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Balanced Scorecards: A Relational Contract Approach

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ABSTRACT

Reward systems based on balanced scorecards often connect pay to an index, that is, a weighted sum of multiple performance measures. We show that such an index contract may indeed be optimal if performance measures are nonverifiable so that the contracting parties must rely on self-enforcement. Under commonly invoked assumptions (including normally distributed measurements), we show that the weights in the index reflect a tradeoff between distortion and precision for the measures. The efficiency of the contract improves with higher precision of the index measure, because this strengthens incentives, and correlations between measurements may for this reason be beneficial. There is a caveat, however, because the index contract is not necessarily optimal for very precise measurements, although it is shown to be asymptotically optimal. We also consider hybrid measurements, and show that the principal may want to include verifiable performance measures in the

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relational index contract in order to improve incentives, and that this has noteworthy implications for the formal contract.

JEL codes: D86, J33, M52

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1. *Introduction*

Very few jobs can be measured along one single dimension; employees usually multitask. This creates challenges for incentive providers: If the firm only rewards a subset of dimensions or tasks, agents will have incentives to exert efforts only on those tasks that are rewarded, and ignore others. A solution for the firm is to add more metrics to the compensation scheme, but this usually implies some form of measurement problem, leading either to more noise or distortions, or to the use of nonverifiable (subjective) performance measures.

The latter is often implemented by the use of a balanced scorecard (BSC). Kaplan and Norton's [1992, 1996] highly influential concept began with a premise that exclusive reliance on verifiable financial performance measures was not sufficient, as it could distort behavior and promote effort that is not compatible with long-term value creation. Their main ideas were indebted to the canonical multitasking models of Holmström and Milgrom [1991] and Baker [1992]. However, their approach was more practical, guiding firms in how to design performance measurement systems that focus not only on short-term financial objectives, but also on long-term strategic goals (Kaplan and Norton [2001]).

While measuring performance is one issue, the question of how to reward performance is a different one. As noted by Budde [2007], there is a general understanding that efficient incentives must be based on multiple performance measures, including nonverifiable ones. Still, the implementation is a matter of controversy. Reward systems based on BSC often connect pay to an index, that is, a weighted sum of multiple performance measures, but there is also variety regarding to what extent index contracts are used (see WorldatWork and Deloitte Consulting LLP [2014], and section 5 later in this paper for a discussion). There is apparently no formal incentive model that actually derives this kind of index contract as an optimal solution in settings with nonverifiable measures.[1] In fact, Kaplan and Norton [1996] were sceptical to compensation formulas that calculated incentive compensation directly via a sum of weighted metrics. Rather, they proposed to establish different bonuses for a whole set of

[1] Banker and Datar [1989] derive conditions under which a contract based on a linear aggregate of verifiable performance measures is optimal in a standard moral hazard problem with a risk-averse agent.

critical performance measures, more in line with the original ideas of Holmström and Milgrom [1991] and Feltham and Xie [1994].

Despite the large literature following the introduction of BSC (see Hogue [2014], for a review), and the massive use of scorecards in practice, it appears that the index contracts that BSC firms often prescribe lack a formal contract theoretic justification.[2] We take some steps to fill the gap. Our starting point is that the performance measures are nonverifiable. This means that the incentive contract cannot be enforced by a third party and thus needs to be self-enforcing—or what is commonly termed "relational." Incentive contracts used by firms, including performance measures based on BSCs, often include nonverifiable qualitative assessments of performance (see Ittner, Larcker, and Meyer [2003], Gibbs et al. [2004], Kaplan and Gibbons [2015]). Moreover, even if some performance measures in principle are verifiable, the costs and uncertainty of taking the contract to court may be so high that the parties in practice need to rely on self-enforcement (see MacLeod [2007] and references therein).

In the now large literature on self-enforcing relational contracts, relatively few papers have considered relational contracts with multitasking agents (prominent papers include Baker, Gibbons, and Murphy [2002], Budde [2007], Schottner [2008], Mukherjee and Vasconcelos [2011], Ishihara [2016], Ishihara [2020]). We on the one hand generalize this literature in some dimensions (to an arbitrary number of tasks with stochastic measurements that are possibly correlated and/or distorted), and on the other hand invoke assumptions (notably normally distributed measurements) that make the model quite tractable.[3]

We first show that the optimal relational contract between a principal and a multitasking agent turns out to be an index contract, or what one may call a BSC. That is, the agent gets a bonus if a weighted sum of performance outcomes on the various tasks (an index) exceeds a hurdle. This is in contrast to the optimal contract in, for example, Holmström and Milgrom [1991], where the agent gets a bonus on each task. The important difference from Holmström and Milgrom is that we consider a relational contracting setting where the size of the bonus is limited by the principal's temptation to renege (rather than risk considerations). In such a setting, the marginal incentives to exert effort on each task are higher with index contracts than with bonuses awarded on each task.

[2] According to Hogue [2014], among the more than 100 papers published on BSC theory, only a handful have used principal agent theory to analyze BSC. See also Hesford et al. [2007] for a review.

[3] Our paper is indebted to the seminal literature on relational contracts. The concept of relational contracts was first defined and explored by legal scholars (Macaulay [1963], Macneil [1978]), whereas the formal literature started with Klein and Leffler [1981]. MacLeod and Malcomson [1989] provide a general treatment of the symmetric information case, whereas Levin [2003] generalizes the case of asymmetric information. The relevance of the relational contract approach to management accounting and performance measurement is discussed in Glover [2012] and Baldenius, Glover, and Xue [2016].

The performance measures within a scorecard may well be correlated. We point out that such correlations will affect the efficiency of the contract and we show that the efficiency of the index contract depends on how correlations affect the precision of the overall scorecard measure. In particular, an index contract with nonnegative weights on all relevant measures will work even better if the measures are negatively correlated. The reason is that negative correlation reduces the variance of the overall performance measure (the index) in such cases. This is beneficial in our setting, not because a more precise measure reduces risk—because the agent is assumed to be risk neutral—but because it strengthens, for any given bonus level, the incentives for the agent to provide effort.[4]

Besides being affected by noise, performance measures are normally also to various degrees distorted, implying that incentives on these measures promote actions that are not perfectly aligned with the firm's true objective. Many firms end up with rewarding performance according to such distorted measures, as long as the performance can be measured precisely. That is, the firm may prefer distorted, but precise performance measures, rather than well-aligned, but vague and imprecise measures. They can find support for this strategy in classic incentive theory where performance measures are verifiable and contracts are court enforceable (e.g., Datar, Kulp, and Lambert [2001]).

A natural solution to this measurement problem may be to rely on subjective performance measures that are better aligned with the true objective, and make the contract self-enforcing. However, as we show in this paper, even in relational contracts, where there are no requirements regarding verifiability, and thus presumably greater scope for subjective and well-aligned performance measures, it may still be optimal to let precision weigh more heavily than alignment in incentive provision.

Our analysis reveals that the optimal weights in the scorecard index reflect a tradeoff between distortion and precision, implying that a measure, which is well aligned with the firm's true objective, may nevertheless get a small weight in the index if that measure is to a large extent affected by noise and therefore highly imprecise. Again, this is not because of risk considerations, but because of incentive effects from the overall precision of the index.

We also consider the case where some measures are verifiable, and some are not. We show that the principal will include verifiable measures in the relational index contract in order to strengthen incentives.[5] This resembles

[4] Similar effects appear in Kvaløy and Olsen [2019], which analyzes relational contracts and correlated performances in a model with multiple agents, but single tasks.

[5] Our analysis of this issue presumes short-term explicit (court-enforced) contracts. Watson, Miller, and Olsen [2020] present a general theory for interactions between relational and court-enforced contracts when the latter are long term and renegotiable, and show that optimal contracts are then nonstationary. Implications of this for the contracting problems considered in this paper are left for future research.

BSCs seen in practice, which often include both verifiable measures such as sales or financial accounting data, and nonverifiable (subjective) measures (see, e.g., Kaplan and Norton [2001], Ittner, Larcker, and Meyer [2003]). By including a verifiable measure in the relational contract, the variance of the performance index may be reduced, which again strengthens incentives. We also show that the verifiable performance measure is taken into the index as a benchmark, to which the other performances are compared. Moreover, the principal will still offer an explicit bonus contract on the verifiable measure, but this bonus is generally affected by the optimal relational index contract.[6] We show that, in spite of the agent being risk neutral, the optimal bonus will, via this link with the relational contract, vary with the variance of the verifiable measure.

A paper closely related to ours is Budde [2007], which was the first to investigate BSCs within a relational contracting framework. Specifically, it analyzes incentive effects of a scorecard scheme based on a set of balanced performance measures under both explicit and relational contracts. The paper is important, as it shows that BSC types of contracts can provide undistorted incentives in settings with no noise and sufficient congruity/alignment between performance measures and the "true" value added. We extend and complement Budde in several aspects. First, and unlike us, Budde assumes at the outset that the available measurement system is "balanced," "minimal," and without noise. These assumptions imply, among other things, that from an observation of the measurements, one can perfectly deduce the agent's action. This means that the action is in essence observable, and simple forcing contracts for the agent are then feasible (and optimal).[7] We extend Budde by allowing for both "unbalanced" and noisy measurements. Actions then cannot be deduced from observations, which means that there is a real hidden action problem, and the characterization of optimal (relational) incentive contracts becomes an essential task. This characterization is an important part of our paper.

The main focus in Budde's paper is the extent to which a relational contract can supplement an explicit contract to achieve a first-best allocation, in a setting where an explicit contract alone cannot do so because of misalignment between the measures that are verifiable and the true value. The assumptions on the total measurement system imply that the first best can

[6] Our model thus complements the influential papers by Baker, Gibbons, and Murphy [1994] and Schmidt and Schnitzer [1995], on the interaction between relational and explicit contracts. Whereas their results are driven by differences in fallback options created by the explicit contracts, our results stem from correlation between the tasks and (or) misalignment between measurements and true values.

[7] The paper allows for noisy observations in settings with verifiable measurements, and briefly discusses general noisy observations in a final section. The discussion concludes that "… a subtle tradeoff between the benefits of risk diversification and congruity has to be considered" and ". . a detailed investigation of this tradeoff requires considerable analysis" (Budde [2007, p. 533]). We provide such an investigation here.

always be achieved if the parties are sufficiently patient.[8] This is generally not the case under our relaxed assumptions. We thus complement Budde's analysis by characterizing optimal relational contracts and second-best allocations under more realistic assumptions about the performance measurement system, especially regarding the measurements' precision, and regarding the interaction between verifiable and nonverifiable measures. Interestingly, an index contract—a scorecard—then emerges as the optimal (relational) contract.

The rest of the paper is organized as follows: In section 2, we present the basic model and a preliminary result. In section 3, we introduce distorted performance measures and present our main results for relational contracts in this setting. They show that an optimal such contract takes the form of a BSC (index) contract, where the weights on the measures in the index reflect a tradeoff between distortion and precision. These results rely on some assumptions, including validity of the "first-order approach" (FOA). It turns out that this approach is invalid if measurements are very precise, and although a characterization of optimal contracts is therefore lacking for such environments, we show that index contracts will nevertheless perform well and in fact become asymptotically optimal when measurement noise vanishes. In section 4, we extend the model to include both verifiable and nonverifiable performance measures, and we present novel results regarding (1) whether and how the relational contract will depend on a verifiable measure and (2) in what way and to what extent bonus elements of the formal contract will be affected by the relational contract. Section 5 presents some empirical evidence on firms' use of BSCs, and section 6 concludes.

2. *Model*

First, we present the basic model between a principal and a multitasking agent. Consider an ongoing economic relationship between a risk-neutral principal and a risk-neutral agent. Each period the agent takes an n-dimensional action $\boldsymbol{a} = (a_1, \ldots, a_n)'$, generating a gross value $v(\boldsymbol{a})$ for the principal, a private cost $c(\boldsymbol{a})$ for the agent, and a set of $m \leq n$ stochastic performance measurements $\boldsymbol{x} = (x_1, \ldots, x_m)'$. These measurements are observable, but not verifiable, with joint density, conditional on action $f(\boldsymbol{x}, \boldsymbol{a})$. Only the agent observes the action. The gross value $v(\boldsymbol{a})$ is not observed (as is the case if this is, e.g., expected revenue for the principal, conditional on the agent's action). We assume $v(\boldsymbol{a})$ to be increasing in each a_i and concave, and $c(\boldsymbol{a})$ to be increasing in a each a_i and strictly convex with $c(\mathbf{0}) = 0$ and gradient vector (marginal costs) $\nabla c(\mathbf{0}) = \mathbf{0}$. The total surplus

[8] The paper characterizes the minimal critical discount factor necessary to achieve the first-best, and importantly shows that this entails restricting informal incentives to that part of the first-best action that cannot be induced by a formal contract. Moreover, all unverifiable measures should be used in the relational contract.

(per period) in the relationship is $v(\boldsymbol{a}) - c(\boldsymbol{a})$. The parties have a common discount factor $\delta \in (0, 1)$.

Given observable (but not verifiable) measurements, the agent is in each period promised a bonus $\beta(\boldsymbol{x})$ from the principal. Specifically, the stage game proceeds as follows: (1) The principal offers the agent a contract consisting of a fixed payment w and a bonus $\beta(\boldsymbol{x})$. (2) If the agent accepts, he chooses some action $\boldsymbol{a}$, generating performance measure $\boldsymbol{x}$. If the agent declines, nothing happens until the next period. (3) The parties observe performance $\boldsymbol{x}$, the principal pays w and chooses whether or not to honor the full contract and pay the specified bonus. (4) The agent chooses whether or not to accept the bonus he is offered. (5) The parties decide whether to continue or break off the relationship. Outside options are normalized to zero.

As shown by Levin [2002], [2003], we may assume trigger strategies and stationary contracts. The parties honor the contract only if both parties honored the contract in the previous period, and they break off the relationship and take their respective outside options otherwise. To prevent deviations, the self-enforced discretionary bonus payments must be bounded above and below. As is well known, the range of such self-enforceable payments is defined by the future value of the relationship, hence we have a dynamic enforceability condition given by

$$0 \leq \beta(\boldsymbol{x}) \leq \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})), \text{all feasible } \boldsymbol{x}. \tag{1}$$

The optimal relational contract maximizes the surplus $v(\boldsymbol{a}) - c(\boldsymbol{a})$ subject to this constraint and the agent's incentive compatibility (IC) constraint. The latter is

$$\boldsymbol{a} \in \arg\max_{\tilde{\boldsymbol{a}}} E(\beta(\boldsymbol{x})|\tilde{\boldsymbol{a}}) - c(\tilde{\boldsymbol{a}}),$$

with first-order conditions (subscripts denote partials)

$$0 = \frac{\partial}{\partial a_i} E(\beta(\boldsymbol{x})|\,\boldsymbol{a}) - c_i(\boldsymbol{a}) = \int \beta(\boldsymbol{x}) f_{a_i}(\boldsymbol{x}, \boldsymbol{a}) - c_i(\boldsymbol{a}),\, i = 1, \cdots n.$$

A standard approach to solve this problem is to replace the global incentive constraint for the agent with the local first-order conditions. It is well known that this may or may not be valid, depending on the circumstances (see, e.g., Hwang [2016] and Chi and Olsen [2018]). We will in this paper mostly assume that it is valid, and subsequently state conditions for which this is true. So we invoke the following:

Assumption A. *The FOA is valid.*

Unless explicitly noted otherwise, we will take this assumption for granted in the following. We then have an optimization problem that is linear in the bonuses $\beta(\boldsymbol{x})$. The optimal bonuses will then have a bang-bang structure, and hence be either maximal or minimal, depending on the outcome $\boldsymbol{x}$. Introducing the likelihood ratios

$$l_{a_i}(\boldsymbol{x}, \boldsymbol{a}) = f_{a_i}(\boldsymbol{x}, \boldsymbol{a})/f(\boldsymbol{x}, \boldsymbol{a}),$$

we obtain the following:

Lemma 1. *There is a vector of multipliers $\boldsymbol{\mu}$ such that (at the optimal action $\boldsymbol{a} = \boldsymbol{a}^*$) the optimal bonus is maximal for those outcomes $\boldsymbol{x}$, where $\Sigma_i \mu_i l_{a_i}(\boldsymbol{x}, \boldsymbol{a}) > 0$, and it is zero otherwise, that is,*

$$\beta(\boldsymbol{x}) = \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})) \text{ if } \Sigma_i \mu_i l_{a_i}(\boldsymbol{x}, \boldsymbol{a}) > 0,$$

and $\beta(\boldsymbol{x}) = 0$ *if* $\Sigma_i \mu_i l_{a_i}(\boldsymbol{x}, \boldsymbol{a}) < 0$.

The lemma says that there is an index $\tilde{y}(\boldsymbol{x}) = \Sigma_i \mu_i l_{a_i}(\boldsymbol{x}, \boldsymbol{a})$, with $\boldsymbol{a} = \boldsymbol{a}^*$ being the optimal action, such that the agent should be paid a bonus if and only if this index is positive, and the bonus should then be maximal. This index, which takes the form of a weighted sum of the likelihood ratios for the various action elements, is in this sense an optimal performance measure for the agent.

The index is basically a scorecard for the agent's performance, and because it is optimal, it is (more or less by definition) balanced. That is, the available measures are used in a balanced way to construct the scorecard so as to achieve the highest feasible surplus for the parties. In the following, we will introduce further assumptions to analyze its properties. As we will then demonstrate, these properties include, among other things, an optimal balance between precision and alignment for the various measures.

3. Scorecards and Distorted Measures

Following Baker [1992], Feltham and Xie [1994], and the often used modeling approach in the management accounting literature (e.g., Datar, Kulp, and Lambert [2001], Huges, Zhang, and Xie [2005], Budde [2007], [2009]), we will from now on assume that the measurements $\boldsymbol{x}$ are potentially distorted and given by

$$\boldsymbol{x} = \boldsymbol{Q}'\boldsymbol{a} + \boldsymbol{\varepsilon}, \tag{2}$$

where $\boldsymbol{Q}'$ is an $m \times n$ matrix of rank $m \leq n$, and $\boldsymbol{\varepsilon} \sim N(0, \boldsymbol{\Sigma})$ is multinormal with covariance matrix $\boldsymbol{\Sigma} = [s_{ij}]$ (i.e., $\boldsymbol{x} \sim N(\boldsymbol{Q}'\boldsymbol{a}, \boldsymbol{\Sigma})$).[9] Let $\boldsymbol{q}_1, \ldots, \boldsymbol{q}_m$ be the column vectors of $\boldsymbol{Q}$, so we have $E(x_i|\boldsymbol{a}) = \boldsymbol{q}_i'\boldsymbol{a}$, $i = 1 \ldots m$. As is common in much of this literature, we assume multinormal noise for tractability. The likelihood ratios for this distribution are linear in $\boldsymbol{x}$, and this implies that the optimal performance index $\Sigma_i \mu_i l_{a_i}(\boldsymbol{x}, \boldsymbol{a})$ identified in the previous lemma is also linear in $\boldsymbol{x}$. In particular, the vector of likelihood ratios is given by the gradient $\nabla_{\boldsymbol{a}} \ln f(\boldsymbol{x}; \boldsymbol{a}) = \boldsymbol{Q}\boldsymbol{\Sigma}^{-1}(\boldsymbol{x} - \boldsymbol{Q}'\boldsymbol{a})$. Hence, defining vector $\boldsymbol{\tau}$ by $\boldsymbol{\tau}' = \boldsymbol{\mu}'\boldsymbol{Q}\boldsymbol{\Sigma}^{-1}$, the index can be written as $\Sigma_i \mu_i l_{a_i}(\boldsymbol{x}, \boldsymbol{a}^*) = \boldsymbol{\tau}'(\boldsymbol{x} - \boldsymbol{Q}'\boldsymbol{a}^*)$,

[9] Budde [2007] in addition assumes "balance," which implies that the first-best action can be implemented by linear bonuses when measurements are verifiable. For the main results (on relational contracts), measurements are also assumed to be noise free.

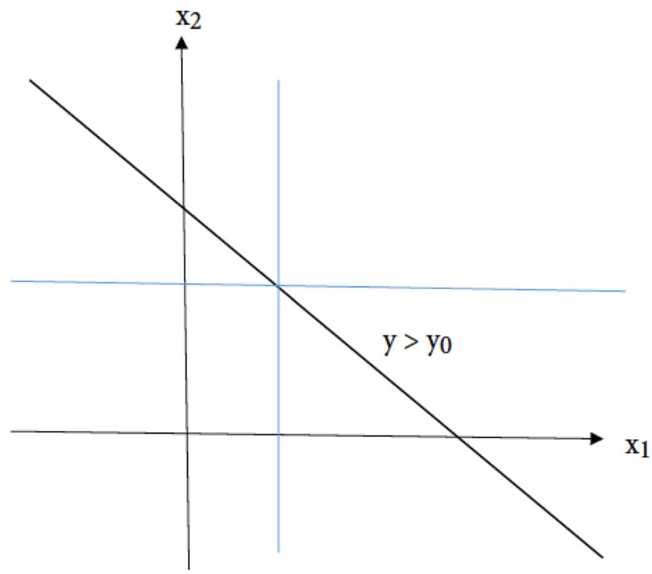

FIG 1.—Structure of the optimal index contract.

where the expression in accordance with Lemma 1 is evaluated at $\boldsymbol{a} = \boldsymbol{a}^*$. So we have the following:

Proposition 1. *In the multinormal case, there is a vector* $\boldsymbol{\tau}$ *and a performance index* $\tilde{y} = \Sigma_j \tau_j x_j$ *such that the agent is optimally paid a bonus if and only if the index exceeds a hurdle* ($\tilde{y}_0$)*. The hurdle is given by the agent's expected performance in this setting* ($\tilde{y}_0 = \Sigma_j \tau_j E(x_j|\boldsymbol{a}^*)$)*, and the bonus, when paid, is maximal:* $\beta(\boldsymbol{x}) = \frac{\delta}{1-\delta}(v(\boldsymbol{a}^*) - c(\boldsymbol{a}^*))$.

This result parallels Levin's [2003] characterization of the single-task case, where the agent optimally gets a bonus if his performance on the single task exceeds a hurdle. Here, in the multitask case, the principal offers an *index* $\tilde{y} = \Sigma_j \tau_j x_j$, that is, a weighted sum of performance outcomes on the various tasks, such that the agent gets a bonus if and only if this index exceeds a hurdle $\tilde{y}_0$. The optimal hurdle is given as the similar weighted sum of optimal expected performances. Hence, performance x_i is compared to expected performance, given (equilibrium) actions. If the weighted sum of performances exceeds what is expected, then the agent obtains the bonus.

Figure 1 illustrates the structure of the optimal bonus scheme. The index and its hurdle define a hyperplane delineating outcomes "above" the plane from those "below," where the former are rewarded with full and maximal bonus, whereas the latter yield no bonus at all. This is clearly different from a structure with separate bonuses and hurdles on each task. Such a structure is illustrated by the blue lines in the figure. In the two-dimensional case this structure defines four regions in the space of outcomes; where either zero, one, or two bonuses are paid, respectively. The analysis shows that the structure defined by the index is better, and in fact optimal.

Proposition 1 characterizes the type of bonus scheme that will be optimal. The next step is to characterize the parameters of the scheme, that is, the weights $\boldsymbol{\tau}$ and the hurdle $\tilde{y}_0$ that will generate optimal actions. We now turn to this.

Given the index $\tilde{y} = \boldsymbol{\tau}'\boldsymbol{x}$ with hurdle $\tilde{y}_0$, and the bonus $\beta = b$ being paid for $\tilde{y} > \tilde{y}_0$, the agent's performance-related payoff is $b\Pr(\boldsymbol{\tau}'\boldsymbol{x} > \tilde{y}_0|\boldsymbol{a}) - c(\boldsymbol{a})$. Using the normal distribution, we find (see the appendix for details) that the agent's first-order conditions for actions at their equilibrium levels ($\boldsymbol{a} = \boldsymbol{a}^*$) then satisfy

$$(b\phi_0/\tilde{\sigma})\boldsymbol{Q\tau} = \nabla c(\boldsymbol{a}^*), \tag{3}$$

where $\phi_0 = 1/\sqrt{2\pi}$ is a parameter of the distribution, and $\tilde{\sigma}$ is the standard deviation of the performance index:

$$\tilde{\sigma} = SD(\tilde{y}) = (\boldsymbol{\tau}'\boldsymbol{\Sigma\tau})^{1/2}.$$

Note that incentives, given by the marginal revenues on the left-hand side of (3), are inversely proportional to the standard deviation $\tilde{\sigma}$. All else equal, a more precise performance index (lower $\tilde{\sigma}$) will thus enhance the effectiveness of a given bonus in providing incentives to the agent. This indicates that more precise measurements will be beneficial in this setting, and that this will occur not because of reduced risk costs (there are none, by assumption) but because of enhanced incentives. The monetary bonus is constrained by self-enforcement, and other factors that enhance its effectiveness will then be beneficial. We return to this below.

The optimal bonus paid for qualifying performance is the maximal one, so

$$b = \frac{\delta}{1-\delta}(v(\boldsymbol{a}^*) - c(\boldsymbol{a}^*)). \tag{4}$$

For given action $\boldsymbol{a}^*$, the elements b and $\boldsymbol{\tau}$ of the optimal incentive scheme will be given by these relations.

On the other hand, optimal actions must maximize the surplus $v(\boldsymbol{a}) - c(\boldsymbol{a})$ subject to these conditions. To characterize the associated optimization program for actions, it is convenient to introduce modified weights in the performance index, namely, a weight vector $\boldsymbol{\theta}$ given by

$$\boldsymbol{\theta} = (b\phi_0/\tilde{\sigma})\boldsymbol{\tau}.$$

Because $\boldsymbol{\theta}$ is just a scaling of $\boldsymbol{\tau}$, that is, $\boldsymbol{\theta} = k\boldsymbol{\tau}$, $k > 0$, the performance index can be expressed in terms of $\boldsymbol{\theta}$ as $y = \boldsymbol{\theta}'\boldsymbol{x}$, and the agent is then given a bonus if this index exceeds its expected value $y_0 = \boldsymbol{\theta}'E(\boldsymbol{x}|\boldsymbol{a}^*)$. Note that the agent's first-order condition (3) then takes the form $\boldsymbol{Q\theta} = \nabla c(\boldsymbol{a}^*)$.

Observe also from the definitions of $\boldsymbol{\theta}$ and $\tilde{\sigma}$ that the modified index has variance $\boldsymbol{\theta}'\boldsymbol{\Sigma\theta} = (b\phi_0/\tilde{\sigma})^2\boldsymbol{\tau}'\boldsymbol{\Sigma\tau} = \phi_0^2 b^2$, meaning that its standard deviation is proportional to the bonus b. The index will thus be enforceable (for a given action $\boldsymbol{a}$) if and only if its standard deviation satisfies the following constraint:

$$\frac{\delta}{1-\delta}(v(\boldsymbol{a})-c(\boldsymbol{a})) \geq \left(\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta}\right)^{1/2}/\phi_0. \tag{5}$$

The optimal action $\boldsymbol{a}^*$ must solve the problem of maximizing $v(\boldsymbol{a})-c(\boldsymbol{a})$ subject to this constraint and the IC constraint. The latter is here represented by the agent's fist-order condition, and we can then state the following result:

Proposition 2. *In the multinormal case, the optimal action $\boldsymbol{a}^*$ solves the following problem:*

$$\max_{\boldsymbol{a},\boldsymbol{\theta}}(v(\boldsymbol{a})-c(\boldsymbol{a}))$$

subject to $\boldsymbol{Q}\boldsymbol{\theta}=\nabla c(\boldsymbol{a})$ and the enforcement constraint (5).

The proposition shows that the general problem of finding a payment function and action can be reduced to the much simpler problem of finding a vector of weight parameters and an action. The optimal solution yields action $\boldsymbol{a}^*$ and associated weight parameters $\boldsymbol{\theta}^*$ for the performance index. These weights are (from $\boldsymbol{Q}\boldsymbol{\theta}^*=\nabla c(\boldsymbol{a}^*)$) given by $\boldsymbol{\theta}^*=(\boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'\nabla c(\boldsymbol{a}^*)$.

There are two sources for deviations from the first-best action in this setting, and they are reflected in the two constraints in the optimization problem. The first is because of distorted primary measures $\boldsymbol{x}$, and will be relevant when the vector of marginal costs at the first-best actions ($\boldsymbol{a}^{FB}$) cannot be written as $\nabla c(\boldsymbol{a}^{FB})=\boldsymbol{Q}\boldsymbol{\theta}$, for any $\boldsymbol{\theta}$, that is, when this vector does not belong to the space spanned by (the column vectors of) $\boldsymbol{Q}$.[10] Implications of distorted measures will be discussed below.

The second source is self-enforcement, which is reflected in the dynamic enforcement constraint (5). The expression $(\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta})^{1/2}$ on the right-hand side of this constraint is the standard deviation of the performance index $y=\boldsymbol{\theta}'\boldsymbol{x}$. It can be written as $(\Sigma_i\Sigma_j s_{ij}\theta_i\theta_j)^{1/2}$, where $s_{ij}=cov(x_i,x_j)$. It is clear that any variation in $\boldsymbol{\Sigma}$ that reduces this expression will relax the constraint, and hence allow for a higher total surplus. In particular, any reduction of a variance in $\boldsymbol{\Sigma}$ will have this effect and, provided $\boldsymbol{\theta}$ has no negative elements, so will any reduction of a covariance in $\boldsymbol{\Sigma}$.

It is also noteworthy that, provided $\boldsymbol{\theta}$ has no negative elements, then positive correlations among elements in the measurement vector $\boldsymbol{x}$ will be detrimental for the surplus, whereas negative correlations will be beneficial. This follows because, all else equal, the former increases and the latter reduces the variance of the performance index.

The economic mechanism behind these effects is that, although the index $\boldsymbol{\theta}^{*\prime}\boldsymbol{x}$ with standard deviation $\sigma^*=(\boldsymbol{\theta}^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\theta}^*)^{1/2}$ can implement action $\boldsymbol{a}^*$ (by satisfying IC and the enforcement constraint) with the bonus

[10] This possibility is precluded in Budde [2007] by the requirement of measurements being balanced.

$b^* = \sigma^*/\phi_0$, the same index could, with a lower standard deviation, say $\hat{\sigma} < \sigma^*$, implement the same action with a lower bonus, namely, the bonus $\hat{b} = \hat{\sigma}/\phi_0$. The lower bonus satisfies IC[11] for action $\boldsymbol{a}^*$, but yields slack in the enforcement constraint, and incentives could then be increased to induce a higher surplus.

From the enforcement constraint (5), it may appear that any action $\boldsymbol{a}$ will satisfy this constraint if the standard deviation of the performance index on the right-hand side is sufficiently small; and hence that the constraint becomes irrelevant if measurements are sufficiently precise. The reason for this is that a very precise index yields strong marginal incentives, and hence implies that just a low monetary bonus is required to satisfy the agent's first-order condition for choosing action $\boldsymbol{a}$. The result in Proposition 2 builds, however, on the assumption that this FOA is valid; but as we will now point out, this is generally not the case for sufficiently precise measurements.[12]

The FOA replaces global IC constraints for the agent with a local one, and is only valid if the action ($\boldsymbol{a}^*$) derived this way is in fact a global optimum for the agent under the given incentive scheme. Observe that, by choosing action $\boldsymbol{a}^*$, the agent gets a bonus if the index $y = \boldsymbol{\theta}^{*\prime}\boldsymbol{x}$ exceeds its expected value, an event that occurs with probability $\frac{1}{2}$. The agent's expected revenue is then $b/2$, and this must strictly exceed the cost $c(\boldsymbol{a}^*)$ in order for the agent to be willing to choose action $\boldsymbol{a}^*$. This is so because by alternatively choosing action $\boldsymbol{a} = \boldsymbol{0}$, the agent incurs zero costs but still obtains the bonus with some (small) positive probability. Because the bonus b cannot exceed the future value of the relationship, we then see that the following condition is necessary:

$$\frac{\delta}{1-\delta}(v(\boldsymbol{a}^*) - c(\boldsymbol{a}^*)) > 2c(\boldsymbol{a}^*). \tag{6}$$

If a solution identified by the program in Proposition 2 does not satisfy this condition, it is not a valid solution.

Observe that if $\boldsymbol{a}^*, \boldsymbol{\theta}^*$ is a solution from Proposition 2 with the enforcement constraint binding, the necessary condition (6) implies a lower bound for the standard deviation of the performance index: $(\boldsymbol{\theta}^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\theta}^*)^{1/2} > 2c(\boldsymbol{a}^*)\phi_0$. It can be shown (proofs available online) that there is in fact $\sigma_0^* > 0$ such that $\boldsymbol{a}^*$ is an optimal choice for the agent, and the FOA is thus valid, if and only if $\boldsymbol{\theta}^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\theta}^* \geq \sigma_0^{*2}$. Moreover, for a quadratic cost function ($c(\boldsymbol{a}) = \boldsymbol{a}'\boldsymbol{a}/2$), this condition will be fulfilled if $\boldsymbol{a}^*$ satisfies a stricter version of (6), where the right-hand side is replaced by $c(\boldsymbol{a}^*)/\phi_0$ with $1/\phi_0 = \sqrt{2\pi} \approx 2.5$. A sufficient condition for the approach employed in Proposition 2 to be valid in this case is thus that the solution entails a cost

[11] If index $\boldsymbol{\theta}^{*\prime}\boldsymbol{x}$ has standard deviation $\hat{\sigma}$, the marginal revenues from bonus $\hat{b} = \hat{\sigma}/\phi_0$ are from (3) equal to $\boldsymbol{Q}\boldsymbol{\theta}^*$ and hence satisfy IC for action $\boldsymbol{a}^*$.

[12] Similar issues arise in other contexts, including tournaments, and have been noted in the literature on this subject, following Lazear and Rosen [1981].

for the agent that is no larger than 40% of the entire value of the future relationship.

3.1 DISTORTIONS, ALIGNMENT, AND PRECISION

We now discuss implications of distorted performance measures in the present setting. Such measures have been studied extensively for the case when these measures are verifiable, see, for example, Feltham and Xie [1994], Baker [1992], Datar, Kulp, and Lambert [2001], Budde [2007]; and particularly in settings where value- and cost-functions are linear and quadratic, respectively:

$$v(\boldsymbol{a}) = \boldsymbol{p}'\boldsymbol{a} + v_0 \text{ and } c(\boldsymbol{a}) = \boldsymbol{a}'\boldsymbol{a}/2. \tag{7}$$

Here, $\nabla c(\boldsymbol{a}) = \boldsymbol{a}$ and the first-best action, characterized by marginal cost being equal to marginal value, is given by $\boldsymbol{a}^{FB} = \boldsymbol{p}$. If we now neglect the dynamic enforceability constraint (5) in Proposition 2, we are led to maximize the surplus $\boldsymbol{p}'\boldsymbol{a} - \boldsymbol{a}'\boldsymbol{a}/2$ subject to $\boldsymbol{a} = \boldsymbol{Q}\boldsymbol{\theta}$. This maximization yields $\boldsymbol{\theta} = (\boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'\boldsymbol{p}$ and action, here denoted by $\boldsymbol{a}_0^*$ given by $\boldsymbol{a}_0^* = \boldsymbol{Q}(\boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'\boldsymbol{p}$. The best action, subject only to the agent's IC constraint $\boldsymbol{a} = \boldsymbol{Q}\boldsymbol{\theta}$, is thus generally distorted relative to the first-best action.

Remark. The solution $\boldsymbol{a}_0^*$ just derived is also the optimal solution in a setting where the measurements $\boldsymbol{x}$ are verifiable and the agent is rewarded with a linear incentive scheme $\tilde{\boldsymbol{\beta}}'\boldsymbol{x} + \alpha$. This is the setting studied in several papers on distorted measures, and the literature has introduced indicators to measure the degree of distortion. One such indicator is the ratio of second-best to first-best surplus (as in Budde [2007]), which for the the second-best solution just derived (and with $v_0 = 0$) amounts to $\boldsymbol{a}_0^{*\prime}\boldsymbol{a}_0^*/(\boldsymbol{p}'\boldsymbol{p})$. In particular, when the measure $\boldsymbol{x}$ is one-dimensional, so $\boldsymbol{Q}$ is a vector, say $\boldsymbol{Q} = \boldsymbol{q} \in R^n$, the ratio is $(\boldsymbol{p}'\boldsymbol{q}/|\boldsymbol{p}||\boldsymbol{q}|)^2$ and is thus a measure of the alignment between vectors $\boldsymbol{p}$ and $\boldsymbol{q}$. Then, the first-best can be attained only if the two vectors are perfectly aligned ($\boldsymbol{q} = k\boldsymbol{p}$, $k \neq 0$).

In the case of nonverifiable measurements $\boldsymbol{x}$, which is the case analyzed in this paper, the solution must also respect the dynamic enforcement constraint, represented by (5) in the last proposition. When this constraint binds, the action $\boldsymbol{a}_0^*$ is generally no longer feasible. Moreover, because the stochastic properties of the measurements, represented by the covariance matrix $\boldsymbol{\Sigma}$, affects the constraint, they will also affect the solution.

This leads to a tradeoff between alignment and precision when it comes to incentive provision. To highlight the tradeoff, suppose there is a measure, which is well aligned with the marginal value vector $\boldsymbol{p}$, but which is very imprecise in the sense of having a large variance, and another measure, which is not as well aligned with $\boldsymbol{p}$, but is quite precise. In a setting with verifiable measures (and no risk aversion), the optimal solution would then entail strong incentives on the first measure and weak incentives on the second one. This solution, however, could imply a large variance for the performance index, and hence quite possibly be infeasible under self-enforcement by violating the enforcement constraint (5). The constraint may thus imply weaker incentives on measures that are well aligned but imprecise, and stronger incentives on measures that are less well aligned but more precise.

To analyze these issues, we consider the optimization problem in Proposition 2. Say that the enforcement constraint is *strictly binding* if the Lagrange multiplier (shadow cost) in this problem is nonzero. We then obtain the following result.

Corollary 1. *Let* $v(\boldsymbol{a}) = \boldsymbol{p}'\boldsymbol{a}$ *and* $c(\boldsymbol{a}) = \frac{1}{2}\boldsymbol{a}'\boldsymbol{a}$. *An optimal solution in Proposition 2 with the enforcement constraint strictly binding then satisfies*

$$\boldsymbol{\theta}^* = (\psi^*\boldsymbol{\Sigma} + \boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'\boldsymbol{p}$$

with $\psi^* > 0$.

The tradeoff between distortion and precision is captured in this expression for $\boldsymbol{\theta}^*$, and can be nicely illustrated by considering measurements that are uncorrelated and for which the associated vectors in $\boldsymbol{Q}$ are orthogonal, that is, $\boldsymbol{q}_i'\boldsymbol{q}_j = 0$ for all $i \neq j$. Then, the formula yields

$$\theta_i^* = \frac{\boldsymbol{q}_i'\boldsymbol{p}}{\psi^* s_{ii} + \boldsymbol{q}_i'\boldsymbol{q}_i}, i = 1, \cdots, m.$$

All else equal, a measure with better alignment (larger $\boldsymbol{q}_i'\boldsymbol{p}$) will optimally have a larger weight in the index; but also, all else equal, so will a measure with higher precision (smaller variance s_{ii}). A highly precise, but not so well-aligned measure may thus get a larger weight than a measure that is better aligned, but quite imprecise.

Remark. A formally similar tradeoff between distortion and precision arises in multitasking models with verifiable measurements and a risk-averse agent, such as Feltham and Xie [1994] or Datar, Kulp, and Lambert [2001]. We should keep in mind, however, that the tradeoffs arise from two very distinct phenomena in the two settings, namely, from the requirements of self-enforcement and the costs of risk exposure, respectively. Moreover, although comparative static results are relatively straightforward in the Feltham-Xie setting, they are less straightforward here. For example, we cannot conclude directly from the last displayed formula that θ_i^* is decreasing in the variance s_{ii}, because ψ^* is endogenous and hence also depends on s_{ii}.

It turns out that a two-step procedure is fruitful for deriving comparative statics results. In the first step, consider the problem of finding an index that implements a given surplus V with minimal variance, that is, the problem

$$\min \boldsymbol{\theta}'\Sigma\boldsymbol{\theta} \text{ s.t. } \nabla c(\boldsymbol{a}) = \boldsymbol{Q}\boldsymbol{\theta} \text{ and } v(\boldsymbol{a}) - c(\boldsymbol{a}) \geq V.$$

Let $\hat{\boldsymbol{\theta}}(\boldsymbol{\Sigma}, V)$ be the optimal solution and $m(\boldsymbol{\Sigma}, V)$ the minimal value. Observe that for $V > v(0)$, the last constraint here must bind, because otherwise $\boldsymbol{a} = \mathbf{0}$ and $\boldsymbol{\theta} = \mathbf{0}$ would solve the minimization problem.

Next observe that if $(\boldsymbol{\theta}^*, \boldsymbol{a}^*)$ is a solution to the problem in Proposition 2 with the enforcement constraint strictly binding and with surplus $V^* = v(\boldsymbol{a}^*) - c(\boldsymbol{a}^*)$, then we must have

$$\boldsymbol{\theta}^* = \hat{\boldsymbol{\theta}}(\boldsymbol{\Sigma}, V^*).$$

If this was not true, there would be $(\boldsymbol{a}, \boldsymbol{\theta})$ satisfying the two constraints in the minimization problem and $\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta} < \boldsymbol{\theta}^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\theta}^*$. Because the enforcement constraint in Proposition 2 would then be slack, a higher surplus than V^* would be feasible.

From the last formula, we now have

$$\frac{\partial \theta_i^*}{\partial s_{ii}} = \frac{\partial \hat{\theta}_i}{\partial s_{ii}} + \frac{\partial \hat{\theta}_i}{\partial V}\frac{\partial V^*}{\partial s_{ii}}. \tag{8}$$

This (Slutsky-type) formula shows that the effect on the weight θ_i^* in the optimal index can be decomposed in two effects: first an effect induced from a change in s_{ii} with the value V^* held constant; and second, an effect generated by the change in V^* induced by the change in s_{ii}.

It turns out that the first effect, that is, the "own effect" on the weight $\hat{\theta}_i$ of an increase in the variance s_{ii}, has the opposite sign of $\hat{\theta}_i$, and is thus negative if $\hat{\theta}_i$ is positive. This follows from the minimal value $m(\boldsymbol{\Sigma}, V)$ being concave[13] in $\boldsymbol{\Sigma}$ and the envelope property, which implies

$$0 \geq \frac{\partial^2 m}{\partial s_{ii}^2} = \frac{\partial}{\partial s_{ii}}\hat{\theta}_i^2 = 2\hat{\theta}_i\frac{\partial \hat{\theta}_i}{\partial s_{ii}}.$$

Regarding the second effect in the decomposition (8), we know from the discussion following Proposition 2 that the value V^* is decreasing in a variance s_{ii}. We thus have $\frac{\partial V^*}{\partial s_{ii}} \leq 0$, but it appears that the sign of $\frac{\partial \hat{\theta}_i}{\partial V}$ may depend on the parameters, and hence that the total effect in (8) cannot be unambiguously signed. For the linear-quadratic case with uncorrelated and orthogonal measurements, however, we can show that $\frac{\partial \hat{\theta}_i}{\partial V}$ has the opposite sign of $\frac{\partial \hat{\theta}_i}{\partial s_{ii}}$, which then implies that the two terms representing the two effects in (8) have equal signs. Thus, we have the following result.

Proposition 3. *Let $v(\boldsymbol{a}) = \boldsymbol{p}'\boldsymbol{a}$ and $c(\boldsymbol{a}) = \frac{1}{2}\boldsymbol{a}'\boldsymbol{a}$, and assume that the measurements are uncorrelated and that $\boldsymbol{q}_i'\boldsymbol{q}_j = 0$ for all $i \neq j$. An optimal solution in Proposition 2 with the enforcement constraint strictly binding then satisfies*

$$\theta_i^* \frac{\partial \theta_i^*}{\partial s_{ii}} \leq 0, i = 1, \cdots, n.$$

[13] Concavity of m follows by observing that if $k \in (0, 1)$ and $\boldsymbol{\Sigma} = k\boldsymbol{\Sigma}_1 + (1-k)\boldsymbol{\Sigma}_2$, then $\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta} = k\boldsymbol{\theta}'\boldsymbol{\Sigma}_1\boldsymbol{\theta} + (1-k)\boldsymbol{\theta}'\boldsymbol{\Sigma}_2\boldsymbol{\theta}$, and hence $\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta} \geq km(\boldsymbol{\Sigma}_1, V) + (1-k)m(\boldsymbol{\Sigma}_2, V)$ holds for any $\boldsymbol{\theta}$ that is admissible in the minimization problem.

The absolute value of the weight θ_i^* on measurement x_i in the optimal index will thus be decreasing in the measurement's variance s_{ii}.

The tradeoffs between distortion and precision that we have analyzed in this section imply that scorecards must be constructed to find the best balance between these effects. Scorecards can be based on nonverifiable measures, and among those it may be possible to find one that is well aligned with the principal's true (marginal) values. This does not mean, however, that such a measure should be given a large weight in the scorecard index. If the measure is highly imprecise, a large weight on this measure may make the relational contract nonsustainable. Then, it will be better to shift more weight to measures that are more precise, even if they may be less well aligned with the principal's true value.

3.2 VERY PRECISE MEASUREMENTS

We have noted that the FOA used to derive Proposition 2 may be invalid if measurements are noisy, but very precise, and we thus lack a characterization of optimal incentive schemes for such settings. On the other hand, the optimal scheme for an environment with no noise is known (e.g., Budde [2007]). In this subsection, we show that *if V^{NF} is the optimal surplus in a setting with no noise, then any surplus value $V < V^{NF}$ can be implemented with an index contract if the measurements are sufficiently precise.* Index contracts (scorecards) are in this sense at least approximately optimal for sufficiently precise measurements.

3.2.1. Measurements Without Noise. As a reference case, we first consider measurements with no noise, that is, of the form

$$\boldsymbol{x} = \boldsymbol{Q}'\boldsymbol{a}.$$

We have then that an action $\boldsymbol{a}$ can be implemented by some bonus scheme $\beta(\boldsymbol{x})$ if and only if

$$\nabla c(\boldsymbol{a}) = \boldsymbol{Q}\boldsymbol{\gamma} \tag{9}$$

for some $\boldsymbol{\gamma} \in R^m$. The condition is necessary because, if $\boldsymbol{a}$ generating measurement $\boldsymbol{x} = \boldsymbol{Q}'\boldsymbol{a}$ is optimal for the agent, then it must be cost-minimizing among all actions that generate the same $\boldsymbol{x}$. So it must solve $\min_{\tilde{\boldsymbol{a}}} c(\tilde{\boldsymbol{a}})$ subject to $\boldsymbol{x} = \boldsymbol{Q}'\tilde{\boldsymbol{a}}$, and hence satisfy the first-order condition (9) with Lagrange multiplier $\boldsymbol{\gamma}$. On the other hand, if $\boldsymbol{a}$ satisfies (9), it is a cost-minimizing action generating measurement $\boldsymbol{x} = \boldsymbol{Q}'\boldsymbol{a}$, and will be chosen by the agent under a bonus scheme with $\beta(\boldsymbol{x}) \geq c(\boldsymbol{a})$ and $\beta(\tilde{\boldsymbol{x}}) = 0$, $\tilde{\boldsymbol{x}} \neq \boldsymbol{x}$.

Being discretionary, bonuses must respect a dynamic enforcement constraint. Because the minimal bonus to implement an action $\boldsymbol{a}$ is its cost $c(\boldsymbol{a})$, the constraint here takes the form (as in Budde [2007]):

$$c(\boldsymbol{a}) \leq \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})). \tag{10}$$

The optimal contract in this setting thus maximizes the surplus $v(\boldsymbol{a}) - c(\boldsymbol{a})$ subject to (9) and (10). Let $\boldsymbol{a}^{NF}$ denote the optimal action and V^{NF} the maximal surplus in this noise-free environment. In the following, we will assume that the enforcement constraint binds and implies a surplus V^{NF} strictly less than the optimal surplus obtained without the constraint, thus $V^{NF} < V_0^* = \max\{v(\boldsymbol{a}) - c(\boldsymbol{a}) | \nabla c(\boldsymbol{a}) = \boldsymbol{Q\theta}, \boldsymbol{\theta} \in R^m\}$.

3.2.2. Measurements with noise. Consider again noisy measurements, and recall that the approach behind Proposition 2 is valid only if the solution (action $\boldsymbol{a}^*$) satisfies condition (6). This condition is stricter than condition (10). This implies that, although noise-free measurements can be seen as a limiting case of noisy measurements when all variances go to zero, a valid solution from Proposition 2 can generally not converge to $\boldsymbol{a}^{NF}$.

It may be noted that Chi and Olsen [2018] have found that for settings with a univariate action, an index contract derived from the likelihood ratio is still optimal even when the FOA is not valid. The only required modification is that the threshold for the index must be adjusted, taking into account not only a local IC constraint for the agent, but also nonlocal ones, which will be binding. It is an open question whether a similar property holds in settings with multivariate actions.

In the setting of this paper, however, we can show that for noisy but sufficiently precise measurements, any surplus $V < V^{NF}$ can be obtained by means of an index contract. This does not mean that such a contract is optimal, but it will at least be approximately optimal for such measurements. Specifically, we will consider actions that satisfy

$$2c(\boldsymbol{a}) \geq \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})) > c(\boldsymbol{a}), \tag{11}$$

plus $\nabla c(\boldsymbol{a}) = \boldsymbol{Q\theta}$ for some $\boldsymbol{\theta} \in R^m$. Such an action will be feasible for the optimization problem with noise-free measurements, but not optimal in that problem, because the enforcement constraint (10) does not bind. Hence, it generates a surplus $V < V^{NF}$, but the action $\boldsymbol{a}$ can be chosen such that V is arbitrarily close to V^{NF}.

The first inequality in (11) implies that the necessary condition (6) for FOA to be valid is violated, hence $\boldsymbol{a}$ cannot be implemented by the scheme applied in Proposition 2. Recall that this is a consequence of the scheme being designed such that, for the desired action, the agent's expected revenue falls short of his costs.[14]

It seems intuitive that this problem can be alleviated by modifying the hurdle so as to make it less demanding for the agent to qualify for the bonus. On the other hand, such a modification will also negatively affect the agent's marginal incentives. It turns out that, if the measurements are sufficiently precise, a modification of the hurdle can achieve both goals: sufficiently strong incentives and a sufficiently large payoff for the agent,

[14] The hurdle for the index is set to maximize marginal incentives, but this implies that the probability to obtain the bonus is 1/2, and the first inequality in (11) then implies a negative payoff for the agent, relative to choosing action $\boldsymbol{a} = \boldsymbol{0}$.

so that the desired action can be implemented. This is formally stated as follows.

Proposition 4. *Let action $\boldsymbol{a}$ satisfy $2c(\boldsymbol{a}) \geq \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})) > c(\boldsymbol{a})$ and $\nabla c(\boldsymbol{a}) = \boldsymbol{Q\theta}$, for some $\boldsymbol{\theta} \in R^m$. There is $\sigma_0 > 0$ with the following property: If $\boldsymbol{\Sigma}$ satisfies $\boldsymbol{\theta}'\boldsymbol{\Sigma\theta} \equiv \sigma^2 < \sigma_0^2$, then there is a hurdle $\kappa(\sigma) < E(\boldsymbol{x}'\boldsymbol{\theta}|\boldsymbol{a})$ such that action $\boldsymbol{a}$ is implemented by the index $\boldsymbol{x}'\boldsymbol{\theta}$ with hurdle $\kappa(\sigma)$ and bonus $b = \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a}))$. Moreover, $\kappa(\sigma) \to E(\boldsymbol{x}'\boldsymbol{\theta}|\boldsymbol{a})$ as $\sigma \to 0$.*

Observe that the second condition in this proposition requires that $\nabla c(\boldsymbol{a})$ belongs to the span of $\boldsymbol{Q}$, and that our assumptions regarding $\boldsymbol{Q}$ imply that there is then a unique $\boldsymbol{\theta}$ that satisfies the condition, namely, $\boldsymbol{\theta} = (\boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'\nabla c(\boldsymbol{a})$.

Now recall that an action $\boldsymbol{a}$ satisfying the two conditions in the proposition generates a surplus V smaller than the optimal surplus with no noise (V^{NF}), but that $\boldsymbol{a}$ can be chosen such that V is arbitrarily close to V^{NF}. An immediate consequence of the proposition is then the following:

Corollary 2. *Any surplus $V < V^{NF}$ can be obtained by means of an index contract, provided measurements are sufficiently precise.*

The proposition also implies that if an index contract generates a surplus V that is close to V^{NF}, and this contract is optimal in the class of index contracts, then FOA must necessarily be violated; and hence some nonlocal incentive constraint must bind.[15]

This implies that characterizing the optimal (linear) index contract can be technically challenging in this setting. Of course this applies also for the overall optimal contract, because it must have nonlocal incentive constraints binding as well. (Otherwise it would be characterized by Proposition 2, and thus be an index contract with only a local constraint binding.) We leave these issues as topics for future research.

4. *Nonverifiable and Verifiable Measurements*

We have so far focused on nonverifiable measurements. But incentive schemes, at least for top management, will typically also include verifiable financial performance measures. Consider then a situation where there are both nonverifiable and verifiable measurements available. To simplify the exposition, we will assume that there is one verifiable measure (x_0) in addition to the nonverifiable measures ($\boldsymbol{x}$) considered above. The latter depends stochastically on effort as in (2) and the former is assumed to have a similar representation:

$$x_0 = \boldsymbol{q}_0'\boldsymbol{a} + \varepsilon_0,$$

[15] The optimal action yielding surplus V must be close to the action $\boldsymbol{a}^{NF}$ yielding surplus V^{NF}, and because the latter action by our assumptions satisfies (10) with equality and thus violates the necessary condition (6) for FOA to be valid, the former action must also violate this condition.

where $\boldsymbol{q}_0 \in R^n$ and ε_0 is normally distributed noise generally correlated with the noise variables $\boldsymbol{\varepsilon}$ in $\boldsymbol{x}$. (More precisely, the vector $(\varepsilon_0, \boldsymbol{\varepsilon})$ is multinormal.)

The agent can now be incentivized by a court-enforced (explicit) bonus $b_0 x_0$ on the verifiable measure and a discretionary (relational) bonus $\beta(x_0, \boldsymbol{x})$ depending on the entire measurement vector $(x_0, \boldsymbol{x})$. This setting allows us to analyze two interesting issues, namely, (1) whether and under what conditions the relational contract will in fact depend on the verifiable measure x_0 and (2) in what way and to what extent the formal bonus b_0 will be affected by the relational contract.

We consider a case where only short-term explicit contracts are feasible, which allows us to confine attention to stationary contracts.[16] In each period, the agent will then choose action $\boldsymbol{a}$ to maximize $E(b_0 x_0 + \beta(x_0, \boldsymbol{x})|\boldsymbol{a}) - c(\boldsymbol{a})$, yielding first-order conditions[17]

$$\int (b_0 x_0 + \beta(x_0, \boldsymbol{x}))\, f_{a_i}(x_0, \boldsymbol{x}, \boldsymbol{a}) - c_i(\boldsymbol{a}) = 0,\ i = 1, \cdots, n.$$

Returning to the assumption that FOA is valid, the principal then maximizes the total surplus $v(\boldsymbol{a}) - c(\boldsymbol{a})$ subject to these constraints and the dynamic enforcement constraint. We assume as before that the parties separate if the relational contract is broken. The enforcement constraint is then the same as (1), just with $\boldsymbol{x}$ now replaced by the entire measurement vector $(x_0, \boldsymbol{x})$.

From the same principles as before, it follows that the agent should be given the discretionary bonus if and only if an index exceeds a hurdle, and from the normal distribution, it follows that this index is linear in the measurements; $y = \Sigma_{i=0}^m \theta_i x_i \equiv \theta_0 x_0 + \boldsymbol{\theta}'\boldsymbol{x}$, and moreover that the hurdle is $y_0 = E(\Sigma_{i=0}^m \theta_i x_i | \boldsymbol{a}^*)$, where $\boldsymbol{a}^*$ is the equilibrium action. As in section 3, we may normalize the weights such that if the magnitude of the bonus is b, the performance index has variance $var(\Sigma_{i=0}^m \theta_i x_i) = b^2 \phi_0^2$. This leads to the following first-order condition for the agent at the equilibrium action $(\boldsymbol{a} = \boldsymbol{a}^*)$:

$$(b_0 + \theta_0)\boldsymbol{q}_0 + \boldsymbol{Q\theta} = \nabla c(\boldsymbol{a}),$$

and the following dynamic enforcement condition:

$$\frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})) \geq (var(\theta_0 x_0 + \boldsymbol{\theta}'\boldsymbol{x}))^{1/2}/\phi_0.$$

The principal maximizes the total surplus $v(\boldsymbol{a}) - c(\boldsymbol{a})$ subject to these constraints.

[16] Watson, Miller, and Olsen [2020] analyze long-term renegotiable court-enforced contracts, and show that it will generally be optimal to renegotiate these contracts each period when in combination with relational contracts.

[17] Here, we use $f(x_0, \boldsymbol{x}, \boldsymbol{a})$ to denote the joint density of all measurements, conditional on action.

Because the court-enforced bonus b_0 can be chosen freely, while the elements $\theta_0, \boldsymbol{\theta}$ of the discretionary bonus scheme are constrained by self-enforcement, we see that θ_0 should be chosen so as to minimize the variance appearing in the enforcement constraint. (If not, then for given $\boldsymbol{\theta}$**,** we could modify b_0 and θ_0 so that the IC constraint holds and the enforcement constraint becomes slack.)

The variance is minimized for $\theta_0 = -cov(x_0, \boldsymbol{\theta}'\boldsymbol{x})/s_{00}$, where $s_{00} = var(x_0)$, and this implies in turn that the performance index takes the form

$$\theta_0 x_0 + \boldsymbol{\theta}'\boldsymbol{x} = \Sigma_{i=1}^{m} \theta_i \left(x_i - \frac{cov(x_0, x_i)}{s_{00}} x_0 \right). \tag{12}$$

This shows that for correlated measurements, performance on the verifiable measure is taken into the index as a benchmark, to which the other performances are compared.

The bonus is awarded when the index exceeds its expected value. As verified in the appendix, this condition can equivalently be formulated as

$$\Sigma_{i=1}^{m} \theta_i (x_i - E(x_i | x_0, \boldsymbol{a}^*)) > 0. \tag{13}$$

Performance x_i is thus compared to expected performance, given (equilibrium) actions and the outcome on the verifiable measure. If the performance exceeds what is expected, given this information, then it contributes positively to making the index exceed the hurdle, and thus for the agent to obtain the bonus.

By benchmarking the agent's performance on the nonverifiable measures to her performance on the verifiable one, the precision of the performance index is increased, which strengthens incentives without violating the dynamic enforcement constraint. The surplus can then be increased.

It turns out that this way of structuring the relational contract has interesting implications for the formal bonus (b_0) on the verifiable measure. To see this, consider the minimized index variance, which is $var(\boldsymbol{\theta}'\boldsymbol{x}) - (cov(x_0, \boldsymbol{\theta}'\boldsymbol{x}))^2/s_{00}$. The last term equals $(\Sigma_{i=1}^{m} \theta_i \rho_{0i} s_{ii}^{1/2})^2$, where $\rho_{0i} = corr(x_0, x_i)$, and if we define the matrix $\tilde{\boldsymbol{\Sigma}}$ to have elements $s_{ij} - \rho_{0i}\rho_{0j}(s_{ii}s_{jj})^{1/2}$, we can then write[18]

$$\min_{\theta_0} var(\theta_0 x_0 + \boldsymbol{\theta}'\boldsymbol{x}) = var(\boldsymbol{\theta}'\boldsymbol{x}) - (\Sigma_{i=1}^{m} \theta_i \rho_{0i} s_{ii}^{1/2})^2 = \boldsymbol{\theta}'\tilde{\boldsymbol{\Sigma}}\boldsymbol{\theta}. \tag{14}$$

Observe that the minimized index variance is, for given correlations, independent of s_{00}, the variance of the verifiable measure x_0. As we verify below, this implies that the optimal action and associated surplus is independent of s_{00}, which in turn has noteworthy implications for the formal bonus (b_0) on x_0. For in order to maintain an optimal action independent of s_{00}, the effective incentive $b_0 + \theta_0$ on the verifiable measure must remain constant,

[18] The matrix $\tilde{\boldsymbol{\Sigma}}$ is the covariance matrix for the variables $\tilde{x}_i = x_i - \rho_{0i}(s_{ii}/s_{00})^{1/2} x_0$, $i = 1, \ldots, m$.

which requires that the formal bonus b_0 must compensate for any variations in the relational bonus θ_0 on this measure. Because the latter varies with the variance s_{00}, so must therefore the former.

To analyze this, define $\tilde{b}_0 = b_0 + \theta_0$ as the effective (net) incentive on x_0, implying that the IC constraint takes the form $\tilde{b}_0 \boldsymbol{q}_0 + \boldsymbol{Q\theta} = \nabla c(\boldsymbol{a})$. Next define $S(\boldsymbol{\theta})$ as the maximal feasible surplus for given weights $\boldsymbol{\theta}$ on the nonverifiable measures:

$$S(\boldsymbol{\theta}) = \max_{\tilde{b}_0, \boldsymbol{a}} \{ v(\boldsymbol{a}) - c(\boldsymbol{a}) | \tilde{b}_0 \boldsymbol{q}_0 + \boldsymbol{Q\theta} = \nabla c(\boldsymbol{a}) \}. \tag{15}$$

The optimal weights $\boldsymbol{\theta}^*$ are then determined by the following program:

$$\max_{\theta} S(\boldsymbol{\theta}) \text{ s.t. } \frac{\delta}{1-\delta} S(\boldsymbol{\theta}) \geq (\boldsymbol{\theta}' \tilde{\Sigma} \boldsymbol{\theta})^{1/2} / \phi_0. \tag{16}$$

Now we see from (14) that the optimal $\boldsymbol{\theta}^*$ is, for given correlations, independent of the variance s_{00}. It follows that the optimal action $\boldsymbol{a}^*$ and the optimal effective bonus $\tilde{b}_0^*$ are also independent of s_{00}. The relational bonus on x_0 is $\theta_0^* = -cov(x_0, \boldsymbol{\theta}^{*\prime} \boldsymbol{x}) / s_{00}$, and the formal bonus is then

$$b_0 = \tilde{b}_0^* - \theta_0^* = \tilde{b}_0^* + cov(x_0, \boldsymbol{\theta}^{*\prime} \boldsymbol{x}) / s_{00}. \tag{17}$$

The last term in the formula equals $\Sigma_{i=1}^m \theta_i^* \rho_{0i} (s_{ii}/s_{00})^{1/2}$ and is, for given correlations, decreasing (increasing) in s_{00} if the covariance $cov(x_0, \boldsymbol{\theta}^{*\prime} \boldsymbol{x})$—or equivalently the correlation $corr(x_0, \boldsymbol{\theta}^{*\prime} \boldsymbol{x})$—is positive (negative). The same is therefore true for the formal bonus. Hence, in spite of the agent being risk neutral, the optimal formal bonus on the verifiable measure does in this setting generally depend on the variance of this measure.

We summarize our findings so far in the following proposition.

Proposition 5. *For correlated measurements, performance on the verifiable measure x_0 is taken into the index as a benchmark, to which performances on the nonverifiable measures are compared. The optimal index awards a bonus when performance exceeds what is expected, conditional on x_0, as stated in (13). The optimal action $\boldsymbol{a}^*$ is, for given correlations, independent of the variance $var(x_0)$. To achieve this, the formal bonus on the verifiable measure must decrease (increase) with the measure's variance when the correlation $corr(x_0, \boldsymbol{\theta}^{*\prime} \boldsymbol{x})$ is positive (negative).*

The proposition implies that, when there is predominantly positive (negative) correlations between the verifiable measure and the nonverifiable ones, and the weights in the index $\boldsymbol{\theta}^*$ are positive, then the optimal formal bonus must decrease (increase) with $var(x_0)$. The precise criterion for whether the bonus is decreasing or increasing is the sign of the correlation (or equivalently the sign of the covariance) between the measure x_0 and the index $\boldsymbol{\theta}^{*\prime} \boldsymbol{x}$ that comprises the nonverifiable measures. This index is endogenous, but its weights—and hence the sign of the correlation—are independent of $var(x_0)$.

Regarding variations in correlations, we see from $\tilde{\boldsymbol{\Sigma}}$ defined in (14) that if all of the correlation coefficients between the verifiable and the nonverifiable measures have the same sign, then the stronger these correlations are, the smaller the variance $\boldsymbol{\theta}'\tilde{\boldsymbol{\Sigma}}\boldsymbol{\theta}$ is if all elements of $\boldsymbol{\theta}$ are nonnegative. This will then relax the enforcement constraint and increase the surplus. Stronger correlations, either all positive or all negative, between the verifiable and each nonverifiable measure, will thus increase the surplus in such a case.

To analyze further effects of variations in correlations we consider the linear-quadratic case ($v(\boldsymbol{a}) = \boldsymbol{p}'\boldsymbol{a}$ and $c(\boldsymbol{a}) = \frac{1}{2}\boldsymbol{a}'\boldsymbol{a}$), and follow the procedure leading to the optimization in (16). The IC constraint is here $\tilde{b}_0\boldsymbol{q}_0 + \boldsymbol{Q}\boldsymbol{\theta} = \boldsymbol{a}$, where as before $\tilde{b}_0$ is the effective bonus on the verifiable measure x_0. Using this to substitute for $\boldsymbol{a}$, and maximize the surplus with respect to $\tilde{b}_0$, we find that the optimal effective bonus, conditional on $\boldsymbol{\theta}$ is[19]

$$\tilde{b}_0 = (\boldsymbol{p}'\boldsymbol{q}_0 - \boldsymbol{\theta}'\boldsymbol{Q}'\boldsymbol{q}_0)/\boldsymbol{q}_0'\boldsymbol{q}_0. \tag{18}$$

We see that, except if $\boldsymbol{q}_0$ is orthogonal to all the columns of $\boldsymbol{Q}$, that is, $\boldsymbol{Q}'\boldsymbol{q}_0 = \boldsymbol{0}$, the optimal effective bonus $\tilde{b}_0$ will depend on $\boldsymbol{\theta}$ and hence be different from the optimal bonus for the verifiable measure alone.

The formal bonus on the verifiable measure is, as in (17), given as $b_0 = \tilde{b}_0 + cov(x_0, \boldsymbol{\theta}'\boldsymbol{x})/s_{00}$, and is generally influenced by the relational contract through both terms. A positive covariance $cov(x_0, \boldsymbol{\theta}'\boldsymbol{x})$ calls for a larger formal bonus, whereas a positive alignment ($\boldsymbol{\theta}'\boldsymbol{Q}'\boldsymbol{q}_0 > 0$) calls for a smaller one (via its effect on $\tilde{b}_0$). Both of these elements are endogenous, and depend on the primitives of the model. For a given scorecard index with positive weights ($\theta_i > 0$), we see that the covariance term $cov(x_0, \boldsymbol{\theta}'\boldsymbol{x})$ is increasing in each of the correlation coefficients (ρ_{0i}) between the verifiable and the nonverifiable measures. Higher such correlations will thus have a positive effect on the formal bonus b_0 in this situation, but this effect may be counteracted (or strengthened) by endogenous responses in the scorecard weights $\boldsymbol{\theta}$.

A full comparative analysis is challenging, but with extra assumptions we can obtain some instructive results. Say that the measurements $\boldsymbol{x}$ have symmetric noise terms if they have equal variances ($s_{ii} = s_{11}$) and equal correlations ($corr(x_i, x_j) \equiv \rho_{ij} = \rho_{12}$ and $corr(x_0, x_i) \equiv \rho_{0i} = \rho_{01}$, $i, j = 1 \ldots m$).[20] Then we can show the following:

Proposition 6. *For the linear-quadratic model, assume the surplus $S(\boldsymbol{\theta})$ in (15) is strictly concave and symmetric in $(\theta_1, \ldots, \theta_m)$, with $S_{\theta_i}(0) > 0$, $i = 1 \ldots m$. Suppose also that the nonverifiable measurements $\boldsymbol{x}$ have symmetric noise terms. Then, if $\Sigma_{i=1}^m \boldsymbol{q}_i'\boldsymbol{q}_0 = 0$, the formal bonus b_0 will increase if the common correlation coefficient ρ_{0i} increases. If $\Sigma_{i=1}^m \boldsymbol{q}_i'\boldsymbol{q}_0 > 0$, there is $\rho^M > 0$ such that if the common correlation coefficient ρ_{0i} increases, the formal bonus b_0 increases for $\rho_{0i} < \rho^M$ and possibly decreases for $\rho_{0i} > \rho^M$.*

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[19] The first-order condition for this maximization is $(\boldsymbol{p} - \boldsymbol{a})'\boldsymbol{q}_0 = 0$, with $\boldsymbol{a} = \tilde{b}_0\boldsymbol{q}_0 + \boldsymbol{Q}\boldsymbol{\theta}$.

[20] The covariance matrix must be positive definite, which implies the following restriction on the parameters: $1 - \rho_{01}^2 + (m-1)(\rho_{12} - \rho_{01}^2) > 0$.

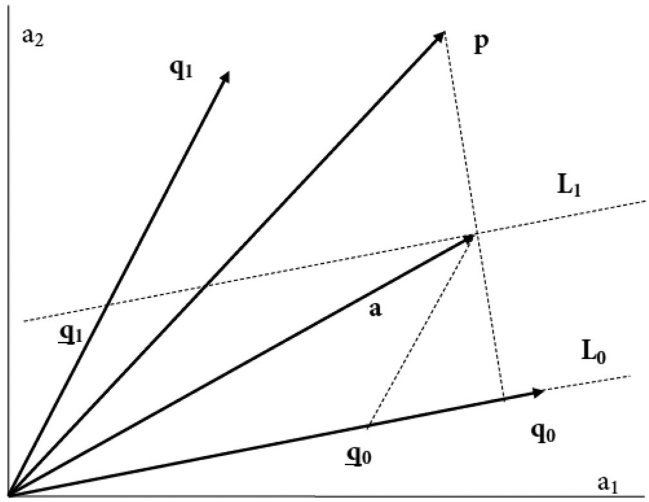

FIG 2.—Illustration of incentives on verifiable and nonverifiable measurements.

If there is orthogonality between the verifiable measure and the nonverifiable ones, the formal bonus b_0 increases with increasing correlations $corr(x_0, x_i)$, and is thus minimal for strong negative correlations and maximal for strong positive ones. At least under symmetry, [21] this monotone effect on b_0, which stems from adjustments to compensate for changes in the scorecard index weight θ_0, is thus not reversed by endogenous responses in the weights $\boldsymbol{\theta}$.

If there is positive alignment, the monotonicity is also preserved for correlations below some upper positive bound. In this case, the formal bonus must also be adjusted to accommodate an adjustment in the effective bonus $\tilde{b}_0$, but this response will not dominate for correlations below the bound. In the setting considered here, there is thus always a range of correlations, including all admissible negative ones, where the formal bonus increases when correlations increase. Put differently, for negative correlations the formal bonus will decrease if these correlations become stronger (i.e., larger in absolute value).

The results in this subsection are illustrated in Figure 2 for the case of one verifiable (x_0) and one nonverifiable measure (x_1), with associated vectors $\boldsymbol{q}_0$ and $\boldsymbol{q}_1$, respectively. If only x_0 is available, only action vectors on the line L_0 can be implemented. The optimal action minimizes the distance [22] to $\boldsymbol{p}$, and is thus the projection of $\boldsymbol{p}$ on the line L_0, defined by action $\boldsymbol{a}_0 = b_0\boldsymbol{q}_0$ such that $(\boldsymbol{p} - \boldsymbol{a}_0)'\boldsymbol{q}_0 = 0$. When also x_1 is available, action vectors on a parallel line such as L_1 (given by $\boldsymbol{a} = \theta_1\boldsymbol{q}_1 + \tilde{b}_0\boldsymbol{q}_0$) can be implemented. This

[21] The model is continuous in the parameters, and the results should therefore be robust to at least small deviations from symmetry.

[22] This yields maximal surplus because of the identity $\boldsymbol{p}'\boldsymbol{a} - \frac{1}{2}\boldsymbol{a}'\boldsymbol{a} = \frac{1}{2}|\boldsymbol{p}|^2 - \frac{1}{2}|\boldsymbol{p} - \boldsymbol{a}|^2$.

allows for implementation of action vectors with a smaller distance to $\boldsymbol{p}$, and hence a larger surplus. The optimality condition for the effective bonus $\tilde{b}_0$ on the verifiable measure still implies $(\boldsymbol{p}-\boldsymbol{a})'\boldsymbol{q}_0 = 0$, and hence that $\boldsymbol{p}-\boldsymbol{a}$ should be orthogonal to $\boldsymbol{q}_0$ (and line L_0). The figure makes clear that, unless $\boldsymbol{q}_0$ and $\boldsymbol{q}_1$ are orthogonal, the bonus $\tilde{b}_0$ defining the component $\underline{\mathbf{q}}_0 = \tilde{b}_0\boldsymbol{q}_0$ of action $\boldsymbol{a}$ will be different from the optimal bonus when only measure x_0 is available. Moreover, unless x_0 and x_1 are stochastically independent, the formal bonus b_0 must also be adjusted to compensate for the incentive element θ_0 on x_0 in the scorecard.

The correlation $corr(x_0, x_1) = \rho_{01}$ will affect the enforcement constraint for the scorecard via the variance term on the right-hand side of (16). This variance is here $\theta_1^2 s_{11}(1-\rho_{01}^2)$, and we see that a stronger correlation (positive or negative) will relax the constraint and allow for a larger weight θ_1. As the variance is nonmonotone in ρ_{01}, the optimal weight θ_1 will also be non-monotone; and specifically decreasing for $\rho_{01} < 0$ and increasing for $\rho_{01} > 0$. It can be seen, however, that the weight θ_0 on the verifiable measure will be monotone and decreasing.[23] When $\boldsymbol{q}_0$ and $\boldsymbol{q}_1$ are orthogonal, the formal bonus will then be monotone increasing in ρ_{01}, because it only compensates for changes in θ_0 in that case.

Under positive alignment ($\boldsymbol{q}_0'\boldsymbol{q}_1 > 0$) as illustrated in the figure, we see that if θ_1 increases and the line L_1 consequently shifts up, the effective bonus $\tilde{b}_0$ on x_0 must decrease. This implies that, when the correlation ρ_{01} is positive and increasing, there are two opposing effects on the formal bonus $b_0 = \tilde{b}_0 - \theta_0$, as the effective bonus $\tilde{b}_0$ and the weight θ_0 in the scorecard both decrease. If the correlation is negative, however, the two effects go in the same direction, as θ_1 then decreases and consequently $\tilde{b}_0$ increases with increasing ρ_{01}. This accords with the results in the last proposition above.

5. BSCs in Practice

There are three key ingredients of the optimal incentive schemes that we deduce in this paper: The use of thresholds to qualify for bonus payments, the use of weighted indexes to measure performance, and the combination of verifiable and nonverifiable measures as basis for bonus payments. All these features are quite common in practice:

With respect to thresholds, firms usually combine a performance threshold with some linear “incentive zone,” that is, there is typically a performance threshold that needs to be met in order to achieve a bonus, and then there is a linear payment for performance in a range up to a maximum level (see Merchant, Stringer, and Shantapriyan [2018]). Many scholars have been critical to the use of thresholds, as it may induce various sorts of gaming and reduce the agents' efforts if they are too far from

[23] We have $\theta_0 = -cov(x_0, \theta_1 x_1)/s_{00} = -\rho_{01}\theta_1\sqrt{s_{11}/s_{00}}$, and this expression is a decreasing function of ρ_{01} because θ_1 is an increasing function of ρ_{01}^2.

the incentive zone (see, e.g., Murphy and Jensen [2011]). Levin [2003] showed, however, that a scheme with a single threshold and no linear incentive zone provides the strongest incentives for a unidimensional action. This paper shows that Levin's [2003] insight also turns out to be valid for multidimensional actions when a number of performance measures (often referred to as key performance indicators—KPIs) are optimally combined in one single index contract.[24]

Indeed, empirical evidence shows that incentive plans with restricted incentive zones are common in practice. A survey of more than 350 publicly traded companies by WorldatWork and Deloitte Consulting LLP [2014] showed that 75% of the companies reported use of performance thresholds. Pearl Meyer & Partners [2013] found in a survey of 130 firms that 71% used thresholds and maximums. Colucci, Peek, and Engel [2015] studied a sample of 100 statements produced by Fortune 500 companies and found that only 37% of the companies disclosed that they used a threshold. According to Merchant, Stringer, and Shantapriyan [2018], however, the difference here appears to be caused by the lack of public disclosure of the details of the incentive contracts by firms that use thresholds. (Indeed, empirical evidence on details of the incentive contracts are generally quite limited, and one may find more details in consulting reports than in academic scholarly work.)

The large study by WorldatWork and Deloitte Consulting LLP [2014] shows how common it is to use indexes in combination with thresholds. In determining the payout amount against performance achievement, 41% of the firms reported to review their employees' performance "holistically," that is, performance measures were part of an overall scorecard, where a minimum score is required to trigger the pay out of bonuses. More precisely, 41% of 350 publicly traded U.S. firms in the survey answered yes to the following option: "Performance measures are part of an overall scorecard that is used to determine the payout amount. If a minimum overall score is not achieved, no award is paid even if some goals were achieved." In comparison, 27% reported that performance measures are evaluated and paid separately, that is, if one goal is achieved, an award is paid regardless of whether any other performance goals are achieved.

The combination of verifiable and nonverifiable measures is also very common, in particular for firms adopting scorecards/index contracts. As underscored in the review by Lueg and Carvalho e Silva [2013], the KPIs of the BSC systems differ from conventional performance measurement systems as they combine financial and nonfinancial KPIs. Traditionally the KPIs have been built around four perspectives in the scheme: (1) a financial perspective, covering how performance is measured by shareholders; (2) a customer perspective, showing how the organization creates value for

[24] If, contrary to our model, outcomes can be observed and actions can be adjusted within each period, incentive zones may be useful to discourage gaming behavior.

its customers; (3) internal business processes, explaining at which processes the organization must develop in order to satisfy its customers and shareholders; and (4) a so-called "learning and growth perspective," addressing the internal capabilities and information systems necessary to improve processes and customer relationships. In particular, the latter two perspectives are open for subjectivity and nonverifiable measures, as, for example, underscored in the seminal work by Ittner, Larcker, and Meyer [2003].

Although our model can contribute to explain common features of BSCs seen in practice, it also produces new empirical predictions that—to our best knowledge—have not yet been tested. In particular, the model offers novel insights on the relationship between verifiable and nonverifiable measures. A key prediction is that when performance measures are correlated, then performance on a verifiable measure is taken into the index as a benchmark, to which the other performances are compared. More specifically: by including such a verifiable measure in the relational contract, the precision of the performance index can be increased, which in turn strengthens incentives.

This result bears resemblance to the argument for why relative performance evaluation (RPE) may be beneficial in conventional settings with verifiable measures: By conditioning an agent's performance on other agents' performances, one may increase precision and strengthen incentives (Holmström [1979], 1982]). Interestingly, this idea, along with several other ideas from incentive theory, appears to have started as normative prescriptions rather than positive descriptions. Indeed, the moderate use of incentives based on RPE in practice has been regarded as a puzzle (Murphy [1999]), but such incentives have become more common the last 20 years (see, e.g., Meridian Compensation Partners LLC [2019]). Some of our new results in this paper can also be seen with a similar perspective, namely as normative advice on incentive design, and/or as new hypotheses awaiting to be tested. [25]

6. *Conclusion*

Employees are often evaluated along many dimensions, and at least some of the performance measures will typically be nonverifiable to a third party. They may also be misaligned with (distorted from) the true values for the

[25] Normative arguments on incentive design rest on the assumption that firms are not always able to implement optimal management practices. As documented by Bloom and Reenen [2010] and Bloom et al. [2019], the large productivity differences one can observe between firms and countries, can to a large extent be explained by differences in management practice, including differences in incentive design. Interestingly, they find that the educational background of managers matters, and that there exists informational barriers that makes it hard to implement best practice: "New management practices are often complex and hard to introduce without the assistance of employees or consultants with prior experience of these innovations. Firms learn from the experiences (good and bad) of others in experimenting with different practices, so not all will adopt immediately" (Bloom and Reenen [2010], p. 220).

principal, and be stochastically dependent. The aim of this paper is to study this environment: Optimal incentives for multitasking agents whose performance measures are nonverifiable and potentially distorted and correlated. We extend and generalize the received literature in some important dimensions (to an arbitrary number of tasks with stochastic measurements that are possibly correlated and/or distorted), and we invoke assumptions (normally distributed measurements) that make the model quite tractable.

We show that under standard assumptions the optimal relational contract is an index contract. That is, the agent gets a bonus if a weighted sum of performance outcomes on the various tasks (an index) exceeds a hurdle. The weights reflect a tradeoff between precision and distortion for the various measures. The efficiency of this contract improves with higher precision of the index measure, because this strengthens incentives. Correlations between measurements may be beneficial for this reason.

We point out that for very precise, but still noisy measurements, the standard FOA breaks down, and we show that, although index contracts may no longer be optimal in such settings, they can be adjusted to become asymptotically optimal.

We also show that the principal may want to include verifiable performance measures in the relational index contract in order to improve incentives. These are then included as benchmarks, to which the other performances are compared. This implies in turn that the formal contract is influenced by the relational contract, and we show that, despite the agent being risk neutral, the bonus on a verifiable measure may be decreasing or increasing in the variance of this measure, depending on the sign of correlations between the measure and the nonverifiable measures.

Stronger correlations, either all positive or all negative, between the verifiable and each nonverifiable measure, will increase the surplus when the optimal index has positive weights. These correlations will also influence the formal contract, and we identify two channels of influence, associated with alignment and stochastic dependence, respectively, that may yield opposing effects. For a symmetric case we show that there is always a range of correlations, including all admissible negative ones, where the formal bonus increases when correlation coefficients increase (from negative values to zero).

The index contracts in our model bear resemblance to key features of the performance measurement system known as BSCs. Reward systems based on BSC typically include non-verifiable measures and often connect pay to an index. In that sense, our paper provides at contract theoretic rationale for this way of implementing BSC schemes. However, although the scheme we present is a bonus contract with just one threshold (or hurdle), scorecards in practice may have several thresholds and bonus levels, where the size of the bonus depends on the score. Future research can extend the model we present to incorporate, for example, risk aversion or limited liability, in order to study under which broader conditions the index contract is optimal, and what kind of index contracts that are optimal under various model specifications.

APPENDIX A: PROOFS

Proof of Lemma 1. The lemma follows directly from the Lagrangian for the problem, which takes the form

$L = v(\boldsymbol{a}) - c(\boldsymbol{a}) + \Sigma_i \mu_i (\int \beta(\boldsymbol{x}) f_{a_i}(\boldsymbol{x}, \boldsymbol{a}) - c_i(\boldsymbol{a})) \quad + \int \lambda(\boldsymbol{x}) (\frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a})) - \beta(\boldsymbol{x}))$.

At the optimal action $\boldsymbol{a} = \boldsymbol{a}^*$, the optimal bonus satisfies

$\frac{\partial L}{\partial \beta(\boldsymbol{x})} = \Sigma_i \mu_i f_{a_i}(\boldsymbol{x}, \boldsymbol{a}) - \lambda(\boldsymbol{x}) = 0$ if $\beta(\boldsymbol{x}) > 0, \leq 0$ if $\beta(\boldsymbol{x}) = 0$.

Hence, we have:

If $\Sigma_i \mu_i f_{a_i}(\boldsymbol{x}, \boldsymbol{a}) > 0$ then $\lambda(\boldsymbol{x}) > 0$ and hence $\beta(\boldsymbol{x}) = \frac{\delta}{1-\delta}(v(\boldsymbol{a}) - c(\boldsymbol{a}))$.

If $\Sigma_i \mu_i f_{a_i}(\boldsymbol{x}, \boldsymbol{a}) < 0$ then $\frac{\partial L}{\partial \beta(\boldsymbol{x})} < 0$ and hence $\beta(\boldsymbol{x}) = 0$ (implying $\lambda(\boldsymbol{x}) = 0$). □

Verification of (3). Given that $\tilde{y} = \boldsymbol{\tau}'\boldsymbol{x}$ is normal with expectation $E(\tilde{y}|\boldsymbol{a})$ and variance $\tilde{\sigma}^2 = \boldsymbol{\tau}'\boldsymbol{\Sigma}\boldsymbol{\tau}$, we have that $\frac{\tilde{y} - E(\tilde{y}|\boldsymbol{a})}{\tilde{\sigma}}$ is a standard normal variable, and hence

$$\Pr(\tilde{y} > \tilde{y}_0 | \boldsymbol{a}) = 1 - \Phi(\frac{\tilde{y}_0 - E(\tilde{y}|\boldsymbol{a})}{\tilde{\sigma}}), \tag{A.1}$$

where $\Phi(\cdot)$ is the standard normal CDF. Because $E(\tilde{y}|\boldsymbol{a}) = \boldsymbol{\tau}'\boldsymbol{Q}'\boldsymbol{a}$ has gradient $\nabla_{\boldsymbol{a}} E(\tilde{y}|\boldsymbol{a}) = \boldsymbol{Q}\boldsymbol{\tau}$, we then obtain

$$\nabla_{\boldsymbol{a}} \Pr(\tilde{y} > \tilde{y}_0 | \boldsymbol{a}) = \phi(\frac{\tilde{y}_0 - E(\tilde{y}|\boldsymbol{a})}{\tilde{\sigma}}) \frac{1}{\tilde{\sigma}} \boldsymbol{Q}\boldsymbol{\tau},$$

where $\phi = \Phi'$ is the standard normal density. This verifies (3), because $\tilde{y}_0 = E(\tilde{y}|\boldsymbol{a}^*)$.

Proof of Corollary 1. Observe that for the linear-quadratic case, the Lagrangean for the optimization problem in Proposition 2 can be written as $(1 + \tilde{\lambda})(\boldsymbol{p}'\boldsymbol{a} - \frac{1}{2}\boldsymbol{a}'\boldsymbol{a}) - \tilde{\lambda}\frac{1-\delta}{\delta\phi_0}(\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta})^{1/2}$ with $\boldsymbol{a} = \boldsymbol{Q}\boldsymbol{\theta}$, and where the multiplier $\tilde{\lambda}$ is nonnegative. The first-order conditions for the optimal $\boldsymbol{\theta}^*$ therefore include

$$\boldsymbol{Q}'(\boldsymbol{p} - \boldsymbol{Q}\boldsymbol{\theta}^*) - \frac{\tilde{\lambda}}{1 + \tilde{\lambda}} \frac{1 - \delta}{\delta\phi_0} (\boldsymbol{\theta}^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\theta}^*)^{-1/2} \boldsymbol{\Sigma}\boldsymbol{\theta}^* = \boldsymbol{0}.$$

By assumption the multiplier $\tilde{\lambda}$ is nonzero and thus positive here. Defining $\psi^* = \frac{\tilde{\lambda}}{1+\tilde{\lambda}} \frac{1-\delta}{\delta\phi_0} (\boldsymbol{\theta}^{*\prime}\boldsymbol{\Sigma}\boldsymbol{\theta}^*)^{-1/2}$, we then obtain the formula in the corollary. □

Proof of Proposition 3. We will show that $\hat{\theta}_i \frac{\partial \hat{\theta}_i}{\partial V} \geq 0$. Because $\hat{\theta}_i \frac{\partial \hat{\theta}_i}{\partial s_{ii}} \leq 0$ and $\frac{\partial V^*}{\partial s_{ii}} \leq 0$, this implies from (8) that $\hat{\theta}_i \frac{\partial \theta_i^*}{\partial s_{ii}} \leq 0$, which verifies the formula in Proposition 3 because $\hat{\theta}_i = \theta_i^*$ when $V = V^*$.

Consider the Lagrangean for the optimization problem that defines $\hat{\boldsymbol{\theta}}$:

$L = -\boldsymbol{\theta}'\Sigma\boldsymbol{\theta} + \lambda(\boldsymbol{p}'\boldsymbol{a} - \boldsymbol{a}'\boldsymbol{a}/2 - V), \boldsymbol{a} = \boldsymbol{Q}\boldsymbol{\theta}$.

The first-order conditions are

$(-2\boldsymbol{\Sigma} - \lambda \boldsymbol{Q}'\boldsymbol{Q})\hat{\boldsymbol{\theta}} + \lambda \boldsymbol{Q}'\boldsymbol{p} = \boldsymbol{0},$
$\boldsymbol{p}'\boldsymbol{Q}\hat{\boldsymbol{\theta}} - \frac{1}{2}\hat{\boldsymbol{\theta}}'\boldsymbol{Q}'\boldsymbol{Q}\hat{\boldsymbol{\theta}} - V = 0.$

Differentiating this system with respect to V yields

$$\begin{bmatrix} -2\boldsymbol{\Sigma} - \lambda \boldsymbol{Q}'\boldsymbol{Q} & \boldsymbol{Q}'(\boldsymbol{p} - \boldsymbol{Q}\hat{\boldsymbol{\theta}}) \\ (\boldsymbol{p} - \boldsymbol{Q}\hat{\boldsymbol{\theta}})'\boldsymbol{Q} & 0 \end{bmatrix} \begin{bmatrix} \nabla_V \hat{\boldsymbol{\theta}} \\ \frac{\partial \lambda}{\partial V} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix}. \tag{A.2}$$

By Cramer's rule we have

$$\frac{\partial \hat{\theta}_1}{\partial V} = \frac{1}{D} D_1,$$

where D is the determinant of the bordered Hessian in (A.2), and D_1 is the determinant of the same matrix with the first column replaced by the column on the right-hand side of (A.2). The determinant D has the same sign as $(-1)^m$.

For uncorrelated and orthogonal measurements we have

$$-2\Sigma - \lambda \boldsymbol{Q}'\boldsymbol{Q} = diag\{d_1, \cdots, d_m\},\ d_i = -2s_{ii} - \lambda \boldsymbol{q}_i'\boldsymbol{q}_i < 0, i = 1 \cdots m.$$

Using this special structure to compute the determinant D_1 (by expansion along the first column and then the first row), we obtain

$$D_1 = (-1)\boldsymbol{q}_1'(\boldsymbol{p} - \boldsymbol{Q}\hat{\boldsymbol{\theta}})\, d_2 \cdot \ldots \cdot d_m,$$

and thus

$$\hat{\theta}_1 \frac{\partial \hat{\theta}_1}{\partial V} = \frac{-1}{D} \hat{\theta}_1 \boldsymbol{q}_1'(\boldsymbol{p} - \boldsymbol{Q}\hat{\boldsymbol{\theta}})\, d_2 \cdot \ldots \cdot d_m.$$

Because each d_i is negative, their product has the same sign as $(-1)^{m-1}$, and hence we see that $-d_2 \cdot \ldots \cdot d_m / D$ is positive. Finally, for uncorrelated and orthogonal measurements, we see from the first-order conditions that we have

$\lambda \boldsymbol{q}_1'(\boldsymbol{p} - \boldsymbol{Q}\widehat{\theta}) = 2s_{11}\widehat{\theta_1}.$

For $V > 0$, we must have $\lambda > 0$ because $\lambda = 0$ will imply $\hat{\boldsymbol{\theta}} = \boldsymbol{0}$ and thus $\boldsymbol{a} = \boldsymbol{0}$ and $V = 0$. Hence, we see that $\hat{\theta}_1 \boldsymbol{q}_1'(\boldsymbol{p} - \boldsymbol{Q}\hat{\boldsymbol{\theta}}) = 2s_{11}\hat{\theta}_1^2/\lambda > 0$, which now implies $\hat{\theta}_1 \frac{\partial \hat{\theta}_1}{\partial V} \geq 0$. The same argument obviously holds for any $i > 1$, and the proof is then complete.

Proof of Proposition 4. We will in this proof denote the given $\boldsymbol{a}$ and $\boldsymbol{\theta}$ by $\tilde{\boldsymbol{a}}$ and $\tilde{\boldsymbol{\theta}}$, respectively. We thus consider $\tilde{\boldsymbol{a}}$ and $\tilde{\boldsymbol{\theta}}$ that satisfy $2c(\tilde{\boldsymbol{a}}) \geq \frac{\delta}{1-\delta}(v(\tilde{\boldsymbol{a}}) - c(\tilde{\boldsymbol{a}})) > c(\tilde{\boldsymbol{a}})$ and $\nabla c(\tilde{\boldsymbol{a}}) = \boldsymbol{Q}\tilde{\boldsymbol{\theta}}$.

We will consider the index $y = \boldsymbol{x}'\tilde{\boldsymbol{\theta}}$ with a hurdle $\kappa < E(y|\tilde{\boldsymbol{a}})$, and with bonus b paid for qualifying performance $(y > \kappa)$. The bonus is

$$b = \frac{\delta}{1-\delta}(v(\tilde{\boldsymbol{a}}) - c(\tilde{\boldsymbol{a}})).$$

The proof will show that the hurdle κ can be chosen such that this index scheme implements $\tilde{\boldsymbol{a}}$, provided the index has a sufficiently low variance.

By assumption, we have $c(\tilde{\boldsymbol{a}}) < b$. Choose $\xi_0 > 0$ and σ_0 such that

$$c(\tilde{\boldsymbol{a}}) = (\Phi(\xi_0) - \Phi(-\xi_0))b \text{ and } \sigma_0 = b\phi(-\xi_0),$$

where Φ is the standard normal CDF and ϕ is its density. The index $y = \boldsymbol{x}'\tilde{\boldsymbol{\theta}}$ has variance $\sigma^2 = \tilde{\boldsymbol{\theta}}'\boldsymbol{\Sigma}\tilde{\boldsymbol{\theta}}$, and assume now $\sigma < \sigma_0$. Define $\xi > \xi_0$ by

$$\sigma = b\phi(-\xi),$$

and let the hurdle for the index be

$$\kappa = E(y|\tilde{\boldsymbol{a}}) - \xi\sigma = \tilde{\boldsymbol{\theta}}'\boldsymbol{Q}'\tilde{\boldsymbol{a}} - \xi\sigma.$$

The agent's payoff from an action $\boldsymbol{a}$ is then $b(1 - \Phi(\frac{\kappa - E(y|\boldsymbol{a})}{\sigma})) - c(\boldsymbol{a})$ with gradient $b\frac{1}{\sigma}\phi(\frac{\kappa - \tilde{\boldsymbol{\theta}}'\boldsymbol{Q}'\boldsymbol{a}}{\sigma})\boldsymbol{Q}\tilde{\boldsymbol{\theta}} - \nabla c(\boldsymbol{a})$. It follows that action $\tilde{\boldsymbol{a}}$ satisfies the first-order condition for an optimum, because we have $\kappa - \tilde{\boldsymbol{\theta}}'\boldsymbol{Q}'\tilde{\boldsymbol{a}} = -\xi\sigma$, $b\frac{1}{\sigma}\phi(-\xi) = 1$ and $\boldsymbol{Q}\tilde{\boldsymbol{\theta}} = \nabla c(\tilde{\boldsymbol{a}})$. Because $\xi > 0$, we can also verify that the Hessian at $\tilde{\boldsymbol{a}}$ is positive definite, hence action $\tilde{\boldsymbol{a}}$ is a local optimum for the agent under the given incentive scheme.

It remains to show that $\tilde{\boldsymbol{a}}$ is a global optimum. Observe that for any action $\boldsymbol{a}$ the agent's expected income depends only on the action via the expected index value $e = E(y|\boldsymbol{a})$. For an index with hurdle κ this expected income is $b(1 - \Phi(\frac{\kappa - e}{\sigma}))$. Let now $C(e)$ be the minimal cost to obtain a given expected value e,

$$C(e) = \min_{\boldsymbol{a}} c(\boldsymbol{a}) \text{ s.t. } \boldsymbol{a}'\boldsymbol{Q}\tilde{\boldsymbol{\theta}} = e, \tag{A.3}$$

and consider the payoff

$$u(e) = b\left(1 - \Phi\left(\frac{\kappa - e}{\sigma}\right)\right) - C(e).$$

Let $\tilde{e} = E(y|\tilde{\boldsymbol{a}}) = \tilde{\boldsymbol{\theta}}'\boldsymbol{Q}'\tilde{\boldsymbol{a}}$, and note that by the definition of κ we have $\frac{\kappa - \tilde{e}}{\sigma} = -\xi$, which implies

$$u(\tilde{e}) = b(1 - \Phi(-\xi)) - c(\tilde{\boldsymbol{a}}).$$

Here we have used the fact that $C(\tilde{e}) = c(\tilde{\boldsymbol{a}})$, by virtue of $\tilde{\boldsymbol{a}}$ being the cost-minimizing action to generate expectation $\tilde{e}$. It is now clear that if $u(e) \leq u(\tilde{e})$ for all feasible e, then action $\tilde{\boldsymbol{a}}$ is an optimal choice for the agent. (If not, there exists an action $\boldsymbol{a}$ yielding a higher payoff. This payoff is $u(e)$, where $e = \boldsymbol{a}'\boldsymbol{Q}\tilde{\boldsymbol{\theta}}$, and thus $u(e) > u(\tilde{e})$, a contradiction.) To show that $u(e)$ is maximal at $e = \tilde{e}$, we first establish a fact about the cost function $C(e)$.

Claim. *$C'(e)$ is increasing in e and $C'(0) = 0$.*

To verify the claim, observe that because $\boldsymbol{Q}\tilde{\boldsymbol{\theta}} = \nabla c(\tilde{\boldsymbol{a}})$, the first-order conditions for the cost minimization problem defining $C(e)$ are

$$\nabla c(\widehat{\boldsymbol{a}}) = \gamma\nabla c(\tilde{\boldsymbol{a}}) \text{ and } e = \widehat{\boldsymbol{a}}'\nabla c(\tilde{\boldsymbol{a}}), \tag{A.4}$$

where $\hat{\boldsymbol{a}} = \hat{\boldsymbol{a}}(e)$ is the optimal action and γ is a Lagrange multiplier. Differentiation with respect to e yields

$$\boldsymbol{H}(\widehat{\boldsymbol{a}})\boldsymbol{d}\widehat{\boldsymbol{a}} = d\gamma\nabla c(\tilde{\boldsymbol{a}}) \text{ and } \nabla c(\tilde{\boldsymbol{a}})'\boldsymbol{d}\widehat{\boldsymbol{a}} = de,$$

where $\boldsymbol{H}(\boldsymbol{a}) = [c_{ij}(\boldsymbol{a})]$ is the Hessian of the cost function $c(\boldsymbol{a})$. Hence, $\boldsymbol{d\hat{a}} = \boldsymbol{H}(\hat{\boldsymbol{a}})^{-1}\nabla c(\tilde{\boldsymbol{a}})d\gamma$ and so

$$\frac{d\gamma}{de} = (\nabla c(\tilde{\boldsymbol{a}})'\boldsymbol{H}(\hat{\boldsymbol{a}})^{-1}\nabla c(\tilde{\boldsymbol{a}}))^{-1} > 0,$$

where the inequality follows from $\boldsymbol{H}$ being positive definite. From the envelope property, we have $C'(e) = \gamma$ and so $C''(e) = \frac{d\gamma}{de} > 0$. Next, because $\hat{\boldsymbol{a}} = \boldsymbol{0}$ is optimal for $e = 0$, we have $\gamma(0) = 0$ (by virtue of $\gamma = \tilde{\boldsymbol{a}}'\nabla c(\hat{\boldsymbol{a}})/\tilde{\boldsymbol{a}}'\nabla c(\tilde{\boldsymbol{a}})$) and therefore $C'(0) = \gamma(0) = 0$. This verifies the claim.

Now we return to showing $u(e) \leq u(\tilde{e})$ for all feasible e. Consider first $e < \tilde{e}$. Because $u'(0) > 0$ (by virtue of $C'(0) = 0$), we have $u(e) \leq u(\tilde{e})$ for all $e \in [0, \tilde{e}]$ if $u(\cdot)$ has no local maximum in the interior of the interval. So suppose $u(\cdot)$ has a local maximum at some $e^0 \in (0, \tilde{e})$. Then $u'(e^0) = 0$ and so $b\frac{1}{\sigma}\phi(\frac{\kappa - e^0}{\sigma}) = C'(e^0)$. Because $C'(e^0) < C'(\tilde{e})$, and $\tilde{e}$ is also a local maximum, we then have $\phi(\frac{\kappa - e^0}{\sigma}) < \phi(\frac{\kappa - \tilde{e}}{\sigma})$. Because $\phi(\cdot)$ is symmetric around zero, this implies $\kappa - e^0 > \tilde{e} - \kappa$ and hence, by definition of $\kappa = \tilde{e} - \xi\sigma$, that $\kappa - e^0 > \xi\sigma$. This yields

$$u(e^0) = b(1 - \Phi(\frac{\kappa - e^0}{\sigma})) - C(e^0) \leq b(1 - \Phi(\xi)),$$

and hence

$$u(\tilde{e}) - u(e^0) \geq b(1 - \Phi(-\xi)) - c(\tilde{\boldsymbol{a}}) - b(1 - \Phi(\xi)).$$

The last expression is increasing in ξ and is (by definition of ξ_0) zero for $\xi = \xi_0$. Hence $u(\tilde{e}) - u(e^0) \geq 0$, because $\xi > \xi_0$. This verifies $u(e) \leq u(\tilde{e})$ for all feasible $e < \tilde{e}$.

Now consider $e > \tilde{e}$. Then, $u(e) < u(\tilde{e})$ because we have $u'(e) < u'(\tilde{e}) = 0$ when $e > \tilde{e}$. This follows because $C'(e)$ is increasing, and because $\phi(\frac{\kappa - e}{\sigma})$ is decreasing in e when $e > \tilde{e}$, because $\tilde{e} > \kappa$ and thus $\kappa - e < 0$. This verifies $u(e) < u(\tilde{e})$ for $e > \tilde{e}$.

We finally verify that $\kappa \to E(y|\tilde{\boldsymbol{a}})$ when $\sigma \to 0$. From the definition of κ and ξ we have $E(y|\tilde{\boldsymbol{a}}) - \kappa = \xi\sigma = \xi\phi(-\xi)b$, where $\xi \to \infty$ when $\sigma \to 0$. The density $\phi(\cdot)$ has the property that $\xi\phi(-\xi) \to 0$ when $\xi \to \infty$, and this completes the proof.

Verification of (13). With $\rho_{0i} = corr(x_0, x_i)$, the index (12) can be written as $\Sigma_{i=1}^{m}\theta_i(x_i - r_i x_0)$, with $r_i = \rho_{0i}(s_{ii}/s_{00})^{1/2}$, $i = 1, \ldots, m$. The hurdle for the index is its expected value $\Sigma_{i=1}^{m}\theta_i(e_i^* - r_i e_0^*)$, where $e_i^* = E(x_i|\boldsymbol{a}^*)$, $i = 0, \ldots, m$. Because $e_i^* + r_i(x_0 - e_0^*)$ is the conditional expectation of x_i, given x_0 (and $\boldsymbol{a}^*$), it follows that we can write the condition for the index to pass the hurdle as (13).

Proof of Proposition 6. Substituting for $\tilde{b}_0$ from (18) we find that the surplus $S(\boldsymbol{\theta})$ is here

$$S(\boldsymbol{\theta}) = \frac{1}{2\boldsymbol{q}_0'\boldsymbol{q}_0}(\boldsymbol{p}'\boldsymbol{q}_0 - \boldsymbol{\theta}'\boldsymbol{Q}'\boldsymbol{q}_0)^2 + \boldsymbol{p}'\boldsymbol{Q}\boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\theta}'\boldsymbol{Q}'\boldsymbol{Q}\boldsymbol{\theta}.$$

By our assumptions, the expression on the right-hand side of the constraint in (16) is symmetric in $(\theta_1, \ldots, \theta_m)$, as we have from (14) $\boldsymbol{\theta}'\tilde{\boldsymbol{\Sigma}}\boldsymbol{\theta} = \Sigma_{i,j=1}^{m}\theta_i\theta_j(s_{ij} - \rho_{0i}\rho_{0j}(s_{ii}s_{jj})^{1/2})$ and thus

$$\boldsymbol{\theta}'\tilde{\boldsymbol{\Sigma}}\boldsymbol{\theta} = \Sigma_{i=1}^{m}\theta_i^2 s_{11}(1-\rho_{01}^2) + 2\Sigma_{i>j=1}^{m}\theta_i\theta_j s_{11}(\rho_{12}-\rho_{01}^2).$$

As $S(\boldsymbol{\theta})$ is also symmetric, the optimal solution $\boldsymbol{\theta}^*$ is then symmetric with $\theta_i^* = \theta_1^*$ determined by the binding enforcement constraint:

$$\frac{\delta\phi_0}{1-\delta}S(\theta_1, \ldots, \theta_1) = |\theta_1|(m(1-\rho_{01}^2) + m(m-1)(\rho_{12}-\rho_{01}^2))^{1/2}\sqrt{s_{11}}.$$

Because $S(\boldsymbol{\theta})$ is concave and $S_{\theta_i}(0) > 0$, we must have $\theta_1^* > 0$. We also see that θ_1^* is an increasing function of ρ_{01}^2, and hence that we may write

$$\frac{\partial\theta_1^*}{\partial\rho_{01}} = \rho_{01}\Theta(\rho_{01}^2), \text{where } \Theta(\rho_{01}^2) > 0.$$

Because now $cov(x_0, \boldsymbol{\theta}^{*\prime}\boldsymbol{x})/s_{00} = \theta_1^* m\rho_{01}\sqrt{s_{11}/s_{00}}$, we have from (17) and (18):

$$b_0 = (\frac{\boldsymbol{p}'\boldsymbol{q}_0}{\boldsymbol{q}_0'\boldsymbol{q}_0} - \frac{\theta_1^*\Sigma_{i=1}^{m}\boldsymbol{q}_i'\boldsymbol{q}_0}{\boldsymbol{q}_0'\boldsymbol{q}_0}) + \rho_{01}\theta_1^* m\sqrt{s_{11}/s_{00}},$$

The statements in the proposition then follow from the last two displayed formulas. Specifically, we can write $b_0 = (k_0 - \theta_1^* k_1) + \rho_{01}\theta_1^* k_2$ and $\frac{\partial b_0}{\partial\rho_{01}} = \theta_1^* k_2 + (-k_1 + \rho_{01}k_2)\rho_{01}\Theta(\rho_{01}^2)$ with $k_2 > 0$, and we see that the derivative is positive for all ρ_{01} if $k_1 = 0$, and positive for all $\rho_{01} \leq 0$ if $k_1 > 0$.

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