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Fatigue design challenges: Recent linear and nonlinear models

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Fatigue design challenges: Recent linear and nonlinear models

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Abstract. Reliable fatigue design rules affect the proactive identification of safety parameters in engineering structures. Numerous fatigue crack initiation and propagation models, linear and nonlinear, have been developed for designing purposes or estimation of the remaining life of aging structures. Depending on the adopted assumptions, the accuracy varies for different loading histories, loading types, and materials. Semi empirical models are simple but yield significant inaccuracies. Models with better theoretical basis provide better accuracy, but implementation in real conditions is problematic. In the present work, a review of author's recent fatigue crack initiation and propagation models based on physical mechanisms is presented and improvements are proposed. Verification of the models on test results is provided and discussed.

1. Introduction

Successful structural design and data driven safety assessment require reliable fracture mechanics tools for fatigue damage accumulation modelling. Palmgren and Miner were the first who attempted to calculate the fatigue damage [1, 2]. Due to its simplicity, their fatigue damage accumulation rule is widely used. However, the Palmgren-Miner's rule is inconsistent with the nonlinear material damage mechanisms [3, 4]. Depending on the type of loading history, the Miner's rule can overestimate or underestimate the fatigue damage accumulation prediction [5]. Its accuracy is better for loading spectra containing high stress levels. The above rule is linear and does not take into account the loading sequence effect and the material damage accumulation theory. The Miner's rule can be used for crack initiation only. Manson and Halford [6, 7] have improved the linear fatigue damage rule. Their double-linear damage model (DLDR) preserves (to some extend) the simplicity of Palmgren-Miner's rule and takes into account two different fatigue mechanisms. They have proposed two fatigue stages in order to model both the crack initiation and crack propagation phases.

However, the load interaction effects [8, 9] and the mixed mode fatigue crack propagation [10-12] are not accounted for. Known nonlinear models are based on continuum damage theory [13], isodamage straight lines [e.g. 14, 15], and Manson-Halford's concept [16]. Further literature review on nonlinear damage accumulation models has been published recently [17]. For crack propagation modelling the Paris rule [18] is well known. However, this old rule is linear, and therefore it does not take into account the load interaction effects [19, 20]. Modification of this rule has been attempted by Wheeler [21] and Willenborg [22] in order to improve the fatigue crack growth prediction for loading histories containing overloads. However, the modelling of the retardation effects has semi empirical basis. Elber [23-25] is the first who modelled the crack closure as a predominant mechanism for overload-induced fatigue crack growth retardation. The proposed model yields accurate results for structural elements subjected to conditions where the crack closure mechanism is predominant (large overload-induced plastic zones).

However, apart from the crack closure mechanism due to residual plastic deformation, more than four other material mechanisms affecting the fatigue crack growth are known, i.e. strain hardening of the material within the overload crack-tip plastic zone [e.g. 26, 27], plastic blunting and re-sharpening [28], fracture surface roughness which causes contact between the crack faces at non-zero loads reducing the effective ΔK [29-31], and crack branching [9, 10].

In the present work, the new crack initiation theory of the S-N fatigue damage envelope [32], developed by the author, will be implemented on Al-2024 specimens subjected to high-low (H-L) and low-high (L-H) two-stage loading. Moreover, the overload-induced crack growth retardation model [26, 27, 33, 34] based on strain hardening, will be improved to take into account both overload and underload effects.

2. Fatigue crack initiation

A recent advance in fatigue crack initiation modelling is based on the proposal that the area bounded by the S and N axes and the S-N curve can provide a damage map for the material [32].

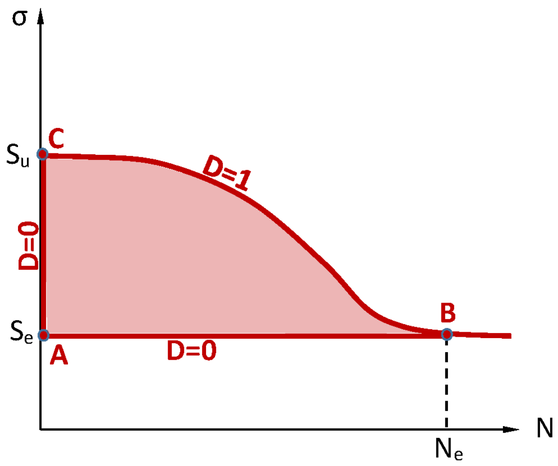

Figure 1. The S-N damage envelope.

The boundary AB of the envelope (Fig. 1) corresponding to the endurance limit S_e should be considered as an isodamage line with damage D=0. The boundary AC (Fig. 1) corresponds to number of loading cycles N=0. Therefore, it should also be considered as an isodamage line with D=0. Moreover, the boundary CB is the S-N curve corresponds to crack initiation, i.e. D=1. Finite element analysis performed in [32] has derived a damage map and isodamage lines. The damage zones for Al-2024 have been derived with the aid of ANSYS [32, 35] using the dimensionless parameters in (1) and (2) for stress and loading cycles and are demonstrated in Figure 2.

$$\sigma_i^* = \frac{\sigma_i - S_e}{S_u - S_e} \qquad 0 \le \sigma_i^* \le 1 \tag{1}$$

$$n_i^* = \frac{n}{N_e} \qquad 0 \le n_i^* \le 1 \tag{2}$$

The colours of the damage zones correspond to $Log(D)$. Therefore, taking into account the vertical legend of colors for $Log(D)$, the isodamage lines correspond to values $D = 10^{-10}, 10^{-8.88889}, ..., 10^{0}$. Using the results of Fig. 2, damage curves D vs n / N_f have been derived in Fig. 3 for stress levels $\sigma^* = 0.088,\ 0.248,\ 0.408,\ 0.888$.

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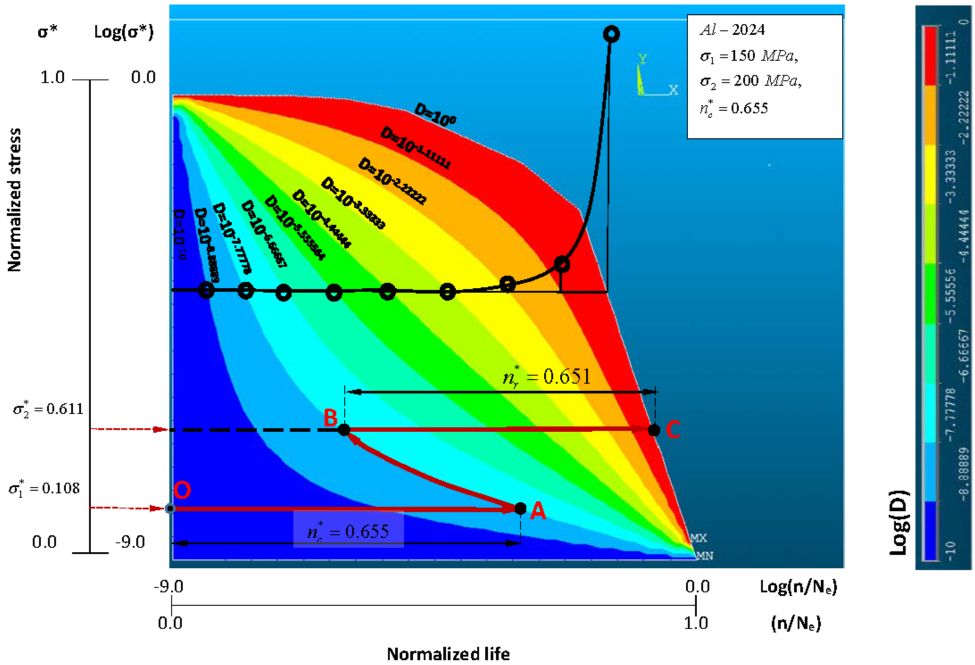

Figure 2. Damage zones for Al-2024.

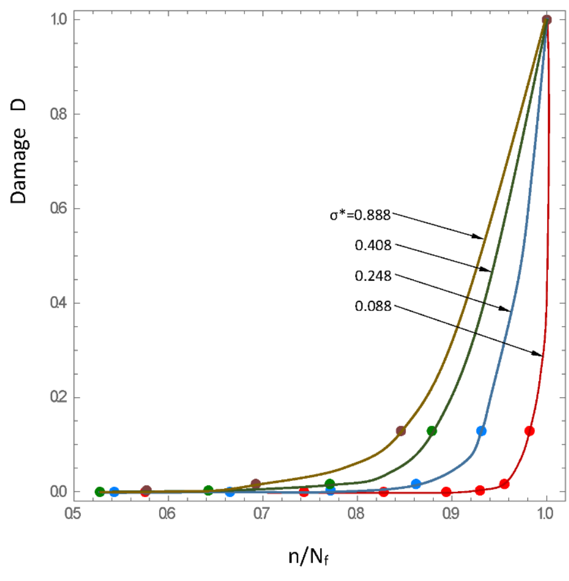

Figure 3. Damage curves for Al-2024.

The damage envelope in Figure 2 can be used for the calculation of the remaining life for two-stage loading (Figure 4). For a loading block with stress level $\sigma_1 = 150MPa$ (i.e. $\sigma_1^* = 0.108$) and loading period $n_c^* = 0.655$ (consumed life), the accumulated damage is $D = 10^{-7.77778}$ (point A in Figure 2). The continuation of the loading for a higher stress $\sigma_2 = 200MPa$ (i.e. $\sigma_2^* = 0.611$) should start from the already accumulated damage $D = 10^{-7.77778}$ corresponding to point B (same isodamage line with point A). The remaining life is demonstrated by the segment BC, i.e. $n_r^* = 0.651$. This procedure is followed by a number of loading data H/L and L/H borrowed by the ref. [36] and the results are shown in Figure 4.

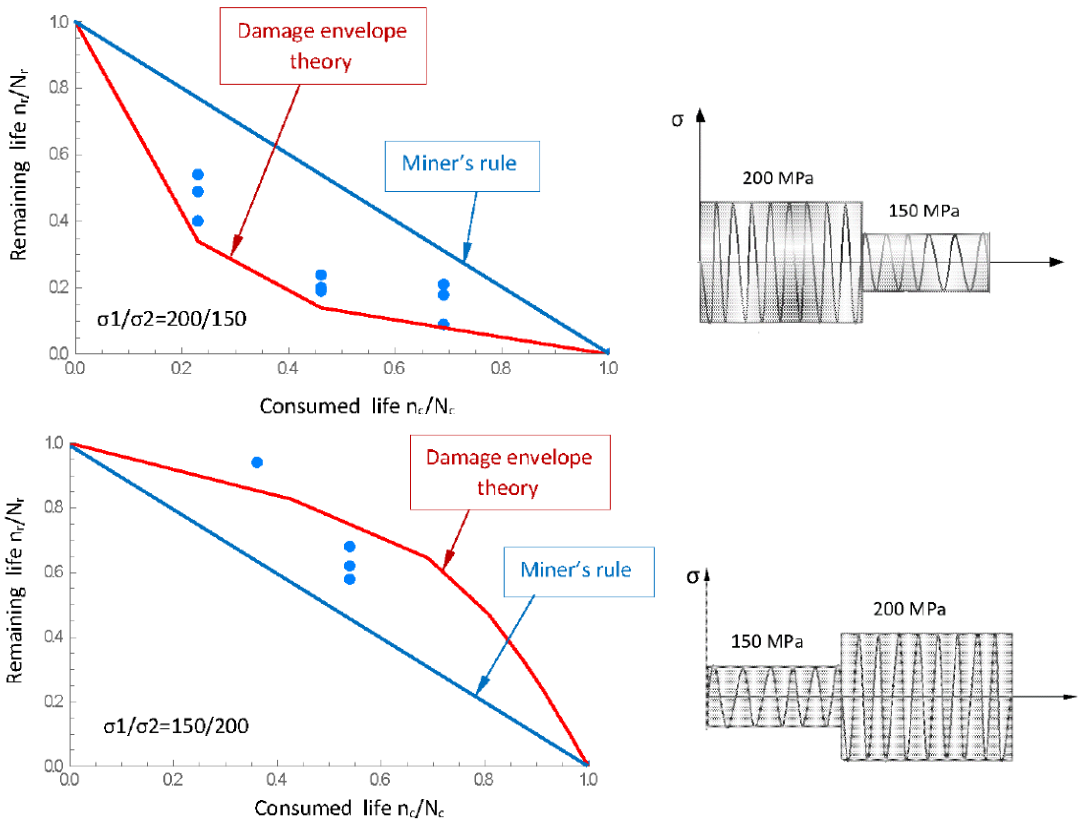

Figure 4. Remaining life predictions for Al-2024 specimens subjected to H/L and L/H two-stage loading

The results indicate that the S-N damage envelope theory provides successful predictions of the remaining life for these loading cases. Unlike the above nonlinear predictions, Miner's rule has overestimated the remaining life for H/L loads, and has underestimated it for the inverse loading sequence.

3. Fatigue crack propagation

Variable amplitude loading sequences are associated with transient effects in fatigue crack growth. Overloads yield significant retardation in the fatigue crack growth rate, and underloads reduce the above effect. Among the proposed tools for fatigue crack growth prediction for variable amplitude loading, Wheeler's [21], Whillenborg's [22], and Elber's [23] models are the most known, and have been used for further improvements [37-40]. Wheeler's and Whillenborg's models have semi empirical basis. Elber's model and its improvements take into account the overload retardation effect due to the crack closure mechanism [41-44] and provide accurate predictions for loading cases where the crack closure mechanism is dominating the fatigue crack growth. The first attempt for modelling the effect of the material hardening within the overload plastic zone took place in 1995 [26, 27]. In the original version of this research, the crack growth rate after an overload is correlated to the actual yield stress of the hardened material within the overload plastic zone. The main idea is based on the assumption that the baseline plastic zone in the crack tip (Figure 5) is subjected to low-cycle-fatigue conditions due to high strain amplitude [26, 27, 33, 34].

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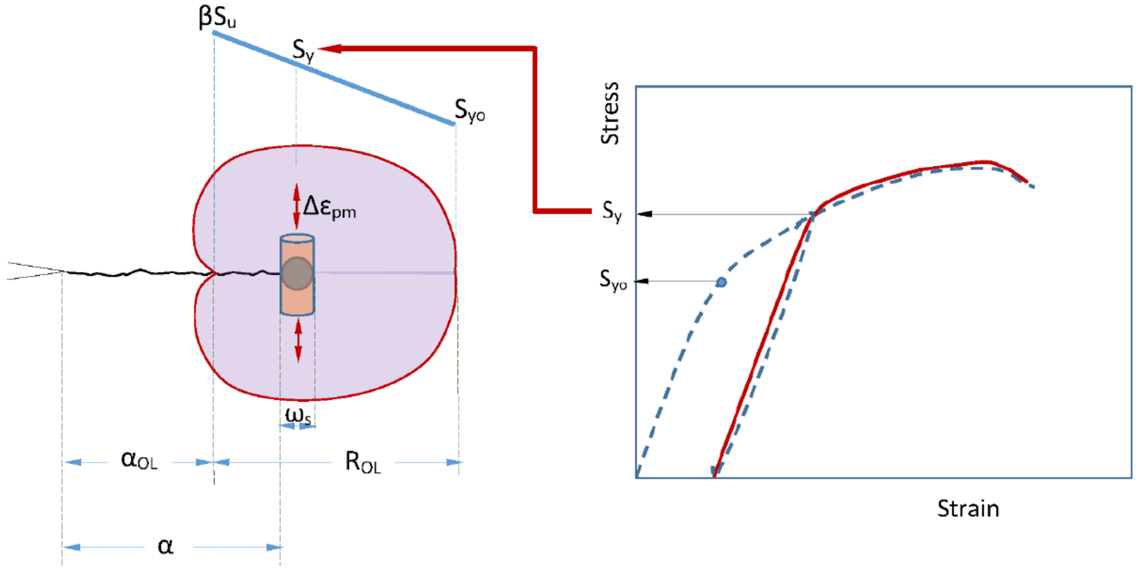

Figure 5. Strain hardening-induced fatigue crack growth retardation model.

The fatigue crack growth rate da/dN is approximated by the formula

$$\frac{da}{dN} = \frac{\omega_s}{\Delta N} \tag{3}$$

where ω_s is the plastic zone size (Figure 5) of the constant amplitude loading $\Delta\sigma$ (Figure 6), and ΔN is the required number of cycles for crack growth by a length ω_s. With the aid of Coffin-Manson rule for low cycle, and following fracture mechanics analysis, the following crack growth retardation model [26, 27] is proposed:

$$\frac{da}{dN} = \frac{\pi}{32}\left(\frac{1}{2K_{cr}^2}\right)^{1/m} \frac{\Delta K^{2(1+1/m)}}{S_y^2} \tag{4}$$

where S_y is the actual yield stress of the hardened material within the overload plastic zone (Fig. 5). Depending on the location of the crack tip, the S_y of the hardened material takes values $S_y = S_u$ for $a = a_{OL}$ (immediately after the overload), and $S_y = S_{yo}$ for $a = a_{OL} + R_{OL}$ when the retardation effect ends.

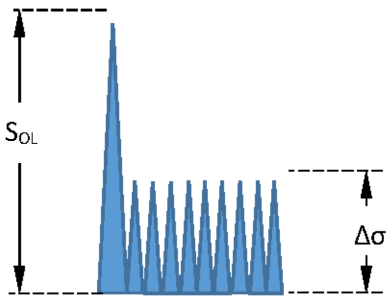

Figure 6. Constant amplitude loading containing an overload.

A simple linear distribution of the S_y between the boundary values S_u, S_{yo} is assumed, and a retardation factor is derived, as given in (5).

$$\lambda = \left(\frac{S_{yo}}{S_y}\right)^2 \qquad \left(S_{yo} / S_u\right)^2 \leq \lambda \leq 1 \tag{5}$$

Since the power of the ratio $\left(S_{yo} / S_y\right)$ is 2, the effect of the strain hardening seems to dominate the overload-induced retardation. However, the above model does not take into account the effect of underloads. It is well known that underloads eliminate the retardation effect of overloads [e.g. 45]. An underload with stress $\sigma_{UL} = -\sigma_{OL}$ can cancel the retardation effect of the overload. Therefore, a correction factor in (6) is proposed to adjust the value of the parameter S_u in Figure 5.

$$\beta = \left(\frac{S_{yo}}{S_u} - 1\right)\frac{|\sigma_{UL}|}{|\sigma_{OL}|} + 1 \tag{6}$$

Therefore,

$$\beta S_u = \begin{cases} S_{yo} & for\ \sigma_{UL} = -\sigma_{OL} \\ S_u & for\ \sigma_{UL} = 0 \end{cases} \tag{7}$$

For $\sigma_{UL} = -\sigma_{OL}$ the value of the maximum yield stress of the hardened material (Figure 5) is $\beta S_u = S_{yo}$. In this case $\lambda = \left(S_{yo} / \beta S_u\right)^2 = 1$. Therefore, no retardation effect takes place. For $\sigma_{UL} = 0$, eq. (6) yields $\beta = 1$. Therefore, $\lambda = \left(S_{yo} / S_u\right)^2$ meaning that considerable retardation occurs. Taking into account the above improvement, the model is applied to the random loading history M90 (0.27) borrowed by ref. [46]. The above loading contains overloads and underloads. The modified rainflow counting method described in [26, 27] is implemented in order to transform the irregular history to a loading history containing full cycles. The theoretical predictions are correlated to test results [46], indicating good agreement (Figure 7).

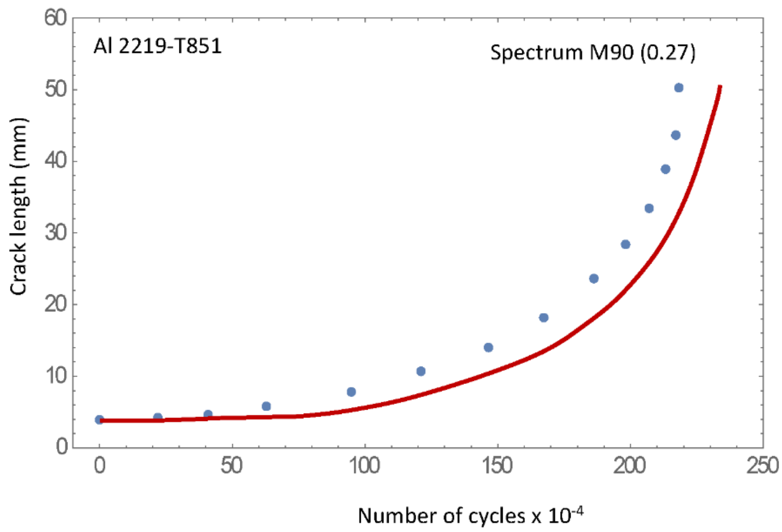

Figure 7. Correlation of the prediction of the proposed method to test results borrowed from ref. [46].

4. Conclusions

1. Fatigue design challenges in aerospace industry have been discussed.
2. Recent advances in fatigue crack initiation and propagation prediction have been presented.
3. Author's recent models and their improvements have been implemented to predict the fatigue crack initiation and propagation in variable amplitude loading cases.
4. Verification of the S-N damage envelope theory for crack initiation, and the strain hardening model for crack propagation on test results for Al-2024 and Al-2219 has been provided.

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