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PRESSURE BUILD-UP IN CLOSED WELLS DURING KICK MIGRATION AND FLUID COMPRESSIBILITY EFFECTS

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ABSTRACT

If a kick is migrating in a closed well, this will lead to pressures building up in the well. It has earlier been shown that for Non-Newtonian fluids, suspension effects will make it impossible to deduce a unique gas velocity from the pressure build-up behavior.

In this work, it will be shown that also for Newtonian fluids, the pressure build-up will depend on both kick size and well volumes. Both very small kicks sizes typically seen in MPD operations and larger kick sizes handled in conventional well control operations will be considered. It will be demonstrated that both the shape of the pressure build-up and the final pressure levels achieved will vary significantly. It is especially when considering very small kick sizes that one starts to see large changes in the profile of the pressure build-up. The main reason for the differences is related to the fact that the liquid phase is compressible and this will again have consequences for how much a gas kick can expand and what pressures it can bring to surface.

An analytical model will be developed that shows directly which parameters have impact on the pressure build-up behavior. Simple closure laws for gas density, fluid density and gas slip will be chosen. The model will be verified against two transient models which are based on the Drift-Flux formulation. It is demonstrated that the pressure build-up and final pressure level will depend on initial kick volume, initial fluid volume, liquid compressibility and fluid density. The effect of numerical diffusion when comparing the two transient models will also briefly be discussed.

The purpose of the paper is to increase fundamental knowledge about two phase flow dynamics and show that an analytical model for the situation considered here can give results that are comparable with the results achieved with more complex transient flow models.

Keywords: gas kick migration, pressure build-up in closed well, liquid compressibility, gas expansion.

NOMENCLATURE

A_{cs}	Cross sectional area of annular wellbore
BHP	Bottomhole pressure
BOP	Blow Out Preventer
K	Flow parameter in the gas slip relation
L_f	Initial height of fluid column
L_w	Depth of well
L_x	Gas kick migration distance
M_f	Fluid mass
M_g	Gas kick mass
P	Pressure
P'	Hydrostatic pressure corresponding to h_{ave}
P_{ex}	Increase in wellhead pressure
P_g	Initial pressure at bottom of the liquid column
P_o	Initial wellhead pressure
S	Gas migration velocity
V_f	Initial fluid volume
V_g	Initial gas kick volume
V_{gf}	Final gas kick volume
V_w	Wellbore volume
WHP	Wellhead pressure

a_f	Sound velocity in fluid
a_g	Sound velocity in gas
g	Acceleration due to gravity
h_{ave}	The well depth where $\rho_f(h_{ave}) = \rho_{fave}$
v_f	Fluid flow velocity
v_g	Gas flow velocity
v_{mix}	Mixture velocity
ΔP	Change in the pressure
ΔV	Change in the volume
α_f	Fluid volume fraction
α_g	Gas volume fraction
ρ_g	Gas density
ρ_f	Fluid density
ρ_{fo}	Fluid density at P_o
ρ_{fave}	Initial average fluid density in well

1. INTRODUCTION

When a kick is taken, the normal procedure is to close the well to let the well pressure build up until the influx stops. Then a kill circulation is initiated. However, if the well is kept closed and water-based mud is used, the gas can migrate until it reaches the BOP. This process will lead to increased pressures in the well. A conservative assumption is to assume that the drilling fluid in the well is incompressible and by applying Boyles law, a worst-case scenario will be that the pressure of the influx at bottom is brought to the BOP. However, in reality the drilling fluid is slightly compressible, and this will give room for some slight gas expansion. When considering Non-Newtonian fluids, small gas bubbles can be suspended in the drilling fluid, and this will also have a significant impact on the compressibility of the system and reduce the pressure build-up in the well. This was first discussed in [1]. They also presented an analytical formula for the pressure build-up rate. A similar formula was also presented in [2] where the suspension effects was not accounted for. This formula shows that the pressure build-up rate is a function of wellbore elasticity, mud volume, kick volume and the compressibility of both the drilling fluid and the influx gas. An application of this model can be seen in [3]

Transient simulation results demonstrating the effect on the pressure build-up effect using different gas migration velocities with or without the Non-Newtonian suspension mechanism have been presented in [4, 5]

In a quite recent work [6], full scale experiments related to gas kick migration was performed. Here it was shown that the gas did not bring the pressure to surface and that final pressure levels in a closed well will depend on the gas volume in the well. Their experimental results also indicated that the pressure builds up more rapidly in the later stage of the migration.

Simulation work in a master thesis project also indicated that the final pressure levels will depend on the kick size [7]. In [8], a commercial software was used to study how different parameters would affect the pressure build up and final pressure level.

In this work, an attempt will be made to show how the pressure build-up and final pressure levels will change when considering a large range of kick sizes as well as different hole sizes. First an analytical model will be derived that shows which parameters the pressure build-up and final pressure levels will depend on. Only simple closure laws will be used. Then this model will be compared against two transient models based on the Drift-Flux formulation. Suspension effects related to Non-Newtonian fluids will not be considered.

The motivation for studying both quite small kick sizes and more normal kick sizes is related to application of Backpressure Managed Pressure Drilling. In this drilling system, one can use a bottomhole pressure that can be quite close to the pore pressure, so it is easier to take a kick. However, the kicks will normally be small in size since this drilling system is equipped with better kick detection technologies. Normally, small kick sizes will be circulated directly out but if the kick size is above a certain limit, the BOP will be closed. The Rotating control device mounted on top of this system cannot handle very large pressures compared to what a BOP can.

To be able to predict the pressure increase and final pressure levels when a kick migrates is important from an operational point of view. The weakest part of the well is usually the formation just beneath the last set casing shoe. So, if a kick is just allowed to migrate on its own for a long period in closed conditions, there is a certain risk that the formation will break down after a certain time period. Procedures must be initiated before this happens.

2. Pressure Build-Up in a Compressible Fluid due to Kick Migration in a Closed Well

In a closed wellbore and compressible fluid system, the pressure builds-up when a kick migrates to the surface. The situation before and the situation after kick migration are shown in Fig.1.

In the following an analytical model will be derived that calculates the pressure build-up as a function of how long distance a single bubble has travelled from bottom. This model will also show which parameters that will have impact on the pressure build-up.

We consider a vertical well having depth L_w and cross-sectional area A_{cs}. The wellbore volume V_w will be assumed to remain fixed. The initial swabbed gas kick enters the bottom of the well as a single bubble with an initial volume V_g and mass M_g. It covers the whole cross-sectional area. The initial fluid volume is then given as: $V_f = A_{cs}L_w - V_g$ and the mass of this fluid is M_f. The fluid depth to the top of the gas bubble is $L_f = V_f/A_{cs}$.

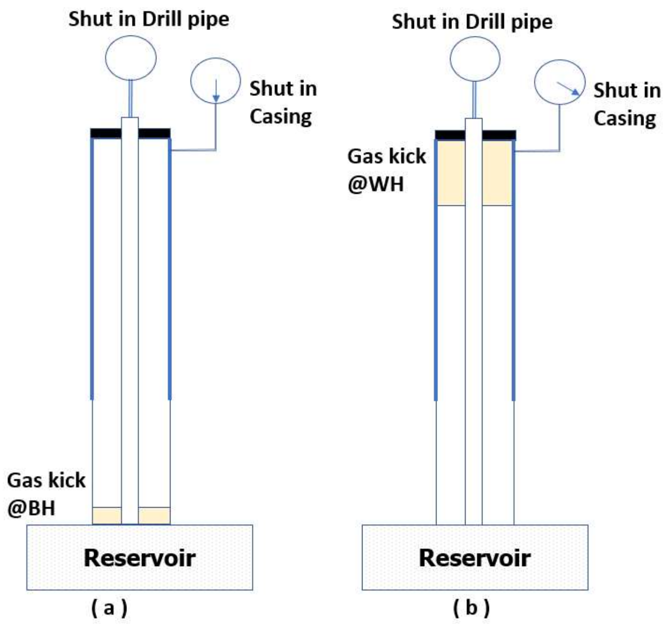

FIGURE 1: THE SITUATION BEFORE AND AFTER KICK MIGRATION

We will only consider simple models for the fluid and gas density under isothermal conditions.

The fluid density ρ_f is given as:

$$\rho_f = \rho_{fo} + \frac{P - P_o}{a_f^2} \tag{1}$$

Here, ρ_{fo} is the fluid density at surface (1000 kg/m^3), P is the pressure, P_o is the surface pressure (100000 Pa). The speed of sound in the fluid a_f is 1500 m/s.

The compressibility c is defined as $\frac{1}{\rho}\frac{\partial \rho}{\partial p}$ which gives $c = \frac{1}{\rho a_f^2}$.

The gas kick density (ρ_g) is determined as:

$$\rho_g = \frac{P}{a_g^2} \tag{2}$$

Here, a_g is the sound velocity in gas (316 m/s), P is the gas pressure.

The average density of fluid (ρ_{fave}) in the wellbore is defined from the total fluid mass (M_f) and the volume of the fluid (V_f) as:

$$\rho_{fave} = \frac{M_f}{V_f} \tag{3}$$

The hydrostatic pressure (P') corresponding to the average density can be obtained from the fluid density equation as:

$$P' = a_f^2(\rho_{fave} - \rho_{fo}) + P_o \tag{4}$$

The challenge with estimating the average fluid density is the fact that for a compressible fluid, the density is a function of the pressure, and the pressure is a function of the density as well.

To estimate the pressure, P', using a hydrostatic pressure formula with the surface fluid density ρ_{fo}, one can introduce a height h_{ave} which is defined by:

$$h_{ave} = \frac{(P' - P_0)}{g\rho_{fo}} \tag{5}$$

Using a numerical model [9], an estimate for the height was given by $h_{ave} = 0.503L_f$ which is approximately half of the fluid column.

The next aim is to determine the pressure at the bottom of the fluid column (i.e., at the top of the gas column), P_g. This will represent the initial pressure of the kick. This can be achieved with the following steps:

1. Obtain the pressure, P', at h_{ave} using ρ_{fo}:
   $P' = \rho_{fo} g h_{ave} + P_o = 0.503 L_f \rho_{fo} g + P_o$
2. Compute the average fluid density:
   $$\rho_{fave} \approx \rho_{fo} + \frac{(P' - P_o)}{a_f^2}$$
3. Compute the pressure at the top of the kick:
   $P_g \approx \rho_{fave} g L_f + P_o$

2.1 Conservation of Liquid Mass

In the closed well, the fluid mass M_f is constant throughout the kick migration process. The average fluid density in the well is given as:

$$\rho_{fave} = \frac{M_f}{V_f} = \rho_{fo} + \frac{(P' - P_o)}{a_f^2} \tag{6}$$

If the wellhead pressure (WHP) increases by ΔP_f, the volume decreases by ΔV_f, Eq. 6 can be re-written as:

$$M_f = (V_f - \Delta V_f)\left\{\rho_{fo} + \frac{(P' + \Delta P_f - P_o)}{a_f^2}\right\} \tag{7}$$

2.2 Conservation of Gas Mass

Similarly, the mass of gas M_g is constant. Using Eq. 2, the gas density of the bubble can be defined as:

$$\rho_g = \frac{M_g}{V_g} = \frac{P_g}{a_g^2} \tag{8}$$

In order to conserve mass, if the pressure decreases by ΔP_g, the volume increases by ΔV_g during gas migration in the wellbore. This gives:

$$M_g = (V_g + \Delta V_g)\frac{(P_g - \Delta P_g)}{a_g^2} \tag{9}$$

2.3 Gas Migration

Consider that the gas migrated a certain distance, L_x, up the well. The gas sees a column of fluid above, which is L_x less than before. The new pressure at the top of the gas, P_g', can now be expressed as:

$$P_g' = \rho_{fave} g (L_f - L_x) + P_o + P_{ex} = P_g - \rho_{fave} g L_x + P_{ex} \quad (10)$$

where $(P_o + P_{ex})$ is the new wellhead pressure. P_{ex} is the increase in the wellhead pressure. Note that L_x can take any value between 0 and L_f.

As the gas kick migrates in the wellbore, the change in the gas pressure is calculated by subtracting the new gas pressure (i.e., after it has been migrated with distance, L_x) from the previous/old value. The change in gas pressure can be calculated as:

$$\Delta P_g = \rho_{fave} g (L_f - L_x) + P_o + P_{ex} - \rho_{fave} g L_f - P_o \quad (11)$$

$$\Delta P_g = -\rho_{fave} g L_x + P_{ex} \quad (12)$$

When the gas migrates, the fluid column decreases, which decreases the pressure. This causes the gas to expand. But the total volume in the well must remain the same, thus the wellhead pressure must increase, causing the gas volume to increase less and the fluid volume to decrease a little.

As the gas migrates, it divides the fluid region into two, one above and one below the gas section. We assume that the error in ignoring this and treating the fluid as one, is insignificant.

As outlined in Appendix A, using conservation of fluid mass, we obtain the fluid volume reduction as:

$$\Delta V_f = \frac{V_f P_{ex}}{a_f^2 \rho_{fave} + P_{ex}} \quad (13)$$

Similarly, using conservation of mass of gas presented in Appendix B, the gas volume expansion is given as:

$$\Delta V_g = \frac{V_g (\rho_{fave} g L_x - P_{ex})}{(P_g - \rho_{fave} g L_x + P_{ex})} \quad (14)$$

The change in pressure in the wellbore due to the upward migration of the gas causes fluid compression and gas expansion. Since the total volume of the system is also conserved, this requires that $\Delta V_g - \Delta V_f = 0$.

Therefore, equating the gas volume expansion (Eq.14) with the volume of liquid compression (Eq. 13) (i.e., $\Delta V_g = \Delta V_f$), we can obtain a quadratic equation for P_{ex} as outlined in Appendix C:

$$(C + 1)P_{ex}^2 - (A - B - CD)P_{ex} - AB = 0 \quad (15)$$

From Eq. 15, the increase in the wellhead pressures P_{ex} is given as:

$$P_{ex}(L_x) = \frac{(A - B - CD) + \sqrt{(A - B - CD)^2 + 4(C + 1)AB}}{2\,(C + 1)} \quad (16)$$

Where,
$A = \rho_{fave}\, g\, L_x$
$B = a_f^2\, \rho_{fave},$ which is the inverse of compressibility of the liquid
$C = V_f / V_g$
$D = P_g - \rho_{fave}\, g\, L_x$
$V_f = V_w - V_g$
L_x is the distance the gas bubble has migrated.

Eq. 16 is the solution of the quadratic equation (QE) for the build-up pressure when the single bubble gas front has migrated a distance L_x from its original position. This analytical model will be given the name QE for the rest of this paper. The wellhead pressure is equal to $P_{ex} + P_o$. From the model, it can be observed that the wellhead pressure depends on initial kick size, initial pressure at top of the gas bubble, initial fluid volume in the well and the fluid density description. It can be noted that pressure on top of the gas bubble depends on the average fluid density.

Figure 2 shows the pressure build-up for a small (0.4 m^3) and a large kick (4.0 m^3). The quadratic form of the pressure build-up becomes more evident for the small kick size. Here the pressure build-up is slow in the beginning, but it accelerates when the gas front position gets closer to surface. If we consider the quadratic equation (Eq.15), it can be noted that the coefficient for the second order term is given by the expression $(\frac{V_f}{V_g} + 1)$.

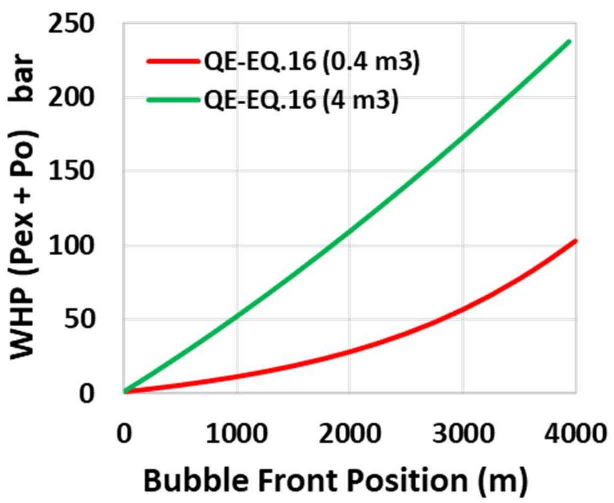

FIGURE 2: PRESSURE BUILD-UP FOR 0.4 M^3-AND 4.0 M^3 INITIAL KICK VOLUME

The value of this will grow when the initial kick size is reduced but also when the well volume is increased. This will make the pressure build-up more parabolic in form. This behavior has been observed both in field data [10] and in full scale experiments [6].

Figure 3 shows how the pressure in the fluid and the pressure at top of the gas bubble changes as the gas bubble migrates upwards. As the gas bubble moves upwards, it will expand, and it is chosen to use the gas volume expansion on the x axis to represent how far the kick has migrated. The blue vertical line to the right represents when the kick has reached the wellhead.

The pressure change in the pressure at the top of the gas bubble is shown on the negative y axis while the pressure change in the fluid is displayed on the positive y axis. The right end value of the upper graph also represents the final wellhead pressure if one adds P_o.

Figure 3 shows the situation for a small kick size (0.4 m^3) and Figure 4 shows the situation for a large kick size (4.0 m^3). For the small kick size, it is observed that the reduction in gas pressure is much larger, and it has a more parabolic form than what is seen for the larger kick size. The reason for the larger reduction is the fact that the gas volume expansion relative to the initial gas volume ratio is larger compared to what will be the situation for the larger kick.

It can also be noted that the change in absolute wellhead pressure (P_{ex}) displayed on the positive y axis is lower for the smaller kick size. The increase in wellhead pressure is caused by fluid compression and the reduction in fluid volume is the same as the gas volume expansion. When comparing the x axis in Figure 3 and Figure 4, it is seen that the relative expansion of the 4 m^3 kick is much less than that of the 0.4 m^3 kick. The 0.4 m^3 kick becomes more "compressible" which leads to a smaller value for P_{ex}. The net result is that the final wellhead pressure will become larger if the kick size is increased.

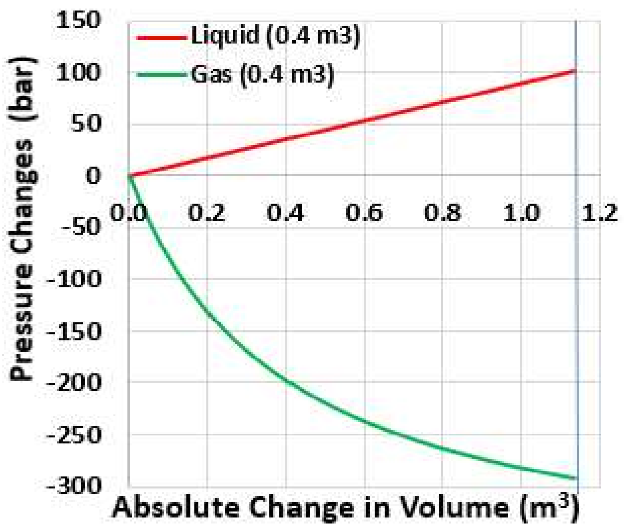

FIGURE 3: PRESSURE CHANGE VS. VOLUME CHANGE 0.4 M³ KICK

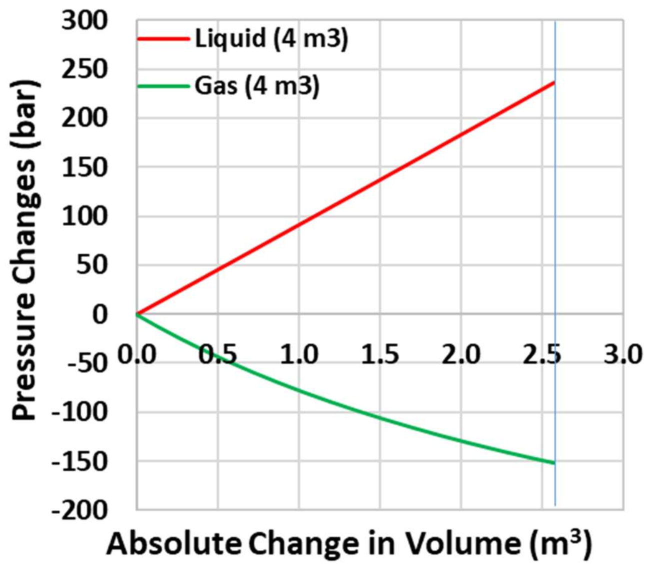

FIGURE 4: PRESSURE CHANGE VS. VOLUME CHANGE 4.0 M³ KICK

3. DRIFT FLUX MODEL

Here the Drift-Flux model (DF) will be presented along with two numerical solution approaches.

3.1 Mathematical Formulation

A Drift-Flux model will be used for describing the pressure build up in a closed well during kick migration. This model consists of two mass conservation laws for each of the phases and a mixture momentum equation. A uniform and vertical geometry will be assumed. We will assume that we have a Newtonian fluid, i.e., implying that there is no gas suspension effects present where small gas bubbles can be trapped in the fluid. There will be no mass transfer between the phases. The following three equations expresses the conservation laws:

Conservation of mass of liquid:

$$\frac{\partial}{\partial t}(\alpha_f \rho_f) + \frac{\partial}{\partial z}(\alpha_f \rho_f v_f) = 0 \tag{17}$$

Conservation of mass of gas:

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \frac{\partial}{\partial z}(\alpha_g \rho_g v_g) = 0 \tag{18}$$

Conservation of mixture momentum:

$$\frac{\partial}{\partial t}(\alpha_f \rho_f v_f + \alpha_g \rho_g v_g) + \frac{\partial}{\partial z}(\alpha_f \rho_f v_f^2 + \alpha_g \rho_g v_g^2 + P) = -F_{fric} - (\alpha_f \rho_f + \alpha_g \rho_g) g \tag{19}$$

$$\alpha_g + \alpha_f = 1 \tag{20}$$

Here subscripts *f* and *g* refer to the fluid and gas phase. Phase volume fractions, phase velocities, phase densities and pressure are represented by the variables α, v, ρ and P. The gravity acceleration is given by g. The frictional pressure loss is represented by $F_{fric.}$

Since a mixture momentum equation is used, a gas slip relation has to be supplied to supply the missing information about how fast gas travels relative to liquid. The following model is used:

$$v_g = Kv_{mix} + S = K\left(\alpha_f v_f + \alpha_g v_g\right) + S \qquad (21)$$

Here we have adopted K = 1.2 and S = 0.55 m/s which are typical values for a slug flow regime [4]. However, the S variable can be represented more exactly by using different formulas for the different flow patterns that can be present [11]. However, here only a simple model has been adopted.

For the liquid and gas density, the models specified by Eq.1 and Eq.2 will be used.

For the friction model, a very simple model was implemented and a reference for this is given in [7]. One should note that the friction will not have any significant impact on the simulated results.

3.2 AUSMV Scheme

The well is discretized into 50 cells and the explicit AUSMV scheme was used to progress the solution forward in time. A short description of the scheme is presented in [12]. Reference [13] provides details of how the fluxes between the cells shall be calculated. In order to reduce numerical diffusion, the slope limiter concept was used.

3.2 High-Resolution Method (HighRes)

This method, as described in [9], uses the shooting method and is in many ways similar to [14]. The same physics subroutines to compute density, viscosity, slip relations, etc. are used. The difference is that in [9] each numerical cell is divided into a large number of sub cells. The flow, the masses and the pressures are calculated for each sub cell. The fluid passes through several sub cells in one time step. This nearly eliminates the numerical diffusion and simplifies the computation within each cell.

The forward stepping makes slip conditions difficult. This is solved by dividing each iteration into a forward v_{mix} step and then a redistribution step for the region(s) affected by the slip. The redistribution step uses the slip relations to re-position the fluid and the gas masses.

The High-Resolution Method is also able to simulate dispersion between mud fronts by applying diffusion operators in the cells at each time step. Physical dispersion is a function of the Reynolds number, i.e., the radial variation of the fluid velocity in the pipes.

This method discretizes the well into 4000 sub cells.

4. SIMULATION RESULTS AND DISCUSSION

4.1 Simulation Setup

A 4000-meter-deep vertical well will be considered. A 12 ¼ x 5-inch geometry will first be considered where the outer diameter is 0.31115 m and the inner diameter is 0.127 m.

In all the models to be used, the mass of the kicks will be the same. We will consider kick masses of 40, 80, 160, 320, 400, 800, 1600 and 3200 kg. However, for a given gas mass, the volume of the kick when the well is shut in will vary slightly when comparing the two transient models based on the Drift-Flux model. In the AUSMV scheme, the kick is introduced more gradually, it is allowed to migrate upwards before shut in and this will allow some gas expansion before the well is closed. In the HighRes method, the influx will be a single bubble at the bottom of the well at shut in.

Hence, the initial kick volume will be slightly different at the start of shut in. Hence in the remaining discussion in the paper, we will denote the kick sizes as 0.1, 0.2, 0.4, 0.8, 1, 2, 4 and 8 m^3 although the real kick volume in the different models will deviate somehow from these exact values.

4.2 Effect of Kick Size on Pressure Build-Up

Figure 5 and Figure 6 show the pressure at bottom and top of the well as function of time for five selected kick sizes when the kicks migrate in the closed wellbore. Here the Drift-Flux model in combination with the AUSMV scheme was used. It can be observed that the smallest kick gives a much lower pressure build- up than the largest kick. One can also note that the smaller kicks have a more evident parabolic form for the pressure build-up where the pressure build-up is slower in the beginning and then accelerates more in the end. These results are in accordance with what was seen from the analytical model.

In Fig. 7, the kick volume development vs time is shown, and it is demonstrated that the gas kicks will expand slightly which is caused by the fact that the liquid is compressible

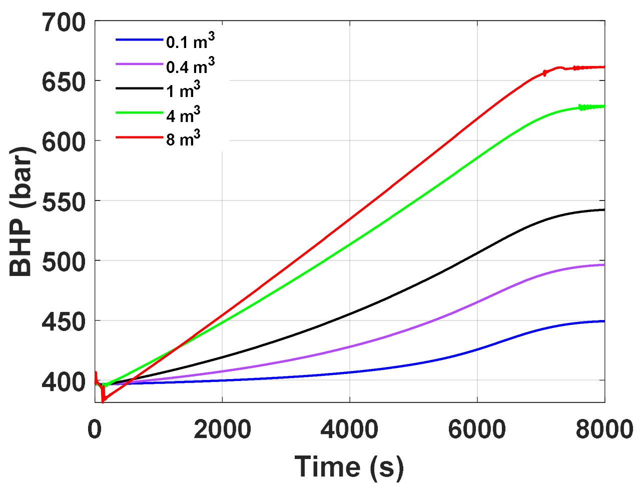

FIGURE 5: PRESSURE AT BOTTOM OF WELL VS. TIME

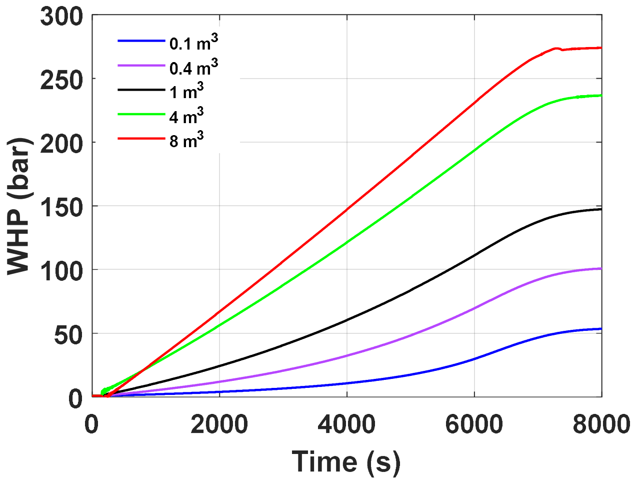

FIGURE 6: PRESSURE AT TOP OF WELL VS. TIME

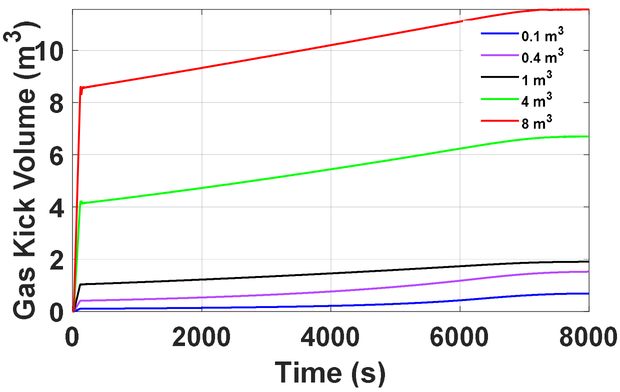

FIGURE 7: GAS KICK VOLUME IN WELL VS. TIME

4.3 Comparison of Models

Figure 8 shows the predicted wellhead pressure using the simple analytical QE model (Eq. 16) and the two transient models based on the Drift-Flux formulation (HighRes and AUSMV) for three different kick sizes. The models give quite similar results. The results from the QE model and HighRes model are almost identical. The final stabilized pressure is slightly larger for the QE model and the HighRes method compared to what is seen for the AUSMV scheme. This is most evident for the largest kick. The main reason for this is that the kick is initially more spread out in the AUSMV scheme and has migrated longer up in the well before the well is closed. For the two other models, the kick is considered as a single bubble concentrated at bottom of the well. This will cause some differences in the initial kick pressure when the kick start migrating in closed in conditions.

One can also observe a more gradual transition between the pressure build-up and the final constant pressure level when using the AUSMV scheme. This is caused by numerical diffusion that tends to smear out the transition zone between the gas kick and the pure liquid in front. But it is also caused by the fact that the kick is more spread out in the beginning. For the HighRes method there is almost no numerical diffusion and the transition is quite sharp from a pressure increase to a constant pressure level. The comparison shows that the analytical QE model can predict the pressure build-up quite well and it gives results that are in accordance with results achieved with the more complex Drift-Flux model. The fact that it predicts slightly larger pressure is just an advantage since one should be conservative when it comes to pressure predictions.

Figure 9 shows the volume expansion of the three kicks when comparing the analytical QE model with the Drift-Flux model based on the AUSMV scheme. Here one can note some differences in the beginning showing that the kick in the AUSMV is introduced more gradually during a time period (150 seconds) while the QE model starts out with the kick placed as a single bubble at bottom. However, the gas volume development after this is quite similar when comparing the models.

Figure 10 shows the final stable wellhead pressure using the three different models for a range of kick sizes. This shows more clearly that the models give very similar results for smaller kick sizes but that the QE model and the HighRes method tend to be more conservative for larger kick sizes. Figure 11 shows the corresponding comparison of the final kick volumes. The results are quite similar. This shows that the simple analytical model can be a reliable tool for predicting both the build-up pressure development and the gas volume expansion taking place.

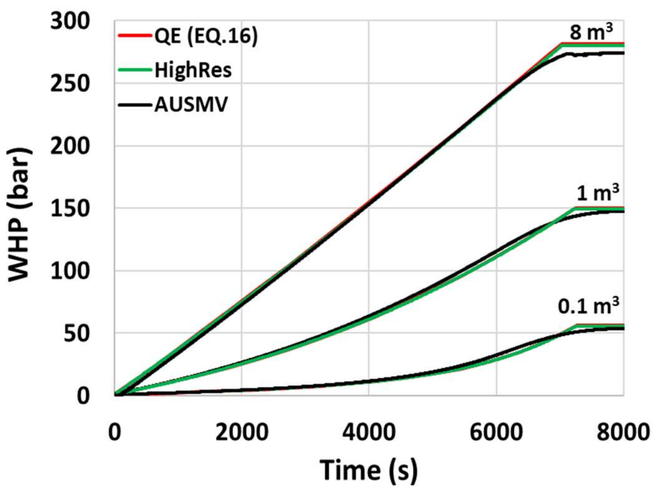

FIGURE 8: COMPARISON BETWEEN QE -AND DF MODELS

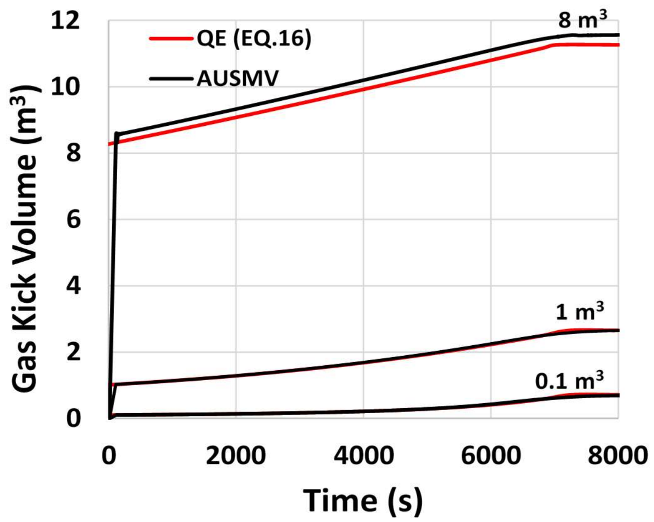

FIGURE 9: COMPARISON BETWEEN QE -AND DF MODELS

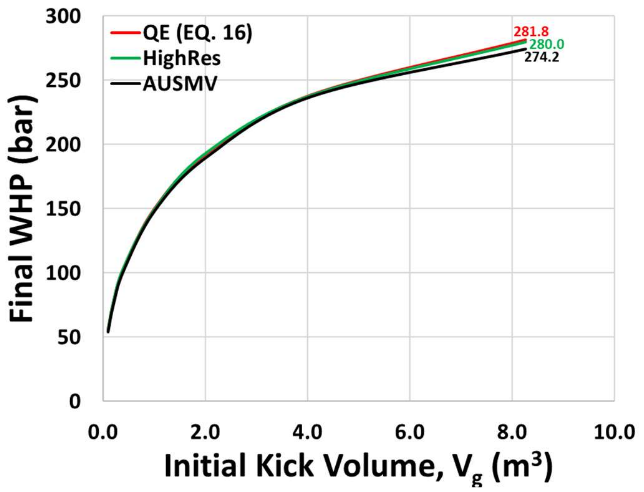

FIGURE 10: FINAL WHP FOR DIFFERENT INITIAL KICK VOLUMES

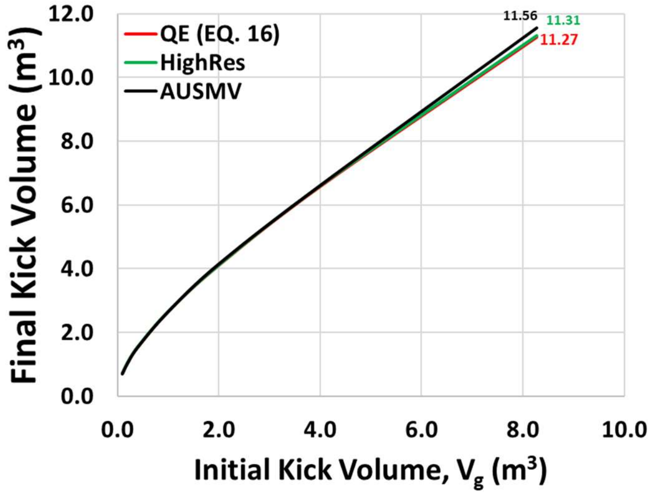

FIGURE 11: FINAL KICK VOLUMES VS. INITIAL KICK VOLUMES.

4.4 Effect of Wellbore Geometry

In the following, the QE model was used to predict final wellhead pressure and final kick volume V_{gf} for three different well volumes. The inner diameter of the annulus was kept fixed to 5 inch but the outer diameter was varied between 8.5, 12.25 and 17.5 inches mimicking typical hole sizes. The ratio V_{gf}/V_g express the relative gas expansion of the kick. Figure 12 shows the final wellhead pressures and relative gas expansion for various kick sizes in the three hole sizes. For a given kick volume size, it is seen that when the well volume increases, the final wellhead pressure will be reduced. The kick will also expand more relative to its initial size which express that the pressure in the gas bubble will be reduced more. Since there is more available liquid volume that can be compressed, a lower pressure increase is needed to create space for the gas expansion. When inspecting Eq. 16, coefficient C, one can see directly that the final pressure level will depend both on initial kick size and initial fluid volume.

Full scale experiments have shown that the final pressure level depends on the kick size [6]. The QE model is in accordance with these observations.

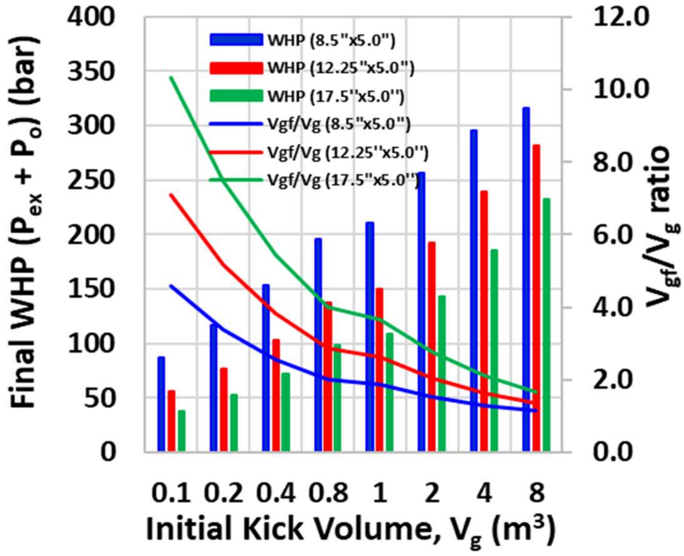

FIGURE 12: PRESSURE BUILD-UP FOR THREE DIFFERENT WELL VOLUMES

5. CONCLUSION

A simple analytical model for predicting the pressure build-up when a kick migrates in a closed wellbore has been developed. It has been validated against simulated results achieved with the more complete Drift-Flux model and it was shown to be able to predict the pressure build-up quite well for the closure laws assumed here.

The model shows that the pressure build-up and the final pressure levels achieved will depend on initial kick volume, initial fluid volume and the fluid density description. The dependency on kick volume is in accordance with what has been shown in full scale experiments [6].

The model shows that the pressure build-up has a parabolic form which becomes more evident for small kick sizes and large initial liquid volumes.

The analytical model was compared with the Drift-Flux model where two different numerical methods were used (AUSMV scheme and HighRes method). For the AUSMV scheme, the final wellhead pressure was slightly lower (especially for larger kick volumes) and the transition to the final steady state pressure was more gradual. The reason for this is that the kick initially is more spread out and has migrated longer up in the well before shut in. In addition, there is some numerical diffusion. The HighRes method has almost no numerical diffusion and the kick is treated as a single bubble from the start.

The analytical model will be easier for engineers to work with and it can predict the final wellhead pressure quite well being slightly more conservative.

APPENDIX

Appendix A: Fluid Volume Contraction

Using the fluid density equation (Eq. 4) in the main body of the paper, as the fluid volume compressed, the pressure on the fluid will increases.

$$M_f = (V_f - \Delta V_f)\left\{\rho_{fo} + \frac{(P' + P_{ex} - P_O)}{a_f^2}\right\} \quad (A1)$$

$$M_f = V_f\left\{\rho_{fo} + \frac{(P' - P_O)}{a_f^2}\right\} + \frac{V_f P_{ex}}{a_f^2} - \Delta V_f\left\{\rho_{fo} + \frac{(P' - P_O)}{a_f^2}\right\} - \frac{\Delta V_f P_{ex}}{a_f^2} \quad (A2)$$

$$M_f = M_f + \frac{V_f P_{ex}}{a_f^2} - \Delta V_f \frac{M_f}{V_f} - \frac{\Delta V_f P_{ex}}{a_f^2} \quad (A3)$$

$$M_f = M_f + \frac{V_f P_{ex}}{a_f^2} - \Delta V_f\left(\rho_{fave} + \frac{P_{ex}}{a_f^2}\right) \quad (A4)$$

This leads to:

$$\Delta V_f = \frac{V_f P_{ex}}{(a_f^2 \rho_{fave} + P_{ex})} \quad (A5)$$

Appendix B: Gas Volume Expansion

Using the gas density equation (Eq.6) in the main body of the paper, as gas volume expands the gas pressure will decreases. We therefore obtain

$$M_g = (V_g + \Delta V_g)\frac{(P_g - \rho_{fave} g L_x + P_{ex})}{a_g^2} \quad (B1)$$

$$M_g = \frac{V_g P_g}{a_g^2} - \frac{V_g(\rho_{fave} g L_x - P_{ex})}{a_g^2} + \frac{\Delta V_g(P_g - \rho_{fave} g L_x + P_{ex})}{a_g^2} \quad (B2)$$

$$M_g = M_g - \frac{V_g(\rho_{fave} g L_x - P_{ex})}{a_g^2} + \frac{\Delta V_g(P_g - \rho_{fave} g L_x + P_{ex})}{a_g^2} \quad (B3)$$

This leads to

$$V_g(\rho_{fave} g L_x - P_{ex}) = \Delta V_g(P_g - \rho_{fave} g L_x + P_{ex}) \quad (B4)$$

$$\Delta V_g = \frac{V_g(\rho_{fave} g L_x - P_{ex})}{(P_g - \rho_{fave} g L_x + P_{ex})} \quad (B5)$$

Appendix C: Absolute Wellhead Pressure

Since $\Delta V_f = \Delta V_g$ we can create one equation with only P_{ex} as the unknown. Equating A5 and B5,

$$\frac{V_g(\rho_{fave}\, g L_x - P_{ex})}{(P_g - \rho_{fave}\, g L_x + P_{ex})} = \frac{V_f P_{ex}}{(a_f^2 \rho_{fave} + P_{ex})} \quad (C1)$$

$$V_g(\rho_{fave} g L_x - P_{ex})(a_f^2 \rho_{fave} + P_{ex}) = V_f P_{ex}(P_g - \rho_{fave} g L_x + P_{ex}) \quad (C2)$$

Or

$$(\rho_{fave} g L_x - P_{ex})(a_f^2 \rho_{fave} + P_{ex}) = \frac{V_f}{V_g} P_{ex}(P_g - \rho_{fave} g L_x + P_{ex}) \quad (C3)$$

If we rewrite:

$A = \rho_{fave}\, g\, L_x$; $B = a_f^2\, \rho_{fave}$; $C = V_f / V_g$; $D = P_g - \rho_{fave}\, g\, L_x$

Then

$$(A - P_{ex})(B + P_{ex}) = CP_{ex}(D + P_{ex}) \quad \text{(C4)}$$

or

$$(C + 1)P_{ex}^2 - (A - B - CD)P_{ex} - AB = 0 \quad \text{(C5)}$$

Solving the quadratic equation, we get the increase in the wellhead pressure as:

$$P_{ex}(L_x) = \frac{(A - B - CD) + \sqrt{(A - B - CD)^2 + 4(C + 1)AB}}{2\,(C + 1)} \quad \text{(C6)}$$

The wellhead pressure thus becomes, $P_{WH}(L_x) = P_{ex}(L_x) + P_o$.

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