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Characterising the robustness of coupled power-law networks

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ABSTRACT

Many networks exhibit a power-law configuration, where the number of connections each node has follows a power-law distribution, including the Internet, terrorist cells, species relationships and infrastructure. Given the prevalence of power-law networks, studying the effects of disruptions on their performance is of interest. Previous work has investigated the influence of network topology on the effects of random node failures for independent networks. Many networks depend on others to function and thus, exploring the influence of network topology on the effects of failures in interdependent networks is of interest. The present paper extends the previous work to coupled power-law network systems. For a set of randomly generated coupled systems, each containing two networks, we investigate the significant topological factors for different dependency types. Failures in the coupled networks are simulated and the effects on the system performance are analysed by performing a beta regression. The results are consistent across the dependency types, with the most influential topological factors being mean nodal degree and factors relating to the dependency type. The results are also compared with those of the independent networks and their potential relevance to the design of interdependent networks is indicated, for example, their use within an infrastructure setting.

1. Introduction

It is well established that to model and evaluate the robustness (or vulnerability) of critical infrastructure, the dependencies that exist between infrastructure systems need to be accounted for [7,32,34]. Over the years, there have been many methods suggested for how to model dependencies between infrastructures, including the use of agent based and network based approaches [29]. Network models are based on a network representation of the important components of each infrastructure, represented as nodes, and the connections between the components within the same network, as well as between the different networks, represented as edges. The edges between nodes of different networks represent the dependencies between the different infrastructures.

Infrastructure networks are a special case of the broader class of interdependent networks. For example, the metabolic pathways of different species in an ecosystem can be interdependent (e.g., one species depends on an output from another species as an input). Similarly, economies, when represented as networks of consumers and producers, are strongly interdependent across regions within a country and across different countries.

There have been many differing methods suggested for modelling the dependencies between infrastructures using network models. Some examples of the different methods are given by Parshani et al. [31], Gaogao et al. [19] Jiang et al. [20], and Cheng and Cao [10]. The main structural differences between the models can be characterised by whether the infrastructures are fully or partially dependent (i.e., if each node has at least one dependency to a node in the other network or only a fraction of the nodes do) and if components with dependencies have single or multiple dependencies (each dependent node has one or more than one dependency) [17].

For both independent and interdependent networks, percolation theory has been used to find analytical solutions to disruptions across an array of different network types and dependency methods [7,10,18]. Such papers show the number or fraction of nodes removed in the initial disruption that lead to complete collapse of the investigated system. This can be used as a measure of the system's robustness and to compare the robustness of different system models [18,20]. However, this measure does not convey information about what happens to network performance at lower levels of node removals and does not directly provide information about the relative importance of different topological properties of the network in terms of their influence on network robustness.

Network flow models are an extension of the network models that

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include the addition of load to the nodes and/or edges of the network. The load represents the amount of commodity present at each node and or/edge. Each node and/or edge is also assigned a maximum capacity. When a disruption occurs the load of any failed nodes and edges is redistributed throughout the remaining functional network components. The reassignment of the load can lead to additional failures if the load of nodes or edges exceeds their maximum capacity [14,38].

Scala et al. [35] investigated the inclusion of physical flow to both independent and interdependent networks, with a focus on how edge overload affected the robustness of the networks. They used a mean field model to redistribute the load of failed edges throughout the system, that is, they assumed when an edge failed its load was redistributed evenly throughout the existing edges within the network.

The addition of commodity flow within networks is useful when looking into the cascading mechanisms between specific infrastructure network types, such as electric power and telecommunication. The interaction between the types of infrastructure can be explored to see how the redistribution of commodity flow can influence the cascading effects of disruptions [22,38]. One conclusion from the literature is that the inclusion of network flow shows an increased level of cascading effects [42], while others argue that including "smart" interactions (which occur due to buffers within real dependent infrastructure systems) between the two networks decreases the cascading effects within the interdependent power-communication system [22].

The use of network flow models is effective when studying a specific system, such as one including an electric power system. However, when investigating the effects of dependencies between general infrastructure networks, the type of infrastructure is not specified, and thus the flow of the commodity cannot be included. Instead the structure of the networks can be explored. The effects of network structure, or topology, on the robustness of independent networks have previously been investigated [3,23]. Different topological factors can be calculated, which capture particular structural features of a network.

Four of these topological factors are nodal degree, path length, betweenness centrality and clustering coefficient. Nodal degree specifies the number of edges connected to each node. Path length provides the shortest path between each nodal pair within a network. Here the shortest path is considered as the path that traverses the least number of edges. Betweenness centrality indicates the extent to which a node lies on the shortest path between two other nodes within the network [27]. Clustering coefficient (also referred to as transitivity) indicates the how likely it is for the neighbours of a node to also be neighbours, where if an edge exists between two nodes, then they are neighbours. Clustering coefficient gives an indication of local redundancy within a network.

Alipour et al. [3] used topological based and reliability based measures to identify weak nodes within power transmission networks. The topology based measures included factors such as nodal degree and betweenness centrality. The reliability based measures incorporates what the author refers to as the reliability of the edges within the system. To do this, a weight is assigned to each edge that represents the probability that the edge is functional. The topological factors are then calculated including the weights of the edges. They also compared the robustness of the independent power transmission networks to random and targeted attacks, using efficiency as a measure of robustness. Efficiency is defined as the inverse of the average of the shortest paths between each nodal pair within the network. The targeted attacks were simulated by removing the most central nodes of the network. The most central nodes are defined as those who had the highest cumulative rank score in relation to the reliability based measure, i.e., the greater the value of each reliability based measure a node has the lower it is ranked.

LaRocca and Guikema [23] provide a general overview of the topological factors that have a significant influence on the robustness of independent networks when random failures occur. The focus of the paper was the robustness of networks, of which the nodal degree followed a power-law distribution with exponential cut-off. Here, robustness was defined as the percentage of functional nodes after disruptions. Their findings show that the following topological factors are significant when characterising the robustness of independent networks: mean nodal degree, mean betweenness centrality, mean clustering coefficient, standard deviation of clustering coefficient and standard deviation of path length. However, the influence of the topology on the robustness of interdependent networks has not been explored. In this paper, characterising the robustness of networks with topological factors is extended to the case of coupled network systems.

LaRocca et al. [24] compared the use of network topology and network flow models to simulate electric power networks. They concluded that using only network topology as performance measures for particular power networks under specific disruption scenarios provides poor estimates of system performance, relative to when commodity flow is taken into account. However, they also find that an average of some performance measures, such as largest connected subgraph, may capture the average behaviour of the system when random failures occur. If investigating the effects of disruption to a specific system that includes at least one infrastructure for which the flow of the commodity can be modelled, then the use of a physical flow model is more appropriate than a network theoretic model. However, this paper aims to give an overview for any type of networks within a coupled system and thus does not include physical flow. The inclusion of flow limits the connections within an individual network to all be of the same type of connection, e.g. physical if the flow of a commodity (e.g. power of water) or of information. By not including physical flow, the connections within the model can represent different types of connections, rather than just one.

To extend the work of LaRocca and Guikema [23] the present paper aims to provide a general overview of which topological factors are important when random disruptions occur in coupled network systems for a variety of different dependency types. The various dependency types allow for the investigation of both dependent and interdependent coupled systems. The 2000 coupled network systems are generated such that each system consists of two networks, both of which are scale-free networks that follow a power-law distribution with exponential cut-off. The two networks present in each coupled system are referred to as Network A and Network B. The dependencies between the two networks are directional (or unidirectional), i.e., if node i in Network A depends on node j in Network B, node j does not necessarily depend on node i in Network A.

In our analysis, the robustness of Network A is explored when random disruptions occur within the coupled system. Robustness here is considered as the percentage of functional nodes after a disruption occurs. The analysis aims to advance the understanding of how the robustness is affected within a short time frame after the initial disruption. All initial failures occur within Network B, thus investigating the first order effects of a disruption on Network A. A first order effect refers to the effect of a disruption that initiates in Network B and affects Network A through the dependencies Network A has on Network B [34]. After the disruptions are simulated within the coupled system, a beta regression is performed to provide an overview of which topological factors are significant in characterising the robustness of Network A. A comparison of the significant topological factors across the different types of dependencies modelled is made, as well as a comparison to the significant factors reported for independent networks in LaRocca and Guikema [23].

The remainder of this paper is structured as follows: Section 2 provides an overview of the different network related terminology used throughout the paper. The methods used to generate and analyse the coupled network systems are outlined in Section 3, with the results of the regression analysis are presented in Section 4. A discussion of the findings is given in Section 5, followed by the conclusion in Section 6.

2. Network terminology

Networks consisting of nodes and edges can be used to construct a simplified representation of an infrastructure system. The nodes represent important components of the system and the edges represent the connections between such components. The network or graph can be denoted as $G = \{V, E\}$, where V is the set of vertices or nodes in the network and E is the set of edges, which form connections between the nodes. The size of the network, N, is equal to the number of vertices [5]. The edges in a network can either be directed or undirected. When the edges are directed, the direction of each edge is specified and can only be traversed in the specified direction. When the edges are undirected, the edges can be traversed in either direction. For simplicity, the networks generated to be included the coupled systems in this paper are undirected.

Barabási and Albert [4] first observed that the nodal degree of some networks can be described as following a power-law distribution, given by:

$$P(k) \sim k^{-\gamma}$$

such that $P(k)$ is the probability that a node is connected to k neighbours and γ is some constant. It has since been suggested that the power-law distribution with exponential cut-off is more accurate as it takes into account the physical cost of adding additional edges to a node, providing an upper limit to the number of edges a node can have. The power-law distribution with exponential cut-off is given as:

$$P(k) \sim k^{-\gamma} e^{-(k/K)}$$

where K is the cut-off at which it becomes too costly to add additional edges to a node [2, 26].

It has recently been questioned if power-law networks are as prevalent in the real world as the mountain of literature stating this would have us believe. Broido and Clauset [6] investigated if the best fitting power-law distribution for the nodal degree of 3662 simple graphs (constructed from 928 real-world networks) was better than alternative (non-scale-free) distributions. They use the term scale-free networks to refer to networks which nodal degree follows a power-law distribution. Likelihood ratio tests were compared for the best fitting model from four alternative degree distributions. One such distribution they compared was the power-law with exponential cut-off, where 56% of the results favoured the power-law distribution with exponential cut-off. This result led Broido and Clauset [6, p. 5] to state “a majority of networks favor the power law with cutoff model, indicating that finite-sized effects may be common”. This topic of discussion will likely gain much attention in the near future, and may lead to a different underlying degree distribution to be proposed. However, for the time being, the power-law distribution with exponential cut-off is one of the better methods to use when constructing simulated networks.

2.1. Network topology

The structure or topology of a network can be described using different network parameters. Four such parameters that are particularly useful for characterising the network structure are: nodal degree, betweenness centrality, clustering coefficient and path length. Each of these four topological parameters can be calculated for any network [23].

2.1.1. Nodal degree

The degree, k, of any node in an undirected network is the number of edges connected to the node. The mean nodal degree of the network is expressed as

$$k = \frac{1}{N} \sum_{i \in V} k_i$$

where V is the set of nodes in the network, and k_i is the degree of node i.

2.1.2. Path length

The length of the shortest path for each pair of nodes within a network is calculated as the least number of edges traversed to get from one node in the pair to the other. The shortest path from node i to node j in a network is denoted as p_{ij}. For undirected graphs $p_{ij} = p_{ji}$. For the remainder of the paper, the set of shortest paths between each nodal pair in a network is denoted as L.

2.1.3. Betweenness centrality

For each node i in the network, the betweenness centrality is defined as:

$$Bc_i = \sum_a \sum_b \frac{p_{aib}}{p_{ab}}, \quad a \neq b \neq i,$$

where p_{aib} is the number of shortest paths from node a to node b that pass through node i, and p_{ab} is the total number of shortest paths from node a to node b.

2.1.4. Clustering coefficient

The clustering coefficient of a node specifies how connected its neighbours are to each other and is an indication of local redundancy in the network. The neighbours of a node is the set of nodes to which it is connected to. For node i, which has k_i neighbours, the clustering coefficient is defined as:

$$Cc_i = \frac{2E_i}{k_i(k_i - 1)}$$

where E_i is the number of edges between the neighbours of node i.

2.2. Giant connected component and source node clusters

When disruptions occur to a network, the network can fragment into several clusters. The largest connected cluster present after the network fragments is referred to as the Giant Connected Component (GCC). The relative size of the GCC is the percentage of nodes within the GCC [9,10,36]. The relative size of the GCC can be used as a measure of network performance after disruption has occurred [17,22]. We acknowledge that this is an imperfect measure of network robustness, especially given that it does not account for source and sink nodes or for the physics of network flows. However, this simple, widely-used measure, provides an initial view of the influence of topological factors on the topological robustness of a network.

Source nodes can also be included into a network. Source nodes represent components of the network that must be functioning in order for the network to be functional. When a disruption occurs within a network containing source nodes, only the clusters that contain source nodes are considered functional.

2.3. Network dependencies

Connections between different networks can also be formed to generate a system of dependent networks. These connections represent the dependencies that exist between different infrastructure networks, for example the dependency a water network has on an electricity network to power electric pumps [13]. To distinguish between the edges within each network and between the networks the terms intra-connections and inter-connections are used. Intra-connections refer to the connections or edges between two nodes within the same network. Inter-connections refer to the connections or edges between two different networks, i.e. the dependencies between the networks.

For the remainder of the article, all intra-connections are assumed to be undirected and all inter-connections are assumed to be directed. This is representative of situations such as a drinking water network and its dependency on a power network. The water within the network can flow in both directions, such that the intra-connections are undirected. However, some components of the water network, such as the pumps, rely on electricity to function and thus the dependency is directional from the power network to the water network. Another example is a transportation network and its dependency on a power network. Within the transportation network traffic flows in both directions, whereas the dependency is directed from the power network to the transportation network, for example, to signals within the transportation network that requires electricity. The power network can also be dependent on the transportation network, for example, the transportation of fuel (e.g., coal) or spare parts, but not necessarily on the component that depends on the power network, such as the signals.

For coupled system where the networks have partial dependency (i.e., only a percentage of the nodes in the network depend on another), the influence of additional variables on the robustness of the system are considered. These variables are the percentage of nodes in the network which are dependent on another network, denoted *Dp*, and the intra-nodal degree (number of intra-connections a node has) of these dependent nodes, which is denoted as *Dk*. When source nodes are included in the coupled systems, the influence of the additional variable of the source nodes' intra-nodal degree is also considered, and denoted as *Sk*.

3. Methods

A total of 4000 networks were generated following the process outlined in Section 3.1 before being sorted into pairs to give 2000 coupled network systems. The two networks within each system are referred to as Network A and Network B. Different types of dependencies between the two networks were explored and are described in Section 3.2.1. For each dependency type, failure scenarios were simulated within the 2000 coupled systems and the robustness of Network A was recorded. More information on simulating the failure scenarios is given in Section 3.3. To characterise the robustness of Network A from the topological factors of the coupled network system a beta regression analysis was performed as described in Section 3.4.

3.1. Generating networks

The 4000 networks were generated using the preferential attachment variation algorithm presented by LaRocca and Guikema [23]. This algorithm assigns the degree of each node from the power-law distribution with exponential cut-off before assigning intra-connections preferentially, based on nodal degree. All intra-connections are assumed to be undirected.

An assortment of simulated networks was produced using combinations of different network sizes and parameter groups for the nodal degree distribution. Five different power-law distributions with exponential cut-off were used to assign the nodal degree of the networks. The parameter groups of the five power-law distributions used are shown in Table 1. These distributions are the same as those used previously by LaRocca and Guikema [23] and were chosen as they represented nodal degree distributions exhibited by real-world networks studied in Albert and Barabási [2]. Twenty different network sizes ranging from 100 to 1000 nodes were chosen from a uniform distribution and can be seen in Table 2. Therefore, for each combination of network size and nodal degree distribution 40 networks were generated.

Table 1
Power-law parameters used for generating networks.

Power-law distribution parameters γ	*K*
1.1	40
1.7	200
2.0	900
2.1	400
2.4	2000

Table 2
Summary of generated networks.

Number of nodes	Number of degree distributions	Number of networks	Number of nodes	Number of degree distributions	Number of networks
100	5	40	485	5	40
133	5	40	509	5	40
142	5	40	536	5	40
232	5	40	547	5	40
249	5	40	690	5	40
350	5	40	697	5	40
361	5	40	752	5	40
448	5	40	862	5	40
464	5	40	896	5	40
467	5	40	1000	5	40

After generating the networks, the mean, minimum, maximum and standard deviation of the four topological factors of each network was calculated. A summary can be seen in Table 3.

3.2. Generating coupled network systems

The 4000 networks generated were then paired such that each pair of networks, referred to as Network A and Network B, were the same size and of the same parameter group for the nodal degree distribution. Each pair was used to form a coupled network system, resulting in 2000 systems. The two networks within each coupled system were assumed to occupy the same spatial area. The layout of each network was decided using the layout.graphopt function in the igraph R package [12]. This assigned each node a Cartesian (x, y) coordinate.

The inclusion of source nodes within the coupled network systems was also explored to see if their presence caused a change in which topological factors were significant to network robustness. When source nodes were present in the coupled system, a random subset of nodes in Network B were chosen to represent these source nodes. The size of the subset was varied at 2%, 5% and 10% of the network's size. These relatively low percentages of source nodes are representative of systems such as infrastructure where the large majority of nodes are demand points and demand is met by a relatively small number of major source nodes; for example, natural gas networks [15, 33, 37], electric power systems [1, 39–41] and water distribution systems [21, 25, 28]. The analysis could be extended to networks with much higher percentages of source nodes, but this is not explored in this paper.

3.2.1. Forming dependencies

For each type of dependency, Network A is always dependent on Network B, however Network B was either independent (did not depend on Network A) or was dependent on Network A. For each dependency type, a subset of nodes in Network A is randomly chosen to depend on Network B. This subset is denoted as A_D. Each node in A_D depends on the closest node in Network B (based on Euclidean distance). This allows multiple nodes in A_D to be dependent on the same node in Network B. The method of forming dependencies based on geographic proximity is used by Dueñas-Osorio et al. [13] when modelling the interdependent power and water system of Shelby County, Tennessee and Ouyang et al. [30] to simulate coupled power and water systems with features similar to those of real infrastructure.

For dependency types that include Network B depending on Network A, the dependencies Network A has on Network B are first formed, using the method described in the previous paragraph. Next a random subset of nodes in Network B is chosen to depend on Network A. This subset is denoted as B_D. Each node in B_D is dependent on the closest node in Network A (based on Euclidean distance) that is not present in the subset A_D. This allows for multiple nodes in B_D to depend on the same node in Network A.

Table 3
Summary of the topological characteristics of the generated networks, separated into Network A and Network B.

Parameter	Within-network measure	Network A Mean	Min	Max	Std dev	Network B Mean	Min	Max	Std dev
Network size (N)		496	100	1000	253.5	496	100	1000	253.5
Degree (k)	Mean	5.35	2.34	12.94	2.35	5.37	2.49	12.44	2.37
	Minimum	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.00
	Maximum	372	39	999	241	374	42	998	241
	Std dev	20.50	6.77	37.18	6.24	20.62	6.83	36.94	6.23
Betweenness centrality (Bc)	Mean	706	95	2948	535	700	95	2953	528
	Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Maximum	214,499	1766	995,119	236,109	216,959	1752	993,561	236,791
	Std dev	9049	368	31,468	7608	9131	344	31,419	7633
Clustering coefficient (Cc)	Mean	0.31	0.04	0.66	0.11	0.31	0.03	0.69	0.12
	Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Maximum	1.00	0.90	1.00	0.00	1.00	1.00	1.00	0.00
	Std dev	0.40	0.11	0.49	0.08	0.40	0.10	0.49	0.08
Path length (L)	Mean	2.36	1.96	4.00	0.53	2.35	1.95	3.98	0.52
	Minimum	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.00
	Maximum	4.75	2.00	14.00	2.20	4.74	2.00	17.00	2.32
	Std dev	0.46	0.06	1.38	0.33	0.46	0.06	1.90	0.33

Table 4
Summary of dependency types considered.

Type of dependency Network A has on Network B	Type of dependency Network B has on Network A	Percentage of source nodes in Network B
Fixed, 10%	–	–
Fixed, 30%	–	–
Fixed, 50%	–	–
Fixed, 100%	–	–
Fixed, 50%	Fixed, 50%	–
Random	–	–
Random	–	2
Random	–	5
Random	–	10
Random	Random	–
Random	Random	2
Random	Random	5
Random	Random	10

The size of the dependent subsets A_D and B_D vary for each dependency type. An overview of the size of the dependent subsets is given in Table 4. When the percentage of dependency is referred to as fixed, this means that each of the 2000 coupled systems have the same fixed percentage of dependent nodes. When Network B was independent, 10%, 30%, 50% and 100% of dependency levels (of Network A on Network B) were considered. These levels were picked such that a range of levels that could be observed by infrastructure systems were covered. When both Network A and B were dependent on each other, a fixed percentage of 50% was considered, though this could be extended in future work. When the percentage is referred to as random, the percentage of nodes to be chosen for the subset(s) A_D (and B_D) is randomly assigned to Network A (and Network B) in each of the 2000 coupled network systems. The percentage of dependent nodes is assigned using a uniform distribution with a range from 1/N% to 100%, where N is the size of the network, for each dependent network. This provides a range of dependency from only one node being dependent in a network to the network being fully dependent.

3.3. Simulating failures

Each failure scenario was simulated by randomly choosing a subset of nodes in Network B to fail. The percentage of nodes randomly chosen to initially fail in Network B was investigated at the 10%, 25% and 50% level. These failed nodes were then removed from the network and the cascading effect throughout the coupled system was observed. For each dependency type, 100 failure scenarios were run for each of the 2000 coupled network systems. The percentage of nodes functional in Network A was averaged over the 100 failure scenarios run on each coupled network system and recorded. Two different methods were used to simulate the cascading effects of the initial disruption. When the coupled network systems did not contain source nodes, only nodes in the GCC of Network A were considered as functional. When source nodes were present in the coupled network system, only nodes that could be reached from source nodes after disruption were considered as functional. A more in-depth explanation to the two methods used to simulate the cascade effects are given in Sections 3.3.1 and 3.3.2.

3.3.1. Giant connected component (source nodes not present)

When source nodes were not present in the coupled system, only nodes present in the GCC were considered as functional. The initial disruption removed a percentage of nodes in Network B, causing the network to fracture into clusters. Of these clusters, only the largest, the GCC, is considered as functional and thus all nodes outside the GCC are also considered as failed. Any nodes in A_D that depend on failed nodes in Network B also fail and are removed from Network A. This causes Network A to fragment into clusters. As with Network B, only the largest cluster, the GCC, of Network A is considered functional and all nodes outside of the GCC are also considered as failed. Any nodes in B_D that depend on nodes in Network A which have failed are also considered failed. This process iterates until an equilibrium is reached (no additional node failures occur). In the dependency types where Network B is independent, B_D will be an empty set and thus the failures of Network A will not affect Network B and the system will reach equilibrium after any nodes outside of the GCC of Network A are considered as failed.

3.3.2. Source node clusters (source nodes present)

When source nodes are present in the coupled system, the initial failures occur within Network B, the failed nodes are removed and the network fragments as with the method described in Section 3.3.1. However, with the inclusion of source nodes, only the clusters which contain source nodes will be considered as functional and all nodes outside of these clusters are also considered as failed. As before, any nodes in $A_D$which depend on failed nodes in Network B fail and Network A fragments into clusters. The set of functioning dependent nodes in Network A is denoted as A_{Df}. Now only clusters that have input from Network B are functional. This means that only clusters containing the nodes in A_{Df} are functional. Nodes outside of these functional clusters are also considered as failed. Any nodes in B_D that depend on failed nodes in Network A are now considered as failed, causing further fragmentation to Network B. As before, this process iterates until the

Table 5
Significant covariates when Network A is dependent on Network B and Network B is independent and the effect of change of these covariates. The sign indicates if the covariate has a positive or negative influence on the percentage of nodes considered functional in Network A after random failures in Network B. The colour indicates the covariate coefficient value with the darker the colour indicating the further the value is from 0.

Type of dependency Network A has on Network B		Fixed, 10%			Fixed, 30%			Fixed, 50%			Fixed, 100%			Random			Random			Random			Random		
Percentage of source nodes in Network B		0			0			0			0			0			2			5			10		
Percentage of initial failures in Network B		10	25	50	10	25	50	10	25	50	10	25	50	10	25	50	10	25	50	10	25	50	10	25	50
Topology of Network A only in regression model	*k*, mean A	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	k, std dev A		-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+		+	+	-		
	Bc, mean A	-											+				+	+	+						
	Bc, max A										+	+					-	-	-			-			
	Cc, mean A		+	+	+	+	+	+	+	+		+	+					-	-			-			
	L, std dev A	-	-	-	-	-	-	-	-	-	-	-	-	-	-				+						
	Dp, A													-	-	-	-	-	-	-	-	-	-	-	-
	Dk, mean A	-	-	-	-	-	-	-	-	-				-	-	-	-	-	-	-	-	-	-	-	-
Topology of Networks A and B in regression model	*k*, mean A	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	Bc, max A																-	-	-			-			
	Cc, mean A		+	+																					
	L, std dev A	-	-	-	-	-		-	-		-	-		-	-		-	-	-			-			
	Dp, A													-	-	-	-	-	-	-	-	-	-	-	-
	Dk, mean A	-	-	-	-	-	-	-	-	-				-	-	-	-	-	-	-	-	-	-	-	-
	k, std dev B	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+		+	+	-		
	Bc, mean B	-																+	+						+
	Cc, mean B				+	+	+	+	+	+		+	+						-						+
	L, std dev B										-	-					+	+	+			+			
	Sk, mean B																	+	+						

system reaches an equilibrium. Again, in the dependency types where Network B is independent, the set B_D will be empty and the failures will not cascade back into Network B.

3.4. Regression model

After simulating the various failure scenarios, regression analyses were performed on the recorded outcomes. The analyses present the significant topological measures of the coupled network system that affect the robustness of Network A. For each method of forming dependencies, and each percentage of initial node failures in Network B, two regression analyses were completed, one that included the topological factors of Network A only and one including the topological factors of both Networks A and B. In real-world situations the two different infrastructures are commonly owned by different private companies that do not share infrastructure data for safety and security reasons. Therefore, if the owner or management of Network A wanted a general overview of the most important topological factors to consider in relation to robustness of random failure events they would be able to have an good overview of their own structure but would likely have little or no information regarding the topological structure of the network they are dependent on.

The dependent variable for the regression was the average percentage of nodes in Network A considered functional after a random disruption occurs in Network B over the 100 failure scenarios. The beta regression model was chosen as the dependent variable was in the range (0, 1). The beta regression model was proposed by Ferrari and Cribari-Neto [16] for instances when the dependent variable follows a beta distribution. The beta density they suggest for the regression model is a parameterisation of the beta density to account for a regression structure where the dependent variable is an average of the response and is given as

$$f(y;\mu;\phi) = \frac{\Gamma(\phi)}{\Gamma(\mu\phi)\Gamma((1-\phi)\mu)} y^{\mu\phi-1}(1-y)^{(1-\mu)\phi-1},\ 0 < y\langle 1,\ \phi\rangle 0$$

and the mean and variance of y are

$$E(y) = \mu$$

and

$$Var(y) = \frac{V(\mu)}{1+\ \phi}.$$

The parameter estimation is performed using the maximum likelihood method. For our analysis the logit link function was used.

When only the topology of Network A is considered, the independent variables were the mean, minimum, maximum and standard deviation of the four topology factors (shown in Table 3) as well as the percentage of dependency and mean nodal degree of dependent nodes, when applicable. When considering the topology of Network A and Network B the independent variables also included the mean, minimum, maximum and standard deviation of the four topological factors for Network B, as well as the percentage of dependency, mean nodal degree of dependent nodes and mean nodal degree of source nodes, when applicable.

Any of the within network topological factors that have a standard deviation of zero in Table 3 were removed from the data set as they do not impact the results. After removing variables with a standard deviation of zero, the Variance Inflation Factor (VIF) method was used to remove multicollinear variables. The VIF of each variable gives an indication of how well each variable can be explained by a combination of the other variables. A VIF of 1 indicates the variable is not explainable with the others, with a larger VIF indicating a larger degree of redundancy with the other variables. The variable with the largest VIF was removed iteratively until all variables had a VIF value of less than 10. For the regression models which only included the topological factors of Network A and for the model including the topological factors of both Networks A and B, the following variables of Network A were

Table 6
Significant covariates when Network A and Network B are interdependent and the effect of change of these covariates. The sign indicates if the covariate has a positive or negative influence on the percentage of nodes considered functional in Network A after random failures in Network B. The colour indicates the covariate coefficient value with the darker the colour the further the value is from 0.

Type of interdependency Networks A and B have		Fixed, 50%			Random			Random			Random			Random		
Percentage of source nodes in Network B		0			0			2			5			10		
Percentage of initial failures in Network B		10	25	50	10	25	50	10	25	50	10	25	50	10	25	50
Topology of Network A only in regression model	*k*, mean A	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	k, std dev A	-	-	-	-	-	-	+	+	+			+			+
	Bc, mean A							+	+	+				-	-	
	Bc, max A							-	-	-	+	+				
	Cc, mean A	+	+	+			+		-	-	+	+				
	L, std dev A	-	-	-	-	-				+						
	Dp, A				-	-	-	-	-	-	-	-	-	-	-	-
	Dk, mean A	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Topology of Networks A and B in regression model	*k*, mean A	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	Bc, max A							-	-	-			-			
	Cc, mean A			+			+				+					
	L, std dev A	-	-		-	-		-	-	-			-			
	Dp, A				-	-	-	-	-	-	-	-	-	-	-	-
	Dk, mean A	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	k, std dev B	-	-	-	-	-	-	+	+	+		+	+			+
	Bc, mean B								+	+				-	-	-
	Cc, mean B	+	+	+				-	-	-		+				
	L, std dev B	-			-	-			+	+			+			+
	Dp, B				-	-	-	-	-	-	-	-	-	-	-	-
	Dk, mean B	-	-			-		+	+						+	+
	Sk, mean B							+	+	+						

removed due to multicollinearity: maximum nodal degree, betweenness centrality standard deviation, clustering coefficient standard deviation, mean path length and maximum path length. Additionally, for the regression model including topological factors from both Networks A and B the variables maximum nodal degree and mean betweenness centrality of Network A were removed due to multicollinearity as well as the following variables from Network B: mean nodal degree, maximum nodal degree, maximum betweenness centrality, betweenness centrality standard deviation, clustering coefficient standard deviation, mean path length and maximum path length.

After removing variables due to multicollinearity, the remaining variable were normalised before fitting a beta regression model using the betareg R package [11]. After fitting the initial beta regression model, the least significant variable was removed iteratively, until all remaining variables were significant at the $\alpha = 0.05$ level. The results of the regression analysis are shown in Section 4.

4. Results

The results of the beta regression analyses are shown in Tables 5 and 6. Table 5 contains the results for regression analyses relating to the dependent coupled systems (i.e., Network A depends on Network B and Network B is independent). Table 6 contains the results for the regression analyses relating to the interdependent coupled systems (i.e. Networks A depends on Network B and Network B depends on Network A). The full results of the beta regression models are given in Appendix A.

Each column of Tables 5 and 6 represents the result of the regression analysis for a dependency type and percentage of initial failures occurring in Network B. For example, the first column in Table 5 shows the results for when, in each of the 2000 coupled network systems, 10% of nodes in Network A are dependent on Network B, Network B has no source nodes and 10% of nodes in Network B are randomly chosen to fail initially. If a topological factor was significant in a beta regression model, then the cell in the corresponding column is shaded and contains either a positive or negative sign. The sign indicates if the topological factor has a positive of negative influence on the robustness of Network A, and the shading indicates how strong of an influence it has, the darker the shading the more influential the factor is (i.e., the further the covariate coefficient is from 0). Table 7 shows the values associated with the levels of shading for both Tables 5 and 6 (the same scale has been used to shade both Tables 5 and 6). If a factor has a positive influence on the robustness of Network A, this indicates the greater the values of the topological factor the more robust Network A is. When a factor has a negative influence on the robustness of Network A this means the greater the value of the topological factor the less robust Network A is.

Table 7
Reference for the covariate coefficient values represented in Tables 5 and 6.

Coefficient Value	-1.4	-1.2	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
Colour													

4.1. General observations

The level or percentage of dependency Network A had on Network B (*Dp*, A) was always significant (when included in the applicable regression models) and has a great negative on the robustness of Network A. Given that all initial failures occur in Network B, it seems intuitive that the more dependent Network A is on Network B, the greater the cascading effects will be in Network A. The mean intra-nodal degree of dependent nodes in Network A (*Dk*, mean A) consistently has a negative effect on the robustness of Network A. The structure of power-law networks is described as containing hubs [8]. The greater the mean nodal degree of dependent nodes, the more likely it is for the central nodes of the hubs to be dependent on Network B. When one of the central nodes of a hub fails, the network is more likely to fragment into many clusters that contain only a small number of nodes. Therefore, the higher the intra-nodal degree of dependent nodes in Network A, the greater the chance that a central node of a hub fails, and thus the less robust the network is when initial failures occur in Network B.

The mean nodal degree of Network A (*k*, mean A) is significant in every regression model with a positive influence on the robustness of Network A. This is expected as the greater the mean nodal degree, the more edges or connections are present in the network. This increases the chance of alternative pathways within the networks, increasing the redundancy of the network.

4.2. Dependent coupled systems

Table 5 shows the results for the dependent coupled systems, that is when Network A depends on Network B and Network B is independent. The top section of Table 5 shows the regression results when only the topological factors of Network A were included as covariates in the regression model. The bottom section of Table 5 shows the results when both the topological factors of Networks A and B were included in the regression model.

4.2.1. Topological factors of Network A only

When Network A has a fixed partial dependency on Network B, the first three columns in Table 5, the two most influential topological factors are the mean nodal degree (*k*, mean A) and the mean intra-nodal degree of dependent nodes (*Dk*, mean A). The mean nodal degree has a positive influence on the robustness of Network A, whereas the mean intra-nodal degree of dependent nodes has a negative influence. For Network A fully dependent on Network B (100% dependency), as shown in column four, it can be seen that the mean nodal degree (*k*, mean A) still has a positive influence on the robustness of Network A, but is less influential compared to when Network A is partially dependent. The standard deviation of both nodal degree and path length (*k*, std dev A and *L*, std dev A) have a weak negative influence on the robustness of Network A for all fixed dependency types. The mean clustering coefficient (*Cc*, mean A) has a weak positive influence on the robustness of Network A.

When the level of dependency is randomly assigned to each of the 2000 coupled systems, the percentage of dependent nodes in Network A (*Dp*, A) becomes the most influential factor, with a negative influence on the robustness of the network. The mean nodal degree and mean intra-nodal degree of dependent nodes (*k*, mean A and *Dk*, mean A) consistently have a positive and negative influence, respectively, on the robustness of Network A, however to a lesser extent than when the dependency level is fixed.

4.2.2. Topological factors of Network A and Network B

The topological factors with the greatest influence when the topological factors of both networks are included in the regression model are consistent of those when only the factors of Network A are considered. For fixed levels of dependency the mean nodal degree of Network A (*k*, mean A) has the greatest positive influence on the robustness of Network A and the mean intra-nodal degree of dependent nodes in Network A (*Dk*, mean A) has the greatest negative influence. When the level of dependency is randomly assigned to each coupled system again the percentage of dependency (*Dp*, A) becomes the most influential factor, with a negative influence on the robustness of Network A. The nodal degree standard deviation of Network A (*k*, std dev A) is no longer significant, however path length standard deviation of Network A (*L*, std dev A) is sometimes significant, mainly when the initial percentage of node failures is 10% and 25%, again with a negative influence on the robustness of Network A.

When source nodes are not present in the model the nodal degree standard deviation of Network B (*k*, std dev B) is significant with a weak negative influence on the robustness of Network A. When source nodes are present the nodal degree standard deviation of Network B (*k*, std dev B) is sometimes significant, mostly with a weak positive influence on the robustness of Network B. However, the inclusion of source nodes within the coupled system does not change which topological factors are the most influential on the robustness of the network.

4.3. Interdependent coupled systems

Table 6 shows the result for interdependent coupled systems, that is when Network A and B both depend on each other. The top section of Table 6 shows the regression results when only the topological factors of Network A are included in the regression model. The bottom section of Table 6 shows the results when the topological factors of both networks were included in the regression model.

4.3.1. Topological factors of Network A only

The first column in Table 6 shows the results when both Network A and Network B had a fixed level of dependency at 50%. Similar to the results for fixed levels of dependency in Table 5, the mean nodal degree and mean intra-nodal degree of dependent nodes in Network A (*k*, mean A and *Dk*, mean A) have the greatest influence on the robustness of Network A. Again, the mean nodal degree (*k*, mean A) has a positive influence and the mean intra-nodal degree of dependent nodes (*Dk*, mean A) has a negative influence. The remaining columns in Table 6 show the results when the level of dependency was randomly assigned to Networks A and B separately, with the level of dependency for Network A (*Dp*, A) included in the regression model. Again, this now becomes the most influential factor, with a negative influence on the robustness of Network A. The influence of the mean nodal degree and mean intra-nodal degree (*k*, mean A and *Dk*, mean A) are still influential with a positive and negative influence, respectively.

For a fixed 50% dependency and random dependency when no source nodes are present both the standard deviation of the nodal degree and path length (k, std dev A and L, std dev A) have a weak negative influence on the robustness of Network A. When source nodes are present the influence of both nodal degree standard deviation and path length standard deviation (k, std dev A and L, std dev A) have a weak influence, when significant, but now both have a positive influence on the robustness of Network A.

4.3.2. Topological factors of Network A and Network B

Comparing the bottom section on Table 6 with that of Table 5, the results look similar, with the main difference being that now that Network B depends on Network A the percentage of dependency Network B has on Network A (Dp, B) is now included in the model, and is significant with a negative influence on the robustness of Network A. The mean intra-nodal degree of the dependent nodes in Network B (Dk, mean B) is significant in some of the regression models, however the influence it has is not as great as the mean intra-nodal degree of the dependent nodes in Network A (Dk, mean A).

Nodal degree standard deviation of Network A (k, std dev A) is no longer significant for any regression models. However, path length standard deviation of Network A (L, std dev A) is still sometimes significant, with a weak negative influence when significant. Nodal degree standard deviation of Network B (k, std dev B) is often significant, with a weak influence on the robustness of Network A. When source nodes are not present in Network B this influence is negative, but becomes positive when source nodes are present in Network B.

5. Discussion

In the analysis presented, the first order effects of a disruption within a coupled system have been explored for different structures of coupled systems. Across the various methods of forming dependencies (both dependent and interdependent systems) as well as two different methods of simulating failures, the majority of the results were consistent.

The most influential factors across all the coupled network structures investigated are the mean nodal degree of Network A, the mean intra-nodal degree of dependent nodes in Network A and, when applicable, the percentage of dependency Network A has on Network B. It is worth noting that of the three most influential factors, two were in relation to the dependency Network A has on Network B. However, this analysis only covers scenarios where initial failures occurred in Network B and so these results are to be expected.

The analysis which included the topological factors of Network B (in addition to those of Network A) concluded some addition factors were significant, but have only minor influence on the robustness of Network A. This suggests that even for interdependent networks, the most important topological factors when characterising the robustness are those relating to the network's own structure.

The most influential factor was the percentage of dependency Network A had on Network B. This has a negative effect on the robustness of a network in relation to first order effects. All initial disruptions occurred within Network B, and so, the more nodes in Network A depending on Network B, the more likely it is for failures to cascade into Network A. Increased percentage of dependency increases the number of paths available for the disruption to cascade from Network B to Network A.

The mean nodal degree of Network A has a positive influence on the network's robustness, however, the mean nodal degree of dependent nodes in Network A has a negative influence. The positive influence of the mean nodal degree can be attributed to the fact that the higher the mean nodal degree a network has, the more intra-connections are present, increasing the likelihood of available paths between the nodes, and so, increasing the redundancy of the network. The negative influence of the mean intra-nodal degree of dependent nodes in Network A is intuitive. Any dependent node in Network A fails if the node it depends on fails. If the dependent nodes have a high intra-degree, when they fail they have a greater potential to affect the robustness of Network A.

Some factors were significant over the different system structures, but their effect on the robustness of the network changed. For example, when source nodes are present in Network B, the standard deviation of the nodal degree of Network A has a positive influence. However, when neither network within the coupled system contains source nodes, the standard deviation of Network A's nodal degree has a negative effect on its robustness. The mean clustering coefficient of Network A also changes from having a positive influence when source nodes not are present in Network B, to a negative influence when source nodes are present in the coupled system.

The change in the influence of the clustering coefficient may be due to the different methods of assessing which nodes are functional for the different coupled system structures. When source nodes are not present within the system, the GCC method is used to assess which nodes are functional after disruption. When this method is used the more connections between a neighbourhood of nodes, the less likely the neighbourhood is to fragment when disruptions occur, leaving a cluster with a high population. However, when source nodes are present, a node is only functional if there is a path available from any source node to that node. If neighbourhoods of nodes are highly connected, they may be reliant on only a small number of nodes in the neighbourhood to receive input from the source nodes. When these nodes fail, the other members of the neighbourhood will no longer have a path from a source node to itself, causing the entire neighbourhood to fail.

When comparing the results of the coupled system analysis to those found by LaRocca and Guikema [23], some factors which were significant for independent networks were no longer significant for dependent networks. Other factors remained significant but the influence of the factors on the robustness of the network changed. LaRocca and Guikema [23] found that the mean clustering coefficient was the most influential topological factor for independent networks when 10% and 25% of nodes initially failed. When 50% and 75% nodes initially failed in an independent network the mean nodal degree was the most influential factor. However, when looking at the robustness of a network in a coupled system, the influence of the mean nodal degree is always more influential that the mean clustering coefficient. This suggests that for first order disruptions the overall redundancy of the network is more important than the local redundancy.

These results can be used alongside those of LaRocca and Guikema [23] to provide some direction on which topological factors should be given more focus on when planning improvements or developing new networks. The influence of the significant topological factors shown by LaRocca and Guikema [23] for failures within a network and those presented in this paper for first order disruptions can be used together to plan the structure of networks, such as infrastructure, so that it is robust to disruptions that both directly affect it and, through dependencies, indirectly affect it.

If a new network is being designed, attention should be given to the level of dependency. Our results show that for each dependency type we investigated, the higher the level of dependency, the less robust the network is to first order disruptions. This suggests that the level of dependency a network has should be low as possible.

The nodes which have dependencies should also be carefully considered. Our results show that the greater the nodal degree of the components that are dependent on another network, the less robust the network is to first order disruptions. This suggests that dependent nodes should have the fewest number of intra-connection possible. However, in reality, the components which have dependencies are guided by functionality. In this case, the results can be considered when deciding how to increase redundancy within the network. For example, in a water supply network, the dependency on the power network is through pumps within the network. The nodal degree of the dependent pumps could be taken into consideration when deciding where to improve redundancy, such as the addition of a back-up generator.

This extension of LaRocca and Guikema [23] to interdependent networks has covered a range of coupled network structures to provide a generalised overview of the important topological factors for characterising robustness of a dependent network, however, there are numerous ways of modelling dependencies between networks, as well as multiple failure scenarios. The results give a general overview of the important topological factors for a network present in a coupled infrastructure system, where each dependent node has one and only one dependency, concerning the first order effects of a random disruption.

The results presented in this paper highlight to networks, such as infrastructure, that even though they depend on another infrastructure, the most influential factors are primarily those attributed within their own structure, or topology. Therefore, changes to their own structure can help to increase their robustness to random failures in the dependent networks. Although when applicable, the percentage of dependency was the most influential topological factor, the dependency of one infrastructure on another is defined by the need for the input (or the utility) that the infrastructure produces and thus is not easy to change to increase the robustness of the dependent infrastructure. Therefore, the more important topological factors to consider when designing or improving infrastructure are nodal degree and the intranodal degree of the dependent networks. The topology of components (or nodes) with dependencies on other networks are shown to be important and thus gives an indication that providing some redundancy into the infrastructure, such as back-up generators for those dependent on the power network, for example, could improve their own robustness.

6. Conclusion

In conclusion we find that the most influential topological factors associated with the robustness of coupled power-law networks with exponential cut-off are those related to the dependency the network has on the network in which the disruption originates. These factors are the percentage of dependency and the mean nodal degree of the dependent nodes in the coupled power-law network system. However, in networks such as infrastructure the dependency an infrastructure has on another, and which components need input from another infrastructure is determined by the operational needs of the network and thus is difficult to change. The mean nodal degree of the network has also shown to be very influential on the robustness of the network, with the greater the mean degree the more robust the network was to first order effects of a disruption. Although a variety of dependency types have been explored, the results remained consistent over the different coupled network structures. The results provide a general overview of the most influential topological factors for a coupled network system and can be used as a basis of which topological factors should be considered by, for example, infrastructure owners or management when developing or improving their infrastructure.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ress.2019.106560.

Appendix A

Tables A.1–A.6 show the full beta regression results for the various regression analyses performed as part of the current paper.

Table A.1
Full beta regression results for fixed dependency types when topology of Network A only are included in the regression analysis.

Dependency type	10% initial failures Topology measure	Co-efficient	Std error	p value	25% initial failures Topology measure	Co-efficient	Std error	p value	50% initial failures Topology measure	Co-efficient	Std error	p value
Network A fixed 10% dependency Network B independent	Intercept	3.904	0.007	0.000	Intercept	2.991	0.007	0.000	Intercept	2.328	0.007	0.000
	k, mean A	0.460	0.010	0.000	k, mean A	0.500	0.013	0.000	k, mean A	0.554	0.014	0.000
	Bc, mean A	−0.049	0.008	0.000	k, std dev A	−0.030	0.007	0.000	k, std dev A	−0.029	0.008	0.000
	L, std dev A	−0.041	0.011	0.000	Cc, mean A	0.036	0.009	0.000	Cc, mean A	0.034	0.010	0.000
	Dk, mean A	−0.491	0.006	0.000	L, std dev A	−0.064	0.014	0.000	L, std dev A	−0.066	0.016	0.000
					Dk, mean A	−0.562	0.006	0.000	Dk, mean A	−0.652	0.007	0.000
Network A fixed 30% dependency Network B independent	Intercept	2.712	0.005	0.000	Intercept	1.741	0.005	0.000	Intercept	0.964	0.006	0.000
	k, mean A	0.581	0.013	0.000	k, mean A	0.634	0.012	0.000	k, mean A	0.769	0.015	0.000
	K, std dev A	−0.042	0.006	0.000	K, std dev A	−0.037	0.006	0.000	K, std dev A	−0.032	0.007	0.000
	Cc, mean A	0.021	0.007	0.003	Cc, mean A	0.023	0.007	0.001	Cc, mean A	0.031	0.008	0.000
	L, std dev A	−0.052	0.012	0.000	L, std dev A	−0.041	0.011	0.000	L, std dev A	−0.031	0.013	0.020
	Dk, mean A	−0.509	0.009	0.000	Dk, mean A	−0.576	0.008	0.000	Dk, mean A	−0.729	0.010	0.000
Network A fixed 50% dependency Network B independent	Intercept	2.168	0.004	0.000	Intercept	1.129	0.004	0.000	Intercept	0.232	0.005	0.000
	k, mean A	0.607	0.013	0.000	k, mean A	0.689	0.012	0.000	k, mean A	0.894	0.015	0.000
	K, std dev A	−0.049	0.005	0.000	K, std dev A	−0.046	0.004	0.000	K, std dev A	−0.045	0.006	0.000
	Cc, mean A	0.012	0.006	0.040	Cc, mean A	0.015	0.005	0.005	Cc, mean A	0.020	0.007	0.002
	L, std dev A	−0.048	0.010	0.000	L, std dev A	−0.038	0.009	0.000	L, std dev A	−0.026	0.011	0.015
	Dk, mean A	−0.498	0.011	0.000	Dk, mean A	−0.581	0.010	0.000	Dk, mean A	−0.790	0.012	0.000
Network A fixed 100% dependency Network B independent	Intercept	1.396	0.003	0.000	Intercept	0.184	0.003	0.000	Intercept	−1.176	0.004	0.000
	k, mean A	0.158	0.006	0.000	k, mean A	0.178	0.006	0.000	k, mean A	0.233	0.007	0.000
	K, std dev A	−0.078	0.007	0.000	K, std dev A	−0.086	0.007	0.000	K, std dev A	−0.102	0.006	0.000
	Bc, max A	0.017	0.007	0.013	Bc, max A	0.017	0.007	0.010	Bc, mean A	0.021	0.008	0.009
	L, std dev A	−0.067	0.006	0.000	Cc, mean A	0.012	0.004	0.005	Cc, mean A	0.037	0.006	0.000
					L, std dev A	−0.054	0.007	0.000	L, std dev A	−0.049	0.010	0.000
Network A fixed 50% dependency Network B fixed 50% dependency	Intercept	1.941	0.005	0.000	Intercept	0.932	0.005	0.000	Intercept	0.092	0.006	0.000
	k, mean A	0.634	0.016	0.000	k, mean A	0.746	0.016	0.000	k, mean A	0.977	0.018	0.000
	K, std dev A	−0.061	0.006	0.000	K, std dev A	−0.053	0.006	0.000	K, std dev A	−0.051	0.007	0.000
	Cc, mean A	0.019	0.007	0.006	Cc, mean A	0.027	0.007	0.000	Cc, mean A	0.036	0.008	0.000
	L, std dev A	−0.081	0.012	0.000	L, std dev A	−0.058	0.012	0.000	L, std dev A	−0.026	0.013	0.045
	Dk, mean A	−0.528	0.013	0.000	Dk, mean A	−0.641	0.013	0.000	Dk, mean A	−0.887	0.015	0.000

Table A.2
Full beta regression results for fixed dependency types when the topology of both Network A and Network B are included in the regression analysis.

	10% initial failures				25% initial failures				50% initial failures			
Dependency type	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value
Network A fixed 10% dependency Network B independent	Intercept	3.904	0.007	0.000	Intercept	2.991	0.007	0.000	Intercept	2.328	0.007	0.000
	k, mean A	0.467	0.011	0.000	k, mean A	0.492	0.013	0.000	k, mean A	0.547	0.014	0.000
	L, std dev A	−0.065	0.012	0.000	Cc, mean A	0.038	0.009	0.000	Cc, mean A	0.037	0.009	0.000
	Dk, mean A	−0.491	0.006	0.000	L, std dev A	−0.053	0.013	0.000	L, std dev A	−0.056	0.014	0.000
	k, std dev B	−0.017	0.008	0.029	Dk, mean A	−0.563	0.006	0.000	Dk, mean A	−0.652	0.007	0.000
	Bc, mean B	−0.032	0.010	0.001	k, std dev B	−0.026	0.007	0.000	k, std dev B	−0.025	0.007	0.000
Network A fixed 30% dependency Network B independent	Intercept	2.712	0.005	0.000	Intercept	1.741	0.005	0.000	Intercept	0.964	0.006	0.000
	k, mean A	0.568	0.012	0.000	k, mean A	0.622	0.011	0.000	k, mean A	0.746	0.011	0.000
	L, std dev A	−0.034	0.010	0.001	L, std dev A	−0.024	0.009	0.011	Dk, mean A	−0.729	0.010	0.000
	Dk, mean A	−0.509	0.009	0.000	Dk, mean A	−0.576	0.008	0.000	k, std dev B	−0.025	0.006	0.000
	k, std dev B	−0.037	0.005	0.000	k, std dev B	−0.032	0.005	0.000	Cc, mean B	0.044	0.006	0.000
	Cc, mean B	0.030	0.006	0.000	Cc, mean B	0.034	0.006	0.000				
Network A fixed 50% dependency Network B independent	Intercept	2.168	0.004	0.000	Intercept	1.129	0.004	0.000	Intercept	0.232	0.005	0.000
	k, mean A	0.593	0.013	0.000	k, mean A	0.678	0.012	0.000	k, mean A	0.874	0.012	0.000
	L, std dev A	−0.030	0.008	0.000	L, std dev A	−0.023	0.007	0.001	Dk, mean A	−0.789	0.012	0.000
	Dk, mean A	−0.497	0.011	0.000	Dk, mean A	−0.580	0.010	0.000	k, std dev B	−0.039	0.005	0.000
	k, std dev B	−0.043	0.004	0.000	k, std dev B	−0.041	0.004	0.000	Cc, mean B	0.029	0.005	0.000
	Cc, mean B	0.019	0.005	0.000	Cc, mean B	0.019	0.005	0.000				
Network A fixed 100% dependency Network B independent	Intercept	1.396	0.003	0.000	Intercept	0.184	0.003	0.000	Intercept	−1.176	0.004	0.000
	k, mean A	0.146	0.005	0.000	k, mean A	0.162	0.006	0.000	k, mean A	0.210	0.004	0.000
	L, std dev A	−0.029	0.006	0.000	L, std dev A	−0.017	0.006	0.007	k, std dev B	−0.078	0.004	0.000
	k, std dev B	−0.058	0.004	0.000	k, std dev B	−0.064	0.004	0.000	Cc, mean B	0.044	0.004	0.000
	L, std dev B	−0.026	0.006	0.000	Cc, mean B	0.016	0.004	0.000				
					L, std dev B	−0.020	0.006	0.002				
Network A fixed 50% dependency Network B fixed 50% dependency	Intercept	1.941	0.005	0.000	Intercept	0.932	0.005	0.000	Intercept	0.092	0.006	0.000
	k, mean A	0.659	0.018	0.000	k, mean A	0.758	0.017	0.000	k, mean A	0.958	0.015	0.000
	L, std dev A	−0.047	0.010	0.000	L, std dev A	−0.039	0.010	0.000	Cc, mean A	0.024	0.011	0.022
	Dk, mean A	−0.529	0.013	0.000	Dk, mean A	−0.639	0.013	0.000	Dk, mean A	−0.886	0.015	0.000
	k, std dev B	−0.059	0.006	0.000	k, std dev B	−0.046	0.005	0.000	k, std dev B	−0.044	0.006	0.000
	Cc, mean B	0.021	0.007	0.003	Cc, mean B	0.033	0.006	0.000	Cc, mean B	0.023	0.011	0.033
	L, std dev B	−0.030	0.011	0.007	Dk, mean B	−0.030	0.011	0.006				
	Dk, mean B	−0.032	0.011	0.005								

Table A.3
Full beta regression results for when Network A has random dependency types, Network B is independent and the topology of Network A only is included in the regression analysis. .

	10% initial failures				25% initial failures				50% initial failures			
Dependency group	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value
Network A random dependency Network B independent	Intercept	2.347	0.009	0.000	Intercept	1.319	0.009	0.000	Intercept	0.397	0.010	0.000
	k, mean A	0.128	0.013	0.000	k, mean A	0.147	0.013	0.000	k, mean A	0.264	0.011	0.000
	k, std dev A	−0.035	0.009	0.000	k, std dev A	−0.037	0.010	0.000	k, std dev A	−0.029	0.010	0.002
	L, std dev A	−0.039	0.013	0.003	L, std dev A	−0.040	0.014	0.005	Dp, A	−1.064	0.011	0.000
	Dp, A	−0.693	0.008	0.000	Dp, A	−0.814	0.009	0.000	Dk, mean A	−0.453	0.020	0.000
	Dk, mean A	−0.037	0.007	0.000	Dk, mean A	−0.057	0.007	0.000				
Network A random dependency Network B independent with 2% source nodes	Intercept	2.430	0.008	0.000	Intercept	1.312	0.008	0.000	Intercept	0.178	0.010	0.000
	k, mean A	0.230	0.013	0.000	k, mean A	0.277	0.016	0.000	k, mean A	0.332	0.023	0.000
	k, std dev A	0.105	0.015	0.000	k, std dev A	0.238	0.017	0.000	k, std dev A	0.495	0.026	0.000
	Bc, mean A	0.025	0.010	0.015	Bc, mean A	0.052	0.016	0.001	Bc, mean A	0.076	0.024	0.001
	Bc, max A	−0.061	0.016	0.000	Bc, max A	−0.134	0.018	0.000	Bc, max A	−0.246	0.022	0.000
	Dp, A	−0.576	0.008	0.000	Cc, mean A	−0.042	0.013	0.002	Cc, mean A	−0.102	0.017	0.000
	Dk, mean A	−0.162	0.011	0.000	Dp, A	−0.618	0.008	0.000	L, std dev A	0.083	0.027	0.002
					Dk, mean A	−0.211	0.012	0.000	Dp, A	−0.705	0.010	0.000
									Dk, mean A	−0.300	0.016	0.000
Network A random dependency Network B independent with 5% source nods	Intercept	2.512	0.008	0.000	Intercept	1.446	0.008	0.000	Intercept	0.431	0.009	0.000
	k, mean A	0.214	0.011	0.000	k, mean A	0.254	0.012	0.000	k, mean A	0.353	0.014	0.000
	Dp, A	−0.622	0.008	0.000	k, std dev A	0.016	0.007	0.033	k, std dev A	0.143	0.019	0.000
	Dk, mean A	−0.143	0.012	0.000	Dp, A	−0.695	0.008	0.000	Bc, max A	−0.066	0.019	0.001
					Dk, mean A	−0.169	0.012	0.000	Cc, mean A	−0.034	0.010	0.001
									Dp, A	−0.832	0.009	0.000
									Dk, mean A	−0.251	0.013	0.000
Network A random dependency Network B independent with 10% source nodes	Intercept	2.533	0.009	0.000	Intercept	1.482	0.008	0.000	Intercept	0.516	0.009	0.000
	k, mean A	0.166	0.011	0.000	k, mean A	0.191	0.011	0.000	k, mean A	0.318	0.013	0.000
	k, std dev A	−0.019	0.007	0.010	Dp, A	−0.687	0.008	0.000	Dp, A	−0.843	0.010	0.000
	Dp, A	−0.616	0.008	0.000	Dk, mean A	−0.125	0.011	0.000	Dk, mean A	−0.258	0.014	0.000
	Dk, mean A	−0.108	0.011	0.000								

Table A.4
Full beta regression results for when Networks A and B have random dependency type and the topology of Network A only is included in the regression analysis.

Dependency group	10% initial failures				25% initial failures				50% initial failures			
	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value
Network A random	Intercept	1.969	0.012	0.000	Intercept	0.994	0.011	0.000	Intercept	0.148	0.011	0.000
dependency	k, mean A	0.384	0.022	0.000	k, mean A	0.453	0.022	0.000	k, mean A	0.548	0.017	0.000
Network B random	k, std dev A	−0.072	0.012	0.000	k, std dev A	−0.071	0.012	0.000	k, std dev A	−0.054	0.011	0.000
dependency	L, std dev A	−0.096	0.018	0.000	L, std dev A	−0.078	0.018	0.000	Cc, mean A	0.040	0.011	0.000
	Dp, A	−0.935	0.012	0.000	Dp, A	−1.111	0.012	0.000	Dp, A	−1.369	0.013	0.000
	Dk, mean A	−0.284	0.019	0.000	Dk, mean A	−0.338	0.018	0.000	Dk, mean A	−0.475	0.017	0.000
Network A random	Intercept	2.043	0.013	0.000	Intercept	0.952	0.012	0.000	Intercept	−0.164	0.013	0.000
dependency	k, mean A	0.254	0.024	0.000	k, mean A	0.338	0.026	0.000	k, mean A	0.343	0.032	0.000
Network B random	k, std dev A	0.111	0.021	0.000	k, std dev A	0.260	0.026	0.000	k, std dev A	0.571	0.036	0.000
dependency with 2%	Bc, mean A	0.046	0.014	0.001	Bc, mean A	0.072	0.023	0.001	Bc, mean A	0.078	0.032	0.014
source nodes	Bc, max A	−0.075	0.023	0.001	Bc, max A	−0.161	0.026	0.000	Bc, max A	−0.283	0.030	0.000
	Dp, A	−0.768	0.012	0.000	Cc, mean A	−0.045	0.019	0.020	Cc, mean A	−0.125	0.023	0.000
	Dk, mean A	−0.219	0.023	0.000	Dp, A	−0.874	0.013	0.000	L, std dev A	0.127	0.037	0.001
					Dk, mean A	−0.285	0.022	0.000	Dp, A	−1.005	0.015	0.000
									Dk, mean A	−0.317	0.023	0.000
Network A random	Intercept	2.054	0.013	0.000	Intercept	1.038	0.012	0.000	Intercept	0.077	0.012	0.000
dependency	k, mean A	0.228	0.018	0.000	k, mean A	0.309	0.019	0.000	k, mean A	0.499	0.020	0.000
Network B random	Bc, max A	0.026	0.011	0.023	Bc, max A	0.038	0.012	0.002	k, std dev A	0.122	0.012	0.000
dependency with 5%	Cc, mean A	0.044	0.011	0.000	Cc, mean A	0.047	0.012	0.000	Dp, A	−1.090	0.014	0.000
source nodes	Dp, A	−0.787	0.012	0.000	Dp, A	−0.919	0.013	0.000	Dk, mean A	−0.359	0.021	0.000
	Dk, mean A	−0.153	0.018	0.000	Dk, mean A	−0.202	0.019	0.000				
Network A random	Intercept	2.139	0.012	0.000	Intercept	1.148	0.011	0.000	Intercept	0.254	0.011	0.000
dependency	k, mean A	0.235	0.016	0.000	k, mean A	0.394	0.019	0.000	k, mean A	0.536	0.018	0.000
Network B random	Bc, mean A	−0.050	0.012	0.000	Bc, mean A	−0.042	0.012	0.001	k, std dev A	0.037	0.011	0.001
dependency with 10%	Dp, A	−0.810	0.012	0.000	Dp, A	−0.939	0.012	0.000	Dp, A	−1.143	0.013	0.000
source nodes	Dk, mean A	−0.159	0.014	0.000	Dk, mean A	−0.339	0.020	0.000	Dk, mean A	−0.488	0.020	0.000

Table A.5
Full beta regression results for when Network A has random dependency types, Network B is independent and the topology of Network A and Network B is included in the regression analysis.

Dependency group	10% initial failures				25% initial failures				50% initial failures			
	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value	Topology measure	Co-efficient	Std error	p value
Network A random	Intercept	2.347	0.009	0.000	Intercept	1.319	0.009	0.000	Intercept	0.397	0.010	0.000
dependency	k, mean A	0.122	0.012	0.000	k, mean A	0.142	0.013	0.000	k, mean A	0.264	0.011	0.000
Network B	L, std dev A	−0.031	0.012	0.012	L, std dev A	−0.032	0.013	0.017	Dp, A	−1.064	0.011	0.000
independent	Dp, A	−0.693	0.008	0.000	Dp, A	−0.814	0.009	0.000	Dk, mean A	−0.453	0.020	0.000
	Dk, mean A	−0.037	0.007	0.000	Dk, mean A	−0.057	0.007	0.000	k, std dev B	−0.032	0.010	0.001
	k, std dev B	−0.035	0.008	0.000	k, std dev B	−0.036	0.009	0.000				
Network A random	Intercept	2.431	0.008	0.000	Intercept	1.312	0.008	0.000	Intercept	0.179	0.010	0.000
dependency	k, mean A	0.221	0.016	0.000	k, mean A	0.240	0.017	0.000	k, mean A	0.306	0.023	0.000
Network B	Bc, max A	−0.061	0.015	0.000	Bc, max A	−0.122	0.017	0.000	Bc, max A	−0.210	0.021	0.000
independent with 2%	L, std dev A	−0.050	0.015	0.001	L, std dev A	−0.077	0.017	0.000	L, std dev A	−0.122	0.021	0.000
source nodes	Dp, A	−0.575	0.008	0.000	Dp, A	−0.615	0.008	0.000	Dp, A	−0.701	0.010	0.000
	Dk, mean A	−0.162	0.011	0.000	Dk, mean A	−0.210	0.012	0.000	Dk, mean A	−0.295	0.016	0.000
	k, std dev B	0.126	0.016	0.000	k, std dev B	0.234	0.018	0.000	k, std dev B	0.458	0.024	0.000
	L, std dev B	0.084	0.016	0.000	Bc, mean B	0.058	0.018	0.001	Bc, mean B	0.089	0.025	0.000
					L, std dev B	0.121	0.022	0.000	Cc, mean B	−0.052	0.016	0.001
					Sk, B	0.018	0.009	0.034	L, std dev B	0.228	0.027	0.000
									Sk, B	0.037	0.010	0.000
Network A random	Intercept	2.512	0.008	0.000	Intercept	1.446	0.008	0.000	Intercept	0.431	0.009	0.000
dependency	k, mean A	0.214	0.011	0.000	k, mean A	0.254	0.012	0.000	k, mean A	0.324	0.018	0.000
Network B	Dp, A	−0.622	0.008	0.000	Dp, A	−0.695	0.008	0.000	Bc, max A	−0.061	0.017	0.000
independent with 5%	Dk, mean A	−0.143	0.012	0.000	Dk, mean A	−0.169	0.012	0.000	L, std dev A	−0.048	0.017	0.004
source nodes					k, std dev B	0.016	0.007	0.024	Dp, A	−0.832	0.009	0.000
									Dk, mean A	−0.250	0.013	0.000
									k, std dev B	0.149	0.018	0.000
									L, std dev B	0.088	0.018	0.000
Network A random	Intercept	2.534	0.008	0.000	Intercept	1.482	0.008	0.000	Intercept	0.516	0.009	0.000
dependency	k, mean A	0.167	0.011	0.000	k, mean A	0.191	0.011	0.000	k, mean A	0.296	0.015	0.000
Network B	Dp, A	−0.616	0.008	0.000	Dp, A	−0.687	0.008	0.000	Dp, A	−0.842	0.010	0.000
independent with 10%	Dk, mean A	−0.109	0.011	0.000	Dk, mean A	−0.125	0.011	0.000	Dk, mean A	−0.256	0.013	0.000
source nodes	k, std dev B	−0.021	0.007	0.005					Bc, mean B	0.039	0.014	0.005
									Cc, mean B	0.037	0.012	0.002

Table A.6
Full beta regression results for when Networks A and B have random dependency types and the topology of Network A and Network B is included in the regression analysis.

Dependency group	10% initial failures Topology measure	Co-efficient	Std error	p value	25% initial failures Topology measure	Co-efficient	Std error	p value	50% initial failures Topology measure	Co-efficient	Std error	p value
Network A random	Intercept	2.045	0.010	0.000	Intercept	1.031	0.010	0.000	Intercept	0.159	0.010	0.000
dependency	k, mean A	0.421	0.018	0.000	k, mean A	0.486	0.021	0.000	k, mean A	0.551	0.017	0.000
Network B random	L, std dev A	−0.050	0.016	0.002	L, std dev A	−0.043	0.017	0.013	Cc, mean A	0.039	0.011	0.000
dependency	Dp, A	−0.974	0.010	0.000	Dp, A	−1.121	0.010	0.000	Dp, A	−1.362	0.012	0.000
	Dk, mean A	−0.319	0.016	0.000	Dk, mean A	−0.362	0.015	0.000	Dk, mean A	−0.486	0.016	0.000
	k, std dev B	−0.073	0.010	0.000	k, std dev B	−0.069	0.010	0.000	k, std dev B	−0.055	0.010	0.000
	L, std dev B	−0.065	0.016	0.000	L, std dev B	−0.036	0.017	0.037	Dp, B	−0.168	0.010	0.000
	Dp, B	−0.293	0.008	0.000	Dp, B	−0.249	0.009	0.000				
					Dk, mean B	−0.025	0.013	0.048				
Network A random	Intercept	2.120	0.010	0.000	Intercept	0.997	0.011	0.000	Intercept	−0.145	0.013	0.000
dependency	k, mean A	0.325	0.021	0.000	k, mean A	0.364	0.025	0.000	k, mean A	0.362	0.030	0.000
Network B random	Bc, max A	−0.049	0.017	0.003	Bc, max A	−0.132	0.021	0.000	Bc, max A	−0.221	0.026	0.000
dependency with 2%	L, std dev A	−0.040	0.017	0.016	L, std dev A	−0.088	0.021	0.000	L, std dev A	−0.106	0.027	0.000
source nodes	Dp, A	−0.839	0.010	0.000	Dp, A	−0.916	0.011	0.000	Dp, A	−1.011	0.014	0.000
	Dk, mean A	−0.244	0.019	0.000	Dk, mean A	−0.302	0.019	0.000	Dk, mean A	−0.326	0.021	0.000
	k, std dev B	0.094	0.016	0.000	k, std dev B	0.236	0.024	0.000	k, std dev B	0.479	0.030	0.000
	Cc, mean B	−0.036	0.011	0.002	Bc, mean B	0.058	0.026	0.024	Bc, mean B	0.124	0.031	0.000
	Dp, B	−0.297	0.009	0.000	Cc, mean B	−0.048	0.016	0.003	Cc, mean B	−0.086	0.020	0.000
	Dk, mean B	0.020	0.010	0.040	L, std dev B	0.071	0.027	0.008	L, std dev B	0.173	0.033	0.000
	Sk, B	0.022	0.009	0.014	Dp, B	−0.274	0.010	0.000	Dp, B	−0.245	0.013	0.000
					Dk, mean B	0.023	0.010	0.024	Sk, B	0.058	0.013	0.000
					Sk, B	0.032	0.010	0.002				
Network A random	Intercept	2.141	0.010	0.000	Intercept	1.090	0.010	0.000	Intercept	0.098	0.011	0.000
dependency	k, mean A	0.235	0.014	0.000	k, mean A	0.307	0.016	0.000	k, mean A	0.448	0.025	0.000
Network B random	Cc, mean A	0.031	0.009	0.001	Dp, A	−0.967	0.011	0.000	Bc, max A	−0.051	0.023	0.027
dependency with 5%	Dp, A	−0.867	0.010	0.000	Dk, mean A	−0.214	0.016	0.000	L, std dev A	−0.068	0.022	0.002
source nodes	Dk, mean A	−0.171	0.015	0.000	k, std dev B	0.023	0.010	0.021	Dp, A	−1.103	0.013	0.000
	Dp, B	−0.326	0.009	0.000	Cc, mean B	0.029	0.010	0.004	Dk, mean A	−0.359	0.019	0.000
					Dp, B	−0.293	0.010	0.000	k, std dev B	0.175	0.024	0.000
									L, std dev B	0.121	0.024	0.000
									Dp, B	−0.223	0.011	0.000
Network A random	Intercept	2.222	0.010	0.000	Intercept	1.198	0.009	0.000	Intercept	0.275	0.010	0.000
dependency	k, mean A	0.234	0.012	0.000	k, mean A	0.422	0.016	0.000	k, mean A	0.537	0.020	0.000
Network B random	Dp, A	−0.874	0.009	0.000	Dp, A	−0.979	0.010	0.000	Dp, A	−1.153	0.012	0.000
dependency with 10%	Dk, mean A	−0.164	0.010	0.000	Dk, mean A	−0.392	0.016	0.000	Dk, mean A	−0.531	0.018	0.000
source nodes	Bc, mean B	−0.042	0.009	0.000	Bc, mean B	−0.031	0.010	0.002	k, std dev B	0.081	0.017	0.000
	Dp, B	−0.313	0.008	0.000	Dp, B	−0.277	0.009	0.000	Bc, mean B	−0.063	0.020	0.001
					Dk, mean B	0.021	0.009	0.024	L, std dev B	0.082	0.026	0.001
									Dp, B	−0.215	0.011	0.000

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