increase by a factor of ten if the N was increased by a factor of ten.

Because there currently are no standards which recommends what N should be set to with regards to kick tolerance calculations, simulations with various values for N was performed in order to see how the results would vary for each simulation.

Simulations were performed five times for each case, and the failure probabilities in fraction were noted. To express variations in the simulations, the difference between the largest probability and smallest probability was used. This difference and the CPU times for the simulations can be found in Table 6-15 below.

	N = 1,000,000		N = 100,000		N = 10,000	
Model	**Difference in probability**	**CPU time [s]**	**Difference in probability**	**CPU time [s]**	**Difference in probability**	**CPU time [s]**
Single bubble	N/A	N/A	$1.8*10^{-3}$	345.116	$6*10^{-3}$	73.789
Analytical	$3.7*10^{-4}$	65.646	$7.2*10^{-4}$	6.646	$3.6*10^{-3}$	0.724
Transient flow	N/A	N/A	$3*10^{-5}$	334.256	$1*10^{-4}$	49.513

Table 6-15 Change in failure probability and CPU times for various number of MCS

The transient flow model has the smallest variations in failure probability for each N. Therefore this model appears to be the most stable in regards to the number of Monte Carlo simulations used. At 100,000 simulations, the difference is only 0.00003, which means that the kick tolerance for this case can be set as low as 0.01% in order to keep the variations smaller than the tolerance requirement. To make the model even more reliable, the number of Monte Carlo simulations can be increased, but in order to do this, the model needs to be improved with respect to how to optimize the calculations.

As the transient models has substantial CPU times compared to the analytical, it is challenging to increase N without making any changes to the scripts. E.g., if the transient flow model would be further revised it might be possible to shorten the CPU time, and consequently making it possible to increase N and the stability of the results.

7 Conclusion

7.1 Single bubble simulations

When looking at the single bubble model to see where the kick is located when maximum casing shoe pressure occurs, the results are what to be expected. When simulating with a relatively short open hole section, the maximum pressure occurs when the kick is located at the BHA. However, when simulating with a very long hole section, a large kick size and a short BHA, the casing shoe pressure due to kick expansion is slightly higher than that of kick located at the BHA. Therefore, it is not unreasonable to neglect the gas expansion, as the analytical model does, as it only impacts the results in the exaggerated cases.
From this, a proposed working method is: first, use a transient kick simulator to calculate the maximum casing shoe pressure for the well, and identify the location of the kick as it occurs. If the kick is located at the BHA, a non-transient model can be used to simulate. But, if maximum pressure is caused by gas expansion, a transient model should be used to account for the expansion effect.

7.2 Comparing the three flow models

As a trend from all simulations, the single bubble model can be considered conservative compared to the other two. This could be expected, as the single bubble model assumes a gas fraction of one, which in itself is considered to be a conservative approach.

The transient flow model gives the least conservative results. With an exception of two specific cases, it gave the lowest failure probabilities compared to the other two models.

The analytical model gives results closer to the transient flow model than the single bubble does, due to the fact that it has a lower gas fraction. The gas fraction can be assumed to be a major component to calculating the kick tolerance.

When looking at results from the short hole section with the 12 m^3 kick, Figure 6-3 and Figure 6-4, it can be observed that the analytical and the transient flow model will approach one another. This can be explained by the fact that a large kick gives a larger gas fraction in the transient model than a smaller kick would.

From the analytical model it can be seen that by increasing the BHA length, the failure probability will also increase. This can be directly observed due to the fact that the analytical model is static.

It is apparent that the effect of increasing the kick size by has a much larger effect than increasing the BHA length with regards to the failure probabilities calculated in 6.2.

When calculating failure probability, choosing an appropriate distribution for the fracture pressure is crucial. As can be seen in Figure 5-10 to Figure 5-12, a slight change in the mean value or the standard deviation has a big impact on the failure probability.

7.3 Monte Carlo simulations

When using this probabilistic method to estimate kick tolerance, the tolerance needs to be quantified. As a main principle, the tolerance cannot be smaller than the variation in the failure fraction. If this is not taken into account, the simulation results are not stable enough to be trusted. In that case, the number of Monte Carlo simulations needs to be increased.

The transient flow model appears to be most stable when it comes to the tolerance. At 100,000 Monte Carlo simulations, the difference between highest and lowest failure rate was $3*10^{-5}$. This should be sufficient in order to set the tolerance at 10^{-4}. When looking at reliability based casing design, the recommended tolerance is 10^{-6} to 10^{-5} for high consequence incidents, and 10^{-3} to 10^{-2} for low consequence incidents [1, p. 12]. So to set the kick tolerance limit, the consequence level of a kick has to be evaluated.

For comparison, the blowout risk probability is usually considered to be around 10^{-4} [19], [20].

My recommendation for the number of Monte Carlo simulations when using the scripts provided in Appendix A, would be to have at least 100,000 for the single bubble and the transient flow model, and at least 1,000,000 for the analytical.

7.4 Recommendations for future work

As an additional investigation as to see how good the models are, a sensitivity analysis could be performed for all three models in order to see which parameters has the biggest impact on the result. This type of analysis has been done for the analytical model in [2, p. 12].

To further look into the importance of the gas concentration, an analysis could be done with the analytical model by varying the gas fraction to see its effect on the results.

A further comparison between probabilistic calculation of kick tolerances and reliability based casing design could be done. A standard has been made for reliability based design [21], and before probabilistic calculations of kick tolerances can be put into use a standard or guide should be made. This should include recommendation as to how to set the accepted tolerance.

As a continuation of this work, Monte Carlo simulations could be implemented for flow models made for oil-based mud with kinematics included.

As can be observed from Table 6-15, the CPU times for the transient models are much larger than that of the analytical model. Because the analytical model does not include the gas expansion, the transient models have the biggest potential to describe kick tolerances properly. To further improve the codes, the CPU times can be reduced, and consequently making them more useable and attractive.

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Appendix A

A.1 Flow models

A.1.1 Single bubble model

```
% Transient single bubble model
% Used to simulate pressure at casing shoe when a kick is taken in a
well
% Main author Rev7: Dalila Gomes, Research Fellow, University of
Stavanger
% Revised by: Martine Kristoffersen, MSc student, University of
Stavanger, 2020

clear
clc

%% Adding constants, and setting start conditions
in_to_m = 0.0254; % Conversion factor, inches to meters
Pa_to_bar = 0.00001; % Conversion factor, pascal to bar
L_min_to_m3_second = (1/(60*1000)); % Conversion factor, L/min to m^3/s

Pbh = 94200000; %(pore pressure + safety margin); % Bottom hole
pressure [Pa]
well_depth = 5000; % Total well depth [m]
shoe_depth = 4500; % Depth of the casing shoe [m]
BHA_depth = 4850; % Depth of the bottom hole assembly [m]
BHA_length = well_depth-BHA_depth; % Length of BHA [m]
V_kick_bottom = 4; % Total kick volume at bottom [m^3]
endtime = 60000; % Total simulation time [s]
dt = 5; % Time step [s]
no_steps = endtime/dt; % Number of time steps
influx_time = 100; % Time for kick to entirely enter well [s]
g = 9.81; % Gravitational constant [m/s^2]
d_DS = 5*in_to_m; % Outer diameter of drill string [m]
d_BHA = 8.5*in_to_m; % Outer diameter of BHA [m]
d_open_hole = 12.25*in_to_m; % Diameter of hole below 13 3/8" csg shoe
[m]
do = d_open_hole; % Outside diameter below shoe, initially [m]
di = d_BHA; % Initially, this is the inner diameter, due to the BHA [m]

A_BHA = pi/4*(d_open_hole^2-d_BHA^2); % Area outside BHA
V_BHA = A_BHA*BHA_length; % Volume outside BHA
```

```matlab
A_DS = pi/4*(d_open_hole^2-d_DS^2); % Area outside drill string

Tbh = 373; % Temperature at bottom [Kelvin]
Tsur = 323; % Temperature at surface [Kelvin]
Tgrad = (Tbh-Tsur)/well_depth; % Temperature gradient in [K/m]
T(1) = Tbh; % Temperature at time=0 [Kelvin]
romud = 1920; % Mud density [Kg/m^3]
mudrate = 315; % Mud pump rate (kill rate) [L/min]

topkick = 0; % Start value of topkick
time = 0; % Start time [s]
timeplot(1) = time; % Add time=0 to time vector used to plot pressures
[s]
rog(1) = rogas(Pbh,Tbh); % Gas density [Kg/m3] at time=0 (methane kick
considered). Use rogas.m function to calculate density of gas.
Pchoke(1) = Pbh-romud*g*well_depth; % SICP at time=0 [Pa]
Pcas(1) = Pbh-romud*g*(well_depth-shoe_depth); % Pressure at the casing
shoe at time=0 [Pa]

Kick_EndePos(1) = well_depth; % Bottom position of kick at time=0
Kick_TopPos(1) = well_depth; % Top position of kick at time=0
Kick_Height(1) = 0; % Height of kick at time=0
V_Kick_Values(1) = 0; % Kick volume at time=0

gg = 0.554; % Gas gravity of methane, this gives worst case scenario
Tb = Tbh;
zbhp = zfactor(gg,Pbh,Tbh); % Use zfactor.m function to calculate
zfactor at bottom

%% for-loop to calculate both pressures at each timestep dt
for i = 1:no_steps
    time = time+dt; % Time in seconds. Increases by dt for each
iteration.
    timeplot(i+1) = time; % Used to add each time step to time vector
used in plots.

    % 1a) Kick around BHA only.
    if (time<=influx_time)

        A = A_BHA;
        V_kick = V_kick_bottom*time/influx_time; % Kick volume (ramping
up until entire kick is in the well)
        h_kick = V_kick/A; % Kick height
```

```
        if h_kick>BHA_length
            Vexcess =  V_kick-V_BHA;
            h_kick = BHA_length + Vexcess/A_DS;
        end


        h_underkick = 0; % Height of mud under the kick.
        h_abovekick = well_depth-h_kick-h_underkick; % Height of mud
above the kick
        Pb = Pbh;
        Tb = Tbh;
        topkick = h_kick; % Distance from bottom of well to top of
kick.


    else
        if (topkick<BHA_length) % In this case, the kick is only around
the BHA
            h_underkick = (mudrate*L_min_to_m3_second)*(time-
influx_time)/A; % Height of mud under the kick in m
            Pb = Pbh-romud*g*h_underkick; % Pressure at the bubble
(intersection)
            Tb = Tbh-Tgrad*h_underkick;
            zb = zfactor(gg,Pb,Tb); % Compressibility factor at the
bubble
            V_kick = (Pbh*V_kick_bottom*zb*Tb)/(zbhp*Tbh*Pb); %
Updating volume (real gas)
            h_kick = V_kick/A; % Updating the kick height
            h_abovekick = well_depth-h_kick-h_underkick;%height of mud
above the kick
            topkick = h_underkick+h_kick;


     % 1b) Kick in transition zone between BHA/DS and also at DS only
        elseif (topkick>BHA_length) %in this case, or is the transition
zone, or only around the drill string
                if (h_underkick>BHA_length) %in this case, the kick is
only around the drill string
                    A=pi/4*(d_open_hole^2-d_DS^2); %flow area
                %   hunderkick=(mudrate/60/1000)*(time-100)/A;% height
of mud under the kick in m

h_underkick=BHA_length+(mudrate*L_min_to_m3_second)*(time-
time_kickatBHA)/A;
                    if (h_underkick>well_depth)
                       h_underkick=well_depth;
                    end
```

```
                    Pb=Pbh-romud*g*h_underkick; % Pressure at the
bubble (intersection)
                    Tb = Tbh-Tgrad*h_underkick;
                    zb=zfactor(gg,Pb,Tb); % Compressibility factor at
the bubble
                    V_kick=(Pbh*V_kick_bottom*zb*Tb)/(zbhp*Tbh*Pb);%
Updating volume (real gas)
                    if V_kick <=0
                        V_kick = 0;
                    end
                    h_kick=V_kick/A; % Updating the kick height
                    if h_kick < 0
                        h_kick = 0;
                    end
                    h_abovekick=well_depth-h_kick-h_underkick; % Height
of mud above the kick
                       if  h_abovekick<0
                           h_abovekick=0;
                           h_kick=well_depth-h_underkick;
                       end
                       if h_kick<0
                               h_kick=0; % Here there was an error
                       end

                    topkick=h_underkick+h_kick;
                    if topkick<0 || topkick>= well_depth
                        topkick = well_depth;
                    end
                else %in this case (hunderkick<BHA_length), the kick is
in the transition zone = part around the BHA and part around the drill
string
                    h_underkick=(mudrate*L_min_to_m3_second)*(time-
influx_time)/A_BHA; % h under the kick (this is around BHA)
                    Pb=Pbh-romud*g*h_underkick; % Pressure at the
bubble (intersection)
                    Tb = Tbh-Tgrad*h_underkick; % Temperature at the
bubble (intersection)
                    zb=zfactor(gg,Pb,Tb); % Compressibility factor at
the bubble
                    V_kick=(Pbh*V_kick_bottom*zb*Tb)/(zbhp*Tbh*Pb);%
Updating volume (real gas)
                    h_kick_BHA=(well_depth-BHA_depth)-h_underkick;
                    V_BHA=A_BHA*h_kick_BHA;
                    V_DS=V_kick-V_BHA;
                    h_DS =V_DS/A_DS;
```

```
                    topkick=(well_depth-BHA_depth)+h_DS;

                    h_kick=h_kick_BHA+h_DS;
                    h_abovekick = well_depth-h_kick-h_underkick;
                    time_kickatBHA = time;
                end
        end
    end

    T(i+1) = T(i)-Tgrad*mudrate*L_min_to_m3_second*dt/A; % New
temperature at the bubble
    rog(i+1) = rogas(Pb,Tb); % New density of the gas (bubble)
    Pchoke(i+1) = Pbh-((h_abovekick+h_underkick)*g*romud)-
(h_kick*g*rog(i+1)); % Updating Pchoke

    h_kick_BHA = h_underkick+h_kick;

    if h_kick_BHA < 0 % Fixes bug for when kick reaches top of well
        h_kick_BHA = well_depth;
    end
    l_openhole = well_depth-shoe_depth;

    % 2a) Kick below or at casing shoe
    if (h_kick_BHA<l_openhole)
        Pcas(i+1) = Pbh-(h_abovekick-shoe_depth)*g*romud-
h_underkick*g*romud-h_kick*g*rog(i+1); % Pcas when the kick is below
(or at) the casing shoe
    else

    % 2b) Kick passing the shoe
        if (h_underkick<l_openhole)
            h_kick1 = l_openhole-h_underkick;
            Pcas(i+1) = Pbh-h_underkick*g*romud-h_kick1*g*rog(i+1); %
Kick is passing the shoe

    % 2c) Kick entirely above shoe
        else
        Pcas(i+1) = Pbh-romud*g*(well_depth-shoe_depth); % Pcas after
the kick passed the shoe
        end

    end
    Kick_EndePos(i+1) = well_depth-h_underkick;
```

```
    TopPos = well_depth-topkick;
    if h_kick == 0 % To fix bug which would decrease top position of
kick after top of kick has reached top of well
        TopPos = 0;
    end
    Kick_TopPos(i+1) = TopPos;
    Kick_Height(i+1) = h_kick;
    V_Kick_Values(i+1) = V_kick;

end

%% Converting pressure unit
Pchoke = Pchoke*Pa_to_bar; % Change pressure unit from Pa to bar
Pcas = Pcas*Pa_to_bar; % Change pressure unit from Pa to bar
maximum_Pcas = max(Pcas); % Finds maximum value of Pcas
i_max_Pcas = find(Pcas==maximum_Pcas); % Finds which time iteration
gives max Pcas
time_max_Pcas = i_max_Pcas*dt-5; % Calculates at what time we have max
Pcas

%% Plot choke and casing pressure
figure('Name','Pchoke')
plot(timeplot,Pchoke) % Plots choke pressure vs. timesteps
title('Choke Pressure')
xlabel('Time [s]')
ylabel('Pressure [bar]')

figure('Name','Pcas')
plot(timeplot,Pcas) % Plots pressure at casing shoe vs. timesteps
title('Pressure at casing shoe')
xlabel('Time [s]')
ylabel('Pressure [bar]')

%% Other plots
figure('Name','Top and bottom position of kick')
plot(timeplot,Kick_TopPos,'red')
hold on
plot(timeplot,Kick_EndePos, 'blue')
title('Top and bottom position of kick')
xlabel('Time [s]')
ylabel('Height ]m]')
```

```
set(gca,'Ydir','reverse')
hold off
legend({'Top Position','End Position'},'Location','northwest')

figure('Name','Kick Height')
plot(timeplot,Kick_Height)
title('Kick Height')
xlabel('Time [s]')
ylabel('Kick Height [m]')

figure('Name','Gas volume')
plot(timeplot,V_Kick_Values)
title('Volume of kick (zoomed)')
xlabel('Time [s]')
ylabel('Gas volume [m^3]')
axis([0 60000 0 80])
```

A.1.2 Single bubble model – Monte Carlo loop implemented

```
% Transient single bubble model
% Used to simulate pressure at casing shoe when a kick is taken in a
well.
% Monte Carlo Simulations are used to provide values for bottom hole
% pressure.
%
% Author Rev7: Dalila Gomes, Research Fellow, University of Stavanger
% Revised by: Martine Kristoffersen, MSc student, University of
Stavanger,
% 2020

clear
clc
close all

t = cputime;
tic,

%% Constants, and setting start conditions
in_to_m = 0.0254; % Conversion factor, Inches to meters
Pa_to_bar = 0.00001; % Conversion factor, Pascal to bar
L_min_to_m3_second = (1/(60*1000)); % Conversion factor, L/min to m^3/s

well_depth = 5000; % Total well depth [m]
shoe_depth = 4500; % Depth of the casing shoe [m]
BHA_depth = 4950; % Depth of the bottom hole assembly [m]
BHA_length = well_depth-BHA_depth; % Length of BHA [m]
V_kick_bottom = 2; % Total kick volume at bottom [m^3]
endtime = 6000; % Total simulation time [s]
dt = 10; % Time step [s]
no_steps = endtime/dt; % Number of time steps
influx_time = 100; % Time for kick to entirely enter well [s]
g = 9.81; % Gravitational constant [m/s^2]
d_DS = 5*in_to_m; % Outer diameter of drill string [m]
d_BHA = 8.5*in_to_m; % Outer diameter of BHA [m]
d_open_hole = 12.25*in_to_m; % Diameter of hole below 13 3/8" csg shoe
[m]
do = d_open_hole; % Outside diameter below shoe, initially [m]
di = d_BHA; % Initially, this is the inner diameter, due to the BHA [m]

A_BHA = pi/4*(d_open_hole^2-d_BHA^2); % Area outside BHA
```

```
V_BHA = A_BHA*BHA_length; % Volume outside BHA
A_DS = pi/4*(d_open_hole^2-d_DS^2); % Area outside drill string

Tbh = 373; % Temperature at bottom [Kelvin]
Tsur = 323; % Temperature at surface [Kelvin]
Tgrad = (Tbh-Tsur)/well_depth; % Temperature gradient in [K/m]
T(1) = Tbh; % Temperature at time=0 [Kelvin]
mudrate = 315; % Mud pump rate (kill rate) [L/min]

N=100000 % number of Monte Carlo Simulations
Pcasmc=zeros(1,N);
Pfracmc=zeros(1,N);
count = 0;

%% Monte Carlo Loop
for mc=1:N % MCS: Pore pressure, safety margin
sm = trianglerand(8,10,12,1); % Safety margin using function
trianglerand.m
Ppore = trianglerand(915,932,950,1); % Pore pressure using function
trianglerand.m
mc
Pbh = (Ppore + sm)/Pa_to_bar; % Bottom hole pressure [Pa]
romud = 1920; % Mud density [Kg/m^3]
topkick = 0; % Start value of topkick
time = 0; % Start time [s]
timeplot(1) = time; % Add time=0 to time vector used to plot pressures
[s]
rog(1) = rogas(Pbh,Tbh); % Gas density [Kg/m3] at time=0 (methane kick
considered). Use rogas.m function to calculate density of gas.
Pchoke(1) = Pbh-romud*g*well_depth; % SICP at time=0 [Pa]
Pcas(1) = Pbh-romud*g*(well_depth-shoe_depth); % Pressure at the casing
shoe at time=0 [Pa]

Kick_EndePos(1) = well_depth; % Bottom position of kick at time=0
Kick_TopPos(1) = well_depth; % Top position of kick at time=0
Kick_Height(1) = 0; % Height of kick at time=0
V_Kick_Values(1) = 0; % Kick volume at time=0

gg = 0.554; % Gas gravity of methane, this gives worst case scenario
Tb = Tbh; % Start temperature of bubble is same as T at bottom hole
zbhp = zfactor(gg,Pbh,Tbh); % Use zfactor.m function to calculate
zfactor at bottom
```

```
%% for-loop to calculate both pressures at each timestep dt
for i = 1:no_steps
    time = time+dt; % Time in seconds. Increases by dt for each
iteration.
    timeplot(i+1) = time; % Used to add each time step to time vector
used in plots.

    % 1a) Kick around BHA only.
    if (time<=influx_time)

        A = A_BHA;
        V_kick = V_kick_bottom*time/influx_time; % Kick volume (ramping
up until entire kick is in the well)
        h_kick = V_kick/A; % Kick height

        if h_kick>BHA_length
            Vexcess =  V_kick-V_BHA;
            h_kick = BHA_length + Vexcess/A_DS;
        end

        h_underkick = 0; % Height of mud under the kick.
        h_abovekick = well_depth-h_kick-h_underkick; % Height of mud
above the kick
        Pb = Pbh;
        Tb = Tbh;
        topkick = h_kick; % Distance from bottom of well to top of
kick.

    else
        if (topkick<BHA_length) % In this case, the kick is only around
the BHA
            h_underkick = (mudrate*L_min_to_m3_second)*(time-
influx_time)/A; % Height of mud under the kick in m
            Pb = Pbh-romud*g*h_underkick; % Pressure at the bubble
(intersection)
            Tb = Tbh-Tgrad*h_underkick;
            zb = zfactor(gg,Pb,Tb); % Compressibility factor at the
bubble
            V_kick = (Pbh*V_kick_bottom*zb*Tb)/(zbhp*Tbh*Pb); %
Updating volume (real gas)
            h_kick = V_kick/A; % Updating the kick height
            h_abovekick = well_depth-h_kick-h_underkick;%height of mud
above the kick
            topkick = h_underkick+h_kick;
     % 1b) Kick in transistion zone between BHA/DS and also at DS only
```

```
        elseif (topkick>BHA_length) %in this case, or is the transition
zone, or only around the drill string
                if (h_underkick>BHA_length) %in this case, the kick is
only around the drill string
                    A=pi/4*(d_open_hole^2-d_DS^2); %flow area
                %   hunderkick=(mudrate/60/1000)*(time-100)/A;% height
of mud under the kick in m

h_underkick=BHA_length+(mudrate*L_min_to_m3_second)*(time-
time_kickatBHA)/A;
                    if (h_underkick>well_depth)
                       h_underkick=well_depth;
                    end
                    Pb=Pbh-romud*g*h_underkick; % Pressure at the
bubble (intersection)
                    Tb = Tbh-Tgrad*h_underkick;
                    zb=zfactor(gg,Pb,Tb); % Compressibility factor at
the bubble
                    V_kick=(Pbh*V_kick_bottom*zb*Tb)/(zbhp*Tbh*Pb);%
Updating volume (real gas)
                    if V_kick <=0
                        V_kick = 0;
                    end
                    h_kick=V_kick/A; % Updating the kick height
                    if h_kick < 0
                        h_kick = 0;
                    end
                    h_abovekick=well_depth-h_kick-h_underkick; % Height
of mud above the kick
                       if  h_abovekick<0
                           h_abovekick=0;
                           h_kick=well_depth-h_underkick;
                       end
                       if h_kick<0
                               h_kick=0; % Here there was an error
                       end
                    topkick=h_underkick+h_kick;
                    if topkick<0 || topkick>= well_depth
                        topkick = well_depth;
                    end
                else %in this case (hunderkick<BHA_length), the kick is
in the transition zone = part around the BHA and part around the drill
string
                    h_underkick=(mudrate*L_min_to_m3_second)*(time-
influx_time)/A_BHA; % h under the kick (this is around BHA)
```

```
                    Pb=Pbh-romud*g*h_underkick; % Pressure at the
bubble (intersection)
                    Tb = Tbh-Tgrad*h_underkick; % Temperature at the
bubble (intersection)
                    zb=zfactor(gg,Pb,Tb); % Compressibility factor at
the bubble
                    V_kick=(Pbh*V_kick_bottom*zb*Tb)/(zbhp*Tbh*Pb);%
Updating volume (real gas)
                    h_kick_BHA=(well_depth-BHA_depth)-h_underkick;
                    V_BHA_kick=A_BHA*h_kick_BHA;
                    V_DS=V_kick-V_BHA_kick;
                    h_DS =V_DS/A_DS;
                    topkick=(well_depth-BHA_depth)+h_DS;

                    h_kick=h_kick_BHA+h_DS;
                    h_abovekick = well_depth-h_kick-h_underkick;
                    time_kickatBHA = time;
                end
        end
    end

    %T(i+1) = T(i)-Tgrad*mudrate*L_min_to_m3_second*dt/A; % New
temperature at the bubble

    T(i+1)= Tbh-Tgrad*h_underkick;
    rog(i+1) = rogas(Pb,Tb); % New density of the gas (bubble)
    Pchoke(i+1) = Pbh-((h_abovekick+h_underkick)*g*romud)-
(h_kick*g*rog(i+1)); % Updating Pchoke

    h_kick_BHA = h_underkick+h_kick;

    if h_kick_BHA < 0 % Fixes bug for when kick reaches top of well
        h_kick_BHA = well_depth;
    end
    l_openhole = well_depth-shoe_depth;
    % 2a) Kick below or at casing shoe
    if (h_kick_BHA<l_openhole)
        Pcas(i+1) = Pbh-(h_abovekick-shoe_depth)*g*romud-
h_underkick*g*romud-h_kick*g*rog(i+1); % Pcas when the kick is below
(or at) the casing shoe
    else
    % 2b) Kick passing the shoe
        if (h_underkick<l_openhole)
            h_kick1 = l_openhole-h_underkick;
```

```
            Pcas(i+1) = Pbh-h_underkick*g*romud-h_kick1*g*rog(i+1); %
Kick is passing the shoe
    % 2c) Kick entirely above shoe
        else
        Pcas(i+1) = Pbh-romud*g*(well_depth-shoe_depth); % Pcas after
the kick passed the shoe
        end


    end
    Kick_EndePos(i+1) = well_depth-h_underkick;


    TopPos = well_depth-topkick;
    if h_kick == 0 % To fix bug which would decrease top position of
kick after top of kick has reached top of well
        TopPos = 0;
    end
    Kick_TopPos(i+1) = TopPos;
    Kick_Height(i+1) = h_kick;
    V_Kick_Values(i+1) = V_kick;


end
% Converting pressure unit
Pchoke = Pchoke*Pa_to_bar; % Change pressure unit from Pa to bar
Pcas = Pcas*Pa_to_bar; % Change pressure unit from Pa to bar
maximum_Pcas = max(Pcas); % Finds maximum value of Pcas
i_max_Pcas = find(Pcas==maximum_Pcas); % Finds which time iteration
gives ut max Pcas
time_max_Pcas = i_max_Pcas*dt-5; % Calculates at what time we have max
Pcas
Pcasmc(1,mc)=maximum_Pcas; % Finds maximum value of Pcas
Pfracmc(1,mc)=normrnd(883,3); % Normal distribution to determine
fracture pressure value
if Pcasmc(1,mc) >= Pfracmc(1,mc) % Registers number of cases where Pcas
is larger than Pfrac
    count = count + 1;
end
end % End of Monte Carlo loop


toc,
e = cputime-t;


disp('Probability of failure, in percent:')
prob = (count/N)*100 % Failure probability in percent
```

```
%% Plot probability density functions

e=min(Pfracmc(1,:));
f=max(Pfracmc(1,:));
s=e:1:f;
[c,d]=hist(Pfracmc(1,:),s); % Creates histogram for Pfrac

h=min(Pcasmc(1,:));
f=max(Pcasmc(1,:));
w=h:1:f;
[a,b]=hist(Pcasmc(1,:),w); % Creates histogram for Pcas
disp('Mean, P10, P90 values of max casing shoe pressure:')
mean_value = mean(Pcasmc(1,:))
P10 = prctile(Pcasmc(1,:),10);
P90 = prctile(Pcasmc(1,:),90);

figure('Name', 'Monte Carlo Simulations, Pcas + Pfrac')
plot(b,a/N) % Plots casing pressure curve
hold on
plot(d,c/N) % Plots fracture pressure curve
hold off
legend({'Casing shoe pressure','Fracture
pressure'},'Location','northeast')
title('Casing shoe pressure vs. Fracture pressure')
xlabel('Pressure [bar]')
ylabel('PDF')
x0 = 830
x1 = 900
y0 = 0;
y1 = 0.15;
axis([x0 x1 y0 y1])
N
V_kick_bottom
BHA_length

figure('Name','Pcas')
plot(timeplot,Pcas) % Plots pressure at casing shoe vs. timesteps
title('Pressure at casing shoe')
xlabel('Time [s]')
ylabel('Pressure [bar]')
```

A.1.3 Transient flow model – Monte Carlo loop implemented

```
% Transient two-phase code based on AUSMV scheme: Gas and Water
% The code assumes uniform geometry and the code is partially
vectorized.
%
% Author: Kjell Kåre Fjelde, Professor, University of Stavanger
% Revised by: Martine Kristoffersen, MSc student, University of
Stavanger, 2020

clear
clc

t = cputime; % Used to calculate CPU time, start
tic,

N = 100000 % Number of Monte Carlo simulations
kickvol = 2 % Kick size [m^3]

% Geometry data/ Must be specified
welldepth = 5000; % Total well depth [m]
shoedepth = 4500; % Shoe depth [m]
BHA_length = 50; % BHA length [m]
nobox = 50; % Number of boxes in the well
nofluxes = nobox+1;
dx = welldepth/nobox; % Box length
shoe_box = (welldepth-shoedepth)/dx; % Box number at shoe depth

% Welldepth array
x(1)= -1.0*welldepth+0.5*dx;
for i=1:nobox-1
 x(i+1)=x(i)+ dx;
end

countprob=0;% Counter for prob intersection pcas/pfrac
pcasmc=zeros(1,N);
pfracmc=zeros(1,N);

for mc=1:N
mc
ppore=trianglerand(91500000,93200000,95000000,1); % Pore pressure with
triangular distribution [sg]. Corresponds to (915,932,950,1) in bar
```

```
sm=trianglerand(800000,1000000,1200000,1); % Safety margin with
triangular distribution [Pa]. Corresponds to (8,10,12,1) in bar

pconstbottom=ppore+sm;

dt= 30;  % Timestep [s]
dtdx = dt/dx;
time = 0.0;
endtime = 1000; % Time for end of simulation [s]
nosteps = endtime/dt;  %Number of total timesteps
timebetweensavingtimedata = 30;  % How often in s we save data vs time
for plotting [s]
nostepsbeforesavingtimedata = timebetweensavingtimedata/dt;

% Slip parameters used in the gas slip relation. vg =Kvmix+S
k = 1.2;
s = 0.5;

% Temperature distribution

tempbot = 100+273; % Temperature at bottom of well [Kelvin]
temptop = 50+273; % Temperature at surface [Kelvin]
tempgrad = (tempbot-temptop)/welldepth;
temp(1)=tempbot-dx/2*tempgrad;
for i = 1:nobox-2
  temp(i+1)=temp(i)-dx*tempgrad;
end
temp(nobox)=temp(nobox-1)-dx*tempgrad;


% Viscosities (Pa*s)/Used in the frictional pressure loss model.
viscl = 0.001; % Liquid phase
viscg = 0.0000182; % Gas phase

g = 9.81; % Gravitational constant

% Define and initialize flow variables

% Here we specify the outer and inner diameter, flow area, boxvolume

   for i = 1:nobox
    do(i) = 12.25*(2.54/100); % Outer diameter of casing [m]
```

```
    di(i) = 5*(2.54/100); % Outer diameter of drill string [m]
    area(i) = pi/4*(do(i)*do(i)- di(i)*di(i));
    vol(i)=area(i)*dx;
   end

   % Below, Includes BHA volume. (Defined for 50 boxes.)
   for i = 1:2 % i = 1:2 corresponds to changing the two bottom boxes
    do(i) = 12.25*(2.54/100);
    di(i) = 8.5*(2.54/100);
    area(i) = pi/4*(do(i)*do(i)- di(i)*di(i));
    if BHA_length <= 100 && i == 1
        vol(i)=(BHA_length/dx)*area(i)*dx+(1-
(BHA_length/dx))*area(3)*dx;
    elseif BHA_length <= 100 && i == 2
        vol(i) = vol(3);
    elseif BHA_length > 100 && i == 1
        vol(i) = area(i)*dx;
    else % BHA_length > 100m && i == 2
        vol(i) = 0.5*area(i)*dx+0.5*area(3)*dx;
    end
   end

% Initialization of slope limiters.
  for i = 1:nobox
    sl1(i)=0;
    sl2(i)=0;
    sl3(i)=0;
    sl4(i)=0;
    sl5(i)=0;
    sl6(i)=0;
  end

% Now comes the initialization of the physical variables in the well.
% Below we initialize pressure and fluid densities. We start from top
of
% the well and calculated downwards. The calculation is done twice with
% updated values to get better approximation. Only hydrostatic
% considerations.

% Boundary condition at outlet:
pbondout = 100000; % Pressure at surface
% Initialize pressure and densities inside boxes
```

```
p(nobox)= pbondout+0.5*9.81*dx*rholiq(pbondout,temptop);  % Pressure
dl(nobox)=rholiq(p(nobox),temp(nobox));  % Liquid density
dg(nobox)=rogas(p(nobox),temp(nobox));   % Gas density

% Initialize the gas and liquid volume fractions:

for i=1:nobox
  eg(i)=0;  % Gas volume fraction
  ev(i)=1-eg(i); % Liquid volume fraction
end

for i=nobox-1:-1:1
p(i)=p(i+1)+dx*9.81*(ev(i+1)*dl(i+1)+eg(i+1)*dg(i+1));
dl(i)=rholiq(p(i),temp(i));
dg(i)=rogas(p(i),temp(i));
end

 for i=nobox-1:-1:1
  rhoavg1= (ev(i+1)*dl(i+1)+eg(i+1)*dg(i+1));
  rhoavg2= (ev(i)*dl(i)+eg(i)*dg(i));
  p(i)=p(i+1)+dx*9.81*(rhoavg1+rhoavg2)*0.5;
  dl(i)=rholiq(p(i),temp(i));
  dg(i)=rogas(p(i),temp(i));

 end

% Initialize phase velocities, masses, inside boxes
% The basic assumption is static fluid, one phase liquid.

for i = 1:nobox
  vl(i)=0; % Liquid velocity new time level.
  vg(i)=0; % Gas velocity at new time level
  liqmassnew(i)=dl(i)*ev(i)*vol(i); % Liquid mass in box
  gasmassnew(i)=dg(i)*eg(i)*vol(i); % Gas mass in box
  fricgrad(i)=0;
  hydgrad(i)=g*(dl(i)*ev(i)+eg(i)*dg(i));
end

% Define boundary cell variables (mixture velocites and pressures)
```

```
for i = 1:nobox
 vm(i)= 0; % mixture velocity at inlet of box
 vp(i)= 0; % mixture velocity at outlet of box
end

 pm(1)=p(1)+dx/2*g*(ev(1)*dl(1)+eg(1)*dg(1));
 pp(1)=(p(1)+p(2))/2;
 pp(nobox)=pbondout;
 pm(nobox)=(p(nobox)+p(nobox-1))/2;

for i = 2:nobox-1
 pm(i)=(p(i-1)+p(i))/2;       % pressure at inlet of box
 pp(i)=(p(i)+p(i+1))/2;       % pressure at outlet of box
end

% Section where we also initialize values at old time level
for i=1:nobox
  dlo(i)=dl(i);
  dgo(i)=dg(i);
  po(i)=p(i);
  ego(i)=eg(i);
  evo(i)=ev(i);
  vlo(i)=vl(i);
  vgo(i)=vg(i);
  liqmassold(i)=liqmassnew(i);
  gasmassold(i)=gasmassnew(i);
  pmold(i)=pm(i);
  ppold(i)=pp(i);
  vmold(i)=vm(i);
  vpold(i)=vp(i);
end

%  Main program. Here we will progress in time. First som
intializations
% and definitions to take out results. The for loop below runs until
the
% simulation is finished.

% Start conditions
countsteps = 0;
counter=0;
printcounter = 1;
```

```
pin(printcounter) = (p(1)+dx*0.5*hydgrad(1))/100000;
hyd(printcounter)=pin(printcounter);
pout(printcounter)= pbondout/100000;
pnobox(printcounter)= p(nobox)/100000;
pcas(printcounter)=pp(shoe_box)/100000; % Calculates casing pressure @
shoe
liquidmassrateout(printcounter) = 0;
gasmassrateout(printcounter)=0;
timeplot(printcounter)=time;
kickvolume=0;

pcasmax=0;

for i = 1:nosteps
   countsteps=countsteps+1;
   counter=counter+1;
   time = time+dt;

% Then a section where specify the boundary conditions.
% Here we specify the inlet rates of the different phases at the
% bottom of the pipe in kg/s. We interpolate to make things smooth.
% It is also possible to change the outlet boundary status of the well
% here. First we specify rates at the bottom and the pressure at the
outlet
% in case we have an open well. This is a place where we can change the
% code to control simulations.

% In the example below, we take a gas kick and then circulate this
% out of the well without closing the well. (how you not should perform
% well control)

XX = 10;  %Liquidrate kg/s
%YY = 2;  %Gasrate kg/s

masskick=kickvol*rogas(pconstbottom,tempbot);
influxperiod = 100; % How long time the kick takes
influxmassrate = masskick/influxperiod;

XX = 10;  %Liquidrate kg/s
YY = influxmassrate;  %Gasrate kg/s

if (time<10)
```

```
  pm(1)=pconstbottom;
  inletligmassrate=0.0;
  inletgasmassrate=0.0;

elseif((time>=10)&&(time<=20))
  pm(1)=pconstbottom;
  inletligmassrate=XX*(time-10)/10;  % Interpolation, ramping up
liqrates
  inletgasmassrate=YY*(time-10)/10;  % Interpolation, ramping up
gasrates
elseif((time>20)&&(time<=(20+influxperiod-10)))
  pm(1)=pconstbottom;
  inletligmassrate=XX;
  inletgasmassrate=YY;
elseif((time>(20+influxperiod-10))&&(time<20+influxperiod))
  pm(1)=pconstbottom;
  inletligmassrate=XX;
  inletgasmassrate=YY-YY*(time-(20+influxperiod-10))/10;
else
  pm(1)=pconstbottom;
  inletligmassrate=XX;
  inletgasmassrate=0;
end

  qmi(1) = inletligmassrate;
  qfi(1) = inletgasmassrate;
  vsl = qmi(1)/rholiq(pm(1),temp(1))/area(1);
  vsg = qfi(1)/rogas(pm(1),temp(1))/area(1);
  vm(1)= vsl+vsg;

kickvolume = kickvolume+inletgasmassrate/dgo(1)*dt;

% Introduce slopelimiters directly on old masses.

  for j = 2:nobox-1
    sl1(j)=minmod(liqmassold(j-1),liqmassold(j),liqmassold(j+1),dx);
    sl2(j)=minmod(gasmassold(j-1),gasmassold(j),gasmassold(j+1),dx);
  end
   sl1(1)=sl1(2);
   sl2(1)=sl2(2);
   sl1(nobox)=sl1(nobox-1);
```

```
   sl2(nobox)=sl2(nobox-1);

for j = 1:nobox
 vmix=(vm(j)+vpold(j))*0.5;

 friction=dx*dpfric(vmix,evo(j),ego(j),dlo(j),dgo(j), ...
           po(j),do(j),di(j),viscl,viscg);
 % if (time<130)
  %  friction = 0;
  %end
 hydrostatic = dx*g*(dlo(j)*evo(j)+dgo(j)*ego(j));
 pp(j)=pm(j)-hydrostatic-friction;
 pmean = (pp(j)+pm(j))*0.5;
 dlmean=rholiq(pmean,temp(j));
 dgmean=rogas(pmean,temp(j));

 qmout=(liqmassold(j)+qmi(j)*dt-dlmean*vol(j)*evo(j))/dt;
 qfout=(gasmassold(j)+qfi(j)*dt-dgmean*vol(j)*ego(j))/dt;
 vslout=qmout/dlmean/area(j);
 vsgout=qfout/dgmean/area(j);
 vp(j)=vslout+vsgout;
 vgout=k*vp(j)+s;
 vlout=(vgout-k*ego(j)*vgout-s)/(k*evo(j));
  liqmassfluxout=liqmassold(j)*vlout*dt/dx;
  gasmassfluxout=gasmassold(j)*vgout*dt/dx;

 liqmassfluxout=(liqmassold(j)+0.5*dx*sl1(j))*vlout*dt/dx;
 gasmassfluxout=(gasmassold(j)+0.5*dx*sl2(j))*vgout*dt/dx;

 liqmassnew(j) = liqmassold(j)+qmi(j)*dt-liqmassfluxout;
 gasmassnew(j)= gasmassold(j)+qfi(j)*dt-gasmassfluxout;
 volumegas = gasmassnew(j)/dgmean;
 eg(j)=volumegas/vol(j);
 ev(j)=1-eg(j);

 for jj=1:4
   vmix=(vm(j)+vp(j))*0.5;
   friction=dx*dpfric(vmix,ev(j),eg(j),dlmean,dgmean,...
           pmean,do(j),di(j),viscl,viscg);

    %if (time<130)
```

```
     %friction = 0;
     %end
    hydrostatic = dx*g*(dlmean*ev(j)+dgmean*eg(j));
    pp(j)=pm(j)-hydrostatic-friction;
    pmean = (pp(j)+pm(j))*0.5;
    dlmean=rholiq(pmean,temp(j));
    dgmean=rogas(pmean,temp(j));
    qmout=(liqmassold(j)+qmi(j)*dt-dlmean*vol(j)*ev(j))/dt;
    qfout=(gasmassold(j)+qfi(j)*dt-dgmean*vol(j)*eg(j))/dt;
    vslout=qmout/dlmean/area(j);
    vsgout=qfout/dgmean/area(j);
    vp(j)=vslout+vsgout;

    vgout=k*vp(j)+s;
    vlout=(vgout-k*eg(j)*vgout-s)/(k*ev(j));
    % Note We must check if gas concentation becomes 1. The above
command
    % will then divide by zero
    liqmassfluxout=liqmassold(j)*vlout*dt/dx;
    gasmassfluxout=gasmassold(j)*vgout*dt/dx;

    liqmassfluxout=(liqmassold(j)+0.5*dx*sl1(j))*vlout*dt/dx;
    gasmassfluxout=(gasmassold(j)+0.5*dx*sl2(j))*vgout*dt/dx;

    liqmassnew(j) = liqmassold(j)+qmi(j)*dt-liqmassfluxout;
    gasmassnew(j)= gasmassold(j)+qfi(j)*dt-gasmassfluxout;
    volumegas = gasmassnew(j)/dgmean;
    eg(j)=volumegas/vol(j);
    ev(j)=1-eg(j);
  end
  % Find remaining variables and prepare for computation in next box:
    fricgrad(j)=friction/dx;
    hydgrad(j)=hydrostatic/dx;
    p(j)= pmean;
    dl(j)=dlmean;
    dg(j)=dgmean;
    vmix = (vm(j)+vp(j))*0.5;
    vg(j)=k*vmix+s;
    vl(j)= (vg(j)-k*eg(j)*vg(j)-s)/(k*ev(j));
    % Note We must check if gas concentation becomes 1. The above
command
    % will then divide by zero
```

```
   if (j<nobox)
    vm(j+1)=vp(j)*area(j)/area(j+1);
    pm(j+1)=pp(j);
%     qmi(j+1)=qmout;
%    qfi(j+1)=qfout;
    qmi(j+1)=liqmassfluxout/dt;
    qfi(j+1)=gasmassfluxout/dt;
   end
end

% Code that finds the accumulated quantities which
% shall be plotted vs time.

      sumfric=0;
      sumhyd=0;
      liqmass=0;
      gasmass=0;
      gasvol=0;
      for j=1:nobox

      sumfric=sumfric+fricgrad(j)*dx;
      sumhyd=sumhyd+hydgrad(j)*dx;
      liqmass=liqmass+liqmassnew(j);
      gasmass=gasmass+gasmassnew(j);
      gasvol = gasvol+eg(j)*vol(j);

      end

%   Save maximum casingshoe pressure
   pcasmax = max(pcasmax,pp(shoe_box)/100000);

% Old values are now set equal to new values in order to prepare
% computation of next time level.

   po=p;
   dlo=dl;
   dgo=dg;
   vlo=vl;
   vgo=vg;
   ego=eg;
   evo=ev;
```

```
   liqmassold=liqmassnew;
   gasmassold=gasmassnew;
   pmold=pm;
   ppold=pp;
   vmold=vm;
   vpold=vp;

% Section where we save some timedependent variables in arrays.
% e.g. the bottomhole pressure. They will be saved for certain
% timeintervalls defined in the start of the program in order to ensure
% that the arrays do not get to long!

  if (counter>=nostepsbeforesavingtimedata)
    printcounter=printcounter+1;
    time;

    % Outlet massrates vs time

liquidmassrateout(printcounter)=dl(nobox)*ev(nobox)*vl(nobox)*area(nobo
x);

gasmassrateout(printcounter)=dg(nobox)*eg(nobox)*vg(nobox)*area(nobox);

    % Hydrostatic and friction pressure in well vs time in bar!
    hyd(printcounter)=sumhyd/100000;
    fric(printcounter)=sumfric/100000;

    % Volume of gas in well vs time
    volgas(printcounter)=gasvol;

    % Total phase masses in the well vs time
    massgas(printcounter)=gasmass;
    massliq(printcounter)=liqmass;
    % pout defines the exact pressure at the outletboundary!
    pout(printcounter)=(p(nobox)-0.5*dx*...
    (dlo(nobox)*evo(nobox)+dgo(nobox)*ego(nobox))*g-
dx*0.5*fricgrad(nobox))/100000;
    % pin defines the exact pressure at the bottom boundary
    pin(printcounter)=
(p(1)+0.5*dx*(dlo(1)*evo(1)+dgo(1)*ego(1))*g+0.5*dx*fricgrad(1))/100000
;
    pnobox(printcounter)=p(nobox)/100000;
    pcas(printcounter)=pp(shoe_box)/100000;
```

```
    % Time variable
    timeplot(printcounter)=time;

    counter = 0;

  end
end

% end of stepping forward in time.

pcasmc(1,mc)= pcasmax;

pfracmc(1,mc)=normrnd(883,3); %fracture pressure at last set shoe, 13
3/8

if pcasmc(1,mc) >= pfracmc(1,mc) % Checks if Pcas exceeds Pfrac
    countprob=countprob+1; % Add one to the count if Pcas>Pfrac
end

end % End of Monte Carlo loop

disp('Probability of failure:')
prob=(countprob/N)*100 % Probability of failure in percent

% Printing of results section

countsteps; % Marks number of simulation steps.
pcasmax;


% Plot commands for variables vs time. The commands can also
% be copied to command screen where program is run for plotting other
% variables.

toc,
e = cputime-t; % Used to calculate CPU time, start

%% Probability of casing shoe pressure
disp('Mean, P10, P90:')
mean_value = mean(pcasmc(1,:))
P10 = prctile(pcasmc(1,:),10);
```

```
P90 = prctile(pcasmc(1,:),90);

%% Plot maximum casing shoe pressure for all simulations

% Creates histogram from Pcas values
e = min(pcasmc(1,:));
f = max(pcasmc(1,:));
w = [e:1:f];
[a,b] = hist(pcasmc(1,:),w);

% Creates histogram from Pfrac values
h=min(pfracmc(1,:));
f=max(pfracmc(1,:));
s=h:1:f;
[c,d]=hist(pfracmc(1,:),s);

figure(2)
plot(b,a/N);
hold on
plot(d,c/N);
hold off
legend({'Casing shoe pressure','Fracture
pressure'},'Location','northeast')
title('Casing shoe pressure vs. Fracture pressure')
xlabel('Pressure |bar]')
ylabel('PDF')
x0 = 830;
x1 = 900;
y0 = 0;
y1 = 0.15;
axis([x0 x1 y0 y1])

%% Plot Pcas vs. time
figure('Name','Pcas')
plot(timeplot,pcas) % Plots pressure at casing shoe vs. timesteps
title('Pressure at casing shoe')
xlabel('Time [s]')
ylabel('Pressure [bar]')
```

A.1.4 Analytical model – Monte Carlo loop implemented

```
% Analytical model used to calculate casing shoe pressure
% Monte Carlo method is implemented to compensate for uncertainty in
pore
% pressure + safety margin.
%
% Author: Dalila Gomes, Research Fellow, University of Stavanger
% Revised by: Martine Kristoffersen, MSc student, University of
Stavanger,
% 2020

clc
clear
close all

t = cputime;
tic,

N = 1000000; % Number of Monte Carlo simulations
Vkick = 2; % Kick size [m^3]
T = 100 + 273; % Reservoir temperature [Kelvin]
g = 0.0981; % Gravitational constant
well_depth = 5000; % Planned TD of 12 1/4" hole section [m]
l_bha = 50; % Length of BHA [m]
shoe_depth = 4500; % Depth of shoe calculated from surface [m]

% Geometry of well:
% OD 0m-well_depth= 12.25"
% ID 0m-(well_depth-l_BHA) = 5" (drill string OD)
% ID (well_depth-l_BHA)-well_depth = 8.5" (BHA OD)

l_oh = well_depth-shoe_depth-l_bha; % Lenght of open hole section w/o
BHA
do_oh = 12.25*0.0254; % [m]
di_DS = 5*0.0254; % [m]
di_bha = 8.5*0.0254; % [m]
Abha = pi/4*(do_oh^2-di_bha^2); % [m^2]
Vbha = Abha*l_bha; % [m^3]
Aoh = pi/4*(do_oh^2-di_DS^2);
Voh = Aoh*l_oh;
FG = 0; % Fristion gradient
```

```
% Create vectors/start conditions
Pbot = 0;
DPbha = 0;
DPoh = 0;
Pcas = 0;
Pfrac = 0;
probcount = 0;

%% Monte Carlo loop
for mc=1:N

    Ppore=trianglerand(915,932,950,1); % Pore pressure @ bottom of well
[bar]
    romud=1.92; % Mud density [sg]
    SM=trianglerand(8,10,12,1); % Safety margin [m]

    Pporepascal =(Ppore+SM)*10^5; % Total pressure at bottom [Pa]
    rogass = rogas(Pporepascal,T); % Density of gas [kg/m^3]
    rogass=rogass/1000; % Convertion of gas density [kg/m^3] to [sg]

    Pbot(mc)=Ppore+SM;

    alphaG=trianglerand(0.3,0.5,0.7,1); % Gas fraction in cross-
sectional
    % area
    %alphaG = 1; % If gas is a single bubble
    alphaL=1-alphaG;

% First calculate across  BHA component
if (Vkick > alphaG*Vbha) % Entire BHA section is filled up by gas
    DPbha(mc)=g*(alphaG*rogass+alphaL*romud)*l_bha+FG*l_bha;

else
     Hkick=Vkick/(Abha*alphaG);
     Hnokick=l_bha-Hkick;

DPbha(mc)=(g*(alphaG*rogass+alphaL*romud)*Hkick)+(g*romud*Hnokick)+(FG*
l_bha);
       % Only parts of BHA filled with kick
end

%Then calculate across openhole/pipe component
```

```
if (Vkick > alphaG*Vbha )      % We have more kick that also covers the
upper region above BHA.
    Vkickexcess=Vkick-alphaG*Vbha    ;
    Hkick2=Vkickexcess/(Aoh*alphaG) ;

    if (Hkick2 > l_oh) % Include this in case kick is so large that it
passes the shoe.
        Hkick2=l_oh;
    end

    Hnokick2=l_oh-Hkick2;

DPoh(mc)=(g*(alphaG*rogass+alphaL*romud)*Hkick2)+(g*romud*Hnokick2)+(FG
*l_oh);

else
    DPoh(mc)=(g*romud*l_oh)+(FG*l_oh);

end

%Then calculate max casing shoe pressure using
Pcas(mc)=Pbot(mc)-DPbha(mc)-DPoh(mc);
Pfrac(mc)=normrnd(883,3); % Fracture pressure [bar] at last set shoe
(13 3/8")


if Pcas(mc)>=Pfrac(mc) % Check if casing shoe pressure is larger than
fracture pressure
    probcount=probcount+1;
end
end % End of Monte Carlo loop

toc,
e = cputime-t;

%% Probability of casing shoe pressure
disp('Probability of failure, in percent:')
PROB=(probcount/N)*100 % Probability of failure in %
Mean = mean(Pcas) % Mean value of casing shoe pressure
P10 = prctile(Pcas,10); % P10 value of casing shoe pressure
P90 = prctile(Pcas,90); % P90 value of casing shoe pressure

%% Plot max casing shoe pressure vs. fracture pressure
```

```
% Plot PDF and CDF

e=min(Pfrac(1,:));
f=max(Pfrac(1,:));
s=e:1:f;
[c,d]=hist(Pfrac(1,:),s); % Creates histogram for Pfrac

h=min(Pcas(1,:));
f=max(Pcas(1,:));
w=h:1:f;
[a,b]=hist(Pcas(1,:),w); % Creates histogram for Pcas

figure('Name', 'Monte Carlo Simulations, Pcas + Pfrac')
plot(b,a/N) % Plots casing pressure curve
hold on
plot(d,c/N) % Plots fracture pressure curve
hold off
legend({'Casing shoe pressure','Fracture
pressure'},'Location','northeast')
title('Casing shoe pressure vs. Fracture pressure')
xlabel('Pressure [bar]')
ylabel('PDF')
x0 = 850
x1 = 910
y0 = 0;
y1 = 0.15;
axis([x0 x1 y0 y1])
```

A.2 Functions used in A.1 scripts

A.2.1 rogas.m: Function used to calculate density of gas bubble (methane)

```
function rhog = rogas(pressure,temp)

% pressure - Pascal
% temp     - Kelvin

M = 16.04; % molar mass methane g/mol
Mair = 28.96; % molar mass air g/mol
R = 8.314;   % Universal gas constant J/(molxK)

gamma = M/Mair;

z = zfactor(gamma,pressure,temp);
%z = 1;

rhog = M/(R*temp)*pressure/z;  %% g/m3
rhog = rhog/1000;              %% kg/m3
```

A.2.2 zfactor.m: Function used to calculate z factor

```
function Zgas = zfactor(gg,pp,tt)
% gg - gas gravity
% pp - pressure pascal
% tt - temperature Kelvin

% pressure in psi
p = pp/100000*14.5;
% temperature in Fahrenheit
t = tt*9/5-459.67;

A1=0.31506237;
A2=-1.0467099;
A3=-0.57832729;
A4=0.53530771;
A5=-0.61232032;
A6=-0.10488813;
A7=0.68157001;
A8=0.68446549;

ppc=702.5-50*gg; % Critical pressure (psi)
tpc=167+316.67*gg; % Critical temperature (Rankine)

ppr=p/ppc; % reduced pressure
tpr=(t+459.67)/tpc; % reduced temperature

Z=1;
error = 999;
iter=0;

while(error>0.001)
  iter=iter+1;
  if(iter>100)
   disp('stop')
  end

  ropr=0.27*ppr/Z/tpr;
  Z1=1+(A1+A2/tpr+A3/tpr/tpr/tpr)*ropr;
  Z1=Z1+(A4+A5/tpr)*ropr*ropr;
```

```
  Z1=Z1+(A5*A6*ropr*ropr*ropr*ropr*ropr)/tpr;
  Z1=Z1+(A7*ropr*ropr/tpr/tpr/tpr)*(1+A8*ropr*ropr)* ...
    exp(-A8*ropr*ropr);
  error=2*abs((Z-Z1)/(Z+Z1));
  Z=(Z1+Z)/2;
end
iter;
Zgas=Z;

end
```

A.2.3 trianglerand.m: Function for triangular distribution in Monte Carlo simulations

```
function f = trianglerand(xstart,mostlik,xstop,N)
% TRIANGLERAND Random numbers from a triangle distribution.
%    R = trianglerand(min,mostlikely,max,N) returns a vector of N draws
from a triangular distribution starting at min, maxpoint at mostlikely
and endpoint at max.
%   Copyright 2003 RF - Rogaland Research
%   Author: Øystein Arild


a = mostlik-xstart;
b = xstop-xstart;


h1 = 2/a;
m1 = h1/a;


A1 = a/b;
p = A1;


f_ = (rand(N,1) < p);
ind1 = find(f_==1);
ind2 = find(f_==0);
N1 = length(ind1);


if (a == b)
  u = rand(N,1);
  f = sqrt(2*m1*u)/m1;
else
  u = rand(N1,1);
  f1 = sqrt(2*m1*u)/m1;


  h2 = 2/(b-a);
  m2 = -h2/(b-a);
  beq=h2;
  u = rand(N-N1,1);
  f2 = a+(-beq+sqrt(beq*beq+2*m2*u))/m2;
  f(ind1) = f1;
  f(ind2) = f2;
  f = f';
end
f = f + xstart;
```

Appendix B

B.1 Results from single bubble simulation according to chapter 6.1

		Kick size			
		2 m³	4 m³	8 m³	12 m³
BHA length	50m	573.3518	578.3913	589.3256	600.9874
	100m	573.4220	581.4077	591.4866	601.5656
	150m	573.4492	581.5472	594.5030	604.5819

Table B-1 Maximum casing shoe pressure in bar, long hole section & 12.25" hole

		Kick size			
		2 m³	4 m³	8 m³	12 m³
BHA length	50m	581.5493	596.8923	626.9816	655.8849
	100m	584.4540	597.8027	626.9783	655.8804
	150m	586.3305	600.7174	627.3946	655.8872

Table B-2 Maximum casing shoe pressure in bar, long hole section & 8.5" hole

		Kick size			
		2 m³	4 m³	8 m³	12 m³
BHA length	50m	855.8798	860.9193	870.9983	881.0772
	100m	855.9500	863.9357	874.0146	884.0936
	150m	855.9772	864.0752	877.0310	887.1099

Table B-3 Maximum casing shoe pressure in bar, short hole section & 12.25" hole

		Kick size			
		2 m³	4 m³	8 m³	12 m³
BHA length	50m	864.0773	877.4159	904.0931	927.6620
	100m	866.9920	880.3307	907.0079	927.6620
	150m	868.8585	883.2454	909.9226	927.6620

Table B-4 Maximum casing shoe pressure in bar, short hole section & 8.5" hole