University of Stavanger

Faculty of Science and Technology

MASTER'S THESIS

Study program/ Specialization: Petroleum Engineering / Well Engineering	Spring semester, 2015 Open
Writer: **Erzuah Samuel**	……………………………………… (Writer's signature)

Faculty supervisor (s): **Kjell Kåre Fjelde & Mesfin Agonafir Belayneh**

External supervisor(s): **Eric Cayeux**

Thesis title:

Hook load measurement correction with non-uniform movement

Credits (ECTS): 30

Key words: Hook load Acceleration Tension Friction Load cell (Weight indicator)	Pages: 145 + enclosure: 23 Stavanger, 15/06/2015

ACKNOWLEDGEMENTS

My sufficiency is not of my own but from God. I ascribe all the glory to God for His mighty hands in my life.

I will like to express my profound gratitude to my internal supervisors Kjell Kåre Fjelde and Mesfin Agonafir Belayneh for their selfless dedication and enthusiasm in this thesis. I am really grateful and blessed to have you as my supervisors. I am also dumb-founded by the exceptional mentorship of my external supervisor Eric Cayeux. Despite your busy schedules, you still made time to guide me throughout the entire research. I really enjoyed working under you.

I will also like to appreciate the centre for Drilling and Wells for Improved Recovery (DrillWell) for providing the needed resources for safe and efficient execution of this thesis. To all the employees who worked relentlessly to ensure a successful completion of this thesis especially Sonja Moi, Gunnstein Sælevik and Robert Ewald, I say Kudos.

I will also like to thank my parents and siblings for their love, support, encouragement and prayers that kept me on my toes. Thanks to all my friends who have a keen interest in my success especially Jieyin Liu. Yin, I thank God to have you as a friend and I am grateful for your help and encouragement.

Finally, whoever worked behind the scenes to make this thesis a success, I salute you all and without your help, this could not have materialised

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Abstract

In the recent years, oil and gas exploration and production are being carried out in extremely harsh and challenging environmental conditions. Hence, accurate prediction of the hook load is essential in order to minimise the Non- Productive Time (NPT) during the drilling operation stages. With accurate prediction of hook load, undesirable drilling problems such as buckling, stuck pipe, tensile failure can be minimised if not completely eradicated.

There are numerous factors affecting the hook load prediction such as, the weight per unit length (W/l) of the drill pipe used, the density of the drilling mud used, the friction in the well, the weight per unit length (W/l) of the drilling line used, just to mention but a few. These above factors will not be discussed in-depth in this thesis but rather this thesis aims at developing a mathematical model to incorporate into the existing models, the effect of acceleration on hook load prediction.

There are numerous hook load prediction models in the oil and gas industry such as the industry accepted hook load prediction model, the Luke and Juvkam-Wold hook load prediction model and the Cayeux et al hook load prediction model. The rationale behind this thesis is to understand these existing hook load prediction models and further develop them by incorporating the effect of acceleration. These existing models gives a good prediction of the hook load measurements but the accuracy can be improved by taking into account that the efficiency of each sheave might not be same and also taking into consideration the effect of acceleration. The extended models will be analysed using hypothetical data.

After analysing the extended models using the hypothetical data, it was discovered that during non-uniform movement of the travelling equipment the sum of the tensions in the supporting lines are not the same as the hook load (W). Hence, the position for the load cell placement is very essential to ensure accurate hook load measurement.

- During hoisting with non-uniform movement of the travelling equipment, the sum of the tensions in the supporting lines always exceeds the hook load (W) value with the discrepancy between them being influenced by the acceleration (a) of the travelling equipment.

- Hence, the minimum expected hook load (W) value during hoisting is during non-uniform movement of the travelling equipment with high $\frac{a}{g}$ ratio and vice-versa.
- During lowering with non-uniform movement of the travelling equipment, the hook load (W) measurement always exceeds the sum of the tensions in the supporting lines with the disparity between them also influenced by the acceleration (a) of the travelling equipment.
- Hence, the maximum hook load (W) measurement during lowering occurs when the travelling equipment is undergoing non-uniform movement with high $\frac{a}{g}$ ratio and vice-versa.
- Finally, it was observed that even though the dead line is non-rotating, its efficiency is not perfect($e_{dl} \neq 1$). The efficiency of each sheave from the extended Cayeux et al hook load prediction model was used as an input for the extended Luke and Juvkam model. It was observed that the extended Cayeux et al hook load prediction model (which served as the experimental data) produces approximately the same results as the rotating (Active) dead line sheave hook load prediction model but deviates from the non-rotating (inactive) dead line sheave counterpart. The degree of the deviation depends on the coefficient of friction (the efficiency of each sheave). Hence, it can be inferred that the dead line sheave is not perfect. This can be confirmed with experimental data.

Nomenclature

W_a	Weight per unit length of the drill pipe in air
ρ_{dp}	Density of the drill pipe used
W_{ads}	Weight of the drillstring in air
W_{bds}	Weight the drillstring in mud (Buoyed drillstring weight)
h_{TVD}	True Vertical Depth of the well
β	Buoyancy factor (upward force) on the drillstring
ρ_s	Density of steel
ρ_o	Density of the mud outside the drillstring
ρ_i	Density of the mud inside the drillstring
A_i	Inner cross-sectional area of the drill pipe
A_o	Outer cross-sectional area of the drill pipe
A_s	Cross-sectional area of the drill pipe (Steel)
e	Efficiency of each sheave
F_d	Derrick load, mL/t^2, Ibf
F_{dl}	Dead line tension, mL/t^2, Ibf
F_{fl}	Fast-line tension, mL/t^2, Ibf
n	Number of lines between the crown block and the travelling block
W	Hook load, mL/t^2, Ibf
F_{net}	The net force on the system
m_T	Total mass of the travelling equipment
m_{dp}	Mass of the drillstring
m_{tb}	Mass of the travelling block
m_{dl}	Mass of the drill-line
a	Acceleration due to non-uniform movement of the travelling equipment
F_{Down}	Force acting downwards
v	Final velocity
dv	Change in velocity
u	Initial velocity
ds	Change in position
s_2	Next position of the travelling equipment

s_1 Current position of the travelling equipment

s_0 Previous position travelled by the travelling equipment

dt Change in time

t_2 Time reading corresponding to position s_2

t_1 Current time reading corresponding to position s_1

t_0 Previous time reading corresponding to position s_0

M_A Actual Mechanical Advantage (MA) with friction

M_I Ideal Mechanical Advantage (MA) without friction

α_1 Azimuth at the initial position (position 1)

α_2 Azimuth at the next position (position 2)

γ Hook load correction factor during non-uniform movement

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1 Introduction

Currently, the Oil and Gas industry are now exploring in harsh and challenging environmental conditions. These challenging environments need special equipment and operational procedures and hence, this leads to increased cost as compared to the non-challenging environments. Hence, there is the need to optimize the drilling operations thereby minimizing the operational cost. One way to achieve this is to reduce the Non-Productive Time (NPT) to the barest minimum thereby saving rig time which will result in the reduction in the Operational expenditure (OPEX) especially for ultra-deep water drilling operations.

As every tangible entity in the world produces a shadow when light falls on it, so are the hook load measurements the "shadow" of the actual downhole condition as depicted by Cayeux et al [1]. Hence, accurate prediction of the hook load is essential to identify the deteriorating down hole conditions due to ledges, tight hole due to swelling clay or mobile formations such as salt, poor hole cleaning (cutting transport challenges) just to mention a few. i.e. Accurate hook load measurements are important for predicting well friction. If these problems are identified ahead of time, appropriate measures can be taken thereby minimizing NPT.

In addition to the above, during drilling weight on bit (WOB) is applied to the bit before we can drill ahead. Hence, it is important to accurately predict the hook load in order not to exceed the buckling limit when applying the WOB. On the other hand, if the tensile limit of the string is exceeded due to over-pull, it can also result in tensile failure and hence accurate prediction of the hook load is indispensable in the drilling operation.

1.1 Background

In order to accurately predict the hook load, various models have been developed such as the Luke and Juvkam-Wold model [2], the industry accepted model [3] and the Cayeux et al hook load prediction model [4].

The hook load (W) is literally the force exerted by the drillstring suspension point in the travelling equipment. In this thesis we assume vertical well and hence the well friction was neglected. i.e. The hook load remains constant for a given drillstring weight. Below is a schematic illustrating a typical block and tackle hoisting system.

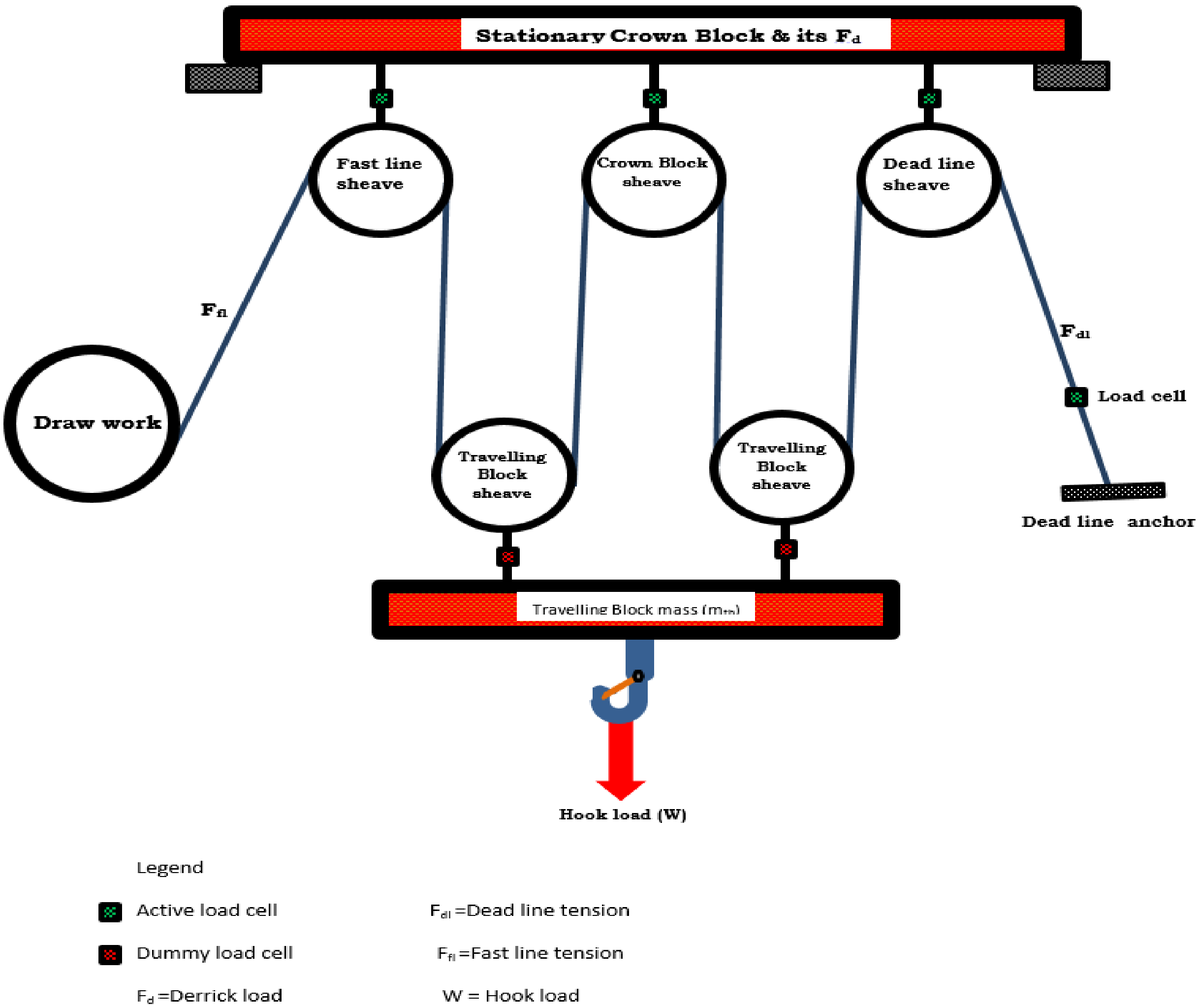

Figure 1: Shows a schematic of a typical block and tackle hoisting system

In the Luke and Juvkam-Wold model, they identified the effect of the load cell position on the accuracy of the hook load measurements. According to their model, if the load cell is positioned at the dead line it will measure the lowest line tension during hoisting since the line tension decreases from the fast line (F_{fl}) towards the dead line (F_{dl}). This results in lowest hook load measurements during hoisting and it is therefore not representative of the actual downhole drilling condition.

On the other hand during lowering, the line tension decreases from the dead line (F_{dl}) towards the fast line (F_{fl}) and hence the dead line (F_{dl}) experiences the highest tension while the fast line (F_{fl}) experiences the least. With the load cell positioned at the dead line, the highest tension is recorded during lowering which is counter intuitive since the hook load (W) has the same direction as the acceleration due to gravity (g).

The challenge with the Luke and Juvkam-Wold model is that they assumed constant sheave efficiency (e) for all the rotating sheaves which might not be necessarily true. It was also based on constant velocity and hence, no effect of acceleration of the travelling equipment was taken into account.

On the other hand, the accepted industry method for predicting hook load (W) is either too low during hoisting or too high during lowering. This discrepancy can be attributed to the fact that the industrial approach assumes a perfect block and tackle system with no frictional losses. In this case, the efficiency of each sheave is not only constant as suggested by Luke and Juvkam but perfect (i.e. e =100% or e =1). This is a conservative approach and impractical.

Both the industry accepted model and the Luke and Juvkam-Wold model are based on the efficiency of the sheaves. Unlike the aforementioned models, Cayeux et al model [4] is based on the coefficient of friction (μ) at the sheave axle during rotation. Cayeux et al model utilizes the Stribeck friction coefficient (μ_s) at the sheave axle instead of the Coulomb friction model (μ_a) in order to account for the effect of changing from static friction (striction) to kinematic friction and vice-versa. The limitation of the Cayeux et al model is that it was also based on constant velocity of the travelling equipment and hence the effect of acceleration was not incorporated into the model.

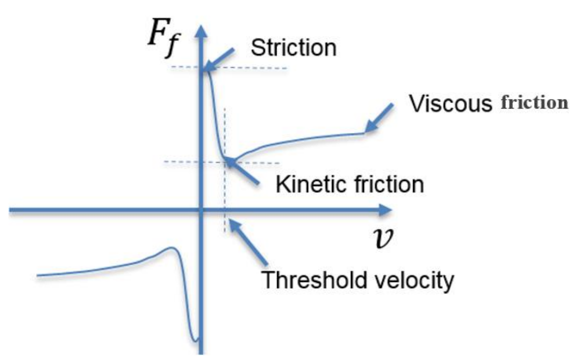

Figure 2: Schematic showing the transition from static coefficient of friction to dynamic coefficient of friction and vice-versa by courtesy of Cayeux et al [4]

The beauty of using the Cayeux et al hook load prediction model is that, the sheave efficiency (e) which is a global effect due to the rotation of the sheave is not utilized in their model. According Cayeux et al, the sheave efficiency depends on the applied load, the elasticity of the drill line, block position and direction of movement of the travelling equipment (whether hoisting or lowering) as illustrated below

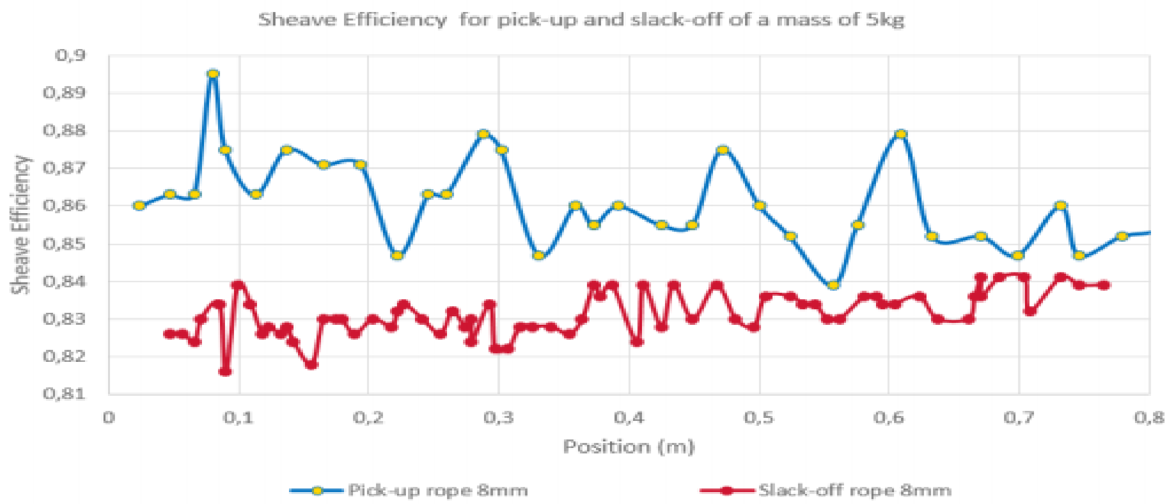

Figure 3: Shows the variation in sheave efficiency as a function of block position and direction of movement of the travelling equipment (hoisting or lowering) by courtesy of Cayeux et al [4]

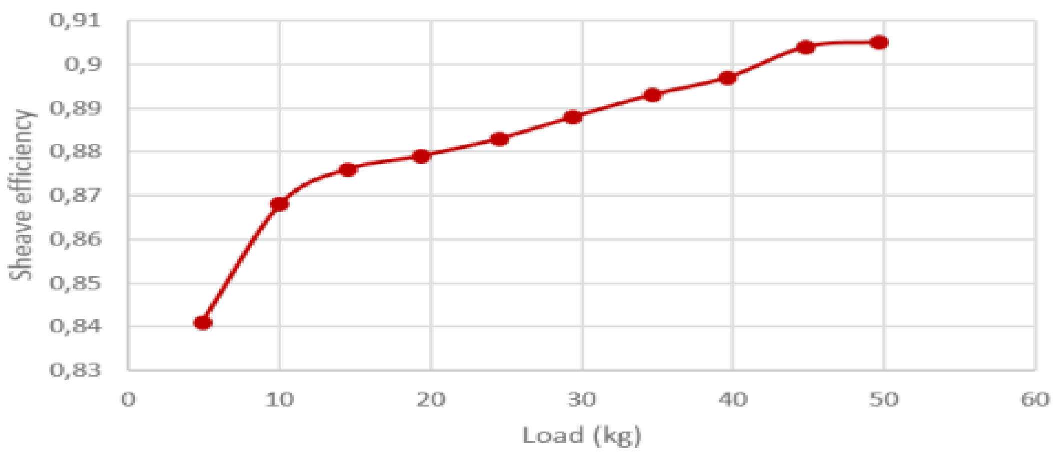

Figure 4: Shows variation in the sheave efficiency as a function of the applied load by courtesy of Cayeux et al [4]

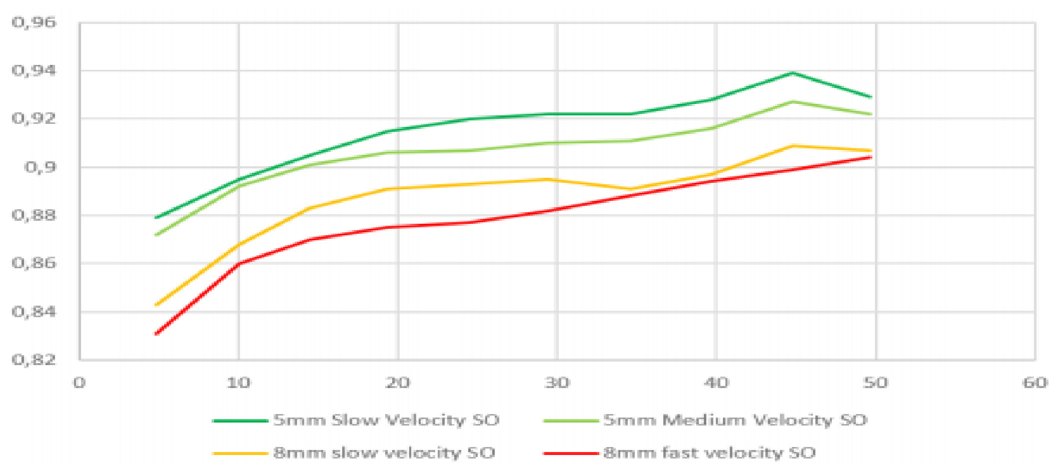

Figure 5: Shows variation in the average sheave efficiency during hoisting and lowering for different applied loads, different elasticity of the drill-line and at different speed of the travelling equipment by courtesy of Cayeux et al [4]

The Cayeux et al hook load prediction model also account for the effect of stick-slip which is prominent at very low velocity. According to Cayeux et al [4], the stick-slip condition is not limited to the dead line sheave. The combination of static friction at the level of the sheave axle and the drill-line elasticity may result in the pulley not rotating [4]. In addition, the Cayeux et al hook load prediction model also account for the effect of each sheave weight (F_W), the centrifugal force (F_C), the coefficient of friction (μ_a) at the sheave bearing, the direction of rotation of each sheave etc. Hence, improving the accuracy of the hook load prediction.

1.2 Objectives

This thesis aims at incorporating into the existing hook load prediction models, the effect of varying the travelling equipment velocity (i.e. non-uniform movement) on the hook load measurements with emphasis on fixed installations (Platform wells) and a vertical wellbore. Below are some of the contributions to the existing models;

- Incorporating into the industry accepted model [3], the effects of acceleration during non-uniform movement
- Incorporating into the Luke and Juvkam model [2], the effect of acceleration during non-uniform movement of the travelling equipment for;
    i. Both Inactive and Active dead line sheave during either hoisting or lowering.
    ii. Both varying sheave efficiency and constant sheave efficiency during either hoisting or lowering.
- Incorporating into the Cayeux et-al hook load prediction model [4] the effect of acceleration during non-uniform movement for either hoisting or lowering. Below are some of the other contributions to the Cayeux et al hook load prediction model.
    i. Cayeux et al proposed two line tension relations for the crown block sheaves and that of the travelling block sheaves during

hoisting with uniform movement of the travelling equipment. In this thesis, a generalised line tension relation during hoisting has been developed for both the crown block sheaves and the travelling block sheaves and with the effect of the non-uniform movement of the travelling equipment also taken into consideration.

ii. In a similar vein during lowering, a generalised line tension relation has also been developed for both the crown block sheaves and the travelling block sheave from the line tension relations proposed by Cayeux et al during lowering. In addition, the effect of non-uniform movement of the travelling equipment has also been incorporated into the generalised line relation.

iii. These generalised line tension relations during either hoisting or lowering were then combined to get the sum of the tensions in the supporting lines. After which Newton's second law of motion was apply to obtain the extended Cayeux et al hook load prediction model.

2 GENERAL OVERVIEW OF THE DRILLING SYSTEM

2.1 Types of drilling rigs

Drilling rigs can be categorised into two (2) main groups based on the location in which it is being used. i.e. Either land rigs for onshore use or marine rigs for offshore use.

2.1.1 Land rigs

The land rig can also be categorised into two main subgroups namely

i. Conventional rigs such as medium land rig
ii. Mobile rig such as Portable mast

2.1.2 Marine rigs

The marine rigs can also be subdivided into two (2) major categories namely

i. Bottom supported rigs such as Jack up, platform etc.
ii. Floating rigs such as semi-submersible and drillship.

The model in this thesis is developed for either a land rig or an offshore bottom supported rig such as the platform rig.

2.2 The drilling system

The drilling system is made up five (5) essential systems which make it possible to drill ahead. These systems include;

i. Power system
ii. Circulation system
iii. Rotary system
iv. Well control system
v. Hoisting system

2.2.1 Power system

All living things require some form of energy such as food in order to undertake their daily activities. Likewise, the drilling system requires electrical power in order to drill ahead. This electric power is either transmitted from a nearby onshore electric power station using power lines or by generating it at the rig site using internal-combustion diesel engines (power plant) [9]. There are two

(2) types of internal-combustion diesel engines depending on the mode in which the generated power is transmitted to the other rig systems, namely a diesel-electric type and a direct-drive type.

The Diesel-electric type refers to an internal-combustion diesel engine in which the main rig engines are used to generate the required electric power but in the Direct-drive rigs, the electrical power is transmitted from the internal combustion engines by utilizing belts, gears, chains clutches instead of using motors and generators to accomplish the electric power transmission [9].

The hoisting system, the circulation system and the rotation system are the three main systems that place high demand on the power system. The power system forms an integral part of the drilling system.

2.2.2 Circulation system

The circulation system is essential with respect to cutting transport thereby minimizing the downhole problems such as stuck pipe, high well friction as a result of cutting bed formation etc. In addition, the drilling mud which is an integral component of the circulation system also helps to lubricate the bit thereby minimizing bit wear. The circulation system is made up of the following components,

i. Mud pumps which can be either duplex pump or triplex pump
ii. Flow lines
iii. Drill pipe
iv. Nozzles
v. Mud pits and tanks (e.g. settling tank, mixing tank, suction tank)
vi. Mud mixing equipment (mud mixing hopper)
vii. Contaminant removal equipment (e.g. shale shaker, desander, desilter, degasser etc.)

2.2.3 Rotary system

For the past decades, the oil and gas industry has moved from the percussion (hammer) drilling into a more efficient and a reliable drilling technique called the rotary drilling technique. The rotary system is used to provide bit rotation in order to drill ahead. The rotary system is either top drive based or rotary table based depending on the mechanical device that provides the required torque to the drillstring in order to drill ahead. The top drive rotary system is composed of the top drive and the drillstring while the rotary table based rotary system consists of the following components;

i. Swivel
ii. Kelly
iii. Rotary table
iv. Drillstring

2.2.4 Well control system

The well control systems are very important in ensuring the integrity of the well at all times by preventing uncontrolled inflow, cross flow or outflow from the wellbore to the external environment. The well barrier during drilling as stipulated in NORSOK D-010 (Rev. 4, June 2013) has the drilling mud (fluid column) as the primary barrier. The secondary barrier elements with shearable string includes,

i. In-situ formation
ii. Casing cement
iii. Casing
iv. Wellhead
v. High pressure riser
vi. Drilling BOP

2.2.5 Hoisting system

The hoisting system is used to either raise or lower pipe into and out of the well. In addition, it is also used to provide the required weight on bit (WOB) on the drillstring during drilling. Currently, there are three (3) types of hoisting systems used in the oil and gas industry. It includes;

a) Ram-rig hoisting system
b) Rack and pinion hoisting system
c) The conventional draw-work hoisting system

2.2.5.1 RAM-RIG HOISTING SYSTEM

In the ram-rig hoisting system, hydraulic power supplied by the hydraulic power unit (HPU) to the two hydraulic cylinders also known as rams provides the required power for either hoisting or lowering. The HPU is made up of eight (8) to fourteen (14) variable displacement pumps with equal hoisting capacity. Each pump is driven by a constant speed alternating current (AC) motor and hence, each pump can give full hoisting force but at lower speed thereby conserving enough power for drilling activities. The hydraulic oil forms an integral component of the HPU. In addition to the HPU and the rams, other components of the ram-rig includes, guide tower (ram-guide), top drive, the travelling yoke, the lifting wires and equalizer assembly.

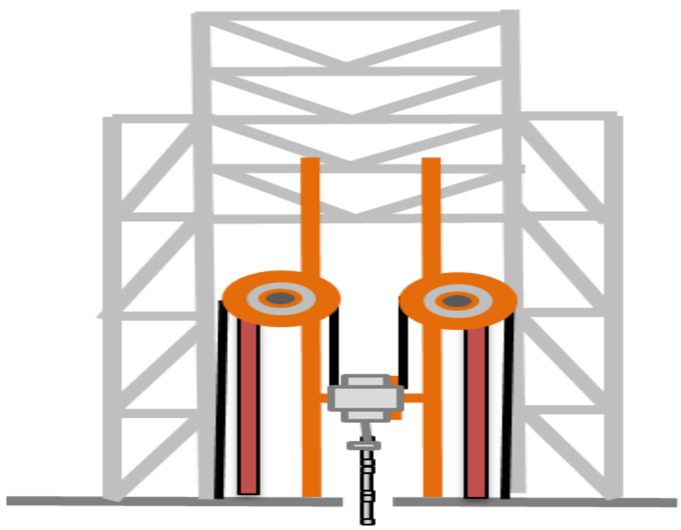

Figure 6: Shows a ram-rig by courtesy of Cayeux et al [4]

2.2.5.2 RACK AND PINION HOISTING SYSTEM

The rack and pinion hoisting system as its name implies is composed of a pinion and a rack. In this type of hoisting system, a rotational motion from the pinion is transformed into a linear motion along the rack thereby permitting hoisting or lowering depending on the direction of rotation of the pinion. This principle is utilized by the jack-up rigs when it is being raised or lowered.

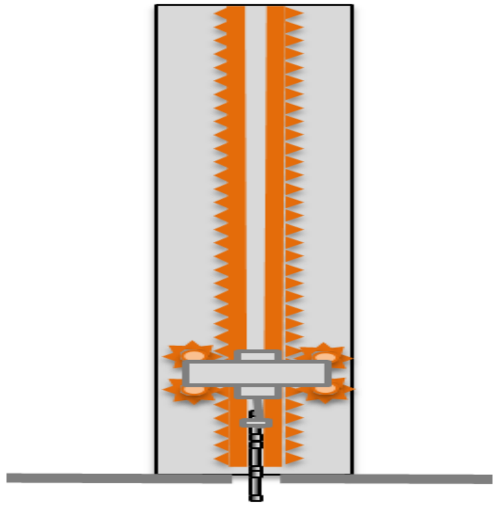

Figure 7: Shows a rack and pinion rig by courtesy of Cayeux et al [4]

2.2.5.3 Conventional draw work hoisting system

This is the oldest hoisting technique used in the industry and with the draw work supplying the required hoisting power. The hoisting power is then transmitted through the drilling lines to the travelling block in order to either raise or lower the drillstring.

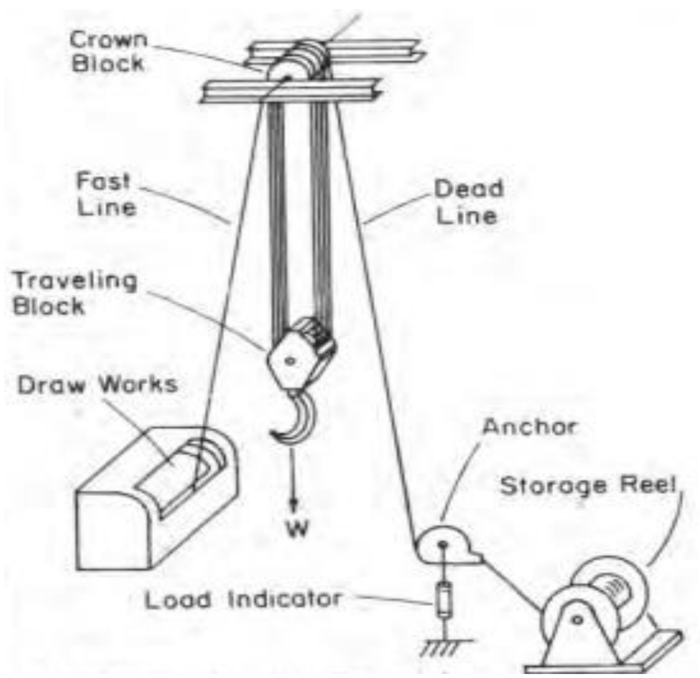

Figure 8: shows a conventional draw work hoisting system by courtesy of Bourgoyne et al 1986 [9]

This type of hoisting system will be employed in this thesis. The conventional hoisting system is composed of the following components

i. Derrick and substructure
ii. Crown Block
iii. Traveling Block
iv. Drilling Line
v. Draw-works

2.2.5.3.1 Derrick and substructure

The derrick is a steel tower that provides mechanical support for the crown block, the traveling block and the drillstring. It also provides vertical clearance for running in hole (RIH) or pulling out of hole (POOH) during the drilling operations. Hence, the greater the vertical clearance, the longer the drillstring length that can be handled thereby saving rig time. Derricks are rated based on their wind load and their compressive load capacities.

The substructure on the other hand elevates the derrick thereby providing working space below the derrick floor for installing the BOP (Blowout Preventer) and other surface equipment. The derrick is positioned above the substructure and hence the substructure must be able to withstand the entire derrick load together with its maximum drillstring weight during RIH or POOH. The design of the substructure depends on the equipment to be installed on it such as the Blow-out preventer (BOP) and it also depend on the local soil condition at the installation point.

2.2.5.3.2 Crown block

In the conventional rotary drilling, the block and tackle arrangement is used to increase the mechanical advantage (MA) of the pulley system. The stationary block at the top of the derrick is referred as the Crown block. The crown block consists of a group of pulleys which may be built into the derrick structure. Below is an illustration of the crown block and its sheaves arranged in a stacked form.

Figure 9: shows a the crown block sheave arrangements, to the left is the zoomed-out view and to the right is the zoomed-in view by courtesy of directional drilling technology blog [6]

2.2.5.3.3 TRAVELLING BLOCK

The moveable block which runs between the crown block and the drill floor is referred to as the traveling block. These pulleys are arranged in a stack form and covered in a protective housing to withstand the corrosive environment as illustrated below

Figure 10: shows the travelling block. To the left is the travelling block sheave in its protective housing while to the right shows an opened protective travelling block housing by courtesy of directional drilling technology blog [6]

2.2.5.3.4 DRILLING LINE

The applied tension from the draw work is transmitted through a steel drill-line that connects the crown block sheaves to the travelling block sheaves in order to either raise or lower the drillstring. Failure of the drill-line can lead to catastrophic events such as injuries to personnel, loss of drillstring downhole thereby resulting in fishing operation etc. Hence, it is essential not to exceed the tensile limit of the drill-line during the drilling operations. This can be achieved using the slip-and-cut maintenance program to get rid of the worn-out sections of the drill-line with time depending on the ton-mile covered. Accurate record of the ton-mile is essential to ensure an effective slip-and-cut maintenance program.

2.2.5.3.5 DRAW-WORK

The draw work serves as the heart of the drilling system and it is used to run equipment into and out of the well. In other words, the draw-work provides both the hoisting and the braking power needed to either raise or lower the drillstring.

Figure 11: shows a conventional draw work hoisting system by courtesy of directional drilling technology blog [6]

The draw work is composed of the following components

i. Drum
ii. Brakes
iii. Transmission
iv. Cathead

Drum

The drum transmits the required torque needed for either hoisting or lowering of the drillstring. The drum is also used to store the drill-line required to move the traveling block between the crown block and the drill floor. i.e. The hoisting drum is used to spool the drill-line in order to raise or lower the drillstring.

Brakes

The brakes are used to halt and sustain further movement of the drum by applying the brake lever. There are two types of auxiliary brakes namely hydrodynamic brake and electromagnetic brake. In the hydrodynamic type, water is impelled to the direction opposite to the direction of the drum rotation thereby halting the drum movement whereas the electromagnetic brakes utilizes two opposing magnetic fields in order to stop and maintain the drum from any further movement. Water cooling system is also used to cool down the heat generated during braking.

Transmission

The draw work transmission is responsible for changing the direction and speed of the travelling block thereby permitting either hoisting or lowering of the drillstring.

Catheads

Catheads are attached to both ends of the draw works to transmit the required electric power needed for the draw work operation. Friction catheads rotate continuously and thereby aiding in hoisting. The torque required to screw or unscrew the pipe is provided by the second catheads which is positioned between the friction catheads and the draw works housing.

2.3 Heave compensation system

In the olden days, oil and gas exploration was limited only to onshore operations due to lack of technologies. With the dawning of advanced and reliable technologies, the exploration of oil and gas has been extended to harsh and challenging environmental conditions such as the offshore environment.

The effect of the heaves on the drilling operations such as the tension measurements became a major concern for the striving industry during the early offshore exploration activities. Hence, there was the need to decouple the dynamics of the drilling rig from the drilling system. This necessitated the introduction of heave compensator in the 1970 by Vetco offshore Inc. The purpose of the heave compensator is to minimise the load variation on the drill bit due to the heave effects during drilling operations. There are two (2) major types of heave compensation used in the oil and gas industry namely, Passive and Active heave compensation.

2.3.1 Passive heave compensation

This type of compensation is usually crown block based. i.e. The compensator is located at the crown of the derrick. This crown mounted compensator is used to decouple the drillstring from the dynamics of the entire drilling system due to the heaves effect and it is usually pneumatic in nature. i.e. It utilizes the compressibility of gas usually nitrogen to provide the needed compensation. The passive heave compensator is made up of gas (air) which also serves as an accumulator due to its compressibility, cylinder and piston assembly. The principle behind the passive heave compensation is that as the load exerts a downward force on the piston, the air inside the cylinder is compressed until the pressure-force that is build-up inside the cylinder becomes equal to the external load that is exerted on it.

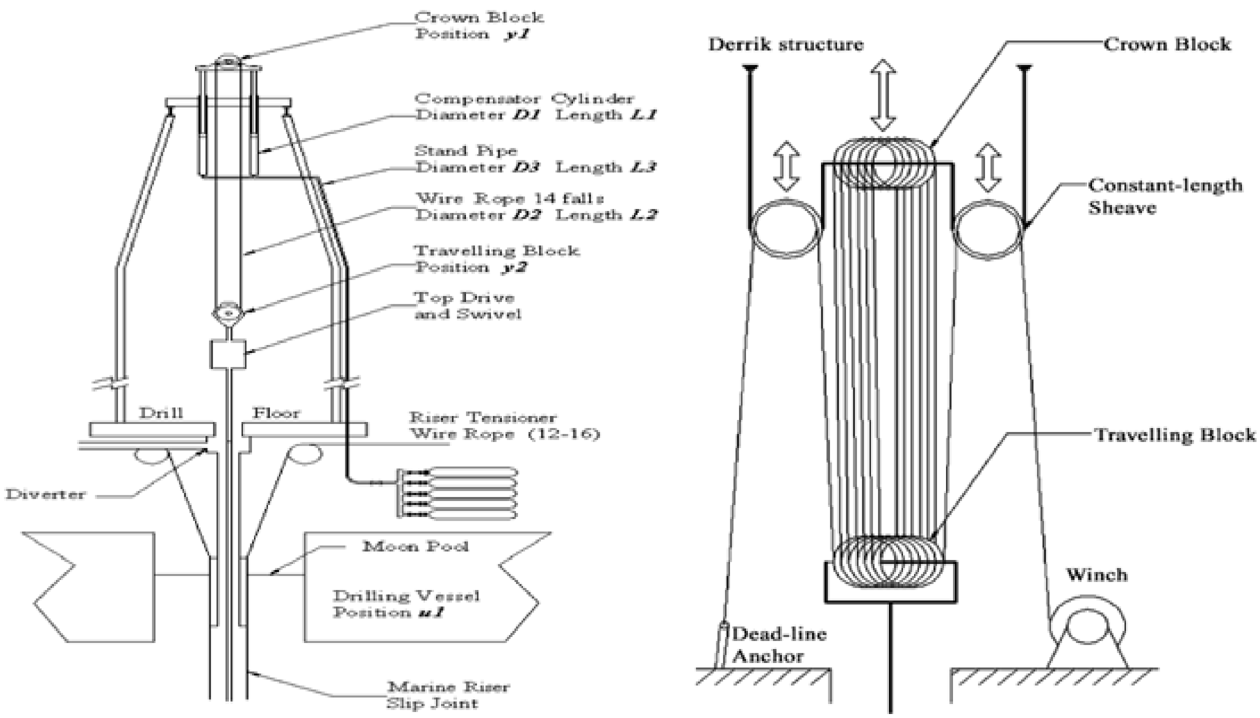

Figure 12: shows Passive heave compensation by courtesy of Hatleskog and Dunnigan (2007). To the left is the zoomed-out view of the Passive heave compensation while to the right is the zoomed-in view of the Passive heave compensation [7]

2.3.2 Active heave compensation

Active heave compensation is usually achieved at the winch level with the help of the hydraulic piston and the reference signal. There are three (3) types of Active heave compensation (AHC) namely, Rotative Active Heave Compensation (RAHC), Primary Controlled Active Heave Compensation (PAHC) and Linear Active Heave Compensation (LAHC)

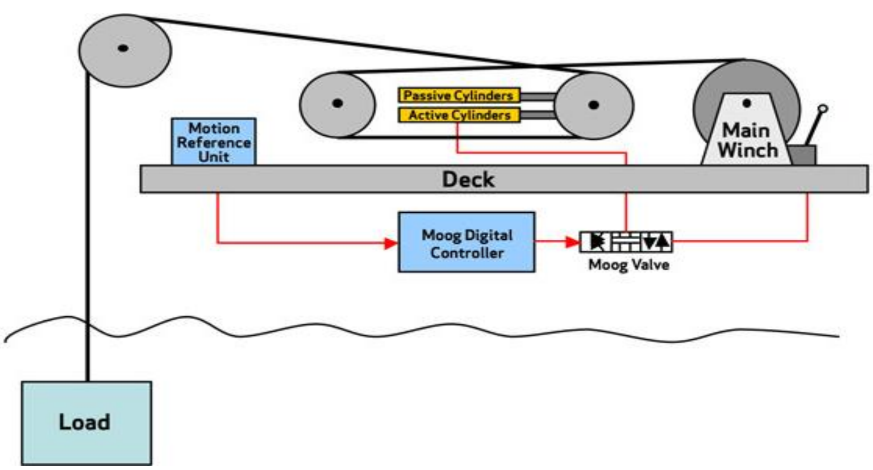

Figure 13: Is a schematic of a Rotative Active Heave Compensation (RAHC) by kind courtesy of offshoreteknikk [8]

3 HOOK LOAD THEORY

Hook load (W) is the total downward force on the hook of the top drive and it includes the buoyed weight of the drillstring, friction in the well etc. According to the Luke and Juvkam-Wold hook load prediction model [2], the hook load (W) during constant velocity of the travelling equipment is equal to the sum of the tensions in the drilling lines supporting the total downward force.

3.1 FACTORS AFFECTING HOOK LOAD MEASUREMENT

The hook loads measurements are affected by a number of factors among these are as follows

i. Weight of the drillstring
ii. Buoyancy effect
iii. Well friction
iv. Load cell position

3.1.1 WEIGHT OF THE DRILLING STRING

The weight in air of the drillstring (weight per unit length) will have a direct effect on the hook load measurements. The weight in air (W_a) of the drillstring is given by the relation

$$W_a = \rho_{dp} A_s g \qquad [1]$$

where W_a is the weight per unit length of the drill pipe in air

ρ_{dp} is the density of the drillpipe used

A_s is the cross-sectional area of the drill pipe

g is the acceleration due to gravity

The total weight of the drillstring (W_{ads}) in air is given by the relation

$$\Rightarrow W_{ads} = W_a h_{TVD} = \rho_{dp} A_s g \, h_{TVD} \qquad [2]$$

Hence, the density of the drill pipe used (ρ_{dp}), its cross–sectional area (A_s) and the true vertical depth (h_{TVD}) of the well will directly affect the weight of the drillstring.

3.1.2 Buoyancy

Archimedes principle states that when a body is partially or fully immersed in a fluid, it displaces its own weight of fluid in which it flows. The weight of fluid displaced is equal to the upward force (buoyancy factor) on that body. When the densities of the fluid inside and outside the drill string are different, the buoyancy force is given by the relation

$$\beta = \frac{\rho_s - \left(\rho_o \frac{A_o}{A_s} - \rho_i \frac{A_i}{A_s}\right)}{\rho_s} \qquad [3]$$

where β is the buoyancy factor (upward force) on the drillstring

ρ_s is the density of steel

ρ_o is the density of the mud outside the drillstring

ρ_i is the density of the mud inside the drillstring

A_i is the inner area of the drillstring

A_o is the outer area of the drillstring

A_s is the cross-sectional area of the drillstring (Steel)

In drilling operations, the density of the mud inside and outside the drillstring is approximately the same neglecting temperature and pressure effects.

i.e. $\rho_i = \rho_o = \rho_{mud} = Constant$ and hence Eqn (3) becomes,

$$\Rightarrow \beta = \frac{\rho_s - \left(\rho_o \frac{A_o}{A_s} - \rho_i \frac{A_i}{A_s}\right)}{\rho_s} = \frac{\rho_s - \left(\rho_{mud} \frac{A_o}{A_s} - \rho_{mud} \frac{A_i}{A_s}\right)}{\rho_s}$$

$$\Rightarrow \beta = \frac{\rho_s - \rho_{mud} \left(\frac{A_o}{A_s} - \frac{A_i}{A_s}\right)}{\rho_s} = \frac{\rho_s - \rho_{mud} \left(\frac{A_o - A_i}{A_s}\right)}{\rho_s}$$

But $A_o - A_i = A_s$

$$\Rightarrow \beta = \frac{\rho_s - \rho_{mud} \left(\frac{A_o - A_i}{A_s}\right)}{\rho_s} = \frac{\rho_s - \rho_{mud} \left(\frac{A_s}{A_s}\right)}{\rho_s}$$

$$\Rightarrow \beta = \frac{\rho_s - \rho_{mud}}{\rho_s} = 1 - \frac{\rho_{mud}}{\rho_s} \qquad [4]$$

Hence, the buoyed weight (W_{bds}) of the drillstring in the well is given by the relation

$$W_{bds} = \beta W_{ads} = \beta W_a h_{TVD} = \beta \rho_{dp} A_{cs} g\, h_{TVD} \qquad [5]$$

3.1.3 Well friction

The well friction has very high influence on the hook load measurement. The well friction is often depicted in the torque and drag measurements and its values varies for varying well section i.e. It has different values for the build-up section, sail section and drop-off section. Since, the well friction models are not the main focus for this thesis, we shall take a quick look at some of the soft string well friction models developed by Aadnøy and Andersen [10].

3.1.3.1 Torque and drag in sail section

According to Aadnøy and Andersen [10], the torque and drag model is based on Coulomb friction model and it is given by the relation

$F_2 = F_1 + w\Delta s(cos\alpha \pm \mu sin\alpha) = F_1 + mg(cos\alpha \pm \mu sin\alpha)$ [6]

where F_2 is the Force at the top of the drillstring

F_1 is the Force at the bottom of the drillstring

"+" represents hoisting of the drillstring

"-" represents lowering of the drillstring

The rotation friction which is also referred to as torque and it is given by the relation

$T = \mu w\Delta srsin\alpha$ [7]

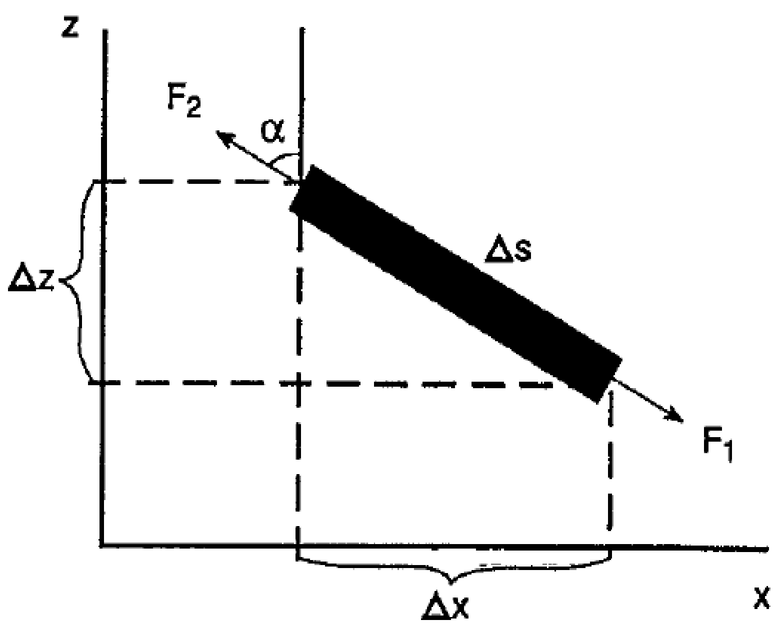

Figure 14: Shows the drag on a drillstring in the sail section by courtesy of Aadnøy and Andersen [10]

3.1.3.2 Torque and drag in build-up section

According to Aadnøy and Andersen [10], the torque and drag in the build-up section for both hoisting and lowering of the drillstring is given by the relation

i. Hoisting (Pulling) of string is given by the relation

$$F_2 = F_1 e^{-\mu(\alpha_2-\alpha_1)} - wR(sin\alpha_2 - e^{-\mu(\alpha_2-\alpha_1)} sin\alpha_1) \quad [8]$$

ii. Lowering of string is also given by the relation

$$F_2 = F_1 e^{\mu(\alpha_2-\alpha_1)} - \frac{wR}{1+\mu^2}\Big((1-\mu^2)\big(sin\alpha_2 - e^{-\mu(\alpha_2-\alpha_1)} sin\alpha_1\big) - 2\mu\big(cos\alpha_2 - e^{-\mu(\alpha_2-\alpha_1)} cos\alpha_1\big)\Big) \quad [9]$$

iii. Torque in the build-up bend

$$T = \mu r\big((F_1 + wRsin\alpha_1)abs(\alpha_2-\alpha_1)\big) + 2\mu wRr\,(cos\alpha_2 - cos\alpha_1) \quad [10]$$

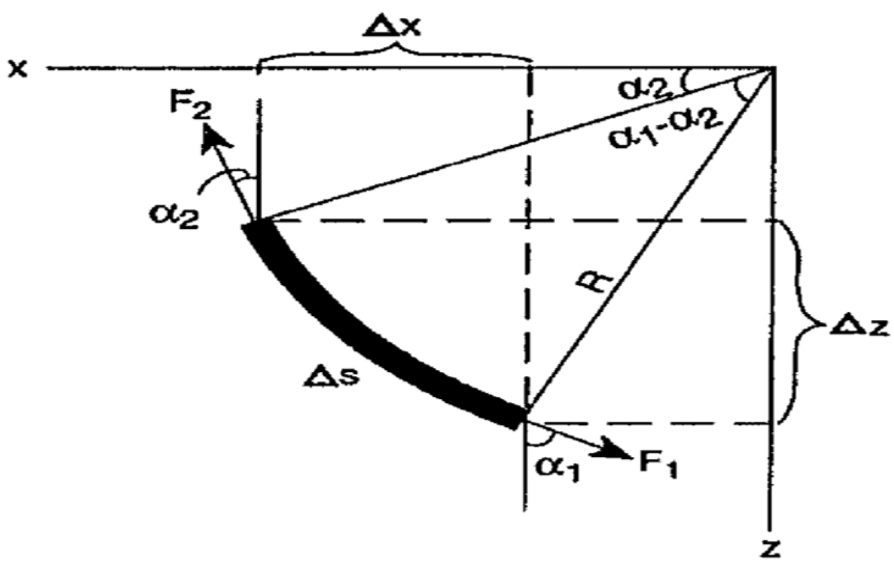

Figure 15: Shows the torque and drag in a build-up section by courtesy of Aadnøy and Andersen [10]

Torque and drag in drop-off section

The torque and drag in the drop-off section for both hoisting and lowering of the drillstring as suggested by Aadnøy and Andersen [10] is given by the relation

i. Hoisting (Pulling) of string is given by the relation

$$F_2 = F_1 e^{\mu(\alpha_2-\alpha_1)} + \frac{wR}{1+\mu^2}\Big((1-\mu^2)\big(sin\alpha_2 - e^{-\mu(\alpha_2-\alpha_1)} sin\alpha_1\big) - 2\mu\big(cos\alpha_2 - e^{-\mu(\alpha_2-\alpha_1)} cos\alpha_1\big)\Big) \quad [11]$$

ii. Lowering of string is also given by the relation

$$F_2 = F_1 e^{-\mu(\alpha_2-\alpha_1)} + wR(sin\alpha_2 - e^{-\mu(\alpha_2-\alpha_1)} sin\alpha_1) \quad [12]$$

iii. Torque in the drop-off bend

$$T = \mu r\big((F_1 + wRsin\alpha_1)(\alpha_2 - \alpha_1)\big) - 2\mu rwR\,(cos\alpha_2 - cos\alpha_1) \quad [13]$$

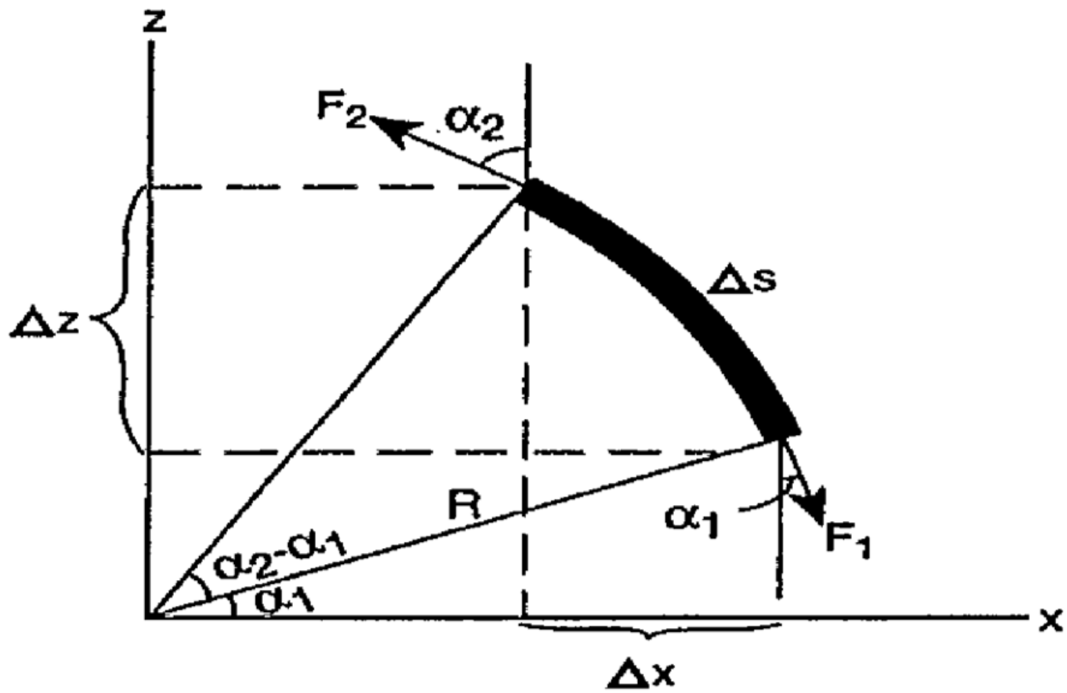

Figure 16: Shows the torque and drag in a drop-off section by courtesy of Aadnøy and Andersen [10]

Load cell position

The load cell position is essential in ensuring accurate hook load prediction. The accepted industry practice is to position the load cell at the dead line. This usually results in discrepancy in the actual hook load (W) measurements as compared to the expected values as described by Luke and Juvkam-Wold [2]. On the other hand, a direct and a more accurate hook load measurement can also be achieved using an Instrumented Internal Blow-out Preventer (IIBOP) as depicted Wylie et al [11]. The only challenge with the latter approach is that it can only be installed on some few top drives that can accommodate an IBOP. The load cell position is extremely important during non-uniform movement of the travelling equipment since the sum of the forces in the supporting lines is not the same as the hook load (W). i.e. Either the hook load exceeds the sum of the tensions in the supporting lines or vice-versa. Hence, the best position for the load cell placement is just above the top of the drillstring as suggested by Wylie et al [11]. Below is a schematic of some of the possible load cell sensor positions.

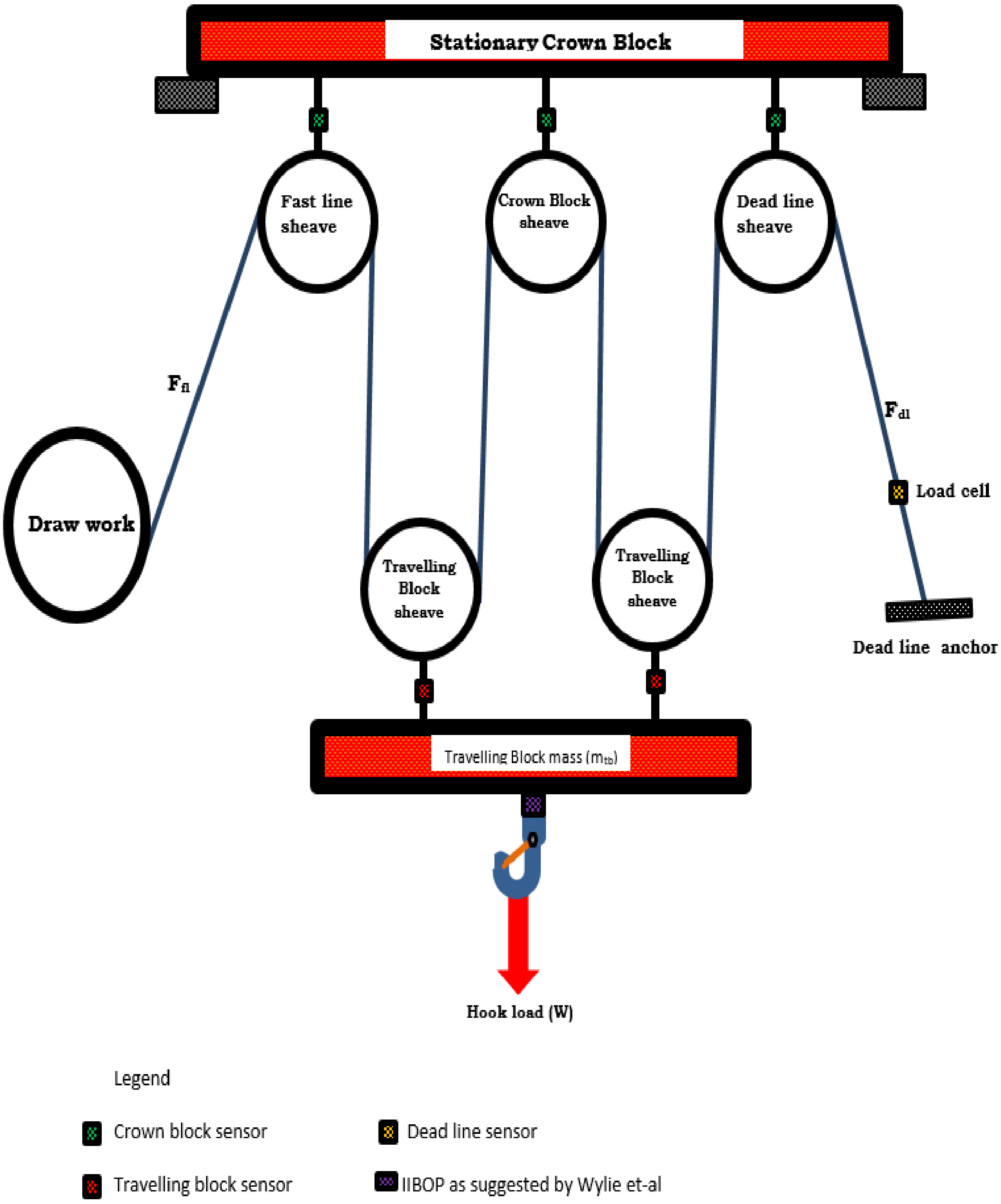

Figure 17: Shows some of the possible load cell positions for measuring hook load (W)

3.1.4 Other forces affecting the hook load measurement

Cayeux et al [4] also described the effect of other parameters on the accuracy of the hook load prediction. These sources of discrepancy include;

i. The tension exerted by both the mud hose and the umbilical connected to the top-drive. The magnitude of the force exerted by the mud hose depends on the position of the travelling block, the volume of the mud hose filled with the drilling mud and the density of the drilling mud used.
ii. The additional force exerted by the dolly on the drilling line during retraction with the magnitude of the force determined by the dolly position during the retraction.
iii. The friction between the dolly and its rails.
iv. The efficiency of each rotating sheave which depends on the velocity of the travelling equipment and the applied load.
v. The position of the travelling equipment which depends on the length of the drilling line that is spooled out from the drum and the elasticity of the drilling line (i.e. The effect of the drilling line weight).

3.2 EXISTING MODELS

There are numerous hook load prediction models but only three (3) of these models will be considered in this thesis. These models includes the industry accepted method [3], the Luke and Juvkam-Wold model [2] and Cayeux et al hook load model [4]. These models were derived based on constant velocity of the travelling equipment and hence need improvement to account for non-uniform movement of the travelling equipment if the need arises. The average sheave efficiency (e) as suggested by Luke and Juvkam-Wold [2] is 0.9 while Cayeux et al [4] also suggested that for both hoisting and lowering, the average sheave efficiency (e) over 0.8 m for 5 kg load and 50 kg load are 0.84 and 0.905 respectively. Below is a schematic of the block and tackle hoisting system and its sheave efficiency (e).

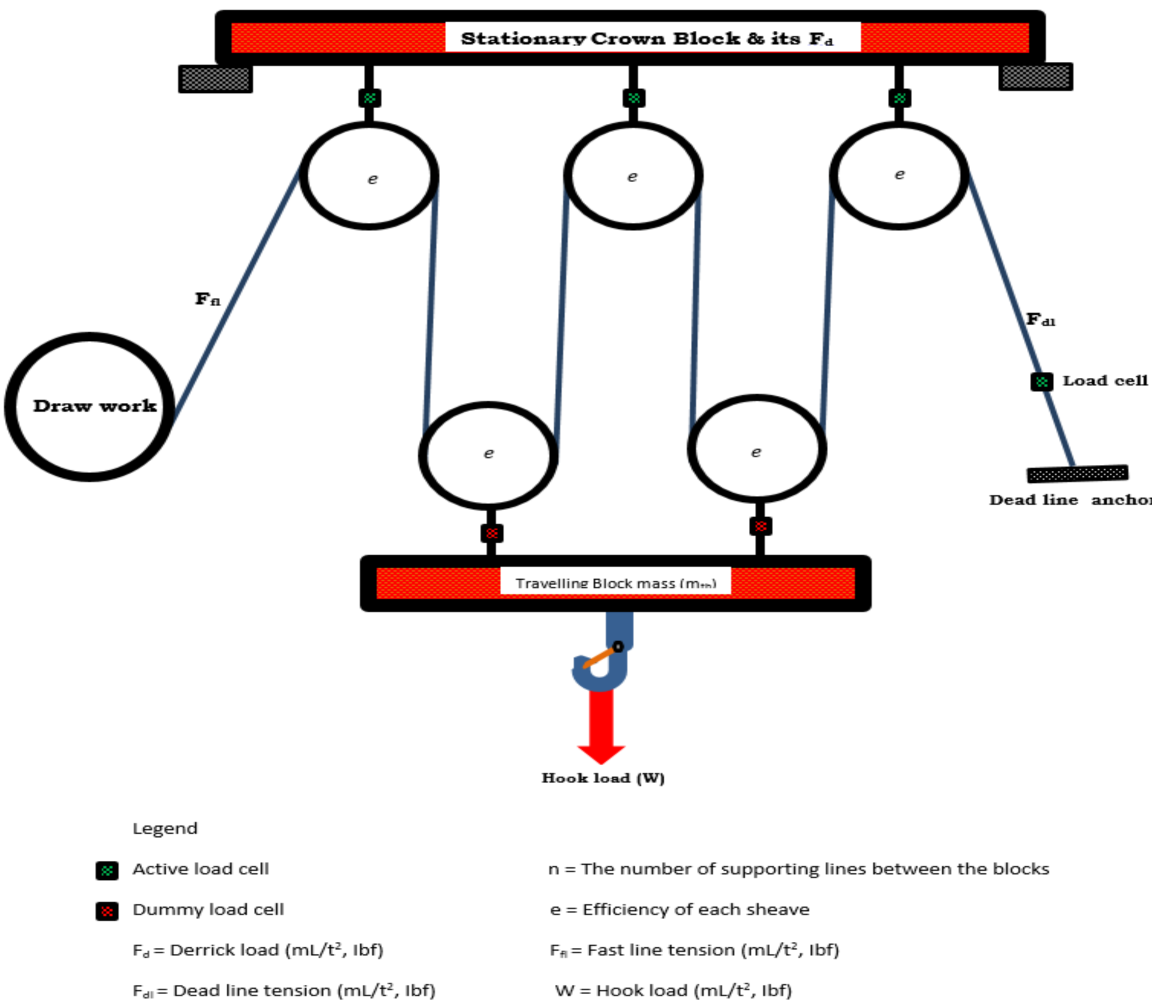

Figure 18: Show a block and tackle hoisting system and its constant sheave efficiency as proposed by Luke and Juvkam- Wold

3.2.1 Industry accepted model

The industry relies on a conservative approach which does not reflect the actual downhole conditions and hence resulting in either too low hook load measurements during hoisting or too high measurements during lowering. Below is the industry accepted relations for the hook load (W) prediction and the derivations are given in Appendix-A.

3.2.1.1 Accepted industry method for derrick and hook load prediction

The basic assumption behind this model is that it is based on perfect transmission of line tension (i.e. e =1) and hence the tensions in the lines remains constant. On the other hand, the relationship between the fast line tension (F_{fl}) and that of the dead line (F_{dl}) is based on constant sheave efficiency (e) and inactive (non-rotating) dead line sheave assumptions.

$$F_d = F_{dl}(\mathrm{n}+2) \quad \text{[A-1]}$$

$$F_d = \frac{W}{n}(\mathrm{n}+2) \quad \text{[A-2]}$$

$$W = nF_{dl} \quad \text{[A-3]}$$

3.2.1.2 Hoisting

$$F_{fl} = \frac{F_{dl}}{e^n} \quad \text{[A-4]}$$

3.2.1.3 Lowering

$$F_{fl} = e^n F_{dl} \quad \text{[A-5]}$$

3.2.2 Luke and juvkam-wold model

Unlike the industry accepted model which is based on perfect transmission of line tension (i.e. e=1), the Luke and Juvkam model is based on imperfect transmission of the line tension ($i.e.\ e \neq 1$) but the efficiency of each sheave is assumed to be constant (i.e. e = constant) as illustrated in figure (18). i.e. The line tension varies from line to line.

Luke and Juvkam also predicted two (2) types of the hook load model which depends on whether the dead line sheave is rotating (Active dead line sheave) or non-rotating (Inactive dead line) sheave. The rotating (active) dead line sheave assumption is questionable since one end of the dead line is fixed to the dead line anchor thus preventing any rotation, though stick-slip may occur due to elongation in the line as suggested by Cayeux et al [4]. Hence, the Inactive dead line sheave is the most practical assumption to use. Below is the Luke and Juvkam-wold hook load prediction relations for both active and in-active dead line sheave (Derivations are given in Appendix-B)

3.2.2.1 Hook load prediction for non-rotating dead line sheave

This model was based on constant sheave efficiency (e) and uniform movement of the travelling equipment and hence there was no acceleration effect on the hook load measurements. In addition, the weight of the drill-line is negligible as compared to the tensions in the line.

3.2.2.1.1 Hoisting

$$F_{fl} = \frac{F_{dl}}{e^n} \qquad \text{[B-1]}$$

$$F_{fl} = \frac{W(1-e)}{e\,(1-e^n)} \qquad \text{[B-2]}$$

$$W = \frac{F_{dl}\, e(1-\frac{1}{e^n})}{(e-1)} \qquad \text{[B-3]}$$

$$F_d = \frac{F_{dl}}{(1-e)}\left(1 + \left(\frac{1}{e^n}\right) - 2e\right) \qquad \text{[B-4]}$$

3.2.2.1.2 Lowering

$$F_{fl} = e^n F_{dl} \qquad \text{[B-5]}$$

$$F_{fl} = \frac{We^n(1-e)}{(1-e^n)} \qquad \text{[B-6]}$$

$$W = F_{dl}\frac{(1-e^n)}{(1-e)} \qquad \text{[B-7]}$$

$$F_d = \frac{F_{dl}\,(2-e-e^{n+1})}{(1-e)} \qquad \text{[B-8]}$$

3.2.2.2 Hook load prediction for rotating dead line sheave

3.2.2.2.1 Hoisting

$$F_{fl} = \frac{F_{dl}}{e^{n+1}} \qquad \text{[B-9]}$$

$$F_{fl} = \frac{W(1-e)}{e\,(1-e^n)} \qquad \text{[B-10]}$$

$$W = \frac{F_{dl}\,(1-e^{n})}{(1-e)e^{n}} \qquad [B\text{-}11]$$

$$F_{d} = \frac{F_{dl}\,(1-e^{n+2})}{(1-e)e^{n+1}} \qquad [B\text{-}12]$$

3.2.2.2.2 Lowering

$$F_{fl} = e^{n+1}F_{dl} \qquad [B\text{-}13]$$

$$F_{fl} = \frac{W\ (1-e)\ e^{n}}{(1-e^{n})} \qquad [B\text{-}14]$$

$$W = F_{dl}\frac{e(1-e^{n})}{(1-e)} \qquad [B\text{-}15]$$

$$F_{d} = \frac{F_{dl}\,(1-e^{n+2})}{(1-e)} \qquad [B\text{-}16]$$

3.2.3 Cayeux et al model

Unlike the previously discussed models above which utilizes the efficiency (e) of each rotating sheave to predict the tension in the lines, Cayeux et al hook load prediction model [4] is based on the coefficient of friction (μ) at the pulley axle. The advantage of using the Cayeux et al model is that the sheave efficiency (e) which depends on the coefficient of friction at the sheave axle is not required in order to accurately predict the hook load. The model also account for the effect of the centrifugal forces on each sheave (F_c), the effect of the weight (F_w) of each sheave on the hook load prediction, the effect of the coefficient of friction at the sheave axle and the effect of the tension in the drilling lines.

Both Coulomb friction model and Stribeck friction models were used. The beauty of using the Stribeck friction model(μ_s) over the coulomb friction model (μ_a) is that it accounts for the transition from static to dynamic conditions and vice-versa. Below is a schematic of the forces acting on the crown block sheave.

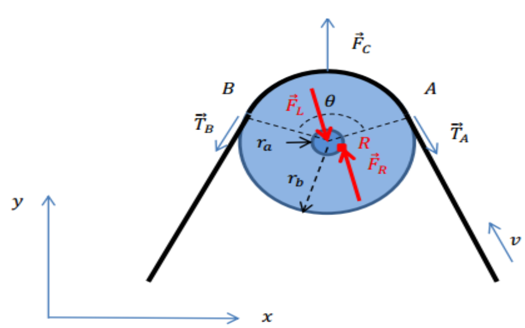

Figure 19: Shows the forces on the crown block sheave by courtesy of Cayeux et al [4]

3.2.3.1 Crown block sheave

Cayeux et al predicted the line tension relations for both the crown block sheaves and that of the travelling block sheaves for either hoisting or lowering.

3.2.3.1.1 Hoisting

$$T_B = -\frac{r_b T_A + \mu_a r_a T_A - 2\bar{\lambda}_m \mu_a \dot{\omega}^2 {r_b}^2 r_a + m_p g \mu_a r_a}{\mu_a r_a - r_b} \quad \text{[C-1]}$$

3.2.3.1.2 Lowering

$$T_B = -\frac{-r_b T_A + \mu_a r_a T_A - 2\bar{\lambda}_m \mu_a \dot{\omega}^2 {r_b}^2 r_a + m_p g \mu_a r_a}{\mu_a r_a + r_b} \quad \text{[C-2]}$$

3.2.3.2 Travelling block sheave

Similarly considering the travelling block, the line tensions relation as predicted by Cayeux et al for the sheaves in the travelling block is given by

3.2.3.2.1 Hoisting

$$T_B = \frac{-r_b T_A - \mu_a r_a T_A + 2\bar{\lambda}_m \mu_a \dot{\omega}^2 {r_b}^2 r_a + m_p g \mu_a r_a}{\mu_a r_a - r_b} \quad \text{[C-3]}$$

3.2.3.2.2 Lowering

$$T_B = \frac{r_b T_A - \mu_a r_a T_A + 2\bar{\lambda}_m \mu_a \dot{\omega}^2 {r_b}^2 r_a + m_p g \mu_a r_a}{\mu_a r_a + r_b} \quad \text{[C-4]}$$

Where

r_b = radius of each sheave [L](m)

r_a = radius of each sheave axle [L](m)

μ_a = friction coefficient between the sheave and its axle [dimensionless]

$\bar{\lambda}_m$ = linear weight of the drill line $[ML^{-1}]$(Kg/m)

m_p = mass of the pulley [M] (Kg)

g = acceleration due to gravity $[LT^{-2}]$ (m/s2)

$\dot{\omega}$ = angular velocity of each sheave $[T^{-1}]$ (rad/s)

T_A = line tension at contact point A , as illustrated in figure (19)

T_B = line tension at contact point B, as illustrated in figure (19)

4 EXTENDED MODELS

The existing models utilizes Newton's second law of motion but assumed constant velocity of the travelling equipement and hence there is no acceleration effect. The acceleration effect will be incorporated into all the three (3) existing models after which hypothetical data will be used to confirm if indeed non-uniform movement of the travelling equipment has an effect on the hook load measurement by comparing the data with and without acceleration effect in the model. All the three models will also be compared with each other to determine which model has the most accurate hook load prediction. In addition to investigating the acceleration effect, this work seeks to investigate the validity of the constant sheave efficiency (e) assumption as proposed by Luke and Juvkam-Wold.

4.1 PROPOSED MODEL

From Newton's second law of motion, the resultant or the net force acting on the travelling equipment is equal to its rate of change of the moment. It is mathematically given as

Ie $i.e.\ \sum F_{net} = \frac{\partial m_T v}{\partial t} = \frac{m_T v - m_T u}{\partial t} = \frac{m_T(v-u)}{\partial t} = m_T a$ [δ- 1A]

$m_T = m_{dp} + m_{tb} + m_{dl} \approx m_{dp} + m_{tb}$ [δ- 2]

where

m_T = Total mass of the travelling equipment

m_{dp} = mass of drill pipe

m_{dl} = mass of the drill line

m_{tb} = mass of travelling block with its pulley

$v =$ final velocity

$u =$ initial velocity

For simplicity, it is assumed that the mass of the drill line (m_{dl}) is negligible compared with the mass of the drill pipe (m_{dp}) and that of the travelling block (m_{tb}) and hence it can be neglected in the total mass (m_T) calculation.

Below is a schematic showing the total mass of the travelling equipment (m_T) and the direction of the resultant force during hoisting or lowering

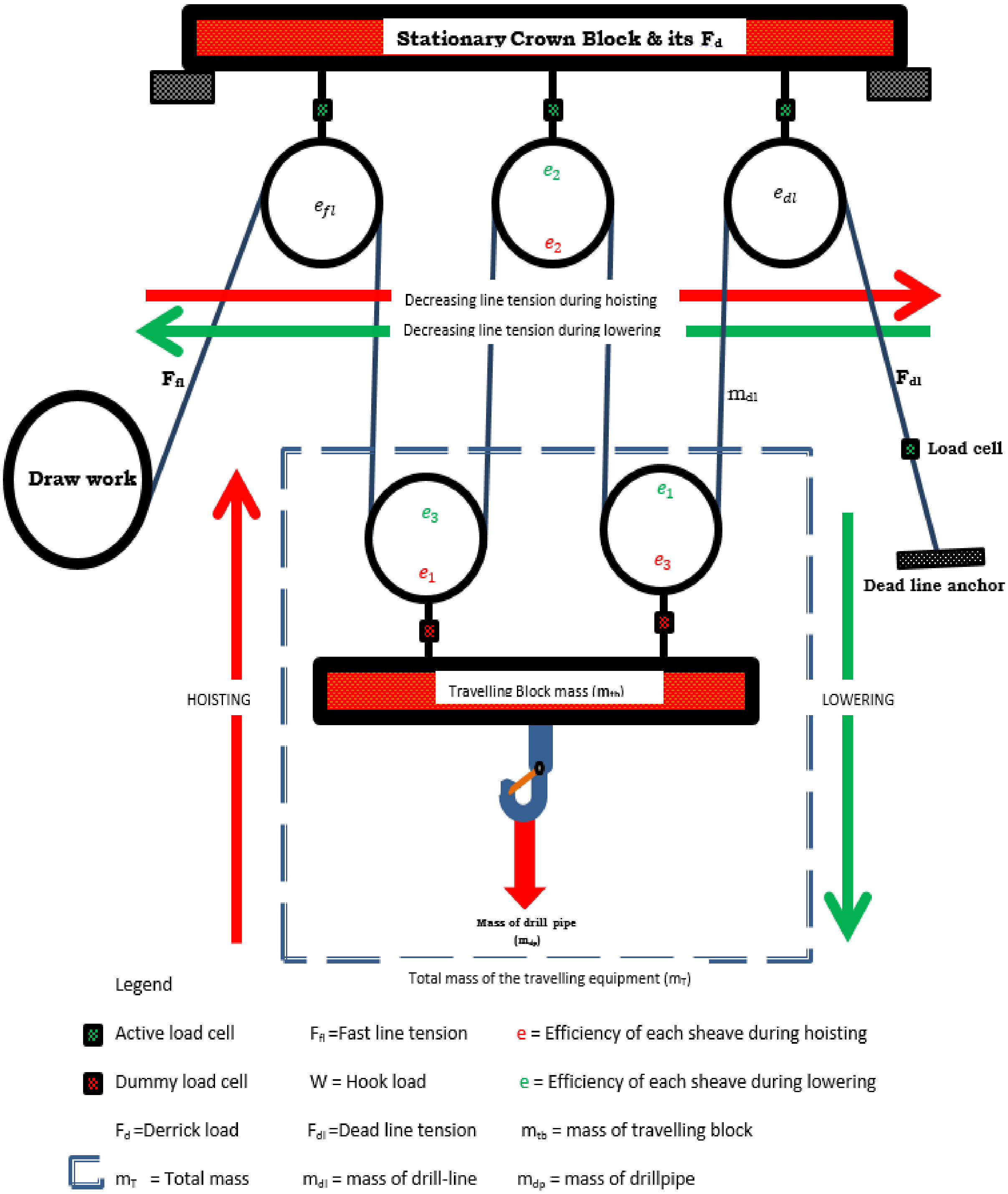

Figure 20: Shows the total mass of the travelling equipment and the direction of the resultant force during either hoisting or lowering

Considering the travelling equipment (i.e. the combined mass of the travelling block and the mass of the drillpipe neglecting the mass of the drill-line) for three (3) discrete positions as illustrated in the figure below

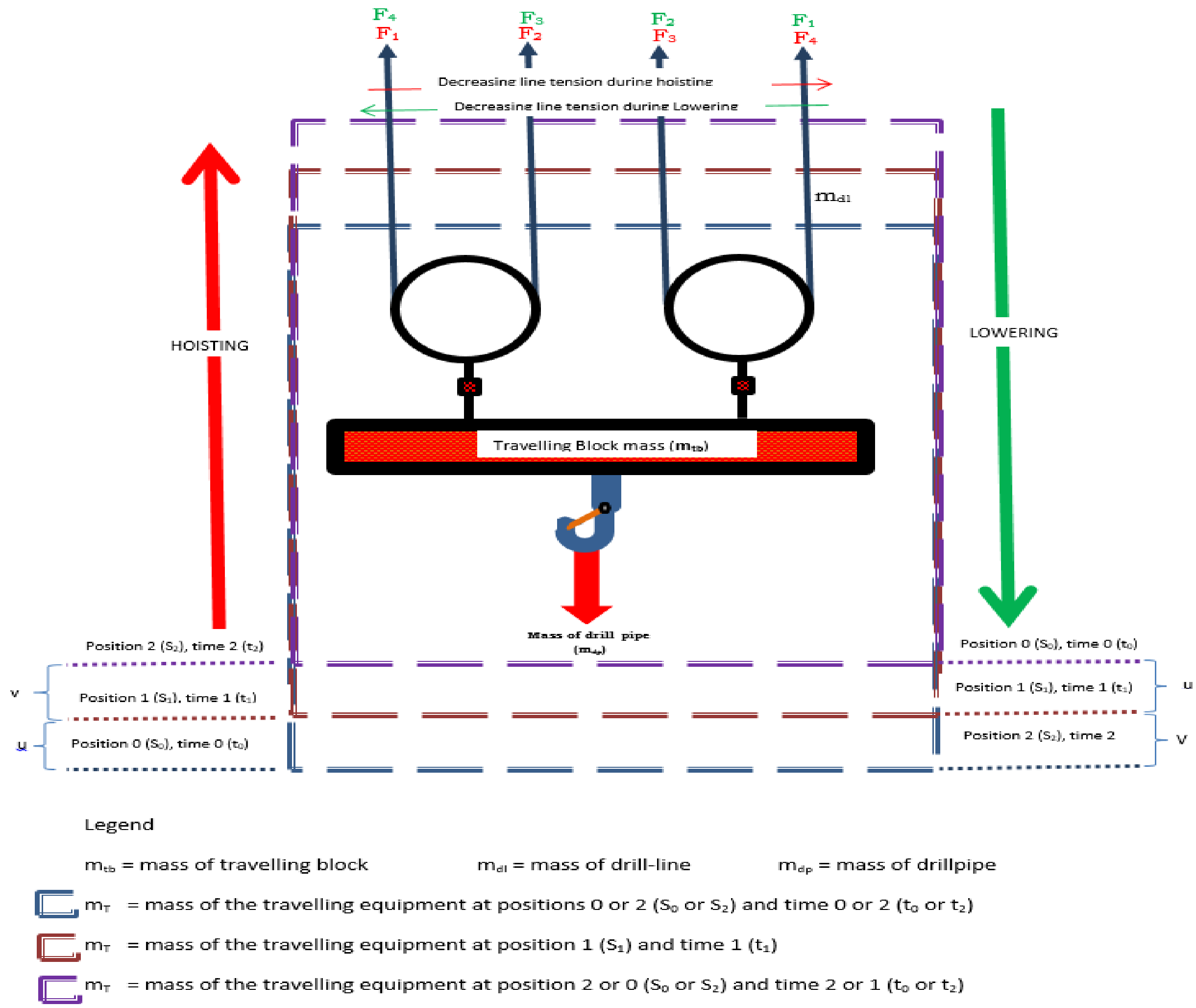

Figure 21: Shows the net forces on the travelling equipment for either hoisting or lowering

Substituting Eqn [δ-2] into Eqn [δ-1A] gives

$$\Rightarrow \sum F_{net} = (m_{dp} + m_{tb})a \qquad [\delta\text{- 1B}]$$

But acceleration (a) is also given by the relation

$$a = \frac{dv}{dt} = \frac{v-u}{t_2-t_1} \qquad [\delta\text{- 3A}]$$

$$v = \frac{ds}{dt} = \frac{s_2-s_1}{t_2-t_1} \qquad [\delta\text{- 4}]$$

$$u = \frac{ds}{dt} = \frac{s_1-s_o}{t_1-t_0} \qquad [\delta\text{- 5}]$$

Substituting Eqn [δ- 4]and Eqn [δ- 5] into Eqn [δ- 3A] gives

$$a = \frac{dv}{dt} = \frac{v-u}{t_2-t_1} = \frac{(\frac{s_2-s_1}{t_2-t_1})-(\frac{s_1-s_o}{t_1-t_0})}{t_2-t_1} \qquad [\delta\text{- 3B}]$$

$$a = \frac{1}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \qquad [\delta\text{- 3C}]$$

$$a = \frac{1}{(t_2-t_1)^2}\left[(s_2 - s_1) - \left(\frac{t_2-t_1}{t_1-t_0}\right)(s_1 - s_o)\right] \quad [δ\text{- }3D]$$

Substituting Eqn[δ- 3C] into Eqn [δ- 1B] gives

$$\sum F_{net} = \left(\frac{(m_{dp}+ m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \quad [D\text{-}1A]$$

Alternatively, $W = m_T\, g = (m_{dp} + m_{tb})g = F_{Down}$ [δ- 6A]

$$m_T = m_{dp} + m_{tb} = \frac{W}{g} \quad [δ\text{- }6B]$$

Substituting Eqn[δ- 6B] into Eqn [D-1A] gives

$$\sum F_{net} = \frac{W}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \quad [D\text{-}1B]$$

HOISTING

During hoisting the sum of the upward forces exceeds that of the downward force as illustrated below

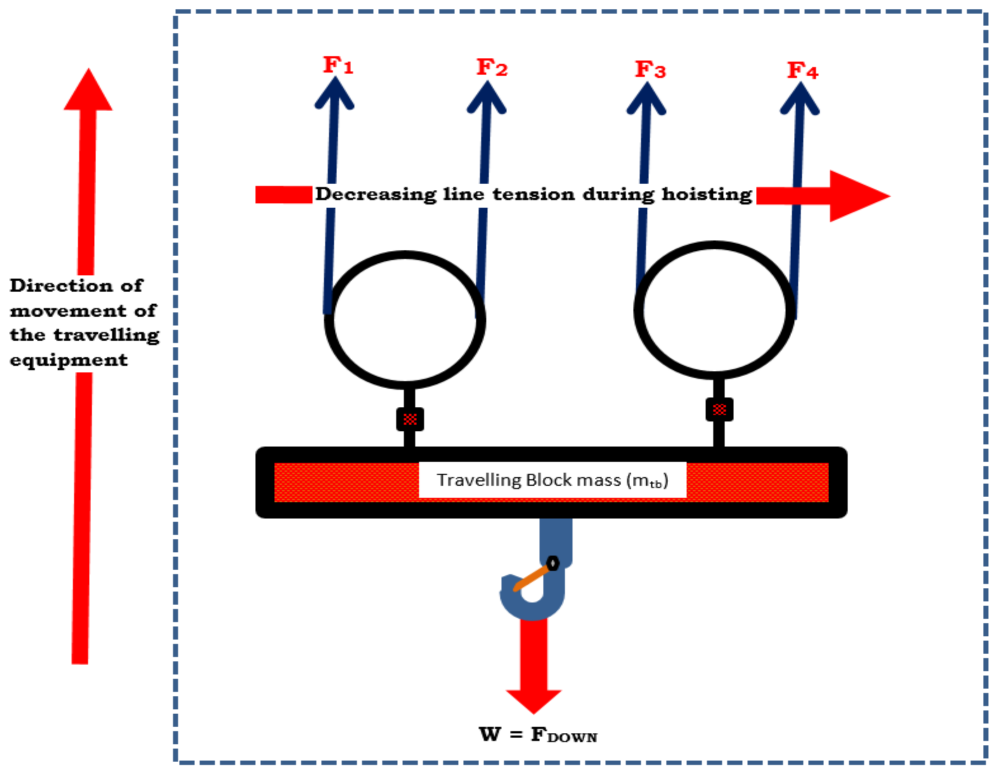

Figure 22: Shows the net force on the travelling equipment during hoisting

Hence, Eqn [D-1A]becomes

$$(F_1 + F_2 + F_3 + \cdots + F_n) - F_{Down} = \frac{(m_{dp}+ m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \quad [D\text{-}2A]$$

Substituting Eqn [δ- 6A] into Eqn [D-2A] gives

$$(F_1 + F_2 + F_3 + \cdots + F_n) - W = \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$W = (F_1 + F_2 + F_3 + \cdots + F_n) - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[D-2B]}$$

Eqn [D-2B] is the hook load relation during hoisting for both uniform and non-uniform movement

Alternatively, substituting Eqn [δ- 6B] into Eqn [D-2B] gives

$$W = (F_1 + F_2 + F_3 + \cdots + F_n) - \frac{w}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$W + \frac{w}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] = (F_1 + F_2 + F_3 + \cdots + F_n)$$

$$W\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) = (F_1 + F_2 + F_3 + \cdots + F_n)$$

$$W = \frac{(F_1 + F_2 + F_3 + \cdots + F_n)}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad \text{[D-2C]}$$

LOWERING

In a similar vein, during lowering the sum of downward forces exceeds that of the the upward forces as illustrated below

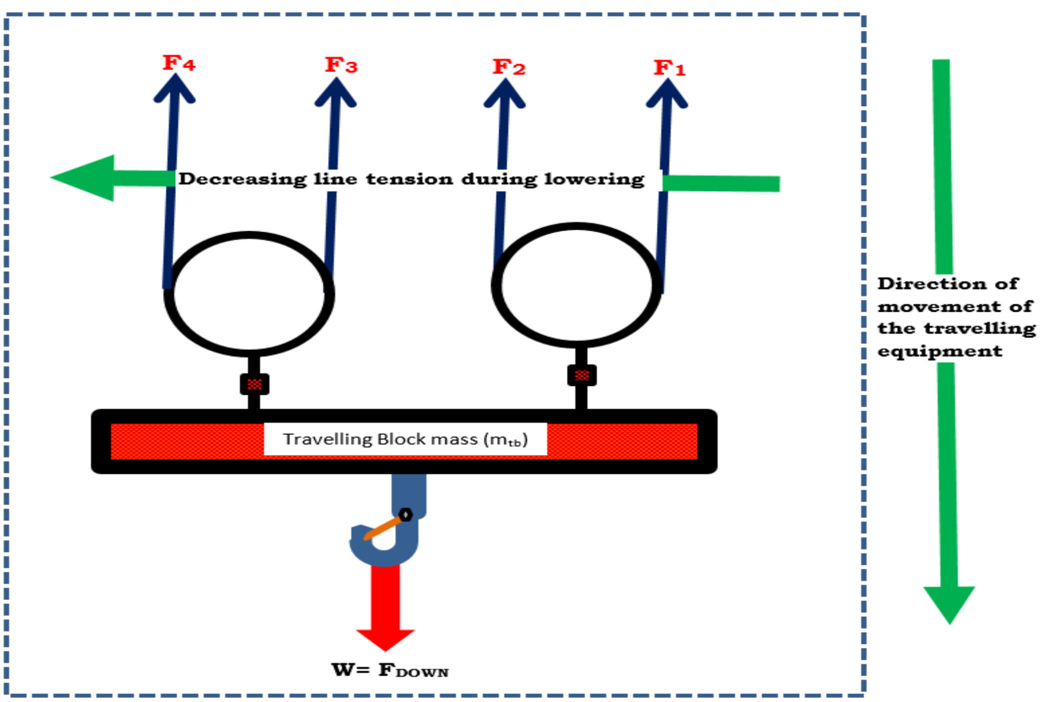

Figure 23: Shows the net force on the travelling equipment during lowering

Eqn [D-1A] becomes

$$\Rightarrow F_{Down} - (F_1 + F_2 + F_3 + \cdots + F_n) = \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[D-3A]}$$

Substituting Eqn [δ- 6A] into Eqn [D-3A] gives the net downward force as

$$W - (F_1 + F_2 + F_3 + \cdots + F_n) = \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$\Rightarrow W = (F_1 + F_2 + F_3 + \cdots + F_n) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[D-3B]}$$

Alternatively, substituting Eqn [δ- 6B] into Eqn [D-3B] gives

$$W = (F_1 + F_2 + F_3 + \cdots + F_n) + \frac{W}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$W - \frac{W}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] = (F_1 + F_2 + F_3 + \cdots + F_n)$$

$$W\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) = (F_1 + F_2 + F_3 + \cdots + F_n)$$

$$W = \frac{(F_1 + F_2 + F_3 + \cdots + F_n)}{\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad \text{[D-3C]}$$

From Eqn [D-2B] and Eqn [D-3B] the effect of the non-uniform movement of the travelling equipment is given by

$$A_{cc} = m_T\left(\frac{v - u}{t_2 - t_1}\right) = \pm\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[δ- 7A]}$$

Similarly, from Eqn [D-2C] and Eqn [D-3C] the effect of the non-uniform movement is also given by

$$A_{cc} = \left(1 \pm \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \qquad \text{[δ- 7B]}$$

During constant velocity of the travelling equipment (i.e. uniform movement), the final velocity (v) is the same as the initial velocity (u) and hence, there is no effect of acceleration on the hook load (W) measurements.

4.2 EXTENSIONS OF THE INDUSTRY ACCEPTED MODEL TO ACCOUNT FOR THE EFFECT OF ACCELERATION DURING NON-UNIFORM MOVEMENT

The industry accepted hook load prediction model assumes a perfect transmission of line tension from the fast line (F_{fl}) towards the dead line (F_{dl}) and vice-versa. $\left(i.e. e = 1 \text{ and hence } F_{fl} = F_{dl}\right)$. The hook load relation for both hoisting and lowering during uniform movement of the travelling equipment is given by Eqn [A-3] as

Hook load $(W) = nF_{dl} = nF_{fl}$

4.2.1 Hoisting with non-uniform movement

Since the industry accepted hook load prediction model assumes perfect transmission of line tension $(i.e.\ e = 1)$ as illustrated in figure (20) above

$\Rightarrow F_{fl} = F_1 = F_2 = F_3 = \cdots = F_{n-1} = F_n = F_{dl}$

Hence for "n" supporting lines between the travelling block and the crown block the sum of the tensions in the supporting lines is given by

$$F_1 + F_2 + F_3 + \cdots + F_n = F_{dl} + F_{dl} + F_{dl} + \cdots + F_{dl} = nF_{dl} \qquad [ς\text{-}1]$$

Substituting Eqn [ς-1] into Eqn [D-2B], which is the hook load relation during hoisting gives

$$W = nF_{dl} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad [E\text{-}1A]$$

Similarly substituting Eqn [ς-1] into Eqn [D-2C] gives,

$$W = \frac{nF_{dl}}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad [E\text{-}1B]$$

4.2.2 Lowering with non-uniform movement

Similarly since the industry accepted hook load prediction model assumes a perfect transmission of line tension, substituting the sum of the tensions in the supporting lines relation (Eqn [ς-1]) into the hook load relation during lowering (Eqn [D-3B]) gives

$$W = nF_{dl} + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad [E\text{-}2A]$$

Similarly, substituting Eqn [ς-1] into Eqn [D-3C] gives

$$W = \frac{nF_{dl}}{\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad [E\text{-}2B]$$

4.3 Extension of Luke and Juvkam-Wold model to incorporate the effect of acceleration for non-uniform movement of the travelling equipment

Luke and Juvkam-Wold derived the hook load prediction model for two (2) different types of dead line sheaves namely, active (rotating) dead line sheave and inactive (non-rotating) dead-line sheave. Their model was based on both constant sheave efficiency $(i.e.\ e_1 = e_2 = e_3 = e = constant)$ and also with the assumption that the travelling equipment undergoes uniform movement$(i.e.\,\mathrm{u} = \mathrm{v} = constant)$. In this thesis, we seek to account for the effect of acceleration on the hook load measurement during non-uniform movement of the travelling equipment and also confirm the constant sheave efficiency assumption as proposed by Luke and Juvkam-Wold. This will be achieved by equipping each sheave with a load cell and hence, the efficiency of each sheave can be determined. Hook load prediction models will be developed for both varying sheave efficiency assumption and that of constant sheave efficiency.

4.3.1.1 Inactive (non-rotating) dead line sheave derivation

Hook load relation during hoisting for non-uniform movement and varying sheave efficiency

During hoisting, maximum tension occurs in the fast line (F_{fl}), while the minimum tension occur in the dead line (F_{dl}) i.e. The tension decreases from the fast line towards the dead line $(ie.\,F_{fl} \geq\ F_{dl})$.

The efficiency for each sheave is given by Eqn (α) as (given in appendix A)

$$e = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} \qquad [\alpha]$$

Considering the fast line sheave (First sheave in the crown block (from the direction of the draw work) and from Eqn (α), its efficiency (e_1) is given by

$$e_1 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{fl}}$$

$$\Rightarrow F_1 = e_1 F_{fl} \qquad [\Upsilon\text{-}1]$$

Similarly, the efficiency (e_2) of the next sheave in the travelling block is also given by

$$e_2 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = e_2 F_1 = e_2(e_1 F_{fl}) = e_2 e_1 F_{fl} = \prod_{i=1}^{2} e_i F_{fl} \qquad [\Upsilon\text{-}2]$$

Also, considering the efficiency (e_3) of the next sheave in the crown block gives

$$e_3 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = e_3 F_2 = e_3(e_2 e_1 F_{fl}) = e_3 e_2 e_1 F_{fl} = \prod_{i=1}^{3} e_i F_{fl} \qquad [\Upsilon\text{-}3]$$

Hence, for "n" number of lines between the travelling block and the crown block, the relationship between the tension in each line and the applied fast line tension (F_{fl}) is given by

$$F_n = \prod_{i=1}^{n} e_i F_{fl} \qquad [\Upsilon\text{-}4]$$

I. ACTIVE DEAD LINE SHEAVE

Considering rotating dead line sheave, its efficiency (e_d) becomes

$$e_d = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{dl}}{F_n}$$

$$\Rightarrow F_{dl} = e_d F_n = e_d\left(\prod_{i=1}^{n} e_i F_{fl}\right) = e_d \prod_{i=1}^{n} e_i F_{fl} \qquad [\Upsilon\text{-}5A]$$

II. INACTIVE DEAD LINE SHEAVE

Similarly, considering non-rotating dead line sheave in the crown block, it is assumed that there is perfect transmission of tension $(i.e.\ \ e_d = 100\% = 1)$

$$\Rightarrow F_n = F_{dl} = \prod_{i=1}^{n} e_i F_{fl} \qquad [\Upsilon\text{-}5B]$$

$$F_{fl} = \frac{F_{dl}}{\prod_{i=1}^{n} e_i} \qquad [\Upsilon\text{-}5C]$$

With the assumption of varying sheave efficiency and from the relationship between each of the lines with respect to the fast line Eqn [ϒ-5B], the sum of the tension in the supporting lines gives

$$\sum_1^n F_i = F_{fl}[\, e_1 + e_2 e_1 + e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i] \qquad [\Upsilon\text{-}6]$$

Substituting Eqn [ϒ-6] into the hook load relation during hoisting Eqn [D-2B] gives

$$W = F_{fl}[\, e_1 + e_2 e_1 + e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i] - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad [\Upsilon\text{-}7]$$

FAST LINE (F_{fl}) AND DEAD LINE TENSION (F_{dl})

Making F_{fl} the subject of Eqn [ϒ-7] gives

$$F_{fl} = \frac{1}{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}\left(W + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \quad \text{[F-1A]}$$

Substituting Eqn [ϒ-5C] into Eqn [F-1A] gives

$$\frac{F_{dl}}{\prod_{i=1}^{n} e_i} = \frac{1}{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}\left(W + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$F_{dl} = \frac{\prod_{i=1}^{n} e_i}{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}\left(W + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \quad \text{[F-1B]}$$

HOOK LOAD (W) RELATIONS

From Eqn [F-1A], the hook load is given by

$$W = F_{fl}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \quad \text{[F-2A]}$$

Similarly, From Eqn [F-1B], the hook load is given by

$$W = \frac{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)F_{dl}}{\prod_{i=1}^{n} e_i} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \quad \text{[F-2B]}$$

Alternatively, the derrick load (F_d) is given by the relation

$$\mathrm{F}_d = \mathrm{F}_{fl} + \mathrm{W} + \mathrm{F}_{dl}$$

$$\Rightarrow \mathrm{W} = \mathrm{F}_d - \mathrm{F}_{fl} - \mathrm{F}_{dl} = \mathrm{F}_d - (\mathrm{F}_{fl} + \mathrm{F}_{dl}) \quad \text{[F-2C]}$$

Substitituing the relationship between the dead line and the fast line tension (Eqn [ϒ-5C]) during hoisting into Eqn [F-2C] gives

$$\mathrm{W} = \mathrm{F}_d - (\mathrm{F}_{fl} + \mathrm{F}_{dl}) = \mathrm{F}_d - \left(\frac{F_{dl}}{\prod_{i=1}^{n} e_i} + \mathrm{F}_{dl}\right) = \mathrm{F}_d - F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right)$$

$$W = \mathrm{F}_d - F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right) \quad \text{[F-2D]}$$

Also substituting Eqn [ϒ-6] into Eqn [D-2C] gives

$$W = \frac{F_{fl}[e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i]}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \quad \text{[F-2E]}$$

Substitituing the relationship between the dead line and the fast line tension (Eqn [ϒ-5C]) during hoisting into Eqn [F-2E] gives

$$W = \frac{F_{dl}[e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i]}{\prod_{i=1}^{n} e_i\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \quad \text{[F-2F]}$$

Eqn [F-2A], Eqn [F-2B], Eqn [F-2C], Eqn [F-2D], Eqn [F-2E] and Eqn [F-2F] are the hook load (W) relations during hoisting for inactive dead line sheave and with non-uniform movement and varying sheave efficiency.

DERRICK LOAD (F_d) RELATIONS

From Eqn [F-2D], the derrick load is given by the relation

$$\Rightarrow F_d = W + F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right) \qquad \text{[F-3A]}$$

Substitute Eqn [F-2A] into Eqn [F-3A] relation gives

$$\Rightarrow F_d = W + F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right)$$

$$F_d = F_{fl}[\, e_1 + e_2e_1 + e_3e_2e_1 + .. + \prod_{i=1}^{n} e_i] - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] + F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right)$$

[F-3B]

Substituting Eqn [F-2B] into Eqn [F-3A] relation gives

$$\Rightarrow F_d = \frac{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)F_{dl}}{\prod_{i=1}^{n} e_i} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] + F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right)$$

$$F_d = F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + \frac{\prod_{i=1}^{n} e_i}{\prod_{i=1}^{n} e_i} + \frac{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}{\prod_{i=1}^{n} e_i}\right) - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$F_d = \frac{F_{dl}}{\prod_{i=1}^{n} e_i}(1 + \prod_{i=1}^{n} e_i + (e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \qquad \text{[F-3C]}$$

Substituting Eqn [F-2E] into Eqn [F-3A] gives

$$F_d = \frac{F_{fl}[\, e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i]}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right) \qquad \text{[F-3D]}$$

Finally, substituting Eqn [F-2F] into Eqn [F-3A] relation gives

$$F_d = \frac{F_{dl}[\, e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i]}{\prod_{i=1}^{n} e_i\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{\prod_{i=1}^{n} e_i} + 1\right) \qquad \text{[F-3E]}$$

4.3.1.2 HOISTING WITH NON-UNIFORM MOVEMENT AND CONSTANT SHEAVE EFFICIENCY

FAST LINE (F_{fl}) AND DEAD LINE TENSION (F_{dl}) RELATIONS

If we assume a constant sheave efficiency (e) as proposed by Luke an Juvkam i.e $e_1 = e_2 = e_3 = e_4 = \cdots = e_n = e = Constant$, Eqn [F-1A]becomes

$$F_{fl} = \frac{1}{(e + e^2 + e^3 + \cdots + e^n)}\left(W + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$F_{fl} = \frac{(1-e)}{\mathrm{e}\,(1-e^n)}\left(W + \frac{(m_{dp}+ m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \qquad [F\text{-}1A_1]$$

For uniform movement, Eqn [F-1A₁] satisfies Eqn [B-2] in the Luke and Juvkam model

Similarly Eqn [F-1B] becomes

$$F_{dl} = \frac{e^n}{(e + e^2 + e^3 + \cdots + e^n)}\left(W + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$F_{dl} = \frac{e^n(1-e)}{\mathrm{e}\,(1-e^n)}\left(W + \frac{(m_{dp}+ m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \qquad [F\text{-}1B_1]$$

HOOK LOAD (W) RELATIONS

Assuming a constant sheave efficiency (e), Eqn [F-2A] becomes

$$W = F_{fl}(e + e^2 + e^3 + \cdots + e^n) - \frac{(m_{dp}+ m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$W = F_{fl}\frac{\mathrm{e}\,(1-e^n)}{(1-e)} - \frac{(m_{dp}+ m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \qquad [F\text{-}2A_1]$$

Also considering Eqn [F-2B] gives

$$W = \frac{(e + e^2 + e^3 + \cdots + e^n)F_{dl}}{e^n} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$W = \frac{e\,(1-e^n)F_{dl}}{e^n(1-e)} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$W = \frac{F_{dl} e\,(1-\frac{1}{e^n})}{(e-1)} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[F-2B}_1\text{]}$$

For constant velocity, Eqn [F-2B$_1$] satisfies Eqn [B-3] in the Luke and Juvkam model

With the same constant sheave efficiency assumption Eqn [F-2D] becomes

$$W = F_d - F_{dl}\left(\frac{1}{e^n} + 1\right) \qquad \text{[F-2D}_1\text{]}$$

Assuming a constant sheave efficiency (e), Eqn [F-2E] becomes

$$W = \frac{F_{fl}(e + e^2 + e^3 + \cdots + e^n)}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)}$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1-e^n)}{(1-e)}$$

$$W = \frac{e\,(1-e^n)F_{fl}}{(1-e)\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad \text{[F-2E}_1\text{]}$$

With the same constant sheave efficiency assumption Eqn [F-2F] becomes

$$W = \frac{F_{dl}(e + e^2 + e^3 + \cdots + e^n)}{e^n\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)}$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1-e^n)}{(1-e)}$$

$$W = \frac{e\,(1-e^n)F_{dl}}{e^n(1-e)\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad \text{[F-2F}_1\text{]}$$

DERRICK LOAD (F_d) RELATIONS

Similarly, Eqn [F-3A] becomes

$$F_d = W + F_{dl}\left(\frac{1}{e^n} + 1\right) \qquad \text{[F-3A}_1\text{]}$$

Also, Eqn [F-3B] also result in

$$F_d = F_{fl}(e + e^2 + e^3 + \cdots + e^n) - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] + F_{dl}\left(\frac{1}{e^n} + 1\right)$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1-e^n)}{(1-e)}$$

$$F_d = \frac{e\,(1-e^n)}{(1-e)}F_{fl} - \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] + F_{dl}\left(\frac{1}{e^n} + 1\right) \qquad \text{[F-3B}_1\text{]}$$

Similarly, Eqn [F-3C] also result in

$$F_d = \frac{F_{dl}}{e^n}(1 + e^n + (e + e^2 + e^3 + \cdots + e^n)) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_o}\right)\right]\right)$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1 - e^n)}{(1 - e)}$$

$$F_d = \frac{F_{dl}}{e^n}\left(1 + e^n + \frac{e\,(1 - e^n)}{(1 - e)}\right) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$F_d = F_{dl}\left(\frac{1}{e^n} + 1 + \frac{e\,(\frac{1}{e^n} - 1)}{(1 - e)}\right)\frac{(1 - e)e^n}{(1 - e)e^n} - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$F_d = \frac{F_{dl}}{(1 - e)e^n}((1 - e) + e^n - e^{n+1} + e\,(1 - e^n)) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$F_d = \frac{F_{dl}}{(1 - e)e^n}(1 - e + e^n - e^{n+1} + e - e^{n+1}) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$F_d = \frac{F_{dl}}{(1 - e)e^n}(1 + e^n - 2e^{n+1}) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$$F_d = \frac{F_{dl}}{(1 - e)}\left(1 + \frac{1}{e^n} - 2e\right) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \qquad [F\text{-}3C_1]$$

If we assume constant velocity of the travelling equipment, Eqn [F-3C1] satisfies Eqn [B-4] in the Luke and Juvkam model

Similarly, Eqn [F-3D] also result in

$$F_d = \frac{F_{fl}(e + e^2 + e^3 + \cdots + e^n)}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{e^n} + 1\right)$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1 - e^n)}{(1 - e)}$$

$$F_d = \frac{e\,(1 - e^n)F_{fl}}{(1 - e)\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{e^n} + 1\right) \qquad [F\text{-}3D_1]$$

Finally, Eqn [F-3E] also result in

$$F_d = \frac{F_{dl}(e + e^2 + e^3 + \cdots + e^n)}{e^n\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{e^n} + 1\right)$$

$$But, \qquad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1 - e^n)}{(1 - e)}$$

$$F_d = \frac{e\,(1-e^n)F_{dl}}{e^n(1-e)\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_0}{t_1-t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{e^n}+1\right) \quad \text{[F-3E}_1\text{]}$$

4.3.1.3 LOWERING WITH NON-UNIFORM MOVEMENT AND VARYING SHEAVE EFFICIENCY

During lowering, maximum tension occurs in the dead line (F_{dl}) while the fast line (F_{fl}) records the least tension .i.e. The tension decreases from the dead line towards the fast line *ie*. $F_{fl} \leq F_{dl}$

Considering the dead line sheave (First sheave in the crown block from the direction of the dead line anchor), Eqn (α) becomes

$$e_1 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{dl}}$$

$$\Rightarrow F_1 = e_1 F_{dl}$$

For non-rotating dead line sheave, it is assumed that there is no work done against friction and hence the efficiency of the dead line sheave is assumed to be 100% ($e_1 = 100\%$)

$$\Rightarrow F_1 = F_{dl} \quad \text{[Ꝁ-1]}$$

Similarly, the efficiency of the next sheave in the travelling block is given by,

$$e_2 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = e_2 F_1 = e_2\,(F_{dl}) = e_2\,F_{dl} \quad \text{[Ꝁ-2]}$$

Also, considering the efficiency of the next sheave in the crown block gives,

$$e_3 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = e_3 F_2 = e_3\,(e_2\,F_{dl}) = e_3 e_2\,F_{dl} = \prod_{i=2}^{3} e_i\,F_{dl} \quad \text{[Ꝁ-3]}$$

i = 2 since e_1 is 100% (ie $e_1 = 1$)

Hence, for n number of lines between the travelling blocks and the crown block, the general line tension reduction from the dead line towards the fast line is given by the relation

$$F_n = \prod_{i=2}^{n} e_i\,F_{dl} \quad \text{[Ꝁ-4]}$$

Finally, considering the efficiency (e_{fl}) of the fast line sheave in the crown block gives,

$$e_{fl} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{fl}}{F_n}$$

$$\Rightarrow F_{fl} = e_{fl}F_n = e_{fl}\left(\prod_{i=2}^{n} e_i\, F_{dl}\right) = e_{fl}\prod_{i=2}^{n} e_i\, F_{dl}$$

$$F_{fl} = e_{fl}\prod_{i=2}^{n} e_i\, F_{dl} \qquad [\text{Ꞔ-5A}]$$

$$\Rightarrow F_{dl} = \frac{F_{fl}}{e_{fl}\prod_{i=2}^{n} e_i} \qquad [\text{Ꞔ-5B}]$$

With the assumption of varying sheave efficiency and from the relationship between each of the lines with respect to the dead line Eqn [Ꞔ-4], the sum of the tension in the supporting lines become

$$\sum_1^n F_i = F_{dl}\left(1 + e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \prod_{i=2}^{n} e_i\right) \qquad [\text{Ꞔ -6}]$$

HOOK LOAD (W) RELATIONS

Substituting Eqn [Ꞔ-6] into the hook load relation during lowering (Eqn [D-3B]) gives

$$W = F_{dl}\left(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \prod_{i=2}^{n} e_i\right) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_0}{t_1-t_0}\right)\right] \qquad [\text{F-4A}]$$

Substituting the relation between the fast line tension and the dead line tension during lowering (Eqn [Ꞔ -5B]) into Eqn [F-4A] gives

$$W = \frac{F_{fl}}{e_f\prod_{i=2}^{n} e_i}\left(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \prod_{i=2}^{n} e_i\right) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_0}{t_1-t_0}\right)\right] \qquad [\text{F-4B}]$$

Alternatively, the derrick load is given by the relation

$$\mathrm{F}_d = \mathrm{F}_{fl} + \mathrm{W} + \mathrm{F}_{dl}$$

$$\Rightarrow \mathrm{W} = \mathrm{F}_d - \mathrm{F}_{fl} - \mathrm{F}_{dl} = \mathrm{F}_d - (\mathrm{F}_{fl} + \mathrm{F}_{dl}) \qquad [\text{F-4C}]$$

Substitituing the relationship between the dead line tension and that of the fast line tension (Eqn[Ꞔ-5A]) during lowering into Eqn [F-4C] gives

$$\mathrm{W} = \mathrm{F}_d - \mathrm{F}_{fl} - \mathrm{F}_{dl} = \mathrm{F}_d - \left(e_f\prod_{i=2}^{n} e_i\, F_{dl} + \mathrm{F}_{dl}\right)$$

$$W = \mathrm{F}_d - F_{dl}\left(e_f\prod_{i=2}^{n} e_i + 1\right) \qquad [\text{F-4D}]$$

Finally, Substituting Eqn [Ꞔ-6] into the hook load relation during lowering (Eqn [D-3C]) gives

$$W = \frac{F_{dl}(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \prod_{i=2}^{n} e_i)}{\left(1 - \frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \quad \text{[F-4E]}$$

Eqn [F-4A], Eqn [F-4B], Eqn [F-4C], Eqn [F-4D] and Eqn [F-4E] are the hook load (W) relations during lowering for inactive dead line sheave with non-uniform movement and varying sheave efficiency.

DERRICK LOAD (F_d) RELATIONS

From Eqn [F-4D], the derrick load is given as

$$F_d = W + F_{dl}\,(e_f \textstyle\prod_{i=2}^{n} e_i + 1) \quad \text{[F-5A]}$$

Substituting Eqn [F-4A] into Eqn [F-5A] gives

$$F_d = F_{dl}(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \textstyle\prod_{i=2}^{n} e_i) + \frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] +$$

$$F_{dl}\,(e_f \textstyle\prod_{i=2}^{n} e_i + 1)$$

$$F_d = F_{dl}\left((1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \textstyle\prod_{i=2}^{n} e_i) + (e_f \prod_{i=2}^{n} e_i + 1)\right) + \frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \quad \text{[F-5B]}$$

Similarly, substituting Eqn [F-4B] into Eqn [F-5A] gives

$$F_d = \frac{F_{fl}}{e_f \prod_{i=2}^{n} e_i}(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \textstyle\prod_{i=2}^{n} e_i) + \frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right] +$$

$$F_{dl}\,(e_f \textstyle\prod_{i=2}^{n} e_i + 1) \quad \text{[F-5C]}$$

Finally, substituting Eqn [F-4E] into Eqn [F-5A] gives

$$F_d = \frac{F_{dl}(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \prod_{i=2}^{n} e_i)}{\left(1 - \frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}\,(e_f \textstyle\prod_{i=2}^{n} e_i + 1)$$

$$F_d = F_{dl}\left(\frac{(1+e_2 + e_3e_2 + e_4e_3e_2 + \cdots + \prod_{i=2}^{n} e_i)}{\left(1 - \frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + e_f \textstyle\prod_{i=2}^{n} e_i + 1\right) \quad \text{[F-5D]}$$

Eqn [F-5A], Eqn [F-5B], Eqn [F-5C] and Eqn [F-5D] are the derrick load (F_d) relations during lowering for inactive dead line sheave with non-uniform movement and varying sheave efficiency.

4.3.1.4 Lowering with non-uniform movement and constant sheave efficiency

For simplicity, If we assume a constant sheave efficiency (e) as proposed by Luke and Juvkam i.e $e_1 = e_2 = e_3 = e_4 = \cdots = e_n = e = Constant$, Eqn [F-4A] becomes

$$W = F_{dl}(1 + e + e^2 + e^3 \ldots + e^{n-1}) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$But \quad 1 + e + e^2 + e^3 \ldots + e^n = \frac{(1 - e^n)}{(1 - e)}$$

$$W = \frac{F_{dl}(1 - e^n)}{(1 - e)} + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad [F\text{-}4A_1]$$

If we assume constant velocity of the travelling equipment, Eqn [F-4A$_1$] reduces to Eqn [B-7] as proposed by Luke and Juvkam

Similarly, Eqn [F-4B] becomes

$$W = \frac{F_{fl}}{e(e^{n-1})}(1 + e + e^2 + e^3 \ldots + e^{n-1}) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$But \quad 1 + e + e^2 + e^3 \ldots + e^n = \frac{(1 - e^n)}{(1 - e)}$$

$$W = \frac{F_{fl}(1 - e^n)}{e^n(1 - e)} + \frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad [F\text{-}4B_1]$$

In a similar vein Eqn [F-4D] also becomes,

$$W = \mathrm{F}_d - F_{dl}\,(e(e^{n-1}) + 1) = \mathrm{F}_d - F_{dl}\,(e^n + 1) \qquad [F\text{-}4D_1]$$

Finally Eqn [F-4E] also becomes,

$$W = \frac{F_{dl}(1 + e + e^2 + e^3 \ldots + e^{n-1})}{\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)}$$

$$But \quad 1 + e + e^2 + e^3 \ldots + e^n = \frac{(1 - e^n)}{(1 - e)}$$

$$W = \frac{(1 - e^n)F_{dl}}{(1 - e)\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad [F\text{-}4E_1]$$

Eqn [F-4A$_1$], Eqn [F-4B$_1$], Eqn [F-4C$_1$], Eqn [F-4D$_1$] and Eqn [F-4E$_1$] are the hook load (W) relations during lowering for inactive dead line sheave with non-uniform movement and assuming constant sheave efficiency (e) as proposed by Luke and Juvkam.

DERRICK LOAD (F_d) RELATIONS

Also considering a contant sheave efficiency (e) as proposed by Luke and Juvkam, Eqn [F-5A] becomes

$$\mathrm{F}_d = W + F_{dl}\,(e(e^{n-1}) + 1) = W + F_{dl}\,(e^n + 1) \qquad \text{[F-5A}_1\text{]}$$

Similarly Eqn [F-5B] becomes

$$\mathrm{F}_d = F_{dl}\big((1 + e + e^2 + e^3 \ldots + e^{n-1}) + (e(e^{n-1}) + 1)\big) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$But \quad 1 + e + e^2 + e^3 \ldots + e^n = \frac{(1 - e^n)}{(1 - e)}$$

$$\mathrm{F}_d = F_{dl}\left(\frac{(1 - e^n)}{(1-e)} + e^n + 1\right) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$\mathrm{F}_d = F_{dl}\frac{((1-e))}{(1-e)}\left(\frac{(1 - e^n)}{(1-e)} + e^n + 1\right) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$\mathrm{F}_d = \frac{F_{dl}}{(1-e)}\left(\frac{(1 - e^n)(1-e)}{(1-e)} + (1 - e)e^n + 1(1 - e)\right) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$\mathrm{F}_d = \frac{F_{dl}}{(1-e)}(1 - e^n + e^n - e^{n+1} + 1 - e) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$\mathrm{F}_d = \frac{F_{dl}}{(1-e)}(2 - e - e^{n+1}) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[F-5B}_1\text{]}$$

If we assume a uniform movement of the travelling equipment, Eqn [F-5B$_1$] is the same Eqn [B-8] as proposed by Luke and Juvkam-Wold.

Also assuming a constant sheave efficiency for Eqn [F-5C] gives

$$\mathrm{F}_d = \frac{F_{fl}}{e(e^{n-1})}(1 + e + e^2 + e^3 \ldots + e^{n-1}) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] + F_{dl}\,(e(e^{n-1}) + 1)$$

$$But \quad 1 + e + e^2 + e^3 \ldots + e^n = \frac{(1 - e^n)}{(1 - e)}$$

$$\mathrm{F}_d = \frac{F_{fl}(1 - e^n)}{e^n(1-e)} + F_{dl}\,(e^n + 1) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[F-5C}_1\text{]}$$

Finally, assuming a constant sheave efficiency for Eqn [F-5D] become

$$\mathrm{F}_d = F_{dl}\left(\frac{(1 + e + e^2 + e^3 \ldots + e^{n-1})}{\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + (e(e^{n-1}) + 1)\right)$$

$$But \quad e = 1 + e + e^2 + e^3 \ldots + e^n = \frac{(1 - e^n)}{(1-e)}$$

$$\mathrm{F}_d = F_{dl}\left(\frac{(1 - e^n)}{(1-e)\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + e^n + 1\right) \qquad \text{[F-5D}_1\text{]}$$

Eqn [F-5A1], Eqn [F-5B1], Eqn [F-5C1] and Eqn [F-5D1] are the derrick load (F_d) relations during lowering for inactive dead line sheave during non-uniform movement of the travelling block and assuming constant sheave efficiency (e) as proposed by Luke and Juvkam.

4.3.2 Active (rotating) dead line sheave derivation

4.3.2.1 Hoisting with non-uniform movement and varying sheave efficiency

During hoisting, maximum tension occurs in the fast line (F_{fl}), while the minimum tension occurs in the dead line (F_{dl}).i.e. The tension decreases from the fast line towards the dead line, $ie. \mathrm{F}_{fl} \geq \mathrm{F}_{dl}$

Considering the fast line sheave (First sheave in the crown block from the direction of the drum) and from Eqn (α), its efficiency e_1 is given by

$$e_1 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{fl}}$$

$$\Rightarrow F_1 = e_1 F_{fl} \qquad [\Omega\text{-}1]$$

Similarly, the efficiency (e_2) of the next sheave in the travelling block is also given by

$$e_2 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = e_2 F_1 = e_2(e_1 F_{fl}) = e_2 e_1 F_{fl} = \prod_{i=1}^{2} e_i F_{fl} \qquad [\Omega\text{-}2]$$

Also, considering the efficiency (e_3) of the next sheave in the crown block gives

$$e_3 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = e_3 F_2 = e_3(e_2 e_1 F_{fl}) = e_3 e_2 e_1 F_{fl} = \prod_{i=1}^{3} e_i F_{fl} \qquad [\Omega\text{-}3]$$

Hence, for "n" number of lines between the travelling block and the crown block, the relationship between the tension in each line and the applied fast line tension (F_{fl}) is given by

$$F_n = \prod_{i=1}^{n} e_i F_{fl} \qquad [\Omega\text{-}4]$$

Finally, considering the efficiency of the dead line sheave (e_d) in the crown block gives

$$e_d = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{dl}}{F_n}$$

$$\Rightarrow F_{dl} = e_d F_n = e_d\left(\prod_{i=1}^{n} e_i\, F_{fl}\right) = e_d \prod_{i=1}^{n} e_i\, F_{fl}$$

For active dead line sheave since it is rotating and hence imperfect transmission of tension ($i.e.\ e_d \neq 1$) $\Rightarrow F_n \neq F_{dl}$

$$F_{fl} = \frac{F_{dl}}{e_d \prod_{i=1}^{n} e_i} \qquad [\Omega\text{-}5]$$

If we assume varying sheave efficiency and from the relationship between each of the lines with respect to the fast line (Eqn [Ω-4]) the sum of the tension in the supporting lines is given by

$$\sum_1^n F_i = F_{fl}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) \qquad [\Omega\text{-}6]$$

Substituting Eqn [Ω-6] into Eqn [D-2B] gives

$$W = F_{fl}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \qquad [\Omega\text{-}7]$$

FAST LINE (F_{fl}) AND DEAD LINE TENSION (F_{dl}) RELATIONS

From Eqn [Ω-7], making F_{fl} the subject gives

$$F_{fl} = \frac{1}{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}\left(W + \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)\right) \qquad [G\text{-}1A]$$

Substituting Eqn [Ω-5] into Eqn [G-1A] gives

$$\frac{F_{dl}}{e_d \prod_{i=1}^{n} e_i} = \frac{1}{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}\left(W + \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)\right)$$

$$F_{dl} = \frac{e_d \prod_{i=1}^{n} e_i}{(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}\left(W + \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)\right) \qquad [G\text{-}1B]$$

HOOK LOAD (W) RELATIONS

From Eqn [G-1A], the hook load is given by

$$W = F_{fl}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \qquad [G\text{-}2A]$$

Similarly, From Eqn [G-1B], the hook load is given by

$$W = \frac{F_{dl}}{e_d \prod_{i=1}^{n} e_i}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \qquad [G\text{-}2B]$$

Alternatively, the derrick load (F_d) is given by the relation

$F_d = F_{fl} + W + F_{dl}$

$$\Rightarrow W = F_d - F_{fl} - F_{dl} = F_d - (F_{fl} + F_{dl}) \quad \text{[G-2C]}$$

Substitituting the relationship between the dead line and the fast line tension (Eqn [Ω-5]) during hoisting into Eqn [G-2C] gives

$$W = F_d - F_{fl} - F_{dl} = F_d - (\frac{F_{dl}}{e_d \prod_{i=1}^{n} e_i} + F_{dl})$$

$$W = F_d - F_{dl}(\frac{1}{e_d \prod_{i=1}^{n} e_i} + 1) \quad \text{[G-2D]}$$

Substituting Eqn [Ω-6] into Eqn [D-2C] gives

$$W = \frac{F_{fl}(e_1 + e_2 e_1 + e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i)}{\left(1 + \frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \quad \text{[G-2E]}$$

Finally, substituting Eqn [Ω-5] into Eqn [G-2E] gives

$$W = \frac{F_{dl}(e_1 + e_2 e_1 + e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i)}{e_d \prod_{i=1}^{n} e_i \left(1 + \frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \quad \text{[G-2F]}$$

Eqn [G-2A], Eqn [G-2B], Eqn [G-2C], Eqn [G-2D], Eqn [G-2E] and Eqn [G-2F] are the hook load (W) relations during hoisting for active dead line sheave with non-uniform movement of the travelling equipment and varying sheave efficiency.

DERRICK LOAD (F_d) RELATIONS

From Eqn [G-2D], the derrick load is given by the relation

$$F_d = W + F_{dl}(\frac{1}{e_d \prod_{i=1}^{n} e_i} + 1) \quad \text{[G-3A]}$$

Substituting Eqn [G-2A] into Eqn [G-3A] gives

$$F_d = F_{fl}(e_1 + e_2 e_1 + e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i) + F_{dl}(\frac{1}{e_d \prod_{i=1}^{n} e_i} + 1) - \left(\frac{(m_{dp} + m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \quad \text{[G-3B]}$$

Similarly substitute Eqn [G-2B] into Eqn [G-3A] relation gives

$F_d =$

$$\frac{F_{dl}}{e_d\prod_{i=1}^{n} e_i}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) + F_{dl}(\frac{1}{e_d\prod_{i=1}^{n} e_i} + 1) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)$$

$F_d =$

$$F_{dl}\left(\frac{1}{e_d\prod_{i=1}^{n} e_i}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) + \left(\frac{1}{e_d\prod_{i=1}^{n} e_i} + 1\right)\right) - \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right) \quad \text{[G-3C]}$$

Substituting Eqn [G-2E] into Eqn [G-3A] relation gives

$$F_d = \frac{F_{fl}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}{\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}(\frac{1}{e_d\prod_{i=1}^{n} e_i} + 1) \quad \text{[G-3D]}$$

Finally, substituting Eqn [G-2F] into Eqn [G-3A] relation gives

$$F_d = \frac{F_{dl}(e_1 + e_2e_1 + e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}{e_d\prod_{i=1}^{n} e_i\left(1 + \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} + F_{dl}(\frac{1}{e_d\prod_{i=1}^{n} e_i} + 1) \quad \text{[G-3E]}$$

Eqn [G-3A], Eqn [G-3B], Eqn [G-3C], Eqn [G-3D] and Eqn [G-3E] are the derrick load (F_d) relations during hoisting for active dead line sheave during non-uniform movement with varying sheave efficiency.

4.3.2.2 HOISTING WITH NON-UNIFORM MOVEMENT AND ASSUMING A CONSTANT SHEAVE EFFICIENCY

FAST LINE (F_{fl}) AND DEAD LINE TENSION (F_{dl}) RELATIONS

For simplicity, lets assume a constant sheave efficiency (e) as proposed by Luke and Juvkam i.e $e_1 = e_2 = e_3 = e_4 = \cdots = e_n = e = Constant$, Eqn [G-1A] becomes

$$F_{fl} = \frac{1}{(e + e^2 + e^3 + \cdots + e^n)}\left(W + \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)\right)$$

$$But,\ e + e^2 + e^3 + \cdots + e^n = \frac{e(1 - e^n)}{(1 - e)}$$

$$F_{fl} = \frac{(1 - e)}{e(1 - e^n)}\left(W + \left(\frac{(m_{dp} + m_{tb})}{(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)\right) \quad \text{[G-1A}_1\text{]}$$

For uniform motion, Eqn [G-1A1] reduces to Eqn [B-10] as proposed by Luke and Juvkam.

Similarly considering Eqn [G-1B]

$$F_{dl} = \frac{ee^n}{(e+e^2+e^3+\cdots+e^n)}\left(W + \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)\right)$$

$$But, \quad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$F_{dl} = \frac{e^n}{\frac{(1-e^n)}{(1-e)}}\left(W + \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)\right)$$

$$F_{dl} = \frac{e^n(1-e)}{(1-e^n)}\left(W + \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)\right) \qquad \text{[G-1B}_1\text{]}$$

HOOK LOAD (W) RELATIONS

Also considering a constant sheave efficiency (e) as proposed by Luke and Juvkam, Eqn [G-2A] become

$$W = F_{fl}(e + e^2 + e^3 + \cdots + e^n) - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$But, \; e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$W = \frac{\mathrm{e}\,(1-e^n)F_{fl}}{(1-e)} - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \qquad \text{[G-2A}_1\text{]}$$

Similarly Eqn [G-2B] become

$$W = \frac{F_{dl}}{ee^n}(e + e^2 + e^3 + \cdots + e^n) - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$But, \quad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$W = \frac{F_{dl}}{ee^n}\left(\frac{\mathrm{e}\,(1-e^n)}{(1-e)}\right) - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$W = \frac{F_{dl}(1-e^n)}{e^n(1-e)} - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \qquad \text{[G-2B}_1\text{]}$$

If we assume constant velocity of the travelling equipment Eqn [G-2B$_1$] satisfies Eqn [B-11] as proposed by Luke and Juvkam.

Eqn [G-2D] also becomes,

$$\mathrm{W} = \mathrm{F}_d - F_{dl}\left(\frac{1}{ee^n} + 1\right) = \mathrm{F}_d - F_{dl}\left(\frac{1}{e^{n+1}} + 1\right) \qquad \text{[G-2D}_1\text{]}$$

Assuming constant sheave efficiency, Eqn [G-2E] becomes

$$W = \frac{F_{fl}(e+e^2+e^3+\cdots+e^n)}{\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)}$$

$$But,\ e+e^2+e^3+\cdots+e^n\ = \frac{\text{e}\,(1-e^n)}{(1-e)}$$

$$W = \frac{\text{e}\,(1-e^n)F_{fl}}{(1-e)\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \qquad \text{[G-2E}_1\text{]}$$

Finally, assuming constant sheave efficiency, Eqn [G-2F] becomes

$$W = \frac{F_{dl}(e+e^2+e^3+\cdots+e^n)}{ee^n\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \qquad \text{[G-2F]}$$

$$But,\qquad e+e^2+e^3+\cdots+e^n\ = \frac{\text{e}\,(1-e^n)}{(1-e)}$$

$$W = \frac{(1-e^n)F_{dl}}{e^n(1-e)\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \qquad \text{[G-2F}_1\text{]}$$

Eqn [G-2A1], Eqn [G-2B1], Eqn [G-2D1], Eqn [G-2E1] and Eqn [G-2F1] are the hook load relations for an active dead line sheave during non-uniform movement of the travelling equipment when assuming constant sheave efficiency as proposed by Luke and Juvkam.

DERRICK LOAD (F_d) RELATIONS

With the assumption of a constant sheave efficiency (e), Eqn [G-3A]

$$\text{F}_d = \text{W} + F_{dl}\left(\frac{1}{ee^n}+1\right) = \text{W} + F_{dl}\left(\frac{1}{e^{n+1}}+1\right) \qquad \text{[G-3A}_1\text{]}$$

Similarly, Eqn [G-3B] becomes

$$\text{F}_d = F_{fl}(e+e^2+e^3+\cdots+e^n) + F_{dl}\left(\frac{1}{ee^n}+1\right) - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$But,\ e+e^2+e^3+\cdots+e^n\ = \frac{\text{e}\,(1-e^n)}{(1-e)}$$

$$\text{F}_d = F_{fl}\frac{\text{e}\,(1-e^n)}{(1-e)} + F_{dl}\left(\frac{1}{e^{n+1}}+1\right) - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \qquad \text{[G-3B}_1\text{]}$$

Also, considering Eqn [G-3C] gives

$$\text{F}_d = F_{dl}\left(\frac{1}{ee^n}(e+e^2+e^3+\cdots+e^n) + \left(\frac{1}{ee^n}+1\right)\right) - \left(\frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$But,\ e+e^2+e^3+\cdots+e^n\ = \frac{\text{e}\,(1-e^n)}{(1-e)}$$

$$F_d = F_{dl}\ \frac{e^{n+1}(1-e)}{e^{n+1}(1-e)}\left(\frac{(1-e^n)}{e^n(1-e)} + \left(\frac{1}{e^{n+1}} + 1\right)\right) - \left(\frac{(m_{dp} + m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$F_d = F_{dl}\ \frac{1}{e^{n+1}(1-e)}(e(1-e^n) + (1-e) + e^{n+1}(1-e)) - \left(\frac{(m_{dp} + m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$F_d = F_{dl}\ \frac{1}{e^{n+1}(1-e)}(e - e^{n+1} + 1 - e + e^{n+1} - e^{n+2})) - \left(\frac{(m_{dp} + m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)$$

$$F_d = F_{dl}\ \frac{(1-e^{n+2})}{e^{n+1}(1-e)} - \left(\frac{(m_{dp} + m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right) - \left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right) \qquad [G\text{-}3C_1]$$

Eqn [G-3D] becomes

$$F_d = \frac{F_{fl}(e+e^2+e^3+\cdots+e^n)}{\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{ee^n} + 1\right)$$

$$But,\ e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1-e^n)}{(1-e)}$$

$$F_d = \frac{e\,(1-e^n)F_{fl}}{(1-e)\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{e^{n+1}} + 1\right) \qquad [G\text{-}3D_1]$$

Finally, Eqn [G-3E] becomes

$$F_d = \frac{F_{dl}(e+e^2+e^3+\cdots+e^n)}{ee^n\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{ee^n} + 1\right)$$

$$But,\quad e + e^2 + e^3 + \cdots + e^n = \frac{e\,(1-e^n)}{(1-e)}$$

$$F_d = \frac{F_{dl}\,(1-e^n)}{e^n(1-e)\left(1+\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}\left(\frac{1}{e^{n+1}} + 1\right) \qquad [G\text{-}3E_1]$$

If we assume constant velocity of the travelling equipment Eqn [G-3C$_1$] and Eqn [G-3E$_1$] reduces to Eqn [B-12] as suggested by Luke and Juvkam

4.3.2.3 LOWERING WITH NON-UNIFORM MOVEMENT AND VARYING SHEAVE EFFICIENCY

During lowering, maximum tension occurs in the dead line (F_{dl}) while the fast line (F_{fl}) records the least tension .i.e. The tension decreases from the dead line towards the fast line, $ie.\ F_{fl} < F_{dl}$

Considering the dead line sheave (First sheave in the crown block from the direction of the dead line anchor), Eqn (α) becomes

$$e_1 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{dl}}$$

$\Rightarrow F_1 = e_1 F_{dl}$

For rotating dead line sheave, it is assumed that there is work done against friction and hence the efficiency of the dead line sheave (e_d) is assumed to be less than100% ($e_1 = e_d \neq 100\% \neq 1$)

$\Rightarrow F_1 = e_1 F_{dl}$ [€-1]

Similarly, the efficiency of the next sheave in the travelling block is given by,

$$e_2 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$\Rightarrow F_2 = e_2 F_1 = e_2\,(e_1 F_{dl}) = e_2\, e_1 F_{dl} = \prod_{i=1}^{2} e_i F_{dl}$ [€-2]

Also, considering the efficiency of the next sheave in the crown block gives,

$$e_3 = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$\Rightarrow F_3 = e_3 F_2 = e_3\,(e_2\, e_1 F_{dl}) = e_3 e_2\, e_1 F_{dl} = \prod_{i=1}^{3} e_i F_{dl}$ [€-3]

Hence, for "n" number of lines between the travelling blocks and the crown block, the relationship between each line tension and the dead line is given by

$F_n = \prod_{i=1}^{n} e_i\, F_{dl}$ [€-4]

Finally, considering the efficiency (e_{fl}) of the fast line sheave in the crown block gives,

$$e_{fl} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{fl}}{F_n}$$

$F_{fl} = e_{fl} F_n = e_{fl}\,(\prod_{i=1}^{n} e_i\, F_{dl}) = e_{fl} \prod_{i=1}^{n} e_i\, F_{dl}$

$\Rightarrow F_{fl} = e_{fl} \prod_{i=1}^{n} e_i\, F_{dl}$ [€-5A]

$F_{dl} = \frac{F_{fl}}{e_{fl} \prod_{i=1}^{n} e_i}$ [€-5B]

If we assume varying sheave efficiency and from the relationship between the tension in each of the lines with respect to the dead line (Eqn [€-4]) the sum of the tension in the supporting lines is given by

$\Rightarrow \sum_1^n F_i = F_{dl}(e_1 + e_2 e_1 + e_3 e_2 e_1 + e_4 e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i)$ [€-6]

HOOK LOAD (W) RELATIONS

Substituting Eqn [€-6] into Eqn [D-3B] gives

$$W = F_{dl}(e_1 + e_2e_1 + e_3e_2e_1 + e_4e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

[G-4A]

Substituting the fast line tension and the dead line tension relation during lowering (Eqn[€-5B]) into Eqn [G-4A] gives

$$W = \frac{F_{fl}}{e_f \prod_{i=1}^{n} e_i}(e_1 + e_2e_1 + e_3e_2e_1 + e_4e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

[G-4B]

Alternatively, the derrick load is given by the relation

$$\mathrm{F}_d = \mathrm{F}_{fl} + \mathrm{W} + \mathrm{F}_{dl}$$

$$W = \mathrm{F}_d - \mathrm{F}_{fl} - \mathrm{F}_{dl} = \mathrm{F}_d - (\mathrm{F}_{fl} + \mathrm{F}_{dl}) \qquad \text{[G-4C]}$$

Substituting Eqn[€-5A] into Eqn [G-4C] gives,

$$\mathrm{W} = \mathrm{F}_d - \mathrm{F}_{fl} - \mathrm{F}_{dl} = \mathrm{F}_d - (e_f \prod_{i=1}^{n} e_i F_{dl} + \mathrm{F}_{dl})$$

$$\mathrm{W} = \mathrm{F}_d - F_{dl}(e_f \prod_{i=1}^{n} e_i + 1) \qquad \text{[G-4D]}$$

Finally, substituting Eqn [€-5] gives into Eqn [D-3C] gives

$$W = \frac{F_{dl}(e_1 + e_2e_1 + e_3e_2e_1 + e_4e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i)}{\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]\right)} \qquad \text{[G-4E]}$$

Eqn [G-4A], Eqn [G-4B], Eqn [G-4C], Eqn [G-4D] and Eqn [G-4E] are the hook load (W) relations during lowering for an active dead line sheave

DERRICK LOAD (F_d) RELATIONS

From Eqn [G-4D], the derrick load (F_d)is given by

$$\mathrm{F}_d = \mathrm{W} + F_{dl}(e_f \prod_{i=1}^{n} e_i + 1) \qquad \text{[G-5A]}$$

Substituting Eqn [G-4A] into Eqn [G-5A] gives

$$\mathrm{F}_d = F_{dl}(e_1 + e_2e_1 + e_3e_2e_1 + e_4e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) + F_{dl}(e_f \prod_{i=1}^{n} e_i + 1) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right]$$

$$\mathrm{F}_d = F_{dl}((e_1 + e_2e_1 + e_3e_2e_1 + e_4e_3e_2e_1 + \cdots + \prod_{i=1}^{n} e_i) + e_f \prod_{i=1}^{n} e_i + 1) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_o}{t_1 - t_0}\right)\right] \qquad \text{[G-5B]}$$

Similarly substituting Eqn [G-4B] into Eqn [G-5A] gives

$$\mathrm{F}_d = \frac{F_{fl}}{e_f \prod_{i=1}^{n} e_i}(e_1 + e_2 e_1 + e_3 e_2 e_1 + e_4 e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i) + F_{dl}(e_f \prod_{i=1}^{n} e_i + 1) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_0}{t_1 - t_0}\right)\right] \qquad \text{[G-5C]}$$

Finally, substituting Eqn [G-4E] into Eqn [G-5A] gives

$$\mathrm{F}_d = \frac{F_{dl}(e_1 + e_2 e_1 + e_3 e_2 e_1 + e_4 e_3 e_2 e_1 + \cdots + \prod_{i=1}^{n} e_i)}{\left(1 - \frac{1}{g(t_2 - t_1)}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_0}{t_1 - t_0}\right)\right]\right)} + F_{dl}(e_f \prod_{i=1}^{n} e_i + 1) \qquad \text{[G-5D]}$$

Eqn [G-5A], Eqn [G-5B], Eqn [G-5C] and Eqn [G-5D] are the derrick load (F_d) relations during lowering for active dead line sheave with non-uniform movement and varying sheave efficiency.

4.3.2.4 LOWERING WITH NON-UNIFORM MOVEMENT AND ASSUMING A CONSTANT SHEAVE EFFICIENCY

HOOK LOAD (W) RELATIONS

For simplicity, If we assume a constant sheave efficiency (e) as proposed by Luke and Juvkam i.e $e_1 = e_2 = e_3 = e_4 = \cdots = e_n = e = Constant$, Eqn [G-4A] becomes

$$W = F_{dl}(e + e^2 + e^3 + \cdots + e^n) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_0}{t_1 - t_0}\right)\right]$$

$$But, \quad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$W = \frac{\mathrm{e}\,(1 - e^n) F_{dl}}{(1 - e)} + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_0}{t_1 - t_0}\right)\right] \qquad \text{[G-4A}_1\text{]}$$

If we assume a uniform movement of the travelling equipment, Eqn [G-4A₁] satisfies Eqn[B-15] as suggested by Luke and Juvkam.

Similarly eqn [G-4B] also becomes

$$W = \frac{F_{fl}}{e e^n}(e + e^2 + e^3 + \cdots + e^n) + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_0}{t_1 - t_0}\right)\right]$$

$$But, \quad e + e^2 + e^3 + \cdots + e^n = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$W = \frac{F_{fl}(1 - e^n)}{e^n(1 - e)} + \frac{(m_{dp} + m_{tb})}{t_2 - t_1}\left[\left(\frac{s_2 - s_1}{t_2 - t_1}\right) - \left(\frac{s_1 - s_0}{t_1 - t_0}\right)\right] \qquad \text{[G-4B}_1\text{]}$$

Also Eqn [G-4D] becomes

$$\mathrm{W} = \mathrm{F}_d - F_{dl}(e e^n + 1) = \mathrm{F}_d - F_{dl}(e^{n+1} + 1) \qquad \text{[G-4D}_1\text{]}$$

Finally, Eqn [G-4E] becomes

$$W = \frac{F_{dl}(e+e^2+e^3+\cdots+e^n)}{\left(1-\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)}$$

$$But,\ e+e^2+e^3+\cdots+e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$W = \frac{\mathrm{e}\,(1-e^n)F_{dl}}{(1-e)\left(1-\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} \qquad \text{[G-4E}_1\text{]}$$

DERRICK LOAD (F_d) RELATIONS

Assuming a constant sheave, Eqn [G-5A] becomes

$$\mathrm{F}_d = \mathrm{W} + F_{dl}(ee^n+1) = \mathrm{W} + F_{dl}(e^{n+1}+1) \qquad \text{[G-5A}_1\text{]}$$

Similarly Eqn [G-5B] becomes,

$$\mathrm{F}_d = F_{dl}((e+e^2+e^3+\cdots+e^n)+(ee^n+1)) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]$$

$$But,\ e+e^2+e^3+\cdots+e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$\mathrm{F}_d = F_{dl}\left(\frac{\mathrm{e}\,(1-e^n)}{(1-e)}+e^{n+1}+1\right) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]$$

$$\mathrm{F}_d = F_{dl}\frac{1}{(1-e)}(\mathrm{e}\,(1-e^n)+e^{n+1}(1-e)+(1-e)) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]$$

$$\mathrm{F}_d = F_{dl}\frac{1}{(1-e)}(\mathrm{e}-e^{n+1}+e^{n+1}-e^{n+2}+1-e) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]$$

$$\mathrm{F}_d = \frac{F_{dl}}{(1-e)}(1-e^{n+2}) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \qquad \text{[G-5B}_1\text{]}$$

Also, considering a constant sheave efficiency for Eqn [G-5C] gives

$$\mathrm{F}_d = \frac{F_{fl}}{ee^n}(e+e^2+e^3+\cdots+e^n) + F_{dl}(ee^n+1) + \frac{(m_{dp}+m_{tb})}{t_2-t_1}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]$$

$$But,\quad e+e^2+e^3+\cdots+e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$\mathrm{F}_d = \frac{F_{fl}\,(1-e^n)}{e^n(1-e)} + F_{dl}(e^{n+1}+1) + \frac{(m_{dp}+m_{tb})}{(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right] \qquad \text{[G-5C}_1\text{]}$$

Finally, considering a constant sheave efficiency for Eqn [G-5D] gives

$$\mathrm{F}_d = \frac{F_{dl}(e+e^2+e^3+\cdots+e^n)}{\left(1-\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}(ee^n+1)$$

$$But,\quad e+e^2+e^3+\cdots+e^n = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$$F_d = \frac{e\,(1-e^n)F_{dl}}{(1-e)\left(1-\frac{1}{g(t_2-t_1)}\left[\left(\frac{s_2-s_1}{t_2-t_1}\right)-\left(\frac{s_1-s_o}{t_1-t_0}\right)\right]\right)} + F_{dl}(e^{n+1}+1) \qquad \text{[G-5D}_1\text{]}$$

If we assume uniform movement of the travelling equipment, Eqn [G-5B$_1$] and Eqn [G-5D$_1$] satisfies Eqn [B-16] as proposed by Luke and Juvkam.

4.4 Extension of Cayeux et-al hook load prediction model to account for the effect of acceleration due to non-uniform movement of the travelling equipment

Unlike the previously discussed hook load prediction models, Cayeux et-al hook load prediction model [4] is based on the coefficient of friction (μ) at the sheave axle instead of utilizing the efficiency (e) of each sheave. Both Coulomb kinetic friction coefficient (μ_a) and stribeck friction (μ_s) model were used. The advantage of using the stribeck friction model is that it compensates for the transition from the static to dynamic coefficient of friction and vice-versa [4].

With the exception of the weight $\left(\overrightarrow{F_W}\right)$ of the sheaves which always acts downwards, the tensions in the line (T_A or T_B) and the centrifugal force $\left(\overrightarrow{F_C}\right)$ always acts in the opposite direction to each other for the sheaves in the crown block and that of the travelling block.

In addition, the sheaves in the travelling block rotate in the same direction as that of the draw work but opposite to that of the crown block sheaves. Using the right hand rule for determining the angular acceleration vector, we will assume clock-wise rotation of each sheave as negative ("-") while anti-clock-wise rotation as positive ("+") as illustrated in the schematic below

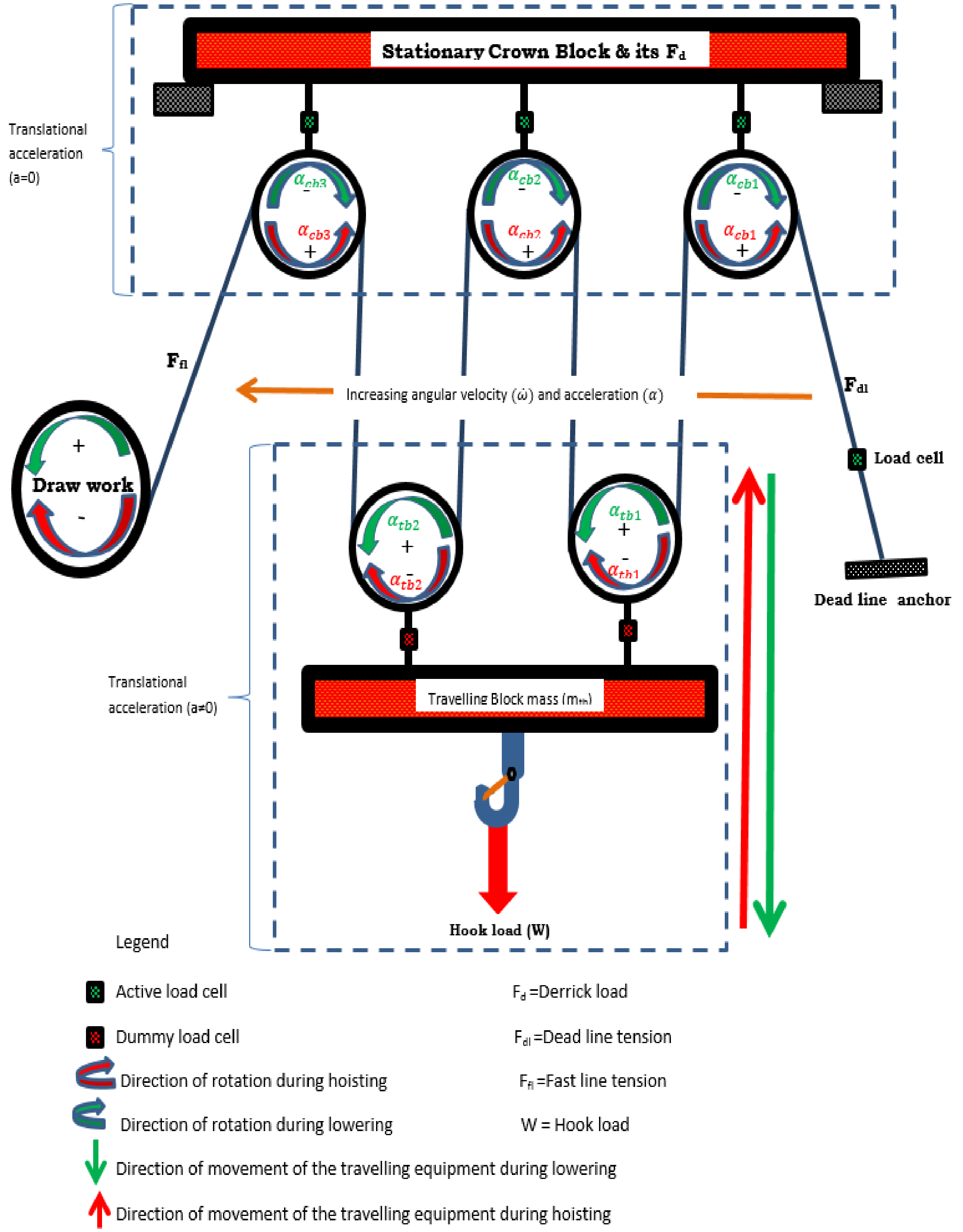

Figure 24: shows the direction of rotation of the sheave for both the crown block sheaves and the travelling block sheaves during hoisting and lowering

We will consider the forces and torque on each sheave for both the crown block and the travelling block.

4.4.1 Forces on each sheave

Below is an illustration of the forces on each sheave for a typical block and tackle hoisting system.

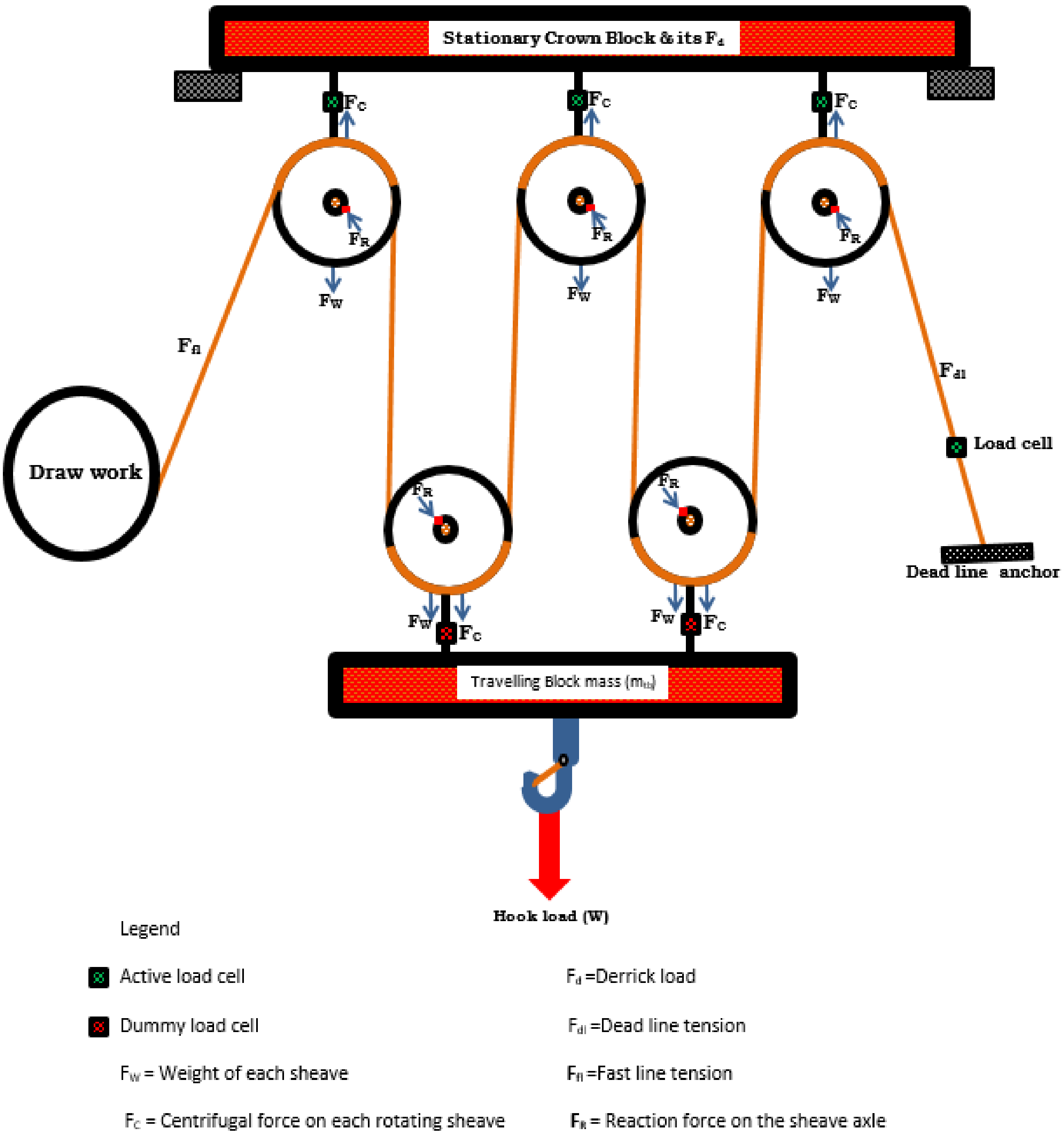

Figure 25: Shows the centrifugal force (F_C) , weight of the each sheave (F_W) and the reaction force (F_R) on the block and tackle hoisting system

Based on Coulomb friction model(μ_a), the magnitude of the torque due to the friction at the sheave axle is given by $\|\overrightarrow{M_f}\| = \mu_a r_a \|\overrightarrow{F_R}\|$

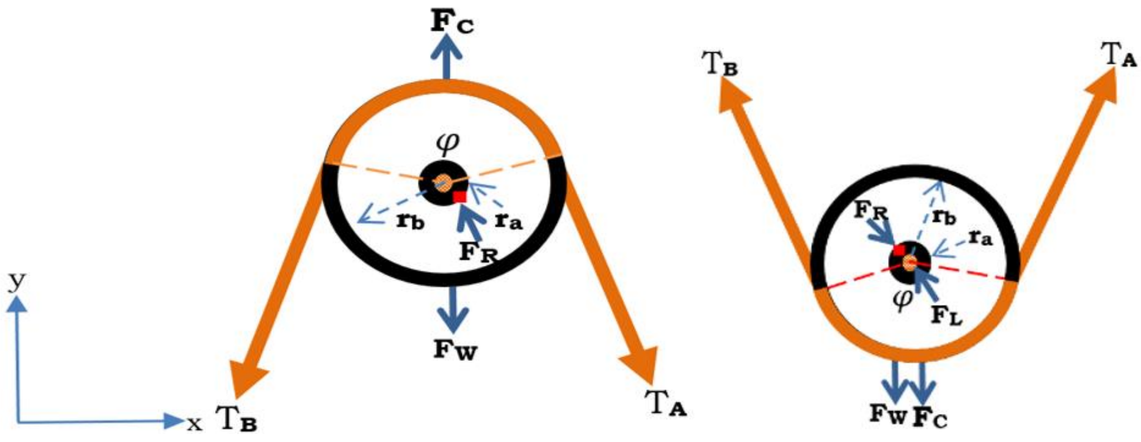

Figure 26: Shows the applied load(F_L) and its corresponding reaction force (F_R) on a crown block and a travelling block sheaves respectively

The net force on each sheave during non-uniform movement of the travelling equipment is given by

$$\overrightarrow{F_L} + \overrightarrow{F_R} = m_p\vec{a}$$

$$\Rightarrow \overrightarrow{F_R} = m_p\vec{a} - \overrightarrow{F_L} = m_p\vec{a} + (-\overrightarrow{F_L}) \qquad \text{[H-1A]}$$

From figure (26), the applied load ($\overrightarrow{F_L}$) on each sheave is given by the relation

$$\overrightarrow{F_L} = \overrightarrow{F_W} + \overrightarrow{F_C} + \overrightarrow{T_B} + \overrightarrow{T_A} \qquad \text{[β-1A]}$$

The magnitude of the applied load $\|\overrightarrow{F_L}\|$ becomes

$$\|-\overrightarrow{F_L}\| = \|\overrightarrow{F_L}\| = \sqrt{(F_{LH})^2 + (F_{LV})^2} \qquad \text{[β-1B]}$$

Where F_{LH} and F_{LV} are the horizontal and the vertical components of the line tensions respectively.

Substituting Eqn [β-1B] into Eqn [H-1A] gives the magnitude of the reaction force ($\|\overrightarrow{F_R}\|$) as

$$\|\overrightarrow{F_R}\| = m_p\vec{a} + \|\overrightarrow{F_L}\| = \pm m_p a + \sqrt{(F_{LH})^2 + (F_{LV})^2} \qquad \text{[H-1B]}$$

4.4.2 Torque on each sheave

There is no contribution to the net torque by either the weight of each sheave $(\overrightarrow{F_W})$ Or the centrifugal force $(\overrightarrow{F_C})$ since their line of action is through the center of the sheave.

The direction of the net torque on each sheave is always in the direction of the maximum line tension and hence for $T_A > T_B$, the net torque on the sheave becomes,

$$r_b\,(T_A - T_B) \pm \mu_a r_a \|\overrightarrow{F_R}\| = I(\pm\,\alpha) = \pm\,I\alpha \qquad \text{[H-2A]}$$

Since the crown block sheaves rotate in the opposite direction to the direction of rotation of the travelling block sheaves, this is accounted for in the angular acceleration $(\pm\,\alpha)$. Based on the right hand rule to determine the direction of the angular acceleration(α), we assume anti-clockwise rotation as positive ("+") while clockwise is negative ("-"). In addition, the direction of the reaction force $(\overrightarrow{F_R})$ differ for the crown block sheaves and that of the travelling block.

Substituting Eqn [H-1B] into Eqn [H-2A] gives

$$r_b\,(T_A - T_B) \pm \mu_a r_a\left(\pm m_p a + \sqrt{(F_{LH})^2 + (F_{LV})^2}\right) = \pm\,I\alpha \qquad \text{[H-2B]}$$

Eqn [H-2B] is the generalized torque relation for both the crown block sheaves and that of the travelling block sheaves during both hoisting and lowering.

4.4.3 Forces and torque the crown block sheave

The crown block sheaves undergo only rotational motion but not translational motion since the crown block is stationary. During non-uniform movement of the travelling equipment, there will be no effect of the translational acceleration $(i.e.\ \vec{a} = 0)$ on the crown block sheaves' reaction forces. The crown block sheaves experiences angular acceleration during the non-uniform movement (i.e. $\alpha \neq 0$) as illustrated in figure (25).

The generalized net torque relation, Eqn [H-2B] reduces to

$$r_b\,(T_A - T_B) \pm \mu_a r_a\left(\sqrt{(F_{LH})^2 + (F_{LV})^2}\right) = \pm\,I\alpha \qquad \text{[H-3A]}$$

From figure (26), the horizontal (F_{LH}) and the vertical (F_{LV}) component of the line tension of the crown block sheave is given by

$$F_{LH} = (T_A - T_B)\cos(\frac{\varphi}{2}) \quad [\beta\text{-}2A_1]$$

$$F_{LV} = -m_p g + \sin(\frac{\varphi}{2})(2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} - T_A - T_B) \quad [\beta\text{-}2B_1]$$

where $\dot{\omega}_{rs}$ is the angular velocity of each rotating sheave

For simplicity, let's assume that the angle (φ) subtended by $\overrightarrow{T_A}$ and $\overrightarrow{T_B}$ as illustrated in figure (26) is 180^0 ($i.e.\ \varphi = 180^0$). Hence, Eqn [β-2A1] and Eqn [β-2B1] becomes

$$\Rightarrow F_{LH} = (T_A - T_B)\cos(\frac{180}{2}) = (T_A - T_B)\cos(90) = 0 \quad [\beta\text{-}2A_2]$$

$$F_{LV} = -m_p g + \sin\left(\frac{180}{2}\right)\left(2\bar{\lambda}_m {r_b}^2 \dot{\omega}^2{}_{rs} - T_A - T_B\right)$$

$$F_{LV} = -m_p g + \sin(90)(2\bar{\lambda}_m {r_b}^2 \dot{\omega}^2{}_{rs} - T_A - T_B)$$

$$F_{LV} = -m_p g + 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} - T_A - T_B \quad [\beta\text{-}2B_2]$$

Substitute Eqn [β-2A2] and [β-2B2] into Eqn [H-3A] gives

$$r_b\,(T_A - T_B) \pm \mu_a r_a(\sqrt{(0)^2 + \left(-m_p g + 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} - T_A - T_B\right)^2}) = \pm I\alpha$$

$$r_b\,(T_A - T_B) \pm \mu_a r_a\left(-m_p g + 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} - T_A - T_B\right) = \pm I\alpha \quad [H\text{-}3B]$$

During hoisting the line tension reduces from the fast line (F_{fl}) towards the dead line (F_{dl}) i.e. ($F_{fl} > F_1 > F_2 > F_3 > F_4 > F_5 > \cdots > F_{dl}\ or\ T_A > T_B$)

$$r_b\,(T_A - T_B) - \mu_a r_a\left(m_p g - 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B\right) = +I\alpha \quad [H\text{-}3B_1]$$

Similarly, for the non-rotating deadline sheave in the crown block with no angular velocity or acceleration($i.e.\ \dot{\omega} = o\ , \alpha = 0$), Eqn [H-3B1] becomes

$$r_b\,(T_A - T_B) - \mu_a r_a(m_p g + T_A + T_B)) = 0 \quad [H\text{-}3B_2]$$

During lowering since the line tension reduces from dead line (F_{dl}) toward the fast line(F_{fl}). ($i.e.\ F_{dl} > F_1 > F_2 > F_3 > F_4 > F_5 > \cdots > F_{fl}\ or\ T_B > T_A$). The friction moment at the sheave bearing always oppose the direction of the net torque, and hence the net torque relation during lowering for both the rotating and non-rotating crown block sheave is given by

$$r_b\,(T_B - T_A) + \mu_a r_a\left(-m_p g + 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} - T_B - T_A\right) = -I\alpha \quad [H\text{-}3C_1]$$

$$r_b\,(T_B - T_A) + \mu_a r_a(-m_p g - T_B - T_A) = 0 \quad [H\text{-}3C_2]$$

4.4.4 Forces and torque on the travelling block sheave

Unlike the crown block sheave, the travelling block sheaves undergo both rotational and translational motion since they are mobile. Hence during non-uniform movement of the travelling equipment, the travelling block sheaves will experience both translational acceleration($i.e. \, a \neq 0$) and rotational acceleration ($\alpha \neq 0$) effects. In addition, all the sheaves in the travelling block rotates unlike the crown block in which the dead line sheave is non-rotating (inactive).

From the generalised net torque relation, the net toque for the travelling block sheave becomes

$$r_b\,(T_A - T_B) \pm \mu_a r_a(\pm m_p a + \sqrt{(F_{LH})^2 + (F_{LV})^2}) = \pm\, I\alpha \qquad \text{[H-4A}_1\text{]}$$

From figure (26), the horizontal (F_{LH}) and vertical (F_{LV}) component of the line tension for the travelling block sheave is given as

$$F_{LH} = (T_A - T_B)\cos(\tfrac{\varphi}{2}) \qquad [\beta\text{-3A}_1]$$

$$F_{LV} = -m_p g + \sin(\tfrac{\varphi}{2})(-2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B) \qquad [\beta\text{-3B}_1]$$

where $\dot{\omega}_{rs}$ is the angular velocity of the rotating sheaves

For simplicity, let's assume that the angle subtended by $\overrightarrow{T_A}$ and $\overrightarrow{T_B}$ is 180^0 ($i.e.\ \varphi = 180^0$) and hence Eqn [β-3A1] and Eqn [β-3B1] becomes

$$\Rightarrow F_{LH} = (T_A - T_B)\cos(\tfrac{180}{2}) = (T_A - T_B)\cos(\,90\,) = 0 \qquad [\beta\text{-3A}_2]$$

$$F_{LV} = -m_p g + \sin\left(\frac{180}{2}\right)\left(-2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B\right)$$

$$F_{LV} = -m_p g + \sin(\,90)(-2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B)$$

$$F_{LV} = -m_p g - \, 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B \qquad [\beta\text{-3B}_2]$$

Substituting Eqn [β-3A2] and [β-3B2] into Eqn [H-4A1] gives

$$r_b\,(T_A - T_B) \pm \mu_a r_a(\pm m_p a + \sqrt{(0)^2 + \left(-m_p g - \, 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B\right)^2}) = \pm\, I\alpha$$

$$r_b\,(T_A - T_B) \pm \mu_a r_a(\pm m_p a - m_p g - \, 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B) = \pm\, I\alpha \qquad \text{[H-4A}_2\text{]}$$

During hoisting, the translational acceleration is positive ($+a$) and the friction moment also opposes the direction of the net torque. Eqn [H-4A2] becomes

$$r_b\,(T_A - T_B) - \mu_a r_a\big(m_p a - m_p g - 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B\big) = -\,I\alpha$$

$$r_b\,(T_A - T_B) + \mu_a r_a\big(-m_p a + m_p g + 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} - T_A - T_B\big) = -\,I\alpha \qquad \text{[H-4B]}$$

Eqn [H-4B] is the net torque on each of the travelling block sheave during hoisting ($F_{fl} > F_1 > F_2 > F_3 > F_4 > F_5 > \cdots > F_{dl}$ or $T_A > T_B$).

Similarly since the friction moment always impose the direction of the net torque and the translational acceleration is negative direction ($-a$) during lowering $\big(F_{dl} > F_1 > F_2 > F_3 > F_4 > F_5 > \cdots > F_{fl}$ or $T_B > T_A\big)$ Eqn [H-4A2] becomes

$$r_b\,(T_B - T_A) - \mu_a r_a\big(-m_p a - m_p g - 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{rs} + T_A + T_B\big) = +\,I\alpha \qquad \text{[H-4C]}$$

4.4.5 Hoisting

During hoisting, the line tension decreases from the fast line (F_{fl}) towards the dead line(F_{dl}). The fast line is always in motion and hence its line tension cannot be measured directly. Hence, the static dead line tension (F_{dl}) will be used as our reference line tension during hoisting instead of the fast line tension(F_{fl}). During hoisting, the line tension increases from the dead line (F_{dl}) toward the fast line ($i.e. F_{dl} < F_1 < F_2 < F_3 < F_4 < \cdots < F_{fl}$) as illustrated in the figure (27) below.

The net torque on the dead line sheave (sheave A) in the crown block is given by Eqn [H-3B1] as (See Appendix C for derivation)

$For\ F_1 > F_{dl}$

$$r_b\,(F_1 - F_{dl}) - \mu_a r_a\big(m_p g - 2\bar{\lambda}_m {r_b}^2\ \dot{\omega}^2{}_{cb1} + F_1 + F_{dl}\big) = +\,I\alpha_{cb1}$$

$$F_1(r_b - \mu_a r_a) - m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2\, \mu_a r_a\, \dot{\omega}^2{}_{cb1} - F_{dl}(\mu_a r_a + r_b) = I\alpha_{cb1}$$

$$F_1 = \frac{-1}{(\mu_a r_a - r_b)}\big(I\alpha_{cb1} + F_{dl}(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2\, \mu_a r_a\, \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a\big) \qquad \text{[γ-1A]}$$

For simplicity, let $x = \mu_a r_a - r_b$ and $y = \mu_a r_a + r_b$

Substituting the x and y into the Eqn [γ-1A] gives

$$F_1 = \frac{-1}{x}\big(I\alpha_{cb1} + F_{dl}y - 2\bar{\lambda}_m {r_b}^2\, \mu_a r_a\, \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a\big) \qquad \text{[γ-1B]}$$

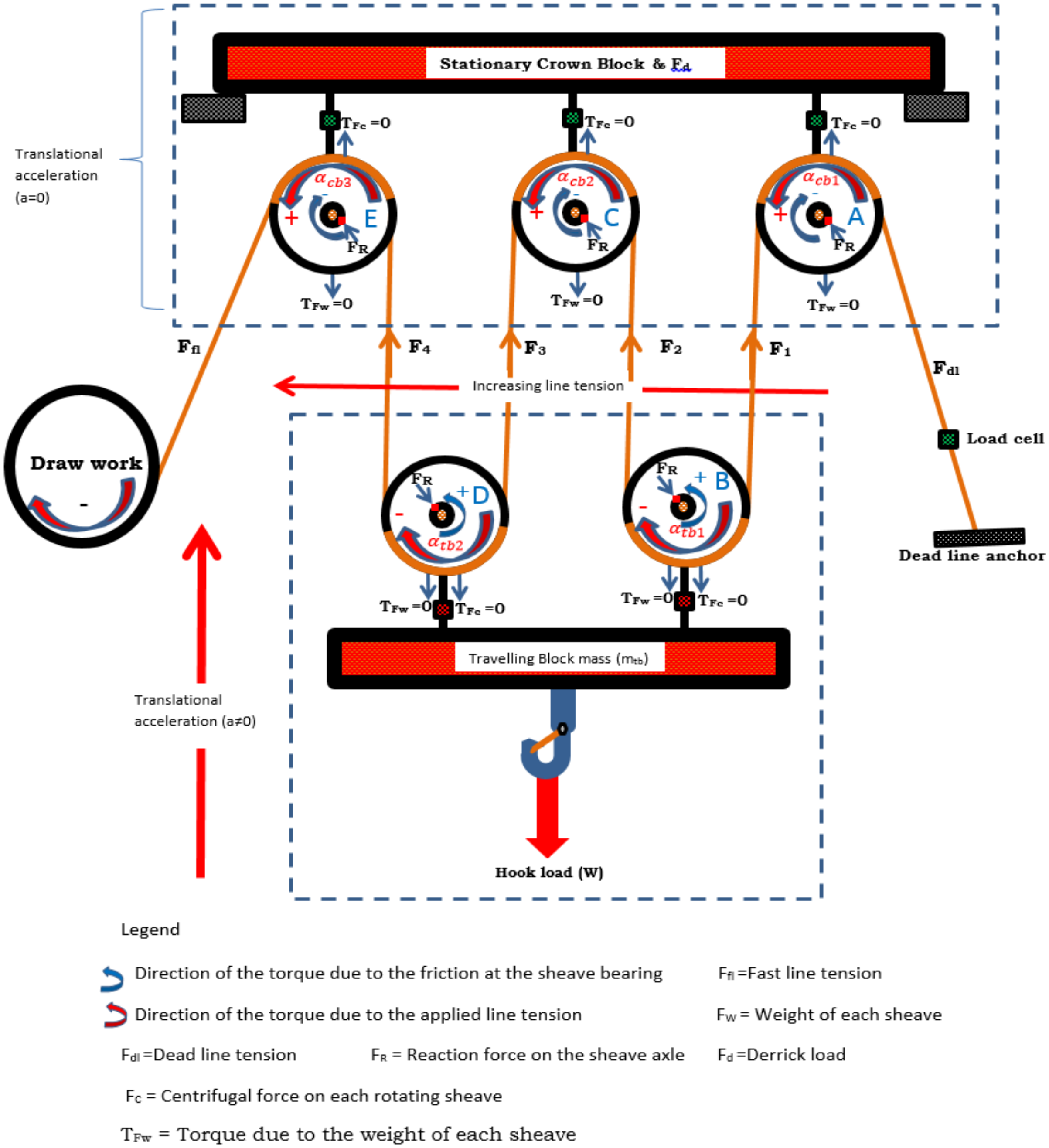

Figure 27: The forces and torques on both the crown block and the travelling block sheaves during hoisting

Similarly considering the net torque in the next sheave (sheave B) in the travelling block, Eqn [H-4B] becomes

$For\ F_2 > F_1$

$$r_b\,(F_2 - F_1) + \mu_a r_a\left(-m_p a + m_p g + 2\bar{\lambda}_m r_b^{\,2}\,\dot{\omega}^2{}_{tb1} - F_2 - F_1\right) = -\,I\alpha_{tb1}$$

$$F_2(r_b - \mu_a r_a) - m_p a \mu_a r_a + m_p g \mu_a r_a + 2\bar{\lambda}_m r_b^{\,2} \mu_a r_a\,\dot{\omega}^2{}_{tb1} - F_1(\mu_a r_a + r_b) = -I\alpha_{tb1}$$

$$F_2 = \frac{1}{(\mu_a r_a - r_b)}\left(I\alpha_{tb1} - m_p a\mu_a r_a - F_1(\mu_a r_a + r_b) + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\, \dot{\omega}^2{}_{tb1} + m_p g\mu_a r_a\right) \quad [\gamma\text{-2A}]$$

For simplicity, let $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$ and substituting x and y into Eqn [γ-2A] gives

$$F_2 = \frac{1}{x}\left(I\alpha_{tb1} - m_p a\mu_a r_a - F_1 y + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\, \dot{\omega}^2{}_{tb1} + m_p g\mu_a r_a\right) \quad [\gamma\text{-2B}]$$

Substituting Eqn [γ-1B] into Eqn [γ-2B] and multiplying through the resulting equation by $\frac{x}{x}$ gives

$$F_2 = \frac{1}{x^2}\left(-m_p a\mu_a r_a x + I(\alpha_{cb1} y + \alpha_{tb1} x) + F_{dl} y^2 + 2\bar{\lambda}_m {r_b}^2\, \mu_a r_a\, (-\dot{\omega}^2{}_{cb1} y + \dot{\omega}^2{}_{tb1} x) + m_p g\mu_a r_a(y + x)\right) \quad [\gamma\text{-2C}]$$

Considering the net torque on the next sheave (sheave C) in the crown block, Eqn [H-3B₁] becomes

For $F_3 > F_2$

$$r_b\,(F_3 - F_2) - \mu_a r_a\left(m_p g - 2\bar{\lambda}_m {r_b}^2\ \dot{\omega}^2{}_{cb2} + F_3 + F_2\right) = + I\alpha_{cb2}$$

$$F_3(r_b - \mu_a r_a) - m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb2} - F_2(\mu_a r_a + r_b) = I\alpha_{cb2}$$

$$F_3 = \frac{-1}{(\mu_a r_a - r_b)}\left(I\alpha_{cb2} + F_2(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right) \quad [\gamma\text{-3A}]$$

For simplicity, substituting $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$ into Eqn [γ-3A] gives

$$F_3 = \frac{-1}{x}\left(I\alpha_{cb2} + F_2 y - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right) \quad [\gamma\text{-3B}]$$

Substitute Eqn [γ-2C] into Eqn [γ-3B] and multiplying through the resulting equation by $\frac{x^2}{x^2}$ gives

$$F_3 = \frac{-1}{x^3}\left(-m_p a\mu_a r_a xy + I(\alpha_{cb1} y^2 + \alpha_{tb1} xy + \alpha_{cb2} x^2) + F_{dl} y^3 + 2\bar{\lambda}_m {r_b}^2\, \mu_a r_a\, (-\dot{\omega}^2{}_{cb1} y^2 + \dot{\omega}^2{}_{tb1} xy - \dot{\omega}^2{}_{cb2} x^2) + m_p g\mu_a r_a(y^2 + xy + x^2)\right) \quad [\gamma\text{-3C}]$$

Similarly considering the net torque in the next sheave (sheave D) in the travelling block, Eqn [H-4B] becomes

For $F_4 > F_3$

$$r_b\,(F_4 - F_3) + \mu_a r_a\left(-m_p a + m_p g + 2\bar{\lambda}_m {r_b}^2\, \dot{\omega}^2{}_{tb2} - F_4 - F_3\right) = - I\alpha_{tb2}$$

$$F_4(r_b - \mu_a r_a) - m_p a\mu_a r_a + m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{tb2} - F_3(\mu_a r_a + r_b) = - I\alpha_{tb2}$$

$$F_4 = \frac{1}{(\mu_a r_a - r_b)}\left(I\alpha_{tb2} - m_p a\mu_a r_a - F_3(\mu_a r_a + r_b) + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a\right) \quad [\gamma\text{-4A}]$$

For simplicity, substituting $x = \mu_a r_a - r_b$ and $y = \mu_a r_a + r_b$ into Eqn [γ-4A] gives

$$F_4 = \frac{1}{x}\left(I\alpha_{tb2} - m_p a \mu_a r_a - F_3 y + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \ \dot{\omega}^2{}_{tb2} + m_p g \mu_a r_a\right) \qquad [γ\text{-}4B]$$

Substitute Eqn [γ-3C] into Eqn [γ-4B] and multiplying through the resulting equation by $\frac{x^3}{x^3}$ gives

$$F_4 = \frac{1}{x^4}\left(-m_p a \mu_a r_a (xy^2 + x^3) + I(\alpha_{cb1} y^3 + \alpha_{tb1} x y^2 + \alpha_{cb2} x^2 y + \alpha_{tb2} x^3) + F_{dl} y^4 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a (-\dot{\omega}^2{}_{cb1} y^3 + \dot{\omega}^2{}_{tb1} x y^2 - \dot{\omega}^2{}_{cb2} x^2 y + \dot{\omega}^2{}_{tb2} x^3) + m_p g \mu_a r_a (y^3 + xy^2 + x^2 y + x^3)\right) \qquad [γ\text{-}4C]$$

Hence for "n" number of lines between the crown block and the travelling block, the general relation for the increase in the line tension from the dead line (F_{dl})towards the fast line (F_{fl}) is given by

$$F_n = \left(\frac{-1}{x}\right)^n \left(-m_p a \mu_a r_a \left(\sum_{k=1}^{r} x^{2k-1} y^{n-2k}\right) + I\left(\sum_{k=0}^{q=n-1} \alpha_{(1+k)} y^{q-k} x^k\right) + F_{dl} y^n + m_p g \mu_a r_a \left(\sum_{k=0}^{q=n-1} y^{q-k} x^k\right) + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left\{\sum_{k=0}^{q=n-1} (-)^{k+1} \dot{\omega}^2{}_{(k+1)} y^{q-k} x^k\right\}\right) \qquad [γ\text{-}5A]$$

where

q = n-1 (i.e. the number of supporting lines minus 1)

r = the number of travelling block sheaves between the dead line and the line of interest.

$\dot{\omega}_{(k+1)}$ *and* $\alpha_{(1+k)}$ represent the numbering of the angular velocity and the angular acceleration of each sheave from the dead line sheave in the crown block through the travelling block sheave as illustrated in figure (27)

4.4.6 Hook load (W) during hoisting with non-uniform movement of the travelling equipment

During hoisting, the sum of the upward forces exceed that of the downward forces as illustrated in figure (22)

$$\Rightarrow \sum_{i=1}^{n} F_i - F_{Down} = m_T\, a \qquad [I\text{-}1A]$$

$$But\ \ F_{Down} = W = (m_{dp} + m_{tb})g = m_T\, g \qquad [δ\text{-}1A]$$

$$m_T = m_{dp} + m_{tb} = \frac{W}{g} \qquad [δ\text{-}1B]$$

Substituting Eqn [δ-1A] into Eqn [I-1A] gives

$$\sum_{i=1}^{n} F_i - W = (m_{dp} + m_{tb})a$$

$$W = \sum_{i=1}^{n} F_i - (m_{dp} + m_{tb})a \qquad [I\text{-}1B]$$

Alternatively, substituting Eqn [δ-1B] into Eqn [I-1B] gives

$$W = \sum_{i=1}^{n} F_i - \frac{W}{g}a$$

$$W + \frac{W}{g}a = \sum_{i=1}^{n} F_i$$

$$W\left(1 + \frac{a}{g}\right) = W\left(\frac{g+a}{g}\right) = \sum_{i=1}^{n} F_i$$

$$\Rightarrow W = \left(\frac{g}{g+a}\right)\sum_{i=1}^{n} F_i \qquad [I\text{-}1C]$$

4.4.7 The relationship between the travelling block velocity (V_{tb}) and the velocity of the line opposite the dead line (V_{dlo})

The velocity of the travelling block (V_{tb}) is assumed to be the same as the velocity of the line opposite the dead line(V_{dlo}), $i.e. V_{tb} = V_{dlo}$ as illustrated in the figure (28) below

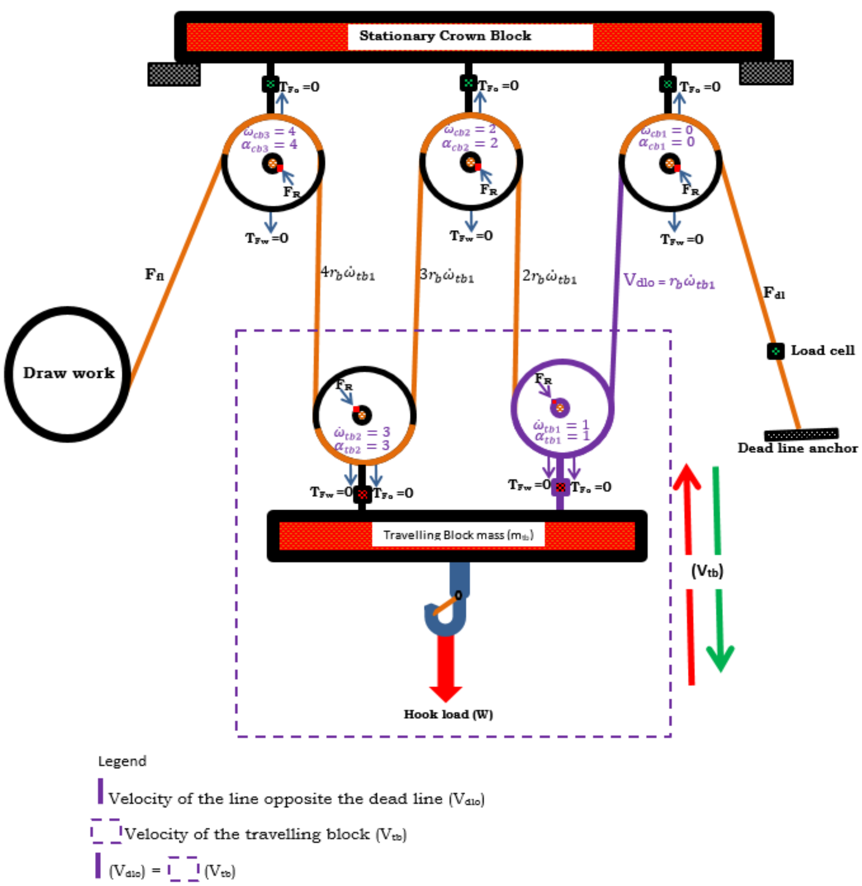

Figure 28: Shows the relationship between the travelling block velocity $(V_{tb})$and the angular velocity of the first sheave $(\dot{\omega}_{tb1})$ in the travelling block connected by the line opposite the dead line

The relationship between the linear velocity of the line opposite the dead line(V_{dlo}), and its corresponding angular velocity for the first rotating sheave in the travelling block is given by

$$\dot{\omega}_{tb1} = \frac{V_{dlo}}{r_b} = \frac{V_{tb}}{r_b} \qquad [\Theta\text{-1}]$$

Similarly, the angular acceleration (α_{tb1}) of the first rotating sheave in the travelling block becomes

$$\alpha_{tb1} = \frac{\partial \dot{\omega}_{tb1}}{\partial t} = \frac{\partial(\frac{V_{tb}}{r_b})}{\partial t} = \frac{1}{r_b}\frac{\partial V_{tb}}{\partial t} = \frac{a}{r_b}$$

$$\alpha_{tb1} = \frac{a}{r_b} \qquad [\Theta\text{-2}]$$

From figure (28) above, the relationship between the angular velocity ($\dot{\omega}$) of all the rotating sheaves relative to that of the first sheave in the travelling block ($\dot{\omega}_{tb1}$) that is connected by the line opposite the dead line is given by

$$\alpha_{cb1} = 0\alpha_{tb1} \quad \& \quad \dot{\omega}_{cb1} = 0\dot{\omega}_{tb1} \qquad [\theta\text{-1A}]$$

$$\alpha_{tb1} = \alpha_{tb1} \quad \& \quad \dot{\omega}_{tb1} = \dot{\omega}_{tb1} \qquad [\theta\text{-1B}]$$

$$\alpha_{cb2} = 2\alpha_{tb1} \quad \& \quad \dot{\omega}_{cb2} = 2\dot{\omega}_{tb1} \qquad [\theta\text{-1C}]$$

$$\alpha_{tb2} = 3\alpha_{tb1} \quad \& \quad \dot{\omega}_{tb2} = 3\dot{\omega}_{tb1} \qquad [\theta\text{-1D}]$$

$$\alpha_{cb3} = 4\alpha_{tb1} \quad \& \quad \dot{\omega}_{cb3} = 4\dot{\omega}_{tb1} \qquad [\theta\text{-1E}]$$

Substituting Eqn [Θ-1A] into Eqn [γ-1B] gives the value of F_1 as

$$F_1 = \frac{-1}{x}\left(F_{dl}y + m_p g\mu_a r_a\right) \qquad [\gamma\text{-1B}_1]$$

Similarly, substituting Eqn [Θ-1A] and Eqn [Θ-1B] into Eqn [γ-2C] gives the value of F_2 as

$$F_2 = \frac{1}{x^2}\left(-m_p a\mu_a r_a x + I\alpha_{tb1}(x) + F_{dl}y^2 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}(x) + m_p g\mu_a r_a(y + x)\right)$$

[γ-2C1]

Also, substituting Eqn [Θ-1A], Eqn [Θ-1B] and Eqn [Θ-1C] into Eqn [γ-3C] gives the value of F_3 as

$$F_3 = \frac{-1}{x^3}\left(-m_p a\mu_a r_a xy + I(\alpha_{tb1}xy + 2\alpha_{tb1}x^2) + F_{dl}y^3 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a (\dot{\omega}^2{}_{tb1}xy - (2\dot{\omega}_{tb1})^2 x^2) + m_p g\mu_a r_a(y^2 + xy + x^2)\right)$$

$$F_3 = \frac{-1}{x^3}\left(-m_p a\mu_a r_a xy + I\alpha_{tb1}(xy + 2x^2) + F_{dl}y^3 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}(xy - 4x^2) + m_p g\mu_a r_a(y^2 + xy + x^2)\right) \qquad [\gamma\text{-3C}_1]$$

Finally, substituting Eqn [ϴ-1A], Eqn [ϴ-1B], Eqn [ϴ-1C] and Eqn [ϴ-1D] into Eqn [γ-4C] gives the value of F_4 as

$$F_4 = \frac{1}{x^4}\left(-m_p a \mu_a r_a (xy^2 + x^3) + I(\alpha_{tb1} xy^2 + 2\alpha_{tb1} x^2 y + 3\alpha_{tb1} x^3) + F_{dl} y^4 + 2\bar{\lambda}_m r_b{}^2\, \mu_a r_a\, (\dot{\omega}^2{}_{tb1} xy^2 - (2\dot{\omega}_{tb1})^2 x^2 y + (3\dot{\omega}_{tb1})^2 x^3) + m_p g \mu_a r_a (y^3 + xy^2 + x^2 y + x^3)\right)$$

$$F_4 = \frac{1}{x^4}\left(-m_p a \mu_a r_a (xy^2 + x^3) + I\alpha_{tb1}(xy^2 + 2x^2 y + 3x^3) + F_{dl} y^4 + 2\bar{\lambda}_m r_b{}^2\, \mu_a r_a \dot{\omega}^2{}_{tb1}\, (xy^2 - 4x^2 y + 9x^3) + m_p g \mu_a r_a (y^3 + xy^2 + x^2 y + x^3)\right) \quad [γ\text{-}4C_1]$$

By substituting Eqn [ϴ-1] and Eqn [ϴ-2] into Eqn [γ-1B₁], Eqn [γ-2C₁], Eqn [γ-3C₁] and Eqn [γ-4C₁] gives

$$F_1 = \frac{-1}{x}\left(F_{dl} y + m_p g \mu_a r_a\right) \quad [γ\text{-}1B_2]$$

$$F_2 = \frac{1}{x^2}\left(-m_p a \mu_a r_a x + I\left(\frac{a}{r_b}\right)x + F_{dl} y^2 + 2\bar{\lambda}_m r_b{}^2\, \mu_a r_a \left(\frac{V_{tb}{}^2}{r_b{}^2}\right)x + m_p g \mu_a r_a (y + x)\right) \quad [γ\text{-}2C_2]$$

$$F_3 = \frac{-1}{x^3}\left(-m_p a \mu_a r_a xy + I\left(\frac{a}{r_b}\right)(xy + 2x^2) + F_{dl} y^3 + 2\bar{\lambda}_m r_b{}^2\, \mu_a r_a \left(\frac{V_{tb}{}^2}{r_b{}^2}\right)(xy - 4x^2) + m_p g \mu_a r_a (y^2 + xy + x^2)\right) \quad [γ\text{-}3C_2]$$

$$F_4 = \frac{1}{x^4}\left(-m_p a \mu_a r_a (xy^2 + x^3) + I\left(\frac{a}{r_b}\right)(xy^2 + 2x^2 y + 3x^3) + F_{dl} y^4 + 2\bar{\lambda}_m r_b{}^2\, \mu_a r_a \left(\frac{V_{tb}{}^2}{r_b{}^2}\right)(xy^2 - 4x^2 y + 9x^3) + m_p g \mu_a r_a (y^3 + xy^2 + x^2 y + x^3)\right) \quad [γ\text{-}4C_2]$$

Hence for "n" number of lines between the crown block and the travelling block, the general increase in the line tension from the dead line (F_{dl}) towards the fast line (F_{fl}) during hoisting becomes

$F_n =$

$$\left(\frac{-1}{x}\right)^n \Bigg(-m_p a\mu_a r_a\left(\sum_{k=1}^{r} x^{2k-1}\, y^{n-2k}\right) + \frac{Ia}{r_b}\left(\sum_{k=0}^{q=n-1} k y^{q-k} x^k\right) + F_{dl} y^n +$$

$$2\bar{\lambda}_m\, \mu_a r_a {V_{tb}}^2 \left(\sum_{k=0}^{q=n-1} (-1)^{k+1} (k^2 y^{q-k} x^k)\right) + m_p g\mu_a r_a\left(\sum_{k=0}^{q=n-1} y^{q-k} x^k\right)\Bigg) \qquad [\gamma\text{-5B}]$$

But $I = \frac{1}{2} m_p\left({R_1}^2 + {R_2}^2\right)$

where R_1 and R_2 are the inner and outer radii of the each sheave. In this thesis, we assume that R_1 and R_2 are constant for each sheave.

Substituting the relation for I into Eqn [γ-5B]

$$F_n = \left(\frac{-1}{x}\right)^n \Bigg(a\left(-m_p\mu_a r_a\left(\sum_{k=1}^{r} x^{2k-1}\, y^{n-2k}\right) + \frac{m_p({R_1}^2+{R_2}^2)}{2r_b}\left(\sum_{k=0}^{q=n-1} k y^{q-k} x^k\right)\right) + F_{dl} y^n +$$

$${V_{tb}}^2 \left(2\bar{\lambda}_m\, \mu_a r_a\right)\left(\sum_{k=0}^{q=n-1} (-1)^{k+1} (k^2 y^{q-k} x^k)\right) + m_p g\mu_a r_a\left(\sum_{k=0}^{q=n-1} y^{q-k} x^k\right)\Bigg) \qquad [\gamma\text{-5C}]$$

During uniform movement of the travelling block, the translational acceleration is zero (i.e. a = 0). Hence, Eqn [γ-5C] becomes

$F_n =$

$$\left(\frac{-1}{x}\right)^n \left(F_{dl} y^n + {V_{tb}}^2 \left(2\bar{\lambda}_m\, \mu_a r_a\right)\left(\sum_{k=0}^{q=n-1} (-1)^{k+1} (k^2 y^{q-k} x^k)\right) + m_p g\mu_a r_a\left(\sum_{k=0}^{q=n-1} y^{q-k} x^k\right)\right)$$

[γ-5D]

For simplicity let

$A = -m_p a\mu_a r_a$ = The torque as a result of the acceleration effect on each of the travelling block sheave's reaction force, during non-uniform movement of the travelling equipment

$B = I\left(\frac{a}{r_b}\right) = \frac{1}{2} m_p\left({R_1}^2 + {R_2}^2\right)\frac{a}{r_b} = \frac{m_p a\left({R_1}^2+{R_2}^2\right)}{2r_b}$ = The torque due to the angular acceleration on each of the rotating sheave, during non-uniform movement of the travelling equipment

$C = 2\bar{\lambda}_m {r_b}^2\, \mu_a r_a \left(\frac{{V_{tb}}^2}{{r_b}^2}\right) = 2\bar{\lambda}_m\, \mu_a r_a {V_{tb}}^2$ = The torque due to the centrifugal force on each rotating sheave

$D = m_p g\mu_a r_a$= The torque due to the weight of each sheave

Substituting these relations A, B, C and D into Eqn [γ-1B2], Eqn [γ-2C2], Eqn [γ-3C2] and Eqn [γ-4C2] results in

$$F_1 = \frac{-F_{dl}y}{x} + \frac{-D}{x} \qquad [\gamma\text{-}1B_3]$$

Similarly F_2 becomes

$$F_2 = \frac{1}{x^2}(Ax + Bx + F_{dl}y^2 + Cx + D(y + x))$$

$$F_2 = \frac{A}{x} + \frac{B}{x} + \frac{F_{dl}y^2}{x^2} + \frac{C}{x} + \left(\frac{Dy}{x^2} + \frac{D}{x}\right) \qquad [\gamma\text{-}2C_3]$$

Also F_3 becomes

$$F_3 = \frac{-1}{x^3}(Axy + B(xy + 2x^2) + F_{dl}y^3 + C(xy - 4x^2) + D(y^2 + xy + x^2))$$

$$F_3 = \frac{-Ay}{x^2} + \left(\frac{-By}{x^2} + \frac{-2B}{x}\right) + \frac{-F_{dl}y^3}{x^3} + \left(\frac{-Cy}{x^2} + \frac{4C}{x}\right) + \left(\frac{-Dy^2}{x^3} + \frac{-Dy}{x^2} + \frac{-D}{x}\right) \qquad [\gamma\text{-}3C_3]$$

Finally F_4 becomes

$$F_4 = \frac{1}{x^4}(A(xy^2 + x^3) + B(xy^2 + 2x^2y + 3x^3) + F_{dl}y^4 + C(xy^2 - 4x^2y + 9x^3) + D(y^3 + xy^2 + x^2y + x^3))$$

$$F_4 = \left(\frac{Ay^2}{x^3} + \frac{A}{x}\right) + \left(\frac{By^2}{x^3} + \frac{2By}{x^2} + \frac{3B}{x}\right) + \frac{F_{dl}y^4}{x^4} + \left(\frac{Cy^2}{x^3} - \frac{4Cy}{x^2} + \frac{9C}{x}\right) + \left(\frac{Dy^3}{x^4} + \frac{Dy^2}{x^3} + \frac{Dy}{x^2} + \frac{D}{x}\right)$$
[γ- 4C3]

SUM OF THE FORCES IN THE SUPPORTING LINES

For simplicity, let us assume the number of supporting lines is four (n = 4)

$$\Rightarrow \sum_{i=1}^{n=4} F_i = F_1 + F_2 + F_3 + F_4$$

Also since addition is commutative, the sum of the forces will be performed sheave-wise (sheave by sheave) and also term by term (A, B, C, D and F_{dl}) bases.

TORQUE DUE TO THE TRANSLATIONAL ACCELERATION EFFECT ON EACH OF THE TRAVELLING BLOCK SHEAVE'S REACTION FORCE (A)

The total torque (A_{Total}) as a result of the acceleration effect on each of the travelling block sheave's reaction force, during non-uniform movement of the travelling equipment is given by

$$A_{Total} = A_{tb1} + A_{tb2}$$

$$A_{Total} = \left(\frac{A}{x}\right) + \left(\frac{-Ay}{x^2}\right) + \left(\frac{Ay^2}{x^3} + \frac{A}{x}\right)$$

The total torque contribution from the first rotating sheave in the travelling block (A_{tb1}) is given by

$$A_{tb1} = \frac{A}{x} + \frac{-Ay}{x^2} + \frac{Ay^2}{x^3} = \frac{A}{x}\left(1 + \frac{-y}{x} + \frac{y^2}{x^2}\right) = \frac{A}{x}GS$$

$$But \quad GS = 1 + \frac{-y}{x} + \frac{y^2}{x^2} = \frac{1\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)}{1-\frac{-y}{x}} = \frac{\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)}{\frac{(x+y)}{x}} = \frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)$$

$$\Rightarrow A_{tb1} = \frac{A}{x}\frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) = \frac{A}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) \qquad [α\text{-}1A]$$

Similarly, the total torque contribution from the second rotating travelling block sheave (A_{tb2}) is becomes

$$A_{tb2} = \frac{A}{x}$$

$$A_{tb2} = \frac{A}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right) \qquad [α\text{- }1B]$$

From Eqn [α-1A] and Eqn [α-1B], A_{Total} becomes

$$A_{Total} = A_{tb1} + A_{tb2} = \frac{A}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) + \frac{A}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right)$$

$$= \frac{A}{(x+y)}\left(\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) + \left(1 - \left(\frac{-y}{x}\right)^{n-3}\right)\right)$$

$$A_{Total} = \frac{A}{(x+y)}\left(\sum_{k=0}^{r-1}\left(1 - \left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) \qquad [α\text{- }2]$$

Where “r” is the number of rotating sheave in the travelling block for “n” number of supporting lines $\left(i.e.\ r = \frac{n}{2}\right)$

TORQUE DUE TO THE ANGULAR ACCELERATION ON EACH OF THE ROTATING SHEAVE (B)

The total torque (B_{TOTAL}) from each of the rotating sheave during non-uniform movement of the travelling equipment is given as

$$B_{TOTAL} = B_{total\_cb1} + B_{total\_tb1} + B_{total\_cb2} + B_{total\_tb2}$$

For non-rotating dead line sheave the angular acceleration is zero $\left(i.ie\ B_{total_{cb1}} = 0\right)$

$$\Rightarrow B_{TOTAL} = B_{total\_tb1} + B_{total\_cb2} + B_{total\_tb2}$$

$$\Rightarrow B_{TOTAL} = \left(\frac{B}{x}\right) + \left(\frac{-By}{x^2} + \frac{-2B}{x}\right) + \left(\frac{By^2}{x^3} + \frac{2By}{x^2} + \frac{3B}{x}\right)$$

Considering the torque due to the total angular acceleration from the first sheave in the travelling block (B_{total_tb1}) gives

$$B_{total\_tb1} = \frac{B}{x} + \frac{-By}{x^2} + \frac{By^2}{x^3} = \frac{B}{x}\left(1 + \frac{-y}{x} + \frac{y^2}{x^2}\right) = \frac{B}{x}GS$$

$$But \quad GS = 1 + \frac{-y}{x} + \frac{y^2}{x^2} = \frac{1\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)}{1 - \frac{-y}{x}} = \frac{\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)}{\frac{(x+y)}{x}} = \frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)$$

$$\Rightarrow B_{total\_tb1} = \frac{B}{x}\frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) = \frac{B}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) \qquad [\alpha\text{- 3A}]$$

Similarly, considering the torque due to the total angular acceleration from the second sheave in the crown block (B_{total_cb2}) gives

$$B_{total\_cb2} = \frac{-2B}{x} + \frac{2By}{x^2} = \frac{2B}{x}\left(-1 + \frac{y}{x}\right) = \frac{2B}{x}GS$$

$$But \quad GS = -1 + \frac{y}{x} = \frac{-1\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)}{1 - \frac{-y}{x}} = \frac{-\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)}{\frac{(x+y)}{x}} = \frac{-x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)$$

$$\Rightarrow B_{total\_cb2} = \frac{2B}{x}\frac{-x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) = \frac{-2B}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) \qquad [\alpha\text{- 3B}]$$

Finally, considering the torque due to the total angular acceleration resulting from the second sheave in the travelling block (B_{total_tb2}) becomes

$$B_{total\_tb2} = \frac{3B}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right) \qquad [\alpha\text{- 3C}]$$

From Eqn [α-3A], Eqn [α-3B] and Eqn [α-3C], B_{TOTAL} becomes

$$\Rightarrow B_{TOTAL} = \frac{B}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) + \frac{-2B}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) + \frac{3B}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right)$$

$$\Rightarrow B_{TOTAL} = \frac{B}{(x+y)}\left(\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) - 2\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) + 3\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right)\right)$$

$$\Rightarrow B_{TOTAL} = \frac{B}{(x+y)}\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1 - \left(\frac{-y}{x}\right)^{n-(k+1)}\right) \qquad [\alpha\text{- 4}]$$

TORQUE DUE TO THE CENTRIFUGAL FORCE ON EACH OF THE ROTATING SHEAVE (C)

The total contribution due to the centrifugal force on each of the rotating sheave is given by

$C_{TOTAL} = C_{total\_cb1} + C_{total\_tb1} + C_{total\_cb2} + C_{total\_tb2}$

But for the non-rotating dead line sheave(C_{total_cb1}), its angular velocity is zero $(i.e.\, \dot{\omega}_{tb1} = 0)$n and hence no centrifugal force contribution to the total torque $(i.e.\, C_{total\_cb1} = 0)$

$\Rightarrow C_{TOTAL} = C_{total\_tb1} + C_{total\_cb2} + C_{total\_tb2}$

$$C_{TOTAL} = \left(\frac{C}{x}\right) + \left(\frac{-Cy}{x^2} + \frac{4C}{x}\right) + \left(\frac{Cy^2}{x^3} - \frac{4Cy}{x^2} + \frac{9C}{x}\right)$$

Considering the total contribution to the torque by the first sheave in the travelling block (C_{total_tb1}) gives

$$C_{total\_tb1} = \frac{C}{x} + \frac{-Cy}{x^2} + \frac{Cy^2}{x^3} = \frac{C}{x}\left(1 + \frac{-y}{x} + \frac{y^2}{x^2}\right) = \frac{C}{x}GS$$

$$But \quad GS = 1 + \frac{-y}{x} + \frac{y^2}{x^2} = \frac{1\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)}{1 - \frac{-y}{x}} = \frac{\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)}{\frac{(x+y)}{x}} = \frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right)$$

$$C_{total\_cb1} = \frac{C}{x}\frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) = \frac{C}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) \qquad [\alpha\text{- 5A}]$$

Similarly, considering the total contribution to the torque by the second sheave in the crown block (C_{total_cb2}) gives

$$C_{total\_cb2} = \frac{4C}{x} + \frac{-4Cy}{x^2} = \frac{4C}{x}\left(1 + \frac{-y}{x}\right) = \frac{4C}{x}GS$$

$$But \quad GS = 1 + \frac{-y}{x} = \frac{1\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)}{1 - \frac{-y}{x}} = \frac{\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)}{\frac{(x+y)}{x}} = \frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)$$

$$\Rightarrow C_{total\_cb2} = \frac{4C}{x}\frac{x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) = \frac{4C}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) \qquad [\alpha\text{- 5B}]$$

Finally, considering the total contribution to the torque by the second sheave in the travelling block (C_{total_tb2}) becomes

$$\Rightarrow C_{total\_tb2} = \frac{9C}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right) \qquad [\alpha\text{- 5C}]$$

From Eqn [α-5A], Eqn [α-5B] and Eqn [α-5C], the total contribution to the torque (C_{TOTAL}) becomes

$$\Rightarrow C_{TOTAL} = \frac{C}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)+\frac{4C}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n-2}\right)+\frac{9C}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n-3}\right)$$

$$C_{TOTAL} = \frac{C}{(x+y)}\left(\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)+4\left(1-\left(\frac{-y}{x}\right)^{n-2}\right)+9\left(1-\left(\frac{-y}{x}\right)^{n-3}\right)\right)$$

$$\Rightarrow C_{TOTAL} = \frac{C}{(x+y)}\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) \quad [\text{a- }6]$$

TORQUE DUE TO THE WEIGHT OF EACH SHEAVE (D)

The total contribution from the weight of each sheave on the torque (D_{TOTAL}) is given by

$$D_{TOTAL} = D_{total\_cb1} + D_{total\_tb1} + D_{total\_cb2} + D_{total\_tb2}$$

$$\Rightarrow D_{TOTAL} = \left(\frac{-D}{x}\right)+\left(\frac{Dy}{x^2}+\frac{D}{x}\right)+\left(\frac{-Dy^2}{x^3}+\frac{-Dy}{x^2}+\frac{-D}{x}\right)+\left(\frac{Dy^3}{x^4}+\frac{Dy^2}{x^3}+\frac{Dy}{x^2}+\frac{D}{x}\right)$$

Considering the total contribution from the weight of the first sheave in the crown block (D_{total_cb1}) to the total torque gives

$$D_{total\_cb1} = \frac{-D}{x}+\frac{Dy}{x^2}+\frac{-Dy^2}{x^3}+\frac{Dy^3}{x^4} = \frac{D}{x}\left(-1+\frac{y}{x}+\frac{-y^2}{x^2}+\frac{y^3}{x^3}\right) = \frac{D}{x}GS$$

$$But \quad GS = -1+\frac{y}{x}+\frac{-y^2}{x^2}+\frac{y^3}{x^3} = \frac{-1\left(1-\left(\frac{-y}{x}\right)^n\right)}{1-\frac{-y}{x}} = \frac{-\left(1-\left(\frac{-y}{x}\right)^n\right)}{\frac{(x+y)}{x}} = \frac{-x}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^n\right)$$

$$\Rightarrow D_{total\_cb1} = \frac{D}{x}\frac{-x}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^n\right) = \frac{-D}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^n\right) \quad [\text{a- }7\text{A}]$$

Similarly, considering the total contribution from the weight of the first sheave in the travelling block (D_{total_tb1}) to the total torque gives

$$D_{total\_tb1} = \frac{D}{x}+\frac{-Dy}{x^2}+\frac{Dy^2}{x^3} = \frac{D}{x}\left(1+\frac{-y}{x}+\frac{y^2}{x^2}\right) = \frac{D}{x}GS$$

$$But \quad GS = 1+\frac{-y}{x}+\frac{y^2}{x^2} = \frac{1\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)}{1-\frac{-y}{x}} = \frac{\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)}{\frac{(x+y)}{x}} = \frac{x}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n-1}\right)$$

$$\Rightarrow D_{total\_tb1} = \frac{D}{x}\frac{x}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n-1}\right) = \frac{D}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n-1}\right) \quad [\text{a- }7\text{B}]$$

Also considering the total contribution from the weight of the second sheave in the crown block (D_{total_cb2}) to the total torque becomes

$$D_{total\_cb2} = \frac{-D}{x} + \frac{Dy}{x^2} = \frac{D}{x}\left(-1 + \frac{y}{x}\right) = \frac{D}{x} GS$$

$$But \quad GS = -1 + \frac{y}{x} = \frac{-1\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)}{1 - \frac{-y}{x}} = \frac{-\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)}{\frac{(x+y)}{x}} = \frac{-x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right)$$

$$\Rightarrow D_{total\_cb2} = \frac{D}{x}\frac{-x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) = \frac{-D}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) \qquad [\alpha\text{- 7C}]$$

Finally considering the total contribution from the weight of the second sheave in the travelling block (D_{total_tb2}) to the total torque becomes

$$\Rightarrow D_{total\_tb2} = \frac{D}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right) \qquad [\alpha\text{- 7D}]$$

From Eqn [α-7A], Eqn [α-7B], Eqn [α-7C] and Eqn [α-7D], the total contribution from the weight of each sheave on the torque (D_{TOTAL}) is given by

$$\Rightarrow D_{TOTAL} = \frac{-D}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n}\right) + \frac{D}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) + \frac{-D}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) + \frac{D}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n-3}\right)$$

$$D_{TOTAL} = \frac{D}{(x+y)}\left(-\left(1 - \left(\frac{-y}{x}\right)^{n}\right) + \left(1 - \left(\frac{-y}{x}\right)^{n-1}\right) - \left(1 - \left(\frac{-y}{x}\right)^{n-2}\right) + \left(1 - \left(\frac{-y}{x}\right)^{n-3}\right)\right)$$

$$\Rightarrow D_{TOTAL} = \frac{D}{(x+y)}\sum_{k=0}^{n-1}(-1)^{k+1}\left(1 - \left(\frac{-y}{x}\right)^{n-k}\right) \qquad [\alpha\text{- 8}]$$

TORQUE DUE TO THE DEAD LINE CONTRIBUTION

The total contribution from the dead line tension on the total torque is given as

$$F_{dl_{TOTAL}} = \frac{-F_{dl}y}{x} + \frac{F_{dl}y^2}{x^2} + \frac{-F_{dl}y^3}{x^3} + \frac{F_{dl}y^4}{x^4} = \frac{F_{dl}y}{x}\left(-1 + \frac{y}{x} + \frac{-y^2}{x^2} + \frac{y^3}{x^3}\right) = \frac{F_{dl}y}{x} GS$$

$$But \quad GS = -1 + \frac{y}{x} + \frac{-y^2}{x^2} + \frac{y^3}{x^3} = \frac{-1\left(1 - \left(\frac{-y}{x}\right)^{n}\right)}{1 - \frac{-y}{x}} = \frac{-\left(1 - \left(\frac{-y}{x}\right)^{n}\right)}{\frac{(x+y)}{x}} = \frac{-x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n}\right)$$

$$\Rightarrow F_{dl_{TOTAL}} = \frac{F_{dl}y}{x}\frac{-x}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n}\right) = \frac{-F_{dl}y}{(x+y)}\left(1 - \left(\frac{-y}{x}\right)^{n}\right) \qquad [\alpha\text{- 9}]$$

From Eqn [α-2], Eqn [α-4], Eqn [α-6], Eqn [α-8], and Eqn [α-9], the total tensions in the line supporting the hook load becomes

$$\Rightarrow \sum_{i=1}^{n} F_i = F_1 + F_2 + F_3 + F_4 = A_{TOTAL} + B_{TOTAL} + C_{TOTAL} + D_{TOTAL} + F_{dl_{TOTAL}}$$

$$\sum_{i=1}^{n} F_i = \frac{A}{(x+y)}\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + \frac{B}{(x+y)}\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) + \frac{C}{(x+y)}\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) + \frac{D}{(x+y)}\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right) + \frac{-F_{dl}y}{(x+y)}\left(1-\left(\frac{-y}{x}\right)^{n}\right)$$

$$\sum_{i=1}^{n} F_i = \frac{1}{(x+y)}\left[A\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + B\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) + C\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) + D\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right) - F_{dl}y\left(1-\left(\frac{-y}{x}\right)^{n}\right)\right]$$

Substituting the value of A, B, C and D into the above equation gives

$$\sum_{i=1}^{n} F_i = \frac{1}{(x+y)}\left[-m_p a\mu_a r_a\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + \frac{m_p a({R_1}^2+{R_2}^2)}{2r_b}\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) + 2\bar{\lambda}_m \mu_a r_a {V_{tb}}^2\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right) + m_p g\mu_a r_a\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right) - F_{dl}y\left(1-\left(\frac{-y}{x}\right)^{n}\right)\right]$$

$$\sum_{i=1}^{n} F_i = a\left(\left(\frac{-m_p\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + \frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\left(\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right)\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right) + \left(\frac{m_p g\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{y}{(x+y)}\right)\left(1-\left(\frac{-y}{x}\right)^{n}\right) \quad [\alpha\text{- }10]$$

The hook load (W) relation during hoisting for non-uniform movement of the travelling block is given by Eqn [I-1B] as

$W = \sum_{i=1}^{n} F_i - (m_{dp} + m_{tb})a$

Substitute Eqn [α-10] into Eqn [I-1B]

$$\Rightarrow W = a\left(\frac{-m_p\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + a\left(\frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m\,\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right) + \left(\frac{m_p g\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{y}{(x+y)}\right)\left(1-\left(\frac{-y}{x}\right)^{n}\right) - (m_{dp} + m_{tb})a$$

$$\Rightarrow W = a\left(\left(\frac{-m_p\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right) - (m_{dp} + m_{tb})\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m\,\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right) + \left(\frac{m_p g\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{y}{(x+y)}\right)\left(1-\left(\frac{-y}{x}\right)^{n}\right) \qquad [α\text{ -11A}]$$

Alternatively, the hook load (W) during hoisting for non-uniform movement of the travelling block is given by Eqn [I-1C] as

$$W = \frac{\sum_{i=1}^{n} F_i}{\left(1+\frac{a}{g}\right)} = \frac{1}{\left(\frac{g+a}{g}\right)}\sum_{i=1}^{n} F_i = \frac{g}{(g+a)}\sum_{i=1}^{n} F_i$$

$$W = \frac{g}{(g+a)}\left(a\left(\left(\frac{-m_p\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-y}{x}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+2}(k+1)\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right)\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m\,\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-y}{x}\right)^{n-(k+1)}\right)\right) + \left(\frac{m_p g\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-y}{x}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{y}{(x+y)}\right)\left(1-\left(\frac{-y}{x}\right)^{n}\right)\right) \qquad [α\text{- 11B}]$$

Where $x = \mu_a r_a - r_b$ and $y = \mu_a r_a + r_b$

$x + y = \mu_a r_a - r_b + \mu_a r_a + r_b = 2\mu_a r_a$

Hence, μ_a can be as small as possible but should not be equal to 0 (i.e. $\mu_a = 0.00000001$ *but* $\mu_a \neq 0$) since $\frac{1}{\mu_a} = \frac{1}{0} = undefined$

Eqn [α -11A] and Eqn [α -11B] are the extended Cayeux et al hook load (W) relations during hoisting with non-uniform movement of the travelling equipment.

4.5 Lowering

During lowering, the line tension decreases from the dead line (F_{dl}) towards the fast line (F_{fl}) as illustrated in figure (29) below.

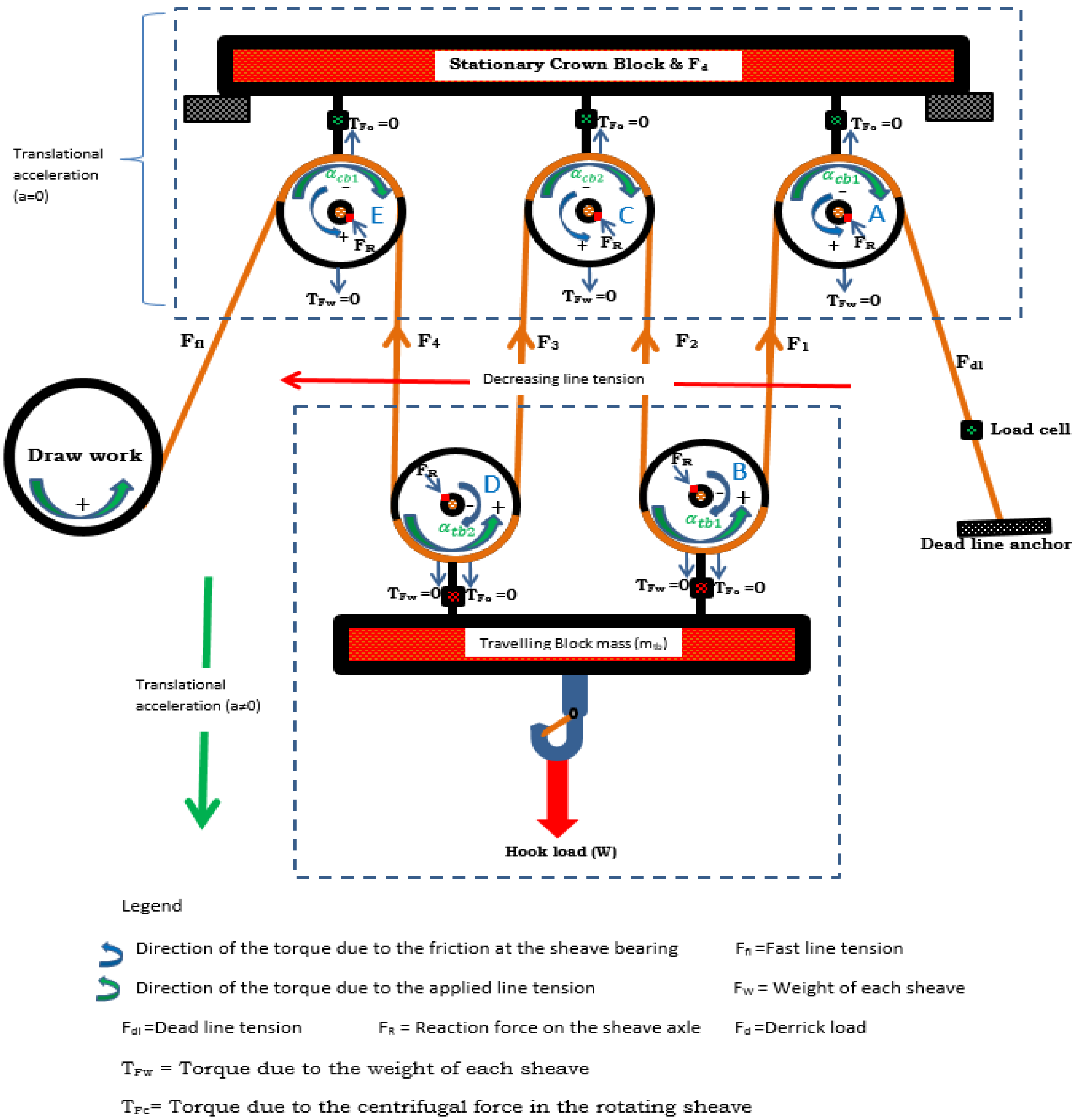

Figure 29: Shows the forces and torques on both the crown block and the travelling block sheaves during lowering

Considering the dead line sheave (sheave A) in the crown block, the net the torque is given by Eqn [H-3C₁] as (see Appendix C for derivation)

$For\ \ F_{dl} > F_1$

$$r_b\,(F_{dl} - F_1\,) + \mu_a r_a\big(-m_p g + \ 2\bar{\lambda}_m {r_b}^2\ \dot{\omega}^2{}_{cb1} - F_{dl} - F_1\big) = \ -I\alpha_{cb1}$$

$$-F_1(\mu_a r_a + r_b\,) - m_p g \mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb1} - F_{dl}(\mu_a r_a - r_b\,) = \ -I\alpha_{cb1}$$

$$F_1 = \frac{-1}{(\mu_a r_a + r_b\,)}\big(-I\alpha_{cb1} + F_{dl}(\mu_a r_a - r_b\,) - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a\big) \qquad [\delta\text{- 1A}]$$

For simplicity, let $x = \mu_a r_a - r_b\ and\ \ y = \mu_a r_a + r_b$

$$F_1 = \frac{-1}{y}\big(-I\alpha_{cb1} + F_{dl}x - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a\big) \qquad [\delta\text{- 1B}]$$

Similarly considering the net torque in the next sheave (sheave B) in the travelling block is given by Eqn [H-4C] as

$For\ \ F_1 > F_2$

$$r_b\,(F_1 - \ F_2) - \mu_a r_a(-m_p a - (m_p g + \ 2\bar{\lambda}_m {r_b}^2\ \dot{\omega}^2{}_{tb1} - F_1 - F_2)) = + I\alpha_{tb1}$$

$$-F_2(\mu_a r_a + \ r_b) = \ I\alpha_{tb1} - m_p a \mu_a r_a + F_1(\mu_a r_a - r_b\,) - m_p g \mu_a r_a - \ 2\bar{\lambda}_m {r_b}^2\ \mu_a r_a \dot{\omega}^2{}_{tb1}$$

$$F_2 = \frac{1}{(\mu_a r_a + \ r_b)}\big(-\ I\alpha_{tb1} + m_p a \mu_a r_a - F_1(\mu_a r_a - r_b\,) + m_p g \mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2\ \mu_a r_a \dot{\omega}^2{}_{tb1}\big)$$

[δ- 2A]

For simplicity, let $x = \mu_a r_a - r_b\ and\ \ y = \mu_a r_a + r_b$

$$F_2 = \frac{1}{y}\big(-\ I\alpha_{tb1} + m_p a \mu_a r_a - F_1 x + m_p g \mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2\ \mu_a r_a \dot{\omega}^2{}_{tb1}\big) \qquad [\delta\text{- 2B}]$$

Substituting Eqn [δ-1B] into Eqn [δ-2B] and multiply through the resulting equation by $\frac{y}{y}$ gives

$$F_2 = \frac{1}{y^2}\big(-\ I\alpha_{tb1}y + m_p a \mu_a r_a y - I\alpha_{cb1}x + F_{dl}x^2 - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ \dot{\omega}^2{}_{cb1}x + m_p g \mu_a r_a x + m_p g \mu_a r_a y + \ 2\bar{\lambda}_m {r_b}^2\ \mu_a r_a \dot{\omega}^2{}_{tb1}y\big)$$

$$F_2 = \frac{1}{y^2}\big(m_p a \mu_a r_a y - I(\alpha_{cb1}x + \alpha_{tb1}y) + F_{dl}x^2 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\ (-\dot{\omega}^2{}_{cb1}x + \dot{\omega}^2{}_{tb1}y) + m_p g \mu_a r_a(x + y)\big) \qquad [\delta\text{- 2C}]$$

Similarly considering the net torque in the next sheave (sheave C) in the crown block is given by Eqn [H-3C₁] as

$For\ \ F_2 > F_3$

$$r_b\,(F_2 - F_3\,) + \mu_a r_a\big(-m_p g + \ 2\bar{\lambda}_m {r_b}^2\ \dot{\omega}^2{}_{cb2} - F_2 - F_3\big) = \ -I\alpha_{cb2}$$

$-F_3(\mu_a r_a + r_b) - m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} + F_2(r_b - \mu_a r_a) = -I\alpha_{cb2}$

$F_3 = \frac{-1}{(\mu_a r_a + r_b)}\left(-I\alpha_{cb2} + F_2(\mu_a r_a - r_b) - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right)$ [δ- 3A]

For simplicity, let $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$

$F_3 = \frac{-1}{y}\left(-I\alpha_{cb2} + F_2 x - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right)$ [δ- 3B]

Substituting Eqn [δ-2C] into Eqn [δ-3B] and multiplying through the resulting equation by $\frac{y^2}{y^2}$ gives

$F_3 = \frac{-1}{y^3}\big(m_p a\mu_a r_a yx - I(\alpha_{cb1}x^2 + \alpha_{tb1}yx + \alpha_{cb2}y^2) + F_{dl}x^3 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, (-\dot{\omega}^2{}_{cb1}x^2 + \dot{\omega}^2{}_{tb1}yx - \dot{\omega}^2{}_{cb2}y^2) + m_p g\mu_a r_a(x^2 + xy + y^2)\big)$ [δ- 3C]

Also, considering the net torque in the next sheave (sheave D) in the travelling block, Eqn [H-4C] becomes

For $F_3 > F_4$

$r_b(F_3 - F_4) - \mu_a r_a(-m_p a - (m_p g + 2\bar{\lambda}_m {r_b}^2 \, \dot{\omega}^2{}_{tb2} - F_3 - F_4)) = +I\alpha_{tb2}$

$-F_4(\mu_a r_a + r_b) + m_p a\mu_a r_a + m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{tb2} + F_3(r_b - \mu_a r_a) = I\alpha_{tb2}$

$F_4 = \frac{1}{(\mu_a r_a + r_b)}(-I\alpha_{tb2} + m_p a\mu_a r_a - F_3(\mu_a r_a - r_b) + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a)$

[δ- 4A]

For simplicity, let $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$

$F_4 = \frac{1}{y}(-I\alpha_{tb2} + m_p a\mu_a r_a - F_3 x + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a)$ [δ- 4B]

Substituting Eqn [δ-3C] into Eqn [δ-4B] and multiplying through by $\frac{y^3}{y^3}$ gives

$F_4 = \frac{1}{y^4}(m_p a\mu_a r_a(yx^2 + y^3) - I(\alpha_{cb1}x^3 + \alpha_{tb1}yx^2 + \alpha_{cb2}y^2x + \alpha_{tb2}y^3) + F_{dl}x^4 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, (-\dot{\omega}^2{}_{cb1}x^3 + \dot{\omega}^2{}_{tb1}yx^2 - \dot{\omega}^2{}_{cb2}y^2x + \dot{\omega}^2{}_{tb2}y^3) + m_p g\mu_a r_a(x^3 + x^2y + xy^2 + y^3))$ [δ- 4C]

Hence for "n" number of lines between the crown block and the travelling block, the general relation for the line tension reduction from the dead line(F_{dl}) towards the fast lines (F_{fl}) is given by

$F_n =$

$$\left(\frac{-1}{y}\right)^n \left(m_p a \mu_a r_a (\textstyle\sum_{k=1}^{r} y^{2k-1}\, x^{n-2k}) - I(\sum_{k=0}^{q=n-1} \alpha_{1+k} x^{q-k} y^k) + F_{dl} x^n + m_p g \mu_a r_a (\sum_{k=0}^{q=n-1} x^{q-k} y^k) + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \{\sum_{k=0}^{q=n-1} (-1)^{k+1} \dot{\omega}^2{}_{(k+1)} x^{q-k} y^k\}\right) \qquad [\delta\text{- } 5A]$$

where

q = n-1 (i.e. the number of supporting lines minus 1)

r = the number of travelling block sheaves between the dead line and the line of interest

$\dot{\omega}_{(k+1)}$ *and* $\alpha_{(1+k)}$ represent the numbering of the angular velocity and the angular acceleration respectively for each sheave from the dead line sheave in the crown block through the travelling block sheave as illustrated in figure (29)

4.5.1 Hook load (W) during lowering with non-uniform movement of the travelling equipment

During lowering the sum of the downward forces (F_{Down}) exceeds that of the upward force ($\sum_{i=1}^{n} F_i$) as illustrated in figure (23)

$$\Rightarrow F_{Down} - \textstyle\sum_{i=1}^{n} F_i = m_T\, a \qquad [I\text{-}2A]$$

$$F_{Down} = W = (m_{dp} + m_{tb})g = m_T\, g \qquad [\delta\text{- } 2A]$$

$$\Rightarrow m_T = m_{dp} + m_{tb} = \frac{W}{g} \qquad [\delta\text{- } 2B]$$

Substituting Eqn [δ-2A] into Eqn [I-2A] gives

$$W - \sum_{i=1}^{n} F_i = (m_{dp} + m_{tb})a$$

$$W = \textstyle\sum_{i=1}^{n} F_i + (m_{dp} + m_{tb})a \qquad [I\text{-}2B]$$

Alternatively, substituting Eqn [δ-2B] into Eqn [I-2B] gives

$$W = \textstyle\sum_{i=1}^{n} F_i + \frac{W}{g} a$$

$$W - \frac{W}{g} a = \textstyle\sum_{i=1}^{n} F_i$$

$$W\left(1 - \frac{a}{g}\right) = \textstyle\sum_{i=1}^{n} F_i$$

$$W = \frac{\sum_{i=1}^{n} F_i}{\left(1-\frac{a}{g}\right)} = \frac{\sum_{i=1}^{n} F_i}{\left(\frac{g-a}{g}\right)} = \left(\frac{g}{g-a}\right) \textstyle\sum_{i=1}^{n} F_i \qquad [I\text{-}2C]$$

4.5.2 Relationship between the angular acceleration and the angular velocity of each rotating sheave relative to that of the first sheave in the travelling block

With an assumption that the dead line is non-rotating and using the relationship between the travelling block velocity (V_{tb}) and the angular velocity of the first rotating sheave ($\dot{\omega}_{tb1}$) in the travelling block which is connected to the dead line sheave by the line opposite the dead line as depicted in Figure (28). The angular velocity and acceleration of all the rotating sheave will be determined relative to that of the first sheave in the travelling block ($\dot{\omega}_{tb1}$).

Substituting Eqn [ϴ-1A] into Eqn [δ-1B] gives the value of F_1 as

$$F_1 = \frac{-1}{y}\left(F_{dl}x + m_p g \mu_a r_a\right) \quad [\delta\text{- } 1B_1]$$

Substituting Eqn [ϴ-1A] and Eqn [ϴ-1B] into Eqn [δ-2C] gives the value of F_2

$$F_2 = \frac{1}{y^2}\left(m_p a \mu_a r_a y - I\alpha_{tb1}(y) + F_{dl}x^2 + 2\bar{\lambda}_m r_b{}^2 \mu_a r_a\, \dot{\omega}^2{}_{tb1}(y) + m_p g \mu_a r_a (x+y)\right)$$

[δ- 2C1]

Similarly, substituting Eqn [ϴ-1A], Eqn [ϴ-1B] and Eqn [ϴ-1C] into Eqn [δ-3C] gives the value of F_3 as

$$F_3 = \frac{-1}{y^3}\left(m_p a \mu_a r_a yx - I(\alpha_{tb1}yx + 2\alpha_{tb1}y^2) + F_{dl}x^3 + 2\bar{\lambda}_m r_b{}^2 \mu_a r_a\, (\dot{\omega}^2{}_{tb1}yx - (2\dot{\omega}_{tb1})^2 y^2) + m_p g \mu_a r_a (x^2 + xy + y^2)\right)$$

$$F_3 = \frac{-1}{y^3}\left(m_p a \mu_a r_a yx - I\alpha_{tb1}(yx + 2y^2) + F_{dl}x^3 + 2\bar{\lambda}_m r_b{}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}\,(yx - 4y^2) + m_p g \mu_a r_a (x^2 + xy + y^2)\right) \quad [\delta\text{- } 3C_1]$$

Finally, substituting Eqn [ϴ-1A], Eqn [ϴ-1B], Eqn [ϴ-1C] and Eqn [ϴ-1D] into Eqn [δ-4C] gives the value of F_4 as

$$F_4 = \frac{1}{y^4}(m_p a \mu_a r_a (yx^2 + y^3) - I(\alpha_{tb1}yx^2 + 2\alpha_{tb1}y^2x + 3\alpha_{tb1}y^3) + F_{dl}x^4 + 2\bar{\lambda}_m r_b{}^2 \mu_a r_a\, (\dot{\omega}^2{}_{tb1}yx^2 - (2\dot{\omega}_{tb1})^2 y^2 x + (3\dot{\omega}_{tb1})^2 y^3) + m_p g \mu_a r_a (x^3 + x^2y + xy^2 + y^3))$$

$$F_4 = \frac{1}{y^4}(m_p a \mu_a r_a (yx^2 + y^3) - I\alpha_{tb1}(yx^2 + 2y^2x + 3y^3) + F_{dl}x^4 + 2\bar{\lambda}_m r_b{}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}\,(yx^2 - 4y^2x + 9y^3) + m_p g \mu_a r_a (x^3 + x^2y + xy^2 + y^3)) \quad [\delta\text{- } 4C_1]$$

Substituting Eqn [Θ-1] and Eqn [Θ-2] into Eqn [δ -1B1], Eqn [δ -2C1], Eqn [δ -3C1] and Eqn [δ -4C1] gives

$$F_1 = \frac{-1}{y}\left(F_{dl}x + m_p g\mu_a r_a\right) \qquad [\delta\text{- }1B_2]$$

$$F_2 = \frac{1}{y^2}\left(m_p a\mu_a r_a y - I\left(\frac{a}{r_b}\right)y + F_{dl}x^2 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a \left(\frac{{V_{tb}}^2}{{r_b}^2}\right)y + m_p g\mu_a r_a(x+y)\right)$$

[δ- 2C2]

$$F_3 = \frac{-1}{y^3}\left(m_p a\mu_a r_a yx - I\left(\frac{a}{r_b}\right)(yx + 2y^2) + F_{dl}x^3 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a \left(\frac{{V_{tb}}^2}{{r_b}^2}\right)(yx - 4y^2) + m_p g\mu_a r_a(x^2 + xy + y^2)\right) \qquad [\delta\text{- }3C_2]$$

$$F_4 = \frac{1}{y^4}(m_p a\mu_a r_a(yx^2 + y^3) - I\left(\frac{a}{r_b}\right)(yx^2 + 2y^2x + 3y^3) + F_{dl}x^4 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a \left(\frac{{V_{tb}}^2}{{r_b}^2}\right)(yx^2 - 4y^2x + 9y^3) + m_p g\mu_a r_a(x^3 + x^2y + xy^2 + y^3)) \qquad [\delta\text{- }4C_2]$$

Hence for "n" number of lines between the travelling block and the crown block, the general line tension reduction from the dead line towards the fast line during lowering is given by

$$F_n = \left(\frac{-1}{y}\right)^n \left(m_p a\mu_a r_a\left(\sum_{k=1}^{r} y^{2k-1}\, x^{n-2k}\right) - \frac{Ia}{r_b}\left(\sum_{k=0}^{q=n-1} k\, x^{q-k}y^k\right) + F_{dl}x^n + 2\bar{\lambda}_m\mu_a r_a {V_{tb}}^2\left(\sum_{k=0}^{q=n-1}(-1)^{k+1}(k^2x^{q-k}y^k)\right) + m_p g\mu_a r_a\left(\sum_{k=0}^{q=n-1} x^{q-k}y^k\right)\right) \qquad [\delta\text{- }5B]$$

But $I = \frac{1}{2}m_p\left({R_1}^2 + {R_2}^2\right)$

Where R_1 and R_2 are the inner and outer radii of the each sheave. In this thesis, we assume that R_1 and R_2 remain constant for each sheave.

Substituting the relation for I into Eqn [δ-5B]

$$F_n = \left(\frac{-1}{y}\right)^n \left(m_p a\mu_a r_a\left(\sum_{k=1}^{r} y^{2k-1}\, x^{n-2k}\right) - \frac{m_p a({R_1}^2 + {R_2}^2)}{2r_b}\left(\sum_{k=0}^{q=n-1} k\, x^{q-k}y^k\right) + F_{dl}x^n + {V_{tb}}^2(2\bar{\lambda}_m\mu_a r_a)\left(\sum_{k=0}^{q=n-1}(-1)^{k+1}(k^2x^{q-k}y^k)\right) + m_p g\mu_a r_a\left(\sum_{k=0}^{q=n-1} x^{q-k}y^k\right)\right)$$

$$F_n = \left(\frac{-1}{y}\right)^n \left(a \left(m_p \mu_a r_a (\sum_{k=1}^{r} y^{2k-1}\, x^{n-2k}) - \frac{m_p({R_1}^2+{R_2}^2)}{2r_b} \left(\sum_{k=0}^{q=n-1} k\, x^{q-k} y^k\right) \right) + F_{dl} x^n + {V_{tb}}^2 (2\bar{\lambda}_m \mu_a r_a) \left(\sum_{k=0}^{q=n-1} (-1)^{k+1} (k^2 x^{q-k} y^k)\right) + m_p g \mu_a r_a \left(\sum_{k=0}^{q=n-1} x^{q-k} y^k\right) \right) \qquad [\delta\text{-5C}]$$

Hence during uniform movement of the travelling equipment, the translational acceleration is zero (i.e. a=0). Eqn [δ-5C] becomes

$$F_n = \left(\frac{-1}{y}\right)^n \left(F_{dl} x^n + {V_{tb}}^2 (2\bar{\lambda}_m \mu_a r_a) \left(\sum_{k=0}^{q=n-1} (-1)^{k+1} (k^2 x^{q-k} y^k)\right) + m_p g \mu_a r_a \left(\sum_{k=0}^{q=n-1} x^{q-k} y^k\right) \right)$$

[δ-5D]

For simplicity let

$A = m_p a \mu_a r_a$ = The torque as a result of the acceleration effect on each of the travelling block sheave's reaction force, during non-uniform movement of the travelling equipment

$B = I\left(\frac{a}{r_b}\right) = \frac{1}{2} m_p \left({R_1}^2 + {R_2}^2\right) \frac{a}{r_b} = \frac{m_p a({R_1}^2+{R_2}^2)}{2r_b}$ = The torque due to the angular acceleration on each of the rotating sheave, during non-uniform movement of the travelling equipment

$C = 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(\frac{{V_{tb}}^2}{{r_b}^2}\right) = 2\bar{\lambda}_m \mu_a r_a {V_{tb}}^2$ = The torque due to the centrifugal force on each rotating sheave

$D = m_p g \mu_a r_a$= The torque due to the weight of each sheave

Substituting these relations A, B, C and D into Eqn [δ -1B₂], Eqn [δ -2C₂], Eqn [δ -3C₂] and Eqn [δ -4C₂] gives

F_1 becomes

$$F_1 = \frac{-F_{dl} x}{y} + \frac{-D}{y} \qquad [\delta\ \text{-1B}_3]$$

Similarly F_2 becomes

$$F_2 = \frac{1}{y^2}(Ay - By + F_{dl} x^2 + Cy + D(x+y))$$

$$F_2 = \frac{A}{y} - \frac{B}{y} + \frac{F_{dl} x^2}{y^2} + \frac{C}{y} + \left(\frac{Dx}{y^2} + \frac{D}{y}\right) \qquad [\delta\ \text{-2C}_3]$$

Also F_3 becomes

$$F_3 = \frac{-1}{y^3}(Ayx - B(yx + 2y^2) + F_{dl}x^3 + C\,(yx - 4y^2) + D(x^2 + xy + y^2))$$

$$F_3 = \frac{-Ax}{y^2} + \left(\frac{Bx}{y^2} + \frac{2B}{y}\right) + \frac{-F_{dl}x^3}{y^3} + \left(\frac{-Cx}{y^2} + \frac{4C}{y}\right) + \left(\frac{-Dx^2}{y^3} + \frac{-Dx}{y^2} + \frac{-D}{y}\right) \qquad [\delta\ \text{-}3C_3]$$

Finally F_4 becomes

$$F_4 = \frac{1}{y^4}(A(yx^2 + y^3) - B(yx^2 + 2y^2x + 3y^3) + F_{dl}x^4 + C\,(yx^2 - 4y^2x + 9y^3) +$$

$$D(x^3 + x^2y + xy^2 + y^3))$$

$$F_4 = \left(\frac{Ax^2}{y^3} + \frac{A}{y}\right) + \left(\frac{-Bx^2}{y^3} + \frac{-2Bx}{y^2} + \frac{-3B}{y}\right) + \frac{F_{dl}x^4}{y^4} + \left(\frac{Cx^2}{y^3} - \frac{4Cx}{y^2} + \frac{9c}{y}\right) + \left(\frac{Dx^3}{y^4} + \frac{Dx^2}{y^3} + \frac{Dx}{y^2} + \frac{D}{y}\right)$$

$[\delta$ - $4C_3]$

SUM OF THE FORCES IN THE SUPPORTING LINES

For simplicity, let's assume the number of supporting lines between the crown block and the travelling block is four (n = 4)

$$\Rightarrow \sum_{i=1}^{n} F_i = F_1 + F_2 + F_3 + F_4$$

Also since addition is commutative, the sum of the forces will be performed sheave-wise (sheave by sheave) and term by term (A, B, C, D and F_{dl}) approach.

TORQUE DUE TO THE TRANSLATIONAL ACCELERATION EFFECT ON EACH OF THE TRAVELLING BLOCK SHEAVE'S REACTION FORCE (A)

The total torque(A_{Total}) as a result of the translational acceleration effect on the reaction force on each of the travelling block sheave is given by

$$A_{Total} = A_{tb1} + A_{tb2}$$

$$A_{Total} = \left(\frac{A}{y}\right) + \left(\frac{-Ax}{y^2}\right) + \left(\frac{Ax^2}{y^3} + \frac{A}{y}\right)$$

Total torque due to the translation acceleration effect on the first sheave in travelling block (A_{tb1}) is given by

$$A_{tb1} = \frac{A}{y} + \frac{-Ax}{y^2} + \frac{Ax^2}{y^3} = \frac{A}{y}\left(1 + \frac{-x}{y} + \frac{x^2}{y^2}\right) = \frac{A}{y}GS$$

$$But \quad GS = 1 + \frac{-x}{y} + \frac{x^2}{y^2} = \frac{1\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)}{1-\frac{-x}{y}} = \frac{\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)}{\frac{(y+x)}{y}} = \frac{y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)$$

$$\Rightarrow A_{tb1} = \frac{A}{y}\frac{y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) = \frac{A}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) \qquad [\mathbb{C}\text{-1A}]$$

Similarly, the torque due to the translation acceleration effect on the second rotating travelling block sheave (A_{tb2}) is given by

$$A_{tb2} = \frac{A}{y}$$

$$\Rightarrow A_{tb2} = \frac{A}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right) \qquad [\mathbb{C}\text{-1B}]$$

From Eqn [ℂ-1A] and Eqn [ℂ-1B], the total torque contribution from the translation acceleration effect on all the sheaves in the travelling block becomes

$$A_{Total} = \frac{A}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + \frac{A}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right)$$

$$A_{Total} = \frac{A}{(x+y)}\left(\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + \left(1-\left(\frac{-x}{y}\right)^{n-3}\right)\right) = \frac{A}{(x+y)}\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) \qquad [\mathbb{C}\text{-2}]$$

Where “r” is the number of sheaves in the travelling block between the dead line and the line of interest. Hence, for “n” number of supporting lines, $r = \frac{n}{2}$

TORQUE DUE TO THE ANGULAR ACCELERATION ON EACH OF THE ROTATING SHEAVE (B)

The total torque from all the rotating sheave (B_{TOTAL}) during non-uniform movement of the travelling equipment is given as

$$B_{TOTAL} = B_{total\_cb1} + B_{total\_tb1} + B_{total\_cb2} + B_{total\_tb2}$$

For non-rotating dead line sheave the angular acceleration is zero $(i.ie\ B_{total_{cb1}} = 0)$

$$\Rightarrow B_{TOTAL} = B_{total\_tb1} + B_{total\_cb2} + B_{total\_tb2}$$

$$\Rightarrow B_{TOTAL} = \left(\frac{-B}{y}\right) + \left(\frac{Bx}{y^2} + \frac{2B}{y}\right) + \left(\frac{-Bx^2}{y^3} + \frac{-2Bx}{y^2} + \frac{-3B}{y}\right)$$

Considering the torque due to the total angular acceleration resulting from the first travelling block sheave (B_{total_tb1}) gives

$$B_{total\_tb1} = \frac{-B}{y} + \frac{Bx}{y^2} + \frac{-Bx^2}{y^3} = \frac{B}{y}\left(-1 + \frac{x}{y} + \frac{-x^2}{y^2}\right) = \frac{B}{y} GS$$

$$But \quad GS = -1 + \frac{x}{y} + \frac{-x^2}{y^2} = \frac{-1\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)}{1-\frac{-x}{y}} = \frac{-\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)}{\frac{(y+x)}{y}} = \frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)$$

$$\Rightarrow B_{total\_tb1} = \frac{B}{y}\frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) = \frac{-B}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) \qquad [\mathbb{C}\text{- 3A}]$$

Considering the torque due to the total angular acceleration resulting from the second sheave in the crown block (B_{total_cb2}) gives

$$B_{total\_cb2} = \frac{2B}{y} + \frac{-2Bx}{y^2} = \frac{2B}{y}\left(1 + \frac{-x}{y}\right) = \frac{2B}{y} GS$$

$$But \quad GS = 1 + \frac{-x}{y} = \frac{1\left(1-\left(\frac{-x}{y}\right)^{n-2}\right)}{1-\frac{-x}{y}} = \frac{\left(1-\left(\frac{-x}{y}\right)^{n-2}\right)}{\frac{(y+x)}{y}} = \frac{y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right)$$

$$\Rightarrow B_{total\_cb2} = \frac{2B}{y}\frac{y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) = \frac{2B}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) \qquad [\mathbb{C}\text{-3B}]$$

Finally, considering the torque due to the total angular acceleration resulting from the second sheave in the travelling block (B_{total_tb2}) becomes

$$B_{total\_tb2} = \frac{-3B}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right) \qquad [\mathbb{C}\text{-3C}]$$

From Eqn [ℂ-3A], Eqn [ℂ-3B] and Eqn [ℂ-3C], the total torque due to the total angular acceleration (B_{TOTAL}) from all the rotating sheave

$$\Rightarrow B_{TOTAL} = \frac{-B}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + \frac{2B}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) + \frac{-3B}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right)$$

$$\Rightarrow B_{TOTAL} = \frac{B}{(x+y)}\left(-1\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + 2\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) - 3\left(1-\left(\frac{-x}{y}\right)^{n-3}\right)\right)$$

$$\Rightarrow B_{TOTAL} = \frac{B}{(x+y)} \sum_{k=0}^{n-2} (-1)^{k+1} (k+1) \left(1 - \left(\frac{-x}{y}\right)^{n-(k+1)}\right) \quad [\mathbb{C}\text{-4}]$$

TORQUE DUE TO THE CENTRIFUGAL FORCE ON EACH OF THE ROTATING SHEAVE (C)

The total contribution to the torque from the centrifugal force (C_{TOTAL}) is given by

$$C_{TOTAL} = C_{total\_cb1} + C_{total\_tb1} + C_{total\_cb2} + C_{total\_tb2}$$

But for non-rotating dead line sheave(C_{total_cb1}), its angular velocity is zero ($i.e.\, \dot{\omega}_{tb1} = 0$) and hence no centrifugal force contribution to the total torque ($i.e.\, C_{total\_cb1} = 0$)

$$\Rightarrow C_{TOTAL} = C_{total\_tb1} + C_{total\_cb2} + C_{total\_tb2}$$

$$C_{TOTAL} = \left(\frac{C}{y}\right) + \left(\frac{-Cx}{y^2} + \frac{4C}{y}\right) + \left(\frac{Cx^2}{y^3} + \frac{-4Cx}{y^2} + \frac{9c}{y}\right)$$

Considering the total contribution to the torque by the first sheave in the travelling block (C_{total_tb1}) gives

$$C_{total\_tb1} = \frac{C}{y} + \frac{-Cx}{y^2} + \frac{Cx^2}{y^3} = \frac{C}{y}\left(1 + \frac{-x}{y} + \frac{x^2}{y^2}\right) = \frac{C}{y} GS$$

$$But \quad GS = 1 + \frac{-x}{y} + \frac{x^2}{y^2} = \frac{1\left(1 - \left(\frac{-x}{y}\right)^{n-1}\right)}{1 - \frac{-x}{y}} = \frac{\left(1 - \left(\frac{-x}{y}\right)^{n-1}\right)}{\frac{(y+x)}{y}} = \frac{y}{(x+y)}\left(1 - \left(\frac{-x}{y}\right)^{n-1}\right)$$

$$\Rightarrow C_{total\_tb1} = \frac{C}{y}\frac{y}{(x+y)}\left(1 - \left(\frac{-x}{y}\right)^{n-1}\right) = \frac{C}{(x+y)}\left(1 - \left(\frac{-x}{y}\right)^{n-1}\right) \quad [\mathbb{C}\text{-5A}]$$

Similarly, considering the total contribution to the torque by the second sheave in the crown block (C_{total_cb2}) gives

$$C_{total\_cb2} = \frac{4C}{y} + \frac{-4Cx}{y^2} = \frac{4C}{y}\left(1 + \frac{-x}{y}\right) = \frac{4C}{y} GS$$

$$But \quad GS = 1 + \frac{-x}{y} = \frac{1\left(1 - \left(\frac{-x}{y}\right)^{n-2}\right)}{1 - \frac{-x}{y}} = \frac{\left(1 - \left(\frac{-x}{y}\right)^{n-2}\right)}{\frac{(y+x)}{y}} = \frac{y}{(x+y)}\left(1 - \left(\frac{-x}{y}\right)^{n-2}\right)$$

$$C_{total\_cb2} = \frac{4C}{y}\frac{y}{(x+y)}\left(1 - \left(\frac{-x}{y}\right)^{n-2}\right) = \frac{4C}{(x+y)}\left(1 - \left(\frac{-x}{y}\right)^{n-2}\right) \quad [\mathbb{C}\text{-5B}]$$

Finally, considering the total contribution to the torque by the second sheave in the travelling block (C_{total_tb2}) becomes

$$\Rightarrow C_{total\_tb2} = \frac{9C}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right) \qquad [\mathbb{C}\text{-5C}]$$

From Eqn [ℂ-5A], Eqn [ℂ-5B] and Eqn [ℂ-5C], the total torque due to the centrifugal force (C_{TOTAL}) from all the rotating sheave is given by

$$\Rightarrow C_{TOTAL} = \frac{C}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + \frac{4C}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) + \frac{9C}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right)$$

$$\Rightarrow C_{TOTAL} = \frac{C}{(x+y)}\left(\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + 4\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) + 9\left(1-\left(\frac{-x}{y}\right)^{n-3}\right)\right)$$

$$\Rightarrow C_{TOTAL} = \frac{C}{(x+y)}\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) \qquad [\mathbb{C}\text{-6}]$$

TORQUE DUE TO THE WEIGHT OF EACH SHEAVE (D)

The total contribution to the torque from the weight of each of the sheave (D_{TOTAL}) is given by

$$D_{TOTAL} = D_{total\_cb1} + D_{total\_tb1} + D_{total\_cb2} + D_{total\_tb2}$$

$$D_{TOTAL} = \left(\frac{-D}{y}\right) + \left(\frac{Dx}{y^2}+\frac{D}{y}\right) + \left(\frac{-Dx^2}{y^3}+\frac{-Dx}{y^2}+\frac{-D}{y}\right) + \left(\frac{Dx^3}{y^4}+\frac{Dx^2}{y^3}+\frac{Dx}{y^2}+\frac{D}{y}\right)$$

Considering the total contribution from the weight of the first sheave in the crown block (D_{total_cb1}) to the net torque gives

$$D_{total\_cb1} = \frac{-D}{y}+\frac{Dx}{y^2}+\frac{-Dx^2}{y^3}+\frac{Dx^3}{y^4} = \frac{D}{y}\left(-1+\frac{x}{y}+\frac{-x^2}{y^2}+\frac{x^3}{y^3}\right) = \frac{D}{y}GS$$

$$But \quad GS = -1+\frac{x}{y}+\frac{-x^2}{y^2}+\frac{x^3}{y^3} = \frac{-1\left(1-\left(\frac{-x}{y}\right)^n\right)}{1-\frac{-x}{y}} = \frac{-\left(1-\left(\frac{-x}{y}\right)^n\right)}{\frac{(y+x)}{y}} = \frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^n\right)$$

$$\Rightarrow D_{total\_cb1} = \frac{D}{y}\frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^n\right) = \frac{-D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^n\right) \qquad [\mathbb{C}\text{-7A}]$$

Similarly, considering the total contribution from the weight of the first sheave in the travelling block (D_{total_tb1}) to the net torque gives

$$D_{total\_tb1} = \frac{D}{y}+\frac{-Dx}{y^2}+\frac{Dx^2}{y^3} = \frac{D}{y}\left(1+\frac{-x}{y}+\frac{x^2}{y^2}\right) = \frac{D}{y}GS$$

$$But \quad GS = 1+\frac{-x}{y}+\frac{x^2}{y^2} = \frac{1\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)}{1-\frac{-x}{y}} = \frac{\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)}{\frac{(y+x)}{y}} = \frac{y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right)$$

$$D_{total\_tb1} = \frac{D}{y}\frac{y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) = \frac{D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) \quad [\mathbb{C}\text{-7B}]$$

Also considering the total contribution from the weight of the second sheave in the crown block (D_{total_cb2}) to the total torque becomes

$$D_{total\_cb2} = \frac{-D}{y} + \frac{Dx}{y^2} = \frac{D}{y}\left(-1+\frac{x}{y}\right) = \frac{D}{y}GS$$

$$But \quad GS = -1 + \frac{x}{y} = \frac{-1\left(1-\left(\frac{-x}{y}\right)^{n-2}\right)}{1-\frac{-x}{y}} = \frac{-\left(1-\left(\frac{-x}{y}\right)^{n-2}\right)}{\frac{(y+x)}{y}} = \frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right)$$

$$D_{total\_cb2} = \frac{D}{y}\frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) = \frac{-D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) \quad [\mathbb{C}\text{-7C}]$$

Finally considering the total contribution from the weight of the second sheave in the travelling block (D_{total_tb2}) to the total torque becomes

$$\Rightarrow D_{total\_tb2} = \frac{D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right) \quad [\mathbb{C}\text{-7D}]$$

From Eqn [ℂ-7A], Eqn [ℂ-7B], Eqn [ℂ-7C] and Eqn [ℂ-7D], the total torque due to the weight of all the sheaves (D_{TOTAL}) is given by

$$\Rightarrow D_{TOTAL} = \frac{-D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n}\right) + \frac{D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-1}\right) + \frac{-D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-2}\right) + \frac{D}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n-3}\right)$$

$$D_{TOTAL} = \frac{D}{(x+y)}\left(-\left(1-\left(\frac{-x}{y}\right)^{n}\right) + \left(1-\left(\frac{-x}{y}\right)^{n-1}\right) - \left(1-\left(\frac{-x}{y}\right)^{n-2}\right) + \left(1-\left(\frac{-x}{y}\right)^{n-3}\right)\right)$$

$$\Rightarrow D_{TOTAL} = \frac{D}{(x+y)}\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right) \quad [\mathbb{C}\text{-8}]$$

DEAD LINE CONTRIBUTION TO THE TOTAL TORQUE

The total contribution to the net torque by the dead line $\left(F_{dl_{TOTAL}}\right)$ is also given by

$$F_{dl_{TOTAL}} = \frac{-F_{dl}x}{y} + \frac{F_{dl}x^2}{y^2} + \frac{-F_{dl}x^3}{y^3} + \frac{F_{dl}x^4}{y^4} = \frac{F_{dl}x}{y}\left(-1+\frac{x}{y}+\frac{-x^2}{y^2}+\frac{x^3}{y^3}\right) = \frac{F_{dl}x}{y}GS$$

$$But \quad GS = -1 + \frac{x}{y} + \frac{-x^2}{y^2} + \frac{x^3}{y^3} = \frac{-1\left(1-\left(\frac{-x}{y}\right)^{n}\right)}{1-\frac{-x}{y}} = \frac{-\left(1-\left(\frac{-x}{y}\right)^{n}\right)}{\frac{(y+x)}{y}} = \frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n}\right)$$

$$F_{dl_{TOTAL}} = \frac{F_{dl}x}{y}\frac{-y}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n}\right) = \frac{-F_{dl}x}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n}\right) \quad [\mathbb{C}\text{-}9]$$

From Eqn [ℂ -2], Eqn [ℂ -4], Eqn [ℂ -6], Eqn [ℂ -8], and Eqn [ℂ -9], the total tensions in the lines supporting the hook load becomes

$$\Rightarrow \sum_{i=1}^{n} F_i = F_1 + F_2 + F_3 + F_4 = A_{TOTAL} + B_{TOTAL} + C_{TOTAL} + D_{TOTAL} + F_{dl_{TOTAL}}$$

$$\sum_{i=1}^{n} F_i = \frac{A}{(x+y)}\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + \frac{B}{(x+y)}\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + \frac{C}{(x+y)}\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + \frac{D}{(x+y)}\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right) + \frac{-F_{dl}x}{(x+y)}\left(1-\left(\frac{-x}{y}\right)^{n}\right)$$

$$\sum_{i=1}^{n} F_i = \frac{1}{(x+y)}\left[A\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + B\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + C\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + D\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right) - F_{dl}x\left(1-\left(\frac{-x}{y}\right)^{n}\right)\right]$$

Substituting the relations of A, B, C and D into the above equation gives

$$\sum_{i=1}^{n} F_i = \frac{1}{(x+y)}\left[m_p a \mu_a r_a\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p a\left({R_1}^2+{R_2}^2\right)}{2r_b}\right)\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + {V_{tb}}^2\left(2\bar{\lambda}_m\,\mu_a r_a\right)\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + m_p g \mu_a r_a\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right) - F_{dl}x\left(1-\left(\frac{-x}{y}\right)^{n}\right)\right]$$

$$\sum_{i=1}^{n} F_i = a\left(\left(\frac{m_p\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p\left({R_1}^2+{R_2}^2\right)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right)\right)\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m\,\mu_a r_a}{(x+y)}\right)\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + \left(\frac{m_p g\mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{x}{(x+y)}\right)\left(1-\left(\frac{-x}{y}\right)^{n}\right) \quad [\mathbb{C}\text{-}10]$$

The hook load relation during lowering for non-uniform movement of the travelling block is given by Eqn [I-2B] as

$$W = \sum_{i=1}^{n} F_i + (m_{dp} + m_{tb})a$$

$$W = a\left(\left(\frac{m_p \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right)\right)\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m \mu_a r_a}{(x+y)}\right)\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + \left(\frac{m_p g \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{x}{(x+y)}\right)\left(1-\left(\frac{-x}{y}\right)^{n}\right) + (m_{dp} + m_{tb})a$$

$$W = a\left(\left(\frac{m_p \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right)\right) + (m_{dp} + m_{tb})\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m \mu_a r_a}{(x+y)}\right)\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right) + \left(\frac{m_p g \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{x}{(x+y)}\right)\left(1-\left(\frac{-x}{y}\right)^{n}\right) \qquad [\mathbb{C}\text{-11A}]$$

Alternatively, the hook load (W) during lowering for non-uniform movement of the travelling equipment is given by Eqn [I-2C] as

$$W = \frac{\sum_{i=1}^{n} F_i}{\left(1-\frac{a}{g}\right)} = \frac{\sum_{i=1}^{n} F_i}{\left(\frac{g-a}{g}\right)} = \left(\frac{g}{g-a}\right)\sum_{i=1}^{n} F_i$$

Substituting Eqn [ℂ-10] into Eqn [I-2C] gives

$$W = \left(\frac{g}{g-a}\right)\left(a\left(\left(\frac{m_p \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{r-1}\left(1-\left(\frac{-x}{y}\right)^{n-(2k+1)}\right)\right) + \left(\frac{m_p({R_1}^2+{R_2}^2)}{2r_b(x+y)}\right)\left(\sum_{k=0}^{n-2}(-1)^{k+1}(k+1)\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right)\right)\right) + {V_{tb}}^2\left(\frac{2\bar{\lambda}_m \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-2}(k+1)^2\left(1-\left(\frac{-x}{y}\right)^{n-(k+1)}\right)\right) + \left(\frac{m_p g \mu_a r_a}{(x+y)}\right)\left(\sum_{k=0}^{n-1}(-1)^{k+1}\left(1-\left(\frac{-x}{y}\right)^{n-k}\right)\right) - F_{dl}\left(\frac{x}{(x+y)}\right)\left(1-\left(\frac{-x}{y}\right)^{n}\right)\right) \qquad [\mathbb{C}\text{-11B}]$$

Where $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$

$x + y = \mu_a r_a - r_b + \mu_a r_a + r_b = 2\mu_a r_a$

Hence, μ_a can be as small as possible but should not be equal to 0 (i.e. $\mu_a = 0.000001$ $\mu_a \neq 0$) since $\frac{1}{\mu_a} = \frac{1}{0} = undefined$

Eqn [ℂ-11A] and Eqn [ℂ-11B] are the extended Cayeux et al hook load (W) relations during hoisting for non-uniform movement of the travelling equipment.

5 Analysis of the Extended Hook Load Prediction Models Using Hypothetical Data

The analysis of the extended models will be done with hypothetical data. The output of the extended Cayeux et al hook load prediction model for a given coefficient of friction will be used as input for analysing the extended Luke and Juvkam-Wold model as illustrated in the figure (30) below.

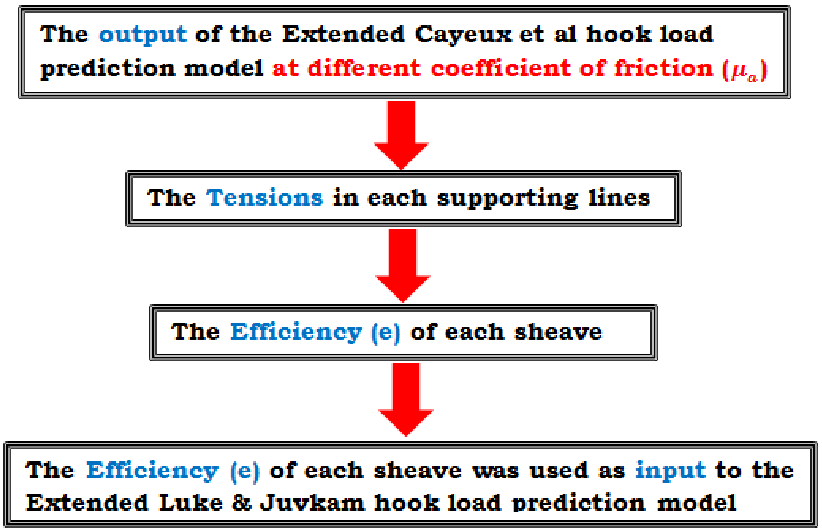

Figure 30: Schematic illustrating how the output of the extended Cayeux et al hook load prediction model was used as input to the extended Luke and Juvkam hook load prediction model

In addition, two (2) hook load prediction methods were developed for each extended model. The first approach is a function of the sum of the tensions in the supporting lines($\sum_{i=1}^{n} F_i$), the mass of the drill pipe(m_{dp}), the mass of the travelling block(m_{tb}), and the acceleration (a) of the travelling equipment as given in the relation below.

$$W = \sum_{i=1}^{n} F_i \pm a(m_{dp} + m_{tb})$$

The sum of the tensions in the supporting lines can easily be determined from the dead line tension(F_{dl}). Since hypothetical data was used in analysing the extended models, it will be very difficult to predict the dead line tension (F_{dl}) that will correspond to a given travelling equipment mass($m_T = m_{dp} + m_{tb}$). Hence, the first approach cannot be analysed using the hypothetical data.

On the other hand, the second hook load (W) prediction approach is also a function of the sum of the tensions in the supporting lines($\sum_{i=1}^{n} F_i$), the acceleration (a) of the travelling equipment and the acceleration due to gravity(g) as illustrated in the relation below.

$$W = \frac{\sum_{i=1}^{n} F_i}{\left(1 \pm \frac{a}{g}\right)}$$

Similarly, the sum of the tensions in the supporting lines can easily be determined from the dead line tension(F_{dl}). Since the mass of the travelling equipment (m_T) has already been incorporated into the hook load (W), the hypothetical data can be used to analyse the extended model.

The hook load measurements during non-uniform movement of the travelling equipment for both hoisting and lowering will be performed for five (5) different acceleration of the travelling equipment values of a = 0 m/s^2, a = 0.5 m/s^2, a = 1.0 m/s^2, a = 1.5 m/s^2, a = 2.1 m/s^2. They will then be compared with the sum of the tensions in the supporting lines ($\sum_{i=1}^{n} F_i$) during constant movement of the travelling equipment (i.e. a = 0 m/s^2) based on the extended Cayeux hook load prediction model as illustrated in table (1) below.

Acceleration values (a), m/s^2	**Dead line tensions (F_{dl}), N**	**Acceleration values (a), m/s^2**	**Dead line tensions (F_{dl}), N**
0	600		1400
0.5	700		1500
1.0	800		1600
1.5	900		1700
2.1	1000		1800
	1100		1900
	1200		2000
	1300		

Table 1: Shows different acceleration (a) of the travelling equipment values and different dead line tensions (Fdl) used to analysed the extended models.

5.1 ANALYSIS OF THE EXTENDED INDUSTRY ACCEPTED MODEL

5.1.1 HOISTING

The industry assumes a perfect transmission of line tension (i.e. e = 1). The hook load values during hoisting with non-uniform movement of the tavelling equipment will be compared with the sum of the tension in the supporting lines during uniform movement as illustrated below.

Legend	Name of Equation
W (Eqn [E-1B]) at different acceleration	The Extended Industry accepted hook load prediction model during hoisting
$\Sigma(F_1+F_2+F_3+F_4)$ at a = 0 m/s²	The sum of the tensions in the supporting lines during hoisting based on the Industry accepted hook load prediction model.

HOOK LOAD MEASUREMENT FOR $e = 1$

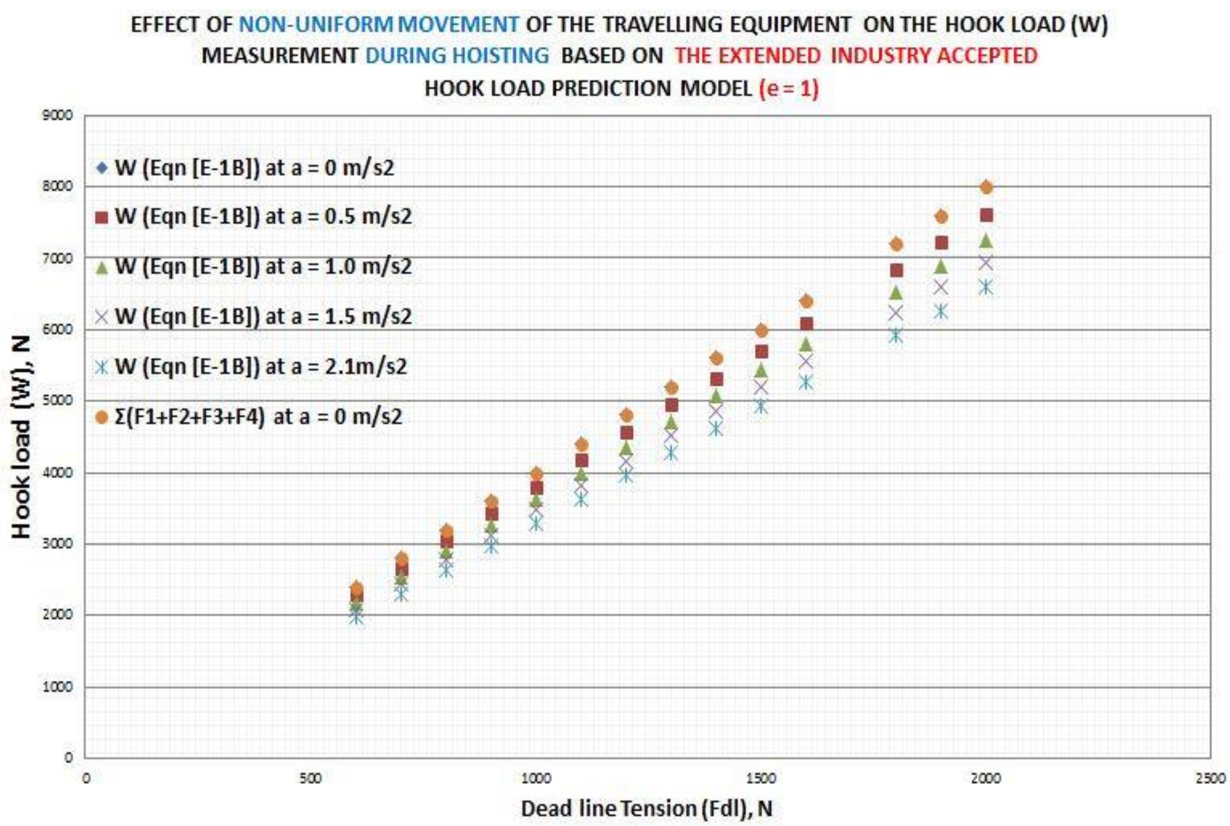

Figure 31: Shows the extended Industry accepted hook load value during hoisting with non-uniform movement of the travelling equipment

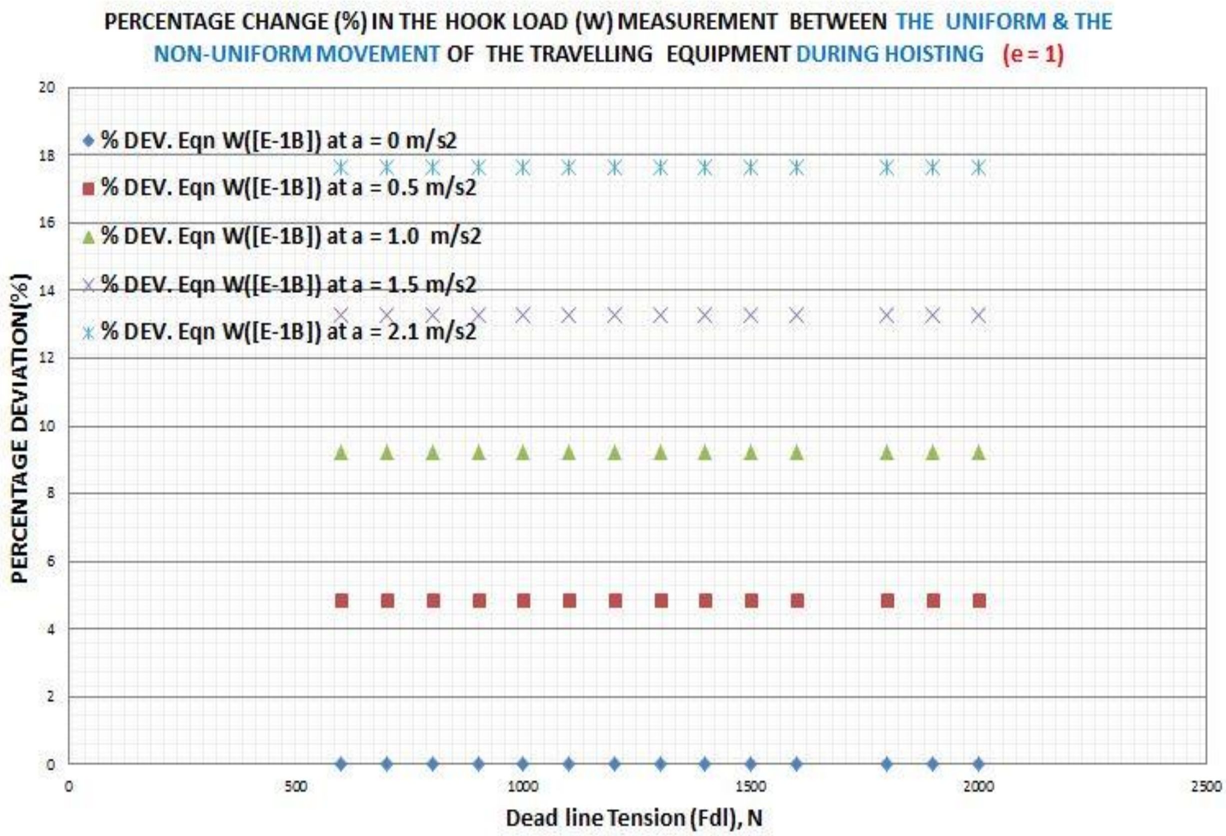

Figure 32: Shows the percentage deviation of the extended Industry accepted hook load values during hoisting with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement also based on the Industry accepted hook load prediction model

COMMENT: It can be observed that during uniform movement of the travelling equipment, the sum of the tensions in the supporting lines is the same as the hook load (W). During non-uniform movement, the hook load decreases with increasing acceleration (a) of the travelling equipment. Hence, the higher the acceleration (a), the higher the deviation of the non-uniform hook load values from the sum of the tensions in the supporting lines during uniform movement of the travelling equipment.

5.1.2 Lowering

Still with the assumption that the transmission of the line tension is perfect (i.e. e =1) as proposed in the industry accepted hook load model, the extended industry accepted hook load values during lowering with non-uniform movement of the tavelling equipment will be compared with the sum of the tension in the supporting lines during uniform movement also based on the industry accepted hook load prediction model. The relations used in the analysis are given below.

Legend	Name of Equation
W (Eqn [E-2B]) at different acceleration	The Extended Industry accepted hook load prediction model during lowering
$\Sigma(F_1+F_2+F_3+F_4)$ at a = 0 m/s^2	The sum of the tensions in the supporting lines during lowering based on the Industry accepted hook load prediction model

HOOK LOAD MEASUREMENT FOR $e = 1$

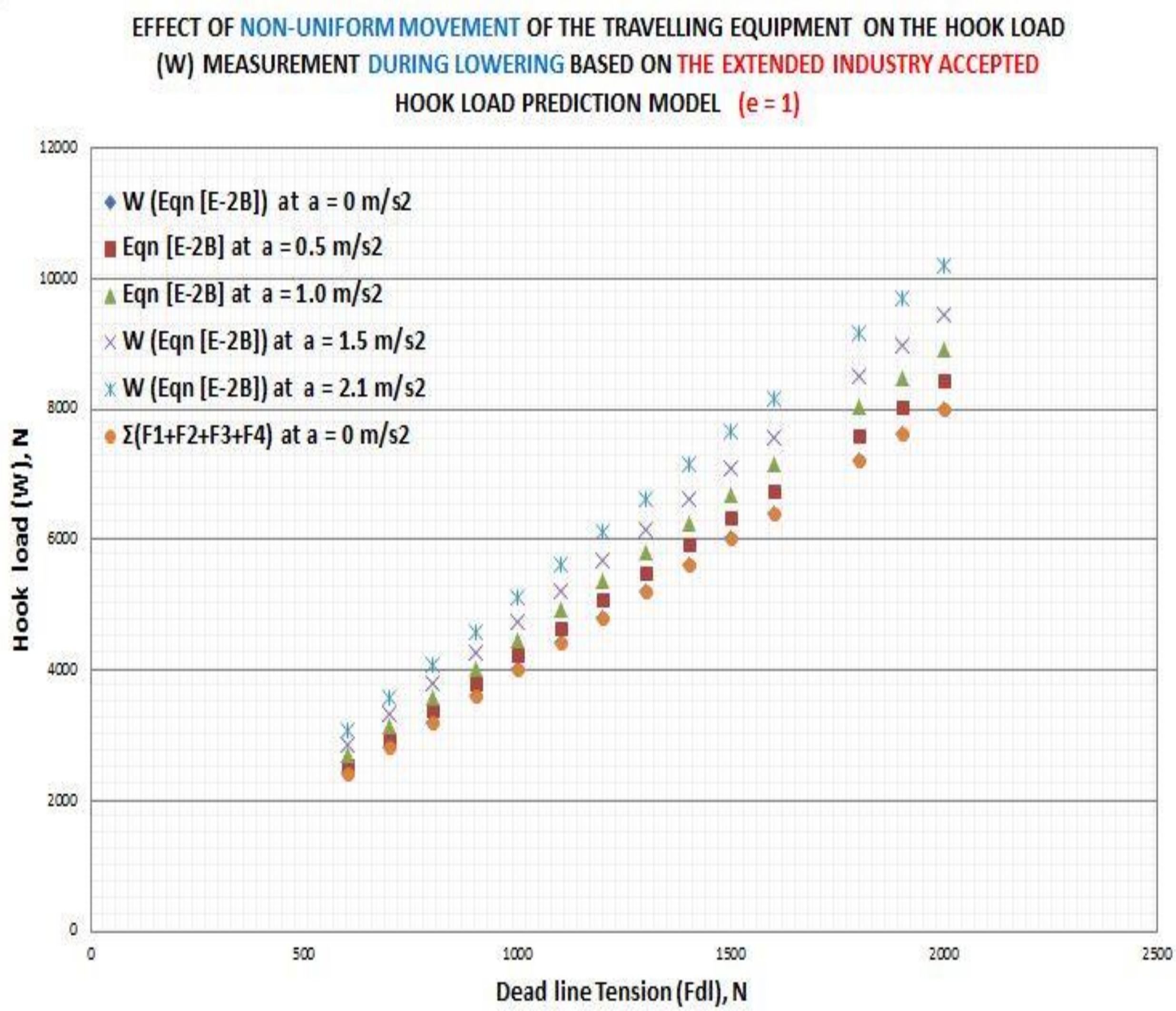

Figure 33: Shows the extended Industry accepted hook load values during lowering with non-uniform movement of the travelling equipment

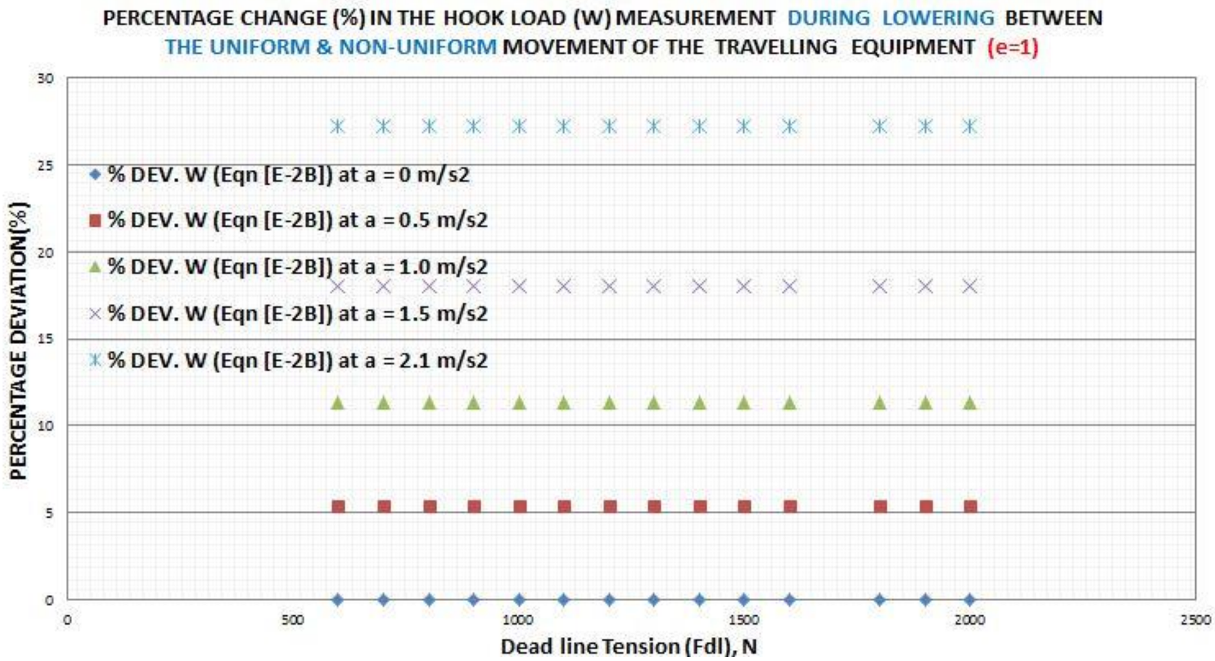

Figure 34: Shows the percentage deviation of the extended Industry accepted hook load values during lowering with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement also based on the Industry accepted hook load prediction model

COMMENT: For either hoisting or lowering, it can be observed that during uniform movement, the hook load measurement is the same as the sum of the tensions in the supporting lines. Hence, no deviation between the two hook load values as illustrated by figure (32) and figure (34) respectively. During lowering with non-uniform movement of the travelling equipment, the hook load (W) always exceeds the sum of the tensions in the supporting lines unlike during hoisting as illustrated in figure (33) and figure (31) respectively. The higher the acceleration (a) of the travelling equipment, the higher the hook load values become and vice-versa.

5.2 Analysis of the extended Cayeux et-al hook load prediction model

In the analysis of the extended Cayeux et al hook load prediction model, the effect of the coefficient of friction (μ_a) on the sheave efficiency (e) will be analysed first. After which the relationship between the tensions in the lines relative to the dead line tension during either hoisting or lowering will also be performed. Finally, the hook load during hoisting and lowering will be analysed respectively.

5.2.1 Effect of the coefficient of friction on the sheave efficiency

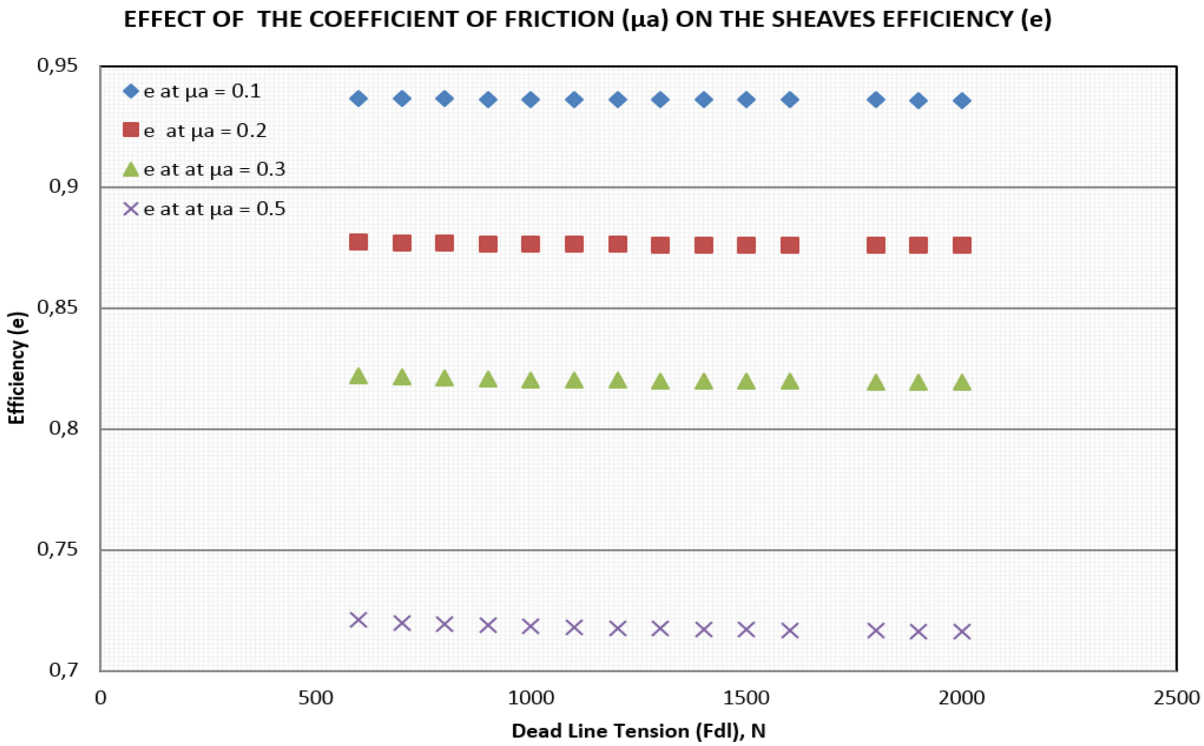

Figure 35: Shows the effect of the coefficient of friction on the efficiency of each sheave based on the extended Cayeux et al hook load prediction model

COMMENT: The higher the coefficient of friction (μ_a) at the sheave axle, the lower the efficiency (e) of the sheave becomes and vice-versa.

5.3 Hoisting

The relationship between the tensions in the lines relative to the dead line tension (F_{dl}) will be analysed first after which the hook load (W) analysis will also be carried out.

5.3.1 Tensions in the line during uniform movement of the travelling equipment

The extended Cayeux et al line tension relations during hoisting with uniform movement of the travelling equipment was compared with the original Cayeux et al line tension relations at three (3) different coefficient of friction $\mu_a = 0.0$, $\mu_a = 0.1$ & $\mu_a = 0.3$ to illustrate the effect of coefficient of friction on the tensions in the lines. Below are the equations used in the analysis.

Legend	Name of Equation
Eqn [γ-5D]	The Extended Cayeux et al line tension relation during hoisting
Eqn [C-1] & Eqn [C-3]	The Original Cayeux et al line tension relations during hoisting for the both the crown block & the travelling block sheaves respectively

A. TENSION IN THE LINES FOR $\mu_a = 0.0$

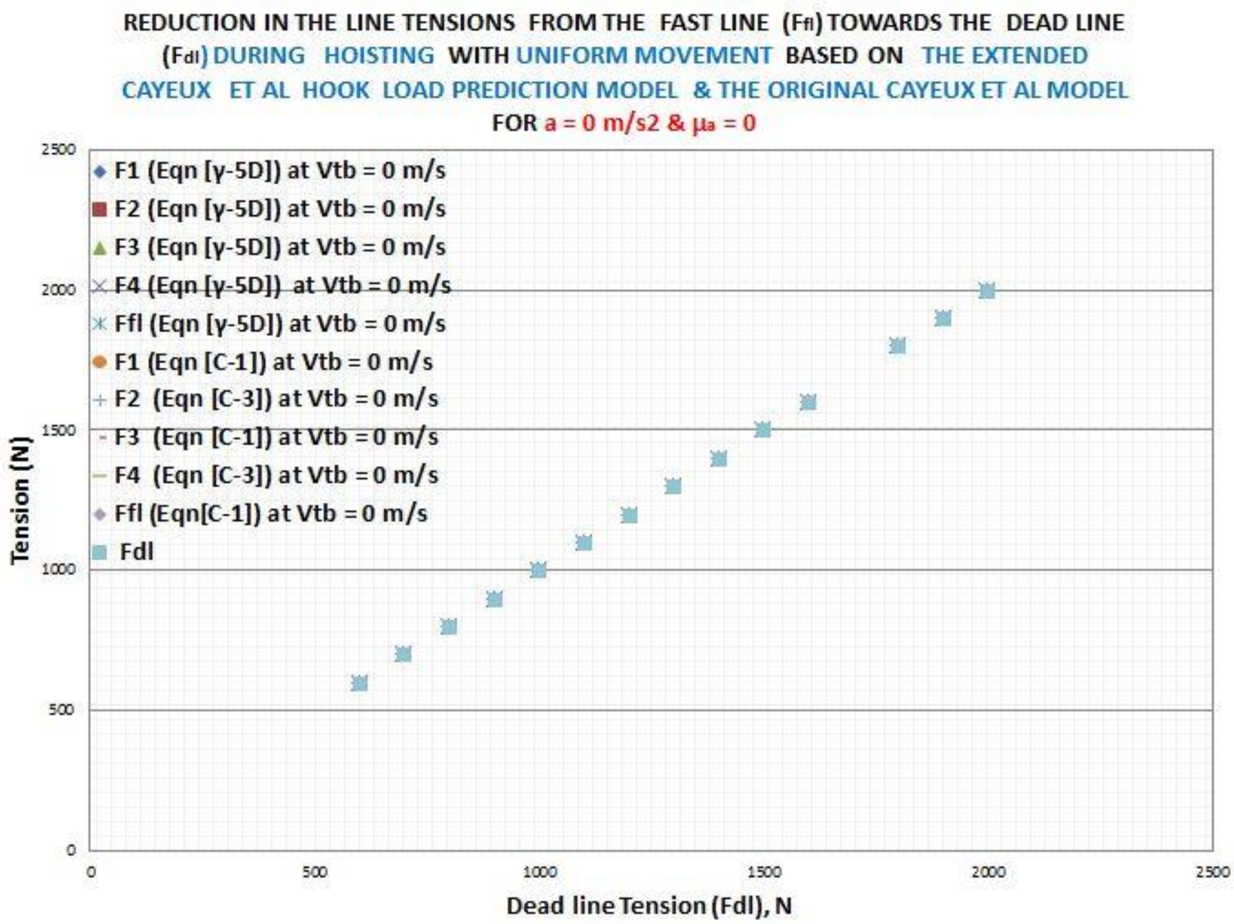

Figure 36: Shows the tensions in the lines with perfect transmission of the line tension ($\mu_a = 0$)

COMMENT: For perfect transmission of the line tensions, there is no work done against friction and hence, the fast line tension (F_{fl}) is the same as the dead line tension (F_{dl}). (i.e. $F_{fl} - F_{dl} = 0$). In addition, the extended Cayeux et al line tension relation output overlaps with the original Cayeux et al line tension relation output.

B. TENSION IN THE LINE FOR $\mu_a = 0.1$

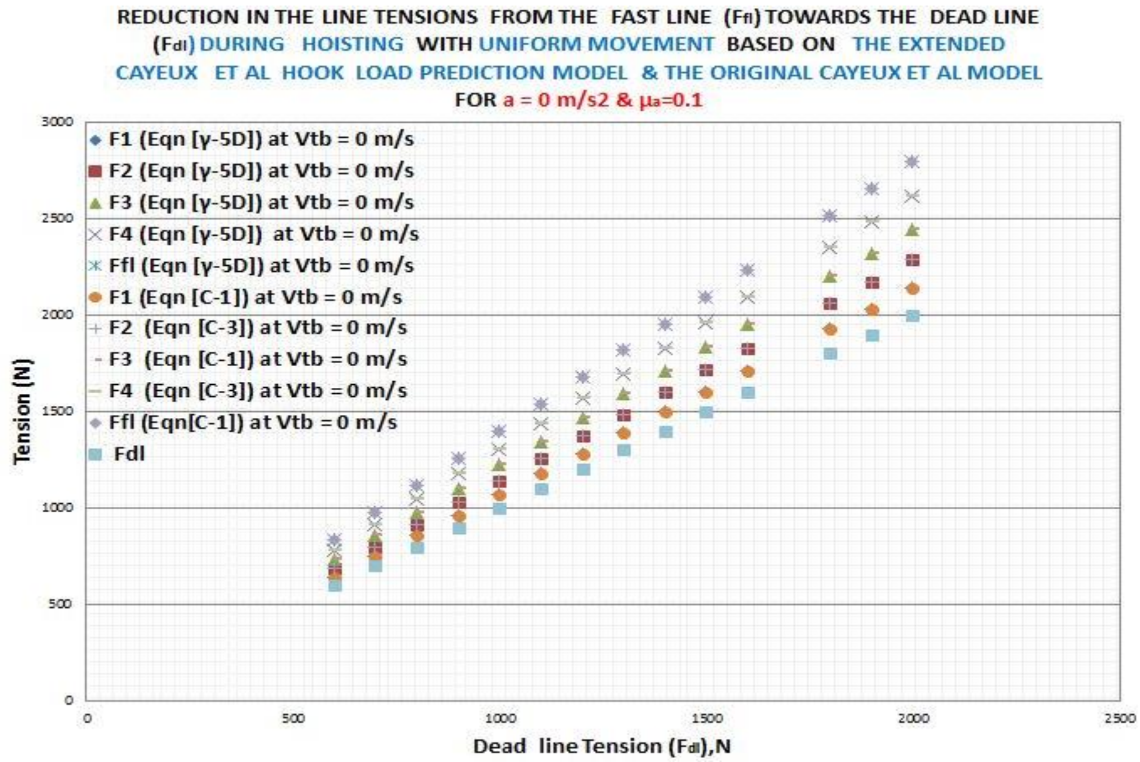

Figure 37: Shows the tensions in the lines during slightly imperfect transmission of the line tension ($\mu_a = 0.1$)

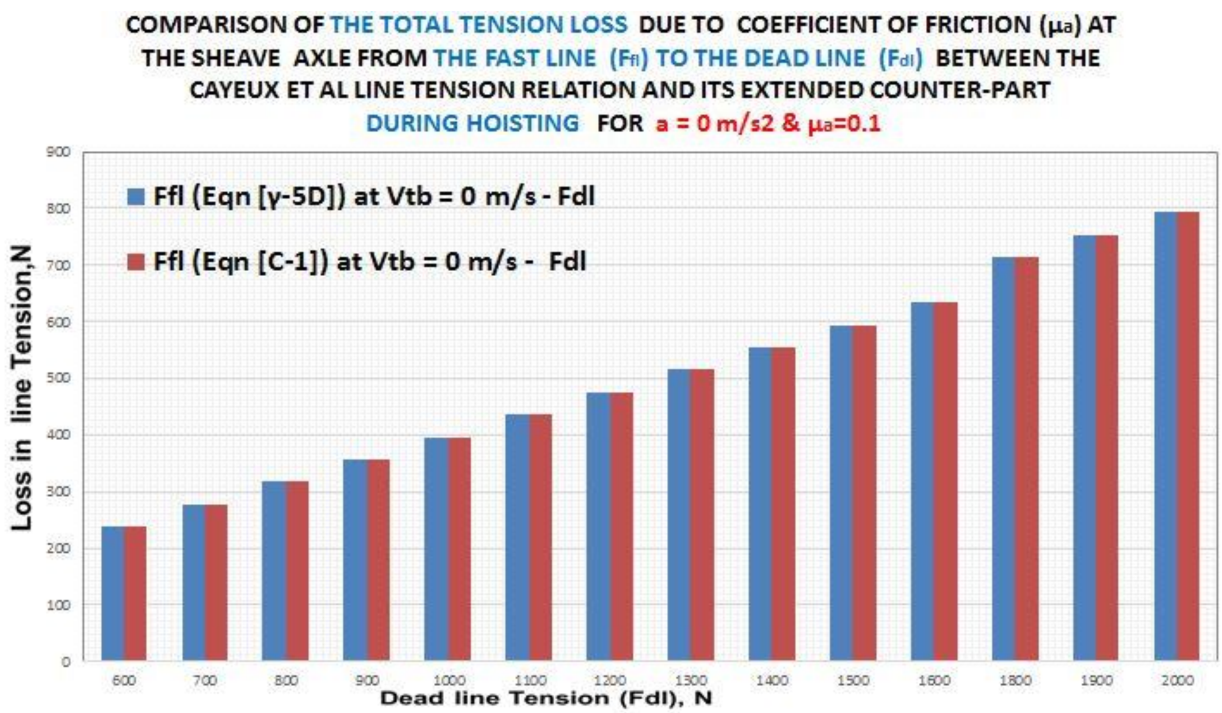

Figure 38: Shows the total tension loss from the fast line (F_{fl}) to the dead line (F_{dl}) during imperfect transmission of the line tension ($\mu_a = 0.1$)

COMMENT: For imperfect transmission of the line tension, there is work done against friction. Hence, the friction needs to be overcome before the load can be raised thereby resulting in higher tensions in the lines as compared to when there is perfect transmission of tensions in the lines. The tension decreases from the fast line (F_{fl}) towards the dead line (F_{dl}). (i.e. $F_{fl} - F_{dl} \neq 0$). In addition, the extended Cayeux et al line tension relation produces the same loss in line tension as its original counterpart.

C. TENSION IN THE LINES FOR $\mu_a = 0.3$

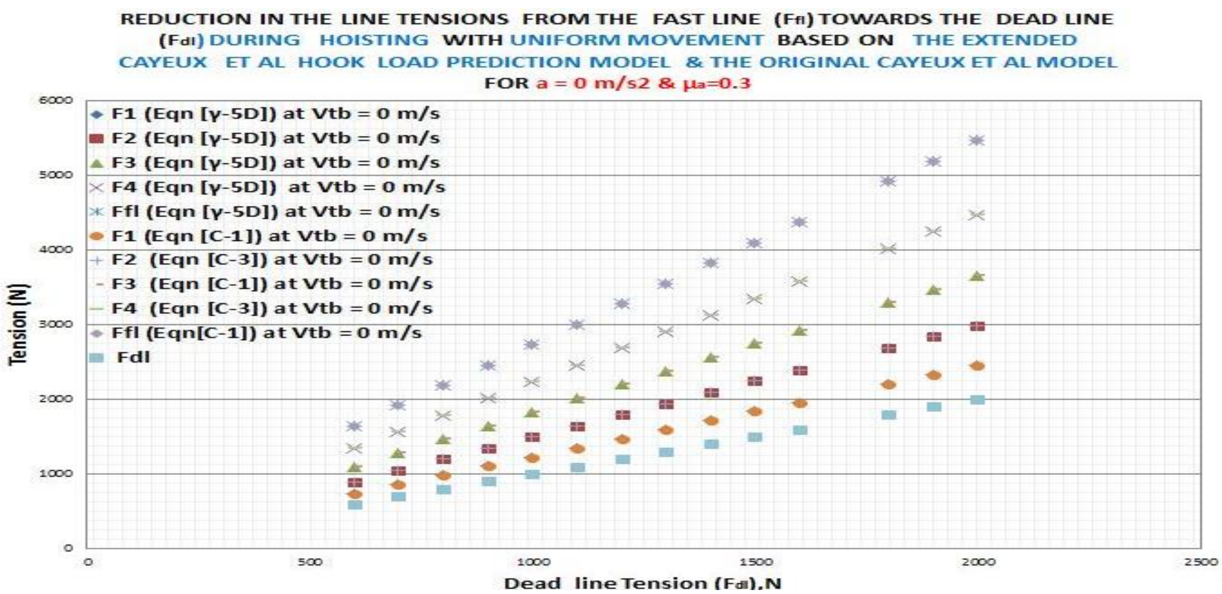

Figure 39: Shows the tensions in the lines during imperfect transmission of the line tension ($\mu_a = 0.3$)

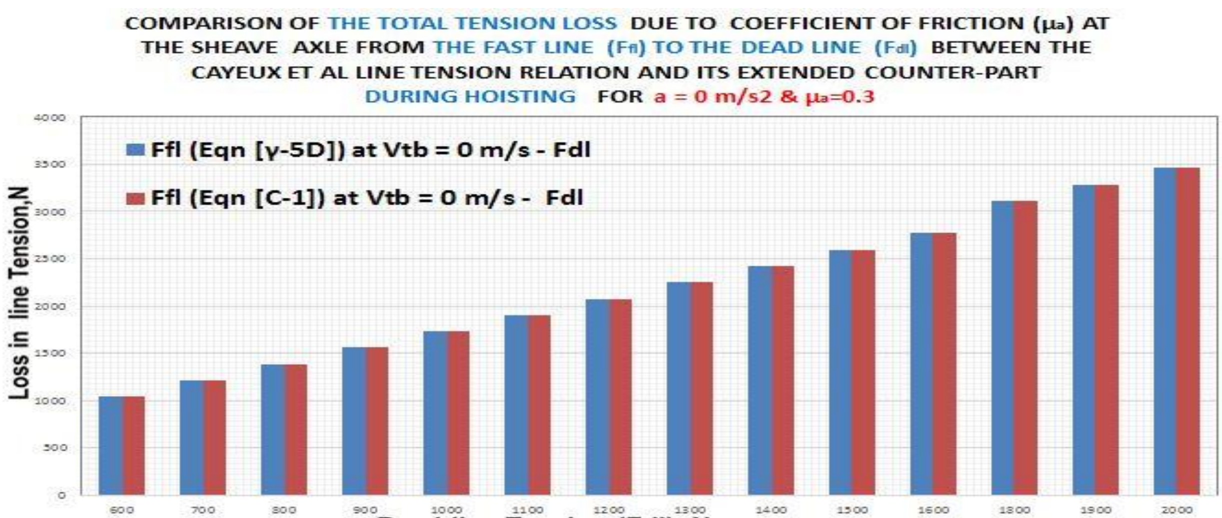

Figure 40: Shows the total tension loss from the fast line (F_{fl}) to the dead line (F_{dl}) during imperfect transmission of the line tension ($\mu_a = 0.3$)

COMMENT: For imperfect transmission of the tensions in the lines, the higher the coefficient of friction at the sheave axle, the higher the work done against friction. Hence, the higher the reduction in the line tensions from the fast line (F_{fl}) towards the dead line (F_{dl}) and vice-versa. In addition, the extended Cayeux et al line tension relation match perfectly with its original counterpart since both models produced the same loss in line tension for a given dead line tension (F_{dl}) and coefficient of friction as illustrated in the figure (40) above.

5.3.1.1 HOISTING WITH NON-UNIFORM MOVEMENT OF THE TRAVELLING EQUIPMENT

The extended Cayeux et al hook load prediction model will be analysed for five (5) different acceleration (a) values at different coefficients of friction and compared with the sum of the tensions in the supporting lines during hoisting with uniform movement of the tavelling equipment also based on the extended Cayeux et al hook load prediction model. The relations used in the analysis are as illustrated in the table below.

Legend	Name of Equation
W (Eqn [α-11B]) at different acceleration	The Extended Cayeux et al hook load prediction model during hoisting
$\Sigma(F_1+F_2+F_3+F_4)$ at $a = 0\ m/s^2$	The sum of the tensions in the supporting lines during hoisting based on the extended Cayeux et al hook load prediction model

A. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.1$

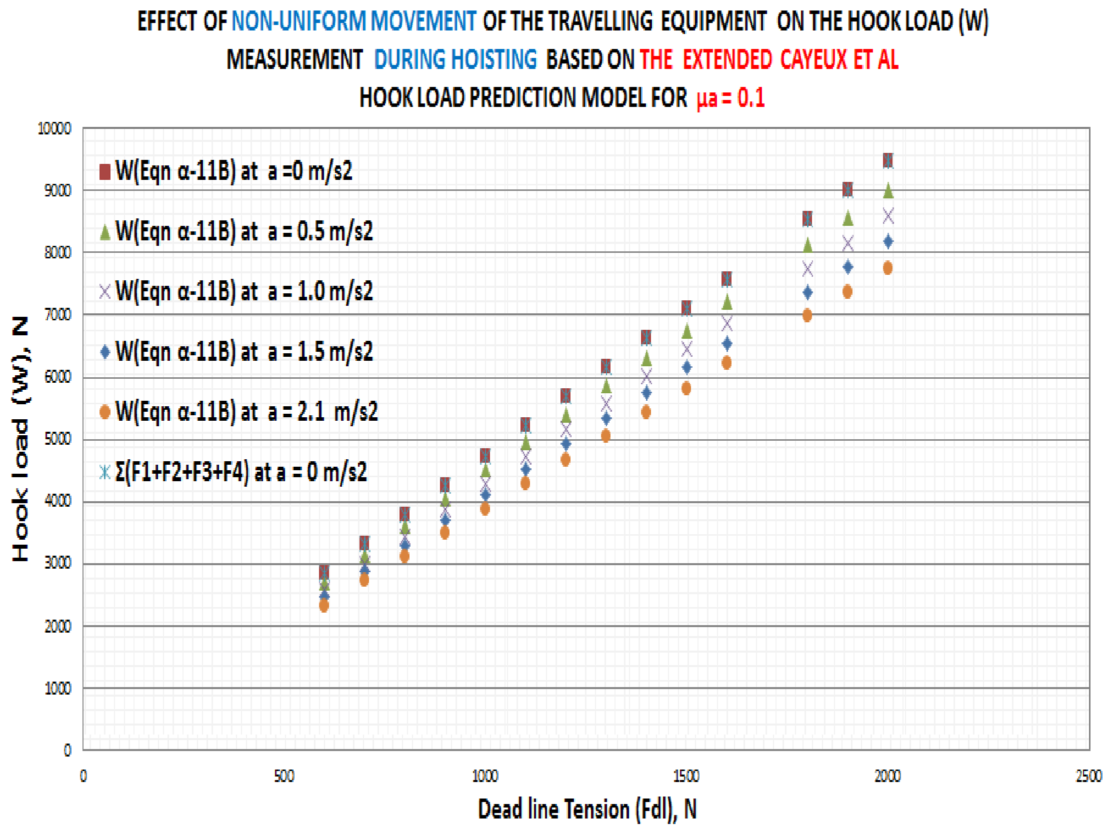

Figure 41: Shows the extended Cayeux et al hook load value during hoisting with non-uniform movement of the travelling equipment for $\mu_a = 0.1$

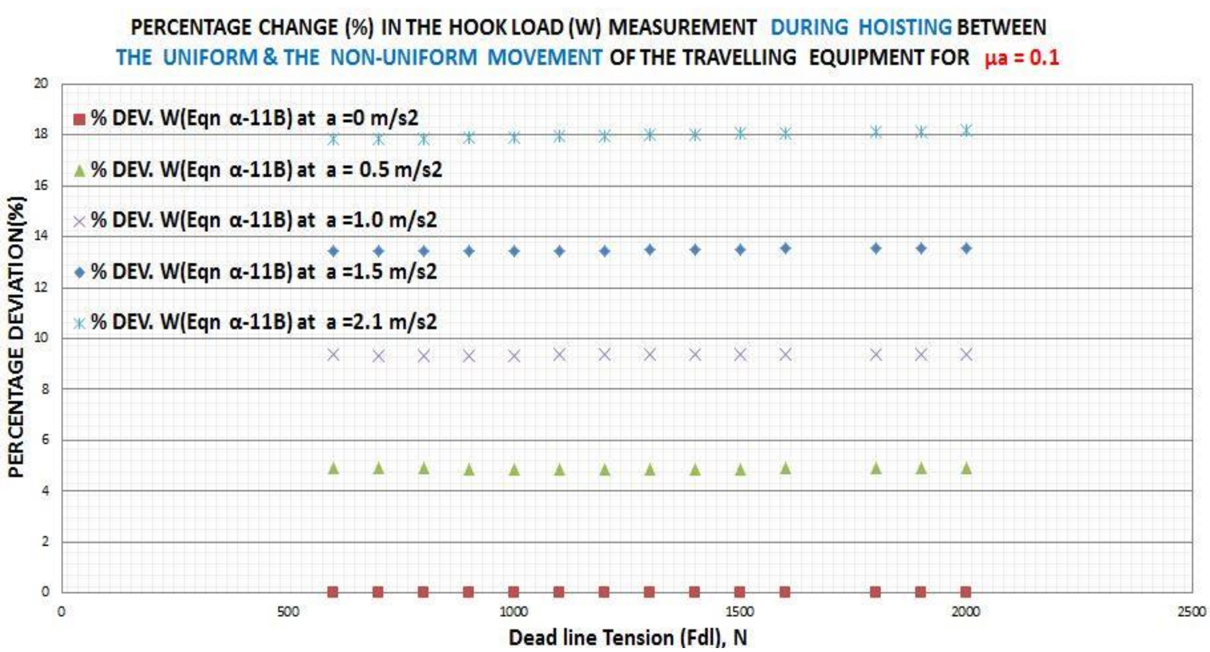

Figure 42: Shows the percentage deviation of the extended Cayeux et al hook load value during hoisting with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $\mu_a = 0.1$

COMMENT: It can be observed that the maximum hook load (W) value during hoisting occurs when there is uniform movement of the travelling equipment. During uniform movement of the travelling equipment, the hook load value overlaps with the sum of the tension in the supporting lines with the deviation between the models being 0% as illustrated in figure (41) and figure (42) respectively. On the other hand, during non-uniform movement of the travelling equipment, the hook load decreases with increasing acceleration (a) of the travelling equipment. Hence, the higher the acceleration, the higher the deviation of the non-uniform hook load value from its corresponding uniform counterpart.

B. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.3$

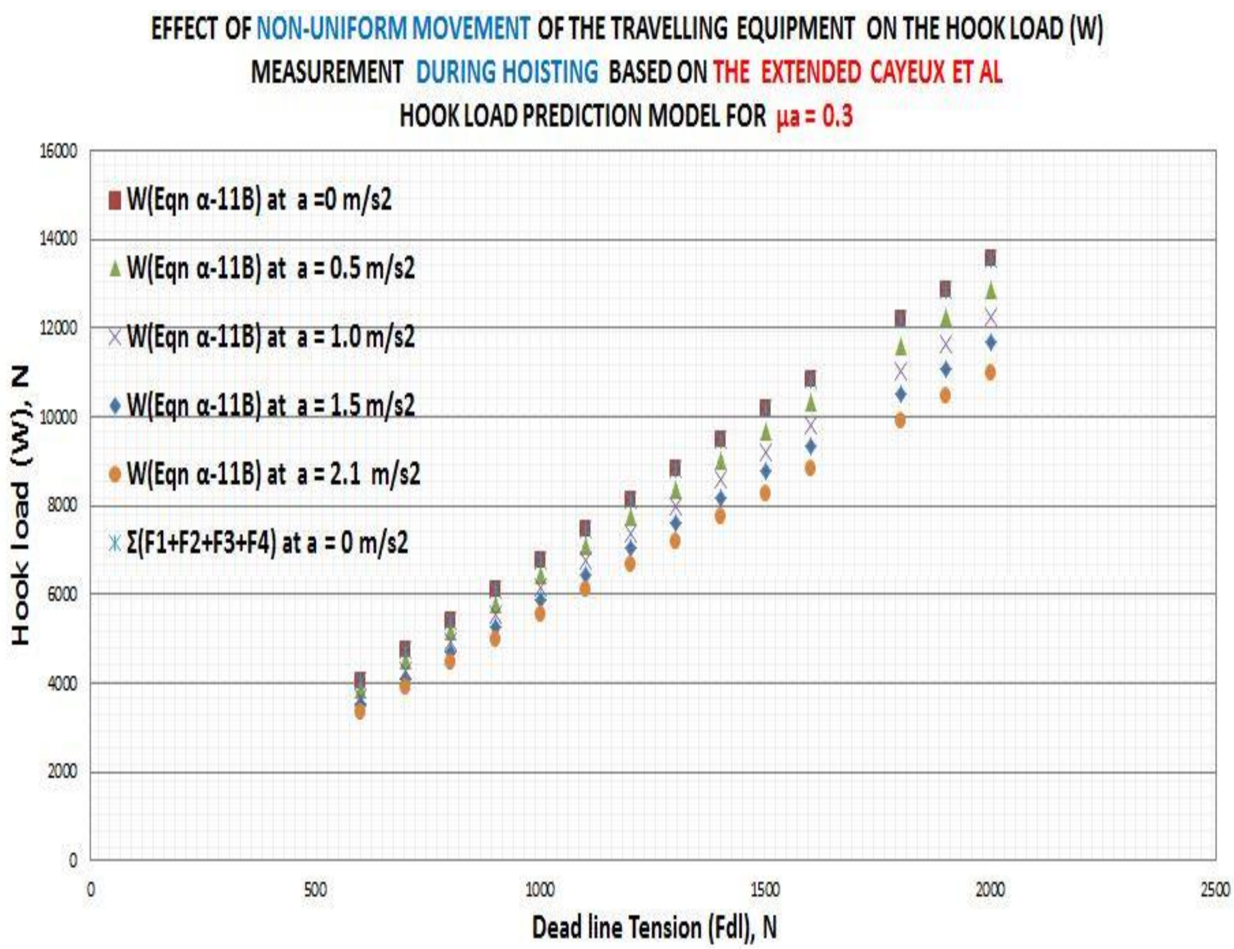

Figure 43: Shows the extended Cayeux et al hook load value during hoisting with non-uniform movement of the travelling equipment for $\mu_a = 0.3$

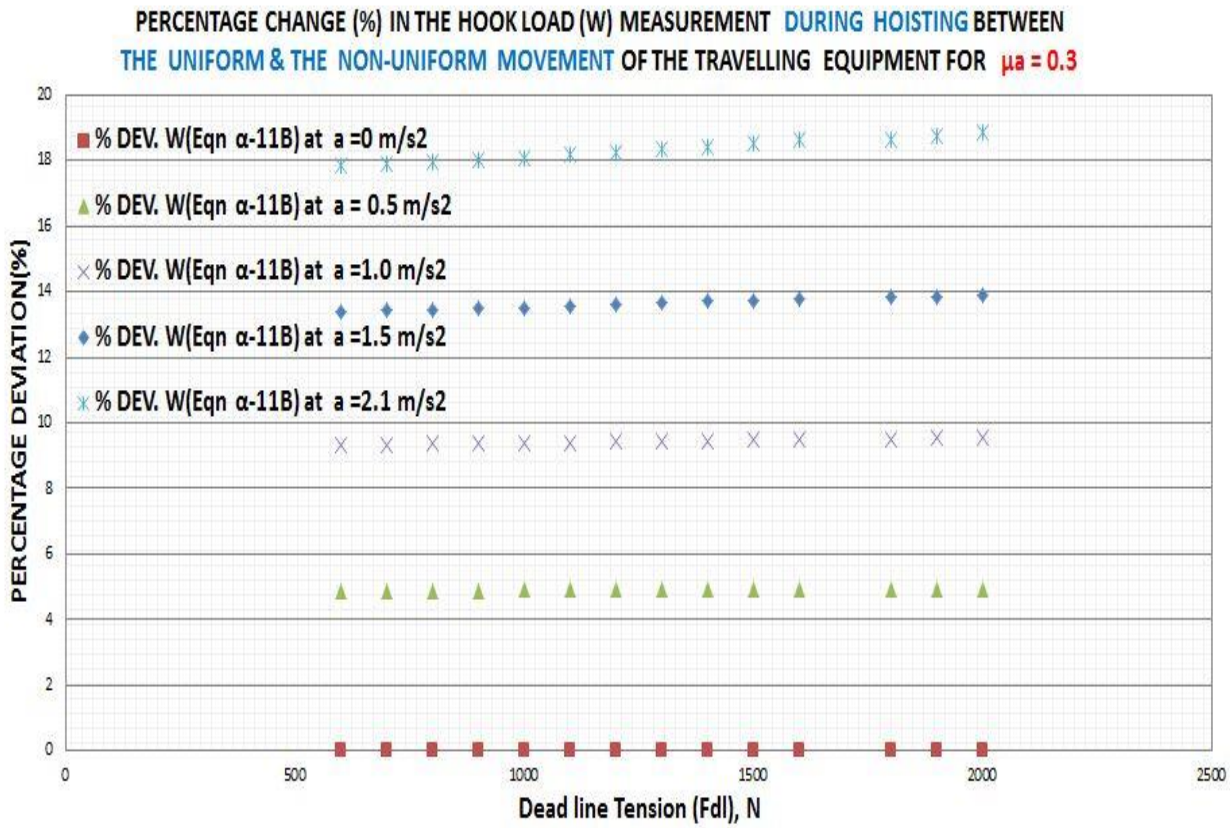

Figure 44: Shows the percentage deviation of the extended Cayeux et al hook load value during hoisting with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement also based on the extended Cayeux et al hook load prediction model for $\mu_a = 0.3$

COMMENT: From figure (41) and figure (43), it can be observed that for F_{dl} = 2000N and a=0m/s^2, it can be observed that the hook load value increases from approximately 9500N for μ_a=0.1 to approximately 14000N for μ_a=0.3. In a similar vein, considering the same dead line tension (F_{dl} = 2000N) and μ_a= 0.3, it can also be observed that for a = 0.5 m/s^2 corresponds to a hook load value of approximately 13000N while a =1.0 m/s^2 corresponds to a hook load value of about 12500 N as illustrated in figure (43). It can be inferred that the coefficient of friction has higher effect on the hook load value than the effect due to the acceleration. Hence, the higher the coefficient of friction, the higher the work done against friction before the load can be raised even though the hook load also decreases marginally with increase in acceleration (a) of the travelling equipment.

5.4 Lowering

Like the hoisting analysis, the relationship between the tensions in the lines relative to the dead line tension (F_{dl}) will be analysed first after which the hook load analysis will also be performed.

5.4.1 Tensions in the line during lowering with uniform movement of the travelling equipment

The analysis of the extended Cayeux et al line tension relations during lowering with uniform movement of the travelling equipment will be carried for three (3) different coefficients of friction ($\mu_a = 0.0$, $\mu_a = 0.1$ & $\mu_a = 0.3$) and then compared with their original counterpart for a given coefficient of friction. This is to illustrate the effect of the coefficient of friction on the tensions in the lines during lowering.

Legend	Name of Equation
Eqn [δ-5D]	The Extended Cayeux et al line tension relation during lowering with uniform movement
Eqn [C-2] & Eqn [C-4]	The Original Cayeux et al line tension relations during lowering for the crown block sheaves & the travelling block sheaves respectively

A. TENSION IN THE LINES FOR $\mu_a = 0.0$

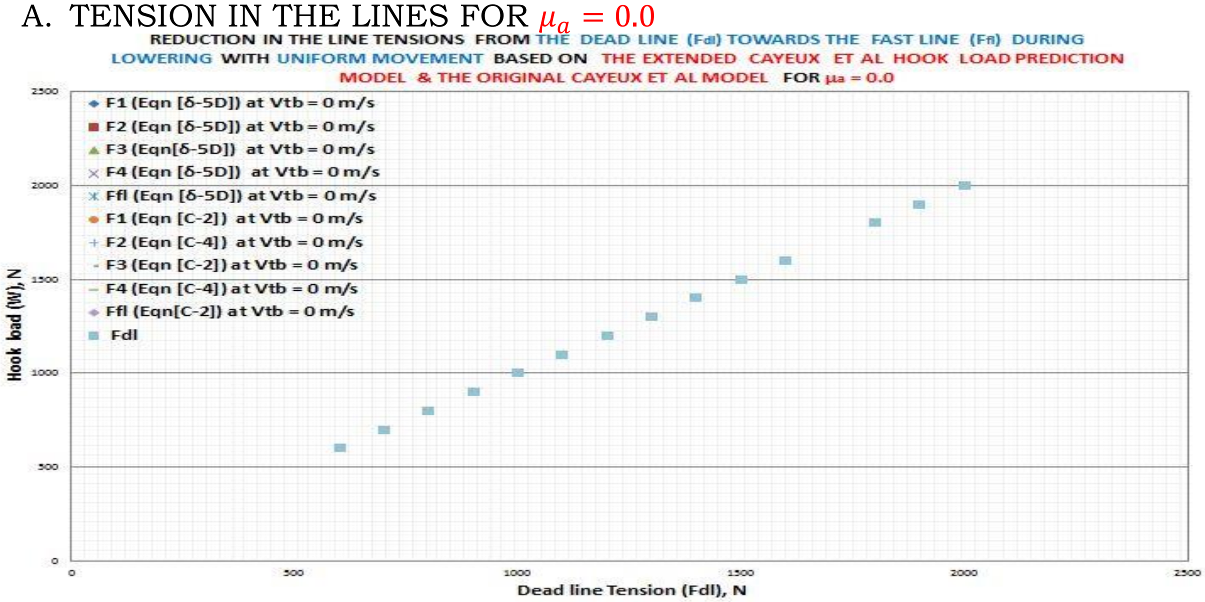

Figure 45: Shows the tensions in the lines with perfect transmission of the line tension ($\mu_a = 0$)

COMMENT: For perfect transmission of the line tension, there is no loss in the line tension from the dead line (F_{dl}) towards the fast line tension (F_{fl}). (i.e. F_{dl} - F_{fl} = 0). In addition, the extended Cayeux et al line tension relation during uniform movement of the travelling equipment produces exactly the same output as its original counterpart.

B. TENSION IN THE LINES FOR $\mu_a = 0.1$

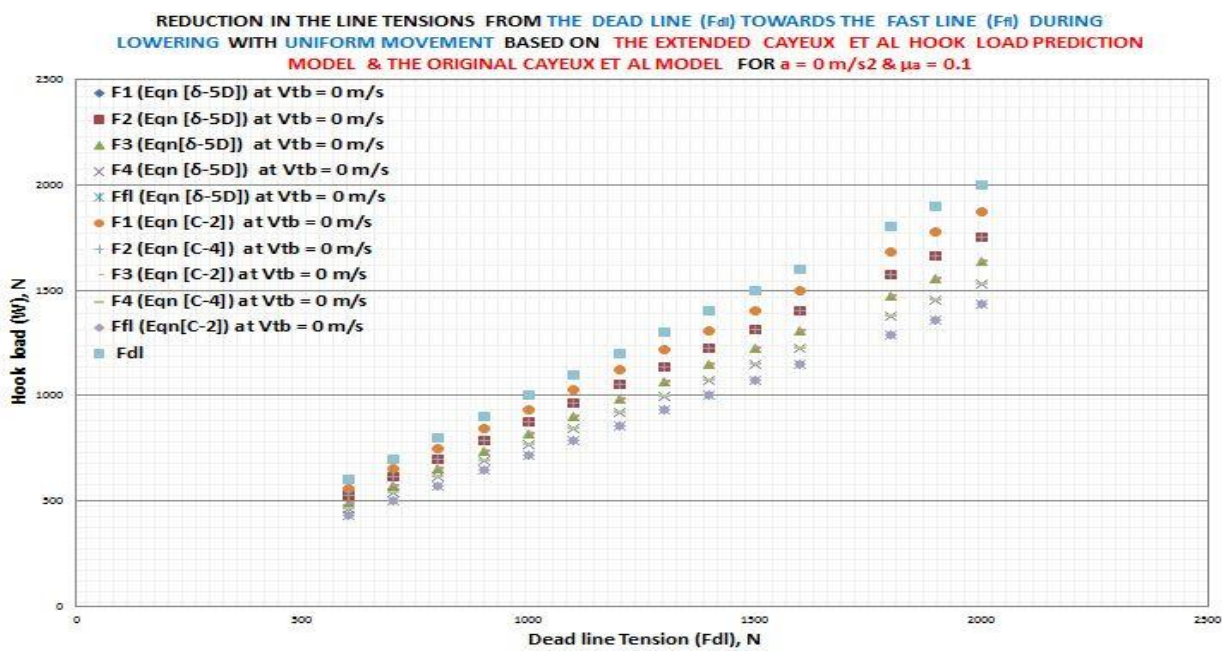

Figure 46: Shows the extended Cayeux et al hook load values during lowering with uniform movement of the travelling equipment for $\mu_a = 0.1$

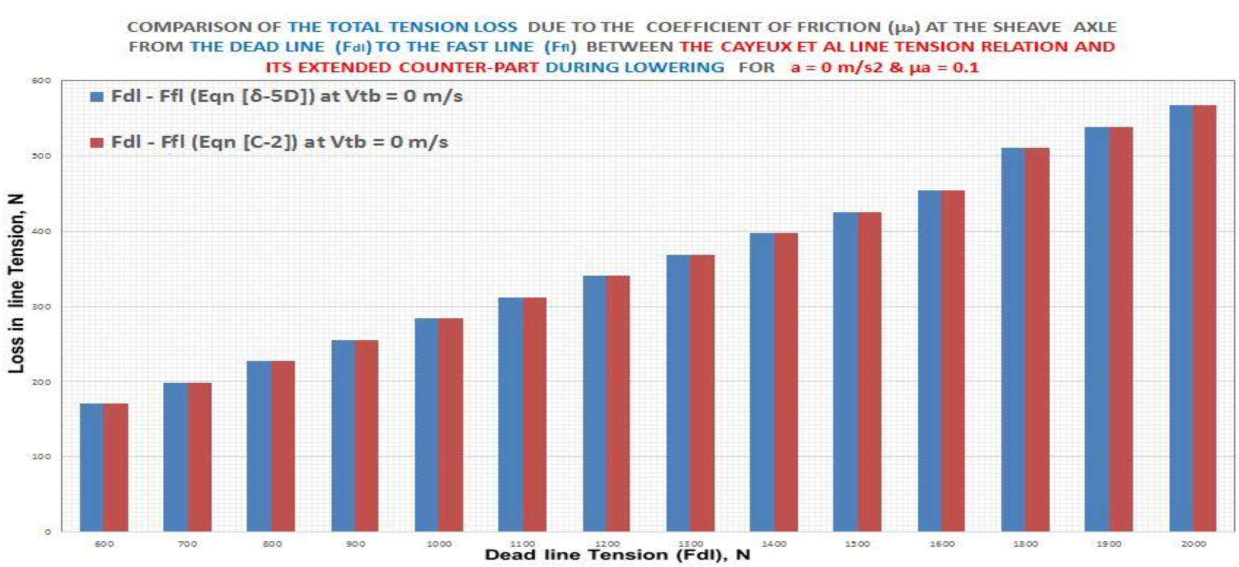

Figure 47: Shows the total tension loss from the dead line (F_{dl}) to the fast line (F_{fl}) during imperfect transmission of the line tension ($\mu_a = 0.1$)

COMMENT: During imperfect transmission of the line tension, there is work done against friction. Hence, there is loss in the line tension from the dead line (F_{dl}) towards the fast line (F_{fl}) depending on the magnitude of the coefficient of friction(μ_a). The extended Cayeux et al line tension relation output overlaps with its original counterpart during uniform movement of the travelling equipment.

C. AT TENSION IN THE LINES FOR $\mu_a = 0.3$

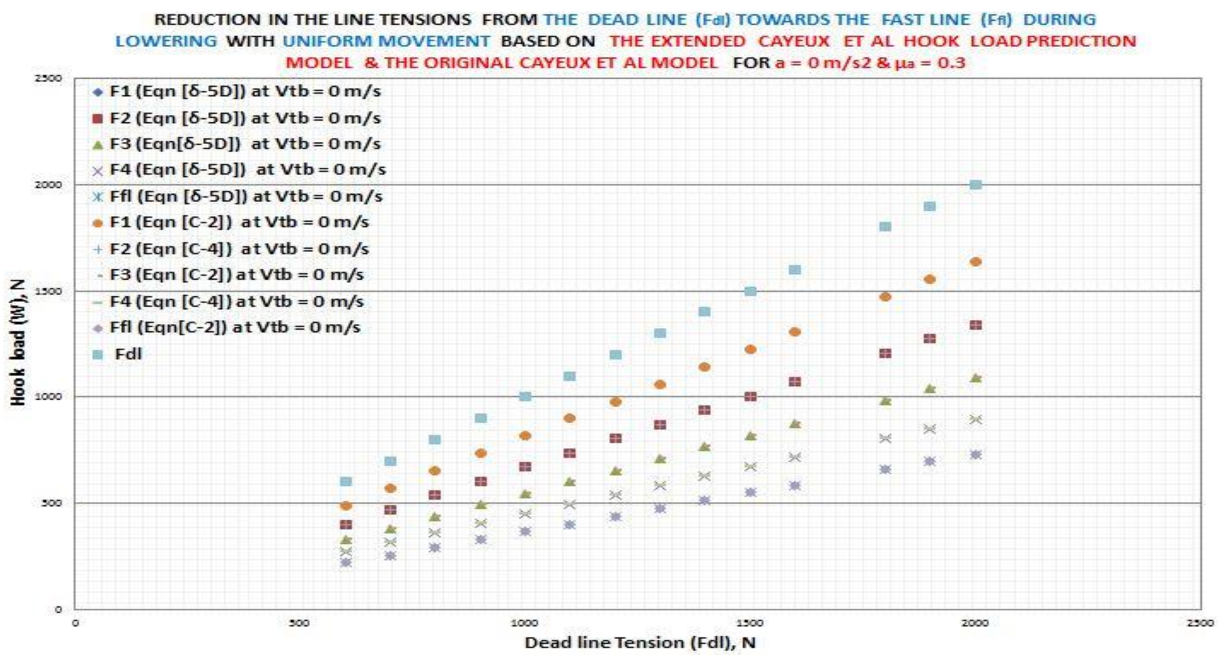

Figure 48: Shows the extended Cayeux et al hook load value during lowering with uniform movement of the travelling equipment for $\mu_a = 0.3$

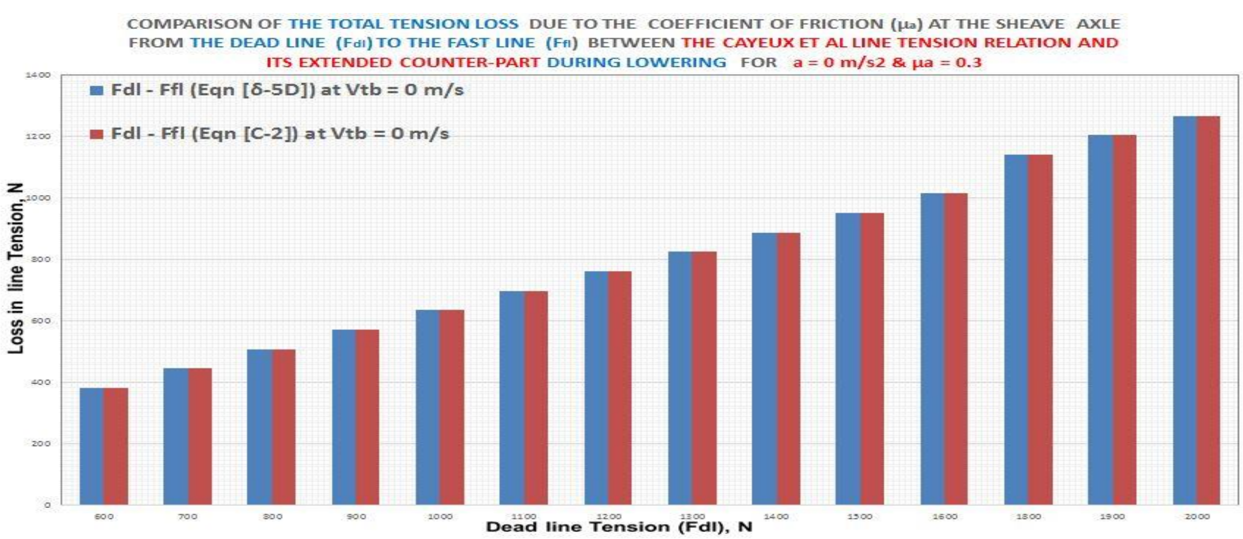

Figure 49: Shows the total tension loss from the dead line (F_{dl}) to the fast line (F_{fl}) during imperfect transmission of the line tension ($\mu_a = 0.3$)

COMMENT: It can be seen that the higher the coefficient of friction, the higher the work done against friction. Thereby resulting in higher loss in the line tension from the dead line (F_{dl}) towards the fast line (F_{fl}) as illustrated in figure (47) and figure (49).

5.4.1.1 HOOK LOAD MEASUREMENT DURING LOWERING WITH NON-UNIFORM MOVEMENT OF THE TRAVELLING EQUIPMENT

The analysis of the extended Cayeux et al hook load prediction model during lowering with non-uniform movement of the travelling equipment will also be analysed for five (5) different acceleration (a) values with varying coefficient of friction. The outcome will then be compared with the sum of the tensions in the supporting lines during lowering with uniform movement of the tavelling equipment also based on the extended Cayeux et al hook load prediction model. Below are the equations used in this analysis.

Legend	Name of Equation
W (Eqn [ℂ-11B]) at different acceleration	The Extended Cayeux et al hook load prediction model during lowering
$\Sigma(F_1+F_2+F_3+F_4)$ at a = 0 m/s²	The sum of the tensions in the supporting lines during lowering based on the extended Cayeux et al hook load prediction model

A. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.1$

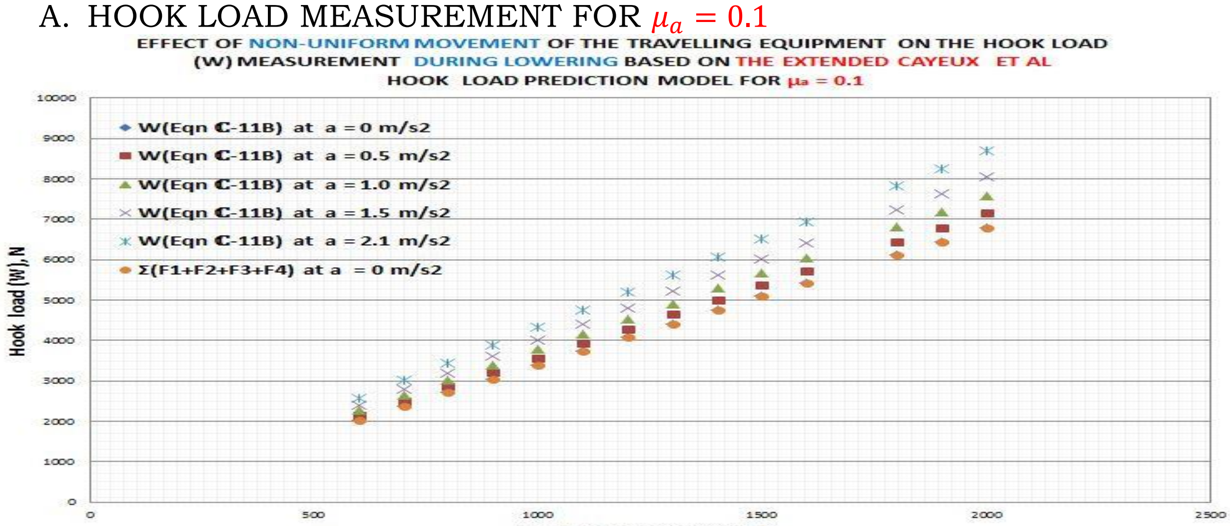

Figure 50: Shows the extended Cayeux et al hook load values during lowering with non-uniform movement of the travelling equipment for $\mu_a = 0.1$

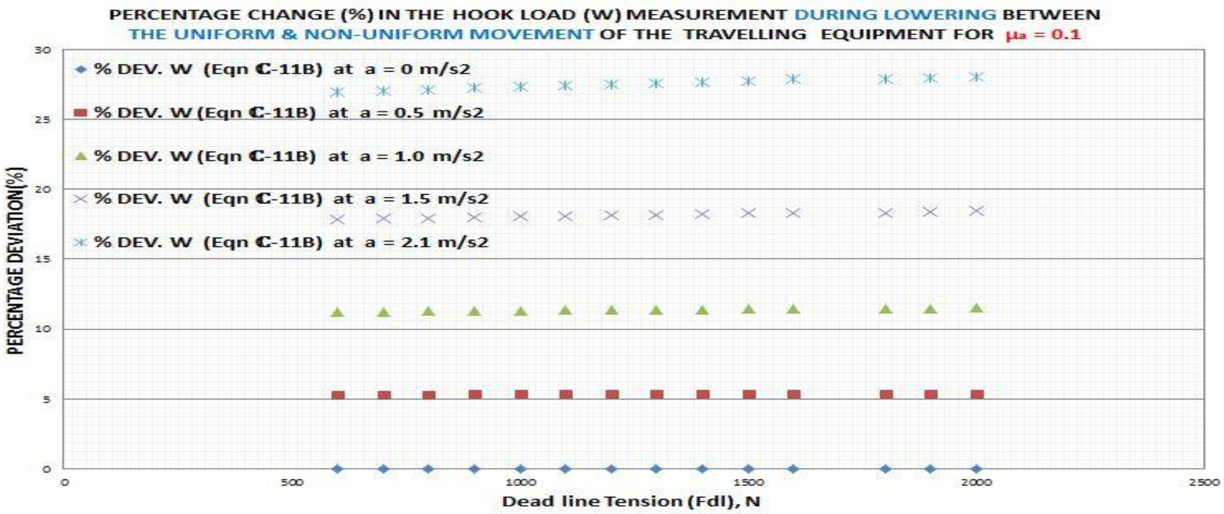

Figure 51: Shows the percentage deviation of the extended Cayeux et al hook load values during lowering with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement also based on the extended Cayeux et al hook load prediction model for $\mu_a = 0.1$

COMMENT: It can be observed that the minimum hook load (W) measurement during lowering occurs when there is uniform movement of the travelling equipment. During non-uniform movement of the travelling equipment, the hook load increases with increasing acceleration (a) of the travelling equipment. The higher the acceleration (a), the higher the deviation of the non-uniform hook load measurement from its corresponding uniform counterpart.

B. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.3$

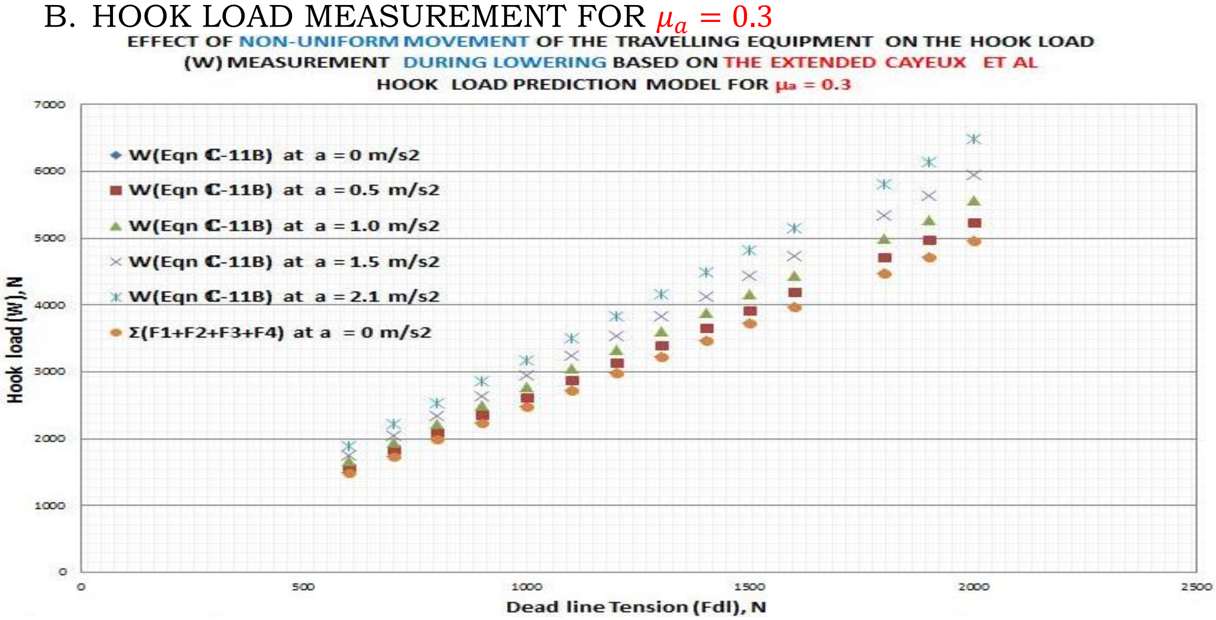

Figure 52: Shows the extended Cayeux et al hook load values during lowering with non-uniform movement of the travelling equipment for $\mu_a = 0.3$

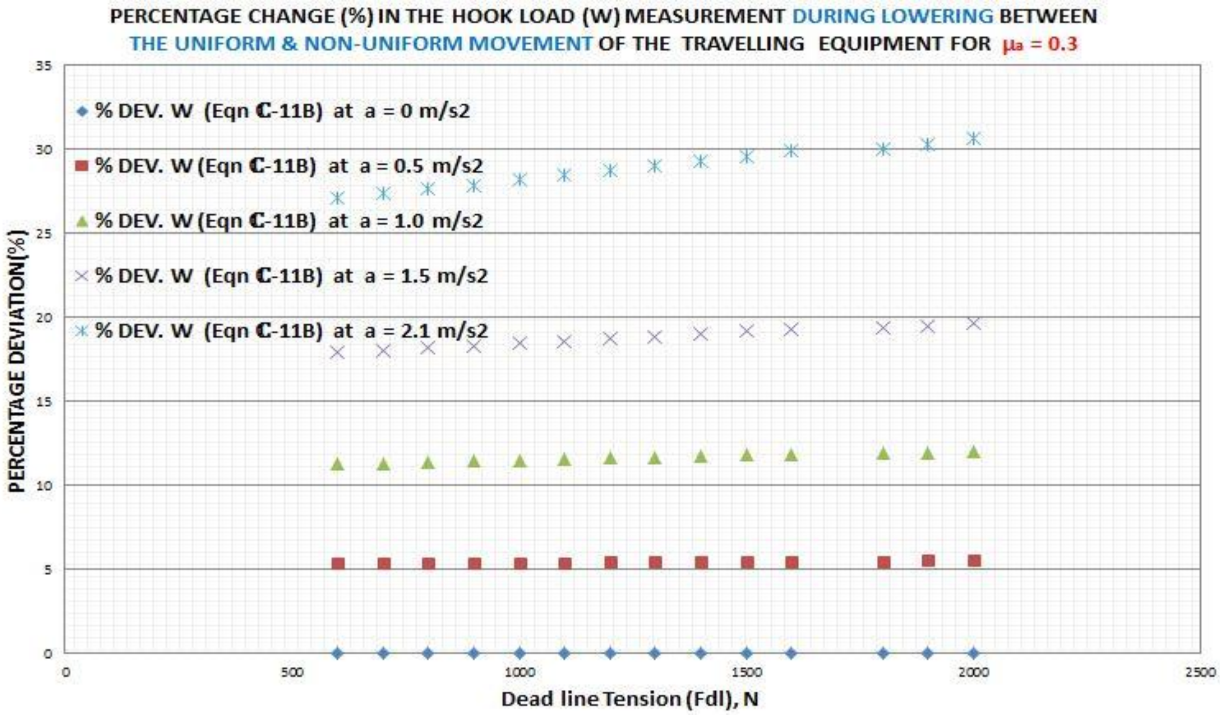

Figure 53: Shows the percentage deviation of the extended Cayeux et al hook load values during lowering with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement also based on the extended Cayeux et al hook load prediction model for $\mu_a = 0.3$

COMMENT: It can be seen that hook load value for a given coefficient of friction and dead line tension (F_{dl}) is lower during lowering than during hoisting. For instance, for μ_a = 0.3 for either hoisting or lowering during uniform movement of the travelling equipment (i.e. a = 0m/s^2) and for F_{dl} = 2000N, the hook load values for both hoisting and lowering are approximately 14000N and 5000N respectively as illustrated in figure (43) and figure (52) respectively. This is because during hoisting, the frictional force due to the coefficient of friction at the sheave axle need to be overcome before the load can be raised and hence resulting in higher hook load (W) value. This is analogous to rolling an object up an inclined plane, the higher the coefficient of friction along the inclined plane, the higher the effort required and vice-versa.

On the other hand, when rolling an object down an inclined plane, the load will only begin to slide down the inclined plane when the frictional force due to the coefficient of friction along the inclined plane has been exceeded. Hence, the hook load value during lowering will apparently be less than during hoisting.

5.5 Analysis of the extended Luke and Juvkam-Wold model with hypothetical data

The output of the extended Cayeux et al hook load prediction model at different coefficient of friction (μ_a) will be used as input to the extended Luke and Juvkam model as illustrated in figure (30).

Although, the extended Luke and Juvkam hook load prediction models were developed for both constant sheave efficiency and varying sheave efficiencies, only the constant sheave efficiency models can be verified. This is because, both the Original Cayeux et al and its extended counterpart which serves as the experimental data to the extended Luke and Juvkam hook load prediction model were also based on constant coefficient of friction. Hence, the extended Luke and Juvkam varying sheave efficiency hook load prediction model can only be verified using experimental data.

5.5.1 Hoisting with Luke & Juvkam inactive (Non-rotating) dead line sheave hook load prediction model

Below are the equations used in the analysis.

Legend	Name of Equation
W (Eqn [F-2F_1]) at different acceleration	The Luke and Juvkam (Inactive dead line sheave) hook load prediction model during hoisting
$\Sigma(F_1+F_2+F_3+F_4)$ at a = 0 m/s^2	The sum of the tensions in the supporting lines during hoisting based on the extended Cayeux et al hook load prediction model

A. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.001\ OR\ e = 0.999$

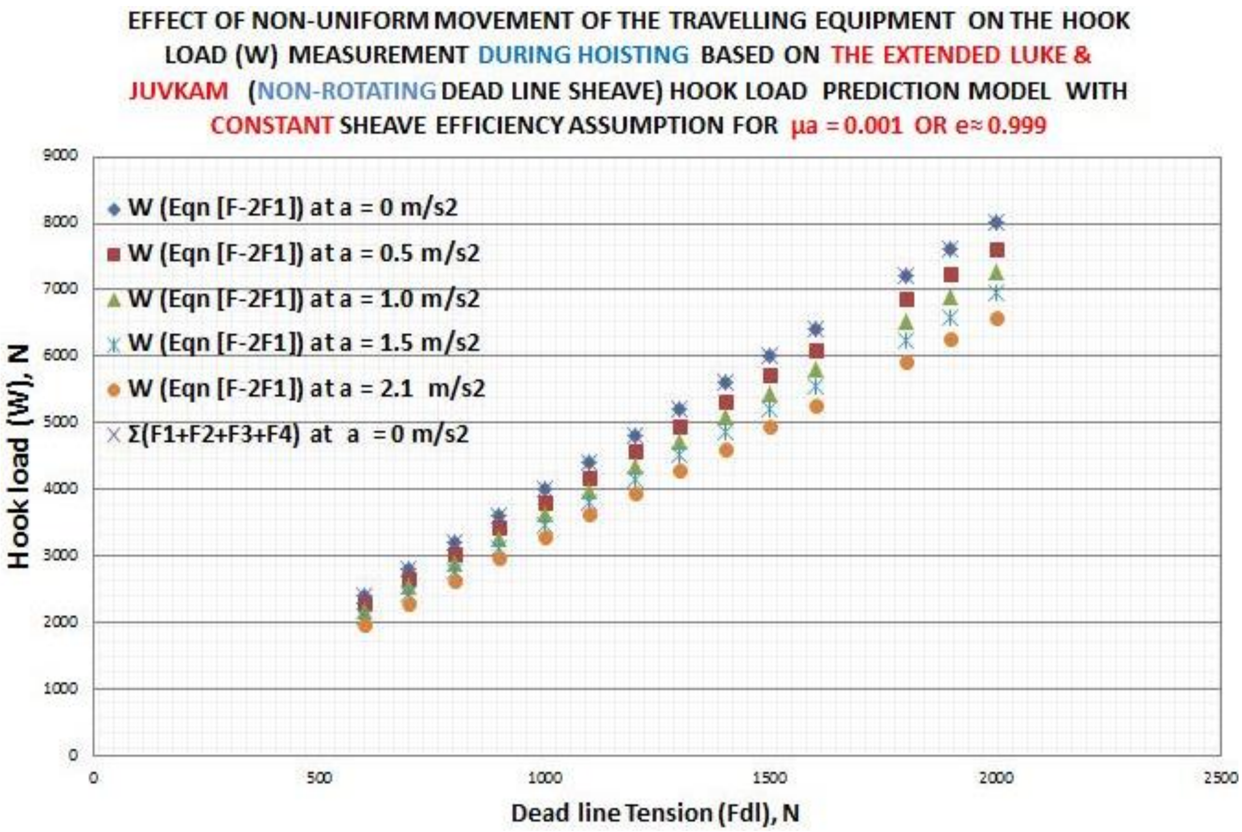

Figure 54: Shows the extended Luke and Juvkam hook load values during hoisting with non-uniform movement of the travelling equipment assuming constant sheave efficiency $e \approx 1$ which corresponds to $\mu_a = 0.001$

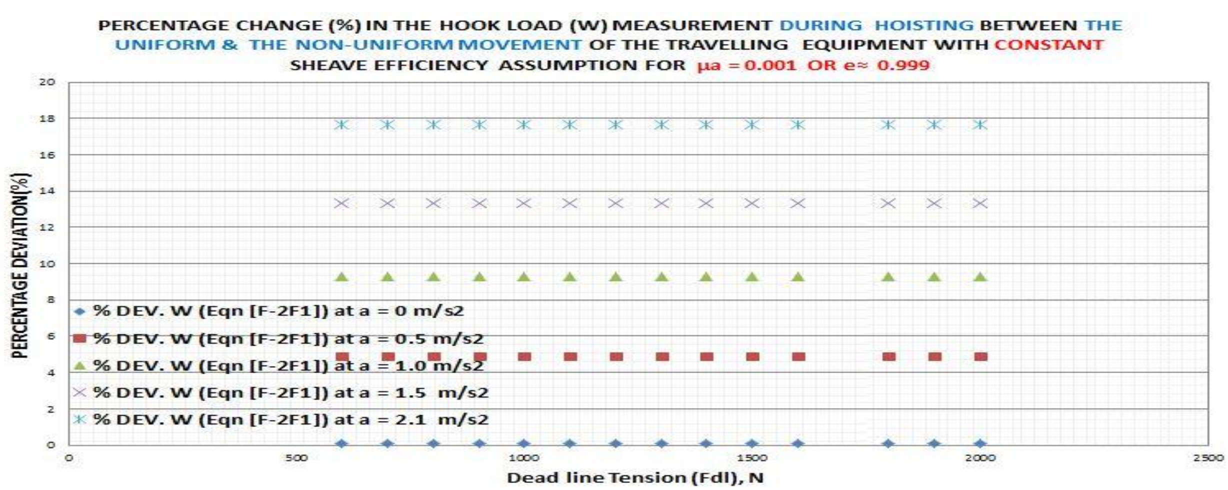

Figure 55: Shows the percentage deviation of the extended Luke and Juvkam hook load values during hoisting with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 1$ $(\mu_a = 0.001)$

COMMENT: During uniform movement, the sum of the tensions in the supporting lines is the same as the hook load. On the other hand, during non-uniform movement of the travelling equipment, the hook load decreases with increasing acceleration (a) of the travelling equipment. The higher the acceleration of the travelling equipment, the lower the hook load values become. Hence, the higher the deviation from the sum of the tensions in the supporting line during uniform movement based on the extended Cayeux et al hook load prediction model.

B. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.3 \; OR \; e = 0.818$

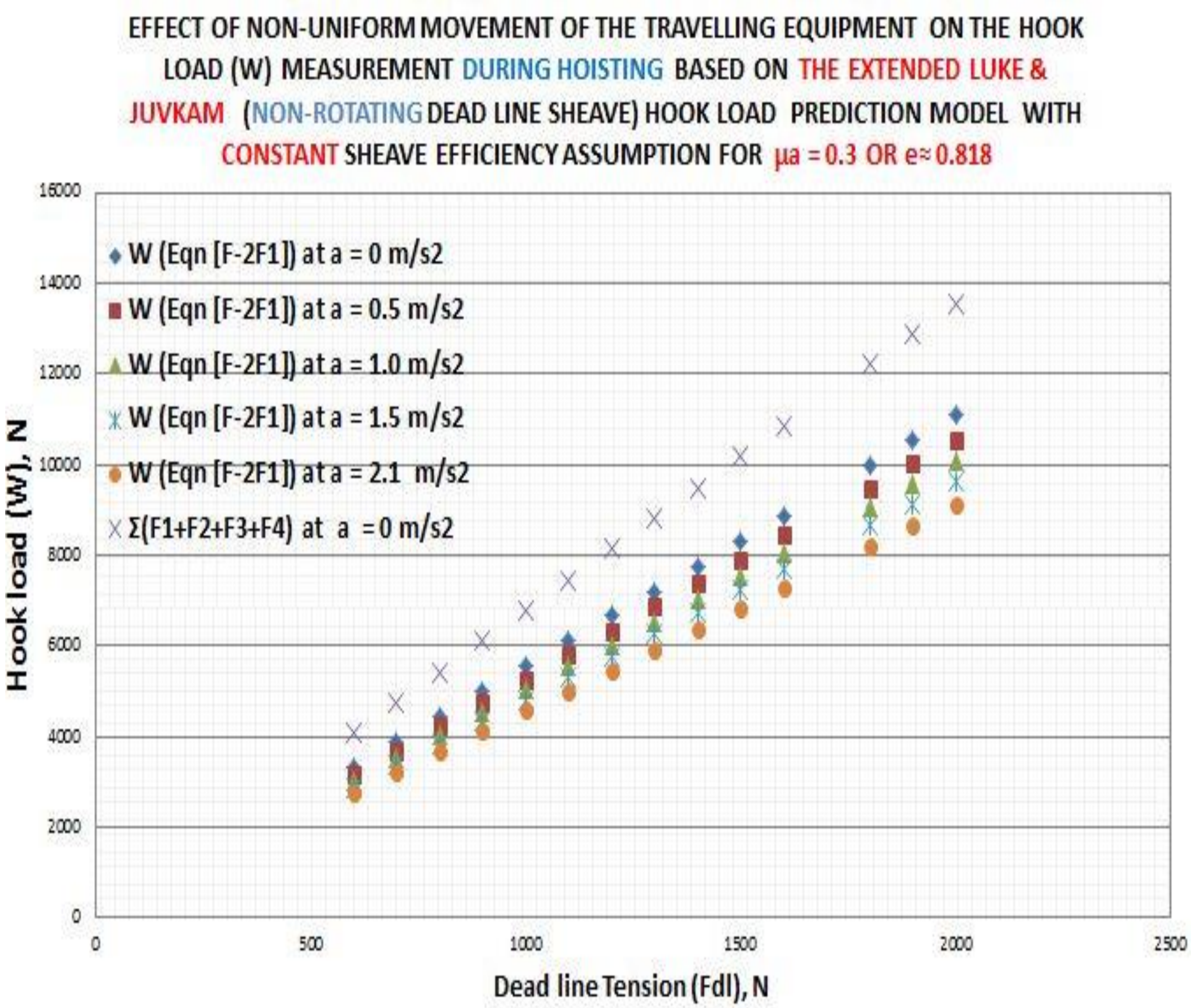

Figure 56: Shows the extended Luke and Juvkam hook load values during hoisting with non-uniform movement of the travelling equipment assuming constant sheave efficiency $e \approx 0.818$ which corresponds to $\mu_a = 0.3$

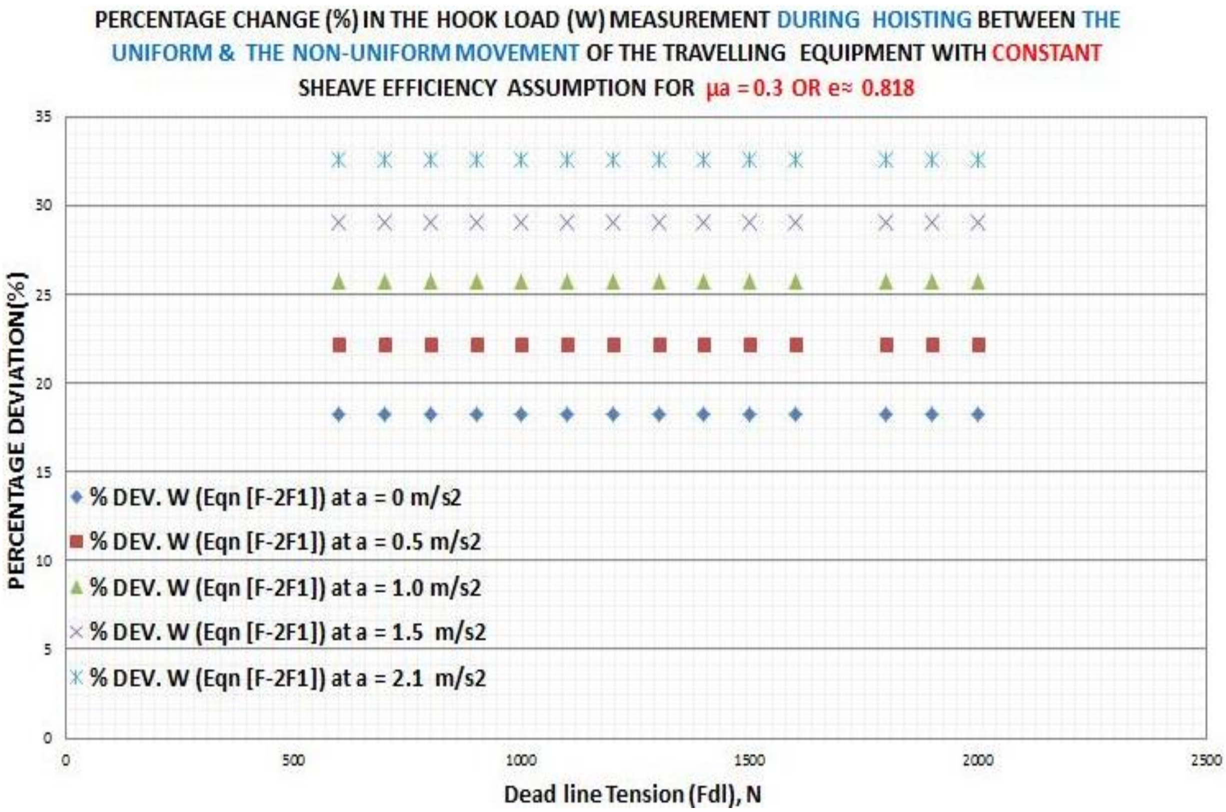

Figure 57: Shows the percentage deviation of the extended Luke and Juvkam hook load values during hoisting with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.818$ ($\mu_a = 0.3$)

COMMENT: During uniform movement of the travelling equipment, the sum of the tensions in the supporting lines was expected to be the same as the hook load value with 0% deviation as illustrated in figure (54) and figure (55) respectively. But during uniform movement with high coefficient of friction ($\mu_a = 0.3$), the hook load value is not the same as the sum of the tensions in the supporting lines based on the extended Cayeux et al hook load prediction model as seen in figure (56) and figure (57). This due to the effect of the perfect transmission of the line tension for the inactive dead line sheave ($e_{dl} = 1$) as proposed by Luke and Juvkam. i.e. If we assume a perfect transmission of the line tension with each sheave efficiency approximately 1 ($\mu_a = 0.001$) , since this efficiency is approximately the same as the inactive dead line sheave assumption ($e_{dl} = 1$) proposed by Luke and Juvkam, the two model produces identical results during uniform movement as depicted in figure (54) and figure (55).

The discrepancies between the two hook load values during uniform movement becomes evident during imperfect transmission of the line tension when the efficiency of the sheaves are less than inactive dead line sheave assumption ($e_{dl} = 1$). i.e. The higher the disparity between the actual sheave efficiency from the inactive dead line sheave ($e_{dl} = 1$) as proposed by Luke and Juvkam, the higher the deviation between the two models as illustrated in figure (56) and figure (57).

5.5.2 Lowering with Luke & Juvkam inactive (non-rotating) dead line sheave hook load prediction model

5.5.2.1 Inactive dead line sheave

Below are the relations used in the analysis and how it was carried out.

Legend	Name of Equation
W (Eqn [F-4E$_1$]) at different acceleration	The Luke and Juvkam (Inactive dead line sheave) hook load prediction model during lowering
Σ(F_1+F_2+F_3+F_4) at a = 0 m/s^2	The sum of the tensions in the supporting lines during lowering based on the extended Cayeux et al hook load prediction model

A. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.001$ OR $e = 0.999$

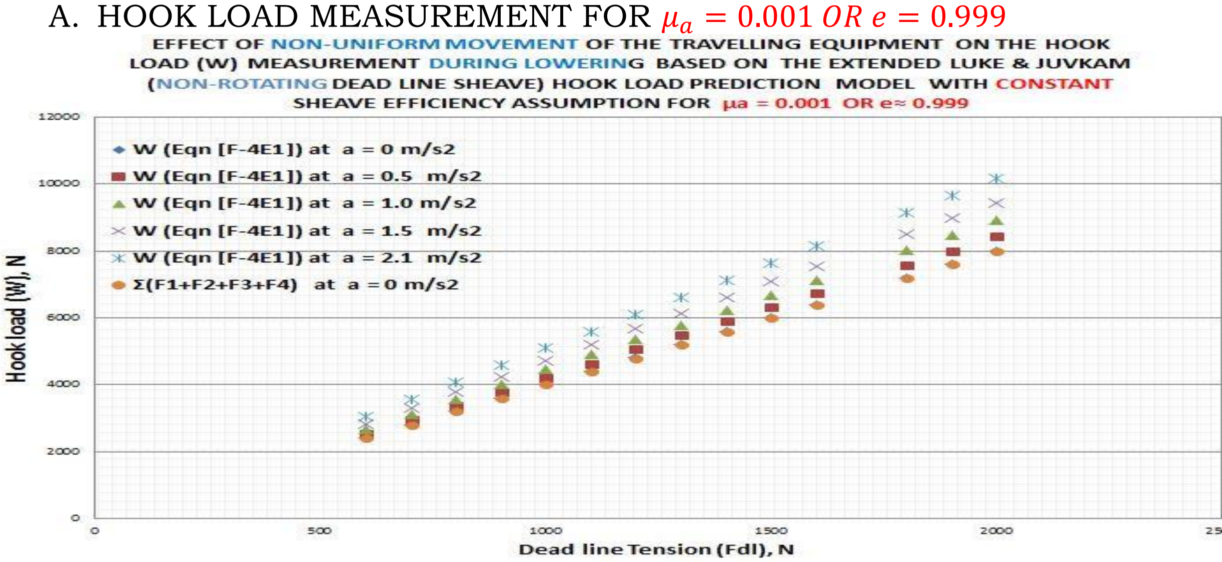

Figure 58: Shows the extended Luke and Juvkam hook load measurement during lowering with non-uniform movement of the travelling equipment assuming constant sheave efficiency $e \approx 1$ which corresponds to $\mu_a = 0.001$

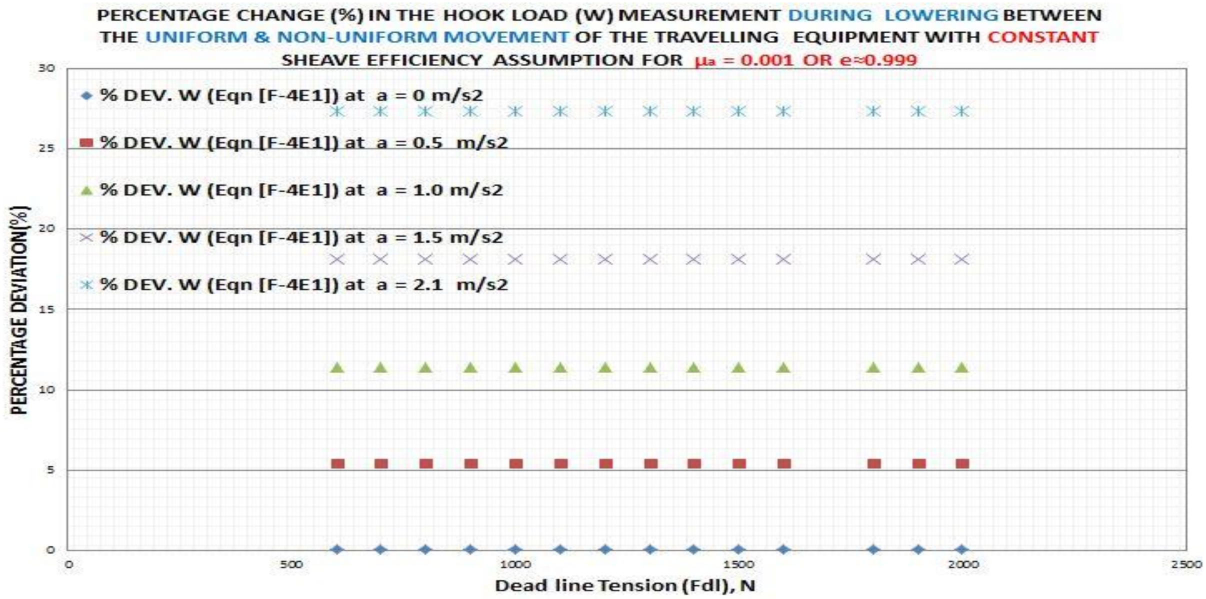

Figure 59: Shows the percentage deviation of the extended Luke and Juvkam hook load values during lowering with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 1$ ($\mu_a = 0.001$)

COMMENT: During lowering, the minimum hook load value occurs during uniform movement of the travelling equipment. The hook load values increases with increasing acceleration of the travelling equipment.

B. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.3$ OR e=0.818

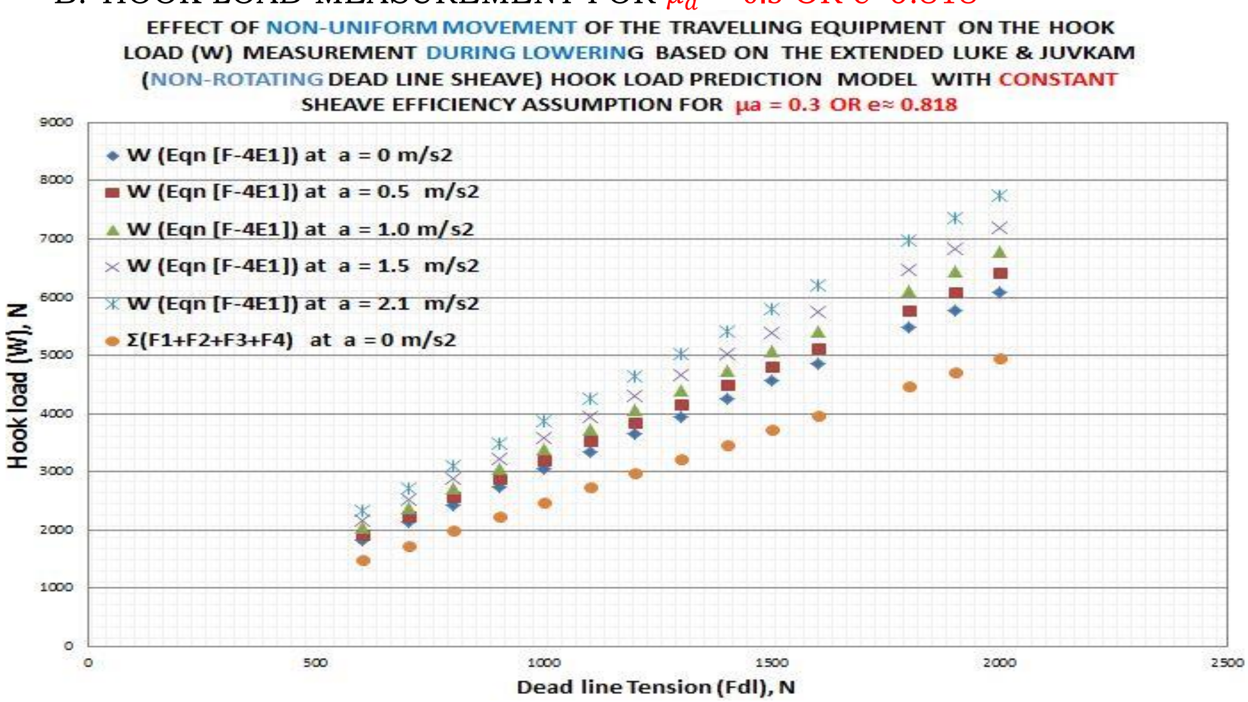

Figure 60: Shows the extended Luke and Juvkam hook load measurement during lowering with non-uniform movement of the travelling equipment assuming constant sheave efficiency $e \approx 0.818$ which corresponds to $\mu_a = 0.3$

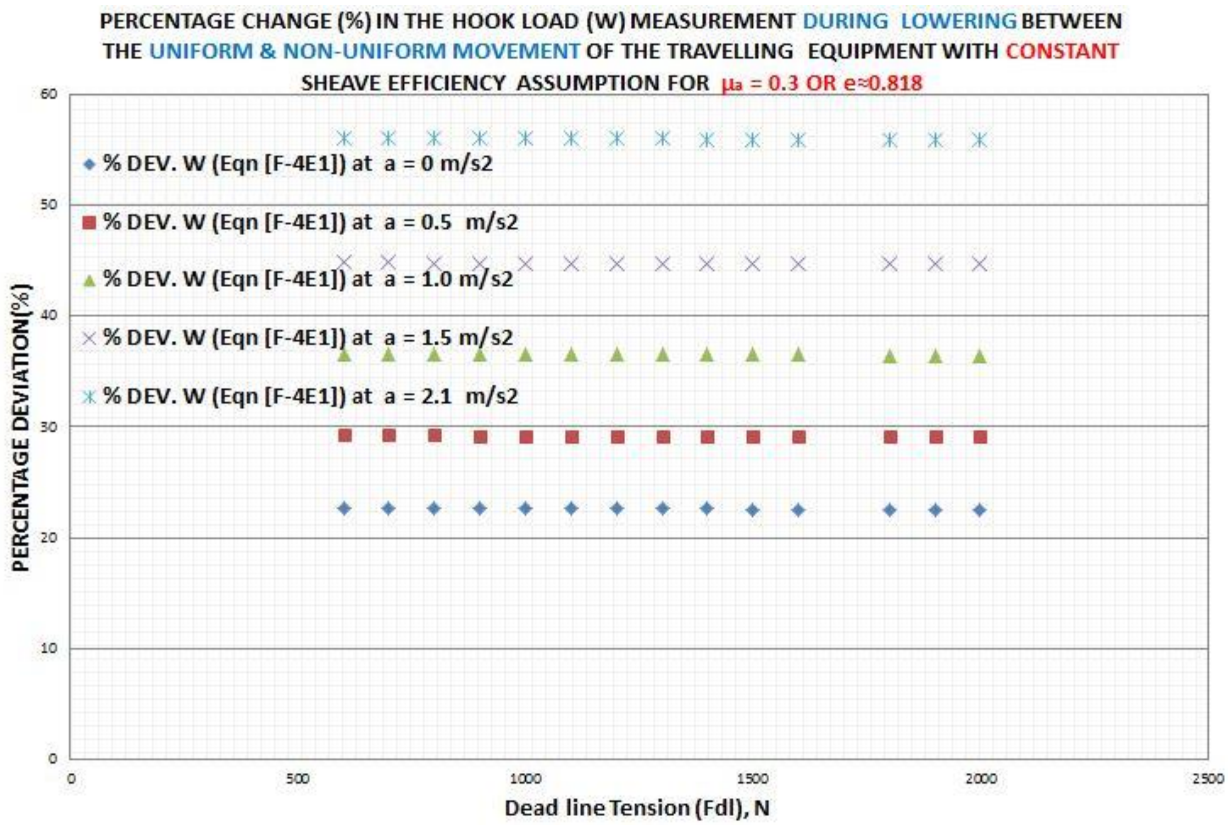

Figure 61: Shows the percentage deviation of the extended Luke and Juvkam hook load values during lowering with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.818$ ($\mu_a = 0.3$)

COMMENT: It can be concluded that the higher the coefficient of friction, the smaller the hook load value becomes during lowering since the friction bears some of the weight of the load. For example, during lowering with uniform movement of the travelling equipment (i.e. a = 0m/s^2) and for F_{dl} = 2000N, for a given μ_a = 0.001 corresponds to a hook load value of 8000N as illustrated in figure (58) while μ_a = 0.3 also corresponds to a hook load value of 6000N as depicted in figure (60). In addition, even though the hook load increases with increasing the acceleration of the travelling equipment, the effect due to the coefficient of friction has a more pronounced effect on the hook load values than the effect due to the acceleration of the travelling equipment.

5.6 Comparison of all the extended models

The extended models will be compared with each other to determine their response under a given condition.

5.6.1 Comparison of all the extended model during hoisting with both uniform & non-uniform movement of the travelling equipment

The comparison of the extended model during hoisting will be carried out using their respective equations as illustrated in the table below

Legend	Name of Equation
W (Eqn [F-2F_1]) at different acceleration	The Extended Luke and Juvkam (Inactive dead line sheave) hook load prediction model during hoisting
W (Eqn [G-2F_1]) at different acceleration	The Extended Luke and Juvkam (Active dead line sheave) hook load prediction model during hoisting
W (Eqn [I-1C]) OR W (Eqn [α-11B]) at different acceleration	The Extended Cayeux et al hook load prediction model during hoisting
W (Eqn [E-1B]) at different acceleration	The Extended Industry accepted hook load prediction model during hoisting
$\Sigma(F_1+F_2+F_3+F_4)$ at $a = 0\ m/s^2$	The sum of the tensions in the supporting lines during hoisting based on the extended Cayeux et al hook load prediction model

A. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.001$ & a = 0 m/s^2

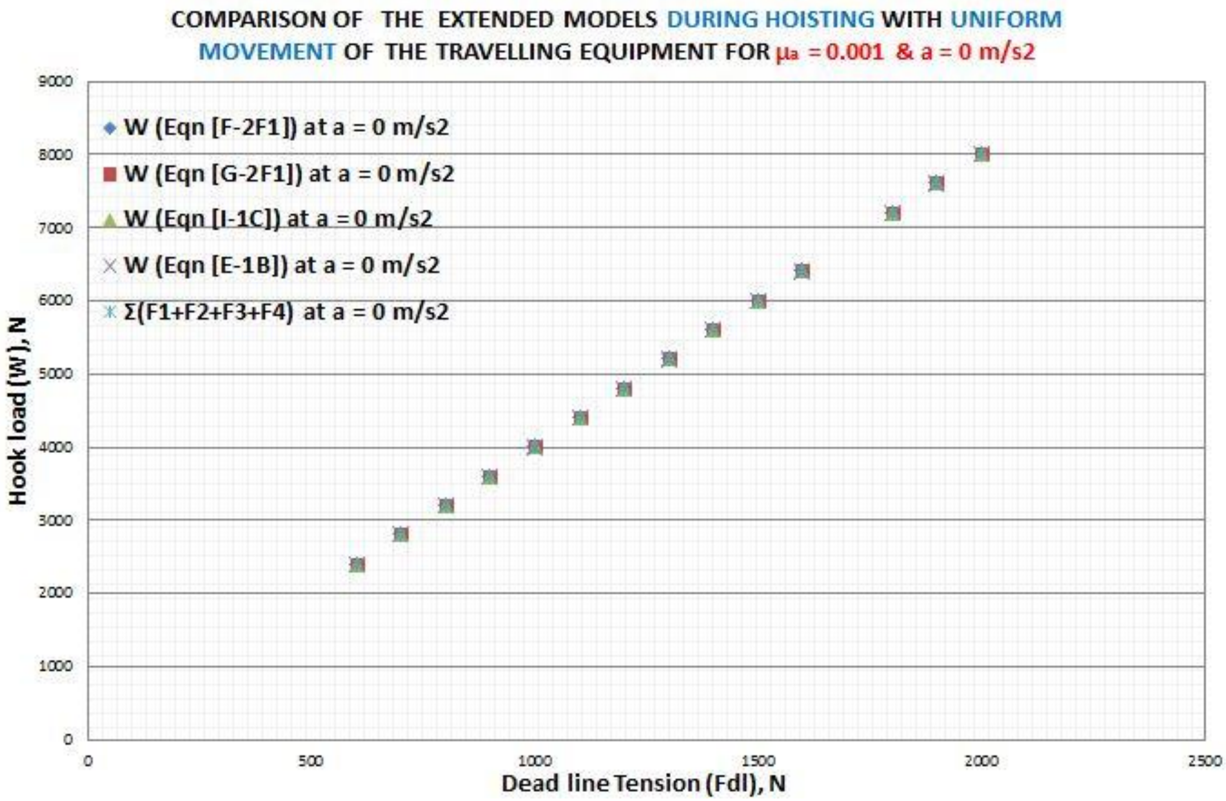

Figure 62: Shows the comparison of all the extended hook load prediction models during hoisting with uniform movement of the travelling equipment and the sum of the tensions in the supporting lines also during uniform movement based on the extended Cayeux et al hook load prediction model.

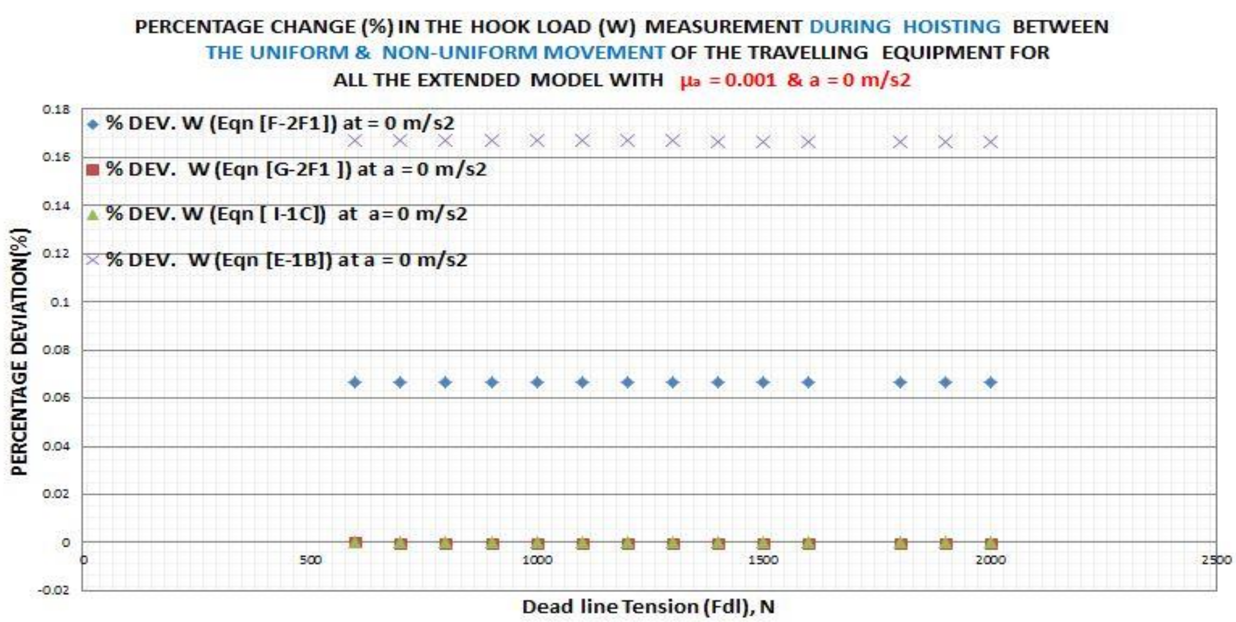

Figure 63: Shows the percentage deviation of all the extended hook load values during hoisting with uniform movement of the travelling equipment from the sum of the tensions in the supporting lines also during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 1$ ($\mu_a = 0.001$) & a = 0 m/s2

COMMENT: It can be observed that if we assume perfect transmission of line tension during uniform movement of the travelling equipment, all the extended models overlap with each other resulting in negligible deviation of each model from the sum of the tensions in the supporting lines based on the extended Cayeux et al hook load prediction model also during uniform movement.

B. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.1$ & a = 0 m/s^2

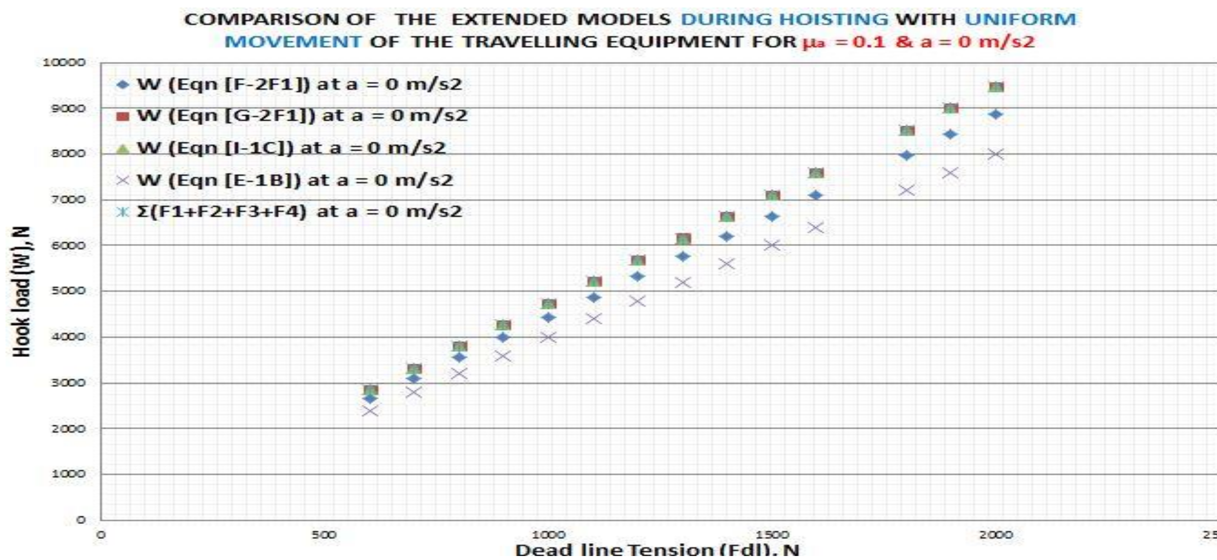

Figure 64: Shows the comparison of all the extended hook load prediction models during uniform movement of the traveling equipment and the sum of the tensions in the supporting lines also during uniform movement of the travelling equipment based on the extended Cayeux et al hook load prediction model for $e \approx 0.94$ ($\mu_a = 0.1$) & a = 0 m/s2

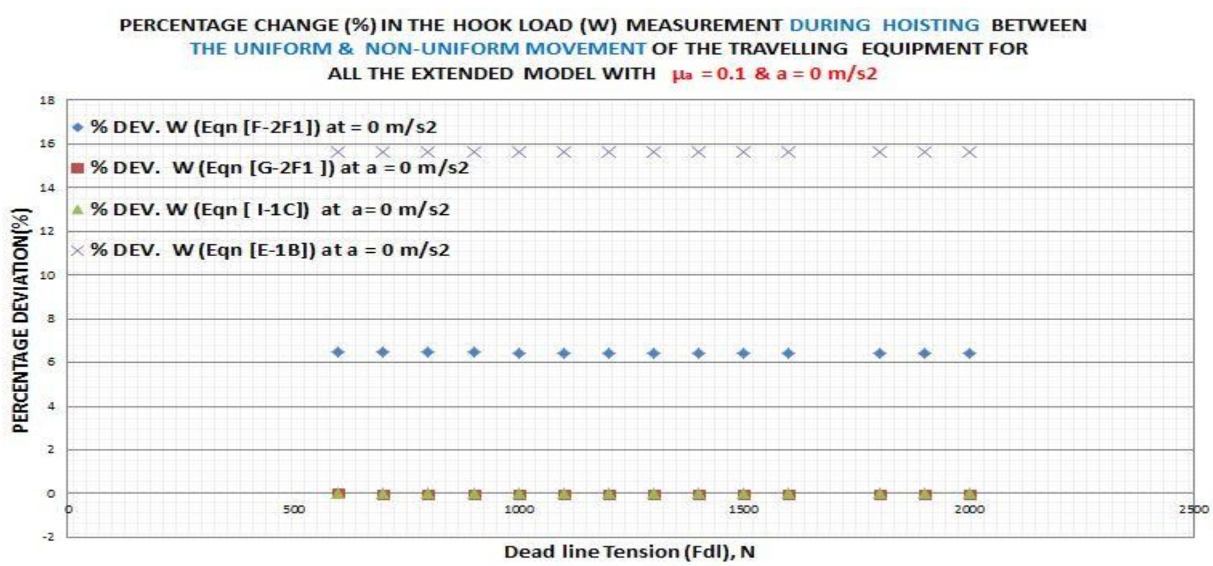

Figure 65: Shows the percentage deviation of all the extended hook load value during hoisting with uniform movement of the travelling equipment from the sum of the tensions in the supporting lines also during uniform movement of the travelling equipment based on the extended Cayeux et al hook load prediction for $e \approx 0.94$ ($\mu_a = 0.1$) & a = 0 m/s2

COMMENT: It can be observed that even at constant velocity of the travelling block, the industry accepted hook load prediction model under predict the hook load values during hoisting. In addition, due to the perfect sheave efficiency assumption of the non-rotating dead line sheave, the extended Luke and Juvkam inactive dead line sheave hook load prediction model also tends to underestimate the hook load value during imperfect line tension transmission. For example, from figure (63), it can be observed that the extended Luke and Juvkam Inactive dead line sheave hook load prediction model for μ_a= 0.001 deviated 0.07% from the sum of the tensions in the supporting lines based on the extended Cayeux et al hook load prediction model while for μ_a= 0.1 also corresponds to about 6.3% deviation as illustrated in figure (63) and figure (65) respectively.

Finally, since the output of the extended Cayeux et al hook load prediction model was used to calibrate the extended Luke and Juvkam model, the extended Luke and Juvkam Active dead line sheave hook load value always overlaps with that of the extended Cayeux et hook load value as depicted in figure (64) and figure (65).

C. HOOK LOAD MEASUREMENT FOR μ_a = 0.3 & a = 0m/s^2

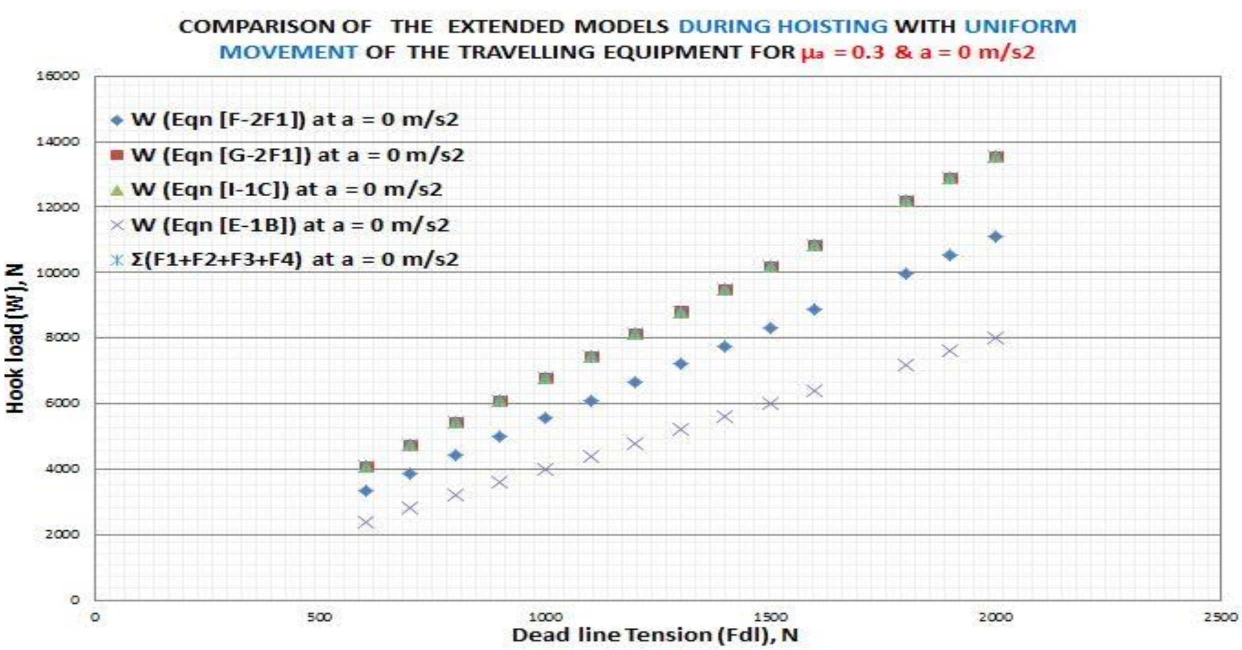

Figure 66: Shows the comparison of all the extended hook load prediction models during uniform movement of the traveling equipment and the sum of the tensions in the supporting lines based

on the extended Cayeux et al hook load prediction also during uniform movement of the travelling equipment for $e \approx 0.83$ ($\mu_a = 0.3$) & a = 0 m/s2

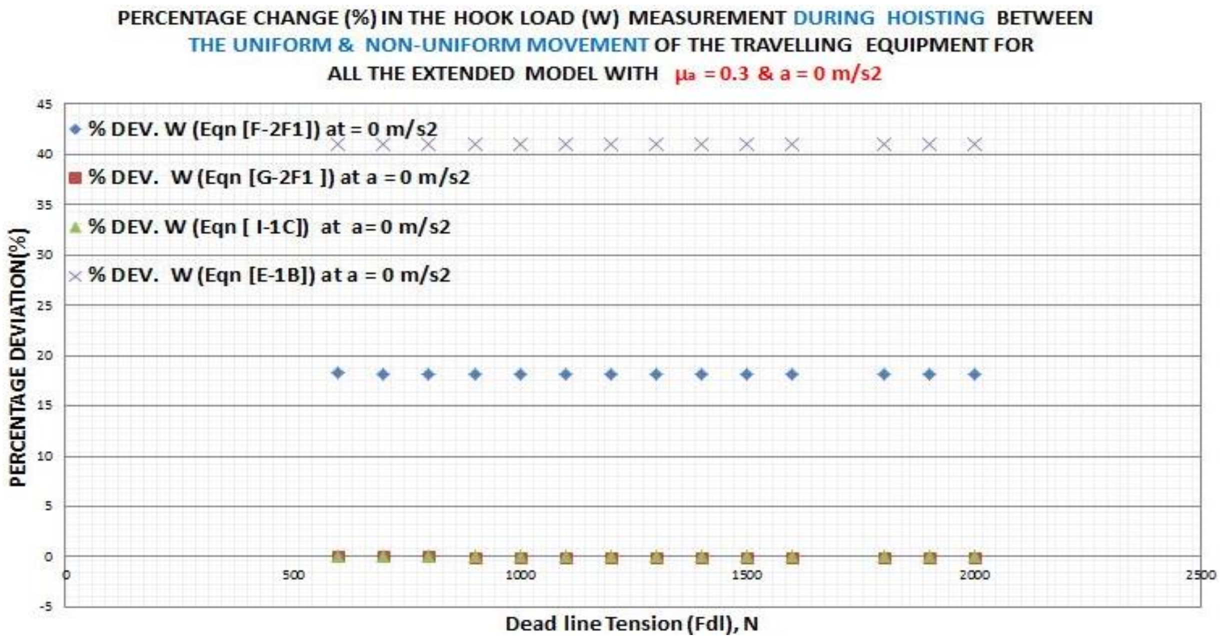

Figure 67: Shows the percentage deviation of all the extended hook load measurement during hoisting with uniform movement of the travelling equipment from the sum of the tensions in the supporting lines also during uniform movement of the travelling equipment based on the extended Cayeux et al hook load prediction for $e \approx 0.83$ ($\mu_a = 0.3$) & a = 0 m/s²

COMMENT: It can be seen that during uniform movement of the travelling equipment, the deviation of the extended industry accepted hook load prediction model increases with increasing coefficient of friction (decreasing sheave efficiency).

D. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.3$ & a = 1.5 m/s²

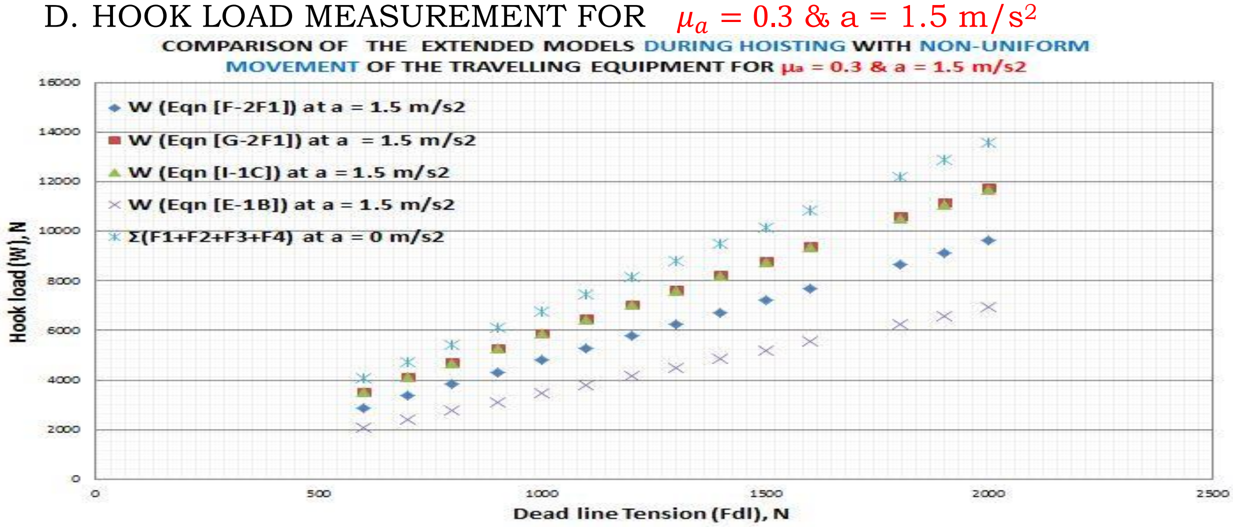

Figure 68: Shows the comparison of all the extended hook load prediction models during non-uniform movement of the travelling equipment and the sum of the tensions in the supporting lines during uniform movement of the travelling equipment based on the extended Cayeux et al hook load prediction for $e \approx 0.83$ ($\mu_a = 0.3$) & a = 1.5 m/s²

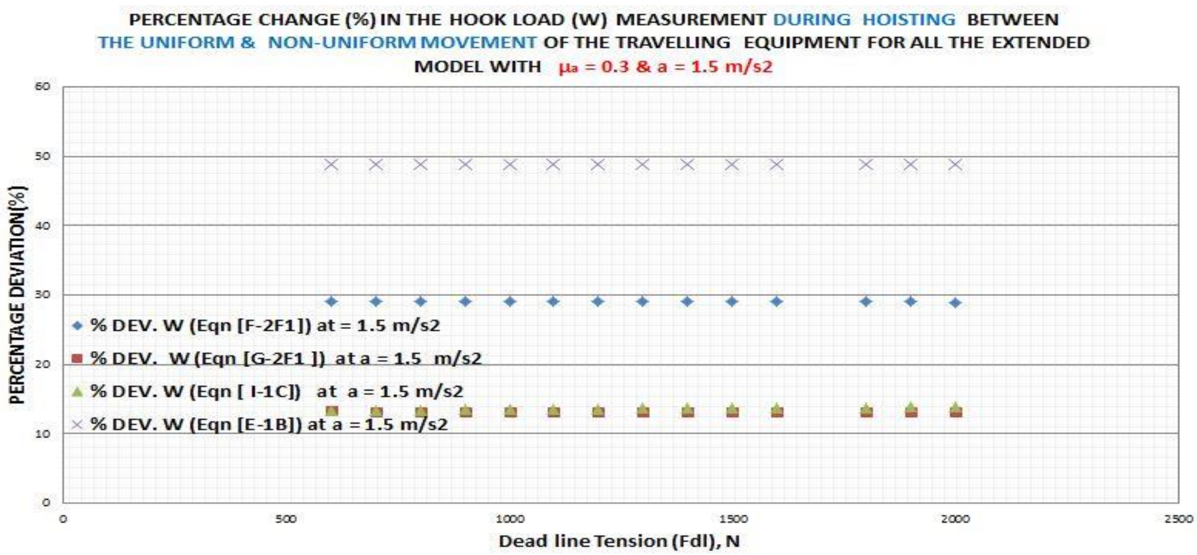

Figure 69: Shows the percentage deviation of all the extended hook load values during hoisting with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement of the travelling equipment based on the extended Cayeux et al hook load prediction for $e \approx 0.83$ ($\mu_a = 0.3$) & a = 1.5 m/s²

COMMENT: It can be observed that the effect due to the non-uniform movement of the travelling equipment on the hook load measurement is negligible compared to the effect due to the coefficient of friction at the sheave axle.

5.6.2 Comparison of all the extended models during lowering with both uniform & non-uniform movement of the travelling equipment

Similarly, the comparison of the extended models during lowering will be carried out using their respective equations as illustrated in the table below.

Legend	Name of Equation
W (Eqn [F-4E$_1$]) at different acceleration	The Extended Luke and Juvkam (Inactive dead line sheave) hook load prediction model during lowering
W (Eqn [G-4E$_1$]) at different acceleration	The Extended Luke and Juvkam (Active dead line sheave) hook load prediction model during lowering
W (Eqn [I-2C] OR W (Eqn [ℂ-11B]) at different acceleration	The Extended Cayeux et al hook load prediction model during lowering

W (Eqn [E-2B]) at different acceleration	The Extended Industry accepted hook load prediction model during lowering
Σ(F_1+F_2+F_3+F_4) at a = 0 m/s²	The sum of the tensions in the supporting lines during lowering based on the extended Cayeux et al hook load prediction model

A. HOOK LOAD MEASUREMENT FOR μ_a = 0.001 & a = 0 m/s²

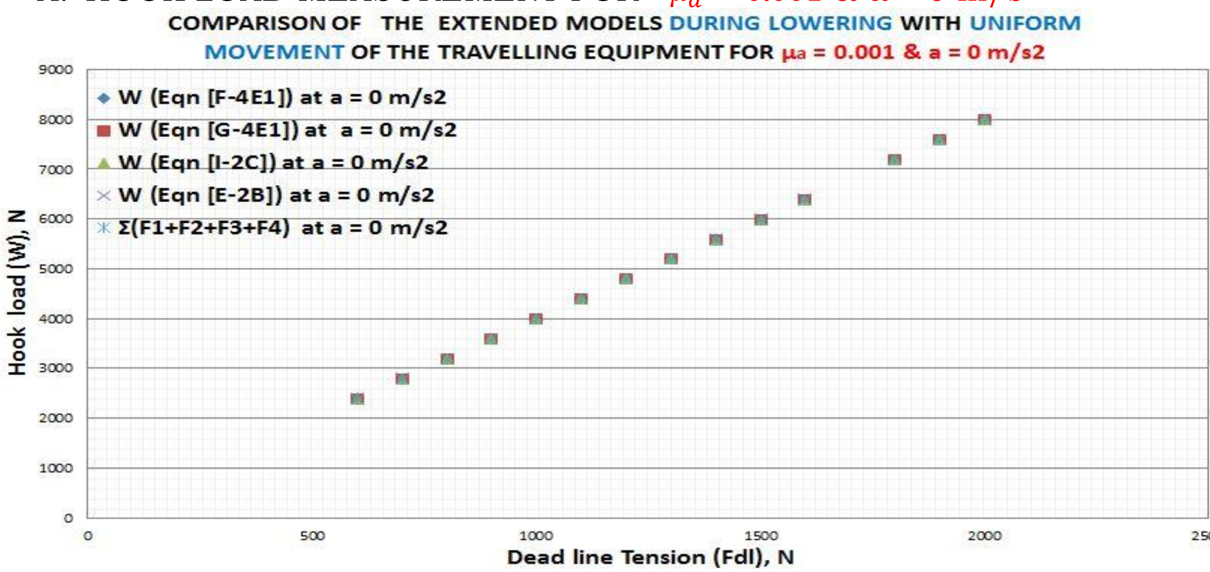

Figure 70: Shows the comparison of all the extended hook load prediction models during lowering with uniform movement of the traveling equipment and the sum of the tensions in the supporting lines also under uniform movement based on the extended Cayeux et al hook load prediction model

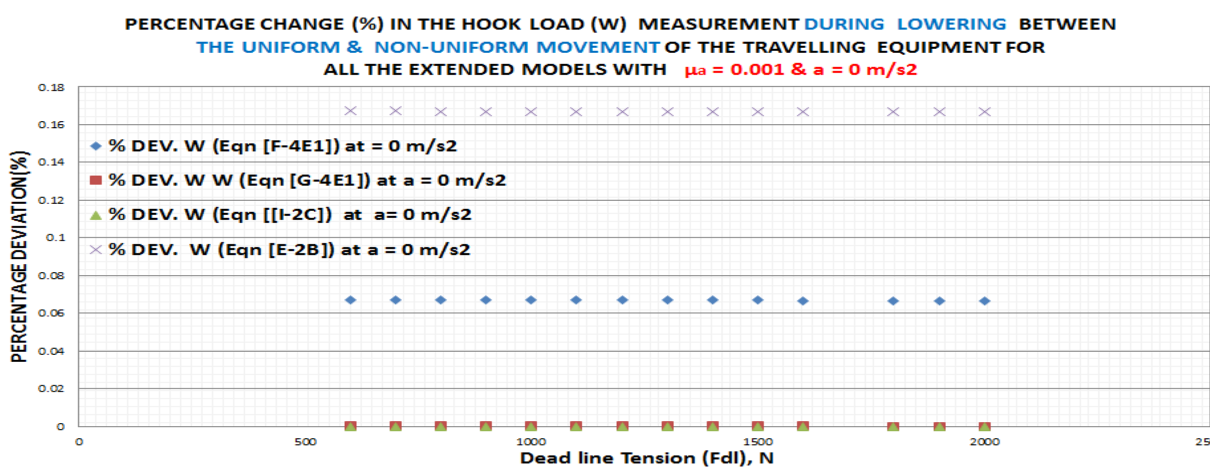

Figure 71: Shows the percentage deviation of all the extended hook load values during lowering with uniform movement of the travelling equipment from the sum of the tensions in the supporting lines also under uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 1$ ($\mu_a = 0.001$) & a = 0 m/s²

COMMENT: It can be observed that if we assume perfect transmission of line tension during uniform movement of the travelling equipment, all the extended models overlap with each other resulting in negligible deviation of each model from the sum of the tensions in the supporting lines during uniform movement of the travelling equipment based on the extended cayeux et al hook load prediction model.

B. HOOK LOAD MEASUREMENT FOR $\mu_a = 0.1$ & a = 0 m/s²

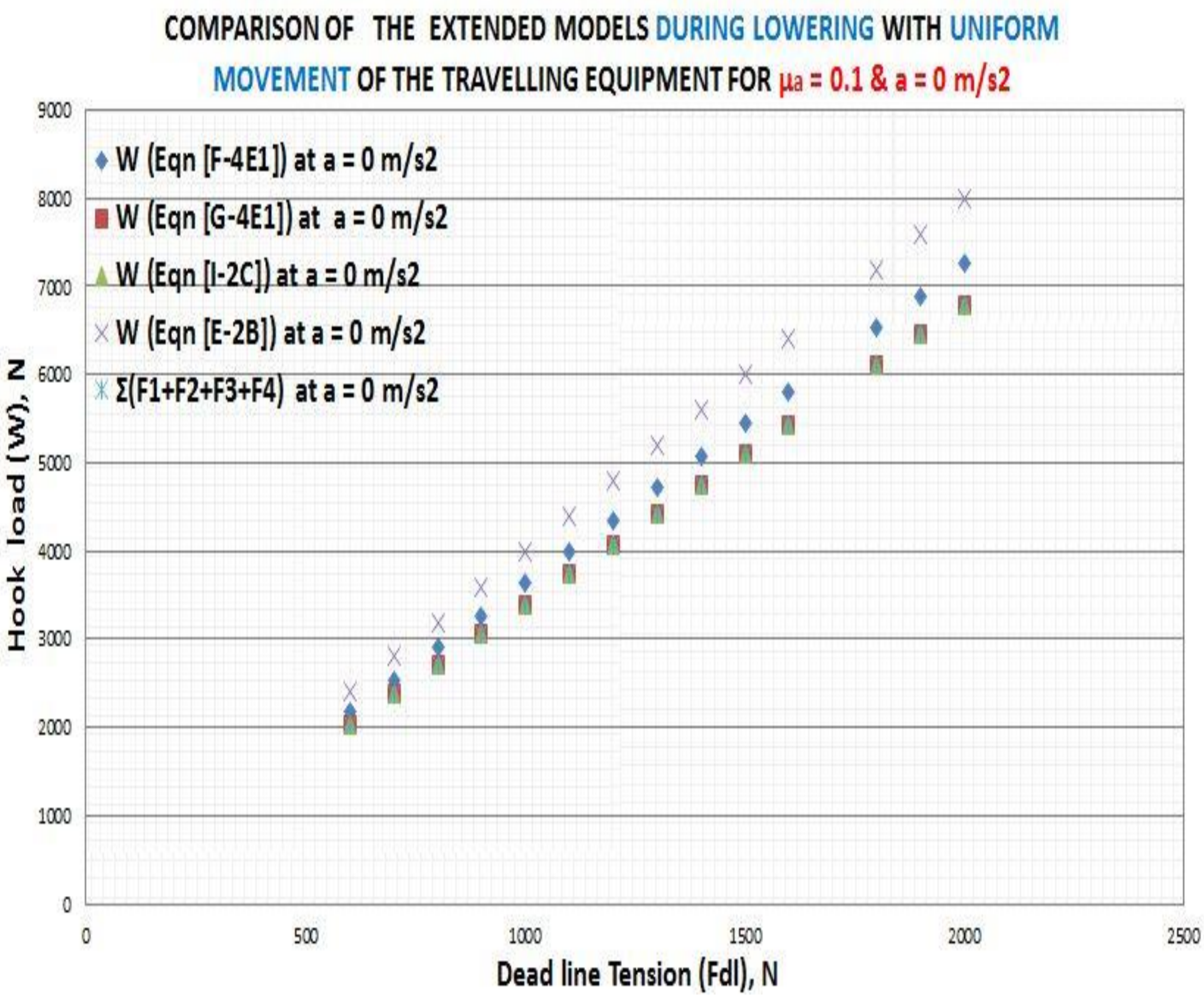

Figure 72: Shows the comparison of all the extended hook load prediction models during lowering with uniform movement of the travelling equipment and the sum of the tensions in the supporting lines also during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.94$ $(\mu_a = 0.1)$ & a = 0 m/s²

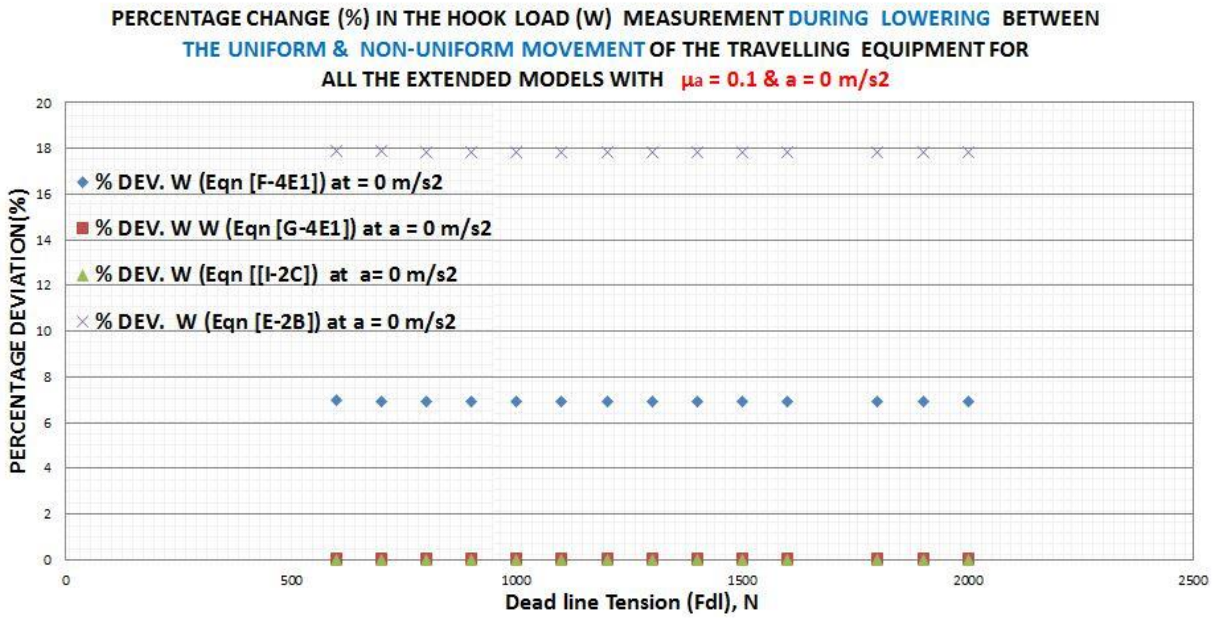

Figure 73: Shows the percentage deviation of all the extended hook load measurement during lowering with uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.94$ ($\mu_a = 0.1$) & a = 0 m/s²

COMMENT: During uniform movement of the travelling equipment, the extended industry accepted hook load prediction model tends to over predict the hook load value during lowering. The magnitude of the deviation is proportional to the coefficient of friction at the sheave axle. For example during uniform movement (a = 0m/s²) and for μ_a=0.001, the deviation of the extended industry accepted hook load prediction model from the sum of the tension in the supporting lines based on the extended Cayeux et al hook load prediction model was 0.17% and at μ_a=0.1, the deviation was 18% as depicted in figure (71) and figure (73) respectively.

In addition, it can be observed that since the extended Luke and Juvkam hook load prediction models were calibrated with the output of the extended Cayeux et al hook load prediction model, it is not surprising that the extended Cayeux et al hook load values overlap with the extended Luke and Juvkam Active dead line sheave hook load values.

Finally, comparing the extended Luke and Juvkam Inactive dead line sheave hook load values to the Active counterpart, the effect of the dead line sheave efficiency perfect ($e_{dl} = 1$) transmission of the line tension becomes evident.

From figure (73), the deviation of the extended Luke and Juvkam Inactive dead line sheave hook load values from the sum of the tension in the supporting lines based on the extended Cayeux et al hook load prediction model was about 7% and the deviation for the Active counterpart was 0%.

C. HOOK LOAD MEASUREMENT FOR μ_a = 0.3 & a = 0m/s^2

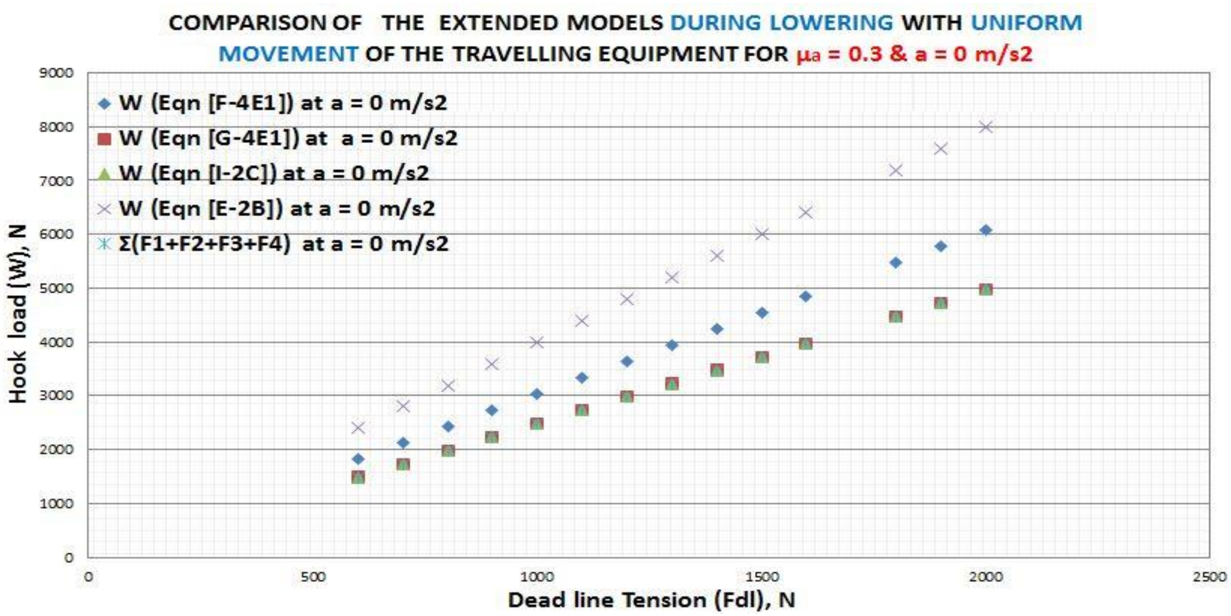

Figure 74: Shows the comparison of all the extended hook load prediction models during lowering with uniform movement of the traveling equipment and the sum of the tensions in the supporting lines also during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.83$ $(\mu_a = 0.3)$ & a = 0 m/s²

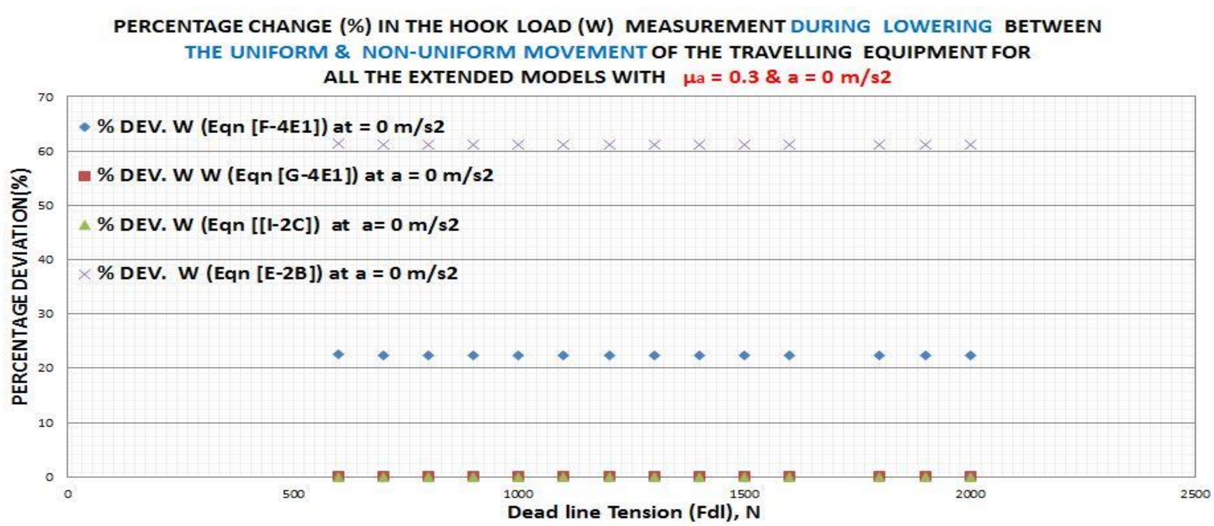

Figure 75: Shows the percentage deviation of all the extended hook load values during lowering with uniform movement of the travelling equipment from the sum of the tensions in the supporting lines during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.818$ $(\mu_a = 0.3)$ & a = 0 m/s²

COMMENT: The magnitude of the over prediction of the industry accepted hook load value depends on the coefficient of friction at the sheave axle.

For μ_a=0.1, the deviation of the extended industry accepted hook load values from the sum of the tension in the supporting lines based on the extended Cayeux et al hook load prediction model was 18% while for μ_a=0.3, the deviation was about 60% as illustrated in figure (73) and figure (75) respectively.

D. HOOK LOAD MEASUREMENT FOR μ_a = 0.3 & a = 1.5 m/s^2

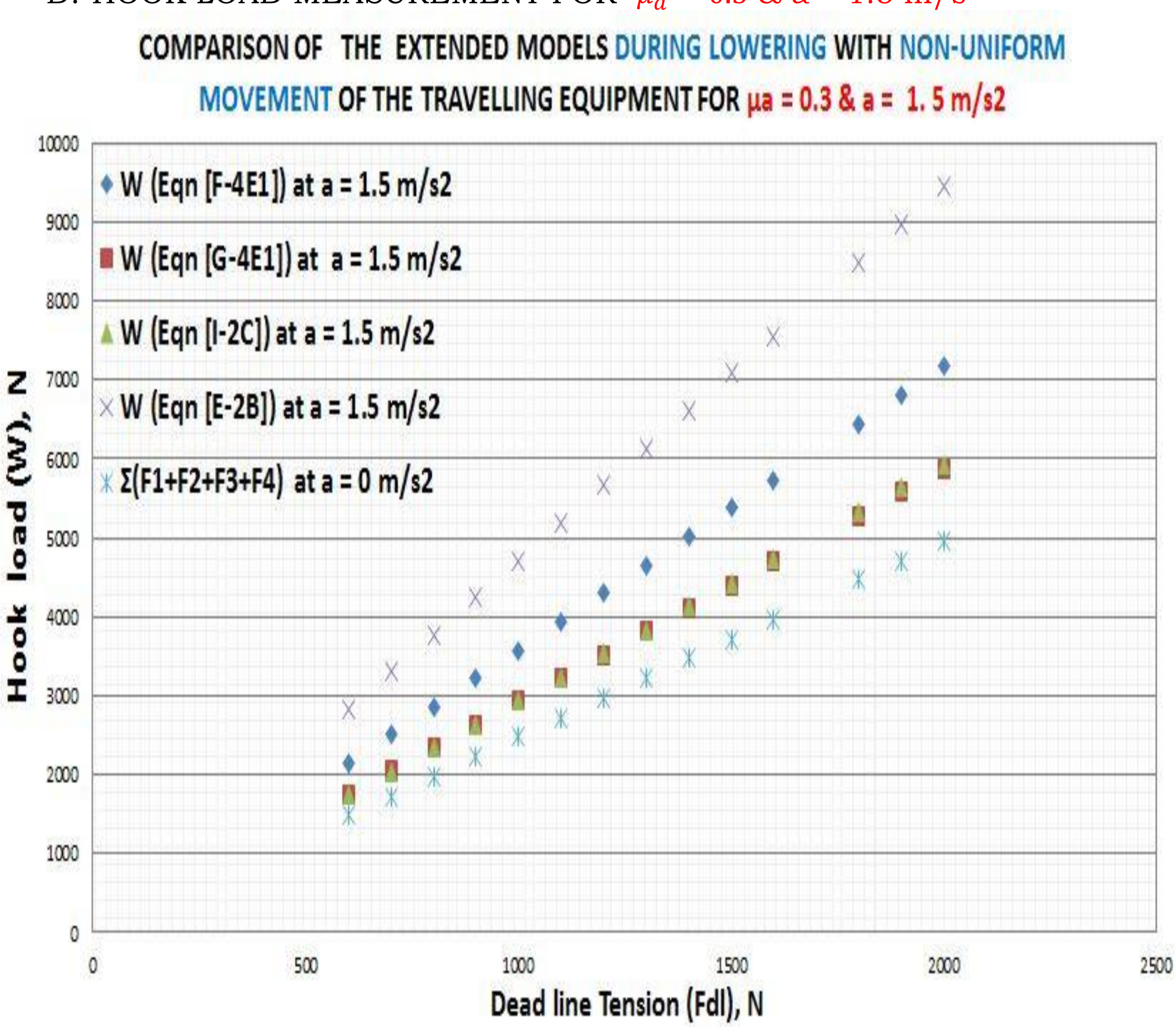

Figure 76: Shows the comparison of all the extended hook load prediction models during lowering with non-uniform movement of the traveling equipment and the sum of the tensions in the supporting also during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.818$ $(\mu_a = 0.3)$ & a = 1.5 m/s²

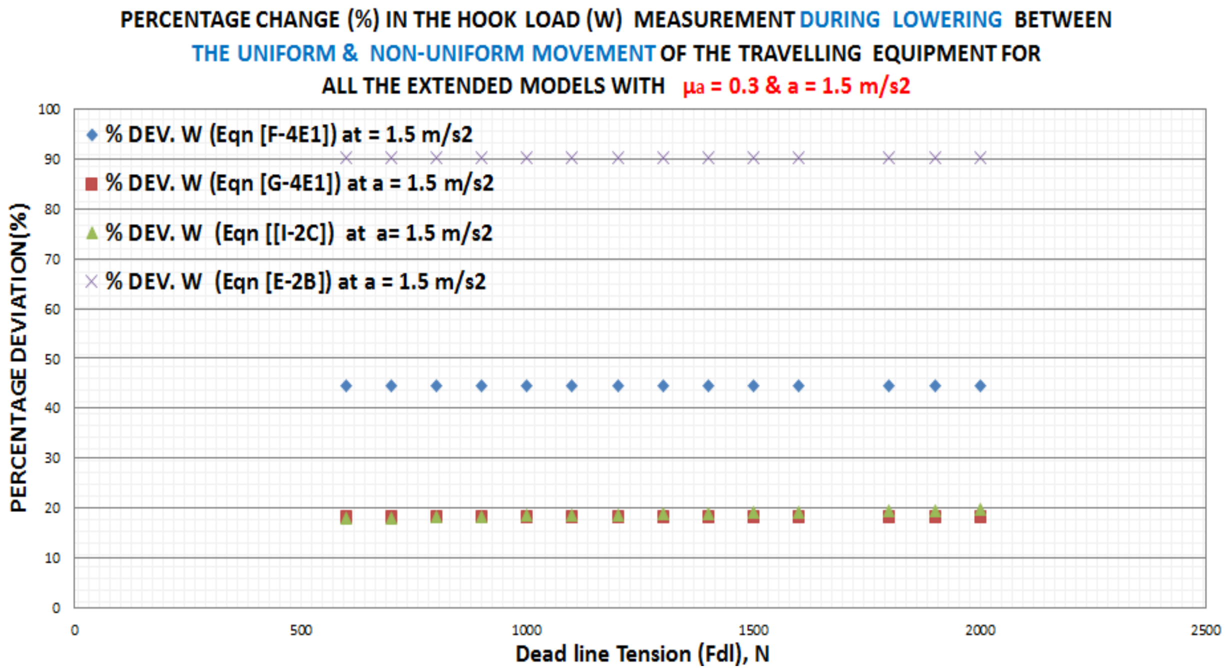

Figure 77: Shows the percentage deviation of all the extended hook load value during lowering with non-uniform movement of the travelling equipment from the sum of the tensions in the supporting during uniform movement based on the extended Cayeux et al hook load prediction model for $e \approx 0.83$ $(\mu_a = 0.3)$ & a = 1.5 m/s²

COMMENT: The over prediction of the hook load measurement during lowering based on the extended industry accepted hook load prediction model is even worse during non-uniform movement of the travelling equipment. The higher the acceleration of the travelling equipment, the higher the over prediction of the hook load value becomes. The extended Cayeux et al hook load prediction model which is based on non-rotating dead line sheave assumption always overlaps with the extended Luke and Juvkam Active (rotating) dead line sheave hook load prediction model. This is due to the fact that the output of the extended Cayeux et al hook load prediction model was used as input to the extended Luke and Juvkam Active dead line sheave hook load prediction model. If the above extended models are analysed with experimental data, their hook load values might not be identical since they are based on different assumptions.

Hence, it can be inferred that although the dead line sheave does not rotate, its efficiency might not be perfect. (i.e. $e_{dl} \neq 1$).

6 Summary & Conclusion

6.1 Summary of all the extended model

Below is a brief description on how the extended models were developed during non-uniform movement of the travelling equipment.

I. Newton's second law of motion was applied on the travelling equipment during either hoisting or lowering with the inherent assumptions behind each particular model still taken into consideration during the extended hook load prediction model. Assumptions such a perfect line tension transmission (i.e. e = 1) for the case of the industry accepted hook load prediction model, the constant sheave efficiency (e = constant) assumption for the case of the Luke and Juvkam hook load prediction model etc. were still taken into account in their respective models.

II. After applying Newton Second law of motion, the downward force exerted by the drillstring suspension point in the travelling equipment which literally represents the hook load (W) is made the subject of the equation.

III. This relation then becomes the extended hook load prediction model during non-uniform movement.

6.2 Summary of how the extended Cayeux et al hook load prediction model was developed

Cayeux et al developed a model for the tensions in the line for both the crown block sheaves and the travelling block sheaves during uniform movement of the travelling equipment. Below is a brief description on how the extended Cayeux et al hook load prediction models were developed from the line tension relations.

I. During hoisting the Cayeux et al line tension relation for both the crown block sheave and the travelling block sheaves are given by Eqn [C-1] and Eqn [C-3] respectively while during lowering they are respectively given by Eqn [C-2] and Eqn [C-4].

II. During hoisting, a generalized line tension relation (Eqn [γ-5A]) was developed for both the crown block sheaves and the travelling block sheaves from Eqn [C-1] and Eqn [C-3] while a generalized line tension relation (Eqn [δ- 5A]) was also developed from Eqn[C-2] and Eqn [C-4] for both the crown block sheaves and the travelling block sheaves to account for the tensions in the lines during lowering.

III. From the relationship between the angular parameter (angular velocity and angular acceleration) of all the rotating sheaves relative to the velocity of the travelling equipment, the generalized line tension relations (Eqn [γ-5A] & Eqn [δ- 5A]) respectively become Eqn [γ-5C] & Eqn [δ- 5C].

IV. From the generalized line tension relations during either hoisting (Eqn [γ-5C]) or lowering (Eqn [δ- 5C]), the sum of the tensions in the supporting lines ($\sum_{i=1}^{n} F_i$) during the non-uniform movement respectively become Eqn [α- 10] and Eqn [ℂ-10].

V. Applying Newton's law of motion on the travelling equipment and the sum of the tensions in the supporting lines during hoisting (Eqn [α-10]), the extended Cayeux et al hook load (W) prediction model during hoisting becomes Eqn [α- 11A] or Eqn [α-11B] as illustrated in the figure (78) below.

VI. Similarly, applying the Newton's second law of motion on the travelling equipment and from the sum of the tensions in the supporting lines during lowering (Eqn [ℂ-10]), the extended Cayeux et al hook load (W) prediction model during lowering also becomes Eqn [ℂ-11A] or Eqn [ℂ-11B] as depicted in the figure (78) below.

NB. Special attention is needed when summing up the tension in the supporting lines. Below is a brief description on how the summation was carried out.

a. The tensions in the lines are due to the contribution from various terms such as the centrifugal force on each rotating sheave, the weight of each sheave, the angular acceleration of each rotating sheave, effect of translational acceleration (a) on the travelling block sheaves' reaction forces and effect of the dead line tension (F_{dl}).

b. For simplicity, the weight of each sheave will be used as a case study. For example, using the dead line as the reference point, the weight of the dead line sheave will affect the tensions in the subsequent lines depending on its magnitude. Hence, each subsequent line will experience a "ripple effect" from the weight of the dead line sheave depending on its magnitude.
c. Each of these "ripple effects" from each sheave forms a Geometric series with the subsequent lines.
d. Adding all the contributions from each sheave gives the total contribution to the sum of the tensions in the supporting lines by the sheaves' weight.
e. This procedure will then be performed for all the remaining terms (the centrifugal force, the angular acceleration of each rotating sheave etc.) to get their respective total contributions to the sum of the tensions in the supporting lines.
f. Adding all these contributions from each term gives the sum of the tensions in the supporting lines during non-uniform movement.
g. Newton's second law of motion was then applied to the travelling equipment to obtain the extended Cayeux et al hook load (W) prediction models for either hoisting or lowering.

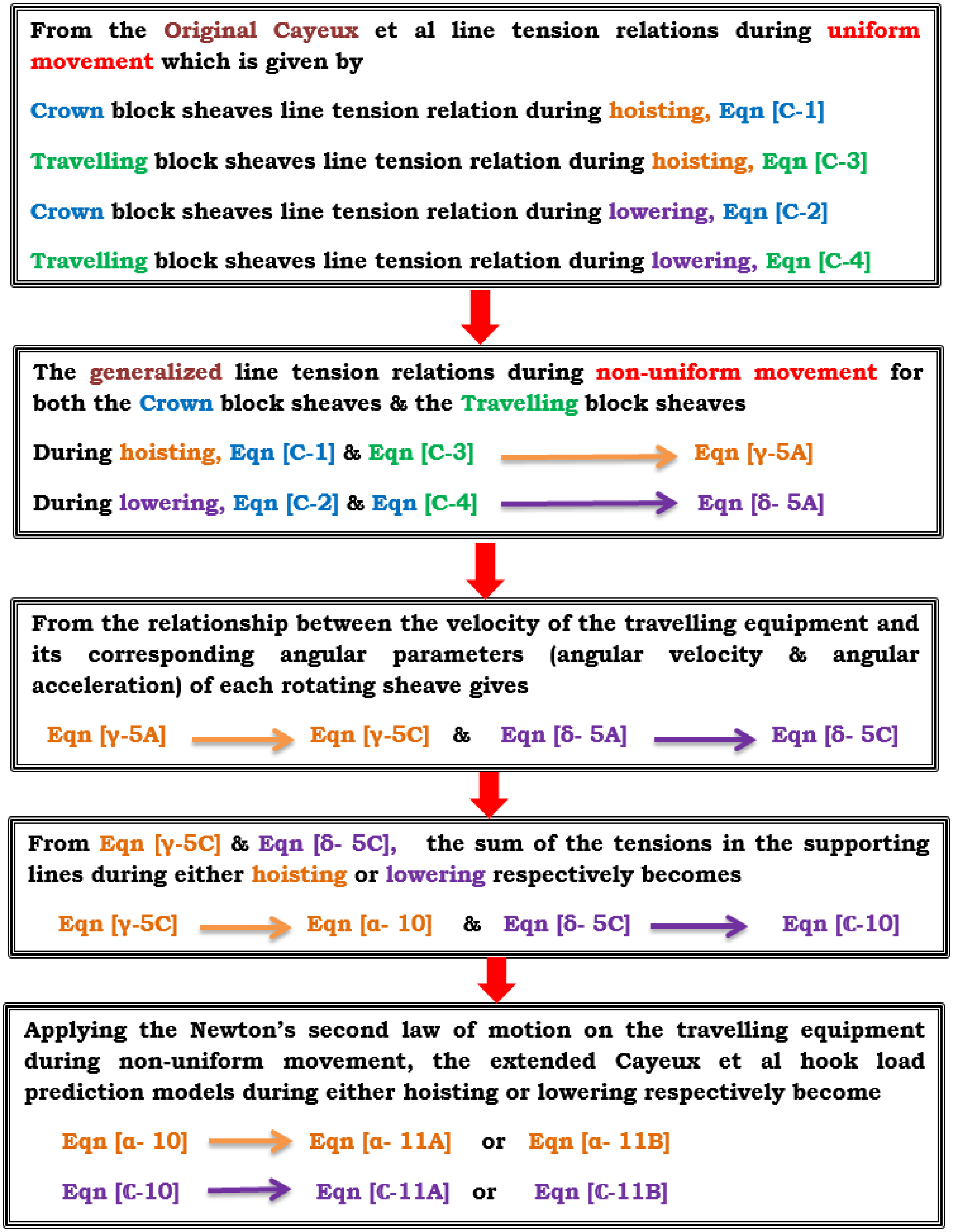

Figure 78: Schematic illustrating how the extended Cayeux et al hook load prediction model during non-uniform movement of the travelling block was obtained

6.3 Conclusion

The second hook load (W) prediction approach during non-uniform movement of the travelling equipment is analogous to the effect of buoyancy factor (β) on the weight of a body that is partially or fully immersed in a fluid as stipulated by Archimedes principle. The buoyed weight (W_{bd}) of the body in a fluid is given by

$$W_{bd} = \beta W_{air} = \left(1 - \frac{\rho_{mud}}{\rho_s}\right) W_{air}$$

In a similar vein, the second hook load prediction approach during non-uniform movement of the travelling equipment is also given by

$$W = \left(\frac{1}{1 \pm \frac{a}{g}}\right) \sum_{i=1}^{n} F_i = \left(\frac{g}{g \pm a}\right) \sum_{i=1}^{n} F_i$$

Comparing the hook load relation during non-uniform movement of the travelling equipment to the buoyed weight of a body immersed in a fluid, the correction factor (γ) to compensate for the non-uniform movement of the travelling equipment is given by

$\gamma = \left(\frac{1}{1 \pm \frac{a}{g}}\right)$ = Non-uniform movement of the travelling equipment correction factor

The non-uniform movement of the travelling equipment correction factor depends on the ratio between the translational acceleration (a) to the gravitational acceleration (g).

During hoisting (+), the higher the $\frac{a}{g}$ ratio, the smaller the non-uniform movement correction factor (γ) becomes. This results in a smaller hook load value as compared to its uniform equivalent. The minimum expected hook load value during hoisting with non-uniform movement of the travelling equipment occurs when the translational acceleration (a) of the travelling equipment attains its maximum value.

On the other hand during lowering (-) the higher the $\frac{a}{g}$ ratio, the higher the non-uniform movement of the travelling equipment correction factor (γ) becomes. Hence, the higher the hook load value becomes and vice-versa. The maximum expected hook load value during lowering with non-uniform movement of the travelling equipment occurs when the translational acceleration (a) approaches the acceleration due to free fall or acceleration due to gravity (g). $(i.e\ \ a \approx g)$

In addition, during non-uniform movement of the travelling equipment, the position for the placement of the load cell is very important since the sum of the tensions in the supporting lines is not the same as the hook load $i.e. W \neq \sum_{i=1}^{n} F_i$. For a smaller $\frac{a}{g}$ ratio, the effect of the non-uniform movement of the travelling equipment on the hook load measurement is negligible. Hence, the indirect hook load measurement with the load cell positioned at the dead line can be used although the direct hook load measurements remains the best option as illustrated in figure (17).

On the other hand for a higher $\frac{a}{g}$ ratio, the difference between the sum of the tensions in the supporting lines and that of the hook load increases. An indirect hook load measurement with the load positioned at the dead line will result in large discrepancy between the actual and the measured hook load. Hence, a direct hook load measurement with the load cell positioned just above the drillstring connection point is very essential for accurate hook load measurement as suggested by Wylie et al [11] using an Instrumented Internal Blow-Out Preventer (IIBOP) as illustrated in figure (17).

Furthermore, during imperfect transmission of the line tension $(i.e.\ \mu_a > 0\ or\ e < 1)$, the hook load value during hoisting increases with increasing coefficient of friction (μ_a) while the hook load value during lowering decreases with increasing coefficient of friction (μ_a). Considering an inclined plane analogy, the higher the coefficient of friction along the inclined plane, the higher the effort required to roll an object up the inclined plane and vice-versa. On the other hand, before an object can be rolled down an inclined plane, the frictional force along the inclined plane must be exceeded by the force applied (weight of the object). Hence, the coefficient of friction bears some of the weight of the object during lowering. The higher the coefficient of friction at the sheave axle, the lower the hook load value during lowering becomes and vice-versa. Hence, the hook load value during lowering will apparently be less than during hoisting.

Moreover, during imperfect transmission of the line tension for either uniform or non-uniform movement of the travelling equipment , the extended industry accepted hook load prediction model tends to underestimate the hook load value during hoisting while it overestimate the hook load value during lowering due to its inherent perfect sheave efficiency ($e = 1$) assumption. This problem becomes worse during non-uniform movement of the travelling equipment. With reference to both the extended Cayeux et al hook load prediction model and the extended Luke and Juvkam Active dead line sheave hook load prediction model which always overlap with each other during either hoisting or lowering since the output of the former model was used as an input for the latter model. The extended Luke and Juvkam Inactive dead line sheave hook load prediction model also tend to either slightly underestimate the hook load value during hoisting or slightly overestimate it during lowering when compared with its Active dead line sheave counterpart. This might be due to the intrinsic perfect dead line sheave efficiency ($e_{dl} = 1$) assumption as suggested by Luke and Juvkam-Wold.

Finally, although the dead line sheave is not rotating, it should be aware that its efficiency is not perfect($e_{dl} \neq 1$). This can be further investigated with experimental data.

7 FUTURE WORK

Experimental data can be used to confirm all the extended models. With respect to the extended Luke and Juvkam-Wold model, if each sheave is equipped with a load cell, the tensions in the supporting lines can easily be determined. Hence, the efficiency (e) of each sheave can also be determined to compare the constant sheave efficiency assumption as proposed by Luke and Juvkam-Wold with the extended varying sheave efficiency counterpart.

With reference to the extended Cayeux et al hook load prediction model, it could be extended to account for the effect of the drill-line elasticity during non-uniform movement of the travelling equipment.

In addition, both the Cayeux et al hook load prediction model and its extended equivalent were based on constant coefficient of friction (μ_a) at the sheave axle. Hence, an experimental data can be used to confirm this assumption. If the coefficient of friction (μ_a) at the sheave axle is not constant, experimental data can be used to determine possible range for the coefficient of friction (μ_a).

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APPENDIX

SOME IMPORTANT DEDUCTIONS FROM LUKE AND JUVKAM-WOLD MODEL

The weight of each sheave and the drilling lines are negligible compared to the hook load (w) and the tensions in the lines and hence the derrick load (F_d) is given by the relation

Derrick load $(Fd) = Fastline\ tension$ (Ffl) + Hook load(W) + Deadline(Fdl)

$$F_d = F_{fl} + W + F_{dl} \quad [1]$$

Similarly, the sum of all the tensions in the lines supporting the hook load is equal to the hook load (W) when the block is travelling with constant velocity and is given by

i.e. $Hook\ load\ (W) = Line\ Tension\ (F_1) + Line\ Tension\ (F_2) + \cdots + Line\ Tension\ (F_n)$

$$W = F_1 + F_2 + F_3 + \cdots + F_n \quad [2]$$

Maximum tension occurs at the Fast line (F_{fl}) during hoisting (raising of the block) while the dead line (F_{dl}) records the least tension $ie\ F_{fl} \geq F_{dl}$

Conversely during lowering, maximum tension occurs in the dead line (F_{dl}) since more drilling lines are spooled out of the draw work resulting in the reduction of line tension from the fastline (F_{fl}) towards the deadline (F_{dl}). $ie\ F_{fl} \leq F_{dl}$

In the block and tackle pulley system all the sheave rotate with the exception of the dead line sheave in the crown block which may or may not rotate.

Hence, if the dead line sheave does not rotate, it is considered as an Inactive dead line sheave and the number of rotating pulley (m) is the same as the number of lines (n) between the crown block and the travelling block. $ie.m = n$

On the other hand, if the dead line sheave in the crown block rotates, it is considered as an Active dead line sheave and the number of rotating pulleys

(m) is not equal to the number of lines (n) between the travelling block and the crown block but rather the n + 1 since there will be reduction in the line tension between the dead line and the nth (the last) due to the rotation of the dead line sheave. $ie.\ m \neq n \quad rather \quad m = n + 1$

APPENDIX A

The industry accepted hookload prediction is based the assumption that the efficiency of each sheave is perfect (i.e. e = 100%) There is perfect transmission of tension from the fast line (F_{fl}) to the dead line (F_{dl})

$$\mathrm{F}_{fl} = \mathrm{F}_1 = \mathrm{F}_2 = \mathrm{F}_3 = \cdots = \mathrm{F}_n = \mathrm{F}_{dl} = Constant \qquad [3]$$

Substituting Eqn [3] into Eqn [2] gives

$$W = F_{dl} + F_{dl} + F_{dl} + \cdots + F_{dl}$$

$$W = nF_{dl} \qquad [4A]$$

Again, substituting Eqn [4A] into Eqn [1] gives

$$\mathrm{F}_d = \mathrm{F}_{fl} + \mathrm{nF}_{dl} + \mathrm{F}_{dl}$$

From Eqn [3], $F_{fl} = F_{dl}$. Substituting this relation into the above equation gives

$$\mathrm{F}_d = F_{dl} + \mathrm{n}F_{dl} + F_{dl}$$

$$Fd = F_{dl}(\mathrm{n} + 2) \qquad [5]$$

From Eqn [4A], the dead line tension is given by

$$F_{dl} = \frac{w}{n} \qquad [4B]$$

Substituting Eqn [4B] into Eqn [5] gives

$$F_d = \frac{w}{n}(\mathrm{n} + 2) \qquad [6]$$

RELATIONSHIP BETWEEN THE FAST LINE TENSION (F_{fl}) AND THE DEAD LINE TENSION (F_{dl}) WHEN RAISING THE BLOCK (HOISTING)

This was based on an Inactive dead line sheave and hence the dead line sheave provides a perfect transmission of tensions .i.e. $\mathrm{F_n} = \mathrm{F_{dl}}$

But the efficiency (e) is given by

$$\text{Efficiency (e)} = \frac{\text{Actual Mechanical Advantage (with friction)}}{\text{Ideal Mechanical Advantage (without friction)}} = \frac{M_A}{M_I}$$

Mechanical Advantage (MA) is also given by

$$MA = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)}$$

For ideal pulley without frictional losses, $F_{fl} = F_1 = F_2 = F_3 = F_4$ ….=…. F_{dl}

$$\Rightarrow M_I = \frac{F_1}{F_{fl}} = \frac{F_{fl}}{F_{fl}} = 1$$

$$\Rightarrow \text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} \qquad [\alpha]$$

During hoisting, tension decrease from the fast line towards the dead line, $ie. F_{fl} \geq F_{dl}$

Considering the fast line sheave in the crown block, its efficiency is given by

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{fl}}$$

$$\Rightarrow F_1 = eF_{fl} \qquad [\alpha\text{-}1]$$

Similarly, the efficiency of the next sheave in the travelling block is given by

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = eF_1 = e(eF_{fl}) = e^2 F_{fl} \qquad [\alpha\text{-}2]$$

Also, considering the efficiency of the next sheave in the crown block gives

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = eF_2 = e(e^2 F_2) = e^3 F_{fl} \qquad [\alpha\text{-}3]$$

Hence, the general form of the reduction in the applied fast line tension (F_{fl}) is given by

$$F_n = e^n F_{fl} \qquad [\alpha\text{-}4]$$

But for Inactive dead line sheave, the tension in the dead line is the same as the tension in the nth line supporting the hook load (w) since it is does not rotate .i.e. $F_n = F_{dl}$

$$\Rightarrow F_{dl} = F_n = e^n F_{fl}$$

$$\Rightarrow F_{fl} = \frac{F_{dl}}{e^n} \qquad [7]$$

RELATIONSHIP BETWEEN THE FAST LINE TENSION (F_{fl}) AND THE DEAD LINE TENSION (F_{dl}) DURING LOWERING OF THE BLOCK

During lowering of the block, maximum tension occurs in the dead line (F_{dl}) while the fastline (F_{fl}) records the least tension. $ie\ \ F_{fl} \leq \ F_{dl}$

Considering the dead line sheave, its efficiency is given by

$$e = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{dl}}$$

$$\Rightarrow F_1 = eF_{dl}$$

But for inactive dead line sheave, the efficiency is 100% (e =1)

$$\Rightarrow F_1 = F_{dl} \qquad [\beta\text{-}1]$$

Similarly, the efficiency of the next sheave in the travelling block is also given by

$$e = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = eF_1 = e(F_{dl}) = eF_{dl} \qquad [\beta\text{-}2]$$

Also, the efficiency of the next sheave in the crown block becomes,

$$e = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = eF_2 = e(eF_{dl}) = e^2F_{dl} \qquad [\beta\text{-}3]$$

Hence, the tension in the nth line supporting the hook load is given by

$$F_n = e^{n-1}F_{dl} \qquad [\beta\text{-}4]$$

During lowering of the block, the least line tension is the fast line and hence the efficiency of the fast line sheave is given by

$$e = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{fl}}{F_n}$$

$$F_{fl} = eF_n = e(e^{n-1}F_{dl}) = e^{n-1+1}F_{dl} = e^nF_{dl}$$

$$F_{fl} = e^nF_{dl} \qquad [8]$$

APPENDIX B

The Luke and Juvkam-Wold model also based their prediction on constant sheave efficiency assumption which might not necessarily be the case.

i.e. $F_{fl} \neq F_1 \neq F_2 \neq F_3 \neq F_4 \neq F_{dl}$

Luke and Juvkam-Wold modelled the hook load prediction for both non-rotating (Inactive) dead line sheave and rotating (Active) dead line sheave.

A. INACTIVE DEAD LINE SHEAVE DERIVATIONS

I. HOISTING

During hoisting, maximum tension occurs in the fast line (F_{fl}), while the minimum tension occur in the dead line (F_{dl}).i.e. The tension decreases from the fast line towards the dead line, $ie. F_{fl} \geq F_{dl}$

Considering the fast line sheave (First sheave in the crown block from the direction of the drum) and from Eqn (α), its efficiency is given by

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{fl}}$$

$$\Rightarrow F_1 = eF_{fl} \qquad [\delta\text{-}1]$$

Similarly, the efficiency of the next sheave in the travelling block is also given by

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = eF_1 = e\left(eF_{fl}\right) = e^2 F_{fl} \qquad [\delta\text{-}2]$$

Also, considering the efficiency of the next sheave in the crown block gives

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = eF_2 = e(e^2 F_2) = e^3 F_{fl} \qquad [\delta\text{-}3]$$

Hence, for "n" number of lines between the travelling block and the crown block, the relationship between the tension in each line and the applied fast line tension (F_{fl}) is given by

$$F_n = e^n F_{fl} \qquad [\delta\text{-}4]$$

For inactive dead line sheave since it is not rotating and hence has perfect transmission of tension (e =100%)

$$\Rightarrow F_n = F_{dl} = e^n F_{fl}$$

$$F_{fl} = \frac{F_{dl}}{e^n} \qquad [9]$$

From Eqn (2), the hook load is given by

$W = F_1 + F_2 + F_3 + \cdots + F_n$

During hoisting, the relationship between the fast line (F_{fl}) and each of the lines is given by Eqn (δ-4) as

$F_n = e^n F_{fl}$

$\Rightarrow W = e^1 F_{fl} + e^2 F_{fl} + e^3 F_{fl} + e^4 F_{fl} + \cdots + e^n F_{fl}$

$W = F_{fl}(\mathrm{e} + e^2 + e^3 + e^4 + \cdots + e^n) = F_{fl} S$

But $S = \mathrm{e} + e^2 + e^3 + e^4 + \cdots + e^n$ is the sum of a geometric series and it is given by the relation

$S = \frac{\mathrm{a}_1\,(1-r^n)}{(1-r)}$

Where a_1 = the first term of the sequence = e and r = the common ratio = $\frac{e^2}{e} = e$

$$S = \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$$

$W = F_{fl} S = F_{fl} \frac{\mathrm{e}\,(1-e^n)}{(1-e)}$ [10]

$F_{fl} = W \frac{(1-e)}{\mathrm{e}\,(1-e^n)}$ [11]

Substituting Eqn (9) into Eqn (10) gives

$$W = \frac{F_{dl}}{e^n} \frac{\mathrm{e}\,(1-e^n)}{(1-e)} = F_{dl} \frac{\mathrm{e}\,(\frac{1}{e^n} - 1)}{(1-e)} = F_{dl} \frac{\mathrm{e}\,(\frac{1}{e^n} - 1)}{(1-e)} \frac{-1}{-1}$$

$W = F_{dl} \frac{\mathrm{e}\,(1-\frac{1}{e^n})}{(e-1)}$ [12]

From Eqn (1), the derrick load is given by

$\mathrm{F}_d = \mathrm{F}_{fl} + \mathrm{n}F_{dl} + F_{dl}$

Substituting Eqn [9] and Eqn [12] into Eqn [1] gives

$F_d = \frac{F_{dl}}{e^n} + F_{dl} \frac{\mathrm{e}\,(1-\frac{1}{e^n})}{(e-1)} + \mathrm{Fdl} = F_{dl}\,(\frac{1}{e^n} + \frac{\mathrm{e}\left(1-\frac{1}{e^n}\right)}{(e-1)} + 1)$

$$F_d = F_{dl}\left(\frac{1}{e^n} + \frac{e\left(1 - \frac{1}{e^n}\right)}{(e-1)} + 1\right) \times \frac{e^n(e-1)}{e^n(e-1)}$$

$$F_d = \frac{F_{dl}}{(e-1)}\left(\frac{(e-1)}{e^n} + \frac{e^n \times e\left(1 - \frac{1}{e^n}\right)}{e^n} + \frac{e^n(e-1)}{e^n}\right)$$

$$F_d = \frac{F_{dl}}{(e-1)}\left(\frac{2e^{n+1} - e^n - 1}{e^n}\right) = \frac{F_{dl}}{(e-1)}\left(2e - 1 - \frac{1}{e^n}\right) \times \frac{-1}{-1}$$

$$F_d = \frac{F_{dl}}{(1-e)}\left(1 + \left(\frac{1}{e^n}\right) - 2e\right) \qquad [13]$$

II) LOWERING

During lowering, maximum tension occurs in the dead line (F_{dl}) while the fast line (F_{fl}) records the least tension .i.e. The tension decreases from the dead line towards the fast line, $ie. F_{fl} \leq F_{dl}$

Considering the dead line sheave (First sheave in the crown block from the direction of the dead line anchor), Eqn (ɑ) becomes

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{dl}}$$

$$\Rightarrow F_1 = eF_{dl}$$

For non-rotating dead line sheave, it is assumed that there is no work done against friction and hence the efficiency of the dead line sheave is assumed to be 100% (e = 100% = 1)

$$\Rightarrow F_1 = F_{dl} \qquad [Ƙ\text{-}1]$$

Similarly, the efficiency of the next sheave in the travelling block is given by,

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = eF_1 = e\ (F_{dl}) = e\ F_{dl} \qquad [Ƙ\text{-}2]$$

Also, considering the efficiency of the next sheave in the crown block gives,

$$\text{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = eF_2 = e\ (eF_{dl}) = e^2 F_{dl} \qquad [Ƙ\text{-}3]$$

Hence, for n number of lines between the travelling blocks and the crown block, the general line tension reduction from the dead line towards the fast line is given by the relation

$$F_n = e^n F_{dl} \qquad [Ƙ\text{-}4]$$

For inactive dead line sheave, during lowering the fast line is the nth (last) line and hence the relationship between the dead line and the fast line is given by

$\Rightarrow F_n = F_{fl} = e^n F_{dl}$

$F_{fl} = e^n F_{dl}$ [14A]

From Eqn [2], the hook load (W) is given by

$W = F_1 + F_2 + F_3 + \cdots + F_n$

But during lowering, the relationship between the dead line (F_{dl}) and each of the lines is given by Eqn [Ꝁ-4] as

$F_n = e^n F_{dl}$

For inactive dead line sheave $F_1 = F_{dl}$ since it is not rotating and hence perfect transmision of tension and hence the hook load (W) becomes,

$W = F_{dl} + eF_{dl} + e^2F_{dl} + e^3F_{dl} + e^4F_{dl} + \cdots + e^nF_{dl} = F_{dl}\,(1 + e + e^2 + e^3 + e^4 + .. + e^n)$

But $S = 1 + e + e^2 + e^3 + e^4 + .. + e^n$ is the sum of a geometric series and it is given by

$S = \frac{a_1\,(1-r^n)}{(1-r)}$

Where a_1 = the first term of the sequence = 1 and r = the common ratio = $\frac{e}{1} = e$

$$S = \frac{1\,(1-e^n)}{(1-e)} = \frac{(1-e^n)}{(1-e)}$$

$$\Rightarrow W = F_{dl}S = F_{dl}\frac{(1-e^n)}{(1-e)}$$

$W = F_{dl}\frac{(1-e^n)}{(1-e)}$ [15]

From Eqn [14A], the relationship between the the fast line and the dead line tension can also be written as

$F_{dl} = \frac{F_{fl}}{e^n}$ [14B]

Substituting Eqn [14B] into Eqn [15] gives

$$W = \frac{F_{fl}}{e^n}\frac{(1-e^n)}{(1-e)}$$

$F_{fl} = W\frac{e^n(1-e)}{(1-e^n)}$ [16]

From Eqn (1), the derrick load is given by

$F_d = F_{fl} + \mathrm{W} + F_{dl}$

Substituting Eqn (15) and Eqn (16) into Eqn (1) gives

$$F_d = e^n F_{dl} + F_{dl}\frac{(1-e^n)}{(1-e)} + F_{dl} = F_{dl}\left(e^n + \frac{(1-e^n)}{(1-e)} + 1\right) = F_{dl}\left(e^n + \frac{(1-e^n)}{(1-e)} + 1\right) \times \frac{(1-e)}{(1-e)}$$

$$F_d = F_{dl}\left(\frac{e^n(1-e) + (1-e^n) + (1-e)}{(1-e)}\right) = \frac{F_{dl}}{(1-e)}(e^n - e^{n+1} + 1 - e^n + 1 - e)$$

$$F_d = \frac{F_{dl}}{(1-e)}(2 - e - e^{n+1}) \qquad [17]$$

B. ACTIVE DEAD LINE SHEAVE DERIVATIONS

I. DURING HOISTING

During hoisting, maximum tension occurs in the fast line (F_{fl}), while the minimum tension occurs in the dead line (F_{dl}).i.e. The tension decreases from the fast line towards the dead line, $ie. F_{fl} \geq F_{dl}$

Considering the fast line sheave (First sheave in the crown block from the direction of the drum) and from Eqn (α), its efficiency is given by

$$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{fl}}$$

$$\Rightarrow F_1 = eF_{fl} \qquad [Ϛ\text{-}1]$$

Similarly, the efficiency of the next sheave in the travelling block is also given by

$$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$$

$$\Rightarrow F_2 = eF_1 = e(eF_{fl}) = e^2 F_{fl} \qquad [Ϛ\text{-}2]$$

Also, considering the efficiency of the next sheave in the crown block gives

$$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$$\Rightarrow F_3 = eF_2 = e(e^2 F_2) = e^3 F_{fl} \qquad [Ϛ\text{-}3]$$

Hence, for “n” number of lines between the travelling block and the crown block, the tension in the nth line (the last line to the dead line) its tension is given by

$$F_n = e^n F_{fl} \qquad [Ϛ\text{-}4]$$

But for inactive dead line sheave due to rotation, there is no perfect transmission of tension

$(ie\ e \neq 100\% \neq 1) \qquad \Rightarrow F_{dl} \neq F_4$

Finally, considering the efficiency of the deadline sheave in the crown block gives

$$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{dl}}{F_n}$$

$$\Rightarrow F_{dl} = eF_n = e\left(e^n F_{fl}\right) = e^{n+1} F_{fl} \qquad [ς\text{-}5]$$

$$F_{fl} = \frac{F_{dl}}{e^{n+1}} \qquad [18]$$

From Eqn [2], the hook load is given by

$$W = F_1 + F_2 + F_3 + \cdots + F_n$$

During hoisting, the relationship between the fast line (F_{fl}) and each of the lines is given by Eqn (δ-4) as

$$F_n = e^n F_{fl}$$

$$\Rightarrow W = eF_{fl} + e^2 F_{fl} + e^3 F_{fl} + e^4 F_{fl} + \cdots + e^{n+1} F_{fl} = F_{fl}\,(e + e^2 + e^3 + e^4 + \cdots + e^{n+1})$$

But $S = e + e^2 + e^3 + e^4 + \cdots + e^{n+1}$ is the sum of a geometric series and it is given by the relation

$$S = \frac{\mathrm{a}_1\,(1 - r^n)}{(1 - r)}$$

Where a_1 = the first term of the sequence = e and r = the common ratio = $\frac{e^2}{e} = e$

$$S = \frac{\mathrm{e}\,(1 - e^n)}{(1 - e)}$$

$$W = F_{fl} S = F_{fl} \frac{\mathrm{e}(1 - e^n)}{(1 - e)}$$

$$F_{fl} = W \frac{(1 - e)}{e\,(1 - e^n)} \qquad [19]$$

Substituting Eqn [18] into Eqn [19] gives

$$\frac{F_{dl}}{e^{n+1}} = W \frac{(1 - e)}{e\,(1 - e^n)}$$

$$W = \frac{F_{dl}}{e^{n+1}} \frac{e\,(1 - e^n)}{(1 - e)} = \frac{F_{dl}}{e^n} \frac{(1 - e^n)}{(1 - e)}$$

$W = \frac{F_{dl}}{e^n}\frac{(1-e^n)}{(1-e)}$ [20]

From Eqn (1), the derrick load is given by

$F_d = F_{fl} + \mathrm{W} + F_{dl}$

Substituting Eqn (20) and Eqn (18) into Eqn (1) gives

$F_d = \frac{F_{dl}}{e^{n+1}} + \frac{F_{dl}}{e^n}\frac{(1-e^n)}{(1-e)} + F_{dl} = F_{dl}\,(\frac{1}{e^{n+1}} + \frac{1}{e^n}\frac{(1-e^n)}{(1-e)} + 1)\frac{e^{n+1}(1-e)}{e^{n+1}(1-e)}$

$$F_d = \frac{F_{dl}}{(e-1)}\,(\frac{(1-e)}{e^{n+1}} + \frac{e\,(1-e^n)}{e^{n+1}} + \frac{e^{n+1}(1-e)}{e^{n+1}})$$

$F_d = \frac{F_{dl}}{(1-e)}\left(\frac{1-e+e-e^{n+1}+e^{n+1}-e^{n+2}}{e^{n+1}}\right) = \frac{F_{dl}}{(1-e)}\left(\frac{1-e^{n+2}}{e^{n+1}}\right)$

$F_d = \frac{F_{dl}}{(1-e)}\left(\frac{1-e^{n+2}}{e^{n+1}}\right)$ [21]

II) DURING LOWERING

During lowering, maximum tension occurs in the dead line (F_{dl}) while the fast line (F_{fl}) records the least tension .i.e. The tension decreases from the dead line towards the fast line, $ie. F_{fl} \leq F_{dl}$

Considering the dead line sheave (First sheave in the crown block from the direction of the dead line anchor), Eqn (α) becomes

$$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_1}{F_{dl}}$$

$\Rightarrow F_1 = eF_{dl}$ [Y-1]

For rotating dead line sheave, due to its rotation its efficiency less than 100% (e ≠ 100%)

$\Rightarrow F_1 \neq F_{dl}$

Similarly, the efficiency of the next sheave in the travelling block is given by,

$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_2}{F_1}$

$\Rightarrow F_2 = eF_1 = e\,(eF_{dl}) = e^2F_{dl}$ [Y– 2]

Also, considering the efficiency of the next sheave in the crown block gives,

$$\mathrm{e} = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_3}{F_2}$$

$\Rightarrow F_3 = eF_2 = e\,(e^2F_{dl}) = e^3F_{dl}$ [Y -3]

Hence, for "n" number of lines between the travelling block and the crown block, the tension in the nth line (the last line to the fast line) its tension is given by

$F_n = e^n F_{dl}$ [Y – 4]

Finally, considering the efficiency of the fast line sheave in the crown block is given by,

$$e = M_A = \frac{Output\ force\ (F_O)}{Input\ force\ (F_I)} = \frac{F_{fl}}{F_n}$$

$$\Rightarrow F_{fl} = eF_n = e\,(e^n F_{dl}) = e^{n+1} F_{dl}$$

$F_{fl} = e^{n+1} F_{dl}$ [22]

From Eqn [2], the hook load (W) is given by

$$W = F_1 + F_2 + F_3 + \cdots + F_n$$

But during lowering, the relationship between the dead line (F_{dl}) and each of the lines is given by Eqn [Y – 4] as

$$F_n = e^n F_{dl}$$

For active dead line sheave $F_1 \neq F_{dl}$ since it is rotating resulting in imperfect transmision of tension and hence the hook load (W) becomes,

$$W = eF_{dl} + e^2 F_{dl} + e^3 F_{dl} + e^4 F_{dl} + \cdots + e^n F_{dl} = F_{dl}\,(e + e^2 + e^3 + e^4 + .. e^n + e^{n+1})$$

But $S = e + e^2 + e^3 + e^4 + .. + e^n + e^{n+1}$ is the sum of a geometric series and it is given by

$$S = \frac{a_1\,(1-r^n)}{(1-r)}$$

Where a_1 = the first term of the sequence = e and r = the common ratio = $\frac{e^2}{e} = e$

$$S = \frac{e\,(1-e^n)}{(1-e)}$$

$$\Rightarrow W = F_{dl} S = F_{dl} \frac{e(1-e^n)}{(1-e)}$$

$W = F_{dl} \frac{e(1-e^n)}{(1-e)}$ [23]

From Eqn [22], the relationship between the the fast line and the dead line tension can be re- written as

$F_{dl} = \frac{F_{fl}}{e^{n+1}}$ [22B]

Substituting Eqn [22B] into Eqn [23] gives

$$W = \frac{F_{fl}}{e^{n+1}} \frac{\mathrm{e}(1-e^n)}{(1- e)}$$

$$F_{fl} = W \frac{e^{n+1}\,(1- e)}{\mathrm{e}(1-e^n)} = W \frac{e^n\,(1- e)}{(1-e^n)}$$

$F_{fl} = W \frac{e^n\,(1-e)}{(1-e^n)}$ [24]

From Eqn [1], the derrick load is given by

$$F_d = F_{fl} + \mathrm{W} + F_{dl}$$

Substituting Eqn [23] and Eqn [22] into Eqn [1] gives

$$F_d = e^{n+1}F_{dl} + F_{dl}\frac{\mathrm{e}(1-e^n)}{(1- e)} + F_{dl} = F_{dl}\left(e^{n+1} + \frac{\mathrm{e}(1-e^n)}{(1- e)} + 1\right)\frac{(1- e)}{(1- e)}$$

$$F_d = \frac{F_{dl}}{(1-e)}\left(e^{n+1}\,(1- e) + \mathrm{e}(1-e^n) + (1- e)\right)$$

$$F_d = \frac{F_{dl}}{(1-e)}\,(e^{n+1} - e^{n+2} + e - e^{n+1} + 1 - e\,) = \frac{F_{dl}}{(1-e)}(1-e^{n+2}\,)$$

$F_d = F_{dl}\frac{(1-e^{n+2}\,)}{(1-e)}$ [25]

APPENDIX C

I. HOISTING

From figure (27), the net torque on the dead line sheave (sheave A) in the crown block is given by Eqn [H-3B$_1$] as

$For\ F_1 > F_{dl}$

$$r_b\,(F_1 - F_{dl}) - \mu_a r_a\big(m_p g - 2\bar{\lambda}_m r_b{}^2\ \dot{\omega}^2{}_{cb1} + F_1 + F_{dl}\big) = +\,I\alpha_{cb1}$$

$$r_b\,F_1 - r_b\,F_{dl} - m_p g\mu_a r_a + 2\bar{\lambda}_m r_b{}^2\,\mu_a r_a\,\dot{\omega}^2{}_{cb1} - \mu_a r_a F_1 - \mu_a r_a F_{dl} = I\alpha_{cb1}$$

$$r_b\,F_1 - \mu_a r_a F_1 - m_p g\mu_a r_a + 2\bar{\lambda}_m r_b{}^2\,\mu_a r_a\,\dot{\omega}^2{}_{cb1} - \mu_a r_a F_{dl} - r_b\,F_{dl} = I\alpha_{cb1}$$

$$F_1(r_b - \mu_a r_a) - m_p g\mu_a r_a + 2\bar{\lambda}_m r_b{}^2\,\mu_a r_a\,\dot{\omega}^2{}_{cb1} - F_{dl}(\mu_a r_a + r_b\,) = I\alpha_{cb1}$$

$$F_1 = \frac{1}{(r_b - \mu_a r_a)} \frac{-1}{-1} \left(I\alpha_{cb1} + F_{dl}(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a \right)$$

$$F_1 = \frac{-1}{(\mu_a r_a - r_b)} \left(I\alpha_{cb1} + F_{dl}(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a \right) \qquad [\gamma\text{-1A}]$$

For simplicity, let $x = \mu_a r_a - r_b \; and \; y = \mu_a r_a + r_b$

Substituting the x and y into the Eqn [γ-1A] gives

$$F_1 = \frac{-1}{x} \left(I\alpha_{cb1} + F_{dl} y - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a \right) \qquad [\gamma\text{-1B}]$$

Similarly considering the net torque in the next sheave (sheave B) in the travelling block, Eqn [H-4B] becomes

$For\ F_2 > F_1$

$$r_b (F_2 - F_1) - \mu_a r_a \left(m_p a - m_p g - 2\bar{\lambda}_m {r_b}^2 \dot{\omega}^2{}_{tb1} + F_2 + F_1 \right) = -I\alpha_{tb1}$$

$$r_b F_2 - r_b F_1 - m_p a \mu_a r_a + m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} - \mu_a r_a F_2 - \mu_a r_a F_1 = -I\alpha_{tb1}$$

$$r_b F_2 - \mu_a r_a F_2 - m_p a \mu_a r_a + m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} - \mu_a r_a F_1 - r_b F_1 = -I\alpha_{tb1}$$

$$F_2 = \frac{1}{(r_b - \mu_a r_a)} \frac{-1}{-1} \left(-I\alpha_{tb1} + m_p a \mu_a r_a - m_p g \mu_a r_a - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} + F_1(\mu_a r_a + r_b) \right)$$

$$F_2 = \frac{1}{(\mu_a r_a - r_b)} \left(I\alpha_{tb1} - m_p a \mu_a r_a - F_1(\mu_a r_a + r_b) + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} + m_p g \mu_a r_a \right) \qquad [\gamma\text{-2A}]$$

For simplicity, let $x = \mu_a r_a - r_b \; and \; y = \mu_a r_a + r_b$ and substituting them into the above equation gives

$$F_2 = \frac{1}{x} \left(I\alpha_{tb1} - m_p a \mu_a r_a - F_1 y + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} + m_p g \mu_a r_a \right) \qquad [\gamma\text{-2B}]$$

Substitute Eqn [γ-1B] into Eqn [γ-2B] gives

$$F_2 = \frac{1}{x} \Big(I\alpha_{tb1} - m_p a \mu_a r_a + \{ \frac{y}{x} \left(I\alpha_{cb1} + F_{dl} y - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a \right) \} +$$
$$2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} + m_p g \mu_a r_a \Big)$$

Multiply through the equation by $\frac{x}{x}$ gives

$$F_2 = \frac{1}{x^2} \Big(I\alpha_{tb1} x - m_p a \mu_a r_a x + y \left(I\alpha_{cb1} + F_{dl} y - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a \right) +$$
$$2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} x + m_p g \mu_a r_a x \Big)$$

$$F_2 = \frac{1}{x^2} \Big(I\alpha_{tb1} x - m_p a \mu_a r_a x + I\alpha_{cb1} y + F_{dl} y^2 - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{cb1} y + m_p g \mu_a r_a y +$$
$$2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} x + m_p g \mu_a r_a x \Big)$$

$F_2 = \frac{1}{x^2}\left(-m_p a\mu_a r_a x + I(\alpha_{cb1}y + \alpha_{tb1}x) + F_{dl}y^2 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(-\dot{\omega}^2{}_{cb1}y + \dot{\omega}^2{}_{tb1}x\right) + m_p g\mu_a r_a(y + x)\right)$ [γ-2C]

Considering the net torque in the next sheave (sheave C) in the crown block, Eqn [H-3B1] becomes

$F_3 > F_2$

$r_b (F_3 - F_2) - \mu_a r_a\left(m_p g - 2\bar{\lambda}_m {r_b}^2 \dot{\omega}^2{}_{cb2} + F_3 + F_2\right) = +I\alpha_{cb2}$

$r_b F_3 - r_b F_2 - m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2} - \mu_a r_a F_3 - \mu_a r_a F_2 = I\alpha_{cb2}$

$r_b F_3 - \mu_a r_a F_3 - m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2} - \mu_a r_a F_2 - r_b F_2 = I\alpha_{cb2}$

$$F_3 = \frac{1}{(r_b - \mu_a r_a)}\frac{-1}{-1}\left(I\alpha_{cb2} + F_2(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right)$$

$F_3 = \frac{-1}{(\mu_a r_a - r_b)}\left(I\alpha_{cb2} + F_2(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right)$ [γ-3A]

For simplicity, substituting $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$ into Eqn [γ-3A] gives

$F_3 = \frac{-1}{x}\left(I\alpha_{cb2} + F_2 y - 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\right)$ [γ-3B]

Substitute Eqn [γ-2C] into Eqn [γ-3B] gives

$F_3 = \frac{-1}{x}\Big(I\alpha_{cb2} + \{\frac{y}{x^2}\left(-m_p a\mu_a r_a x + I(\alpha_{cb1}y + \alpha_{tb1}x) + F_{dl}y^2 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(-\dot{\omega}^2{}_{cb1}y + \dot{\omega}^2{}_{tb1}x\right) + m_p g\mu_a r_a(y + x)\right)\} - 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\Big)$

Multiply through the above equation by $\frac{x^2}{x^2}$ gives

$F_3 = \frac{-1}{x^3}\Big(I\alpha_{cb2}x^2 + y\left(-m_p a\mu_a r_a x + I(\alpha_{cb1}y + \alpha_{tb1}x) + F_{dl}y^2 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(-\dot{\omega}^2{}_{cb1}y + \dot{\omega}^2{}_{tb1}x\right) + m_p g\mu_a r_a(y + x)\right) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2}x^2 + m_p g\mu_a r_a x^2\Big)$

$F_3 = \frac{-1}{x^3}\Big(I\alpha_{cb2}x^2 - m_p a\mu_a r_a xy + I(\alpha_{cb1}y^2 + \alpha_{tb1}xy) + F_{dl}y^3 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(-\dot{\omega}^2{}_{cb1}y^2 + \dot{\omega}^2{}_{tb1}xy\right) + m_p g\mu_a r_a(y^2 + xy) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a \dot{\omega}^2{}_{cb2}x^2 + m_p g\mu_a r_a x^2\Big)$

$F_3 =$

$\frac{-1}{x^3}\left(-m_p a\mu_a r_a xy + I(\alpha_{cb1}y^2 + \alpha_{tb1}xy + \alpha_{cb2}x^2) + F_{dl}y^3 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(-\dot{\omega}^2{}_{cb1}y^2 + \dot{\omega}^2{}_{tb1}xy - \dot{\omega}^2{}_{cb2}x^2\right) + m_p g\mu_a r_a(y^2 + xy + x^2)\right)$ [γ- 3C]

Similarly considering the net torque in the next sheave (sheave D) in the travelling block, Eqn [H-4B] becomes

$For\ F_4 > F_3$

$$r_b\,(F_4 - F_3) - \mu_a r_a\big(m_p a - m_p g - 2\bar{\lambda}_m {r_b}^2\,\dot{\omega}^2{}_{tb2} + F_4 + F_3\big) = -I\alpha_{tb2}$$

$$r_b\,F_4 - r_b\,F_3 - m_p a\mu_a r_a + m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} - \mu_a r_a F_4 - \mu_a r_a F_3 = -I\alpha_{tb2}$$

$$F_4(r_b - \mu_a r_a) - m_p a\mu_a r_a + m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} - F_3(\mu_a r_a + r_b) = -I\alpha_{tb2}$$

$$F_4 = \frac{1}{(r_b - \mu_a r_a)}\frac{-1}{-1}\big(-I\alpha_{tb2} + m_p a\mu_a r_a + F_3(\mu_a r_a + r_b) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} - m_p g\mu_a r_a\big)$$

$$F_4 = \frac{1}{(\mu_a r_a - r_b)}\big(I\alpha_{tb2} - m_p a\mu_a r_a - F_3(\mu_a r_a + r_b) + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a\big)$$

[γ- 4A]

For simplicity, substituting $x = \mu_a r_a - r_b$ and $y = \mu_a r_a + r_b$ into Eqn [γ-4A] gives

$$F_4 = \frac{1}{x}\big(I\alpha_{tb2} - m_p a\mu_a r_a - F_3 y + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a\big) \qquad [γ\text{-4B}]$$

Substitute Eqn [γ-3C] into Eqn [γ-4B] gives

$$F_4 = \frac{1}{x}\Big(I\alpha_{tb2} - m_p a\mu_a r_a + \{\frac{y}{x^3}\big(-m_p a\mu_a r_a xy + I(\alpha_{cb1}y^2 + \alpha_{tb1}xy + \alpha_{cb2}x^2) + F_{dl}y^3 + 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a\,(-\dot{\omega}^2{}_{cb1}y^2 + \dot{\omega}^2{}_{tb1}xy - \dot{\omega}^2{}_{cb2}x^2) + m_p g\mu_a r_a(y^2 + xy + x^2)\big)\} + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a\Big)$$

Multiply through the above equation by $\frac{x^3}{x^3}$ gives

$$F_4 = \frac{1}{x^4}\big(I\alpha_{tb2}x^3 - m_p a\mu_a r_a x^3 + \{y\big(-m_p a\mu_a r_a xy + I(\alpha_{cb1}y^2 + \alpha_{tb1}xy + \alpha_{cb2}x^2) + F_{dl}y^3 + 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a\,(-\dot{\omega}^2{}_{cb1}y^2 + \dot{\omega}^2{}_{tb1}xy - \dot{\omega}^2{}_{cb2}x^2) + m_p g\mu_a r_a(y^2 + xy + x^2)\big)\} + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2}x^3 + m_p g\mu_a r_a x^3\big)$$

$$F_4 = \frac{1}{x^4}\big(-m_p a\mu_a r_a(xy^2 + x^3) + I(\alpha_{cb1}y^3 + \alpha_{tb1}xy^2 + \alpha_{cb2}x^2y + \alpha_{tb2}x^3) + F_{dl}y^4 + 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a\,(-\dot{\omega}^2{}_{cb1}y^3 + \dot{\omega}^2{}_{tb1}xy^2 - \dot{\omega}^2{}_{cb2}x^2y + \dot{\omega}^2{}_{tb2}x^3) + m_p g\mu_a r_a(y^3 + xy^2 + x^2y + x^3)\big) \qquad [γ\text{-4C}]$$

Hence for “n” number of lines between the crown block and the travelling block, the general relation for the increase in the line tension from the dead line (F_{dl}) towards the fast line (F_{fl}) during hoisting is given by

$$F_n = \left(\frac{-1}{x}\right)^n\ \big(-m_p a\mu_a r_a(\textstyle\sum_{k=1}^{r} x^{2k-1}\,y^{n-2k}) + I(\sum_{k=0}^{q=n-1}\alpha_{1+k}y^{q-k}x^k) + F_{dl}y^n + m_p g\mu_a r_a(\sum_{k=0}^{q=n-1} y^{q-k}x^k) + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\{\sum_{k=0}^{q=n-1}(-1)^{k+1}\dot{\omega}^2{}_{(k+1)}y^{q-k}x^k\}\big) \qquad [γ\text{-5A}]$$

where

q = n-1 (i.e. the number of supporting lines minus 1)

r = the number of travelling block sheave between the dead line and the line of interest.

$\dot{\omega}_{(k+1)}$ *and* $\alpha_{(1+k)}$ are the numbering of the angular velocity and the angular acceleration of each sheave from the dead line sheave in the crown block through the travelling block sheave as illustrated in figure (27)

LOWERING

During lowering, the line tension decreases from the dead line towards the fast line.

Considering the dead line sheave (sheave A) in the crown block, the net the torque is given by Eqn [H-3C1] as

$For\ \ F_{dl} > \ F_1$

$r_b\,(F_{dl} - F_1\,) + \mu_a r_a\big(-m_p g + \ 2\bar{\lambda}_m {r_b}^2\,\dot{\omega}^2{}_{cb1} - F_{dl} - F_1\big) = \ -I\alpha_{cb1}$

$r_b\,F_{dl} - r_b\,F_1 {-} m_p g \mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2 \mu_a r_a\,\dot{\omega}^2{}_{cb1} - F_{dl}\mu_a r_a - F_1\mu_a r_a = \ -I\alpha_{cb1}$

$-F_1(\mu_a r_a + r_b\,){-}m_p g\mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2\mu_a r_a\,\dot{\omega}^2{}_{cb1} - F_{dl}(\mu_a r_a - r_b\,) = \ -I\alpha_{cb1}$

$F_1 = \frac{-1}{(\mu_a r_a + r_b\,)}\big(-I\alpha_{cb1} + F_{dl}(\mu_a r_a - r_b\,) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a\,\dot{\omega}^2{}_{cb1} + m_p g\mu_a r_a\big)$ [δ-1A]

For simplicity, let $x = \mu_a r_a - r_b$ *and* $y = \mu_a r_a + r_b$

$F_1 = \frac{-1}{y}\big(-I\alpha_{cb1} + F_{dl}x - 2\bar{\lambda}_m {r_b}^2\mu_a r_a\,\dot{\omega}^2{}_{cb1} + m_p g\mu_a r_a\big)$ [δ-1B]

Similarly considering the net torque in the next sheave (sheave B) in the travelling block is given by Eqn [H-4C] as

$For\ \ F_1 > \ F_2$

$r_b\,(F_1 - \ F_2) - \mu_a r_a(-m_p a - (m_p g + \ 2\bar{\lambda}_m {r_b}^2\,\dot{\omega}^2{}_{tb1} - F_1 - F_2)) = +\,I\alpha_{tb1}$

$r_b\,F_1 - \ r_b F_2 + m_p a\mu_a r_a + m_p g\mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a \dot{\omega}^2{}_{tb1} - F_1\mu_a r_a - F_2\mu_a r_a = \ I\alpha_{tb1}$

$-F_2\mu_a r_a - \ r_b F_2 + m_p a\mu_a r_a + m_p g\mu_a r_a + \ 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a \dot{\omega}^2{}_{tb1} + r_b\,F_1 - F_1\mu_a r_a = \ I\alpha_{tb1}$

$-F_2(\mu_a r_a + \ r_b) = \ I\alpha_{tb1} - m_p a\mu_a r_a + F_1(\mu_a r_a - r_b\,) - m_p g\mu_a r_a - \ 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a \dot{\omega}^2{}_{tb1}$

$F_2 = \frac{-1}{(\mu_a r_a + \,r_b)}\big(I\alpha_{tb1} - m_p a\mu_a r_a + F_1(\mu_a r_a - r_b\,) - m_p g\mu_a r_a - \ 2\bar{\lambda}_m {r_b}^2\,\mu_a r_a \dot{\omega}^2{}_{tb1}\big)$

$$F_2 = \frac{1}{(\mu_a r_a + r_b)}\left(- I\alpha_{tb1} + m_p a \mu_a r_a - F_1(\mu_a r_a - r_b) + m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}\right)$$

[δ-2A]

For simplicity, let $x = \mu_a r_a - r_b \, and \;\; y = \mu_a r_a + r_b$

$$F_2 = \frac{1}{y}\left(- I\alpha_{tb1} + m_p a \mu_a r_a - F_1 x + m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}\right) \quad [\delta\text{-2B}]$$

Substituting Eqn [δ-1B] into Eqn [δ-2B] gives

$$F_2 = \frac{1}{y}\Big(- I\alpha_{tb1} + m_p a \mu_a r_a + \{\frac{x}{y}\left(-I\alpha_{cb1} + F_{dl} x - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a\right)\} +$$

$$m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1}\Big)$$

Multiply through the above equation by $\frac{y}{y}$ gives

$$F_2 = \frac{1}{y}\Big(- I\alpha_{tb1} y + m_p a \mu_a r_a y + x\left(-I\alpha_{cb1} + F_{dl} x - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb1} + m_p g \mu_a r_a\right) +$$

$$m_p g \mu_a r_a y + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} y\Big)$$

$$F_2 = \frac{1}{y^2}\Big(- I\alpha_{tb1} y + m_p a \mu_a r_a y - I\alpha_{cb1} x + F_{dl} x^2 - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb1} x + m_p g \mu_a r_a x +$$

$$m_p g \mu_a r_a y + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \dot{\omega}^2{}_{tb1} y\Big)$$

$$F_2 = \frac{1}{y^2}\Big(m_p a \mu_a r_a y - I(\alpha_{cb1} x + \alpha_{tb1} y) + F_{dl} x^2 + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \left(-\dot{\omega}^2{}_{cb1} x + \dot{\omega}^2{}_{tb1} y\right) +$$

$$m_p g \mu_a r_a (x + y)\Big) \quad [\delta\text{-2C}]$$

Similarly considering the net torque in the next sheave (sheave C) in the crown block is given by Eqn [H-3C1] as

$For \;\; F_2 > \; F_3$

$$r_b\,(F_2 - F_3) + \mu_a r_a\left(-m_p g + 2\bar{\lambda}_m {r_b}^2 \, \dot{\omega}^2{}_{cb2} - F_2 - F_3\right) = -I\alpha_{cb2}$$

$$r_b\, F_2 - r_b\, F_3 - m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} - F_2 \mu_a r_a - F_3 \mu_a r_a = -I\alpha_{cb2}$$

$$-F_3(\mu_a r_a + r_b) - m_p g \mu_a r_a + 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} + F_2(r_b - \mu_a r_a) = -I\alpha_{cb2}$$

$$F_3 = \frac{-1}{(\mu_a r_a + r_b)}\left(-I\alpha_{cb2} + F_2(\mu_a r_a - r_b) - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} + m_p g \mu_a r_a\right) \quad [\delta\text{-3A}]$$

For simplicity, let $x = \mu_a r_a - r_b \, and \;\; y = \mu_a r_a + r_b$

$$F_3 = \frac{-1}{y}\left(-I\alpha_{cb2} + F_2 x - 2\bar{\lambda}_m {r_b}^2 \mu_a r_a \, \dot{\omega}^2{}_{cb2} + m_p g \mu_a r_a\right) \quad [\delta\text{-3B}]$$

Substitute Eqn [δ-2C] into Eqn [δ-3B] gives

$F_3 = \frac{-1}{y}\Big(-I\alpha_{cb2} + \{\frac{x}{y^2}\big(m_p a\mu_a r_a y - I(\alpha_{cb1}x + \alpha_{tb1}y) + F_{dl}x^2 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ (-\dot{\omega}^2{}_{cb1}x +$

$\dot{\omega}^2{}_{tb1}y) + m_p g\mu_a r_a(x+y)\big)\} - \ 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{cb2} + m_p g\mu_a r_a\Big)$

Multiply through the above equation by $\frac{y^2}{y^2}$ gives

$F_3 = \frac{-1}{y^3}\big(-I\alpha_{cb2}y^2 + \{x\big(m_p a\mu_a r_a y - I(\alpha_{cb1}x + \alpha_{tb1}y) + F_{dl}x^2 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ (-\dot{\omega}^2{}_{cb1}x +$

$\dot{\omega}^2{}_{tb1}y) + m_p g\mu_a r_a(x+y)\big)\} - \ 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{cb2}y^2 + m_p g\mu_a r_a y^2\big)$

$F_3 = \frac{-1}{y^3}\big(m_p a\mu_a r_a yx - I(\alpha_{cb1}x^2 + \alpha_{tb1}yx + \alpha_{cb2}y^2) + F_{dl}x^3 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ (-\dot{\omega}^2{}_{cb1}x^2 +$

$\dot{\omega}^2{}_{tb1}yx - \dot{\omega}^2{}_{cb2}y^2) + m_p g\mu_a r_a(x^2 + xy + y^2)\big)$ [δ-3C]

Considering the net torque in the next sheave (sheave D) in the travelling block, Eqn [H-4C] becomes

$For\ \ F_3 >\ F_4$

$r_b\,(F_3 -\ F_2) - \mu_a r_a(-m_p a - (m_p g +\ 2\bar{\lambda}_m {r_b}^2\ \dot{\omega}^2{}_{tb2} - F_3 - F_4)) = +\,I\alpha_{tb2}$

$r_b\,F_3 -\ r_b F_4 + m_p a\mu_a r_a + m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} - \mu_a r_a F_3 - \mu_a r_a F_4 =\ I\alpha_{tb2}$

$-F_4(\mu_a r_a +\ r_b) + m_p a\mu_a r_a + m_p g\mu_a r_a + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + F_3(r_b\ - \mu_a r_a) =\ I\alpha_{tb2}$

$F_4 = \frac{-1}{(\mu_a r_a +\ r_b)}(\,I\alpha_{tb2} - m_p a\mu_a r_a + F_3(\mu_a r_a - r_b\,) - 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} - m_p g\mu_a r_a)$

$F_4 = \frac{1}{(\mu_a r_a + r_b)}(-\,I\alpha_{tb2} + m_p a\mu_a r_a - F_3(\mu_a r_a - r_b\,) + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a)$

[δ-4A]

For simplicity, let $x = \mu_a r_a - r_b\ and\ \ y = \mu_a r_a + r_b$

$F_4 = \frac{1}{y}(-\,I\alpha_{tb2} + m_p a\mu_a r_a - F_3 x + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a)$ [δ-4B]

Substitute Eqn [δ-3C] into Eqn [δ-4B] gives

$F_4 = \frac{1}{y}(-\,I\alpha_{tb2} + m_p a\mu_a r_a + \Big\{\frac{x}{y^3}\Big(m_p a\mu_a r_a yx - I(\alpha_{cb1}x^2 + \alpha_{tb1}yx + \alpha_{cb2}y^2) + F_{dl}x^3 +$

$2\bar{\lambda}_m {r_b}^2\mu_a r_a\ (-\dot{\omega}^2{}_{cb1}x^2 + \dot{\omega}^2{}_{tb1}yx - \dot{\omega}^2{}_{cb2}y^2) + m_p g\mu_a r_a(x^2 + xy + y^2)\Big)\Big\} +$

$2\bar{\lambda}_m {r_b}^2\mu_a r_a\ \dot{\omega}^2{}_{tb2} + m_p g\mu_a r_a)$

Multiply through the above equation by $\frac{y^3}{y^3}$ gives

$$F_4 = \frac{1}{y^4}\Big(- I\alpha_{tb2}y^3 + m_p a\mu_a r_a y^3 + x\Big(m_p a\mu_a r_a yx - I(\alpha_{cb1}x^2 + \alpha_{tb1}yx + \alpha_{cb2}y^2) + F_{dl}x^3 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\,(-\dot{\omega}^2{}_{cb1}x^2 + \dot{\omega}^2{}_{tb1}yx - \dot{\omega}^2{}_{cb2}y^2) + m_p g\mu_a r_a(x^2 + xy + y^2)\Big) + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\,\dot{\omega}^2{}_{tb2}y^3 + m_p g\mu_a r_a y^3\Big)$$

$$F_4 = \frac{1}{y^4}(m_p a\mu_a r_a(yx^2 + y^3) - I(\alpha_{cb1}x^3 + \alpha_{tb1}yx^2 + \alpha_{cb2}y^2x + \alpha_{tb2}y^3) + F_{dl}x^4 + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\,(-\dot{\omega}^2{}_{cb1}x^3 + \dot{\omega}^2{}_{tb1}yx^2 - \dot{\omega}^2{}_{cb2}y^2x + \dot{\omega}^2{}_{tb2}y^3) + m_p g\mu_a r_a(x^3 + x^2y + xy^2 + y^3)) \quad [\delta\text{-}4C]$$

Hence for "n" number of lines between the crown block and the travelling block, the general relation for the line tension reduction is given by

$$F_n = \left(\frac{-1}{y}\right)^n \left(m_p a\mu_a r_a(\textstyle\sum_{k=1}^{r} y^{2k-1}\, x^{n-2k}) - I(\sum_{k=0}^{q=n-1} \alpha_{1+k}x^{q-k}y^k) + F_{dl}x^n + m_p g\mu_a r_a(\sum_{k=0}^{q=n-1} x^{q-k}y^k) + 2\bar{\lambda}_m {r_b}^2\mu_a r_a\left\{\sum_{k=0}^{q=n-1}(-1)^{k+1}\dot{\omega}^2{}_{(k+1)}x^{q-k}y^k\right\}\right) \quad [\delta\text{-}5A]$$

Where

q = n-1 (i.e. the number of supporting lines minus 1)

r = the number of travelling block sheave between the fast line and the line of interest

$\dot{\omega}_{(k+1)}$ *and* $\alpha_{(1+k)}$ represent the numbering of the angular velocity and the angular acceleration of each sheave from the dead line sheave in the crown block through the travelling block sheave as illustrated in figure (29).

APPENDIX D

Below are some of the calculations performed during hoisting for some of the extended models.

Times	Position	Vtb	Acceleration (a)	Fdl	ΣF1	Σ(F1+F2)	Σ(F1+F2+F3)	Σ(F1+F2+F3+F4) at a = 0 m/s2	W (Eqn I-1C)	W(Eqna-11B) at a = 0 m/s2	% DEV. W(Eqna-11B) at a = 0 m/s2
0.1	0	0	0	600	642.05563	1327.715	2061.3368	2844.877267	2844.87727	2844.877267	0
0.2	0	0	0	700	748.95218	1548.88	2404.6514	3318.765297	3318.7653	3318.765297	0
0.3	0	0	0	800	855.84873	1770.045	2747.966	3792.653328	3792.65333	3792.653328	0
0.4	0	0	0	900	962.74529	1991.21	3091.2806	4266.541358	4266.54136	4266.541358	0
0.5	0	0	0	1000	1069.6418	2212.376	3434.5952	4740.429388	4740.42939	4740.429388	0
0.6	0	0	0	1100	1176.5384	2433.541	3777.9098	5214.317419	5214.31742	5214.317419	0
0.7	0	0	0	1200	1283.4349	2654.706	4121.2245	5688.205449	5688.20545	5688.205449	0
0.8	0	0	0	1300	1390.3315	2875.872	4464.5391	6162.093479	6162.09348	6162.093479	0
0.9	0	0	0	1400	1497.228	3097.037	4807.8537	6635.98151	6635.98151	6635.98151	0
1	0	0	0	1500	1604.1246	3318.202	5151.1683	7109.86954	7109.86954	7109.86954	0
1.1	0	0	0	1600	1711.0211	3539.367	5494.4829	7583.75757	7583.75757	7583.75757	0
1.2	0	0	0	1800	1924.8143	3981.698	6181.1121	8531.533631	8531.53363	8531.533631	0
1.3	0	0	0	1900	2031.7108	4202.863	6524.4267	9005.421661	9005.42166	9005.421661	0
1.4	0	0	0	2000	2138.6074	4424.029	6867.7413	9479.309692	9479.30969	9479.309692	0
Times	Position	Vtb	Acceleration (a)	Fdl	ΣF1	Σ(F1+F2)	Σ(F1+F2+F3)	Σ(F1+F2+F3+F4) at a = 0.5 m/s2	W (Eqn I-1C)	W(Eqna-11B) at a = 0.5 m/s2	% DEV. W(Eqna-11B) at a = 0.5 m/s2
0.1	0.01	0.1	0.5	600	642.05563	1326.951	2061.3365	2843.316536	2705.38052	2705.380517	4.90343648
0.2	0.03	0.15	0.5	700	748.95218	1548.113	2404.6302	3317.13392	3156.2119	3156.211897	4.8980083
0.3	0.06	0.2	0.5	800	855.84873	1769.273	2747.9155	3790.923045	3607.01639	3607.01639	4.894645553
0.4	0.1	0.25	0.5	900	962.74529	1990.432	3091.1925	4264.683911	4057.79399	4057.793995	4.892660013
0.5	0.15	0.3	0.5	1000	1069.6418	2211.59	3434.461	4738.416519	4508.54471	4508.544712	4.891638651
0.6	0.21	0.35	0.5	1100	1176.5384	2432.746	3777.7212	5212.120868	4959.26854	4959.268541	4.891318589
0.7	0.28	0.4	0.5	1200	1283.4349	2653.901	4120.9731	5685.796959	5409.96548	5409.965483	4.891524548
0.8	0.36	0.45	0.5	1300	1390.3315	2875.055	4464.2165	6159.444791	5860.63554	5860.635537	4.892135171
0.9	0.45	0.5	0.5	1400	1497.228	3096.207	4807.4516	6633.064364	6311.2787	6311.278703	4.893063764
1	0.55	0.55	0.5	1500	1604.1246	3317.358	5150.6783	7106.655679	6761.89498	6761.894981	4.894246746
1.1	0.66	0.6	0.5	1600	1711.0211	3538.507	5493.8967	7580.218735	7212.48437	7212.484372	4.895636429
1.2	0.78	0.65	0.5	1800	1924.8143	3980.821	6180.4212	8527.641562	8113.94547	8113.945475	4.894643501
1.3	0.91	0.7	0.5	1900	2031.7108	4201.967	6523.6228	9001.148101	8564.48109	8564.48109	4.896390066
1.4	1.05	0.75	0.5	2000	2138.6074	4423.112	6866.8161	9474.626381	9014.98982	9014.989817	4.89824565

Table 2: Shows the hook load calculation during hoisting with non-uniform movement of the travelling equipment base on the extended Cayeux et al hook load prediction model.

Times	Position	Vtb	Acceleration (a)	Fdl	F1	F2	F3	F4	Σ(F1+F2+F3+F4) at a = 0 m/s2	W (Eqn [E-1B]) at a = 0 m/s2	% DEV. Eqn W([E-1B]) at a = 0 m/s2
0.1	0	0	0	600	600	600	600	600	2400	2400	0
0.2	0	0	0	700	700	700	700	700	2800	2800	0
0.3	0	0	0	800	800	800	800	800	3200	3200	0
0.4	0	0	0	900	900	900	900	900	3600	3600	0
0.5	0	0	0	1000	1000	1000	1000	1000	4000	4000	0
0.6	0	0	0	1100	1100	1100	1100	1100	4400	4400	0
0.7	0	0	0	1200	1200	1200	1200	1200	4800	4800	0
0.8	0	0	0	1300	1300	1300	1300	1300	5200	5200	0
0.9	0	0	0	1400	1400	1400	1400	1400	5600	5600	0
1	0	0	0	1500	1500	1500	1500	1500	6000	6000	0
1.1	0	0	0	1600	1600	1600	1600	1600	6400	6400	0
1.2	0	0	0	1800	1800	1800	1800	1800	7200	7200	0
1.3	0	0	0	1900	1900	1900	1900	1900	7600	7600	0
1.4	0	0	0	2000	2000	2000	2000	2000	8000	8000	0
Times	Position	Vtb	Acceleration (a)	Fdl	F1	F2	F3	F4	Σ(F1+F2+F3+F4) at a = 0.5 m/s2	W (Eqn [E-1B]) at a = 0.5 m/s2	% DEV. Eqn W([E-1B]) at a = 0.5 m/s2
0.1	0.01	0.1	0.5	600	600	600	600	600	2400	2283.570316	4.851236823
0.2	0.03	0.15	0.5	700	700	700	700	700	2800	2664.165369	4.851236823
0.3	0.06	0.2	0.5	800	800	800	800	800	3200	3044.760422	4.851236823
0.4	0.1	0.25	0.5	900	900	900	900	900	3600	3425.355474	4.851236823
0.5	0.15	0.3	0.5	1000	1000	1000	1000	1000	4000	3805.950527	4.851236823
0.6	0.21	0.35	0.5	1100	1100	1100	1100	1100	4400	4186.54558	4.851236823
0.7	0.28	0.4	0.5	1200	1200	1200	1200	1200	4800	4567.140633	4.851236823
0.8	0.36	0.45	0.5	1300	1300	1300	1300	1300	5200	4947.735685	4.851236823
0.9	0.45	0.5	0.5	1400	1400	1400	1400	1400	5600	5328.330738	4.851236823
1	0.55	0.55	0.5	1500	1500	1500	1500	1500	6000	5708.925791	4.851236823
1.1	0.66	0.6	0.5	1600	1600	1600	1600	1600	6400	6089.520843	4.851236823
1.2	0.78	0.65	0.5	1800	1800	1800	1800	1800	7200	6850.710949	4.851236823
1.3	0.91	0.7	0.5	1900	1900	1900	1900	1900	7600	7231.306001	4.851236823
1.4	1.05	0.75	0.5	2000	2000	2000	2000	2000	8000	7611.901054	4.851236823

Table 3: Shows the hook load calculation during hoisting with non-uniform movement of the travelling equipment base on the extended Industry accepted hook load prediction model.

Below are some of the calculations performed during lowering for some of the extended models.

Times	Position	Vtb	Acceleration (a)	Fdl	ΣF1	Σ(F1+F2)	Σ(F1+F2+F3)	Σ(F1+F2+F3+F4) at a = 0 m/s2	W (Eqn I-2C) at a = 0 m/s2	W(Eqn C-11B) at a = 0 m/s2	% DEV. W (Eqn C-11B) at a = 0 m/s2
0.1	0	0	0	600	560.657	1085.776296	1576.383631	2035.971784	2035.971784	2035.971784	0
0.2	0	0	0	700	654.206	1266.83769	1839.312032	2375.48545	2375.48545	2375.48545	0
0.3	0	0	0	800	747.754	1447.899084	2102.240434	2714.999115	2714.999115	2714.999115	0
0.4	0	0	0	900	841.303	1628.960479	2365.168835	3054.512781	3054.512781	3054.512781	0
0.5	0	0	0	1000	934.851	1810.021873	2628.097236	3394.026447	3394.026447	3394.026447	0
0.6	0	0	0	1100	1028.4	1991.083267	2891.025637	3733.540112	3733.540112	3733.540112	0
0.7	0	0	0	1200	1121.95	2172.144662	3153.954038	4073.053778	4073.053778	4073.053778	0
0.8	0	0	0	1300	1215.5	2353.206056	3416.88244	4412.567444	4412.567444	4412.567444	0
0.9	0	0	0	1400	1309.04	2534.267451	3679.810841	4752.081109	4752.081109	4752.081109	0
1	0	0	0	1500	1402.59	2715.328845	3942.739242	5091.594775	5091.594775	5091.594775	0
1.1	0	0	0	1600	1496.14	2896.390239	4205.667643	5431.10844	5431.10844	5431.10844	0
1.2	0	0	0	1800	1683.24	3258.513028	4731.524446	6110.135772	6110.135772	6110.135772	0
1.3	0	0	0	1900	1776.79	3439.574422	4994.452847	6449.649437	6449.649437	6449.649437	0
1.4	0	0	0	2000	1870.33	3620.635817	5257.381248	6789.163103	6789.163103	6789.163103	0
Times	Position	Vtb	Acceleration (a)	Fdl	ΣF1	Σ(F1+F2)	Σ(F1+F2+F3)	Σ(F1+F2+F3+F4) at a = 0.5 m/s2	W (Eqn I-2C) at a = 0.5 m/s2	W(Eqn C-11B) at a = 0.5 m/s2	% DEV. W (Eqn C-11B) at a = 0.5 m/s2
0.1	0.01	0.1	0.5	600	560.657	1085.067048	1576.509389	2034.701902	2143.976977	2143.976977	5.304847234
0.2	0.03	0.15	0.5	700	654.206	1266.131668	1839.456937	2374.27864	2501.790919	2501.790919	5.317038234
0.3	0.06	0.2	0.5	800	747.754	1447.197579	2102.412144	2713.880608	2859.631446	2859.631446	5.327159393
0.4	0.1	0.25	0.5	900	841.303	1628.26478	2365.375009	3053.507804	3217.498556	3217.498556	5.335900908
0.5	0.15	0.3	0.5	1000	934.851	1809.333271	2628.345533	3393.16023	3575.392251	3575.392251	5.343676807
0.6	0.21	0.35	0.5	1100	1028.4	1990.403052	2891.323716	3732.837885	3933.312529	3933.312529	5.35075052
0.7	0.28	0.4	0.5	1200	1121.95	2171.474124	3154.309557	4072.540768	4291.259392	4291.259392	5.35729764
0.8	0.36	0.45	0.5	1300	1215.5	2352.546486	3417.303057	4412.268881	4649.232838	4649.232838	5.363439719
0.9	0.45	0.5	0.5	1400	1309.04	2533.620139	3680.304216	4752.022223	5007.232869	5007.232869	5.369263573
1	0.55	0.55	0.5	1500	1402.59	2714.695082	3943.313033	5091.800795	5365.259484	5365.259484	5.374832859
1.1	0.66	0.6	0.5	1600	1496.14	2895.771315	4206.329509	5431.604595	5723.312683	5723.312683	5.38019532
1.2	0.78	0.65	0.5	1800	1683.24	3257.910232	4732.282045	6110.94729	6439.139948	6439.139948	5.384564078
1.3	0.91	0.7	0.5	1900	1776.79	3438.989046	4995.313839	6450.801548	6797.246314	6797.246314	5.389391787
1.4	1.05	0.75	0.5	2000	1870.33	3620.06915	5258.353291	6790.681036	7155.379265	7155.379265	5.394128213

Table 4: Shows the hook load calculation during lowering with non-uniform movement of the travelling equipment base on the extended Cayeux et al hook load prediction model.

Times	Position	Vtb	Acceleration (a)	Fdl	F1	F2	F3	F4	Σ(F1+F2+F3+F4) at a = 0 m/s2	W (Eqn [E-2B]) at a = 0 m/s2	% DEV. W (Eqn [E-2B]) at a = 0 m/s2
0.1	0	0	0	600	600	600	600	600	2400	2400	0
0.2	0	0	0	700	700	700	700	700	2800	2800	0
0.3	0	0	0	800	800	800	800	800	3200	3200	0
0.4	0	0	0	900	900	900	900	900	3600	3600	0
0.5	0	0	0	1000	1000	1000	1000	1000	4000	4000	0
0.6	0	0	0	1100	1100	1100	1100	1100	4400	4400	0
0.7	0	0	0	1200	1200	1200	1200	1200	4800	4800	0
0.8	0	0	0	1300	1300	1300	1300	1300	5200	5200	0
0.9	0	0	0	1400	1400	1400	1400	1400	5600	5600	0
1	0	0	0	1500	1500	1500	1500	1500	6000	6000	0
1.1	0	0	0	1600	1600	1600	1600	1600	6400	6400	0
1.2	0	0	0	1800	1800	1800	1800	1800	7200	7200	0
1.3	0	0	0	1900	1900	1900	1900	1900	7600	7600	0
1.4	0	0	0	2000	2000	2000	###	2000	8000	8000	0
Times	Position	Vtb	Acceleration (a)	Fdl	F1	F2	F3	F4	Σ(F1+F2+F3+F4) at a = 0.5 m/s2	Eqn [E-2B] at a = 0.5 m/s2	% DEV. W (Eqn [E-2B]) at a = 0.5 m/s2
0.1	0.01	0.1	0.5	600	600	600	600	600	2400	2528.940059	5.372502458
0.2	0.03	0.15	0.5	700	700	700	700	700	2800	2950.430069	5.372502458
0.3	0.06	0.2	0.5	800	800	800	800	800	3200	3371.920079	5.372502458
0.4	0.1	0.25	0.5	900	900	900	900	900	3600	3793.410088	5.372502458
0.5	0.15	0.3	0.5	1000	1000	1000	1000	1000	4000	4214.900098	5.372502458
0.6	0.21	0.35	0.5	1100	1100	1100	1100	1100	4400	4636.390108	5.372502458
0.7	0.28	0.4	0.5	1200	1200	1200	1200	1200	4800	5057.880118	5.372502458
0.8	0.36	0.45	0.5	1300	1300	1300	1300	1300	5200	5479.370128	5.372502458
0.9	0.45	0.5	0.5	1400	1400	1400	1400	1400	5600	5900.860138	5.372502458
1	0.55	0.55	0.5	1500	1500	1500	1500	1500	6000	6322.350147	5.372502458
1.1	0.66	0.6	0.5	1600	1600	1600	1600	1600	6400	6743.840157	5.372502458
1.2	0.78	0.65	0.5	1800	1800	1800	1800	1800	7200	7586.820177	5.372502458
1.3	0.91	0.7	0.5	1900	1900	1900	1900	1900	7600	8008.310187	5.372502458
1.4	1.05	0.75	0.5	2000	2000	2000	###	2000	8000	8429.800197	5.372502458

Table 5: Shows the hook load calculation during lowering with non-uniform movement of the travelling equipment base on the extended Industry accepted hook load prediction model.