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ORIGINAL PAPER

Teaching the concept of zero in a Malawi primary school: illuminating the language and resource challenge

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Abstract

In this paper we discuss findings of a study that investigated the resources and language that teachers in Malawi use to teach the concept of zero. In Malawi primary schools, textual resources available to teachers are mainly the curriculum materials in the form of syllabus, teacher guides and learner textbooks. The syllabus and teacher guides are in English while the learner textbooks are in Chichewa as teaching is in Chichewa or other local language in the first 4 years of primary school. We used the Mediating Primary Mathematics framework (Venkat and Askew in Educ Stud Math 97:71–92, 2018) and a qualitative case study of two teachers to explore the resources used, how the teachers interacted with the resources and how they moved between the two languages. Our findings include that the language and resources that the teachers used provided affordances as well as constraints for learning the concept of zero. We identified two types of challenges for the teachers; that of naming and that of representing the concept of zero. We discuss what the Malawi context illuminates about teaching zero in post-colonial multilingual settings.

Keywords Chichewa · Resources · Language · Malawi · Mediating Primary Mathematics · Multilingual · Zero

1 Introduction

Zero is an important concept that needs to be taught to children in their early years of learning mathematics. In most cases children are introduced to zero as they learn to count and recognise numbers using concrete objects, where the absence of an object is referred to as zero (Barton, 2020). The development of the concept of zero as a number is then expected to move from zero as absence to zero as a number (Barton, 2020). This move is challenging for many teachers and children (Russell & Chernoff, 2011) even in contexts where resources are not limited and the language of instruction and curriculum materials is the same. In Malawi, curriculum materials provided by the Ministry of Education are teaching syllabus, learner textbook and corresponding teacher guide for each grade. The learner textbooks and teacher guides are the main textual resources that teachers use in teaching mathematics, and for many teachers, the only textual resources they have.

The Malawi language in Education policy states that language of instruction should be in the local language in the first 4 years of primary schools (Standard 1[1] to 4), and in English thereafter (Mjaya et al., 2006). The most common is Chichewa which is the national language and is spoken by the largest proportion of the population (Mjaya et al., 2006). The teaching syllabus and teacher guide are in English while the learner textbooks for standard 1–4 are in Chichewa. Consequently, teachers work across the two languages in their teaching, and in areas where the local language is not Chichewa, the teachers work across three languages. In addition to the language challenges, schools in Malawi have limited teaching and learning resources. It is therefore important to investigate the teaching of the concept of zero in such contexts, in particular the resources and the language that teachers use. In this paper we explore and discuss how teachers introduce and explain the concept of zero in standard 1. We focus on how teachers use resources and language to convey the concept and we identify what is made available to learn and what is constrained. The paper is guided by the

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research question: what resources and language do teachers use to mediate the concept of zero to young learners, and what challenges do they face?

2 Literature review

2.1 Teaching and learning of zero

Some challenges about teaching and learning of zero have been reported. One of the challenges arises from the dual contradicting mathematical and philosophical viewpoints of zero. From a mathematical viewpoint, zero is a bona fide cardinal number just like the other whole numbers, representing the quantity of something and the position in a structure (Barton, 2020). From a philosophical viewpoint, zero represent nothingness or non-being. Thus, the philosophical comparison between zero and other numbers is that we can experience two or three objects, while there is nothing to experience for zero objects therefore difficult to conceptualise (Barton, 2020). Another challenge is that zero does not fit in being used as a counting number but only as a natural number. Furthermore, since zero which is regarded as "nothing" needs to become "something", then the empty set needs to become a mental object for learners and zero has to be understood both as an empty set and a numerical concept requiring abstract thinking (Nieder, 2016). This suggests that for children to master the zero as a numerical concept, they should have opportunities to engage in tasks that will not only recognise the number zero and name it but also to illustrate its cardinality and position in a structure of numbers.

2.2 Language in the teaching of zero

Krajcsi et al. (2021) differentiate natural language and mathematical language for learning numbers. Natural language being the language of numbers that children learn at home, usually in form of representations like number of objects. Mathematical language being the scientific meanings of the numbers including their cardinality.

Working with mathematical language of Arabic notation and verbal count tasks ranging from 0 to 5, Wellman and Miller (1986) found that children's use of zero was delayed as compared to their understanding of other natural numbers, and they concluded that children face challenges in working with zero than the other natural numbers. However, when working with natural language tasks such as "give me five sweets", Bialystok and Codd (2000) found that pre-schoolers could understand the concept of zero like any other natural numbers. Krajcsi et al. (2021) argue that the contradiction in the two results rose from methodological and interpretational differences, the children's performance was influenced by the linguistic formulation of the tasks. To confirm their argument, Krajcsi et al. (2021) tested Hungarian preschoolers understanding of zero using a range of tasks whose aims included investigating the role of linguistic form on children's understanding of zero. The findings revealed that children are not familiar with mathematical form of language as such, they need to be taught these forms. Krajcsi et al. (2021) suggest that both the "natural" and "mathematical" versions of the task should be taught to children. These studies imply that the importance of language in teaching of zero cannot be over emphasised.

2.3 Concrete materials in the teaching of zero

We think of concrete materials as supporting material in teaching where the mathematics and the concrete material initially is separated from each other. As stated by Nührenbörger and Steinbring (2008) "a relation between them can only be productively constructed by the learner's own consideration and interpretation in order to actively develop mathematical understanding and insights" (p. 160). In our review of the literature, we found much research on the use of concrete materials as a resource in the teaching of number sense to young learners. However, we have not been able to identify explicit research investigating the use of different types of concrete materials in the teaching of the concept of zero. A review of online teaching resources from different countries around the world provided several examples on how concrete material can be used in the teaching of zero. They share, however, a common feature that they focus on the empty set, typically represented with a collection of boxes or jars containing different number of items where the box with no items illustrates the concept of zero (e.g., LearnBright, 2022; Twinkl, 2022). The teacher guide for Montessori schools gives examples on games where children pick a card with a hidden number and then they repeat an activity the number of times given on the card, like clapping or jumping. When the zero number is given, the children should do nothing (Montesorri, 2022). Measurement examples follow to show zero as having a magnitude, and the position of zero is illustrated on the number line (e.g., LearnBright, 2022; Twinkl, 2022). This move from the empty set to a position on the number line is complex with language entailments for teaching and learning.

2.4 Language and the teaching and learning of mathematics in Malawi

In Malawi, research has revealed that language affects the teaching and learning of mathematics. The research has mostly been conducted in upper primary and secondary schools where the language of instruction is English—the colonial language and not the home language of the learners and teachers. The main challenge is that many learners are not yet fluent in English to understand and communicate in the lessons. This makes it difficult for many learners to access the mathematics, as research has shown. For example, Kachaso (1988) found that learners perform better in mathematics tasks when presented as computations than when the same were presented as word problems in English. Later in the same study Kachaso (1988) compared two groups of standard 7 learners; one group taught mathematics in English while the other was taught in Chichewa and the word problems also in Chichewa. He found that the Chichewa group outperformed the English group in the word problems. This confirmed that English was the difficulty and barrier to the learners accessing the mathematics. Although word problems in contexts familiar to learners can aid their understanding of the mathematical computation required, the context is not accessible for learners that are not fluent in the language. This was confirmed by Chilora et al. (2003) who also compared learners' performance on mathematics word problems and similar problems in numerical computation form. Unlike Kachaso (1988), they tested young learners in standard 2–4, where teaching is in Chichewa. The word problems were presented in Chichewa and in contexts familiar to the learners. Furthermore, the problems were read out loud to the learners. Contrary to Kachaso's (1988) findings whose word problems were in English, Chilora et al. (2003) found that learners in all the classes performed better in the Chichewa word problems than the numerical computation forms. They explain that presenting the problems in contexts and language that the learners understand facilitated their understanding and ability to problem solving.

Another challenge learners face in Malawi is to understand the vocabulary used in mathematics. Kazima (2006) found that learners understanding of probability vocabulary such as *likely, unlikely, certain,* and *impossible*, varied and in some cases the learners' meanings were completely different from the mathematics meanings. Furthermore, the learners' meanings were influenced by their home languages. For example, *certain* was very difficult for the learners because in their home language there is no word or concept of certainty (Kazima, 2006). Kazima et al. (2015) found that this difficulty persists up to university. They studied university students' meanings of mathematics vocabulary used in logic such as *implies*, and *if and only if*, and also found that the influence of home language limited their understanding of the terms.

Studies on teachers' practices have found that teachers rely on code switching between English and Chichewa or other home language to communicate in the classroom. This has been found to be effective by Kaphesi (2002), similar to studies elsewhere for example in South Africa (Setati et al, 2008; Webb & Webb, 2008).

Teaching in early years of primary although in the home language also faces challenges, one of which is the translation of mathematical vocabulary. For example, multiplication is translated as *kuchulukitsa* which literally means making more. This is not mathematically precise, as Kazima (2023) pointed out, multiplication by numbers less than one does not make more. Number names in Chichewa can also be a resource because the structure of the words illustrate place value clearly. However, zero has no number name. The words used for zero (*mulibe* or *palibe*) are equivalent to empty or nothing. The number name *zilo*, borrowed from the English zero is also used, and is often used interchangeably with the words *mulibe* or *palibe*. Unlike the other number names, *zilo* has no meaning in Chichewa. This points to the importance of teacher education in preparing teachers for teaching using the home languages. However, studies in mathematics teacher education in Malawi have found that English is used in all lessons and that teachers are not properly prepared to teach in the home languages (Chitera, 2009). The discussion above informs us that the language of teaching and learning zero is crucial to learners' mathematical understanding of the concept.

3 Theoretical framework: Mediating Primary Mathematics

This study was guided by the Mediating Primary Mathematics (MPM) framework (Venkat & Askew, 2018). The framework is rooted in the Vygotsky's sociocultural theory, which posit that cultural experiences offer the context for learning (Kozulin, 2003). A key aspect of the theory is that learning is achieved through mediation by a human mediating agent using cultural tools (Kozulin, 2003; Wertsch, 2017). Cultural tools are means of mediation which include resources in a practice, in particular language (Adler, 2001). In the sociocultural perspective, the teacher is the only mediating agent in the classroom (Wertsch, 2017). Therefore, how the teacher uses resources and language in teaching is crucial for learning. As Venkat and Adler (2020) explain, the sociocultural theory views mathematics as a network of scientific concepts that are interconnected. Therefore, it views teaching mathematics as aiming to gradually provide more and more sophisticated and generalised thinking that constitute progression in the discipline (Venkat & Adler, 2020). The extent of the progression made available to learn largely depends on the teacher's mediation. Thus, we found the MPM framework suitable for our study because it describes teachers' means of mediation and pays specific attention to the mediation of mathematics to young learners. Furthermore, the school context in Malawi is similar to the context of the South African schools that led to the development of the framework. The schools had limited resources and the teachers had knowledge gaps in mathematics and pedagogy (Venkat & Askew, 2018).

The MPM recognizes everyday concepts in Mathematics and that they are often spontaneous and without connections. Hence the MPM focuses on scientific concepts because they are interconnected. The framework argues that mathematics should be taught as scientific concepts with connections, thus emphasises the sociocultural theory view of mathematics. The MPM concentrates on the nature of the mathematics that is made available to learn and enables a detailed exploration of the quality of primary mathematics teaching. The framework identifies four overarching means of mediation, which Venkat and Askew (2018) call strands: (i) Tasks and examples, (ii) Artefacts, (iii) Inscriptions, and (iv) Talk and gesture.

3.1 Tasks and examples

This strand includes all the examples and tasks that are used in the lesson. Venkat and Askew (2018) argue that although tasks and examples are often considered as objects requiring mediation in literature, they are considered as a mediating strand in the MPM framework. They emphasise that it is the work of the teacher to make relationships and connections explicit among examples so that learners can notice the mathematical structures and make generalisations, thus move towards scientific concepts. The framework examines how the tasks and examples strand is mediated by the other three strands; artefacts, inscriptions, and talk and gesture.

3.2 Artefacts

This strand includes all physical objects that the teacher uses, which exist before and after the lesson (Askew, 2019). Venkat and Askew (2018) describe a "structured artefact" as one where "the material nature of the artefact presents some possibilities for learners to attend to structure and relations even if the teacher does not make these explicit" (p. 81).

3.3 Inscriptions

Venkat and Askew (2018) define inscriptions as what the teacher writes or draws during the lesson. Thus, they do not consider charts that were prepared before the lesson as inscriptions but as artefacts. When teaching mathematics, inscriptions are useful for recording and facilitating moves beyond the presence of physical objects and also facilitating the move from the everyday and concrete to scientific and symbolic.

3.4 Talk and gesture

The talk and gesture strand dominates many mathematics lessons. These are verbal explanations and motions that the teacher gives during the lesson to clarify mathematical concepts or procedures. The talk and gesture strand has three sub strands namely; (i) talk and gesture for generating solutions to problems, (ii) talk and gesture for building mathematical connections, and (iii) talk and gesture for advancing learning connections.

3.5 MPM as the analytical framework

In using the MPM as our analytical framework, we considered the tasks and examples strand as mediated by the other three strands as the framework suggests. For artefacts and inscriptions, we analysed the mediation by identifying their presence and use, that is what artefacts and what inscriptions were used, and how they were used. We coded presence of an artefact or inscription as structured if it offered potential to convey mathematical structure without the teacher making it explicit, and unstructured if there was no such potential. Furthermore, we coded how the teacher used the artefacts or inscriptions as structured or unstructured if the way the artefact or inscription was used offered or did not offer potential for conveying mathematical structure, respectively. For talk and gesture, we identified the substrands of the talk. In addition, we went further than the MPM and identified the words that the teacher used and analysed whether the word use offered or did not offer potential for mathematical learning. We went beyond the framework because MPM does not deal with different languages or specific word use. Unlike the MPM, we also emphasised on the everyday concepts and whether there were moves to the scientific concepts. We added this because it is central to what our data reveals. We did not use the levels of MPM because our purpose was not to measure and compare teaching across time or across the teachers. Our aim was to describe the teaching and identify whether it afforded or constrained learning of the scientific concept of zero.

4 Methodology

4.1 Selection of school and teachers

The motivation for this paper originates from a larger study that explored the teaching of addition in the first 4 years of primary school in Malawi. It was a case study of four teachers teaching at one school. The school was purposely selected because it is a relatively high-performing school and the assumption was that a well-performing school would provide rich data on teacher mediation of mathematics. For this paper, two out of the four teachers were selected because they were teaching the first 2 years; standard 1 (pseudonym Ms Banda) and standard 2 (pseudonym Ms Phiri). During the first cycle of data collection, Ms Banda had seven years teaching experience, and had taught standard 1 for 4 years.

By the second cycle she had 11 years teaching experience of which 8 years were in teaching mathematics in standard 1. Ms Phiri was in her first year of teaching during the first cycle, she was teaching mathematics in standard 2 and had no experience of teaching standard 1. During the second cycle she had 4 years teaching experience and less than 1 year experience of teaching mathematics in standard 1.

4.2 Data collection

During the first cycle, both teachers were observed teaching the entire topic of addition of whole numbers, as that was the focus of the larger study. We observed a total of six lessons by Ms Banda in standard 1 and three lessons by Ms Phiri in standard 2. During the second cycle, we observed Ms Banda and Ms Phiri teach one lesson each on introducing zero to standard 1 classes. All the lessons were video recorded. After the lessons we conducted in-depth interviews with each teacher and audio recorded the interviews. The in-depth interview was semi-structured guided by the MPM strands and the research question. The interview questions were posed in English then explained further in Chichewa. The teachers were free to respond in English or Chichewa. Both teachers used a mixture of English and Chichewa.

We included the standard 1 syllabus, learner textbook and teacher guide as part of our data because these are textual resources that the teachers used.

4.3 Data analysis

All the lesson videos and interview audios were transcribed and the authors read through the transcripts several times. Each lesson transcript was segmented into episodes. An episode was determined by the beginning and end of an activity/task that the teacher focused on within that section of the lesson. The segmentation of the lessons into episodes was done by two of the authors. They first did independently then they discussed and agreed on the episodes. The interviews were segmented into sections according to the questions/issues that were discussed. For example, there were sections about artefacts, inscriptions, talk and gestures. After the segmentation, we conducted both theory driven and conventional content analysis (Hsieh & Shannon, 2005). The theory driven was informed by the MPM framework where the strands and their descriptive levels were used deductively as coding categories. For example, artefacts and inscriptions were coded as structured or unstructured and further coded as used in structured or unstructured ways. For the conventional content analysis, we immersed ourselves into the transcripts of lesson episodes and interview sections to gain new insights (Hsieh & Shannon, 2005). We then developed codes that we could search for in the transcripts using ATLAS.ti qualitative data analysis software. For example, for each strand, we created codes for the nature, use and rationale of the observed resource. The same codes were applied to the textbook and teacher guide but was done manually.

5 Findings and Discussion

5.1 Teaching the concept of zero in the Malawi curriculum materials

In the Malawi primary curriculum, zero is introduced in standard 1, where teaching is in Chichewa or other local language. Zero is taught within the first topic of the standard 1 mathematics curriculum. The topic is called "counting up to 5" and is divided into six subtopics of the numbers 0–5 in the order of 1, 2, 3, 0, 4, 5 (Malawi Institute of Education, 2006). For each of the numbers there are four activities that the curriculum materials suggest the teacher does with the children, which are: introducing the number, recognising the symbol, tracing the number and writing the number (Soko et al., 2012). The curriculum materials suggest teaching children the numbers 1, 2 and 3 in that order before they are introduced to zero. The activities for zero follow the same pattern; introducing 0, recognising the symbol 0, tracing 0, and writing 0 (Soko et al., 2012). After the introduction of zero, the curriculum materials suggest introduction of the numbers with sum not exceeding 5, and subtraction of numbers within the number range 0–5. In these topics the children experience operating on zero; adding zero, adding to zero, and subtracting zero (Soko et al., 2012).

5.1.1 Artefacts suggested in curriculum materials

In addition to pictures in the learners' textbooks, the teacher guide suggests concrete materials such as bottle tops, stones, sticks, boxes and number trays (Soko et al., 2012). The teacher guide also suggests the use of empty transparent bottle and empty basket to demonstrate nothing and relate that to zero. According the MPM framework, the number trays and pictures are structured because they present possibilities for learners to see structure even without the teacher making these explicit (Venkat & Askew, 2018). The rest are unstructured, because on their own they do not present the possibility to see structure. However, they have potential to be used in structured ways.

5.1.2 Inscriptions suggested in curriculum materials

The teacher guide encourages teachers to draw and write on the chalkboard during the lessons to illustrate 1, 2, and 3 objects then no objects to represent zero. The teacher guide also suggests that teachers help learners trace the symbol 0 before they learn to write it, by demonstrating tracing 0 and

Table 1 Transcript of part of Ms Banda's episode 3

Line	Chichewa	English translation	MPM substrands
502	**T:** Bokosi la nambala 4 masamba munalibe, mitengonso munatani kodi?	**T:** In box number 4 there were no leaves, what about sticks?	
503	**C:** Munalibemo	**C:** There was nothing in there	
504	**T:** Munalibemo, eti?	**T:** There was nothing in there, not so?	
505	**C:** Ee!	**C:** Yes!	
506	**T:** Eya! Ndiye masamba mulibe! Mitengo mulibe! "Mulibe!" Olo kunena kuti "Palibe!". Ndiye kuti pamenepo ndi chani kodi? Nambala yake timanena kuti chani kodi Zilo! Timati ndi chani kodi?	**T:** Yes! There are no leaves. There are no sticks. "Nothing in there" or we can say "nothing there". So here what is the number? What do we call the number? Zero! What do we call the number?	Talk for building mathematical connections
507	**C:** Zilo!	**C:** Zero!	

later demonstrating writing 0 on the chalkboard. The inscriptions suggested only refer to the writing of numbers and not mediating the concept of zero.

5.1.3 Talk and gesture suggested in curriculum materials

The syllabus and teacher guide which are in English use the word "nothing" to refer to zero and suggest the translation *palibe* (literal meaning: there is nothing) or *mulibe* (literal meaning: nothing in there) when teaching in Chichewa. Furthermore, the teacher guide suggests using the gesture of empty hands to illustrate zero as nothing.

The suggested talk and gesture affords the learning of zero as nothing and constrains the learning of zero as a number. Also the suggested artefacts and inscriptions mostly afford learning of zero as nothing while zero as a number is constrained. Since the curriculum materials are the main textual resources that teachers use, the question of how they bring resources and use language in their lessons arises.

5.2 Analysis of lessons

We start by giving descriptive summaries of the lessons on introducing the concept of zero and lessons on addition of numbers involving zero. The summaries include some transcripts of episodes we use as illustrative examples. The transcripts are in Chichewa and English translation, which was done by the two authors that analysed the data. They are both Chichewa first language speakers. T, C and L represent Teacher, Children and Learner, respectively.

5.2.1 Descriptive summary of Ms Banda's lesson introducing zero

The lesson was organised in a way that the learners would first identify the numbers, 1, 2, and 3, that they had learnt before and then introduce the concept of zero. The lesson started with a number song. In the first episode, Ms Banda introduced her lesson by informing the learners that they are going to learn a new symbol.

She started the second episode by asking learners in turns to pick cards bearing the numbers 2 and 3. The learners picked the correct cards, but faced difficulties in orienting the written numbers when showing the cards to the class.

In the third episode, Ms Banda worked with a box having four compartments. She placed one leaf, two leaves, three leaves, and no leaf in the first, second, third, and fourth compartments respectively. She then asked learners to come to the front to pick and show the number of leaves in each compartment; and the learners presented the number of leaves as 1, 2, 3 and "nothing". The teacher showed the compartment with nothing to the class. During the second part of the third episode, Ms Banda replaced the leaves with sticks in the four compartments. As she did with leaves, she asked learners to come to the front in turns to pick and show the number of sticks in the compartments. Again she showed the class the fourth compartment that was described as having "nothing" by the learners. The teacher ended the episode by emphasising that "nothing" corresponds to the number zero (see Table 1).

During the fourth episode, Ms Banda introduced the learners to the symbolic representation of zero written on cards. The learners were then asked to relate anything that looks similar to 0. Some learners mention the letter O, but Ms Banda led them to parts of a bicycle; and the learners mention a tyre.

In the fifth and final episode Ms Banda asked learners to clap hands three times, one time, then zero times. Some leaners clapped their hands when asked to do so zero times. She explained that clapping zero times means that they would not clap.

5.2.2 Descriptive summary of Ms Phiri's lesson introducing zero

Ms Phiri started the first episode of the lesson by a number song. She asked the learners to sing numbers from 1 up to 5.

Table 2 Transcript of part of Ms Phiri episode 4

Line	Chichewa	English translation	MPM sub-strands
252	**T:** Eya. Nde apa ndanyamula mabotolo angati?	**T:** *[Shows two transparent plastic bottles, one filled with water and the other empty]* So here how many bottles am I carrying?	
254	**C:** Awiri!	**C:** Two!	
255	**T:** Mabotolo awiri. Mubotolo umu muli chani?	**T:** Two bottles. What is in this bottle? *[Shows the bottle filled with water]*	
257	**C:** Madzi!	**C:** Water!	
258	**T:** Muli chani umu?	**T:** What is in here?	
259	**C:** Madzi!	**C:** Water!	
260	**T:** Nanga mubotolo umu muli chani?	**T:** What about this one what is in this bottle? *[Shows the empty bottle]*	
262	**C:** Mulibe kanthu!	**C:** There is nothing!	

Table 3 Transcript of Ms Phiri episode 5

Line	Chichewa	English translation	MPM substrands
331	**T:** Nde palibe kanthu ameneyuyu ku masamu timamutcha kuti Zilo. Tiyeni aliyense anene	**T:** So this 'nothing' in mathematics is called zero. Let us all say it	An attempt of talk for building mathematical connections
332	**C:** Zilo!	**C:** Zero!	
333	**T:** Aliyense anene, ndiyambe kunena ineyo. Zilo. Zilo. Aliyense anene Zilo	**T:** Everyone should say it, I will start. Zero. Zero. Everyone should say it	
334	**C:** Zilo!	**C:** Zero!	

In the second episode, Ms Phiri reviewed the numbers 1, 2, and 3 by asking the learners in turns to write the numbers on the chalkboard. The learners made up to three attempts when writing 2. She then asked the learners to say the numbers in the sequential order following her pointer.

In the third episode, Ms Phiri put the learners in groups and placed one stone, two stones, three stones, and no stone in circles labelled a, b, c, and d respectively, that she drew on the floor for each group. Learners quickly recognised the number of stones in circles a, b, and c, but struggled in identifying d, they confused the letter d with b. So, when asked the contents of d, they responded by giving the contents of b. After the teacher's mediation of the difference between the letters, the learners responded by saying d had nothing.

In the fourth episode, Ms Phiri continued identification of "nothing" using a bottle filled with water and an empty bottle; holding a bottle in one hand and nothing in the other; letting one learner hold a duster and the other hold nothing; putting a learner on a position and thereafter shift the learner from that position. In each case Ms Phiri asked "what is here?" and the class responded by saying that the empty bottle, the bare hands, and the empty position all had nothing (see Table 2).

In the fifth episode, Ms Phiri drew on the chalkboard pictures of one cup, two cups, three cups, and no cup in boxes labelled a, b, c, and d then asked learners to identify what is in each box. The learners identified the boxes a having one cup, two cups, three cups, and nothing, respectively. She explained that "nothing" is called "zero" in mathematics, and asked the learners to say zero after her up to four times (see Table 3).

5.2.3 Descriptive summary of lessons on addition with zero

All lessons on addition of numbers in standard 1 were taught by Ms Banda during the first cycle of the data collection. All examples and tasks were addition of two numbers with sum not exceeding 5, and used the "combine and count all" strategy. In Table 4 we show all the examples and tasks across all the lessons and episodes. We then show transcript of one episode as an illustrative examples of how the teacher mediated addition with zero.

5.2.4 Mediation of zero through tasks and examples

The two teachers followed the methodological sequence for introducing zero as presented in the curriculum

Table 4 Examples and tasks used in lessons on addition

	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Lesson 5	Lesson 6
Episode 1	+ and = signs	+ and = signs	+ and = signs	+ sing, 2 + 2	2 + 2	+ and = signs
Episode 2	2 + 1	2 + 3	2 + 1	2 + 0	1 + 0	0 + 1, 1 + 3
Episode 3	4 + 1	2 + 1, 1 + 1, 3 + 0	3 + 2	2 + 1, 3 + 1, 4 + 1, 1 + 1, 5 + 0, 1 + 2, 3 + 0, 2 + 2, 2 + 3, 4 + 0	2 + 2, 4 + 0, 1 + 1, 3 + 2, 1 + 3, 0 + 4, 2 + 1, 2 + 0	1 + 1, 4 + 0, 2 + 2, 3 + 1, 5 + 0, 3 + 2, 1 + 2, 1 + 3, 0 + 1, 0 + 2, 2 + 1
Episode 4	2 + 2	2 + 1, 3 + 1, 1 + 1	2 + 2	3 + 2, 0 + 5, 2 + 1	3 + 1, 4 + 1, 0 + 3	1 + 4, 2 + 0, 3 + 1

materials, where the first three numbers; 1, 2, and 3, are introduced, followed by zero. Both teachers also used examples of empty containers or empty spaces to represent zero as suggested by the curriculum materials. During interviews both teachers confirmed that they follow the curriculum materials in teaching zero. For example, in response to the question of how she decided on the examples and tasks, Ms Phiri said:

> Learners' book guide us how to teach. There are some examples there. In teacher guide there are also instructions which guide us with how we can teach.

For the addition lessons, Table 4 shows all the examples and tasks during the six lessons. The first addition with zero was 3 + 0 in lesson 2, then more occurred in lessons 4, 5 and 6. During the interview, Ms Banda was asked how she selected the examples and tasks and her response was:

> When it comes to selection…we just take any numbers. Why? Aah, what can I say? Okay. *[Sighs]* What you should bear in mind is that when they add the number should not exceed five. They should be between these numbers, one up to five. It can be any number; any numbers.

Ms Banda's response suggests that the examples were chosen randomly. However, the example spaces in lessons 4–6 contained both adding zero, e.g., 3 + 0, and adding to zero, e.g., 0 + 2. Besides, in each of the three lessons 4–6, the example spaces included adding zero to a number and adding the same number to zero (e.g., 5 + 0 and 0 + 5 in lesson 4), which had the potential of highlighting variant and invariant features associated with the addition of zero. Furthermore, looking at the learner textbooks and the teacher guides, we notice that the examples and tasks are similar and sometimes identical to what Ms Banda presented. This reflects the influence of the textual curriculum materials on teachers' selections of resources to use in their lessons. This is important to note because as observed, the textual materials have affordances and constraints.

5.2.5 Mediation of zero using artefacts

Ms Banda worked with a compartmentalised box and the cardinality of the set of objects in each compartment. She used the box as suggested in the teacher guide; having no items in the fourth compartment after one, two, and three items in the first three compartments in that order. Ms Phiri worked with stones placed in circles drawn on the floor for each group of learners. Similar to Ms Banda, the order was 1, 2, 3, 0, which is also the order suggested by the teacher guide. This arrangement is likely to constrain the opportunity for the learners to discern the relative position of 0 with respect to the numbers 1, 2, 3. Using the same resources, it was possible for the teachers to arrange the quantities of objects in the sequence 3, 2, 1, 0 or 0, 1, 2, 3. As such, the way both teachers worked with the artefact would constrain possibilities for building generality. In the MPM framework, Ms Banda's compartmentalised box could be described as being structured but the teacher used it in unstructured ways. The teacher guide was silent on how to use the box, it was left to the teacher.

Ms Phiri also used two transparent bottles; one bottle filled with water and the other empty. The use of a bottle filled with a liquid (an uncountable substance) and an empty bottle as a way of introducing zero emphasises the meaning of zero as empty or absence, which is supported by informal everyday language, but will likely limit the understanding of zero as a number. Thus, the everyday concept of zero as emptiness could not be generalised to the scientific concept of zero as a number. According to the MPM framework the bottles as artefacts are unstructured and were used in an unstructured way. A structured way of using the bottles would be to have countable items in one bottle and the empty bottle to illustrate zero items. It is important to note that the use of a transparent bottle to demonstrate nothing inside is recommended by the teacher guide. The teacher guide only suggests showing nothing and not the contrast of something in another bottle.

Looking at the use of artefacts by the two teachers, we see that leaves, sticks and stones provided opportunities for learning the concept of zero. This is because counting the number of objects in the compartments or circles, then relating no objects to zero has potential to convey the concept. The transparent bottles as artefacts were used in a way that constrained the learning of zero as a number.

During interviews the two teachers also mentioned plates, baskets and charts that can be used as artefacts. To explain her use of bottles Ms Phiri said:

> We use different resources … in my lesson, I used bottles. I took two bottles. In the other bottle there was water. Another bottle was empty. Yeah. So, I asked learners what is in this bottle? They said there is water, what is in another bottle? They said there is nothing.

Ms Phiri did not see the limitation of using water. In contrast, Ms Banda although she did not use the bottle, she mentioned it as one of the artefacts that can be used as a container for countable objects. She explained:

> There are so many resources that we can use but it should have … only one thing which cannot have anything inside. We can use bottles, to say, okay, let's put stones inside the bottle.

Ms Banda's suggestion of having stones inside a bottle would make its use become structured.

Artefacts in the lessons on addition included concrete materials such as leaves, sticks, stones and bottle tops, which were used as counters. The most common were bottle tops or sticks that were framed using a string and curved stick. When demonstrating addition using framed counters, the counters are pushed from one end to other while counting then all are counted to determine the sum. Ms Banda demonstrated zero by sliding her fingers across the string without any counter (see lines 304). When adding zero the absence of concrete counters represented zero. The artefacts were used in structured ways which could lead to generalisation of zero as a number.

5.2.6 Mediation of zero using inscriptions

Both teachers mainly used chalkboard inscriptions for reviewing the numbers 1, 2, 3 and for addressing learner errors. Ms Banda used chalkboard inscriptions for contrasting what is 3 and what is not 3, when reviewing with learners the numbers 1–3 before introducing zero. While Ms Phiri used chalkboard inscriptions for differentiating the labels b and d when identifying the circles drawn on the floor. Ms Phiri used more inscriptions in her lesson than Ms Banda. She drew four circles labelled a, b, c, d on the floor for each group of learners where she placed stones. Then she drew four circles a, b, c, d on the chalkboard. Ms Phiri also drew cups in boxes on the chalkboard representing 1, 2, 3 and 0. This emphasised the counting of objects in boxes, where no object was related to zero, and generalisation of zero as a number.

Regarding the inscriptions of the symbol zero, the teachers indicated during interviews that the writing of zero comes after the introductory lesson. This is in line with what the curriculum materials suggest.

In all the addition lessons, there were no instances where Ms Banda worked with inscriptions aimed at mediating the concept of zero. Zero appeared in inscriptions as one of the addends in some examples. For instance, in lesson 6 she introduced vertical addition using the example of $0+1$. She initially asked learners to attempt writing $0+1$ vertically before demonstrating the presentation on the chalkboard, and emphasising that the answer had to be aligned vertically with the addends. This offered generalisation of zero as a number, and like other numbers, having its own symbol 0. Thus, potential for learners to move their everyday concept of zero as emptiness to the scientific concept of zero as a number.

5.2.7 Mediation of zero through talk and gesture

Across the lessons, two substrands of talk and gesture were identified. Talk and gesture for generating solutions to problems was identified in lessons of addition involving zero (e.g., line 304, Table 5). Talk for making mathematical connections was identified when teachers related no objects to zero (e.g., line 506, Table 1). However, most of the talk that attempted to make mathematical connections was weak because it focused on emptiness rather than "no objects" (e.g. lines 331–333, Table 3), which constrained the learning of zero. Talk for building learning connections to the concept of zero was identified only once across all the lessons. This was in episode 5 of Ms Banda's lesson when she asked learners to clap zero times and some learners clapped. The teacher talk addressing the error and explaining that clapping zero times means no clapping is about building learning connections. There were no other incidents where the teachers responded to learner errors or explained avoiding errors or misconceptions to building on learning of the concept of zero.

In the two lessons on introducing zero, the teachers and learners used Chichewa number names when referring to the numbers 1, 2 and 3. They used the words *mulibe* (nothing in there) or *palibe* (nothing there) to refer to zero. The borrowed word *zilo*, which has no meaning in Chichewa, was used as the number name after the introduction of zero as nothing. During the activities where learners counted leaves and sticks (Ms Banda) or stones and drawings of cups (Ms Phiri), both teachers emphasised "nothing" rather than "no objects". This constrained the learning of zero as a number. Emphasising on "no objects" would make the link to zero objects easier.

Interestingly during interviews Ms Banda said:

Table 5 Transcript on 2 + 0 from lesson 4 episode 2

Line	Chichewa	English translation	MPM substrands
296	**T:** Tawerenga two, eti?	**T:** We have counted two, alright?	Talk for generating solutions to problems
297	**C:** Ee!	**C:** Yes!	
298	**T:** Ndiye akuti tiwerenge zina zingati?	**T:** Thereafter, we are expected to count how many more? *[Pointing at 0 on chart with 2 + 0 written]*	
299	**C:** Zilo!	**C:** Zero!	
300	**T:** Aah! zilondi zingati?	**T:** Aah! zero is how many?	Talk for building mathematical connections
301	**C:** Palibe!	**C:** Nothing!	
302	**T:** Palibe! eti?	**T:** Nothing! Right?	
303	**C:** Ee!	**C:** Yes!	
304	**T:** Ndiye tiyeni titenge ziloyo tiphatikize ku two kuja! Tiye tiphatikize!	**T:** Now, let us take that zero and add to the two we got earlier! let us add! *[Moves her fingers along an empty string towards the previously counted two counters]*	Talk and gesture for generating solutions to problems and for building mathematical connections

I was saying during the lesson, to say *mulibe, mulibe kanthu, mulibe masamba* [nothing, there is nothing, there are no leaves]… that is what? Zero".

She seems to understand the importance of emphasising "no objects" although during the lesson she did not emphasise as much as she claimed in the interview.

Both teachers used empty hands gestures to represent nothing and relate to zero. The hand gesture is also suggested in the teacher guide. Furthermore, it is familiar in everyday communication in Malawi to show empty hands to emphasise that there is nothing. The concept of nothing is familiar to learners, thus using the empty hands gesture conveys easily the concept of zero as nothing. However, it is the everyday concept of zero and not the scientific concept. More emphasis on no objects as explained above would help move the everyday concept to the scientific concept.

Ms Banda also used handclaps in her last episode. The use of hand clapping is similar to what is in the resources of Montessori schools and also used in other different contexts (Montesorri, 2022). Relating the number of claps to numbers, and no clapping to zero claps is a good way of emphasising zero as a number. Therefore, it is important for the teacher to explain and mediate this gesture effectively. It is not surprising that some learners clapped when asked to clap zero times because as has been observed by other researchers such as Barton (2020), it is easier to understand for example, no bananas, than to say zero bananas. In this case some learners could not relate zero clap to no clap.

In the addition lessons, Ms Banda also used empty hands gesture when referring to zero. To add zero, she used the gesture of sliding fingers across the framed counter string indicated sliding nothing to the other side as an addend. This conveyed well the concept of adding zero as adding nothing. Linking this gesture of nothing to the symbol 0 written on addition task such as 2 + 0, as Ms Banda did (see line 298 in Table 5), offers the generalisation of zero as a number. Thus, moving from the everyday to scientific concept.

5.3 Moving between the two languages

As was evident during the lessons, both teachers used the teacher guide which is in English and the learner textbook which is in Chichewa. The teachers' lesson plans were written in English while the teaching is in Chichewa. During the interview, we asked the teachers how they find this movement between the languages. Ms Banda explained as follows:

> I feel very difficult. Why? It could be easy when they could have write what? Teacher guide in Chichewa also. Why? We are teaching in Chichewa. And even when we are writing a lesson plan, we write in what? In English. Eh we write lesson plan in English but when we come here, … we then convert to what? To Chichewa. It is very difficult.

As can be seen, Ms Banda found the work of moving across the languages challenging. On the contrary, Ms Phiri did not see this as a problem, and this is what she had to say:

> To my side, if the word is difficult to know, or I don't understand that word; I ask my friends, what does this word mean? How can I teach this? I am not understanding it. So, we help each other… Yeah, it's okay [*referring to the movement between the languages*], because we consult.

Ms Phiri acknowledges the challenge but does not take it as a difficulty. She sees the movement as something that she and other teachers can do because they consult each other.

It is interesting that the more experienced teacher is the one that sees huge difficulty in moving between the languages while the less experienced teacher sees it as not a difficulty. It is likely that the experience of Ms Banda has made her more aware of the role of language in teaching mathematics and the importance of having precise meanings and mathematically accurate words in teaching. While the less experienced teacher probably does not yet see and appreciate the complexity of language in teaching mathematics. The teacher guide seems also to assume easy movement between English and the teaching in Chichewa or other local language. It is silent on how to explain the concept of zero as a number and beyond relating it to nothing. The fact that transparent bottles are suggested for use to show empty as nothing and relate to zero without further explanation about having countable objects is misleading because the bottles would ordinarily have liquid if not empty. This leaves to the teacher the work of understanding the limitation as well as deciding on what words to use to convey the concept of zero as a number. Choosing the appropriate Chichewa words to use is important because as Krajcsi et al. (2021) explain, natural language helps children's development of number sense and mathematical language. More could be done about how the teachers worked with the two languages, but was not within the scope of this study. Our data relied on what the teachers said they do, and we did not have the analytical tools within our framework to analyse more deeply the language use. This is something further work can move forward.

6 Conclusion and implications

In this paper we set out to explore the resources and language that two teachers used to mediate the concept of zero to standard 1 learners. Our aim is to show what the setting of Malawi illuminates about teaching of zero in multilingual post-colonial contexts. From our findings we see that the teaching of the concept of zero was mostly located in the everyday concept of zero as absence or emptiness. Yet, from the sociocultural perspective, zero is a scientific concept with a position in the structure of numbers and is connected to other scientific concepts. Moving from the everyday to the scientific concept seems to be the main difficulty in teaching and learning of zero. Our findings related to similar findings from other different contexts, where zero is taught as nothing or empty or absence of something (Barton, 2020; Nieder, 2016). What is limited is the move beyond zero as nothing to zero as a number; moving from the philosophical viewpoint to the mathematical viewpoint (Barton, 2020). This move can be made using resources and language carefully planned to convey the number sense of zero, its position in the structure of numbers and its links to other scientific concepts. The language and resources that teachers use are the cultural tools (Adler, 2001) that can enable this move from the everyday to the scientific concept. From a sociocultural perspective and as the MPM framework emphasises, the presence of the cultural tools is important but more so how they are used.

Looking at the findings from the two teachers' lessons, we see that the artefacts used were locally available resources such as stones, leaves, sticks and self-made boxes thus easy for the teachers to access and easy for the learners to relate. We also see that the artefacts used in other contexts in developed countries include counters and containers which serve the same purpose of counting number of counters in containers, where empty container is referred to as having zero counters (Learnbrigth, 2022; Twinkl, 2022). This emphasises that the type of artefacts is not important but what teachers do with them to convey the mathematics. To be effective in offering opportunities to see generalisations and mathematical structure, the artefacts need to be used in structured ways (Venkat & Askew, 2018).

We have seen that the textual curriculum materials offer affordances as well as constraints. In particular the teacher guide suggests the use of specific artefacts such as an empty bottle, without explicitly stating how they should be used or the logic of using such artefacts. Consequently, as observed in this study, some teachers might use the empty bottle artefact without realising that they should contrast this to a bottle containing countable items such as stones so that learners can establish the description of zero as a quantity of objects rather than as nothing inside. This points to the implications for textual curriculum materials that they should not only name the suggested artefacts, but also need to describe explicitly the function of the artefacts, the logic of using the artefacts, and how to use in a structured way. Such explicit descriptions will support teaching and learning towards understanding the scientific concepts.

We have also seen that mediation through talk and gesture is crucial. From the two teachers' lessons, there was limited talk and gesture for building strong mathematical connections, while talk and gesture for advancing learning connections was almost invisible. This is not surprising because the concept of zero was mostly presented as an everyday concept. The teachers' talk and gesture presented opportunities for learners to learn zero as nothing but constrained the opportunities for learning zero as a number. As noted from literature this is a challenge for many teachers (Barton, 2020). For the Malawi setting, the challenge is enhanced because there is no number name for zero in Chichewa or other local languages. During the lessons teachers related the Chichewa number names of 1–3 to their mathematical English names, making the learners get the sense of "oneness", "twoness" and "threeness". This was not done for zero because there are no words in Chichewa to convey "zeroness" in the mathematical sense. The borrowed number name *zilo* has no meaning in Chichewa and does not fit in the structure of the other Chichewa number names. Krajcsi et al. (2021) argue that natural language is easier for children than mathematical language. We argue that if the natural language does not have the words, then it becomes more complex. This adds to the complexity of language in teaching zero in Malawi and other similar contexts.

Another complexity for the teachers is that of moving between different languages; English in the teacher guide and Chichewa in learner textbook, and the requirement to write lesson plan in English and teach in Chichewa. The assumption is that teachers can do this easily, but we see that it is complex. Planning lessons in English while using textual resource in English might be easy but implementing the lesson in Chichewa is not a straightforward move. Even more so for those teaching in a third language. This points to the need for teacher education to prepare teachers in interacting with the textual resources and the use of local languages in teaching. It has been found that teacher education in Malawi does not prepare teachers for teaching in local languages (Chitera, 2009). More research is required to further understand this complexity and find ways of supporting teachers.

Our study has some limitations, two of which stem from our use of the MPM framework. Firstly, we designed our in-depth interview around the framework and its strands. While that helped us get the teachers' motivation for the choices they made around the strands, it did not probe further into their use of resources and language. We suggest that for further work going forward there should be more probing to get the data required for deeper analysis than was possible in this paper. Secondly, the framework enabled us to get access to what the teachers were doing in teaching zero but it could not get us insight into the language issues. We could only access what we were able to observe and what the teachers said they do. Going forward we suggest further analysis that would really understand how the resources and language intersect. To do this the data collection would need to be carefully designed in terms of what to be probed and what frameworks to use to illuminate this.

Nevertheless, our study allows us to understand how the teachers mediate the concept of zero through the strands of MPM, how they use curriculum materials and how they perceive their movement between English in curriculum materials and teaching in Chichewa. There has been work done on the concept of zero (e.g., Barton, 2020), on translanguaging (e.g., Planas & Chronaki, 2021) and on resources (e.g., Trouche, et al., 2019). However, we have not found work that has tried to link the three. Our work offers that and has illuminated the complexity. Our work contributes as a starting point on linking resources, language and mathematical concepts like zero. There is definitely much more to do, in particular deeper analysis of the language use which is beyond what was possible in this paper. For example, a sociolinguistic analysis would be illuminating. This can be taken forward in further research.

Other limitations include that we had only two teachers. Therefore, we do not claim generalisability of the findings to all teachers in Malawi. However, we do learn from them the complexity of teaching the concept of zero in Chichewa and using textual resources in two languages. What the Malawi setting illuminates is "relatable" to other similar post-colonial settings where colonial languages continue to be used for teacher education and textual resources for teachers even when teaching is in local language.

From our findings, it is intriguing that the experienced teacher seems more aware of the complexity of language and her awareness appears to inform her selection of resources and language to use. Selection of resources to use and what not to use from suggested textual resources is an important part of teachers' planning. This further suggests implications for teacher education. Mathematics teacher education can bring out this awareness and how to manage the complexities.

In this paper we focused on the concept of zero, but the issues raised apply to other mathematics concepts. There is a huge role that teacher education can play in preparing and supporting teachers in their use of resources and language, including how to interact with the resources while moving across languages. Teacher education can also inform development of curriculum materials so that what is suggested in the textual materials that teachers use can offer affordances that establish learning of scientific concepts. This applies not only to Malawi but also other similar settings around the world.

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