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A Game Theoretic Model of Adversaries and Media Manipulation

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Abstract: A model is developed for two players exerting media manipulation efforts to support each of two actors who interact controversially. Early evidence may support one actor, while the full evidence emerging later may support the other actor. Exerting effort when the full evidence exceeds (falls short off) the early evidence is rewarded (punished) with lower (higher) unit effort cost. Properties and simulations are presented to illustrate the players' strategic challenges when altering eight model parameters, i.e., a player's unit effort cost, stake in the interaction, proportionality parameter scaling the strength of reward or punishment, time discount parameter, early evidence, full evidence, contest intensity, and evidence ratio intensity. Realizing the logic of the model may aid understanding on how to handle the difference between early and full evidence of controversies, in which players have an ideological stake.

Keywords: media; game; adversaries; players; contest; manipulation; spin control

1. Introduction

In 1983, 90% of the American media was owned by 50 companies, while in 2011 only six companies (General Electric, News Corp, Disney, Viacom, Time Warner, and CBS) owned the same 90% of the American media [1]. Since the year 1900s, independent news media has become subject to more concentrated control. With fewer and more powerful players, ideological impact becomes a more prominent concern. Economies of scale, and possibly other factors, may enable individual players to impose their ideological views more fiercely, without being compromised by a plurality of multifarious other players. Reporting objectively, truthfully, and with ideologically neutrality, is challenging. Tribe [2] suggests that tools in policy science are themselves ideologically biased. Levins [3] and Nagy, Fairbrother, Etterson, and Orme-Zavaleta [4] suggest that truth may emerge by intersecting independent lies. (That is, various independent models may together resemble truth.)

One widely reported controversial media scenario, which led to subsequent lawsuits, was between high school student Nicholas Sandmann from northern Kentucky Covington Catholic High School and the 64-year old native American Nathan Phillips at the Lincoln Memorial in Washington D.C., USA [5,6]. Sandmann wore a Make America Great Again hat and was in a group of fellow students on an annual school trip to attend the pro-life March for Life, combined with sightseeing. They waited for a bus to Kentucky. Phillips was beating a drum and chanting, while partaking in a group of Native American marchers attending the Indigenous Peoples March. Phillips was locking eyes with a smiling Sandmann a few inches apart. Early selective videos of the interaction on 18 January 2019 led most of the media to criticize Sandmann and the students for potentially provoking Phillips. It turned out that many videos and audio recordings of the interaction existed given the presence of many people at the prominent location. As accumulated and more full evidence of the interaction emerged, a view gradually arose that, potentially, the story was the opposite of that originally reported, and that Phillips was potentially provoking Sandmann. This remarkable turn of events caused the media to subsequently react in many different manners, hypothetically, in accordance with their ideological position. Examples of the media's reactions, from one extreme to the other, were to retract and apologize, rewrite, reinterpret due to new evidence, ignore, retain the original account with some rewriting, or retain the original account with no adjustment.

Motivated by the potential ideological concerns of media organizations, and scenarios, such as the one sketched above, this article develops a model of the background where two adversarial actors interact controversially. The model is technically a one-period model, but accounts for early evidence of the interaction emerging in period 1, and full accumulated evidence emerging in period 2. Before period 1 two adversarial actors interact controversially. Incomplete early evidence emerges. Two media organizations are the players. Each player supports one of the actors ideologically, and exerts manipulation efforts including spin control to persuade media consumers that the actor he represents is righteous and should not be blamed. Media manipulation effort is interpreted broadly to include competition, verbal fighting, etc. Hirshleifer [7] interprets fighting as a metaphor, i.e., "falling also into the category of interference struggles are political campaigns, rent-seeking maneuvers for licenses and monopoly privileges [8], commercial efforts to raise rivals' costs [9], strikes and lockouts, and litigation—all being conflictual activities that need not involve actual violence." The early and full evidence, and a variety of characteristics of the two players are incorporated into the game they play.

Hardly any literature exists on the phenomenon. Allcott and Gentzkow [10] analyze fake news and social media in the 2016 US election. Blom and Hansen [11], Zannettou, Chatzis, Papadamou, and Sirivianos [12], and Khoja [13] consider clickbait news. Kshetri and Voas [14] examine fake news within an economic perspective.

The article is organized as follows. Section 2 presents the model. Section 3 analyzes the model. Section 4 illustrates with an example. Section 5 concludes.

2. The model

Appendix A shows the nomenclature. Consider two actors 1 and 2 which are adversaries interacting controversially. An actor may be an individual, a group, or any collective unit. Player i is a media media organization which reports on both actors and their interaction, while identifying with actor i, $i = 1, 2$, see Figure 1.

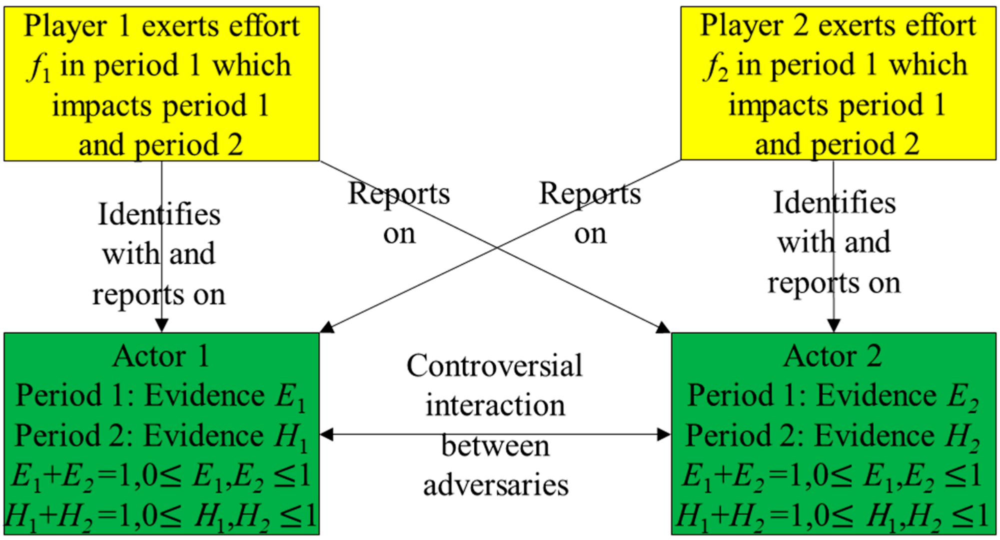

Figure 1. Two adversaries actor 1 and actor 2 interacting controversially, reported on by player 1 (media organization 1) which identifies with actor 1, and player 2 (media organization 2), which identifies with actor 2.

2.1. Period 1

Assume that some early evidence E_i, $0 \leq E_i \leq 1$, $E_1 + E_2 = 1$, is available before period 1 supporting player i and actor i. Early evidence E_i may be preliminary videos and audios of the interaction, neutral witness accounts, etc. Early evidence E_i may be incomplete and capture only part of the truth, or may be framed so that it reports a falsehood, e.g. if the beginning and end of a video are deleted. Player i in period 1 exerts media manipulation effort f_i at unit cost a_i to manipulate the perception of early evidence E_i so that it is perceived to be higher. Effort f_i is player i's only strategic choice variable.

The two players (i.e., the two media organizations) are in a contest modeled with the common ratio form contest success function [15]. Player i earns a proportion of the stake, expressing the degree to which actor i is perceived not to be at fault in the interaction. Player i has a probability p_i of success, $0 \leq p_i \leq 1$, $p_1 + p_2 = 1$. Analogous to Hirshleifer and Osborne's [16] Litigation Success Function, [1] where we apply early evidence E_i instead of truth (which is unavailable), we consider the Media Manipulation Success Function ratio,

$$\frac{p_1}{p_2} = \left(\frac{f_1}{f_2}\right)^{\alpha}\left(\frac{E_1}{E_2}\right)^{\gamma} \tag{1}$$

where α, $\alpha \geq 0$, is the contest intensity, and γ, $\gamma \geq 0$, is the early evidence ratio intensity with an interpretation analogous to α. The right-hand side of (1) contains the media manipulation effort ratio f_1/f_2 raised to α, and the early evidence ratio E_1/E_2 raised to γ. When $\alpha = 0$ or $f_1 = f_2$, the success ratio p_1/p_2 equals the early evidence ratio E_1/E_2 raised to γ. When $0 \leq \alpha \leq 1$, exerting less effort f_i than the other player has disproportional advantage. When $\alpha = 1$, exerting effort f_i has proportional advantage. When $\alpha > 1$, exerting more effort f_i than the other player has disproportional advantage. When $\alpha = \infty$, exerting slightly more effort f_i than the other player gives a "winner-takes-all" situation.

Since E_1 and E_2 are parameters, $\left(\frac{E_1}{E_2}\right)^{\gamma}$ is also a parameter where the early evidence ratio intensity γ depends on culture, law, the nature of the interaction, the time and place of the interaction, the identities of actors 1 and 2 and players 1 and 2, and how the early evidence E_i is generated, presented, and framed. Equation (1) is rewritten as a contest success probability,

$$p_i = \frac{f_i^{\alpha} E_i^{\gamma}}{f_1^{\alpha} E_1^{\gamma} + f_2^{\alpha} E_2^{\gamma}} \tag{2}$$

for player i. Assuming that player i has a stake J_i in the interaction, his expected utility in period 1 is,

$$U_i = p_i J_i - a_i f_i = \frac{f_i^{\alpha} E_i^{\gamma}}{f_1^{\alpha} E_1^{\gamma} + f_2^{\alpha} E_2^{\gamma}} J_i - a_i f_i \tag{3}$$

where (2) has been inserted.

2.2. Period 2

The early evidence E_i in period 1 is incomplete. Assume that additional evidence becomes available in period 2, causing the accumulated or full evidence H_i supporting player i, $0 \leq H_i \leq 1$, $H_1 + H_2 = 1$. The full evidence H_i in period 2 includes early evidence E_i from period 1, and additional evidence in period 2. Thus $H_i = E_i$ means either that no new evidence becomes available in period 2, or that the new available evidence in period 2 does not alter how the evidence impacts the support of players 1 and 2. In contrast, $H_i \neq E_i$ means more ($H_i > E_i$) or less ($H_i < E_i$) evidence supporting player i in period 2. The full evidence H_i may e.g., include the beginning and end of the video missing in

[1] See Hausken, Levitin, and Levitin [17] for an application of the contest success function to lawsuits, and Hausken and Levitin [18] for an application of the contest success function to risk analysis.

the period 1 early evidence E_i, or additional videos or audio. Although, the full evidence H_i becomes available in period 2, we assume that the players make no strategic choices in period 2.

Modeling the full evidence H_i in player i's expected utility for period 2 involves three characteristics and no strategic choice variables for period 2. First, since $H_i > E_i$ means that more evidence becomes available, supporting player i, player i should be rewarded proportionally to $H_i - E_i$. Since player i was disadvantaged with low early evidence E_i in the contest success function in (3), higher full evidence H_i should advantage player i. In contrast, since $H_i < E_i$ means that the full evidence H_i supports player i to a lower extent, player i should be punished proportionally to $H_i - E_i$. Second, a proportionality parameter Q_i, $Q_i \geq 0$, is introduced to scale the strength of the reward or punishment $H_i - E_i$. Third, proportionality with player i's effort f_i is assumed since the reward or punishment $H_i - E_i$ should be higher (lower) if player i's effort f_i is higher (lower). Letting these three characteristics operate multiplicatively, player i's utility in period 2 is:

$$V_i = -Q_i(E_i - H_i)f_i. \tag{4}$$

The full evidence H_i becomes available after early evidence E_i. The time duration may be a few days, a few seconds or many years, depending on the phenomenon. Hence a time discount parameter δ_i, $0 \leq \delta_i \leq 1$, is introduced, where $\delta_i = 0$ means no emphasis on the future so that the full evidence H_i is irrelevant, and $\delta_i = 1$ means that the future is as important as the present. Letting the future be more important than the present is possible, but probably more uncommon. In other words, as in Equation (5) by Hausken [19], assume that player i has time discount parameter δ_i, $0 \leq \delta_i \leq 1$. Consequently, player i's expected utility over the two periods is,

$$W_i = U_i + \delta_i V_i = \frac{f_i^\alpha E_i^\gamma}{f_1^\alpha E_1^\gamma + f_2^\alpha E_2^\gamma} J_i - (a_i + \delta_i Q_i(E_i - H_i))f_i. \tag{5}$$

where U_i and V_i are inserted from (3) and (4). The term $a_i + \delta_i Q_i(E_i - H_i)$ can be interpreted as player i's actual unit effort cost (accounting for the reward or punishment) when $a_i + \delta_i Q_i(E_i - H_i) \geq 0$, and as player i's actual unit effort benefit (accounting for substantial reward) when $a_i + \delta_i Q_i(E_i - H_i) \leq 0$. The latter unit effort benefit is uncommon, and arises when $H_i \geq E_i + \frac{a_i}{\delta_i Q_i}$, which means that the accumulated evidence H_i supporting player i in period 2 overwhelms the early evidence E_i supporting player i in period 1. This uncommon event causes infinite effort f_i. The following Assumption 1 excludes this uncommon event: Assumption 1: $H_i \leq E_i + \frac{a_i}{\delta_i Q_i}$.

3. Analyzing the Model

Since the players make no strategic choices in period 2, the game is technically a one-period game with strategic choice variables f_1 and f_2 in period 1. Differentiating player i's expected utility in (5) with respect to his free choice variable f_i in period 1, and equating with zero, gives:

$$\begin{aligned} \frac{\partial W_1}{\partial f_1} &= \frac{\alpha E_1^\gamma E_2^\gamma f_1^{\alpha-1} f_2^\alpha}{\left(f_1^\alpha E_1^\gamma + f_2^\alpha E_2^\gamma\right)^2} J_1 - (a_1 + \delta_1 Q_1(E_1 - H_1)) = 0, \\ \frac{\partial W_2}{\partial f_2} &= \frac{\alpha E_1^\gamma E_2^\gamma f_1^\alpha f_2^{\alpha-1}}{\left(f_1^\alpha E_1^\gamma + f_2^\alpha E_2^\gamma\right)^2} J_2 - (a_2 + \delta_2 Q_2(E_2 - H_2)) = 0. \end{aligned} \tag{6}$$

Solving (6) when Assumption 1 is satisfied gives:

$$f_1 = \frac{J_1(a_2 + \delta_2 Q_2(E_2 - H_2))}{J_2(a_1 + \delta_1 Q_1(E_1 - H_1))} f_2,\ f_2 = \frac{\frac{\alpha E_2^\gamma J_2^{1+\alpha}(a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^{1+\alpha}}}{\left(1 + \frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\right)^2}. \tag{7}$$

The second order derivatives, inserting (7), are satisfied as negative, i.e.,

$$\begin{aligned}\frac{\partial^2 W_1}{\partial f_1^2} &= -\frac{(a_1+\delta_1 Q_1(E_1-H_1))^{2-\alpha}}{\alpha E_1^\gamma E_2^\gamma J_1^{1+\alpha} J_2^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\\ &\times\left(E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha + E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha\right)\\ &\times\left(E_2^\gamma J_2^\alpha (1-\alpha)(a_1+\delta_1 Q_1(E_1-H_1))^\alpha + E_1^\gamma J_1^\alpha (1+\alpha)(a_2+\delta_2 Q_2(E_2-H_2))^\alpha\right),\\ \frac{\partial^2 W_2}{\partial f_2^2} &= -\frac{(a_2+\delta_2 Q_2(E_2-H_2))^{2-\alpha}}{\alpha E_1^\gamma E_2^\gamma J_1^\alpha J_2^{1+\alpha} (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}\\ &\times\left(E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha + E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha\right)\\ &\times\left(E_2^\gamma J_2^\alpha (1+\alpha)(a_1+\delta_1 Q_1(E_1-H_1))^\alpha + E_1^\gamma J_1^\alpha (1-\alpha)(a_2+\delta_2 Q_2(E_2-H_2))^\alpha\right)\end{aligned} \tag{8}$$

Inserting (7) into (5), when Assumption 1 is satisfied, gives the player's expected utilities:

$$\begin{aligned} W_1 &= \frac{\left(1+(1-\alpha)\frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\right) J_1}{\left(1+\frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\right)^2},\\ W_2 &= \frac{\frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\left(1-\alpha+\frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\right) J_2}{\left(1+\frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\right)^2}\end{aligned} \tag{9}$$

which are positive when,

$$\alpha \le 1 + Min\left\{\frac{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}, \frac{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha}{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}\right\} \tag{10}$$

which is a lenient restriction on the contest intensity α. For equivalent players (10) simplifies to $\alpha \le 2$. When (10) is not satisfied, assume without loss of generality that $\frac{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha}{E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha} \ge 1$, which means that player 2 is disadvantaged e.g., due to a higher unit effort cost a_2. To avoid negative expected utility, player 2 exerts zero effort f_2 earning zero expected utility W_2. If player 2 chooses zero effort f_2, player 1 cannot choose arbitrarily low but positive effort, f_1, which would not be an equilibrium, since player 2 would deviate by choosing some positive effort. Hence, we assume that player 1 chooses an effort f_1 which deters player 2 from receiving positive expected utility W_2. This is accomplished by solving player 1's first order condition $\frac{\partial W_1}{\partial f_1} = 0$, i.e., the first Equation in (6), together with $W_2 = 0$ determined by (5). These are two equations with two unknown f_1 and f_2 which are solved to yield,

$$f_1 = \left(\frac{f_2^\alpha E_2^\gamma}{E_1^\gamma}\left(\frac{J_2}{(a_2+\delta_2 Q_2(E_2-H_2))f_2} - 1\right)\right)^{1/\alpha} \tag{11}$$

which cannot be solved analytically, but is illustrated numerically in the next section.

Property 1. For the interior solution when Assumption 1 is satisfied, $\alpha = 1$, $i = 1,2$, $i \ne j$, $\frac{\partial f_i}{\partial a_i} \le 0$, $\frac{\partial f_j}{\partial a_i} \le 0$ when $E_i^\gamma J_i\left(a_j+\delta_j Q_j\left(E_j-H_j\right)\right) \le E_j^\gamma J_j(a_i+\delta_i Q_i(E_i-H_i))$, $\frac{\partial W_i}{\partial a_i} \le 0$, $\frac{\partial W_j}{\partial a_i} \ge 0$. Proof. Appendix B Equations (A1), (A2), (A3), (A4).

Intuitively, a higher unit effort cost a_i in period 1 discourages player i causing lower effort f_i and lower expected utility W_i. In contrast, higher a_i causes higher expected utility W_j for player j, and lower effort f_j when the specified inequality is satisfied. The specified inequality is more easily satisfied when a_i, J_j and H_j are high, which advantage player j.

Property 2. For the interior solution when Assumption 1 is satisfied, $\alpha = 1$, $i = 1,2$, $i \neq j$, $\frac{\partial f_i}{\partial J_i} \geq 0$, $\frac{\partial f_j}{\partial J_i} \geq 0$ when $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \leq E_j^{\gamma} J_j(a_i + \delta_i Q_i(E_i - H_i))$, $\frac{\partial W_i}{\partial J_i} \geq 0$, $\frac{\partial W_j}{\partial J_i} \leq 0$. Proof. Appendix B Equations (A5), (A6), (A7), (A8).

Higher stake J_i in the interaction in period 1 for player i induces higher effort f_i and higher expected utility W_i. In contrast, higher J_i causes lower expected utility W_j for player j, and higher effort f_j when the same inequality as in Property 1 is satisfied.

Property 3. For the interior solution when Assumption 1 is satisfied, $\alpha = 1$, $i = 1,2$, $i \neq j$, $\frac{\partial f_i}{\partial Q_i} \geq 0$ when $E_i \leq H_i$, $\frac{\partial f_j}{\partial Q_i} \geq 0$ when $E_i \leq H_i$ and $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \leq E_j^{\gamma} J_j(a_i + \delta_i Q_i(E_i - H_i))$, $\frac{\partial W_i}{\partial Q_i} \geq 0$ when $E_i \leq H_i$, $\frac{\partial W_j}{\partial Q_i} \leq 0$ when $E_i \leq H_i$. Proof. Appendix B Equations (A9), (A10), (A11), (A12).

The impact of player i's proportionality parameter Q_i depends on whether the early evidence E_i supporting player i in period 1 is lower or higher than the accumulated evidence H_i supporting player i in period 2. When $E_i \leq H_i$, so that player i benefits from transitioning from period 1 to period 2, higher Q_i induces player i to exert higher effort f_i and he receives higher expected utility W_i. In contrast, player j receives lower expected utility W_j, and exerts higher effort f_j when the same inequality as in Properties 1 and 2 is satisfied.

Property 4. For the interior solution when Assumption 1 is satisfied, $\alpha = 1$, $i = 1,2$, $i \neq j$, $\frac{\partial f_i}{\partial \delta_i} \geq 0$ when $E_i \leq H_i$, $\frac{\partial f_j}{\partial \delta_i} \geq 0$ when $E_i \leq H_i$ and $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \leq E_j^{\gamma} J_j(a_i + \delta_i Q_i(E_i - H_i))$, $\frac{\partial W_i}{\partial \delta_i} \geq 0$ when $E_i \leq H_i$, $\frac{\partial W_j}{\partial \delta_i} \leq 0$ when $E_i \leq H_i$. Proof. Appendix B Equations (A13), (A14), (A15), (A16).

The impact of player i's time discount parameter δ_i is equivalent to the impact of the proportionality parameter Q_i, except that δ_i is confined to the interval $0 \leq \delta_i \leq 1$, while Q_i is unbounded from above, i.e., $Q_i \geq 0$.

Property 5. For the interior solution when Assumption 1 is satisfied, $\alpha = \gamma = 1$, $i = 1,2$, $i \neq j$, $\frac{\partial f_i}{\partial E_i} \geq 0$ when $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \leq E_j^{\gamma} J_j(a_i - \delta_i Q_i(E_i + H_i))$, $\frac{\partial f_j}{\partial E_i} \geq 0$ when $a_i \geq \delta_i Q_i H_i$ and $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \leq E_j^{\gamma} J_j(a_i + \delta_i Q_i(E_i - H_i))$, $\frac{\partial W_i}{\partial E_i} \geq 0$ when $a_i \geq \delta_i Q_i H_i$, $\frac{\partial W_j}{\partial E_i} \leq 0$ when $a_i \geq \delta_i Q_i H_i$. Proof. Appendix B Equations (A17), (A18), (A19), (A20).

Property 5 assumes the intermediate value $\gamma = 1$ for the early evidence ratio intensity in period 1, to simplify the analysis. More early evidence E_i supporting player i in period 1 causes higher expected utility W_i for player i and lower expected utility W_j for player j when $a_i \geq \delta_i Q_i H_i$, i.e., when player i's unit effort cost a_i is high compared with his discount parameter δ_i, proportionality parameter Q_i, and the accumulated evidence H_i supporting him in period 2. More early evidence E_i supporting player i in period 1 causes higher effort f_i for player i when the same inequality as in Properties 1,2,3,4 is satisfied. Higher E_i also causes higher effort f_j for player j when player i is disadvantaged with $a_i \geq \delta_i Q_i H_i$, and the same inequality as in Properties 1,2,3,4 is satisfied.

Property 6. For the interior solution when Assumption 1 is satisfied, $\alpha = \gamma = 1$, $i = 1,2$, $i \neq j$, $\frac{\partial f_i}{\partial H_i} \geq 0$, $\frac{\partial f_j}{\partial H_i} \geq 0$ when $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \leq E_j^{\gamma} J_j(a_i + \delta_i Q_i(E_i - H_i))$, $\frac{\partial W_i}{\partial H_i} \geq 0$, $\frac{\partial W_j}{\partial H_i} \leq 0$. Proof. Appendix B Equations (A21), (A22), (A23), (A24).

Property 6 also assumes the intermediate value $\gamma = 1$ for the early evidence ratio intensity in period 1, to simplify the analysis. Again, and intuitively, more accumulated evidence H_i supporting player i in period 2 causes higher effort f_i expected utility W_i for player i and lower expected utility W_j for player j. Higher H_i causes higher effort f_j for player j when same inequality as in Properties 1,2,3,4,5 is satisfied.

Property 7. For the interior solution when Assumption 1 is satisfied, $i = 1,2$, $i \neq j$, $\frac{\partial f_i}{\partial \alpha} \geq 0$ and $\frac{\partial f_j}{\partial \alpha} \geq 0$ when $E_1^{\gamma} J_1^{\alpha}(a_2 + \delta_2 Q_2(E_2 - H_2))^{\alpha}\left(1 + \alpha Ln\left(\frac{J_2(a_1+\delta_1 Q_1(E_1-H_1))}{J_1(a_2+\delta_2 Q_2(E_2-H_2))}\right)\right) + E_2^{\gamma} J_2^{\alpha}(a_1 + \delta_1 Q_1(E_1 - H_1))^{\alpha} \left(1 + \alpha Ln\left(\frac{J_1(a_2+\delta_2 Q_2(E_2-H_2))}{J_2(a_1+\delta_1 Q_1(E_1-H_1))}\right)\right)$, $\frac{\partial W_i}{\partial \alpha} \geq 0$ when $E_1^{\gamma} J_1^{\alpha}(a_2 + \delta_2 Q_2(E_2 - H_2))^{\alpha} + E_2^{\gamma} J_2^{\alpha}(a_1 + \delta_1 Q_1(E_1 - H_1))^{\alpha} + \left(E_1^{\gamma} J_1^{\alpha}(1+\alpha)(a_2 + \delta_2 Q_2(E_2 - H_2))^{\alpha} + E_2^{\gamma} J_2^{\alpha}(1-\alpha)(a_1 + \delta_1 Q_1(E_1 - H_1))^{\alpha}\right) Ln\left(\frac{J_2(a_1+\delta_1 Q_1(E_1-H_1))}{J_1(a_2+\delta_2 Q_2(E_2-H_2))}\right) \leq 0$,

$\frac{\partial W_j}{\partial \alpha} \le 0$ when $E_1^{\gamma} J_1^{\alpha} (a_2 + \delta_2 Q_2 (E_2 - H_2))^{\alpha} + E_2^{\gamma} J_2^{\alpha} (a_1 + \delta_1 Q_1 (E_1 - H_1))^{\alpha} + \left(E_1^{\gamma} J_1^{\alpha} (1-\alpha)(a_2 + \delta_2 Q_2 (E_2 - H_2))^{\alpha} + E_2^{\gamma} J_2^{\alpha} (1+\alpha)(a_1 + \delta_1 Q_1 (E_1 - H_1))^{\alpha}\right) \; Ln\left(\frac{J_1(a_2+\delta_2 Q_2(E_2-H_2))}{J_2(a_1+\delta_1 Q_1(E_1-H_1))}\right) \ge 0$. Proof. Appendix B Equations (A25), (A26), (A27), (A28).

Property 8. For the interior solution when Assumption 1 is satisfied, $\alpha = 1$, $i = 1,2$, $i \ne j$, $\frac{\partial f_i}{\partial \gamma} \le 0$ and $\frac{\partial f_j}{\partial \gamma} \le 0$ when $Ln(E_1) \le Ln(E_2)$ and $E_i^{\gamma} J_i\left(a_j + \delta_j Q_j\left(E_j - H_j\right)\right) \le E_j^{\gamma} J_j(a_i + \delta_i Q_i(E_i - H_i))$, $\frac{\partial W_i}{\partial \gamma} \le 0$ and $\frac{\partial W_j}{\partial \gamma} \ge 0$ when $Ln(E_1) \le Ln(E_2)$. Proof. Appendix B Equations (A29), (A30), (A31), (A32).

4. Illustrating the Solution

Referring to the scenario in the introduction, in this section we can think of actor 1 as Catholic Kentucky high school student, Nicholas Sandmann, and actor 2 as native American, Nathan Phillips. We can think of player 1 as the parts of the media that supported or identified ideologically with Nicholas Sandmann. Possible examples are the Covington Catholic High School newspaper and local media institutions in Covington, Kentucky, or various catholic media outlets. We can think of player 2 as the parts of the media that supported or identified ideologically with Nathan Phillips. Possible examples are native American media outlets, and the media institutions that Sandmann filed lawsuits against.

Figure 2 illustrates the solution with the benchmark parameter values $a_i = J_i = Q_i = \alpha = \gamma = \delta_i = 1$, $E_1 = 0.1$, $H_1 = 0.8$, which imply $E_2 = 0.9$, $H_2 = 0.2$, $i = 1,2$. That is, actor 1 supported by player 1 is assigned substantial fault expressed as low $E_1 = 0.1$ in period 1, and much less fault expressed as low $H_1 = 0.8$ in period 2, and vice versa for actor 2 supported by player 2. That causes low actual unit effort cost $a_1 + \delta_1 Q_1 (E_1 - H_1) = 0.3$ for player 1, and high actual unit effort cost $a_2 + \delta_2 Q_2 (E_2 - H_2) = 1.7$ for player 2, at the benchmark. Player 2 nevertheless receives the highest expected utility $W_2 > W_1$ at the benchmark since the early evidence ratio $E_1/E_2 = 1/9$ favors player 2 in the benchmark contest. In each of the eight panels one parameter value varies, while the other parameter values are kept at their benchmarks.

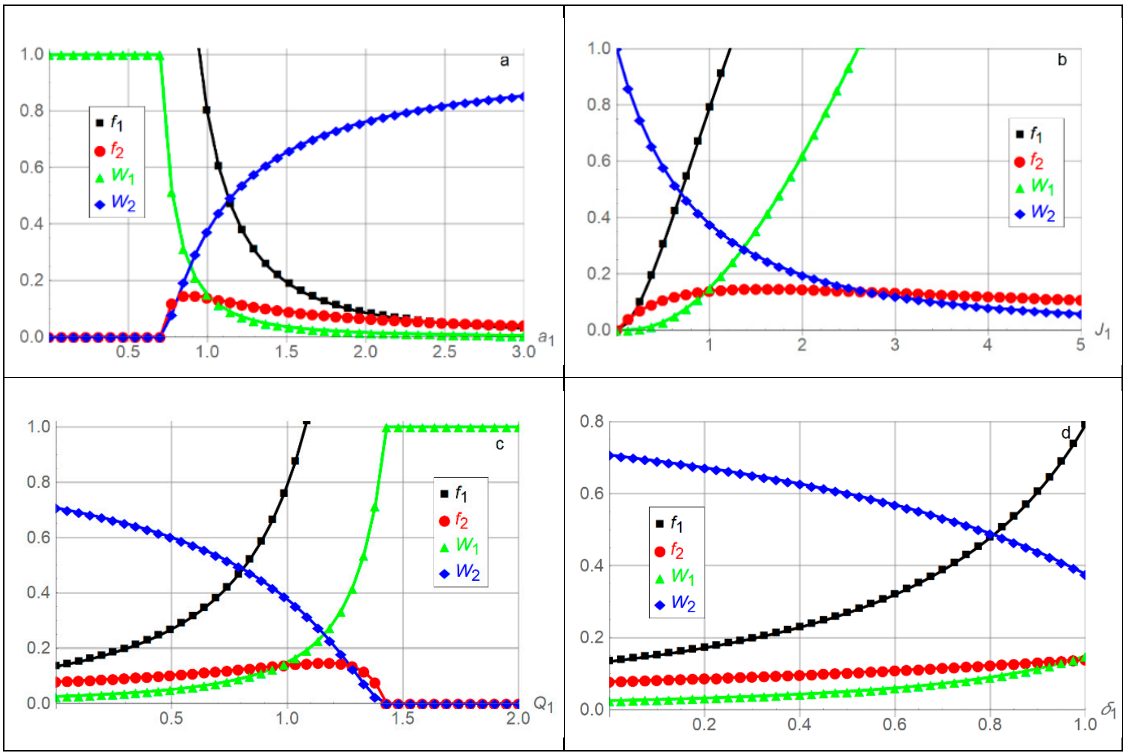

Figure 2. *Cont.*

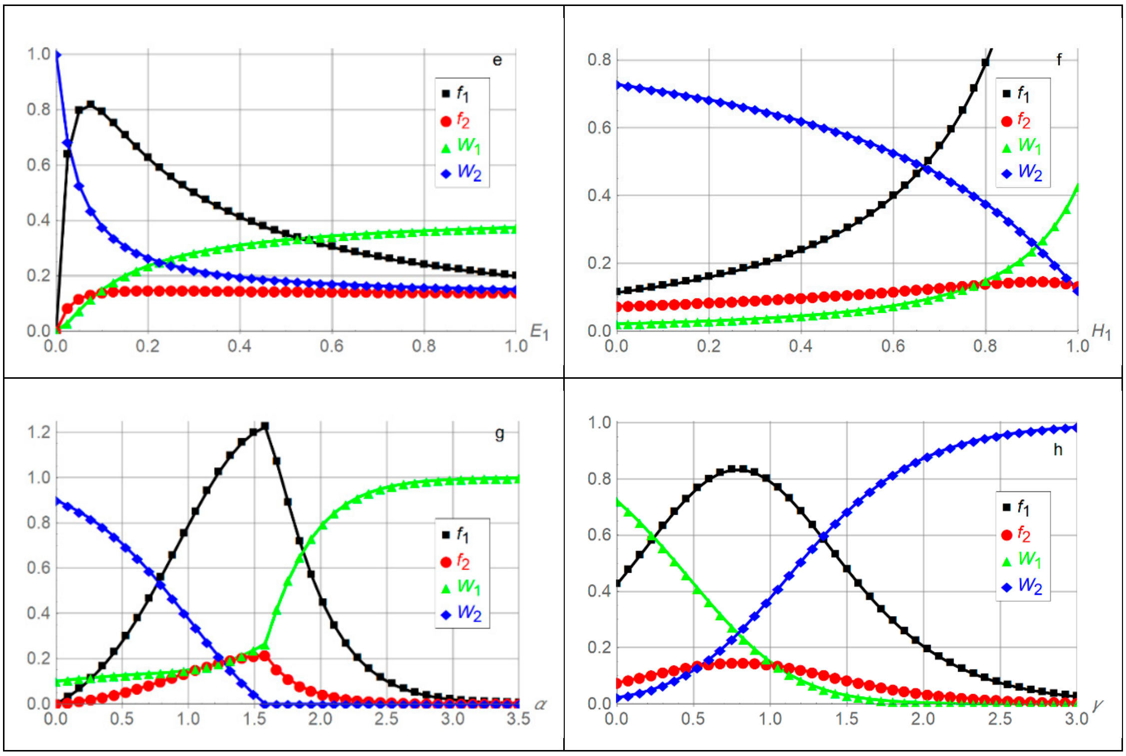

Figure 2. Media manipulation efforts f_1 and f_2 and expected utilities W_1 and W_2 for players 1 and 2 as functions of a_1, J_1, Q_1, δ_1, E_1, H_1, α, and γ relative to the benchmark parameter values $a_i = J_i = Q_i = \alpha = \gamma = \delta_i = 1$, $E_1 = 0.1$, $H_1 = 0.8$, which imply $E_2 = 0.9$, $H_2 = 0.2$, $i = 1, 2$

In Figure 2 panel a, a high unit effort cost a_1 for player 1 causes low effort f_1 and low expected utility W_1 for player 1, $\lim_{a_1 \infty} f_1 = \lim_{a_1 \infty} W_1 = 0$, and high expected utility $\lim_{a_1 \infty} W_2 = J_2 = 1$ for player 2. Player 2 is advantaged when a_1 is high, which does not induce a need to exert high effort f_2, and $\lim_{a_1 \infty} f_2 = 0$. As a_1 decreases, the players have equal unit effort costs when $a_1 = a_2 = 1$. Then, player 1 exerts high effort f_1 and receives low expected utility W_1, and vice versa, player 2 exerts low effort f_2 and receives higher expected utility W_2. Although, player 1 is rewarded with a low actual unit effort cost 0.3 at the benchmark, the contest is not sufficiently beneficial for player 1. That is, player 2 is advantaged at the benchmark. As a_1 decreases below 1 to $a_1 = 0.88$, the players receive equal expected utilities $W_1 = W_2 = 0.25$. Decreasing a_1 further to $a_1 = 0.7$ causes player 1's actual unit effort cost to be $a_1 + \delta_1 Q_1 (E_1 - H_1) = 0$, enabling player 1 to exert arbitrarily high effort f_1 at no cost. Hence for $a_1 \leq 0.7$, player 1 receives expected utility $W_1 = J_1 = 1$, and player 2 exerts effort $f_2 = 0$ and receives expected utility $W_2 = 0$.

In Figure 2 panel b, increasing stake J_1 in the interaction in period 1 for player 1 causes increasing effort f_1 and increasing expected utility W_1 for player 1. From (7), $\lim_{J_1 \infty} f_1 = \frac{0.9}{0.17} \approx 5.29$, see Appendix C Figure A1. From (9), $\lim_{J_1 \infty} W_1 = \infty$. In contrast, player 2's effort f_2 is inverse U shaped as J_1 increases. This common phenomenon arises because player 2 is advantaged when J_1 is low, receiving high expected utility W_2, and disadvantaged when J_1 is high, receiving low expected utility W_2. Inserting into (7) and (9), $\lim_{J_1 \infty} f_2 = \lim_{J_1 \infty} W_2 = 0$. If $J_1 = J_2$, rather than J_1 varies along the horizontal axis in panel b, $\lim_{J_1 \infty} f_1 = \lim_{J_1 \infty} W_1 = \lim_{J_1 \infty} f_2 = \lim_{J_1 \infty} W_2 = \infty$, where player 2 is advantaged throughout since player 2 is advantaged at the benchmark $J_1 = J_2 = 1$.

In Figure 2 panel c, decreasing the proportionality parameter Q_1 which scales the strength of the reward $H_1 - E_1 = 0.7$ for player 1 has an impact similar to increasing player 1's unit effort cost a_1 in Figure 2 panel a, due to the opposite roles Q_1 and a_1 play in player 1's actual unit effort cost $a_1 + \delta_1 Q_1 (E_1 - H_1)$ when $E_1 - H_1 = -0.7$ is negative. Hence when Q_1 is low, f_1, W_1, and f_2 are low, and player 2's expected utility W_2 is high. As the proportionality parameter Q_1 increases, player 1 benefits from the lower actual unit effort cost which causes higher effort f_1 and higher expected utility W_1 for player 1, and lower expected utility W_2 for player 2. As in Figure 2 panels a and b, player 2's effort f_2 is inverse U shaped due to being advantaged when Q_1 is low and disadvantaged when Q_1 is high. When Q_1 increases to $Q_1 = 1.43$, player 1's actual unit effort cost decreases to $a_1 + \delta_1 Q_1(E_1 - H_1) = 0$, enabling player 1 to exert arbitrarily high effort f_1 at no cost. Hence for $Q_1 \geq 1.43$, player 1 receives expected utility $W_1 = J_1 = 1$, and player 2 exerts effort $f_2 = 0$ and receives expected utility $W_2 = 0$.

In Figure 2 panel d, varying player 1's time discount parameter δ_1 between 0 and 1 when $Q_1 = 1$ is equivalent to varying player 1's proportionality parameter Q_1 between 0 and 1 when $\delta_1 = 1$, since δ_1 and Q_1 only occur as $\delta_1 Q_1$. Figure 2 panel d highlights how disadvantaged player 1 becomes by being shortsighted expressed with low δ_1. That is, player 1 does not envision the reward flowing from $E_1 - H_1 = -0.7$ being negative, exerts low effort f_1 and receives low expected utility W_1. In contrast, player 2, endowed with the benchmark discount parameter $\delta_2 = 1$, benefits from player 1's shortsightedness when δ_1 is low, and receives high expected utility W_2.

In Figure 2 panel e, decreasing player 1's early evidence E_1 supporting player 1 in period 1 below the already low benchmark $E_1 = 0.1$ increases his reward $H_1 - E_1$, but decreases the evidence ratio E_1/E_2 in (1) and (5). Player 1 gets less incentive to conduct media manipulation effort since E_1/E_2 is low, but can manipulate the media more cheaply since his actual unit effort cost $a_1 + \delta_1 Q_1(E_1 - H_1)$ is low. Hence decreasing E_1 below $E_1 = 0.1$ causes player 1's effort f_1 to be inverse U-shaped and eventually decrease towards zero, while his expected utility W_1 decreases to zero, and player 2's expected utility increases to $W_2 = J_2 = 1$ when $E_1 = 0$. In contrast, increasing player 1's early evidence E_1 above his benchmark $E_1 = 0.1$ decreases his reward $H_1 - E_1$, causing his actual unit effort cost $a_1 + \delta_1 Q_1(E_1 - H_1)$ to increase. When $E_1 > H_1$, the reward becomes a punishment, since the early evidence E_1 supporting player 1 in period 1 is higher than the accumulated evidence H_1 supporting player 1 in period 2. That gives a higher actual unit effort cost causing player 1's effort f_1 to decrease. As E_1 increases, player 2's effort f_2 is slightly inverse U shaped, player 1's expected utility W_1 increases, and player 2's expected utility W_2 decreases.

In Figure 2 panel f, increasing player 1's accumulated evidence H_1 supporting player 1 in period 2 above the already high benchmark $H_1 = 0.8$, increases his reward $H_1 - E_1$, which decreases his actual unit effort cost $a_1 + \delta_1 Q_1(E_1 - H_1)$. Hence his effort f_1 increases, reaching $f_1 = 2.26$ when $H_1 = 1$ (outside what is plotted in panel f), and his expected utility W_1 increases. As H_1 increases, player 2's effort f_2 is slightly inverse U shaped, and his expected utility W_2 decreases. In contrast, decreasing player 1's accumulated evidence H_1 below his benchmark $H_1 = 0.8$ decreases his reward $H_1 - E_1$, causing his actual unit effort cost $a_1 + \delta_1 Q_1(E_1 - H_1)$ to increase. Hence, his effort f_1 and expected utility W_1 decrease, while player 2's effort f_2 decreases and his expected utility W_2 increases. As H_1 decreases below $E_1 = 0.1$, player 1's actual unit effort cost increases above his unit effort cost a_1 since $H_1 - E_1$ becomes a punishment.

In Figure 2 panel g, decreasing the contest intensity α below the benchmark $\alpha = 1$ causes player 1 to exert lower effort f_1 and receive lower expected utility W_1. A lower α causes efforts f_1 and f_2 to have lower impact on the contest, which becomes more egalitarian, and 100% egalitarian with no impact on the contest when $\alpha = 0$. Hence, player 1's advantage of a lower actual unit effort cost 0.3 than 1.7 for player 2 at the benchmark gradually gets eroded. Hence, lower α causes higher expected utility W_2 for player 2, sustained by decreasing effort f_2. In contrast, increasing α above the benchmark $\alpha = 1$ causes higher effort f_1 and expected utility W_1 for player 1, and higher effort f_2 and lower expected utility W_2 for player 2, up to when $\alpha = 1.58$. Higher contest intensity α is usually characterized by both players exerting higher efforts, which is costly. The efforts f_1 and f_2 cannot increase without bounds. At some point the weakest player reaches his limit. Thus player 2's expected utility is $W_2 = 0$ when $\alpha \geq 1.58$. Player 2 may exert zero effort or some positive effort f_2 when $\alpha \geq 1.58$, as long as his expected utility W_2 is not negative. As discussed in the previous section, if player 2 chooses zero effort f_2, player 1 cannot choose an arbitrarily low, but positive effort f_1, which would not be an equilibrium, since player 2 would deviate by choosing some positive effort. Applying (11), Figure 2 panel g for $\alpha \geq 1.58$ is determined numerically. Continuous efforts f_1 and f_2 are ensured through $\alpha = 1.58$. As α increases above $\alpha = 1.58$, decreasing effort f_1 by player 1 suffices to deter player 2 from exerting effort f_2 to obtain positive expected utility W_2. Thus player 1's expected utility W_1 increases concavely, $\lim_{\alpha\infty} W_1 = J_1 = 1$, while $\lim_{\alpha\infty} f_1 = \lim_{\alpha\infty} f_2 = \lim_{\alpha\infty} W_2 = 0$.

In Figure 2 panel h, increasing the early evidence ratio intensity γ from zero is beneficial for player 2, $\lim_{\gamma\infty} W_2 = J_2 = 1$, and not beneficial for player 1, $\lim_{\gamma\infty} W_1 = 0$, accompanied by $\lim_{\gamma\infty} f_1 = \lim_{\gamma\infty} f_2 = 0$. To see this, inserting the benchmark parameter values when γ varies into (7) and (9) gives,

$$f_1 = \frac{17}{3} f_2,\ f_2 = \frac{0.3 \times 9^{\gamma}}{1.7^2 \left(1 + \frac{3 \times 9^{\gamma}}{17}\right)^2}, W_1 = \frac{1}{\left(1 + \frac{3 \times 9^{\gamma}}{17}\right)^2}, W_2 = \frac{\left(\frac{3 \times 9^{\gamma}}{17}\right)^2}{\left(1 + \frac{3 \times 9^{\gamma}}{17}\right)^2} \quad (12)$$

where the inverse early evidence ratio $E_2/E_1 = 9$ raised to γ, i.e., $(E_2/E_1)^{\gamma} = 9^{\gamma}$, favors player 2 in terms of higher expected utility $W_2 > W_1$ at the benchmark when $\gamma = 1$. Player 2 is favored increasingly when $\gamma > 1$, and decreasingly when $\gamma < 1$. Both players' efforts f_1 and f_2 are inverse U shaped in γ since one player is advantaged when the other is disadvantaged, and vice versa, with the highest media manipulation efforts f_1 and f_2 for intermediate γ. In other words, for low early evidence ratio intensity, the fact that player 1 is subject to low early evidence is ameliorated, his effort matters less, and he receives high expected utility. In contrast, high early evidence ratio intensity amplifies how player 1 is subject to low early evidence, giving his lower expected utility.

5. Conclusions

A model is developed for two adversarial actors, which interact controversially. Early incomplete evidence emerges about which actor is at fault. A game is analyzed between two media organizations, as the players identifying ideologically with each of the two actors. Each player exerts manipulation efforts to support the actor he represents in a contest with the other player. We consider the two actors by comparison with the scenario in the introduction and simulation section, as high school student Nicholas Sandmann and native American Nathan Phillips, who interacted controversially January 18, 2019 at the Lincoln Memorial in Washington D.C., USA. We can think of the two players as two media organizations, which try to report the facts from the interaction, but additionally have ideological or other preferences that induce them to report favorably on the actor they identify with and support.

The game is technically a one-period game, where each player exerts one effort, but accounts for early evidence emerging in period 1 and full evidence emerging in period 2. If the full evidence equals the early evidence, each player's unit effort cost has a fixed value. If the full evidence supports an actor more (less) than the early evidence, the player identifying with that actor is rewarded (punished) with a lower (higher) unit effort cost proportional to the strength of the additional (decreased) support and proportional to a time discount parameter. The article illustrates each player's strategic challenge in determining the amount of media manipulation effort to exert, while accounting for the difference between the early and full evidence, the unit cost, and various other parameters.

To specify the model's implications, properties are developed for the model's eight parameters, which are illustrated with simulations relative to a benchmark. Without the loss of generality, actor 1 is supported by player 1 and is assumed to be substantially at fault, based on the early evidence, and much less at fault based on the full evidence. The impacts of player 1's unit effort cost and stake in the interaction are discussed. Higher proportionality parameter, scaling the strength of the reward to player 1 for being disadvantaged with low early evidence, and higher time discount parameter for player 1, cause higher effort and expected utility for player 1, and inverse U shaped effort and lower expected utility for player 2. Inverse U shapes are common when one player decreases his effort when either, advantaged or disadvantaged, and exerts high effort when being neither, advantaged nor disadvantaged. Increasing the early evidence supporting player 1 from zero causes inverse U shaped efforts for both players, increasing expected utility for player 1, and decreasing expected utility for player 2. Increasing the full evidence supporting player 1 from zero causes increasing effort and expected utility for player 1, and decreasing expected utility for player 2. Increasing the contest intensity with the given benchmark is shown to increase both players' efforts until a point where the disadvantaged player is deterred and receives zero expected utility. One implication for scenarios, such as the one between Nicholas Sandmann and Nathan Phillips, which gained widespread coverage and some degree of intensity, is that media organizations supporting one of the actors may potentially deter, outcompete, or silence the opposing media organizations. Finally, by increasing early evidence ratio intensity with the given benchmark demonstrates a common example where both players' efforts are inverse U shaped, player 1's expected utility decreases, and player 2's expected utility increases. The prevalence of inverse *U* shaped results illustrates how the interaction between media organizations as players may often be characterized by one player or the other being advantaged, which may compromise, objectively, the neutral and ideology-free reporting.

The article provides a tool for media organizations, analysts, consumers, regulators, researchers, and regular people to better understand how adversarial interaction may play out in today's continuously evolving media landscape. Realizing the interests of each player and actor, how each player and actor interact, and how new information becomes available over time, as illustrated in this article, may potentially enable everyone involved to contribute to mutually beneficial future media development. Future research should apply the model to more than two adversarial actors, more than two media players, more than two time periods, different kinds of information, and incorporate the role of media owners, regulators, advertisers, and consumers.

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Appendix A. Nomenclature

Parameters

a_i Player i's unit effort cost in period 1, $i = 1, 2$
Q_i Proportionality parameter which scales the strength of reward or punishment $E_i - H_i$, $i = 1, 2$
J_i Player i's stake in the interaction in period 1, $i = 1, 2$
δ_i Player i's time discount parameter, $i = 1, 2$
E_i Early evidence supporting player i in period 1, $i = 1, 2$
H_i Accumulated evidence supporting player i in period 2, $i = 1, 2$
α Contest intensity in period 1
γ Early evidence ratio intensity in period 1

Strategic choice variable

f_i Player i's media manipulation effort in period 1, $i = 1, 2$

Dependent variables

p_i Player i's probability of success in period 1, $i = 1, 2$
U_i Player i's expected utility in period 1, $i = 1, 2$
V_i Player i's utility in period 2, $i = 1, 2$
W_i Player i's expected utility over the two periods, $i = 1, 2$

Appendix B. Proof of Properties 1–8

$$\frac{\partial f_1}{\partial a_1} = -\frac{2E_1^{\gamma}E_2^{2\gamma}J_1^2J_2^2(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A1}$$

$$\frac{\partial f_2}{\partial a_1} = \frac{E_1^{\gamma}E_2^{\gamma}J_1J_2^2\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))-E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A2}$$

$$\frac{\partial W_1}{\partial a_1} = -\frac{2E_1^{2\gamma}E_2^{\gamma}J_1^3J_2(a_2+\delta_2Q_2(E_2-H_2))^2}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A3}$$

$$\frac{\partial W_2}{\partial a_1} = \frac{2E_1^{\gamma}E_2^{2\gamma}J_1J_2^3(a_1+\delta_1Q_1(E_1-H_1))(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A4}$$

$$\frac{\partial f_1}{\partial J_1} = \frac{2E_1^{\gamma}E_2^{2\gamma}J_1J_2^2(a_1+\delta_1Q_1(E_1-H_1))(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A5}$$

$$\frac{\partial f_2}{\partial J_1} = \frac{E_1^{\gamma}E_2^{\gamma}J_2^2(a_1+\delta_1Q_1(E_1-H_1))\left(E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))-E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))\right)}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A6}$$

$$\frac{\partial W_1}{\partial J_1} = \frac{E_1^{2\gamma}J_1^2(a_2+\delta_2Q_2(E_2-H_2))^2\left(3E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))+E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))\right)}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A7}$$

$$\frac{\partial W_2}{\partial J_1} = -\frac{2E_1^{\gamma}E_2^{2\gamma}J_2^3(a_1+\delta_1Q_1(E_1-H_1))^2(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A8}$$

$$\frac{\partial f_1}{\partial Q_1} = -\frac{2E_1^{\gamma}E_2^{2\gamma}J_1^2J_2^2\delta_1(E_1-H_1)(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A9}$$

$$\frac{\partial f_2}{\partial Q_1} = \frac{E_1^{\gamma}E_2^{\gamma}J_1J_2^2\delta_1(E_1-H_1)\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))-E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A10}$$

$$\frac{\partial W_1}{\partial Q_1} = -\frac{2E_1^{2\gamma}E_2^{\gamma}J_1^3J_2\delta_1(E_1-H_1)(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A11}$$

$$\frac{\partial W_2}{\partial Q_1} = \frac{2E_1^{\gamma}E_2^{2\gamma}J_1J_2^3\delta_1(E_1-H_1)(a_1+\delta_1Q_1(E_1-H_1))(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A12}$$

$$\frac{\partial f_1}{\partial \delta_1} = -\frac{2E_1^{\gamma}E_2^{2\gamma}J_1^2J_2^2Q_1(E_1-H_1)(a_2+\delta_2Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A13}$$

$$\frac{\partial f_2}{\partial \delta_1} = \frac{E_1^{\gamma}E_2^{\gamma}J_1J_2^2Q_1(E_1-H_1)\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))-E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)}{\left(E_1^{\gamma}J_1(a_2+\delta_2Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1Q_1(E_1-H_1))\right)^3} \tag{A14}$$

$$\frac{\partial W_1}{\partial \delta_1} = -\frac{2E_1^{2\gamma}E_2^{\gamma}J_1^3 J_2 Q_1(E_1-H_1)(a_2+\delta_2 Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))\right)^3} \quad (A15)$$

$$\frac{\partial W_2}{\partial \delta_1} = \frac{2E_1^{\gamma}E_2^{2\gamma}J_1 J_2^3 Q_1(E_1-H_1)(a_1+\delta_1 Q_1(E_1-H_1))(a_2+\delta_2 Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))\right)^3} \quad (A16)$$

Equations (A17), (A18), (A19), (A20) assume $\gamma = 1$.

$$\frac{\partial f_1}{\partial E_1} = -\frac{E_2 J_1^2 J_2(a_2+\delta_2 Q_2(E_2-H_2))(E_1 J_1(a_2+\delta_2 Q_2(E_2-H_2))-E_2 J_2(a_1-\delta_1 Q_1(E_1+H_1)))}{(E_1 J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2 J_2(a_1+\delta_1 Q_1(E_1-H_1)))^3} \quad (A17)$$

$$\frac{\partial f_2}{\partial E_1} = \frac{E_2 J_1 J_2^2(a_1-\delta_1 Q_1 H_1)(E_2 J_2(a_1+\delta_1 Q_1(E_1-H_1))-E_1 J_1(a_2+\delta_2 Q_2(E_2-H_2)))}{(E_1 J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2 J_2(a_1+\delta_1 Q_1(E_1-H_1)))^3} \quad (A18)$$

$$\frac{\partial W_1}{\partial E_1} = \frac{2E_1 E_2 J_1^3 J_2(a_1-\delta_1 Q_1 H_1)(a_2+\delta_2 Q_2(E_2-H_2))^2}{(E_1 J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2 J_2(a_1+\delta_1 Q_1(E_1-H_1)))^3} \quad (A19)$$

$$\frac{\partial W_2}{\partial E_1} = -\frac{2E_2^2 J_1 J_2^3(a_1-\delta_1 Q_1 H_1)(a_1+\delta_1 Q_1(E_1-H_1))(a_2+\delta_2 Q_2(E_2-H_2))}{(E_1 J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2 J_2(a_1+\delta_1 Q_1(E_1-H_1)))^3} \quad (A20)$$

Equations (A21), (A22), (A23), (A24) assume $\gamma = 1$.

$$\frac{\partial f_1}{\partial H_1} = \frac{2E_1^{\gamma}E_2^{2\gamma}J_1^2 J_2^2 \delta_1 Q_1(a_2+\delta_2 Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))\right)^3} \quad (A21)$$

$$\frac{\partial f_2}{\partial H_1} = \frac{E_1^{\gamma}E_2^{\gamma}J_1 J_2^2 \delta_1 Q_1\left(E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))-E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))\right)}{\left(E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))\right)^3} \quad (A22)$$

$$\frac{\partial W_1}{\partial H_1} = \frac{2E_1^{2\gamma}E_2^{\gamma}J_1^3 J_2 \delta_1 Q_1(a_2+\delta_2 Q_2(E_2-H_2))^2}{\left(E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))\right)^3} \quad (A23)$$

$$\frac{\partial W_2}{\partial H_1} = -\frac{2E_1^{\gamma}E_2^{2\gamma}J_1 J_2^3 \delta_1 Q_1(a_1+\delta_1 Q_1(E_1-H_1))(a_2+\delta_2 Q_2(E_2-H_2))}{\left(E_1^{\gamma}J_1(a_2+\delta_2 Q_2(E_2-H_2))+E_2^{\gamma}J_2(a_1+\delta_1 Q_1(E_1-H_1))\right)^3} \quad (A24)$$

$$\frac{\partial f_1}{\partial \alpha} = \frac{\begin{array}{c} E_1^{\gamma}E_2^{\gamma}J_1^{\alpha+1}J_2^{\alpha}(a_1+\delta_1 Q_1(E_1-H_1))^{\alpha-1}(a_2+\delta_2 Q_2(E_2-H_2))^{\alpha} \\ \times\left(\begin{array}{c} E_1^{\gamma}J_1^{\alpha}(a_2+\delta_2 Q_2(E_2-H_2))^{\alpha}\left(1+\alpha Ln\left(\frac{J_2(a_1+\delta_1 Q_1(E_1-H_1))}{J_1(a_2+\delta_2 Q_2(E_2-H_2))}\right)\right) \\ +E_2^{\gamma}J_2^{\alpha}(a_1+\delta_1 Q_1(E_1-H_1))^{\alpha}\left(1+\alpha Ln\left(\frac{J_1(a_2+\delta_2 Q_2(E_2-H_2))}{J_2(a_1+\delta_1 Q_1(E_1-H_1))}\right)\right) \end{array}\right) \end{array}}{\left(E_1^{\gamma}J_1^{\alpha}(a_2+\delta_2 Q_2(E_2-H_2))^{\alpha}+E_2^{\gamma}J_2^{\alpha}(a_1+\delta_1 Q_1(E_1-H_1))^{\alpha}\right)^3} \quad (A25)$$

$$\frac{\partial f_2}{\partial \alpha} = \frac{\begin{array}{c} E_1^{\gamma}E_2^{\gamma}J_1^{\alpha}J_2^{\alpha+1}(a_1+\delta_1 Q_1(E_1-H_1))^{\alpha}(a_2+\delta_2 Q_2(E_2-H_2))^{\alpha-1} \\ \times\left(\begin{array}{c} E_1^{\gamma}J_1^{\alpha}(a_2+\delta_2 Q_2(E_2-H_2))^{\alpha}\left(1+\alpha Ln\left(\frac{J_2(a_1+\delta_1 Q_1(E_1-H_1))}{J_1(a_2+\delta_2 Q_2(E_2-H_2))}\right)\right) \\ +E_2^{\gamma}J_2^{\alpha}(a_1+\delta_1 Q_1(E_1-H_1))^{\alpha}\left(1+\alpha Ln\left(\frac{J_1(a_2+\delta_2 Q_2(E_2-H_2))}{J_2(a_1+\delta_1 Q_1(E_1-H_1))}\right)\right) \end{array}\right) \end{array}}{\left(E_1^{\gamma}J_1^{\alpha}(a_2+\delta_2 Q_2(E_2-H_2))^{\alpha}+E_2^{\gamma}J_2^{\alpha}(a_1+\delta_1 Q_1(E_1-H_1))^{\alpha}\right)^3} \quad (A26)$$

$$\frac{\partial W_1}{\partial \alpha} = \frac{\begin{array}{c} -E_1^\gamma E_2^\gamma J_1^{\alpha+1} J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha \\ \times\left(\begin{array}{c} \times\Big\{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha + E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha \\ +\Big(E_1^\gamma J_1^\alpha (1+\alpha)(a_2+\delta_2 Q_2(E_2-H_2))^\alpha + E_2^\gamma J_2^\alpha (1-\alpha)(a_1+\delta_1 Q_1(E_1-H_1))^\alpha\Big) \\ \times Ln\left(\frac{J_2(a_1+\delta_1 Q_1(E_1-H_1))}{J_1(a_2+\delta_2 Q_2(E_2-H_2))}\right)\Big\} \end{array} \right) \end{array}}{\left(E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha + E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha\right)^3} \quad \text{(A27)}$$

$$\frac{\partial W_2}{\partial \alpha} = \frac{\begin{array}{c} -E_1^\gamma E_2^\gamma J_1^{\alpha} J_2^{\alpha+1} (a_1+\delta_1 Q_1(E_1-H_1))^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha \\ \times\left(\begin{array}{c} \times\Big\{E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha + E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha \\ +\Big(E_1^\gamma J_1^\alpha (1-\alpha)(a_2+\delta_2 Q_2(E_2-H_2))^\alpha + E_2^\gamma J_2^\alpha (1+\alpha)(a_1+\delta_1 Q_1(E_1-H_1))^\alpha\Big) \\ \times Ln\left(\frac{J_1(a_2+\delta_2 Q_2(E_2-H_2))}{J_2(a_1+\delta_1 Q_1(E_1-H_1))}\right)\Big\} \end{array} \right) \end{array}}{\left(E_1^\gamma J_1^\alpha (a_2+\delta_2 Q_2(E_2-H_2))^\alpha + E_2^\gamma J_2^\alpha (a_1+\delta_1 Q_1(E_1-H_1))^\alpha\right)^3} \quad \text{(A28)}$$

$$\frac{\partial f_1}{\partial \gamma} = \frac{\begin{array}{c} E_1^\gamma E_2^\gamma J_1^2 J_2 (a_2+\delta_2 Q_2(E_2-H_2)) \\ \times\Big(E_2^\gamma J_2(a_1+\delta_1 Q_1(E_1-H_1)) - E_1^\gamma J_1(a_2+\delta_2 Q_2(E_2-H_2))\Big)(Ln(E_1)-Ln(E_2)) \end{array}}{\Big(E_1^\gamma J_1(a_2+\delta_2 Q_2(E_2-H_2)) + E_2^\gamma J_2(a_1+\delta_1 Q_1(E_1-H_1))\Big)^3} \quad \text{(A29)}$$

$$\frac{\partial f_2}{\partial \gamma} = \frac{\begin{array}{c} E_1^\gamma E_2^\gamma J_1 J_2^2 (a_1+\delta_1 Q_1(E_1-H_1)) \\ \times\Big(E_2^\gamma J_2(a_1+\delta_1 Q_1(E_1-H_1)) - E_1^\gamma J_1(a_2+\delta_2 Q_2(E_2-H_2))\Big)(Ln(E_1)-Ln(E_2)) \end{array}}{\Big(E_1^\gamma J_1(a_2+\delta_2 Q_2(E_2-H_2)) + E_2^\gamma J_2(a_1+\delta_1 Q_1(E_1-H_1))\Big)^3} \quad \text{(A30)}$$

$$\frac{\partial W_1}{\partial \gamma} = \frac{2E_1^{2\gamma} E_2^\gamma J_1^3 J_2 (a_1+\delta_1 Q_1(E_1-H_1))(a_2+\delta_2 Q_2(E_2-H_2))^2 (Ln(E_1)-Ln(E_2))}{\Big(E_1^\gamma J_1(a_2+\delta_2 Q_2(E_2-H_2)) + E_2^\gamma J_2(a_1+\delta_1 Q_1(E_1-H_1))\Big)^3} \quad \text{(A31)}$$

$$\frac{\partial W_2}{\partial \gamma} = -\frac{2E_1^{\gamma} E_2^{2\gamma} J_1 J_2^3 (a_1+\delta_1 Q_1(E_1-H_1))^2 (a_2+\delta_2 Q_2(E_2-H_2)) (Ln(E_1)-Ln(E_2))}{\Big(E_1^\gamma J_1(a_2+\delta_2 Q_2(E_2-H_2)) + E_2^\gamma J_2(a_1+\delta_1 Q_1(E_1-H_1))\Big)^3} \quad \text{(A32)}$$

Appendix C. Supplement

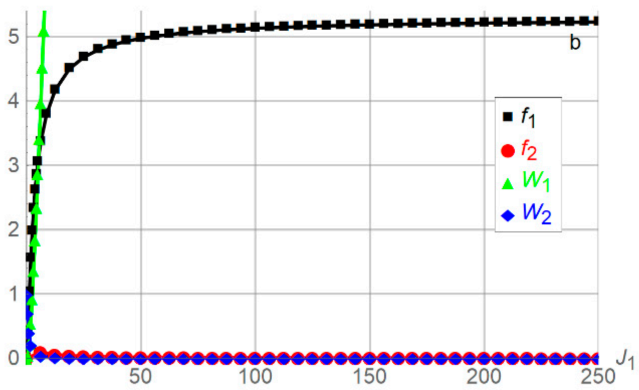

Figure A1. Supplementing Figure 2 panel b to illustrate $\lim_{J_1 \infty} f_1 = \frac{0.9}{0.17} \approx 5.29$, the plotranges along the horizontal and vertical axes are extended. Plotted are the media manipulation efforts f_1 and f_2 and the expected utilities W_1 and W_2 for players 1 and 2 as functions of J_1 relative to the benchmark parameter values $a_i = J_i = Q_i = \alpha = \gamma = \delta_i = 1$, $E_1 = 0.1$, $H_1 = 0.8$, which imply $E_2 = 0.9$, $H_2 = 0.2$, $i = 1, 2$.

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