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Good mathematics teaching as constructed in Norwegian teachers' discourses

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This paper explores the notion of good mathematics teaching as constructed in the discourses of practicing Norwegian mathematics teachers. Analyses of data from group interviews show that the teachers tend to conceptualize good mathematics teaching in terms of structuring lessons, differentiating in accordance with individual students' different needs, mathematical communication between teacher and students, as well as teachers' use of tasks and resources. In addition to this, the teachers emphasize student engagement and students' learning when discussing good mathematics teaching. Possible implications for these findings are discussed.

Introduction

A continually growing body of research investigates what constitutes good mathematics teaching (Franke, Kazemi, & Battey, 2007). To better support teachers in learning to carry out ambitious teaching practices fundamental for supporting children's learning of mathematics (e.g., Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010), the focus of in-service teacher education has recently shifted from developing pre-service teachers' knowledge toward developing teaching practices (Zeichner, 2012). Although the question of what constitutes good mathematics teaching practices has been examined in numerous studies, the question of how to define good mathematics teaching continues to remain unresolved (Cai, Kaiser, Perry, & Wong, 2009; Franke et al., 2007; Krainer, 2005; Li, 2011). In their overview of research on mathematics teaching and classroom practices, Franke et al. (2007) highlight creating mathematical classroom discourse, developing norms and building relationships that support mathematical learning as three core features of good mathematics teaching, but no universal definition has been developed to this date. Attempts to define good mathematics teaching seem to depend on the views of mathematics teacher educators and mathematics teachers (Cai et al., 2009; Li, 2011). Since attempts to define good mathematics teaching can be regarded as a process of establishing norms, the views of teachers and teacher educators are arguably important

(Krainer, 2005). The views about good teaching also influence teachers' decision-making (Krainer, 2005), and they might thus influence the development and adaptation of common ideas and recommendations across countries as well as sharing of visions of effective classroom practice (Givvin, Jacobs, Hollingsworth, & Hiebert, 2009).

There are many ways to denote the way teachers talk about good mathematics teaching, but a common approach is to study teachers' views about good mathematics teaching through analysis of their discourses (Franke et al., 2007; Hemmi & Ryve, 2015). In the present study, we analyze data from Norwegian teachers' group discussions with a focus on how they construct the notion of "good mathematics teaching" in their discourses. Our approach to analyze data draws upon the study by Hemmi and Ryve (2015) of how Finnish and Swedish teacher educators conceptualize effective mathematics teaching. Where these researchers focused on teacher educators, we focus on practicing teachers. We address the following research question: What aspects of good mathematics teaching constitute a group of Norwegian mathematics teachers' discourse? To answer this question, we analyze the discussions given by 20 Norwegian teachers in focus-group interviews at the end of the first day in a professional development project. To our knowledge, few studies have examined Norwegian teachers' construction of good mathematics teaching from studying their discourses in focused discussions.

Methodology

The study presented in this paper is part of a larger project called "Mastering Ambitious Mathematics Teaching" (MAM). In this project a model for school-based professional development of in-service mathematics teachers have been developed along with resources for teachers. The model as well as the resources was originally developed to be used in pre-service teacher education. The model has repeated enactment of specifically designed instructional activities to be used in the teachers' instruction as a point of departure, and all the activities focus on numbers and operations. The activities are developed to learn in, from, and for teaching practice (see e.g., Kazemi & Wæge, 2015; Lampert et al., 2010; Valenta & Wæge, 2017).

Twenty teachers participated in the part of the MAM project that is presented in this paper. Our focus is not on investigating differences among teachers with different background, but we provide some background information to inform the readers. The participating teachers work at 10 different schools, and they teach fifth, sixth or seventh grade. Their age range vary from 23 to 59 years, their teaching experience vary from one to 30 years, and their formal education in mathematics/mathematics education vary between 15 ECTS and 120 ECTS (i.e. master's degree). The overall design and selected population makes it possible to

draw conclusions in relation to these teachers only and we cannot make any claims about the Norwegian teacher population in general. The participating teachers are volunteer participants in the MAM project, which might imply that they are more concerned about mathematics teaching than many other teachers might be. In our efforts to learn more about how good mathematics teaching is constructed in these teachers' discourses, we arranged three focus-group interviews. The interviews had six or seven participants and lasted from 44 to 51 minutes. After some introductory questions, the following main questions served as point of departure for the discussions: 1) How would you characterize a good mathematics lesson? and 2) How would you characterize what for you is a "normal" mathematics lesson?

For the purpose of this paper, the focus-group discussion related to these questions were analyzed by using content analysis (Hsieh & Shannon, 2005). Two researchers (authors 1 and 3 of this paper) coded all the data material independently. Both researchers developed individual codes and grouped them into categories, in an iterative process including several cycles of analysis. The two researchers then reconciled and agreed upon categories and corresponding codes. The codes and categories were shared with two other researchers (authors 2 and 4 of this paper) who coded the data material using these codes and categories to validate the coding. Some minor adjustments to the codes were made during this process, but the categories listed remained the same:

1. Teacher's instruction/role
2. Structure in lessons
3. Differentiation
4. Communication
5. Use of tasks and resources
6. Student engagement
7. Students' learning

Categories 1–5 refer to the teachers' actions, but categories 3–5 also include the students' actions. Categories 6 and 7 refer to the students only, focusing on their engagement and learning. When seen in relation, the seven categories indicate a shared responsibility for good mathematics teaching by teachers and student. Examples of codes for the category of Teacher's instruction/role (1) are to: a) be a guide, not a lecturer, b) find a way to present the content in engaging way, c) using precise mathematical language, d) work in depth with concepts, e) predict student response, f) find a way to respond to students' thinking, g) build on students' thinking towards the learning goal, h) ask good questions, and i) use resources critically.

As can be seen from the next section, these categories are partly overlapping. An example is the category of Teacher's instruction/role (1) and the category of Communication (4): Parallel to highlighting mathematical communication as

central for students' learning (4), the teacher´s role as facilitator of such discussions (1) is emphasized. Although these categories are partly overlapping, we stick to these since they all emerged in the coding process from the content analysis, and they refer to the similar aspects, but in partly different ways (Hsieh & Shannon, 2005). The seven categories illustrate the teachers' own conceptualizations of good mathematics teaching, constructed from the teachers' discourses.

Results

Our analysis reveals that the group of Norwegian teachers tend to conceptualize good mathematics teaching in terms of paying attention to their own roles as facilitators in the classroom, structuring lessons, differentiating in accordance with individual students' various needs, the mathematical communication between teacher and students, as well as teachers' use of tasks and resources. In addition to this, the teachers emphasize student engagement and student learning when discussing good mathematics teaching. In the following, examples from each of these conceptualizations will be presented.

Teacher's instruction/role

The teachers express in the interviews that it is important to present the mathematical content in an engaging way. They want to be facilitators and stimulate for mathematical discussions by responding to students' thinking, build on students' initiatives, and guide them towards the learning goals. For instance, in one of the group discussions, a teacher says that "formative assessment should be a part of our teaching all the time, to stimulate and help them [the students] further." Formative assessment is however, also described as challenging. The teachers stress the importance of working in depth with mathematical concepts like multiplicative structures and emphasize the use of a precise mathematical language in lessons. In one of the interviews, prediction of students' responses is highlighted. In another interview, teachers describe challenges of teaching. One teacher states that it is challenging to pose good questions in the classroom conversation, while another finds it challenging to summarize lessons in a constructive way due to lack of time.

Structure in lessons

The teachers dwell on the importance of having a good structure in mathematics lessons. They suggest that mathematics teachers must have clear content goals for the lessons. These goals should be made explicit in the beginning of the lesson in a way that directs the students' attention towards the content in focus and support their learning of the content, without reducing opportunities for thinking and exploration. They also make a point of varying the lessons, for instance introductions, work stations and discussions. At the end of a lesson, teachers

should sum up and make connections to the learning goals. One teacher argues that this is especially important when working with inquiry-based tasks: "One can really 'stray from the subject' without a goal in this kind of teaching."

Differentiation

Another aspect that pointed out by the teachers as important in good mathematics teaching is differentiation. This is exemplified by one of the teachers who states that it is important "to reach all students, find tasks that are suitable for everybody, both those students that strive in mathematics and those who are high-achieving." The teachers find it important to allow all students to participate, either by using tasks that can be worked on in different ways or by organizing the students in groups where they can work on differentiated tasks. Differences between students are conceived as challenging, but the teachers maintain that differences can also be an asset, since different students' ways of thinking can come up. In one of the group discussions, a teacher says that, "oftentimes, students are cleverer to explain to each other than I am as a teacher, since I often use a more difficult language in my explanations." In one of the other group discussions, a teacher gives an example of a high-achieving student who had investigated the commutative law and made "a guest lecture" for her fifth-grade students. This teacher also expresses that other high-achieving students have been investigating "other things", as she expressed it, and such mathematical inputs are valuable both for these students themselves and for the other students in her class.

Communication

The teachers agree that mathematical discussions are central for students' learning and therefore important for good mathematics teaching. For instance, one of the teachers contends that, "discussions are important, no matter what type of activity. They are important for students' learning, and they are important for the teacher to get an impression of students' understanding." The teachers stress that communication in a mathematics classroom must be two-ways. Students must participate actively in discussions and explain to each other, and teachers have to elicit and respond to students' ideas.

Use of tasks and resources

In their conceptualization of good mathematics teaching, the teachers express that it is important to introduce mathematical tasks that are motivating for their students. One teacher recalls an example of a task that was motivating for his students: finding patterns to come up with a recursive formula. In addition to being motivating, the teachers suggest that tasks should be open and stimulate different approaches to reaching a solution or stimulate to find different solutions. Another teacher tells that she could present a task for her students and say, "help me to solve it!" Other teachers suggest that a good mathematical task is open for

differentiation. Different uses of games with cards, dices and computers are mentioned as teaching resources in the interviews.

Student engagement

"Good mathematics teaching can be recognized when all students say 'No!' when you tell them that the lesson is finished", a teacher suggests in the group discussion. Student engagement is presented as an important characteristic of good mathematics teaching by the teachers. They describe student engagement as active participation, eagerness to solve a given problem, listening and trying to understand. The teachers stress that hard work and effort, followed by gradual mastery, is decisive for student engagement and for the quality of mathematics teaching.

Students' learning

The teachers characterize good mathematics teaching as teaching that provides students with opportunities to think, be creative, discover, use their knowledge in new problems, and develop understanding. One of the teachers declares that, "it is great to see students using strategies we have been working on before in new situations. That is a good mathematics lesson." For student learning, the teachers emphasize concentrated work on problems, explaining to others and listening to other students' explanations. Finally, the teachers suggest that good mathematics teaching supports students' learning of a way to work in mathematics, use of mathematical terminology and knowing certain facts by heart.

Discussion

Several recent studies investigate teachers' discourse of good mathematics teaching in different contexts (e.g., Krainer, 2005; Li, 2011). Our study adds to this discussion and thereby contributes to the ongoing efforts to conceptualize good mathematics teaching (e.g., Cai et al., 2009; Franke et al., 2007; Givvin et al., 2009). From our analysis of focus-group interviews of 20 Norwegian mathematics teachers, we notice that the teachers conceptualize good mathematics teaching in terms of structuring lessons, differentiating in accordance with individual students' different needs, two-way mathematical communication between teacher and students, as well as teachers' use of tasks and resources. It was also shown that the teachers emphasize student engagement and student learning when discussing good mathematics teaching, and this corresponds with results from international studies (e.g., Li, 2011).

Some conceptualizations of good mathematics teaching found in our study correspond with findings from similar studies in other Nordic contexts (e.g., Hemmi & Ryve, 2015). For instance, the teachers express that they want to be facilitators and build their teaching on individual students' thinking and initiatives. Like in the Finnish teacher education context (Hemmi & Ryve, 2015), the

Norwegian teachers emphasize the structure of the mathematics lesson and in-depth work with mathematical concepts by using a precise mathematical language in lessons. The Norwegian teachers also seem to agree with the Swedish and Finnish teacher educators about enabling individual students to participate, while simultaneously viewing differences among students as an asset. Moreover, mathematical discussions are emphasized as an important element of good mathematics teaching. This corresponds with a larger body of research that highlights creation of mathematical classroom discourse as a core feature of good mathematics teaching (Franke et al., 2007).

There are also some differences between the conceptualizations of good mathematics teaching found in the Norwegian context and previous findings in other countries. For instance, Hemmi and Ryve (2015) suggest that Swedish teacher educators emphasize constructivist thinking and student-centered teaching, and that their interpretations in this respect are extreme, but the Norwegian teachers do not have a similar emphasis in their conceptualizations as shown in the emphasis on the importance of teachers' role. In terms of differentiation, there appear to be some nuances in definitions across countries. In the Swedish teacher education discourse, differentiation is operationalized referring to letting all students work at their own pace and level, whereas in the Finnish context, keeping the group of students within the same mathematical area and at the same time support and challenge individual students are highlighted. Furthermore, the 20 Norwegian teachers contend that it is important to introduce mathematical tasks that are motivating for their students, or tasks which are open for differentiation. This aspect of these Norwegian teachers' conceptualization of good mathematics teaching appears closer to what is found in the Finnish teacher education context (Hemmi & Ryve, 2015), where problem-solving and inquiry approaches are emphasized. In the Swedish context, however, they emphasize spontaneous everyday situations and thematic work.

The conceptualization of good mathematics teaching that seems to emerge from the present study – balancing the communication between the teacher and the students – appears to be somewhere between the contexts in Finland and Sweden. Whereas the Finnish discourse described the teacher as "a very proactive agent in the classroom" (Hemmi & Ryve, 2015, p. 515), the Swedish discourse concentrated on "basing teaching on students' thinking, ideas and interests" (Hemmi & Ryve, 2015, p. 511). The Norwegian image of good mathematics teaching seems to be found in between these two. Whereas the aspects of teacher's instruction/role, the structure in lessons, differentiation, communication and use of tasks and resources in the discourse mainly refer to the teachers' actions, the aspects of student engagement and students learning refer mainly to the students. The Norwegian teachers describe student engagement as an important characteristics of good mathematics teaching. Active participation, eagerness to

solve a given problem, willingness to listen and try to understand, as well as hard work, are described as important prerequisites for good mathematics teaching. In the discussions, part of the responsibility for the quality of mathematics teaching is thus given to the students. This is in line with previous studies in the Norwegian context (Fauskanger, 2016), but differs from findings in Sweden and Finland (Hemmi & Ryve, 2015). However, in line with Swedish teacher educators, the Norwegian teachers also characterize good mathematics teaching as giving opportunities for students to think, be creative and discover. The Norwegian teachers also contend that the responsibility for engagement is supposed to be shared among teachers and students (cf. Fauskanger, 2017). Such a shared responsibility is also what constitutes the Norwegian teachers' discourse about student learning. The teachers are, however, responsible for helping their students to learn mathematics, the Norwegian teachers say.

When comparing with results from international studies outside the Nordic context, it appears that the focus on student learning is always at the center. Views about the role of the teacher, however, seem to differ across countries. Whereas US mathematics teachers emphasize classroom management (e.g., Cai et al., 2009), mathematics teachers in countries like China seem to focus more on teachers' preparation, content knowledge and understanding of textbook contents (Cai et al., 2009; Li, 2011). The Norwegian mathematics teachers in our study do not emphasize classroom management, and their views appear different from those of US teachers in this respect. Unlike Chinese teachers, however, these Norwegian teachers do not emphasize teachers' knowledge, preparation and understanding of textbook content (cf. Li, 2011). Hemmi and Ryve (2015) report that Swedish and Finnish teacher educators stress the importance of teacher knowledge but practicing teachers in these countries might have different views.

Conclusion

By providing some perspectives of Norwegian teachers' views of good mathematics teaching, the results from this study add to the body of literature on views of good mathematics teaching (e.g., Cai et al., 2009; Givving et al., 2009; Hemmi & Ryve, 2015; Li, 2011). The Norwegian mathematics teachers in our study share some views of good mathematics teaching with teachers and educators from other countries, but their views also differ from findings in international studies in certain respects. We notice in particular that these Norwegian teachers emphasize a shared responsibility for engagement and learning among teachers and students, and they want to facilitate good mathematical discussions by using tasks and activities that enable differentiation among students.

Although we have described our sample as "Norwegian teachers", we do not claim that the results from this study are representative for the entire population of Norwegian teachers. The participants in this project are special, in that they are

volunteer participants in the MAM project, which implies that they are more concerned about mathematics teaching than many other teachers are. Having said this, we believe that the results from this study may indicate some views of mathematics teaching that are characteristic for the Norwegian context. Like Givvin et al. (2009), we believe that variations in teachers' views about mathematics teaching across countries may relate to the cultural differences in teaching itself, and the findings from our study seem to correspond with observations of mathematics teaching in Norway.

Since attempts to define good mathematics teaching can be regarded as a process of establishing norms (Franke et al., 2007), and since views influence decision-making (Krainer, 2005), mathematics teachers' views of good mathematics teaching are arguably important. Further research may be useful to investigate if the views of good mathematics teaching reported in this study correspond with the views of a larger population of Norwegian mathematics teachers. In addition, we suggest that it may be useful to explore similarities and differences between the views of teachers and teacher educators in the Norwegian context, since there may be cross-professional differences even within countries. Researching good mathematics teaching as constructed in teachers' discourses can contribute to a better understanding of teachers' views and thus allow teacher educators to tailor their in-service education.

References

Cai, J., Kaiser, G., Perry, B., & Wong, N.-Y. (Eds.) (2009). *Effective mathematics teaching from teachers' perspectives. National and cross-national studies*. Rotterdam: Sense Publishers.

Fauskanger, J. (2017). Kunnskap nødvendig for effektiv matematikkundervisning – slik lærere selv ser det. *Norsk Pedagogisk Tidsskrift, 101*(1), 41–52.

Fauskanger, J. (2016). Matematikklæreres oppfatninger om ingrediensene i god matematikkundervisning. *Acta Didactica Norge, 10*(3), Art. 5.

Franke, M.L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F.K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 225–256). Charlotte, NC: Information Age Publishing.

Givvin, K.B., Jacobs, J., Hollingsworth, H., & Hiebert, J. (2009). What is effective math teaching? International educators' judgments of mathematics lessons from the TIMSS 1999 Video Study. In Cai J., Kaiser G., Perry B. & Wong N.-Y. (Eds.), *Effective mathematics teaching from teachers' perspectives: National and cross-national studies* (pp. 37–69). Rotterdam: Sense Publishers.

Hemmi, K., & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education, 18*(6), 501–521.

Hsieh, H.-F., & Shannon, S.E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research, 15*(9), 1277–1288.

Kazemi, E., & Wæge, K. (2015). Learning to teach within practice-based methods course. *Mathematics Teacher Education and Development, 17*(2), 125–145.

Krainer, K. (2005). Editorial. What is "good" mathematics teaching, and how can research inform practice and policy? *Journal of Mathematics Teacher Education, 8*(2), 75–81.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M.K. Stein & L. Kucan (Eds.), *Instructional explanations in the disciplines* (pp. 129–141). New York, NY: Springer.

Li, Y. (2011). Elementary teachers' thinking about a good mathematics lesson. *International Journal of Science and Mathematics Education, 9*(4), 949–973.

Valenta, A. & Wæge, K. (2017). Rehearsals in work with in-service mathematics teachers. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the tenth congress of the European Society for Research in Mathematics Education* (pp. 3881–3888). Dublin, Ireland: DCU Institute of Education and ERME.

Zeichner, K. (2012). The turn once again toward practice-based teacher education. *Journal of Teacher Education, 63*(5), 376–382.