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Abstract

Inflation is a sudden and near exponential expansion of space in early universe. Inflation solved flatness problem, horizon problem and monopole problem that persisted in standard big bang cosmology. It was not only useful in solving these problem but also accounted for origin of structure in universe that we observe today. Direct observation of this phenomenon is not possible but its occurrence is confirmed by present cosmological observation.

In this thesis we will investigate inflation driven by scalar field via equation of motion on classical level and with quantum corrected ones as well.First we will solve classical and quantum corrected evolution of field in different cosmological background and eventually compare results using graphs.

Similarly we will explore some aspects of curvature perturbation.It is now widely accepted that curvature perturbation is dominant cause of structure formation in universe. The field responsible for curvature mechanism is called curvaton. It evolves during inflation as sub-dominant field but is independent of field driving inflation. Curvaton is subdominant during inflation so adiabatic (curvature) perturbation is first achieved by isocurvature(entropy) perturbation.After the end of inflation curvaton starts to oscillate during radiation dominated era and converts isocurvature perturbation into curvature perturbation.We will examine various potential by taking into account all the constraints and also use the fact that the model we use generates curvature perturbation of observed limit ,$\zeta \sim 10^{-5}$.

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Units and Convention

Throughout the thesis we will use following units and convention

We will use natural units where the speed of light ,the Boltzmann constant and the Planck constant are set to unity i.e.

$c \equiv k_B \equiv \hbar$=1

Reduced Planck mass is defined as follows

$M_{pl}^2 = \frac{\hbar c}{8\pi G}$=$\frac{1}{8\pi G}$

G is Newton's constant

Sign convention for Minkowski metric is

$\eta_{\mu\nu}$=diag (1,-1,-1,-1)

Similarly Riemann tensor is defined as

$R^{\delta}{}_{abc}$=$\Gamma^{\delta}{}_{ac,b}$-$\Gamma^{\delta}{}_{ab,c}$+$\Gamma^{\delta}{}_{db}\Gamma^{d}{}_{ca}$-$\Gamma^{\delta}{}_{dc}\Gamma^{d}{}_{ba}$

where $\Gamma^{d}{}_{ca}$ is called Christoffel symbol and is defined as

$\Gamma^{d}{}_{ca}$=$\frac{1}{2}g^{db}(g_{bc,a} + g_{ba,c} - g_{ca,b})$

Fourier convention

$f_k(t) = \int \frac{d^3x}{(2\pi)^3} f(x,t)e^{-ik.x}$

Chapter 1

Introduction

1.1 Inflation

Inflation is a sudden and near exponential expansion of space in early universe. The period of inflation was quite short and lasted for about fraction of a second. Big Bang cosmology successfully explained primordial abundances of elements, red shift of the distant galaxy and origin of cosmic microwave background but it was unable to explain flatness problem and horizon problem [1] . Introduction of idea of inflation made it possible to resolve these issues.

1.2 Flatness Problem

Flatness problem is associated with standard big bang cosmology. Universe is balanced between positively curved closed universe and negatively curved open universe resulting it to be spatially flat. This balance was finer at the time of big bang. Small deviation would have resulted in the different fate of universe. For example if it was not fined tuned and was deviated by small value from being flat then the universe would have re-collapsed for positively curved universe or would have expanded so rapidly that it seems to be devoid of matter for negatively curved universe [2].

We have Friedmann eqution(refer section 2.4 for Friedmann Equation) for homogeneous and isotropic universe as

$$H^2 = \frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2} \tag{1.1}$$

where

H is Hubble parameter

M_{pl} is Planck mass
ρ is energy density of the universe
k is curvature of space
a is scale factor
If $k = 0$ i.e. universe is spatially flat then equation (1.1) reduces to

$$H^2 = \frac{\rho_c}{3M_{pl}^2} \tag{1.2}$$

where ρ_c is called critical density.
Using equation (1.1) and (1.2)

$$1 = \frac{\rho}{\rho_c} - \frac{k}{a^2 H^2} \tag{1.3}$$

where

$$H = \frac{\dot{a}}{a} \tag{1.4}$$

$$\Omega = \frac{\rho}{\rho_c} \tag{1.5}$$

$\dot{a}$ is rate of change of scale factor and Ω is relative density.So

$$1 - \Omega = -\frac{k}{\dot{a}^2} \tag{1.6}$$

According to standard Big Bang cosmology rate of change of scale factor is decreasing with time, which means universe is decelerating. This causes $|\Omega|$ to deviate away form one.This known as Flatness problem.

1.3 Horizon Problem

This problem arises due to the fact that on large scale we measure universe to be homogeneous and isotropic.
Friedmann-Robertson-Walker metric for homogeneous and isotropic universe is [3]

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \; d\phi^2\right] \tag{1.7}$$

where $a(t)$ is scale factor and k measures curvature of space.

$$k = \begin{cases} > 0 & \text{closed universe} \\ < 0 & \text{open universe} \\ = 0 & \text{spatially flat} \end{cases}$$

For $k = 0$ equation (1.7) can be re-written as

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \; d\phi^2\right] \tag{1.8}$$

As light follow null geodesics i.e. $ds = 0$ and θ, ϕ are constant so

$$0 = dt^2 - a^2(t)\left[dr^2\right] \tag{1.9}$$

$$r = \int \frac{dt}{a(t)} \tag{1.10}$$

Then radius of the causally connected parts of the universe at the time of recombination compared to co-moving radius at present is given by

$$\int_0^{t_{dec}} \frac{dt}{a(t)} << \int_{t_{rec}}^{t_0} \frac{dt}{a(t)} \tag{1.11}$$

This expression implies that much larger part of the universe is visible today than was visible at the time of recombination.There was not enough time in the past for photon to communicate between two opposite direction of the universe and maintain thermal equilibrium. Today we measure temperature of background radiation as 2.73 K irrespective of direction in which we measure.Almost isotropic temperature and homogeneous distribution of matter in universe is characterized by Horizon problem associated with standard big bang cosmology.

Flatness and Horizon problem was solved by introduction of idea of inflation.According to which early universe went through an accelerating phase and universe expanded by at least factor of e^{60}. Before inflation started universe was casually connected which resulted in isotropy. Likewise enormous expansion of space during inflation washed out all the inhomogenities.

Inflation not only solves flatness and horizon problem but also proven to be seed for structure formation that we see in universe. Recently main aspect of inflation is that it has provided mechanism of density perturbation which is root for structure formation [4]. Small variation in energy density due to quantum uncertainties was amplified by gravity over billions of years to form galaxies, clusters and super cluster of galaxy. This perturbation in energy density is imprinted in the cosmic microwave background radiation which was first detected by COBE satellite [5]. Although it is successful in explaining various cosmological observations however its nature and origin is still unexplained [6].Most of the model explaining inflation are based on scalar field ϕ called inflaton. Scalar field dominated by potential energy creates negative pressure. This negative pressure makes gravity to be repulsive. Gravitational repulsion is believed to have caused inflation. Density perturbation and tensor perturbation are generated by quantum fluctuation of this scalar field (inflaton) [7].At the time of inflation physical scale grow faster than Hubble radius. But in case of radiation and matter dominated era growth of physical scale is slower. So quantum fluctuation generated during inflation cross horizon twice. Thus quantum fluctuation generated during inflation undergo transition from quantum to classical [8].Wavelength of fluctuations which are smaller than Hubble radius cross horizon, for first time, become classical and freeze out. Secondly, during decelerated stage both in radiation dominated era or matter dominated era these fluctuation again re-enter horizon and provide base for structure formation .Though there are number of models explaining inflation but the common feature of these model is that the quantum fluctuation is Gaussian and scale invariant spectrum. [9, 10].

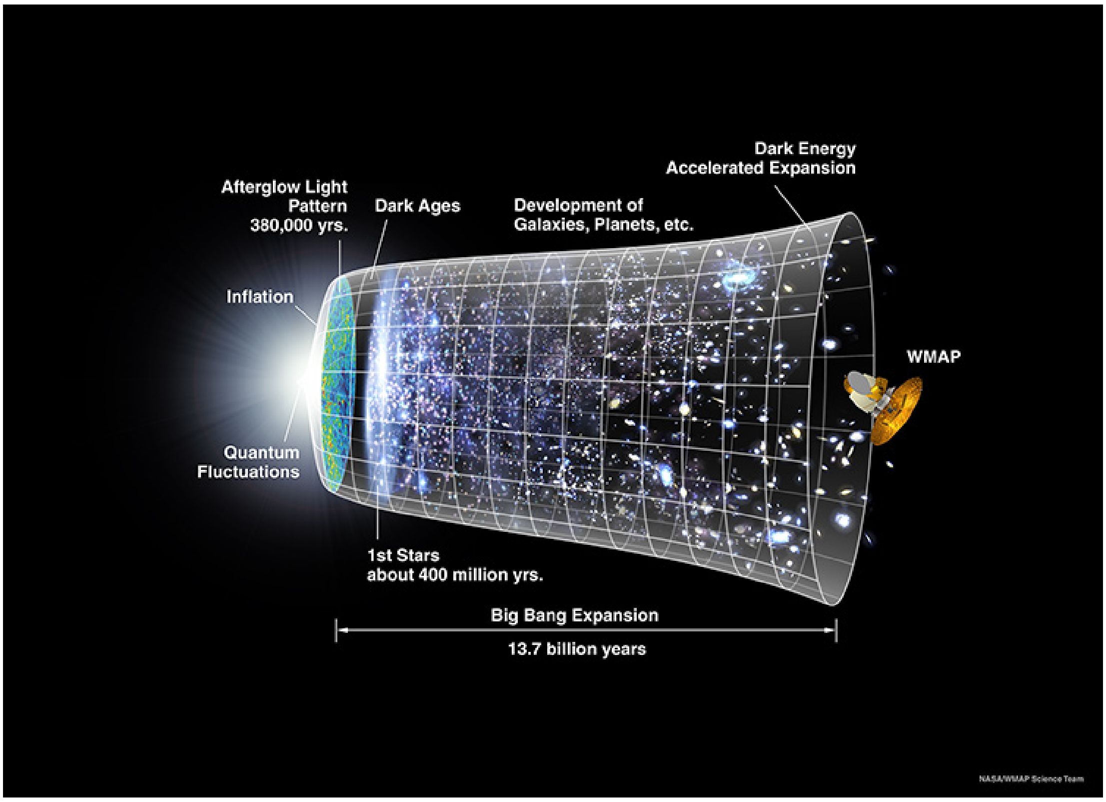

Figure 1.1: Time line of universe

Reference http://wmap.gsfc.nasa.gov/media/060915/index.html (Accessed on 13-04-2015)

1.4 Layout of thesis

Chapter two includes overview of cosmic microwave background radiation, slow roll inflation with discussion of different parameter associated with it. Also we will take effective action and equation of motion from paper by Markkanen and Anders Tranberg for two scalar field model [11].Under the condition FRW space, minimal coupling to gravity, no self interaction and allowing only one field to evolve (i.e. σ) we obtain quantum corrected dynamics of spectator field,σ [12] . These equations of motion are solved under different cosmological background. Finally the dynamics are compared graphically with classical evolution of the field.
Chapter third is on curvature perturbation. It is now widely believed that curvature mechanism is root for structure formation. Quantum fluctuation during inflation induces adiabatic density perturbation and results in curvature perturbation. But quantum fluctuations generated during inflation is independent of inflation. Its evolution is guided by separate scalar field called curvaton. Cosmological observation show that the anisotropies at decoupling was of order 10^{-5}. Here we will explore different potential starting from the quadratic one to potential with self interaction and figure out numerical range of parameters which satisfies the observed value of curvature perturbation with underlying constraints which are imposed by current precise measurement.

Chapter 2

Quantum corrected field dynamics

2.1 History of universe

Present understanding of physics and our hypothesis claim that universe began some 14 billion years ago as a Big Bang scenario. In fact Big Bang theory does not reveal how universe came to existence for the first time .Physical laws that we have today does not probe nature beyond Planck time. So at the time of Big Bang space, time and energy were already into existence [13]. Inflation followed after this which lasted for 10^{-34} seconds. As inflaton field decayed universe was filled with radiation. Baryonic matter were formed within first seconds and as the universe expanded, simultaneously cooled, these baryonic matter combined to form nucleons which is termed as Nucleosynthesis [14].After this it took nearly 380,000 years to form neutral atom. By this time temperature of universe was about 3000k [15].This was followed by cosmic "dark ages".It is named so because luminous stars and galaxies were not into existence and most of the matter in universe was in the form of dark matter and few percentage of ordinary matter which was mostly hydrogen and helium.After dark ages universe went through a phase called Epoch of reionization.During this period gravitational attraction caused dark matter to collapse to form halo-like structure and ordinary matter was also pulled into it to form first stars and galaxy.High energy radiation was released during this process which caused normal matter to ionize.This phenomenon is cosmic re-ionization.Presently universe is 13.8 billion years old and is dominated by dark energy.Short overview of History of universe is presented in the table below.

Event	Time	Temperature
Inflation	$10^{-34}s$	-
Baryogenesis	-	-
EW phase transition	20 ps	100 GeV
QCD phase transition	20 μs	150 MeV
Dark Matter Freeze-out	-	-
Neutron Decoupling	1 s	1 MeV
Electron-positron annihilation	6 s	500 KeV
Big Bang Nucleosynthesis	3 min	100 Kev
Matter Radiation equality	60 Kyr	0.75 MeV
Recombination	$260 - 380$ Kyr	$0.26 - 0.33$ eV
Photon Decoupling	380 Kyr	$0.23 - 0.28$ eV
Reionization	$100 - 400$ Myr	$2.6 - 7.0$ meV
Dark energy-matter equality	9 Gyr	0.33 meV
Present	13.8 Gyr	0.24 meV

Table 2.1: Thermal history of Universe [16]

2.2 Cosmic Microwave Background Radiation

As we know early universe was hot, dense and full of particle and radiation. All the fundamental particles of standard model were moving freely which is known as plasma. Due to very high temperature, combination of these fundamental particles to form atoms was impossible. As soon as they try to combine (i.e. if electron tries to orbit around the nucleus) high energy photon would smash it apart. Due to this hot and dense universe mean free path of photon was extremely short. So as we try look universe beyond the surface of last scattering it is opaque. But as universe expanded , wavelength of radiation also got stretched which resulted in decrease of temperature. When temperature was about 3000 K nucleons and electrons combined to form neutral atoms. This made radiation to decouple from the particle. After that radiation traveled freely but got stretched as universe expanded. Presently we detect it in microwave range and entitled as cosmic microwave background radiation. This has become fossil remnant of the universe. CMB radiation has black body distribution which peaks at 2.73 K. This value is obtained by subtracting all the radiation coming from the local sources like galaxy, sun, stars as well as neglecting the radiation like x-ray, Gamma ray ,infrared radiation, dust etc. COBE, WMAP, and Planck are the main satellite missions which are lunched to measure this radiation.

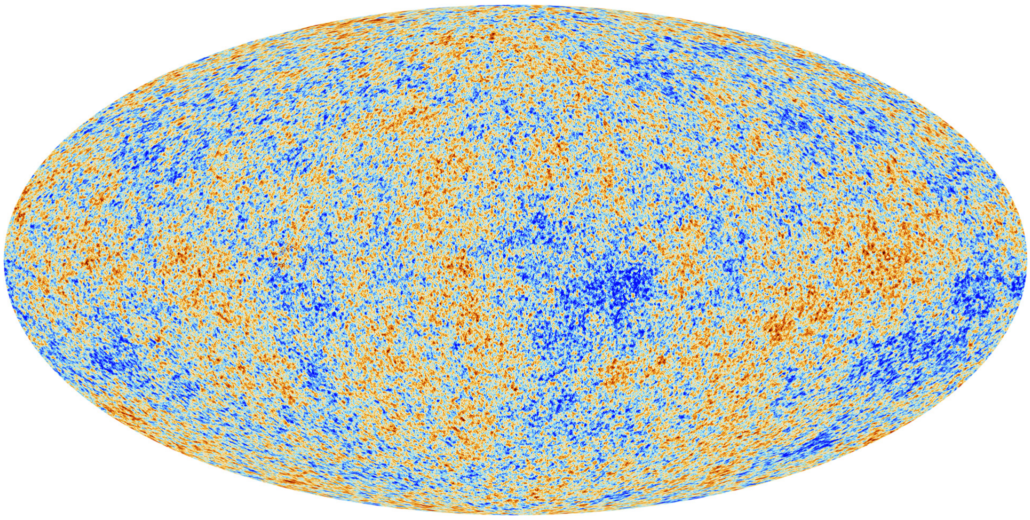

Figure 2.1: Plank CMB

Credit, ESA and the Planck Collaboration (Released 21/03/2013 12:00 pm)(ESA, European space agency)
Web reference: http://www.esa.int/spaceinimages/Images/2013/03/Planck_CMB (Accessed on 16-04-2015)
In the figure above blue and red regions (CMB anisotropy) are due to gravitational red-shift also known as Sachs-Wolfe effect. It is dependent on the Newtonian Potential (ϕ). Higher the value of ϕ more will be red-shift. So red regions represent a denser and higher temperature region compared to blue region.For adiabatic perturbation, mathematically it can be expressed as

$$\frac{\Delta T}{T} = \frac{1}{3}\phi \tag{2.1}$$

(2.2)

On top of this smooth background there are small fluctuations , 1 out of 100,000.These deviation in temperature is vital in formation of structure like galaxies, cluster of galaxy and super cluster of galaxy [1]. From this fluctuation what we are trying to figure out is
a) What do these deviations really look like?
b) How do they come?
These deviations have origin way back from inflation. Quantum fluctuation of inflaton

field is believed to have produced these deviation. This also suggests that inflaton field had different value at different point of universe. These fluctuations imprint themselves in CMB map. These fluctuations in field couple through the gravitational field and the gravitational field then determine through the Einstein equation how matter and radiation move.

2.3 CMB Power Spectrum

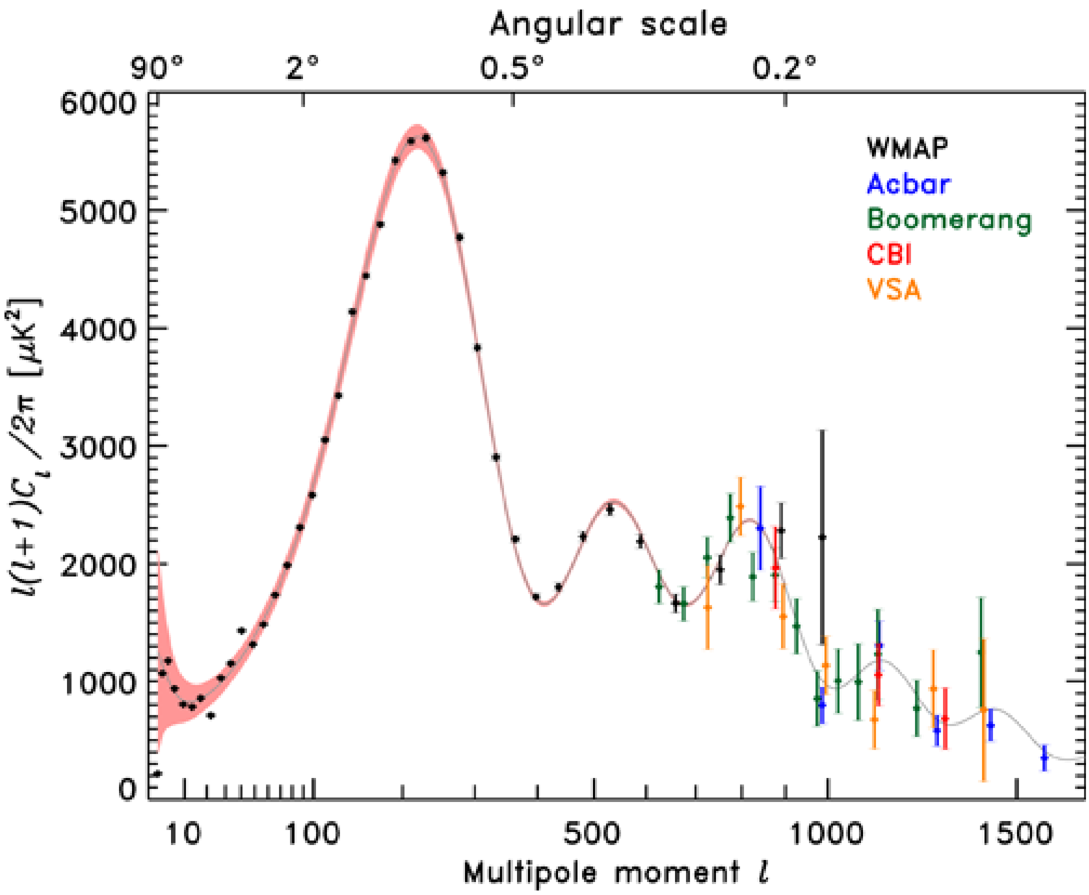

Figure 2.2: The CMB power spectrum modeled with data from WMAP (2006), ACBAR (2004), BOOMERanG (2005), CBI(2004), and VSA (2004) experiments.

Web Reference:http://physics.stackexchange.com/questions/155508/angular-power-spectrum-of-cmb (Accessed on 18-04-2015)

In CMB map we are interested in temperature anisotropies. These anisotropies are projected in 2D spherical surface so can be expanded in spherical harmonics as [17]

$$\frac{\Delta T}{T} = T(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi) \tag{2.3}$$

(2.4)

where a_{lm} are multipole coefficient and given by

$$a_{lm} = \int_{\theta=-\pi}^{\pi} \int_{\phi=0}^{2\pi} T(\theta, \phi) Y_{lm}^{*} d\Omega \tag{2.5}$$

l is multipole moment and measures angular scale and is given by

$$l = \frac{\pi}{\theta} \tag{2.6}$$

This shows that for higher value of l,θ is small. For each value of l there are $(2l + 1)$ values of m. So amount of anisotropy for multipole moment l is given by

$$C_l = |a_{lm}|^2 \tag{2.7}$$

C_l is known as power spectrum.
The plot between l and $l(l+1)C_l$ is known as CMB power spectrum. C_l measures measures temperature anisotropies of two point separated by an angle,θ. $l = 0$ corresponds to point separated from itself by 360^0 so it is zero.Similarly dipole term $l = 1$ i.e. $(\theta = 180^0)$ is effected by our motion across the space therefore CMB photon will suffer Doppler effect and is discarded from power spectrum. So higher values of l are taken into account for CMB power spectrum.
When photon got decoupled from the surface of last scattering there was temperature in-homogeneity. Photons which got scattered by same electron when met each other, scattered radiation was polarized. As this phenomenon occurred at very last seconds of recombination quadrupole polarization of CMB is in small fraction. The peaks in the CMB power spectrum depend on various factors like baryon density, Hubble constant, density of matter and cosmological constant [18, 19] The plot is complex but reveals much information about the universe. Careful examination of the spectrum gives the information about the density, curvature and the matter content of the universe.

2.4 Einstein and Friedmann Equation

Einstein equation can be obtained by using principle of least action.Rigorous derivation of Einstein equation can be found in [11, 15, 20].We will consider scalar field φ which couples to standard Einstein gravity with Freidmann-Robertson-Walker type metric.Line element of FRW metric is given by

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = dt^2 - a^2(t) d\mathbf{x}^2 \tag{2.8}$$

Action integral for gravitation can be written as

$$S_G = \frac{1}{2\kappa}\int \mathcal{L}[g_{\mu\nu}]\sqrt{-g}d^4x \tag{2.9}$$

where

$$\mathcal{L}[g_{\mu\nu}] = R - 2\Lambda \tag{2.10}$$

where R is called Ricci scalar and Λ is called cosmological constant.
Similarly action integral for matter and energy is given by

$$S_m = \int \mathcal{L}_m[\varphi, g_{\mu\nu}]\sqrt{-g}d^4x \tag{2.11}$$

where $\mathcal{L}_m[\varphi, g_{\mu\nu}]$ is Lagrangian density of energy and matter given by

$$\mathcal{L}_m[\varphi, g_{\mu\nu}] = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + V(\varphi, g_{\mu\nu}) \tag{2.12}$$

so total action is sum of S_G and S_m i.e.

$$S = S_G + S_m \tag{2.13}$$

Now using principle of least action and varying with respect to metric $g_{\mu\nu}$,Einstein equation can be obtained

$$\frac{\delta S[\varphi, g_{\mu\nu}]}{\delta g_{\mu\nu}} = 0 \tag{2.14}$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{2.15}$$

Also,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \tag{2.16}$$

is called Einstein tensor.Similarly,

$$\kappa = \frac{1}{M_{pl}^2} = \frac{1}{8\pi G} \tag{2.17}$$

$R_{\mu\nu}$ is called Ricci tensor
G is universal gravitational constant
$T_{\mu\nu}$ is called energy momentum tensor.
Λ is cosmological constant
For perfect fluid in the orthonormal basis comoving with the fluid energy momentum

tensor takes the form

$$T_{\mu\nu} = diag(\rho, -p, -p, -p) \tag{2.18}$$

where ρ is total energy density of fluid
p is pressure associated with fluid component.
Equation (2.15)can be reduced to two independent equations.Because of isotropy Einstein tensor($G_{\mu\nu}$) and Ricci tensor($R_{\mu\nu}$) can be written as

$$R_{00} = -3\frac{\ddot{a}}{a} \tag{2.19}$$

$$R_{ij} = \left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\left(\frac{k}{a^2}\right)\right) \tag{2.20}$$

$$R = \frac{6}{a^2}\left(a\ddot{a} + 2\dot{a}^2 + k\right) \tag{2.21}$$

Similary, Einstein tensor can be written as

$$G_{00} = 3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a}^2\right] \tag{2.22}$$

$$G_{0i} = 0 \tag{2.23}$$

$$G_{ij} = -\left(2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) \tag{2.24}$$

Using above sets of relations in Einstein equation we get Friedmann equation as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} \tag{2.25}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{2.26}$$

For isotropic and homogeneous universe model which expands adiabatically and with $\Lambda = 0$ above Friedmann equation can be reduced to

$$\rho a^{3(\omega+1)} = \rho_0 \tag{2.27}$$

ρ_0 is the present density of universe
$\omega = -1$ corresponds to vacuum energy density. This also indicates that density of vacuum energy is constant throughout the history of universe regardless of scale factor a. Similarly $\omega = 0$ corresponds to dust (pressure less matter) and matter density scales as a^{-3} as universe expands. Likewise, $\omega = \frac{1}{3}$ corresponds to radiation and radiation density scales as a^{-4} as universe expands. This also indicates that universe was radiation dominated at the beginning and as universe expanded it was taken over by matter and now it is

dominated by vacuum energy. The variation of different composition of universe and relative change with red shift is shown in graph below. At red-shift about 1100 radiation was dominant energy component and at low value of red shift the dark energy is major component of energy density

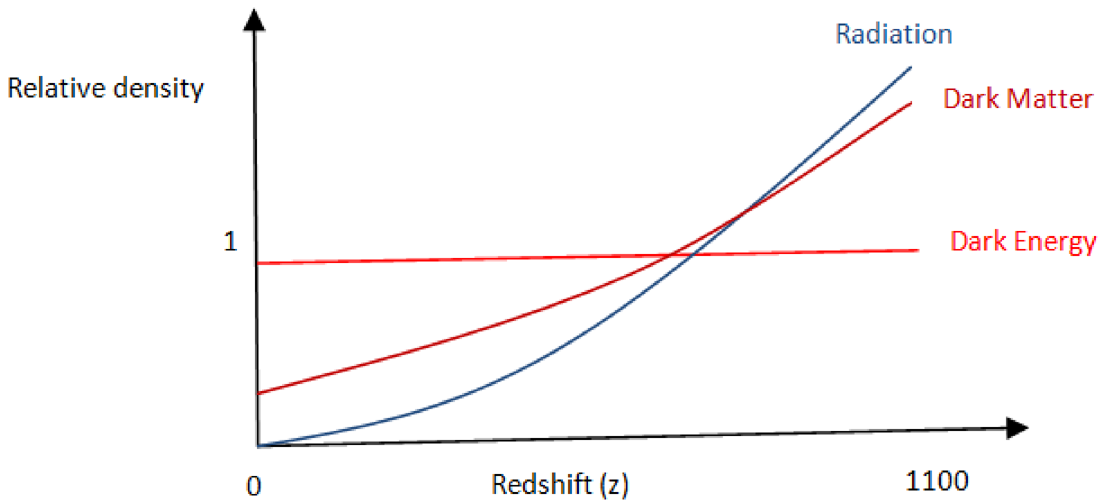

Figure 2.3: Variation of content of universe with red-shift

2.5 Slow-Roll inflation model

Gaussian, adiabatic and scale invariant spectrum of primordial fluctuation are generic prediction of most inflationary models. These predictions are also supported by precise measurement from various experiments therefore inflation has set a firm ground in cosmological evolution. Experimental observation is not only validating theory but also constraining different models of inflation. Among wide variety of model for inflation slow-roll inflation is robust to generic prediction. Flat potential of slow roll inflation leads to slowly varying Hubble parameter which provides sufficient number of e-folds, scale invariant spectrum and gaussianity [7]. Simplest model of inflation can be explained in term of single scalar field. Scalar field represent particle with zero spin. In homogeneous universe scalar field is only function of time. Effective energy density and pressure of homogeneous scalar field are obtained by comparing with energy momentum tensor of perfect fluid as follows [3]

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{2.28}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{2.29}$$

First term of these equation can be termed as kinetic energy and second term can be regarded as the potential energy.
When we substitute this equation in Friedmann equations with $\Lambda = 0$ and for flat universe i.e $\kappa = 0$ we get

$$H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \tag{2.30}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \tag{2.31}$$

where $'$ denotes derivative with respect to ϕ. Similarly.

$$\frac{\ddot{a}}{a} = -\frac{1}{3M_{pl}^2}\left(\dot{\phi}^2 - V(\phi)\right) \tag{2.32}$$

But for accelerating universe

$$\ddot{a} > 0 \Rightarrow \dot{\phi}^2 < V(\phi) \tag{2.33}$$

which leads to following condition

$$\ddot{\phi} << 3H\dot{\phi} \tag{2.34}$$

Second term of equation (2.31) represents friction.This also slows down evolution of scalar field.
With the aid of equation (2.33) and (2.34) equations (2.30) and (2.31) can be written as

$$H^2 = \frac{1}{3M_{pl}^2}V(\phi) \tag{2.35}$$

$$3H\dot{\phi} = -V'(\phi) \tag{2.36}$$

Relation (2.33) shows that for inflation to occur potential energy must dominate the kinetic energy. This is realized in slow roll inflation where potential is flat such that scalar field rolls slowly. This slow rolling flat potential helps in providing the quantum fluctuation to scalar field. As field rolls down the hill it oscillates at minimum of potential with release of energy and subsequent production of plasma.This phenomenon is commonly known as Reheating. Now the theory again reunites with the original hot big-bang model and evolution of universe continues [21].

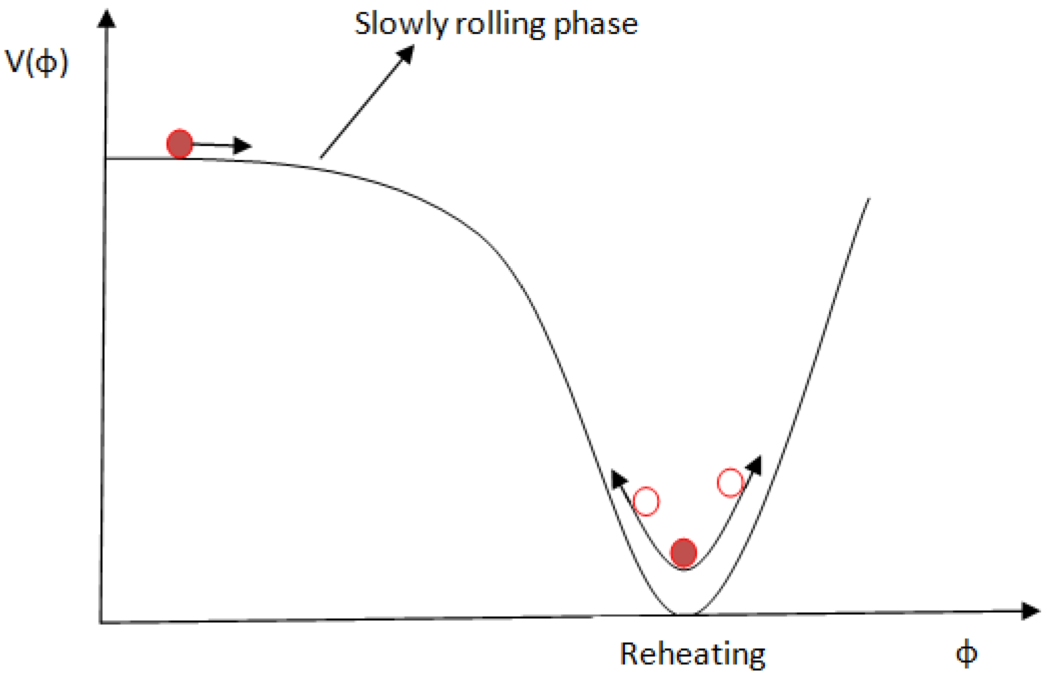

Figure 2.4: Slow Roll inflation model

Differentiating equation (2.36),we get

$$\ddot{\phi} = (-\eta + \epsilon)H\dot{\phi} \tag{2.37}$$

where,

$$\eta = M_{pl}^2 \frac{V''(\phi)}{V(\phi)} \tag{2.38}$$

$$\epsilon = \frac{M_{pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right) \tag{2.39}$$

are called slow roll parameter and satisfy slow roll condition if $|\eta| << 1$, $|\epsilon| << 1$.
Slow roll parameter ϵ can also be related to rate of change of Hubble parameter as follows
Differentiating equation (2.35) with respect to time(t),we get

$$\dot{H} = \frac{V'(\phi)}{3M_{pl}^2}\frac{\dot{\phi}}{2H} \tag{2.40}$$

$$\Rightarrow \frac{\dot{H}}{H} = \frac{V'(\phi)}{3M_{pl}^2}\frac{\dot{\phi}}{2H^2} \tag{2.41}$$

Using relation (2.35) in equation (2.41)

$$\frac{\dot{H}}{H} = \frac{V'(\phi)}{V(\phi)}\frac{\dot{\phi}}{2} \tag{2.42}$$

$$\Rightarrow \frac{\dot{H}}{H^2} = \frac{V'(\phi)}{V(\phi)}\frac{\dot{\phi}}{2H} \tag{2.43}$$

Substituting value of $\dot{\phi}$ from equation (2.36) in equation (2.43)

$$\frac{\dot{H}}{H^2} = -\frac{M_{pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \tag{2.44}$$

From relation (2.39) equation (2.44) can be rewritten as

$$\dot{H} = -\epsilon H^2 \tag{2.45}$$

$$\tag{2.46}$$

As $|\epsilon| << 1$,this shows that Hubble parameter changes slowly during slow roll inflation.

2.6 Amount of Inflation

Slow roll parameter can be used to calculate amount of inflation required to solve flatness problem and horizon problem. This is defined as [22]

$$N = log\left[\frac{a(t)}{a(t_0)}\right] \tag{2.47}$$

where $a(t)$ and $a(t_0)$ represent final and initial value of scale factor respectively. In slow roll approximation , potential is almost constant so relation (2.35) shows that the value of Hubble parameter is also constant.Constant Hubble parameter during inflation implies that equation (1.4) has exponential solution . This leads to exponential expansion and number of e-folding is given by

$$N = \int_{t_0}^{t} H dt \tag{2.48}$$

$$N = \frac{1}{M_{pl}^2}\int_{\phi(t)}^{\phi(t_0)} \frac{V(\phi)}{V'(\phi)} d\phi \tag{2.49}$$

Using relation (2.39) in equation (2.49)

$$N = \frac{1}{M_{pl}^2}\int_{\phi(t)}^{\phi(t_0)} \frac{d\phi}{\sqrt{2\epsilon}} \tag{2.50}$$

60 e-folding of inflation is required to solve horizon problem.

2.7 Classical and Quantum Corrected field Dynamics

Quantum field theory in curved space-time (QFTCST) is the theory of quantum fields propagating in a background classical curved space-time. QFTCST is expected to provide to some extent a reasonable description of quantum phenomenon in area where the effect of curved space time may be significant but effect of quantum gravity itself may be neglected. So, quantum field theory in curved space time is applicable in describing early universe. As quantum theory of gravity does not exist presently therefore basic principle of quantum field theory in classical general theory of relativity is adopted [12, 23]. The action for the two scalar field model as calculated in [12] as follows

$$\begin{aligned} S_m[\phi, \sigma, g_{\mu\nu}] = \int d^4x\sqrt{g}\Big[& -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \eta_\phi\Box\phi^2 - \frac{m_\phi}{2}\phi^2 - \frac{1}{2}\xi_\phi R\phi^2 \\ & - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \eta_\sigma\Box\sigma^2 - \frac{m_\sigma}{2}\sigma^2 - \frac{1}{2}\xi_\sigma R\sigma^2 - \frac{g\phi^2\sigma^2}{4} - \frac{\lambda_\sigma\sigma^4}{4!} - \frac{\lambda_\phi\phi^4}{4!}\Big] \end{aligned} \tag{2.51}$$

where,

$$\Box = \Delta^\mu\Delta_\mu = |g|^{-\frac{1}{2}}\partial_\mu(|g|^{\frac{1}{2}}\partial^\mu) \tag{2.52}$$

For special case of FRW ,minimal coupling to gravity ,no self interaction and using symmetry of the potential so that only one field evolves the effective Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{eff} = -\frac{1}{2}\partial_\mu\partial^\mu - \frac{m_\sigma^2}{2}\sigma^2 + \Lambda + \alpha R + \frac{1}{64\pi^2}\{ & \frac{1}{24}(R - 3g\sigma^2)(R - 3g\sigma^2 - 4m_\phi^2) \\ & + \left[-(m_\phi^2 - \frac{R}{6} + \frac{g\sigma^2}{2})^2 + \frac{G}{180}\right] log\left(\frac{m_\phi^2 - \frac{R}{6} + \frac{g\sigma^2}{2}}{m_\sigma^2}\right)\} \end{aligned} \tag{2.53}$$

Here G is called Gauss-Bonnet density and is defined as

$$G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} \tag{2.54}$$

where, R is Ricci scalar
$R^{\mu\nu}$ is Ricci Tensor
$R^{\mu\nu\rho\sigma}$ is Riemann tensor and
$\alpha = \frac{1}{16\pi G_N}$,$G_N$ is Newton Gravitational constant
Equation of motion is derived using principle of least action. Now equation of motion

is solved for matter domination, radiation domination and in de- sitter space (i.e. the positive value of the cosmological constant). The equation of motion thus obtained is

$$\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2\sigma = \frac{1}{64\pi^2}\left[\frac{g\sigma}{2}(2m_\phi^2 - R - 3g\sigma^2) + g\sigma\left(\frac{\frac{G}{180} - (m_\phi^2 - \frac{R}{6} + \frac{g\sigma^2}{2})^2}{m_\phi^2 - \frac{R}{6} + \frac{g\sigma^2}{2}}\right) - 2g\sigma(m_\phi^2 - \frac{R}{6} + \frac{g\sigma^2}{2})log\left(\frac{m_\phi^2 - \frac{R}{6} + \frac{g\sigma^2}{2}}{m_\phi^2}\right)\right] \tag{2.55}$$

Classical evolution of field is obtained if all terms in right side of equation are neglected

2.7.1 Classical Solution

i)De Sitter space

$$\ddot{\sigma} + 3H\dot{\sigma} + m_\sigma^2\sigma = 0 \tag{2.56}$$

H is Hubble parameter defined as $H = \frac{\dot{a}_d[t]}{a_d[t]}$
For de Sitter space the scale factor evolves as

$$a_d(t) = a_0 e^{H_0 t} \tag{2.57}$$

So for de Sitter space Hubble parameter is

$$H = H_0 \tag{2.58}$$

In order to solve equation (2.56) we set $\frac{H_0}{m_\sigma} = \frac{1}{\sqrt{2}}$ and initial condition as $\frac{\sigma[0]}{m_\sigma} = n$ and $\dot{\sigma}[0] = 0$ where $n = 1, 10, 20, 30, 40$. The resultng solution will then be

$$\frac{\sigma[t]}{m_\sigma} = ne^{-\sqrt{2}m_\sigma t}\left(-1 + 2e^{\frac{m_\sigma t}{\sqrt{2}}}\right) \tag{2.59}$$

$$\tag{2.60}$$

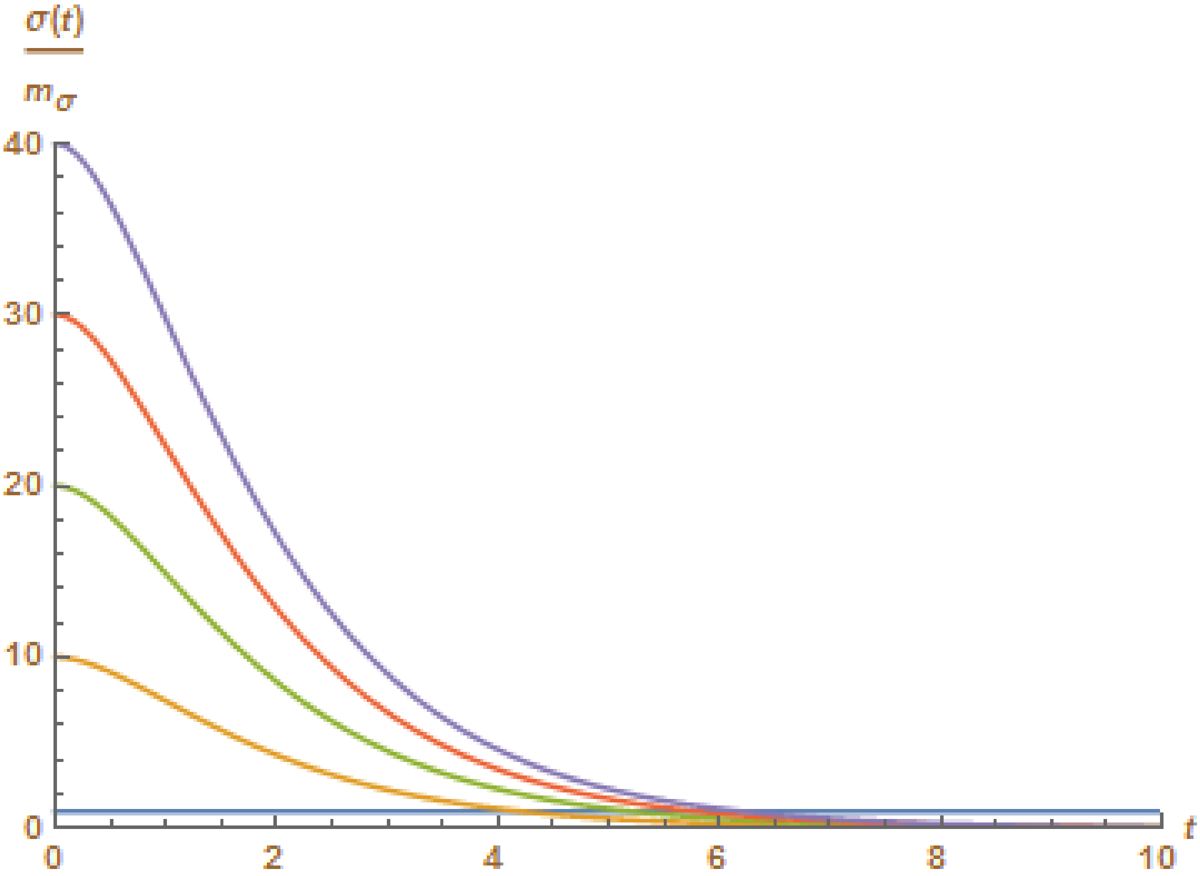

Figure 2.5: Evolution of scalar field ,$\sigma[t]$ in de Sitter space

ii)Radiation Domination

In case of radiation domination scale factor evolves as

$$a_r(t) = a_0 \left(\frac{t + t_0}{t_0} \right)^{\frac{1}{2}} \tag{2.61}$$

So Hubble parameter is

$$H_r = \frac{1}{2(t + t_0)} \tag{2.62}$$

Now solution to equation (2.56) under initial condition $\frac{\sigma[0]}{m_\sigma} = n$ and $\dot{\sigma}[0] = 0$ is

$$\sigma[t] = n \frac{2^{1/4} m_\sigma^{3/4} J_{\frac{1}{4}}[m_\sigma t] \Gamma[\frac{5}{4}]}{t^{\frac{1}{4}}} \tag{2.63}$$

where $n = 1, 10, 20, 30, 40$

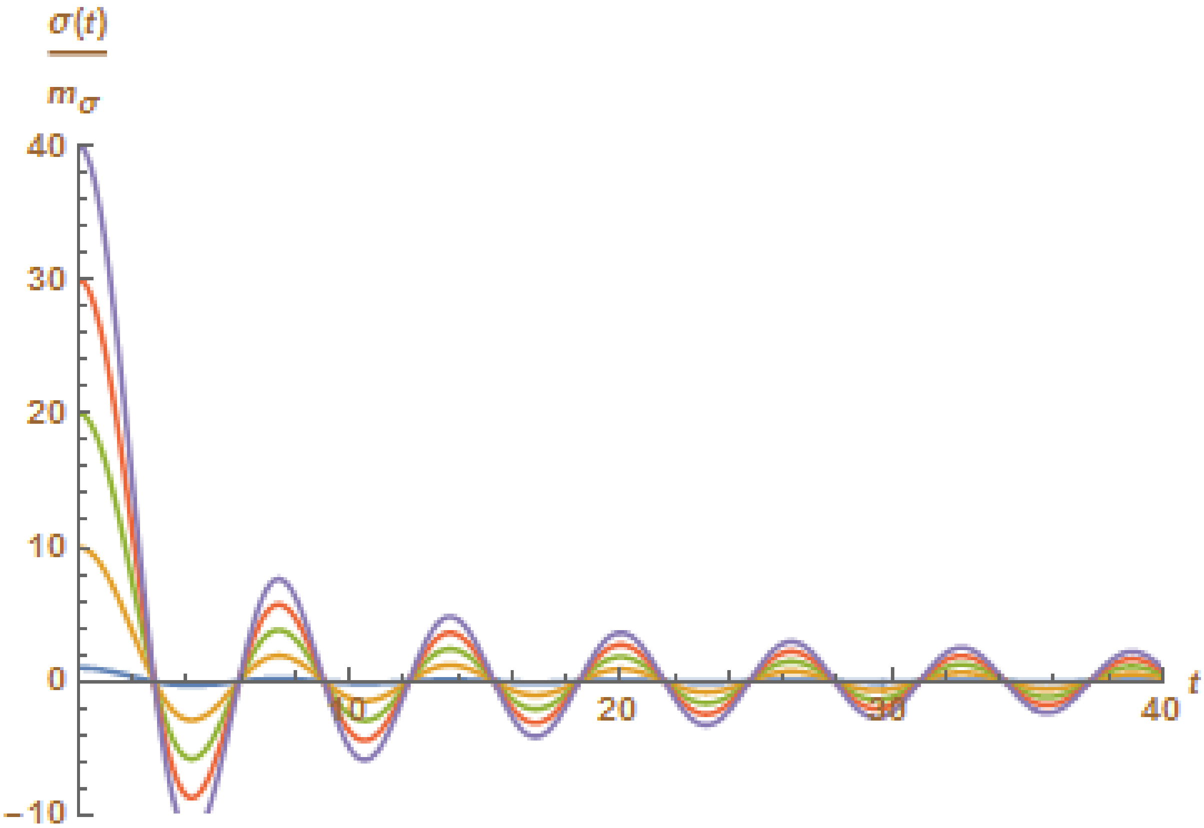

Figure 2.6: Evolution of scalar field, $\sigma[t]$ in radiation domination

iii)Matter Domination

In case of matter domination scale factor evolves as

$$a_r(t) = a_0 \left(\frac{t+t_0}{t_0}\right)^{\frac{2}{3}} \tag{2.64}$$

So Hubble parameter is

$$H_r = \frac{2}{3(t+t_0)} \tag{2.65}$$

Now solution to equation (2.56) under initial condition $\frac{\sigma[0]}{m_\sigma} = n$ and $\dot{\sigma}[0] = 0$ is

$$\sigma[t] = n\frac{e^{-\sqrt{-m_\sigma^2}t}\left(-1+e^{2\sqrt{-m_\sigma^2}t}\right)m_\sigma}{-2\sqrt{-m_\sigma^2}t} \tag{2.66}$$

where $n = 1, 10, 20, 30, 40$

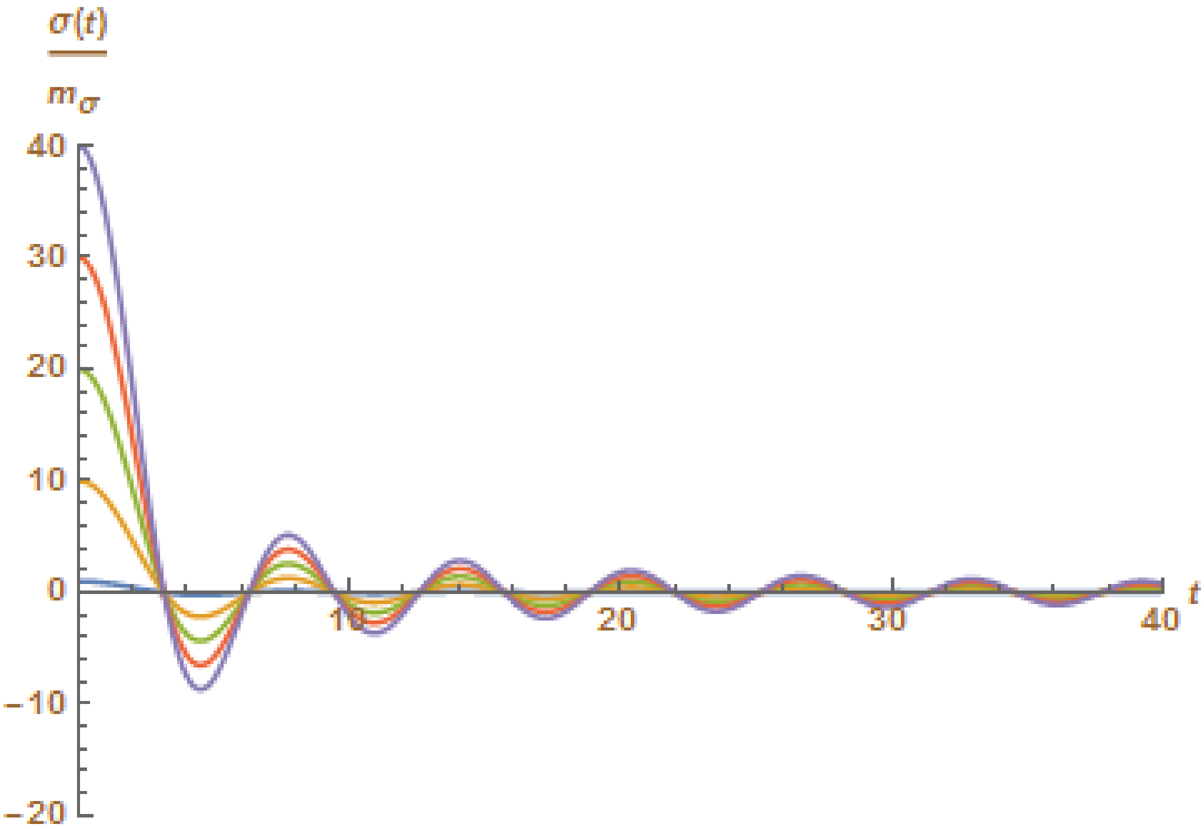

Figure 2.7: Evolution of scalar field ,$\sigma[t]$ in matter domination

2.7.2 Quantum Corrected Solution

i)De Sitter space

For Quantum corrected dynamics in case of de Sitter space we take equation (2.55) and solve it numerically in different cosmological background with underlying following assumption.

$$\frac{m_\phi}{m_\sigma} = 2, g = 1, \frac{H_0}{m_\sigma} = \frac{1}{\sqrt{2}} \tag{2.67}$$

Equation (2.55) also has logarithmic term which cannot be negative.This is ensured by following condition.

$$\frac{R}{6m_\phi^2} = \frac{1}{4} \tag{2.68}$$

Also higher order operator R and G are defined as

$$R = \left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) \tag{2.69}$$

$$G = \frac{24\dot{a}^2\ddot{a}}{a^3} \tag{2.70}$$

Using relation (2.57),(2.69),(2.70) we can relate higher order operator R and G with Hubble parameter in De Sitter space as

$$R_d = 12H_0^2 \tag{2.71}$$

$$G_d = 24H_0^4 \tag{2.72}$$

Now applying relation (2.67),(2.68),(2.71),(2.72) to equation (2.55) and solving it numerically under the initial condition $\frac{\sigma[o]}{m_\sigma} = n$ and $\sigma'[0] = 0$,where $n = 1, 10, 20, 30, 40$,following plot is obtained

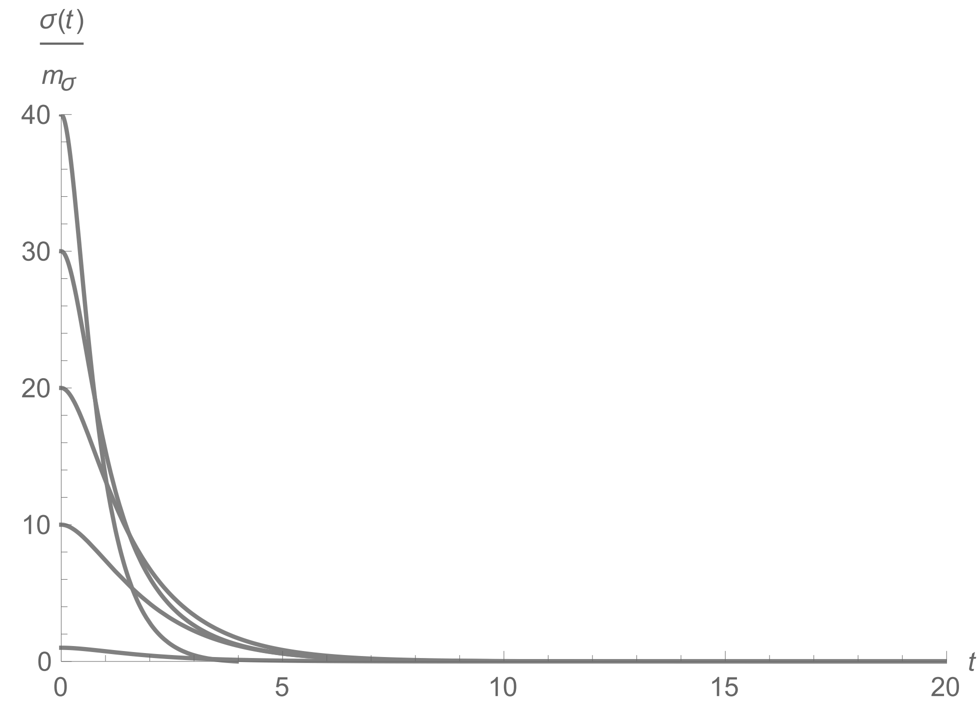

Figure 2.8: Quantum corrected evolution of field, $\sigma[t]$ in De Sitter space

Now we can superimpose classical evolution of field with quantum corrected ones and following plot is obtained. Blue lines represent classical solutions and black lines represent quantum corrected ones.

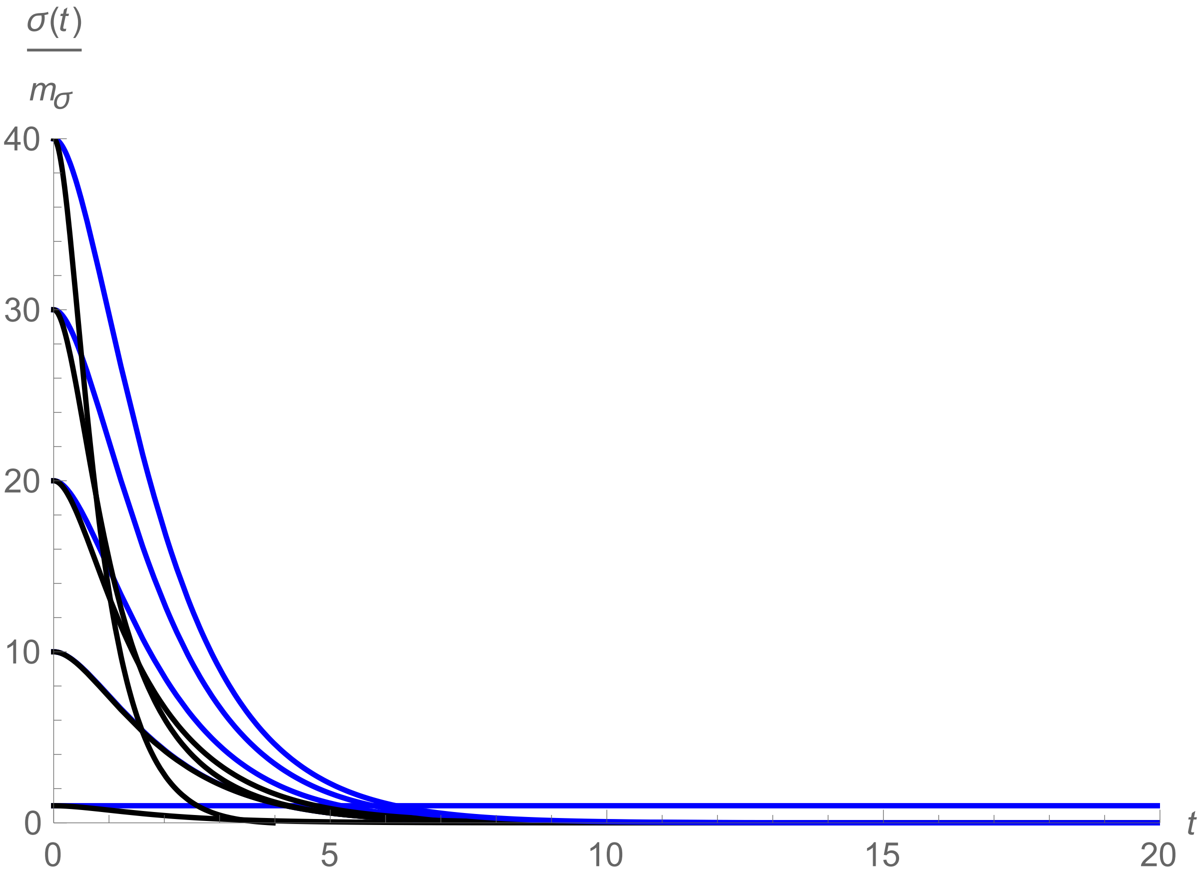

Figure 2.9: Classical and quantum corrected evolution of field, $\sigma[t]$ in de-sitter space.

ii) Radiation Domination

Making use of relation (2.61),(2.69) and (2.70), higher order operator R and G can be written as

$$R_r = 0, G_r = -\frac{3}{2(t+t_0)^4} \tag{2.73}$$

Finally using relations (2.73) and (2.62) in equation (2.55) along with initial conditions $\frac{\sigma[0]}{m_\sigma} = n$ and $\sigma'[0] = 0$, where $n = 1, 10, 20, 30, 40$ following plot is obtained.

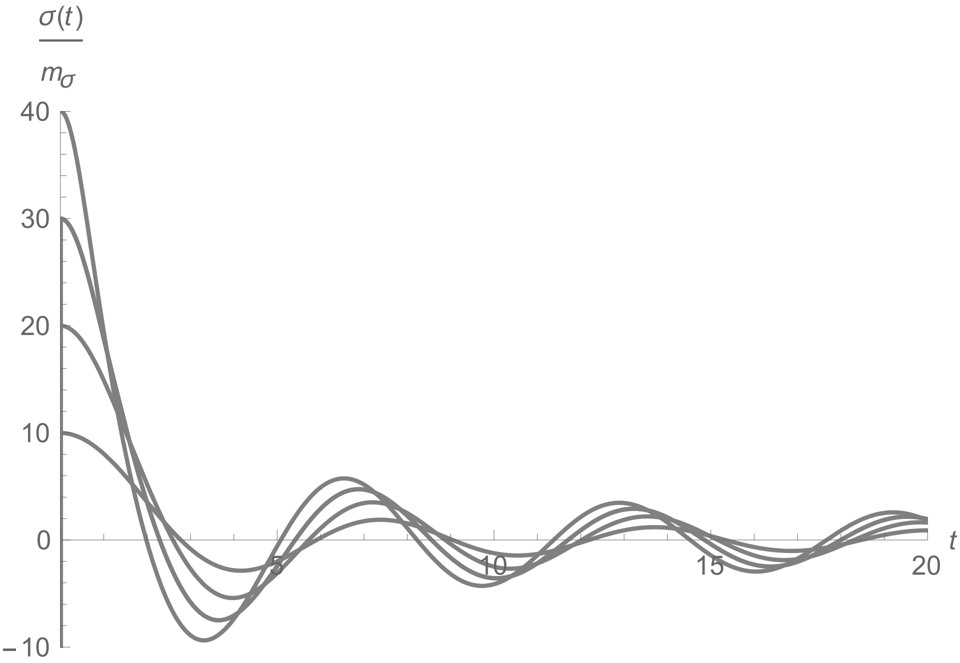

Figure 2.10: Quantum corrected evolution of field,$\sigma[t]$ in radiation domination

Also we can superimpose both classical and quantum corrected evolution of field to generate following plots.

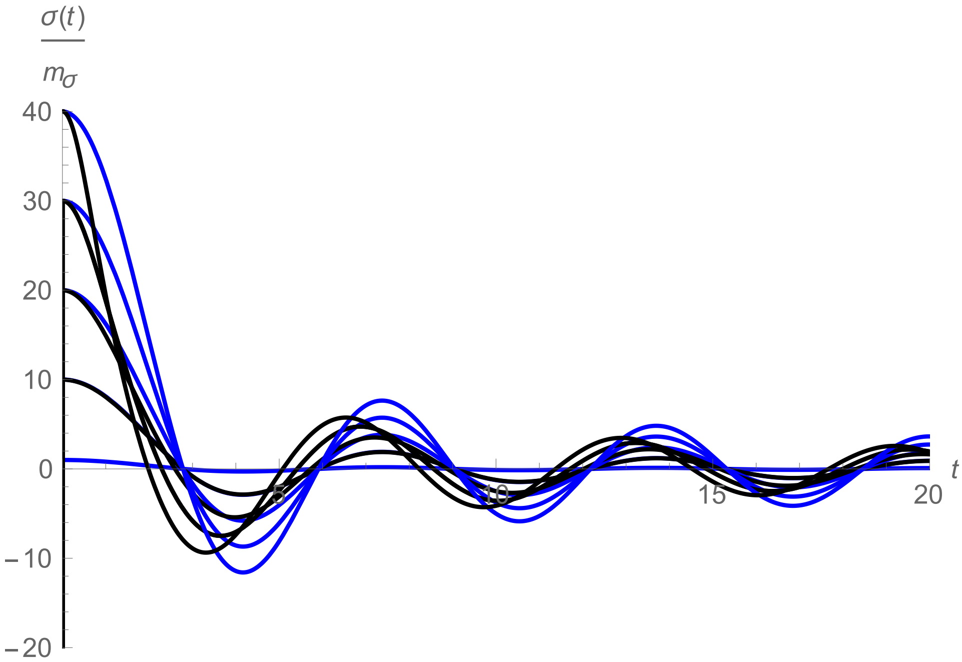

Figure 2.11: Classical and quantum corrected evolution of field, $\sigma[t]$ in radiation dominated universe

Blue lines represent classical ones and black lines that of quantum corrected ones.

iii) Matter Domination

In case of matter domination higher order operators R_m and G_m can be calculated by making use of equations (2.64),(2.69) and (2.70).

$$R_m = \frac{4}{3(t+t_0)^2}, G_m = -\frac{64}{27(t+t_0)^2} \tag{2.74}$$

Now using relation (2.65) and (2.74) in equation (2.55) ,we get differential equation which can be solved numerically under initial condition $\sigma[0]/m_\sigma = n$ and $\sigma'[0] = 0$ to obtain following plot.

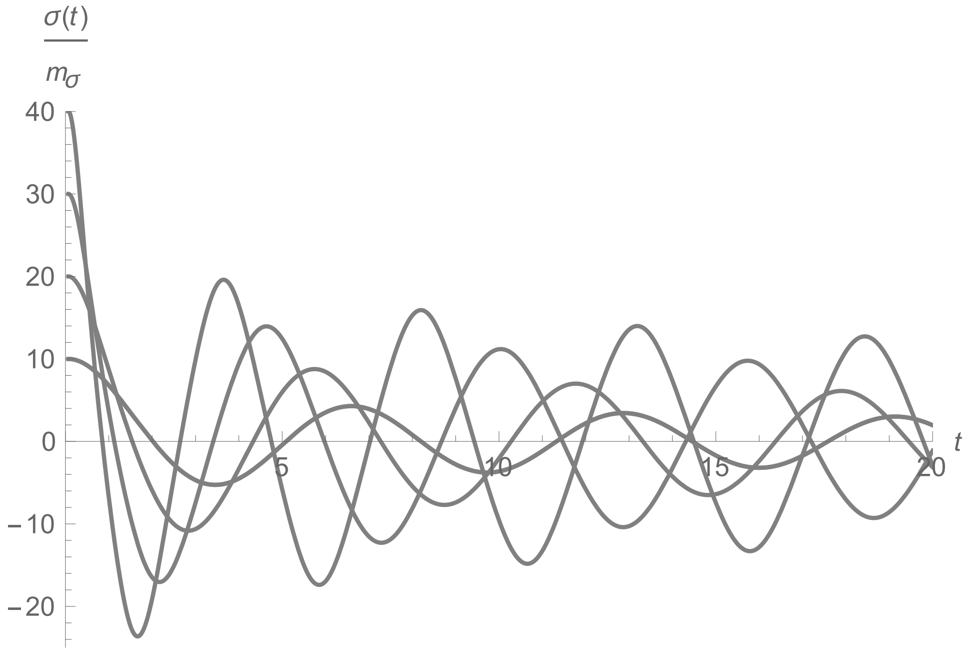

Figure 2.12: Quantum corrected evolution of field ,$\sigma[t]$ in matter dominated universe.

Now the classical and quantum corrected evolution of field can be combined and is realized as follows.

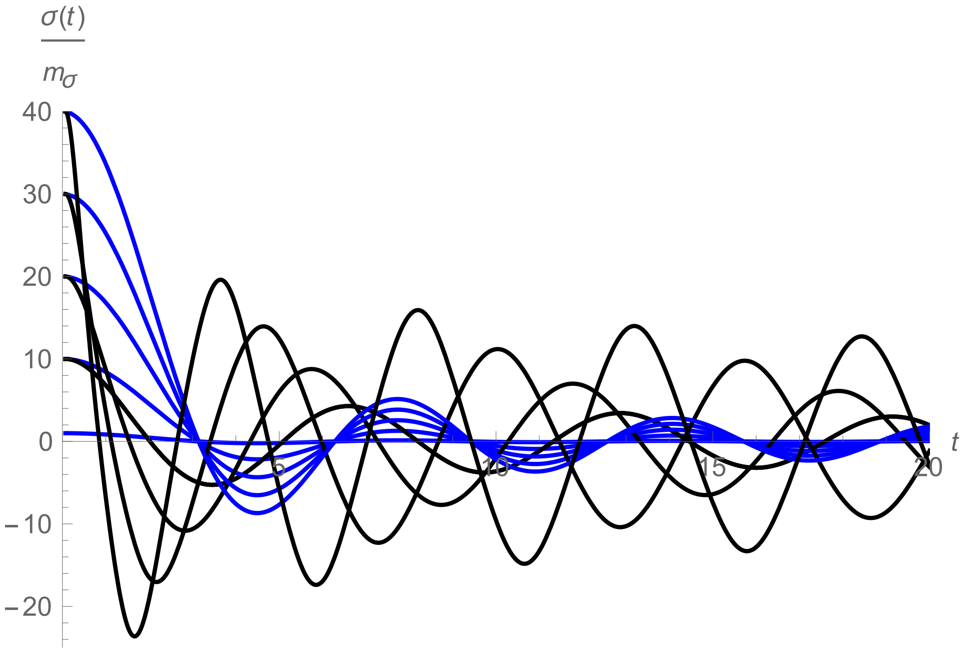

Figure 2.13: Classical and quantum corrected evolution of field, $\sigma[t]$ in matter dominated universe.

Blue lines represent classical ones and black lines that of quantum corrected ones.

Chapter 3

Curvature Mechanism

3.1 Introduction

Basic principle of cosmology states that on very large scale (>100Mpc) universe is homogenous and isotropic. But on small scale (1Kpc to 100Mpc) there are in-homogeneities. CMB observation suggest that during decoupling anisotropies was one part in 10^5. If this is so then amplitude of in-homogeneities were smaller in early epoch of the universe. In order to explain this issue of anisotropy seen in CMB spatial curvature perturbation was introduced.Curvature perturbation during inflation is completely independent of slowly rolling inflation field. It was sub-dominant during inflation.This field is called curvaton,σ. Curvaton creates curvature perturbation in two different stages. First, during inflation Hubble parameter is almost constant so wavelength of fluctuation generated during inflation cross the horizon become classical and decouple from microphysical processes. Upon re-entering the horizon as universe undergoes through various cosmological phases these classical perturbations seed in-homogeneities which generate structure during gravitational collapse [7, 24–26].

Initial density fluctuation can be written as combination of curvature perturbation and iso-curvature perturbation i.e. these two perturbation are orthogonal to each other [27]. Curvature perturbation is also known as adiabatic perturbation and isocurvature perturbation is called entropy perturbation.Both of these perturbation can be generated during inflation which involves one and more than one scalar field. Adiabatic perturbation is associated with perturbation in total energy density whereas isocurvature perturbation is related to relative number density fluctuations in different particle species present in the system [28, 29].

Adiabatic perturbation can be written as [30]

$$\frac{\delta\rho_i}{\rho_i} = \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} \tag{3.1}$$

where i represent any non-relativistic matter and γ represents radiation .Adiabatic curvature perturbation is generated by both inflaton and curvaton. But with the curvaton present, this is achieved first by an isocurvature perturbation.
Isocurvature perturbation can be stated as

$$\frac{\delta\rho_B}{\rho_B} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} \neq 0 \tag{3.2}$$

The simplest case of generating curvature perturbaiton is by fluctuation of inflaton itself but this puts constraint in models of inflation.So the concept of curvaton was purposed by David H Lyth and David Wands.As curvaton is independent of model of inflation and is subdominant during inflation so it can only produce isocurvature perturbation. After the end of inflation curvaton starts to oscillate during radiation dominated era and converts isocurvature perturbation into curvature perturbation [26]
Consider a curvaton field σ coupled to another to another field ϕ, with the action

$$S = \int d^4xa^3 \left[\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{m_\sigma^2\sigma^2}{2} + \frac{g}{4}\sigma^2\phi^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m_\phi^2\phi^2}{2}\right] \tag{3.3}$$

Where a is the scale factor of FRW metric .Ignoring quantum and thermal correction lead to an effective action of the form

$$S = \int d^4xa^3 \left[\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{m_\sigma^2\sigma^2}{2}\right] \tag{3.4}$$

The classical equation of motion for this action is

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dV[\sigma]}{d\sigma} = 0 \tag{3.5}$$

We are interested in the perturbation of σ so

$$\sigma(x) = \sigma + \delta\sigma(x) \tag{3.6}$$

σ is the unperturbed part whose evolution is guided by equation (3.5) .The perturbed field obeys the equation

$$\delta\ddot{\sigma}_k + 3H\dot{\delta\sigma_k} + \left(\left(\frac{k}{a}\right)^2 + V_{\sigma\sigma}\right)\delta\sigma_k = 0 \tag{3.7}$$

Here subscript, $\sigma{=}\frac{\partial}{\partial\sigma}$
Let curvaton potential be sufficiently flat during inflation i.e.

$$V_{\sigma\sigma} << H^2 \tag{3.8}$$

This condition when applied to equation (3.7) leads to

$$\delta\ddot{\sigma}_k + 3H\delta\dot{\sigma}_k + \left(\left(\frac{k}{a}\right)^2\right)\delta\sigma_k = 0 \tag{3.9}$$

Now if the wavelength of the fluctuation is within the horizon i.e.
$\lambda << H^{-1} \iff k >> aH$ then equation (3.9) reduces to

$$\delta\ddot{\sigma}_k + \left(\left(\frac{k}{a}\right)^2\right)\delta\sigma_k = 0 \tag{3.10}$$

This equation is similar to harmonic oscillator and solution to this equation is some oscillating function. So as long as the fluctuations are within horizon they oscillate.
Now consider the case when wavelength of fluctuations crosses the horizon i.e.
$\lambda >> H^{-1} \iff k << aH$ then equation (3.9) reduces to

$$\delta\ddot{\sigma}_k + 3H\delta\dot{\sigma}_k = 0 \tag{3.11}$$

Here due to frictional term solution of $\delta\sigma_k$ is constant. This implies that as modes cross horizon they freeze.
The power spectrum associated with the fluctuations $\delta\sigma_k$ is given by

$$P^{\frac{1}{2}}_{\delta\sigma} = \frac{H_*}{2\pi} \tag{3.12}$$

here * represent horizon exit
If condition (3.8) is applied then H_* is almost constant so the observed spectrum is almost scale invariant.
Now as the curvaton field starts to oscillate in some radiation dominated era then total energy density is given by

$$\rho = 3H^2M_{pl}^2 \tag{3.13}$$

Also decay rate of curvaton is given by

$$\Gamma = \gamma\frac{g^2m}{64\pi^2} \tag{3.14}$$

Where g is coupling constant and γ represents number of available channel to which curvaton may couple. For our calculation we have taken it to be unity.

Curvature perturbation on uniform slices of radiation and matter is separately given by

$$\zeta_r = \frac{1}{4}\frac{\delta\rho_r}{\rho_r} \tag{3.15}$$

$$\zeta_\sigma = \frac{1}{3}\frac{\delta\rho_\sigma}{\rho_\sigma} \tag{3.16}$$

Using these relations the curvature perturbation can be written as

$$\zeta = \frac{4\rho_\sigma\zeta_r + 3\rho_\sigma\zeta_\sigma}{4\rho_r + 3\rho_\sigma} \tag{3.17}$$

If ζ_r is considered negligible then

$$\zeta = \delta r \tag{3.18}$$

where $\delta = \frac{\delta\rho_\sigma}{\rho_\sigma}$ is called density contrast and

$$r = \frac{\rho_\sigma}{4\rho_r + 3\rho_\sigma} \tag{3.19}$$

Finally we get the curvature perturbation as

$$\zeta = \frac{H_* r}{3\pi\sigma_*} \tag{3.20}$$

also further approximate that

$$r = \frac{\rho_\sigma}{4M_{pl}^2 H^2} \tag{3.21}$$

where,ρ_σ is given by

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V[\sigma] \tag{3.22}$$

3.2 Curvaton Perturbation with Different Potential

We have the classical evolution of curvaton as

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dV[\sigma]}{d\sigma} = 0 \tag{3.23}$$

If we assume that the expansion of the universe is determined by a separate radiation energy density component ρ_γ, in which case

$$\rho_\gamma(t) = \rho_\gamma^* \left(\frac{a(t_*)}{a(t)}\right)^4, a(t) = \left(1 + \frac{t}{t_0}\right)^{\frac{1}{2}}, H(t) = \frac{H_*}{a^2(t)} \tag{3.24}$$

where * label refers to the initial time $t_* = 0$ and $H_* = \frac{1}{2t}$. We define number of e-fold as $N = log\left[\frac{a(t)}{a(0)}\right]$ then we can deduce that

$$H(N) = H_* e^{-2N}, H(N)\frac{d}{dN} = \frac{d}{dt} \tag{3.25}$$

This relation can be used to rewrite equation (3.23) as differential equation in N instead of t

$$\sigma'' + 3\sigma' + \frac{dV[\sigma]}{d\sigma}\frac{e^{4N}}{H_*^2} = 0 \tag{3.26}$$

where $'$ represents $\frac{d}{dN}$
We choose a starting value $\sigma = \sigma_*$, $H = H_*$, $N = 0$ and an end point N_{end} defined by the decay time of the curvaton

$$\Gamma = H_{end} = H_* e^{-2N} \Longrightarrow N_{end} = -log\left[\frac{\Gamma}{H_*}\right] \tag{3.27}$$

Using the Friedmann equation and the fraction r

$$3M_{pl}^2 H^2 = \rho_\gamma, r(\sigma_{end}) \equiv r_{dec} \simeq \frac{\rho_\sigma^{end}}{4M_{pl}^2 H_{end}^2} \tag{3.28}$$

where,

$$\rho_\sigma^{end} = \left(\frac{1}{2}\dot{\sigma}^2 + V[\sigma]\right)|_{end} = \frac{\Gamma^2}{2}\sigma'(N_{end})^2 + V[\sigma_{end}] \tag{3.29}$$

the curvature perturbation is now given by

$$\zeta = \frac{H_* r_{dec}}{3\pi\sigma_*} = \frac{H_*\left(\frac{\Gamma^2}{2}\sigma'(N_{end})^2 + V[\sigma_{end}]\right)}{12\Gamma^2 M_{pl}^2 \pi\sigma_*} \tag{3.30}$$

Now further we will explore different potential, $V[\sigma]$. Using various form of potential in equation (3.23) we will solve for σ from initial σ_* ,$\sigma' = 0$ until $\sigma(N_{end})$ and compute ζ using (3.30). Also we pick value of H_*, Γ and σ_* to solve equation (3.23). Finally we solve for $\zeta = 10^{-5}$ to find value of σ_* that works. Following constraint are applied while picking up different values of H_*, Γ and σ_*

1) Initial field value must be less than Planck mass i.e. $\sigma_* << M_{pl}$
2) Curvaton must be light during inflation i.e. $\frac{d^2V}{d\sigma^2} << H_*$
3) Initially energy density of the curvaton should be negligible i.e.$r_{dec} < 1$
4) Curvaton should decay before nucleosynthesis i.e. $\Gamma > 10^{-22}$
5) Non-Gaussianity should be less than current observed value i.e. $\frac{\frac{4}{5r_{dec}}}{74} < 1$
This also insures that $\sigma_* >> H_*$
6) The decay of the curvaton must satisfy the condition $10^{-3}\frac{m_\sigma^3}{\Gamma M_{pl}^2} < 1$
7) It must generate the curvature perturbation of $\zeta = 10^{-5}$

3.2.1 Free Theory

Let us take the simplest case where

$$V[\sigma] = \frac{1}{2}m_\sigma^2\sigma^2 \tag{3.31}$$

Using this in equation (3.5),we get

$$\sigma'' + 3\sigma' + \frac{m_\sigma^2}{H_*^2}e^{4N}\sigma = 0 \tag{3.32}$$

$$\zeta = \frac{H_*\left(\frac{\Gamma^2}{2}\sigma'(N_{end})^2 + \frac{m_\sigma^2}{2}\sigma^2\right)}{12\Gamma^2 M_{pl}^2\pi\sigma_*} \tag{3.33}$$

Solution to equation (3.32) under the initial condition $\sigma[0] = \sigma_*$,$\sigma' = 0$ yields

$$\sigma(N) = \frac{e^{-\frac{3N}{2}}\left(\sigma_*\left(\left(m_\sigma J_{\frac{1}{4}}P + (-m_\sigma)J_{-\frac{7}{4}}P + 3H_*J_{-\frac{3}{4}}P\right)J_{\frac{3}{4}}(Q) + \left(-3J_{\frac{3}{4}}P - m_\sigma J_{\frac{7}{4}}P + m_\sigma J_{-\frac{1}{4}}P\right)J_{-\frac{3}{4}}(Q)\right)\right)}{\sqrt{m_\sigma^2}\left(\left(J_{\frac{1}{4}}P - J_{-\frac{7}{4}}P\right)J_{\frac{3}{4}}P + \left(J_{-\frac{1}{4}}P - J_{\frac{7}{4}}P\right)J_{-\frac{3}{4}}P\right)} \tag{3.34}$$

where $P = \left(\frac{\sqrt{m_\sigma^2}}{2H_*}\right)$ and $Q = \sqrt{e^{4N}}P$
From the equation above derivative of $\sigma[N]$ is calculated and value at $N \to N_{end}$ is obtained. Now using the relation (3.33) we can solve for $\zeta = 10^{-5}$ to find value of σ_*.

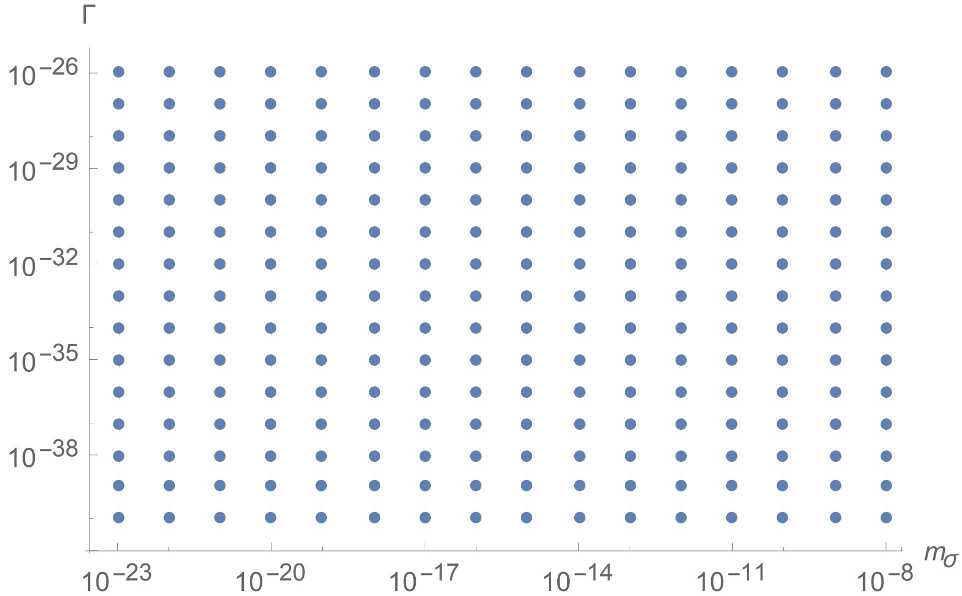

Figure 3.1: Plot between m_σ and Γ for $H_* = 10^{-8}$

[Note all the values are re-scaled with respect to reduced Planck mass]
Figure (3.1) represent all the value of m_σ and Γ that are used for solving equation (3.33). When above stated constraints are applied then plot looks as follows

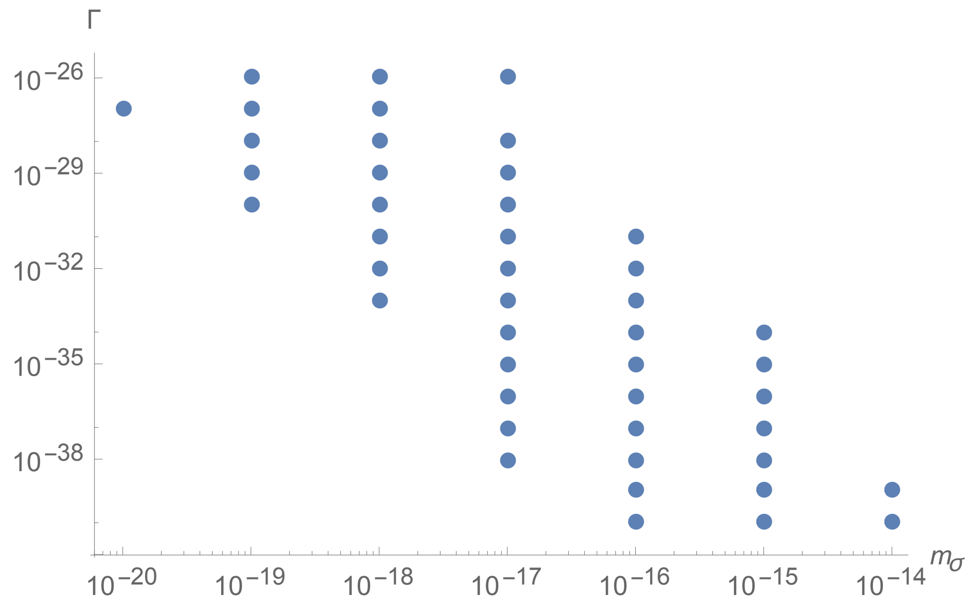

Figure 3.2: Plot between m_σ and Γ for $H_* = 10^{-8}$ when subjected to constraint.

The region only within blue dots satisfy constraint required for curvature perturbation.

3.2.2 Self Interaction

Consider the potential

$$V[\sigma] = \frac{1}{2}m_\sigma^2\sigma^2 + \triangle V[\sigma] + V(\sigma, T) \tag{3.35}$$

where,

$$\triangle V[\sigma] = \frac{\beta g^2\sigma^4}{64\pi^2}\left[log\left(\frac{g\sigma^2}{\mu^2}\right) - \frac{3}{2}\right] \tag{3.36}$$

Here g is coupling constant, β is some number which depends on number of field to which curvaton is coupled. While calculating we have taken $\beta = 1$ and the term $\left[log\left(\frac{g\sigma^2}{\mu^2}\right) - \frac{3}{2}\right]$ is also approximated by unity due to its negligible effect.
Similarly

$$V(\sigma, T) = \alpha g^2 T^2 \sigma^2 \tag{3.37}$$

After the end of inflation universe was dominated by radiation. Hence this term is introduced to account for thermal correction to potential. g is coupling constant and α depends on the number of field to which inflaton is coupled. It is taken as unity for calculation. Temperature after inflation i.e. in radiation domination is given by

$$T^2(N) = T_* e^{-2N}, T_* = \sqrt{H_* M_{pl}} \tag{3.38}$$

We also use the relation

$$\frac{\Gamma}{m_\sigma} = \gamma \frac{g^2}{64\pi^2} \rightarrow \alpha g^2 = \frac{\alpha}{\gamma} 64\pi^2 \frac{\Gamma}{m_\sigma} \equiv \bar{\alpha} \frac{\Gamma}{m_\sigma} \tag{3.39}$$

Now equation of motion reduces to

$$\sigma'' + \sigma' + \frac{m_\sigma^2}{H_*^2} e^{4N} \left(1 + 2\bar{\alpha} \frac{\Gamma H_* M_{pl}}{m_\sigma^3} e^{-2N} + 4\bar{\beta} \frac{\Gamma \sigma^2}{m_\sigma^3} \right) \sigma = 0 \tag{3.40}$$

where, $\bar{\beta} = \frac{\beta}{\gamma}$
Exact solution to this equation was not possible so numerical solution was done by using same values of parameter as used in free theory. Here instead of solving for σ_* we took range of it (values re-scaled with respect to reduced Planck mass) satisfying the constraints $m_\sigma < H_* << \sigma_* < M_{pl}$.Then we checked for values which gave curvature perturbation (ζ) in the order of 10^{-5}. But none of these values were able to generate the required value of curvature perturbation. This means the region marked with blue as in figure (3.2) was not achieved in this condition

3.2.3 Self Interaction All

Similarly, for the potential used by Markkanen in his paper is (see [12]) simplified by assuming that in radiation domination R=0 and neglecting the purely gravitation part and the forth order operator G we get

$$V[\sigma] = \frac{m_\sigma^2}{2} \left(1 + 2\alpha g^2 \frac{T^2}{m_\sigma^2} \right) \sigma^2 + \\ \frac{1}{64\pi^2} \left[\frac{3g\sigma^2(3g\sigma^2 + 4m_\phi^2)}{24} - \left(m_\phi^2 - \frac{g\sigma^2}{2} \right)^2 log \left(1 + \frac{g\sigma^2}{2m_\phi^2} \right) \right] \tag{3.41}$$

Let us define

$$g^2 = 64\pi^2 \frac{\Gamma}{\gamma m_\sigma} \rightarrow \frac{g}{2m_\phi^2} = \frac{4\pi}{\gamma^{\frac{1}{2}} m_\phi^2} \left(\frac{\Gamma}{m_\sigma} \right)^{\frac{1}{2}} \equiv \frac{1}{\bar{m}_\phi^2} \left(\frac{\Gamma}{m_\sigma} \right)^{\frac{1}{2}} \equiv C \tag{3.42}$$

Using potential (3.41) and relation (3.42) equation (3.26) reduces to

$$\sigma'' + 3\sigma' + \frac{m_\sigma^2}{H_*^2}e^{4N}\left(1 + 2\bar{\alpha}\frac{\Gamma H_* M_{pl}}{m_\sigma^3}e^{-2N} + \frac{\bar{m_\phi}^2\Gamma^{\frac{1}{2}}}{\gamma m_\sigma^{\frac{5}{2}}}\left(C\sigma^2 - (1 + C\sigma^2)log[1 + C\sigma^2]\right)\right)\sigma = 0 \tag{3.43}$$

Following the same procedure as followed in self interaction and using same values of parameter as used in free theory, we can determine parameter space which satisfies the condition for curvature perturbation.

Chapter 4

Result and Discussion

First chapter of this thesis was introduction to inflation. In second chapter we took equation of motion as defined by equation (2.55) and solved it under different cosmological background. This equation contains higher order operators R and G which appears due to quantum corrections to inflation and treating inflation in terms of scalar field. Classical equation of motion corresponds to setting right hand side of the equation to zero. First we solved classical equation of motion in case of De-Sitter space (this corresponds to positive cosmological constant), radiation dominated universe and matter dominated universe. Scale factor being dependent on time takes different values in these three different epoch of universe. Hence value of Hubble parameter ,H and higher order operators R and G will have different value of in different cosmological background. This also resulted in different solutions to equation (2.55) at classical and quantum level for the cosmological background we take into account.

Figure (2.5), (2.6), and (2.7) corresponds to classical evolution of field in case of De-sitter space, radiation dominated and matter dominated background respectively. Light blue ,yellow, green, red and blue corresponds to choosing initial value of field as $\frac{\sigma_0}{m_\sigma} = 1, 10, 20, 30, 40$ and $\dot{\sigma}[0] = 0$ We can see from figure that in case of De-Sitter space evolution of field is over-damped (exponentially decreasing) and in case of radiation and matter domination oscillation is damped. Quantum corrected evolution of field is characterized by figures (2.8), (2.9) and (2.10).Exact solution of (2.55) was not possible and was solved numerically under same initial field as in case of classical one but with parameter $\frac{m_\phi}{m_\sigma} = 2$, $\frac{H_0}{m_\sigma} = \sqrt{\frac{1}{2}}$, $g = 1$ and $\frac{R_0}{6m_\phi^2} = \frac{1}{4}$. Behavior of field is almost same in this case as well but are deviated from classical ones which can be observed in figure (2.11), (2.12) and (2.13). Blues lines represent classical ones and black lines that of quantum corrected ones.

In third chapter we studied curvature mechanism using different potential. Curvature mechanism is considered as root cause for structure that we see in universe today. At first we took simple potential with no interaction at all i.e $V[\sigma] = \frac{1}{2}m_\sigma^2\sigma^2$ and solved for $\sigma[N]$. From cosmological observation we know that value of curvature perturbation(ζ) is of order 10^{-5}. Using this fact we solve equation (3.20) and determine value of σ_* that works under the constraint $m_\sigma < H_* << \sigma_* < M_{pl}$. For $H_* = 10^{-8}M_{pl}$ the allowed parameter space which satisfies all the constraint is represented by region within blue dotted spots as in figure (3.2). Similarly we took potential given by equation (3.35) but the region disappeared in this case for same value of parameter we choose to solve free theory.

In second chapter of the thesis we developed classical and quantum evolution of the field with choice of parameter as $\frac{m_\phi}{m_\sigma} = 2$, $\frac{H_0}{m_\sigma} = \sqrt{\frac{1}{2}}$, $g = 1$ and $\frac{R_0}{6m_\phi^2} = \frac{1}{4}$. With these choice of parameter and factor of $\frac{1}{64\pi^2}$ made quantum corrections are relatively small. So it would be interesting to see the effects of quantum corrections with different choice of parameter.Similarly in chapter 3 on curvature perturbation we solved for simple potential without interaction.In this case differential equation had exact solution but when potential as described equation (3.41) was used equation became non-linear and numerical approach is required.Due to lack of time I was not able to analyze it completely. This is also next part where further computation can be done.

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