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The dynamics of terrorist organizations

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ABSTRACT

Terrorist organizations are most often comprised of ideologues, criminal mercenaries, and captive participants. Ideologues provide political purpose and direction and have a strong group commitment. However, every organization needs money to survive. For terrorist organizations this comes through capital support or through criminal activities. Mercenaries serve the organization by providing the latter but have a weak group commitment and may corrupt the organization's ideological purity. Captive participants have neither strong commitments nor strong personal financial interests, but cannot leave without repercussions. Factors are assessed influencing how the composition of an organization evolves through time. The three labor groups value each other differently which impacts their relative strengths. Capital sponsors may view criminal mercenaries as ideologically detrimental to the terrorist organization. Capital sponsoring may cause an ideologically conscious terrorist organization, while lacking capital may cause a criminal organization relying on mercenary labor. If the ideologues lose their commitment, or the mercenaries and captive participants jointly value each other more, the organization may also become criminal or go extinct. The article provides tools for understanding the evolution of terrorist organizations.

1. Introduction

1.1. Background

Most terrorist organizations have a short life span [27]. Those that survive may increase and decrease in significance through time due to their internal composition and external factors. A terrorist organization needs ideological conviction (religion, nationalism, separatism, right wing, left wing, Marxism, animal rights, etc.) and capital to enhance that conviction and run its operations. An ideologically pure terrorist organization may acquire capital from donors. For example, Laskar-e-Taiba is funded by the Pakistani military and Hezbollah by Iran [5,29]. Al-Qaeda was historically funded by individuals in Saudi Arabia and various Gulf countries [26]. Ideologically less pure terrorist organization may acquire capital from crime, e.g. hijacking, hostage taking, illegal drugs, human trafficking, prostitution, money laundering, extortion counterfeiting, etc. For example, the Colombian FARC guerilla had a Marxism ideology, but gradually became criminal through illegal drug trade and kidnapping [2,12]. Abu Sayaaf resorted to kidnapping and extortion [1]. Various spinoffs of the Northern Irish IRA also resorted to crime [7].

Terrorist and crime organizations have differences and similarities [6]. For example, terrorists want publicity while criminals don't. Mexican drug cartels and ISIS in Iraq and Syria, and al-Qaeda in Afghanistan, all behead their enemies. Both the Italian Mafia and various terrorist organizations bomb tourist destinations [9,10,25,30]. However, motivations differ. A terrorist seeks societal change to benefit those with the same conviction, and may even sacrifice his life towards that goal. A criminal seeks material benefit including monetary gain regardless of ideological conviction. To illustrate these phenomena, Hoffman [21] considers the nature of terrorism including nationalist, separatist movements, religious, and single-issue movements. Hoffer [20] list factors that drive the minds of fanatics and mass movements. Gupta [12] considers the birth, growth, transformation, and demise of terrorist organizations.

1.2. Contribution

To capture a terrorist organization's internal dynamics we model three kinds of operatives [11,12]. Ideologues ensure the ideological commitment, and may sacrifice their own interests and even their lives for the organization [20]. Criminal mercenaries (mercenaries for short) provide monetary input through crime, seek individual financial gain, and may have weak commitment to the organization. Captive participants are coerced to support the ideologues and/or mercenaries. They cannot leave the organization except through extreme costs, and may also have weak or no ideological commitment. They may provide safe houses, get-away cars, protection, logistics, or act as guards, watchmen

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or lookouts. Captive participants are forced to serve the terrorist organization's interest. They obtain no financial benefits from membership, and continue as organization members because their defection costs outweigh the benefits.

Mercenaries may join a terrorist organization for many reasons. They may be inspired by the charismatic leader. They may benefit from the presence of ideologues, just as ideologues may benefit from the mercenaries. Mercenaries may also join because they expect selective incentives such as personal benefits including loot, contraband, drugs, rape, etc. [31]. They may furthermore join because the terrorist organization provides an already designed and ready institutionalized organizational structure within which the mercenaries can operate. The organization may have been built at considerable risk and expense by the ideologues. The mercenaries may hope to adapt to it, transform it to their own benefit, and perhaps even exclude the ideologues over time. The Colombian FARC guerilla illustrates how mercenaries may hijack an organization of organized ideologues. The Phillippine Abu Sayyaf organization illustrates how the adoption of ideological slogans and jihadi vocabulary enabled al-Qaeda financial support, terminated by bin Laden through discovering the organization's criminal nature [28]. Mercenaries may prefer many ideologues when they are few, and few ideologues when they are many.

Tensions between ideologues and mercenaries may fracture an organization. In 1972 the more left wing Irish National Liberation Army consisting of many mercenaries splintered from the Provisional Irish Republican Army which opposed illegal drug trade. The Provisional Irish Republican Army observed the degraded reputation in 1975 and executed most of the active members of the Irish National Liberation Army [19,22].

Organizations consisting only of captive participants lack commitment and effectiveness, and disintegrate. Captive participants join to avoid the cost of not participating, and not for the positive payoff which may be non-existent. They increase the organization's payoff through supporting the ideologues and mercenaries in achieving their objectives.

The relative presence of each kind of operative may vary. With only ideologues, the organization needs outside capital to surive. Some organizations such as Hamas, Hezbollah, the Provisional Irish Republican Army and Laskar-i-Taiba succeed in that regard, and dislike crime. Without sponsors, crime may be needed, and mercenaries are recruited. We model organizations that can be positioned along a continuum from pure terrorism to pure crime. A terrorist organization‘s destiny depends on its composition of the three types of operatives, funding, and external factors. Although a terrorist organization may impose havoc when in its prime, most die out (e.g. Japanese Red Army, Baader Meinhof).

Societal pressures and the surroundings within which a terrorist organization operates impacts its evolution. In failed states where the government's ability to impose differential levels of punishment is low, the composition of terrorist organizations is uncertain. Mercenaries, ideologues, or both may flourish. In non-failed states, where strong governments can impose higher costs on ideologues than mercenaries, a terrorist organization may become criminal, by which we mean that the ratio of ideologues to mercenaries decreases. A pure terrorist organization is defined as an organization with no mercenaries. A pure criminal organization is defined as an organization with no ideologues. Many organizations are mixtures between these two extremes.

Other negative inducements are imposed by the terrorist organization's political base. While an organization consisting mostly of mercenaries may yield high functional effectiveness, it may lose the support of its political base, causing it to die or become criminal. This is modeled through how each kind of operative within the terrorist organization is able to evolve in the given environment.

Each terrorist organization has three kinds of operatives, with certain abilities and objectives, which impact how the organization is positioned within the continuum from pure terrorism to pure crime. Outside factors including societal pressures also impact, e.g. whether the organization operates in failed or non-failed states. Other factors are government abilities and priorities such as fighting terrorism and/or crime, and the terrorist organization's political and economic base from which it may recruit members and potentially provide benefits.

To determine how a terrorist organization evolves, how it is composed is essential. Whether it attracts capital or not is essential. Whether it focuses on terrorism or crime is essential. Whether and how it wages war or competes for market share with other terrorist organizations are essential. How and whether multiple governments intervene strategically are essential. This article is to the authors' knowledge the first to model the conditions impacting how a terrorist organization evolves through time accounting for its internal composition and external factors. It may grow, decrease in size, change its composition, and/or die out. This evolution depends on the organization's internal composition modeled with three kinds of operatives (ideologues, mercenaries, captive participants), and on external factors. The external factors are modeled by capital provided by willing sponsors, multiple governments intervening to constrain any of the three kinds of operatives plus the capital flow, and war and competition with other terrorist organizations.

The model is developed in a logically clear way, by adding different aspects of the evolution one-by-one sequentially. Each new development is first justified theoretically to ensure appropriateness and accordance with intuition. Thereafter the model is illustrated with simulations to show that various realizations, with sensitivity analysis, confirm that at least the minimal standard of internal consistency has been achieved.

1.3. Literature

The key novelties of this article described above differs from the affine literature in especially two regards. First, models of the internal structure of a terrorist organization usually don't account for the time dimension. Developing static models, Hausken [13], Hausken and Gupta [16-18], and Hausken et al. [15] model ideologues and mercenaries, thus accounting for terrorism and crime, but not captive participants. Second, terrorism models accounting for the time dimension often do not account for the internal structure of the organization, and typically focus on a variety of different phenomena. Some examples are as follows.

Feinstein and Kaplan [8] analyze a terror organization's short-term attacks in a single period, with low fixed cost and high marginal cost, and longer term attacks over two periods, with high fixed cost but low marginal cost. Longer term attacks require more resources and cause more damage if successful. Udwadia et al. [32] present a dynamic model of terrorism. A population consists of terrorists, those susceptible to both terrorist and pacifist propaganda, and nonsusceptibles (pacifists). Both direct military/police intervention, and nonviolent, persuasive intervention, are incorporated and are analyzed over time. Kaminskiy and Ayyub [23] develop a terrorist population dynamics model. They perform a cost-effectiveness analysis which shows that if the effectiveness of disabling a terrorist cell is getting worse after 2–3 half-lives of a cell, the respective policy should be revised, using risk assessment. Hausken [14] analyzes government intervention against terrorist organizations evolving through time. Bunn [3] presents a mathematical model for measuring the global risk of nuclear theft and terrorism. Finally, Chamberlain [4] presents six sub-models, one to replicate the United States' actions against al-Qa'ida and five to describe how al-Qa'ida recruits new members, trains these recruits, sustains their capabilities, and then executes terrorist attacks.

1.4. Article organization

Section 2 presents a model of a terrorist organization with ideologues and sponsors of capital. Section 3 incorporates criminal mercenaries into the model. Section 4 incorporates criminal captive participants into the model. Section 5 concludes.

2. An ideologically pure model of ideologues and sponsors of capital

2.1. Theoretical analysis

Appendix A shows the nomenclature. Assuming ideological purity, we define I as the amount or stock of labor exerted by ideologues to run a terrorist organization, K as the amount of capital provided by sponsors, and t as time. Over time the ideologue labor I increases with the injection of capital K, subject to depreciation by itself. Analogously, the willingness of sponsors to insert capital K increases with the ideologue labor I, also subject to depreciation by itself. This simple and plausible approach gives two linear first order coupled differential equations which also happen to constitute a linear time-invariant system, i.e.

$$\{\dot{I} = aK - bI, \dot{K} = cI - dK\} \Leftrightarrow \begin{bmatrix} \dot{I} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} -b & a \\ c & -d \end{bmatrix} \begin{bmatrix} I \\ K \end{bmatrix}, I(0) \geq 0, K(0) \geq 0, t \geq 0 \tag{1}$$

where a dot above a variable means time differentiation d/dt, a and c are growth rates, b and d are depreciation rates, and $I(0)$ and $K(0)$ are the initial conditions. All parameters in this section are assumed to be zero or positive. As a first approximation, this article assumes linear terms for most of the variables on the right hand side of the differential equations. Linear terms approximate terms with slight concavity or convexity when the terms are small, and limit the number of parameters. Future research may raise each linear term to an exponent to allow concavity (when the exponent is between zero and one) and convexity (when the exponent is above one), or may incorporate functional forms, e.g. multiple additive or multiplicative terms of multiple orders, for the variables on the right hand side of each differential equation. The ratio a/b expresses how successfully capital is converted into ideologue labor I, and c/d expresses sponsors' willingness to insert capital as a consequence of ideologue labor I. We first characterize the steady state by determining the system's equilibria.

Property 1. *If* $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) = bd - ac \neq 0$, *the steady state is a unique equilibrium in the origin* $\begin{bmatrix} I_s \\ K_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, *where subscript s means steady state.*

Proof. This is a well-known property of linear time-invariant systems, see Khalil [[24], p. 46]. □

Property 2. *If* $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) = 0$, *the steady state is a continuum of equilibrium points.*

Proof. This is a well-known property of linear time-invariant systems, see Khalil [[24], p. 46]. □

We next proceed to discuss stability by analyzing the eigenvalues. Using (1), the eigenvalues are determined by

$$det\left(\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) = det\left(\begin{bmatrix} \lambda + b & -a \\ -c & \lambda + d \end{bmatrix}\right) = (\lambda + b)(\lambda + d) - ac = 0 \tag{2}$$

which is solved to yield two values of λ, i.e.

$$\lambda_1 = \frac{1}{2}(\Omega - b - d), \lambda_2 = -\frac{1}{2}(\Omega + b + d), \Omega \equiv \sqrt{4ac + (b - d)^2} \tag{3}$$

When $\Omega \neq 0$, the solution of a linear time-invariant system such as (1) is of the form

$$\begin{bmatrix} I \\ K \end{bmatrix} = k_1 e^{\lambda_1 t} \begin{bmatrix} \eta_I^{(1)} \\ \eta_K^{(1)} \end{bmatrix} + k_2 e^{\lambda_2 t} \begin{bmatrix} \eta_I^{(2)} \\ \eta_K^{(2)} \end{bmatrix} \tag{4}$$

where k_1 and k_2 are constants and $\begin{bmatrix} \eta_I^{(1)} \\ \eta_K^{(1)} \end{bmatrix}$ and $\begin{bmatrix} \eta_I^{(2)} \\ \eta_K^{(2)} \end{bmatrix}$ are the eigenvectors determined as follows. For $\lambda_1 = \frac{1}{2}(\Omega - b - d)$ we insert $\lambda_1 = \lambda$ into the rightmost matrix in (2) and solve

$$\begin{bmatrix} \frac{1}{2}(\Omega + b - d) & -a \\ -c & \frac{1}{2}(\Omega - b + d) \end{bmatrix} \begin{bmatrix} \eta_I^{(1)} \\ \eta_K^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \eta_I^{(1)} = \frac{2a}{(\Omega + b - d)} \eta_K^{(1)} \tag{5}$$

For $\lambda_2 = -\frac{1}{2}(\Omega + b + d)$ we insert $\lambda_2 = \lambda$ into the rightmost matrix in (2) and solve

$$\begin{bmatrix} \frac{1}{2}(-\Omega + b - d) & -a \\ -c & \frac{1}{2}(-\Omega - b + d) \end{bmatrix} \begin{bmatrix} \eta_I^{(2)} \\ \eta_K^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \eta_I^{(2)} = \frac{-2a}{(\Omega - b + d)} \eta_K^{(2)} \tag{6}$$

Inserting (5) and (6) for $\eta_K^{(1)} = \eta_K^{(2)} = 1$ into (4) gives

$$\begin{bmatrix} I \\ K \end{bmatrix} = k_1 e^{\lambda_1 t} \begin{bmatrix} \frac{2a}{(\Omega + b - d)} \\ 1 \end{bmatrix} + k_2 e^{\lambda_2 t} \begin{bmatrix} \frac{-2a}{(\Omega - b + d)} \\ 1 \end{bmatrix} \tag{7}$$

Applying the initial conditions to (7) yields

$$\begin{bmatrix} I(0) \\ K(0) \end{bmatrix} = k_1 \begin{bmatrix} \frac{2a}{(\Omega + b - d)} \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} \frac{-2a}{(\Omega - b + d)} \\ 1 \end{bmatrix} \tag{8}$$

which is solved to yield

$$k_1 = \frac{(\Omega + b - d)K(0) + 2cI(0)}{2\Omega}, k_2 = \frac{(\Omega - b + d)K(0) - 2cI(0)}{2\Omega} \tag{9}$$

Inserting (9) into (7) and simplifying gives

$$\begin{bmatrix} I \\ K \end{bmatrix} = \frac{e^{\lambda_1 t}}{2\Omega} \begin{bmatrix} (\Omega - b + d)I(0) + 2aK(0) \\ (\Omega + b - d)K(0) + 2cI(0) \end{bmatrix} + \frac{e^{\lambda_2 t}}{2\Omega} \begin{bmatrix} (\Omega + b - d)I(0) - 2aK(0) \\ (\Omega - b + d)K(0) - 2cI(0) \end{bmatrix} \tag{10}$$

Eqs. (4)-(10) assume $\Omega \neq 0$. Solving (1) when $\Omega = 0$ is straightforward. Rewriting (10), and distinguishing between $\Omega \neq 0$ and $\Omega = 0$, give

$$I = \begin{cases} \frac{e^{\lambda_1 t}}{2\Omega}((\Omega - b + d)I(0) + 2aK(0)) + \frac{e^{\lambda_2 t}}{2\Omega}((\Omega + b - d)I(0) - 2aK(0)) & if\ \Omega \neq 0 \\ e^{-dt}(I(0) + atK(0)) & if\ \Omega = 0 \end{cases}$$

$$K = \begin{cases} \frac{e^{\lambda_1 t}}{2\Omega}((\Omega + b - d)K(0) + 2cI(0)) + \frac{e^{\lambda_2 t}}{2\Omega}((\Omega - b + d)K(0) - 2cI(0)) & if\ \Omega \neq 0 \\ e^{-dt}(K(0) + ctI(0)) & if\ \Omega = 0 \end{cases} \tag{11}$$

$$\lambda_1 = \frac{1}{2}(\Omega - b - d), \lambda_2 = -\frac{1}{2}(\Omega + b + d),$$

$$\Omega \equiv \sqrt{4ac + (b - d)^2}, a \geq 0, b \geq 0, c \geq 0, d \geq 0$$

The analysis above implies $\lambda_1 \neq 0 \Leftrightarrow ac \neq bd$, $\lambda_2 < 0$, and $\Omega = 0 \Leftrightarrow \{ac = 0\ and\ b = d\}$. Hence both $ac \neq bd$ and $ac = bd$ may cause and be caused by $\Omega = 0$.

Property 3. *If* $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) \neq 0$, *the system stability specified by the eigenvalues is determined by*

$$\lim_{t\to\infty} I = \begin{cases} 0 \ if \ \{\lambda_1 < 0 \Leftrightarrow ac < bd \ or \ \{\lambda_1 > 0 \ and \ max(I(0), K(0)) = 0\}\} \ and \ \Omega \neq 0 \\ \infty \ if \ \lambda_1 > 0 \Leftrightarrow ac > bd \ and \ max(I(0), K(0)) > 0 \ and \ \Omega \neq 0 \\ 0 \ if \ \Omega = 0 \end{cases},$$
$$\lim_{t\to\infty} K = \begin{cases} 0 \ if \ \{\lambda_1 < 0 \Leftrightarrow ac < bd \ or \ \{\lambda_1 > 0 \ and \ max(I(0), K(0)) = 0\}\} \ and \ \Omega \neq 0 \\ \infty \ if \ \lambda_1 > 0 \Leftrightarrow ac > bd \ and \ max(I(0), K(0)) > 0 \ and \ \Omega \neq 0 \\ 0 \ if \ \Omega = 0 \end{cases} \quad (12)$$

Proof. Appendix B. □

In (12) the solution for $\lambda_1 < 0$ is stable. Assuming $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) \neq 0$ and $\Omega = 0 \Leftrightarrow \{ac = 0 \ and \ b = d\}$ imply $ac = 0 < b = d$, and thus the solution for $\Omega = 0$ is stable. The solution for $\lambda_1 > 0 \ and \ max(I(0), K(0)) = 0$ is unstable since a small perturbation of $I(0)$ or $K(0)$ causes $\lim_{t\to\infty} I = \lim_{t\to\infty} K = \infty$. The solution for $\lambda_1 > 0$ is unstable since infinity is reached.

Property 4. *If* $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) = 0$, *the system stability specified by the eigenvalues is determined by*

$$\lim_{t\to\infty} I = \begin{cases} \frac{d(cI(0) + bK(0))}{c(b+d)} \ if \ \lambda_1 = 0 \Rightarrow a = \frac{bd}{c}, \ and \ \Omega \neq 0 \\ 0 \ if \ \Omega = 0 \end{cases},$$
$$\lim_{t\to\infty} K = \begin{cases} \frac{bK(0) + cI(0)}{(b+d)} \ if \ \lambda_1 = 0 \Rightarrow a = \frac{bd}{c}, \ and \ \Omega \neq 0 \\ 0 \ if \ \Omega = 0 \end{cases} \quad (13)$$

Proof. Appendix B. □

The solution for $\lambda_1 = 0$ is unstable, i.e. a small perturbation of *a, b, c, d* causes the specified stable solutions. Assuming $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) = 0$ and $\Omega = 0 \Leftrightarrow \{ac = 0 \ and \ b = d\}$ imply $ac = 0 = b = d$. The solution for $\Omega = 0$ is unstable since a small perturbation of *a, b, c, d* causes the specified stable solutions.

Let us discuss Properties 1–4 referring to the approach and graphic illustrations by Khalil [24]. First, both eigenvalues are real [23, pp. 38–40]. Second, we always have $\lambda_2 < 0$, whereas λ_1 can be negative, zero, or positive. Third, $\lambda_1 < 0$ corresponds to line 1 in the expressions for $\lim_{t\to\infty} I = 0$ and $\lim_{t\to\infty} K = 0$ in (12) in Property 3. If $\lambda_2 < \lambda_1 < 0$, as assumed by Khalil [[24], p. 38] without loss of generality, $e^{\lambda_2 t}$ tends to zero faster than $e^{\lambda_1 t}$, as shown in Khalil's [23, p. 39] Figures 2.3 and 2.4(a). If $\lambda_1 < \lambda_2 < 0$, $e^{\lambda_1 t}$ tends to zero faster than $e^{\lambda_2 t}$. The steady state solution is $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 0$. Fourth, $\lambda_1 > 0$ corresponds to line 2 in the expressions for $\lim_{t\to\infty} I = \infty$ and $\lim_{t\to\infty} K = \infty$ in (12) in Property 3. Hence $e^{\lambda_1 t} \to \infty$ and $e^{\lambda_2 t} \to 0$ as $t \to \infty$, so that λ_1 is the unstable eigenvalue and λ_2 is the stable eigenvalue [23, p. 39], as shown in Khalil's [23, p. 40] Figures 2.5(a) and 2.5(b). Fifth, $\lambda_1 = 0$ corresponds to line 1 in the expressions for $\lim_{t\to\infty} I$ and $\lim_{t\to\infty} K$ in (13) in Property 4. Khalil [[24], pp. 42–46] refers to such as system as degenerate in some sense, so that the matrix in (1) has a nontrivial null space. That is, the system has an equilibrium subspace rather than an equilibrium point. All trajectories converge to the equilibrium subspace illustrated in Khalil's [[24], p. 43] Figure 2.10(a). Sixth, $\Omega = \sqrt{4ac + (b-d)^2} = 0$ gives $\lambda_1 = \lambda_2 = -d$ since $b = d$ and $ac = 0$. If $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) \neq 0$, this corresponds to line 1 in the expressions for $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 0$ in (12) in Property 3, i.e. a stable equilibrium point, and is illustrated in Khalil's [23, p. 42] Figures 2.8(a) and 2.9(a). Equivalent eigenvalues imply that we do not get asymptotic slow-fast behavior [23, p. 41]. If $\Omega = 0$ and $det\left(\begin{bmatrix} -b & a \\ c & -d \end{bmatrix}\right) = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 0$ is unstable since a small perturbation of *a, b, c, d* causes the specified stable solutions.

The signs of the eigenvalues, and the eigenvectors, are

$$\lambda_1 = \tfrac{1}{2}(\Omega - b - d) > 0 \Rightarrow ac > bd, \begin{bmatrix} \eta_I^{(1)} \\ \eta_K^{(1)} \end{bmatrix} = \begin{bmatrix} \frac{(\Omega - b + d)I(0) + 2aK(0)}{2\Omega} \\ \frac{(\Omega + b - d)K(0) + 2cI(0)}{2\Omega} \end{bmatrix},$$
$$\lambda_2 = -\tfrac{1}{2}(\Omega + b + d) < 0 \ always, \begin{bmatrix} \eta_I^{(2)} \\ \eta_K^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{(\Omega + b - d)I(0) - 2aK(0)}{2\Omega} \\ \frac{(\Omega - b + d)K(0) - 2cI(0)}{2\Omega} \end{bmatrix} \quad (14)$$

Eq. (11) shows that ideologue labor I and capital K remain in the origin $I = K = 0$ with zero initial conditions $I(0) = K(0) = 0$. Since $\lambda_2 < 0$, the second term in I and K decreases exponentially as time t increases. Since λ_1 can be positive or negative, the first term in I and K increases or decreases exponentially as time t increases depending on whether λ_1 is positive ($ac > bd$) or negative ($ac < bd$). The inequality $ac > bd$ is challenging for law enforcement which seeks to constrain terrorism. Property 3 states that the steady state is the origin $I = K = 0$ when $\lambda_1 < 0$, i.e. when capital is unsuccessfully converted into ideologue labor I expressed with low a/b, or sponsors reveal low willingness to insert capital expressed with low c/d. In contrast, the steady state is infinity $I = K = \infty$ when $\lambda_1 > 0$, i.e. successful conversion of capital into ideologue labor I or high sponsor willingness to insert capital. These two steady states are stable, while $\lambda_1 = 0$ is unstable.

When the needs of funds exceed the sponsor's willingness to provide, or an upper limit K_{max} on capital funding exists for other reasons, the solution in (11) has to be checked for the boundary condition $K \leq K_{max}$. If the boundary condition is not satisfied, $K = K_{max}$ is inserted into the first equation in (1) which is solved for I causing

$$I = \frac{aK_{max}}{b} + e^{-bt}\left(I(0) - \frac{aK_{max}}{b}\right) \quad (15)$$

which has steady state $\lim_{t\to\infty} I = \frac{aK_{max}}{b}$ approached asymptotically from I (0).

2.2. Simulations

The time paths from the initial conditions $I(0)$ and $K(0)$ to one of the steady states can vary greatly. The remainder of the article presents benchmark parameter values chosen to illustrate different representative characteristics of the model. A good simple starting point is unity $a = b = c = d = 1$. We thereafter choose 20% higher parameter values for one or two of the parameters to analyze the impact. Empirical support for the parameter values is left to future research. Table 1 shows the eigenvalues λ_1 and λ_2 and eigenvectors $\begin{bmatrix} \eta_I^{(1)} \\ \eta_K^{(1)} \end{bmatrix}$ and $\begin{bmatrix} \eta_I^{(2)} \\ \eta_K^{(2)} \end{bmatrix}$, using (14), for the six panels in Fig. 1 with initial conditions $I(0) = 2$ and $K(0) = 0$. The eigenvalue $\lambda_1 = 0$ (panels a and f), $\lambda_1 > 0$ (panels b and c), and $\lambda_1 < 0$ (panels d and e), cause I and K to approach a constant, eventually increase, and eventually decrease, respectively.

Phase portraits for the six panels in Fig. 1 are straightforward to produce but are omitted due to space considerations. More illuminatingly, to motivate connections to the next sections, we present time developments of I and K. Fig. 1 plots (1) and (11) with the benchmark parameter values $a = b = c = d = 1$ when $I(0) = 2$, $K(0) = 0$, $0 \leq t \leq 7$. This time horizon is chosen to ensure illustrative curvatures on I and K before increase towards infinity. The high initial ideologue labor $I(0) = 2$, enabling startup of the terrorist organization, ensures rapid initial increase of capital K. The absence of initial capital $K(0) = 0$ implies rapid initial decrease of ideologue labor I.

In Fig. 1 panel a, the benchmark parameter values initially decrease ideologue labor I, initially increases capital K, and eventually $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 1$ according to Property 4. In the subsequent panels, the benchmark parameter values are kept except that one or several specified parameter value(s) differ(s) from the benchmark. In panel b, the high $c = 1.2$ increases K, where the sponsors' willingness to insert capital K per time unit increases more than unity with ideologue labor I.

Table 1

Eigenvalues λ_1 and λ_2 and eigenvectors $\begin{bmatrix}\eta_I^{(1)}\\ \eta_K^{(1)}\end{bmatrix}$ and $\begin{bmatrix}\eta_I^{(2)}\\ \eta_K^{(2)}\end{bmatrix}$ for the six panels in Fig. 1 with initial conditions $I(0) = 2$ and $K(0) = 0$. The left column P refers to Panel.

P	a	b	c	d	λ_1	$\eta_I^{(1)}$	$\eta_K^{(1)}$	λ_2	$\eta_I^{(2)}$	$\eta_K^{(2)}$
a	1	1	1	1	0	1	1	-2	1	-1
b	1	1	1.2	1	$\sqrt{\frac{6}{5}}-1$	1	$\sqrt{\frac{6}{5}}$	$-\sqrt{\frac{6}{5}}-1$	1	$-\sqrt{\frac{6}{5}}$
c	1.2	1	1	1	$\sqrt{\frac{6}{5}}-1$	1	$\sqrt{\frac{5}{6}}$	$-\sqrt{\frac{6}{5}}-1$	1	$-\sqrt{\frac{5}{6}}$
d	1	1	1	1.2	$\frac{\sqrt{101}-11}{10}$	$1+\frac{1}{\sqrt{101}}$	$\frac{10}{\sqrt{101}}$	$\frac{-\sqrt{101}-11}{10}$	$1-\frac{1}{\sqrt{101}}$	$\frac{-10}{\sqrt{101}}$
e	1	1.2	1	1	$\frac{\sqrt{101}-11}{10}$	$1-\frac{1}{\sqrt{101}}$	$\frac{10}{\sqrt{101}}$	$\frac{-\sqrt{101}-11}{10}$	$1+\frac{1}{\sqrt{101}}$	$\frac{-10}{\sqrt{101}}$
f	1	1.2	1.2	1	0	$\frac{10}{11}$	$\frac{12}{11}$	$\frac{-11}{5}$	$\frac{12}{11}$	$\frac{-12}{11}$

This eventually also increases ideologue labor I. After the rapid initial changes, ideologue labor I and capital K increase moderately driven by the growth parameter $c = 1.2$, which exceeds the depreciation rate $d = 1.2$. Although $a = b = 1$, ideologue labor I increases moderately driven by increasing capital K. With $I(0) = K(0) = 1$, both I and K start at 1 and are negligibly lower than in Fig. 1 for $t > 2$. With $I(0) = 0$, $K(0) = 2$, I and K are practically interchanged relative to Fig. 1 and are negligibly lower than in Fig. 1 for $t > 2$. With $c < 1$, I and K decrease

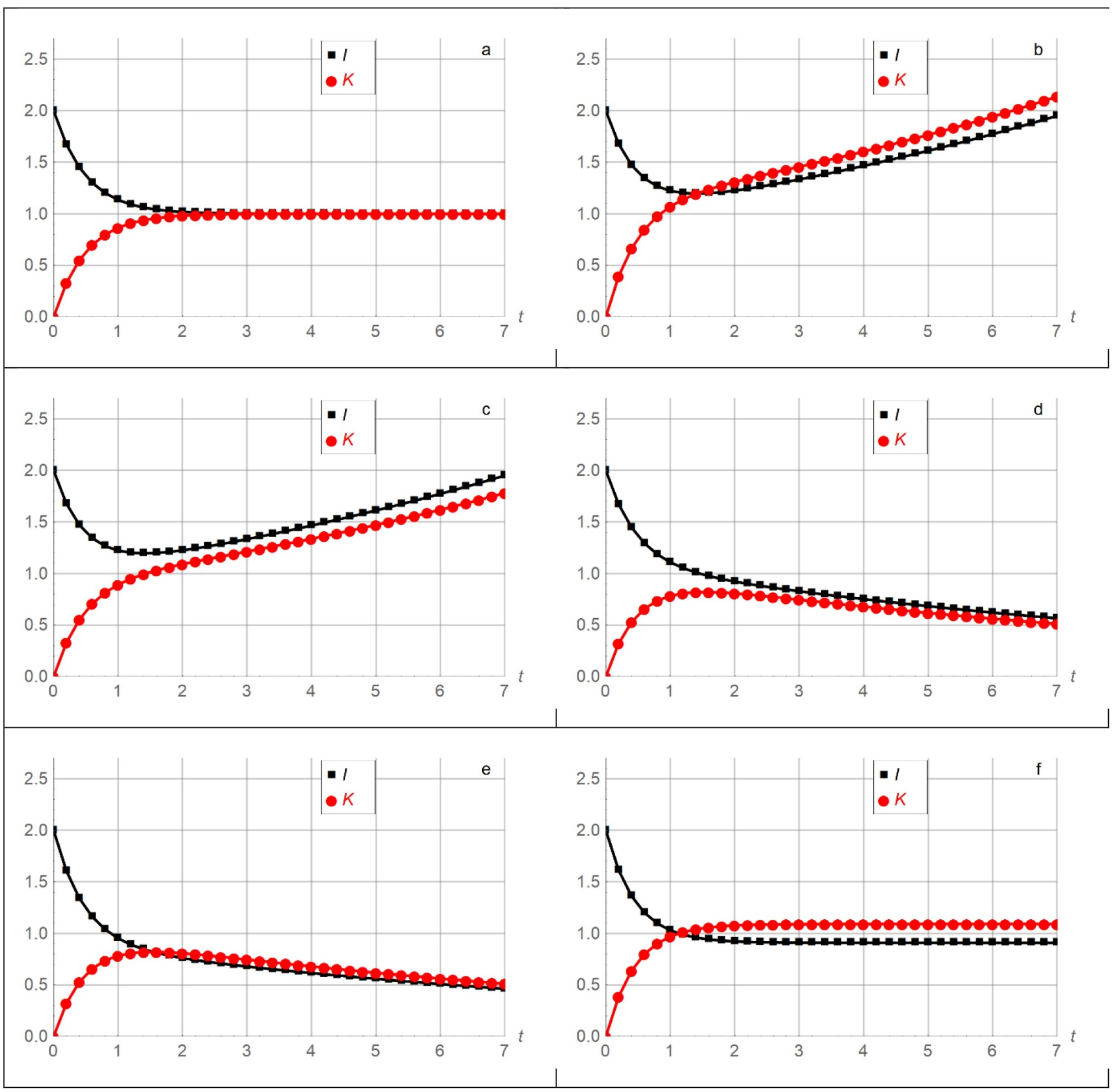

Fig. 1. Ideologue labor I and capital K as functions of time t with benchmark parameter values $a = b = c = d = 1$ when $I(0) = 2$, $K(0) = 0$, $0 \le t \le 7$. Panel a: Benchmark. Panel b: $c = 1.2$. Panel c: $a = 1.2$. Panel d: $d = 1.2$. Panel e: $b = 1.2$. Panel f: $b = c = 1.2$.

toward zero instead of increasing towards infinity.

In Fig. 1 panel c, $a = 1.2$ causes slower increase of K, while I evolves as in panel b (since $ac = 1.2$ and $K(0) = 0$ in (11)). In panel d, the high $d = 1.2$ eventually decreases K, and thus also decreases I. In panel e, the high $b = 1.2$ causes more rapid decrease of I, while K evolves as in panel d (since $(b - d)^2 = 0.\ 2^2$ and $K(0) = 0$ in (11)). In panel f, $b = c = 1.2$ causes $\lim_{t\to\infty} I = \frac{10}{11}$ and $\lim_{t\to\infty} K = \frac{12}{11}$ according to Property 4.

3. Incorporating criminal mercenaries

3.1. Theoretical analysis

As a/b or c/d in (1) decreases, which decreases λ_1, capital becomes less readily available as it gets less efficiently utilized by ideologues, or sponsors become less appreciative of the ideologues' labor I, or a combination of these two factors occurs. Low capital availability may also mean that K_{max} is low, so that $K \le K_{max}$ may not be satisfied, which constrains the sponsor's willingness to fund. Then $K = K_{max}$ and (15) apply, with steady state solution $\lim_{t\to\infty} I = \frac{aK_{max}}{b}$ which may be a lower I than that preferred by the ideologues. To ensure continued financing, when sponsors are lacking or constrained, the ideologues may decide to recruit criminal mercenaries. Mercenaries engage in criminal activities. First, that causes monetary inflows for the ideologues. Second, criminal activities cannot be kept hidden long and the sponsors get deterred. We thus generalize (1) to

$$\dot{I} = aK + eM - \theta KM - bI,\ \dot{K} = cI - fM - dK,$$
$$\dot{M} = gI + hK - \varphi IK - mM,$$
$$I \ge 0,\ K \ge 0,\ M \ge 0 \quad (16)$$

where M is the amount of labor exerted by mercenaries, and e,f,g,h,m,θ,φ are parameters. eM is added to the right hand side of $\dot{I}$ since the ideologues benefit from mercenary labor M. But limits exist to how both K and M can impact I positively. Capital sponsors are deterred by criminal mercenaries which dilute the ideological purity of the terrorist organization. This follows since capital sponsors of terrorism seek to support terrorist ideology. Capital sponsors of terrorism do not seek to support criminality motivated by financial gain. As shown by e.g. Gupta [12]; Hausken [13]; Hausken et al. [15]; Hausken and Gupta [16-18], terrorist organizations degrading themselves to become criminal, such as e.g. FARC, Abu Sayaaf, and various spinoffs of the Northern Irish IRA, decreasingly succeed recruiting ideologically committed sponsors. We thus assume that jointly high K and M is detrimental to ideologue labor I. Hence we subtract θKM from the right hand side of $\dot{I}$. The multiplicative term θKM is large when both K and M are large, and small when either K or M is small. When K is large and M is small, ideologue labor I increases mainly due to K, and θKM does not detract too much. When K is small and M is large, ideologue labor I increases mainly due to M, and θKM also does not detract too much. All parameters in this section and the next section are assumed to be zero or positive, with a possible exception for h. One may envision that capital K is directly detrimental to mercenary labor M, in addition to being detrimental when combined with ideologue labor I.

fM is subtracted from the right hand side of $\dot{K}$ since the sponsors are deterred by mercenary labor M which dilutes ideological purity. To analyze this dilution, Hausken and Gupta [18] develop $J = I/(I + \alpha M)$ as an ideological purity indicator of the terrorist organization, where α is a parameter. When $M = 0$, the terrorist organization has maximal purity $J = 1$. At the limit as M approaches infinity, the terrorist organization has zero purity expressed as $J = 0$. Thus if mercenaries become too numerous, the terrorist organization may turn into a criminal organization. The ideological purity indicator J is a dependent variable, not analyzed further in this article, which straightforwardly specifies the extent to which a terrorist organization preserves its ideology. As a dependent variable J does not impact system dynamics in (16), which depends on the independent variables and parameters in (16).

Assume that mercenary labor M in (16) follows the same logic as ideologue labor I. Afterall, if mercenary labor M gets the upper hand, the organization becomes more criminal, i.e., the ratio I/M of ideologues to mercenaries increases. In contrast, if ideologue labor I gets the upper hand, the organization becomes more focused on terrorism, i.e., I/M decreases. First, mercenary labor M in (16) benefits from the presence of ideologues which provide purpose, meaning, and an organizational structure within which to operate. Second, mercenary labor M benefits from capital K since, at least theoretically, mercenaries may be able to exploit capital inflow K to their advantage just as ideologues are able to exploit capital inflow K to their advantage. Although all parameters in this article are assumed to be zero or positive, we hold the possibility open, without exploring it further, that the parameter h in (16) may be negative. That would mean that mercenary labor M does not benefit from the inflow of capital K, which is then interpreted as a competing money generating function which decreases the need for mercenaries. Third, φIK is subtracted on the right hand side of $\dot{M}$, analogously to subtracting θKM on the right hand side of $\dot{I}$. The analogous logic is that limits exist to how both K and I can impact M positively. High capital sponsoring K benefits ideologue labor I causing mercenary labor M to be less needed. This increases the ideological purity of the terrorist organization. We thus assume that jointly high K and I is detrimental to mercenary labor M. Fourth, mercenary labor M is subject to depreciation by itself, expressed with subtracting mM in (16).

The asymmetry between ideologue labor I and mercenary labor M in (16) arises since cI is positive and fM is subtracted on the right hand side of $\dot{K}$, where c and f are zero or positive. (If c and f were negative, sponsors would favor criminality over terrorism. That is indirectly analyzed in this article by switching the labels ideologue labor I and mercenary labor M.)

If one of the three non-negativity constraints in (16) ceases to hold at time $t = t'$, the independent variable falling below zero is set to zero from that time $t = t'$, and the remaining two equations are solved from time $t = t'$ with non-negativity constraints. For each time $t > t'$, it has to be checked whether the conditions are satisfied for the independent variable to remain at zero. If a time $t = t'' > t'$ is determined when the conditions are not satisfied, the three equations in (16) are solved for $t > t''$. The procedure continues until $t = \infty$. For the time intervals where one independent variable is set to zero, if one of the two non-negativity constraints ceases to hold at time $t = t'''$, the independent variable falling below zero is set to zero from that time $t = t'''$, and the remaining one equation is solved from time $t = t'''$ with the non-negativity constraint. From that time $t = t'''$ it has to be checked whether the conditions are satisfied for the independent variable to remain at zero. If the conditions for one or both variables are not satisfied from time $t = t''''$, one or both variables are reintroduced from time $t = t''''$, and the solution for the two or three variables is determined from time $t = t''''$. The procedure continues until $t = \infty$. This same procedure is applied for all equations with non-negativity constraints in this article.

3.2. Simulations

With mercenaries, (16) is not a linear time-invariant system. Hence characterization of eigenvalues and eigenvectors is impossible. Instead we illustrate trajectory convergence and divergence with the limits $\lim_{t\to\infty} I$, $\lim_{t\to\infty} M$, and $\lim_{t\to\infty} K$, which approach zero, a constant, or infinity, as time t approaches infinity. Fig. 2 illustrates (16) with the benchmark parameter values $a = b = c = d = e = f = g = h = m = 1$, $\theta = 0.3$, $\varphi = 0.6$, $I(0) = 2$, $K(0) = M(0) = 0$ as time t varies from 0 to 7, and from 0 to 220 in panel d. Initial absence of mercenary labor $M(0) = 0$ causes ideologue labor I and capital K initially to evolve as in Fig. 1. Mercenary labor M benefits from the initially high ideologue labor I, and initially from capital K. Increasing mercenary labor M benefits ideologue labor I through the parameter $e = 1$, making the terrorist organization more criminal, and it hurts capital inflow K through the parameter $f = 1$. Assuming $\theta = 0.3$ and $\varphi = 0.6$ means that ideologue labor I is 50% less hurt by high KM than mercenary labor M is hurt by high IK.

In Fig. 2 panel a, the benchmark parameter values cause more favorable evolution of ideologue labor I than in Fig. 1 panel a with no mercenary labor M. Mercenary labor M eventually deters capital funding K which, after an initial maximum, decreases to $K < 10^{-5}$ when $t = 18.0$ due to depreciation by itself and since I and M become more similar where $c = f = 1$ in (16). With $K = 0$ when $t > 18.0$, ideologue

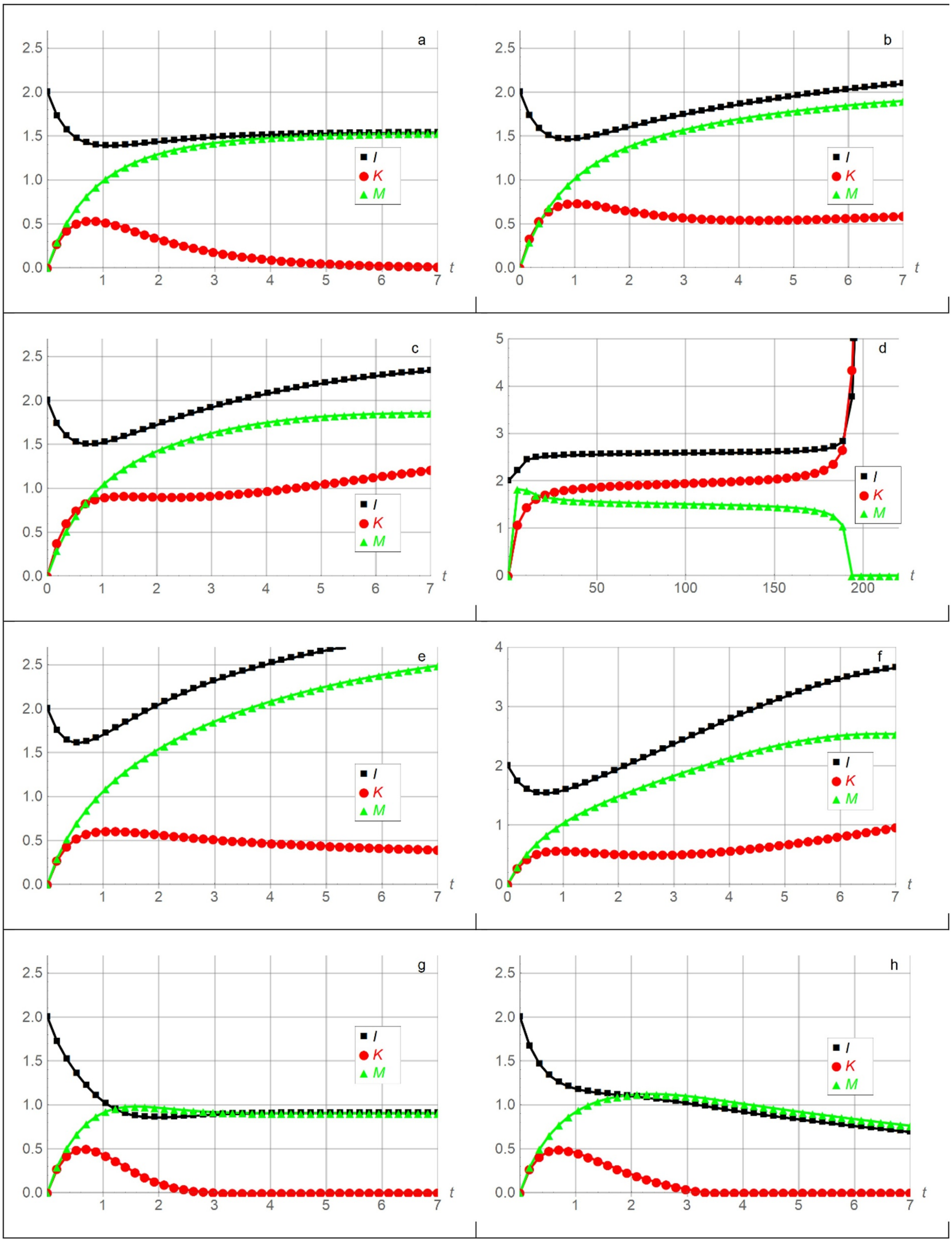

Fig. 2. Ideologue labor I, capital K, and mercenary labor M as functions of time t with the benchmark parameter values $a = b = c = d = e = f = g = h = m = 1$ when $\theta = 0.3$, $\varphi = 0.6$, $I(0) = 2$, $K(0) = M(0) = 0$, $0 \le t \le 7$, and $0 \le t \le 220$ in panel d. Panel a: Benchmark. Panel b: $c = 1.2$. Panel c: $c = 1.343$. Panel d: $c = 1.344$. Panel e: $a = 1.909$. Panel f: $e = 1.4$. Panel g: $\theta = 2$. Panel h: $b = 1.2$. Panel i: $f = 0.4$. Panel j: $d = 1.2$. Panel k: $g = 0.5$. Panel l: $h = 0$. Panel m: $\varphi = 2$. Panel n: $m = 1.2$.

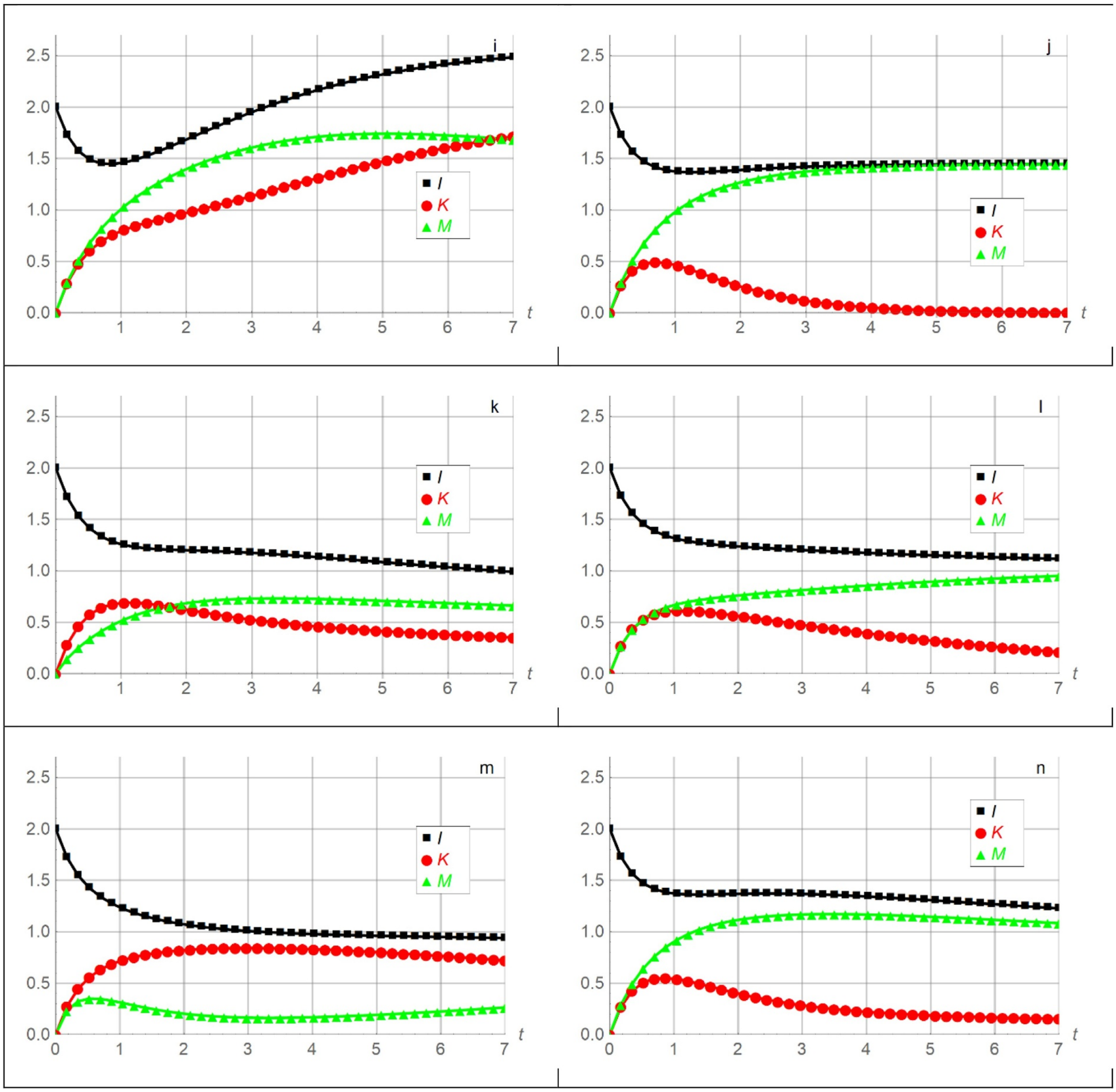

Fig. 2. (*continued*)

labor I eventually reaches a higher value than in Fig. 1 panel a, i.e. $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.54$, determined numerically. Ideologue labor I is initially boosted by both K and M, and is eventually sustained by M, making the terrorist organization more criminal. Ideologue labor I and mercenary labor M eventually reach the same steady state value 1.54 since $K = 0$ in (16) causes $\dot{I} = eM - bI$, $\dot{M} = gI - mM$, where $e = b = g = m = 1$.

In the subsequent panels, the benchmark parameter values are kept except that one or several specified parameter value(s) differ(s) from the benchmark. Sponsors provide more capital K as they appreciate ideologue labor I more. Increasing the parameter c negligibly above $f = 1$ in (16) causes $\lim_{t\to\infty} K$ to increase negligibly above 0. In Fig. 2 panel b, $c = 1.2$ causes more favorable evolution of capital K, eventually reaching $\lim_{t\to\infty} K = 0.77$, $\lim_{t\to\infty} I = 2.32$, and $\lim_{t\to\infty} M = 2.02$. Thus both ideologue labor I and mercenary labor M benefit from the higher $c = 1.2$.

A limit exists for increasing c, since jointly high K and M in (16) is unsustainable. That limit is $c = 1.343$, shown in panel c, where $\lim_{t\to\infty} K = 1.92$, $\lim_{t\to\infty} I = 2.57$, and $\lim_{t\to\infty} M = 1.53$. Panel d assumes $c = 1.344$ showing breakdown of mercenary labor to $M = 0$ when $t > 193.2$, after which ideologue labor I and capital K approach infinity according to Property 3 where $ac > bd$ (and more quickly than in Fig. 1 panel b where $c = 1.2$). When $c = 1.7$, $M = 0$ when $t > 6.36$.

In panel e, $a = 1.909$ causes ideologue labor I to appreciate capital K more than in panel a. This initially causes higher I and K, which also causes more mercenary labor M through positive g and h in (16). The increase of M is eventually detrimental to capital K, causing $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 3.22$. However, also here a limit exists for increasing a since eventually ideologue labor I prefers capital K rather than mercenary labor M. That limit is $a = 1.910$ causing breakdown of mercenary labor to $M = 0$ when $t > 106.2$, after which $\lim_{t\to\infty} I = \lim_{t\to\infty} M = \infty$ as in panel d. Assuming $a = 0$ causes slightly lower capital K evolution than in panel a, and eventually $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.02$.

As e increases negligibly above 1, $\lim_{t\to\infty} I > \lim_{t\to\infty} M$ benefits capital K in (16). In panel f, $e = 1.4$ benefits ideologue labor I more than in panel a. Eventually, $\lim_{t\to\infty} K = 1.33$, $\lim_{t\to\infty} I = 3.33$, $\lim_{t\to\infty} M = 2.00$. Assuming $e = 1.899$ benefits capital K more, i.e. $\lim_{t\to\infty} K = 3.00$, $\lim_{t\to\infty} I = 3.33$, $\lim_{t\to\infty} M = 0.34$. When $e > 1.899$, M becomes negative when $t > 12.77$. As

e decreases negligibly below 1, $\lim_{t\to\infty} I < \lim_{t\to\infty} M$ does not benefit capital K in (16) and is detrimental to ideologue labor I in the long run, causing $\lim_{t\to\infty} K = \lim_{t\to\infty} I = \lim_{t\to\infty} M = 0$.

As θ increases, ideologue labor I suffers, especially when both K and M are high. In panel g, $\theta = 2$ causes I initially to decrease more rapidly than in panel a. Eventually $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 0.90$. With the chosen benchmark parameter values, θ must increase substantially to be detrimental to I. When $\theta = 13.5$, I decreases to 0 when $t > 0.99$. Capital K in (16) cannot grow without ideologue labor I, and K decreases to 0 when $t > 1.19$. Thereafter $\lim_{t\to\infty} M = 0$. Conversely, when $\theta = 0$, ideologue labor I does not suffer when both K and M are high, and $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 2.00$, which are higher I and M than in panel a.

In panel h, $b = 1.2$ causes $K = 0$ when $t > 3.37$, due to ideologue labor I depreciating more strongly by itself. Thus also mercenary labor M eventually decreases after a maximum. Thereafter $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 0$ applying the logic in Property 3 to I and M, which also arises when b negligibly exceeds $b = 1$. When b is negligibly below $b = 1$, capital K eventually decreases, but reaches a strictly positive value as t increases, i.e. $\lim_{t\to\infty} K > 0$. When $b = 0.75$, $\lim_{t\to\infty} K = \lim_{t\to\infty} M = 1.67, \lim_{t\to\infty} I = 3.33$. But that is also the limit. When $b = 0.749$, mercenary labor M collapses to $M = 0$ when $t > 79.2$. Thereafter $\lim_{t\to\infty} K = \lim_{t\to\infty} I = \infty$. The reason is that ideologue labor I does not depreciate sufficiently, and can sustain itself through capital K.

In panel i, $f = 0.4$ expresses that capital sponsors become more tolerant of mercenary labor M. Thus capital K increases more than in panel a, eventually reaching $\lim_{t\to\infty} K = 2.00$, $\lim_{t\to\infty} I = 2.60$, $\lim_{t\to\infty} M = 1.48$. The increase and then decrease through an inverse U shape of mercenary labor M follow due to the jointly high I and K (scaled with the parameter φ). Decreasing f further to $f = 0$, which means that capital sponsors are not negatively impacted by mercenaries, gives a slightly lower maximum for the inverse U shape of M, though eventually $\lim_{t\to\infty} K = \lim_{t\to\infty} I = 3.33$, $\lim_{t\to\infty} M = 0$. Although capital sponsors tolerate mercenary labor M, mercenary labor M gets extinguished by jointly high I and K. This illustrates how a dynamic analysis may possibly reveal unexpected consequences. Conversely, increasing f above $f = 1$ makes capital sponsors less tolerant of mercenary labor M. Compared with panel a, that causes lower capital K, and eventually also lower high I and M. For example, $f = 2$ causes $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.11$, and $f = 10^{10}$ causes $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.00$.

In panel j, $d = 1.2$ causes capital K to depreciate slightly faster than in panel a, $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.45$. When $d = 10^{10}$, depreciation is almost instant, causing $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.00$. In contrast, as d decreases below $d = 1$, K depreciates less. For example, $d = 0.377$ causes $K = 0.62$, $I = 2.15$, $M = 1.93$ when $t = 7$, and $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 2.22$. But that is also the limit below which mercenary labor M collapses as K fails to depreciate sufficiently . When $d = 0.376$, M becomes negative when $t > 93.33$, after which $\lim_{t\to\infty} M = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} K = \infty$.

A terrorist organization can be made less criminal by letting mercenary labor M benefit less from ideologue labor I than in panel a. In panel k, $g = 0.5$ causes M initially to be lower and capital K to be initially higher than in panel a. However, ideologue labor I eventually suffers from lacking support of mercenary labor M, and capital sponsoring K is also insufficient. Thus eventually the terrorist organization collapses expressed with $\lim_{t\to\infty} I = \lim_{t\to\infty} K = \lim_{t\to\infty} M = 0$. This collapse actually occurs when g is negligibly lower than $g = 1$, but then the collapse is slower. In contrast, increasing g negligibly above $g = 1$, causing M to benefit more from I, and K to suffer more than in panel a, eventually causing $\lim_{t\to\infty} K = 0, \lim_{t\to\infty} I = \lim_{t\to\infty} M = \infty$.

In panel l, $h = 0$ prevents mercenary labor M to benefit from capital K. Mercenary labor M then increases more slowly than in panel a, $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.06$. The same occurs as h becomes negative down to $h = -0.522$, causing $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 0.50$. However, $h = -0.523$ causes mercenary labor to break down to $M = 0$ when $t > 16.19$, and thereafter $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 0.80$.

As φ increases, mercenary labor M suffers, especially when both K and I are high. A terrorist organization can be made less criminal by making jointly high ideologue labor I and capital K more detrimental to mercenary labor M. In panel m, $\varphi = 2$ causes M initially to be lower, and K to be higher, than in panel a. Eventually $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 0.82$, which are lower values for I and M than in panel a. When $\varphi = 2.11$, $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 0.78$. However, when $\varphi = 2.12$, mercenary labor M eventually collapses. That is, M decreases to 0 when $t > 3.90$, $\lim_{t\to\infty} M = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 0.95$. Conversely, when $\varphi = 0$, mercenary labor M does not suffer when both I and K are high, and $\lim_{t\to\infty} K = 0$ and $\lim_{t\to\infty} I = \lim_{t\to\infty} M = 1.70$, which are higher I and M than in panel a.

A terrorist organization can be made less criminal by letting mercenary labor M depreciate more strongly by itself. In panel n, $m = 1.2$ causes M initially to be lower and capital K to be initially higher than in panel a, though $\lim_{t\to\infty} K = \lim_{t\to\infty} I = \lim_{t\to\infty} M = 0$. This is limit is reached for all $m > 1$. When m negligibly exceeds $m = 1$, K approaches zero first, and thereafter M and I. When m substantially exceeds $m = 1$, M approaches zero first, and thereafter K and I. When $m = 0.99$, K reaches zero when $t > 7.43$. Thereafter $\lim_{t\to\infty} I = \lim_{t\to\infty} M = \infty$. This limit is also reached when m decreases to zero, but then K reaches zero more quickly.

4. Incorporating captive participants

4.1. Theoretical analysis

Both ideologues and criminal mercenaries need and benefit from captive participants to operate efficiently. We thus generalize (16) to

$$\dot{I} = aK + eM + nC - \theta KM - bI,\ \dot{K} = cI - fM - dK,$$
$$\dot{M} = gI + oC + hK - \varphi IK - mM,\ \dot{C} = pI + qM - rC,$$
$$I \geq 0,\ K \geq 0,\ M \geq 0,\ C \geq 0 \tag{17}$$

where C is the amount of labor exerted by captive participants, and n,o,p,q,r are parameters. Incorporated into nC on the right hand side of $\dot{I}$, and oC on the right hand side of $\dot{M}$, is the cost to the ideologues and mercenaries of handling the captive participants. Thus the net benefit of captive participants contribute to the increase in ideologue labor I and mercenary labor M. On the right hand side of $\dot{C}$, with more ideologue labor I and mercenary labor M, captive participants labor C increases, and C is subject to depreciation by itself. Less mathematically, with more ideologues and mercenaries, more captive participants will be recruited. We assume that sponsors are unaffected by captive participants.

The differential equation for capital K in (17) means that capital K can be the one and only support of ideologue labor I, but cannot be the one and only support of mercenary labor M. This follows from the positive term cI and the negative term $-fM$ on the right hand side of the equation for $\dot{K}$. For a terrorist organization to become criminal, either its sponsors or the organization itself has to somehow become criminally corrupted. Our assumption $c \geq 0$ and $f \geq 0$ ensures that the sponsors are not criminally corrupted. The alternative assumption $f < 0$ would mean that the capital sponsors value criminal mercenaries. The relative sizes of $|f|$ and $|c|$ determine the relative weights assigned to ideologue labor I and mercenary labor M.

4.1.1. Removing mercenary labor M

Without criminal mercenaries, or if the initial conditions or parameters cause the extinction of mercenary labor M, inserting $M = 0$ into (17) causes a pure terrorist organization supported by capital K and captive participants labor C, i.e.

$$\dot{I} = aK + nC - bI,\ \dot{K} = cI - dK,\ \dot{C} = pI - rC,\ I \geq 0,\ K \geq 0,\ C \geq 0 \tag{18}$$

In (18) the dynamics of capital K and captive participants labor C are equivalent, both increasing due to ideologue labor I (regulated by c and p), and both constrained by their own growth (regulated by d and r). Furthermore, K and C impact ideologue labor I equivalently, i.e. additively as $aK + nC$ on the right hand side of $\dot{I}$. Hence insight into (18) flows from our insight in (1) and Section 2 where captive participants labor C is absent. Mathematically, ideologue labor I in (18) benefits equivalently from capital K and captive participants labor C when mercenary labor M is absent. When the practical limitations are understood, this means that K and C to some extent are substitutes. For example, capital K can be converted into captive participants labor C by hiring employees. Conversion from captive participants labor C to capital K is not always practically possible, but captive participants labor C can alleviate capital needs. Hence K and C can to some extent be interpreted as complements. Eq. (18) is analytically solvable. The solution is voluminous and omitted to save space.

4.1.2. Removing ideologue labor I

Without ideologues, or if the initial conditions or parameters cause the extinction of ideologue labor I, inserting $I = 0$ into (17) causes a pure criminal organization supported by captive participants labor C, and temporarily supported by capital K if $K(0) > 0$, i.e.

$$\begin{aligned} \dot{K} &= -fM - dK,\ \dot{M} = oC + hK - mM, \\ \dot{C} &= qM - rC,\ K \geq 0,\ M \geq 0,\ C \geq 0 \end{aligned} \tag{19}$$

If $K(0) > 0$ in (19), $\lim_{t\to\infty} K = 0$ due to the two negative terms on the right hand side of $\dot{K}$. If $K(0) = 0$, $K = 0$ for all $t \geq 0$. Inserting $K = 0$ into (19) gives

$$\dot{M} = oC - mM,\ \dot{C} = qM - rC,\ M \geq 0,\ C \geq 0 \tag{20}$$

which is equivalent to (1) when I,K,a,b,c,d are replaced with M,C,o,m,q,r, respectively. Thus the analysis in Section 2 of ideologues and sponsors applies for the analysis of a pure criminal organization of criminal mercenary labor M supported by captive participants labor C. For example, if $oq > mr$, which corresponds to $ac > bd$ in Section 2, $\lim_{t\to\infty} M = \lim_{t\to\infty} C = \infty$, and $\lim_{t\to\infty} M = \lim_{t\to\infty} C = 0$ if $oq < mr$. For the special event that capital sponsors may exist willing to finance criminal mercenaries, captive participants labor C may be reinterpreted as capital K.

4.2. Simulations

This section illustrates trajectory convergence and divergence of (17) with the limits $\lim_{t\to\infty} I$, $\lim_{t\to\infty} M$, $\lim_{t\to\infty} K$, and $\lim_{t\to\infty} C$, which approach zero, a constant, or infinity, as time t approaches infinity. Fig. 3 illustrates (17) with the benchmark parameter values $a = b = c = d = e = f = g = h = m = r = 1$, $\theta = 0.3$, $\varphi = 0.6$, $n = o = p = q = 0.25$, $I(0) = 2$, $K(0) = M(0) = C(0) = 0$ as time t varies from 0 to 7, from 0 to 30 in panels h and j, and from 0 to 280 in panel i. Initial absence of captive participants labor $C(0) = 0$ causes ideologue labor I and mercenary labor M initially to evolve as in Fig. 2. Captive participants labor C increases slowly because of the low parameter values $p = q = 0.25$.

In Fig. 3 panel a with the benchmark parameter values, captive participants labor C causes more favorable evolution of ideologue labor I and mercenary labor M than in Fig. 2 panel a. In the subsequent panels, the benchmark parameter values are kept except that one or several specified parameter value(s) differ(s) from the benchmark. In panel b, $r = 1.2$ causes higher depreciation of captive participants labor C. In panel c, $n = 1$ in (17) means that the ideologues value captive participants labor C four times more highly. Thus ideologue labor I increases and becomes higher than mercenary labor M, which causes capital K to be higher. In panel d, $o = 1$ means that the mercenaries value captive participants labor C four times more highly. Thus mercenary labor M increases and becomes higher than ideologue labor I, which causes capital K to decrease to zero. Eventually, $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = \lim_{t\to\infty} C = \infty$. In panel e, $p = 1$ means that the captive participants value ideologue labor I four times more highly. Thus captive participants labor C increases more strongly. In panel f, $q = 1$ means that the captive participants value mercenary labor M four times more highly. Thus captive participants labor C increases more strongly.

In panel g, $c = 1.2$ causes capital sponsors to value ideologue labor I more strongly, and thus capital K evolves more favorably, $\lim_{t\to\infty} K = 1.40$, $\lim_{t\to\infty} I = 3.01$, $\lim_{t\to\infty} M = 2.21$, $\lim_{t\to\infty} C = 1.30$. As in Section 3, a limit exists for increasing c, since jointly high K and M in (17) is unsustainable. That limit is $c = 1.244$, shown in panel h, where $\lim_{t\to\infty} K = 1.95$, $\lim_{t\to\infty} I = 2.97$, $\lim_{t\to\infty} M = 1.74$, $\lim_{t\to\infty} C = 1.18$. Ideologue labor I is high, sustained by mercenary labor M (though less than in panel g where $c = 1.2$), sustained by captive participants labor C (which also sustains M), and sustained by capital K, where capital sponsors are at their upper tolerance threshold for the presence of mercenary labor M. Panel i assumes $c = 1.245$ showing breakdown of mercenary labor to $M = 0$ when $t > 258.8$, after which ideologue labor I, capital K, and captive participants labor C approach infinity, $\lim_{t\to\infty} I = \lim_{t\to\infty} K = \lim_{t\to\infty} C = \infty$.

In panel j, $b = g = 0.8$ causes ideologue labor I to depreciate 20% more slowly, and mercenary labor M to value ideologue labor I 20% less. Consequently, I becomes higher than in panel a, and M initially grows more slowly. This positive discrepancy between I and M encourages the capital sponsors, and K grows more strongly than in panel a. This joint growth of I and K deters mercenary labor M, regulated through the parameter φ in (17), causing M to be inverse U shaped and reach $M = 0$ when $t > 14.50$. Thereafter the organization is purely a terrorist organization, with no criminal presence. When $t > 14.50$, also captive participants labor C increases. As time t approaches infinity, $\lim_{t\to\infty} I = \lim_{t\to\infty} K = \lim_{t\to\infty} C = \infty$.

In panel k, $e = m = 0.8$ causes mercenary labor M to depreciate 20% more slowly, and ideologue labor I to value mercenary labor M 20% less. Consequently, M becomes higher than in panel a, and I eventually grows more slowly. This negative discrepancy between I and M discourages the capital sponsors, causing K to become inverse U shaped and reach zero when $t > 2.53$. Since joint growth of M and K does not occur, ideologue labor I increases, while being slightly lower than mercenary labor M, $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = \lim_{t\to\infty} C = \infty$.

Whereas $b = g = 0.8$ in panel j makes the organization a pure terrorist organization, $b = m = 0.8$ in panel k does not make the organization a pure criminal organization, due to the asymmetry between ideologues and criminal mercenaries discussed in Section 4.1. To make the organization relatively more criminal than in panel j, panel l doubles three parameter values and cuts in half three parameter values compared with the benchmark parameter values in panel a. More specifically, panel l doubles the mercenary labor M's valuation o of captive participants labor C to $o = 0.5$, doubles the captive participants labor C's valuation q of mercenary labor M to $q = 0.5$, and doubles the depreciation parameter b for ideologue labor I to $b = 2$. Furthermore, panel l cuts in half the ideologue labor I's valuation of mercenary labor M to $e = 0.5$, cuts in half the ideologue labor I's valuation of captive participants labor C to $n = 0.125$, and cuts in half the depreciation parameter m for mercenary labor M to $m = 0.5$. Capital K becomes inverse U shaped and reaches zero when $t > 1.27$. When $t > 1.27$, altering parameter values in (17) multiplied with K has no impact on the relative criminal orientation of the organization. As t approaches infinity, $\lim_{t\to\infty} K = 0$, $\lim_{t\to\infty} I = \lim_{t\to\infty} M = \lim_{t\to\infty} C = \infty$.

An organization can alternatively become criminal if it disintegrates ideologically. In panel m, $b = 3$ means that ideologue labor I depreciates three times faster than in panel a, and $e = n = 0$ means that ideologue labor I lacks support from both mercenary labor M and captive participants labor C. This may occur if the criminal mercenaries go their own way, or no longer follow directions from the ideologues, and also the captive participants fail to support the ideologues, e.g. because they realize that the ideologues are no longer in charge, or they are induced, coopted, or coerced by the mercenaries to stop supporting the ideologues. Consequently, ideologue labor I quickly decreases from $I(0) = 2$ and approaches zero asymptotically. That causes capital K to be inverse U shaped and reach zero $K = 0$ when $t > 1.25$. Mercenary labor M and captive participants labor C are also inverse U shaped, but approach zero asymptotically, i.e. $\lim_{t\to\infty} I = \lim_{t\to\infty} M = \lim_{t\to\infty} C = 0$. That M and C also approach zero follows from in (20) in Section 4.1.2 which applies when $K = 0$, and applies approximatively when I is negligible, where $oq = 0.25 \times 0.25 = 0.125 < mr = 1 \times 1 = 1$.

In panel n, $b = 3$ and $e = n = 0$ cause ideologue labor I to evolve similarly to panel m, but $o = q = 1.2$ implies $oq = 1.2 \times 1.2 = 1.44 > mr = 1 \times 1 = 1$. The large $o = q = 1.2$ means that mercenary labor M and captive participants labor C value each other more. Hence mercenary labor M and captive participants labor C approach infinity as t approaches infinity, $\lim_{t\to\infty} M = \lim_{t\to\infty} C = \infty$. The large initial increase of M follows from the large initial I and positive K. Capital K is inverse U shaped and reaches zero $K = 0$ when $t > 1.07$, and I approaches zero asymptotically, $\lim_{t\to\infty} I = 0$, while throughout being lower than in panel m.

5. Conclusion

The article models the internal dynamics and composition of terrorist organizations through time with four differential equations. Terrorist organizations are comprised of ideologues providing meaning

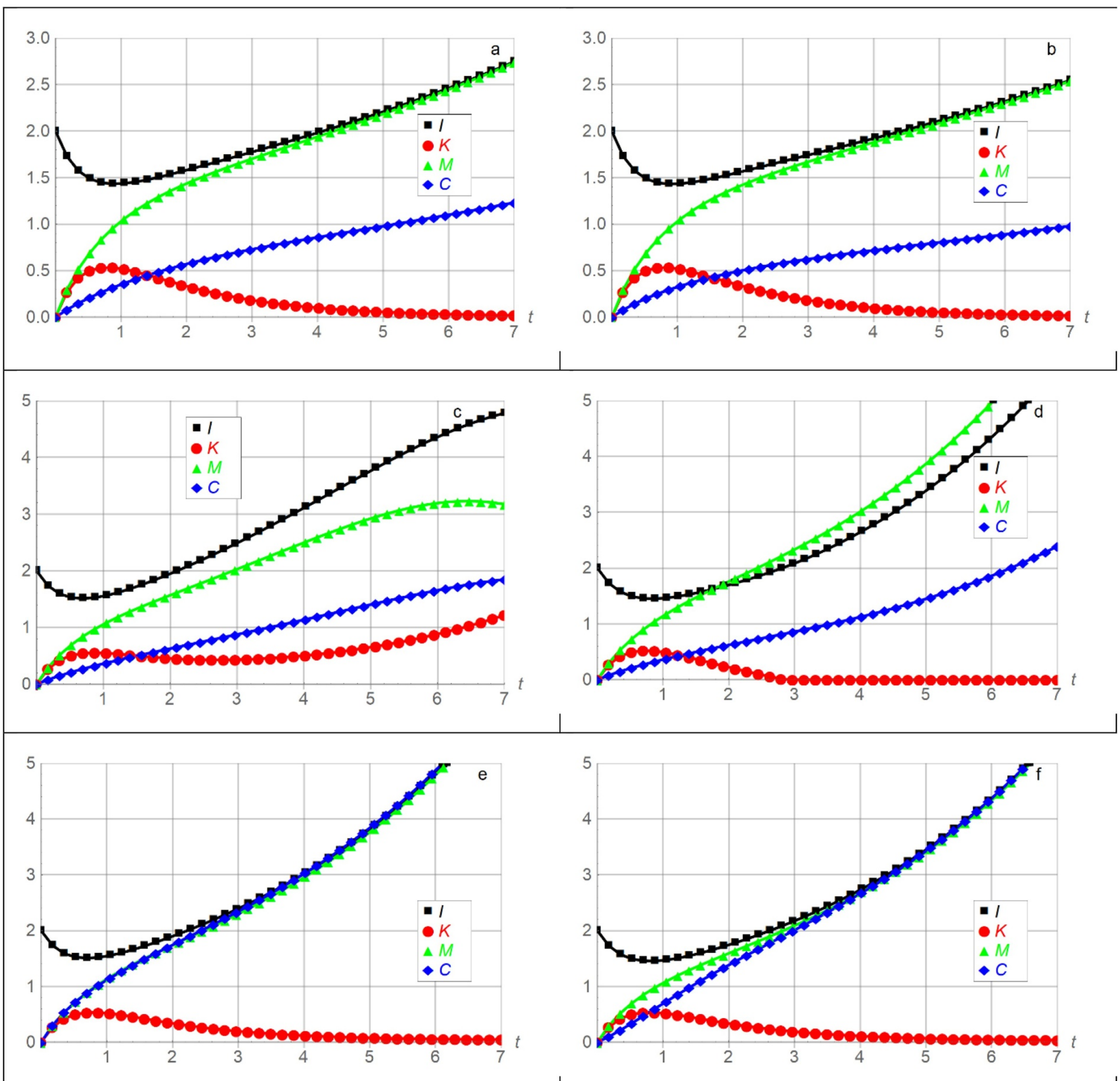

Fig. 3. Ideologue labor I, capital K, mercenary labor M, and captive participants labor C as functions of time t with the benchmark parameter values $a = b = c = d = e = f = g = h = m = r = 1$, $\theta = 0.3$, $\varphi = 0.6$, $n = o = p = q = 0.25$, $I(0) = 2$, $K(0) = M(0) = C(0) = 0$ as time t varies from 0 to 7, from 0 to 30 in panels h and j, and from 0 to 280 in panel i. Panel a: Benchmark. Panel b: $r = 1.2$. Panel c: $n = 1$. Panel d: $o = 1$. Panel e: $p = 1$. Panel f: $q = 1$. Panel g: $c = 1.2$. Panel h: $c = 1.244$. Panel i: $c = 1.245$. Panel j: $b = g = 0.8$. Panel k: $e = m = 0.8$. Panel l: $o = q = e = m = 0.5$, $b = 2$, $n = 0.125$. Panel m: $b = 3$, $e = n = 0$. Panel n: $b = 3$, $e = n = 0$, $o = q = 1.2$.

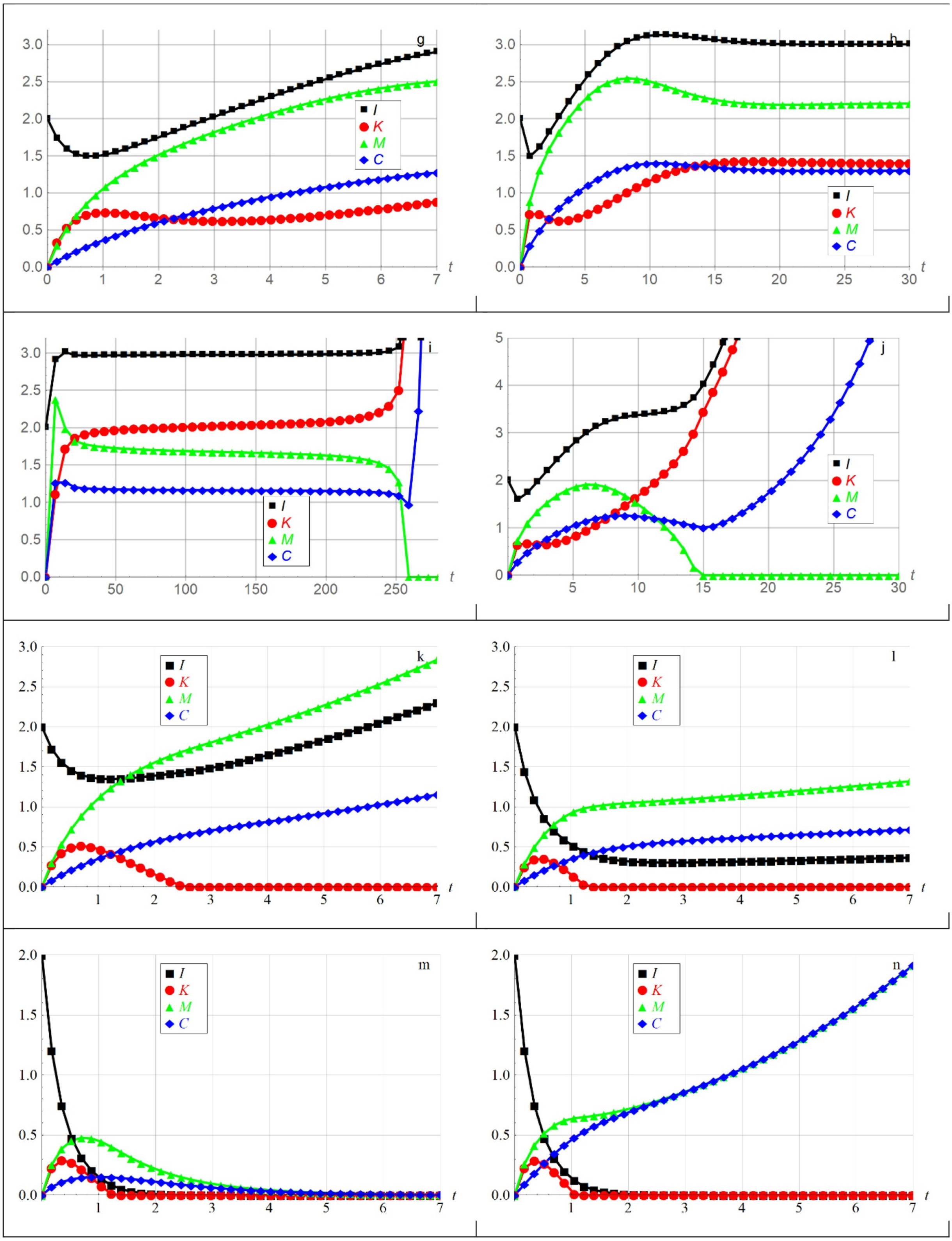

Fig. 3. (*continued*)

and direction. To ensure funding they recruit sponsors. If sponsor recruitment is challenging or impossible, criminal mercenaries are recruited, which may compromise the ideologues' direction and purpose. Captive participants, which cannot defect without repercussions, are recruited to provide various kinds of support.

Labor exerted by ideologues is impacted by various factors. Three positive factors are capital input from sponsors, mercenary labor, and captive participants' labor. One negative factor is the depreciation rate due to the presence of ideologues. Capital provision is positively impacted by ideologue labor, is negatively impacted by competing mercenary labor, and is subject to depreciation by itself. Mercenary labor is positively impacted by ideologue labor and captive participants' labor, and is subject to depreciation by itself. Captive participants' labor is positively by ideologue labor and mercenary labor, and is subject to depreciation by itself.

Factors are identified impacting how the composition of an organization evolves through time, i.e. how the relative strengths of ideologue labor, mercenary labor, and captive participants' labor fluctuate, impacted by capital sponsoring. The three labor groups value each other differently which impacts their relative strengths. Capital sponsors may view criminal mercenaries as ideologically detrimental to the terrorist organization. Organizations with strong capital sponsoring may evolve strong ideologue labor with limited or no mercenary labor, while organizations without capital sponsoring may go extinct or rely on mercenary labor causing a criminal organization.

The internal dynamics are illustrated by theoretical analysis for the pure model of ideologues and capital sponsors in Section 2 which is a linear time-invariant system. If the product of the growth rates of ideologue labor and capital sponsoring exceeds the product of the depreciation rates, expressed with one positive eigenvalue, ideologue labor and capital sponsoring grow, and otherwise decline.

Sections 3 and 4 introduce criminal mercenaries and captive participants. Dynamics and trajectory convergence and divergence of the variables are illustrated with simulations. Capital sponsors valuing ideologues sufficiently highly may drive out criminal mercenaries. Furthermore, capital sponsors who sponsor despite some presence of criminal mercenaries may indirectly drive out criminal mercenaries whose contribution becomes less needed by the organization. If the mercenaries don't value the ideologues sufficiently highly, and the ideologue labor depreciates more slowly, capital sponsors may be recruited that cause the extinction of the mercenaries.

Conversely, if the ideologues don't value the mercenaries sufficiently highly, and the mercenary labor depreciates more slowly, capital sponsoring may cease, while the ideologues and mercenaries continue to coexist, supported by the captive participants. The relative strength of mercenary labor increases if the mercenaries and captive participants jointly value each other more, and these two labor groups are less valued by the ideologues. If the ideologues depreciate more quickly, and fail to get support from the mercenaries and captive participants, the ideologues may go extinct. For the organization to survive as a criminal organization, the product of the growth rates of mercenary labor and captive participants labor must exceed the product of these two labor groups' depreciation rates. Otherwise the organization goes extinct.

The article provides tools for understanding the evolution of the composition and sponsoring of terrorist organizations. This is of interest to researchers and policy makers who can test various hypotheses and policy mechanisms with various assumptions in various societies and environments to decrease the presence and sustainability of terrorist organizations, and alter their composition and sponsor mechanisms. To further support such tools, future research should relate the findings in this article to empirics observed in real terrorist organizations. Empirics should be compiled for how ideologue labor, mercenary labor and captive participants labor evolve over time within terrorist organizations, how capital sponsoring play a role, and how terrorist organizations are born, evolve within the same and different niches, and die. Thereafter the parameters should be estimated, e.g. as illustrated by Hausken [13] compiling empirics from the global terrorism database and the fragile states index.[1] In future research we will analyze how governments intervene to regulate the dynamics of terrorist organizations.

[1] http://www.start.umd.edu/gtd, https://fragilestatesindex.org/, retrieved July 11, 2019.

Conflict of interest

The authors have no conflict of interest.

Appendices

Appendix A. Nomenclature

I Amount or stock of labor exerted by ideologues
K Amount of capital provided by sponsors
M Amount or stock of labor exerted by mercenaries
C Amount or stock of labor exerted by captive participants
t Time
a Growth rate for capital K impacting ideologue labor I
c Growth rate for ideologue labor I impacting capital sponsoring K
b Depreciation rate of ideologue labor I
d Depreciation rate of capital K
e Growth rate for mercenary labor M impacting ideologue labor I
f Depreciation rate of mercenary labor impacting M capital sponsoring K
g Growth rate for ideologue labor I impacting mercenary labor M
h Growth rate for capital K impacting mercenary labor M
m Depreciation rate of mercenary labor M
θ Depreciation rate of the product KM of capital K and mercenary labor M
φ Depreciation rate of the product IK of ideologue labor I and capital K
n Growth rate for captive participants C impacting ideologue labor I
o Growth rate for captive participants C impacting mercenary labor M
p Growth rate for ideologue labor I impacting captive participants C
q Growth rate for mercenary labor M impacting captive participants C
r Depreciation rate of captive participants C

Appendix B. Proof of Properties 3 and 4

Since $\lim_{t\to\infty} e^{-\frac{1}{2}(\Omega+b+d)t} = 0$, the second term in the expressions for I and K in (11) approaches zero as t approaches infinity. Assuming $\Omega - b - d < 0$ implies $ac < bd$ and $\lim_{t\to\infty} e^{\frac{1}{2}(\Omega-b-d)t} = 0$, and then the first term in the expressions for I and K in (11) also approaches zero as t approaches infinity. This proves $\lim_{t\to\infty} I = \lim_{t\to\infty} K = 0$ if $ac < bd$ as a stable solution as stated in (12). This solution also holds when $ac > bd$ and $max(I(0), K(0)) = 0$, since the first term in the expressions for I and K in (11) contains $I(0)$ and $K(0)$ additively, and then I and K never leaves the origin $I = K = 0$. Assuming $ac = bd$ implies $\Omega = b + d$ and $\lim_{t\to\infty} e^{\frac{1}{2}(\Omega-b-d)t} = 1$, which gives the constant finite expressions in (13). Assuming $\Omega - b - d > 0$ implies $ac > bd$ and $\lim_{t\to\infty} e^{\frac{1}{2}(\Omega-b-d)t} = \infty$. To prove $\lim_{t\to\infty} I = \lim_{t\to\infty} K = \infty$ if $ac > bd$ and $max(I(0), K(0)) > 0$, it is sufficient to prove $(\Omega - b + d)I(0) + 2aK(0) > 0$ and $(\Omega + b - d)K(0) + 2cI(0) > 0$. Since $ac > bd$, we can replace ac with bd which causes these two inequalities to simplify to $2dI(0) + 2aK(0) > 0$ and $2bK(0) + 2cI(0) > 0$, which are always satisfied. □

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