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APPLICATION OF THE QUEMADA VISCOSITY MODEL FOR DRILLING FLUIDS

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ABSTRACT

A number of different models are used to describe the shear rate dependent viscosity of drilling fluids. Most, such as the Herschel-Bulkley model, have a purely empirical basis. The Quemada model, while still empirical, is based on physical principles. It is based on the notion that structural units develop in the fluid at low shear rates which are then partially broken down as the applied shear rate increases.

In the current work, drilling fluid rheological data are fitted to the Herschel-Bulkley and the Quemada model. The development of the Quemada model and the calculation of each model parameter are presented. We show that the Quemada model better fits measurements over a wider range of shear rates than the Herschel-Bulkley model. We describe how to select the parameters of the Quemada model. Knowing the difficulty of obtaining a known shear rate for fluids with yield stresses, we discuss how this can affect the quality of the Quemada model fit. Furthermore, in principle, the Quemada model is not applicable in presence a non-zero yield stress. Therefore, we show how to handle the yield stress using a (very high) zero shear rate viscosity.

Keywords: Herschel-Bulkley model, Quemada model, Drilling fluid

NOMENCLATURE

A	cross-sectional area [L^2](m^2)
f	function corresponding to eq. (25)
h	thin slot height [L](m)
k	consistency index [ML^{-1}T^{n-2}](Pa.s^n)
n	flow behavior index [dimensionless]
p	parameter of the Quemada model [dimensionless]
Q	volumetric flowrate [L^{3}T^{-1}](m^3/s)
r_p	pipe radius [L](m)
r_{wb}	wellbore radius [L](m)
s	curvilinear abscissa [L](m)
$\bar{v}$	bulk velocity [LT^{-1}](m/s)
W	thin slot width [L](m)
Greek Letters	
β	model parameter vector
$\dot{\gamma}$	shear rate [T^{-1}](s^{-1})
$\dot{\gamma}_1$	first drawn value for the shear rate [T^{-1}](s^{-1})
$\dot{\gamma}_2$	second drawn value for the shear rate [T^{-1}](s^{-1})
$\dot{\gamma}_3$	third drawn value for the shear rate [T^{-1}](s^{-1})
$\dot{\gamma}_c$	characteristic shear rate [T^{-1}](s^{-1})
$\dot{\gamma}_w$	shear rate at the wall [T^{-1}](s^{-1})
$\dot{\gamma}_{w_N}$	Newtonian wall shear rate [T^{-1}](s^{-1})
η_{eff}	effective viscosity [ML^{-1}T^{-1}](Pa.s)
η_{eff_1}	estimated effective viscosity associated with $\dot{\gamma}_1$ [ML^{-1}T^{-1}](Pa.s)
η_{eff_2}	estimated effective viscosity associated with $\dot{\gamma}_2$ [ML^{-1}T^{-1}](Pa.s)
η_{eff_3}	estimated effective viscosity associated with $\dot{\gamma}_3$ [ML^{-1}T^{-1}](Pa.s)
$\tilde{\eta}_i$	effective viscosity based on measured shear stress [ML^{-1}T^{-1}](Pa.s)
η_0	Newtonian viscosity at shear rate tending to zero [ML^{-1}T^{-1}](Pa.s)
η_∞	Newtonian viscosity at shear rate tending to infinity [ML^{-1}T^{-1}](Pa.s)
τ	shear stress [ML^{-1}T^{-2}](Pa)
$\tilde{\tau}_i$	measured shear stress [ML^{-1}T^{-2}](Pa)
τ_w	shear stress at the wall [ML^{-1}T^{-2}](Pa)
τ_Y	yield stress [ML^{-1}T^{-2}](Pa)
χ^2_η	chi-square function based on effective viscosities [M^{2}L^{-2}T^{-2}](Pa^2.s^2)
χ^2_τ	chi-square function based on sheared stresses [M^{2}L^{-2}T^{-4}](Pa^2)

1. INTRODUCTION

Understanding the rheological behaviour of drilling fluids is crucial for formulating methods for cleaning the wellbore efficiently and maintaining wellbore stability. However, the flow behaviour is not always known for the entire range of relevant shear rates. Modelling the rheological behaviour of drilling fluids, with the available models is a essential, but can sometimes be challenging due to changes in drilling parameters. The drilling industry generally uses simple models like Herschel-Bulkley, but their accuracy and applicability are questionable when the application goes beyond the estimation frictional pressure losses. Therefore, research into alternative rheological models to describe the behaviour of drilling fluids with increased accuracy over a wide range of wall shear rates, is important.

The Quemada rheological model includes internal structural effects [1] and is more complex than Herschel-Bulkley. Quemada (1998) demonstrated that structural models are more suitable for complex fluids such as drilling fluids. His model introduces the concept of shear-dependent structures involving the construction and destruction of structural units (SUs) caused by the interaction of internal and shear forces. Thus, research into the use of this model to characterise the shear-thinning behaviour of drilling fluids, is of interest. Previous work has been conducted on application of the Quemada model to drilling and well fluids. Hodne et al. [2] employed it to describe the properties of cementitious materials for well cementing. Baldino et al. [3] used it to describe the viscous properties of oil and synthetic oil-based drilling fluids. The model has also shown good performance in analysis of hemorheological measurement; that is colloidal system such as blood [4].

Many simple models can be used to describe the viscous behaviour of drilling fluid with reasonable accuracy if the measurements needed to build the viscous model is selected from reasonably relevant shear rates. The accuracy of viscosity predictions can be improved over a larger selection of shear rates by use of the Quemada model. Still, the main benefit of using the Quemada model may not be for direct viscosity application of empirical data. Its strength lies in the inclusion of physical processes like the construction and destruction of the structural units. Hence, it is possible to use a model where altering the fluid composition may have an understandable change in fluid model quantities. This may not be possible in cases where empirical models are used solely on curve fitting. The Herschel-Bulkley model for example, is based on the use of a yield stress which is not straightforward to determine neither by measurements nor by curve fitting.

In many applications, such as dealing with annular frictional pressure losses and drilling automation, estimating frictional pressure losses is the principal objective. It is therefore desirable to measure the viscous properties accurately at the relevant shear rates as described by Cayeux [5,6].

With steady state rheological models incorporating the notion of yield stress, this yield stress value can be fitted using the shear dependent viscosity measurement results. However, if the purpose is to study the effect of fluid properties in applications such as the consolidation of cuttings beds, sag prevention, fluid adhesion to the wall or chemical diffusion processes, the yield stress is a characteristic that depends on the time frame of the problem in question. In some cases, the yield stress is regarded as a constant while in others it is time dependent.

Power and Zamora [7] describe a "low-shear yield point" method for determining drilling fluid yield stress from a standard viscometer. This method is applicable for measurements performed in accordance with API procedures. If more sophisticated measurement equipment and methods are used, then static and dynamic yield stresses can be determined from dynamic viscosity measurements [8]. Yield stresses have also been approximated by shear stress overshoot methods [9].

The Quemada model does not include a yield stress term. However, this effect can be taken into account by employing a constant very low shear rate viscosity. In principle, the value can be arbitrarily high and so the Quemada model can be used as a rheological model for a fluid material. Until recently, there was no published work for estimating pressure losses with a Quemada rheological behaviour. Recently research has been directed towards this [6,10].

2. THE HERSCHEL-BULKLEY MODEL

The Herschel-Bulkley model is a non-linear, three-parameter model. It is one of the simplest models for describing the flow behaviour of drilling fluids with reasonable accuracy over a large range of shear rates [11, 12]. The model is presented in Equation 1:

$$\forall\tau > \tau_\gamma, \tau = \tau_\gamma + k\dot{\gamma}^n \quad (1)$$

where the shear stress (τ) depends on the yield stress (τ_γ), shear rate $(\dot{\gamma})$, and the consistency and flow behaviour indices, k and n respectively. Where k and n are empirical curve fitting parameters. The n-parameter determines the curvature in shear rates of the shear stress for the fluids.

Herschel-Bulkley is a practical model for flow calculations. The difficulty with the model relates to the bookkeeping of the properties. It is not possible to make a library where the fluid recipe is selected based on the required viscosity parameters. The dimension of the flow index, k, is dependent on the dimensionless parameter n. A consequence of this dependency is that the Herschel-Bulkley consistency index, k, cannot be used in a direct comparison between different fluids and, thus, such a comparison is of little practical use [13] unless the relevant shear rate is close to unity. To circumvent the problem of having a consistency index which physical dimension depends on the flow behaviour index, Saasen and Ytrehus [12,14] rewrote Equation (1), by using the dimensionless shear rate and defined it as:

$$\forall\tau > \tau_\gamma, \tau = \tau_\gamma + \tau_s\left(\frac{\dot{\gamma}}{\dot{\gamma}_s}\right)^n \quad (2)$$

This equation can be re-written using the same notation as we will use later for the Quemada model, $\Gamma = (\dot{\gamma}/\dot{\gamma}_s)$:

$$\forall\tau > \tau_\gamma, \tau = \tau_\gamma + \tau_s\Gamma^n \quad (3)$$

$\dot{\gamma}_s$ is not a model parameter. It is a characteristic shear rate representative of the flow conditions in question. The same $\dot{\gamma}_s$ must be used to compare τ_s and n between different Herschel-Bulkley rheological behaviours.

3. THE QUEMADA MODEL

The Quemada model [1], is an extension of the hard-sphere model and a revisited concept of the effective volume fraction (EVF). This extension includes complex fluids assuming that they are monodisperse dispersions of approximately spherical structural units (SUs). These SUs are agglomerates of particles suspended in the fluid. The Quemada model describes how viscosity is affected by the formation and destruction of these SUs by shear. When the fluid is prepared, it may form aggregated flocs of the initial fluid particles, called individual flocs (IFs). At low shear rates (LSRs), the inter-particle forces result in the formation of SUs from the initial fluid particles and/or the IFs. When SUs are formed, they lock up some of the suspending fluid, increasing the EVF of the particles, resulting in increased viscosity. When the shear rate increases, these SUs break apart, subsequently releasing the locked-up fluid, decreasing the EVF, and reducing the viscosity [1]. In accordance with this concept, the viscosity equation is defined as:

$$\eta = \eta_\infty \left[\frac{1+\Gamma^p}{\chi+\Gamma^p}\right]^2 \tag{4}$$

η_∞ in Equation (4) is the steady state infinite-shear viscosity, where the dimensionless shear variable $\Gamma \to \infty$. This variable can be expressed in terms of a relative shear rate $\Gamma = (\dot{\gamma}/\dot{\gamma}_c)$ [1]. The exponent of the dimensionless shear variable, p, has been pre-defined by Quemada to be $0 < p < 1$ [1], and has usually been found to be close to 0.5 in colloidal dispersions [15]. On the contrary to using a characteristic shear rate, $\dot{\gamma}_s$ used to present the Herschel-Bulkley model in terms of dimensionless shear rates, (see Equation (2)), $\dot{\gamma}_c$ is a calculated model parameter.

The model depends on its structural index, χ, defined as:

$$\chi(\phi) = \frac{1-\phi/\phi_0}{1-\phi/\phi_\infty} \equiv \pm\left(\frac{\eta_\infty}{\eta_0}\right)^{\frac{1}{2}} \tag{5}$$

which functions as a rheological index by describing the rheological behaviour of the fluid. For shear-thinning drilling fluids, the structural index is limited to $0 < \chi < 1$. It can be expressed by the limiting zero- and infinite-shear viscosities where $\Gamma \to 0$ and $\Gamma \to \infty$:

$$\begin{cases} \eta_0 = \eta_F \left(1-\dfrac{\phi}{\phi_0}\right)^{-2} \\ \eta_\infty = \eta_F \left(1-\dfrac{\phi}{\phi_\infty}\right)^{-2} \end{cases} \tag{6}$$

Each limiting viscosity depends on its limiting maximum-packing fraction defined as:

$$\begin{cases} \phi_0 = \dfrac{\phi_m}{1+CS_0} \\ \phi_\infty = \dfrac{\phi_m}{1+CS_\infty} \end{cases} \tag{7}$$

These parameters are related to the maximum packing fraction ϕ_m, the compactness factor $C = \varphi^{-1} - 1$ where φ is the SU's mean compactness, and the limiting values of the structure variable. The structure variables are defined as $S_0 = \phi_{A0}/\phi$ and $S_\infty = \phi_{A\infty}/\phi$ when $\Gamma \to 0$ and $\Gamma \to \infty$ respectively, where Quemada [1] describes ϕ_A as the volume fraction of particles in the SUs. For pseudo-plastic behaviour, Quemada [1] confines the limiting maximum packing fractions to $\phi < \phi_0 < \phi_\infty$ and the limiting aggregated volume fractions to $\phi \geq \phi_{A0} \geq \phi_{A\infty}$.

The time-dependency of the structure variable S is defined as:

$$\frac{dS}{dt} = \kappa_A(S_0 - S) - \kappa_D(S - S_\infty) \tag{8}$$

where κ_A and κ_D are shear-dependent constants of construction and destruction of SUs. When $\frac{dS}{dt} = 0$, the equation yields the steady state solution:

$$S_{eq} = \frac{S_0 + S_\infty\theta}{1+\theta} \tag{9}$$

where Quemada [1] assumes θ to be:

$$\theta(\dot{\gamma}) = \frac{\kappa_D}{\kappa_A} = \frac{t_A}{t_D} = (t_c\dot{\gamma})^p = \Gamma^p \tag{10}$$

in concentrated systems. The time t_c is required for dimensional homogeneity and it must be closely related to one of the relaxation times, t_A and/or t_D [1,2].

Some of the parameters ($t_c, \eta_0, \eta_\infty, \phi$, and ϕ_m) can be determined by different methods and needs to be limited with care.

4. THE VISCOSITY MODELS AND YIELD STRESSES

Yield stress is a fundamental property of the Herschel-Bulkley rheological model. It is regarded as a physical property which can be determined through methods other than curve fitting. On the other hand, the Quemada rheological model does not feature a yield stress explicitly in its formulation. Therefore, let us determine an apparent yield stress for Quemada rheological behaviour, defining it as:

$$\tau_0 = \lim_{\dot{\gamma}\to 0} \tau \tag{11}$$

From the definition of effective viscosity, i.e., $\eta = \frac{\tau}{\dot{\gamma}}$, we can estimate the yield stress for the Quemada rheological behaviour:

$$\tau_0 = \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma} \left(\frac{1 + \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^p}{\sqrt{\frac{\eta_\infty}{\eta_0}} + \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^p} \right)^2 \tag{12}$$

$$= \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma} \left(\frac{1}{\sqrt{\frac{\eta_\infty}{\eta_0}}} \right)^2$$

$$= \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma} \frac{\eta_0}{\eta_\infty} = \lim_{\dot{\gamma}\to 0} \dot{\gamma}\eta_0$$

If $\eta_0 \neq \infty$, then $\lim_{\dot{\gamma}\to 0} \dot{\gamma}\eta_0 = 0$. If $\eta_0 = \infty$, then we must re-evaluate the limit, noting that $\sqrt{\frac{\eta_\infty}{\eta_0}} = 0$:

$$\tau_0 = \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma} \left(\frac{1 + \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^p}{\left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^p} \right)^2 \tag{13}$$

$$= \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma} \left(\left(\frac{\dot{\gamma}_c}{\dot{\gamma}}\right)^p \right)^2$$

$$= \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma} \left(\frac{\dot{\gamma}_c}{\dot{\gamma}}\right)^{2p}$$

$$= \lim_{\dot{\gamma}\to 0} \eta_\infty \dot{\gamma}_c{}^{2p} \dot{\gamma}^{1-2p}$$

If $p = \frac{1}{2}$, then $\tau_0 = \eta_\infty \dot{\gamma}_c$, otherwise, $\tau_0 = 0$ when $p < \frac{1}{2}$ and $\tau_0 = \infty$ when $p > \frac{1}{2}$. We can therefore conclude that with the Quemada rheological behaviour the yield stress is either 0 or infinite except for the particular case of $p = \frac{1}{2}$ and $\mu_0 = \infty$ where it takes a strictly positive definite value. This result suggests that the Quemada and Herschel-Bulkley models are fundamentally different, and we must therefore consider the idea of yield stress in more depth.

A review of studies into yield stress reveals controversy around how it should be defined, how it should be measured (directly or indirectly), and whether yield stress even exists. This problem has been summarised in a review article written by Watson [16], describing a play acted by Niall Young and Mads Larsson as Sherlock Holmes and Dr. Watson discussing the yield stress myth. Yield stress is commonly defined as a change between solid- and liquid-like behaviour of a fluid. If shear stress is applied below the yield stress, the fluid exhibits a solid-like behaviour and, when the yield stress is exceeded, it behaves like a liquid. This transition does not necessarily result in complete destruction of the fluid structure when the applied stress exceeds the yield stress. According to Balmforth et al. [17], fluid structures typically exist after the fluid has yielded, resulting in the viscosity being shear-rate dependent.

Blair [18] defined yield stress as "the critical shear stress below which no flow can be observed under the condition of experimentation". On the other hand, Barnes and Walters [19] assert that a true yield stress does not exist and defines what cannot be measured. They came to this conclusion as a result of work with newly developed stress instruments capable of measuring shear rates as low as 10^{-6} s^{-1}.

For our purposes, we assume that yield stress exists and can be approximated through direct and indirect measurements. The indirect method uses the shear stress data at low shear rates (LSRs) to extrapolate a value at zero shear rate. However, this method can be very inaccurate. The LSR data may be inaccurate due to slippage, or the measured shear stress rates exceed the LSR threshold. [16]. One direct method for determining yield stress is the vane method which is assumed to measure yield stress as a physical property of the fluid. As the vane rotates, it believed to stretch the network bond between the particles and aggregates, eventually breaking the bonds. When the majority of these bonds have been broken, the fluid is regarded as having yielded [20]. However, Barnes and Carnali [21] conducted a numerical analysis on vane geometry and showed that no yield stress existed. They claimed that a thixotropic layer, forming at the vane surface, lead to apparent slip. When removing this effect, they produced viscosity curves with zero-shear plateau indications thus demonstrating the absence of a true yield stress. The review by Watson [16] concludes that the language used and the definitions of the of the measurement parameters are critical when studying yield stress related cases.

The conclusion from these analyses is that the yield stress and other viscous parameters reflect time dependent properties of the fluid. An apparent yield stress may be observable for a flow situation that has a finite duration and characterizes a time-dependent property of some fluids. Therefore, Herschel-Bulkley model behaviour describes steady state behaviour of a fluid for a flow situation sufficiently short compared to the time constant of the material behaviour. We might question why a time-dependent parameter is employed to characterize the steady-state rheological behaviour of a fluid. However, if we believe that the steady state rheological behaviour does not have a yield stress, then Herschel-Bulkley reduces to a power-law model which conflicts with most observations of rheometer measurements taken with actual drilling fluids. Here, use of the Quemada rheological model has a an advantage over use of the Herschel-Bulkley model as it avoids use of yield stress and yet has at least three or four parameters, depending on whether zero viscosity is finite or infinite.

Taking the arguments from this section into consideration, both the Herschel-Bulkley and Quemada models must be used with care and preferably only for appropriate time independent rheological behaviour. Specifically, the characteristic time of deformation should be insignificant compared to that of re-structuring the internal configuration of the material. If it is necessary to address problems where the rheological time response is important, then other type of models must be used, like viscoelastic measurements or, simply controlled stress measurements using a cone-and-plate option.

5. EXPERIMENTAL INVESTIGATION

We present rheological measurements of an oil-based drilling fluid in FIGURE 1 taken at 25ºC. The Herschel-Bulkley and Quemada models are fitted to the data. For Herschel-Bulkley the 2.04 Pa yield stress was determined prior to fitting the other parameters. Only the viscosity measurements obtained at lower shear rates than 250 1/s were used. All the Quemada model parameters were found by curve fitting. Based on these measurements, we expect that pressure loss modelling using the two models will produce similar accuracy for hole sizes between 8½" and 17½" diameter. In addition to this visual comparison, we present an analysis of pressure losses in the following sections.

The curves in FIGURE 1 suggest that the Quemada model fits the measured values better than Herschel-Bulkley for shear rates below about 2.5 1/s. The fluid represented by FIGURE 1 is strongly shear thinning and which is very pronounced at the low shear rates. The shear rates at low shear rates for such strongly shear thinning fluids are difficult to determine when using a bob and cup geometrical configuration in the viscometer [22]. Both curves appear to represent the measured data adequately. While there may be relatively large inaccuracies in the low shear rate data, it would seem reasonable to assume that the higher shear rate data, greater than 10 1/s, are likely to be more accurate. Consequently, the models are going to be more reliable under high shear rate conditions, for example, if assessing frictional pressure losses when a concentric cylinder configuration is used for measuring the viscous properties.

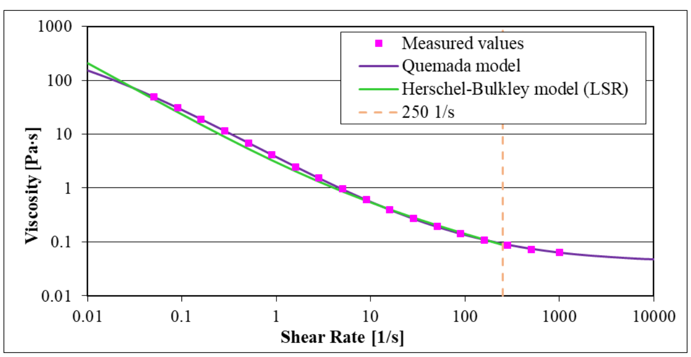

FIGURE 1: COMPARISON BETWEEN THE HERSCHEL-BULKLEY AND THE QUEMADA MODELS.

6. MODEL CALIBRATION

The purpose of calibrating a rheological model is to determine the parameters that fit best to a series of observations. Let us suppose that we have m rheological measurement pairs: $\{\forall i \in [1, m], (\dot{\gamma}_i, \tilde{\tau}_i)\}$. We would like to solve:

$$\arg\min_{\beta} \sum_{i=1}^{m} (\tau(\beta, \dot{\gamma}_i) - \tilde{\tau}_i)^2 \tag{14}$$

where β is a vector of dimension p representing the rheological model parameters. Note that the sum of the square of the differences between the measured and estimated shear stresses is the chi-square function: $\chi_\tau^2 = \sum_{i=1}^{m} (\tau(\beta, \dot{\gamma}_i) - \tilde{\tau}_i)^2$. In the case of the Herschel-Bulkley rheological behaviour, $\beta^t = [\tau_\gamma \quad K \quad n]$ with τ_γ being the yield stress, K being the consistency index and n the flow behavior index. At an extremum value of the sum of square of differences, the partial derivatives with regards to the component of β are all equal to zero:

$$\forall j \in [1, p], \frac{\partial \chi_\tau^2}{\partial \beta_j} = 0 \Leftrightarrow \forall j \in [1, p], \sum_{i=1}^{m} 2 \frac{\partial \tau(\beta, \dot{\gamma}_i)}{\partial \beta_j} (\tau(\beta, \dot{\gamma}_i) - \hat{\tau}_i) = 0 \tag{15}$$

where β_j is the j-component of β. For a Herschel-Bulkley rheological behaviour, the resulting set of equations is non-linear. To overcome that obstacle, it is proposed to linearize the Herschel-Bulkley model by considering the function:

$$\tau - \tau_\gamma = k\dot{\gamma}^n \tag{16}$$

by applying a logarithm function, we obtain:

$$\log(\tau - \tau_\gamma) = \log k + n \log \dot{\gamma} \tag{17}$$

Therefore, if we determine τ_γ, estimation of $\log k$ and n is a simple linear regression. Unfortunately, use of a logarithmic function to linearize a function for the purpose of least square fitting should be confined to data with multiplicative errors. With rheometer measurements, shear stress measurement errors are additive and therefore influence strongly the linear least square fitting, especially for low shear rates. Furthermore, the slightest error in yield stress estimation can lead to very different results for the estimation of the consistency and flow behavior indices as demonstrated by Mullineux [23]. To avoid that problem, Mullineux derived a method that allows us to estimate n directly from the rheometer measurements independently of the determination of the yield stress and the consistency index. When n has been determined, τ_0 and k are estimated using a simple linear regression. The method described by Mullineux is very fast and stable and not particularly biased by measurement errors, except that it cannot be used for determining a material constant type yield stress.

Proposed solutions for calibrating the Quemada mode are based on estimating some of the model parameters under specific conditions, like for instance utilizing the low shear rate measurements to estimate η_0 or the high shear rates to estimate η_∞ [2,3]. Such approaches tend to fall short when there are not enough measurements in the appropriate shear rate ranges. Therefore, it is desirable to find a calibration method that considers all the measurements for estimation of model parameters. The m rheological measurement pairs can be converted to m effective viscosity measurements: $\{\forall i \in [1, m], (\dot{\gamma}_i, \tilde{\eta}_i)\}$ where $\tilde{\eta}_i = \frac{\tilde{\tau}_i}{\dot{\gamma}_i}$. Then the model parameters are calibrated to fit the measurements after solving:

$$\arg\min_{\eta_0, \eta_\infty, \dot{\gamma}_c, p} \sum_{i=1}^{m} \left(\mu_\infty \left(\frac{\dot{\gamma}_c^p + \dot{\gamma}_i^p}{\sqrt{\frac{\eta_\infty}{\eta_0}} \dot{\gamma}_c^p + \dot{\gamma}_i^p} \right)^2 - \eta_i \right)^2 \tag{18}$$

which is achieved by solving the system of nonlinear equations:

$$\begin{cases} \dfrac{\partial \chi_\eta^2}{\partial \eta_0} = 0 \\ \dfrac{\partial \chi_\eta^2}{\partial \eta_\infty} = 0 \\ \dfrac{\partial \chi_\eta^2}{\partial \dot{\gamma}_c} = 0 \\ \dfrac{\partial \chi_\eta^2}{\partial n} = 0 \end{cases} \tag{19}$$

where $\chi_\eta^2 = \sum_{i=1}^{m} \left(\mu_\infty \left(\frac{\dot{\gamma}_c^p + \dot{\gamma}_i^p}{\sqrt{\frac{\eta_\infty}{\eta_0}}\dot{\gamma}_c^p + \dot{\gamma}_i^p} \right)^2 - \tilde{\eta}_i \right)^2$ is the chi-square function based on effective viscosities. The non-linear least square optimization can be solved using a Levenberg-Marquart method [24]. The solution algorithm must start from an initial solution that is not too far from the global minimum. We will now describe a method for estimating an initial solution.

First of all, it should be noted that most drilling fluids have a very large zero shear rate viscosity, and for practical considerations, we can consider that η_0 tends to infinity. For instance, if we consider a power law shear thinning fluid, its rheological behaviour is defined as $\tau = k\dot{\gamma}^n$. Then its effective viscosity tends to infinity as long as $n < 1$ because $\lim_{\dot{\gamma} \to 0} k\dot{\gamma}^{n-1} = \infty$. From this fact, it is very likely that $\sqrt{\frac{\eta_\infty}{\eta\mu_0}}$ tends to zero for the vast majority of drilling fluids, and therefore, the Quemada rheological behaviour reduces to a three-parameter model:

$$\eta_{eff} = \eta_\infty \left(\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p} \right)^2 \tag{20}$$

Let us choose three samples amongst the list of rheological measurements: $\left(\dot{\gamma}_1, \eta_{eff_1}\right)$, $\left(\dot{\gamma}_2, \eta_{eff_2}\right)$ and $\left(\dot{\gamma}_3, \eta_{eff_3}\right)$. Then we can obtain a first expression of an estimated value of the infinite viscosity for the Quemada model:

$$\eta_\infty \left(\frac{\dot{\gamma}_c^p + \dot{\gamma}_1^p}{\dot{\gamma}_1^p} \right)^2 = \eta_{eff_1} \Leftrightarrow \eta_\infty = \eta_{eff_1} \left(\frac{\dot{\gamma}_1^p}{\dot{\gamma}_c^p + \dot{\gamma}_1^p} \right)^2 \tag{21}$$

We inject this expression in the second pair of shear rate and effective viscosity:

$$\eta_{eff_1} \left(\frac{\dot{\gamma}_1^p}{\dot{\gamma}_c^p + \dot{\gamma}_1^p} \right)^2 \left(\frac{\dot{\gamma}_c^p + \dot{\gamma}_2^p}{\dot{\gamma}_2^p} \right)^2 = \eta_{eff_2} \Leftrightarrow \dot{\gamma}_c^{2p} + 2\dot{\gamma}_c^p \dot{\gamma}_2^p + \dot{\gamma}_2^{2p} = \frac{\eta_{eff_2}\dot{\gamma}_2^{2p}}{\eta_{eff_1}\dot{\gamma}_1^{2p}} \left(\dot{\gamma}_c^{2p} + 2\dot{\gamma}_c^p \dot{\gamma}_1^p + \dot{\gamma}_1^{2p} \right) \tag{22}$$

After noting $\dot{\gamma}_c^p = x$, eq. (21) can be rewritten as

$$\left(1 - \frac{\eta_{eff_2}\dot{\gamma}_2^{2p}}{\eta_{eff_1}\dot{\gamma}_1^{2p}} \right) x^2 + 2\left(\dot{\gamma}_2^p - \frac{\mu_{eff_2}\dot{\gamma}_2^{2p}}{\mu_{eff_1}\dot{\gamma}_1^{2p}} \dot{\gamma}_1^p \right) x + \dot{\gamma}_2^{2n} - \frac{\mu_{eff_2}\dot{\gamma}_2^{2p}}{\mu_{eff_1}\dot{\gamma}_1^{2p}} \dot{\gamma}_1^{2p} = 0 \tag{23}$$

which is a second-degree equation in x. When $b^2 - ac \geq 0$, it has two real solutions $x_1 = \frac{-b \pm \sqrt{b^2 - ac}}{a}$, where $a = \left(1 - \frac{\eta_{eff_2}\dot{\gamma}_2^{2p}}{\eta_{eff_1}\dot{\gamma}_1^{2p}} \right)$, $b = \left(\dot{\gamma}_2^p - \frac{\eta_{eff_2}\dot{\gamma}_2^{2p}}{\eta_{eff_1}\dot{\gamma}_1^p} \right)$ and $c = \dot{\gamma}_2^{2p} - \frac{\eta_{eff_2}\dot{\gamma}_2^{2p}}{\eta_{eff_1}}$. We choose the smallest positive solution, if it exists as a negative solution would not make sense. If both solutions are negative, we draw an alternate combination of rheometer measurements until we get at least one positive root. Now, we can express n as a function of $\dot{\gamma}_c$ and x_1.

$$\dot{\gamma}_c^p = x_1 \Leftrightarrow \dot{\gamma}_c = x_1^{\frac{1}{p}} \tag{24}$$

Finally, after injecting that last expression in the equation corresponding to the last pair of shear rate and effective viscosity, we obtain:

$$f(p) = \eta_{eff_1}\dot{\gamma}_1^{2p} \left(x_1^2 + 2x_1\dot{\gamma}_3^p + \dot{\gamma}_3^{2p} \right) - \eta_{eff_3}\dot{\gamma}_3^{2p} \left(x_1^2 + 2x_1\dot{\gamma}_1^p + \dot{\gamma}_1^{2p} \right) = 0 \tag{25}$$

which is an equation of only n. This equation can be solved numerically using, for instance, a Newton-Raphson method. However, eq. (25) may have a pole and a zero, so it is necessary to perform a scan of the interval in order to find an initial value for n that is on the correct side of the pole. After obtaining n, it is easy to calculate an estimated value for $\dot{\gamma}_c$ using eq. (24) and an estimated value for μ_∞ with eq. (21).

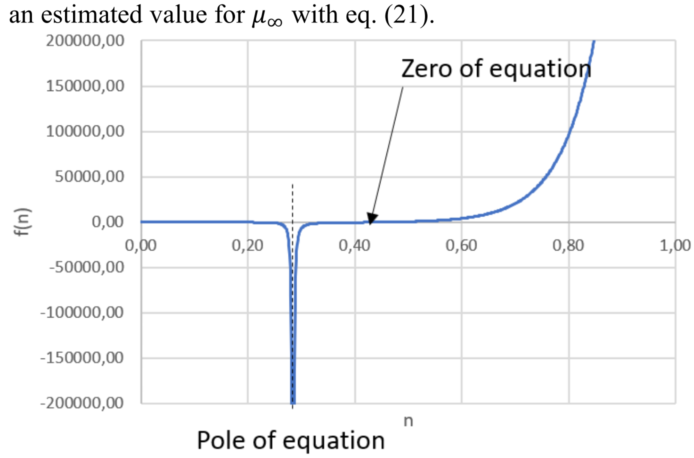

FIGURE 2: EQ. (25) MAY HAVE A POLE AND A ZERO.

Using that initial solution, the application of the Levenberg-Marquart algorithm gives us an estimation of the Quemada rheological model that minimize the chi-square function S.

The results of fitting high precision rheometer data for a KCl / Polymer fluid mass density 1750kg/m^3 measured at 10°C to the Herschel-Bulkley and Quemada models are shown on FIGURE 3a. Herschel-Bulkley is calibrated using the method described by Mullineux and the calibrated parameters are $\tau_0 = 3Pa$, $K = 1.005Pa.s^{0.546}$, $p = 0.546$ and $\chi_\tau^2 = 0.050Pa^2$.

Quemada is calibrated using the method described above and the calibrated parameters are $\eta_0 = 13328853\text{Pa.s}$, $\eta_\infty = 23.6cP$, $\gamma_c = 273.957s$, $p = 0.438$ and $\chi_\tau^2 = 0.006Pa^2$.

FIGURE 3b shows the percentage difference between the modelled shear stresses compared to the measured ones. In this particular example the errors in the Quemada model are within ±0.5% over the whole range of shear rates, while in this particular case the Herschel-Bulkley overestimates shear stress by up to 2% at the low end and underestimates it by almost 1% at high shear rates with > 0.5% exceedances in between.

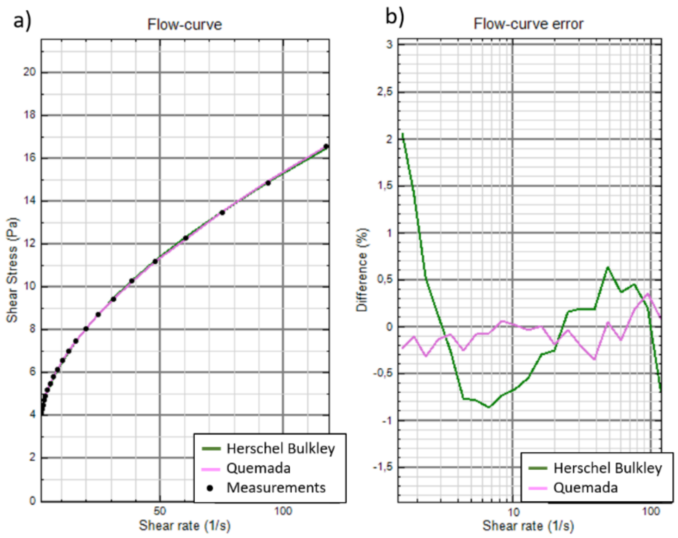

FIGURE 3: EXAMPLE OF MODEL FITTING OF RHEOMETER MEASUREMENTS WITH THE HERSCHEL-BULKLEY AND QUEMADA RHEOLOGICAL MODELS FOR KCL/POLYMER FLUID OF MASS DENSITY 1750KG/M^3, RHEOMETER MEASUREMENTS MADE AT 10°C.

However, it is not always the case that the Quemada rheological behaviour provides a better fit to a rheogram than the Herschel Bulkley model. FIGURE 4a shows the results of fitting Herschel Bulkley and Quemada models to a rheogram measured at 80°C of a KCl/Polymer fluid of mass density 1500kg/m^3. The fitted Herschel Bulkley parameters are: $\tau_0 = 1.1Pa$, $K = 1.339Pa.s^{0.362}$, $n = 0.362$ and $\chi_\tau^2 = 0.013Pa^2$, while the Quemada parameters are: $\eta_0 = 3185605\text{Pa.s}$, $\eta_\infty = 2.3cP$, $\gamma_c = 5334.17s$, $p = 0.402$ and $\chi_\tau^2 = 0.037Pa^2$. In FIGURE 4b, we can see that both models follow a similar error pattern but with the Quemada model showing a slightly larger amplitude.

However, based on 385 rheograms of various drilling fluids measured with a scientific rheometer, statistically the Quemada model provided a better fit than the Herschel-Bulkley model. FIGURE 5 shows the probability distribution of the chi-square after curve fitting of the Herschel-Bulkley and Quemada models on the above mentioned rheograms. The median value for the Quemada model is at 0.01Pa^2 while it is 0.025Pa^2 for Herschel-Bulkley. Also, the minimum chi-square for the Quemada model reaches 0.0005Pa^2 while it is only 0.005Pa^2 for the Herschel-Bulkley rheological behaviour. The maximum chi-square for both models is about the same and around 10Pa^2.

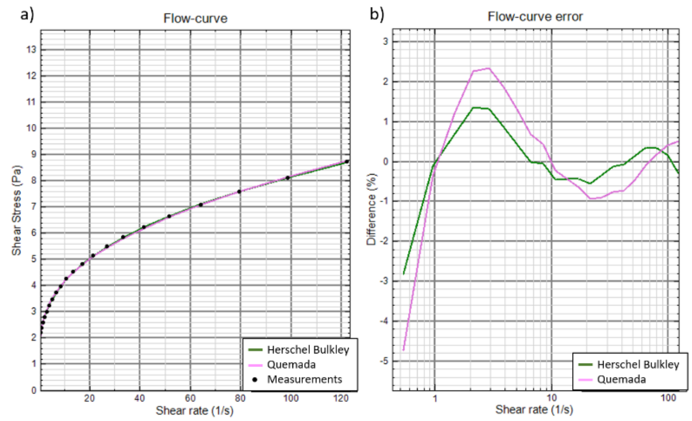

FIGURE 4: EXAMPLE OF MODEL FITTING FOR A KCL-POLYMER FLUID OF MASS DENSITY 1500KG/M^3 WITH RHEOLOGICAL MEASUREMENTS MADE AT 80°C, WHERE THE HERSCHEL-BULKLEY MODEL HAS A LOWER CHI-SQUARE THAN THEN ONE OF THE QUEMADA MODEL.

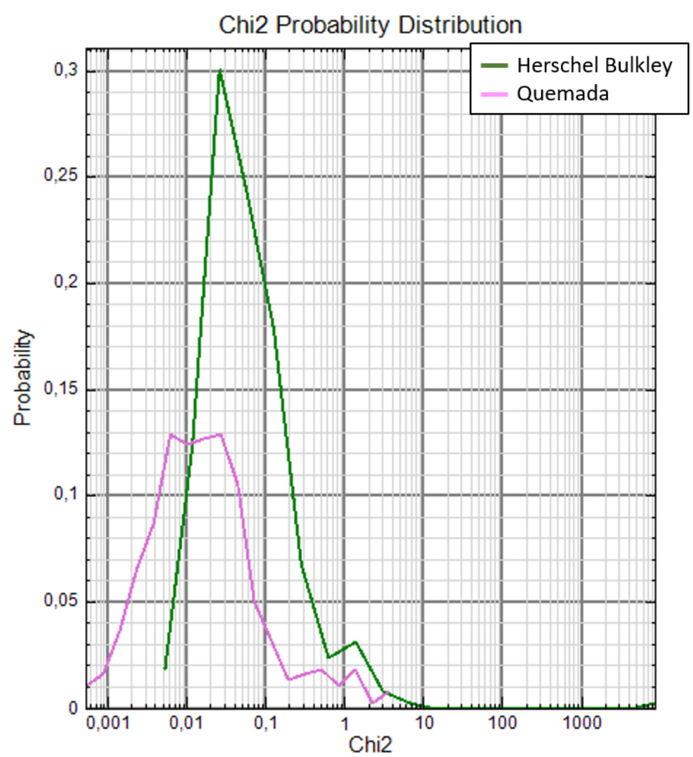

FIGURE 5: PROBABILITY DISTRIBUTION OF THE CHI-SQUARE AFTER CURVE FITTING OF THE HERSCHEL-BULKLEY AND QUEMADA MODELS ON 385 RHEOGRAMS MEASURED WITH A SCIENTIFIC RHEOMETER ON VARIOUS DRILLING FLUIDS.

7. PRESSURE GRADIENTS

The pressure gradient based on the Herschel-Bulkley model, for laminar flow, in a circular pipe has been described by Kelissidis et al. in 2006 [25]. Similarly, the pressure drop in laminar flow of a Herschel-Bulkley fluid in a concentric circular annulus has been described by Founargiotakis et al. in 2008 utilizing the small gap approximation [26]. Both solutions are semi-analytical. The model applies a transfer of the Herschel-Bulkley parameters to a power-law representation of the flow at the actual shear rate of the flow. Hence, a simplified method can be chosen, where power-law parameters are selected at characteristic shear rates of the flow, without a significant loss of accuracy [27].

Cayeux and Leulseged (2020) [6] have described a semi analytical solution to the calculation of the pressure gradient in a circular pipe, in laminar flow regime, of a fluid modelled by the Quemada rheological behaviour when $\mu_0 \gg \mu_\infty$.

We will now describe a semi analytical solution to the calculation of the pressure gradient of the Quemada rheological model for the concentric circular anulus case utilizing the thin slot approximation.

A concentric circular annulus with a small difference between the pipe and borehole diameter, can be regarded as similar to a thin slot, i.e., the flow in between two plates separated by a distance $h = r_{wb} - r_p$, r_{wb} is the wellbore radius and r_p is the pipe radius. We want to preserve the cross-sectional area of the annulus $A = \pi\left(r_{wb}^2 - r_p^2\right)$, and therefore the slot width is: $W = \frac{A}{h} = \pi\left(r_{wb} + r_p\right)$.

Weissenberg, Rabinowitsch, Mooney and Shofield have demonstrated that in laminar flow [28], the flowrate in a slit of a generalized Newtonian fluid can be expressed as:

$$Q = \frac{Wh^2}{2\tau_w^2}\int_0^{\tau_w} \dot{\gamma}\tau d\tau \qquad (26)$$

where τ_w is the shear stress at the wall. A generalized Newtonian fluid has a rheological equation that can be expressed as follows:

$$\tau = \eta_{eff}(\dot{\gamma})\dot{\gamma} \qquad (27)$$

which is how Quemada rheological behaviour is defined. Let us denote I the term $\int_0^{\tau_w} \dot{\gamma}\tau d\tau$. In the case $\eta_0 \gg \eta_\infty$, the differentiation of eq. (20) gives:

$$\frac{d\tau}{d\dot{\gamma}} = \frac{d}{d\dot{\gamma}}\left(\eta_\infty \dot{\gamma}\left(\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p}\right)^2\right) = \eta_\infty\left(2p\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p} + (1-2p)\left(\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p}\right)^2\right) \qquad (28)$$

Therefore, the integral term can be written:

$$I = \eta_\infty^2 \int_0^{\dot{\gamma}_w} \dot{\gamma}^2 \left(\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p}\right)^2 \left(2p\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p} + (1-2p)\left(\frac{\dot{\gamma}_c^p + \dot{\gamma}^p}{\dot{\gamma}^p}\right)^2\right) d\dot{\gamma} \qquad (29)$$

where $\dot{\gamma}_w$ is shear rate at the wall. After integration, we obtain:

$$I = \eta_\infty^2 \frac{a_1 p^4 + a_2 p^3 + a_3 p^2 + a_4 p + a_5}{24p^4 - 150p^3 + 315p^2 - 270p + 81} \qquad (30)$$

where $a_1 = 8\dot{\gamma}_w^3 + 48\dot{\gamma}_c^p\dot{\gamma}_w^{3-p} + 72\dot{\gamma}_c^{2p}\dot{\gamma}_w^{3-2p} + 48\dot{\gamma}_c^{3p}\dot{\gamma}_w^{3-3p} + 12\dot{\gamma}_c^{4p}\dot{\gamma}_w^{3-4p}$, $a_2 = -50\dot{\gamma}_w^3 - 252\dot{\gamma}_c^p\dot{\gamma}_w^{3-p} - 414\dot{\gamma}_c^{2p}\dot{\gamma}_w^{3-2p} - 284\dot{\gamma}_c^{3p}\dot{\gamma}_w^{3-3p} - 72\dot{\gamma}_c^{4p}\dot{\gamma}_w^{3-4p}$, $a_3 = 105\dot{\gamma}_w^3 + 474\dot{\gamma}_c^p\dot{\gamma}_w^{3-p} + 774\dot{\gamma}_c^{2p}\dot{\gamma}_w^{3-2p} + 546\dot{\gamma}_c^{3p}\dot{\gamma}_w^{3-3p} + 141\dot{\gamma}_c^{4p}\dot{\gamma}_w^{3-4p}$, $a_4 = -90\dot{\gamma}_w^3 - 378\dot{\gamma}_c^p\dot{\gamma}_w^{3-p} - 594\dot{\gamma}_c^{2p}\dot{\gamma}_w^{3-2p} - 414\dot{\gamma}_c^{3p}\dot{\gamma}_w^{3-3p} - 108\dot{\gamma}_c^{4p}\dot{\gamma}_w^{3-4p}$, $a_5 = 27\dot{\gamma}_w^3 + 108\dot{\gamma}_c^p\dot{\gamma}_w^{3-p} + 162\dot{\gamma}_c^{2p}\dot{\gamma}_w^{3-2p} + 108\dot{\gamma}_c^{3p}\dot{\gamma}_w^{3-3p} + 27\dot{\gamma}_c^{4p}\dot{\gamma}_w^{3-4p}$.

Since for a slot, shear stress at the wall is:

$$\tau_w = \frac{h}{2}\frac{dp}{ds} \qquad (31)$$

where $\frac{dp}{ds}$ is the pressure loss gradient, we can estimate numerically $\dot{\gamma}_w$ as a function of the pressure loss gradient by solving the following equation:

$$\eta_\infty\left(\frac{\dot{\gamma}_c^p + \dot{\gamma}_w^p}{\dot{\gamma}_w^p}\right)^2 \dot{\gamma}_w = \frac{h}{2}\frac{dp}{ds} \qquad (32)$$

The initial solution for the numerical solver can be taken as the Newtonian wall shear rate $\dot{\gamma}_{w_N} = \frac{12\bar{v}}{h} = \frac{12Q}{\pi h\left(r_{wb}^2 - r_p^2\right)}$, because the bulk velocity is $\bar{v} = \frac{Q}{A}$. The problem is then solved by searching for a value of $\frac{dp}{ds}$ that gives the correct volumetric flowrate Q. A Newton-Raphson algorithm can be used with an initial value estimated using an equivalent power law behaviour.

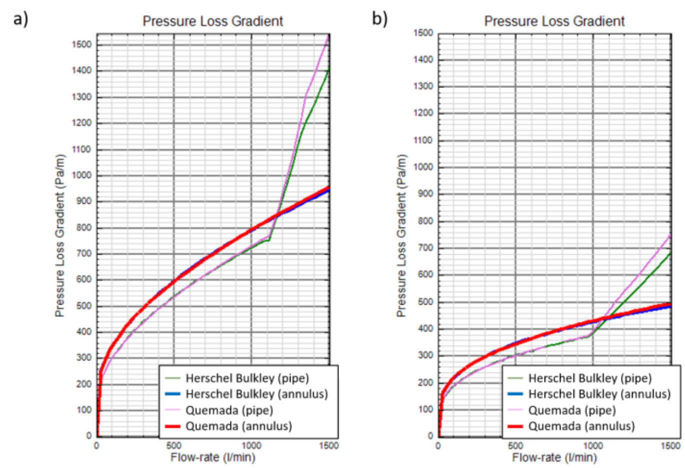

FIGURE 6: A) VISCOUS PRESSURE GRADIENTS CORRESPONDING TO THE FLUID OF FIGURE 3, B) VISCOUS PRESSURE GRADIENTS CORRESPONDING TO THE FLUID OF FIGURE 4.

FIGURE 6 shows the viscous pressure gradients calculated for a 5-in pipe (internal diameter 4.28-in) in a 8½-in borehole for flow-rates varying between 0 and 2000 L/min. FIGURE 6a corresponds to the calculation made with the fluid described by FIGURE 3 and FIGURE 6b corresponds to the fluid depicted by FIGURE 4. The thick lines represent the pressure gradients in the annulus, here considering a concentric configuration. In both cases, the annulus frictions are almost identical whether it is calculated with the Herschel-Bulkley or the Quemada rheological behaviour. Yet, a small discrepancy appears with flowrates above 1600 L/min in the first case. Recall that the Quemada rheological behaviour fit was slightly better than for the Herschel-Bulkley model. The maximum difference reaches 3% at 2000 L/min. In the second case, annulus pressure gradients are only calculated to the limit of laminar flow.

8. CONCLUSION

Both the Quemada and the Herschel-Bulkley models describe the viscosity of drilling fluids adequately. If the properties are measured using a concentric cylinder measurement device, the low shear rate viscosity determination may have large inaccuracy. A consequence is that a yield stress determination may contain large errors.

Measurement points at shear rates in excess of 10 (s^{-1}) are less liable to inaccuracies. Hence, for drilling fluid circulation flow problems such as pressure losses, both the Quemada and the Herschel-Bulkley model give adequate results. Accuracy can be improved by using curve fitting to determine a fluid mechanical yield stress that can be different from the real yield stress of the fluid.

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