Second graders' reflections about the number 24

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Students' written responses to an open task were examined to identify potential indications of emerging number sense. Content analysis indicates that the number of responses given by students varied, with addition tasks being more commonly provided than tasks that involved other operations. Whereas several students refer to place value, no students mention possible applications of the number. From these findings, implications are discussed in terms of the mathematical demands that teachers are faced with when presenting such tasks in a mathematics lesson.

Introduction and theoretical background

Definitions of number sense differ, but they often refer to students' general understanding of numbers and operations, as well as ability to use their understanding in flexible ways to make mathematical judgements (McIntosh, Reys, Reys, Bana, & Farrell, 1997). Number sense is often described as a prerequisite for students' further development of mathematical knowledge (Verschaffel, Greer, & de Corte, 2007). Children's number sense has been investigated for decades (e.g., Gelman & Gallistel, 1978; Verschaffel et al., 2007), and understanding of the place value system is regarded as particularly important in students' development of number sense and eventually in their work with multi-digit numbers (Kilpatrick, Swafford, & Findell, 2001). Students' understanding of place value develops over time, and it influences understanding of multi-digit numbers, which includes a person's general understanding of numbers and operations (Jones et al., 1996). A fully developed number sense enables students to flexibly operate on numbers and develop useful strategies (McIntosh et al., 1992). This includes understanding how numbers are ordered, how different representations of numbers are connected, what effects and mathematical properties different operations have, as well as understanding how the arithmetical operations are related.

Jones et al. (1996) present four core components that constitute the process of developing multi-digit number sense: counting, partitioning, grouping and number relationships. They then distinguish between five different levels for each of the four components: pre-place value (level 1), initial place value (level 2), developing place value (level 3), expanded place value (level 4), and essential place value (level 5). With reference to the competence aims of the national curriculum, we assume that students in grade 2 are in one of the first three levels. Whereas older students develop more advanced counting strategies (Camos, 2003), students at the level of pre-place value count by ones and know how to partition a number in different quantities, for instance $8 = 6 + 2 = 1 + 7$ (Jones et al., 1996). In their work, they indicate that students at these initial levels can tell if a number is bigger or smaller than another number, but they cannot tell how big this difference is. Students with an initial understanding of place value can think in groups and they can count with tens and ones. To rationalize by counting by tens, the students realize they need to group objects. They understand that they can partition two-digit numbers, for example $24 = 15 + 9$, and in addition they understand that grouping facilitate estimation and counting. When the digits' place change, the students understand that it represents different numbers. Students developing place value (level 3) know how to count by tens and ones and are capable of applying it in operations. This level differs from the previous ones because of the ability to think part-part-whole with two-digit numbers. Within grouping, the students can estimate between which tens a sum of two two-digit numbers will be located, and they master operations and comparing simultaneously (Jones et al., 1996).

Thompson (2003) describes two sub-concepts of the place value system: quantity value and column value. One is more important in (written) mental calculation and the other in using standard algorithms. For instance, the two-digit number 24 can be decomposed into 20 and 4, which relates to the quantity value of the number. Mental calculation is mainly based on quantity value. As an example, 24 and 38 can be added as $20 + 30 = 50$ and $4 + 8 = 12$. The sum is $50 + 12 = 62$. Column value is when 24 is considered to consist of two tens and four ones. The standard algorithm for (written) addition focuses on column value by putting tens over tens and ones over ones (two-digit), and then each of the digits are added (Thompson, 2003)

In this paper, we investigate what Grade 2 students' responses to an open task about the number 24 may reveal about their emerging number sense. We consider data material from two classes of Grade 2 students, who were given the open task called "The number of today".

The study

Our examination of Grade 2 students' reflections about the number 24 is part of a larger school-based research project focusing on developing in-service teachers' knowledge. The first author of this paper has supervised the teachers in the planning of the lessons, observed their teaching, collected material from the students and discussed the teaching with the teachers in retrospect. Prior to the study presented in this paper, the teachers participated in a half-day long in-service course focusing on tasks that invite the students into discussions and different solution strategies. The task used in this study is one example.

The data material is collected from two different classes from the same school, referred to as Group A (N=17) and Group B (N=21). The two teachers who taught these groups used the task, “The number of today”, as one of four tasks that the students worked on during a 60 minutes long session. Prior to this lesson, the students have mainly been working with numbers between 0 and 20. Following the textbook (Alseth, Arnås, Kirkegaard, & Røsseland, 2011) they have first focused on the numbers 0–9. After this, they have spent time on the numbers from 10 to 20, which have been partitioned into tens and ones. They have worked with numbers that add up to 10, addition and subtraction of numbers between 0 and 20, and they have encountered the concept of numerical neighbours. According to the competence aims of the national curriculum, they are supposed to know how to “count to 100, divide and compose amounts up to 10, put together and divide groups of ten up to 100, and divide double-digit numbers into tens and ones” (Ministry of Education and Research, 2013, p. 5) by the time they finish Grade 2.

The students, who were in the first semester of 2nd grade (seven years old), worked individually for approximately 15 minutes on each of four different tasks. All four tasks had been introduced in a previous lesson, and the students could therefore start working on them without any further introduction in this lesson. The students had been told by the teachers that the task (which is the focus of this paper) was related to the question of what they know about the number 24. In each group, the students provided written responses on a worksheet. The teachers made some slightly different choices in how the worksheet was designed. In group A, the worksheet was a blank piece of paper with the number 24 on top of it (Figure 1).

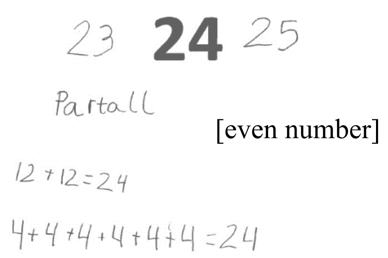

Figure 1. One example of student response from group A (A1).

In group B, the teacher had added eight arrows that were sticking out from the number 24 (Figure 2), but he did not indicate that only eight pieces of information should be provided. The first author of this paper was observing while the students were working on the task. Although various data materials were collected, only the written responses are analyzed for this paper.

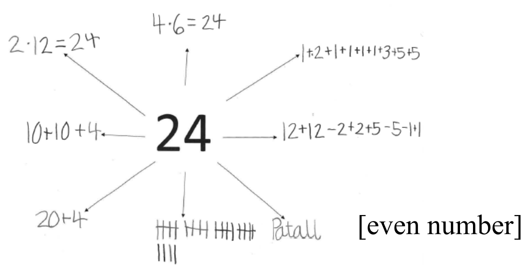

Figure 2. One example of student response from group B (B5).

The students had previously encountered similar tasks in whole-class discussions, and they were now allowed to collaborate and use manipulatives to develop their written responses. Unstructured material like milk caps and structured material like multi-base material were available for the students to use, but few students used the material. The teacher allowed them to work in groups, but most students decided to work individually on the task.

The students' responses were collected immediately after they had worked on the task for 15 minutes. To ensure anonymity, each worksheet was assigned a letter A or B to indicate what group the student was affiliated with and a number to distinguish between students in each group. For instance, A3 is student number 3 in group A. The students' written responses were analyzed using content analysis. We began by identifying how what was written related to aspects highlighted in previous research on children's understanding of number (see theoretical background), specifically. This was followed by a theory driven approach to content analysis (Fauskanger & Mosvold, 2015; Hsieh & Shannon, 2005). The theory driven analysis was based on 1) Thompson's (2003) quantity value and column value, 2) McIntosh et al.'s (1992) aspects of fully developed number sense, and 3) Jones et al.'s (1996) constructs of counting, partitioning, grouping and number relationships.

Findings

The 38 students provide a total of 161 responses. Students in group A provide 61 responses, and students in group B provide 100. Table 1 presents an overview of the different types of responses. Below we discuss these results with a focus on differences among students and groups of students. Examples of student responses are displayed to indicate the variation of responses given to the task.

Only one student (B4) does not provide any response to the task, whereas four students provide eight responses (see e.g., Figure 2). The two groups of students vary in the type of responses they give. In group A, 13 out of 17 students mention concepts or characteristics of the number 24 (e.g., even number, numerical neighbours, number of digits). The students in group B provide responses within all categories, but they have more focus on arithmetic operations than the students in group A. Five students provide examples that involve a combination of arithmetic operations. The two most advanced examples are *10 × 2 + 4* (B12) and *100 – 80 + 4* (B3). The responses contain few errors; 20 of the 38 students do not have any incorrect responses. Five students have two incorrect responses (A13, A16, B7, B16 and B21), but no students have more than two errors. Few responses from a student does not necessarily indicate a lack of knowledge. For instance, B13 only provides three responses, but these responses include three different operations: *12 + 12*, *28 – 4* and *8 × 3*.

The teacher in group B added eight arrows from the number 24 on the worksheet, and this adjustment might have influenced the students' interpretation of the task. For instance, 15 of the 21 students in group B appear to believe that the arrows should point to examples involving arithmetic operations rather than referring to place value. The students have some previous experience with the place value system; seven students—from both groups—draw arrows towards the digits of the number 24 or write about the value of the digits. For instance, students B8 and A15 write about how many tens and ones the number consists of like "2 tens and 4 ones", whereas student A3 write 10 above 2 and 1 above 4 to indicate tens and ones. This corresponds with what is often referred to as column value (Thompson, 2003). There are also examples of quantity value in the students' responses. For instance, student A7 draws an arrow from 2 and wrote 20, and another arrow from 4 and wrote 4. This student also write 10 and 1 over the digits 2 and 4.

Among the responses that include addition, many of these also indicate knowledge of place value. For instance, some students partition the numbers into tens and ones, or group numbers that add up to 10. Such responses are categorized as relating to place value, although they also include addition. Several students include *20 + 4* (six responses) and *10 + 10 + 4* (e.g., A2, A9, B1, B3, B4 and B6, 17 responses). Six students only include *10 + 10 + 4*, whereas two students include *10 + 14*. The responses of these students indicate that they have developed understanding of quantity value (Thompson, 2003).

The responses that include addition also provide other examples of partitioning. Examples are *4 + 5 + 5 + 5 + 5* (B1) and *8 + 2 + 8 + 2 + 4* (B12). These responses indicate ability in partitioning as well as regrouping, which are two important elements of Jones et al.'s (1996) model of number sense. Emerging understanding of place value involves knowing that grouping in ones and tens simplify the arithmetic operations (Jones et al., 1996). Two students' (A3 and B5) responses include tally marks or small circles that are grouped in fives. These are examples of grouping without using numerals and illustrate use of different representations of number (McIntosh et al., 1992). A response like *8 + 2 + 8 + 2 + 4* (B12) indicates understanding that one representation is more useful than another—in particular a representation that involves grouping of tens (cf. McIntosh et al., 1992).

Although addition is the most frequently used arithmetic operation in the responses, there are also examples that involve subtraction, multiplication and division. Some responses also combine arithmetical operations. An interesting example is *12 + 12 – 2 + 2 + 5 – 5 – 1 + 1* (B5). This response indicates knowledge of mathematical properties of operations, including awareness that adding and subtracting the same number does not change the answer. By providing the responses of both *10 × 2 + 4* and *10 + 10 + 4*, B12 indicates understanding of relationships between operations, and this might also be interpreted as indicating emerging understanding of how multiplication can facilitate addition (cf. McIntosh et al., 1992).

In group A, two students wrote down the numerical neighbours 23 and 25, either by writing that 24 is “numerical neighbour of 23 and 25” (student A6), or by writing 23 to the left of 24 and 25 to the right of 24 on the worksheet (A1). No students in group B mention numerical neighbours, and this may be due to the adjustment of the worksheet for group B that may not invite to mentioning numerical neighbours.

Among the students’ responses, only occasional errors occur. For instance, student B16 writes *10 + 10* above the 2 in 24. This is correct, but then the student writes 8 and *4 + 4* above the 4. This might indicate an understanding that two tens automatically mean that there must also be two ones.

Mathematical focus	**Gr. A**	**Gr. B**	**Examples (students)**	**Incorrect examples (students)**
Place value	3	1	2 tens and 4 ones (B8 and A15) Arrows under the number with 20 and 4, and arrows over the number with 10 and 1 (A7)	Arrows from the digit 2 with 10 +10 and 20, arrows from the digit 4 with 4 + 4 and 8 (B16)
Concepts and the number’s characteristics	21	4	Even numbers (B6) Numerical neighbors: 23 and 25 (A6) Two digits (A4)	
Writing digits, reversing	1	2	24, 42 (B1) Wrote that the numbers were reverse. Reversing the number 4 and the number 2 (B2)	
Addition Two different addends Some similar and some different addends Only similar addends	 5 8 12	 13 30 22	3 + 21 (B1), 9 + 15 = 24 (B19), 24 = 2 + 22 (B1) 8 + 2 + 8 + 2 + 4 (B12), 24 = 4 + 5 + 5 + 5 + 5 (B1), 10 + 10 + 4 = 24 (A2) 12 + 12 = 24 (A9), 4 + 4 + 4 + 4 + 4 + 4 = 24 (B6),	24 = 5 + 15 (B2), 12 + 2 (A16) 1 + 2 + 1 + 1 + 1 + 1 + 3 + 5 + 5 (B5), 10 + 3 + 4 + 1 = 24 (A2), 10 + 4 + 10 + 4 = 24 (A4) 4 + 4 + 4 + 4 (B1)
Subtraction	1	5	24 – 0 (B11), 28 – 4 = 24 (A8), 34 – 10 (B17), 25 – 1 (B21)	17 – 4 = 24 (B19), 100 – 86 = 24 (A13)
Multiplication	0	5	4 × 6 = 24 (B5), 24 × 1 (B12), 8 × 3 = 24 (B13)	
Division	0	4	24 ÷ 6 = 4 and 24 ÷ 4 = 6 (B3), We divide by four (B17)	
Combined arithmetical operations	0	5	100 – 80 + 4 = 24 (B3), 10 + 10 + 6 – 2 (B10), 12 + 12 – 2 + 2 + 5 – 5 – 1 + 1 (B5), 10 × 2 + 4 (B12)	
Different representations	2	1	B5	A3
Wrong answers	8	8		
Total	*61*	*100*		

Table 1: Overview of responses to the task, “The number of today is 24”

Concluding discussion

Analysis of students' responses to this open task about the number 24 provide indications of emerging understanding of place value. Many students are able to group and partition the number 24, but we cannot conclude from this study that the other students are lacking understanding in this respect. The students' responses might have been influenced by the way the task was presented, and it is important to consider the possibilities and limitations of a task like this. We will highlight five issues. First, arranging the worksheet like a blank piece of paper with the number 24 on top (group A, Figure 1) or as eight arrows sticking out from the number 24 (group B, Figure 2) might affect the students' responses. With students who fill in responses at the end of each of the eight arrows, the arrows may have restricted them from providing more responses to the task. Second, there is an issue related to the responses students give and if the responses are at a more advanced level than recommended by the curriculum at the actual grade level. For instance, when student B12 responds $10 \times 2 + 4$ and student B3 responds $24 \div 6 = 4$ and $24 \div 4 = 6$, they include multiplication and division in their responses—concepts that are in focus on a later grade level (Ministry of Education Research, 2013). Third, there is an issue of how to interpret the lack of responses from some participants. Some students do not provide any response or one response only, but there is not necessarily a correlation between number of responses to an open task like this and students' knowledge and understanding of place value. Fourth, one might wonder why so few students use the concrete materials that were available or work in groups. Finally, one can ask why no students mentioned anything about applications of the number 24, e.g. that 24^{th} of December is Christmas Eve. The reason can be that this was a written task, and the students may have interpreted it as a task where they were supposed to make arithmetic problems. Following up on the students' responses by adding cognitive interviews might have provided additional information about their number sense. An interview with the teachers about their teaching in advance could also have given answers to some of these questions.

Our focus in this study has been strictly on the students' responses, but the results of our study may also have implications for teachers. Investigations of Grade 2 students' mathematical reflections about the number 24 may indicate some mathematical demands teachers are faced with when facilitating such an open-ended activity. For instance, teachers must interpret students' responses on tasks like these and act upon them—often quickly. A teacher must also figure out what students know and are able to do from looking at their responses to open-ended questions like this. These are some examples of the mathematical demands that are embedded in the work of teaching early number sense. To skilfully carry out the work of teaching, teachers need a professional knowledge that includes—but is not restricted to—knowledge of quantity value and column value (e.g., Thompson, 2003), knowing models for examining important components of number sense like counting, partitioning, grouping and number relationships (e.g., Jones et al., 1996; McIntosh et al., 1992). Such knowledge is required to analyze students' responses and draw out their thinking through carefully selected questions and tasks and to consider and check alternative interpretations of the students' ideas as visible in their written responses.

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