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BACHELOROPPGAVE

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Tittel på bacheloroppgaven: Development of a game as a tool for enhancing learning of control theory	
Emneord: Dynamical systems. Feedback control design. Educational games.	Sidetall: 70 + vedlegg/annet: 11 Stavanger 15.mai 2021

Abstract

Game based learning has emerged as an effective tool for educational purposes, and has shown a positive effect in student motivation. The purpose of this thesis was the design of an educational physical card game, with the aim of enhancing student motivation and perception of basic control theory concepts. With the backdrop of the introductory control theory course ELE320 - Reguleringsteknikk, core concepts such as dynamic systems, system characteristics, transfer functions and feedback control were implemented onto playable cards. The card game was split into two different playable phases: plant analysis and closed feedback control, respectively. Through the use of MATLAB, players can simulate systems and perform control system analysis specified by the game's missions or card effects. Preliminary assessment lead to the conclusion that the card game has the potential to be an engaging educational tool.

Contents

Abbreviations

ODE .... Ordinary Differential Equation

LTI .... Linear Time Invariant

STEM .... Science Technology Engineering Mathematics

PID .... Proportional-Integral-Derivative

NMP .... Non-Minimum Phase

MSD .... Mass-Spring-Damper

BIBO .... Bounded-Input, Bounded-Output

SISO .... Single-input single-output

GM .... Gain Margin

PM .... Phase Margin

Part I
Introduction

1.1 Preface

Control theory is an interdisciplinary branch associated mainly with electrical engineering and mathematics, and deals with dynamic systems and their behavior. A fundamental step in performing system analysis is mathematical modeling of a given system. Here, ordinary differential equations (ODE) are used to describe the behaviour of the particular system, which requires a combination of principles like physics and thermodynamics. Once these ODE's are properly derived, the system dynamics can be formulated into a "state space" form, which acts as a canonical template for analysis. The general purpose of control theory is to control a feedback system so that the output follows a given *reference*. To achieve this, a controller is needed. The controller compares the output signal with the given reference, calculating the difference between them which is called the *error signal*. The error signal is then used to compute the input. The controller's job is to continuously compare the output to the reference, trying to make the error signal converge to zero.

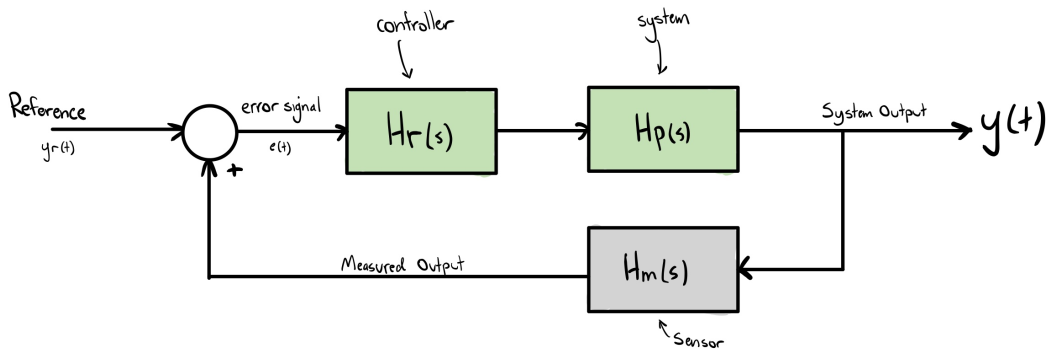

Figure 1: Block diagram feedback loop

In the current curriculum for the course $ELE320-Reguleringsteknikk$, the sensor dynamics are not included in the feedback loop. Figure 1 illustrates the block diagram for the feedback loop, where the sensor dynamics is specified as $H_m(s)$. Thus, it is assumed that the system output equals the measured output.

1.2 Summary

The motivation behind this thesis is the design of a card game as a tool to aid and engage students in the course *ELE320 - Reguleringsteknikk*, with the aim of developing a deeper intuition on the core topics. The report is divided into four main parts. Following this section, the remaining parts of this report are organized as follows:

- **PART I - Introduction:** This part contains the main objective of this thesis and the current state of the art on educational boardgames and their pedagogical effect on students. Then, the general strategy on how the card game should be designed is explored. The card game was split into two separate playable phases, covering the analysis of the plant, and the closed-loop feedback with a controller, respectively.

- **PART II - Phase 1:** In this part, phase 1 is described. A mass-spring-damper system considered for implementation as the first playable system. Furthermore, card effects and missions are designed to complement the game.
- **PART III - Phase 2:** This part follows the same core game design as for the first playable phase, but with emphasis on feedback control.
- **PART IV - Assessment/Conclusive remarks:** In the last part, preliminary assessments of the game is presented. Moreover, conclusive remarks regarding assignment goals are discussed.

1.3 Assignment

Historically, education has consisted mainly of traditional techniques such as classroom lectures on blackboards, oral lectures and assigned exercises. In present times more modern techniques have been implemented to achieve a more effective educational experience. Among these, the coined term gamification has been reported to be a great motivator for students [1]. Games provide clear goals and give a sense of accomplishment, which can engage students and make courses more enjoyable.

Taken from the thesis description:

The main goal of this project is the development of either a boardgame or a (possibly collectable) card game that enhances the learning experience in control theory courses, so that the motivation of the students is enhanced and their overall satisfaction and learning outcome is improved.

The game should be designed in such a way that different simultaneous goals are accomplished:

- *It should have simple mechanics, so that even students who are not experienced with boardgames and card games could use it.*
- *It should be fun and engaging, ideally pushing the students towards playing it often.*
- *It should help in teaching theoretical concepts/notions related to control theory.*
- *It should help in teaching how to apply those concepts for solving problems.*
- *It should help in teaching how to use MATLAB and other similar software.*
- *It should be expandable, so that new "expansions" related to other topics (possibly even not restricted to control theory) could be developed in the future.*

Motivated by the thesis description, this project concerns the design and physical prototype of a card game aimed at enhancing understanding of concepts related control theory courses. This report aims at describing the obtained results.

1.4 State of the art

In spite of their ludic reputation, playing games have a lot in common with scientific research. Playing games forces players to experiment and form hypotheses, then test their hypothesis and draw a conclusion based on the experiments results. This method shares a strong association with the scientific methods that are universally taught [19]. Solving problems is the main aspiration of games. As stated by game designer Raph Koster, *"Fun from games arises out of mastery. It arises out of comprehension. It is the act of solving puzzles that makes games fun. In other words, with games, learning is the drug" [20].* Games can contribute to learning [1], and if structured well, they could assist in educating various concepts and outcomes.

In recent times, there has been a growing attentiveness and interest to how playing games can lead to better health and well being. [5][6]. Indulging in games has reportedly suggested positive effects regarding school engagement and mental health, among others [6]. By playing games it appears that stress and anxiety levels can be reduced through relaxation and a satisfying tensity [8], leaning more toward competitiveness and the strive for success.

When it comes to cognitive abilities, some research has showed that playing games correlates to slowing down cognitive decline that comes naturally with aging, possibly even reducing the probability to develop Alzheimer's and related disorders by stimulating the brain. In a study conducted by Anguera [11], a group of older adults showed improvements in cognitive control and multitasking. Additionally, studies suggest that children diagnosed with dyslexia benefit from visual stimulation when playing video games. According to a study by Sandro Franceschini [12], the short term phonological memory in dyslexic children improved after playing action video games (AVG), as shown in Figure 2.

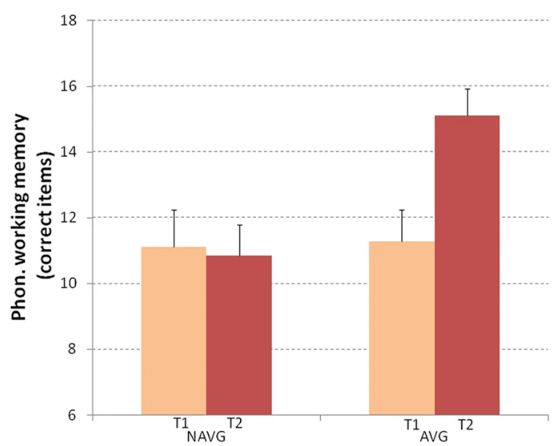

Figure 2: Phonological working memory measured before (T1) and after (T2) NAVG and AVG (Non action/action video game) Improvement observed in playing AVG [12].

Whenever it is by design or just by trial and error, more recent games have started to implement a lot of good practises for those who are interested in learning by generating environments that encourages players to invest a good amount of time in learning. Naturally, spending a lot of time on a particular task leads to learning more about a subject, whether it is games for entertainment or educating purposes. Games in this regard excel at encouragement, because playing games is often driven by interest. People are wired to crave things that result in some kind of reward and a sense of accomplishment. Research shows that playing various games can contribute to basic psychological needs [9]. The work reported a positive impact within autonomy, competence and relatedness to people who spend time on playing games. Furthermore, when indulging in games the brain releases dopamine as reward for achieving goals, reportedly imitating the amount released when using recreational drugs [10].

While there are numerous games that focus solely on entertainment and autonomy for the sake of intrinsic needs, more serious games are emerging. These types of games are intentionally designed on the grounds of educating the players and helping them acquire useful skills. In a study conducted by J. Shawn Jones et al. [14], a test group of students enrolled in a pharmacology course undertook a pre-test and a post-test after playing an educational boardgame about the autonomic nervous system. The participants scores in the tests were compared to assess immediate improvements, along with comparing examination scores between the board game participants (PART) and the rest of their non participating (NPART) classmates. The results from the study, shown in Figure 3, indicate that the boardgame participants scored higher in the post test after using the boardgame. Moreover, the PART group scored higher on the examination than their NPART peers.

Assessment	**PART**	**n**	**NPART**	**n**	**Independent Samples *t* Test *p* value***
Pretest Score	47.1	22	–	0	–
Posttest Score	68.3	20	–	0	–
ANSQ Score	87.6	21	78.8	50	0.01
CTLQ Score	78.0	21	84.0	50	0.16
Examination Score	86.3	21	79.6	50	0.04

*PART vs NPART, independent samples *t* test, $p<0.05$; CTLQ, control examination questions; ANSQ, autonomic nervous system examination questions

Figure 3: Scores (% mean) of PART and NPART students[14]

Board games in education can also serve other purposes than directly educating. Teachers have used board games as an alternative examination method rather than using traditional test methods. A study in Poland was conducted where two groups of 131 students participated in a physics course. The groups were split up in an experimental group and a control group. A boardgame was used by the experimental group to assess their knowledge in the subject [15]. Both groups took a pre-test after completing the courses in waves and vibrations and in optics, respectively. A week after the experimental group´s boardgame test, both groups partook in a post test. Reportedly, as shown in Figure 4, the experimental group scored better in the board game test than in the pre-test, while also scoring higher in the post-test than the control group. Results gathered from the alternative assessment method showed reduced test anxiety as well as being good motivator for further learning.

Variable	Characteristics						
	Mean	95% Confidence interval for mean	Median	Lower quartile	Upper quartile	Standard deviation	
FA^1 [%]	47.05	(44.38; 49.72)	45.41	40.31	54.67	8.00	School 1
GS^1 [%]	69.74	(66.46; 73.03)	69.71	64.58	77.47	9.86	
PT^1 [%]	58.44	(54.29; 62.56)	57.58	48.48	69.70	12.44	
FA^2 [%]	58.58	(53.07; 64.07)	55.37	46.68	68.54	16.25	School 2
GS^2 [%]	80.01	(76.27; 83.76)	79.81	71.56	87.84	11.07	
PT^2 [%]	67.76	(62.56; 72.97)	68.36	56.78	76.82	15.38	

Figure 4: [15] Results from the experimental groups. FA - avg former achievement, GS - score in game, PT - post game test

Opinions expressed by the students on the assessment form were largely positive. Some of the expressed opinions are reported hereafter [15].

- *"This is a good option to test for people who are weaker in calculation. Not everyone is able to solve a complex task, but anyone can learn theory."*
- *"I think that we have learned and invented more during this game than during a written test. It was a very good possibility for integration."*
- *"This form of the test was very good, because you could learn also during the test. It teaches cooperation in the way you could have fun."*

It seems that implementing games, specifically board games, into education is providing a positive basis for learning. According to data gathered from Scopus [16], board games in education are gaining traction in recent years, with relevant published articles increasing each year (see Figure 5).

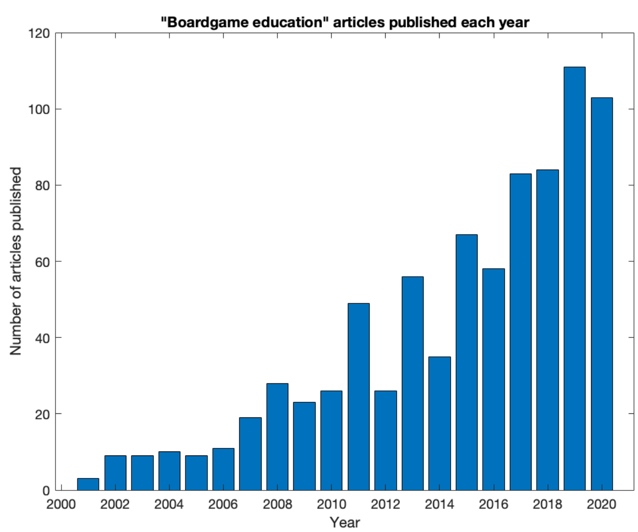

Figure 5: Number of articles related to 'boardgame education' in Scopus.

By narrowing the scope and focusing more on boardgames related to Science, Technology, Engineering and Mathematics (STEM) fields, specifically engineering practices, the perceived notion is that they are harder to come by. In a project by Adam M. Moss et al. [17], a boardgame called Space Tug Skirmish (STS) has been designed for educational use in systems engineering. Students claim to have developed a deeper understanding of core concepts, and to be more easily equipped with applying these concepts in real life problems. Higher education and universities within STEM fields appear to increasingly implement game-based learning. At Cornell University, introductory courses related to computer programming have been introduced through a virtual robot-based simulator using MATLAB [21]. The student's enthusiasm of the course was heightened by holding tournaments towards the end of the semester where the students went head-to-head, each of them controlling a robot with the goal of outlasting each other through fuel consumption.

Aachen University provides another example of university where STEM related games have been implemented. A competitive game has been used for learning mathematical logic related to computer science, with the intent to train collaboration and cooperation [22]. Later evaluation showed that the game was user friendly, and thanks to the presence of multiplayer and competition the game was perceived as fun and motivating by the students.

Gamification of control theory courses seems to be even more scarce, although there are instances where smaller projects have emerged whereas some relatively antiquated.
Introductory control theory courses are usually frequented by students that come from varying study backgrounds within STEM fields, who are often interested in solving real world problems. The groundwork for the courses is often analytical and mathematical in nature, so the gap between more application oriented students and the theoretical nature of the course can become demotivating. Through game-based learning, a stronger connection between theory and application can be established.

The graphical simulation game *DuckMaze* [3] was developed for graduate students in linear control theory courses. This game presents a mass-spring-damper system placed under a body of water, where the systems parameters are modifiable in real time with the goal of controlling the position of the water surface.

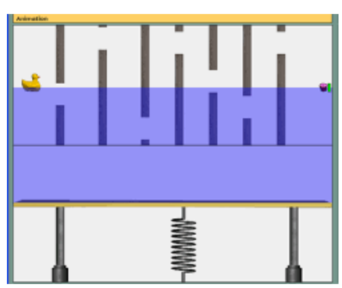

Figure 6: DuckMaze game [3]

Another instance of gamification within control theory is the *PIDstop* game [4], which is an assortment of various systems such as harmonic oscillators, magnetic levitation and hydrodynamic forces. Here, players can simulate controller parameters interactively to gain a deeper intuition as to how PID tuning affects a system.

The University of Stuttgart includes in the basic control theory course a variety of educational games that aim to give the students more real life intuition when it comes to dynamic systems [23]. One game is the *submarine game*, where players manoeuvre a submarine through water while trying to keep it stable. During the initial parts of the course, students have yet to be introduced to closed loop systems so they control the submarine manually in an open-loop architecture, highlighting the advantages of automatic control. Later on, a feedback controller is introduced in the loop and players can experiment with PID tuning. In a study conducted by the professors at the University of Stuttgart [24], students were questioned on their perception of the submarine game. The majority of the students found the game to be entertaining and educational.

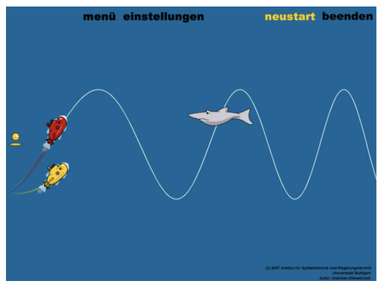

Figure 7: GUI of the submarine game developed by University of Stuttgart [23]

Another game from the University of Stuttgart is the *spaceball* game [23]. Given various levels of difficulty, players have to make a ball follow given trajectories using controller dynamics. On harder levels, solar wind is introduced acting as a disturbance on the dynamic system. Through repetitive simulation, the controller may be improved by analyzing the result data. Players have the opportunity to study disturbance rejection and the robustness of the closed loop system.

It seems that the emerging field of game-based learning that seeks to utilize the gamification of subjects can be an effective tool for educating. Learning through games has shown a positive effect in both motivation and mental well-being. Gamification has the potential to target the origin of the challenges associated with educating students, and even aid more experienced individuals in gaining familiarity with new concepts and ideas [18]. The making of a boardgame to aid basic control theory courses could lead to improvements in the quality of the course, while at the same time engaging students.

2 General strategy

As this project is assumed to be implemented as a tool for the course ELE320, a natural course of action is to implement the curriculum into a card game form that highlights the core concepts of the course. It has been reported that although the topics covered in control theory usually follow a natural progression, if a student falls behind on some topics, learning following topics will quickly become difficult. By playing and experimenting with the card game, the goal is that students will gain a better overview of control theory as a whole.

As per the 2021 syllabus, the course is divided into five main topics. In short:

1. **Modeling of dynamical systems**
   - This part consists of mathematical modeling of electrical and mass systems, as well as basic notations of thermal and mechanical systems
2. **Analysis in the state space domain**
   - Matrix manipulation, state space representations, classification of systems, linearization, solutions to state and output equations
3. **Analysis in the Laplace domain**
   - The Laplace transform and its importance for solving ODEs, transfer functions, system stability, responses, dominant pole approximations, analysis of first and second order systems, the effect of zeros
4. **The frequency response**
   - The notion of frequency response of a system, Bode plots, filters
5. **Feedback control design**
   - Open and closed loop control, PID control, design of PID controllers, analysis of feedback control systems, PID design based on transfer functions.

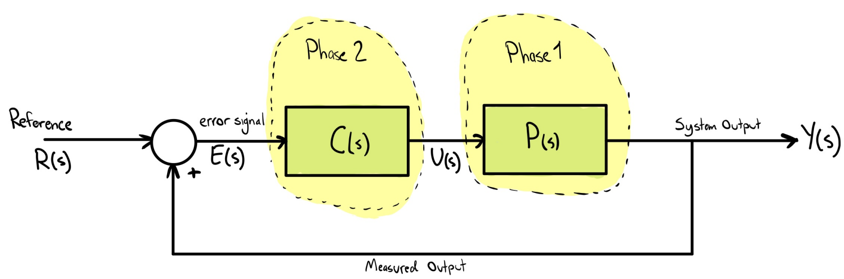

Figure 8: Relationship between the boardgame phases and the elements of a feedback control system.

Based on the curriculum layout, the card game was divided into two separate playable phases. Here, each phase addresses different parts of the curriculum naturally following the layout of the course, illustrated in Figure 8. Phase 1 will address topics 1 to 3, with the main objective being the creation of the system part of the control loop. Furthermore, phase 2 will address the controller part of the loop, covered mainly by topic 5. Topic 4 will not be implemented at this stage, but can be implemented at a later stage due to the game being fairly expandable.

2.1 Core mechanics strategy

As stated in the last section, the card game was split into two phases, addressing the mathematical characterization of the plant and the design of the control system, respectively. As the control theory course is rather linear in nature, with new topics building on previous topics, it would be intuitive to implement them onto composite cards to establish a clearer overall view of the curriculum.

The topics of the course are relatively intertwined, and the main strategy will be to build a complete system using all of the course components to highlight how the subjects interact. By using a solitaire like game mechanic, players will perform a chain of actions connected to the system step by step starting from the mathematical modelling cards to the final step/impulse responses of the system. The cards will have to be built in the correct order, while also corresponding to the matching system chain. How the two phases of the game are implemented is described in the following chapters. The game instruction manual is included in Appendix C.

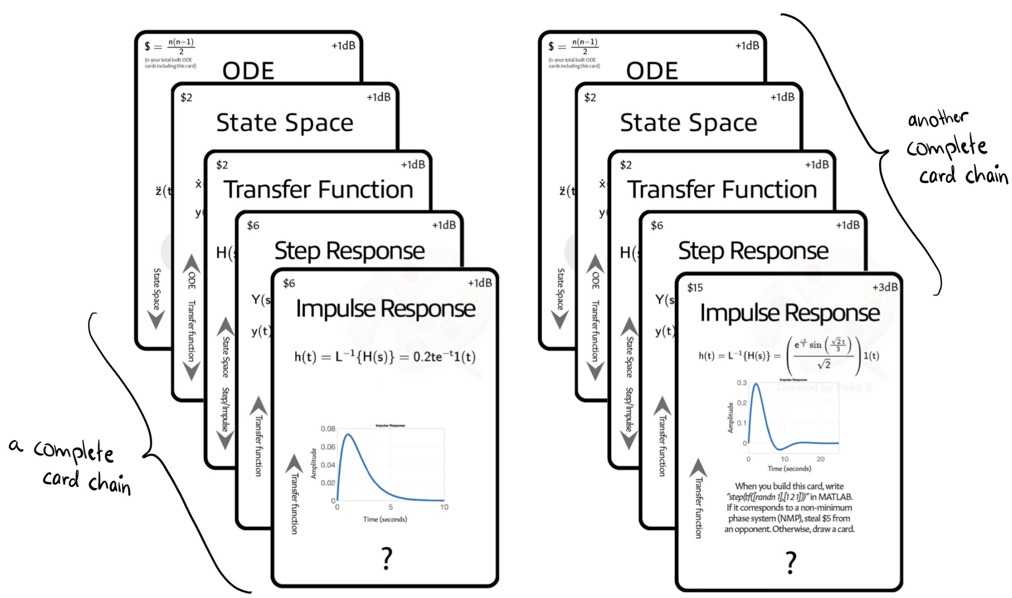

Figure 9: Complete card chains of a system.

By structuring the curriculum into playable cards that form a chain that interconnects subjects, players can hopefully develop a better intuition.

2.2 Core cards design structure

The core card design structure is shown in Figure 10, and is described below:

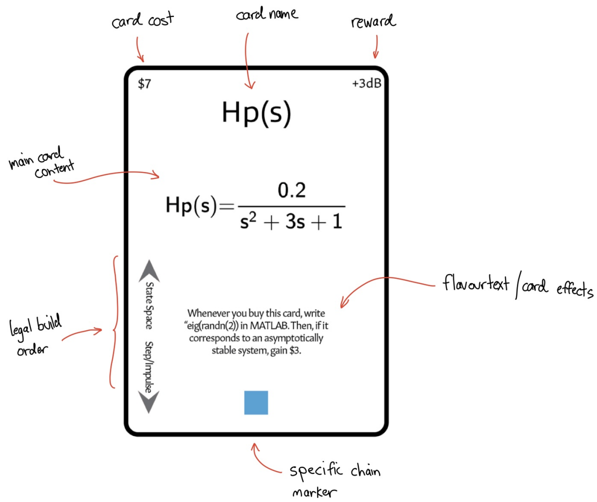

Figure 10: Example of a core card's structure.

- **Card name -** The name of the card. While some cards have the same name, the contents of the card can differ.
- **Card cost/reward -** The cost/reward of the specific card. Players use money that is earned in different ways to buy cards that they want to build. When a card is built, the reward leads to the increasing of the players total decibels. Gaining decibels increases the amount of money the player receives at the beginning of their turn. The currency system is further described in Section 2.3.
- **Main card content -** This part of the core cards contains the mathematical equations of the specific card topic. Based on the systems' ODE parameters, these equations are connected to a specific system. To legally build the chain, these cards must be matched.
- **Flavour text/card effects -** Sometimes, this is the part of the card where the effects that impact the game, appear. Some other times, a flavour text outlined in *italic* appears. This text has no effect on gameplay, as its only role is to provide additional information that allows for more educational depth. The card effects are further described in Section 3.2.
- **Legal build order -** When building a system chain, cards have to be built in a specific order starting from the ODE card. Some visual aid shows which cards are allowed to be built before and after the specified card, indicated by the card names.
- **Specific chain marker -** As described above in *"Main card content"*, this marker indicates which systems fit together. Some cards are marked with a question mark, which means that the players will have to figure out by themselves if that specific card fits the chain or not.

2.3 Currency system

Every round, players earn a passive income that can be increased by acquiring logarithmic decibels (dB) based on the voltage power gain:

$$\$ \text{ per round} = floor(10^{\frac{dB}{20}}) \quad (1)$$

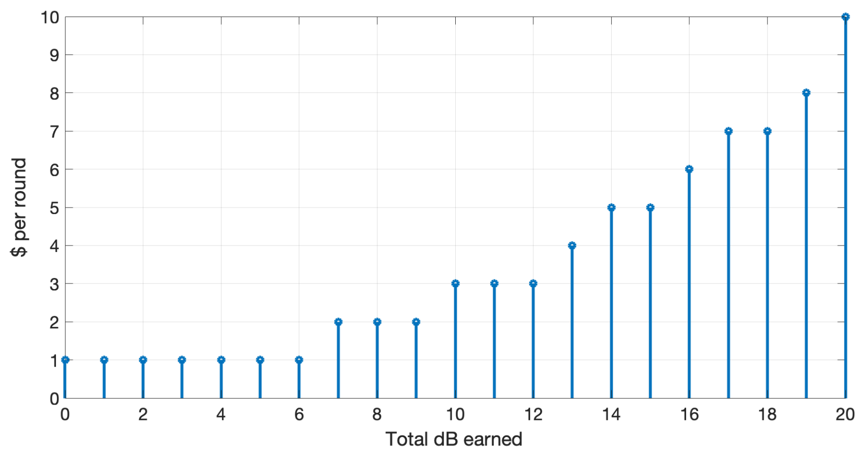

Figure 11: Passive income curve, up to 20dB

The player's income per round is adjusted through building cards, in addition to some card effects.

Part II

Phase 1

3 The plant - Core construction

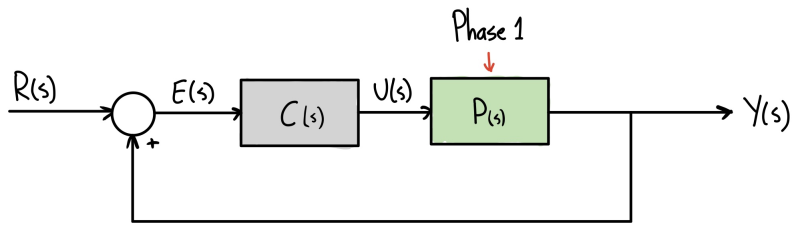

Figure 12: Standard SISO feedback control system, with focus on the system plant.

The first phase of the game will seek to implement the first part of the course *ELE320 - Reguleringsteknikk*, namely the topics:

1. Modeling of dynamical systems
2. Analysis in the state space domain
3. Analysis in the Laplace domain

where the central focus is the analysis of a plant. By establishing a preliminary mind map with emphasis on early fundamental course topics, an overview of what core cards should be implemented can be derived.

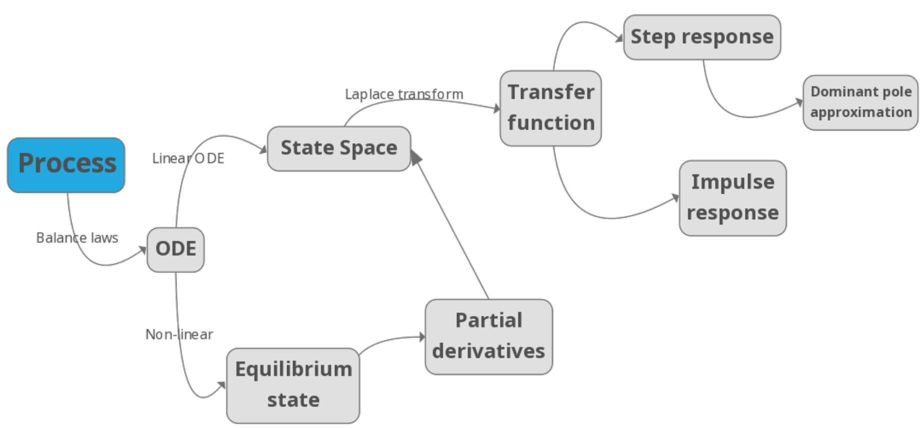

Figure 13: Correlation of fundamental topics in ELE320.

With ease of implementation in mind, the following five topics are chosen to be implemented as core cards for phase 1.

1. Ordinary Differential Equations (ODE)

   - ODEs emerge in many contexts of mathematics. The mathematical description of change is obtained through the use of differentials and derivatives. When obtaining a dynamic mathematical model of a system, the use of differential equations is prominent. If a system is described by a first order differential equation, the need to break it into simpler equations is non-essential. However, if the system is described by higher order derivatives, it can be convenient to rewrite the model as multiple first order equations that each represent the behavior of one variable, due to the first order differential equations being easier to solve. Mathematical modeling of a system is usually introduced early on in control theory courses as it forms the base of subsequent system analysis, and will therefore be introduced as the first card.

2. LTI state space representation

   - Generally, it is difficult to mathematically analyze a system of differential equations in its raw form. A more formal approach for representing linear systems of ODEs is using the state space representation. For Linear Time Invariant (LTI) systems, matrices can be used to represent the state and output equations in a compact manner.

     The system's dynamics can now be described through the state matrix A, the input matrix B, the output matrix C and the feedforward matrix D shown in equation (2) and (3). By using the state space representation, the basis for subsequent analysis is formed. Thus, this will be introduced as a card which follows the ODE card and precedes the transfer function card, which is described next.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2}$$

$$\dot{y}(t) = Cx(t) + Du(t) \tag{3}$$

3. Plant transfer function $P(s)$

   - Until this stage, students have been introduced to the modeling of dynamical systems in the time domain. The next step is obtaining an equivalent representation of an LTI system in the s-plane using the Laplace transform. The reason for the use of the Laplace transform is to simplify mathematical operations when dealing with differential equations. For example, convolution in the time domain becomes multiplication in the s-domain. Transfer functions describe how an input signal is modified when passing through a system, and can be used to describe the response to an arbitrary input signal. The transfer function can explain the system characteristics such as poles and time responses. By using the state space representation of the system the transfer function can be computed as in equation (4), and is thus introduced as the third card in the chain.

$$H(s) = C(sI - A)^{-1}B + D \tag{4}$$

4. Unit step response
    - Now that the transfer function of the system has been introduced, a natural progression would be to look at its output step response. When designing a control system, one of the main concerns is how the system responds to a step input. A step change in the input value acts as a drastic disturbance to the system, and allows for identification of the process model from the step response's data. Typical profiles of the reference signal can be step chains, as the operating point moves from one value to another. Peak time, percent overshoot, settling/rising time and the steady state error are all physical parameters that can be extracted from the step response. Thus, they can be connected mathematically to the transfer function, at least for first and second order systems. The lower order (1st and 2nd) systems are easy to characterize, but is more complicated with higher order systems. One method to make the higher order systems simpler to understand is to approximate the system by a lower order system, known as the dominant pole approximation. This is done by assuming that the slowest pole of the system dominates the systems response, which means that the faster poles can be ignored. Thus, the step response is introduced as a core chain card.
5. Impulse response
    - Unlike the step response, the impulse response is the response of a given system when it is excited by a signal taking infinite amplitude over an infinitesimal amount of time, also called the Dirac delta function. Generally, the impulse response describes how the system reacts as a function of time. The impulse response can be proven to be the inverse Laplace transform of the transfer function. Therefore, given an unknown system, the impulse response can be used as an equivalent description of the system, as the output signal can be obtained by convoluting the input signal with the impulse response.

3.1 People cards - Purpose and structure

At the start of each game the players will choose a person card to play with. These cards, known as the "people cards", include historical figures who played to some degree a core part in developing concepts related to control theory. Each person card includes an effect that broadly parallels that individuals past accomplishments.

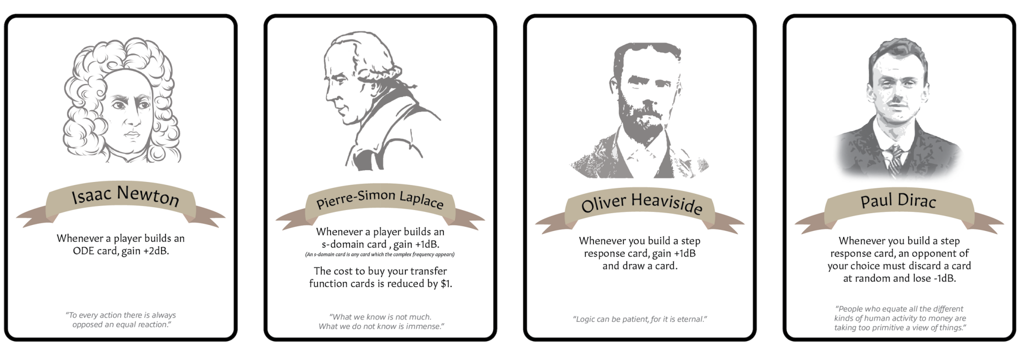

Figure 14: Historical figures associated to control theory.

3.2 Card effects

Card effects are described on some of the cards. Effects can either be triggered by some condition fulfilled when the card is built, or when another action is performed in the game. Figure 15 shows an example of the implementation of effects on the cards, in addition to the indicator for trigger cards which helps remind the player to check for conditions during the game. Although some effects are added only for the sake of increasing the game's mechanical depth and flow, many effects are associated with control theory subjects. The following section describes the implemented control theory effects. All card effects for phase 1 can be inspected on the cards listed in Appendix A.

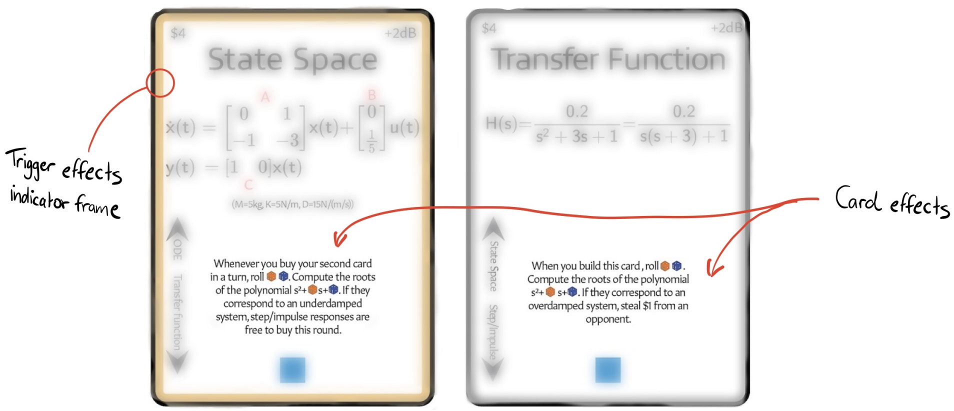

Figure 15: Card effect examples and indicator frame for trigger cards.

3.2.1 Steady-state output effects

- *When you build this card, all players with a built state space card roll a die. Calculate the steady-state output y_o corresponding to the input step signal $u_0(t)1(t)$. The owner of the highest y_o gains $3 and +3dB. If you are that player, you gain $6 and +6 dB instead. Tip:* $y_o = -C \cdot inv(A) \cdot B \cdot u_0$

The steady-state output of a system is defined as the output value when the transients of the system have dissipated, which means that the response has reached its steady state. By using the state space matrices A, B and C on the *State Space* cards, in addition to the input signal, the steady-state output can be calculated by the players.

3.2.2 Damping characteristics effects

- *When you build this card, reveal the top card from your deck. If it relates to an underdamped system, draw that card.*
- *Whenever you buy your second card in a turn, roll 2 dice (A, B). Compute the roots of the polynomial* $s^2 + As + B$*. If they correspond to an overdamped system, steal $1 from an opponent.*
- *Whenever you buy your second card in a turn, roll 2 dice (A, B). Compute the roots of the polynomial* s^2+As+B*. If they correspond to an underdamped system, step/impulse responses are free to buy this round.*

- *Whenever you play this card, write "damp(tf([1],[1 randn 1]))" in MATLAB. If the damping ratio is >0.5, draw a card.*

These effects require players to calculate arbitrary damping ratio coefficients. Furthermore, players will have to acquire an understanding of the damping ratio parameters and how they are classified in order to earn the rewards.

3.2.3 Stability characteristics effects

- *When you buy this card, write "eig(randn(2))" in MATLAB. Then, if it corresponds to the eigenvalues of the state matrix of an asymptotically stable LTI system, steal $3 from an opponent of your choice.*
- *When you buy this card, write "eig(randn(2))" in MATLAB. Then, if it corresponds to the state matrix of an asymptotically stable LTI system, gain $2.*
- *When you buy this card, write "eig(randn(2))" in MATLAB. Then, if it corresponds to the eigenvalues of the state matrix of an unstable LTI system, steal $3 from an opponent.*

The effects correlated to system stability characteristics require players to examine the eigenvalues of a state matrix and what they represent. From a theoretical perspective, the stability of a state matrix can be assessed by looking at its eigenvalues. Asymptotical stability requires all the eigenvalues of A to be negative; if any one of the eigenvalues is positive, the system is classified as unstable. However, repeated eigenvalues on the imaginary axis might also lead to instability, but is not included in the game as it goes beyond the scope of basic control theory courses.

3.2.4 Poles and dominant pole approximation effects

- *When you buy this card, write "eig(randn(3))" in MATLAB. If the dominant pole approximation is 2nd order, steal $2 from an opponent.*
- *Whenever a chain is completed, all players roll two dice (A, B), then compute the plant tf=(1, [A B 1]). The player with the faster pole draws 1 card. If you are that player, you draw 2 cards instead. Tip: pzmap(tf)*

As referenced in Section 3, the dominant poles in a stable system are the poles lying closest to the imaginary axis, as their response components last longer than those corresponding to the poles located further into the left half plane. By only retaining the dominant pole or pole pair, a system can be approximated using a lower order model. For example, an arbitrary transfer function given by:

$$H(s) = \frac{50}{(s+10)^2(s^2+2s+5)} \tag{5}$$

has poles at s = -10, -1 ± 2i. Figure 16 shows the dominant pole approximation by neglecting the fast poles at s = -10. By maintaining the same gain for both transfer functions, the approximated second order transfer function becomes:

$$H(s) \approx \frac{0.5}{(s^2+2s+5)} \tag{6}$$

This card effect makes the player spot the arbitrary dominant poles given by the MATLAB command.

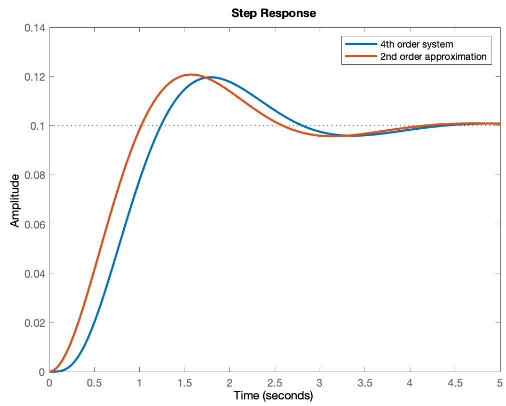

Figure 16: Dominant pole approximation of a 4th order system.

3.2.5 Non-minimum-phase system effects

- *When you build this card, write "step(tf([randn 1],[1 2 1])" in MATLAB. If it corresponds to a non-minimum phase system (NMP), steal \$2 from an opponent. Otherwise, draw a card.*

By playing this effect, the player will compute an arbitrary transfer function and spot if it corresponds to a non-minimum phase (NMP) system. NMP systems are causal and stable systems, whereas its inverse is causal but unstable. This leads to an initial undershoot of the response, which is also classified by the transfer function having a positive zero (see Figure 17).

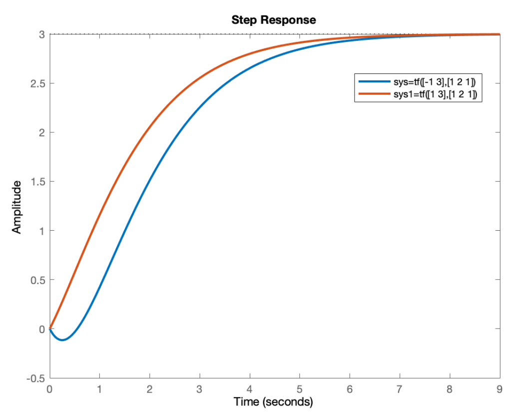

Figure 17: Two systems, which are equal up to a change in the sign of the zero.

4 Implementation of a mass spring damper system

The system chosen for the implementation of a first prototype of the game is the widely known mass-spring-damper (MSD) system. Some students can recognize this mechanical system from control theory lectures, in addition to it being a common case study in basic physics courses. This brings familiarity and can be used for the transition from the physical modeling acquired in previous subjects to the abstract analysis typical of control theory. The following sub-chapters will describe this system in general terms which will then be used for calculating the systems implemented in the cards. By roughly following this described approach, the implementation of additional systems can be done at a later stage.

For the MSD system there will be three different card chains that can be used to familiarize the players with different system characteristics, and the effect of the position of the poles in the complex plane. The card chains include an under-damped system, an over-damped system and a critically damped system, indicated with green triangle markers, blue square markers and red circle markers, respectively.

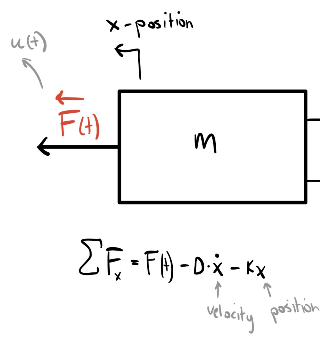

Figure 18: Mass-spring-damper free-body diagram

4.1 Ordinary Differential Equation (ODE)

Newton's laws of motion establishes the basis for modeling mechanical systems. Newton's second law states that the sum of all forces acting on a body equals the product of its mass and acceleration [25]:

$$\sum F(t) = ma(t) = m\ddot{x}(t) \tag{7}$$

In the mechanical system shown in Figure 18, the spring force $F_s(t)$ is proportional to the displacement $x(t)$ of the mass m, while the damping force $F_d(t)$ is proportional to the velocity of the mass, $v(t) = \dot{x}(t)$. The applied force $F(t)$ opposes the force of the spring and damper, and the summed forces are described along the x-axis as:

$$\sum F_x(t) = F(t) - D_d\dot{x}(t) - Kx(t) = m\ddot{x}(t) \tag{8}$$

where $F_s(t) = Kx(t)$ and $F_d(t) = D_d\dot{x}(t)$.

Using the impulse balance law the position of the mass, *x(t)*, can be acquired by:

$$\frac{dI(t)}{dt} = \sum F_{positive}(t) - \sum F_{negative}(t) \tag{9}$$

where *I(t)* is given by $I(t) = mv(t)$, and $v(t) = \dot{x}(t)$. Then, establishing the ODE is done by:

$$\ddot{x}(t) = \frac{1}{m}(F(t) - D_d\dot{x}(t) - Kx(t)) \tag{10}$$

4.2 State space representation

State space representation of LTI ODE's is generally represented as in Equation (2) and (3).

Furthermore, for obtaining the state space representation, the input, state and output variables are expressed as:

- $x_2(t) = \dot{x_1}(t)$ - Velocity
- $y(t) = x_1(t)$ - Output is the position of the mass
- $F(t) =$ *u(t)* - System input

which grants the linear first order differential equation described by:

$$\dot{x}_1(t) = x_2(t) \tag{11}$$

$$\dot{x}_2(t) = \frac{1}{m}(u(t) - D_d \cdot x_2(t) - K \cdot x_1(t)) \tag{12}$$

The state variable x_1*(t)* expresses the potential energy stored by the spring, while the state variable x_2*(t)* describes the kinetic energy stored by the mass m. Given the ODE in equation (46) and (47), a compact representation of the dynamic system can be expressed as:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \frac{-K}{m} & \frac{-D_d}{m} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t) \tag{13}$$

As the output to be controlled is the position of the mass $x_1(t)$, the output equation is:

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \tag{14}$$

4.3 Plant transfer function, H(s)

If the compact matrix representation is known, the transfer function can be derived using the state, input, feedforward and output matrices, as follows:

$$H(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D = \frac{1}{ms^2 + D_d s + K} \tag{15}$$

The standard form of a 2nd order transfer function is given by:

$$H(s) = \frac{H(0)}{(\frac{s}{\omega_0})^2 + \frac{2\zeta}{\omega_0}s + 1} \tag{16}$$

where $H(0)$ is the static gain, ω_0 is the natural frequency and ζ is the damping coefficient. By comparing the MSD transfer function (15) to the standard form (16), the following can be extracted:

$$H(0) = \frac{1}{K} \tag{17}$$

$$\omega_0 = \sqrt{\frac{K}{m}} \tag{18}$$

$$\zeta = \frac{D_d}{2 \cdot \sqrt{K \cdot m}} \tag{19}$$

which will be used for designing MSD systems with specified characteristics.

4.4 Unit step response

In the Laplace domain, the output of a system is given by:

$$Y(s) = H(s)U(s) \tag{20}$$

and the Laplace transform of the unit step function is given by:

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 1 \end{cases} \quad \Rightarrow \quad U(s) = \frac{1}{s} \tag{21}$$

Taking into account (15) and (21), the output of the MSD system becomes:

$$Y(s) = \frac{1}{s(ms^2 + D_d s + K)} \tag{22}$$

The output response *y(t)* can be obtained by taking the inverse Laplace transform of Y(s) after rewriting it through the partial fraction expansion:

$$y(t) = \mathscr{L}^{-1}(Y(s)) \tag{23}$$

The following section describes the computation of the generic partial fraction expansion of $Y(s)$ and the output $y(t)$. For a standard second order system, the transfer function is:

$$\frac{Y(s)}{U(s)} = \frac{H(0)\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{24}$$

Depending on the damping coefficient, different output responses are obtained.

Case i - Underdamped $(0 < \zeta < 1)$

The output response of the normalised second-order system with a unit step input $U(s) = \frac{1}{s}$ becomes:

$$\begin{aligned} Y(s) &= \frac{1}{s} \cdot \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \\ &= \frac{1}{s} \cdot \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \zeta^2\omega_0^2 + \omega_0^2 - \zeta^2\omega_0^2} \\ &= \frac{1}{s} \cdot \frac{\omega_0^2}{(s + \zeta\omega_0)^2 + \omega_0^2(1 - \zeta^2)} \\ &= \frac{1}{s} \cdot \frac{a^2 + b^2}{(s + a)^2 + b^2} \end{aligned} \tag{25}$$

where (a, b) are defined as:

$$\begin{cases} a = \zeta\omega_0 \\ b = \omega_0\sqrt{1 - \zeta^2} \end{cases} \tag{26}$$

Partial fraction expansion of Equation (25) gives:

$$Y(s) = \frac{1}{s} - \frac{s + a}{(s + a)^2 + b^2} - \frac{a}{b} \cdot \frac{b}{(s + a)^2 + b^2} \tag{27}$$

By inverse Laplace transforming Equation (27), the output response in the time domain becomes:

$$y(t) = (1 - e^{-at}\cos bt - \frac{a}{b}e^{-at}\sin bt)1(t) \tag{28}$$

where (a, b) are defined as in equation (26).

Case ii - Overdamped ($\zeta > 1$) The characteristic equation for a standard 2nd order system is $s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$. Therefore, for the overdamped case, it has two real roots at $s = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1}$ The absolute value of the two roots can be defined as:

$$\begin{cases} \alpha_1 = \zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1} \\ \alpha_2 = \zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1} \end{cases} \Rightarrow \begin{cases} \alpha_1\alpha_2 = \omega_0^2 \\ \alpha_1 + \alpha_2 = 2\zeta\omega_0 \end{cases} \tag{29}$$

Thus, the output becomes:

$$\begin{aligned} Y(s) &= \frac{1}{s} \cdot \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \\ &= \frac{1}{s} \cdot \frac{H(0)\alpha_1\alpha_2}{(s + \alpha_1)(s + \alpha_2)} \end{aligned} \tag{30}$$

Furthermore, the inverse Laplace transform of the asymptotic double exponential from Equation (30) becomes:

$$y(t) = \mathscr{L}^{-1}\{\frac{H(0)\alpha_1\alpha_2}{s(s + \alpha_1)(s + \alpha_2)}\} = \Big(H(0) + \frac{H(0)\alpha_2 e^{-\alpha_1 t}}{\alpha_1 - \alpha_2} - \frac{H(0)\alpha_1 e^{-\alpha_2 t}}{\alpha_1 - \alpha_2}\Big)1(t) \tag{31}$$

Case (iii) - Critically damped ($\zeta = 1$)

Finally, for the critically damped case, the characteristic equation contains two repeated real roots. Since $\zeta = 1$, b is equal to 0:

$$\begin{cases} a = \zeta\omega_0 = \omega_0 \\ b = \omega_0\sqrt{1 - \zeta^2} = 0 \end{cases} \tag{32}$$

Therefore, the output becomes:

$$Y(s) = \frac{1}{s} \cdot \frac{H(0)}{(s + a)^2} \tag{33}$$

By taking the partial fraction expansion of Equation (33) then using the inverse Laplace transform, the output in the time domain becomes:

$$Y(s) = \frac{H(0)}{a^2 s} - \frac{H(0)}{a(a + s)^2} - \frac{H(0)}{a^2(a + s)} \Rightarrow y(t) = \Big(\frac{H(0)}{a^2} - \frac{H(0)e^{-at}}{a^2} - \frac{H(0)te^{-at}}{a}\Big)1(t) \tag{34}$$

4.5 Impulse response

The impulse response *h(t)* can be obtained through the inverse Laplace transform of the transfer function. By following a similar procedure as for the unit step response, the subsequent cases are computed.

$$h(t) = \mathscr{L}^{-1}\{H(s)\} \tag{35}$$

Case i - Underdamped $(0 < \zeta < 1)$

The transfer function $H(s)$ can be defined as:

$$H(s) = \frac{1}{ms^2 + D_d s + K} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{a^2 + b^2}{(s+a)^2 + b^2} \tag{36}$$

where:

$$\begin{cases} a = \zeta\omega_0 \\ b = \omega_0\sqrt{\zeta^2 - 1} \end{cases} \tag{37}$$

The impulse response then becomes:

$$h(t) = \mathscr{L}^{-1}\{H(s)\} = \Big(\frac{(a^2 + b^2)e^{-at}\sin bt}{b}\Big)1(t) \tag{38}$$

Case (ii) - Overdamped $(\zeta > 1)$

The two real roots are defined as:

$$\begin{cases} \alpha_1 = \zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1} \\ \alpha_2 = \zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1} \end{cases} \Rightarrow \begin{cases} \alpha_1\alpha_2 = \omega_0^2 \\ \alpha_1 + \alpha_2 = 2\zeta\omega_0 \end{cases} \tag{39}$$

Then, the transfer function becomes:

$$H(s) = \frac{H(0)}{(s + \alpha_1)(s + \alpha_2)} \tag{40}$$

By using the inverse Laplace transform on Equation (40), the impulse response becomes:

$$h(t) = \mathscr{L}^{-1}\{\frac{H(0)}{(s + \alpha_1)(s + \alpha_2)}\} = \Big(\frac{H(0)e^{-\alpha_1 t}}{\alpha_2 - \alpha_1} + \frac{H(0)e^{-\alpha_2 t}}{\alpha_1 - \alpha_2}\Big)1(t) \tag{41}$$

Case (iii) - Critically damped ($\zeta = 1$) Lastly, for the critically damped case, the two repeated roots are defined as:

$$\begin{cases} a = \zeta\omega_0 = \omega_0 \\ b = \omega_0\sqrt{1-\zeta^2} = 0 \end{cases} \tag{42}$$

so the transfer function $H(s)$ is equal to:

$$H(s) = \frac{H(0)}{(s+a)^2} \tag{43}$$

Subsequently, the impulse response becomes:

$$h(t) = \left(\mathscr{L}^{-1}\{H(s)\} = H(0)te^{-at}\right)1(t) \tag{44}$$

4.6 Overdamped MSD system

A second order system is classified as overdamped when the damping coefficient $\zeta > 1$. Overdamped transfer functions have two real poles. From equation (19) the parameters $K{=}5\frac{N}{m}$, $D_d{=}15N\frac{s}{m}$ and M=5kg can be used to obtain a damping coefficient ζ=1.5.

$$\zeta = \frac{D}{2 \cdot \sqrt{K \cdot m}} = \frac{15}{2 \cdot \sqrt{5 \cdot 5}} = 1.5 \tag{45}$$

Using the above parameters, the ODE becomes:

$$\dot{x}_1(t) = x_2(t) \tag{46}$$

$$\dot{x}_2(t) = \frac{1}{5}(u(t) - 15 \cdot x_2(t) - 5 \cdot x_1(t)) \tag{47}$$

With the ODE card being the first card players build in the chain, the variables will be generalized to fit all systems, as shown in Figure 19.

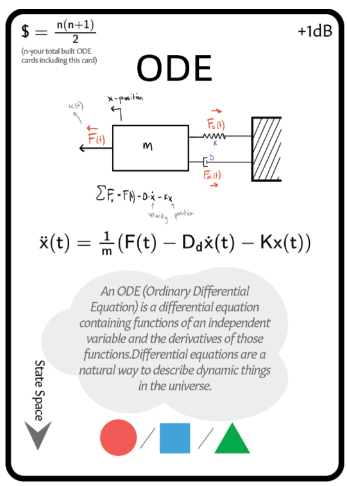

Figure 19: Ordinary Differential Equation (ODE) card.

Given the system variables and equation (13), the state space representation becomes:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \cdot u(t) \tag{48}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{49}$$

The resulting state space card is shown in Figure 20.

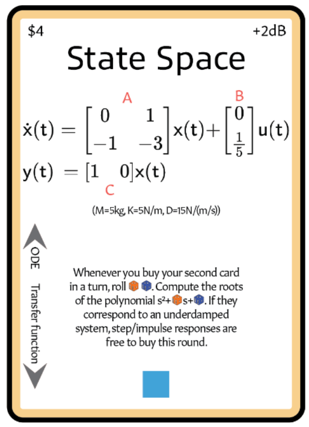

Figure 20: State space card for the overdamped MSD system.

For the plant transfer function $P(s)$, equation (15) gives:

$$P(s) = \frac{1}{s^2 + 15s + 5} = \frac{0.2}{s^2 + 3s + 1} \tag{50}$$

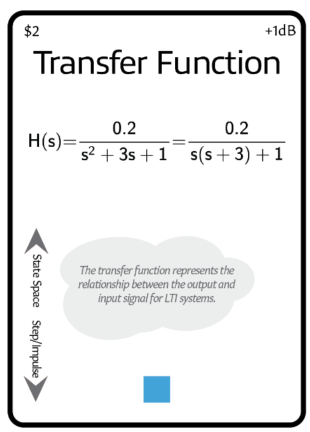

Figure 21: Transfer function card for the overdamped MSD system.

By applying equation (22) with the zero degree polynomial in the denominator equal 1, the unit step response can be calculated as:

$$Y(s) = \frac{0.2}{(s^2 + 3s + 1)s} = -\frac{0.23}{s + 0.38} + \frac{0.0034}{s + 2.61} + \frac{0.2}{s} \tag{51}$$

by using partial fraction expansion. The output response is then given by the inverse Laplace transform of Y(s).

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{-\frac{0.23}{s + 0.38} + \frac{0.0034}{s + 2.61} + \frac{0.2}{s}\} \tag{52}$$

$$= (0.2 + 0.034e^{-2.61t} - 0.23e^{-0.38t})1(t) \tag{53}$$

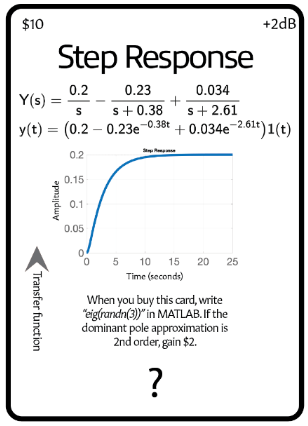

Figure 22: Step response card for the overdamped MSD system.

When computing the impulse response $h(t)$, the same procedure can be followed by applying equation (35).

$$h(t) = \mathscr{L}^{-1}\{Hp(s)\} = \mathscr{L}^{-1}\{\frac{0.2}{s^2 + 3s + 1}\} = \mathscr{L}^{-1}\{\frac{0.089}{s + 0.38} - \frac{0.089}{s + 2.61}\} \tag{54}$$

$$= (0.089e^{-0.38t} - 0.089e^{-2.61t})1(t) \tag{55}$$

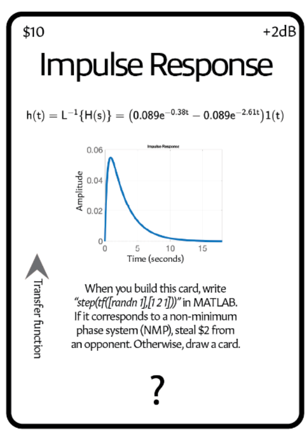

Figure 23: Impulse response card for the overdamped MSD system.

4.7 Critically damped MSD system

In the following sections, only calculations will be exhibited, as the overall procedure is similar to the one already described for the overdamped case. All phase 1 cards are included in Appendix A.

To produce a critically damped system, the damping coefficient must be ζ=1. When a system is critically damped the two poles are equal and real. From equation (19), K=5$\frac{N}{m}$, D_d=10$N\frac{s}{m}$ and M=5kg can be used to construct a critically damped system.

$$\zeta = \frac{D_d}{2 \cdot \sqrt{K \cdot m}} = \frac{10}{2 \cdot \sqrt{5 \cdot 5}} = 1 \tag{56}$$

These variables produce the following equations for ODE, state space representation, transfer function $P(s)$, step response and impulse response, utilizing the same procedure as in Section 4.6.

- ODE:

$$\dot{x}_2(t) = \frac{1}{5}(u(t) - 10 \cdot x_2(t) - 5 \cdot x_1(t)) \tag{57}$$

- State space representation:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \cdot u(t) \tag{58}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{59}$$

- Plant transfer function P(s):

$$P(s) = \frac{1}{s^2 + 10s + 5} = \frac{0.2}{s^2 + 2s + 1} \tag{60}$$

- Step response:

$$Y(s) = \frac{0.2}{(s^2 + 2s + 1)s} = -\frac{0.2}{s+1} + \frac{0.2}{(s+1)^2} + \frac{0.2}{s} \tag{61}$$

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\{-\frac{0.2}{s+1} + \frac{0.2}{(s+1)^2} + \frac{0.2}{s}\} \tag{62}$$

$$= (0.2 - 0.2e^{-t} - 0.2e^{-t} \cdot t)1(t) \tag{63}$$

- Impulse response:

$$h(t) = \mathscr{L}^{-1}\{P(s)\} = \mathscr{L}^{-1}\{\frac{0.2}{s^2 + 2s + 1}\} = (0.2e^{-t} \cdot t)1(t) \tag{64}$$

4.8 Underdamped MSD system

To construct the underdamped system, the damping coefficient must be $0 < \zeta < 1$. When the system is underdamped the two poles split into the imaginary plane becoming complex conjugate. With K=1$\frac{N}{m}$, D_d=2$N\frac{s}{m}$ and M=3kg, Equation (19) gives a damping coefficient $\zeta = \frac{1}{\sqrt{3}}$.

$$\zeta = \frac{D_d}{2 \cdot \sqrt{K \cdot m}} = \frac{2}{2 \cdot \sqrt{1 \cdot 3}} = \frac{1}{\sqrt{3}} \tag{65}$$

Using the same procedure as Section 4.6, the following equations are produced.

- ODE:

$$\dot{x}_2(t) = \frac{1}{3}(u(t) - 2 \cdot x_2(t) - 1 \cdot x_1(t)) \tag{66}$$

- State space representation:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{1}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix} \cdot u(t) \tag{67}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{68}$$

- Plant transfer function P(s):

$$P(s) = \frac{\frac{1}{3}}{s^2 + \frac{2}{3}s + \frac{1}{3}} = \frac{1}{3s^2 + 2s + 1} \tag{69}$$

- Step response:

$$Y(s) = \frac{1}{(3s^2 + 2s + 1)s} = \frac{-2 - 3s}{3s^2 + 2s + 1} + \frac{1}{s} \tag{70}$$

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\{\frac{1}{(3s^2 + 2s + 1)s} = \frac{-2 - 3s}{3s^2 + 2s + 1} + \frac{1}{s}\} \tag{71}$$

$$= (1 - \frac{e^{-\frac{t}{3}} \sin(\frac{\sqrt{2}t}{3}) + \sqrt{2}\cos(\frac{\sqrt{2}t}{3})}{\sqrt{2}})1(t) \tag{72}$$

- Impulse response:

$$h(t) = \mathscr{L}^{-1}\{P(s)\} = \mathscr{L}^{-1}\{\frac{1}{3s^2 + 2s + 1}\} = (\frac{e^{-\frac{t}{3}} \sin(\frac{\sqrt{2}}{3}t)}{\sqrt{2}})1(t) \tag{73}$$

4.9 Mission cards for the MSD system

In addition to the core card chains, the game will include missions that players need to complete during the game. Having to complete objectives throughout the game adds depth and can lay the groundwork for more strategy oriented gameplay, with the intent to increase the replay value. Missions will act as an extra layer of educational potential, increasing the amount of theory oriented mechanics to the game. Additionally, missions make for a simple way to increase possible expandability in the future. Missions will also play a core part in phase 2, see Section 6.7.

While the core chain cards construct the actual system, the missions will have emphasis on system analysis and offer the players more insight into the systems behaviour. Missions force players to not just think about what system components fit together, but also their characteristics.

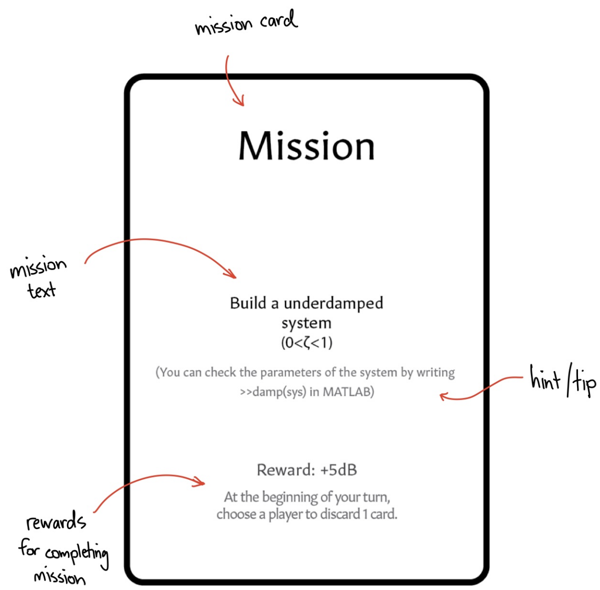

Figure 24: Example structure of a mission card.

For the MSD system, the following missions are introduced:

- Build a system with natural frequency $< 1\ rad/s$.
- Build a critically damped system ($\zeta = 1$).
- Build an under-damped system ($0 < \zeta < 1$).
- Build an over-damped system ($\zeta > 1$).
- Build a system with two real poles.
- Build a system with double poles.
- Build a system with two complex conjugated poles.

5 Example of an additional system: Water heater tank

The following section describes a preliminary sketch of the application to a non-linear dynamic system, although due to time limitation, the cards corresponding to this system have not been implemented at this stage. The purpose is to emphasize the fact that the developed game is easily expandable so that expansions can be developed, related to more complex, in-depth or separate subjects. Therefore, the assembly of these cards are purely for exploratory purposes, and only new subject cards will be calculated.

In the case of a non-linear system, linearization must be applied. Most non-linear models can not be solved directly, thus it is advantageous to look for a linear approximation to the system instead. Equilibrium points, which are the steady states of a system, are important to understand the system. Stable systems settle to an equilibrium state over time, and gives insight about the behaviour of the system. Linearization is used to analyze system behaviour in the proximity of equilibrium points, and is computed with the use of partial derivatives. Thus, for this preliminary system, the equilibrium state and partial derivatives are implemented as core chain cards.

The presented second order system is a closed water heater tank with a heating element *P(t)* as the system input. The in-flow *w(t)* with the temperature $T_i(t)$ equals the outflow *w(t)* with temperature *T(t)*, so the volume is kept constant. The complete system is described by two differential equations related to the dynamics of the tank temperature $T(t)$ and of the heating element temperature $T_h(t)$:

$$\dot{T}(t) = f_1(T_i(t), w(t), T(t), T_h(t)) \tag{74}$$

$$\dot{T_h}(t) = f_2(P(t), T_h(t), T(t)) \tag{75}$$

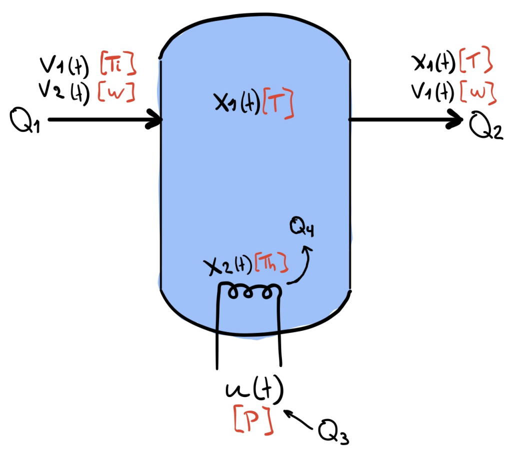

Figure 25: Water heater tank diagram with inputs/outputs.

5.1 Ordinary Differential Equations (ODE)

As this system deals with heat flow, it is characterized by the following energy balance law:

$$\frac{dE(t)}{dt} = \sum Q_{in}(t) - \sum Q_{out}(t) \tag{76}$$

The heat flow coefficients are defined by thermodynamic laws:

- $Q_1(t) = w(t) \cdot c_p \cdot T_i(t)$ - Flow into tank
- $Q_2(t) = w(t) \cdot c_p \cdot T(t)$ - Flow out of tank
- $Q_3(t) = P(t)$ - Heating element, system input
- $Q_4(t) = h_h \cdot A_h(T(t) - T_h(t))$ - Heating element output to tank

where cp is the specific heat capacity, h_h is the thermal conductivity of the heating element and A_h is the area of the heating element.

Using the energy balance law, the following two ODEs can be derived:

$$\dot{T}(t) = \frac{1}{m \cdot c_p}(w(t) \cdot c_p(T_i(t) - T(t)) + h_h \cdot A_h(T_h(t) - T(t)) \tag{77}$$

$$\dot{T_h}(t) = \frac{1}{m_h \cdot c_{ph}}(P(t) - h_h \cdot A_h(T_h(t) - T(t))) \tag{78}$$

Furthermore, for generalization and state space representation, state and input variables are expressed as:

- $x_1(t) = T(t)$ - Tank temperature
- $x_2(t) = T_h(t)$ - Heating element temperature
- $v_1(t) = w(t)$ - Flow disturbance
- $v_2(t) = T_i(t)$ - Inflow temperature disturbance
- $u(t) = P(t)$ - System input

which grant the non-linear second order differential equations described in equation (79) and (80) to be used for calculations further down the chain.

$$\dot{x_1}(t) = \frac{1}{m \cdot c_p}(v_1(t) \cdot c_p(v_2(t) - x_1(t)) + h_h \cdot A_h(x_2(t) - x_1(t))) \tag{79}$$

$$\dot{x_2}(t) = \frac{1}{m_h \cdot c_{ph}}(u(t) - h_h \cdot A_h(x_2(t) - x_1(t))) \tag{80}$$

5.2 Equilibrium state card

For construction of the equilibrium state card, some predetermined values for the system variables need to be known. For the sake of this example, the following constant values will be used to construct a critically damped system($\zeta = 1.05 \approx 1$):

- m = m_h = 20 [kg]
- cp = 4200 [J/kgC]
- cp_h = 460 [J/kgC]
- A_h = 0.02 [m^2]
- h_h = 1500 [J/smC]
- $\bar{x_2}$ = 62 [C]
- $\bar{v_1}$ = 0.075 [w]
- $\bar{v_2}$ = 5.0 [C]

Inserting the given constant values into equation (79) and (80), the equilibrium state becomes:

$$\begin{bmatrix} \bar{x_1} \\ \bar{x_2} \end{bmatrix} = \begin{bmatrix} 9.95°C \\ 62°C \end{bmatrix}, \bar{u} = 1561W \quad (81)$$

5.3 Partial derivatives card

For linearization of the system, the partial derivatives of equation (79) and (80) are computed using the equilibrium state constants:

$$\frac{\partial f_1}{\partial x_1} = \frac{1}{m \cdot cp}(-cp \cdot \bar{v_1} - h_h \cdot A_h) \quad (82)$$

$$\frac{\partial f_1}{\partial x_2} = \frac{h_h \cdot A_h}{m \cdot cp} \quad (83)$$

$$\frac{\partial f_1}{\partial v_1} = \frac{\bar{v_2}}{m} - \frac{\bar{x_1}}{m} \quad (84)$$

$$\frac{\partial f_1}{\partial v_2} = \frac{\bar{v_1}}{m} \quad (85)$$

$$\frac{\partial f_2}{\partial x_1} = \frac{h_h \cdot A_h}{m_h \cdot cp_h} \quad (86)$$

$$\frac{\partial f_2}{\partial x_2} = -\frac{h_h \cdot A_h}{m_h \cdot cp_h} \quad (87)$$

$$\frac{\partial f_2}{\partial u} = \frac{1}{m_h \cdot cp_h} \tag{88}$$

The linearized system and preliminary card becomes:

$$\delta \dot{x_1}(t) = \frac{\partial f_1}{\partial x_1} \cdot \delta x_1(t) + \frac{\partial f_1}{\partial x_2} \cdot \delta x_2(t) + \frac{\partial f_1}{\partial v_1} \cdot \delta v_1(t) + \frac{\partial f_1}{\partial v_2} \cdot \delta v_2(t) \tag{89}$$

$$= -4.1 \cdot 10^{-3} \cdot \delta x_1(t) + 3.8 \cdot 10^{-4} \cdot \delta x_2(t) - 0.2 \cdot \delta v_1(t) + 3.8 \cdot 10^{-3} \cdot \delta v_2(t) \tag{90}$$

$$\delta \dot{x_2}(t) = \frac{\partial f_2}{\partial x_1} \cdot \delta x_1(t) + \frac{\partial f_2}{\partial x_2} \cdot \delta x_2(t) + \frac{\partial f_2}{\partial u} \cdot \delta u(t) \tag{91}$$

$$= 3.3 \cdot 10^{-3} \cdot \delta x_1(t) - 3.3 \cdot 10^{-3} \cdot \delta x_2(t) + 1.1 \cdot 10^{-4} \cdot \delta u(t) \tag{92}$$

5.4 Water heater tank system cards

Subsequently, standard calculations for the rest of the cards in the chain can be made similarly to how it was done for the MSD system. Figure 26 shows how the new non-linear water heater system cards could be shaped into the card chain.

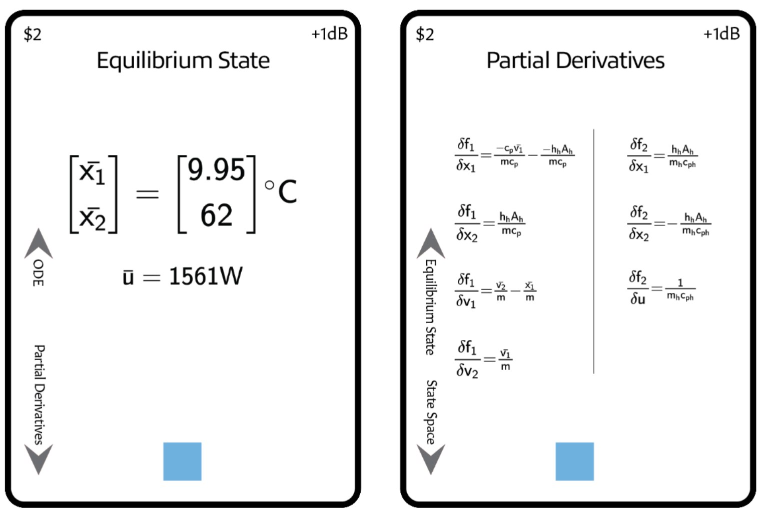

Figure 26: Cards for Equilibrium State and Partial Derivatives

Contrarily to the MSD system, the water heater system can never become underdamped. Potential missions and card effects would differ from focusing on damping coefficients and rather include other requirements, e.g. pole placement specifications. While this system was being considered for implementation, it appeared that the resulting equations and numbers became too complex for game intuition, and therefore new potential systems should strive for more numerical simplicity.

Part III

Phase 2

6 The controller - Core construction

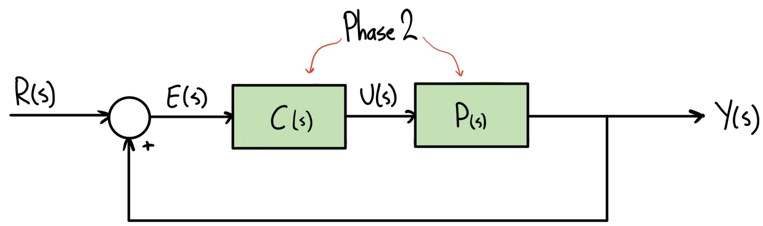

Figure 27: Standard SISO feedback control system, with emphasis on phase 2 objectives.

As the first phase is focused purely on the analysis of a plant, the second phase of the game will act as an independent extension of the plant phase by implementing the closed feedback loop. This phase of the game will implement topic 5 - "*Feedback control design*", as described in Section 2.

By following the same card chain structure as the first phase, the ensuing core cards are chosen to be implemented, and are described in the following chapters:

- Plant transfer function
- P, PI, PD controllers
- Closed loop transfer function $H_{ER}(s)$
- Closed loop transfer function $H_{YR}(s)$
- Routh Hurwitz table

The main objective of phase 2 of the card game is to build and tune system chains that fulfill the mission requirements, by building the correct controller and plant card combinations. How this is achieved depends on the individuals players strategy. Figure 28 shows a conceptual diagram on how the complete card chain will be built.

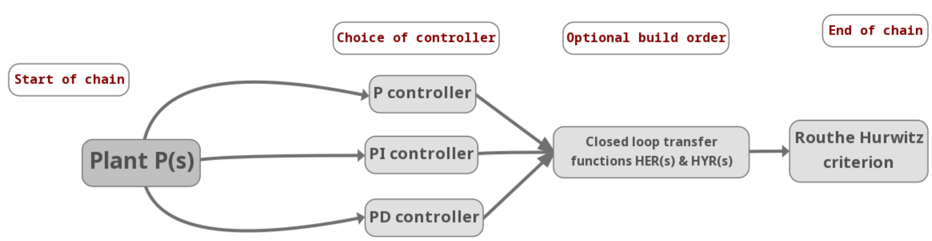

Figure 28: Conceptual card chain sequence for phase 2.

While players are required to build entire card chains to complete objectives, mission cards will state that the systems also have to adhere to a specified response. A complete card chain will only include generic values for controller parameters. Thus, tuning the controller is a vital core mechanic. All controller cards contain parameter values for K_P, K_I and/or K_D that will initially be set using a six sided die when built. Furthermore, these values can be tuned with the help of several game mechanics and card effects, which are further explained in Section 6.8.

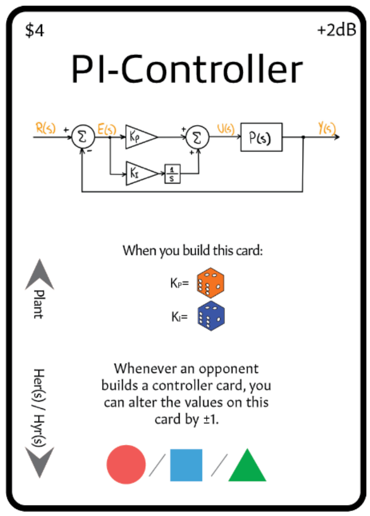

Figure 29: Controller card example, showcasing controller parameters.

For preliminary testing purposes, a MATLAB Simulink file was created that can be utilized by players to simulate more effortlessly and analyze new system parameters, so that the reduction of the game flow is minimized. Here, players can switch between different controllers and plants to simulate the closed feedback loop, along with controller parameter values.

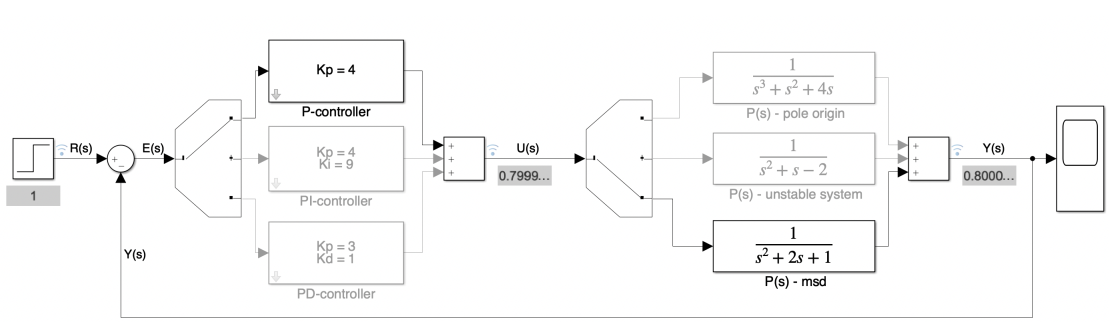

Figure 30: Game tool for simulating system parameters.

6.1 Controller cards

The controller cards are independent cards that only represent the type of controller, and are not directly associated with other unique system cards. Therefore, given an arbitrary plant, the choice of controller is reliant on the players objectives.

For this phase, P, PI and PD controllers are implemented into the game as core cards.

6.1.1 P-controller

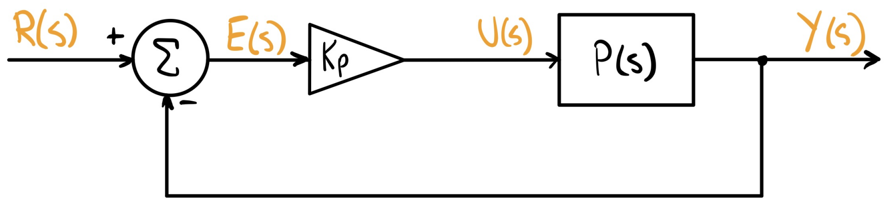

Figure 31: Block diagram of a feedback control system with a proportional controller.

The proportional controller produces an output *u(t)* that is directly proportional to the error signal *e(t)*, which is the difference between the reference and output signal. Furthermore, the output of the proportional controller is the product of the error signal *e(t)* by the proportional gain K_P as follows:

$$u(t) = K_P e(t) \tag{93}$$

The largest drawback with the proportional controller is that it cannot eliminate completely the offset when used to control a plant with no integrator, as it relies on the error signal to generate an output.

By using the Laplace transform, the transfer function $C(s)$ for the proportional controller can be computed as follows:

$$U(s) = K_P \cdot E(s) \Rightarrow C(s) = \frac{U(s)}{E(s)} = K_P \tag{94}$$

Now that the controller transfer function is established, the relationship between the reference *R(s)* and the output *Y(s)*, in addition to the relationship between the reference $R(s)$ and the error $E(s)$ can be computed using block diagram simplification of Figure 31:

$$H_{YR}(s) = \frac{K_P P(s)}{1 + K_P P(s)} \tag{95}$$

$$E(s) = R(s) - Y(s) \Rightarrow H_{ER}(s) = \frac{1}{1 + K_P P(s)} \tag{96}$$

These equations will be used for the closed loop transfer functions $H_{ER}(s)$ and $H_{YR}(s)$ on subsequent core cards.

6.1.2 PI-controller

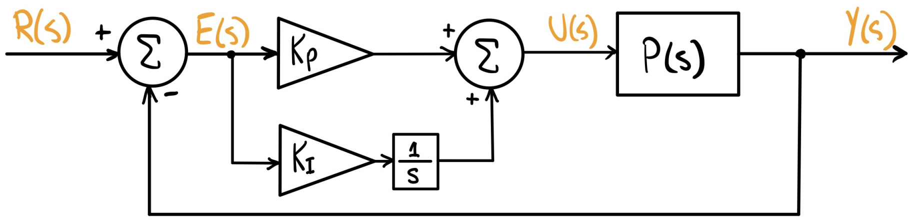

Figure 32: Block diagram of a proportional-integral controller.

The proportional-integral controller is commonly used for controlling non-integrating processes, for which the proportional controller is not enough to achieve a zero steady-state error. For the PI controller, an integral term is introduced. When an integrator is introduced into the loop, it forces the system error *e(t)* to go to zero as the system reaches steady-state.

$$u(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau \tag{97}$$

where K_I is the integral gain. The transfer function $C(s)$ for the controller can be derived using the Laplace transform:

$$U(s) = K_P \cdot E(s) + K_I \cdot \frac{1}{s} \cdot E(s) \tag{98}$$

$$C(s) = \frac{U(s)}{E(s)} = \frac{K_P s + K_I}{s} \tag{99}$$

Furthermore, by using block diagram simplification of Figure 32, the relationship between the reference *R(s)* and the output *Y(s)* can be established as:

$$H_{YR}(s) = \frac{(K_P s + K_I) P(s)}{s + (K_P s + K_I) P(s)} \tag{100}$$

which gives:

$$E(s) = R(s) - Y(s) \Rightarrow H_{ER}(s) = \frac{s}{s + (K_P s + K_I) P(s)} \tag{101}$$

6.1.3 PD-controller

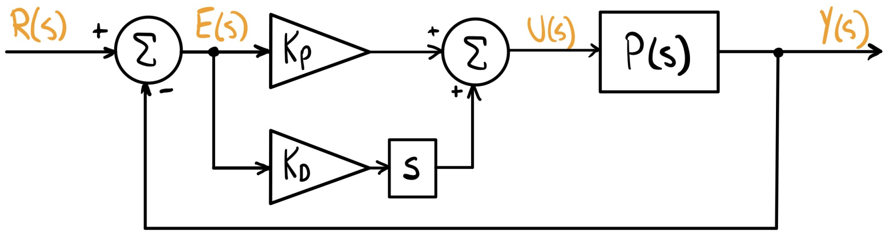

Figure 33: Block diagram of a proportional-derivative controller.

The proportional-derivative controller operates on the current process in addition to predicted future process states. The PD-controller can be thought of as a composition of proportional-only and derivative-only control equations. Thus, the purpose of the derivative control equation is to predict and compensate for the upcoming error. PD-control does not contain the integral term, so it is affected by the same limitations as the P-controller.

$$u(t) = K_P e(t) + K_D \frac{de(t)}{dt} \Rightarrow U(s) = (K_P + K_D s)E(s) \tag{102}$$

$$C(s) = \frac{U(s)}{E(s)} = K_P + K_D s \tag{103}$$

By using block diagram simplification of Figure 33, $H_{YR}(s)$ and $H_{ER}(s)$ are obtained as follows:

$$H_{YR}(s) = \frac{(K_P + K_D s)P(s)}{1 + (K_P + K_D s)P(s)} \tag{104}$$

$$H_{ER}(s) = \frac{1}{1 + (K_P + K_D s)P(s)} \tag{105}$$

6.1.4 P, PI, PD controller cards

The resulting controller cards are shown in Figure 34.

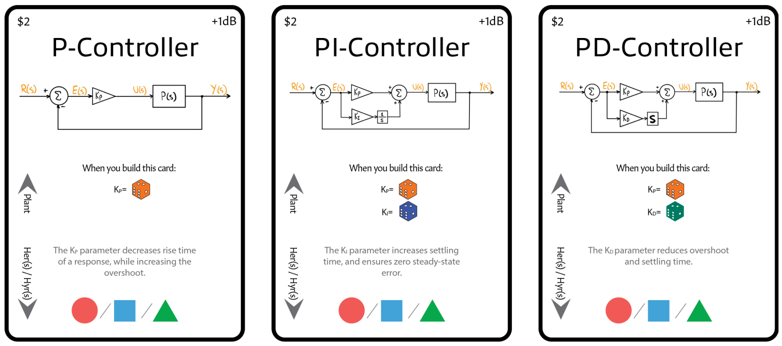

Figure 34: P, PI and PD controller cards.

6.2 Routh Hurwitz Table

The selection of PID parameter values for obtaining the optimal performance from the system process is called tuning, and is a crucial part of closed loop feedback systems. Though there are a number of tuning methods, the introductory PID tuning method for the ELE320 course uses the Routh-Hurwitz criterion, and it is therefore chosen as a core chain card.

For LTI control systems, the Routh-Hurwitz criterion is a mathematical test to check for stability in the system. Given an arbitrary transfer function:

$$H(s) = \frac{N(s)}{D(s)} \tag{106}$$

the characteristic polynomial *D(s)* must have its poles to the left of the imaginary axis in the complex plane in order to achieve a bounded-input bounded-output (BIBO) stable system. If the characteristic polynomial *D(s)* has a root with a positive real part, the output would diverge, whereas the response of a BIBO stable system converges to a steady state.

To illustrate the consequence of an unstable pole, an arbitrary system made up of three separate poles is constructed:

$$H(s) = \frac{1}{s+1} \cdot \frac{1}{s+3} \cdot \frac{1}{s-2} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s-2} \tag{107}$$

By inverse Laplace transforming the partial fraction expanded system, the response of the higher order system can be checked.

$$\mathscr{L}^{-1}(H(s)) = Ae^{t} + Be^{-3t} + Ce^{2t} \tag{108}$$

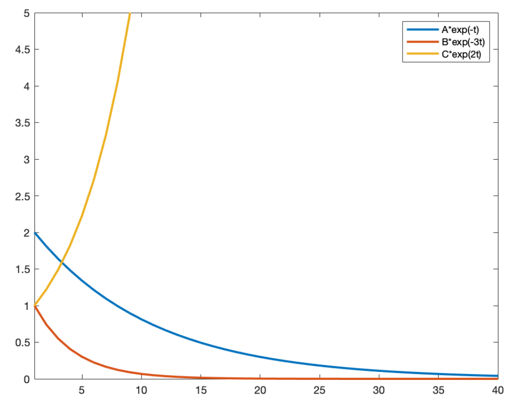

Figure 35: Plot of H(s)

Figure 35 shows how one single unstable pole will make the entire system unstable, even if the other poles are stable. Therefore, the stability of the system can be altered by the roots of the characteristic polynomial. This leads to the Routh-Hurwitz criterion, where the BIBO stability of a transfer function can be checked. The polynomials of the closed-loop transfer functions H_{ER} and H_{YR} will depend on the PID parameters, and by utilizing the Routh-Hurwitz criterion, these parameters can be selected to obtain closed-loop stability.

For an nth degree polynomial:

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + ... + a_1 s + a_0 \tag{109}$$

the Routh-Hurwitz table has the structure provided in Table 1.

a_n	a_{n-2}	a_{n-4}	...
a_{n-1}	a_{n-3}	a_{n-5}	...
b_1	b_2	b_3	...
c_1	c_2	c_3	...
...	...	...	...

Table 1: Routh-Hurwitz table

where b_i and c_i are computed as:

$$b_i = \frac{a_{n-1} \cdot a_{n-2i} - a_n \cdot a_{n-(2i+1)}}{a_{n-1}} \tag{110}$$

$$c_i = \frac{b_1 \cdot a_{n-(2i+1)} - a_{n-1} \cdot b_{i+1}}{b_1} \tag{111}$$

For a closed-loop system to be BIBO stable, the Routh-Hurwitz criterion states that the values in the first column of the table must all be positive. For example, given a closed loop transfer function with a proportional-integral controller:

$$H_{ER}(s) = \frac{s^3 + s^2}{s^3 + s^2 + K_P s + K_I} \tag{112}$$

the table becomes:

1	K_P
1	K_I
$K_P - K_I$	0
K_I	0

so that it becomes clear that BIBO stability of the closed-loop system is achieved when:

$$\begin{cases} K_P - K_I > 0 \\ K_I > 0 \end{cases} \Rightarrow \begin{cases} K_P > K_I \\ K_I > 0 \end{cases} \tag{113}$$

6.3 System 1 - Pole origin plant

For the first card chain, a system with a pole in the origin is considered. This means that the transfer function contains an integrator, which is sufficient to eliminate steady state errors without the need to include an integrator term in the controller. Furthermore, as the player's PID parameter values are decided using a six sided die, the plant must be constructed with this in mind. Therefore, the values for K_P, K_I and K_D should lie in the range 1-6 when BIBO stability is achieved.

To achieve this, the fundamental system plant is initially constructed using generic values:

$$P(s) = \frac{1}{As^3 + Bs^2 + Cs} \tag{114}$$

The plant, in addition to subsequent system plants, will all be constructed by using a proportional-integral controller as a base. Therefore, obtaining suitable controller parameter values for stability conditions becomes more consistent.

By utilizing Equation (101), the closed loop transfer function H_{ER}(s) becomes:

$$H_{ER}(s) = \frac{As^4 + Bs^3 + Cs^2}{As^4 + Bs^3 + Cs^2 + K_P s + K_I} = \frac{N(s)}{D(s)} \tag{115}$$

Then, the characteristic polynomial *D(s)* is inserted in the Routh-Hurwitz table and computed using Equation (110) and (111):

A	C	K_I
B	K_P	0
$C - \frac{AK_P}{B}$	K_I	0
$\frac{B^2 K_I}{AK_P - BC} + K_P$	0	0
K_I	0	0

Assuming that $A > 0$ and $B > 0$, BIBO stability is obtained if:

$$\begin{cases} C - \frac{AK_P}{B} > 0 \\ \frac{B^2 K_I}{AK_P - BC} + K_P > 0 \\ K_I > 0 \end{cases} \Rightarrow \begin{cases} K_P < \frac{BC}{A} \\ K_I < \frac{BCK_P - AK_P^2}{B^2} \\ K_I > 0 \end{cases} \tag{116}$$

By choosing the following values:

$$\begin{cases} A = 1 \\ B = 1 \\ C = 4 \end{cases} \Rightarrow P(s) = \frac{1}{s^3 + s^2 + 4s} \tag{117}$$

the stability conditions become:

$$\begin{cases} K_P < 4 \\ \\ K_I < 4K_P - K_P^2 \end{cases} \tag{118}$$

The resulting transfer function generates stability conditions that operates within the game's boundaries, and is chosen as a core plant card.

6.3.1 P-controller card chain

This section covers the complete card chain for the *pole in the origin* plant expressed in Equation (125) when choosing the proportional controller. The plant transfer function, given the above discussion, is given by:

$$P(s) = \frac{1}{s^3 + s^2 + 4s} \tag{119}$$

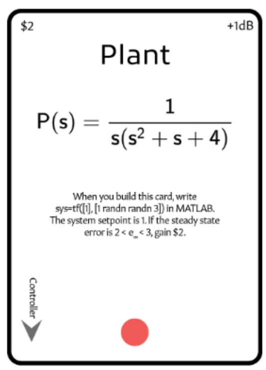

Figure 36: Plant transfer function card.

combined with the proportional controller card produces the closed loop transfer functions H_{YR}(s) and H_{ER}(s) using Equations (95) and (96) respectively:

$$H_{YR}(s) = \frac{K_P P(s)}{1 + K_P P(s)} = \frac{K_P}{s^3 + s^2 + 4s + K_P} \tag{120}$$

$$H_{ER}(s) = \frac{1}{1 + K_P P(s)} = \frac{s^3 + s^2 + 4s}{s^3 + s^2 + 4s + K_P} \tag{121}$$

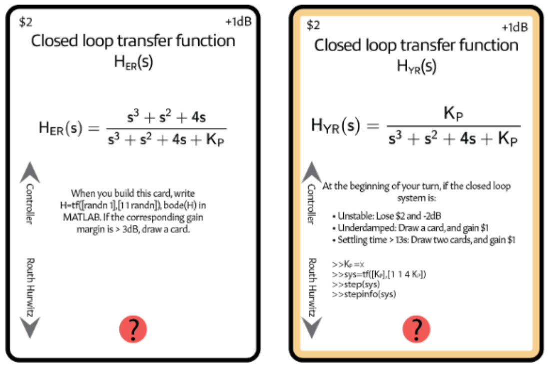

Figure 37: Resulting closed loop transfer function cards.

Furthermore, the Routh Hurwitz criterion card is constructed by inserting the denominator of the characteristic polynomial into the Routh-Hurwitz table, following the same procedure as in Section 6.2:

1	4
1	K_P
4-K_P	0
K_P	0

BIBO stability is achieved when:

$$\begin{cases} 4 - K_P > 0 \\ K_P > 0 \end{cases} \Rightarrow 0 < K_P < 4 \tag{122}$$

The Routh Hurwitz criterion card is presented in Figure 38.

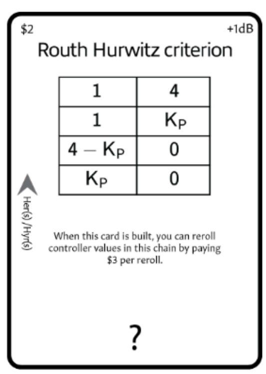

Figure 38: Routh Hurwitz criterion card.

6.3.2 PI-controller card chain

This section covers the calculations for the complete card chain when choosing a proportional-integral controller. The resulting cards are included in Appendix B.

The closed loop transfer function cards H_{YR}(s) and H_{ER}(s) are calculated using Equation (100) and (101):

$$H_{YR}(s) = \frac{(K_P s + K_I)P(s)}{s + (K_P s + K_I)P(s)} = \frac{K_P s + K_I}{s^4 + s^3 + 4s^2 + K_P s + K_I} \quad (123)$$

$$H_{ER}(s) = \frac{s}{s + (K_P s + K_I)P(s)} = \frac{s^4 + s^3 + 4s^2}{s^4 + s^3 + 4s^2 + K_P s + K_I} \quad (124)$$

Then, the Routh Hurwitz criterion card is constructed as in Section 6.2:

1	4	K_I
1	K_P	0
4-K_P	K_I	0
$\frac{K_I}{K_P-4} + K_P$	0	0
K_I	0	0

BIBO stability is achieved when:

$$\begin{cases} 4 - K_P > 0 \\ \frac{K_I}{K_P-4} + K_P > 0 \\ K_I > 0 \end{cases} \Rightarrow \begin{cases} K_P < 4 \\ K_I < 4K_P - K_P^2 \\ K_I > 0 \end{cases} \quad (125)$$

By increasing K_P, the stability criterion varies:

$K_P = 1 \Rightarrow K_I < 4 \cdot 1 - 1^1 \Rightarrow K_I < 3$
$K_P = 2 \Rightarrow K_I < 4 \cdot 2 - 2^2 \Rightarrow K_I < 4$
$K_P = 3 \Rightarrow K_I < 4 \cdot 3 - 3^2 \Rightarrow K_I < 3$

6.3.3 PD-controller card chain

When paired with a proportional-derivative controller, the closed loop transfer function cards H_{YR}(s) and H_{ER}(s) are calculated using Equation (104) and (105):

$$H_{YR}(s) = \frac{(K_P + K_D s)P(s)}{1 + (K_P + K_D s)P(s)} = \frac{K_D s + K_P}{s^3 + 4s^2 + 4s + K_D s + K_P} \tag{126}$$

$$H_{ER}(s) = \frac{1}{1 + (Kp + K_D s)P(s)} = \frac{s(s^2 + 4s + 2)}{s^3 + s^2 + 4s + K_D s + K_P} \tag{127}$$

Lastly, the Routh Hurwitz criterion card is constructed as:

1	4+K_D
1	K_P
$K_D - K_P + 4$	0
K_P	0

BIBO stability is achieved when:

$$\begin{cases} K_D - K_P + 4 > 0 \\ K_P > 0 \end{cases} \Rightarrow 0 < K_P < K_D + 4 \tag{128}$$

6.4 System 2 - Unstable system plant

For the second card chain, an unstable system plant is implemented. The motivation behind implementing an unstable system into the game is to convey to players that with the use of closed loop feedback, unstable plants can be stabilized. As asserted in Section 6.3, the plant should be constructed with game mechanics in mind. By following the same approach as for System 1, the following unstable system plant with generic values is chosen:

$$P(s) = \frac{1}{As^2 + Bs - C} \tag{129}$$

Paired with a proportional-integral controller and utilizing Equation (101), the closed loop transfer function $H_{ER}(s)$ becomes:

$$H_{ER}(s) = \frac{As^3 + Bs^2 - Cs}{As^3 + Bs^2 - Cs + K_P s + K_I} \tag{130}$$

The Routh-Hurwitz table is then computed as:

A	$-C + K_P$
B	K_I
$-\frac{AK_I}{B} - C + K_P$	0
K_I	0

Assuming that $(A, B)>0$, the BIBO stability is decided by:

$$\begin{cases} -\frac{AK_I}{B} - C + K_P > 0 \\ \\ K_I > 0 \end{cases} \Rightarrow 0 < K_I < \frac{BK_P - BC}{A} \tag{131}$$

By selecting the following values for A, B and C, the unstable plant and the stability conditions become:

$$\begin{cases} A = 1 \\ B = 1 \\ C = 2 \end{cases} \Rightarrow P(s) = \frac{1}{s^2 + s - 2} \Rightarrow \begin{cases} 0 < K_I < K_P - 2 \end{cases} \tag{132}$$

The following sub-chapters use the same approach as Section 6.3 for calculations.

6.4.1 P-controller

Equations (95) and (96) produce the following H_{YR}(s) and H_{ER}(s) cards:

$$H_{YR}(s) = \frac{K_P P(s)}{1 + K_P P(s)} = \frac{K_P}{s^2 + s - 2 + K_P} \tag{133}$$

$$H_{ER}(s) = \frac{1}{1 + K_P P(s)} = \frac{s^2 + s - 2}{s^2 + s - 2 + K_P} \tag{134}$$

The denominator of H_{ER}(s) produces the Routh-Hurwitz table:

1	-2+K_P
1	0
K_P -2	0

$$\begin{cases} K_P - 2 > 0 \end{cases} \Rightarrow K_P > 2 \tag{135}$$

6.4.2 PI-controller

Equations (100) and (101) produce the following H_{YR}(s) and H_{ER}(s) cards:

$$H_{YR}(s) = \frac{(K_P s + K_I)P(s)}{s + (K_P s + K_I)P(s)} = \frac{K_P s + K_I}{s^3 + s^2 - 2s + K_P s + K_I} \tag{136}$$

$$H_{ER}(s) = \frac{s}{(K_P s + K_I)P(s)} = \frac{s^3 + s^2 - 2s}{s^3 + s^2 - 2s + K_P s + K_I} \tag{137}$$

The denominator of H_{ER}(s) produces the Routh-Hurwitz table:

1	$K_P - 2$
1	K_I
$K_P - 2 - K_I$	0
K_I	0

$$\begin{cases} K_P - 2 - K_I > 0 \\ K_I > 0 \end{cases} \Rightarrow \begin{cases} K_P > K_I + 2 \\ K_I > 0 \end{cases} \tag{138}$$

6.4.3 PD-controller

Equations (104) and (105) produce the following H_{YR}(s) and H_{ER}(s) cards:

$$H_{YR}(s) = \frac{(K_P + K_D s)P(s)}{1 + (K_P + K_D s)P(s)} = \frac{K_D s + K_P}{s^2 + s + K_D s - 2 + K_P} \tag{139}$$

$$H_{ER}(s) = \frac{1}{1 + (K_P + K_D s)P(s)} = \frac{s^2 + s - 2}{s^2 + K_D s + 2 - 2 + K_P} \tag{140}$$

The denominator of H_{ER}(s) produces the Routh-Hurwitz table:

1	$K_P - 2$
$K_D + 1$	0
$K_P - 2$	0

$$\begin{cases} K_D + 1 > 0 \\ K_P - 2 > 0 \end{cases} \Rightarrow \begin{cases} K_D > -1 \\ K_P > 2 \end{cases} \tag{141}$$

6.5 System 3 - MSD system plant

The last system to be implemented in the game is a version of the MSD system from phase 1. Polynomial values are altered to fit the game mechanics, using the same method as described in Sections 6.3 and 6.4:

$$P(s) = \frac{1}{As^2 + Bs + C} \tag{142}$$

The generic plant is then paired with a proportional-integral controller, and Equation (101) generates the closed loop transfer function $H_{ER}(s)$:

$$H_{ER}(s) = \frac{As^3 + Bs^2 + Cs}{As^3 + Bs^2 + Cs + K_P s + K_I} \tag{143}$$

The Routh-Hurwitz table becomes:

A	C+K_P
B	K_I
$C + K_P - \frac{AK_I}{B}$	0
K_I	0

Then, assuming $(A, B > 0)$, BIBO stability is achieved by:

$$\begin{cases} C + K_P - \frac{AK_I}{B} > 0 \\ K_I > 0 \end{cases} \Rightarrow \begin{cases} K_I < \frac{BC+BK_P}{A} \end{cases} \tag{144}$$

By selecting the following values for A, B and C, the MSD plant and the stability conditions become:

$$\begin{cases} A = 1 \\ B = 2 \\ C = 1 \end{cases} \Rightarrow P(s) = \frac{1}{s^2 + 2s + 1} \Rightarrow \begin{cases} 0 < K_I < 2K_P + 2 \end{cases} \tag{145}$$

6.5.1 P-controller

Equations (95) and (96) produce the following H_{YR}(s) and H_{ER}(s) cards:

$$H_{YR}(s) = \frac{K_P P(s)}{1 + K_P P(s)} = \frac{K_P}{s^2 + 2s + K_P + 1} \tag{146}$$

$$H_{ER}(s) = \frac{1}{1 + K_P P(s)} = \frac{(s+1)^2}{s^2 + 2s + K_P + 1} \tag{147}$$

The denominator of H_{ER}(s) produces the Routh-Hurwitz table:

1	$K_P + 1$
2	0
$K_P + 1$	0

$$K_P + 1 > 0 \Rightarrow K_P > -1 \tag{148}$$

6.5.2 PI-controller

Equations (100) and (101) produce the following H_{YR}(s) and H_{ER}(s) cards:

$$H_{YR}(s) = \frac{(K_P s + K_I)P(s)}{s + (K_P s + K_I)P(s)} = \frac{K_P s + K_I}{s^3 + 2s^2 + s + K_P s + K_I} \tag{149}$$

$$H_{ER}(s) = \frac{s}{s + (K_P s + K_I)P(s)} = \frac{s^3 + 2s^2 + s}{s^3 + 2s^2 + s + K_P s + K_I} \tag{150}$$

The denominator of H_{ER}(s) produces the Routh-Hurwitz table:

1	$K_P + 1$
2	K_I
$K_P - \frac{K_I}{2} + 1$	0
K_I	0

$$\begin{cases} K_P - \frac{K_I}{2} + 1 > 0 \\ K_I > 0 \end{cases} \Rightarrow 0 < K_I < 2K_P + 2 \tag{151}$$

6.5.3 PD-controller

Equations (104) and (105) produce the following H_{YR}(s) and H_{ER}(s) cards:

$$H_{YR}(s) = \frac{(K_P + K_D s)P(s)}{1 + (K_P + K_D s)P(s)} = \frac{K_D s + K_P}{s^2 + 2s + K_D s + K_P + 1} \tag{152}$$

$$H_{ER}(s) = \frac{1}{1 + (K_P + K_D s)P(s)} = \frac{(s+1)^2}{s^2 + 2s + K_D s + K_P + 1} \tag{153}$$

The denominator of H_{ER}(s) produces the Routh-Hurwitz table:

1	$K_P + 1$
$K_D + 2$	0
$K_P + 1$	0

$$\begin{cases} K_P + 1 > 0 \\ K_D + 2 > 0 \end{cases} \Rightarrow \begin{cases} K_P > -1 \\ K_D > -2 \end{cases} \tag{154}$$

6.6 People cards

For this phase of the game a different set of playable people cards was produced, illustrated in Figure 39. Like the people cards in phase 1, these cards grant passive effects.

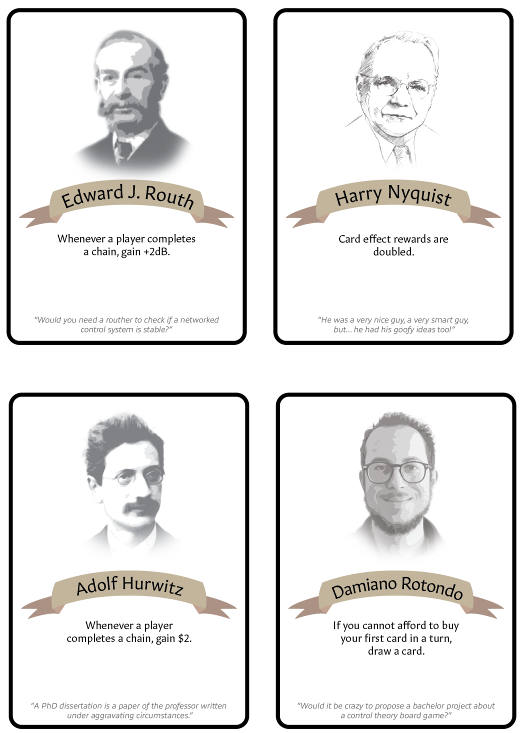

Figure 39: People cards for phase 2.

6.7 Mission cards

Contrarily to the first phase of the game, phase 2 introduces more interactive gameplay. In other words, simply building card chains will not in itself meet the requirements to win the game. The conditions for the closed loop response depend on controller design. These conditions are set by the mission cards. Some missions can take longer to complete than others. However, players will have to complete three missions to win. Thus, balancing mission cards is rather incidental at this stage. Below is the list of missions selected for this phase, with emphasis on closed loop feedback conditions.

- Build a system with no overshoot.
- Build a system with zero steady state error.
- Build a system with settling time < 10s.
- Build a system with zero state error without the use of a PI controller.
- Stabilize an unstable plant.
- Build system with steady state output: $0.1 < y(\infty) < 0.3$.

Figure 40 shows an example of a mission card. On some cards, a subtext is included to assist players.

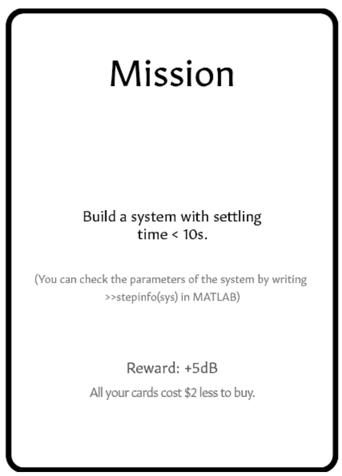

Figure 40: Example of a mission card.

The intention of these missions is to indulge players deeper into system analysis through the use of MATLAB, in addition to intuitive thinking when it comes to understanding the effect that tuning the controller has on the closed loop system.

6.8 Card effects

As introduced in Section 3.2, card effects also play a vital role in phase 2. These include traditional effects that are triggered by buying or building a card. However, passive trigger effects are only included on the *Closed loop transfer function* $H_{YR}(s)$ cards. Lastly, the controller cards have their own set of unique effects. The following subsections explain the different sets of card effects that are introduced for phase 2, excluding card effects that are not interconnected with control theory. All phase 2 card effects and their rewards can be seen on the cards listed in Appendix B.

6.8.1 Controller card effects

Every controller card share the same effect:

- Whenever an opponent builds a controller card, you can alter the values on this card by $\pm\, x$.

Achieving a determined closed loop response is ultimately the main goal of the game. Thus, this effect is widely attainable. The number that players can alter their controller parameters by is decided on the initial cost of the card.

Figure 41: Card effect that all controller cards have in common.

These parameters, though important for completing end game missions, are also relevant for the $H_{YR}(s)$ trigger cards.

6.8.2 Trigger effects - $H_{YR}(s)$ cards

In phase 2 of the game, trigger effects are moved to the closed loop transfer function $H_{ER}(s)$ cards, and are all formulated similarly with the exception of settling time conditions. This makes it so players can pay more attention to the response of their systems that are continuously being tuned through other core chain cards, in addition to some general card effects. Furthermore, these cards will become a main attraction for gaining passive rewards each round. The effect is formulated as follows:

- At the beginning of your turn, if the closed loop system is:
  - Unstable: Lose $2 and -2dB.
  - Underdamped: Draw a card, and gain $1.
  - Settling time > x sec: Draw two cards, and gain $1.

By including punishment for unstable systems, players must strive for closed-loop stability. Furthermore, through several card effects, players can adjust opponent parameters. Therefore, even if a player manages to stabilize their systems temporarily, other players have the chance to destabilize them by changing appropriately the controller's parameters. Hopefully, this will heighten player enthusiasm, while also building a more profound understanding of control theory.

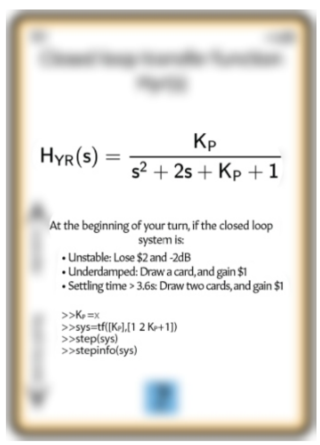

Figure 42: $H_{YR}(s)$ card example

Additionally, the card includes MATLAB functions that will be used by players to test their system response. For the mass-spring-damper system along with a proportional controller, the MATLAB functions become:

```
>>KP = x
>>sys=tf([KP],[1 2 KP+1])
>>step(sys)
>>stepinfo(sys)
```

As stated in the introduction in Section 6, the premade simulink file can also be utilized for simulating systems.

6.8.3 General card effects

This section describes the general effects that are included on the rest of the cards.

(i) Nyquist stability criterion effects

- *When you build this card, write H=1/((s+randn)(s+randn)), nyquist(H) in MATLAB. If the number of anticlockwise encirclements of the origin is equal to the number of unstable poles in H(s), then ... (system will then become stable when the loop is closed)*
- *When you build this card, write H=1/((s+randn)(s+randn)), nyquist(H) in MATLAB. If the closed loop system is BIBO stable, then ...*
- *When you build this card, write H=1/((s+randn)(s+randn)), nyquist(H) in MATLAB. If the closed loop system generates an extra unstable pole, then ...*

These effects requires players to check the stability of a randomized system using the Nyquist stability criterion. By using a graphical technique for open-loop systems, the Nyquist criterion can be applied without explicitly calculating the zeros and poles of the system. The criterion is normally expressed as:

$$N = Z - P \tag{155}$$

and

$$D(s) = 1 + C(s)H(s) \tag{156}$$

where

- Z - number of zeros of $D(s)$ in the right-half plane.
- P - number of poles of the open loop transfer function $C(s)H(s)$ in the right-half plane.

A contour in the s-plane is defined. As s moves along the contour in the clockwise direction, D(s) will encircle the origin in the (Re{D(s)}, Im{D(s)}) plane in the clockwise direction N times.

BIBO stability of the system is achieved if Z=0, so:

$$N = -P = -\text{number of unstable poles in C(s)H(s)} \tag{157}$$

This means that, if the number of unstable poles in $C(s)H(s)$ is the same as the number of anticlockwise encirclements of the origin by the Nyquist plot, BIBO stability is achieved when the loop is closed. If not, the system will be unstable. However, since $D(j\omega) = 1 + P(j\omega)C(j\omega)$, the encirclements of the origin by $D(j\omega)$ corresponds to the encirclements of the point (-1, 0) by $P(j\omega)C(j\omega)$. Therefore, the closed-loop system is BIBO stable if the number of encirclements of the point (-1, 0) by $P(j\omega)C(j\omega)$ in the anticlockwise direction, is equal to the number of unstable poles in $P(s)C(s)$.

(ii) Bode plot effects

- *When you build this card, write sys=tf([randn 1],[1 1 randn]), bode(sys) in MATLAB. If the closed loop system is BIBO stable, then ...*
- *When you build this card, write sys=tf([randn 1],[1 1 randn]), bode(sys) in MATLAB. If the corresponding gain margin is > 3dB, then ...*
- *When you build this card, write sys=tf([randn 1],[1 1 randn]), bode(sys) in MATLAB. If the gain margin is positive, then ...*

These card effects include the use of the Bode plot, which is a commonly used tool in control theory to determine the stability of a system. The players will compute a randomized control system which is then represented in a Bode plot graph. Rewards are given based on the stability characteristics of the control system. A Bode plot consists of two graphs that maps the frequency response of the system: the magnitude and phase plot. The magnitude plot illustrates the magnitude in decibels, while the phase plot illustrates the phase shift in degrees. To determine stability through the Bode stability criterion, the gain and phase margins are used.

In contrast to the Routh-Hurwitz stability criterion, the Bode stability criterion is advantageous in two aspects. First, it provides exact results for systems that have a time delay. Second, the Bode stability criterion measures the relative stability to a system, rather than only telling if the system is stable or not. Moreover, a feedback control system can only become stable if the roots of the characteristic equation is located to the left hand side of the imaginary axis. This also applies to the Bode stability criterion.

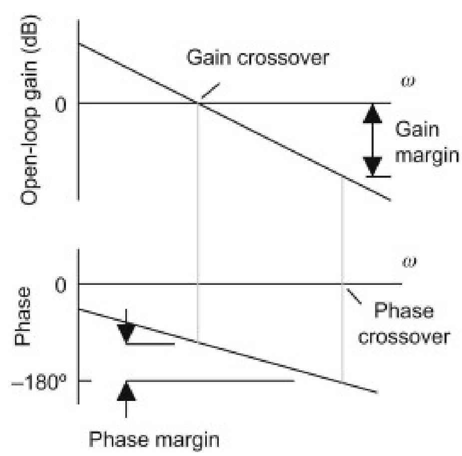

Figure 43: Gain and phase margins illustrated [33]

The gain margin (GM) is referred to as the amount of gain that can be applied to a system without making it unstable, expressed in dB. Thus, the higher the gain margin, the more stable the system becomes. Usually, the gain margin can be read directly from the Bode plot at the phase crossover frequency. This point is where the phase plot is equal to 180°on the x-axis. The G is the vertical distance between 0dB and magnitude curve at the phase crossover frequency. The formula for GM is:

$$GM = 0 - \mathrm{G} \tag{158}$$

where G is the gain around the feedback loop, expressed in dB.

The phase margin (PM) denotes how much the phase can be increased/decreased before the system becomes unstable. The phase margin can be read of the Bode plot by computing the vertical distance between the phase curve and at the x-axis where the frequency of the magnitude plot is equal to 0 dB. This is also known as the gain crossover frequency. The formula for the PM is:

$$PM = \Phi - (-180^\circ) \tag{159}$$

where Φ is the phase lag of the system, and can be read from the y-axis at the gain crossover frequency in the phase plot.

The implemented card effects that are related to Bode plots refer to the Bode stability criterion, where stability conditions are defined as:

- BIBO stable systems: PM and GM should both be positive, or PM>GM.
- Marginally stable system: Both PM and GM are zero, or PM=GM.
- Unstable system: PM or GM are negative, or PM<GM

(iii) Step-response characteristics effects

- *When you build this card, write sys=tf([1],[1 randn 3]) in MATLAB. If the settling time is >10s, then ... (stepinfo(sys))*
- *When you build this card, write sys=tf([1],[1 randn randn 3]) in MATLAB. The system set point is 1. If the steady state error is* $2 < e_\infty < 3$ *, then ...*

These card effects require the player to analyze the randomized system with respect to step-response characteristics. This enforces the use of MATLAB and its analytical toolkits.

Part IV

Assessment and conclusive remarks

7 Asessment

Due to COVID-19 restrictions, assessment circumstances were not ideal. At the time of assessment for phase 1, a maximum of 5 people were allowed to gather at a time. The university was closed for the majority of students, who have resorted to online classes. Therefore, collecting student feedback during the pandemic was sub par.

7.1 Preliminary assessment with faculty members

Before testing the game with students in the classroom, the preliminary prototype of the game was tested with members of the Faculty of Science at University of Stavanger. An early version of Phase 1 was tested with a Professor and a lab engineer. To obtain general feedback on the game, a questionnaire was developed as an evaluation tool to test overall design. The questions and results are listed in table 2.

	Scale (Disagree - Agree)					**Results**
Goals and objectives						
The motive and rationale for the game is fully explained.	1	2	3	4	5	**3.5**
The goals of the game are clearly defined.	1	2	3	4	5	**3.5**
The game encouraged student interaction.	1	2	3	4	5	**5**
The game stimulated discussion of core topics.	1	2	3	4	5	**4.5**
Components and mechanics						
The game rules were clear and easily understood.	1	2	3	4	5	**4**
The duration of the game is reasonable	1	2	3	4	5	**5**
Spending time playing was an effective use of time	1	2	3	4	5	**4**
The amount of cards were reasonable	1	2	3	4	5	**4.5**
Summary						
The game contained relevant material.	1	2	3	4	5	**4.5**
The terminology was appropriate.	1	2	3	4	5	**4.5**
The game was entertaining.	1	2	3	4	5	**4.5**
How much fun was playing this game compared to traditional class work?	1	2	3	4	5	**5**
Did you feel like this game increased your perception of the course?	1	2	3	4	5	**3.5**
I would recommend other students to play this game.	1	2	3	4	5	**5**

Table 2: Assessment questionnaire

Analyzing the small sample of feedback revealed the following results for the early testing. The feedback from both testers answered with an overall satisfaction with playing the game. The goals and objectives were relatively well explained, and player interaction was overall positive. Mechanically, testers believed the game to be easily learned. Game time was approximately 30 minutes, excluding the general discussion regarding the game.

Suggestions for improvement did not directly refer to the educational aspect or the overall game, but rather balancing issues. It was reported that under some circumstances, it was hard to obtain game flow due to the unbalanced currency system and the cost of buying cards. This was subsequently altered after initial testing. Another suggestion was to make a more clear divide between card effects and flavour text, in addition to making trigger effects more simple to spot. Therefore, a yellow border was added to the trigger cards to make it easier to keep track of when these effects came into play. The testers enjoyed the game as an additional tool to learn control theory, and would recommend the game to other students.

These results shows that to a degree, the goals of the assignment described in Section 1.3 were satisfactory.

7.2 Preliminary assessment with students

For the final assessment, four students volunteered as game testers. Prior to testing, these students had finished the exam in *ELE320 - Reguleringsteknikk*. The students formed in pairs of two, with both groups playing one phase each. While one group played, the other group observed and joined in discussions. Initial feedback during playing showed that the students would like to have played the card game prior to the exam, as it served as a good tool to recap core topics while also increasing general intuition on how the subjects are interconnected. Furthermore, the card game stimulated discussions and interaction between the students. The following sections present the assessment of both the phases, as done in Section 7.1.

7.2.1 Group one - Phase 1 testing

The first group tested phase 1 of the game. The results from the questionnaire are listed in Table 3.

	Scale (Disagree - Agree)					**Results**
Goals and objectives						
The motive and rationale for the game is fully explained.	1	2	3	4	5	**4**
The goals of the game are clearly defined.	1	2	3	4	5	**3**
The game encouraged student interaction.	1	2	3	4	5	**4.5**
The game stimulated discussion of core topics.	1	2	3	4	5	**5**
Components and mechanics						
The game rules were clear and easily understood.	1	2	3	4	5	**3**
The duration of the game is reasonable	1	2	3	4	5	**4**
Spending time playing was an effective use of time	1	2	3	4	5	**4**
The amount of cards were reasonable	1	2	3	4	5	**4.5**
Summary						
The game contained relevant material.	1	2	3	4	5	**5**
The terminology was appropriate.	1	2	3	4	5	**4.5**
The game was entertaining.	1	2	3	4	5	**5**
How much fun was playing this game compared to traditional class work?	1	2	3	4	5	**5**
Did you feel like this game increased your perception of the course?	1	2	3	4	5	**4.5**
I would recommend other students to play this game.	1	2	3	4	5	**5**

Table 3: Assessment questionnaire

Following the first assessment of phase 1, some changes were made to balance the pace of the game. These included reducing the cost on some cards, while increasing the dB gain. With these new changes, the game felt more balanced but there was still room for improvement. Generally, the feedback was very positive. The volunteers felt that the game was educational and enjoyable, and a refreshing alternative to traditional class work. When it came to understanding the game rules, there was initially some difficulties. Nonetheless, it became more apparent as the game went on. As the students did not have access to the game manual at the time of testing, it was reported that the goals/rules of the game could be further established.

General comments included expanding the game, allowing for nonlinear systems. Additionally, the students recommended the inclusion of system identification cards.

7.2.2 Group two - Phase 2 testing

The second group tested phase 2 of the game. The results from the questionnaire are listed in Table 4.

	Scale (Disagree - Agree)					Results
Goals and objectives						
The motive and rationale for the game is fully explained.	1	2	3	4	5	**5**
The goals of the game are clearly defined.	1	2	3	4	5	**4**
The game encouraged student interaction.	1	2	3	4	5	**4**
The game stimulated discussion of core topics.	1	2	3	4	5	**4.5**
Components and mechanics						
The game rules were clear and easily understood.	1	2	3	4	5	**4**
The duration of the game is reasonable	1	2	3	4	5	**4**
Spending time playing was an effective use of time	1	2	3	4	5	**4.5**
The amount of cards were reasonable	1	2	3	4	5	**4.5**
Summary						
The game contained relevant material.	1	2	3	4	5	**5**
The terminology was appropriate.	1	2	3	4	5	**5**
The game was entertaining.	1	2	3	4	5	**5**
How much fun was playing this game compared to traditional class work?	1	2	3	4	5	**4.5**
Did you feel like this game increased your perception of the course?	1	2	3	4	5	**4**
I would recommend other students to play this game.	1	2	3	4	5	**5**

Table 4: Assessment questionnaire

The phase 2 feedback from the second group was largely positive. The students enjoyed the mechanic of variable controller parameters, and the ability to disturb their opponent. The initial planning of strategies to complete the missions effectively was reported to be fun, as well as being educational. The volunteers had to think ahead about what system combinations to build. Furthermore, when it was the opposing players turn, the students experimented with simulations, on both their own and their opponents systems.

The students agreed that, at times, a lot of time was spent on manually writing functions into MATLAB. This could be improved by implementing premade MATLAB functions to serve as the card effects, so that players will only need to call that specific function when an effect calls for it. Additionally, the players agreed that some of the people card effects were too strong compared to others.

8 Conclusive remarks

Reports suggest that educational games can be beneficial for student motivation and engagement [24][19]. Students can use various strategies to actively facilitate and challenge their own understanding of educational material. The approach to develop a card game as an instrument for the teacher's didactic toolkit lets the students implement a variety of learning techniques. Developing a suitable card game that will engage students proved to be a demanding task. Furthermore, entering the field of STEM subjects can often be considered as difficult by fresh students. In the limited preliminary assessments of the card game, the four students and the members of the faculty of science at the University of Stavanger enjoyed playing the game. Moreover, the students perception of the course was believed to be somewhat increased based on the assessment. This leads to the conclusion that educational games have the potential be an efficient and engaging learning tool in engineering courses. Realistically, the game should be subject to further assessment, as positive findings could serve as a trigger for wider adaption of similar educational techniques.

There are plenty of opportunities for expanding the game into new topics within the course, possibly even outside the realm of control theory. As mentioned in the assignment introduction, the game was designed with this in mind. For control theory subjects, a preliminary nonlinear system was constructed to serve as an example of expandability. In this case, to construct a stable closed loop feedback system, linearization around an equilibrium point must be carried out. Similarly, an expanded chain of cards could include the frequency response. Sine wave responses could replace the basic step- and impulse responses, and core chain cards could include bode diagrams and the Fourier transform.

Considering the method of building orderly chains where each component is connected, a selection of arbitrary courses where the topics are somewhat interconnected could also be adapted using similar game mechanics as the control theory game.

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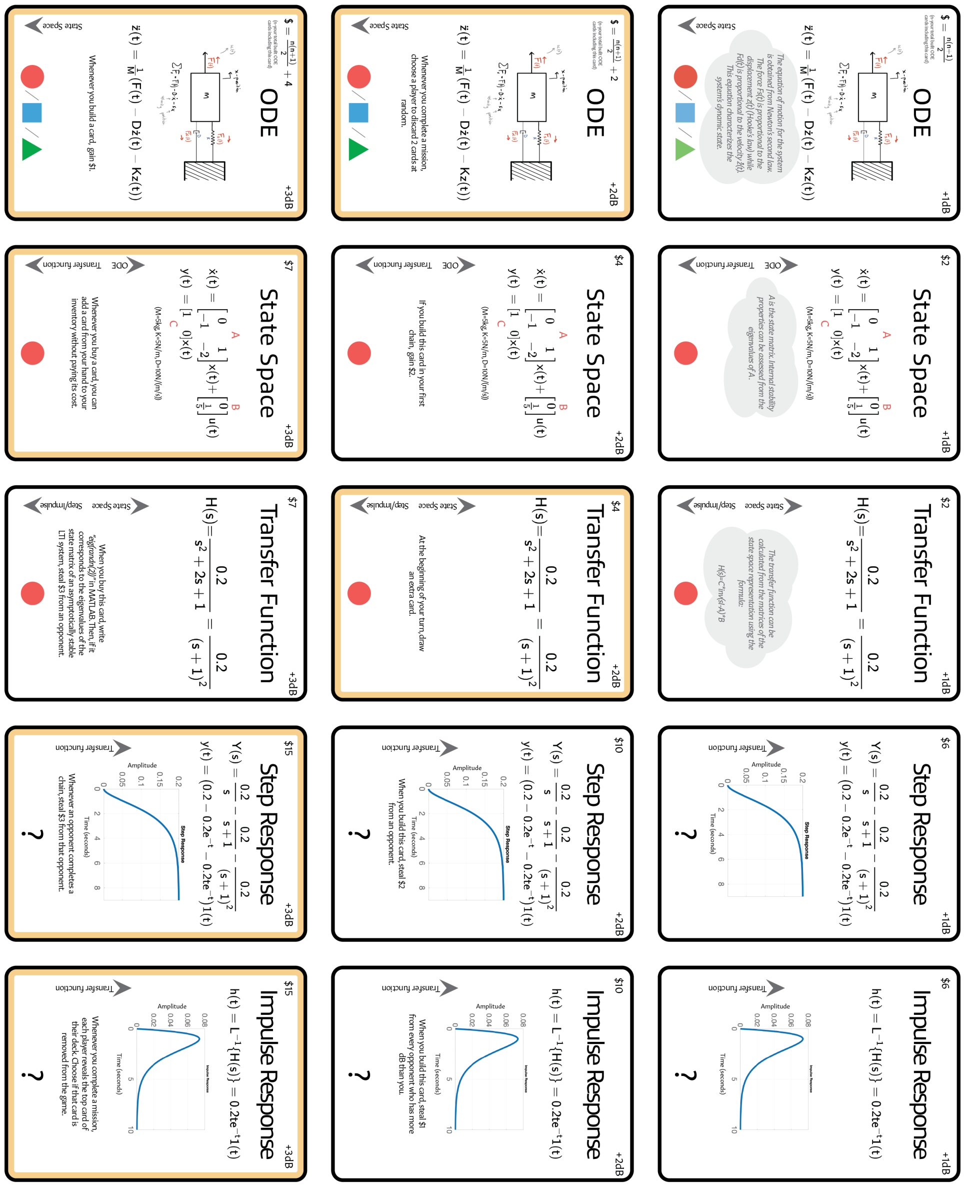
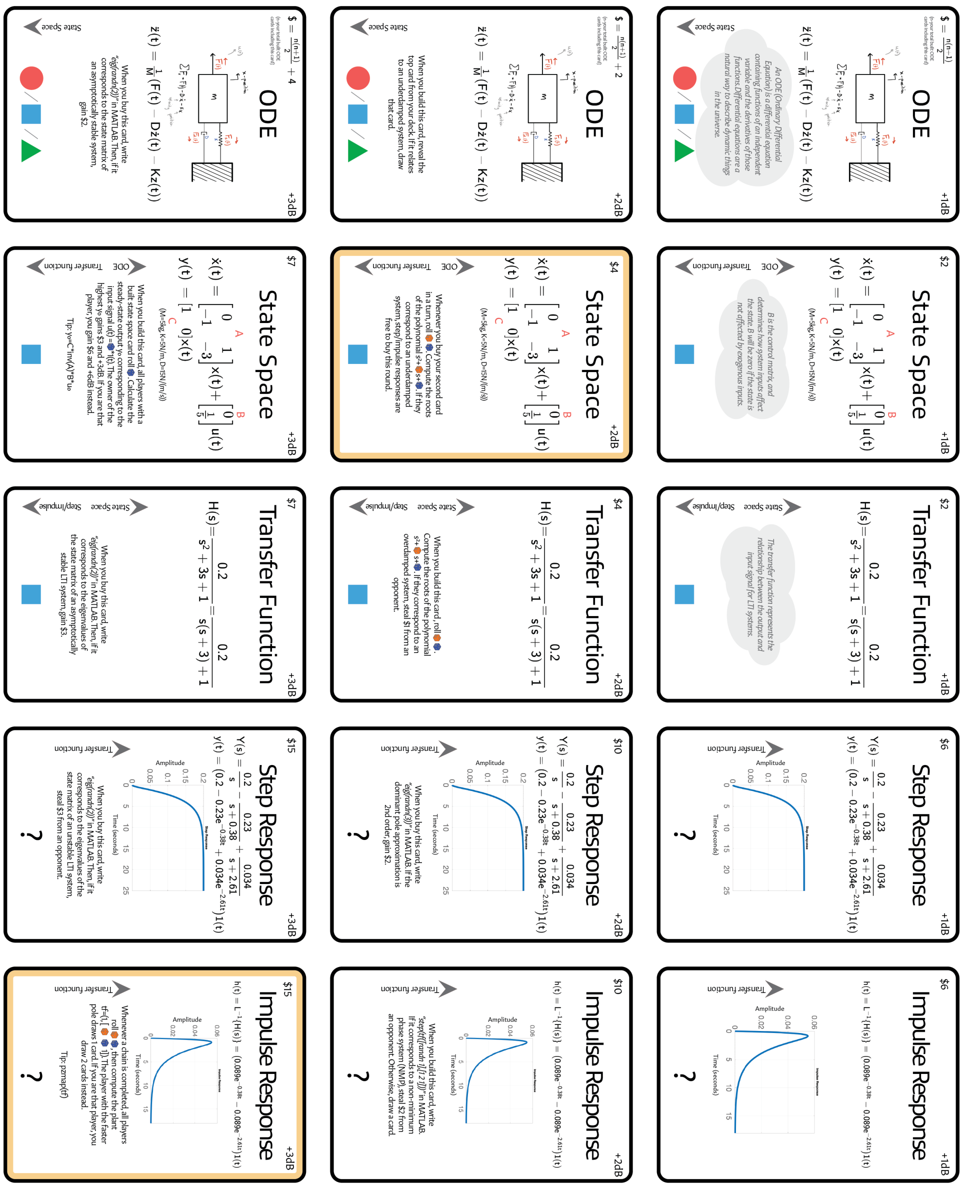
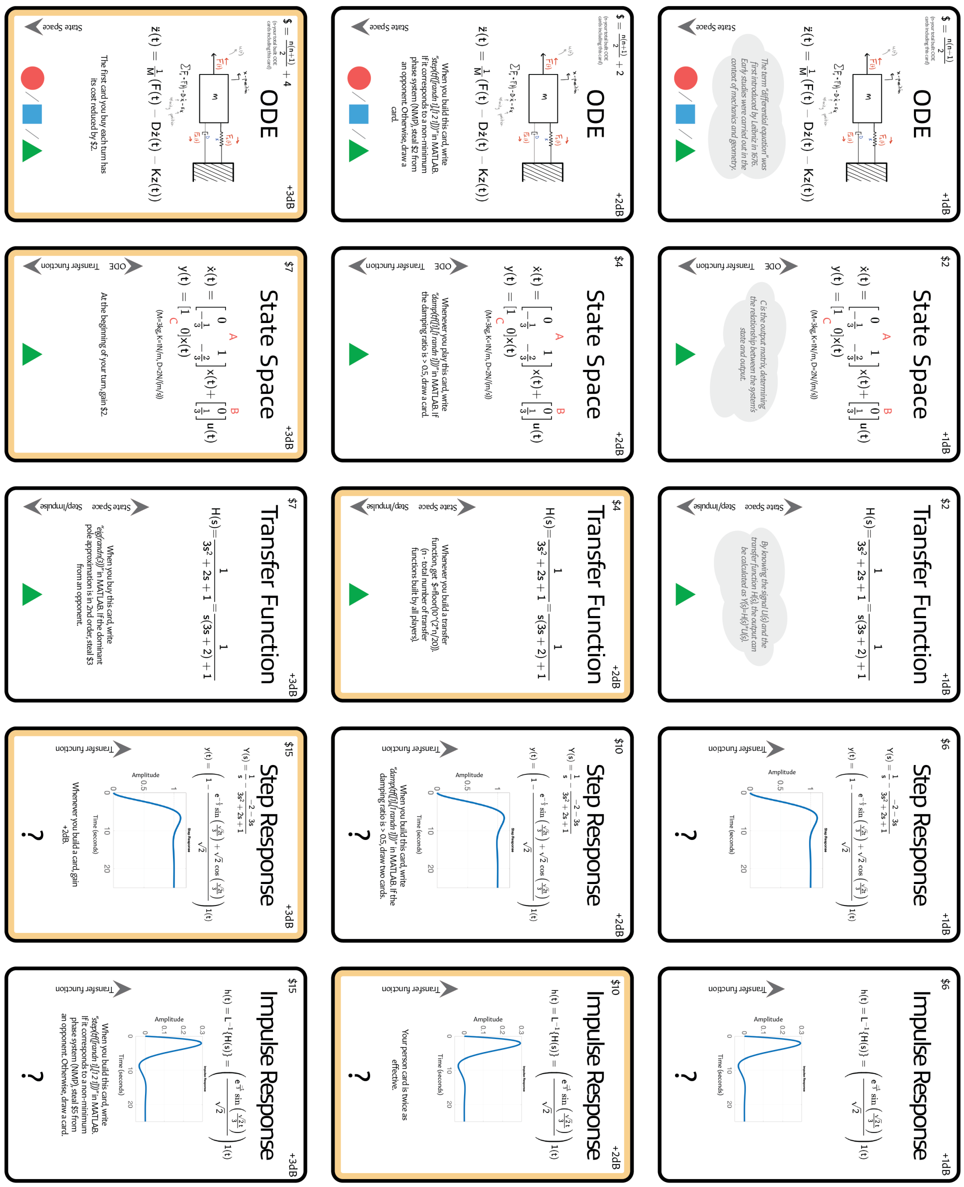
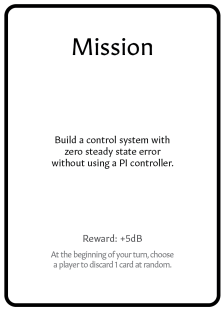
A Appendix - Full list of cards (Phase 1)

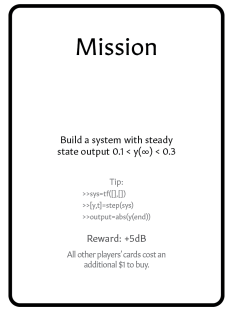

ODE
State Space
Transfer Function
Step Response
Impulse Response
+1dB
+2dB
+3dB
$2
$4
$7
$6
$10
$15
Whenever you build a card, gain $1.
Whenever you complete a mission, choose a player to discard 2 cards at random.
If you build this card in your first chain, gain $2.
Whenever you buy a card, you can add a card from your hand to your inventory without paying its cost.
At the beginning of your turn, draw an extra card.
When you build this card, steal $2 from an opponent.
Whenever an opponent completes a chain, steal $3 from that opponent.
When you build this card, steal $1 from every opponent who has more dB than you.

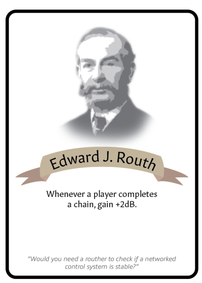

ODE
State Space
Transfer Function
Step Response
Impulse Response

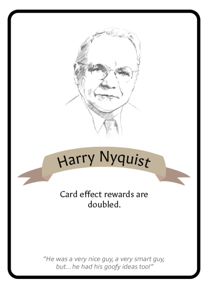

ODE
ODE
ODE
State Space
State Space
State Space
Transfer Function
Transfer Function
Transfer Function
Step Response
Step Response
Step Response
Impulse Response
Impulse Response
Impulse Response

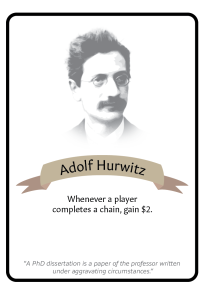
B Appendix - Full list of cards (Phase 2)

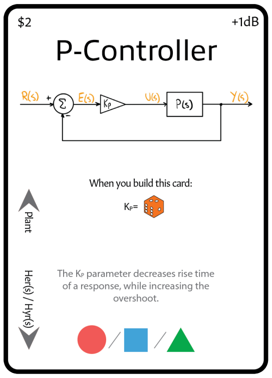


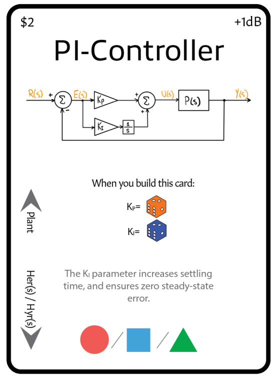


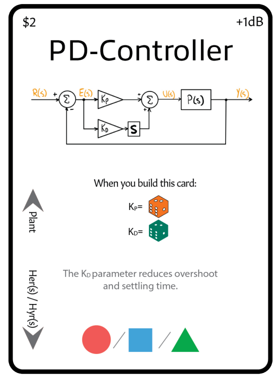


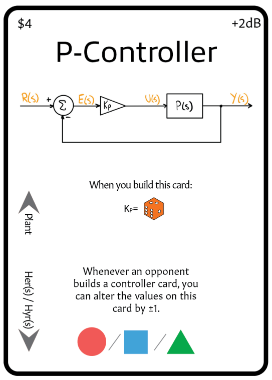


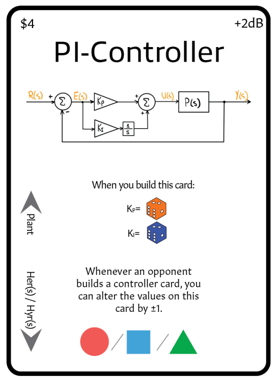


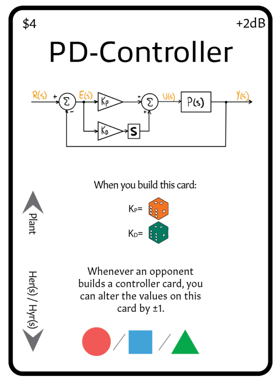

$2
+1dB
P-Controller
When you build this card:
K_P=
Plant
Her(s) / Hyr(s)
The K_P parameter decreases rise time of a response, while increasing the overshoot.

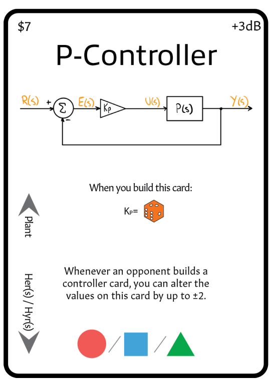

$2
+1dB
PI-Controller
When you build this card:
K_P=
K_I=
Plant
Her(s) / Hyr(s)
The K_I parameter increases settling time, and ensures zero steady-state error.

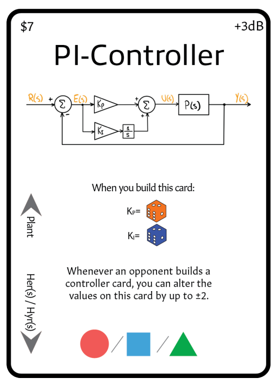

$2
+1dB
PD-Controller
When you build this card:
K_P=
K_D=
Plant
Her(s) / Hyr(s)
The K_D parameter reduces overshoot and settling time.

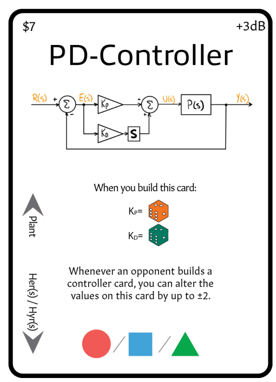

$4
+2dB
P-Controller
When you build this card:
K_P=
Plant
Her(s) / Hyr(s)
Whenever an opponent builds a controller card, you can alter the values on this card by ±1.


$4
+2dB
PI-Controller
When you build this card:
K_P=
K_I=
Plant
Her(s) / Hyr(s)
Whenever an opponent builds a controller card, you can alter the values on this card by ±1.


$4
+2dB
PD-Controller
When you build this card:
K_P=
K_D=
Plant
Her(s) / Hyr(s)
Whenever an opponent builds a controller card, you can alter the values on this card by ±1.


$7
+3dB
P-Controller
When you build this card:
K_P=
Plant
Her(s) / Hyr(s)
Whenever an opponent builds a controller card, you can alter the values on this card by up to ±2.


$7
+3dB
PI-Controller
When you build this card:
K_P=
K_I=
Plant
Her(s) / Hyr(s)
Whenever an opponent builds a controller card, you can alter the values on this card by up to ±2.


$7
+3dB
PD-Controller
When you build this card:
K_P=
K_D=
Plant
Her(s) / Hyr(s)
Whenever an opponent builds a controller card, you can alter the values on this card by up to ±2.

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card)

+1dB

Plant

$$P(s) = \frac{1}{s(s^2+s+4)}$$

When you build this card, write sys=tf([1], [1 randn randn 3]) in MATLAB. The system setpoint is 1. If the steady state error is $2 < e_\infty < 3$, you can alter a parameter of an opposing player's system by ±1, and gain $1.

Controller

$2 +1dB

Closed loop transfer function $H_{ER}(s)$

$$H_{ER}(s) = \frac{s^3+s^2+4s}{s^3+s^2+4s+K_P}$$

Controller

When you build this card, write H=tf([randn 1],[1 1 randn]), bode(H) in MATLAB. If the corresponding gain margin is > 3dB, draw a card.

Routh Hurwitz

?

$3 +2dB

Closed loop transfer function $H_{YR}(s)$

$$H_{YR}(s) = \frac{K_P}{s^3+s^2+4s+K_P}$$

Controller

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 13s: Draw two cards, and gain $1

```
>>KP=x
>>sys=tf([KP],[1 1 4 KP])
>>step(sys)
>>stepinfo(sys)
```

Routh Hurwitz

?

$5 +3dB

Routh Hurwitz criterion

1	4
1	K_P
$4-K_P$	0
K_P	0

Her(s)/Hyr(s)

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

?

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card)

+2dB

Plant

$$P(s) = \frac{1}{s(s^2+s+4)}$$

When you build this card, write sys=tf([randn 1],[1 1 randn]), bode(sys) in MATLAB. If the closed loop system is BIBO stable, steal $2 from an opponent.

Controller

$2 +1dB

Closed loop transfer function $H_{ER}(s)$

$$H_{ER}(s) = \frac{s^4+s^3+4s^2}{s^4+s^3+4s^2+K_Ps+K_I}$$

Controller

If you build this card in your first chain, gain $2.

Routh Hurwitz

?

$3 +2dB

Closed loop transfer function $H_{YR}(s)$

$$H_{YR}(s) = \frac{K_Ps+K_I}{s^4+s^3+4s^2+K_Ps+K_I}$$

Controller

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 40s: Draw two cards, and gain $1

```
>>KP=x
>>KI=x
>>sys=tf([KP KI],[1 1 4 KP KI])
>>step(sys)
>>stepinfo(sys)
```

Routh Hurwitz

?

$5 +3dB

Routh Hurwitz criterion

1	4	K_I
1	K_P	0
$4-K_P$	K_I	0
$\frac{K_I}{K_P-4}+K_P$	0	0
K_I	0	0

Her(s)/Hyr(s)

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

?

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card)

+3dB

Plant

$$P(s) = \frac{1}{s(s^2+s+4)}$$

When you build this card, steal $2 from an opponent, and alter a parameter of that player's system by ±1.

Controller

$2 +1dB

Closed loop transfer function $H_{ER}(s)$

$$H_{ER}(s) = \frac{s(s^2+s+4)}{s^3+s^2+4s+K_Ds+K_P}$$

Controller

When you build this card, draw a card and gain $1.

Routh Hurwitz

?

$3 +2dB

Closed loop transfer function $H_{YR}(s)$

$$H_{YR}(s) = \frac{K_Ds+K_P}{s^3+s^2+4s+K_Ds+K_P}$$

Controller

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 40s: Draw two cards, and gain $1

```
>>KP=x
>>KD=x
>>sys=tf([KD KP],[1 1 4 KD KP])
>>step(sys)
>>stepinfo(sys)
```

Routh Hurwitz

?

$5 +3dB

Routh Hurwitz criterion

1	$4+K_D$
1	K_P
K_D-K_P+4	0
K_P	0

Her(s)/Hyr(s)

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

?

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card) +1dB

Plant

$$P(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

When you build this card, write sys=tf([randn 1],[1 1 randn]), bode(sys) in MATLAB. If the system is stable, gain $2.

Controller

$2 +1dB

Closed loop transfer function $H_{ER}(s)$

$$H_{ER}(s) = \frac{(s+1)^2}{s^2 + 2s + K_P + 1}$$

When you build this card, steal $1 from every opponent who has more dB than you, then alter a parameter value in this chain by ±1.

Controller / Routh Hurwitz

?

$3 +2dB

Closed loop transfer function $H_{YR}(s)$

$$H_{YR}(s) = \frac{K_P}{s^2 + 2s + K_P + 1}$$

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 3.6s: Draw two cards, and gain $1

```
>>KP =x
>>sys=tf([KP],[1 2 KP+1])
>>step(sys)
>>stepinfo(sys)
```

Controller / Routh Hurwitz

?

$5 +3dB

Routh Hurwitz criterion

1	$K_P + 1$
2	0
$K_P + 1$	0

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

Her(s) /Hyr(s)

?

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card) +2dB

Plant

$$P(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

When you build this card, reveal the top card of your deck. If it is a controller card, draw that card. If not, you can alter a parameter of an opposing player's system by ±1.

Controller

$2 +1dB

Closed loop transfer function $H_{ER}(s)$

$$H_{ER}(s) = \frac{s^3 + 2s^2 + s}{s^3 + 2s^2 + s + K_P s + K_I}$$

When you build this card, write s=tf('s'), H=1/((s+randn)(s+randn)), nyquist(H) in MATLAB. If the number of anticlockwise encirclements of the origin is equal to the number of unstable poles in H(s), gain $3. (system will then become stable when the loop is closed)

Controller / Routh Hurwitz

?

$3 +2dB

Closed loop transfer function $H_{YR}(s)$

$$H_{YR}(s) = \frac{K_P s + K_I}{s^3 + 2s^2 + s + K_P s + K_I}$$

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 6.5s: Draw two cards, and gain $1

```
>>KP =x
>>KI =x
>>sys=tf([KP+KI],[1 2 KP+1  KI])
>>step(sys)
>>stepinfo(sys)
```

Controller / Routh Hurwitz

?

$5 +3dB

Routh Hurwitz criterion

1	$K_P + 1$
2	K_I
$K_P - \frac{K_I}{2} + 1$	0
K_I	0

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

Her(s) /Hyr(s)

?

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card) +3dB

Plant

$$P(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

When you build this card, write s=tf('s'), H=1/((s+randn)(s+randn)), nyquist(H) in MATLAB. If the closed loop system is BIBO stable, you can alter a parameter of an opposing player's system by ±1, and gain $1.

Controller

$2 +1dB

Closed loop transfer function $H_{ER}(s)$

$$H_{ER}(s) = \frac{(s+1)^2}{s^2 + 2s + K_D s + K_P + 1}$$

When you build this card, write sys=tf([1], [1 randn 3]) in MATLAB. If the settling time is > 10s, steal $2 from an opponent.

(>>stepinfo(sys))

Controller / Routh Hurwitz

?

$3 +2dB

Closed loop transfer function $H_{YR}(s)$

$$H_{YR}(s) = \frac{K_D s + K_P}{s^2 + 2s + K_D s + K_P + 1}$$

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 4.5s: Draw two cards, and gain $1

```
>>KP =x
>>KD =x
>>sys=tf([KD+KP],[1 KD+2  KP+1])
>>step(sys)
>>stepinfo(sys)
```

Controller / Routh Hurwitz

?

$5 +3dB

Routh Hurwitz criterion

1	$K_P + 1$
$2 + K_D$	0
$K_P + 1$	0

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

Her(s) /Hyr(s)

?

Plant

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card)

+1dB

$$P(s) = \frac{1}{s^2 + s - 2}$$

When you build this card, you can alter a parameter value of an opposing player's system by ±1, and gain +2dB.

Controller

Closed loop transfer function $H_{ER}(s)$

$2 +1dB

$$H_{ER}(s) = \frac{s^2 + s - 2}{s^2 + s - 2 + K_P}$$

When you build this card, write
s=tf('s'),
H=1/((s+randn)(s+randn)),
nyquist(H)
in MATLAB. If the closed loop system has exactly one more unstable pole than H, gain $2.

Controller / Routh Hurwitz

Closed loop transfer function $H_{YR}(s)$

$3 +2dB

$$H_{YR}(s) = \frac{K_P}{s^2 + s - 2 + K_P}$$

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 7.5s: Draw two cards, and gain $1

```
>>KP =x
>>sys=tf([KP],[1 1 -2+KP])
>>step(sys)
>>stepinfo(sys)
```

Controller / Routh Hurwitz

Routh Hurwitz criterion

$5 +3dB

1	$K_P - 2$
1	0
$K_P - 2$	0

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

Her(s)/Hyr(s)

?

Plant

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card)

+2dB

$$P(s) = \frac{1}{s^2 + s - 2}$$

When you build this card, write
s=tf('s'),
H=1/((s+randn)(s+randn)),
nyquist(H)
in MATLAB. If the closed loop system is BIBO stable, you can alter a parameter of an opposing player's system by ±1, and gain $1.

Controller

Closed loop transfer function $H_{ER}(s)$

$2 +1dB

$$H_{ER}(s) = \frac{s^3 + s^2 - 2s}{s^3 + s^2 - 2s + K_P s + K_I}$$

When you build this card, write H=tf([randn 1],[1 1 randn]), bode(H) in MATLAB. If the gain margin is > 3dB, steal $2 from an opponent. Otherwise, draw a card.

Controller / Routh Hurwitz

Closed loop transfer function $H_{YR}(s)$

$3 +2dB

$$H_{YR}(s) = \frac{K_P s + K_I}{s^3 + s^2 - 2s + K_P s + K_I}$$

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 20s: Draw two cards, and gain $1

```
>>KP =x
>>KI =x
>>sys=tf([KP KI],[1 1 -2+KP KI])
>>step(sys)
>>stepinfo(sys)
```

Controller / Routh Hurwitz

Routh Hurwitz criterion

$5 +3dB

1	$K_P - 2$
1	K_I
$K_P - 2 - K_I$	0
K_I	0

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

Her(s)/Hyr(s)

?

Plant

$\$ = \frac{n(n+1)}{2}$ (n-your total built Plant cards including this card)

+3dB

$$P(s) = \frac{1}{s^2 + s - 2}$$

When you buy this card, you can alter a parameter of an opposing player's system by ±1, and draw a card.

Controller

Closed loop transfer function $H_{ER}(s)$

$2 +1dB

$$H_{ER}(s) = \frac{s^2 + s - 2}{s^2 + s + K_D s - 2 + K_P}$$

When you build this card, write H=tf([randn 1],[1 1 randn]), bode(H) in MATLAB. If the system is BIBO stable, you can alter a parameter of an opposing player's system by ±2.

Controller / Routh Hurwitz

Closed loop transfer function $H_{YR}(s)$

$3 +2dB

$$H_{YR}(s) = \frac{K_D s + K_P}{s^2 + s + K_D s - 2 + K_P}$$

At the beginning of your turn, if the closed loop system is:

- Unstable: Lose $2 and -2dB
- Underdamped: Draw a card, and gain $1
- Settling time > 3.5s: Draw two cards, and gain $1

```
>>KP =x
>>KD =x
>>sys=tf([KD KP],[1 KD+1  KP-2])
>>step(sys)
>>stepinfo(sys)
```

Controller / Routh Hurwitz

Routh Hurwitz criterion

$5 +3dB

1	$K_P - 2$
$K_D + 1$	0
$K_P - 2$	0

When this card is built, you can reroll controller values in this chain by paying $3 per reroll.

Her(s)/Hyr(s)

?

C Appendix - Game manual

C.1 Game overview

The main objective of the game is to buy and build subsystems. When a player has finished all 3 missions, the game ends and the player with the most points wins the game.

C.2 Game rules

- Each player starts with $8 at the start of the game.
- Before the game starts, each player chooses a person card to play as.
- When the game starts, place 3 random mission cards from the mission deck onto the table.
- One player starts, going in the clockwise direction.
- When you finish your turn, discard your hand and draw 5 new cards from the deck. Your hand acts as a shop where you can buy the specified cards.
- To buy cards from your hand, you must spend money that you accumulate each round by different methods. The currency system is explained below.
- When you buy cards, they move into your separate inventory pile.
- You can buy and build as many cards as you want each round, as long as you can afford them.
- When building cards onto the card chain, you must build the system in order (for phase 1: ODE → State space → Hp(s) → Impulse and/or step response. The order is indicated on each card).
- The first card chain is free to build, while subsequent chains have an increasing cost as indicated on the first card of the chain.

C.3 Currency system

- Gaining passive income is achieved by collecting dB through various methods.
- Each player starts with 0dB, accumulating as the game goes on.
- Every dB value corresponds with a certain amount of $:

$$\text{\$ per round} = floor(10^{\frac{dB}{20}})$$

- At the beginning of the players turn, the player collects $ equal to that of their dB value.
- Gain +dB by building cards, completing missions or by utility cards.

C.4 Ending the game

- The game ends when a player has finished all 3 missions.
- Each player adds up their total score to see who wins. Points are given as follows:
  - 1 point per $.
  - 1 point per dB.
  - 1 point per built card.
  - 3 points for each complete card chain.
  - 3 points per completed mission.
  - 5 points to the player who first completes 3 missions and ends the game.