

Introduction to the work of TWG19: Mathematics Teaching and Teacher Practice(s)

Reidar Mosvold,¹ Helena Grundén,² Mark Hoover,³ Siún Nic Mhuiri,⁴ and Edyta Nowinska⁵

¹University of Stavanger, Stavanger, Norway; reidar.mosvold@uis.no

²Dalarna University, Falun, Sweden; hgn@du.se

³University of Michigan, Ann Arbor, USA; mhoover@umich.edu

⁴Dublin City University, Dublin, Ireland; siun.nicmhuiri@dcu.ie

⁵Universität Osnabrück, Osnabrück, Germany; enowinska@uni-osnabrueck.de

This introduction for TWG19 offers a brief history of the group and describes past challenges the group has experienced when discussing papers — seeing papers as related and as contributing to a common effort. These challenges led us as TWG19 team leaders to develop three initiatives to support communication among researchers who work in different contexts with different purposes. The initiatives are presented and used to discuss the papers. We conclude with implications for the future.

Introduction

Thematic Working Group 19 (TWG19) is concerned with the study of mathematics teaching and teacher practice(s). From its inception, it has struggled to clarify its focus. TWG19 was created after CERME8 from a division of a TWG titled *From a Study of Teaching Practices to Issues in Teacher Education*. Having grown too large, this group split into three: one targeting teacher education and professional development (TWG18); another targeting teacher knowledge, beliefs, and identity (TWG20); and a third with an uncertain target (TWG19). Participants at CERMEs 9, 10, and 11 spoke of challenges in thinking and talking across papers. Studies had different purposes, were conducted in different contexts, and used different conceptualizations, theories, and methods. How should the group understand the “theme” and how should it make sense of submitted papers? The title was changed from *Mathematics Teacher and Classroom Practices* to *Mathematics Teaching and Teacher Practice(s)* after CERME10 to clarify interest in studying *teaching* rather than *teachers* and *teacher practices* rather than broader *classroom practices*. Still, the distinction between *teaching* and *teacher practice(s)* remains unclear and communicating across papers continues to be challenging.

Perhaps challenges should not be surprising. Despite its long history, research on mathematics teaching remains relatively underdeveloped. Well-developed learning theories abound, but teaching theories are scant. Important frameworks capture key aspects of teaching but fall short of comprehensive, elaborated theories. Two compelling reasons come to mind. First, teaching is extraordinarily complex. It is an involved human activity that navigates other complex human activities, such as learning, communication, child development, and social life in and beyond schools. Second, teaching has a familiarity that breeds a presumption that teaching is little more than common sense. As Lortie (1975) argues, the “apprenticeship of observation,” carried out by everyone from the perspective of being a student for countless hours, renders teaching either invisible or distorted in ways that have to be unlearned. Nor are researchers immune. It is little wonder then that participants struggle to think and talk across the fragmented glimpses of teaching that papers provide.

To meet these challenges, as organizers of TWG19 who are both equal participants and responsible for leading, we have invited participants to experiment with three initiatives:

1. Think, write, and talk more explicitly about the meaning of “teaching”
2. Use five proposed analytic domains to stimulate communication and reflection
3. Contribute to and use shared datasets to provide common referents.

In addition, we have set goals in the spirit of ERME’s commitment to communication, cooperation, and collaboration. First, we aim to give all participants opportunities to develop as scholars. We invest in thoughtful reviews and design sessions for participants to present their work, have colleagues react, and respond to others in turn. Second, we aim to support communication and collaboration among participants by having them explore the meaning of teaching across studies, use the domains to organise and reflect on studies, and study shared data. We offer these as ways to build bridges among diverse studies. Third, we aim to characterise TWG19’s collective effort to study teaching and teacher practice(s), including reflecting on what we are learning and priorities for future work. This is no small feat, and our efforts are nascent at best, but this is an important goal.

Exploring the meaning of teaching

Papers studying teaching often take the concept for granted. The call for proposals for CERME12 asked that papers explicitly say what is meant by “teaching” in the paper. Nevertheless, only five papers did so (all involving an organiser as an author). Mosvold and Wæge offer a practice-based conception grounded in instructional interactions as collaboratively constructed between teacher and students around particular content, and situated in broader environments, where teaching is the work involved in managing these interactions. Grundén offers a linguistics-based conception grounded in sociolinguistics of power relations and power struggles, where teaching is a social, cultural, and political practice of situated and habitual actions and interactions. And Nowinska offers a discipline-based conception grounded in socio-mathematical norms and a developing theory of metacognitive-discursive activities that shape them, where teaching is expert participation in and scaffolding of the doing of mathematics. More implicit definitions differ as well. For example, Holmedal draws on discourse analysis to explore the idea of teaching as identity performance, and Arnesen and Dahl draw on commognition to explore the idea of teaching as expert participation in classroom discourse.

At CERME12, participants considered the meaning of teaching implicated each paper in group discussions and posted their in-the-moment thoughts on a *Padlet*¹ page for the paper. In additional sessions, participants discussed the meaning of teaching across papers and posted thoughts on another page. Discussions appeared meaningful and lively. Posts reflected thinking more deeply and together about what is meant by teaching. One participant reflected on teaching as a complex set of tasks and professional judgements made in real-time, and another wrote about the importance of finding a balance among six dimensions of a teaching-learning process: epistemic, cognitive, mediational, ecological, emotional, and interactional. Others highlighted teaching as creating opportunities to learn or as managing dilemmas. Some raised questions about how values and theories shape views of

¹ *Padlet* is an online digital canvas where users can post text, videos, and images.

teaching. One post asked for strengths and limitations of thinking about teaching as “what teachers do” versus “work to be done,” an idea discussed at CERME11 (Sakonidis et al., 2019). Another suggested that the two might be used alternately by looking for what teachers do in the classroom, analysing and discussing these observations, and then considering the specialised work to be done.

Some participants raised questions about the meaning of teaching in relation to teacher education and teacher development: What do we teach teachers when we teach teaching? Another asked where teachers learn what we as researchers expect them to do. These questions emphasised that how we as researchers and teacher educators think of teaching has consequences for what is possible for prospective and practicing teachers to learn. Participants also pondered whether a choice of how to conceptualise teaching might differ for mathematics teaching versus teaching other subjects; whether norms, values, beliefs, and emotions (as related to mathematics) influence what might be meant by teaching; and how intuition and habits might play a role.

Several insights emerged from attending to the meaning of teaching. First, meaning is not as obvious as often presumed. Furthermore, what is meant by teaching matters for understanding all aspects of a study (questions, methods, and claims). In addition, attending to the meaning of teaching made it easier to think across papers. No one argued for a single, shared meaning, but many saw a need for being clearer about what is meant and how this would help us make sense of research.

Domains of research on teaching

The second initiative is to experiment with using five domains as an analytic frame to locate and discuss studies being conducted. The domains are:

- (i) Consideration of *mathematics* and the central endeavour of extending the subject to students.
- (ii) Organizing and *enacting* the design, interactions, and discourse of teaching and learning.
- (iii) Becoming acquainted with, relating to, and responding to *students* as people and learners.
- (iv) Attending to broader social, cultural, and political *issues* that matter for teaching and learning, including imperatives of social justice.
- (v) Addressing all domains of teaching in a *comprehensive* way, with that as an explicit aim.

The domains are based on the didactical/instructional triangle evident in studies of teaching across intellectual traditions (cf., Brousseau, 1970-1990/2002; Cohen et al., 2003; Jaworski, 1994; Steinbring, 2011; see Goodchild & Sriraman, 2012, for a discussion of this breadth). More recently, attention has been given to how the interactions of teaching and learning are situated in broad socio-political, historical, and cultural environments (cf., Ball, 2018; Jaworski & Potari, 2009). Any study of teaching necessarily involves all domains, but a study may foreground one while maintaining regard for others. Hoover et al. (2022) posit capacity for maintaining mutual regard for all aspects of teaching while focusing on a single aspect as key to professional growth of mathematics teacher educators but propose that this capacity is likely crucial to both doing and understanding teaching. The goal is to support thinking, not to prescribe studies. One can imagine a study investigating the interplay between two domains, with others in the background. Yet, we identify only these five, where a dual-regard paper could be addressed in either. The fifth domain, *comprehensive*, is meant for studies that deliberately attempt to attend to all aspects of teaching together.

The domains were first presented at the virtual meeting of TWG19 in February 2021 and, with some modifications, were used to organise papers and to stimulate discussion at CERME12. The intention is to use the domains to support elaboration and understanding of the contribution of each paper in relation to others. It is also hoped that the domains offer an analytic lens that might characterise our collective research and help make sense of research on teaching and teacher practice. All papers grouped together belonged, at least loosely, to the same domain. After individual papers were discussed, the papers and then the domain itself were discussed. Participants reflected on the types of research questions being asked and what other important questions might be asked for the domain.

At CERME12, papers were presented in the domains of *Mathematics*, *Enactment*, and *Issues*, with no papers submitted in the domains of *Students* or *Comprehensive*. The absence of papers in the *Students* domain was noted by participants but not much discussed.

Mathematics

Studies in this domain foreground mathematics and how it can be extended to students, but the meaning of mathematics and what it means to foreground it vary across studies. Hummes et al. and Lovemore et al. consider teacher engagement in collaborative processes, namely lesson study and a community of practice respectively, with a focus on irrational numbers in the Pythagorean theorem (Hummes et al.) and using music to support the teaching of fractions (Lovemore et al.). Within the context of teaching geometrical patterns, Gray and Kleve examine the demand on mathematical knowledge for teaching when supporting students' agency, identity and access to mathematics. Papadaki and Biza present a method of analysis suitable for investigating teachers' use of opportunities to go beyond the "mathematics of the moment" in the context of teaching geometry. Finally, Adler and Mosvold use shared data to illustrate how the Mathematical Discourse in Instruction framework identifies four tasks of teaching central to making mathematics available to students, regardless of the nature of the pedagogy employed (e.g., traditional or reform).

Studies in this domain are different, yet participants appeared to find it provocative and rewarding to reflect on these papers as located in a common territory that emphasizes mathematical considerations. All are concerned with teaching, but specifically with questions of creating mathematical opportunities for students and recognizing and taking them up as they arise. They focus on discourse, as well as identity, agency, and autonomy, as distinctly mathematical, central to the doing and learning of mathematics. Comments on the *Padlet* for this domain wondered how mathematics teaching is distinct from the teaching of other subjects and how conceptions of mathematics might shape conceptions of mathematics teaching. Considering this domain led to reflections on both the essential role it needs to play in studying teaching and how incomplete it is as a lens.

Enactment

Twelve papers and one poster foreground the enactment of mathematics teaching, with diversity in the focus and intent of individual studies, the underpinning theory, and the nature of the methods employed. Papers involve empirical investigations of teacher actions. In some cases, specific hypotheses are investigated, for example, whether teachers' use of why-questions supports student reasoning (Arnesen & Dahl). In other papers, efforts are made to investigate or describe aspects of teaching. Drawing on data from different classrooms, van Bommel et al. analyse teachers'

communication of learning goals and Sigurjónsson investigates features of teaching in classrooms where cognitive activation was high. In a more fine-grained analysis, Gray et al. use data from a single lesson to report on the role of progressing and focusing actions in students' appropriation of mathematical ideas, while Gobede and Mosvold use data from a single classroom to identify dilemmas of teaching arithmetical notation to young learners. Some papers report descriptions and investigations of teacher or student actions after, or in the context of, interventions to support particular forms of teaching and learning. Kovács-Kószó et al. investigate a teacher's responses to significant moments of student thinking after participation in a professional development programme; Røsseland et al. report on students' participation in mathematics tasks where roles and positions were used to foster explorative talk.

Several papers explicitly aim to build or test theory or methodological approaches. Nowińska illustrates a set of negative discursive activities that may lead to negative socio-mathematical norms, while Svensson and Wester propose a method for identifying socio-mathematical norms through analysis of student and teacher responses in mathematical activity. Drawing on discourse and positioning theory, Drageset and Eidissen develop and test a framework using shared data gathered for TWG19. Their framework describes teacher positions based on an analysis of their interactions. Concepts from theatre practitioners, in addition to organisational and educational theory, inform the framework proposed by McIvor for identifying and analysing the practice of improvisation in the secondary mathematics classroom. Most papers in this domain analyse recordings of mathematics lessons. In contrast, Mosvold and Wæge examine collective planning activities within a professional development initiative to investigate and propose entailments of questioning practices.

Comments about this domain on the *Padlet* reflected on how much of the work in this domain focuses on what is easily observable, both with a sense that this domain is central to teaching yet also with questions about what we might be missing, the less observable. One participant commented that the students seemed to be missing in this domain, and others commented that studies in this domain do not attend to context, or to teaching as influenced by larger structural systems. Another wondered whether a focus on frameworks in different studies might hide similarities across studies.

Issues

Boundaries, influences, and contradictions are highlighted in the set of four papers in this domain. In the context of a collaboration between mathematics teaching and visual art teaching communities, Choutou and Potari use grounded theory to analyse data from group meetings to identify the boundaries that emerged, as well as the ways in which these were handled. In a paper where mathematics teaching is seen as a social, cultural, and political practice, Grundén investigates textbooks as actors in the transformation of the intended curriculum by conducting thematic analysis on focus group discussions among primary school teachers. Holmedal draws on the notion of big D Discourse and figured worlds to unpack the role of teacher identity in navigating contradictions. Finally, Mwale and Jakobsen use the Mathematical Discourse in Instruction framework to investigate teachers' practices when teaching mass.

Padlet posts that reflected on this domain surfaced concerns and new insights. The "name" of this domain, especially shortening it to "issues," is seen as inadequate. It lacks clarity, but participants

have different ways of thinking about this domain and widely agreeable terms are hard to find. Participants discussed and commented on the advantages of “ecology” and “environment.” Part of the concern is about language; part is about meaning. Some participants wondered how broader social, cultural, and political issues matter for doing research as well as teaching. Several commented on the need to acknowledge differences among countries, both for teaching and for research. Another theme in the *Padlet* posts was about how much the teacher and teaching matter for how these issues influence what happens in classrooms. This insight prompted comments about needing to consider methodological tools for addressing such complexity. Prominent across comments was a newfound appreciation for needing to keep this domain in mind. One post described a view of teaching as being about classroom interactions and how the papers in this domain seemed initially unrelated to teaching. The participant went on to write:

Thinking about them in light of this domain, however, at least helped me see how and why these can be considered as studies of mathematics teaching. Whereas many other studies focus on dialogue, moves or interactions, these papers identify and analyze different kinds of structures that might influence teaching and learning. This is a useful way of looking at teaching.

Views of this domain varied, but having it as a domain seemed to help the group see how these studies were contributing to research on teaching and to see the work as having challenging yet important implications for all studies in TWG19.

Sharing data to study teaching

The third initiative is to contribute to and use data that others in the group have shared — for analysis in papers, as illustrations in presentations, or more generally as a common reference point in discussions. For CERME12, five contributions were available. Most included video, along with transcripts, a document that provides information about the context, and a document containing information from the owners of the data about restrictions. Seven papers made use of the shared data for CERME11, and a session was devoted to discussion of these papers. The initiative appeared to create enthusiasm and productive discussions. For CERME12, only two papers used the shared data.

Gathering and preparing data for sharing and reuse can be rewarding but comes with challenges. One is navigating the General Data Protection Regulation (GDPR), which provides directives for protecting personal data. These regulations can discourage, but there are important reasons to work to make data available. Besides reproducibility and replicability of research, as Dewey (1904) points out, to teach is to be a student of teaching. Teaching is a public act, and its study is a foundational professional activity. In part, we need to educate ourselves and others in the role of responsibly generating and sharing records of practice. Responsible data sharing requires careful planning, crafting proper consent, and establishing secure infrastructure for storage and access. Among the benefits of sharing and reusing data, scholars have highlighted the promise of better utilization of data, access to rich and unique data, saved costs for data production, reduced burden for participants, validation and extension of previous work, and increased quality in analysis. In TWG19, we have experienced potential for new and productive research collaboration and having a common ground for improving communication as significant benefits. Going forward, we hope to explore new ways of organising this work.

Implications for the future of TWG19

The focus on the meaning of teaching, the use of domains to organise and examine studies, and the establishment of shared data have provided greater cohesion for TWG19. They are also helping the group realise CERME's commitment to communication, cooperation, and collaboration. Attending to what is meant by "teaching" is pushing participants' thinking regarding individual studies and is occasioning new conversations across studies. Seeing other's thinking about teaching and what it affords for their research adds perspective to our own thinking. Likewise, the use of domains is helping us think together about each study as well as across studies. So far, the domains appear to be functioning without privileging some work over other work. Instead, they are helping us see diverse studies as part of a larger, concerted effort to investigate teaching. Last, while establishing and using shared data is challenging, it remains a promising tool for supporting the work of TWG19.

In addition to these three innovations, the organisers have also been exploring ways of working together. The pandemic forced us into virtual meetings. Designing online sessions was not easy, but several new approaches may be worth keeping. Participants recorded 5-minute presentations beforehand, which seemed to encourage preparation, lead to better and efficient presentations, and helped everyone get familiar with the work. Viewing presentations does not replace reading all papers, but it helps everyone recall what they read and provides a quick way to revisit and remember.

A second novelty was designing sessions for authors to hear reactions from a small group of colleagues. These author sessions were highly structured to engage authors in listening and participants in honing professional skills. After viewing the 5-minute recorded presentation, one volunteer had 3 minutes to describe what they heard, then two other volunteers had 2 minutes each to say what stood out to them as significant about the work. This was followed by time for brief reaction from the author and then discussion of the meaning of teaching evident in the work and its place in the identified domains. Perhaps the skills identified in these sessions should be more scaffolded and the sessions facilitated. Perhaps participants should know ahead of time which papers they will be reacting to, so they might prepare. Notwithstanding potential improvements, participants appeared to appreciate deliberate attention to building professional skills, and authors appeared to appreciate the opportunity to listen to how their work was being understood and taken up.

In this design, individual papers were not presented to the whole group. However, the author sessions were seen as valuable to both authors and participants, and authors received several other types of feedback. Having an organiser comment on a set of papers after everyone had a chance to engage with one paper in a small group seemed to help participants go deeper with the one paper and extend thinking across the papers. In addition, having individual papers discussed in parallel allowed for more whole-group discussions across the papers, where authors again had a chance to hear their paper discussed, albeit in relation to other papers, a specific domain, or meanings of teaching. Last, we created a *Padlet* page for each paper. This allowed authors to ask for pointed feedback and provided all participants with an expedient way of offering comments and raising questions. We now have better ideas about how to support participants' productive use of these online tools and plan to continue exploring their use when we meet in person.

In planning for the conference, we strove to provide opportunities for in-depth discussions of teaching and teachers' practice(s). Our sense is that the three initiatives together with thoughtfully designed sessions contributed to such in-depth discussions. However, we think there is more to do. As we plan for CERME13, we will continue to consider how to design sessions that support each participant while also supporting our collective efforts to advance research on teaching.

References

- Ball, D. L. (2018, April). Just dreams and imperatives: The power of teaching in the struggle for public education [Presidential address]. American Educational Research Association Annual Meeting, New York.
- Brousseau, G. (2002). *Theory of didactical situations in mathematics: Didactiques des mathématiques, 1970–1990* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Trans.). Kluwer. (Original work published ca. 1970–1990)
- Cohen, D. K., Raudenbush, S., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, 25(2), 119–142. <https://doi.org/10.3102/01623737025002119>
- Dewey, J. (1964). The relation of theory to practice in education. In R. Archambault (Ed.), *John Dewey on education* (pp. 313–338). University of Chicago Press. (Original work published 1904)
- Goodchild, S., & Sriraman, B. (2012). Revisiting the didactic triangle: From the particular to the general. *ZDM*, 44(5), 581–585. <https://doi.org/10.1007/s11858-012-0449-3>
- Hoover, M., Dahlgren, M., Mosvold, R., & Goffney, I. M. (2022). *Mathematics teacher educators' thinking about teaching and justice as pivotal to thinking about mathematical knowledge for teaching*. Submitted to the *Journal of Mathematics Teacher Education*.
- Jaworski, B. (1994). *Investigating mathematics teaching: A constructivist enquiry*. Falmer Press.
- Jaworski, B., & Potari, D. (2009). Bridging the macro- and micro-divide: Using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, 72(2), 219–236. <https://doi.org/10.1007/s10649-009-9190-4>
- Lortie, D. (1975). *Schoolteacher: A sociological study*. University of Chicago Press.
- Sakondis, C., Mosvold, R., Drageset, O., Mhuirí, S., & Taylan, R. (2019). Introduction to the papers of TWG19: Mathematics Teaching and Teacher Practice(s). In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 3548–3555). Freudenthal Group & Freudenthal Institute, Utrecht University, and ERME.
- Steinbring, H. (2011). Changed views on mathematical knowledge in the course of didactical theory development: Independent corpus of scientific knowledge or result of social constructions? In T. Rowland & K. Ruthven (Eds.), *Mathematical knowledge in teaching* (pp. 43–64). Springer.